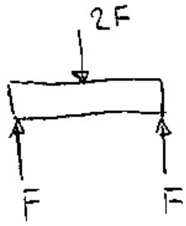
ME 80 STATICS REVIEW

In statics problems, we very often want to determine the values of unknown forces/moments acting on/in static systems.

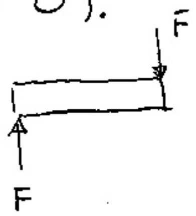
Static systems are those for which the sum of all forces and moments acting on the system is zero.

$\Rightarrow \sum F = 0$
$\sum M = 0$

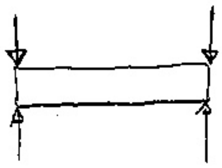
The system must therefore be moving at a constant velocity (often 0).



This is a static system.



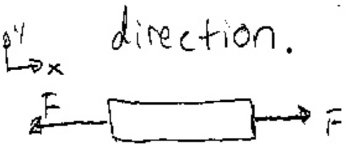
This is not.



This could be, but we don't know w/o values.

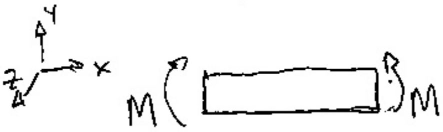
A "load" can refer to a force or a moment.

A force is represented by an arrow with magnitude and direction.

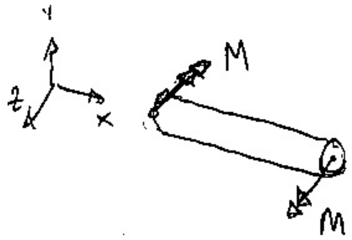


These forces have magnitude F acting in the (+) and (−) x directions

A moment is represented by a curved arrow OR a double-headed arrow with a magnitude and a direction.

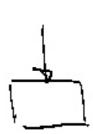


These moments have magnitude M acting in the (+) and (−) z directions. Use right-hand rule (RHR) to determine the direction.



This is the exact same system shown in 3D. The double-headed arrow points in the direction that the curved arrow curls around using RHR.

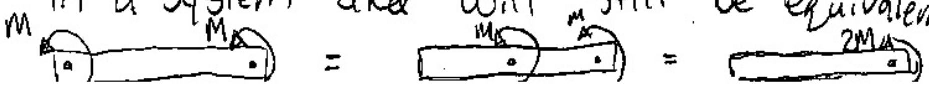
A "point load" is an idealized load applied at a single point



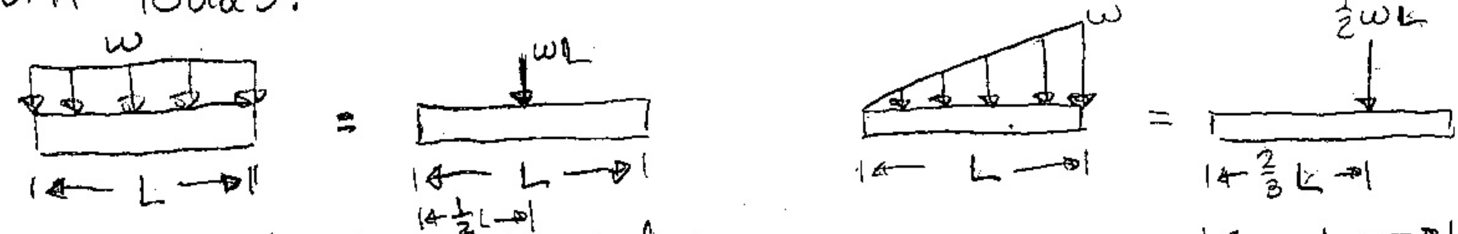
pointforce

pointmoment

Point moments can be moved anywhere in a system and will still be equivalent.



Distributed loads are more realistic. For the purpose of statics (only) math (only), you can resolve them to point loads.



