Evolutionary Dynamics of Coordinated Cooperation

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Abstract

Nowadays, social evolution theory is improving progressively and new models are appearing. We propose here a model that make use of conditional cooperation, more precisely the coordinated cooperation. Indeed, there will be a negotiation phase before playing an action and each player can change his action, defection or cooperation, depending on what the other players are planning to play. We will test our results on a finite population trying to understand why players tend to know the other player's actions. Finally, we will experiment the model in a free-scale network and study the distribution of the strategies.

Introduction

The conditional cooperation is a sort of mechanism with positive assortment as goal achieving. One of the most famous strategy is named Tit-for-Tat (Axelrod and Hamilton (1981); Nowak and Sigmund (1992)). This strategy first chooses to cooperate, then, in subsequent rounds, chooses the action that the other player chose in the previous round. Positive results have shown that if the agents interact and help only those who helped them before then the game will reach a good evolution of cooperation (Nowak and Sigmund (2005)). We will focus here on the Public Goods Games which are a standard of social experiments. They are usually employed to model the behavior of groups of individuals achieving a common goal. The Public Goods Game has the same properties as the prisoner's dilemma game, but describes a public good or a resource from which all may benefit regardless of whether or not they contributed to the good. Agent contributions to Public Goods Game are affected by the contributions of the others. (e.g. Negotiation about the global climate change are a good example, where many hours of discussion are performed before participants finally decide whether or not they will cooperate). It has been demonstrated that individuals choose to match the past contribution decisions and amounts of other contributors to the same public good (Fischbacher and Fehr (2001)).

In this paper we will analyze the role of negotiation in the evolution of cooperation, specially whether it explains or not the emergence and maintenance of conditional cooperators in a linear public goods game. First of all, we will explain in more details the game and the negotiation phase (principle, progress and initial specifications). Then, we will provide our results on finite population for the game with 2 and 3 players. Finally, we will discuss the advantages of our model and its limits. It is important to note that this model is a specific human behavior, we cannot really imagine a negotiation phase between animals for example.

Methods

Public goods game

The game is composed of n players playing with each other. Each player has to choose an action, either to cooperate or defect. If all players choose to cooperate to the *public good*, the payoffs will benefit every player, however, if one of them chooses to defect the payoffs will benefit the defectors. Indeed, each cooperator pays a cost (c) for a public good while defectors do not. Moreover, the total payment is aggregated, multiplied by a factor (r) and equally redistributed to the players. Therefore, the payoffs are different between cooperators and defectors. Assuming there are n players where k are cooperators and $n-k$ are defectors, their respective payoffs are:

$$\begin{aligned} W_C &= -c + \frac{rkc}{n} \\ W_D &= \frac{rkc}{n} \end{aligned} \tag{1}$$

Strategies

During the game, players have strategies. Indeed, the actions are considered coordinated so the player need to determine his own action according to the other players actions. We note a strategy as C_k where k ($0 \leq k \leq n$), is the minimum number of cooperators (except himself) necessary to cooperate. For instance, if a player strategy is C_3, the player will cooperate if at least 3 other players cooperate otherwise he will defect.

Negotiation

The negotiation happens in the following way. First of all, each player have a *thought* which is a temporary action (not definitive) visible by every other player. We assume that a player chooses C with a probability p and D with a probability $1 - p$. The p parameter cannot be defined *a priori* so it is a model parameter. All players thoughts are gathered to obtain a n-tuple which represent the *initial state*. *The initial state* is very important and has an impact on the rest of the game. Then, the negotiation process starts. At each update step, a random player is chosen and given a chance to change his thought (C to D or D to C) according to his strategy. Indeed, the player can change his own thought depending on the other players thoughts. We do as many updates as we need to reach the *stationary state* where the state cannot change anymore since players have their actions set and verified by their conditional strategy. Every game has a stationary state, it has been demonstrated (Ohtsuki (2018)). The final thoughts correspond to the actual actions of the players during the game.

