1. Electrostatics

Electrostatics, as the name implies, is the branch of Physics, which deals with the study of static electric charges or charges at rest. In this chapter, we shall study only those electric phenomena which are related to charges at rest. However, those electric phenomena which are related to charge in motion find no place in electrostatics. The charge in electrostatics plays similar role to that of mass in gravitation. It means the charges in an electrostatic field are analogous to masses in a gravitational field. These charges have forces acting on them and work done in moving a charge against the force of other charge store as potential energy. The ideas are widely used in many branches of electricity and in the theory of atom.

1.1 Frictional electricity (Electrostatics)

The first observation of an electric effect (force) was made in 600 B.C. by Thales of Miletus, a Greek Philosopher. He observed that, when a piece of amber is rubbed with fur, it acquires the property of attracting light objects like bits of paper. In the 1600 AD, William Gilbert discovered that, glass, ebonite etc, also exhibit this attracting property, when rubbed with suitable materials.

The substances which show the electric attractive property on rubbing are said to be 'electrified' or charged or charged with electricity. The terms like electrified, electricity are derived from the Greek word elektron, meaning amber. The electricity produced by friction is called frictional electricity.

If the charges in a body do not move, then, the frictional electricity is also known as Static Electricity or electrostatics.

1.1.1 Two kinds of charges

(i) If a glass rod is rubbed with a silk cloth, it acquires positive charge while the silk cloth acquires an equal amount of negative charge.

(ii) If an ebonite rod is rubbed with fur, it becomes negatively charged, while the fur acquires equal amount of positive charge. This classification of positive and negative charges were termed by American scientist, Benjamin Franklin.

Thus, charging a rod by rubbing does not create electricity, but simply transfers or redistributes the charges in a material.

1.1.2 Like charges repel and unlike charges attract each other – experimental verification.

A charged glass rod is suspended by a silk thread, such that it swings horizontally. Now another charged glass rod is brought near the end of the suspended glass rod. It is found that the ends of the two rods repel each other (Fig 1.1). However, if a charged ebonite rod is brought near the end of the suspended rod, the two rods attract each other (Fig 1.2). The above experiment shows that like charges repel and unlike charges attract each other.

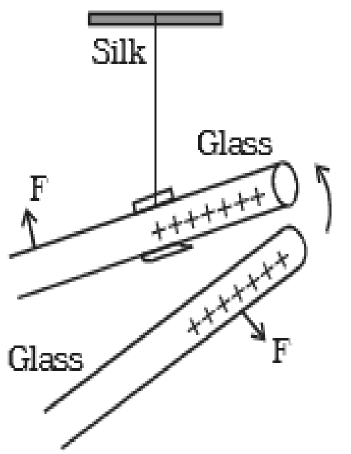


Fig. 1.1 Two charged rods of same sign

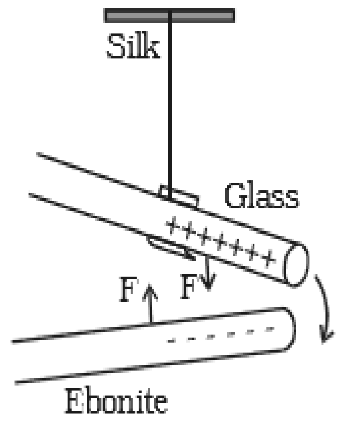


Fig 1.2 Two charged rods of opposite sign

The property of attraction and repulsion between charged bodies have many applications such as electrostatic paint spraying, powder coating, fly–ash collection in chimneys, ink–jet printing and photostat copying (Xerox) etc.

1.1.3 Conductors and Insulators

According to the electrostatic behaviour, materials are divided into two categories : conductors and insulators (dielectrics). Bodies which allow the charges to pass through are called conductors. e.g. metals, human body, Earth etc. Bodies which do not allow the charges to pass through are called insulators. e.g. glass, mica, ebonite, plastic etc.

1.1.4 Basic properties of electric charge

(i) Quantisation of electric charge

The fundamental unit of electric charge (e) is the charge carried by the electron and its unit is coulomb. e has the magnitude 1.6×10^{-19} C.

In nature, the electric charge of any system is always an integral multiple of the least amount of charge. It means that the quantity can take only one of the discrete set of values. The charge, $q = ne$ where n is an integer.

(ii) Conservation of electric charge

Electric charges can neither be created nor destroyed. According to the law of conservation of electric charge, the total charge in an isolated system always remains constant. But the charges can be transferred from one part of the system to another, such that the total charge always remains conserved.

Example (1): Uranium (${}_{92}U^{238}$) can
decay by emitting an alpha particle (${}_{2}He^{4}$ nucleus) and transforming to thorium (${}_{90}Th^{234}$).

$${}_{92}U^{238} \dashrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$$

Total charge before decay = +92e, total charge after decay = 90e + 2e. Hence, the total charge is conserved. i.e. it remains constant.

Example(2): Pair production in which gamma photon (γ) materializes into electron (${}_{-1}^{0}e$) and its anti-particle, positron (${}_{+1}^{0}e$).

$$\gamma \xrightarrow{yields} {}_{-1}^{0}e + {}_{+1}^{0}e$$

Total charge before decay = 0, total charge after decay = -1e + 1e = 0. Hence, the total charge is conserved. i.e. it remains constant.

The reverse phenomenon of pair production which is known as matter annihilation also obeys the law of conservation of charge.

(iii) Additive nature of charge

The total electric charge of a system is equal to the algebraic sum of electric charges located in the system. For example, if two charged bodies of charges +2q, −5q are brought in contact, the total charge of the system is −3q.

(iv) Bitupaz (two) nature of Charge

Although one of the intrinsic property of matter, mass exists in positive nature only. However, Charge another intrinsic property of matter exists in both positive and negative nature.

(v) Invariance of charge with motion

Unlike mass, the magnitude of charge does not change with motion. Note that charge at rest produce electric field only but produce both electric and magnetic field in motion.

Measurement of force:

Two charged objects exert electric force on each other, repulsive in case of like charges and attractive in case of unlike charges. So, a positively charged object will exert a repulsive force upon second positively charged object and push the two objects apart. Similarly, negatively charged object will exert a repulsive force upon second negatively charged object and push the two objects apart. However, a positively charged object will exert an attractive force upon second negatively charged object. Now, the question arises by how much magnitude two charged objects exert force on each other?

The response to this question lies in one of the famous law of electrostatics known as Coulomb's law. Would you like to know it?

Let us study now coulombs Law.

1.1.5 Coulomb's law

The force between two charged objects was studied by Augustine de-Coulomb in 1785.

Coulomb's law states that the force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of distance between them. The direction of forces is along the line joining the two point charges.

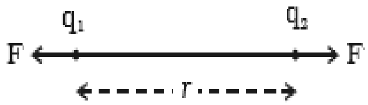


Fig 1.3a Coulomb forces

Let q_1 and q_2 be two point charges placed in air or vacuum at a distance r apart (Fig. 1.3a). Then, according to Coulomb's law,

$$F \propto \frac{q_1 q_2}{r^2} \quad \text{or} \quad F = k\,\frac{q_1 q_2}{r^2}$$

where k is a constant of proportionality. In air or vacuum, $k = \frac{1}{4\pi\varepsilon_o}$, where ε_o is the permittivity of free space (i.e., vacuum) and the value of ε_o is $8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$.

$$F = \frac{1}{4\pi\varepsilon_o}\,\frac{q_1 q_2}{r^2} \qquad \text{...(1)}$$

and $$\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9\ N\ m^2\ C^{-2}$$

In the above equation, if $q_1 = q_2 = 1C$ and $r = 1m$ then,

$$F = (9 \times 10^9)\,\frac{1\times 1}{1^2} = 9 \times 10^9\ N$$

One Coulomb is defined as the quantity of charge, which when placed at a distance of 1 metre in air or vacuum from an equal and similar charge, experiences a repulsive force of 9×10^9 N.

If the charges are situated in a medium of permittivity ε, then the magnitude of the force between them will be,

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \qquad \text{...(2)}$$

Dividing equation (1) by (2)

$$\frac{F}{F_m} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$$

The ratio $\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$, is called the relative permittivity or dielectric constant of the medium. The value of ε_r for air or vacuum is 1.

$$\therefore \quad \varepsilon = \varepsilon_0 \varepsilon_r$$

Since $F_m = \frac{F}{\varepsilon_r}$, the force between two point charges depends on the nature of the medium in which the two charges are situated.

Coulomb's law – vector form

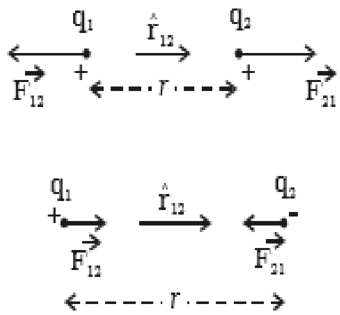


Fig 1.3b Coulomb's law in vector form

If $\vec{F}_{21}$ is the force exerted on charge q_2 by charge q_1 (Fig.1.3b),

$$\vec{F}_{21} = k\frac{q_1 q_2}{r_{12}^2}\ \hat{r}_{12}$$

where $\hat{r}_{12}$ is the unit vector from q_1 to q_2.

If $\vec{F}_{12}$ is the force exerted on q_1 due to q_2,

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{21}^2}\ \hat{r}_{21}$$

where $\hat{r}_{21}$ is the unit vector from q_2 to q_1.

[Both $\hat{r}_{21}$ and $\hat{r}_{12}$ have the same magnitude, and are oppositely directed]

$$\therefore \qquad \vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\ (-\hat{r}_{12})$$

$$\text{or} \qquad \vec{F}_{12} = -k\frac{q_1 q_2}{r_{12}^2}\ \hat{r}_{12}$$

$$\text{or} \qquad \vec{F}_{12} = -\vec{F}_{21}$$

So, the forces exerted by charges on each other are equal in magnitude and opposite in direction.

1.1.6 Principle of Superposition

The principle of superposition is to calculate the electric force experienced by a charge q_1 due to other charges q_2, q_3 .... q_n.

The total force on a given charge is the vector sum of the forces exerted on it due to all other charges.

The force on q_1 due to q_2

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$$

Similarly, force on q_1 due to q_3

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_3}{r_{31}^2}\hat{r}_{31}$$

The total force F_1 on the charge q_1 by all other charges is,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} \,......... + \vec{F}_{1n}$$

Therefore,

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + ...\frac{q_1 q_n}{r_{n1}^2}\hat{r}_{n1}\right]$$

1.2 Electric Field

Electric field due to a charge is the space around the test charge in which it experiences a force. The presence of an electric field around a charge cannot be detected unless another charge is brought towards it.

When a test charge q_o is placed near a charge q, which is the source of electric field, an electrostatic force F will act on the test charge.

Electric Field Intensity (E)

Electric field at a point is measured in terms of electric field intensity. Electric field intensity at a point, in an electric field is defined as the force experienced by a unit positive charge kept at that point.

It is a vector quantity. $|\vec{E}| = \frac{|\vec{F}|}{q_o}$. The unit of electric field intensity is $N\,C^{-1}$.

The electric field intensity is also referred as electric field strength or simply electric field. So, the force exerted by an electric field on a charge is $F = q_o E$.

1.2.1 Electric field due to a point charge

Let q be the point charge placed at O in air (Fig.1.4). A test charge q_o is placed at P at a distance r from q. According to Coulomb's law, the force acting on q_o due to q is

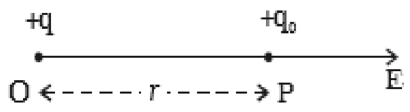


Fig 1.4 Electric field due to a point charge

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q\,q_o}{r^2}$$

The electric field at a point P is, by definition, the force per unit test charge.

$$E = \frac{F}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

The direction of E is along the line joining O and P, pointing away from q, if q is positive and towards q, if q is negative.

In vector notation $\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$, where $\hat{r}$ is a unit vector pointing away from q.

1.2.2 Electric field due to system of charges

If there are a number of stationary charges, the net electric field (intensity) at a point is the vector sum of the individual electric fields due to each charge.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \ldots\ldots \vec{E}_n$$

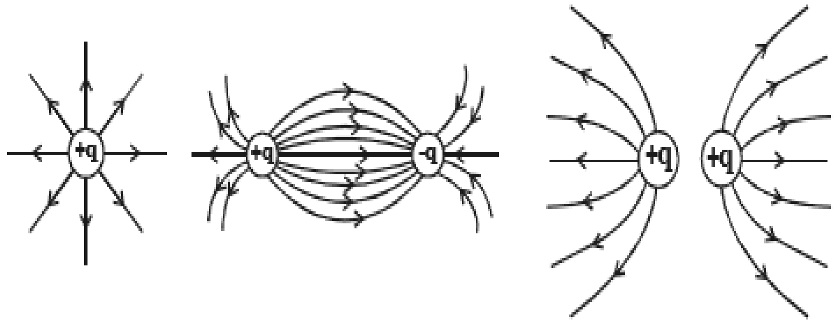
$$= \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + \ldots\ldots\ldots\right]$$

1.2.3 Electric lines of force

The concept of field lines was introduced by Michael Faraday as an aid in visualizing electric and magnetic fields.

Electric line of force is an imaginary straight or curved path along which a unit positive charge tends to move in an electric field.

The electric field due to simple arrangements of point charges are shown in Fig 1.5.



(a) *(b)* *(c)*

Isolated charge *Unlike charges* *Like charges*

Fig1.5 Lines of Forces

Properties of lines of forces:

(i) Lines of force start from positive charge and terminate at negative charge.

(ii) Lines of force never intersect.

(iii) The tangent to a line of force at any point gives the direction of the electric field (E) at that point.

(iv) The number of lines per unit area, through a plane at right angles to the lines, is proportional to the magnitude of E. This means that, where the lines of force are close together, E is large and where they are far apart, E is small.

(v) Each unit positive charge gives rise to $\frac{1}{\varepsilon_o}$ lines of force in free space. Hence number of lines of force originating from a point charge q is $N = \frac{q}{\varepsilon_o}$ in free space.

1.2.4 Electric dipole and electric dipole moment

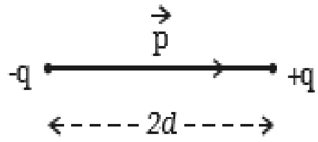


Fig 1.6 Electric dipole

Two equal and opposite charges separated by a very small distance constitute an electric dipole.

Water, ammonia, carbon–dioxide and chloroform molecules are some examples of permanent electric dipoles. These molecules behave like electric dipole, because the centres of positive and negative charge do not coincide and are separated by a small distance.

Two point charges +q and –q are kept at a distance 2d apart (Fig.1.6). The magnitude of the dipole moment is given by the product of the magnitude of the one of the charges and the distance between them.

$\therefore$ Electric dipole moment, $p = q2d$ or $2qd$.

It is a vector quantity and acts from –q to +q. The unit of dipole moment is C m.

1.2.5 Electric field due to an electric dipole at a point on its axial line.

AB is an electric dipole of two point charges –q and +q separated by a small distance $2d$ (Fig 1.7). P is a point along the axial line of the dipole at a distance r from the midpoint O of the electric dipole.

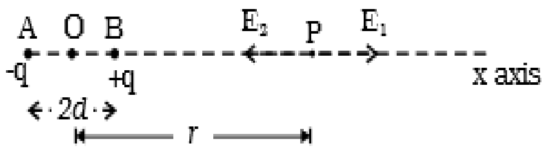


Fig 1.7 Electric field at a point on the axial line

The electric field at the point P due to +q placed at B is,

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-d)^2} \quad \text{(along BP)}$$

The electric field at the point P due to –q placed at A is,

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+d)^2} \text{ (along PA)}$$

E_1 and E_2 act in opposite directions.

Therefore, the magnitude of resultant electric field (E) acts in the direction of the vector with a greater magnitude. The resultant electric field at P is,

$$E = E_1 + (-E_2)$$

$$E = \left[\frac{1}{4\pi\varepsilon_o}\frac{q}{(r-d)^2} - \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+d)^2}\right] \text{along BP.}$$

$$E = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right] \text{ along BP}$$

$$E = \frac{q}{4\pi\varepsilon_o}\left[\frac{4rd}{(r^2-d^2)^2}\right] \text{along BP.}$$

If the point P is far away from the dipole, then d <<r

$$\therefore \quad E = \frac{q}{4\pi\varepsilon_o}\frac{4rd}{r^4} = \frac{q}{4\pi\varepsilon_o}\frac{4d}{r^3}$$

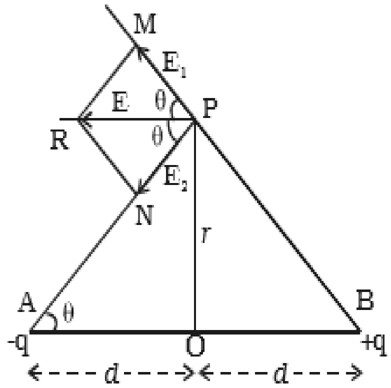
$$E = \frac{1}{4\pi\varepsilon_o}\frac{2p}{r^3} \text{ along BP.}$$

[∵ Electric dipole moment p = q x 2d]

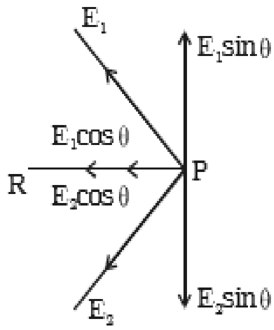
E acts in the direction of dipole moment.

1.2.6 Electric field due to an electric dipole at a point on the equatorial line.

Consider an electric dipole AB. Let $2d$ be the dipole distance and p be the dipole moment. P is a point on the equatorial line at a distance r from the midpoint O of the dipole (Fig 1.8a).



(a) Electric field at a point on equatorial line



(b) The components of the electric field

Fig 1.8

Electric field at a point P due to the charge $+q$ of the dipole,

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{BP^2} \text{ along BP.}$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+d^2)} \text{ along BP} \quad (\because BP^2 = OP^2 + OB^2)$$

Electric field (E_2) at a point P due to the charge $-q$ of the dipole

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{AP^2} \text{ along PA}$$

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+d^2)} \text{ along PA}$$

The magnitudes of E_1 and E_2 are equal. Resolving E_1 and E_2 into their horizontal and vertical components (Fig 1.8b), the vertical components $E_1 \sin\theta$ and $E_2 \sin\theta$ are equal and opposite, therefore they cancel each other.

The horizontal components $E_1 \cos\theta$ and $E_2 \cos\theta$ will get added along PR.