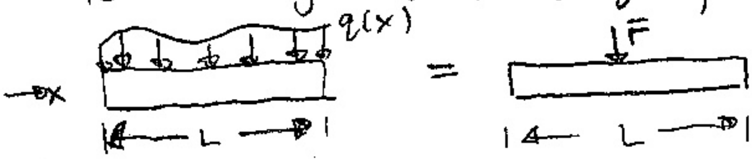
We very rarely see distributed moments, but the same principles apply.

Resolving a distributed force to a point force

A distributed force is in units of $\frac{\text{force}}{\text{distance}}$ (e.g. $\frac{N}{m}$, $\frac{lb}{in.}$, etc)

The resolved force magnitude is equal to the area under the load curve. This gives us units of force ($\frac{N}{m}\cdot m = N$).

For some general load $q(x)$, $F = \int_0^L q(x)\,dx$

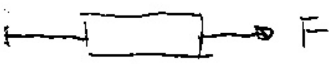


Compare this to the rectangular and triangular loads above. We will see those most frequently.

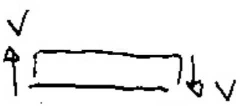
The distributed force acts through the centroid of the load curve. For a rectangular load of length L, this is at $x = \frac{1}{2}L$. For a triangular load of length L, this is at $x = \frac{1}{3}L$ or $x = \frac{2}{3}L$, depending on which side $x=0$ and which side is the "heavier" portion of the triangle.

More generally, $\bar{x} = \frac{1}{A}\int_0^L x\cdot q(x)\,dx$

Two types of forces:



Normal force
Can be in tension$^{(+)}$ or compression$^{(-)}$

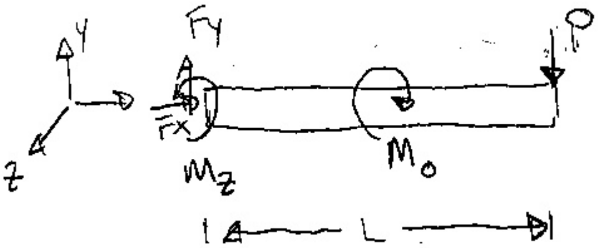


Shear force
can be pos. or neg.

Free Body Diagrams and Equilibrium Equations

FBDs in Statics are isolated systems in equilibrium that show the loads acting on that system. A correct FBD will show:

- all pertinent loads (forces & moments)
- all pertinent dimensions
- a coordinate system
- Static EQ



FIND: the reaction loads
GIVEN: M_o, P, L
In order to solve for unknown loads, we need to write equilibrium equations

$$\sum F_x = \boxed{0 = F_x}$$

$$\sum F_y = 0 = F_y - P \Rightarrow \boxed{F_y = P}$$

Forces are straight forward

$$\sum M_z @_{x=0} = 0 = M_z - M_o - PL \Rightarrow \boxed{M_z = M_o + PL}$$

Moments are more complicated.

Point moments: write the magnitude and sign, just like point forces. Location does not matter as long as the moment is in the diagram

Moments generated by forces about a point: equal to the force magnitude multiplied by the moment arm. Use RHR to determine sign. OR use vectors and cross products: $\vec{M} = \vec{r} \times \vec{F}$

For FBDs in 2D, you can only generate up to 3 EQ equations. This increases to 6 equations per FBD for 3D. You need to generate enough FBDs and equations to be able to solve for all your unknowns.

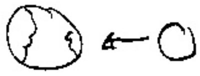
If you have 2 unknowns, you need 2 equations.
" " 3 " , " 3 " etc

Loads come in two flavors: external and internal.

1) External loads consist of:

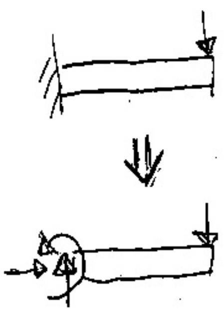
a) body forces

eg. gravity between bodies, EM force, weight (acts through centroid of object)



b) surface loads - distributed loads, idealized point load

c) reaction loads - a special case of surface loads
(or support)

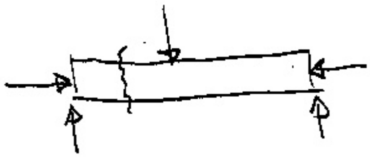


where we replace physical supports with the forces they exert on a system

See table in back for types of supports and the loads they exert.

