Proceeding of the IEEE
International Conference on Automation and Logistics
Zhengzhou, China, August 2012

Automatic Parking Path Optimization Based on Bezier Curve Fitting

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***Abstract* - The autonomous parking system is an intelligent technology to park a car into a small space. More and more studies have addressed this issue. However, the planned path in these studies is not smooth enough. Based on Ackermann steering geometry, the car kinematic model is established in this paper, the collision possibility between car body and obstacles in the parking space are analyzed, and then the bound of start point and collision-free space are presented. Based on this, the parking trajectory is planned and Bezier curve is used to smooth the trajectory which is generated on multi arcs of circumference, finally, the generation of continuous curvature path is realized. In the end, we build the environment of the simulation in MATLAB and the Fuzzy PID control method is used to track the trajectory which is planned in advance. The simulation results show that the car can not only avoid the obstacles effectively, but also move smoothly on turning points for using the Bezier curve. The design can meet the continuity of the parking requirements very well.**

Index Terms - Automatic Parking, Path Planning, Bezier Curve, Trajectory Tracking

I. Introduction

In recent years, with the rapid development of the industry and the improvement of people's living standard, the number of the car has greatly increased. The parking space for the drivers sharply reduced, accidents are especially prone to occur when reversing the car for these less experience drivers. Therefore, the automatic parking problem has become the research focus.

The automatic parking technology is not mature enough in environmental perception, path planning and control method. Our country has just started the research of these aspects. Compared with our country, the foreign research work are carried out much earlier, Mukherjee etc have studied the nonholonomic constraint of the path planning problem [1] and Yanan Zhao etc have put forward some route of parking theory in succession [2-5]; they have done research on the continuous control and the path planning method. The domestic experts Guo Konghui etc have also done researches on the parking theory in recent years [6-7].

The authors would like to thank National Hi-Tech Research Development Program (863 Program) under NO.2009AA044902 and The Major Science Research Project of Henan Province under Grant 91100210300, National Defense Project (JPPT-ZCGX1-1), Major Project of Basic and Advanced Technology Research of Henan Province (No.112300413205), Henan University of Science and Technology Innovation Fund Project for Postgraduates (CXJJ-YJS-Z011), and Education Department of Henan Province Key Science and Technology Research Project (12B510010).

However, the path planned in these studies is not smooth enough and the robust performance is not good, it is difficult to track the path. This paper presented a research on the parallel automatic parking problems and planned the track of the automatic parking, Bezier curve was used to smooth the trajectory which was generated on multi arcs of circumference, and finally, we make a simulation in MATLAB by using Fuzzy PID control strategy to complete the trajectory tracking.

II. The Vehicle Kinematic Model

In the low speed range, we don't consider the impact of the tire slide; the deflection of the wheels meets the relationship named Ackerman Steering any time. In this process, all the wheels make a circular motion around the same instantaneous center and all the following studies are based on this formula. The Fig.1 is the sketch map of vehicle kinematic model, from the map we can conclude:

$$x_a^2 + (y_a - l\cot\phi)^2 = (l\cot\phi - \frac{L_{rw}}{2})^2 . \tag{1}$$

$$x_d^2 + (y_d - l\cot\phi)^2 = (l\cot\phi - \frac{L_{rw}}{2})^2 . \tag{2}$$

Where, (x_a, y_a) and (x_d, y_d) are the coordinates of the car rear which points contacting to the ground; l is the wheelbase of the car; L_{rw} is the wheel tread. In this diagram, we define v as the moving speed of the front wheel center, ϕ is the turning angle of the front wheel; θ is the turning angle of the car body; point *ABCD* stand for the four body vertices and in all parameters we define counterclockwise is positive and clockwise is negative.

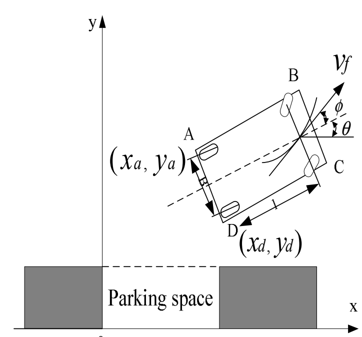

Fig.1 The vehicle kinematic model

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III. Path Planning

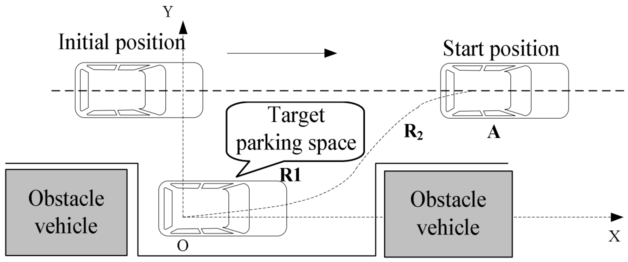

Fig.2 Sketch map of backing a vehicle

As is shown in Fig.2, basing on Formula (1) and (2), we learn that the car trajectory comprise of pieces of circular arc. This paper defines the two tangent arcs which connect the start point and the terminate point as the backing trajectory [8-10]. This makes the reverse operation as simple as possible.

