DEPENDENCE OF TURBULENT BURNING VELOCITY ON TURBULENT REYNOLDS NUMBER AND RATIO OF LAMINAR BURNING VELOCITY TO R.M.S. TURBULENT VELOCITY

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Measurements are reported of premixed hydrogen-air turbulent burning velocities, made by the double kernel method during explosions. Turbulence was created by four high speed fans within the explosion vessel. The method is described for calibrating the system, which is capable of giving high values of turbulent Reynolds numbers. The values obtained are compared with those of many other workers, over a wide range of burning conditions, mixtures and turbulent parameters.

The ratio of turbulent to laminar burning velocity correlates well with both the turbulent Reynolds number and the ratio of laminar burning velocity to r.m.s. turbulent velocity. The use of hydrogen-air mixtures has extended the data on premixed turbulent combustion to regimes with higher values of the last dimensionless ratio. At high values of the ratio there is evidence of a wrinkled laminar flame structure, but at lower values a small scale eddy structure seems to be dominant. There is discussion on these findings, which accord with theoretical expectations.

1. Introduction

At the last Combustion Symposium, Andrews, Bradley and Lwakabamba[1] presented values of turbulent burning velocities, u_t, for premixed gases, and suggested a primary correlation of such values with the laminar burning velocity, u_l, and the Taylor turbulent Reynolds number of the unburnt gas. Arising from their results a secondary influence was suggested, in that values of u_t/u_l tended to be less at a given Reynolds number for those mixtures with a higher value of u_l. To elucidate this possibility, it was decided to measure turbulent burning velocities for mixtures with significantly higher values of laminar burning velocity than previously investigated.

The turbulence was produced by four identical high speed fans, running at the same speed, and symmetrically disposed within an explosion vessel. Such a system has the advantages of producing a significant volume of gas with uniform, isotropic turbulence, at values of turbulent Reynolds number comparable to those achieved in practical combustion chambers.[1,2]

Any correlations which might be proposed between the different parameters should be valid for any system, irrespective of how the turbulence is produced and, for this reason, all known reasonably reliable data, drawn from a variety of rigs, are plotted alongside the present data. This has involved the correlation of several hundred experimental points and, where the required parameters had not been measured directly, the best possible estimating of their values. The problem is further complicated by the difficulties of measuring u_l and u_t accurately and by a certain arbitrariness in derivations of turbulent length scales.

2. Apparatus

The fans and vessel have been described previously.[1] The vessel was of 305 mm diam., 305 mm long, and with 150 mm diam. windows in each end. Four identical eight bladed fans were symmetrically positioned at 90° intervals around the central circumference. Hydrogen-air mixtures, at an initial pressure of one atmosphere, were used in order to obtain higher values of u_l than formerly, and the use of hydrogen necessitated improved shaft sealing, in the form of Leybold Rotary Transmissions, which comprise a seal and bearing housing. A higher power was required to drive the fans, because of the increased resisting torque of the seal. The original air turbines therefore were replaced by 1 horsepower d.c. motors, the speed of each of which could be independently and accurately adjusted via thyristor control of the armature voltage.

Turbulent burning velocities were measured by the double kernel technique, which involves high speed schlieren photography of twin propagating flame kernels, ignited simultaneously at separate spark gaps. At the instant when the edges of the kernels meet, the flame speed of each of the kernels becomes equal to the burning velocity.[3] A laser source with two external lenses was used, in conjunction with a "V" knife edge at the focus, and a Fastax rotating prism camera, to record the flame front propagations.

The two pairs of spark electrodes were mounted horizontally, 40 mm apart. The two gaps were connected in series, in an inductance-capacitance circuit with a stored energy of 1.8J at 5 kV, and this ensured a sufficiently short time interval between the two sparks. Ignition was triggered by a pulse to a three electrode gap.

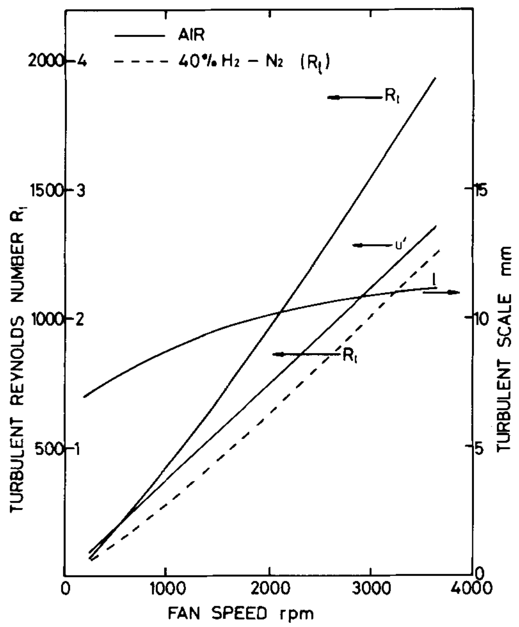

FIG. 1. Turbulent calibration of fan system. R.m.s. turbulent velocity, u', in m sec^{-1}.