Resultant electric field at the point P due to the dipole is

$$E = E_1 \cos\theta + E_2 \cos\theta \text{ (along PR)}$$

$$= 2 E_1 \cos\theta \quad (\because E_1 = E_2)$$

$$E = \frac{1}{4\pi\varepsilon_o} \frac{q}{(r^2 + d^2)} \times 2\cos\theta$$

But $\cos\theta = \dfrac{d}{\sqrt{r^2 + d^2}}$

$$E = \frac{1}{4\pi\varepsilon_o} \frac{q}{(r^2 + d^2)} \times \frac{2d}{(r^2 + d^2)^{1/2}} = \frac{1}{4\pi\varepsilon_o} \frac{q2d}{(r^2 + d^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_o} \frac{p}{(r^2 + d^2)^{3/2}} \qquad (\because p = q2d)$$

For a dipole, d is very small when compared to r

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_o} \frac{p}{r^3}$$

The direction of E is along PR, parallel to the axis of the dipole and directed opposite to the direction of dipole moment.

1.2.7 Electric dipole in a uniform electric field

Consider a dipole AB of dipole moment p placed at an angle θ in an uniform electric field E (Fig.1.9). The charge +q experiences a force qE in the direction of the field. The charge –q experiences an equal force in the opposite direction. Thus the net force on the dipole is zero. The two equal and unlike

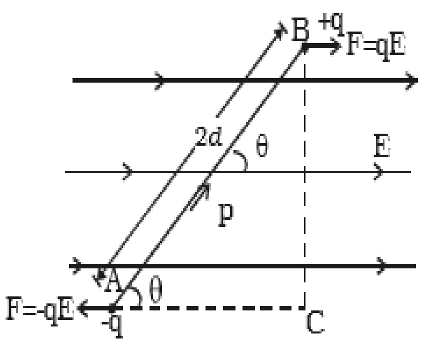


Fig 1.9 Dipole in a uniform field

parallel forces are not passing through the same point, resulting in a torque on the dipole, which tends to set the dipole in the direction of the electric field.

The magnitude of torque is,

τ = One of the forces x perpendicular distance between the forces

$= F \times 2d \sin\theta$

$= qE \times 2d \sin\theta = pE \sin\theta \qquad (\because q \times 2d = P)$

In vector notation, $\vec{\tau} = \vec{p} \times \vec{E}$

Note : If the dipole is placed in a non–uniform electric field at an angle θ, in addition to a torque, it also experiences a force.

1.2.8 Electric potential energy of an electric dipole in an electric field.

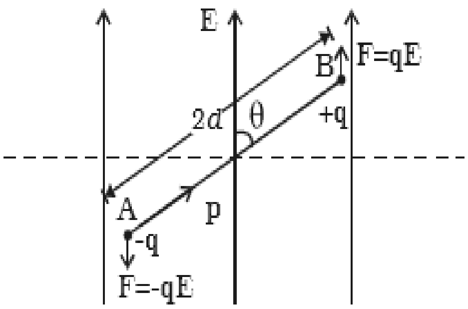


Fig 1.10 Electric potential energy of dipole

Electric potential energy of an electric dipole in an electrostatic field is the work done in rotating the dipole to the desired position in the field.

When an electric dipole of dipole moment p is at an angle θ with the electric field E, the torque on the dipole is

$\tau = pE \sin\theta$

Work done in rotating the dipole through $d\theta$,

$dw = \tau . d\theta$

$= pE \sin\theta . d\theta$

The total work done in rotating the dipole through an angle θ is

$W = \int dw$

$W = pE \int \sin\theta . d\theta = -pE \cos\theta$

This work done is the potential energy (U) of the dipole.

$\therefore \quad U = -pE \cos\theta$

When the dipole is aligned parallel to the field, $\theta = 0^o$

$\therefore U = -pE$

This shows that the dipole has a minimum potential energy when it is aligned with the field. A dipole in the electric field experiences a torque ($\vec{\tau} = \vec{p} \times \vec{E}$) which tends to align the dipole in the field direction, dissipating its potential energy in the form of heat to the surroundings.

Microwave oven

It is used to cook the food in a short time. When the oven is operated, the microwaves are generated, which in turn produce a non–uniform oscillating electric field. The water molecules in the food which are the electric dipoles are excited by an oscillating torque. Hence few bonds in the water molecules are broken, and heat energy is produced. This is used to cook food.

1.3 Electric potential

Let a charge +q be placed at a point O (Fig 1.11). A and B are two points, in the electric field. When a unit positive charge is moved from A to B against the electric force, work is done. This work is the potential difference between these two points. i.e., $dV = W_{A \to B}$.

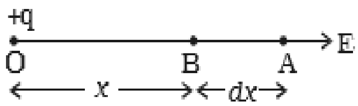


Fig1.11 Electric potential

The potential difference between two points in an electric field is defined as the amount of work done in moving a unit positive charge from one point to the other against the electric force.

The unit of potential difference is volt. The potential difference between two points is 1 volt if 1 joule of work is done in moving 1 Coulomb of charge from one point to another against the electric force.

The electric potential in an electric field at a point is defined as the amount of work done in moving a unit positive charge from infinity to that point against the electric forces.

Relation between electric field and potential

Let the small distance between A and B be dx. Work done in moving a unit positive charge from A to B is $dV = E.dx$.

The work has to be done against the force of repulsion in moving a unit positive charge towards the charge +q. Hence,

$$dV = -E.dx$$

$$E = \frac{-dV}{dx}$$

The change of potential with distance is known as potential gradient, hence the electric field is equal to the negative gradient of potential.

The negative sign indicates that the potential decreases in the direction of electric field. The unit of electric intensity can also be expressed as Vm^{-1}.

1.3.1 Electric potential at a point due to a point charge

Let +q be an isolated point charge situated in air at O. P is a point at a distance r from +q. Consider two points A and B at distances x and $x + dx$ from the point O (Fig.1.12).

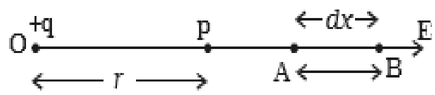


Fig 1.12 Electric potential due to a point charge

The potential difference between A and B is,

$$dV = -E\,dx$$

The force experienced by a unit positive charge placed at A is

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{x^2}$$

$$\therefore \quad dV = -\frac{1}{4\pi\varepsilon_o}\frac{q}{x^2}\,.\,dx$$

The negative sign indicates that the work is done against the electric force.

The electric potential at the point P due to the charge $+q$ is the total work done in moving a unit positive charge from infinity to that point.

$$V = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_o x^2}\,.\,dx = \frac{q}{4\pi\varepsilon_o r}$$

1.3.2 Electric potential at a point due to an electric dipole

Two charges $-q$ at A and $+q$ at B separated by a small distance $2d$ constitute an electric dipole and its dipole moment is p (Fig 1.13).

Let P be the point at a distance r from the midpoint of the dipole O and θ be the angle between PO and the axis of the dipole OB. Let r_1 and r_2 be the distances of the point P from $+q$ and $-q$ charges respectively.

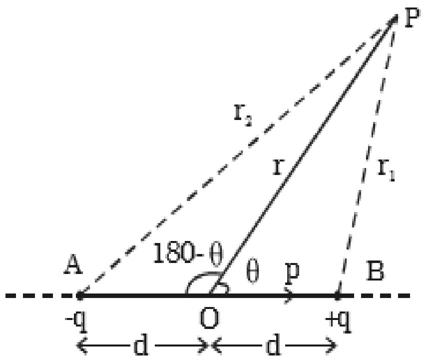


Fig 1.13 Potential due to a dipole

Potential at P due to charge (+q) = $\frac{1}{4\pi\varepsilon_o}\frac{q}{r_1}$

Potential at P due to charge (–q) = $\frac{1}{4\pi\varepsilon_o}\left(-\frac{q}{r_2}\right)$

Total potential at P due to dipole is, V = $\frac{1}{4\pi\varepsilon_o}\frac{q}{r_1}-\frac{1}{4\pi\varepsilon_o}\frac{q}{r_2}$

$$V = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \qquad ...(1)$$

Applying cosine law,

$$r_1^2 = r^2 + d^2 - 2rd\cos\theta$$

$$r_1^2 = r^2\left(1-2d\frac{\cos\theta}{r}+\frac{d^2}{r^2}\right)$$

Since d is very much smaller than r, $\frac{d^2}{r^2}$ can be neglected.

$$\therefore \quad r_1 = r\left(1-\frac{2d}{r}\cos\theta\right)^{\frac{1}{2}}$$

or $$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2d}{r}\cos\theta\right)^{-1/2}$$

Using the Binomial theorem and neglecting higher powers,

$$\therefore \quad \frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{d}{r}\cos\theta\right) \quad \text{...(2)}$$

Similarly,

$$r_2^2 = r^2 + d^2 - 2rd\cos(180 - \theta)$$

or $$r_2^2 = r^2 + d^2 + 2rd\cos\theta.$$

$$r_2 = r\left(1 + \frac{2d}{r}\cos\theta\right)^{1/2} \quad \left(\because \frac{d^2}{r^2} \text{ is negligible}\right)$$

or $$\frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{2d}{r}\cos\theta\right)^{-1/2}$$

Using the Binomial theorem and neglecting higher powers,

$$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{d}{r}\cos\theta\right) \quad \text{...(3)}$$

Substituting equation (2) and (3) in equation (1) and simplifying

$$V = \frac{q}{4\pi\varepsilon_o}\frac{1}{r}\left(1 + \frac{d}{r}\cos\theta - 1 + \frac{d}{r}\cos\theta\right)$$

$$\therefore \quad V = \frac{q\,2d\cos\theta}{4\pi\varepsilon_o\,.\,r^2} = \frac{1}{4\pi\varepsilon_o}\frac{p\,.\cos\theta}{r^2} \quad \text{...(4)}$$

Special cases :

1. When the point P lies on the axial line of the dipole on the side of $+q$, then $\theta = 0$

$$\therefore V = \frac{p}{4\pi\varepsilon_o r^2}$$

2. When the point P lies on the axial line of the dipole on the side of $-q$, then $\theta = 180$

$$\therefore V = -\frac{p}{4\pi\varepsilon_o r^2}$$

3. When the point P lies on the equatorial line of the dipole, then, $\theta = 90^o$,

$$\therefore V = 0$$

1.3.3 Electric potential energy

The electric potential energy of two point charges is equal to the work done to assemble the charges or workdone in bringing each charge or work done in bringing a charge from infinite distance.

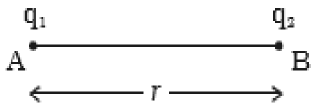


Fig 1.14a Electric potential energy

Let us consider a point charge q_1, placed at A [Fig 1.14a].

The potential at a point B at a distance r from the charge q_1 is

$$V = \frac{q_1}{4\pi\varepsilon_o r}$$

Another point charge q_2 is brought from infinity to the point B. Now the work done on the charge q_2 is stored as electrostatic potential energy (U) in the system of charges q_1 and q_2.

$\therefore$ work done, $w = Vq_2$

$$\text{Potential energy (U)} = \frac{q_1 q_2}{4\pi\varepsilon_o r}$$

Keeping q_2 at B, if the charge q_1 is imagined to be brought from infinity to the point A, the same amount of work is done.

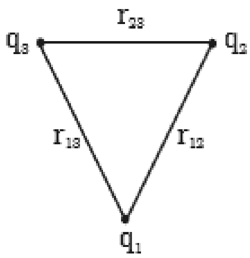


Fig 1.14b Potential energy of system of charges

Also, if both the charges q_1 and q_2 are brought from infinity, to points A and B respectively, separated by a distance r, then potential energy of the system is the same as the previous cases.

For a system containing more than two charges (Fig 1.14b), the potential energy (U) is given by

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

1.3.4 Equipotential Surface

If all the points of a surface are at the same electric potential, then the surface is called an equipotential surface.

(i) In case of an isolated point charge, all points equidistant from the charge are at same potential. Thus, equipotential surfaces in this

1.3.3 Electric potential energy

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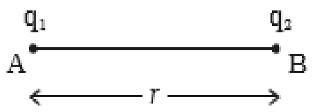

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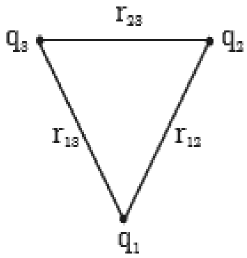

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1.3.4 Equipotential Surface

If all the points of a surface are at the same electric potential, then the surface is called an equipotential surface.

(i) In case of an isolated point charge, all points equidistant from the charge are at same potential. Thus, equipotential surfaces in this area ds is,

$$d\phi = \overline{E} \cdot \overline{ds} = E\, ds \cos\theta$$

The total flux through the closed surface S is obtained by integrating the above equation over the surface.

$$\phi = \oint d\phi = \oint \vec{E} \cdot \overline{ds}$$

The circle on the integral indicates that, the integration is to be taken over the closed surface. The electric flux is a scalar quantity.

Its unit is $N\,m^2 C^{-1}$

1.4.1 Gauss's law

The law relates the flux through any closed surface and the net charge enclosed within the surface. The law states that the total flux of the electric field E over any closed surface is equal to $\frac{1}{\varepsilon_o}$ times the net charge enclosed by the surface.

$$\phi = \frac{q}{\varepsilon_o}$$

This closed imaginary surface is called Gaussian surface. Gauss's law tells us that the flux of E through a closed surface S depends only on the value of net charge inside the surface and not on the location of the charges. Charges outside the surface will not contribute to flux.

1.4.2 Applications of Gauss's Law

i) Field due to an infinite long straight charged wire

Consider an uniformly charged wire of infinite length having a constant linear charge density λ (charge per unit length). Let P be a point at a distance r from the wire (Fig. 1.17) and E be the electric field at the point P. A cylinder of length *l*, radius r, closed at each end by plane caps normal to the axis is chosen as Gaussian surface. Consider a very small area *ds* on the Gaussian surface.

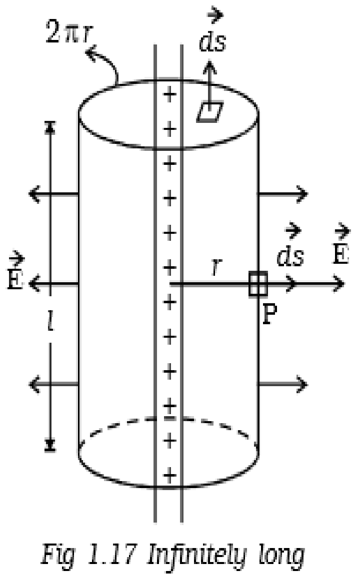


Fig 1.17 Infinitely long straight charged wire

By symmetry, the magnitude of the electric field will be the same at all points on the curved surface of the cylinder and directed radially outward. $\vec{E}$ and $\vec{ds}$ are along the same direction.

The electric flux (ϕ) through curved surface = $\oint E\ ds \cos\theta$

$$\phi = \oint E\ ds \qquad [\because \theta = 0; \cos\theta = 1]$$

$$= E\ (2\pi rl)$$

($\because$ The surface area of the curved part is 2π rl)

Since $\vec{E}$ and $\vec{ds}$ are right angles to each other, the electric flux through the plane caps = 0

$\therefore$ Total flux through the Gaussian surface, $\phi = E.\ (2\pi rl)$

The net charge enclosed by Gaussian surface is, $q = \lambda l$

$\therefore$ By Gauss's law,

$$E\ (2\pi rl) = \frac{\lambda l}{\varepsilon_o} \quad \text{or} \quad E = \frac{\lambda}{2\pi\varepsilon_o r}$$

The direction of electric field E is radially outward, if line charge is positive and inward, if the line charge is negative.

1.4.3 Electric field due to an infinite charged plane sheet

Consider an infinite plane sheet of charge with surface charge density σ. Let P be a point at a distance *r* from the sheet (Fig. 1.18) and E be the electric field at P. Consider a Gaussian surface in the form of cylinder of cross-sectional area A and length *2r* perpendicular to the sheet of charge.

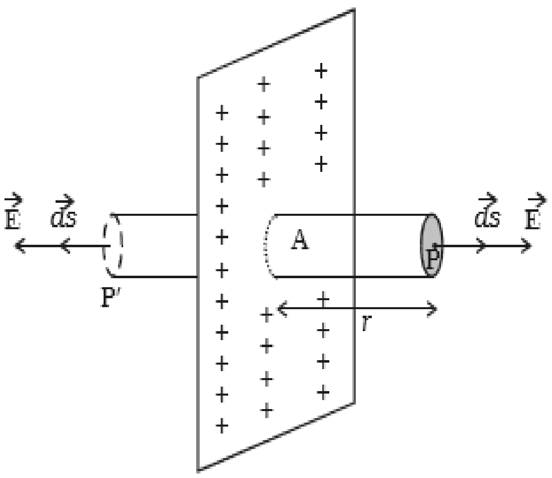


Fig 1.18 Infinite plane sheet

By symmetry, the electric field is at right angles to the end caps and away from the plane. Its magnitude is the same at P and at the other cap at P′.

Therefore, the total flux through the closed surface is given by

$$\phi = \left[\oint E.ds\right]_P + \left[\oint E.ds\right]_{P^1} \qquad (\because \theta = 0, \cos\theta = 1)$$

$$= E A + E A = 2 E A$$

If σ is the charge per unit area in the plane sheet, then the net positive charge q within the Gaussian surface is, $q = \sigma A$

Using Gauss's law,

$$2 E A = \frac{\sigma A}{\varepsilon_o}$$

$$\therefore E = \frac{\sigma}{2\varepsilon_o}$$

1.4.4 Electric field due to two parallel charged sheets

Consider two plane parallel infinite sheets with equal and opposite charge densities +σ and –σ as shown in Fig 1.19. The magnitude of electric field on either side of a plane sheet of charge is $E = \sigma/2\varepsilon_o$ and acts perpendicular to the sheet, directed outward (if the charge is positive) or inward (if the charge is negative).

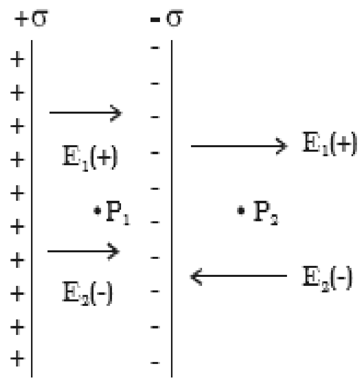


Fig 1.19 Field due to two parallel sheets

(i) When the point P_1 is in between the sheets, the field due to two sheets will be equal in magnitude and in the same direction. The resultant field at P_1 is,

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o} \text{ (towards the right)}$$

(ii) At a point P_2 outside the sheets, the electric field will be equal in magnitude and opposite in direction. The resultant field at P_2 is,

$$E = E_1 - E_2 = \frac{\sigma}{2\varepsilon_o} - \frac{\sigma}{2\varepsilon_o} = 0.$$

1.4.5 Electric field due to uniformly charged spherical shell

Case (i) At a point outside the shell.

