

# Engineering Optics

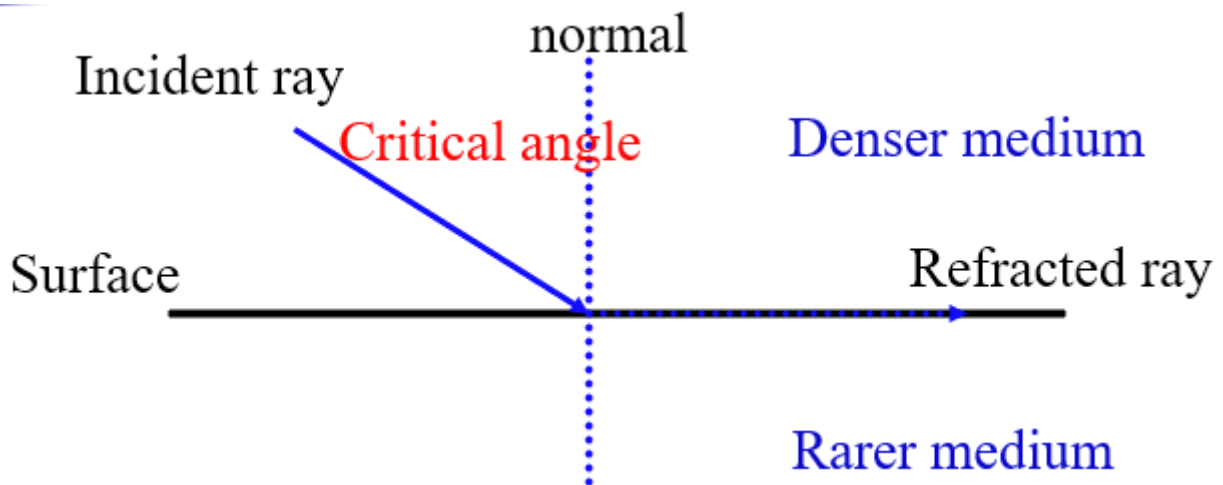
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# 1. Basic laws of geometrical optics and imaging concepts

- The electromagnetic spectrum
  - wave length: short -> long
  - GammaRays | X-Rays | UltraViolet | VisibleLight(380nm-780nm) | InfraRed | MicroWaves | RadioWaves

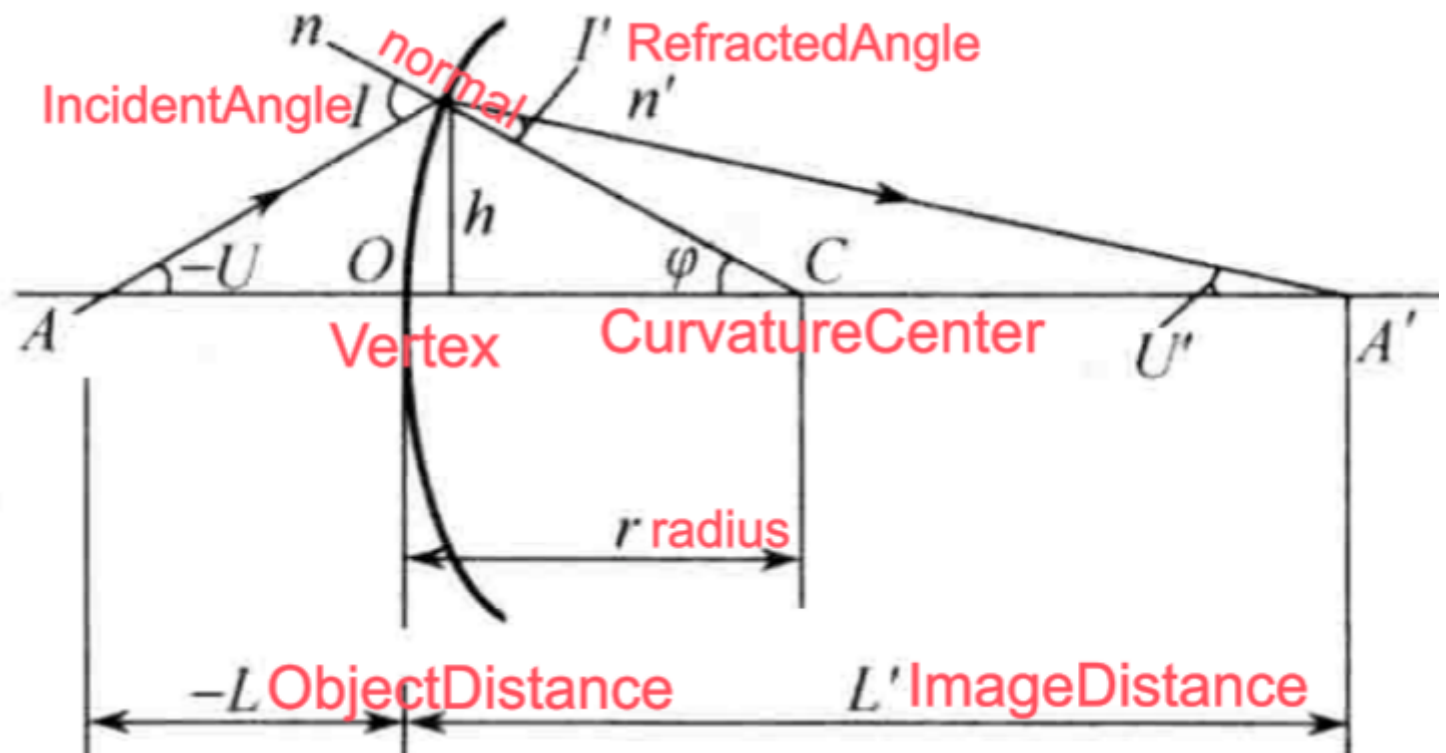
## 1.1 Basic laws

1. **Law of Rectilinear propagation:** Light -> free space / homogeneous, isotropic matter = beeline.
2. **Law of independent propagation:** different lights meet, not affect each other
3. **Law of Reflection:** Light -> reflecting surface and reflected make equal angles
  1.  $I'' = -I$
  2. incoming, outgoing, normal  $\in$  same plane
  3. **Total Internal Reflection**