The parking processes are as follows: the vehicle driving slowly along the road, perceiving the environment information through the ultrasonic sensors and cameras, then it tests whether there is a suitable parking space in considering the shortest space constraints. If not, go on searching until a suitable parking space is found. After finding out a suitable parking space, it builds up the environment map, calculating the scope of the starting position according the perception and position information, and then it generates collision avoidance space, subsequently, the parking trajectory is planned with the constraints of the minimum vehicle turning radius and the direction of rotation speed, finally, the vehicle control system will control the vehicle to track the path that planned in advance, after the driver confirms the vehicle has finished parking, they stop the vehicle.

The performance of the vehicle is an important factor to the path planning. First of all, the minimum turning radius is limited when the vehicle front wheel work in the maximum value, secondly, the changing range of the path curvature should not be more than the biggest constraint value of the angular velocity; otherwise, the planned path can not use for trajectory tracking.

In addition, in the process of reversing the car from the parking starting point to the final parking space, the collision

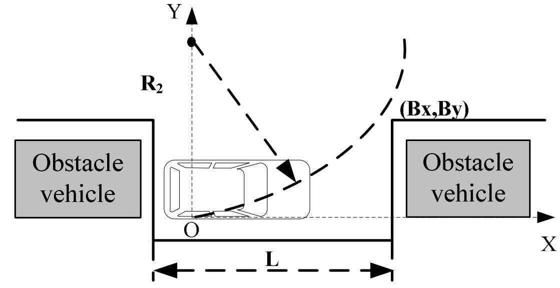

Fig.4 Sketch map of collision avoidance

between vehicle and obstacles are likely to happen in the following three cases, which is shown in Fig.3:

A. Determine the Constraints Space

In order to avoid collision in the parking process, we analyze the possibility of collision and discuss the constraint conditions according to the environment of the vehicle and parking motion characteristics.

Before the discussion on the constraint condition, we define the following parameters:

r is the distance from the rear to the front wheel; d stands for the width of the car; l is the distance from the headway to the rear; (B_x, B_y) is the vertex of the parking space which is most likely to be met for the vehicle.

(1)As is shown in Fig.4, according to the possible collision between the vehicle body vertex C and parking space (B_x, B_y), we determine the length of the collision-free parking space and a maximum value of the turning radius.

From the sketch map, the no-hit-zone and the shortest continuous parking space can be expressed as:

$$L_{\min} = r + \sqrt{2(+d)B_y + l^2 - B_y^2} \, . \tag{3}$$

In the parking process, we define the steering radius as R_2, define the length of parking space measured by the sensors as L and the most possibly collision point as (B_x, B_y), then we obtain the range of no-hit-zone is $[R_{\min}, R_{2\max}]$, where,

$$R_{2\max} = [(L+r)^2 + B_y^2 - l^2] / (2B_y) - d \, . \tag{4}$$

(2)The different unsuitable starting points may make different collisions, they can be divided into two cases: if the starting point is too near, it may occur the lateral collision which is shown in Fig.5, meanwhile, if the starting point is too far, it may occur the collision which is shown in Fig.6:

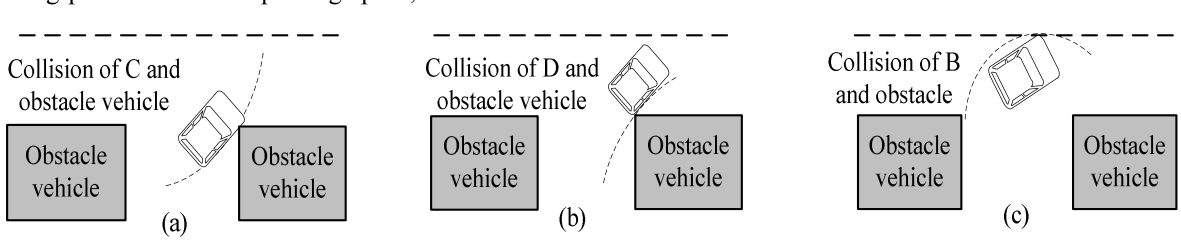

Fig.3 Sketch map of possibility collision

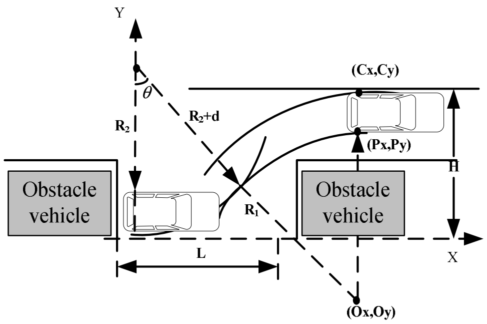

Fig.5 Lateral collision

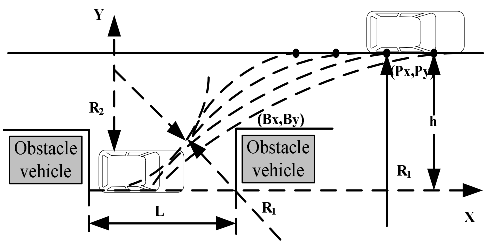

Fig.6 Sketch map of the furthest starting point

As is shown in Fig.5, the collision point is (C_x, C_y), suppose H is the distance between the bottom of the parking space and the collision side of the vehicle, and as is shown in Fig.6, the collision point is (B_x, B_y) ,when the starting point is the farthest point (P_x, P_y), we suppose h is the distance between the bottom of the parking space and the right side of the vehicle. By the above constraint condition, we get the parking starting point range:

$$\begin{cases} P_{x\min} = \sqrt{2(R_{1\min} + R_{\min} + d)\cdot P_y - P_y^2} \\ P_{x\max} = \sqrt{2(R_{1\max} + R_{\min} + d)\cdot P_y - P_y^2} \end{cases}. \tag{5}$$

$$R_{1\min} = \frac{d^2 + l^2 - (H - P_y)^2}{2(H - P_y - d)}. \tag{6}$$

$$R_{1\max} = \frac{B_x}{B_y^2}(h\cdot B_x + \Gamma^{1/2}) + h - \rho. \tag{7}$$

Where,

$$\begin{aligned}\Gamma &= h^2\cdot(B_x^2 + B_y^2) \\ &\quad - B_y\cdot h\cdot[2(R_{\min} + d)(h + B_y) - (B_x^2 + B_y^2)]; \\ \rho &= [2(R_{\min} + d)h + B_x^2 + B_y^2]/(2B_y)\end{aligned}$$

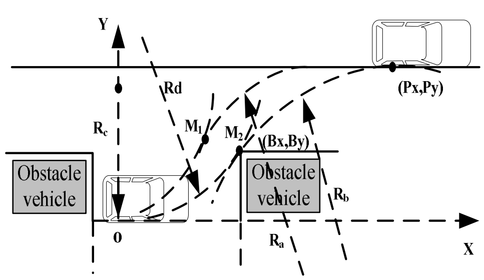

Fig.7 Sketch map of collision constraint space

Then, the initial range of the points for starting parking can be confirmed basically, the vehicle should not knock against only the front vehicle but also the back vehicle, besides, it can't hit bottom spot. As shown in Fig.7, by above analysis, we get a closed area from the four segments of tangent arc, and we take it as the candidate area for parallel parking trajectory planning. Then, we make a circular arc through the point P with the radius of R_a, make a circular arc through the point O with the radius of R_c ($R_c = R_{\min} + d$), and the point M_1 is the tangency point of the two circular arcs, we also make a circular arc through the point O with the radius of R_d, make a circular arc through the point P with the radius of R_b and the tangency point is point M_2. Where, R_a is the biggest radius which should make sure the lateral collision can not be occur, R_c is the minimum turning radius of the vehicle. When the medial point of the rear axle locates in this area and the direction gradient is no more than the gradient of the bottom corresponding points of the circular arc, the vehicle won't knock against the obstacles. The next step we can plan the trajectory in this area after the generation of the collision avoidance constraint space.

B. The Genernation and Optimization of Parking Path

Using the method of Reference [8-10], we planned the parking path as two segments of the tangent arc, and in this way, the trajectory of the vehicle can be expressed. But the path planned out is discontinuous on the tangent point, this means that the vehicle steering wheel in here will appear jump, this should be avoided for a car parking. So, the most important problem about the automatic parking is to look for a smooth path. In actual free curve fitting process, we usually use cubic polynomial B-spline basis function to fitting the operation. In order to make smoother curve, the point on the curve should be as much as possible to march the calculation, however, this will bring a large amount of calculation and the fitting speed of the B-spline curve will be slowly. With this reason, it is difficult to meet the practical needs. In order to avoid this situation, the path optimization based on Bezier Curve is put forward in this paper to smooth the path. Bezier curve has the advantages of small curvature, more control points, fast fitting speed, and the control effect is obvious. According to the Reference [11], Bezier curve can be fit any number of control points; the number of times about Bezier polynomial is related to the position of the control points.