Consider a charged shell of radius R (Fig 1.20a). Let P be a point outside the shell, at a distance r from the centre O. Let us construct a Gaussian surface with r as radius. The electric field E is normal to the surface.

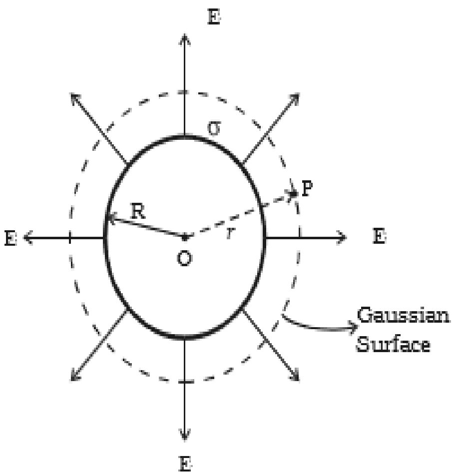


Fig1.20a. Field at a point outside the shell

The flux crossing the Gaussian sphere normally in an outward direction is,

$$\phi = \int_s \overline{E} \cdot \overline{ds} = \int_s E\, ds = E\,(4\pi r^2)$$

(since angle between E and ds is zero)

By Gauss's law, $\quad E \cdot (4\pi r^2) = \dfrac{q}{\varepsilon_o}$

or $\quad E = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q}{r^2}$

It can be seen from the equation that, the electric field at a point outside the shell will be the same as if the total charge on the shell is concentrated at its centre.

Case (ii) At a point on the surface.

The electric field E for the points on the surface of charged spherical shell is,

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{R^2} \; (\because\ r = R)$$

Case (iii) At a point inside the shell.

Consider a point P′ inside the shell at a distance r′ from the centre of the shell. Let us construct a Gaussian surface with radius r′.

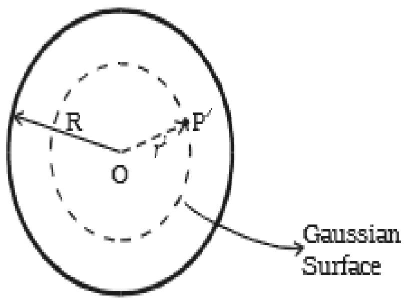


Fig 1.20b Field at a point inside the shell

The total flux crossing the Gaussian sphere normally in an outward direction is

$$\phi = \int_s \vec{E} \cdot \vec{ds} = \int_s E ds = E \times (4\pi r'^2)$$

since there is no charge enclosed by the gaussian surface, according to Gauss's Law

$$E \times 4\pi r'^2 = \frac{q}{\varepsilon_o} = 0 \qquad \therefore E = 0$$

(i.e) the field due to a uniformly charged thin shell is zero at all points inside the shell.

1.4.6 Electrostatic shielding

It is the process of isolating a certain region of space from external field. It is based on the fact that electric field inside a conductor is zero.

During a thunder accompanied by lightning, it is safer to sit inside a bus than in open ground or under a tree. The metal body of the bus provides electrostatic shielding, where the electric field is zero. During lightning the electric discharge passes through the body of the bus.

1.5 Electrostatic induction

It is possible to obtain charges without any contact with another charge. They are known as induced charges and the phenomenon of producing induced charges is known as electrostatic induction. It is used in electrostatic machines like Van de Graaff generator and capacitors.

Fig 1.21 shows the steps involved in charging a metal sphere by induction.

(a) There is an uncharged metallic sphere on an insulating stand.

(b) When a negatively charged plastic rod is brought close to the sphere, the free electrons move away due to repulsion and start pilling up at the farther end. The near end becomes positively charged due to deficit of electrons. This process of charge distribution stops when the net force on the free electron inside the metal is zero (this process happens very fast).

(c) When the sphere is grounded, the negative charge flows to the ground. The positive charge at the near end remains held due to attractive forces.

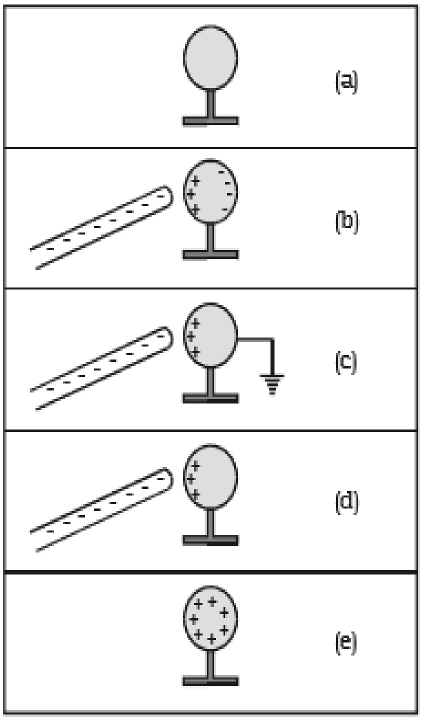


Fig 1.21 Electrostatic Induction

(d) When the sphere is removed from the ground, the positive charge continues to be held at the near end.

(e) When the plastic rod is removed, the positive charge spreads uniformly over the sphere.

1.5.1 Capacitance of a conductor

When a charge q is given to an isolated conductor, its potential will change. The change in potential depends on the size and shape of the conductor. The potential of a conductor changes by V, due to the charge q given to the conductor.

$$q \propto V \text{ or } q = CV$$

i.e. $$C = q/V$$

Here C is called as capacitance of the conductor.

The capacitance of a conductor is defined as the ratio of the charge given to the conductor to the potential developed in the conductor.

The unit of capacitance is farad. A conductor has a capacitance of one farad, if a charge of 1 coulomb given to it, rises its potential by 1 volt.

The practical units of capacitance are μF and pF.

Principle of a capacitor

Consider an insulated conductor (Plate A) with a positive charge '*q*' having potential V (Fig 1.22a). The capacitance of A is C = q/V. When another insulated metal plate B is brought near A, negative charges are induced on the side of B near A. An equal amount of positive charge is induced on the other side of B (Fig 1.22b). The negative charge in B decreases the potential of A. The positive charge in B increases the potential of A. But the negative charge on B is nearer to A than the positive charge on B. So the net effect is that, the potential of A decreases. Thus the capacitance of A is increased.

If the plate B is earthed, positive charges get neutralized (Fig 1.22c). Then the potential of A decreases further. Thus the capacitance of A is considerably increased.

The capacitance depends on the geometry of the conductors and nature of the medium. A capacitor is a device for storing electric charges.

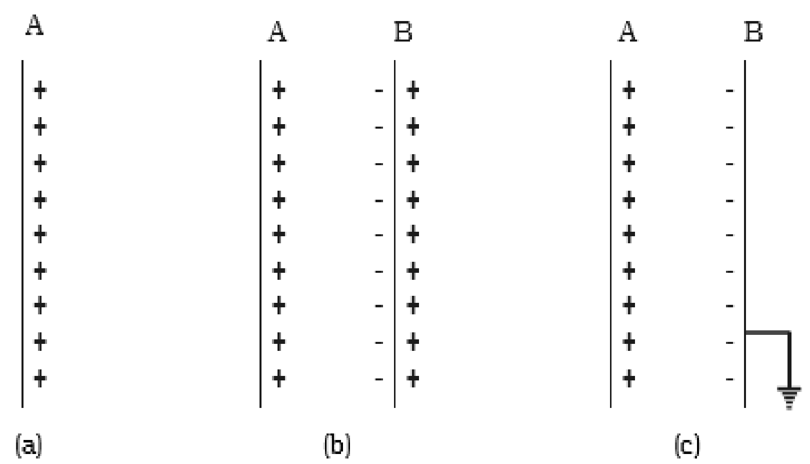


Fig 1.22 Principle of capacitor

1.5.2 Capacitance of a parallel plate capacitor

The parallel plate capacitor consists of two parallel metal plates X and Y each of area A, separated by a distance d, having a surface charge density σ (fig. 1.23). The medium between the plates is air. A charge $+q$ is given to the plate X. It induces a charge $-q$ on the upper surface of earthed plate Y. When the plates are very close to each other, the field is confined to the region between them. The electric lines of force starting from plate X and ending at the plate Y are parallel to each other and perpendicular to the plates.

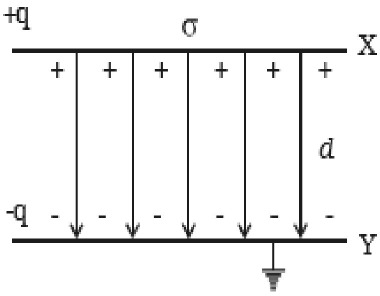


Fig 1.23 Parallel plate capacitor

By the application of Gauss's law, electric field at a point between the two plates is,

$$E = \frac{\sigma}{\varepsilon_o}$$

Potential difference between the plates X and Y is

$$V = \int_d^0 -E\,dr = \int_d^0 -\frac{\sigma}{\varepsilon_o}\,dr = \frac{\sigma d}{\varepsilon_o}$$

The capacitance (C) of the parallel plate capacitor

$$C = \frac{q}{V} = \frac{\sigma A}{\sigma d/\varepsilon_o} = \frac{\varepsilon_o A}{d} \qquad [\text{since, } \sigma = \frac{q}{A}]$$

$$\therefore \quad C = \frac{\varepsilon_o A}{d}$$

The capacitance is directly proportional to the area (A) of the plates and inversely proportional to their distance of separation (d).

1.5.3 Dielectrics and polarisation

Dielectrics

A dielectric is an insulating material in which all the electrons are tightly bound to the nucleus of the atom. There are no free electrons to carry current. Ebonite, mica and oil are few examples of dielectrics. The electrons are not free to move under the influence of an external field.

Polarisation

A nonpolar molecule is one in which the centre of gravity of the positive charges (protons) coincide with the centre of gravity of the negative charges (electrons). Example: O_2, N_2, H_2. The nonpolar molecules do not have a permanent dipole moment.

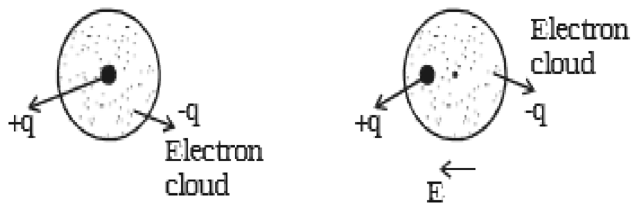
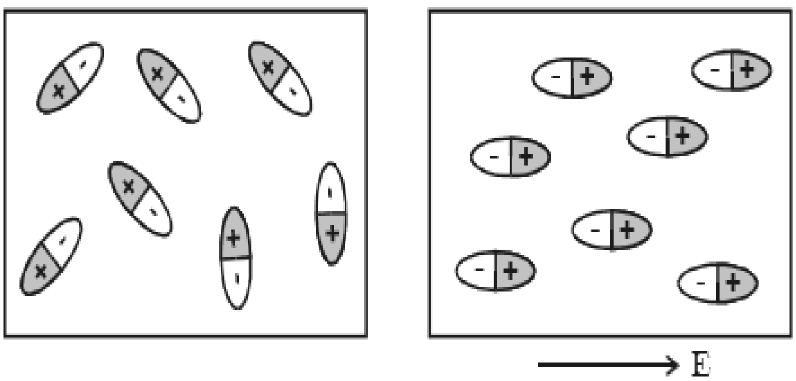


Fig 1.24 Induced dipole

If a non polar dielectric is placed in an electric field, the centre of charges get displaced. The molecules are then said to be polarised and are called induced dipoles. They acquire induced dipole moment p in the direction of electric field (Fig 1.24).

A polar molecule is one in which the centre of gravity of the positive charges is separated from the centre of gravity of the negative charges by a finite distance. Examples : N_2O, H_2O, HCl, NH_3. They have a permanent dipole moment. In the absence of an external field, the dipole moments of polar molecules orient themselves in random directions. Hence no net dipole moment is observed in the dielectric. When an electric field is applied, the dipoles orient themselves in the direction of electric field. Hence a net dipole moment is produced (Fig 1.25).



(a) No field *(b) In electric field*

Fig1.25 Polar molecules

The alignment of the dipole moments of the permanent or induced dipoles in the direction of applied electric field is called polarisation or electric polarisation.

The magnitude of the induced dipole moment p is directly proportional to the external electric field E.

$\therefore$ $p \propto E$ or $p = \alpha E$, where α is the constant of proportionality and is called molecular polarisability.

1.5.4 Polarisation of dielectric material

Consider a parallel plate capacitor with +q and –q charges. Let E_0 be the electric field between the plates in air. If a dielectric slab is introduced in the space between them, the dielectric slab gets polarised. Suppose $+q_i$ and $-q_i$ be the induced surface charges on the face of dielectric opposite to the plates of capacitor (Fig 1.26). These induced charges produce their own field E_i which opposes the electric field E_o. So, the resultant field, $E < E_o$. But the direction of E is in the direction of E_o.

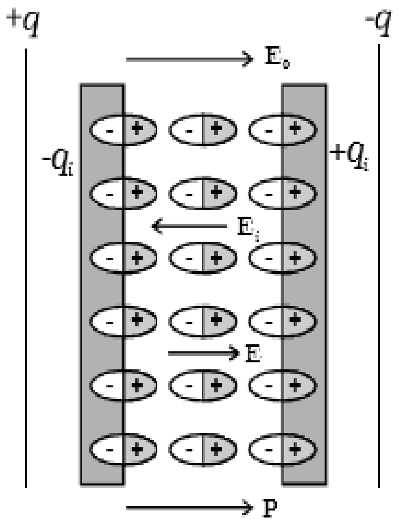


Fig1.26 Polarisation of dielectric material

$$\therefore\ E = E_o + (-E_i)$$

($\because$ E_i is opposite to the direction of E_o)

1.5.5 Capacitance of a parallel plate capacitor with a dielectric medium.

Consider a parallel plate capacitor having two conducting plates X and Y each of area A, separated by a distance d apart. X is given a positive charge so that the surface charge density on it is σ and Y is earthed.

Let a dielectric slab of thick-ness t and relative permittivity ε_r be introduced between the plates (Fig.1.27).

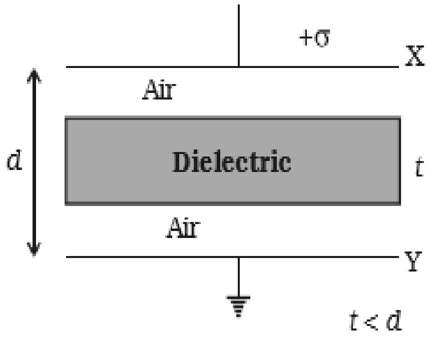


Fig 1.27 Dielectric in capacitor

Thickness of dielectric slab = t

Thickness of air gap = (d–t)

Electric field at any point in the air between the plates,

$$E = \frac{\sigma}{\varepsilon_o}$$

Electric field at any point, in the dielectric slab $E' = \frac{\sigma}{\varepsilon_r \varepsilon_o}$

The total potential difference between the plates, is the work done in crossing unit positive charge from one plate to another in the field E over a distance (d–t) and in the field E′ over a distance t, then

$$V = E(d-t) + E't$$

$$= \frac{\sigma}{\varepsilon_o}(d-t) + \frac{\sigma t}{\varepsilon_o \varepsilon_r}$$

$$= \frac{\sigma}{\varepsilon_o}\left[(d-t) + \frac{t}{\varepsilon_r}\right]$$

The charge on the plate X, q = σA

Hence the capacitance of the capacitor is,

$$C = \frac{q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_o}\left[(d-t)+\frac{t}{\varepsilon_r}\right]} = \frac{\varepsilon_o A}{(d-t)+\frac{t}{\varepsilon_r}}$$

Effect of dielectric

In capacitors, the region between the two plates is filled with dielectric like mica or oil.

The capacitance of the air filled capacitor, $C = \frac{\varepsilon_o A}{d}$

The capacitance of the dielectric filled capacitor, $C' = \frac{\varepsilon_r \varepsilon_o A}{d}$

$$\therefore \quad \frac{C'}{C} = \varepsilon_r \text{ or } C' = \varepsilon_r C$$

since, $\varepsilon_r > 1$ for any dielectric medium other than air, the capacitance increases, when dielectric is placed.

1.5.6 Applications of capacitors.

(i) They are used in the ignition system of automobile engines to eliminate sparking.

(ii) They are used to reduce voltage fluctuations in power supplies and to increase the efficiency of power transmission.

(iii) Capacitors are used to generate electromagnetic oscillations and in tuning the radio circuits.

1.5.7 Capacitors in series and parallel

(i) Capacitors in series

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series (Fig 1.28). Let V be the potential difference applied across the series combination. Each capacitor carries the same amount of charge q. Let V_1, V_2, V_3 be the potential difference across the capacitors C_1, C_2, C_3 respectively. Thus $V = V_1 + V_2 + V_3$

The potential difference across each capacitor is,

$$V_1 = \frac{q}{C_1}; V_2 = \frac{q}{C_2}; V_3 = \frac{q}{C_3}$$

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

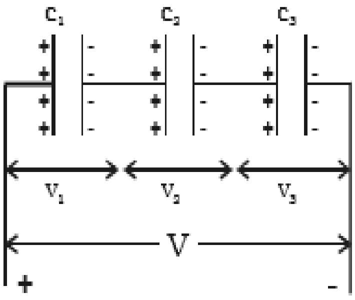


Fig 1.28 Capacitors in series

If C_S be the effective capacitance of the series combination, it should acquire a charge q when a voltage V is applied across it.

i.e. $$V = \frac{q}{C_S}$$

$$\frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\therefore \quad \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

when a number of capacitors are connected in series, the reciprocal of the effective capacitance is equal to the sum of reciprocal of the capacitance of the individual capacitors.

(ii) Capacitors in parallel

Consider three capacitors of capacitances C_1, C_2 and C_3 connected in parallel (Fig.1.29). Let this parallel combination be connected to a potential difference V. The potential difference across each capacitor is the same. The charges on the three capacitors are,

$q_1 = C_1V$, $q_2 = C_2\,V$, $q_3 = C_3V$.

The total charge in the system of capacitors is

$$q = q_1 + q_2 + q_3$$
$$q = C_1V + C_2V + C_3V$$

But $q = C_p.V$ where C_p is the effective capacitance of the system

$$\therefore \quad C_pV = V\,(C_1 + C_2 + C_3)$$
$$\therefore \quad C_P = C_1 + C_2 + C_3$$

Hence the effective capacitance of the capacitors connected in parallel is the sum of the capacitances of the individual capacitors.