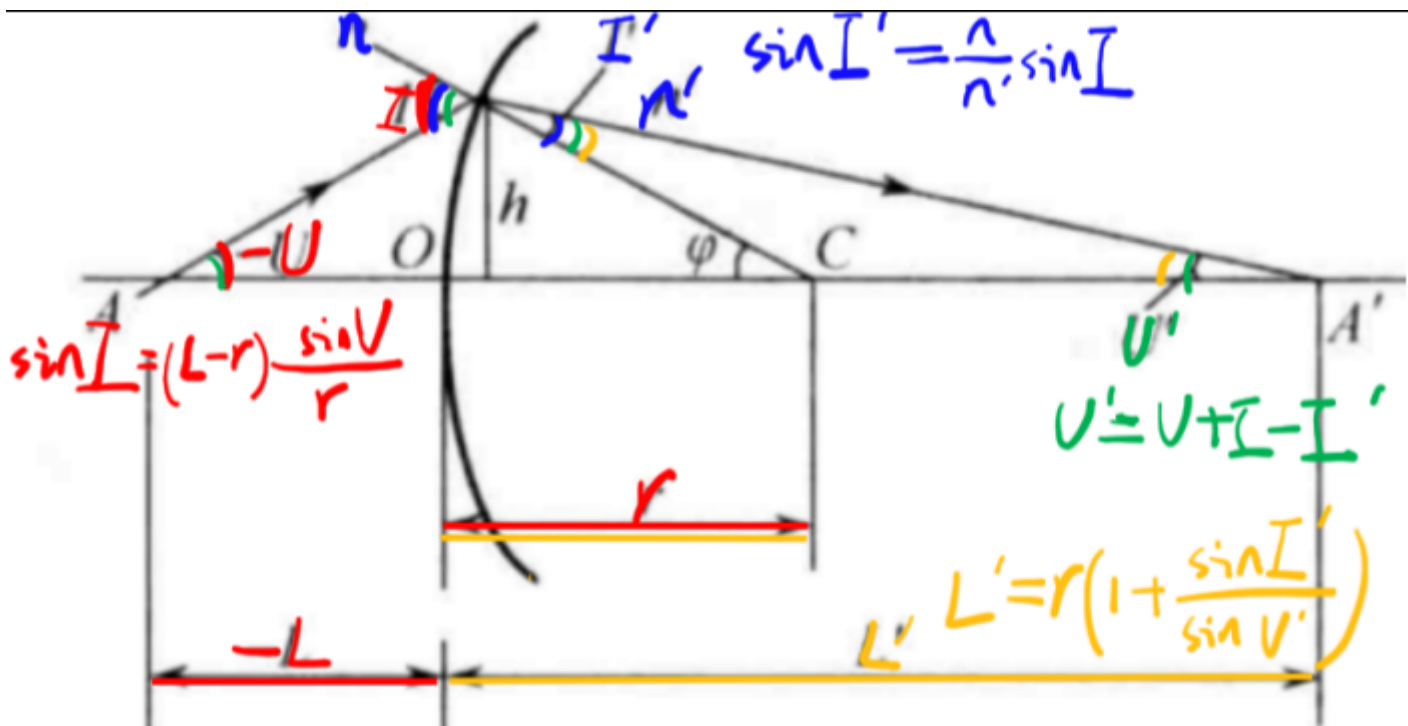
4. **Law of Refraction:** incident angle  $\sim$  normal  $\sim$  refracted angle
    1.  $n \sin I = n' \sin I'$
    2. incident, refracted, normal  $\in$  same plane
- **Fermat' principle:** light takes the path that requires the least time
  - **Malus' principle:** In homogeneous&isotropic matter, Light  $\perp$  WaveFront

## 1.2 Light path calculation



Calculate light path

- $\frac{\sin I}{r-L} = \frac{\sin -U}{r}$
- $n' \sin I' = n \sin I$
- $U' + I' = U + I$
- $\frac{\sin I'}{L'-r} = \frac{\sin U'}{r}$



Paraxial rays

- $i = \frac{l-r}{r}u$
- $i' = \frac{n}{n'}i$
- $u' = u + i - i'$
- $l' = r(1 + \frac{i'}{u'})$
- $l'u' = lu = h$
- $\frac{n'}{l'} - \frac{n}{l} = \frac{n'-n}{r}$

## 1.3 Imaging

### Magnification

- Transverse magnification:  $\beta = \frac{y'}{y} = \frac{n}{n'} \frac{l'}{l}$
- Axial magnification:  $\alpha = \frac{dl'}{dl} = \frac{n'}{n} \beta^2$
- Angular magnification:  $\gamma = \frac{\tan U'}{\tan U} = \frac{n}{n'} \frac{1}{\beta}$
- $\alpha\gamma = \beta$
- Lagrange's invariant:  $J = n'u'y' = nuy$
- Reflection:  $n = -n', \frac{1}{l'} + \frac{1}{l} = \frac{2}{r}$