3. Turbulence Measurements

Measurements of length scale and r.m.s. turbulent velocity, u', were taken in room temperature gases using Disa hot wire anemometry equipment. A variety of mixtures, with different values of kinematic viscosity, ν, were used in the calibration, which will be reported more comprehensively elsewhere.[4] Here, results are briefly presented for air and a 40% H_2-60% N_2 mixture. There were some differences with previous measurements.[1] In the present work turbulent Reynolds numbers, R_l, $(=u'l/\nu)$, are based upon the integral scale of turbulence, l.

This latter was obtained by two point velocity correlation. A correlation coefficient of unity was never attained, possibly because of interference effects between the two wires at small separations.[5] Neither was a coefficient of zero obtained at the largest possible separation within the homogeneous region of turbulence. The lowest value was 0.03, which was also attained when one wire was removed from the vessel and immersed in an independent gas jet. The value of l was obtained by dividing the area under the correlation coefficient separation curve between the maximum and minimum values of the coefficient by the differences between these values. The variation of l with fan speed is shown in Fig. 1, and this was the same for both air and the 40% H_2 mixture.

For measurements of u' in air, wires were calibrated in a Disa calibration rig. For measurements in H_2 mixtures, wires of approximately 1.2 mm length were placed centrally in a long copper tube of 12.7 mm diam. with fully developed laminar flow, and the volumetric flow rate measured. In the explosion vessel there is no mean flow and, because the hot wire is incapable of indicating the direction of flow, the velocity fluctuations are effectively rectified. A Disa d.c. voltmeter, type 55D30, measured the time mean value of the corresponding voltage fluctuations, μ, whilst an r.m.s. voltmeter, type 55D35, measured an r.m.s. value, σ, about the pseudo mean velocity, indicated by μ. The voltage, e, corresponding to the effective r.m.s. velocity to which the wire responded is given by[6]

$$e = (\mu^2 + \sigma^2)^{1/2} \tag{1}$$

It is assumed that the wire does not respond to the velocity component along its length, but that "e" is the response to the two rectangular components of velocity at right angles to it. If u' is the r.m.s. value of any one of these, it can be shown[2] for the present isotropic conditions that the calibration linked to "e"

TABLE I
Experimental results of present work with H_2-air mixtures

Fan Speed rpm	15% H_2 $u_l = 1.01$ m sec^{-1}			20% H_2 $u_l = 1.56$ m sec^{-1}			30% H_2 $u_l = 2.78$ m sec^{-1}			40% H_2 $u_l = 3.48$ m sec^{-1}		
	$\frac{u_t}{u_l}$	$\frac{u_l}{u'}$	R_l	$\frac{u_t}{u_l}$	$\frac{u_l}{u'}$	R_l	$\frac{u_t}{u_l}$	$\frac{u_l}{u'}$	R_l	$\frac{u_t}{u_l}$	$\frac{u_l}{u'}$	R_l
250				1.67	7.8	75	1.37	13.92	67	1.35	17.40	59
500	2.72	2.66	163	2.31	4.11	154	1.87	7.33	138	1.69	9.16	122
1000				3.27	2.08	348	2.37	3.71	311	2.09	4.64	274
1500				3.78	1.38	567	2.66	2.46	507	2.33	3.08	447
2000				4.17	1.04	799	2.91	1.86	713	2.54	2.32	629
2500				4.62	0.84	1030	3.28	1.50	920	2.79	1.87	811
3000				5.03	0.70	1276	3.46	1.25	1139	3.05	1.56	1005
3500				5.51	0.60	1522	3.74	1.07	1359	3.25	1.34	1198

gives $(2)^{1/2}\, u'$. There was some heating of the gas by the rotating fans and allowance for the effect of this on the calibration was made by maintaining a constant overheat ratio.[7]

The variations of u' with fan speed are shown in Fig. 1 and these were the same for both air, ($\nu = 15.7 \times 10^{-6}$ m^2 sec^{-1}), and 40% H_2-N_2, ($\nu = 24.06 \times 10^{-6}$ m^2 sec^{-1}). Also shown in this figure are the variations of R_l for the two mixtures.