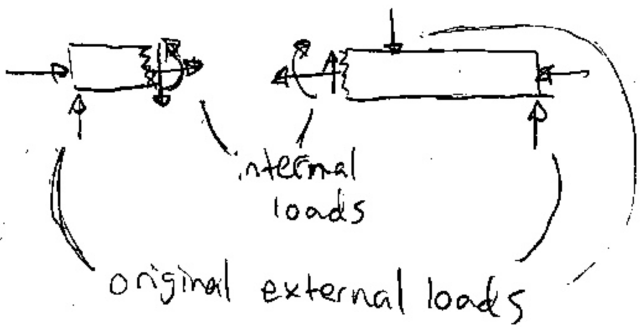
2) Internal loads

When we draw FBDs, we can isolate a section of a system as a new system. Each subsystem still has to be in equilibrium. Internal loads show the action of the rest of the body on the smaller system.



Full system FBD

cut at the section line, look at the two new subsystems.



Internal loads in 2D vs. 3D

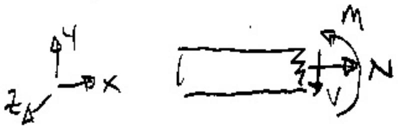
In 2D, we have 3 possible equations we could write:

$\sum F_x = 0$

$\sum F_y = 0$

$\sum M_z = 0$

Therefore, for any given internal face resulting from a "cut," you could potentially have 3 loads:



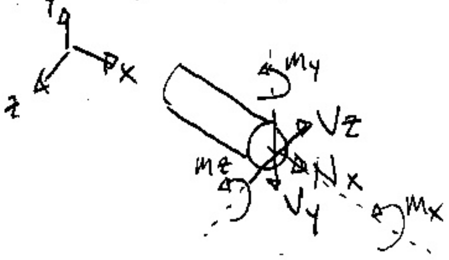
In 3D, we have 6 possible equations we could write:

$\sum F_x = 0 \qquad \sum M_x = 0$

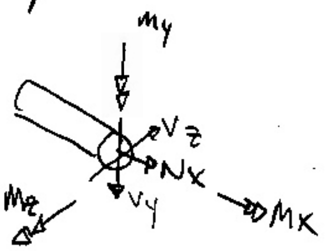
$\sum F_y = 0 \qquad \sum M_y = 0$

$\sum F_z = 0 \qquad \sum M_z = 0$

Therefore, for any given internal face resulting from a cut, you could potentially have 6 loads:



or

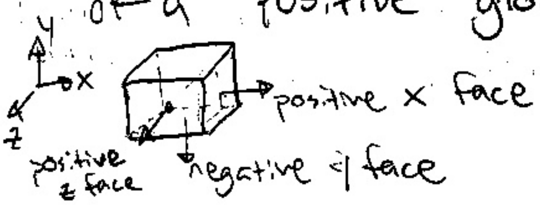


Conventions

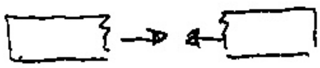
If you follow these conventions in ME 80 / Statics / life, things will be MUCH easier.

1) Faces

A face is positive if its normal vector points in the direction of a positive global axis.



2) Internal loads

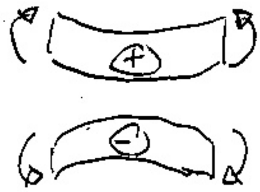


Normal forces are positive (tensile) when pointing away from a face.

Shear forces are positive when pointing down on a positive face or up on a negative face.

Moments are positive when pointing pos. on a pos. face and neg. on a neg. face.