### Image characteristics

- larger | smaller
- upright | inverted
- real | virtual

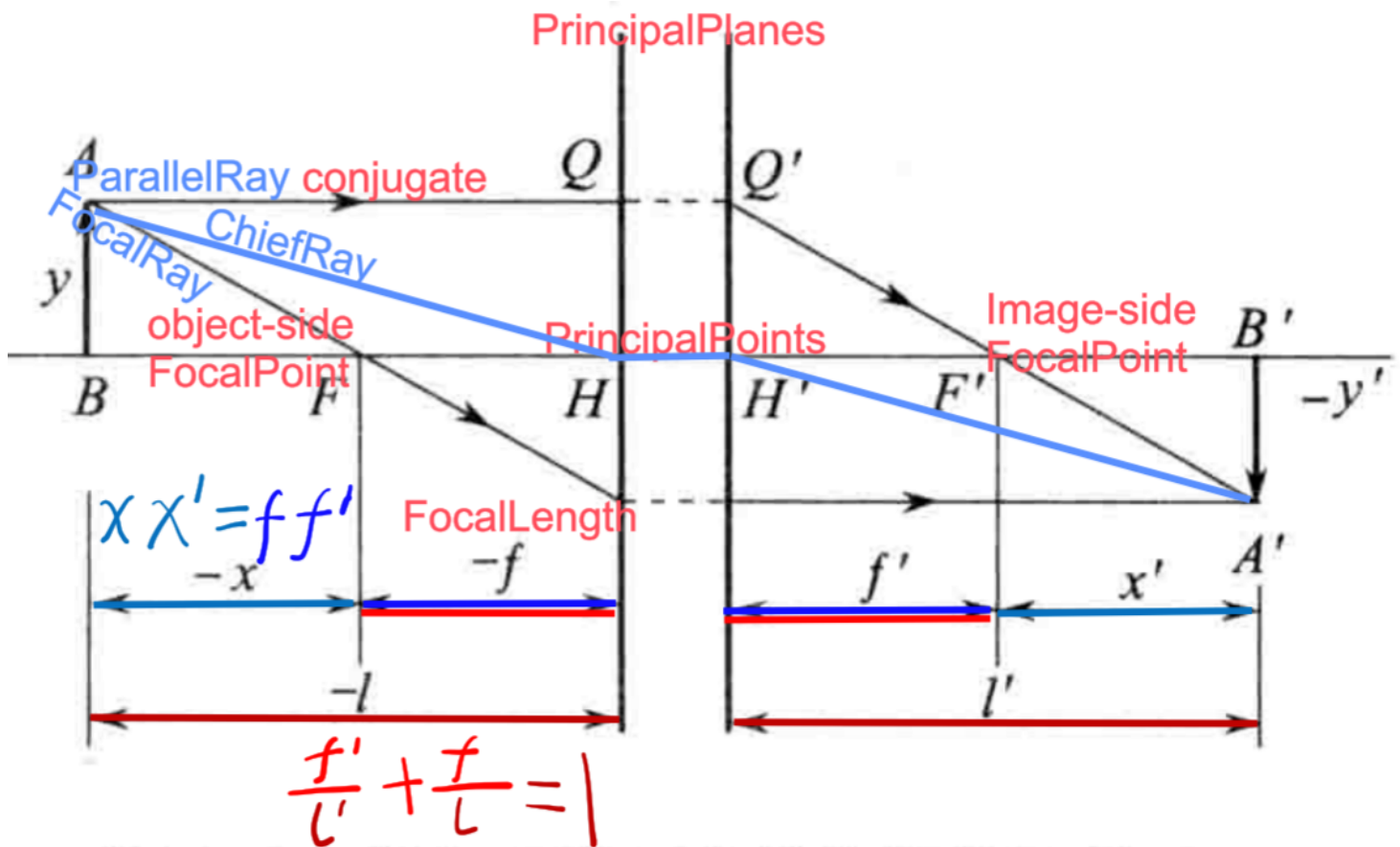
## homework

1. A staff 2m long when held erect casts a shadow 3.4 m long, while a building's shadow is 170 m long. How tall is the building?
  1. similar triangles:  $\frac{x}{170} = \frac{2}{3.4}, x = 100m$
2. Light from a water medium with  $n=1.33$  is incident upon a water-glass interface at an angle of  $45^\circ$ . The glass index is 1.50. What angle does the light make with the normal in the glass?
  1. Law of refraction:  $1.33 \sin 45^\circ = 1.5 \sin \theta, \theta = 38.83^\circ$
3. A goldfish swims 10cm from the side of a spherical bowl of water of radius 20cm. Where does the fish appear to be? Does it appear larger or smaller?
  1.  $\frac{n'}{l'} - \frac{n}{l} = \frac{n'-n}{r}, l' = -8.58cm$
  2.  $\beta = \frac{n}{n'} \frac{l'}{l}, \beta = 1.14 > 1, \text{bigger}$
4. An object is located 2cm to the left of convex end of a glass rod which has a radius of curvature of 1cm. The index of refraction of the glass is  $n=1.5$ . Find the image distance.

1.  $\frac{n'}{l'_1} - \frac{n}{l_1} = \frac{n'-n}{r_1}, l'_1 = -\infty$
2.  $\frac{n}{l'_2} - \frac{n'}{l_2} = \frac{n-n'}{r_2}, l'_2 = 2cm$

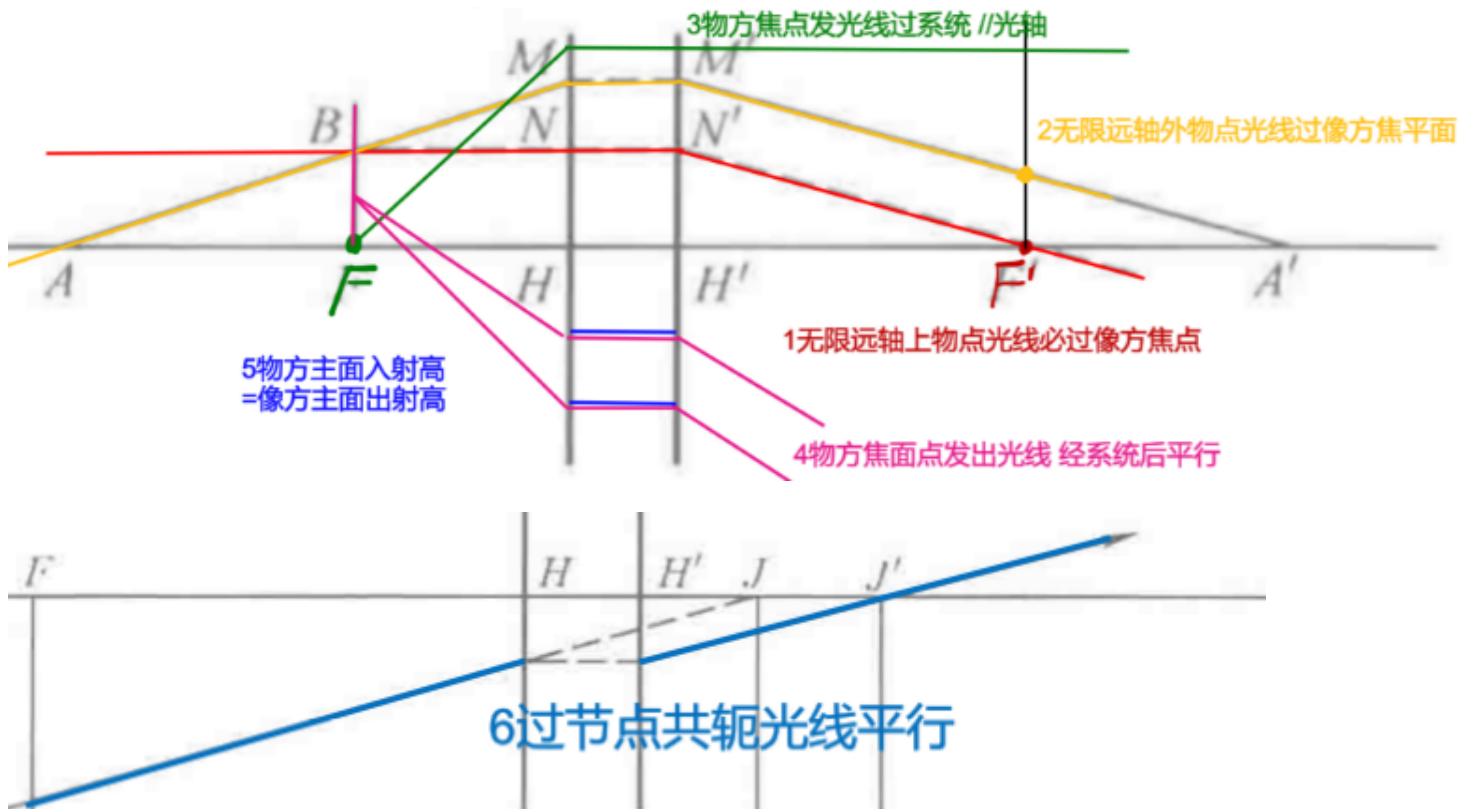
## 2. Perfect optical system

### 2.1 Concept



- Principal plane: conjugate planes,  $\beta = \pm 1$
- Focal plane
  - $\frac{f'}{f} = -\frac{n'}{n}$
- Nodal plane: conjugate planes,  $\gamma = \pm 1$ 
  - $x_J = f', x'_J = f$
- Nodal points: no refraction occurs
  - $n=n'$ , NodalPoints=PrincipalPoints
  - $n \neq n'$ , NodalPoints  $\rightarrow$  bigger  $n$  side

### 2.2 Graphical construction



## 2.3 Image position fomula

- Newton's equation:  $xx' = ff'$
- Gauss' equation:  $\frac{f'}{l'} + \frac{f}{l} = 1$ , same matter  $\frac{1}{l'} - \frac{1}{l} = \frac{1}{f'}$

## 2.4 Magnificaation

- $\beta = -\frac{f}{x} = -\frac{x'}{f'}$
- $\alpha = -\frac{x'}{x} = -\frac{f'}{f}\beta^2$
- $\gamma = \frac{n}{n'}\frac{1}{\beta}$
- $\alpha\gamma = \beta$