4. Experimental Results

(a) *Present Work*

The results are given in Table I and many of these also are shown in the graphs of u_t/u_l, plotted against u_l/u', for different ranges of R_l in Figs. 2 to 6. Other correlating parameters have been suggested.[8] The reason for using R_l as a parameter has been discussed pre-

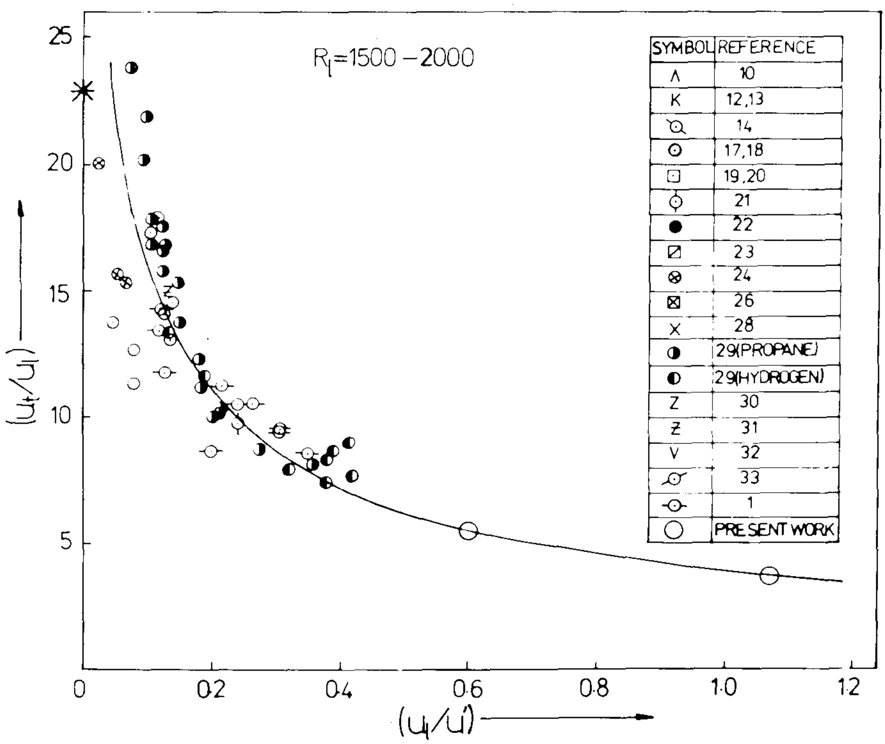

FIG. 2. Variation of u_t/u_l with u_l/u', $R_l = 1500 - 2000$. Asterisked point from Eq. (10).

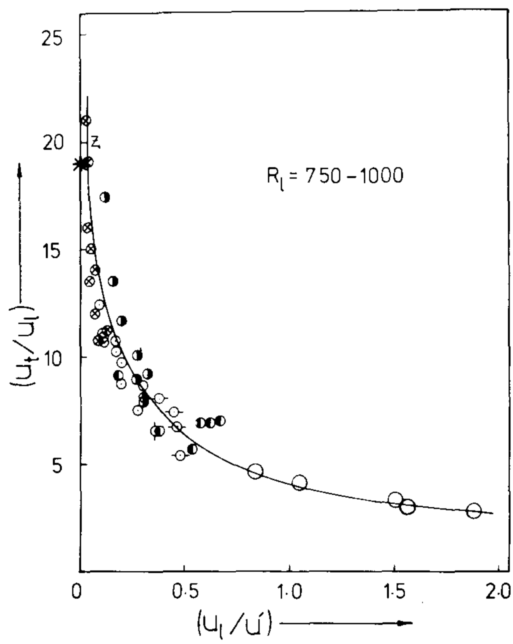

FIG. 3. Variation of u_t/u_l with u_l/u', R_l = 750 – 1000. Key on Fig. 2. Asterisked point from Eq. (10).

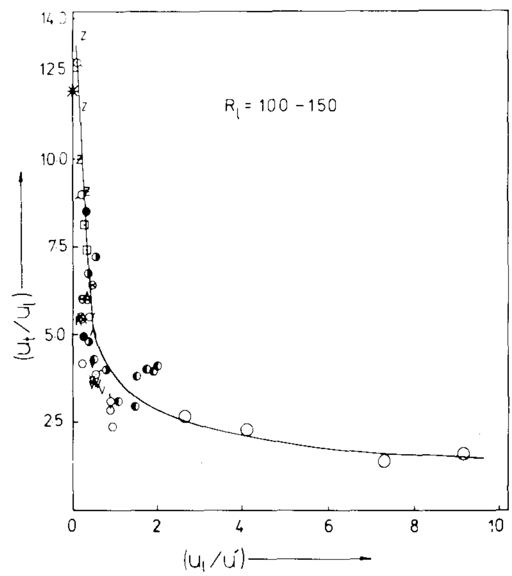

FIG. 5. Variation of u_t/u_l with u_l/u', R_l = 100 – 150. Key on Fig. 2. Asterisked point from Eq. (10).

