

Typing Rules and Evaluation rules

L

June 5, 2023

1 Syntax

$t ::=$	<i>terms</i>
v	
$\text{if term then term else term}$	
succ number	
pred number	
iszero number	
ref term	
$t\ t$	
$\text{wait } t$	
$\text{fork}\{t\}$	
$\text{mutex} < X_1, X_2, \dots, X_n >$	
abstraction	
tag term	
record term	
$\text{abstraction} ::=$	<i>abstraction term</i>
$\lambda x : T. t$	
$\lambda < X_1, X_2, \dots, X_n > x : T. t$	
$\lambda < X_1, X_2, \dots, X_n > [Y]x : T. t$	

$refterm ::=$	<i>terms about ref</i>
$!t$	
$ref\ t$	
$ref\ < X1, X2, ..., Xn >\ t$	
$t := t$	
$thterm ::=$	<i>terms about thread</i>
$wait\ t$	
$fork\{t\}$	
$tagterm ::=$	<i>terms about tags</i>
$< l = t >\ as\ T$	
$case\ t\ of\ < l_i = x_i > \implies t_i^{i \in 1..n}$	
$recodesterm ::=$	<i>terms about recoders</i>
$l_i = t_i^{i \in 1..n}$	
$t.l$	
$v =$	<i>values</i>
$true$	
$false$	
0	
$\lambda x : T. t$	
$string$	
$unit$	
$number$	
$float$	
$record$	
$mutex$	
loc	
tag	
$forkv$	
$< l = v >\ as\ T$	
$l_i = v_i^{i \in 1..n}$	

2 Typing rules

2.1 Fork

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t : T}{\Gamma|\Sigma|\mathbb{L} \vdash \text{fork}\{t\} : \text{Thread } T} \quad (\text{T-FORK})$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t : \text{Thread } T}{\Gamma|\Sigma|\mathbb{L} \vdash \text{wait } t : T} \quad (\text{T-WAIT})$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash \Sigma(p) : T}{\Gamma|\Sigma|\mathbb{L} \vdash p : \text{Thread } T} \quad (\text{T-THREAD})$$

2.2 Mutex

$$\frac{}{\Gamma|\Sigma|\mathbb{L} \vdash \text{mutex} < X_i >^{i \in 1 \dots n} : \text{Mutex } X} \quad (\text{T-MUTEX})$$

2.3 Acquire

$$\frac{\text{max}\{\mathbb{L}\} <_{lex} X \quad \Gamma|\Sigma|\mathbb{L} \vdash t_1 : \text{Mutex } X \quad \Gamma|\Sigma|(\mathbb{L}, X) \vdash t_2 : T}{\Gamma|\Sigma|\mathbb{L} \vdash \text{Acquire } t_1 \ t_2 : T} \quad (\text{T-ACQUIRE})$$

2.4 Abstraction

$$\frac{(\Gamma, x : T_1) | (\mathbb{L} \cup \{X_i\}^{i \in 1 \dots n}) \vdash t : T_2 \quad \text{mam}(\Gamma|\Sigma|\mathbb{L}, T_1) = Y}{\Gamma|\Sigma|\mathbb{L} \vdash \lambda < X_i^{i \in 1 \dots n} > x : T_1.t : T_1 < X_i^{i \in 1 \dots n} > [Y] \rightarrow T_2} \quad (\text{T-ABS})$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash T_1.t : T_1 < X_i^{i \in 1 \dots n} > [Y] \rightarrow T_2 \quad \Gamma|\Sigma|\mathbb{L} \vdash t_2 : T_1}{\Gamma|\Sigma|\mathbb{L} \vdash t_1 \ t_2 : T_2} \quad (\text{T-APP})$$

2.5 Ref

$$\frac{\Sigma(l) : T}{\Gamma|\Sigma|\mathbb{L} \vdash l < X_i^{i \in 1 \dots n} > : Ref < X_i^{i \in 1 \dots n} > T} \quad (\text{T-LOC})$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash v : T}{\Gamma|\Sigma|\mathbb{L} \vdash ref < X_i^{i \in 1 \dots n} > v : Ref < X_i^{i \in 1 \dots n} > T} \quad (\text{T-REF})$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : Source < X_i^{i \in 1 \dots n} > T \quad X_i \in \mathbb{L}^{i \in 1 \dots n}}{\Gamma|\Sigma|\mathbb{L} \vdash !t_1 : T} \quad (\text{T-DEREF})$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : Sink < X_i^{i \in 1 \dots n} > T \quad \Gamma|\Sigma|\mathbb{L} \vdash t_2 : T \quad X_i \in \mathbb{L}^{i \in 1 \dots n}}{\Gamma|\Sigma|\mathbb{L} \vdash t_1 := t_2 : Unit} \quad (\text{T-ASSIGN})$$

