

$$\begin{aligned}
\frac{\sin(x)}{x} &= 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots \\
&= \left(1 - \frac{x}{\pi}\right) \left(1 + \frac{x}{\pi}\right) \left(1 - \frac{x}{2\pi}\right) \left(1 + \frac{x}{2\pi}\right) \left(1 - \frac{x}{3\pi}\right) \left(1 + \frac{x}{3\pi}\right) \dots \\
&= \left(1 - \frac{x^2}{\pi^2}\right) \left(1 - \frac{x^2}{4\pi^2}\right) \left(1 - \frac{x^2}{9\pi^2}\right) \dots \\
&\quad - \left(\frac{1}{\pi^2} + \frac{1}{4\pi^2} + \frac{1}{9\pi^2} + \dots\right) = -\frac{1}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2}
\end{aligned}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

$$\lim_{n \rightarrow \infty} \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2}$$

$$\zeta(2n) = \frac{(2\pi)^{2n}(-1)^{n+1}B_{2n}}{2 \cdot (2n)!} (*)$$

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \left( (-1)^n \frac{4}{n^2} \cos nx \right) \quad (1)$$

$$\therefore \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad (2)$$

$$\begin{aligned}
a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \cos nx \, dx = \frac{2}{\pi} \int_0^{\pi} x^2 \cos nx \, dx \\
&= \frac{2}{\pi} \times \frac{2\pi}{n^2} (-1)^n = (-1)^n \frac{4}{n^2} \\
b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx = 0
\end{aligned}$$

$$\therefore \int_0^{2\pi} x^2 \cos nx \, dx = \left[ \frac{2x}{n^2} \cos nx + \left( \frac{x^2}{n} - \frac{2}{n^3} \right) \sin nx \right] \Big|_0^{2\pi} = \frac{2\pi}{n^2} (-1)^n.$$

$$\therefore \phi = \frac{\sqrt{5}-1}{2}$$