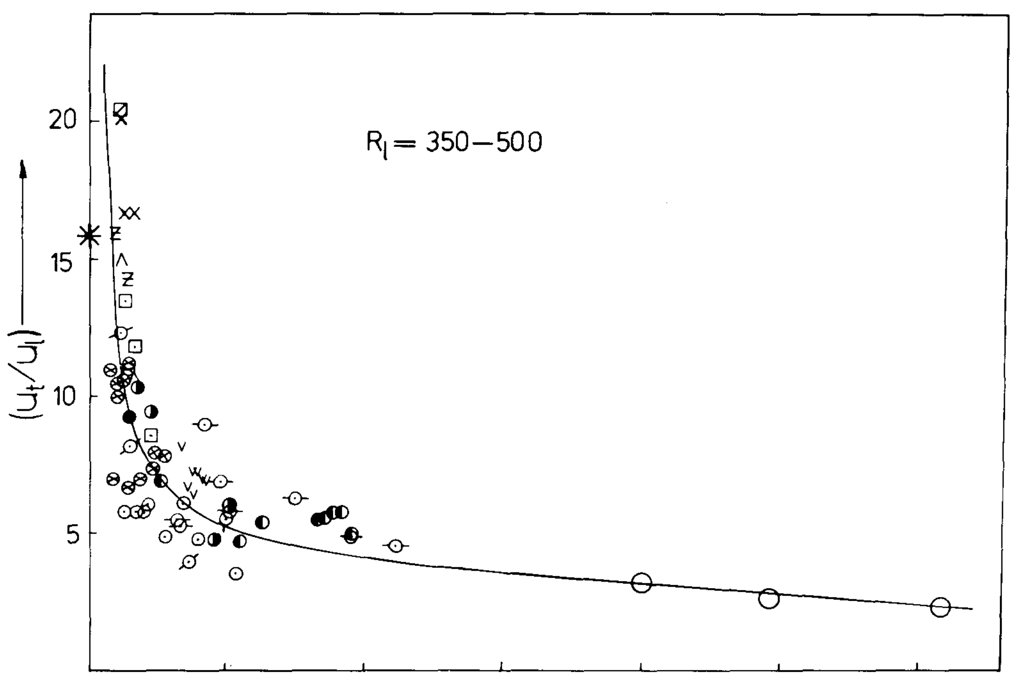

FIG. 4. Variation of u_t/u_l with u_l/u', R_l = 350 – 500. Key on Fig. 2. Asterisked point from Eq. (10).

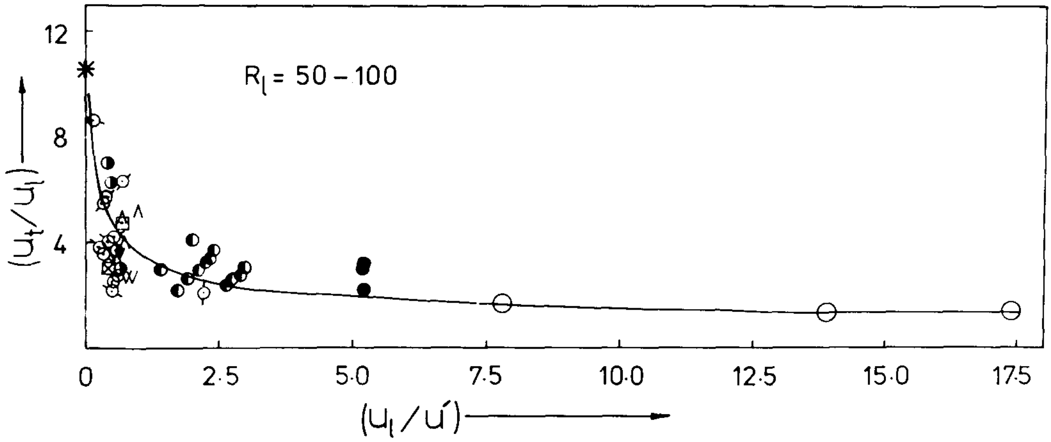

FIG. 6. Variation of u_t/u_l with u_l/u', R_l = 50 – 100. Key on Fig. 2. Asterisked point from Eq. (10).

viously[8,1] and the choice of u_l/u', and not u'/u_l, is preferred due to the existence of a theoretical limiting value of u_t/u_l as $u_l/u' \to 0$. This will become more apparent later.

Alongside the present experimental points, are presented those of previous workers, which are summarised in Table II. For reasons of space, not all the experimental points are shown in these figures. In particular, no points are shown from Refs. 9, 11, 15, 16, 25 and 27. These references did, however, provide some of the points from which were derived the curves in Fig. 7. The "best" curves were drawn through the points by eye with some regard being paid to the possible accuracy of the measurements although, because of the