Example

To illustrate an example of a negotiation stage we will use a simple example. Suppose there are two players (X,Y) that have both a thought D and which respective conditional strategy is C_1 and C_0. We take randomly a player, Y, we see that Y needs no cooperators to cooperate, which is the case, so we change Y thought to C. The new state is (D,C) and it is not stationary since X can change his thought. Next step, we take randomly another player, X, this last needs one cooperator to cooperate, Y is cooperating so X changes his thought to C and the new state is (C,C). This state is stationary. In fact, the player X has enough cooperators so he will keep his thought, and Y is an unconditional cooperator thus the negotiation stage is finished. The following diagram shows the different iterations of the negotiation.

$$\xRightarrow[state]{Initial} (D, D) \xRightarrow{Y chosen} (D, C) \xRightarrow{X chosen} (C, C)$$

The algorithms 1, 2, 3 show respectively the pseudo-code of the negotiation stage, the Public Goods Game and the strategy update process.

Algorithm 1 Negotiation process

assign initial thought to each player
$state \leftarrow$ players thoughts

while stationary state not reached **do**
    $player \leftarrow$ pick_random_player()
    $state \leftarrow player$.update_thought($state$)
end while

Algorithm 2 Public goods game

$n \leftarrow$ number of players for a one-shot game
$M \leftarrow$ population size

$population \leftarrow$ initialize population
for each $game$ **do**
    random_shuffle($population$)
    $groups$ = $population$.split(n)
    for each $group$ in $groups$ **do**
        negotiation_process($group$)
        play_game($group$) ▷ assign payoffs
    end for
    update_process()
end for

Algorithm 3 Update process

$s \leftarrow$ intensity of selection
$\mu \leftarrow$ mutation rate

$rand1 \leftarrow$ get_random_number(0,1)
$rand2 \leftarrow$ get_random_number(0,1)
$player1, player2 \leftarrow$ get_random_players()
$\Delta \leftarrow player2$.get_payoff() - $player1$.get_payoff()

if $rand1 < \mu$ **then**
    $player1$.change_strat(get_random_strat())
else
    if $rand2 < \frac{1}{1+\exp(-s\Delta)}$ **then**
        $player1$.change_strat($player2$.get_strat())
    end if
end if

Population and Evolutionary dynamics

Now that we have explained the *one shot* public goods game we need to apply this to a population of agents. Given a population of size M, n random players are selected and play a one shot public goods game preceded by a negotiation stage and then return to the population. We repeat this a lot of times and therefore each agent obtains an average payoff per game which we will represent by w. The frequencies of the strategy chosen, over time, can be studied by evolutionary dynamics.

Infinite Population

For an infinite population ($M \to \infty$), the following replicator equation describes the evolutionary dynamics of $(n + 1)$ strategies (C_0 to C_n):

$$\dot{x}_k = x_k(w_k - \bar{w}) \tag{2}$$

where, given a strategy C_k, x_k represents its frequency, w_k its average payoff and $\bar{w}$ the average payoff in the population.

Finite Population

For a finite population, the standard models that describe the population evolutionary dynamics are frequency dependent Moran process and pairwise comparison process. In the same way, several n-person games are played and each agent obtains average payoffs. At each updating step, we choose randomly two players from the population and compare the payoff of the first player with the payoff of the second player. We compute Δ which is the difference between the payoff of the second player and the payoff of the first player. The first player copies the second player strategy with probability

$$\frac{1}{1+\exp(-s\times\Delta)} \tag{3}$$

otherwise he keeps his actual strategy. The s parameter is strictly positive and is usually called intensity of selection. This probability comes from the Fermi distribution function in physics (Traulsen et al., 2006, 2007). The bigger the Δ is, the bigger the chance that the first player copies the second player strategy is.

Due to the fact that the population is finite, there is a *fixation* problem. Indeed, once all players have the same strategy, no other strategy can invade the population. To avoid this kind of problem, during the updating step, we use a mutation probability, $\mu > 0$, that makes the first player change his strategy to another random strategy irrespective of the value of Δ.

Two person Public Goods Game: Payoffs

The **Table 1** gives us the stationary states we can reach from a composition of players with the corresponding probability of occurrence. For instance, two C_0 players will always cooperate (because they are both unconditional cooperators) and a C_0 and C_1 player will also always cooperate. In fact, the C_0 player is an unconditional cooperator and thus C_1 player will have enough cooperators to cooperate too. The tricky case is with two C_1 players where the stationary state depends on the initial state. In fact, if the initial state is (C,C) or (D,D) (which happens with probability p^2 and $(1-p)^2$) the final stationary state is trivial to determine, however, if the initial state is (C,D) or (D,C) (which happens with probability $2p(1-p)$) the stationary state depends on the player who changes his thought first. If the player with thought D chooses first, he will see that he has a neighbour playing C and he will change his thought to finally reach the final state (C,C). On the other hand, if the player with thought C chooses first, he will change his thought because no one of the other players wants to cooperate and thus we will reach then the final state (D,D). Therefore, the probability to reach (C,C) is $p^2+2p(1-p)\times\frac{1}{2} = p(p+(1-p)) = p$ and the probability to reach (D,D) is $(1-p)^2+2p(1-p)\times\frac{1}{2} = (1-p)((1-p)+p) = 1-p$.