## 2.5 Combination

$$f = \frac{f_1 f_2}{\Delta}$$

$$l_F = f \left( 1 + \frac{d}{f_2} \right)$$

$$l_H = f \frac{d}{f_2}$$

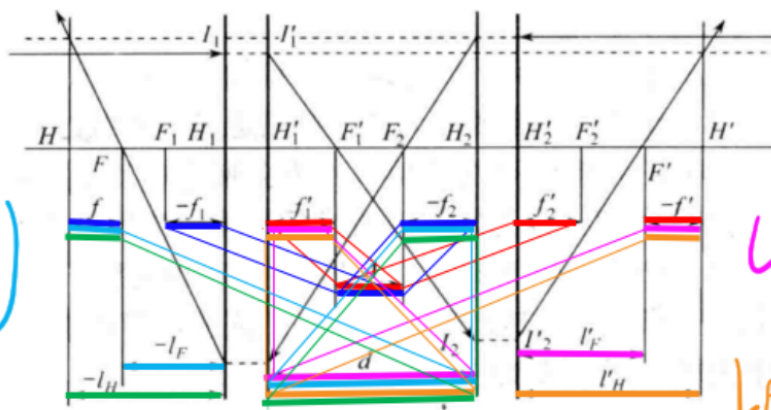


图 2-3 两光组组合

$$f' = - \frac{f_1' f_2'}{\Delta}$$

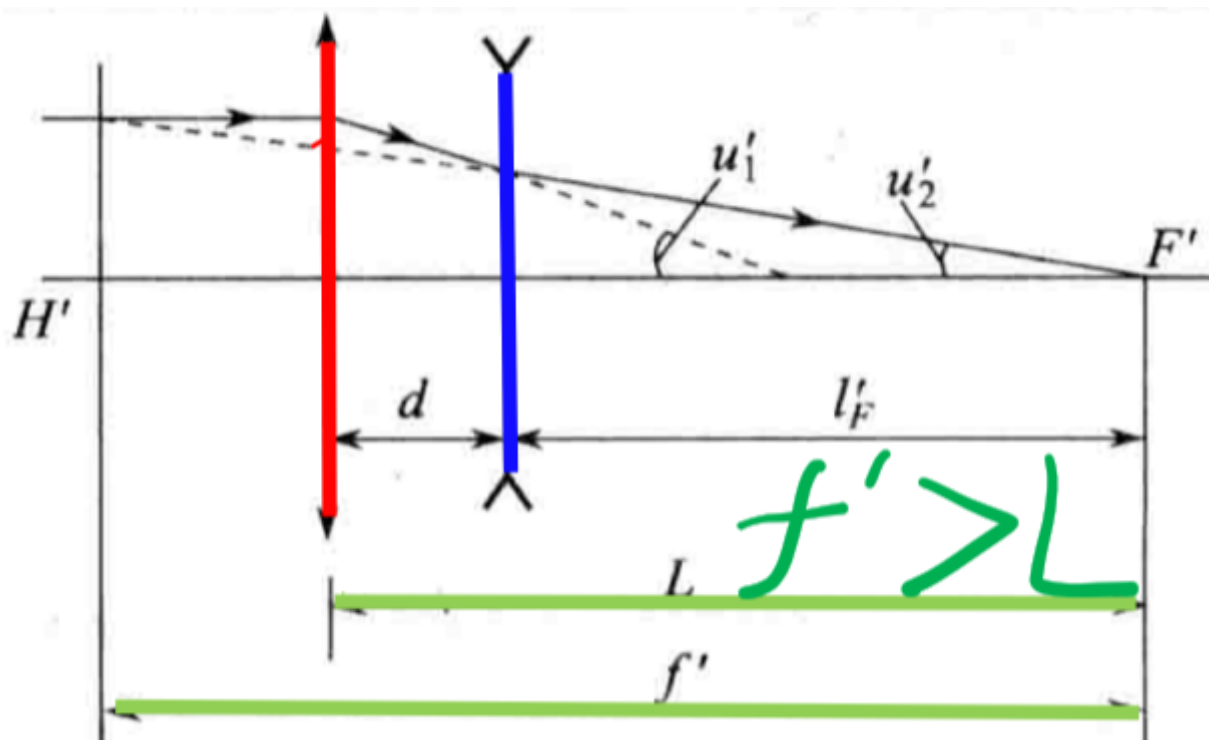
$$l_F' = f' \left( 1 - \frac{d}{f_1'} \right)$$

$$l_H' = -f' \frac{d}{f_1'}$$

- $f' = - \frac{f_1' f_2'}{\Delta} \sim f = \frac{f_1 f_2}{\Delta}$
- $l_F' = f' \left( 1 - \frac{d}{f_1'} \right) \sim l_F = f \left( 1 + \frac{d}{f_2} \right)$
- $l_H' = -f' \frac{d}{f_1'} \sim l_H = f \frac{d}{f_2}$
- focal power:  $\Phi = \Phi_1 + \Phi_2 - d \Phi_1 \Phi_2$

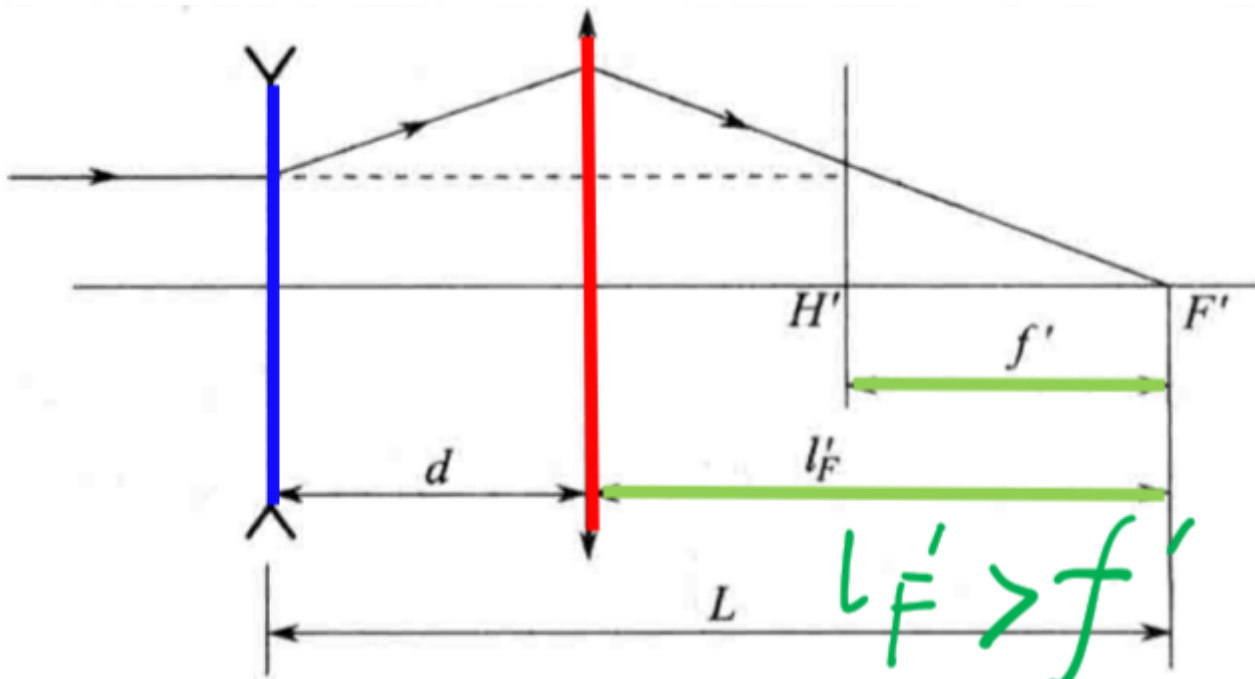
## 2.6 examples

Telephoto group



- used for long focal length lenses to reduce mechanical dimensions

Inverse telephoto group



- used for short focal length & long work length(microscope objective)

## 2.7 Lens

Convex lens not always are plus lens;

Concave lens not always are negative lens.

in air:  $f' = \frac{nr_1r_2}{(n-1)[n(r_2-r_1)+(n-1)d]}$

thin lens  $d \rightarrow 0$ :  $\Phi = (n-1)(\rho_1 - \rho_2)$

## homework

1. An object 1cm high is 30cm in front of a thin lens with a focal length of 10cm. Where is the image?  
Verify your answer by graphical construction of the image.  
1.  $\frac{1}{l'} - \frac{1}{l} = \frac{1}{f'}, l' = 15cm$
2. A lens is known to have a focal length of 30cm in air. An object is placed 50cm to the left of the lens. Locate the image.  
1.  $\frac{1}{l'} - \frac{1}{l} = \frac{1}{f'}, l' = 75cm$
3. The object is a transparent cube, 4mm across, placed 60cm in front of a lens of 20cm focal length. Calculate the transverse and axial magnification and describe what the image looks like?  
1.