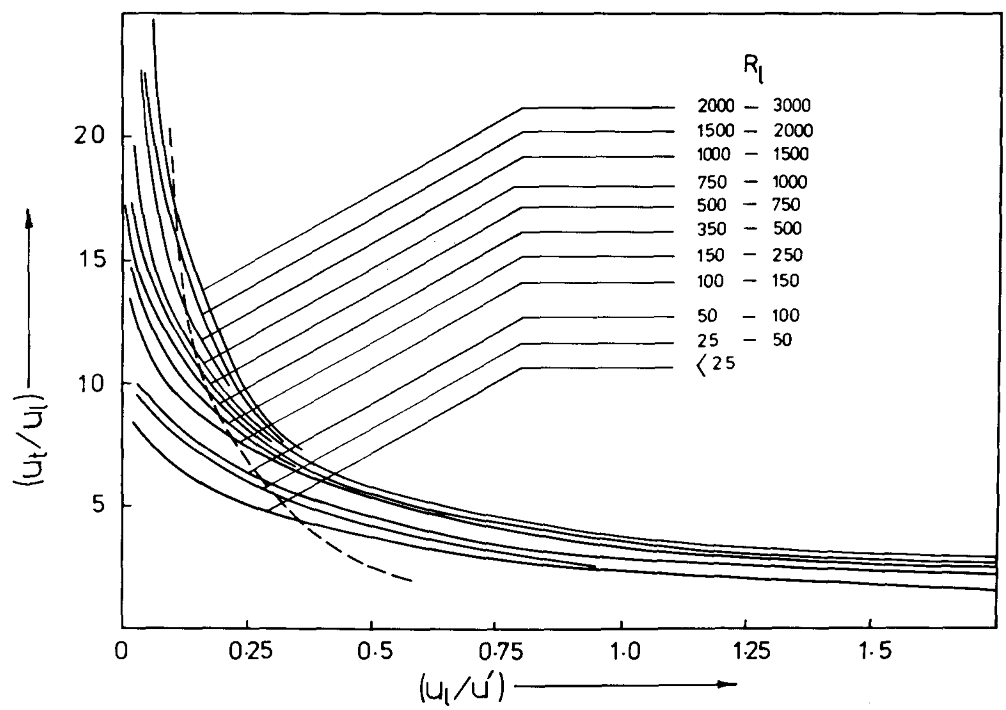

FIG. 7. Variation of u_t/u_l with u_l/u', for different values of R_l. Equality of left and right terms in Eq. (7) indicated by broken curve.

TABLE II
Turbulent burning velocity data of previous workers

Ref.	Mixture	P(atmos)	T(°C)	ν($m^2 s^{-1} \times 10^6$)	u_l(ms^{-1})	u_l Ref.	l(mm)	u'(ms^{-1})	Method
9	Propane-O_2	1	25	11.6-13.2	1.27-2.90	9	Eq. (2), R_l	Eq. (4), 1.12-4.17	Tube, inner cone area
10	City Gas-Air	1	25	16.0	.24-.35	10	Measured, 3.18	Given, $u'/U = 2.3\%$	Nozzle, grid, inner flame area.
11	3 mixtures	1	25	14.72-15.68	.45-1.47	11	Eq. (2), R_l	Eq. (4), .162-2.85	Different size burners, total area method
12	2 mixtures	1	25	15.879	.45, 1.75	12, 34	Eq. (2), R_l	Measured, .4-6.5	Tube, inner angle, $r/R \leq 0.7$
13	Methane-Air	1	25	15.879	0.45	34	Measured, .5-3.0	Measured, .09-3.5	Tube, inner angle
14	2 mixtures	1	25	15.70	.12-.4	14	Given, Ref. 38, .22-1.64	Given, Ref. 39,40, $u'/U = 0.9\%-13.3\%$	Tube, total area of max. luminosity.
15	3 mixtures	1	25	≈15.0	.36-1.24	15	Eq. (2), R_l	Given, $u'/U = 2\%-4.9\%$	Tube, mean flame-surface area
16	Methane-Air	1	25	15.89	.45	34	Eq. (2), R_l	Measured, .04-.37	Tube, "V" flame, angle
17,18	9 mixtures	1	25	13.6-95.3	.29-1.4	17,18	Measured, 2.3-4.8	Measured, .8-8.4	Bomb, fan, expansion ratio
19,20	Benzene-Air	1	167-197	29.62-33.17	.69-.79	19,35	Eq. (6), 1.54-3.30	Given, $u'/U = 5\%-15\%$	Duct, grid, inner area
21	Pentane-Air	1	25	≈15.0	.442	21	Eq. (2), R_l	Eq. (4), .201-1.83	Curve extrapolated to give $U/U_o = 2.5$ at Re = 2500, $u_t/u_l = U/U_o \times 1/2.5$
22	Propane-Air	0.1-0.6	20	26.22-157.32	.46 $P^{-0.5}$, .59-1.45	3, 36	Measured, 2.8	Given, 2.8-5.0	Tube, grid, inner area
23	Benzene-Air	1	17-227	15.27-36.55	.46-.96	23, 35	Eq. (2), R_l	Given, $u'/U = 5\%$	Duct, inner area
24	Benzene-Air	1	100	23.2	.2-.6	24	Eq. (6), 2.67-3.90	Measured, 1.3-16	Pipe, grid, inner area
25	Propane-Air	0.1-1.0	25	15.73-157.32	Given, u_t/u_l	25	Eq. (6), 1.65	Measured, .022-.036	Tube, grid total area method.
26	Methane-Air	1	25	15.89	.32	26	Given, Ref. 38, 0.76-1.27	Given, Ref. 38, .19-.65	Tube, grid, total area method
27	CH_4-O_2-N_2	1	25	16.461	4.5	27	Eq. (4), (2), R_l	Measured, .08-.33	Nozzle, flat flame