Suppose the position of these $n+1$ control points as $p_k = (x_k, y_k, z_k), k = 0,1,2,\cdots,n$, then the Bezier fitting function from p_0 to p_n can be described as:

$$p(u) = \sum_{k=0}^{n} p_k BEZ_{k,\,n(u)}. \tag{8}$$

Where, BEZ_k, $n(u)$ is the blending function of Bezier, it is Bernstein polynomial:

$$BEZ_{k,\,n(u)} = C(n,k)u^k(1-u)^{n-k}. \tag{9}$$

Where, $C(n,k)$ is the binomial coefficient:

$$C(n,k) = \frac{n!}{k!(n-k)!}. \quad (10)$$

Bezier curve has the following characteristics:

(1) This curve is always through the first and the last control points, that is the boundary conditions about the curve on the two end points should be $p(0) = p_0, p(n) = p_n$.

(2) The first derivative parameters of the curve on the end point can be calculated by the coordinates of the control point: $p'(0) = -np_0 + np_1, p'(n) = -np_{n-1} + np_n$.

Thus, the tangent line at the beginning point lie on the connecting line of the first two control points, and the tangent line at the end point lie on the connecting line of the last two control points. The advantage of doing this is: (1) the curve fall within the convex hull of the control points makes sure the polynomial go forward smoothly along with the control points and won't cause swing; (2) if the control points lies in a straight line, Bezier curve is a straight line; (3) the curve can achieve a smooth and continuous plane when joining together.

The movement path of vehicle can be regarded as the curves joining together, in order to ensure the vehicle make smooth movement along the path, we must consider the continuity problems. From the above analysis, Bezier curve that compound three times can achieve a smooth and continuous plane, it represents that two adjacent curve equations have the same first derivative in the intersection point. Therefore, Bezier curve that compound three times can be regarded as the movement of vehicle path. Then we should make the control to the path we planned.

IV. The Simulation Analysis

A suitable control method should be used to track the path planned [12]. At present, the conventional PID controller have been widely used in industrial control operation with its high accuracy, but the robustness of this kind controller is bad, it is difficult to obtain satisfactory effect for these nonlinear and time-varying parameters. The Fuzzy Control method has the advantage of good robustness and can be used in nonlinear and time-varying system; the main shortcoming is the low accuracy. Fuzzy PID control method overcomes these deficiencies; it has the advantage of better robustness and can satisfy the nonlinear system. With using this control method, the control effect is good, the response rate is rapid and the error is small.

As is shown in Fig.8, first of all, the vehicle drives along the road, perceiving the environment information by using all kinds of sensors, then, it integrates the data which obtained from surrounding, meanwhile, it generates the ideal path through related path planning algorithm. The feedback part adopts Fuzzy PID control, it adjusts the deviation through the fuzzy PID regulator, and then after the corresponding change, it inputs the vehicle with steering wheel angle and the wheel speed. The system will get the actual tracking path and compare with the ideal path, it generates deviation ΔX and ΔZ which the Fuzzy PID regulator needed, thus, the closed-loop control formed, which greatly reduce the error of tracking the path.

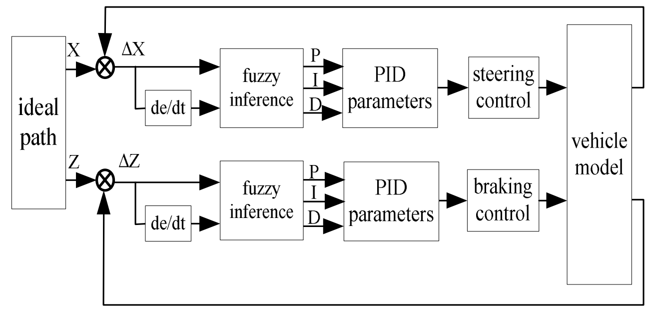

Fig.8 Fuzzy PID control system diagram

We build the simulation diagram in MATLAB based on the kinematics model of the vehicle and establish the simulation environment. The Fuzzy PID control method was used to make the simulation. The parameters are set as follows: the 7.5m length of the parking spaces, the 1.76m wide of the car, the 4.3m length of the car, the car to the front range of the car is1.0m, the minimum turning radius $R_{\min}$ is 3.5m, and the biggest deflection angle of the front wheel is 45 degrees. The simulation diagram is shown in Fig.9.

