

# Typing Rules and Evaluation rules

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## 1 Syntax

$t ::=$	<i>terms</i>
$v$	
$\text{if term then term else term}$	
$\text{succ number}$	
$\text{pred number}$	
$\text{iszero number}$	
$\text{ref term}$	
$t\ t$	
$\text{wait } t$	
$\text{fork}\{t\}$	
$\text{mutex} < X_1, X_2, \dots, X_n >$	
$\text{abstraction}$	
$\text{tag term}$	
$\text{record term}$	
$\text{abstraction} ::=$	<i>abstraction term</i>
$\lambda x : T. t$	
$\lambda < X_1, X_2, \dots, X_n > x : T. t$	
$\lambda < X_1, X_2, \dots, X_n > [Y]x : T. t$	

$refterm ::=$	<i>terms about ref</i>
	$!t$
	$ref\ t$
	$ref\ < X1, X2, \dots, Xn >\ t$
	$t := t$
$thterm ::=$	<i>terms about thread</i>
	$wait\ t$
	$fork\{t\}$
$tagterm ::=$	<i>terms about tags</i>
	$< l = t >\ as\ T$
	$case\ t\ of\ < l_i = x_i > \implies t_i^{i \in 1..n}$
$recodesterm ::=$	<i>terms about recoders</i>
	$l_i = t_i^{i \in 1..n}$
	$t.l$
$v =$	<i>values</i>
	$true$
	$false$
	$0$
	$\lambda x : T. t$
	$string$
	$unit$
	$number$
	$float$
	$record$
	$mutex$
	$loc$
	$tag$
	$forkv$
	$< l = v >\ as\ T$
	$l_i = v_i^{i \in 1..n}$

## 2 Typing rules

### 2.1 Fork

$$\frac{\Gamma|\mathbb{L} \vdash t : T}{fork\{ t \} : Thread\ T} \quad (T-FORK)$$

$$\frac{\Gamma|\mathbb{L} \vdash t : Thread\ T}{wait\ t : T} \quad (T-WAIT)$$

### 2.2 Mutex

$$\frac{}{\Gamma|\mathbb{L} \vdash mutex < X > : Mutex\ X} \quad (T-MUTEX)$$

### 2.3 Acquire

$$\frac{max\{\mathbb{L}\} <_{lex} X \quad \Gamma|\mathbb{L} \vdash t_1 : Mutex\ X \quad \Gamma|(\mathbb{L}, X) \vdash t_2 : T}{\Gamma|\mathbb{L} \vdash Acquire\ t_1\ t_2 : T} \quad (T-ACQUIRE)$$

### 2.4 Abstraction

$$\frac{(\Gamma, x : T_1)|(\mathbb{L} \cup \{X_i\}^{i \in 1 \dots n}) \vdash t : T_2 \quad mam(\Gamma, T_1) = Y}{\Gamma|\mathbb{L} \vdash \lambda < X_i^{i \in 1 \dots n} > x : T_1.t : T_1 < X_i^{i \in 1 \dots n} > [Y] \rightarrow T_2} \quad (T-ABS)$$

$$\frac{\Gamma|\mathbb{L} \vdash T_1.t : T_1 < X_i^{i \in 1 \dots n} > [Y] \rightarrow T_2 \quad \Gamma|\mathbb{L} \vdash t_2 : T_2}{\Gamma|\mathbb{L} \vdash t_1\ t_2 : T_2} \quad (T-APP)$$

## 2.5 Ref

$$\frac{\Gamma|\mathbb{L} \vdash v : T}{\Gamma|\mathbb{L} \vdash \text{ref} < X_i^{i \in 1 \dots n} > v : \text{Ref} < X_i^{i \in 1 \dots n} > T} \quad (\text{T-REF})$$

$$\frac{\Gamma|\mathbb{L} \vdash t_1 : \text{Source} < X_i^{i \in 1 \dots n} > T \quad X_i \in \mathbb{L}^{i \in 1 \dots n}}{\Gamma|\mathbb{L} \vdash !t_1 : T} \quad (\text{T-DEREF})$$

$$\frac{\Gamma|\mathbb{L} \vdash t_1 : \text{Sink} < X_i^{i \in 1 \dots n} > T \quad \Gamma|\mathbb{L} \vdash t_2 : T \quad X_i \in \mathbb{L}^{i \in 1 \dots n}}{\Gamma|\mathbb{L} \vdash t_1 := t_2 : \text{Unit}} \quad (\text{T-ASSIGN})$$

## 3 Subtyping rules

### 3.1 Thread

$$\frac{T_1 <: T_2}{\text{Thread } T_1 <: \text{Thread } T_2} \quad (\text{S-THREAD})$$

