MODULE *beans*

EXTENDS *Integers*

CONSTANTS $R,\ G,\ B$

ASSUME $\land\ R \in 1\,..\,100$
$\land\ G \in 1\,..\,100$
$\land\ B \in 1\,..\,100$
$\land\ R + G + B > 0$

--fair algorithm *beans*{
variable $r = R,\ g = G,\ b = B$;
{ S: **while** (TRUE)
{ **either**
$A1r$: { **await** $(r > 1)$; same color and red
$r := r - 2$; $g := g + 1$; $b := b + 1$;
} ;
or
$A1g$: { **await** $(g > 1)$; same color and green
$g := g - 2$; $r := r + 1$; $b := b + 1$;
} ;
or
$A1b$: { **await** $(b > 1)$; same color and blue
$b := b - 2$; $r := r + 1$; $g := g + 1$;
} ;
or
$A2rg$: { **await** $(r > 0 \land g > 0)$; different color red and green
$r := r - 1$; $g := g - 1$; $b := b + 1$;
} ;
or
$A2bg$: { **await** $(b > 0 \land g > 0)$; different color blue and green
$b := b - 1$; $g := g - 1$; $r := r + 1$;
} ;
or
$A2rb$: { **await** $(r > 0 \land b > 0)$; different color red and blue
$r := r - 1$; $b := b - 1$; $g := g + 1$;
} ;
} end while
} end alg
} * end alg

BEGIN TRANSLATION

VARIABLES $r,\ g,\ b,\ pc$

$vars \triangleq \langle r,\ g,\ b,\ pc \rangle$

$Init \triangleq$ Global variables
$\land\ r = R$
$\land\ g = G$

$$
\begin{array}{l}
\land\ b = B \\
\land\ pc = \text{“S”}
\end{array}
$$

$$
S \triangleq \begin{array}{l}
\land\ pc = \text{“S”} \\
\land\ \lor\ \land\ pc' = \text{“A1r”} \\
\quad\ \lor\ \land\ pc' = \text{“A1g”} \\
\quad\ \lor\ \land\ pc' = \text{“A1b”} \\
\quad\ \lor\ \land\ pc' = \text{“A2rg”} \\
\quad\ \lor\ \land\ pc' = \text{“A2bg”} \\
\quad\ \lor\ \land\ pc' = \text{“A2rb”} \\
\land\ \text{UNCHANGED}\ \langle r,\ g,\ b \rangle
\end{array}
$$

$$
A1r \triangleq \begin{array}{l}
\land\ pc = \text{“A1r”} \\
\land\ (r > 1) \\
\land\ r' = r - 2 \\
\land\ g' = g + 1 \\
\land\ b' = b + 1 \\
\land\ pc' = \text{“S”}
\end{array}
$$

$$
A1g \triangleq \begin{array}{l}
\land\ pc = \text{“A1g”} \\
\land\ (g > 1) \\
\land\ g' = g - 2 \\
\land\ r' = r + 1 \\
\land\ b' = b + 1 \\
\land\ pc' = \text{“S”}
\end{array}
$$

$$
A1b \triangleq \begin{array}{l}
\land\ pc = \text{“A1b”} \\
\land\ (b > 1) \\
\land\ b' = b - 2 \\
\land\ r' = r + 1 \\
\land\ g' = g + 1 \\
\land\ pc' = \text{“S”}
\end{array}
$$

$$
A2rg \triangleq \begin{array}{l}
\land\ pc = \text{“A2rg”} \\
\land\ (r > 0 \land g > 0) \\
\land\ r' = r - 1 \\
\land\ g' = g - 1 \\
\land\ b' = b + 1 \\
\land\ pc' = \text{“S”}
\end{array}
$$

$$
A2bg \triangleq \begin{array}{l}
\land\ pc = \text{“A2bg”} \\
\land\ (b > 0 \land g > 0) \\
\land\ b' = b - 1 \\
\land\ g' = g - 1 \\
\land\ r' = r + 1 \\
\land\ pc' = \text{“S”}
\end{array}
$$

$$A2rb \triangleq \land pc = \text{"A2rb"}$$
$$\land (r > 0 \land b > 0)$$
$$\land r' = r - 1$$
$$\land b' = b - 1$$
$$\land g' = g + 1$$
$$\land pc' = \text{"S"}$$

$$Next \triangleq S \lor A1r \lor A1g \lor A1b \lor A2rg \lor A2bg \lor A2rb$$

$$Spec \triangleq \land Init \land \Box[Next]_{vars}$$
$$\land \mathrm{WF}_{vars}(Next)$$

END TRANSLATION

$$TypeOK \triangleq \land r \in 0 \mathrel{..} 200$$
$$\land g \in 0 \mathrel{..} 200$$
$$\land b \in 0 \mathrel{..} 200$$
$$Termination \triangleq \Diamond((r + g + b) < 2)$$

* Modification History
* Last modified *Tue Sep* 30 22:18:41 *EDT* 2014 by *Siddharth*
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The invariant functions are given as:
1. $(r + g + b) > 0$
2. $r \geq 0$
3. $g \geq 0$
4. $b \geq 0$

The above are verified my model checking. The first invariant is however given as $(r+g+b) > 1$ in the model to avoid stuttering. When the invariant is violated we know that the program has reached a fixed point.

The metric/variant function can be given as the total number of beans:

$No.B = (r + g + b)$ is the metric function.

We observe from the program that the $No.B$ either stays the same or decreases for each action and hence selected as the metric funtion.

Fixed point:

To find the FP, we take the guards and negate it to get:

$r < 2$
$\hat{}g < 2$
$\hat{}b < 2$
$\hat{}(r < 1)$ v $(g < 1)$
$\hat{}(b < 1)$ v $(g < 1)$
$\hat{}(r < 1)$ v $(b < 1)$

But we know that atleast one of them should be $'1'$ from the invariant $(r + g + b) > 0$.

Therefore the fixed points states are given as ($< r >$, $< g >$, $< b >$) :

1. $< 1,0,0 >$
2. $< 0,1,0 >$
3. $< 0,0,1 >$

We also know that the program terminates when $(r + g + b) < 2$, or more specifically when $r + b + g = 1$.(*This can be verified using TLA+ model checking*). This is also given by the metric function $No.B(r + g + b)$ the value of which either decreases or stays the same after every action.

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