**Solution.** From Gauss's equation, we find for the rear surface of the cube (the face closer to the lens)

$$\text{that, } l' = \frac{f l_1}{f' + l_1} = \frac{20 \times (-60)}{20 + (-60)} = 30 \text{ cm}$$

For the front surface (the face farther away from the lens),

$$l'_2 = \frac{f l_2}{f' + l_2} = \frac{20 \times (-60.4)}{20 + (-60.4)} \doteq 29.90099 \text{ cm}$$

The transverse magnification for the rear surface is  $\beta = \frac{+30}{-60} = -0.5$

But the axial magnification is  $\alpha = \frac{\Delta l'}{\Delta l} = \frac{30 - 29.90099}{-60 - (-60.4)} \doteq 0.247525$

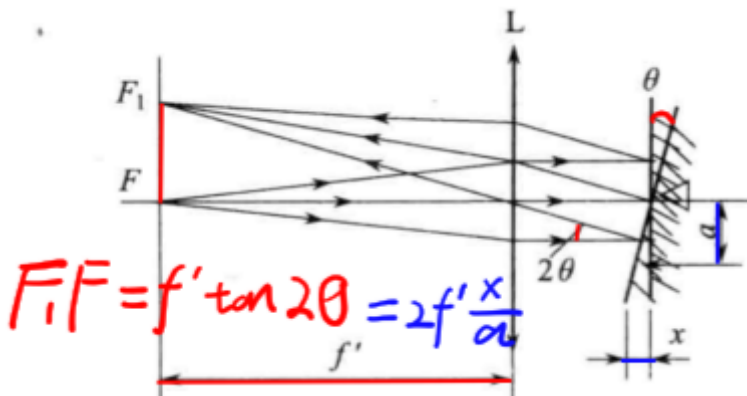
Since  $\alpha \neq \beta$ , so the cube doesn't look like a cube anymore.

4. A biconvex lens is made out of glass of  $n=1.52$ . If one surface has twice the radius of curvature of the other, and if the focal length is 5 cm, what are the two radius?
1.  $r_1 = -2r_2, \Phi = (n - 1)(\rho_1 - \rho_2)$

### 3.Plane&Plane system

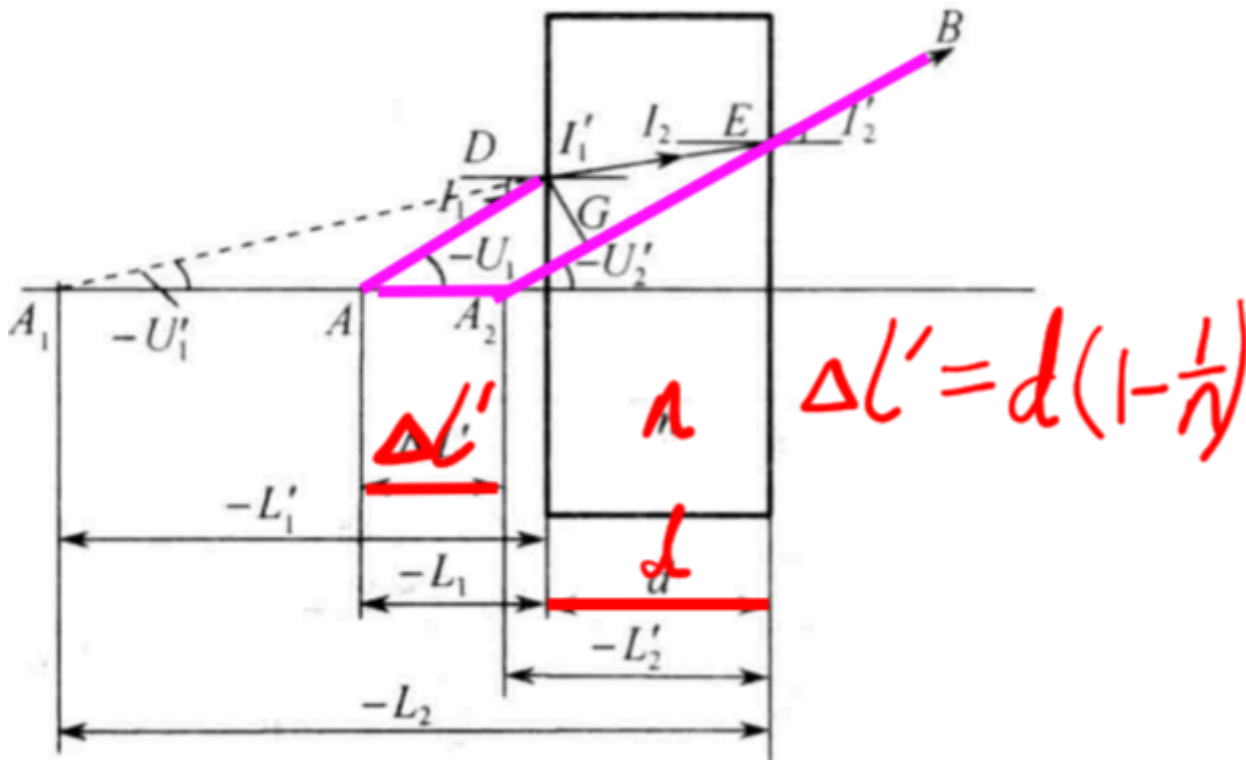
### 3.1 Plane mirror

- the only simple optical element can produce a perfect image
- object distance = image distance, object size = image size, real-virtual in contrast, rotate in contrast
- mirror rotate  $\alpha$ , reflected light rotate  $2\alpha$



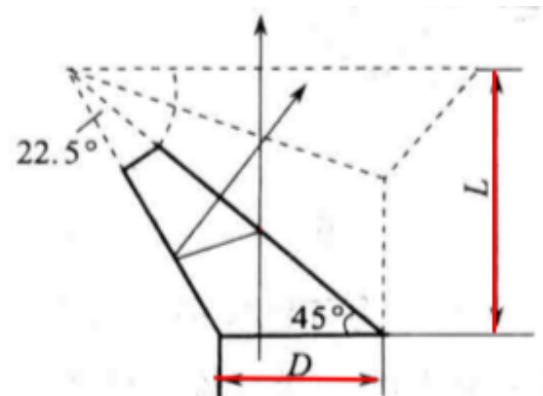
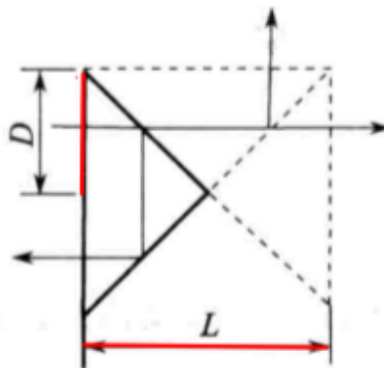
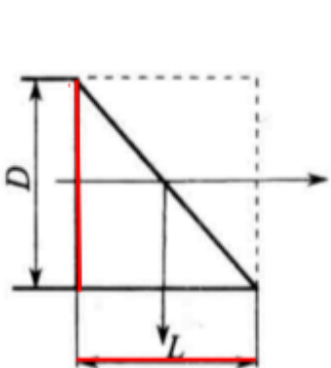
## 3.2 Parallel plate