Composition of players	Stationary states	Probability of occurrence
(C_0,C_0)	(C,C)	1
(C_0,C_1)	(C,C)	1
(C_0,C_2)	(C,D)	1
(C_1,C_1)	(C,C)	p
	(D,D)	1-p
(C_1,C_2)	(D,D)	1
(C_2,C_2)	(D,D)	1

Table 1: Stationary states in the two-person game.

With the **Table 1** and the equation (1) we can define the payoff matrix of a 2-person game (see **Figure 1**). The payoffs are determined from the stationary states reached from a given composition of players. For instance, one C_0 and one C_1 will always cooperate (k=2), therefore, the payoff of each player is $-c+\frac{2rc}{2} = -c+rc$. Concerning the tricky case, the two C_1 players will get a payoff equal to $-c+rc$ with probability p and with probability $1-p$ they will get a null payoff. The average payoff is given by $p(-c+rc)+(1-p)(0) = p(-c+rc)$.

$$\mathbf{A} = \begin{array}{c c} & \begin{array}{ccc} C_0 & C_1 & C_2 \end{array} \\ \begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array} & \begin{pmatrix} -c+rc & -c+rc & -c+\frac{1}{2}rc \\ -c+rc & p(-c+rc) & 0 \\ \frac{1}{2}rc & 0 & 0 \end{pmatrix} \end{array}$$

Figure 1: Payoff matrix for a 2-person game.

Two person Public Goods Game: Dynamics

Here we will study the evolution of a player's strategy according to the values of p.

$$\boxed{p=0}$$

We can see in the payoff matrix that C_1 is less interesting than C_0, but C_0 is invaded by C_2: if the majority of players always cooperate the best decision would be to exploit them. Moreover, when there are no C_0 players then C_1 and C_2 are neutrals. We can conclude that $(x_0^*, x_1^*, x_2^*) = (0,0,1)$ is one stable equilibrium.

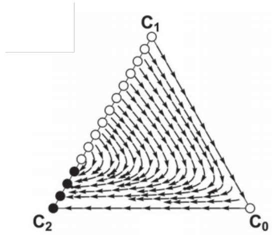

Figure 2: Replicator dynamics of the two-person game played in an infinitely large population with p = 0 and r = 1.6.

$$\boxed{p = 1}$$

For the same reason, C_0 is invaded by C_2 and we can easily observe in the payoff matrix that C_1 is more interesting than C_2. Furthermore, when there are no C_2 players then C_1 and C_0 are neutrals. Thus, $(x_0^*, x_1^*, x_2^*) = (0, 1, 0)$ is one stable equilibrium.

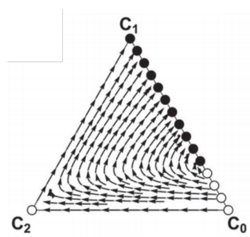

Figure 3: Replicator dynamics of the two-person game played in an infinitely large population with p = 1 and r = 1.6.

$$\boxed{0 < p < 1}$$

In this case, C_0 is invaded by C_2 which is invaded by C_1 which is invaded by C_0. There is one stable equilibrium $(x_0^*, x_1^*, x_2^*) = (\frac{2p(r-1)}{r}, \frac{2-r}{r}, \frac{2(1-p)(r-1)}{r})$.

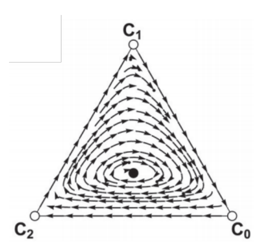

Figure 4: Replicator dynamics of the two-person game played in an infinitely large population with $0 < p < 1$ and r = 1.6.

