Path Planning for Nonholonomic Mobile Robot Based on Bézier Curve

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***Abstract*—This study presents simulation results of nonholonomic mobile robot control with path planning which is based on Bézier Curves. The purpose of this paper is set to develop a simulation model for differential drive wheeled (DDW) robot by using MATLAB/Simulink environment. Fractional order PID structure is used to increase the tracking performance of DDW robot. Simulation results are compared for the various path scenario. It was shown that the proposed control scheme exhibits agreeable performance for the desired path tracking.**

***Index Terms*—Mobile robot, path planning, Bézier curves, fractional order control, simulation.**

I. INTRODUCTION

Nowadays, mobile robots are actively used in defense industry, academic fields, space studies, service sector and many other areas. These robots will continue to be widely used in the future, as they have the ability to work in closed environments and open areas.

If the concepts of time and safety are considered to be important, extensive researches are underway on trajectory planning to minimize energy. Solutions are being sought to optimize traveling time and smooth traveling toward the desired destination problems. Thus, path planner and tracking controller are the basic components of the navigation systems in mobile robots. Path planner which determines the appropriate and smooth way to reach the desired target, helps the robot to overcome some obstacles. Tracking and drive controller allow a robot to travel along the specified trajectory while staying within physical limits [1]. There are several methods which include linear and nonlinear techniques to control the robot in desired path.

Interest in fractional calculus has made a significant impact on the field of control engineering and as a result of this there have been many studies dealing with the analysis and design of control systems [2].

The theoretical discussion on a fractional-order derivative can be traced back to the 1690s, [3, 4]. During the last two decades, it has been increasingly utilized in various science and engineering areas such as, computer, electrical and electronics, mechatronics, robotics, chemistry, medicine, signal processing because of better theoretical understanding and subsequent developments in the computing technologies [5].

The aim of this paper is to study the problem of the stability of a differential drive wheeled robot system with a fractional order controller. In addition, Bézier curves were used to define a path for reference input to the DDW mobile robot. Simulation results show that, the proposed controller scheme has acaptable response for differential robots. Literature survey indicates that Bézier curves are still used in many robotics applications [6-10] .

The rest of the paper is organized as follows: Section 2 reviews the methodology for path planning via Bézier curves and fractional order PID controller structure. Section 3 deals with defination of mobile robot kinematic model. Some simulation examples are presented in Section 4. Section 5 gives some concluding remarks.

II. METHODOLOGY

A. Path Planning via Bézier Curve

In the early 1960s, Bézier Curves were invented by Dr. Pierre Bézier for designing automobile bodies . Since then, Bézier curves have been widely used in graphic illustration, designing and font defination areas. It has proved to be very useful to find the center of mass of a set of point masses.

In recent years these curves, which have been used in the field of robotics, are planning smooth path with selected control points [11-13]. Bézier curves of any degree can be defined with Bernstein polynomials blending function $B_i^n(\varsigma)$ as [13],

$$B_i^n(\varsigma) = \binom{n}{i}(1-\varsigma)^{n-i}\varsigma^i, \qquad i = 0,1,\ldots,n \tag{1}$$

where n is degree of Bézier curve with n+1 control points, $\binom{n}{i}$ is called binomial coefficient and u is an arbitrary parametric

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value in $0 \le \varsigma \le 1$. Thus, equation of Bézier curve can be written as,

$$\mathbf{P}(\varsigma) = \sum_{i=0}^{n} B_i^n(\varsigma)\mathbf{P}_i \tag{2}$$

where $\mathbf{P}_i = (A_i, B_i)$. For instance let's take n=3 to find the 3rd order Bézier curve. According to n=3, $B_0^3 = (1-\varsigma)^3$, $B_1^3 = 3\varsigma(1-\varsigma)^2$, $B_2^3 = 3\varsigma^2(1-\varsigma)$ and $B_3^3 = \varsigma^3$. One can obtain the $\mathbf{P}(u)$ for control points $P_0 = (A_0, B_0)$, $P_1 = (A_1, B_1)$, $P_2 = (A_2, B_2)$ and $P_3 = (A_3, B_3)$ with the Figure 1 as,

$$\begin{aligned}\mathbf{x}(\varsigma) &= A_0(1-\varsigma)^3 + 3A_1\varsigma(1-\varsigma)^2 + 3A_2\varsigma^2(1-\varsigma) + A_3\varsigma^3 \\ \mathbf{y}(\varsigma) &= B_0(1-\varsigma)^3 + 3B_1\varsigma(1-\varsigma)^2 + 3B_2\varsigma^2(1-\varsigma) + B_3\varsigma^3\end{aligned} \tag{3}$$