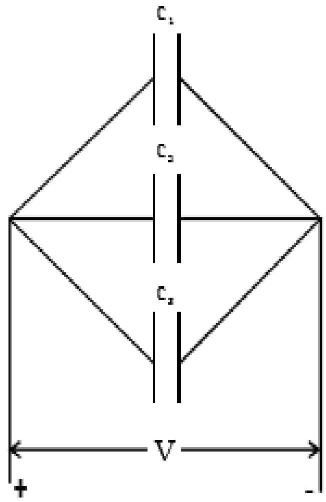


Fig 1.29 Capacitors in parallel

1.5.8 Energy stored in a capacitor

The capacitor is a charge storage device. Work has to be done to store the charges in a capacitor. This work done is stored as electrostatic potential energy in the capacitor.

Let q be the charge and V be the potential difference between the plates of the capacitor. If dq is the additional charge given to the plate, then work done is, $dw = Vdq$

$$dw = \frac{q}{C}dq \qquad \left(\because V = \frac{q}{C}\right)$$

Total work done to charge a capacitor is

$$w = \int dw = \int_0^q \frac{q}{C}\,dq = \frac{1}{2}\frac{q^2}{C}$$

This work done is stored as electrostatic potential energy (U) in the capacitor.

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 \qquad (\because\ q = CV)$$

This energy is recovered if the capacitor is allowed to discharge.

1.5.9 Distribution of charges on a conductor and action of points

Let us consider two conducting spheres A and B of radii r_1 and r_2 respectively connected to each other by a conducting wire (Fig 1.30). Let r_1 be greater than r_2. A charge given to the system is distributed as q_1 and q_2 on the surface of the spheres A and B. Let σ_1, σ_2 be the charge densities on the sphere A and B.

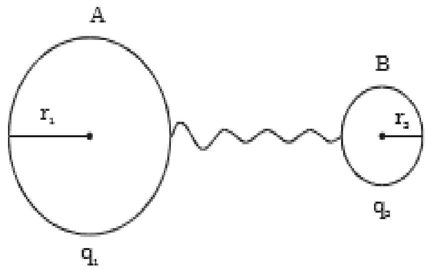


Fig 1.30 Distribution of charges

The potential at A,

$$V_1 = \frac{q_1}{4\pi\varepsilon_o r_1}$$

The potential at B, $\quad V_2 = \frac{q_2}{4\pi\varepsilon_o r_2}$

Since they are connected, their potentials are equal

$$\frac{q_1}{4\pi\varepsilon_o r_1} = \frac{q_2}{4\pi\varepsilon_o r_2} \qquad \begin{bmatrix} \because q_1 = 4\pi r_1^2\sigma_1 \\ and \\ q_2 = 4\pi r_2^2\sigma_2 \end{bmatrix}$$

$$\sigma_1 r_1 = \sigma_2 r_2$$

i.e., σr is a constant. From the above equation it is seen that, smaller the radius, larger is the charge density.

In case of conductor, shaped as in Fig.1.31 the distribution is not uniform. The

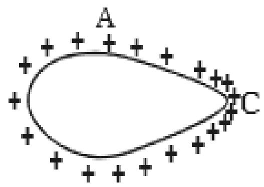


Fig 1.31 Action of point

charges accumulate to a maximum at the pointed end where the curvature is maximum or the radius is minimum. It is found experimentally that a charged conductor with sharp points on its surface, loses its charge rapidly.

The reason is that the air molecules which come in contact with the sharp points become ionized. The positive ions are repelled and the negative ions are attracted by the sharp points and the charge in them is therefore reduced.

Thus, the leakage of electric charges from the sharp points on the charged conductor is known as action of points or corona discharge. This principle is made use of in the electrostatic machines for collecting charges and in lightning arresters (conductors).

1.6 Lightning conductor

This is a simple device used to protect tall buildings from the lightning.

It consists of a long thick copper rod passing through the building to ground. The lower end of the rod is connected to a copper plate buried deeply into the ground. A metal plate with number of spikes is connected to the top end of the copper rod and kept at the top of the building.

When a negatively charged cloud passes over the building, positive charge will be induced on the pointed conductor. The positively charged sharp points will ionize the air in the vicinity. This will partly neutralize the negative charge of the cloud, thereby lowering the potential of the cloud. The negative charges that are attracted to the conductor travels down to the earth. Thereby preventing the lightning stroke from the damage of the building.

Van de Graaff Generator

In 1929, Robert J. Van de Graaff designed an electrostatic machine which produces large electrostatic potential difference of the order of 10^7 V.

The working of Van de Graaff generator is based on the principle of electrostatic induction and action of points.

A hollow metallic sphere A is mounted on insulating pillars as shown in the Fig.1.32. A pulley B is mounted at the centre of the sphere and another pulley C is mounted near the bottom. A belt made of silk moves over the pulleys. The pulley C is driven continuously by an electric motor. Two comb–shaped conductors D and E having number of needles, are mounted near the pulleys. The comb D is maintained at a positive potential of the order of 10^4 volt by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere.

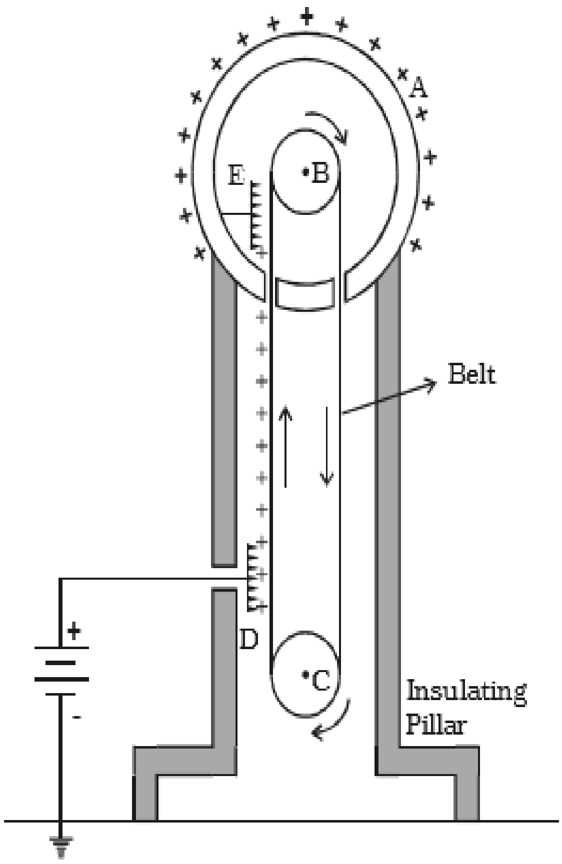


Fig 1.32 Van de Graaff Generator

Because of the high electric field near the comb D, the air gets ionised due to action of points, the negative charges in air move towards the needles and positive charges are repelled on towards the belt. These positive charges stick to the belt, moves up and reaches near the comb E.

As a result of electrostatic induction, the comb E acquires negative charge and the sphere acquires positive charge. The acquired positive charge is distributed on the outer surface of the sphere. The high electric field at the comb E ionises the air. Hence, negative charges are repelled to the belt, neutralises the positive charge on the belt before the belt passes over the pulley. Hence the descending belt will be left uncharged.

Thus the machine, continuously transfers the positive charge to the sphere. As a result, the potential of the sphere keeps increasing till it attains a limiting value (maximum). After this stage no more charge can be placed on the sphere, it starts leaking to the surrounding due to ionisation of the air.

The leakage of charge from the sphere can be reduced by enclosing it in a gas filled steel chamber at a very high pressure.

The high voltage produced in this generator can be used to accelerate positive ions (protons, deuterons) for the purpose of nuclear disintegration.

Solved Problems

1.1 Three small identical balls have charges -3×10^{-12}C, 8×10^{-12}C and 4×10^{-12}C respectively. They are brought in contact and then separated. Calculate (i) charge on each ball (ii) number of electrons in excess or deficit on each ball after contact.

Data : $q_1 = -3 \times 10^{-12}$C, $q_2 = 8 \times 10^{-12}$ C, $q_3 = 4 \times 10^{-12}$ C

Solution : (i) The charge on each ball

$$q = \frac{q_1 + q_2 + q_3}{3} = \left(\frac{-3+8+4}{3}\right) \times 10^{-12}$$

$$= 3 \times 10^{-12} \text{ C}$$

(ii) Since the charge is positive, there is a shortage of electrons on each ball.

$$n = \frac{q}{e} = \frac{3 \times 10^{-12}}{1.6 \times 10^{-19}} = 1.875 \times 10^7$$

$\therefore$ number of electrons = 1.875×10^7.

1.2 Two insulated charged spheres of charges 6.5×10^{-7}C each are separated by a distance of 0.5m. Calculate the electrostatic force between them. Also calculate the force (i) when the charges are doubled and the distance of separation is halved. (ii) when the charges are placed in a dielectric medium water ($\varepsilon_r = 80$)

Data : $q_1 = q_2 = 6.5 \times 10^{-7}$C, r = 0.5 m

Solution : $F = \dfrac{1}{4\pi\varepsilon_o} \quad \dfrac{q_1 q_2}{r^2}$

$$= \frac{9\times10^{9}\times(6.5\times10^{-7})^{2}}{(0.5)^{2}}$$

$= 1.52 \times 10^{-2}$ N.

(i) If the charge is doubled and separation between them is halved then,

$$F_1 = \frac{1}{4\pi\varepsilon_o}\frac{2q_1 2q_2}{\left(r/2\right)^2}$$

F_1 = 16 times of F.

$= 16 \times 1.52 \times 10^{-2}$

$F_1 = 0.24$ N

(ii) When placed in water of $\varepsilon_r = 80$

$$F_2 = \frac{F}{\varepsilon_r} = \frac{1.52\times10^{-2}}{80}$$

$F_2 = 1.9 \times 10^{-4}$ N

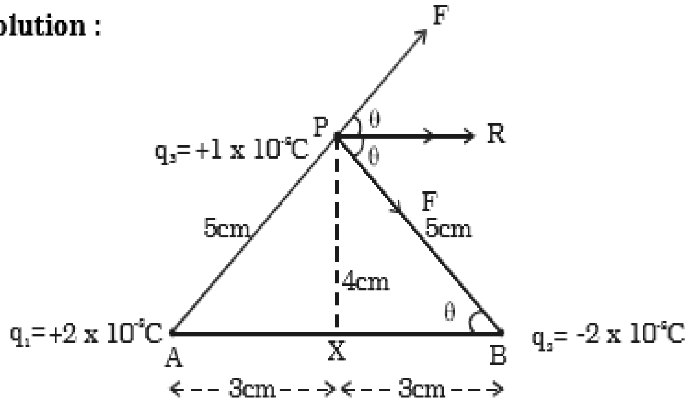
1.3. Two small equal and unlike charges 2×10^{-8}C are placed at A and B at a distance of 6 cm. Calculate the force on the charge 1×10^{-8}C placed at P, where P is 4cm on the perpendicular bisector of AB.

Data : $q_1 = +2 \times 10^{-8}$C, $q_2 = -2 \times 10^{-8}$ C

$q_3 = 1 \times 10^{-8}$ C at P

XP = 4 cm or 0.04 m, AB = 6 cm or 0.06 m

Solution :



From Δ APX, AP = $\sqrt{4^2+3^2}$ = 5 cm or 5×10^{-2} m.

A repels the charge at P with a force F (along AP)

$$F = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r^2} = \frac{9\times10^9\times2\times10^{-8}\times1\times10^{-8}}{(5\times10^{-2})^2}$$

$= 7.2 \times 10^{-4}$ N along AP.

B attracts the charge at P with same F (along PB), because BP = AP = 5 cm.

To find R, we resolve the force into two components

$$R = F\cos\theta + F\cos\theta = 2F\cos\theta$$

$$= 2 \times 7.2 \times 10^{-4} \times \frac{3}{5} \qquad \left[\because \cos\theta = \frac{BX}{PB} = \frac{3}{5}\right]$$

$$\therefore R = 8.64 \times 10^{-4}\ N$$

1.4 Compare the magnitude of the electrostatic and gravitational force between an electron and a proton at a distance r apart in hydrogen atom. (Given : $m_e = 9.11 \times 10^{-31}$ kg ; $m_p = 1.67 \times 10^{-27}$ kg ; $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}; $e = 1.6 \times 10^{-19}$ C)

Solution :

The gravitational attraction between electron and proton is

$$F_g = G\frac{m_e m_p}{r^2}$$

Let r be the average distance between electron and proton in hydrogen atom.

The electrostatic force between the two charges.

$$F_e = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r^2}$$

$$\therefore \frac{F_e}{F_g} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{G m_e m_p} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{G m_e m_p}$$

$$= \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{6.67\times10^{-11}\times9.11\times10^{-31}\times1.67\times10^{-27}}$$

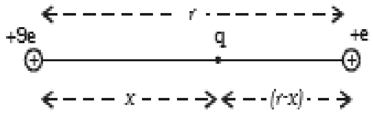
$$\frac{F_e}{F_g} = 2.27 \times 10^{39}$$

This shows that the electrostatic force is 2.27×10^{39} times stronger than gravitational force.

1.5 Two point charges +9e and +1e are kept at a distance of 16 cm from each other. At what point between these charges, should a third charge q to be placed so that it remains in equilibrium?

Data : r = 16 cm or 0.16 m; q_1 = 9e and q_2 = e

Solution : Let a third charge q be kept at a distance x from + 9e and $(r - x)$ from + e



$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}$$

$$= \frac{1}{4\pi\varepsilon o}\frac{9e \times q}{x^2} = \frac{1}{4\pi\varepsilon_o}\frac{q\ e}{(r-x)^2}$$

$$\therefore \frac{x^2}{(r-x)^2} = 9$$

$$\frac{x}{r-x} = 3$$

or $x = 3r - 3x$

$\therefore$ $4x = 3r = 3 \times 16 = 48$ cm

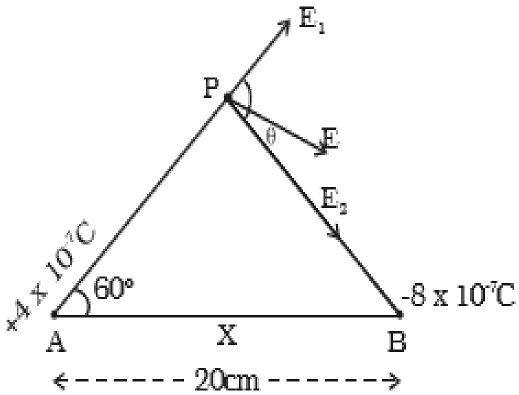
$\therefore$ $x = \frac{48}{4} = 12$ cm or 0.12 m

$\therefore$ The third charge should be placed at a distance of 0.12 m from charge 9e.

1.6 Two charges 4×10^{-7} C and -8×10^{-7}C are placed at the two corners A and B of an equilateral triangle ABP of side 20 cm. Find the resultant intensity at P.

Data : $q_1 = 4 \times 10^{-7}$ C; $q_2 = -8 \times 10^{-7}$ C; r = 20 cm = 0.2 m

Solution :



Electric field E_1 along AP

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r^2} = \frac{9\times10^9\times4\times10^{-7}}{(0.2)^2} = 9\times10^4\ \text{N C}^{-1}$$

Electric field E_2 along PB.

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r^2} = \frac{9\times10^9\times8\times10^{-7}}{0.04} = 18\times10^4\text{N C}^{-1}$$

$$\therefore \quad E = \sqrt{E_1^2+E_2^2+2E_1E_2\cos120^o}$$

$$= 9\times10^4\sqrt{2^2+1^2+2\times2\times1\left(-\tfrac{1}{2}\right)}$$

$$= 9\sqrt{3}\times10^4 = 15.6\times10^4 N\ C^{-1}$$

1.7 Calculate (i) the potential at a point due a charge of 4×10^{-7}C located at 0.09m away (ii) work done in bringing a charge of 2×10^{-9} C from infinity to the point.

Data : $q_1 = 4\times10^{-7}$C, $q_2 = 2\times10^{-9}$ C, r = 0.09 m

Solution :

(i) The potential due to the charge q_1 at a point is

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r}$$

$$= \frac{9\times10^9\times4\times10^{-7}}{0.09} = 4\times10^4\ \text{V}$$

(ii) Work done in bringing a charge q_2 from infinity to the point is

$$W = q_2V = 2\times10^{-9}\times4\times10^4$$

$$W = 8\times10^{-5}\ \text{J}$$

1.8 A sample of HCl gas is placed in an electric field of 2.5×10^4 N C^{-1}. The dipole moment of each HCl molecule is 3.4×10^{-30} C m. Find the maximum torque that can act on a molecule.

Data : $E = 2.5\times10^4$ N C^{-1}, $p = 3.4\times10^{-30}$ C m.

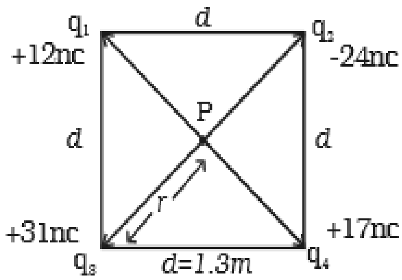
Solution : Torque acting on the molecule

τ = pE sin θ for maximum torque, $\theta = 90^o$

$= 3.4\times10^{-30}\times 2.5\times10^4$

Maximum Torque acting on the molecule is = 8.5×10^{-26} N m.

1.9 Calculate the electric potential at a point P, located at the centre of the square of point charges shown in the figure.



Data : $q_1 = +12\ \text{n C}$;

$q_2 = -24\ \text{n C}$; $q_3 = +31\text{n C}$;

$q_4 = +17\text{n C}$; $d = 1.3\ \text{m}$

Solution :

Potential at a point P is

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{r}+\frac{q_2}{r}+\frac{q_3}{r}+\frac{q_4}{r}\right]$$

The distance $r = \frac{d}{\sqrt{2}} = \frac{1.3}{\sqrt{2}} = 0.919\ \text{m}$

Total charge $= q_1 + q_2 + q_3 + q_4$

$= (12 - 24 + 31 + 17) \times 10^{-9}$

$q = 36 \times 10^{-9}$

$$\therefore \quad V = \frac{9\times10^9\times36\times10^{-9}}{0.919}$$

$V = 352.6\ \text{V}$

1.10 Three charges -2×10^{-9}C, $+3 \times 10^{-9}$C, -4×10^{-9}C are placed at the vertices of an equilateral triangle ABC of side 20 cm. Calculate the work done in shifting the charges A, B and C to A_1, B_1 and C_1 respectively which are the mid points of the sides of the triangle.