Composition of players	Stationary states	Probability of occurrence
(C_0,C_0,C_0)	(C,C,C)	1
(C_0,C_0,C_1)	(C,C,C)	1
(C_0,C_0,C_2)	(C,C,C)	1
(C_0,C_0,C_3)	(C,C,D)	1
(C_0,C_1,C_1)	(C,C,C)	1
(C_0,C_1,C_2)	(C,C,C)	1
(C_0,C_1,C_3)	(C,C,D)	1
(C_0,C_2,C_2)	(C,C,C)	$\frac{1}{2}p^2 + \frac{1}{2}p$
	(C,D,D)	$-\frac{1}{2}p^2 - \frac{1}{2}p + 1$
(C_0,C_2,C_3)	(C,D,D)	1
(C_0,C_3,C_3)	(C,D,D)	1
(C_1,C_1,C_1)	(C,C,C)	$-p^2 + 2p$
	(D,D,D)	$p^2 - 2p + 1$
(C_1,C_1,C_2)	(C,C,C)	$-\frac{1}{2}p^2 + \frac{3}{2}p$
	(D,D,D)	$\frac{1}{2}p^2 - \frac{3}{2}p + 1$
(C_1,C_1,C_3)	(C,C,D)	$-\frac{1}{2}p^2 + \frac{3}{2}p$
	(D,D,D)	$\frac{1}{2}p^2 - \frac{3}{2}p + 1$
(C_1,C_2,C_2)	(C,C,C)	$\frac{1}{2}p^2 + \frac{1}{2}p$
	(D,D,D)	$-\frac{1}{2}p^2 - \frac{1}{2}p + 1$
(C_1,C_2,C_3)	(D,D,D)	1
(C_1,C_3,C_3)	(D,D,D)	1
(C_2,C_2,C_2)	(C,C,C)	p^2
	(D,D,D)	$-p^2 + 1$
(C_2,C_2,C_3)	(D,D,D)	1
(C_2,C_3,C_3)	(D,D,D)	1
(C_3,C_3,C_3)	(D,D,D)	1

Table 2: Stationary states in the three-person game.

Three person Public Goods Game: Payoffs

The **Table 2** gives us the stationary states we can reach from a composition of players with the corresponding probability of occurrence. With this table we can define the payoff matrix of a 3-person game (**Figure 5**). This matrix is defined in the same way as the payoff matrix for 2-players game.

Three person Public Goods Game: Dynamics

Now we will study which strategy dominates who (by pair) for a three person Public Goods Game (C_0, C_1, C_2, C_3). In order to do so, we will determine whether the difference between the average payoff of the first strategy and the second one is positive (First strategy wins) or negative (Second strategy wins). Notice that $1 < r < 3$, $0 < p < 1$, x_k is the frequency of strategy C_k in the population and w_k the frequency of strategy C_k for $k \in (0, 1, 2, 3)$.

On the edge $C_0 - C_1$:

$$w_0 - w_1 = c(1-p)^2(r-1)(1-x_0)^2 > 0$$

$$
A = \begin{array}{c} \\ C_0 \\ C_1 \\ C_2 \\ C_3 \end{array}
\begin{array}{c}
\begin{array}{cccccccccc} C_0C_0 & C_0C_1 & C_0C_2 & C_0C_3 & C_1C_1 & C_1C_2 & C_1C_3 & C_2C_2 & C_2C_3 & C_3C_3 \end{array} \\
\begin{pmatrix}
-c+rc & -c+rc & -c+rc & -c+\frac{2}{3}rc & -c+rc & -c+rc & -c+\frac{2}{3}rc & -c+\frac{p^2+p+1}{3}rc & -c+\frac{1}{3}rc & -c+\frac{1}{3}rc \\
-c+rc & -c+rc & -c+rc & -c+\frac{2}{3}rc & p(2-p)(-c+rc) & \frac{p(3-p)}{2}(-c+rc) & -\frac{p(3-p)}{2}c+\frac{p(3-p)}{3}rc & \frac{p(1+p)}{2}(-c+rc) & 0 & 0 \\
-c+rc & -c+rc & -\frac{p(1+p)}{2}c+\frac{p^2+p+1}{3}rc & \frac{1}{3}rc & \frac{p(3-p)}{2}(-c+rc) & \frac{p(1+p)}{2}(-c+rc) & 0 & p^2(-c+rc) & 0 & 0 \\
\frac{2}{3}rc & \frac{2}{3}rc & \frac{1}{3}rc & \frac{1}{3}rc & \frac{p(3-p)}{3}rc & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}.
$$

Figure 5: Payoff matrix for a 3-person game

So C_0 dominates C_1.