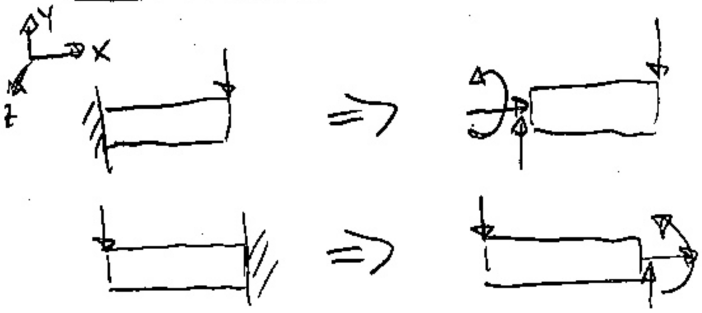
"A positive moment is a happy moment."



3) Reaction loads

There is no convention for reaction loads. Feel free to choose your own, but be consistent for your own sake.

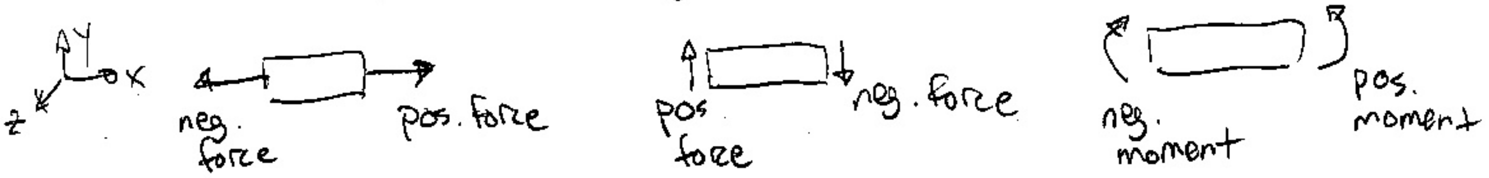
I always draw my reactions globally positive



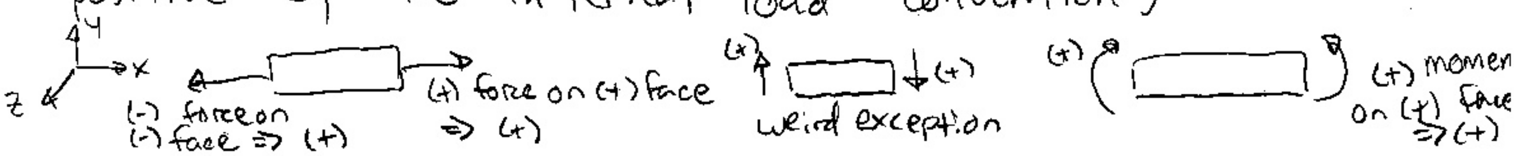
Text

A note on "globally positive" vs. "positive on a face"

- Globally positive means that the load is drawn positive with respect to the global axes.



- "positive on a face" means that the load is considered positive by the internal load conventions



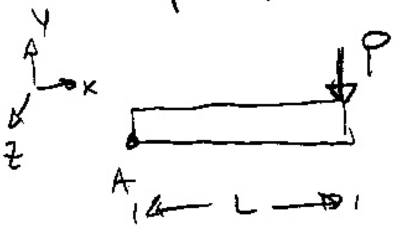
Miscellanea

1) Right-hand rule for moments

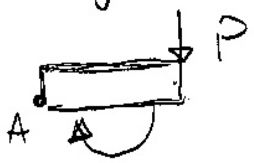
The direction of a moment can be determined using the right-hand rule

curl your fingers in the direction of the arrow and stick your thumb out straight.
The direction of the moment is the direction your thumb points – in this case, out of the page.

You can use the same process to determine the direction of a moment caused by a force about a point.



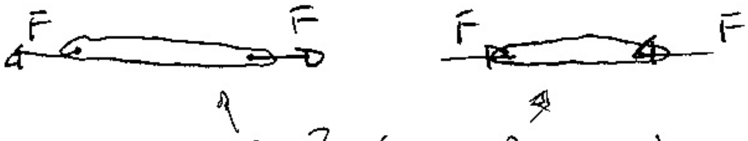
If we want to know the sign of the moment caused by force P about point A, curl your fingers in the direction of the rotation.