Data :

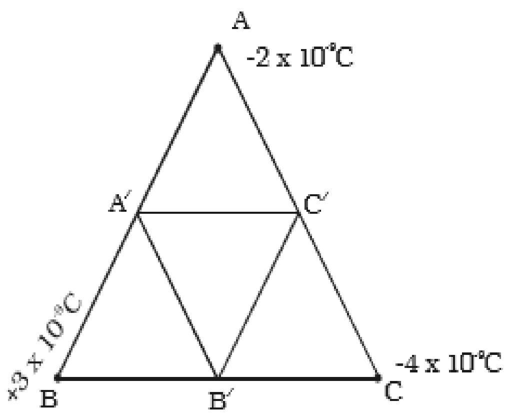
$q_1 = -2 \times 10^{-9}$C;

$q_2 = +3 \times 10^{-9}$C;

$q_3 = -4 \times 10^{-9}$C;

AB = BC = CA = 20cm

= 0.20 m



Solution :

The potential energy of the system of charges,

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1q_2}{r} + \frac{q_2q_3}{r} + \frac{q_3q_1}{r}\right]$$

Work done in displacing the charges from A, B and C to A_1, B_1 and C_1 respectively

$W = U_f - U_i$

U_i and U_f are the initial and final potential energy of the system.

$$U_i = \frac{9\times10^9}{0.20}\ [-6 \times 10^{-18} - 12 \times 10^{-18} + 8 \times 10^{-18}]$$

$$= -4.5 \times 10^{-7}\ J$$

$$U_f = \frac{9\times10^9}{0.10}\ [-6 \times 10^{-18} - 12 \times 10^{-18} + 8 \times 10^{-18}]$$

$$= -9 \times 10^{-7} J$$

$\therefore$ work done $= -9 \times 10^{-7} - (-4.5 \times 10^{-7})$

$W = -4.5 \times 10^{-7} J$

1.11 An infinite line charge produces a field of 9×10^4 N C^{-1} at a distance of 2 cm. Calculate the linear charge density.

Data : $E = 9 \times 10^4$ N C^{-1}, r = 2 cm = 2×10^{-2} m

Solution : $E = \frac{\lambda}{2\pi\varepsilon_o r}$

$\lambda = E \times 2\pi\varepsilon_o r$

$$= 9 \times 10^4 \times \frac{1}{18\times10^9} \times 2 \times 10^{-2} \left(\because 2\pi\varepsilon_o = \frac{1}{18\times10^9}\right)$$

$\lambda = 10^{-7}$ C m^{-1}

1.12 A point charge causes an electric flux of -6×10^3 Nm^2 C^{-1} to pass through a spherical Gaussian surface of 10 cm radius centred on the charge. (i) If the radius of the Gaussian surface is doubled, how much flux will pass through the surface? (ii) What is the value of charge?

Data : $\phi = -6 \times 10^3$ N m^2 C^{-1}; r = 10 cm = 10×10^{-2} m

Solution :

(i) If the radius of the Gaussian surface is doubled, the electric flux through the new surface will be the same, as it depends only on the net charge enclosed within and it is independent of the radius.

$\therefore \quad \phi = -6 \times 10^{8}$ N m^2 C^{-1}

(ii) $\therefore \quad \phi = \frac{q}{\varepsilon_o}$ or $q = -(8.85 \times 10^{-12} \times 6 \times 10^{8})$

$q = -5.31 \times 10^{-8}$ C

1.13 A parallel plate capacitor has plates of area 200 cm^2 and separation between the plates 1 mm. Calculate (i) the potential difference between the plates if 1n C charge is given to the capacitor (ii) with the same charge (1n C) if the plate separation is increased to 2 mm, what is the new potential difference and (iii) electric field between the plates.

Data: d = 1 mm = 1×10^{-3}m; A = 200 cm^2 or 200×10^{-4} m^2 ;

q = 1 nC = 1×10^{-9} C ;

Solution : The capacitance of the capacitor

$$C = \frac{\varepsilon_o A}{d} = \frac{8.85 \times 10^{-12} \times 200 \times 10^{-4}}{1 \times 10^{-3}}$$

$C = 0.177 \times 10^{-9}$ F = 0.177 nF

(i) The potential difference between the plates

$$V = \frac{q}{C} = \frac{1 \times 10^{-9}}{0.177 \times 10^{-9}} = 5.65\ V$$

(ii) If the plate separation is increased from 1 mm to 2 mm, the capacitance is decreased by 2, the potential difference increases by the factor 2

$\therefore$ New potential difference is 5.65×2

= 11.3 V

(iii) Electric field is,

$$E = \frac{\sigma}{\varepsilon_o} = \frac{q}{A.\varepsilon_o} = \frac{1 \times 10^{-9}}{8.85 \times 10^{-12} \times 200 \times 10^{-4}}$$

= 5650 N C^{-1}

1.14 A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance, if the distance between the plates be reduced to half and the space between them is filled with a substance of dielectric constant 6.

Data : $C_o = 8$ pF , $\varepsilon_r = 6$, distance d becomes, d/2 with dielectric

Solution : $C_o = \frac{A\varepsilon_o}{d} = 8\text{pF}$

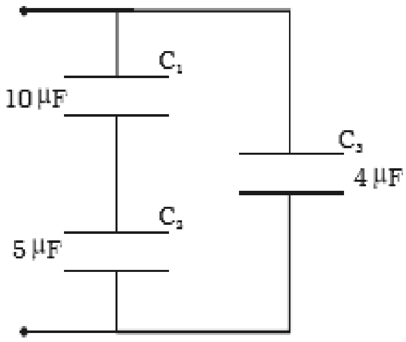
when the distance is reduced to half and dielectric medium fills the gap, the new capacitance will be

$$C = \frac{\varepsilon_r A\varepsilon_o}{d/2} = \frac{2\varepsilon_r A\varepsilon_o}{d}$$

$$= 2\varepsilon_r C_o$$

$$C = 2 \times 6 \times 8 = 96 \text{ pF}$$

1.15 Calculate the effective capacitance of the combination shown in figure.



Data : $C_1 = 10\mu F$; $C_2 = 5\mu F$; $C_3 = 4\mu F$

Solution : (i) C_1 and C_2 are connected in series, the effective capacitance of the capacitor of the series combination is

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{1}{10} + \frac{1}{5}$$

$$\therefore \quad C_S = \frac{10 \times 5}{10 + 5} = \frac{10}{3} \mu F$$

(ii) This C_S is connected to C_3 in parallel.

The effective capacitance of the capacitor of the parallel combination is

$$C_p = C_s + C_3$$

$$= \left(\frac{10}{3} + 4\right) = \frac{22}{3}\,\mu F$$

$C_p = 7.33\ \mu F$

1.16 The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply. How much electrostatic energy is stored by the capacitor?

Data : $A = 90\ cm^2 = 90 \times 10^{-4}\ m^2$; $d = 2.5\ mm = 2.5 \times 10^{-3}\ m$;

$V = 400\ V$

Solution : Capacitance of a parallel plate capacitor

$$C = \frac{\varepsilon_o A}{d} = \frac{8.85 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}}$$

$$= 3.186 \times 10^{-11}\ F$$

Energy of the capacitor $= (\frac{1}{2})\ CV^2$

$$= \frac{1}{2} \times 3.186 \times 10^{-11} \times (400)^2$$

Energy $= 2.55 \times 10^{-6}$ J

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

1.1 A glass rod rubbed with silk acquires a charge of $+8 \times 10^{-12}$C. The number of electrons it has gained or lost

(a) 5×10^{-7} (gained) (b) 5×10^{7} (lost)

(c) 2×10^{-8} (lost) (d) -8×10^{-12} (lost)

1.2 The electrostatic force between two point charges kept at a distance d apart, in a medium $\varepsilon_r = 6$, is 0.3 N. The force between them at the same separation in vacuum is

(a) 20 N (b) 0.5 N

(c) 1.8 N (d) 2 N

1.3 Electic field intensity is 400 V m^{-1} at a distance of 2 m from a point charge. It will be 100 V m^{-1} at a distance?

(a) 50 cm (b) 4 cm

(c) 4 m (d) 1.5 m

1.4 Two point charges +4q and +q are placed 30 cm apart. At what point on the line joining them the electric field is zero?

(a) 15 cm from the charge q (b) 7.5 cm from the charge q

(c) 20 cm from the charge 4q (d) 5 cm from the charge q

1.5 A dipole is placed in a uniform electric field with its axis parallel to the field. It experiences

(a) only a net force

(b) only a torque

(c) both a net force and torque

(d) neither a net force nor a torque

1.6 If a point lies at a distance x from the midpoint of the dipole, the electric potential at this point is proportional to

(a) $\frac{1}{x^2}$ (b) $\frac{1}{x^3}$

(c) $\frac{1}{x^4}$ (d) $\frac{1}{x^{3/2}}$

1.7 Four charges +q, +q, −q and −q respectively are placed at the corners A, B, C and D of a square of side a. The electric potential at the centre O of the square is

(a) $\frac{1}{4\pi\varepsilon o}\frac{q}{a}$ (b) $\frac{1}{4\pi\varepsilon o}\frac{2q}{a}$

(c) $\frac{1}{4\pi\varepsilon o}\frac{4q}{a}$ (d) zero

1.8 Electric potential energy (U) of two point charges is

(a) $\frac{q_1 q_2}{4\pi\varepsilon_o r^2}$ (b) $\frac{q_1 q_2}{4\pi\varepsilon_o r}$

(c) pE cos θ (d) pE sin θ

1.9 The work done in moving 500 μC charge between two points on equipotential surface is

(a) zero (b) finite positive

(c) finite negative (d) infinite

1.10 Which of the following quantities is scalar?

(a) dipole moment (b) electric force

(c) electric field (d) electric potential

1.11 The unit of permittivity is

(a) $C^2 N^{-1} m^{-2}$ (b) $N m^2 C^{-2}$

(c) $H m^{-1}$ (d) $N C^{-2} m^{-2}$

1.12 The number of electric lines of force originating from a charge of 1 C is

(a) 1.129×10^{11} (b) 1.6×10^{-19}

(c) 6.25×10^{18} (d) 8.85×10^{12}

1.13 The electric field outside the plates of two oppositely charged plane sheets of charge density σ is

(a) $\frac{+\sigma}{2\varepsilon o}$ (b) $\frac{-\sigma}{2\varepsilon o}$

(c) $\frac{\sigma}{\varepsilon o}$ (d) zero

1.14 The capacitance of a parallel plate capacitor increases from 5 μf to 60 μf when a dielectric is filled between the plates. The dielectric constant of the dielectric is

(a) 65 (b) 55

(c) 12 (d) 10

1.15 A hollow metal ball carrying an electric charge produces no electric field at points

(a) outside the sphere (b) on its surface

(c) inside the sphere (d) at a distance more than twice

1.16 State Coulomb's law in electrostatics and represent it in vector form.

1.17 What is permittivity and relative permittivity? How are they related?

1.18 Explain the principle of superposition.

1.19 Define electric field at a point. Give its unit and obtain an expression for the electric field at a point due to a point charge.

1.20 Write the properties of lines of forces.

1.21 What is an electric dipole? Define electric dipole moment?

1.22 Derive an expression for the torque acting on the electric dipole when placed in a uniform field.

1.23 What does an electric dipole experience when kept in a uniform electric field and non–uniform electric field?

1.24 Derive an expression for electric field due to an electric dipole (a) at a point on its axial line (b) at a point along the equatorial line.

1.25 Define electric potential at a point. Is it a scalar or a vector quantity? Obtain an expression for electric potential due to a point charge.

1.26 Distinguish between electric potential and potential difference.

1.27 What is an equipotential surface?

1.28 What is electrostatic potential energy of a system of two point charges? Deduce an expression for it.

1.29 Derive an expression for electric potential due to an electric dipole.

1.30 Define electric flux. Give its unit.

1.31 State Gauss's law. Applying this, calculate electric field due to (i) an infinitely long straight charge with uniform charge density (ii) an infinite plane sheet of charge of q.

1.32 What is a capacitor? Define its capacitance.

1.33 Explain the principle of capacitor. Deduce an expression for the capacitance of the parallel plate capacitor.

1.34 What is dielectric ? Explain the effect of introducing a dielectric slab between the plates of parallel plate capacitor.

1.35 A parallel plate capacitor is connected to a battery. If the dielectric slab of thickness equal to half the plate separation is inserted between the plates what happens to (i) capacitance of the capacitor (ii) electric field between the plates (iii) potential difference between the plates.

1.36 Deduce an expression for the equivalent capacitance of capacitors connected in series and parallel.

1.37 Prove that the energy stored in a parallel plate capacitor is $\frac{q^2}{2C}$.

1.38 What is meant by dielectric polarisation?

1.39 State the principle and explain the construction and working of Van de Graaff generator.

1.40 Why is it safer to be inside a car than standing under a tree during lightning?

Problems :

1.41 The sum of two point charges is 6 μ C. They attract each other with a force of 0.9 N, when kept 40 cm apart in vacuum. Calculate the charges.

1.42 Two small charged spheres repel each other with a force of 2×10^{-3} N. The charge on one sphere is twice that on the other. When one of the charges is moved 10 cm away from the other, the force is 5×10^{-4} N. Calculate the charges and the initial distance between them.

1.43 Four charges +q, +2q, +q and −q are placed at the corners of a square. Calculate the electric field at the intersection of the diagonals of the square of side10 cm if $q = 5/3 \times 10^{-9}$C.

1.44 *Two charges 10×10^{-9} C and 20×10^{-9}C are placed at a distance of 0.3 m apart. Find the potential and intensity at a point mid-way between them.*

1.45 *An electric dipole of charges 2×10^{-10}C and -2×10^{-10}C separated by a distance 5 mm, is placed at an angle of 60° to a uniform field of $10Vm^{-1}$. Find the (i) magnitude and direction of the force acting on each charge. (ii) Torque exerted by the field*

1.46 *An electric dipole of charges 2×10^{-6} C, -2×10^{-6} C are separated by a distance 1 cm. Calculate the electric field due to dipole at a point on its. (i) axial line 1 m from its centre (ii) equatorial line 1 m from its centre.*

1.47 *Two charges +q and −3q are separated by a distance of 1 m. At what point in between the charges on its axis is the potential zero?*

1.48 *Three charges +1μC, +3μC and −5μC are kept at the vertices of an equilateral triangle of sides 60 cm. Find the electrostatic potential energy of the system of charges.*

1.49 *Two positive charges of 12 μC and 8 μC respectively are 10 cm apart. Find the work done in bringing them 4 cm closer, so that, they are 6 cm apart.*

1.50 *Find the electric flux through each face of a hollow cube of side 10 cm, if a charge of 8.85 μC is placed at the centre.*

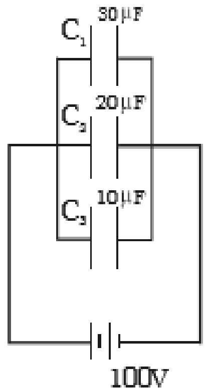
1.51 *A spherical conductor of radius 0.12 m has a charge of 1.6×10^{-7}C distributed uniformly on its surface. What is the electric field (i) inside the sphere (ii) on the sphere (iii) at a point 0.18 m from the centre of the sphere?*

1.52 *The area of each plate of a parallel plate capacitor is 4×10^{-2} sq m. If the thickness of the dielectric medium between the plates is 10^{-3} m and the relative permittivity of the dielectric is 7. Find the capacitance of the capacitor.*

1.53 *Two capacitors of unknown capacitances are connected in series and parallel. If the net capacitances in the two combinations are 6μF and 25μF respectively, find their capacitances.*

1.54 *Two capacitances 0.5 μF and 0.75 μF are connected in parallel and the combination to a 110 V battery. Calculate the charge from the source and charge on each capacitor.*

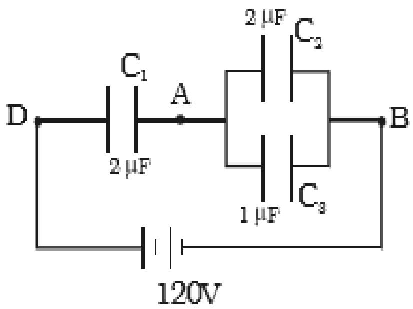
1.55 Three capacitors are connected in parallel to a 100 V battery as shown in figure. What is the total energy stored in the combination of capacitor?



1.56 A parallel plate capacitor is maintained at some potential difference. A 3 mm thick slab is introduced between the plates. To maintain the plates at the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab.

1.57 A dielectric of dielectric constant 3 fills three fourth of the space between the plates of a parallel plate capacitor. What percentage of the energy is stored in the dielectric?

1.58 Find the charges on the capacitor shown in figure and the potential difference across them.



1.59 Three capacitors each of capacitance 9 pF are connected in series (i) What is the total capacitance of the combination? (ii) What is the potential difference across each capacitor, if the combination is connected to 120 V supply?

Answers

1.1 (b)	**1.2** (c)	**1.3** (c)	**1.4** (c)	**1.5** (d)
1.6 (a)	**1.7** (d)	**1.8** (b)	**1.9** (a)	**1.10** (d)
1.11 (a)	**1.12** (a)	**1.13** (d)	**1.14** (c)	**1.15** (c)

1.35 (i) increases (ii) remains the same (iii) remains the same

1.41 $q_1 = 8 \times 10^{-6} C$, $q_2 = -2 \times 10^{-6}$ C

1.42 $q_1 = 33.33 \times 10^{-9} C$, $q_2 = 66.66 \times 10^{-9} C$, $x = 0.1$ m

1.43 $0.9 \times 10^4 Vm^{-1}$

1.44 V = 1800 V, E = 4000 Vm^{-1}

1.45 2×10^{-9}N, along the field, $\tau = 0.866 \times 10^{-11}$ Nm

1.46 360 N/C, 180 N C^{-1}

1.47 x = 0.25 m from +q

1.48 –0.255 J

1.49 5.70 J

1.50 1.67×10^5 Nm^2C^{-1}

1.51 zero, 10^5 N C^{-1}, 4.44×10^4 N C^{-1}

1.52 2.478×10^{-9}F

1.53 $C_1 = 15$ μF, $C_2 = 10$μF

1.54 q = 137.5 μC, $q_1 = 55$ μC, $q_2 = 82.5$ μC

1.55 0.3 J

1.56 $\varepsilon_r = 5$

1.57 50%

1.58 $q_1 = 144 \times 10^{-6}C$, $q_2 = 96 \times 10^{-6}C$, $q_3 = 48 \times 10^{-6}C$
$V_1 = 72$ V, $V_2 = 48$ V

1.59 3 pF, each one is 40 V

2. Current Electricity

The branch of Physics which deals with the study of motion of electric charges is called current electricity. In an uncharged metallic conductor at rest, some (not all) electrons are continually moving randomly through the conductor because they are very loosely attached to the nuclei. The thermodynamic internal energy of the material is sufficient to liberate the outer electrons from individual atoms, enabling the electrons to travel through the material. But the net flow of charge at any point is zero. Hence, there is zero current. These are termed as free electrons. The external energy necessary to drive the free electrons in a definite direction is called electromotive force (emf). The emf is not a force, but it is the work done in moving a unit charge from one end to the other. The flow of free electrons in a conductor constitutes electric current.