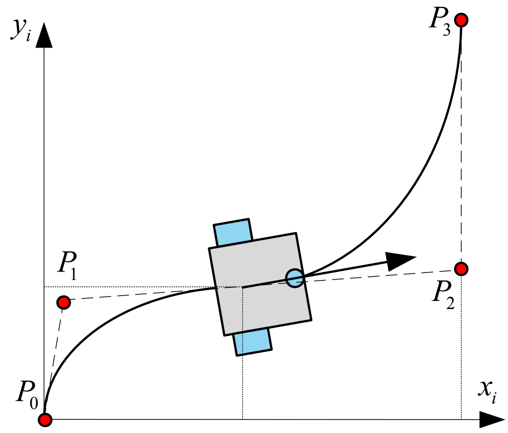

Fig.1. Bézier curve based path planning

Arbitrary parametric value of ς can be used to generate a smooth curve from a starting point to a desired point. A more precise curve can be obtained with a smaller increase. The path given by equation does not consider velocity and it is only parameterized by ς [14].

B. Fractional Order Control Structure

A generalized fractional-order system with input $r(t)$ and output $y(t)$ can be described by a fractional differential equation of the form [15],

$$\begin{aligned}a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \cdots + a_0 D^{\alpha_0} y(t) = \\ b_m D^{\beta_n} r(t) + b_{n-1} D^{\beta_{n-1}} r(t) + \cdots + b_0 D^{\beta_0} r(t)\end{aligned} \tag{4}$$

where $D^{\alpha,\beta}$ denotes the fractional derivative. Equation 4 can be arranged by using a fractional order transfer function as,

$$G(s) = \frac{Y(s)}{R(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} \tag{5}$$

In control engineering perspective, it is possible to apply non-integer sysytmes for control purposes. One of the most important controller structure PID is used for operating control systems in industrial applications. Morever, even a small improvement in PID features could have a relevant impact by using $PI^{\lambda}D^{\mu}$ [3]. $PI^{\lambda}D^{\mu}$ controller has been introduced in 1999 by Podlubny [16]. This paper focuses on the case when the controller is a fractional order $PI^{\lambda}D^{\mu}$ controller for robot tracking control, i.e;

$$C(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{6}$$

In addition, 4th order Continued Fraction Expansion (CFE) method has been used to obtain the integer order approximation of $PI^{\lambda}D^{\mu}$ controller. The basic control structure of navigation control of DDW mobile robot system is shown in Figure 2.

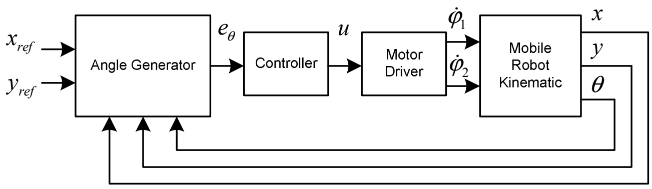

Fig.2. Block diagram of control scheme

The angle generator block generates the desired θ_{ref} angle between x_{ref} and y_{ref} in radian form. Error function e_θ has been obtained from negative feedback. Motor driver block consists two saturation function that ensure the robot satisfies the nonholonomic constraint. This procedure is provided by limiting the outputs between 0 and 1.

$$\begin{aligned}u_{RM} &= \dot{\varphi}_1 = sat(1+u) \\ u_{LM} &= \dot{\varphi}_2 = sat(1-u)\end{aligned} \tag{7}$$

III. System Modelling

In this chapter, differential drive model of mobile robot, will be given.

A. Differential Drive Model

Differential drive model is derived from the kinematics equations that represent basic model of the robot. It is a nonholonomic system where the number of control variables are less than the number of output variables. Wheel rotation speeds are considered as input variables which are called control inputs. Morever, the variables of x, y, θ are generates the robot motion that are used as output variables. In this case, pose of robot can be written as,

$$Q_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}, \tag{8}$$

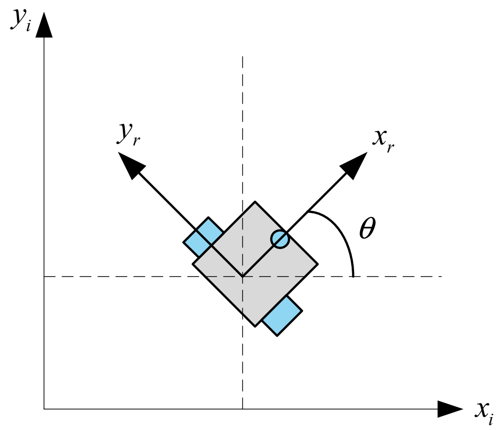

Fig.3. Reference frame

Translational speed which is the average velocity can be written in Equation 9 from the speed of each wheel given in Figure 4a and 4b.