- direction, size is constant
- Axial displacement:  $\Delta l' = d(1 - \frac{1}{n})$



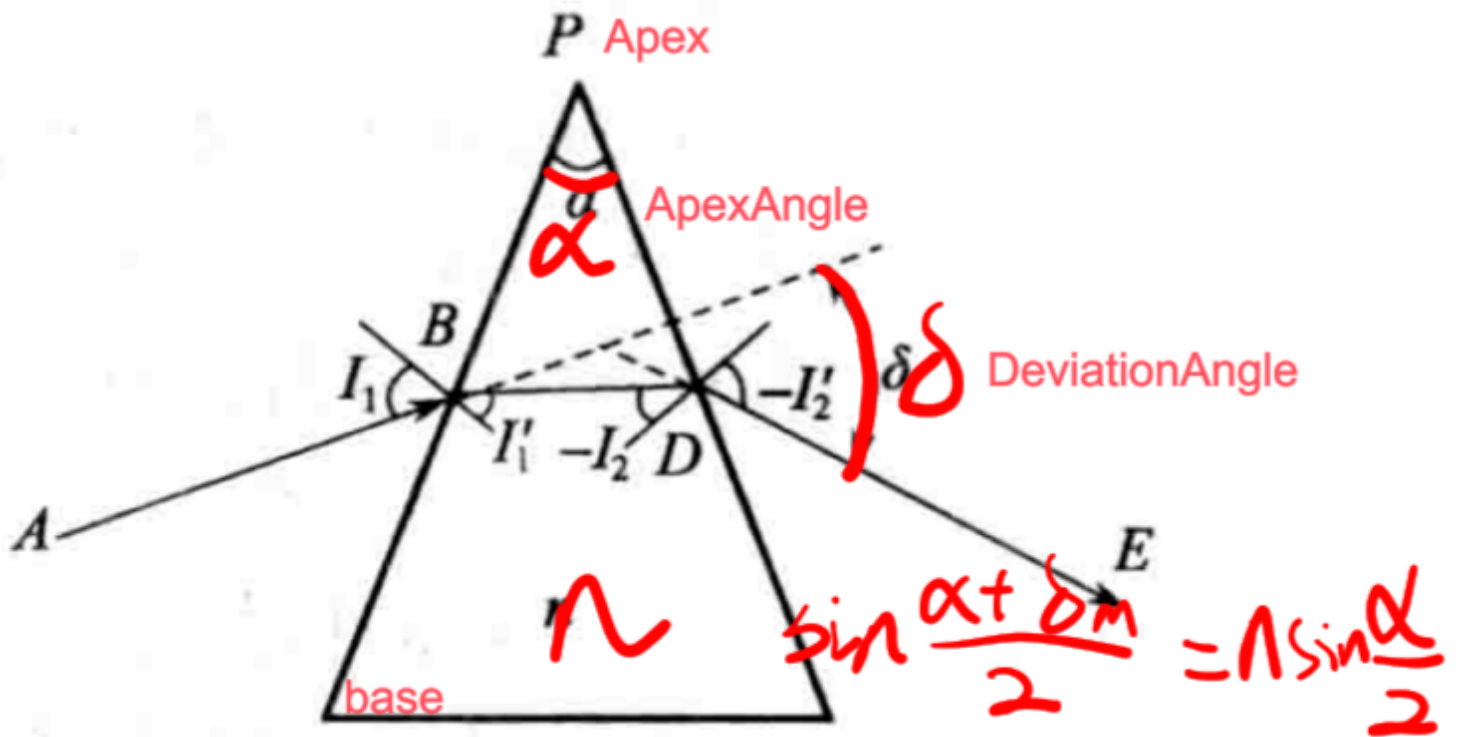
## 3.3 Reflecting prism

- dispersing prisms
- reflecting prisms
  - right-angle prisms
  - roof(Porro) prisms
  - Dove(erecting) prisms
  - pentagonal prisms

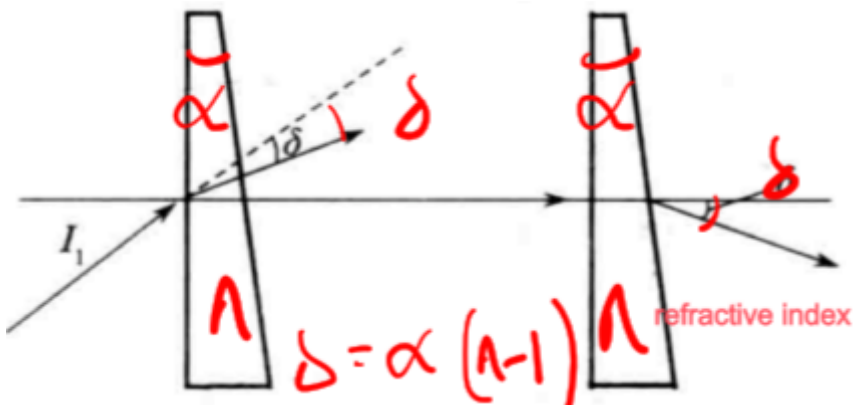


- $L = KD$  D:optical dia, L:plate thickness, K:structure constant
- Chromatic dispersion:  $\lambda \uparrow n \downarrow$

### 3.4 Dispersing prisms & optical wedge



- $\sin \frac{\alpha + \delta_m}{2} = n \sin \frac{\alpha}{2}$



- $\delta = \alpha(n - 1)$
- measure angle:  $\delta = 2\alpha(n - 1) \cos \varphi$
- measure displacement:  $\Delta y = \Delta z \delta$

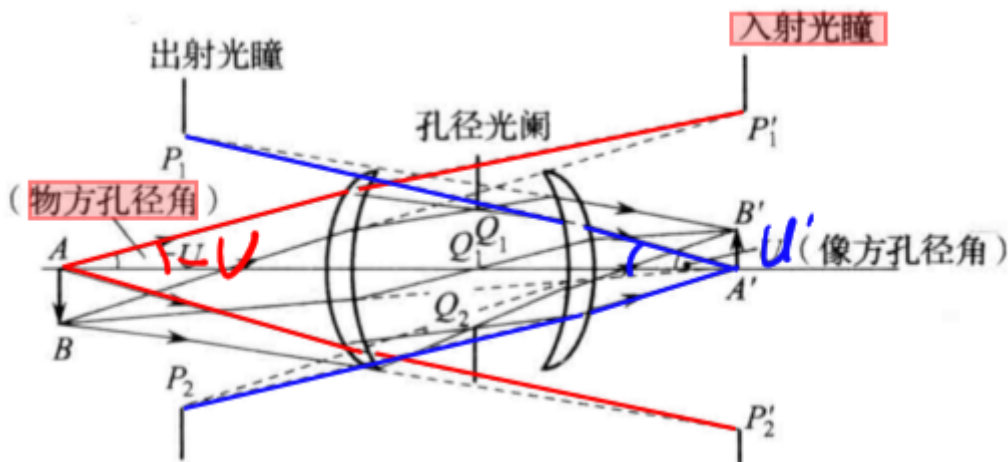
### 3.5 Optical material

- transmission material
  - **optical glass, crytical, plastic**
  - Abbe constant  $\uparrow$  chromatic dispersion  $\downarrow$
  - CrownGlass-low dispersion, FlintGlass-high dispersion
- reflecting material
  - no chromatic dispersion

