Typing Rules and Evaluation rules

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1 Syntax

$$
\begin{array}{rll}
t ::= & & \texttt{terms} \\
& v & \\
& if\ term\ then\ term\ else\ term & \\
& succ\ number & \\
& pred\ number & \\
& iszero\ number & \\
& refterm & \\
& t\ t & \\
& wait\ t & \\
& fork\{t\} & \\
& mutex < X1, X2, ..., Xn > & \\
& abstraction & \\
& tagterm & \\
& recordsterm & \\
abstraction ::= & & \texttt{abstraction term} \\
& \lambda x : T.t & \\
& \lambda < X_1, X_2, ..., X_n > x : T.t & \\
& \lambda < X_1, X_2, ..., X_n > [Y]x : T.t &
\end{array}
$$

$refterm ::=$ `terms about ref`

$!t$

$ref\ t$

$ref < X1, X2, ..., Xn >\ t$

$t := t$

$thterm ::=$ `terms about thread`

$wait\ t$

$fork\{t\}$

$tagterm ::=$ `terms about tags`

$< l = t >\ as\ T$

$case\ t\ of\ < l_i = x_i > \Longrightarrow\ t_i^{i \in 1..n}$

$recodesterm ::=$ `terms about recoders`

$l_i = t_i^{i \in 1..n}$

$t.l$

$v =$ `values`

$true$

$false$

0

$\lambda x : T.t$

$string$

$unit$

$number$

$float$

$record$

$mutex$

loc

tag

$forkv$

$< l = v >\ as\ T$

$l_i = v_i^{i \in 1..n}$

2 Typing rules

2.1 Fork

$$\frac{(\Gamma, tid_2)|\Sigma|\mathbb{L} \vdash t :\ T \qquad lasttid(\Gamma) = tid_1}{\Gamma|\Sigma|\mathbb{L} \vdash fork\{t\ , tid_2\} :\ Thread[tid_1]\ T} \tag{T-Fork}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t :\ Thread[tid]\ T \qquad lasttid(\Gamma) = tid \qquad \mathbb{L} = \emptyset}{\Gamma|\Sigma|\mathbb{L} \vdash wait\ t\ \ :\ \ T} \tag{T-Wait}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash \Sigma(p) :\ T}{\Gamma|\Sigma|\mathbb{L} \vdash < p, tid >: Thread < tid > T} \tag{T-Thread}$$

2.2 Mutex

$$\frac{}{\Gamma|\Sigma|\mathbb{L} \vdash mutex < X_i >^{i \in 1 \ldots n} :\ Mutex\ X} \tag{T-Mutex}$$

2.3 Acquire

$$\frac{max\{\mathbb{L}\} <_{lex} X \qquad \Gamma|\Sigma|\mathbb{L} \vdash t_1 : Mutex\ X \qquad \Gamma|\Sigma|(\mathbb{L}, X) \vdash t_2 : T}{\Gamma|\Sigma|\mathbb{L} \vdash Aacquire\ t_1\ t_2 : T} \tag{T-Acquire}$$

2.4 Abstraction

$$\frac{(\Gamma, x : T_1)|(\mathbb{L} \cup \{X_i\}^{i \in 1 \ldots n}) \vdash t : T_2 \qquad mam(\Gamma|\Sigma|\mathbb{L}, T1) = Y}{\Gamma|\Sigma|\mathbb{L} \vdash \lambda < X_i^{i \in 1 \ldots n} > x : T_1.t :\ T_1 < X_i^{i \in 1 \ldots n} > [Y] \to T_2} \tag{T-Abs}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash T_1.t :\ T_1 < X_i^{i \in 1 \ldots n} > [Y] \to T_2 \qquad \Gamma|\Sigma|\mathbb{L} \vdash t_2 : T_1}{\Gamma|\Sigma|\mathbb{L} \vdash t_1\ t_2 :\ T_2} \tag{T-App}$$

2.5 Ref

$$\frac{\Sigma(l) : T}{\Gamma|\Sigma|\mathbb{L} \vdash l < X_i^{i\in 1\ldots n} >: Ref < X_i^{i\in 1\ldots n} > T} \quad \text{(T-Loc)}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash v : T}{\Gamma|\Sigma|\mathbb{L} \vdash ref < X_i^{i\in 1\ldots n} > v : \ Ref < X_i^{i\in 1\ldots n} > T} \quad \text{(T-Ref)}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : Source < X_i^{i\in 1\ldots n} > T \qquad X_i \in \mathbb{L}^{i\in 1\ldots n}}{\Gamma|\Sigma|\mathbb{L} \vdash !t_1 : T} \quad \text{(T-Deref)}$$