$$\dot{x}_r = r\frac{\dot{\varphi}_1 + \dot{\varphi}_2}{2} \tag{9}$$

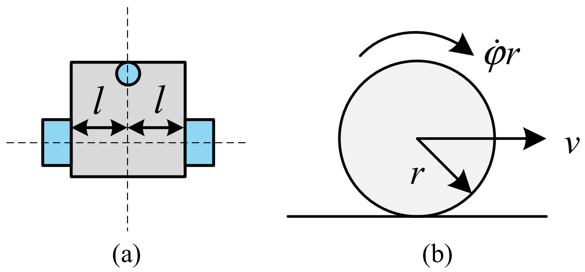

Fig.4. a) Differential drive model b) Wheel model

Figure 4 shows the differential drive model and wheel model of the robot respectively. Wheels have radius r which are placed at distance l from centre line of the robot. Motion between frames is given in Equation 10.

$$\dot{Q}_R = T(\theta)\dot{Q}_I \tag{10}$$

where $T(\theta)$ is the standard orthogonal rotation transformation matrix which can be written as,

$$T(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

Equation 12 gives the total instantaneous rotation,

$$\dot{\theta} = \frac{r}{2l}(\dot{\varphi}_1 - \dot{\varphi}_2), \tag{12}$$

where $\omega_1 = {r\dot{\varphi}_1}/{2l}$ is instantaneous rotation of one wheel, $\dot{\varphi}_1$ and $\dot{\varphi}_2$ are rotation speed of first wheel and second wheel respectively. From Equation 10 and 11, the full model is derived as,

$$\dot{Q}_I = T(\theta)^{-1}\dot{Q}_R, \tag{13}$$

$$\dot{Q}_I = T(\theta)^{-1}\frac{r}{2}\begin{bmatrix} \dot{\varphi}_1 + \dot{\varphi}_2 \\ 0 \\ {(\dot{\varphi}_1 - \dot{\varphi}_2)}/{l} \end{bmatrix}, \tag{14}$$

where $T(\theta)^{-1}$ is the inverse form of Equation 11.

$$T(\theta)^{-1} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{15}$$

Finally differential drive model of the robot has been obtained by substituting Equations 9, 12, 14 into Equation 13 as [1], [17],

$$\dot{Q}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}\frac{r}{2}\begin{bmatrix} \dot{\varphi}_1 + \dot{\varphi}_2 \\ 0 \\ {(\dot{\varphi}_1 - \dot{\varphi}_2)}/{l} \end{bmatrix} \tag{16}$$

IV. Simulation Results

Simulation results were performed to illustrate the effectiveness of the controller and path planner. The procedure of the simulation steps is given in Figure 5. The fractional order PID controller mentioned in Section 2 was implemented in MATLAB/Simulink. Two path scenarios were chosen for the simulation. The length from wheel to the center of the robot is $l = 0.5m$ and radius of the wheel is $r = 0.1m$. The controller parameters were set to; $K_p = 0.2$, $K_i = 0.3$, $K_d = 0.25$, $\lambda = 0.237$ and $\mu = 0.145$.

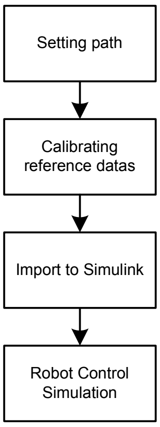

Fig.5. Simulation steps

The first scenario of the path planner is given in Figure 6. There are 6 control points of the 5th order Bézier curve. The obstacles in the map are shown in small rectangular boxes. The red line represents the desired path which has been created by using the blue lines via the control points.