On the edge $C_1 - C_2$:

$$w_1 - w_2 = \frac{c}{2}p(1-p)(r-1)\{2x_1(1-x_1)+1\} > 0$$

So C_1 dominates C_2.

On the edge $C_2 - C_3$:

$$w_2 - w_3 = \frac{cp^2(3-p)^2(2r-3)^2}{9(r-1)}$$

It's positive unless $r = \frac{3}{2}$, this case is not study in this paper.
So C_2 dominates C_3.

On the edge $C_3 - C_0$:

$$w_3 - w_0 = \frac{c}{3}(3-r) > 0$$

So C_3 dominates C_0.

On the edge $C_0 - C_2$:

$$w_0 - w_2 = \frac{c}{3}(1-p)x_2\{3(r-1)x_2 - (2+p)(2r-3)\}$$

Since $\frac{c}{3}(1-p)x_2$ is positive the sign of this expression is given by the sign of $3(r-1)x_2-(2+p)(2r-3)$. When $r < \frac{3}{2}$ this expression is positive. Since $x_2 < 1$ we have:

$$3(r-1)x_2 - (2+p)(2r-3) < 3(r-1) - (2+p)(2r-3)$$

$$0 < 3(r-1)x_2 - (2+p)(2r-3) \leftrightarrow r > \frac{3+3p}{1+2p}$$

On the edge $C_1 - C_3$:

$$w_3 - w_1 = \frac{c}{3}p(1-x_3)\{-3(r-1)x_3 + (2r-3)p + (-3r+6)\}$$

Since $\frac{c}{3}p(1-x_3)$ is positive, the sign of this expression is given by the sign of $-3(r-1)x_3+(2r-3)p+(-3r+6)$.

By applying the same reasoning we obtain:

$$w_0 - w_2 > 0 \leftrightarrow r < \frac{3}{2} \text{ or } r > \frac{6-3p}{3-2p}$$

To summarize: $C_3 \to C_2 \to C_1 \to C_0 \to C_3$.

$$\begin{cases} C_2 \to C_0, & \text{if } \frac{3}{2} < r < \frac{6-3p}{3-2p} \\ C_0 \to C_2, & \text{otherwise} \end{cases}$$

$$\begin{cases} C_3 \to C_1, & \text{if } \frac{3}{2} < r < \frac{3+3p}{1+2p} \\ C_1 \to C_3, & \text{otherwise} \end{cases}$$

Results

Parameters

Let's justify the choice of the parameters for which we obtained the following results. First of all, we will explain the parameters that did not change during the entirety of our tests. The value of *n* is obviously the number of player playing with each other (2 or 3 in our case). *M* is the size of our population, we decided to choose 36 because we needed a multiple of 2 and 3 and furthermore it was a golden mean

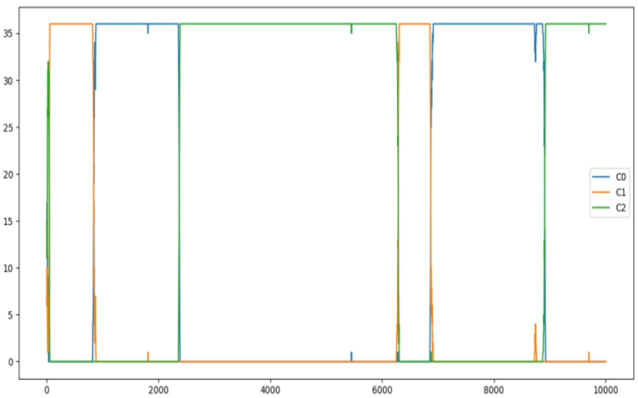

Figure 6: Representation in the population of each strategy depending on the round for a 2-person PGG with $p = 0.5$ and 10000 generations.

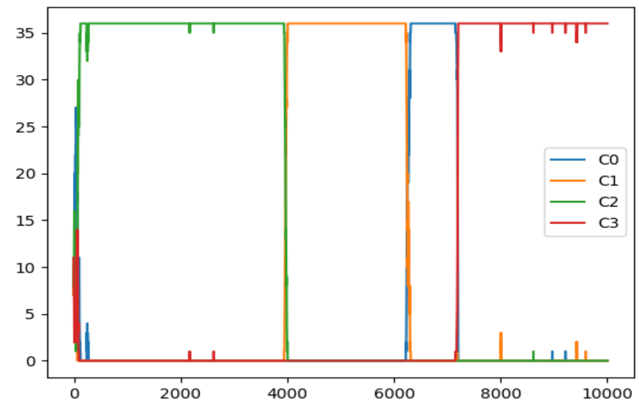

Figure 7: Representation in the population of each strategy depending on the round for a 3-person PGG with $p = 0.5$, $r = 2$ and 10000 generations.