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : Sink < X_i^{i\in 1\ldots n} > T \qquad \Gamma|\Sigma|\mathbb{L} \vdash t_2 : T \qquad X_i \in \mathbb{L}^{i\in 1\ldots n}}{\Gamma|\Sigma|\mathbb{L} \vdash t_1 := t_2 : Unit} \quad \text{(T-Assign)}$$

2.6 Fix

$$\frac{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : T_1 < X_i^{i\in 1\ldots n} > [None] \to T_1}{\Gamma|\Sigma|\mathbb{L} \vdash t_1 : T_1} \quad \text{(T-Fix)}$$

3 Subtyping rules

3.1 Thread

$$\frac{T_1 <: T_2}{Thread\ T_1 \ <: \ Thread\ T_2} \quad \text{(S-Thread)}$$

3.2 Abstraction

$$\frac{T_1 \ <: \ S_1 \qquad S_2 \ <: \ T_2 \qquad Y_1 \geq_{lex} Y_2 \qquad \{X_i\}^{i\in 1\ldots n} \subseteq \{Z_j\}^{j\in 1\ldots m}}{S_1 < X_i^{i\in 1\ldots n} > [Y_1] \to S_2 \ <: \ T_1 < Z_j^{j\in 1\ldots m} > [Y_2] \to T_2} \quad \text{(S-ARROW)}$$

3.3 Ref

$$\frac{T_1 <: T_2 \qquad \{X_i\}^{i\in 1\ldots n} \subseteq \{Z_j\}^{j\in 1\ldots m}}{Source < X_i^{i\in 1\ldots n} > T_1 <: Source < Z_j^{j\in 1\ldots m} > T_2} \quad \text{(S-SOURCE)}$$

$$\frac{T_2 <: T_1 \qquad \{X_i\}^{i\in 1\ldots n} \subseteq \{Z_j\}^{j\in 1\ldots m}}{Sink < X_i^{i\in 1\ldots n} > T_1 <: Sink < X_j^{j\in 1\ldots m} > T_2} \quad \text{(S-SINK)}$$

$$\frac{T_1 <: T_2 \qquad \{X_i\}^{i\in 1\ldots n} \subseteq \{Z_j\}^{j\in 1\ldots m}}{Ref < X_i^{i\in 1\ldots n} > T_1 <: Source < X_i^{i\in 1\ldots n} > T_1} \quad \text{(S-REFSOURCE)}$$

$$\frac{T_2 <: T_1 \qquad \{X_i\}^{i\in 1\ldots n} \subseteq \{Z_j\}^{j\in 1\ldots m}}{Ref < X_i^{i\in 1\ldots n} > T_1 <: Sink < X_i^{i\in 1\ldots n} > T_1} \quad \text{(S-REFSINK)}$$

3.4 Thread

$$\frac{T_1 <: T_2 \qquad fpid_1 = fpid_2}{Thread < fpid_1 > T_1 \ <: \ Thread < fpid_2 > T_2} \quad \text{(S-THREAD)}$$

4 Algorithmic Typing Rules

5 Evaluation Rules

5.1 Threads

$$\frac{\begin{array}{c} threads = \{< t_i, L_i^t, tid_i >\}^{i \in 1 \ldots n} \qquad [t_{id}, \mu, Th, L, L^t] \to [t'_{id}, \mu', Th', L', L^{t'}] \\ id' = next(id, threads') \\ Th' = < t_i, L_i^t, tid_i >^{i \in 1 \ldots id-1} \cup < t'_{id}, L^{t'}, tid_{id} > \cup < t_i, L_i^t, tid_i >^{i \in id+1 \ldots n} \end{array}}{[Th, \mu, id, L, L^t] \to [Th', \mu', id', L', L^{t'}]} \tag{E-Thread}$$

5.2 Wait

$$\frac{threads(p) = v}{[wait\ p, \mu, Th, L, L^t] \to [v, \mu, Th \backslash \{p\}, L, L^t]} \tag{E-Wait}$$

$$\begin{array}{c} [fork\{t, tid_2\}, \mu, Th, l < X_i >^{i \in 1 \ldots n}, L^t] \\ \to [< p, Tid(current) >, \mu, Th \cup \{< t, \emptyset, tid_2 >\}, L, L^t] \end{array} \tag{E-Fork}$$