TABLE II (*continued*)
Turbulent burning velocity data of previous workers

Ref.	Mixture	P(atmos)	T(°C)	ν(m^2 s^{-1} $\times 10^6$)	u_l(ms^{-1})	u_l Ref.	l(mm)	u'(ms^{-1})	Method
28	Propane-Air	1	25	14.72	.28-.46	35	Eq. (2), R_l	Given, u'/u_l	Square tube, "V" flame, gas vel. angle
29	Propane-Air	1	25	14.37-14.85	.27-.46	35	Eq. (5),(3), R_l	Measured, .3-11.0	Slot burner, inner area
29	Hydrogen-Air	1	25	17.83-20.75	.66-2.42	3	Eq. (5),(3), R_l	Measured, .4-11.6	Slot burner, inner area
30	Gasoline-Air	1	150-550	28.1-87.5	.8-2.92	37	Eq. (2), R_l	Eq. (4), 1.47-4.86	Tube, inner area
31	Gasoline-Air	1	150-350	28.1-55.8	.5-1.65	37	Eq. (2), R_l	Measured, 1.0-8.0	Tube, inner area
32	6 mixtures	1	25	15.01-16.07	.89-1.98	32	Measured, .55-1.64	Measured, 1.06-5.70	Opposed flow, flame position.
33	Propane-Air	1	25	14.72	.45	33	Measured, .56-5.08	Measured, .25-3.84	'V' flame, angle
1	2 mixtures	1	25	15.83-15.94	.15-.79	1	Measured (Fig. 1-this paper), 7.7-11.25	Measured (Fig. 1-this paper), .38-2.98	Bomb, fan, double kernel

variety of measured parameters involved, a satisfactory error analysis for each point was not possible.

The present work extends the range of experimental investigations with an approximately three fold increase in maximum values of u_l/u' over previously measured values, up to values of R_l in the region of 1500. Values of u_l for H_2-air were taken from Ref. 3 and these reached a maximum of 3.48 m sec^{-1} for 40% H_2. The use of H_2 made possible the attainment of values of u_l/u' as high as 17.40, at a value of R_l of 59. The lowest value of u_l/u' was 0.60 at a peak value of R_l of 1522.

(b) *Previous Work*

Reference to Table II shows that a significant number of experimenters have not measured the turbulent length scale. An alternative method of finding R_l therefore was adopted in such cases, when circular tube and flat slot burners had been employed. Experimental measurements of turbulent length scales and r.m.s. turbulent velocities in nonreacting flows in tubes and between plates were reviewed[41] and R_l, with l measured in the transverse direction, was correlated with the flow Reynolds numbers, Re and Re_p.

One of the problems with such burners is that u' and l vary across the burner. Powe et al.[42,43] have measured the variation of these parameters across a pipe and Laufer[44] has measured them between parallel plates. It is clear that there is an associated variation in R_l. The procedure adopted in the present work was to estimate the values of these at the plane which is half-way between the centre axis and the walls, as flame measurements have been associated with this region. Such estimates are not facilitated by the increasing departure from isotropic turbulence and the lack of length scale data as the walls are approached. A further complication might arise from the generation of turbulence by the gases discharging from the burner, but any such effect upon the unburnt gases has been neglected.

At half the radius of the tube it was estimated that

$$R_l = 5.927 \times 10^{-8} (\mathrm{Re})^{1.84} \qquad (2)$$

for $R_l > 100$.

Half-way between the centre plane and a plate

$$R_l = 6.121 \times 10^{-6} (\mathrm{Re}_p)^{1.593} \qquad (3)$$

for $R_l > 50$, and with Re_p based upon the distance apart of the plates.

Values of R_l which have been utilised and which are less than these limiting values were all obtained by other means.

Where necessary, values of u' at these same planes were obtained from[41]

$$\frac{u'}{U} = 0.1676\,(\mathrm{Re})^{-0.119} \tag{4}$$

and

$$\frac{u'}{U} = 0.1151\,(\mathrm{Re}_p)^{-0.118} \tag{5}$$

where U is the mean velocity.