Fig.10 is the curvature comparison chart before and after the path optimization. From the chart, we can see that the path become smoother after optimization with using Bezier curve, the simulation results indicate that this method can control the car move effectively in the planning area, the trajectory is smooth and the effect of reversing is ideal, it can satisfy the requirement of parking very well.

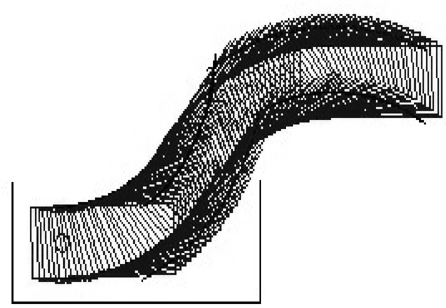

Fig.9 The simulation diagram of the reversing path

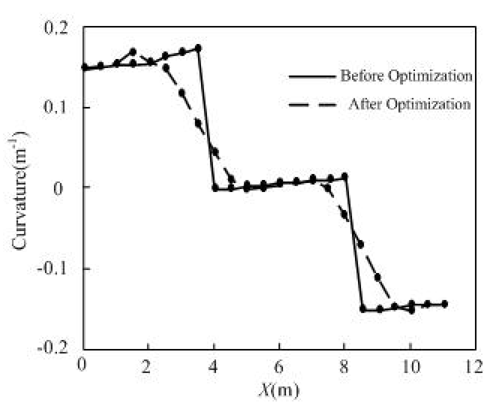

Fig.10 The curvature comparison chart

V. CONCLUSIONS

In considering the vehicle performance constraint and the possible crash bound, this paper plans a collision-free parking path based on the vehicle kinematics model. In order to overcome the discontinuity of the trajectory curvature, we adopt Bezier curve to smooth the trajectory. The Fuzzy PID control strategy is used to build a simulation system, the simulation results show that the car can not only avoid the obstacles effectively, but also move smoothly on turning points by using the Bezier curve to smooth the trajectory, this method can meet the continuity of the parking requirements very well.

REFERENCES

[1] Mukherjee R, Anderson D P. "A surface integral approach to the motion planning of nonholonomic systems," *ASME Journal of Dynamic Systems, Measurement and Control*. 116(9):315−325, 1994.

[2] Yanan Zhao, Emmanuel G. Collins Jr. "Robust automatic parallel parking in tight spaces via fuzzy logic," *Robotics and Autonomous Systems*, 51(2/3):111-127, 2005.

[3] Lee K, Kim D, Chung W, et al. "Car parking control using a trajectory tracking controller," *SICEICASE International Joint Conference*, 2058-2063, 2006.

[4] K. Demirli, M. Khoshnejad. "Autonomous parallel parking of a car-like mobile robot by a neuro-fuzzy sensor-based controller," *Fuzzy sets and systems*, 160(19):2876-2891, 2009.

[5] Tzuu-Hseng S.Li, Ying-Chieh Yeh, Jyun-Da Wu, Ming-Ying Hsiao. "Multifunctional Intelligent Autonomous Parking Controllers for Carlike Mobile Robots," *IEEE Transactions on Industrial Electronics*, 57(5):1687-1700, 2010.

[6] JiangHui, Guo Konghui, Zhang Jianwei. "Design of automatic parallel parking steering controller based on path-planning," *Journal of Jilin University*, 41(02): 293-297, 2011.

[7] Yang Xiyang, YouQingxi, Li Hongxing. "Truck backer-upper based on variable-universe fuzzy controller," *Journal of Beijing Normal University*, 41(4):348-350, 2005.

[8] Song Jinze, Dai Bin, Shan Enzhong, Sun Zhenping, He Hangen. "Trajectory planning approach of autonomous parallel parking with Kino dynamic constraints," *Journal of Central South University*, 40(1):135-141, 2009.

[9] Zhang Qin, Qu Ningxi, Li Yan, Kamiya Yoshitsugu. "Research of the method to generate automatic parking trajectory," *Compute Simulation*, 27(10):148-151, 2010.

[10] Wang Daobin, Liang Huawei, Yang Nina, Gao Hang. "On path planning of two self-parking methods," *Electronic Measurement Technology*, 34(1):27-30, 2011.

[11] Wang Xiaofeng, Li Hui, Chen Kaimin. "An algorithm of simulating objects movement based on Bezier Curves in Virtual Scene," *Microelectronics & Computer*, 26(7): 244-247, 2009.

[12] Pang Xiaofeng, Lu Zhixiong. "Study on Control Algorithms of Automobile Automatic Reverse Motion," *Agricultural Equipment & Vehicle Engineering*, 4:15-19, 2011