The direction your thumb points determines the sign – in this case, it points down the -z axis, so the moment is -PL

2) Two-force members

A two-force member is a bar in pure tension or pure compression



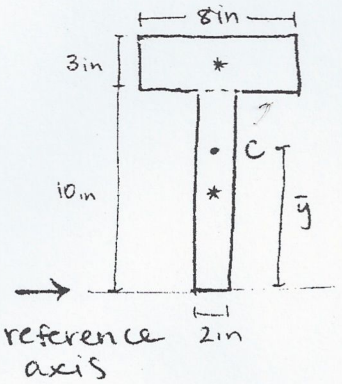
see? two forces!

But, it may not be obvious that a member is a two-force member.

You can easily identify them in complex systems (like trusses) if connected at both ends (only) by pin joints. See truss example.

Centroid Example

Locate the centroid C of the cross-sectional area for the T-beam shown below.



① Establish reference axis (at the bottom is easiest)

② Segment the area into rectangles

③ Locate the centroid for each rectangle (*)

④ Solve using $\bar{y} = \dfrac{\Sigma \tilde{y} A}{\Sigma A}$

$$\bar{y} = \frac{\overbrace{(5in)}^{\tilde{y}}\overbrace{(10in)(2in)}^{A} + \overbrace{(11.5in)}^{\tilde{y}}\overbrace{(3in)(8in)}^{A}}{(10in)(2in) + (3in)(8in)}$$

Basic Procedure for Statics Problems

① Draw a FBD
   - Support reaction table will come in handy for this

② Calculate external forces

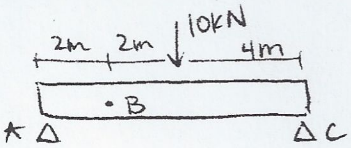
③ Cut long members ⊥ to axis 

④ Add internal forces

- Normal force, N : acts ⊥ to cross-sectional area
- Shear force, V : lies in plane of "
- Torsional Moment or Torque, T : external loads that twist one segment w/ respect to the other
- Bending Moment, M : you'll learn more in ME 80

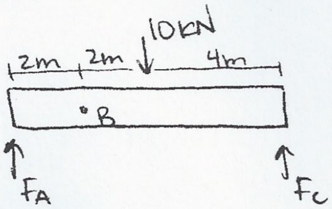
⑤ Find unknowns using equations of equilibrium

Statics Example



Find the forces at B
& moments

① FBD



② Calculate external forces

ΣF_y: $F_A + F_C - 10kN = 0$

ΣF_x: 0

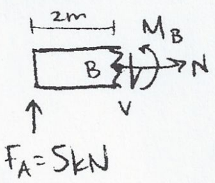
ΣM_A: $-(10kN)(4m) + (F_C)(8m) = 0$

$F_C = 5kN$

$F_A = 5kN$

③ Cut & ④ Add Internal Forces

Solution 1:



⑤ ΣF_x: $\boxed{0 = N}$

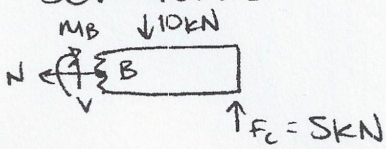
ΣF_y: $F_A - V = 0$

$\boxed{V = 5kN}$

ΣM: $M_B - (5kN)(2m) = 0$

$\boxed{M_B = 10kNm}$

Solution 2:



⑤ ΣF_x: $\boxed{N = 0}$

ΣF_y: $V - 10kN + 5kN = 0$

$\boxed{V = 5kN}$

ΣM: $-M_B - (10kN)(2m) + (5kN)(6m)$

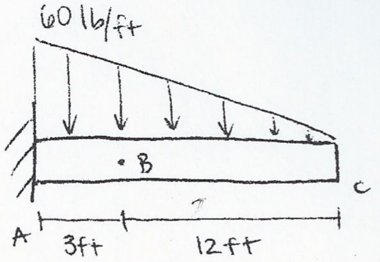
$-M_B - 20kNm + 30kNm = 0$

$\boxed{M_B = 10kNm}$

Same cut, looked at different sides

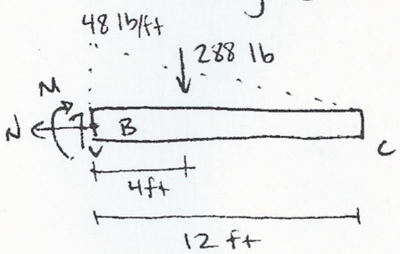
⟶ Still got the same answer

Statics Example



Find the resultant internal loadings acting on the cross-section at point B.

Cut along B



- Find value of distributed load at B using similar triangles

$$\frac{w}{12ft} = \frac{60\,lb/ft}{15\,ft} \qquad w = 48\,lb/ft$$

- idealize as concentrated load acting at the centroid

Magnitude $\frac{1}{2}(48\,lb/ft)(12ft)$
$= 288\,lb$

Centroid $\frac{1}{3}(12ft)$
$= 4ft$

<u>Statics time</u>

$\Sigma F_x = -N_B = 0 \Rightarrow \boxed{N_B = 0}$

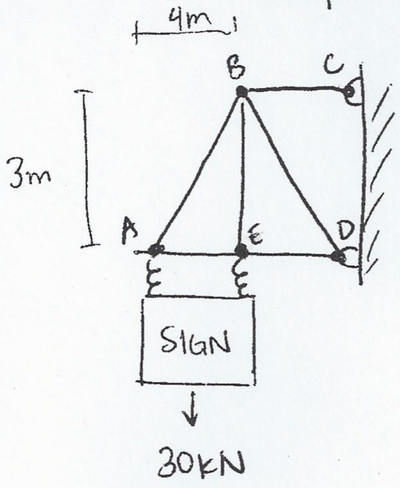
$\Sigma F_y = V_B - 288\,lb = 0$

$\boxed{V_B = 288\,lb}$

$\Sigma M_{@B}: -M_B - (4ft)(288\,lb) = 0$

$\boxed{M_B = -1152\,ft\,lb}$

Truss Example

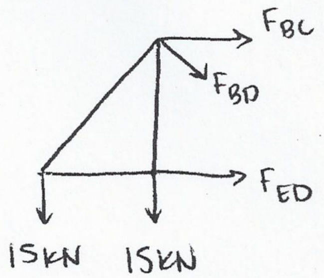


Find the load in BD

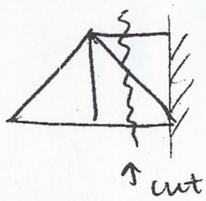
* It's important to identify 2-force members so as not to introduce unnecessary unknown forces. It allows you to simplify the problem.

AB, BE, BD, BC, AE, AD

① FBD & Cut



A cut was made through the segment we're interested in (BD)



Since we know all of the external forces on this side of the cut, we don't even need to solve for the reactant forces on the wall.

We have 3 unknowns → we need 3 equations

ΣM_B : $(15\,kN)(4m) + (F_{ED})(3m)$ → $F_{ED} = -20\,kN$ (compression)

