Typing Rules and Evaluation rules

L

June 5, 2023

1   Syntax

$t ::=$		*terms*
	v	
	if term then term else term	
	succ number	
	pred number	
	iszero number	
	refterm	
	$t\ t$	
	wait t	
	$fork\{t\}$	
	$mutex < X1, X2, ..., Xn >$	
	abstraction	
	tagterm	
	recordsterm	
$abstraction ::=$		*abstraction term*
	$\lambda x : T.t$	
	$\lambda < X_1, X_2, ..., X_n > x : T.t$	
	$\lambda < X_1, X_2, ..., X_n > [Y]x : T.t$	

$$refterm ::= \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad terms\ about\ ref$$

$$!t$$
$$ref\ t$$
$$ref < X1, X2, ..., Xn > \ t$$
$$t := t$$

$$thterm ::= \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad terms\ about\ thread$$

$$wait\ t$$
$$fork\{t\}$$

$$tagterm ::= \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad terms\ about\ tags$$

$$< l = t > \ as\ T$$
$$case\ t\ of\ \ < l_i = x_i > \Longrightarrow\ t_i^{i \in 1..n}$$

$$recodesterm ::= \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad terms\ about\ recoders$$

$$l_i = t_i^{i \in 1..n}$$
$$t.l$$

$$v = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad values$$

$$true$$
$$false$$
$$0$$
$$\lambda x : T.t$$
$$string$$
$$unit$$
$$number$$
$$float$$
$$record$$
$$mutex$$
$$loc$$
$$tag$$
$$forkv$$
$$< l = v > \ as\ T$$
$$l_i = v_i^{i \in 1..n}$$

2

2 Typing rules

2.1 Fork

$$\frac{(\Gamma, tid_2)|\Sigma|\mathbb{L} \vdash t: \ T \qquad lasttid(\Gamma) = tid_1}{\Gamma|\Sigma|\mathbb{L} \vdash fork\{t, tid_2\}: \ Thread[tid_1] \ T} \qquad \text{(T-FORK)}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t: \ Thread[tid] \ T \qquad lasttid(\Gamma) = tid}{\mathbb{L} = \emptyset}{\Gamma|\Sigma|\mathbb{L} \vdash wait \ t \ : \ T} \qquad \text{(T-WAIT)}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash \Sigma(p): \ T}{\Gamma|\Sigma|\mathbb{L} \vdash < p, tid >: Thread < tid > T} \qquad \text{(T-THREAD)}$$

2.2 Mutex

$$\frac{}{\Gamma|\Sigma|\mathbb{L} \vdash mutex < X_i >^{i \in 1...n}: \ Mutex \ X} \qquad \text{(T-MUTEX)}$$

2.3 Acquire

$$\frac{max\{\mathbb{L}\} <_{lex} X \qquad \Gamma|\Sigma|\mathbb{L} \vdash t_1 : Mutex \ X \qquad \Gamma|\Sigma|(\mathbb{L}, X) \vdash t_2 : T}{\Gamma|\Sigma|\mathbb{L} \vdash Aacquire \ t_1 \ t_2 : T} \qquad \text{(T-ACQUIRE)}$$

2.4 Abstraction

$$\frac{(\Gamma, x : T_1)|(\mathbb{L} \cup \{X_i\}^{i \in 1...n}) \vdash t : T_2 \qquad mam(\Gamma|\Sigma|\mathbb{L}, T1) = Y}{\Gamma|\Sigma|\mathbb{L} \vdash \lambda < X_i^{i \in 1...n} > x : T_1.t : \ T_1 < X_i^{i \in 1...n} > [Y] \rightarrow T_2} \qquad \text{(T-ABS)}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash T_1.t : \ T_1 < X_i^{i \in 1...n} > [Y] \rightarrow T_2 \qquad \Gamma|\Sigma|\mathbb{L} \vdash t_2 : T_1}{\Gamma|\Sigma|\mathbb{L} \vdash t_1 \ t_2 : \ T_2} \qquad \text{(T-APP)}$$