2.1 Electric current

The current is defined as the rate of flow of charges across any cross sectional area of a conductor. If a net charge q passes through any cross section of a conductor in time t, then the current $I = q / t$, where q is in coulomb and t is in second. The current I is expressed in ampere. If the rate of flow of charge is not uniform, the current varies with time and the instantaneous value of current i is given by,

$$i = \frac{dq}{dt}$$

Current is a scalar quantity. The direction of conventional current is taken as the direction of flow of positive charges or opposite to the direction of flow of electrons.

2.1.1 Drift velocity and mobility

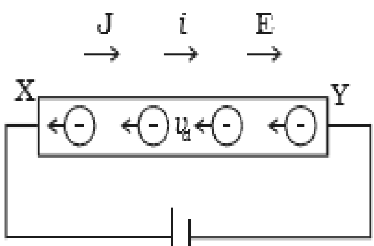


Fig 2.1 Current carrying conductor

Consider a conductor XY connected to a battery (Fig 2.1). A steady electric field E is established in the conductor in the direction X to Y. In the absence of an electric field, the free electrons in the conductor move randomly in all possible directions.

They do not produce current. But, as soon as an electric field is applied, the free electrons at the end Y experience a force F = eE in a direction opposite to the electric field. The electrons are accelerated and in the process they collide with each other and with the positive ions in the conductor.

Thus due to collisions, a backward force acts on the electrons and they are slowly drifted with a constant average drift velocity v_d in a direction opposite to electric field.

Drift velocity is defined as the velocity with which free electrons get drifted towards the positive terminal, when an electric field is applied.

If τ is the average time between two successive collisions and the acceleration experienced by the electron be a, then the drift velocity is given by,

$$v_d = a\tau$$

The force experienced by the electron of mass m is

$$F = ma$$

Hence $a = \frac{eE}{m}$

$$\therefore v_d = \frac{eE}{m}\tau = \mu E$$

where $\mu = \frac{e\tau}{m}$ is the mobility and is defined as the drift velocity acquired per unit electric field. It takes the unit $m^2V^{-1}s^{-1}$. The drift velocity of electrons is proportional to the electric field intensity. It is very small and is of the order of 0.1 cm s^{-1}.

2.1.2 Current density

Current density at a point is defined as the quantity of charge passing per unit time through unit area, taken perpendicular to the direction of flow of charge at that point.

The current density **J** for a current I flowing across a conductor having an area of cross section A is

$$\mathbf{J} = \frac{(q/t)}{A} = \frac{I}{A}$$

Current density is a vector quantity. It is expressed in A m^{-2}

2.1.3 Relation between current and drift velocity

Consider a conductor XY of length L and area of cross section A (Fig 2.1). An electric field E is applied between its ends. Let n be the number of free electrons per unit volume. The free electrons move towards the left with a constant drift velocity v_d.

The number of conduction electrons in the conductor = nAL

The charge of an electron = e

The total charge passing through the conductor $q = (nAL)\ e$

The time in which the charges pass through the conductor, $t = \frac{L}{v_d}$

The current flowing through the conductor, $I = \frac{q}{t} = \frac{(nAL)e}{(L/v_d)}$

$$I = nAev_d \qquad ...(1)$$

The current flowing through a conductor is directly proportional to the drift velocity.

From equation (1), $\frac{I}{A} = nev_d$

$$\boldsymbol{J} = nev_d \qquad \left[\because \boldsymbol{J} = \frac{I}{A}, current\ density\right]$$

2.1.4 Ohm's law

George Simon Ohm established the relationship between potential difference and current, which is known as Ohm's law. The current flowing through a conductor is,

$$I = nAev_d$$

But $v_d = \frac{eE}{m} . \tau$

$$\therefore \quad I = nAe\frac{eE}{m}\tau$$

$$I = \frac{nAe^2}{mL}\tau V \qquad \left[\because E = \frac{V}{L}\right]$$

where V is the potential difference. The quantity $\frac{mL}{nAe^2\tau}$ is a constant for a given conductor, called electrical resistance (R).

$$\therefore \quad I \propto V$$

The law states that, at a constant temperature, the steady current flowing through a conductor is directly proportional to the potential difference between the two ends of the conductor.

(i.e) $I \propto V$ or $I = \frac{1}{R}V$

$\therefore$ $V = IR$ or $R = \frac{V}{I}$

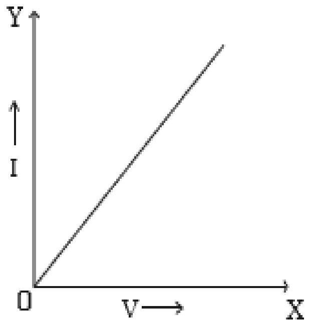


Fig 2.2 V–I graph of an ohmic conductor.

Resistance of a conductor is defined as the ratio of potential difference across the conductor to the current flowing through it. The unit of resistance is ohm (Ω)

The reciprocal of resistance is conductance. Its unit is mho (Ω^{-1}).

Since, potential difference V is proportional to the current I, the graph (Fig 2.2) between V and I is a straight line for a conductor. Ohm's law holds good only when a steady current flows through a conductor.

2.1.5 Electrical Resistivity and Conductivity

The resistance of a conductor R is directly proportional to the length of the conductor l and is inversely proportional to its area of cross section A.

$R \propto \frac{l}{A}$ or $R = \frac{\rho l}{A}$

ρ is called specific resistance or electrical resistivity of the material of the conductor.

If l = 1 m, A = 1 m^2, then ρ = R

The electrical resistivity of a material is defined as the resistance offered to current flow by a conductor of unit length having unit area of cross section. The unit of ρ is ohm–m (Ω m). It is a constant for a particular material.

The reciprocal of electrical resistivity, is called electrical conductivity, $\sigma = \frac{1}{\rho}$

The unit of conductivity is mho m^{-1} (Ω^{-1} m^{-1})

2.1.6 Classification of materials in terms of resistivity

The resistivity of a material is the characteristic of that particular material. The materials can be broadly classified into conductors and insulators. The metals and alloys which have low resistivity of the order of 10^{-6} – 10^{-8} Ω m are good conductors of electricity. They carry current without appreciable loss of energy. Example : silver, aluminium, copper, iron, tungsten, nichrome, manganin, constantan. The resistivity of metals increase with increase in temperature. Insulators are substances which have very high resistivity of the order of 10^{8} – 10^{14} Ω m. They offer very high resistance to the flow of current and are termed non–conductors. Example : glass, mica, amber, quartz, wood, teflon, bakelite. In between these two classes of materials lie the semiconductors (Table 2.1). They are partially conducting. The resistivity of semiconductor is 10^{-2} – 10^{4} Ω m. Example : germanium, silicon.

Table 2.1 Electrical resistivities at room temperature
(NOT FOR EXAMINATION)

Classification	Material	ρ (Ω m)
conductors	silver	1.6×10^{-8}
	copper	1.7×10^{-8}
	aluminium	2.7×10^{-8}
	iron	10×10^{-8}
Semiconductors	germanium	0.46
	silicon	2300
Insulators	glass	10^{10} – 10^{14}
	wood	10^{8} – 10^{11}
	quartz	10^{13}
	rubber	10^{13} – 10^{16}

2.2 Superconductivity

Ordinary conductors of electricity become better conductors at lower temperatures. The ability of certain metals, their compounds and alloys to conduct electricity with zero resistance at very low temperatures is called superconductivity. The materials which exhibit this property are called superconductors.

The phenomenon of superconductivity was first observed by Kammerlingh Onnes in 1911. He found that mercury suddenly showed zero resistance at 4.2 K (Fig 2.3). The first theoretical explanation of superconductivity was given by Bardeen, Cooper and Schrieffer in 1957 and it is called the BCS theory.

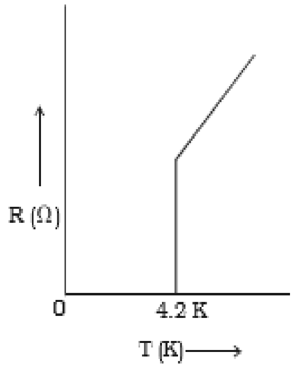


Fig 2.3 Superconductivity of mercury

The temperature at which electrical resistivity of the material suddenly drops to zero and the material changes from normal conductor to a superconductor is called the transition temperature or critical temperature T_C. At the transition temperature the following changes are observed :

(i) The electrical resistivity drops to zero.

(ii) The conductivity becomes infinity

(iii) The magnetic flux lines are excluded from the material.

Applications of superconductors

(i) Superconductors form the basis of energy saving power systems, namely the superconducting generators, which are smaller in size and weight, in comparison with conventional generators.

(ii) Superconducting magnets have been used to levitate trains above its rails. They can be driven at high speed with minimal expenditure of energy.

(iii) Superconducting magnetic propulsion systems may be used to launch satellites into orbits directly from the earth without the use of rockets.

(iv) High efficiency ore–separating machines may be built using superconducting magnets which can be used to separate tumor cells from healthy cells by high gradient magnetic separation method.

(v) Since the current in a superconducting wire can flow without any change in magnitude, it can be used for transmission lines.

(vi) Superconductors can be used as memory or storage elements in computers.

2.3 Carbon resistors

The wire wound resistors are expensive and huge in size. Hence, carbon resistors are used. Carbon resistor consists of a ceramic core, on which a thin layer of crystalline carbon is deposited. These resistors are cheaper, stable and small in size. The resistance of a carbon resistor is indicated by the colour code drawn on it (Table 2.2). A three colour code carbon resistor is discussed here. The silver or gold ring at one end corresponds to the tolerance. It is a tolerable range (±) of the resistance. The tolerance of silver, gold, red and brown rings is 10%, 5%, 2% and 1% respectively. If there is no coloured ring at this end, the tolerance is 20%. The first two rings at the other end of tolerance ring are significant figures of resistance in ohm. The third ring indicates the powers of 10 to be multiplied or number of zeroes following the significant figure.

Table 2.2 Colour code for carbon resistors

Colour	Number
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9

Example :

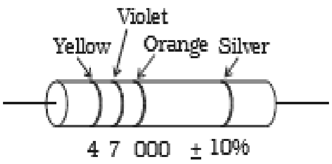


Fig 2.4 Carbon resistor colour code.

The first yellow ring in Fig 2.4 corresponds to 4. The next violet ring corresponds to 7. The third orange ring corresponds to 10^3. The silver ring represents 10% tolerance. The total resistance is $47 \times 10^3 \pm 10\%$ i.e. 47 k Ω, 10%. Fig 2.5 shows 1 k Ω, 5% carbon resistor.

Presently four colour code carbon resistors are also used. For certain critical applications 1% and 2% tolerance resistors are used.

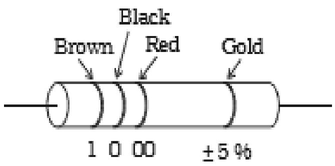


Fig 2.5 Carbon resistor

2.4 Combination of resistors

In simple circuits with resistors, Ohm's law can be applied to find the effective resistance. The resistors can be connected in series and parallel.

2.4.1 Resistors in series

Let us consider the resistors of resistances R_1, R_2, R_3 and R_4 connected in series as shown in Fig 2.6.

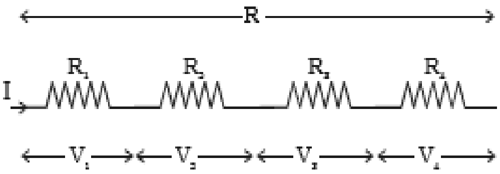


Fig 2.6 Resistors in series

When resistors are connected in series, the current flowing through each resistor is the same. If the potential difference applied between the ends of the combination of resistors is V, then the potential difference across each resistor R_1, R_2, R_3 and R_4 is V_1, V_2, V_3 and V_4 respectively.

The net potential difference $V = V_1 + V_2 + V_3 + V_4$

By Ohm's law

$V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$, $V_4 = IR_4$ and $V = IR_s$

where R_S is the equivalent or effective resistance of the series combination.

Hence, $IR_S = IR_1 + IR_2 + IR_3 + IR_4$ or $R_S = R_1 + R_2 + R_3 + R_4$

Thus, the equivalent resistance of a number of resistors in series connection is equal to the sum of the resistance of individual resistors.

2.4.2 Resistors in parallel

Consider four resistors of resistances R_1, R_2, R_3 and R_4 are connected in parallel as shown in Fig 2.7. A source of emf V is connected to the parallel combination. When resistors are in parallel, the potential difference (V) across each resistor is the same.

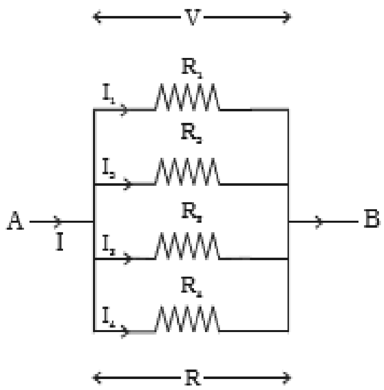


Fig 2.7 Resistors in parallel

A current I entering the combination gets divided into I_1, I_2, I_3 and I_4 through R_1, R_2, R_3 and R_4 respectively,

such that $I = I_1 + I_2 + I_3 + I_4$.

By Ohm's law

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2},\ I_3 = \frac{V}{R_3},\ I_4 = \frac{V}{R_4} \text{ and } I = \frac{V}{R_P}$$

where R_P is the equivalent or effective resistance of the parallel combination.

$$\therefore \frac{V}{R_P} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

Thus, when a number of resistors are connected in parallel, the sum of the reciprocal of the resistance of the individual resistors is equal to the reciprocal of the effective resistance of the combination.

2.5 Temperature dependence of resistance

The resistivity of substances varies with temperature. For conductors the resistance increases with increase in temperature. If R_o is the resistance of a conductor at 0^o C and R_t is the resistance of same conductor at t^o C, then

$$R_t = R_o (1 + \alpha t)$$

where α is called the temperature coefficient of resistance.

$$\alpha = \frac{R_t - R_o}{R_o t}$$

The temperature coefficient of resistance is defined as the ratio of increase in resistance per degree rise in temperature to its resistance at 0^o C. Its unit is per ^{o}C.

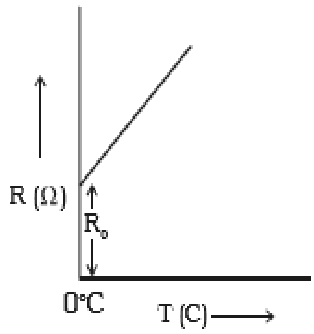


Fig 2.8 Variation of resistance with temperature

The variation of resistance with temperature is shown in Fig 2.8.

Metals have positive temperature coefficient of resistance, i.e., their resistance increases with increase in temperature. Insulators and semiconductors have negative temperature coefficient of resistance, i.e., their resistance decreases with increase in temperature. A material with a negative temperature coefficient is called a thermistor. The temperature coefficient is low for alloys.

2.6 Internal resistance of a cell

The electric current in an external circuit flows from the positive terminal to the negative terminal of the cell, through different circuit elements. In order to maintain continuity, the current has to flow through the electrolyte of the cell, from its negative terminal to positive terminal. During this process of flow of current inside the cell, a resistance is offered to current flow by the electrolyte of the cell. This is termed as the internal resistance of the cell.

A freshly prepared cell has low internal resistance and this increases with ageing.

Determination of internal resistance of a cell using voltmeter

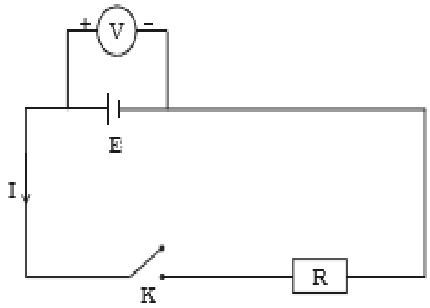


Fig 2.9 Internal resistance of a cell using voltmeter.

The circuit connections are made as shown in Fig 2.9. With key K open, the emf of cell E is found by connecting a high resistance voltmeter across it. Since the high resistance voltmeter draws only a very feeble current for deflection, the circuit may be considered as an open circuit. Hence the voltmeter reading gives the emf of the cell. A small value of resistance R is included in the external circuit and key K is closed. The potential difference across R is equal to the potential difference across cell (V).

The potential drop across R, $V = IR$ ...(1)

Due to internal resistance r of the cell, the voltmeter reads a value V, less than the emf of cell.

Then $V = E - Ir$ or $Ir = E - V$ ...(2)

Dividing equation (2) by equation (1)

$$\frac{Ir}{IR} = \frac{E-V}{V} \quad \text{or} \quad r = \left(\frac{E-V}{V}\right)R$$

Since E, V and R are known, the internal resistance r of the cell can be determined.

2.7 Kirchoff's law

Ohm's law is applicable only for simple circuits. For complicated circuits, Kirchoff's laws can be used to find current or voltage. There are two generalised laws : (i) Kirchoff's current law (ii) Kirchoff's voltage law

Kirchoff's first law (current law)

Kirchoff's current law states that the algebraic sum of the currents meeting at any junction in a circuit is zero.

The convention is that, the current flowing towards a junction is positive and the current flowing away from the junction is negative. Let 1,2,3,4 and 5 be the conductors meeting at a junction O in an electrical circuit (Fig 2.10). Let I_1, I_2, I_3, I_4 and I_5 be the currents passing through the conductors respectively. According to Kirchoff's first law.

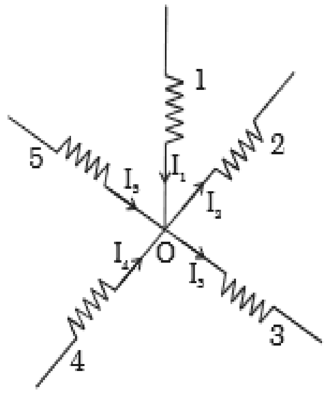


Fig 2.10 Kirchoff's current law

$$I_1 + (-I_2) + (-I_3) + I_4 + I_5 = 0 \quad \text{or} \quad I_1 + I_4 + I_5 = I_2 + I_3.$$

The sum of the currents entering the junction is equal to the sum of the currents leaving the junction. This law is a consequence of conservation of charges.

Kirchoff's second law (voltage law)

Kirchoff's voltage law states that the algebraic sum of the products of resistance and current in each part of any closed circuit is equal to the algebraic sum of the emf's in that closed circuit. This law is a consequence of conservation of energy.

In applying Kirchoff's laws to electrical networks, the direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the problems, the current will be found to have negative sign. If the result is positive, then the assumed direction is the same as actual direction.