Where it was necessary to estimate the value of l in grid-generated turbulence the data of Ref. 45 were utilised in the form

$$\frac{l_y}{b} = 0.115\left(\frac{x}{b}\right)^{0.452} \tag{6}$$

where

x = distance downstream from mid-plane of screen.
b = bar width.
l_y = length scale transverse to direction of flow (x).

5. Discussion

The curves in Figs. 2 to 6 clearly indicate the influence of both R_l and u_l/u' upon the ratio u_t/u_l. The best curves through the experimental points, including those not given on the earlier figures, are collected together in Fig. 7. Bearing in mind all the difficulties in accurately measuring burning velocities and turbulent parameters, the exhibited correlations are significant. Practical combustors operate at the highest values of R_l and with u_l/u' in the region of 0.1.[8] Figure 7 shows this is a regime where an increase in turbulence can increase u_t appreciably, although this may be accompanied by increasingly difficult ignition and stability.

Turbulent flame structure has been discussed elsewhere[8,46] and it is of interest to ascertain possible bounds to a regime in which a wrinkled laminar flame structure can exist.[47,48] A simple criterion for this structure is that the laminar flame thickness must be smaller than the Kolmogorov microscale. This leads,[49] to

$$\frac{u_l}{u'} >> 0.745\, R_l^{-1/4} \tag{7}$$

Equality of the two terms is indicated by the broken curve in Fig. 7. A wrinkled laminar flame structure is probable, only significantly to the right of this curve. The curve, however, does not delineate two distinctively different flame structures.

Schlieren photographs of flames were compared for the two regimes. At high values of R_l with hydrocarbon-air flame values of u_l/u' in the vicinity of 0.1, a pronounced small scale structure was in evidence, in addition to the macro-structure. For a 20% H_2-air flame with R_l = 1522 and u_l/u' = 0.6, which point lies to the right of the curve, a small scale structure also was revealed. At comparable values of R_l, but with hydrogen-air flames with values of u_l/u' greater than unity, only the macro-structure was in evidence. The appearance now was that of a wrinkled laminar flame. Reference to Fig. 7 shows such a flame to be in the regime where this structure might be expected. As u_l/u' tends to larger values, so molecular transport processes overwhelm those of turbulent transport and, in the limit, with u_l/u' infinite, u_t/u_l is unity. The present evidence suggests the wrinkled flame regime occurs for $u_l/u' > 1$.

Outside the regime of the wrinkled laminar flame, the small eddies, that have been observed photographically, probably play an active role in flame propagation. Ciné film measurements reveal them to be moving with very high velocities, in excess of u'. If it is assumed that turbulent predominates over molecular transport, with but small changes in chemical kinetics, a relationship can be derived between u_t/u_l and ϵ/ν, where ϵ is the eddy diffusivity. The treatment of Spalding[50,51] in deriving an analytical expression for u_l may be followed, but with turbulent transport coefficients replacing those of molecular transport. For constant transport parameters and specific heats, and with a turbulent Lewis number of unity, this leads to

$$\frac{u_t}{u_l} = \left(\frac{\epsilon}{\nu}\,\mathrm{Pr}\right)^{1/2} \tag{8}$$

where Pr is the Prandtl number. This assumes that the area under the chemical heat release rate versus reactedness curve, and the shape of the curve, are unchanged by turbulence.

Values of ϵ/ν, a turbulent transport Reynolds number, cannot be obtained theoretically but they have been measured in several experimental studies of non-reacting turbulent transport systems. Correlation of available experimental data[41] suggests, for isotropic turbulence, that

$$\frac{\epsilon}{\nu} = 20.5\, R_l^{0.476} \tag{9}$$

for $R_l > 100$.

Equation (8) essentially is obtained from an analogy between small eddy and molecular transport, but it is deficient in an important respect. Whereas a molecule has a high velocity and can easily move against a flow velocity of u_l, an eddy has a much smaller velocity and cannot so easily move against a flow velocity of u_t. In the limit, as u_t/u' and u_l/u' tend towards zero, the latter motion is always possible and here Eq. (8) is most valid.

Theoretical values of u_t/u_l were found from Eqs. (8) and (9), for different values of R_l, with Pr = 0.7. The two equations give

$$\frac{u_t}{u_l} = 3.788\,(R_l)^{0.238} \tag{10}$$

for $R_l > 100$ and $u_l/u' \to 0$. Values obtained from Eq. (10) have been marked by an asterisk, for the mid range values of R_l, on Figs. 2 to 6, for u_l/u' equal to zero. The agreement with the curves through the experimental points is fair and supports a small scale eddy mechanism for flames in the non-wrinkled régime. The decrease of u_t/u_l from such values, as u_l/u' is increased, is explained by the increasing difficulty for the small eddies to move against the flow.