## 4. Beam limit

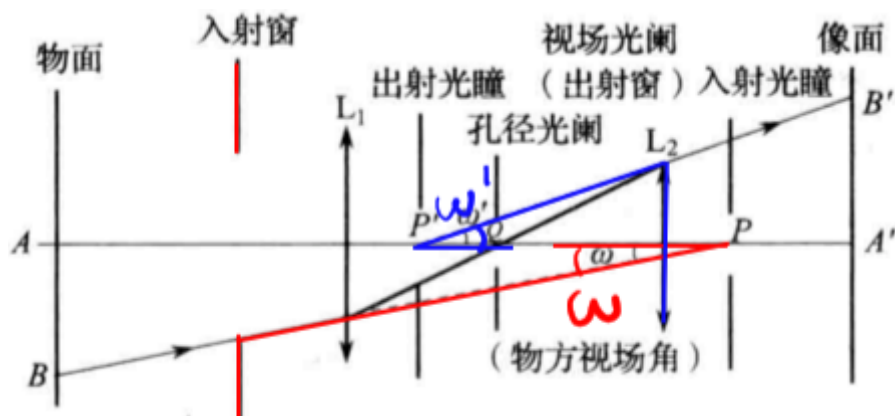
### 4.1 Aperture stop

- Aperture stop: limit the aperture angle
  - Entrance pupil: Aperture stop image according to front optical group
  - Exit pupil: Aperture stop image according to behind optical group
  - Square aperture angle of object: Axis object point -- Entrance pupil  $\angle$  optical axis
  - Square aperture angle of image: Axis image point -- Exit pupil  $\angle$  optical axis



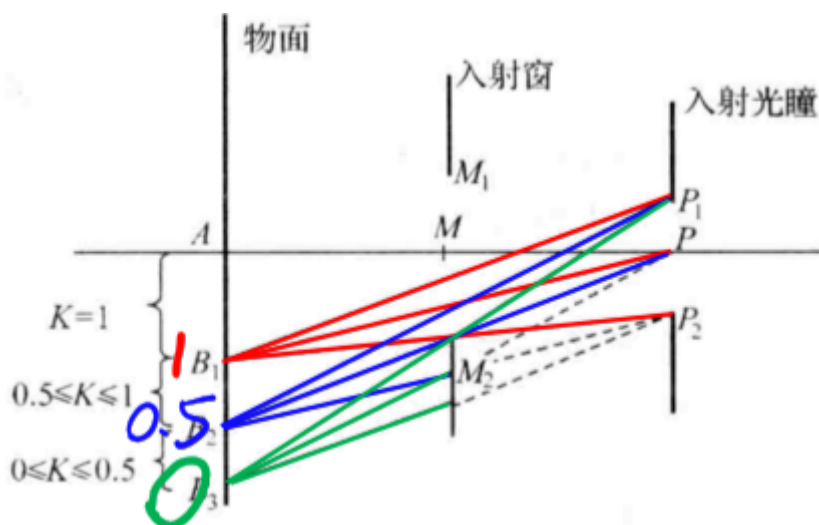
### 4.2 Field stop

- Field stop: limit imaging range
  - Entrance window: Field stop image according to front optical group
  - Exit window: Field stop image according to behind optical group
  - Object square field angle: Entrance pupil center -- Entrance window  $\angle$  optical axis
  - Image square field angle: Exit pupil center -- Exit window  $\angle$  optical axis

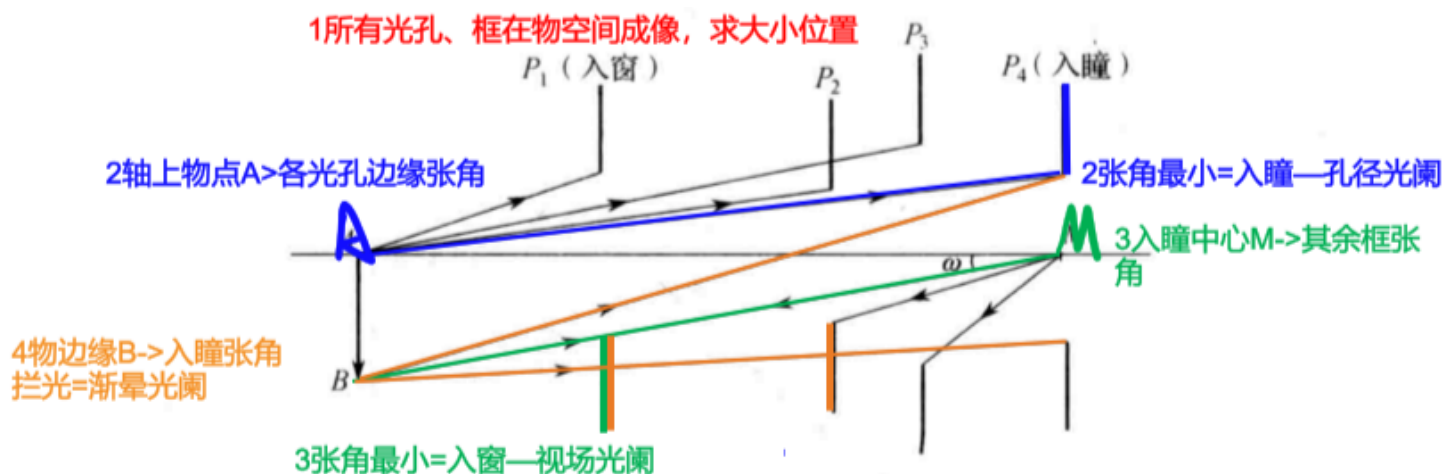


## 4.3 Vignetting

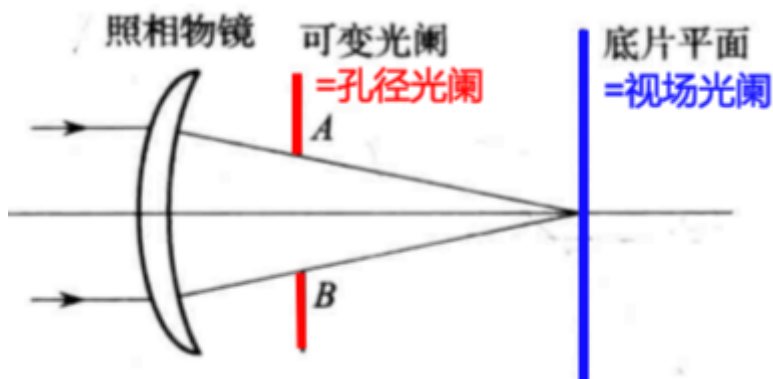
- Vignetting: light full of entrance pupil stopped by others aperture
- Vignetting factor:  $K_\omega = \frac{D_\omega}{D}$