between the time of execution and the fact that we did not want a small number. The σ value which represents the mutation probability was set to 10^{-4}, we wanted a value close to 0 in order that the mutation will occur but with a weak frequency. The s (intensity of selection) was set up to 10^4 (we wanted something very large which can tend to the infinity), increasing the intensity of selection makes some strategies more successful than others. The c value which is the cost a player has to pay when he cooperates was set up to 1. Finally, the r value was set up to 1.6.

Two person Public Goods Game

Most of the time, all the players of the population will have the same strategy. Here we will study the fraction of time during which the population follows a strategy. Suppose q_k the time during which the population follows the strategy C_k divided by the total time of our simulation. Note that $0 \leq q_k \leq 1$ and is defined for $k \in (0, 1, 2)$. For $0 < p < 1$ we get:

$$(q_0, q_1, q_2) = (\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2})$$

The demonstration is given in Ohtsuki (2018). These results can be verified by our empirical data in the **Figure 6**. As we can see , the population here is mainly monomorphic. We can observe that the population follows the strategy C_2 more or less half of the time and that the previous results hold. If the population follows a certain strategy and suddenly there is a mutation, the population starts to change its strategy only if the chosen strategy is more dominant, otherwise it will fail. We plainly notice that just before 2000 for the two players game for example.

Three person Public Goods Game

Here we will simulate many times our model for different values of ours parameters to show the many possible dynamics. At the end of each simulation, we save which strategies the population follows (which is monomorphic). Under the adiabatic limit we obtain these theoretical values for (q_0, q_1, q_2, q_3):

$$\begin{cases} (0.3125, 0.2500, 0.0625, 0.3750) & \text{if } r > \frac{3+3p}{1+2p} \\ (0.2778, 0.2778, 0.1111, 0.3333) & \text{if } \frac{7-3p}{4-2p} < r < \frac{3+3p}{1+2p} \\ (0.1000, 0.1000, 0.2000, 0.6000) & \text{if } \frac{3}{2} < r < \frac{7-3p}{4-2p} \\ (0.0769, 0.1154, 0.2308, 0.5769) & \text{if } r < \frac{3}{2} \end{cases}$$

It is very difficult to verify these values. In fact, when the population is monomorphic, we need to have a mutation (which is unlikely due to the adiabatic limit) and this mutation must lead to a new strategy which dominates the old one. In a 2-person game, this can appear with a probability of $\frac{1}{2}$ but here it is $\frac{1}{3}$ or $\frac{2}{3}$ (depending of the dominating strategy).

The **Figure 7** is an example of a simulation for a three players game. We can verify in this test the dynamics between strategies but not the proportions given by q_k for the reason explained above.

Two person Public Goods Game in Scale-Free Networks

Here we will study the frequency of each strategy in a population of players distributed in a Barabási-Albert (scale free) network. At each round, each player plays with all his neighbors the two person public goods game with negotiation. Then, each player takes randomly one of his neighbors and adopts his strategy if the neighbor has a better average payoff than himself ($s \to \infty$). We consider that there is no mutation, therefore, if a strategy is not present in the population it cannot appear later.

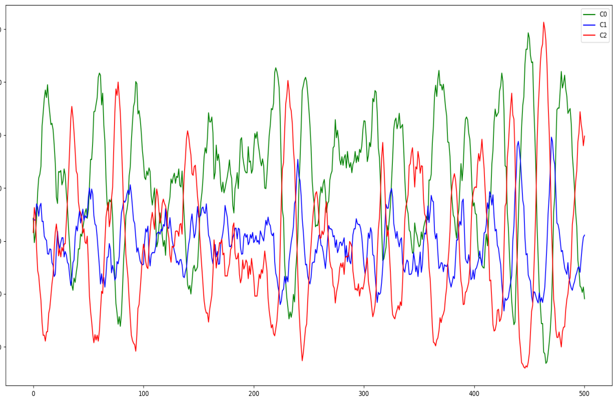

Figure 8: Representation in the population of each strategies depending on the round for a 2-person PGG in a scale-free network with $p = 0.5$, $r = 1.5$ and 500 generations