5.3 Abstraction

$$\frac{[t_1, \mu, Th, L, L^t] \to [t'_1, \mu', Th', L', L^{t'}]}{[t_1\ t_2, \mu, Th, L, L^t] \to [t'_1\ t_2, \mu', Th', L', L^{t'}]} \tag{E-App1}$$

$$\frac{[t_2, \mu, Th, L, L^t] \to [t'_2, \mu', Th', L', L^{t'}]}{[v_1\ t_2, \mu, Th, L, L^t] \to [v_1\ t'_2, \mu', Th', L', L^{t'}]} \tag{E-App2}$$

$$[(\lambda x : T.t_{12})\ v_2, \mu, Th, L, L^t] \to [t_{12}[x \mapsto v_2], \mu, Th, L, L^t] \tag{E-AppAbs}$$

5.4 Reference

$$\frac{[t_1, \mu, Th, L, L^t] \rightarrow [t_1', \mu', Th', L', L^{t'}]}{[ref < X_i >^{i \in 1 \dots n} \;\; t_1, \mu, Th, L, L^t] \rightarrow [ref < X_i >^{i \in 1 \dots n} \;\; t_1', \mu', Th', L', L^{t'}]} \quad \text{(E-Ref)}$$

$$\frac{l \notin dom(\mu)}{[ref < X_i >^{i \in 1 \dots n} \;\; v, \mu, Th, L, L^t] \rightarrow [l < X_i >^{i \in 1 \dots n}, (\mu, l \mapsto v), Th', L', L^{t'}]} \quad \text{(E-RefV)}$$

$$\frac{[t_1, \mu, Th, L, L^t] \rightarrow [t_1', \mu', Th', L', L^{t'}]}{[!t_1, \mu, Th, L, L^t] \rightarrow [!t_1', \mu', Th', L', L^{t'}]} \quad \text{(E-Deref)}$$

$$\frac{\mu(l) = v}{[!l < X_i >^{i \in 1 \dots n}, \mu, Th, L, L^t] \rightarrow [v, \mu, Th, L, L^t]} \quad \text{(E-DerefLoc)}$$

$$\frac{[t_1, \mu, Th, L, L^t] \rightarrow [t_1', \mu', Th', L', L^{t'}]}{[t_1 := t_2, \mu, Th, L, L^t] \rightarrow [t_1' := t_2, \mu', Th', L', L^{t'}]} \quad \text{(E-Assign1)}$$

$$\frac{[t_2, \mu, Th, L, L^t] \rightarrow [t_2', \mu', Th', L', L^{t'}]}{[v := t_2, \mu, Th, L] \rightarrow [v := t_2', \mu, Th', L', L^{t'}]} \quad \text{(E-Assign2)}$$

$$\frac{\mu(l) = v}{[l < X_i >^{i \in 1 \dots n} := v, \mu, Th, L, L^t] \rightarrow [unit, (\mu, l \mapsto v), Th', L', L^{t'}]} \quad \text{(E-AssignV)}$$

5.5 Acquire

$$\frac{[t_1, \mu, Th, L, L^t] \to [t_1', \mu', Th', L', L^{t'}]}{\begin{array}{c}[acquire\ t_1\ t_2, \mu, Th, L^t] \\ \to [acquire\ t_1'\ t_2, \mu', Th', L', L^{t'}]\end{array}} \quad \text{(E-Acquire1)}$$

$$\frac{X \notin L \qquad X \notin L^t \qquad L' = L \cup \{X\} \qquad L^{t'} = L^t \cup \{X\}}{\begin{array}{c}[acquire\ mutex < X_i >^{i \in 1 \ldots n}\ \ t_2, \mu, Th, L, L^t] \\ \to [acquire\ mutex < X_i >^{i \in 1 \ldots n}\ \ t_2, \mu, Th, L', L^{t'}]\end{array}} \quad \text{(E-Acquire)}$$

$$\frac{X \in L^t \qquad [t_2, \mu, Th, L, L^t] \to [t_2', \mu', Th', L', L^{t'}]}{\begin{array}{c}[acquire\ mutex < X_i >^{i \in 1 \ldots n}\ \ t_2, \mu, Th, L, L^t] \\ \to [acquire\ mutex < X_i >^{i \in 1 \ldots n}\ \ t_2', \mu', Th', L', L^{t'}]\end{array}} \quad \text{(E-Acquire2)}$$

$$\frac{X \in L^t \qquad L' = L \backslash \{X\} \qquad L^{t'} = L^t \backslash \{X\}}{\begin{array}{c}[acquire\ mutex < X_i >^{i \in 1 \ldots n}\ \ v_2, \mu, Th, L, L^t] \\ \to [v_2, \mu', Th', L', L^{t'}]\end{array}} \quad \text{(E-AcqRelease)}$$