2.5 Ref

$$\frac{\Sigma(l) : T}{\Gamma|\Sigma|\mathbb{L} \vdash l < X_i^{i \in 1 \ldots n} >: Ref < X_i^{i \in 1 \ldots n} > T} \quad \text{(T-Loc)}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash v : T}{\Gamma|\Sigma|\mathbb{L} \vdash ref < X_i^{i \in 1 \ldots n} > v : \ Ref < X_i^{i \in 1 \ldots n} > T} \quad \text{(T-Ref)}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : Source < X_i^{i \in 1 \ldots n} > T \quad X_i \in \mathbb{L}^{i \in 1 \ldots n}}{\Gamma|\Sigma|\mathbb{L} \vdash !t_1 : T} \quad \text{(T-Deref)}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : Sink < X_i^{i \in 1 \ldots n} > T \quad \Gamma|\Sigma|\mathbb{L} \vdash t_2 : T \quad X_i \in \mathbb{L}^{i \in 1 \ldots n}}{\Gamma|\Sigma|\mathbb{L} \vdash t_1 := t_2 : Unit} \quad \text{(T-Assign)}$$

2.6 Fix

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : T_1 < X_i^{i \in 1 \ldots n} > [None] \rightarrow T_1}{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : T_1} \quad \text{(T-Fix)}$$

3 Subtyping rules

3.1 Thread

$$\frac{T_1 <: T_2}{Thread \ T_1 \ <: \ Thread \ T_2} \quad \text{(S-Thread)}$$

3.2 Abstraction

$$\frac{T_1 \ <: \ S_1 \quad S_2 \ <: \ T_2 \quad Y_1 \geq_{lex} Y_2 \quad \{X_i\}^{i \in 1...n} \subseteq \{Z_j\}^{j \in 1...m}}{S_1 < X_i^{i \in 1...n} > [Y_1] \rightarrow S_2 \ <: \ T_1 < Z_j^{j \in 1...m} > [Y_2] \rightarrow T_2}$$

(S-Arrow)

3.3 Ref

$$\frac{T_1 <: T_2 \quad \{X_i\}^{i \in 1...n} \subseteq \{Z_j\}^{j \in 1...m}}{Source < X_i^{i \in 1...n} > T_1 <: Source < Z_j^{j \in 1...m} > T_2}$$

(S-Source)

$$\frac{T_2 <: T_1 \quad \{X_i\}^{i \in 1...n} \subseteq \{Z_j\}^{j \in 1...m}}{Sink < X_i^{i \in 1...n} > T_1 <: Sink < X_j^{j \in 1...m} > T_2}$$

(S-Sink)

$$\frac{T_1 <: T_2 \quad \{X_i\}^{i \in 1...n} \subseteq \{Z_j\}^{j \in 1...m}}{Ref < X_i^{i \in 1...n} > T_1 <: Source < X_i^{i \in 1...n} > T_1}$$

(S-RefSource)

$$\frac{T_2 <: T_1 \quad \{X_i\}^{i \in 1...n} \subseteq \{Z_j\}^{j \in 1...m}}{Ref < X_i^{i \in 1...n} > T_1 <: Sink < X_i^{i \in 1...n} > T_1}$$

(S-RefSink)

3.4 Thread

$$\frac{T_1 <: T_2}{Thread \ T_1 \ <: \ Thread \ T_2}$$

(S-Thread)

4 Algorithmic Typing Rules

5 Evaluation Rules

5.1 Threads

$$\frac{threads = \{< t_i, L_i^t, tid_i >\}^{i\in 1...n} \quad [t_{id}, \mu, Th, L, L^t] \to [t'_{id}, \mu', Th', L', L^{t'}]}{id' = next(id, threads')} \quad Th' = < t_i, L_i^t, tid_i >^{i\in 1...id-1} \cup < t'_{id}, L^{t'}, tid_{id} > \cup < t_i, L_i^t, tid_i >^{i\in id+1...n}}{[Th, \mu, id, L, L^t] \to [Th', \mu', id', L', L^{t'}]} \quad \text{(E-Thread)}$$

5.2 Wait

$$\frac{threads(p) = v}{[wait\ p, \mu, Th, L, L^t] \to [v, \mu, Th\backslash\{p\}, L, L^t]} \quad \text{(E-Wait)}$$

$$\frac{[fork\{t, tid_2\}, \mu, Th, l < X_i >^{i\in 1...n}, L^t]}{\to [< p, Tid(current) >, \mu, Th \cup \{< t, \emptyset, tid_2 >\}, L, L^t]} \quad \text{(E-Fork)}$$