2.6 Fix

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : T_1 < X_i^{i \in 1 \dots n} > [None] \rightarrow T_1}{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : T_1} \quad (\text{T-FIX})$$

3 Subtyping rules

3.1 Thread

$$\frac{T_1 <: T_2}{Thread\ T_1\ <: Thread\ T_2} \quad (\text{S-THREAD})$$

3.2 Abstraction

$$\frac{T_1 <: S_1 \quad S_2 <: T_2 \quad Y_1 \geq_{lex} Y_2 \quad \{X_i\}^{i \in 1 \dots n} \subseteq \{Z_j\}^{j \in 1 \dots m}}{S_1 < X_i^{i \in 1 \dots n} > [Y_1] \rightarrow S_2 <: T_1 < Z_j^{j \in 1 \dots m} > [Y_2] \rightarrow T_2} \quad (\text{S-ARROW})$$

3.3 Ref

$$\frac{T_1 <: T_2 \quad \{X_i\}^{i \in 1 \dots n} \subseteq \{Z_j\}^{j \in 1 \dots m}}{Source < X_i^{i \in 1 \dots n} > T_1 <: Source < Z_j^{j \in 1 \dots m} > T_2} \quad (\text{S-SOURCE})$$

$$\frac{T_2 <: T_1 \quad \{X_i\}^{i \in 1 \dots n} \subseteq \{Z_j\}^{j \in 1 \dots m}}{Sink < X_i^{i \in 1 \dots n} > T_1 <: Sink < Z_j^{j \in 1 \dots m} > T_2} \quad (\text{S-SINK})$$

$$\frac{T_1 <: T_2 \quad \{X_i\}^{i \in 1 \dots n} \subseteq \{Z_j\}^{j \in 1 \dots m}}{Ref < X_i^{i \in 1 \dots n} > T_1 <: Source < X_i^{i \in 1 \dots n} > T_1} \quad (\text{S-REFSOURCE})$$

$$\frac{T_2 <: T_1 \quad \{X_i\}^{i \in 1 \dots n} \subseteq \{Z_j\}^{j \in 1 \dots m}}{Ref < X_i^{i \in 1 \dots n} > T_1 <: Sink < X_i^{i \in 1 \dots n} > T_1} \quad (\text{S-REFSINK})$$

3.4 Thread

$$\frac{T_1 <: T_2}{Thread \ T_1 \ <: \ Thread \ T_2} \quad (\text{S-THREAD})$$

4 Algorithmic Typing Rules

5 Evaluation Rules

5.1 Threads

$$\frac{\begin{array}{l} threads = \{ \langle t_i, L_i^t \rangle \}^{i \in 1 \dots n} \\ id' = next(id, threads') \end{array} \quad \begin{array}{l} [t_{id}, \mu, Th, L, L^t] \rightarrow [t'_{id}, \mu', Th', L', L^{t'}] \\ Th' = \langle t_i, L_i^t \rangle^{i \in 1 \dots id-1} \cup \langle t'_{id}, L^{t'} \rangle \cup \langle t_i, L_i^t \rangle^{i \in id+1 \dots n} \end{array}}{[Th, \mu, id, L, L^t] \rightarrow [Th', \mu', id', L', L^{t'}]} \quad (\text{E-THREAD})$$

5.2 Wait

$$\frac{threads(p) = v}{[wait\ p, \mu, Th, L, L^t] \rightarrow [v, \mu, Th \setminus \{p\}, L, L^t]} \quad (\text{E-WAIT})$$

$$[fork\ t, \mu, Th, l < X_i \rangle^{i \in 1 \dots n}, L^t] \rightarrow [p, \mu, Th \cup \{ \langle t, \emptyset \rangle \}, L, L^t] \quad (\text{E-FORK})$$