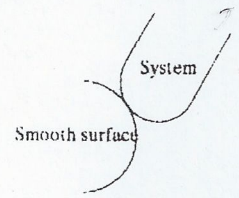
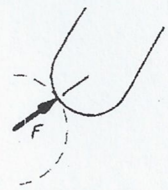
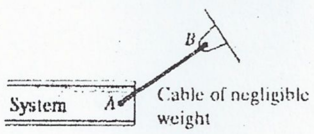
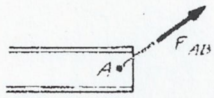
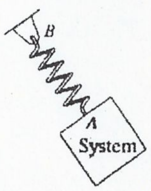
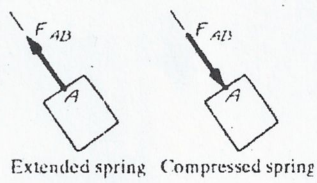
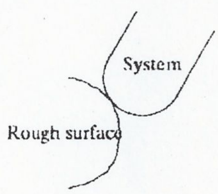
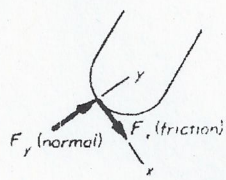
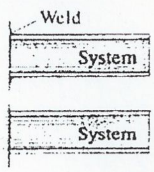
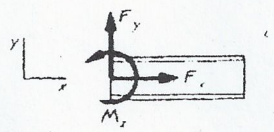
ΣF_y : $-15\,kN - 15\,kN - F_{BD}\sin\theta = 0$ $(\sin\theta = 3/5)$

$\boxed{F_{BD} = -50\,kN}$ (compression)

ΣF_x : $F_{BC} + F_{BD}\cos\theta + F_{ED} = 0$

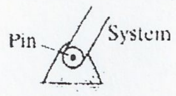
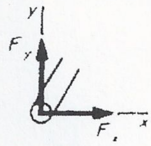
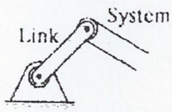
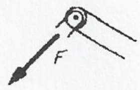
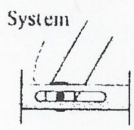
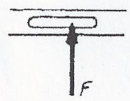
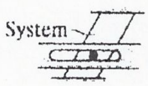
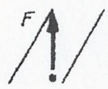
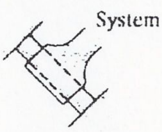
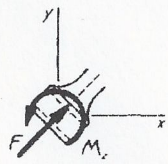
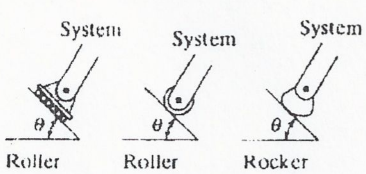
$F_{BC} = 60\,kN$ (tension)

Table 4.1: Standard Supports for Planar Systems

(A) Supports	Description of Loads	(B) Loads to Be Shown on Free-Body Diagram
1. **Normal contact without friction** 	**Force** (F) oriented normal to surface on which system rests. Direction is such that force pushes on system.	F 
2. **Cable, rope, wire** 	**Force** (F) oriented along cable. Direction is such that cable pulls on the system.	F 
3. **Spring** 	**Force** (F) oriented along long axis of spring. Direction is such that spring pulls on system if spring is in tension, and pushes if spring is in compression.	F 
4. **Normal contact with friction** 	**Two forces**, one (F_y) oriented normal to surface on which the system rests so as to push on system, other force (F_x) is tangent to surface.	F_y F_x 
5. **Fixed support** 	**Force** in xy plane represented in terms of components F_x and F_y. **Moment** about z axis (M_z).	F_x, F_y M_z 

(*Continued*)

Table 4.1: (Cont.)

(A) Supports	Description of Loads	(B) Loads to Be Shown on Free-Body Diagram
6 **Pin connection** (pin or hole is part of system) 	**Force** perpendicular to pin represented in terms of components F_x and F_y. Point of application is at center of pin.	F_x, F_y 
7. **Link** 	**Force** (F) oriented along link length; force can push or pull on the system.	F 
8. **Slot-on-pin** (slotted member is part of system) 	**Force** (F) oriented normal to long axis of slot. Direction is such that force can pull or push on system.	F 
9. **Pin-in-slot** (pin is part of system) 	**Force** (F) oriented normal to long axis of slot. Direction is such that force can pull or push on system.	F 
10. **Smooth collar on smooth shaft** 	**Force** (F) oriented perpendicular to long axis of shaft. Direction is such that force can pull or push on system. **Moment** (M_z) about z axis.	F M_z 
11. **Roller or rocker** 	**Force** (F) oriented normal to surface on which system rests. Direction is such that force pushes on system.	F