### 3.2 Abstraction

$$\frac{T_1 <: S_1 \quad S_2 <: T_2 \quad Y_1 \geq_{\text{lex}} Y_2 \quad \{X_i\}^{i \in 1 \dots n} \subseteq \{Z_j\}^{j \in 1 \dots m}}{S_1 < X_i^{i \in 1 \dots n} > [Y_1] \rightarrow S_2 <: T_1 < Z_j^{j \in 1 \dots m} > [Y_2] \rightarrow T_2} \quad (\text{S-ARROW})$$

### 3.3 Ref

$$\frac{T_1 <: T_2 \quad \{X_i\}^{i \in 1 \dots n} \subseteq \{Z_j\}^{j \in 1 \dots m}}{Source < X_i^{i \in 1 \dots n} > T_1 <: Source < Z_j^{j \in 1 \dots m} > T_2} \quad (\text{S-SOURCE})$$

$$\frac{T_2 <: T_1 \quad \{X_i\}^{i \in 1 \dots n} \subseteq \{Z_j\}^{j \in 1 \dots m}}{Sink < X_i^{i \in 1 \dots n} > T_1 <: Sink < Z_j^{j \in 1 \dots m} > T_2} \quad (\text{S-SINK})$$

$$\frac{T_1 <: T_2 \quad \{X_i\}^{i \in 1 \dots n} \subseteq \{Z_j\}^{j \in 1 \dots m}}{Ref < X_i^{i \in 1 \dots n} > T_1 <: Source < X_i^{i \in 1 \dots n} > T_1} \quad (\text{S-REFSOURCE})$$

$$\frac{T_2 <: T_1 \quad \{X_i\}^{i \in 1 \dots n} \subseteq \{Z_j\}^{j \in 1 \dots m}}{Ref < X_i^{i \in 1 \dots n} > T_1 <: Sink < X_i^{i \in 1 \dots n} > T_1} \quad (\text{S-REFSINK})$$

### 3.4 Thread

$$\frac{T_1 <: T_2}{Thread\ T_1\ <: Thread\ T_2} \quad (\text{S-THREAD})$$

## 4 Algorithmic Typing Rules

## 5 Evaluation Rules

### 5.1 Threads

$$\frac{threads = \{t_i\}^{i \in 1 \dots n} \quad \langle t_{id}, \mu, Th, L \rangle \rightarrow \langle t'_{id}, \mu', Th', L' \rangle \quad id' = next(id, threads')}{[Th, \mu, id, L] \rightarrow [Th', \mu', id', L']} \quad (\text{E-THREAD})$$

## 5.2 Wait

$$\frac{threads(p) = v}{< wait\ p, \mu, Th, L > \rightarrow < v, \mu, Th \setminus \{p\}, L >} \quad (\text{E-WAIT})$$

$$< fork\ t, \mu, Th, L > \rightarrow < p, \mu, Th, L > \quad (\text{E-FORK})$$

### 5.3 Mutex

$$\frac{t_1 \rightarrow t'_1}{ref < x > t_1 \rightarrow ref < x > t'_1} \quad (\text{E-REFMUTEX})$$

$$\frac{t_1|\mu \rightarrow t'_1|\mu'}{ref < x > t_1|\mu \rightarrow ref < x > t_1|\mu} \quad (\text{E-REFMUTEX})$$

$$\frac{l \notin dom(\mu)}{ref < x > v|\mu \rightarrow l|\mu, l \mapsto v} \quad (\text{E-REFV})$$

$$\frac{t_1|\mu \rightarrow t'_1|\mu'}{!t_1|\mu \rightarrow !t'_1|\mu} \quad (\text{E-DEREF})$$

$$\frac{\mu(l) = v}{!l|\mu \rightarrow v|\mu} \quad (\text{E-DEREFLOC})$$

$$\frac{t_1|\mu \rightarrow t'_1|\mu'}{t_1 := t_2|\mu \rightarrow t'_1 := t_2|\mu} \quad (\text{E-ASSIGN1})$$

$$\frac{t_2|\mu \rightarrow t'_2|\mu'}{v := t_2|\mu \rightarrow v := t'_2|\mu} \quad (\text{E-ASSIGN2})$$

$$\frac{\mu(l) = v}{l := v|\mu \rightarrow unit|\mu, l \mapsto v} \quad (\text{E-ASSIGNV})$$

$$\frac{\Gamma|\mathbb{L} \vdash t_1 : Ref < X > T \quad \Gamma \vdash t_2 : T \quad X \in \mathbb{L}}{\Gamma|\mathbb{L} \vdash t_1 := t_2 : Unit} \quad (\text{S-REFMUTEX})$$

$$\frac{t_1 \rightarrow t'_1}{lock\ t_1\ t_2 \rightarrow lock\ t'_1\ t_2} \quad (\text{E-LOCK1})$$

$$\frac{t_2 \rightarrow t'_2}{lock\ v_1\ t_2 \rightarrow lock\ v_1\ t'_2} \quad (\text{E-LOCK2})$$

$$lock\ v_1\ v_2 \rightarrow v_2 \quad (\text{E-LOCKV})$$