6. Conclusions

(1) Turbulent combustion has been studied experimentally over a regime with wide variation of u_l/u'. Correlations are shown of u_t/u_l with u_l/u' and R_l.

(2) When $u_l/u' > 1$ a wrinkled flame régime is possible, but otherwise a small scale eddy structure seems important.

(3) Reference to non-reacting turbulent flows suggests a particular relationship between R_l and ϵ/ν. This has been used in turbulent flame theory to give values of u_t/u_l at the limiting condition of $u_l/u' = 0$. Such values are in fair agreement with experiment.

(4) The transition between a small scale eddy and a wrinkled laminar flame mechanism, at constant R_l, can be explained qualitatively.

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COMMENTS

Arthur H. Lefebvre, Purdue University, USA. Although the authors' equation $(u_t/u_l) = (\epsilon \mathrm{Pr}/\nu)^{0.5}$ might appear to have little in common with Ballal and Lefebvre's previously derived expression $(u_t/u_l) = 1 + \text{const.}\ (u'l/u_l\delta_l)$, if one makes the substitutions $\epsilon = u'l$, $\mathrm{Pr} = c_p\ \mu/k$, $\nu = \mu/\rho$ and laminar flame thickness $\delta_l = k/c_p\,\rho\, u_l$, the equation then becomes $(u_t/u_l) = (u'l/u_l\delta_l)^{0.5}$. The close similarity between the two expressions becomes immediately apparent, thus providing useful confirmation of the validity of Ballal and Lefebvre's 3-region model for turbulent flames.[1]

Of the two similar expressions, that of Ballal and Lefebvre has the advantage of predicting that u_t approaches u_l as u' tends to zero, which is exactly what one would expect and is, in fact, borne out by experimental observations.[2,3]

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Authors' Reply. At present we do not suggest a general theoretical expression for u_t/u_l, but only two limit conditions. These are

$$u_t/u_l \rightarrow \left(\frac{\epsilon}{\nu}\,\mathrm{Pr}\right)^{1/2} \text{ as } \frac{u_l}{u'} \rightarrow 0$$

and $u_t/u_l \rightarrow 1$ as $u_l/u' \rightarrow \infty$. We recommend ϵ/ν be obtained from Eq. (9) and not from $\epsilon = u'l$. The experimental results tend to accord with these limits.

Unfortunately, the agreement between the present results and the algebraic expressions of Ballal and Lefebvre[33] is not good, as is shown from considerations of two cases.

Case 1: Kolmogorov scale, η, less than the laminar flame thickness, δ_l. This is the régime to the left of the broken curve in Fig. 7. Reference 33 gives for this régime

$$\frac{u_t}{u_l} = 0.5\left(\frac{u'\delta_l}{u_l\eta}\right) \tag{A}$$

From the expressions given in Ref. 8 it readily can be shown that $\eta \propto l\,\mathrm{R}_l^{-3/4}$. If this is combined with the expression quoted by Professor Lefebvre for δ_l, then

$$\frac{\delta_l}{\eta} \propto \frac{u'}{u_l}\,\mathrm{R}_l^{-1/4} \tag{B}$$

Substitution of Eq. (B) in Eq. (A) gives

$$\frac{u_t}{u_l} \propto \left(\frac{u'}{u_l}\right)^2 \mathrm{R}_l^{-1/4} \tag{C}$$

and u_t/u_l would decrease with increase of R_l at constant value of u_l/u'. This is contrary to the results summarised by the curves in Fig. 7.

Case 2: Kolmogorov scale greater than the laminar flame thickness. This is the régime to the right of the broken curve in Fig. 7. Reference 33 gives for this régime (after a correction to the published length scales by the authors)

$$\left(\frac{u_t}{u_l}\right)^2 = 1 + 0.03\left(\frac{u'l}{u_l\delta_l}\right)^2 \tag{D}$$

With Professor Lefebvre's expression for δ_l this becomes

$$\left(\frac{u_t}{u_l}\right)^2 = 1 + 0.03\,(\mathrm{R}_l\mathrm{Pr})^2 \tag{E}$$

This expression shows no dependence upon u_l/u'. The experimental results for hydrogen-air flames show that even when R_l is large, $u_t/u_l \rightarrow 1$ as $u_l/u' \rightarrow \infty$. For example, measurements, reported in Table I, show, that with $\mathrm{R}_l = 1005$, and $u_l/u' = 1.56$ then $u_t/u_l = 3.05$. On the other hand, Eq. (E) gives u_t/u_l, for this values of R_l and with $\mathrm{Pr} = 0.7$, the impossibly high value of 122.

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Rolf Kleine, University of Karlsruhe, West Germany. Was turbulent flame velocity changing with time during propagation or was there a constant flame velocity? Flame velocity should depend on time theoretically.

Authors' Reply. We have not observed such a change, but this is not to say it does not exist. A larger explosion vessel would be required to study fully any such effect.