5.3 Abstraction

$$\frac{[t_1, \mu, Th, L, L^t] \to [t'_1, \mu', Th', L', L^{t'}]}{[t_1\ t_2, \mu, Th, L, L^t] \to [t'_1\ t_2, \mu', Th', L', L^{t'}]} \quad \text{(E-App1)}$$

$$\frac{[t_2, \mu, Th, L, L^t] \to [t'_2, \mu', Th', L', L^{t'}]}{[v_1\ t_2, \mu, Th, L, L^t] \to [v_1\ t'_2, \mu', Th', L', L^{t'}]} \quad \text{(E-App2)}$$

$$[(\lambda x : T.t_{12})\ v_2, \mu, Th, L, L^t] \to [t_{12}[x \mapsto v_2], \mu, Th, L, L^t] \quad \text{(E-AppAbs)}$$

5.4 Reference

$$\frac{[t_1, \mu, Th, L, L^t] \to [t_1', \mu', Th', L', L^{t'}]}{[ref < X_i >^{i \in 1 \dots n} \; t_1, \mu, Th, L, L^t] \to [ref < X_i >^{i \in 1 \dots n} \; t_1', \mu', Th', L', L^{t'}]}$$
$$\text{(E-Ref)}$$

$$\frac{l \notin dom(\mu)}{[ref < X_i >^{i \in 1 \dots n} \; v, \mu, Th, L, L^t] \to [l < X_i >^{i \in 1 \dots n}, (\mu, l \mapsto v), Th', L', L^{t'}]}$$
$$\text{(E-RefV)}$$

$$\frac{[t_1, \mu, Th, L, L^t] \to [t_1', \mu', Th', L', L^{t'}]}{[!t_1, \mu, Th, L, L^t] \to [!t_1', \mu', Th', L', L^{t'}]}$$
$$\text{(E-Deref)}$$

$$\frac{\mu(l) = v}{[!l < X_i >^{i \in 1 \dots n}, \mu, Th, L, L^t] \to [v, \mu, Th, L, L^t]}$$
$$\text{(E-DerefLoc)}$$

$$\frac{[t_1, \mu, Th, L, L^t] \to [t_1', \mu', Th', L', L^{t'}]}{[t_1 := t_2, \mu, Th, L, L^t] \to [t_1' := t_2, \mu', Th', L', L^{t'}]}$$
$$\text{(E-Assign1)}$$

$$\frac{[t_2, \mu, Th, L, L^t] \to [t_2', \mu', Th', L', L^{t'}]}{[v := t_2, \mu, Th, L] \to [v := t_2', \mu, Th', L', L^{t'}]}$$
$$\text{(E-Assign2)}$$

$$\frac{\mu(l) = v}{[l < X_i >^{i \in 1 \dots n} := v, \mu, Th, L, L^t] \to [unit, (\mu, l \mapsto v), Th', L', L^{t'}]}$$
$$\text{(E-AssignV)}$$

5.5 Acquire

$$\frac{[t_1, \mu, Th, L, L^t] \rightarrow [t'_1, \mu', Th', L', L^{t'}]}{\begin{array}{c}[acquire\ t_1\ t_2, \mu, Th, L^t] \\ \rightarrow [acquire\ t'_1\ t_2, \mu', Th', L', L^{t'}]\end{array}} \quad \text{(E-Acquire1)}$$

$$\frac{X \notin L \quad X \notin L^t \quad L' = L \cup \{X\} \quad L^{t'} = L^t \cup \{X\}}{\begin{array}{c}[acquire\ mutex < X_i >^{i \in 1...n}\ t_2, \mu, Th, L, L^t] \\ \rightarrow [acquire\ mutex < X_i >^{i \in 1...n}\ t_2, \mu, Th, L', L^{t'}]\end{array}} \quad \text{(E-Acquire)}$$

$$\frac{X \in L^t \quad [t_2, \mu, Th, L, L^t] \rightarrow [t'_2, \mu', Th', L', L^{t'}]}{\begin{array}{c}[acquire\ mutex < X_i >^{i \in 1...n}\ t_2, \mu, Th, L, L^t] \\ \rightarrow [acquire\ mutex < X_i >^{i \in 1...n}\ t'_2, \mu', Th', L', L^{t'}]\end{array}} \quad \text{(E-Acquire2)}$$

$$\frac{X \in L^t \quad L' = L \backslash \{X\} \quad L^{t'} = L^t \backslash \{X\}}{\begin{array}{c}[acquire\ mutex < X_i >^{i \in 1...n}\ v_2, \mu, Th, L, L^t] \\ \rightarrow [v_2, \mu', Th', L', L^{t'}]\end{array}}$$

$$\text{(E-AcqRelease)}$$