start

1.Taylor series

$$f\left(x\right)=\sum_{n=0}^{\infty}\frac{f^{(n)}\left(a\right)}{n!}(x-a)^{n} \tag{1}$$

2.Maclaurin series

$$f\left(x\right)=\sum_{n=0}^{\infty}\frac{f^{(n)}\left(0\right)}{n!}x^{n} \tag{2}$$

3. $f\left(x\right)=\arctan\left(x\right)$ to solve $f'(x)$

$$\begin{aligned}y&=f\left(x\right)=\arctan\left(x\right)\\ x&=tan\left(y\right)\end{aligned}$$

$$\begin{aligned}\Longrightarrow dx&=\sec^{2}y*dy\\ f'(x)&=\frac{dx}{dy}=\frac{1}{x^{2}+1}\end{aligned}$$

4.

$$\left(\frac{f\left(x\right)}{g\left(x\right)}\right)'=\frac{f'\left(x\right)g\left(x\right)-f\left(x\right)g'\left(x\right)}{g^{2}\left(x\right)}$$

$$\begin{aligned}
&f^{(1)}\left(x\right)=\frac{1}{x^{2}+1}\\
&f^{(2)}\left(x\right)=\frac{-2x}{\left(x^{2}+1\right)^{2}}\\
&f^{(3)}\left(x\right)=\frac{2\left(3x^{2}-1\right)}{\left(x^{2}+1\right)^{3}}\\
&f^{(4)}\left(x\right)=\frac{-24x\left(x^{2}-1\right)}{\left(x^{2}+1\right)^{4}}\\
&f^{(5)}\left(x\right)=\frac{24\left(5x^{4}-10x^{2}+1\right)}{\left(x^{2}+1\right)^{5}}\\
&...\\
&f^{(n)}\left(x\right)=\frac{1}{2}(-1)^{n}i\left[(-i+x)^{-n}-(i+x)^{-n}\right](n-1)!\\
&...
\end{aligned}$$

$$
\begin{aligned}
k_1 &= \frac{f^{(1)}(0)}{1!} = 1 \\
k_2 &= \frac{f^{(2)}(0)}{2!} = 0 \\
k_3 &= \frac{f^{(3)}(0)}{3!} = \frac{-1}{3} \\
k_4 &= \frac{f^{(4)}(0)}{4!} = 0 \\
k_5 &= \frac{f^{(5)}(0)}{5!} = \frac{1}{5} \\
&...
\end{aligned}
$$

5.get the conclusion, Maclaurin Series.
Gregory's series or Leibniz's series

$$
\begin{aligned}
\because \arctan(x) &= \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1} x^{2n+1} \\
&= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + ... \\
\therefore \arctan(1) &= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + ... = \frac{\pi}{4}
\end{aligned}
$$

6.another solution

$$
\begin{aligned}
\because \arctan(x) &= \int_0^x \frac{1}{1+t^2}\,dt \\
\frac{1}{1+x^2} &= \frac{1}{2}\left(\frac{1}{1-ix} + \frac{1}{1+ix}\right) \\
\therefore \arctan(x) &= \frac{1}{2} i \left[\ln(1-ix) - \ln(1+ix)\right]
\end{aligned}
$$