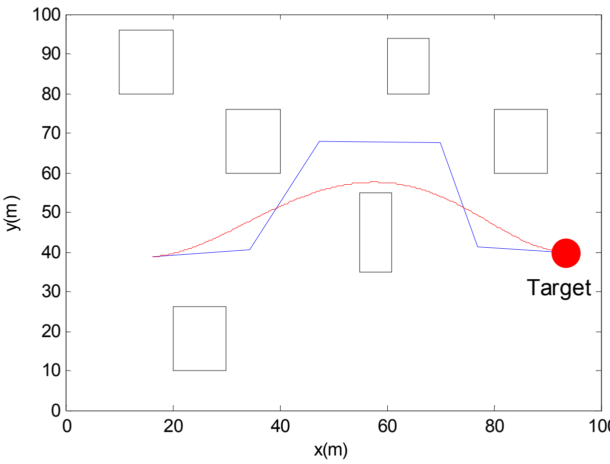

Fig.6. First scenario with 5th order Bézier curve

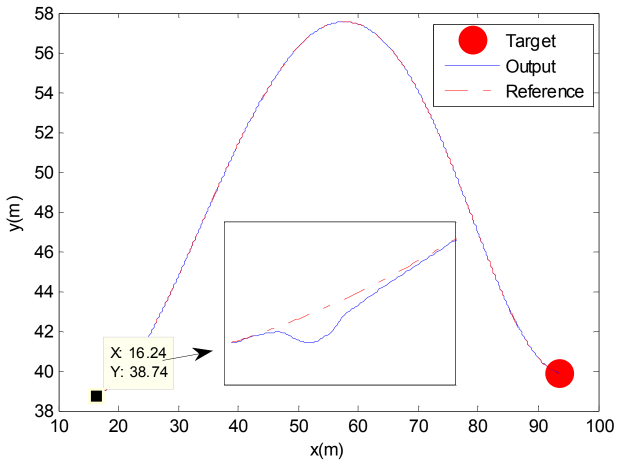

Fig.7. Actual and reference trajectory for first scenario

Figure 7 shows the actual and reference trajectory for first scenario. It is observed that, the proposed controller has enforced the system to reach the target in the direction of the planned path. The second scenario of the path planner is given in Figure 8. There are 10 control points of the 9th order Bézier curve.

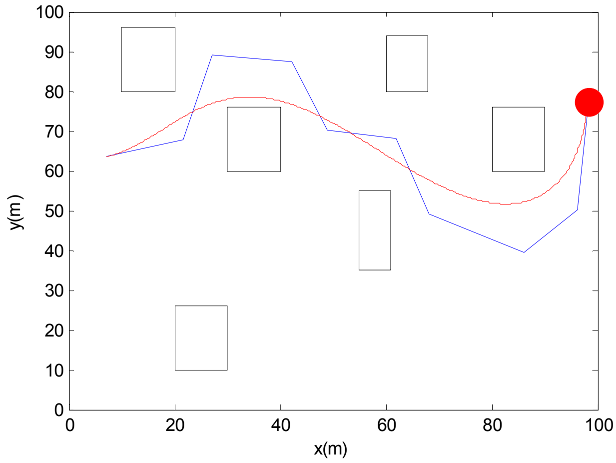

Fig.8. Second scenario with 9th order Bézier curve

Reference trajectory which is in the form of sinusoidal signal is shown in Figure 9. The controller performs satisfactory response in this scenario as in the previous example. Figure 9 indicates that there has been some fluctuation because of the limitations of robot. On the other hand, the DDW robot has good tracking performance with stable features.

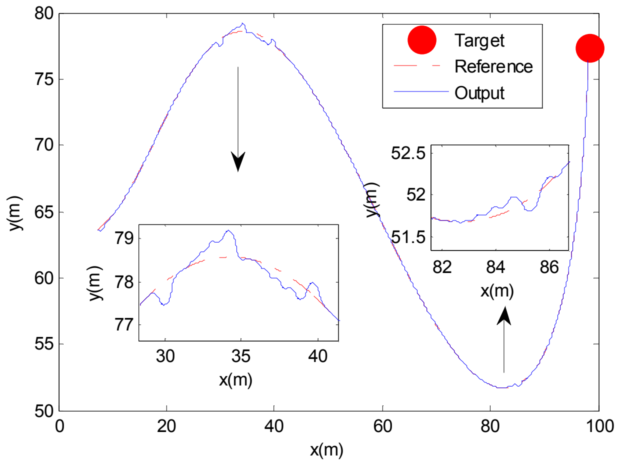

Fig.9. Actual and reference trajectory for second scenario

V. Conclusion

In this paper fractional order PID controller for DDW mobile robot has been presented. A forward kinematic model of the robot was derived with Simulink blocks.