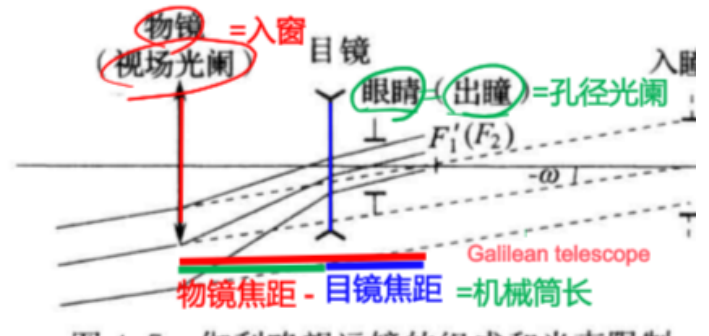
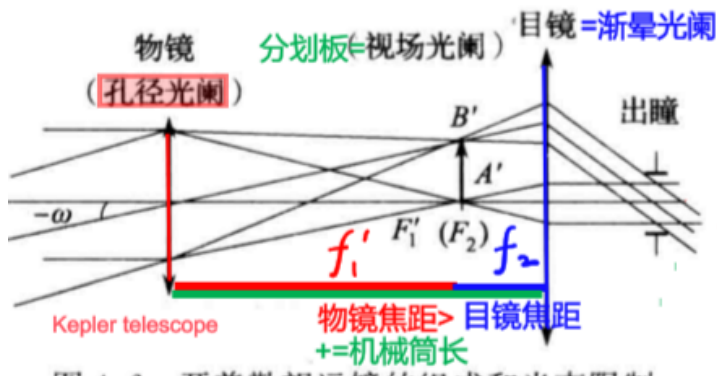
## 4.4 distinguish



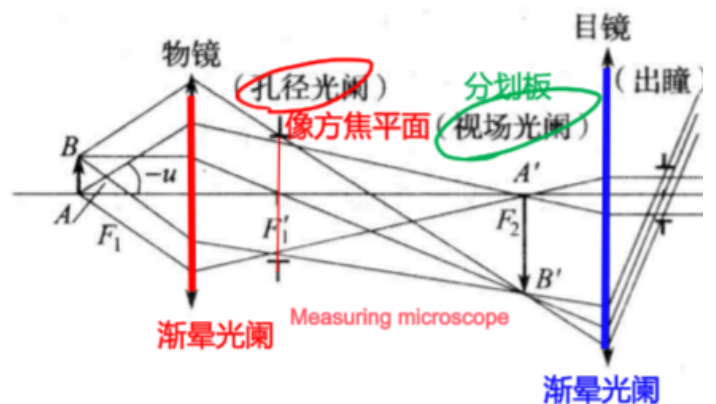
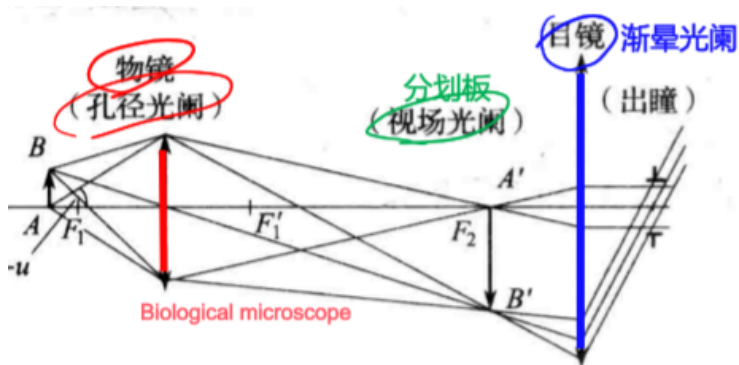
## 4.5 Photographic system



## 4.6 Telescopic system



## 4.7 Microscope system



Object side far center light path: avoid measurement error caused by inaccurate focusing

## 4.8 Field lens

- compress  $h$  at which incoming ray is projected in the subsequent light set

- reduce optical aperture of subsequent optical groups
- no affect to image
- new pupil connection requirements

## 4.9 Depth of field

- $\Delta = \Delta_1 + \Delta_2$
- Vision of the depth of field  $\Delta_1 = \frac{pz}{2a-z}$
- Close shot the depth of field  $\Delta_2 = \frac{pz}{2a+z}$
- disc of confusion  $z, z'$

## homework

1. A stop 8mm in diameter is placed halfway between an extended object and a large-diameter lens of 9cm focal length. The lens projects an image of the object onto a screen 14cm away. What is the diameter of the exit pupil?

1. **Solution:**

As shown in the figure.

First, from the known focal length and the image distance, we can find the object distance:

$$\because \frac{1}{l'} - \frac{1}{l} = \frac{1}{f'}, \quad l' = 14 \text{ cm}, \text{ and } f' = 9 \text{ cm}, \Rightarrow l = -25.2 \text{ cm}$$

The stop is half of  $l$ , so  $l_s = -12.6 \text{ cm}$

The exit pupil is the image of the stop, according to

$$\frac{1}{l'_s} - \frac{1}{l_s} = \frac{1}{f'}, \text{ we can get } l'_s = 31.5 \text{ cm}$$

$$\because \beta = \frac{D_{ex}}{D_{stop}} = \frac{l'_s}{l_s} = \frac{31.5}{-12.6} = -2.5$$

$$\therefore |D_{ex}| = |\beta D_{stop}| = 2.5 \times 0.8 = 2 \text{ cm}$$

**So**, the exit pupil is located at 31.5 cm to the right side of the lens and its diameter is 2 cm.

2. Two lenses, a lens of 12.5cm focal length and a minus lens of unknown power, are mounted coaxially and 8 cm apart. The system is afocal, that is light entering the system parallel at one

side emerges parallel at the other. If a stop 15mm in diameter is placed halfway between the lenses:

1. Where is the entrance pupil?
2. Where is the exit pupil?
3. What are their diameters?

**Solution.** As shown in the figure. For the system to be afocal, the focal points of the two lenses must coincide. Since  $f'_1 = 12.5$  cm , and the two lenses are 8cm apart, so  $f'_2 = -4.5$  cm . The entrance pupil is the image of stop formed by the first lens.

According to Gauss's equation,  $\frac{1}{l'_1} - \frac{1}{l_1} = \frac{1}{f'_1}$

and  $l'_1 = 4$  cm ,  $f'_1 = 12.5$  cm . We get

$$l_1 = \frac{f'_1 l'_1}{f'_1 - l'_1} = \frac{12.5 \times 4}{8.5} = 5.88 \text{ cm}$$

**So** the entrance pupil is 5.88 cm behind the first lens.

$$D_{\text{entrance}} = \frac{D_{\text{stop}}}{\beta_1} = \frac{D_{\text{stop}}}{l'_1/l_1} = \frac{15}{4/5.88} = 22.05 \text{ cm}$$

**So** the diameter of the entrance pupil is 22.05 mm

Similarly, the exit pupil's location and diameter can be calculated as following

$$l'_2 = \frac{f'_2 l_2}{f'_2 + l_2} = \frac{(-4.5) \times (-4)}{(-4.5) + (-4)} = -\frac{18}{8.5} = -2.12 \text{ cm}$$

$$D_{\text{exit}} = |\beta_2| D_{\text{stop}} = \frac{2.12}{4} \times 15 = 7.95 \text{ cm}$$

**So** the exit pupil is located 2.12 cm before the second lens and its diameter is 7.95 mm