5.3 Abstraction

$$\frac{[t_1, \mu, Th, L, L^t] \rightarrow [t'_1, \mu', Th', L', L^{t'}]}{[t_1\ t_2, \mu, Th, L, L^t] \rightarrow [t'_1\ t_2, \mu', Th', L', L^{t'}]} \quad (\text{E-APP1})$$

$$\frac{[t_2, \mu, Th, L, L^t] \rightarrow [t'_2, \mu', Th', L', L^{t'}]}{[v_1\ t_2, \mu, Th, L, L^t] \rightarrow [v_1\ t'_2, \mu', Th', L', L^{t'}]} \quad (\text{E-APP2})$$

$$[(\lambda x : T. t_{12})\ v_2, \mu, Th, L, L^t] \rightarrow [t_{12}[x \mapsto v_2], \mu, Th, L, L^t] \quad (\text{E-APPABS})$$

5.4 Reference

$$\frac{[t_1, \mu, Th, L, L^t] \rightarrow [t'_1, \mu', Th', L', L^{t'}]}{[ref < X_i >^{i \in 1 \dots n} \quad t_1, \mu, Th, L, L^t] \rightarrow [ref < X_i >^{i \in 1 \dots n} \quad t'_1, \mu', Th', L', L^{t'}]} \quad (\text{E-REF})$$

$$\frac{l \notin \text{dom}(\mu)}{[ref < X_i >^{i \in 1 \dots n} \quad v, \mu, Th, L, L^t] \rightarrow [l < X_i >^{i \in 1 \dots n}, (\mu, l \mapsto v), Th', L', L^{t'}]} \quad (\text{E-REFV})$$

$$\frac{[t_1, \mu, Th, L, L^t] \rightarrow [t'_1, \mu', Th', L', L^{t'}]}{[!t_1, \mu, Th, L, L^t] \rightarrow [!t'_1, \mu', Th', L', L^{t'}]} \quad (\text{E-DEREF})$$

$$\frac{\mu(l) = v}{[!l < X_i >^{i \in 1 \dots n}, \mu, Th, L, L^t] \rightarrow [v, \mu, Th, L, L^t]} \quad (\text{E-DEREFLOC})$$

$$\frac{[t_1, \mu, Th, L, L^t] \rightarrow [t'_1, \mu', Th', L', L^{t'}]}{[t_1 := t_2, \mu, Th, L, L^t] \rightarrow [t'_1 := t_2, \mu', Th', L', L^{t'}]} \quad (\text{E-ASSIGN1})$$

$$\frac{[t_2, \mu, Th, L, L^t] \rightarrow [t'_2, \mu', Th', L', L^{t'}]}{[v := t_2, \mu, Th, L, L^t] \rightarrow [v := t'_2, \mu', Th', L', L^{t'}]} \quad (\text{E-ASSIGN2})$$

$$\frac{\mu(l) = v}{[l < X_i >^{i \in 1 \dots n} := v, \mu, Th, L, L^t] \rightarrow [unit, (\mu, l \mapsto v), Th', L', L^{t'}]} \quad (\text{E-ASSIGNV})$$

5.5 Acquire

$$\frac{[t_1, \mu, Th, L, L^t] \rightarrow [t'_1, \mu', Th', L', L^{t'}]}{\begin{array}{l} [acquire\ t_1\ t_2, \mu, Th, L^t] \\ \rightarrow [acquire\ t'_1\ t_2, \mu', Th', L', L^{t'}] \end{array}} \quad (\text{E-ACQUIRE1})$$

$$\frac{X \notin L \quad X \notin L^t \quad L' = L \cup \{X\} \quad L^{t'} = L^t \cup \{X\}}{\begin{array}{l} [acquire\ mutex < X_i >^{i \in 1 \dots n} t_2, \mu, Th, L, L^t] \\ \rightarrow [acquire\ mutex < X_i >^{i \in 1 \dots n} t_2, \mu, Th, L', L^{t'}] \end{array}} \quad (\text{E-ACQUIRE})$$

$$\frac{X \in L^t \quad [t_2, \mu, Th, L, L^t] \rightarrow [t'_2, \mu', Th', L', L^{t'}]}{\begin{array}{l} [acquire\ mutex < X_i >^{i \in 1 \dots n} t_2, \mu, Th, L, L^t] \\ \rightarrow [acquire\ mutex < X_i >^{i \in 1 \dots n} t'_2, \mu', Th', L', L^{t'}] \end{array}} \quad (\text{E-ACQUIRE2})$$

$$\frac{X \in L^t \quad L' = L \setminus \{X\} \quad L^{t'} = L^t \setminus \{X\}}{\begin{array}{l} [acquire\ mutex < X_i >^{i \in 1 \dots n} v_2, \mu, Th, L, L^t] \\ \rightarrow [v_2, \mu', Th', L', L^{t'}] \end{array}} \quad (\text{E-ACQRELEASE})$$