It should be noted that, once the particular direction has been assumed, the same should be used throughout the problem. However, in the application of Kirchoff's second law, we follow that the current in clockwise direction is taken as positive and the current in anticlockwise direction is taken as negative.

Let us consider the electric circuit given in Fig 2.11a.

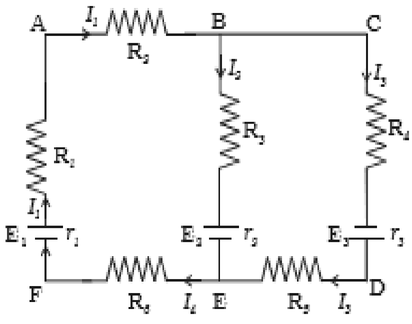


Fig 2.11a Kirchoff's laws

Considering the closed loop ABCDEFA,

$$I_1R_2 + I_3R_4 + I_3r_3 + I_3R_5 + I_4R_6 + I_1r_1 + I_1R_1 = E_1 + E_3$$

Both cells E_1 and E_3 send currents in clockwise direction.

For the closed loop ABEFA

$$I_1R_2 + I_2R_3 + I_2r_2 + I_4R_6 + I_1r_1 + I_1R_1 = E_1 - E_2$$

Negative sIgn in E_2 indicates that it sends current in the anticlockwise direction.

As an illustration of application of Kirchoff's second law, let us calculate the current in the following networks.

Illustration I

Applying first law to the Junction B, (FIg.2.11b)

$$I_1 - I_2 - I_3 = 0$$

$$\therefore \quad I_3 = I_1 - I_2 \quad ...(1)$$

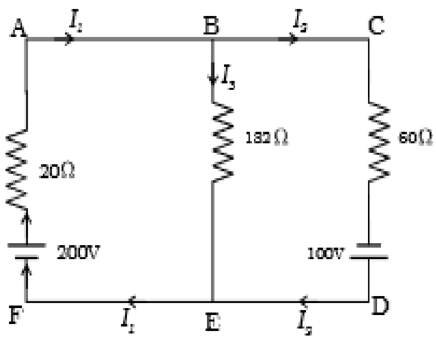


Fig 2.11b Kirchoff's laws

For the closed loop ABEFA,

$$132\ I_3 + 20I_1 = 200 \ ...(2)$$

Substituting equation (1) in equation (2)

$$132\ (I_1 - I_2) + 20I_1 = 200$$

$$152I_1 - 132I_2 = 200 \ ...(3)$$

For the closed loop BCDEB,

$$60I_2 - 132I_3 = 100$$

substituting for I_3,

$$\therefore \quad 60I_2 - 132\ (I_1 - I_2) = 100$$

$$-132I_1 + 192I_2 = 100 \ ...(4)$$

Solving equations (3) and (4), we obtain

$$I_1 = 4.39 \text{ A and } I_2 = 3.54 \text{ A}$$

Illustration 2

Taking the current in the clockwise direction along ABCDA as positive (FIg 2.11c)

$$10\,I + 0.5\,I + 5\,I + 0.5\,I + 8\,I + 0.5\,I + 5\,I + 0.5\,I + 10\,I = 50 - 70 - 30 + 40$$

$$I\,(10 + 0.5 + 5 + 0.5 + 8 + 0.5 + 5 + 0.5 + 10) = -10$$

$$40\,I = -10$$

$$\therefore \quad I = \frac{-10}{40} = -0.25\ A$$

The negative sign indicates that the current flows in the anticlockwise direction.

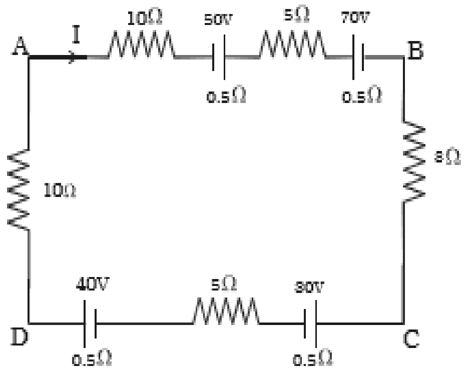


Fig 2.11c Kirchoff's laws

2.7.1 Wheatstone's bridge

An important application of Kirchoff's law is the Wheatstone's bridge (FIg 2.12). Wheatstone's network consists of resistances P, Q, R and S connected to form a closed path. A cell of emf E is connected between points A and C. The current I from the cell is divided into I_1, I_2, I_3 and I_4 across the four branches. The current through the galvanometer is I_g. The resistance of galvanometer is G.

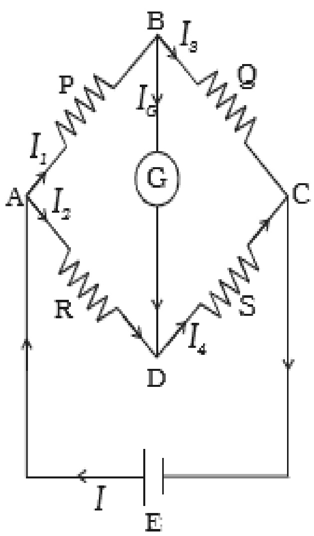


Fig 2.12
Wheatstone's bridge

Applying Kirchoff's current law to junction B,

$$I_1 - I_g - I_3 = 0 \qquad ...(1)$$

Applying Kirchoff's current law to junction D

$$I_2 + I_g - I_4 = 0 \qquad ...(2)$$

Applying Kirchoff's voltage law to closed path ABDA

$$I_1\,P + I_g G - I_2\,R = 0 \qquad ...(3)$$

Applying Kirchoff's voltage law to closed path ABCDA

$$I_1 P + I_3 Q - I_4 S - I_2 R = 0 \qquad ...(4)$$

When the galvanometer shows zero deflection, the points B and D are at same potential and $I_g = 0$. Substituting $I_g = 0$ in equation (1), (2) and (3)

$$I_1 = I_3 \qquad \text{...(5)}$$

$$I_2 = I_4 \qquad \text{...(6)}$$

$$I_1P = I_2R \qquad \text{...(7)}$$

Substituting the values of (5) and (6) in equation (4)

$$I_1P + I_1Q - I_2S - I_2R = 0$$

$$I_1 (P + Q) = I_2 (R+S) \qquad \text{...(8)}$$

Dividing (8) by (7)

$$\frac{I_1(P+Q)}{I_1P} = \frac{I_2(R+S)}{I_2R}$$

$$\therefore \frac{P+Q}{P} = \frac{R+S}{R}$$

$$1+\frac{Q}{P} = 1+\frac{S}{R}$$

$$\therefore \frac{Q}{P} = \frac{S}{R} \quad \text{or} \quad \frac{P}{Q} = \frac{R}{S}$$

This is the condition for bridge balance. If P, Q and R are known, the resistance S can be calculated.

2.7.2 Metre bridge

Metre bridge is one form of Wheatstone's bridge. It consists of thick strips of copper, of negligible resistance, fixed to a wooden board. There are two gaps G_1 and G_2 between these strips. A uniform manganin wire AC of length one metre whose temperature coefficient is low, is stretched along a metre scale and its ends are soldered to two copper strips. An unknown resistance P is connected in the gap G_1 and a standard resistance Q is connected in

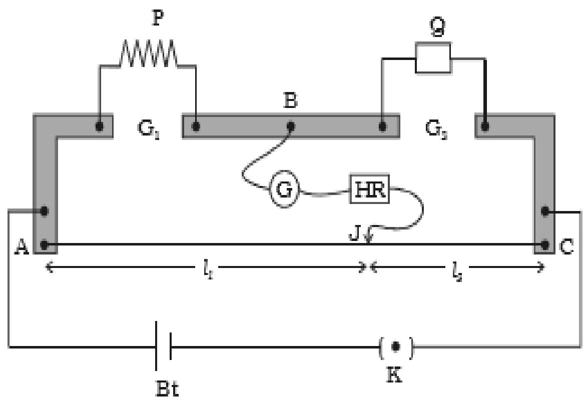


Fig 2.13 Metre bridge

the gap G_2 (Fig 2.13). A metal jockey J is connected to B through a galvanometer (G) and a high resistance (HR) and it can make contact at any point on the wire AC. Across the two ends of the wire, a Leclanche cell and a key are connected.

Adjust the position of metal jockey on metre bridge wire so that the galvanometer shows zero deflection. Let the point be J. The portions AJ and JC of the wire now replace the resistances R and S of Wheatstone's bridge. Then

$$\frac{P}{Q} = \frac{R}{S} = \frac{r.AJ}{r.JC}$$

where r is the resistance per unit length of the wire.

$$\therefore \quad \frac{P}{Q} = \frac{AJ}{JC} = \frac{l_1}{l_2}$$

where AJ = l_1 and JC = l_2

$$\therefore \quad P = Q\frac{l_1}{l_2}$$

Though the connections between the resistances are made by thick copper strips of negligible resistance, and the wire AC is also soldered to such strips a small error will occur in the value of $\frac{l_1}{l_2}$ due to the end resistance. This error can be eliminated, if another set of readings are taken with P and Q interchanged and the average value of P is found, provided the balance point J is near the mid point of the wire AC.

2.7.3 *Determination of specific resistance*

The specific resistance of the material of a wire is determined by knowing the resistance (P), radius (r) and length (L) of the wire using the expression $\rho = \frac{P\pi r^2}{L}$

2.7.4 *Determination of temperature coefficient of resistance*

If R_1 and R_2 are the resistances of a given coil of wire at the temperatures t_1 and t_2, then the temperature coefficient of resistance of the material of the coil is determined using the relation,

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

2.8 Potentiometer

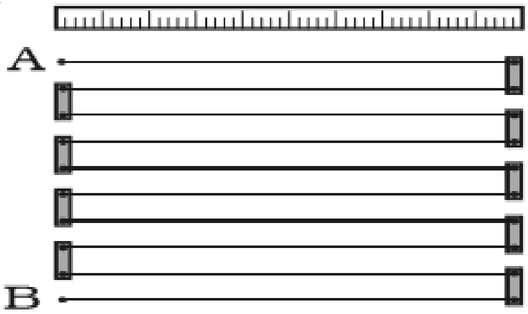


Fig 2.14 Potentiometer

The Potentiometer is an instrument used for the measurement of potential difference (Fig 2.14). It consists of a ten metre long uniform wire of manganin or constantan stretched in ten segments, each of one metre length. The segments are stretched parallel to each other on a horizontal wooden board. The ends of the wire are fixed to copper strips with binding screws. A metre scale is fixed on the board, parallel to the wire. Electrical contact with wires is established by pressing the jockey J.

2.8.1 *Principle of potentiometer*

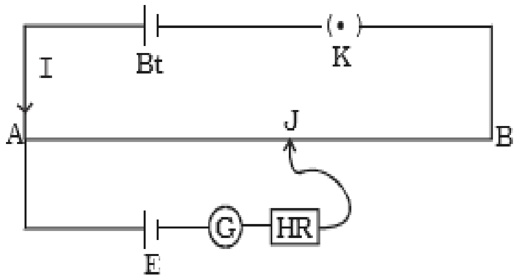


Fig 2.15 Principle of potentiometer

A battery Bt is connected between the ends A and B of a potentiometer wire through a key K. A steady current I flows through the potentiometer wire (Fig 2.15). This forms the primary circuit. A primary cell is connected in series with the positive terminal A of the potentiometer, a galvanometer, high resistance and jockey. This forms the secondary circuit.

If the potential difference between A and J is equal to the emf of the cell, no current flows through the galvanometer. It shows zero deflection. AJ is called the balancing length. If the balancing length is l, the potential difference across AJ = Irl where r is the resistance per unit length of the potentiometer wire and I the current in the primary circuit.

$\therefore$ E = Irl,

since I and r are constants, E α l

Hence emf of the cell is directly proportional to its balancing length. This is the principle of a potentiometer.

2.8.2 Comparison of emfs of two given cells using potentiometer

The potentiometer wire AB is connected in series with a battery (Bt), Key (K), rheostat (Rh) as shown in Fig 2.16. This forms the primary circuit. The end A of potentiometer is connected to the terminal C of a DPDT switch (six way key–double pole double throw). The terminal D is connected to the jockey (J) through a galvanometer (G) and high resistance (HR). The cell of emf E_1 is connected between terminals C_1 and D_1 and the cell of emf E_2 is connected between C_2 and D_2 of the DPDT switch.

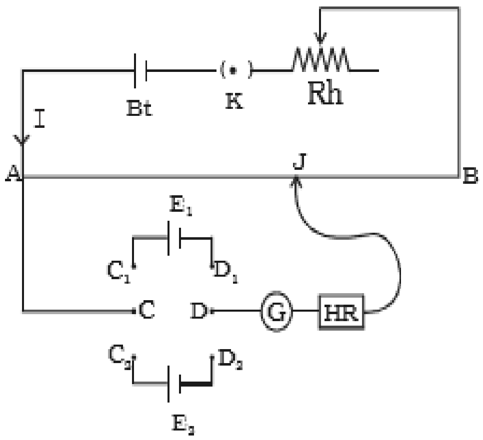


Fig 2.16 comparison of emf of two cells

Let I be the current flowing through the primary circuit and r be the resistance of the potentiometer wire per metre length.

The DPDT switch is pressed towards C_1, D_1 so that cell E_1 is included in the secondary circuit. The jockey is moved on the wire and adjusted for zero deflection in galvanometer. The balancing length is l_1. The potential difference across the balancing length l_1 = Irl_1. Then, by the principle of potentiometer,

$$E_1 = Irl_1 \qquad ...(1)$$

The DPDT switch is pressed towards E_2. The balancing length l_2 for zero deflection in galvanometer is determined. The potential difference across the balancing length is l_2 = Irl_2, then

$$E_2 = Irl_2 \qquad ...(2)$$

Dividing (1) and (2) we get

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

If emf of one cell (E_1) is known, the emf of the other cell (E_2) can be calculated using the relation.

$$E_2 = E_1\frac{l_2}{l_1}$$

2.8.3 Comparison of emf and potential difference

1. The difference of potentials between the two terminals of a cell in an open circuit is called the electromotive force (emf) of a cell. The difference in potentials between any two points in a closed circuit is called potential difference.

2. The emf is independent of external resistance of the circuit, whereas potential difference is proportional to the resistance between any two points.

2.9 Electric energy and electric power.

If I is the current flowing through a conductor of resistance R in time t, then the quantity of charge flowing is, $q = It$. If the charge q, flows between two points having a potential difference V, then the work done in moving the charge is $= V.\ q = V\ It$.

Then, electric power is defined as the rate of doing electric work.

$$\therefore \quad \text{Power} = \frac{Work\ done}{time} = \frac{VIt}{t} = VI$$

Electric power is the product of potential difference and current strength.

Since V = IR, Power = I^2R

Electric energy is defined as the capacity to do work. Its unit is joule. In practice, the electrical energy is measured by watt hour (Wh) or kilowatt hour (kWh). 1 kWh is known as one unit of electric energy.

(1 kWh = 1000 Wh = 1000×3600 J = 36×10^5 J)

2.9.1 Wattmeter

A wattmeter is an instrument used to measure electrical power consumed i.e energy absorbed in unit time by a circuit. The wattmeter consists of a movable coil arranged between a pair of fixed coils in the form of a solenoid. A pointer is attached to the movable coil. The free end of the pointer moves over a circular scale. When current flows through the coils, the deflection of the pointer is directly proportional to the power.

2.10 Chemical effect of current

The passage of an electric current through a liquid causes chemical changes and this process is called electrolysis. The conduction is possible, only in liquids wherein charged ions can be dissociated in opposite directions (Fig 2.17). Such liquids are called electrolytes. The plates through which current enters and leaves an electrolyte are known as electrodes. The electrode towards which positive ions travel is called the cathode and the other, towards which negative ions travel is called anode. The positive ions are called cations and are mostly formed from metals or hydrogen. The negative ions are called anions.

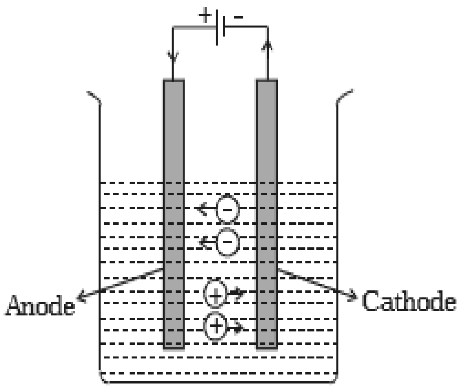


Fig 2.17 Conduction in liquids

2.10.1 Faraday's laws of electrolysis

The factors affecting the quantities of matter liberated during the process of electrolysis were investigated by Faraday.

First Law : The mass of a substance liberated at an electrode is directly proportional to the charge passing through the electrolyte.

If an electric current I is passed through an electrolyte for a time t, the amount of charge (q) passed is It. According to the law, mass of substance liberated (m) is

$$m \propto q \quad \text{or} \quad m = zIt$$

where Z is a constant for the substance being liberated called as electrochemical equivalent. Its unit is kg C^{-1}.

The electrochemical equivalent of a substance is defined as the mass of substance liberated in electrolysis when one coulomb charge is passed through the electrolyte.

Second Law : The mass of a substance liberated at an electrode by a given amount of charge is proportional to the *chemical equivalent of the substance.

If E is the chemical equivalent of a substance, from the second law

$$m \propto E$$

$$\text{*Chemical equivalent} = \frac{\text{Relative atomic mass}}{\text{Valency}} = \frac{\text{mass of the atom}}{1/12 \text{ of the mass } C^{12} \text{ atom x valency}}$$

2.10.2 Verification of Faraday's laws of electrolysis

First Law : A battery, a rheostat, a key and an ammeter are connected in series to an electrolytic cell (Fig 2.18). The cathode is cleaned, dried, weighed and then inserted in the cell. A current I_1 is passed for a time t. The current is measured by the ammeter. The cathode is taken out, washed, dried and weighed again. Hence the mass m_1 of the substance deposited is obtained.

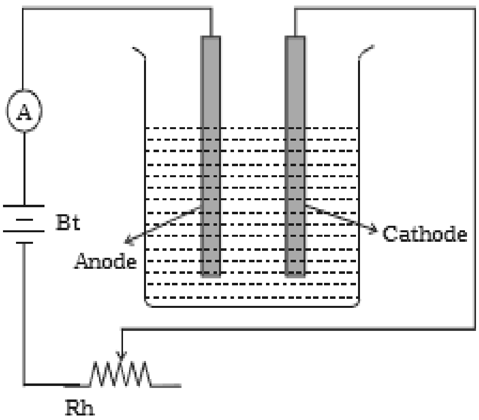


Fig 2.18 Verification of Faraday's first law

The cathode is reinserted in the cell and a different current I_2 is passed for the same time t. The mass m_2 of the deposit is obtained. It is found that

$$\frac{m_1}{m_2} = \frac{I_1}{I_2}$$

$$\therefore \quad m \propto I \qquad ...(1)$$

The experiment is repeated for same current I but for different times t_1 and t_2. If the masses of the deposits are m_3 and m_4 respectively, it is found that

$$\frac{m_3}{m_4} = \frac{t_1}{t_2}$$

$$\therefore \quad m \propto t \qquad ...(2)$$

From relations (1) and (2)

$m \propto It$ or $m \propto q$ Thus, the first law is verified.