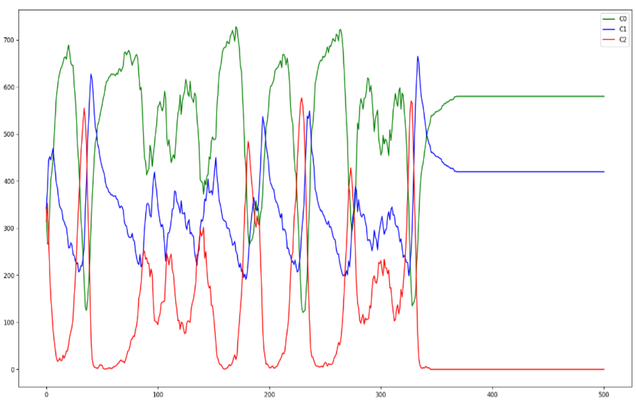

Figure 9: Representation in the population of each strategies depending on the round for a 2-person PGG in a scale-free network with $p = 0.8$, $r = 1.5$ and 500 generations

We have set r to $\frac{3}{2}$, the number of nodes to 1000 and each node has at least 4 neighbors. With $p = 0.5$ we obtained the following distribution (see **Figure 8**). The main difference between this result and the one in **Figure 6** is that the population is never monomorphic. In the other hand, we can verify that C_2 dominates C_0 and C_0 dominates C_1. We have done the same experiment without C_1 and we obtained the following results: the population is directly invaded by C_0 and all players adopt this strategy. Without C_1 there is no cooperation.

With $p = 0.8$ we obtained some interesting results (see **Figure 9**): after some rounds, the C_2 strategy disappears, the population is not monomorphic but remember that, without C_2 players, C_0 and C_1 are same.

By plotting the average number of rounds necessary to avoid C_2 players depending on the value of p we obtained

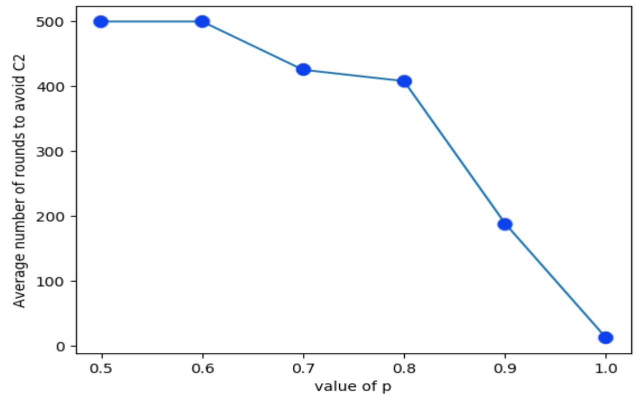

Figure 10: Average number of rounds to avoid C_2 strategy from the population for a 2-person PGG in a scale-free network $r = 1.5$ and 1000 players.

these results (see **Figure 10**). After 500 rounds, the simulation stopped even if some players still follow the C_2 strategy.

To summarize, for $p > 0.6$, most of the time, before the 500^{th} round, the unconditional defector strategy will be avoided. If $p \leq 0.6$, we can think that the population become monomorphic (only C_2) or that more steps will be necessary to avoid C_2 strategy, but it is only a supposition.

Discussion

This article proposes a modified version of the Public Goods Game: in this one the players will first negotiate before playing. In this model, the players can choose to use a strategy of conditional cooperation or defection, those strategies are only possible if there is a negotiation phase before.

This model encourages the presence of coordinate cooperation: the players will have a tendency to cooperate simultaneously and thereby increase their reward.

Furthermore, the presence of conditional cooperators has a major influence on the rate of cooperation of our game. Indeed, by allowing players to have only unconditional strategies (only defect or only cooperate), the dominant strategy will unanimously be the non-cooperation (Hauert (2005)). In our case, the presence of conditional cooperators allows us to avoid the total defection. For instance, the C_1 strategy will dominate the strategy C_2 (for $p \neq 0$).

Finally, the dynamics are very different in a scale free network. If the players interact regarding their position in a scale free network (and not randomly), the population is less likely monomorphic. Notice that what we have done previously is a particular case of this idea but here the graph was fully connected and players interacted with a subset of their neighbours.

Something we have to mention is that our model does not take into account the fact that a player can lie which makes our model less realistic. Another important difference between our model and the other classical Public Goods Game models is that our players change their thoughts asynchronously during the negotiation phase.

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