The Bézier curves, which are used in many fields in the literature, contribute for path planning in this study. The proposed controller has shown fine results in reaching the goal in both path scenarios. In the next studies, it is planned to be used fractional order nonlinear control algorithms and image processing techniques on real time platforms with simultaneous localization and mapping.

REFERENCES

[1] R. Siegwart, I. R. Nourbakhsh, and S. Davide, *Introduction to Autonomous Mobile Robots, Second Edition*. London: MIT Press, 2011.

[2] D. P. Atherton, N. Tan, C. Yeroglu, G. Kavuran, and A. Yüce, "Computation of limit cycles in nonlinear feedback loops with fractional order plants," *Fractional Differentiation and Its Applications (ICFDA), 2014 International Conference on*. pp. 1–6, 2014.

[3] R. Caponetto, G. Dongola, L. Fortuna, and I. Petras, *Fractional Order Systems: Modeling and Control Applications*. Singapore: World Scientific, 2010.

[4] C. A. Monje, Y. Chen, B. M. Vinagre, D. Xue, and V. Feliu, *Fractional-order Systems and Controls*. London: Springer London, 2010.

[5] G. Kavuran, A. Ateş, B. B. Alagöz, and C. Yeroğlu, "Image Processing Based Object Tracking Application with Fractional-Order Model Reference Controller," *Pamukkale Univ Muh Bilim Derg*, vol. 22, no. 8, pp. 659–665, 2016.

[6] X. Yang, Z. Zeng, J. Xiao, and Z. Zheng, "Trajectory planning for RoboCup MSL mobile robots based on Bézier curve and Voronoi diagram," *2015 IEEE International Conference on Information and Automation*. pp. 2552–2557, 2015.

[7] N. Arana-Daniel, A. A. Gallegos, C. López-Franco, and A. Y. Alanis, "Smooth global and local path planning for mobile robot using particle swarm optimization, radial basis functions, splines and Bézier curves," *2014 IEEE Congress on Evolutionary Computation (CEC)*. pp. 175–182, 2014.

[8] J. Jiao, Z. Cao, P. Zhao, X. Liu, and M. Tan, "Bézier curve based path planning for a mobile manipulator in unknown environments," *2013 IEEE International Conference on Robotics and Biomimetics (ROBIO)*. pp. 1864–1868, 2013.

[9] J. A. S. Jayasinghe and A. M. B. G. D. A. Athauda, "Smooth trajectory generation algorithm for an unmanned aerial vehicle (UAV) under dynamic constraints: Using a quadratic Bézier curve for collision avoidance," *2016 Manufacturing & Industrial Engineering Symposium (MIES)*. pp. 1–6, 2016.

[10] X. Zhang, Y. Wang, and Y. Fang, "Vision-based moving target interception with a mobile robot based on motion prediction and online planning," *2016 IEEE International Conference on Real-time Computing and Robotics (RCAR)*. pp. 17–21, 2016.

[11] J. Choi, "Bézier Curve for Trajectory Guidance," *World Congr. Eng. Comput. …*, pp. 0–5, 2008.

[12] J. W. Choi, R. Curry, and G. Elkaim, "Path planning based on bezier curve for autonomous ground vehicles," *Proc. - Adv. Electr. Electron. Eng. - IAENG Spec. Ed. World Congr. Eng. Comput. Sci. 2008, WCECS 2008*, no. 2, pp. 158–166, 2008.

[13] J.-W. Choi, R. Curry, and G. Elkaim, "Piecewise Bézier Curves Path Planning with Continuous Curvature Constraint for Autonomous Driving," in *Machine Learning and Systems Engineering*, S.-I. Ao, B. Rieger, and M. A. Amouzegar, Eds. Dordrecht: Springer Netherlands, 2010, pp. 31–45.

[14] Y. Gil Jin and W. C. Byoung, "Robot Velocity Based Path Planning Along Bezier Curve Path," in *AST 2013*, 2013, pp. 31–34.

[15] I. Podlubny, "Fractional-order systems and fractional-order controllers," *Proc. Conf. Int. Francoph. d'Automatique*, p. 2002, 1994.

[16] I. Podlubny, "Fractional-order systems and PIλDμ-controllers," *IEEE Trans. Automat. Contr.*, vol. 44, no. 1, pp. 208–214, 1999.

[17] H. I. Christensen, "Introduction to Perception and Robotics," *Centre for Autonomous Systems, Kungl Tekniska Högskolan*, 2005. [Online]. Available: https://www.nada.kth.se/kurser/kth/2D1426/slides2005/aut-rob3.pdf. [Accessed: 01-Jan-2017].