Second Law : Two electrolytic cells containing different electrolytes, $CuSO_4$ solution and $AgNO_3$ solution are connected in series with a battery, a rheostat and an ammeter (Fig 2.19). Copper electrodes are inserted in $CuSO_4$ and silver electrodes are inserted in $AgNO_3$.

The cathodes are cleaned, dried, weighed and then inserted in the respective cells. The current is passed for some time. Then the cathodes are taken out, washed, dried and weighed. Hence the masses of copper and silver deposited are found as m_1 and m_2.

It is found that $\frac{m_1}{m_2}=\frac{E_1}{E_2}$, where E_1 and E_2 are the chemical equivalents of copper and silver respectively.

$m \propto E$

Thus, the second law is verified.

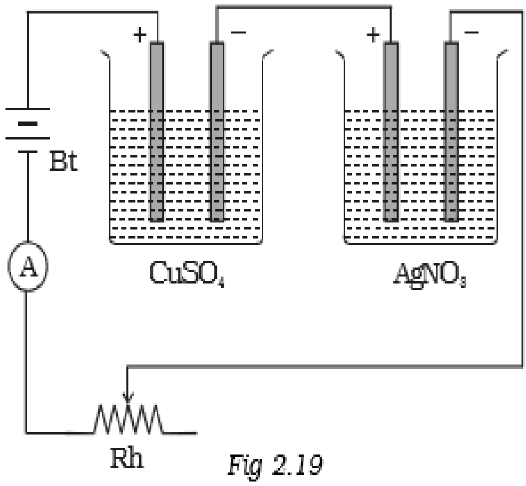


Fig 2.19
Verification of Faraday's second law

2.11 Electric cells

The starting point to the development of electric cells is the classic experiment by Luige Galvani and his wife Lucia on a dissected frog hung from iron railings with brass hooks. It was observed that, whenever the leg of the frog touched the iron railings, it jumped and this led to the introduction of animal electricity. Later, Italian scientist and genius professor Alessandro Volta came up with an electrochemical battery. The battery Volta named after him consisted of a pile of copper and zinc discs placed alternately separated by paper and introduced in salt solution. When the end plates were connected to an electric bell, it continued to ring, opening a new world of electrochemical cells. His experiment established that, a cell could be made by using two dissimilar metals and a salt solution which reacts with atleast one of the metals as electrolyte.

2.11.1 Voltaic cell

The simple cell or voltaic cell consists of two electrodes, one of copper and the other of zinc dipped in a solution of dilute sulphuric acid in a glass vessel (Fig 2.20). On connecting the two electrodes externally, with a piece of wire, current flows

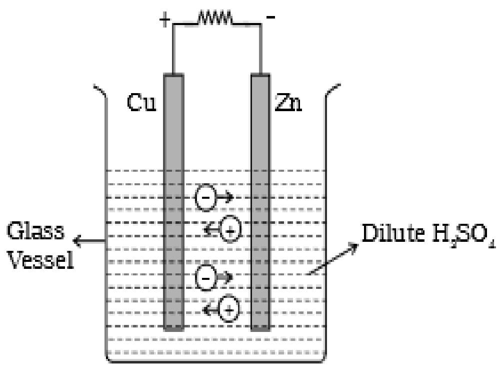


Fig 2.20 Voltaic cell

from copper to zinc outside the cell and from zinc to copper inside it. The copper electrode is the positive pole or copper rod of the cell and zinc is the negative pole or zinc rod of the cell. The electrolyte is dilute sulphuric acid.

The action of the cell is explained in terms of the motion of the charged ions. At the zinc rod, the zinc atoms get ionized and pass into solution as Zn^{++} ions. This leaves the zinc rod with two electrons more, making it negative. At the same time, two hydrogen ions ($2H^+$) are discharged at the copper rod, by taking these two electrons. This makes the copper rod positive. As long as excess electrons are available on the zinc electrode, this process goes on and a current flows continuously in external circuit. This simple cell is thus seen as a device which converts chemical energy into electrical energy. Due to opposite charges on the two plates, a potential difference is set up between copper and zinc, copper being at a higher potential than zinc. The difference of potential between the two electrodes is 1.08V.

2.11.2 Primary Cell

The cells from which the electric energy is derived by irreversible chemical actions are called primary cells. The primary cell is capable of giving an emf, when its constituents, two electrodes and a suitable electrolyte, are assembled together. The three main primary cells, namely Daniel Cell and Leclanche cell are discussed here. These cells cannot be recharged electrically.

2.11.3 Daniel cell

Daniel cell is a primary cell which cannot supply steady current for a long time. It consists of a copper vessel containing a strong solution of copper sulphate (Fig 2.21). A zinc rod is dipped in dilute sulphuric acid contained in a porous pot. The porous pot is placed inside the copper sulphate solution.

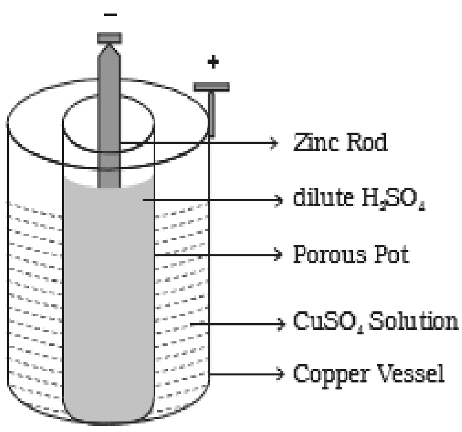


Fig 2.21 Daniel cell

The zinc rod reacting with dilute sulphuric acid produces Zn^{++} ions and 2 electrons.

Zn^{++} ions pass through the pores of the porous pot and reacts with copper sulphate solution, producing Cu^{++} ions. The Cu^{++} ions deposit on the copper vessel. When Daniel cell is connected in a circuit, the two electrons on the zinc rod pass through the external circuit and reach the copper vessel thus neutralizing the copper ions. This constitutes an electric current from copper to zinc. Daniel cell produces an emf of 1.08 volt.

2.11.4 Leclanche cell

A Leclanche cell consists of a carbon electrode packed in a porous pot containing manganese dioxide and charcoal powder (Fig 2.22). The porous pot is immersed in a saturated solution of ammonium chloride (electrolyte) contained in an outer glass vessel. A zinc rod is immersed in electrolytic solution.

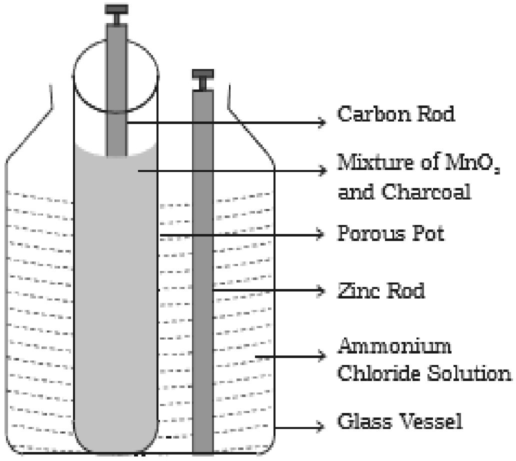


Fig 2.22 Leclanche cell

At the zinc rod, due to oxidation reaction Zn atom is converted into Zn^{++} ions and 2 electrons. Zn^{++} ions reacting with ammonium chloride produces zinc chloride and ammonia gas.

i.e $\quad Zn^{++} + 2\ NH_4Cl \rightarrow 2NH_3 + ZnCl_2 + 2\ H^+ + 2e^-$

The ammonia gas escapes. The hydrogen ions diffuse through the pores of the porous pot and react with manganese dioxide. In this process the positive charge of hydrogen ion is transferred to carbon rod. When zinc rod and carbon rod are connected externally, the two electrons from the zinc rod move towards carbon and neutralizes the positive charge. Thus current flows from carbon to zinc.

Leclanche cell is useful for supplying intermittent current. The emf of the cell is about 1.5 V, and it can supply a current of 0.25 A.

2.11.5 Secondary Cells

The advantage of secondary cells is that they are rechargeable. The chemical reactions that take place in secondary cells are reversible. The active materials that are used up when the cell delivers current can be reproduced by passing current through the cell in opposite direction. The chemical process of obtaining current from a secondary cell is called discharge. The process of reproducing active materials is called charging. The most common secondary cells are lead acid accumulator and alkali accumulator.

2.11.6 Lead – Acid accumulator

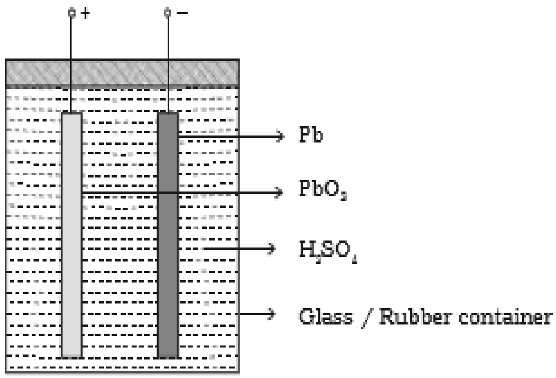


Fig 2.23 Lead - Acid accumulator

The lead acid accumulator consists of a container made up of hard rubber or glass or celluloid. The container contains dilute sulphuric acid which acts as the electrolyte. Spongy lead (Pb) acts as the negative electrode and lead oxide (PbO_2) acts as the positive electrode (Fig 2.23). The electrodes are separated by suitable insulating materials and assembled in a way to give low internal resistance.

When the cell is connected in a circuit, due to the oxidation reaction that takes place at the negative electrode, spongy lead reacting with dilute sulphuric acid produces lead sulphate and two electrons. The electrons flow in the external circuit from negative electrode to positive electrode where the reduction action takes place. At the positive electrode, lead oxide on reaction with sulphuric acid produces lead sulphate and the two electrons are neutralized in this process. This makes the conventional current to flow from positive electrode to negative electrode in the external circuit.

The emf of a freshly charged cell is 2.2 Volt and the specific gravity of the electrolyte is 1.28. The cell has low internal resistance and hence can deliver high current. As the cell is discharged by drawing current from it, the emf falls to about 2 volts. In the process of charging, the chemical reactions are reversed.

2.11.7 Applications of secondary cells

The secondary cells are rechargeable. They have very low internal resistance. Hence they can deliver a high current if required. They can be recharged a very large number of times without any deterioration in properties. These cells are huge in size. They are used in all automobiles like cars, two wheelers, trucks etc. The state of charging these cells is, simply monitoring the specific gravity of the electrolyte. It should lie between 1.28 to 1.12 during charging and discharging respectively.

Solved problems

2.1 If 6.25×10^{18} electrons flow through a given cross section in unit time, find the current. (Given : Charge of an electron is 1.6×10^{-19} C)

Data : $n = 6.25 \times 10^{18}$; $e = 1.6 \times 10^{-19}$ C ; t = 1 s ; I = ?

Solution : $I = \frac{q}{t} = \frac{ne}{t} = \frac{6.25 \times 10^{18} \times 1.6 \times 10^{-19}}{1} = 1$ A

2.2 A copper wire of 10^{-6} m^2 area of cross section, carries a current of 2 A. If the number of electrons per cubic metre is 8×10^{28}, calculate the current density and average drift velocity.

(Given $e = 1.6 \times 10^{-19}$C)

Data : $A = 10^{-6}$ m^2 ; Current flowing I = 2 A ; $n = 8 \times 10^{28}$

$e = 1.6 \times 10^{-19}$ C ; J = ? ; v_d =?

Solution : Current density, $J = \frac{I}{A} = \frac{2}{10^{-6}} = 2 \times 10^6$A/m^2

$J = n\, e\, v_d$

or $v_d = \frac{J}{ne} = \frac{2 \times 10^6}{8 \times 10^{28} \times 1.6 \times 10^{-19}} = 15.6 \times 10^{-5}$ m s^{-1}

2.3 An incandescent lamp is operated at 240 V and the current is 0.5 A. What is the resistance of the lamp ?

Data : V = 240 V ; I = 0.5 A ; R = ?

Solution : From Ohm's law

$$V = IR \quad \text{or} \quad R = \frac{V}{I} = \frac{240}{0.5} = 480\ \Omega$$

2.4 The resistance of a copper wire of length 5m is 0.5 Ω. If the diameter of the wire is 0.05 cm, determine its specific resistance.

Data : l = 5m ; R = 0.5 Ω ; d = 0.05 cm = 5×10^{-4} m ;

$r = 2.5 \times 10^{-4}$ m ; ρ = ?

Solution : $R = \frac{\rho l}{A}$ or $\rho = \frac{RA}{l}$

$$A = \pi r^2 = 3.14 \times (2.5 \times 10^{-4})^2 = 1.9625 \times 10^{-7} m^2$$

$$\rho = \frac{0.5 \times 1.9625 \times 10^{-7}}{5}$$

$$\rho = 1.9625 \times 10^{-8}\ \Omega m$$

2.5 The resistance of a nichrome wire at 0°C is 10 Ω. If its temperature coefficient of resistance is 0.004/°C, find its resistance at boiling point of water. Comment on the result.

Data : At 0°C, R_o = 10 Ω ; α = 0.004/°C ; t = 100°C ;

At t°C, R_t = ?

Solution : $R_t = R_o (1 + \alpha t)$

$= 10 (1 + (0.004 \times 100))$

$R_t = 14\ \Omega$

As temperature increases the resistance of wire also increases.

2.6 Two wires of same material and length have resistances 5 Ω and 10 Ω respectively. Find the ratio of radii of the two wires.

Data : Resistance of first wire R_1 = 5 Ω ;

Radius of first wire = r_1

Resistance of second wire R_2 = 10 Ω

Radius of second wire = r_2

Length of the wires = l

Specific resistance of the material of the wires = ρ

Solution : $R = \frac{\rho l}{A}; A = \pi r^2$

$$\therefore R_1 = \frac{\rho l}{\pi r_1^2};\ R_2 = \frac{\rho l}{\pi r_2^2}$$

$$\frac{R_2}{R_1} = \frac{r_1^2}{r_2^2} \text{ or } \frac{r_1}{r_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{10}{5}} = \frac{\sqrt{2}}{1}$$

$$r_1 : r_2 = \sqrt{2} : 1$$

2.7 If a copper wire is stretched to make it 0.1% longer, what is the percentage change in resistance?

Data : Initial length of copper wire $l_1 = l$

Final length of copper wire after stretching

$$l_2 = l + 0.1\% \text{ of } l$$

$$= l + \frac{0.1}{100} l$$

$$= l\,(1 + 0.001)$$

$$l_2 = 1.001\, l$$

During stretching, if length increases, area of cross section decreases.

Initial volume = $A_1 l_1 = A_1 l$

Final volume = $A_2 l_2 = 1.001\ A_2 l$

Resistance of wire before stretching = R_1.

Resistance after stretching = R_2

Solution : Equating the volumes

$$A_1 l = 1.001\ A_2 l$$

(or) $A_1 = 1.001 A_2$

$$R = \frac{\rho l}{A}$$

$$R_1 = \frac{\rho l_1}{A_1} \text{ and } R_2 = \frac{\rho l_2}{A_2}$$

$$R_1 = \frac{\rho l}{1.001 A_2} \text{ and } R_2 = \frac{\rho 1.001 l}{A_2}$$

$$\frac{R_2}{R_1} = (1.001)^2 = 1.002$$

Change in resistance = (1.002 – 1) = 0.002

Change in resistance in percentage = 0.002 × 100 = 0.2%

2.8 The resistance of a field coil measures 50 Ω at 20°C and 65 Ω at 70°C. Find the temperature coefficient of resistance.

Data : At R_{20} = 50 Ω ; 70°C, R_{70} = 65 Ω ; α = ?

Solution : $R_t = R_o (1 + \alpha t)$

$R_{20} = R_o (1 + \alpha\, 20)$

$50 = R_o (1 + \alpha\, 20)$ ...(1)

$R_{70} = R_o (1 + \alpha\, 70)$

$65 = R_o [1 + \alpha\, 70]$ ...(2)

Dividing (2) by (1)

$$\frac{65}{50} = \frac{1 + 70\alpha}{1 + 20\alpha}$$

$65 + 1300\ \alpha = 50 + 3500\ \alpha$

$2200\ \alpha = 15$

$\alpha = 0.0068\ /\ ^\circ C$

2.9 An iron box of 400 W power is used daily for 30 minutes. If the cost per unit is 75 paise, find the weekly expense on using the iron box.

Data : Power of an iron box P = 400 W

rate / unit = 75 p

consumption time t = 30 minutes / day

cost / week = ?

Solution :

Energy consumed in 30 minutes = Power × time in hours

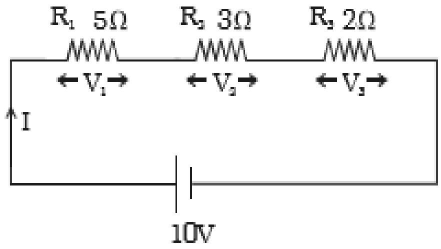
= 400 × ½ = 200 Wh

Energy consumed in one week = 200 × 7 = 1400 Wh = 1.4 unit

Cost / week = Total units consumed × rate/ unit

= 1.4 × 0.75 = Rs.1.05

2.10 Three resistors are connected in series with 10 V supply as shown in the figure. Find the voltage drop across each resistor.



Data : $R_1 = 5\Omega, R_2 = 3\Omega, R_3 = 2\Omega$; V = 10 volt

Effective resistance of series combination,

$R_s = R_1 + R_2 + R_3 = 10\Omega$

Solution : Current in circuit $I = \frac{V}{R_s} = \frac{10}{10} = 1A$

Voltage drop across R_1, $V_1 = IR_1 = 1 \times 5 = 5V$

Voltage drop across R_2, $V_2 = IR_2 = 1 \times 3 = 3V$

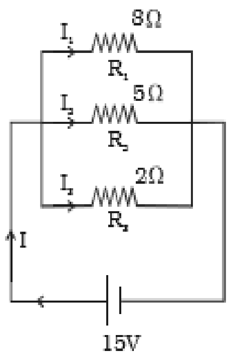
Voltage drop across R_3, $V_3 = IR_3 = 1 \times 2 = 2V$

2.11 Find the current flowing across three resistors 3Ω, 5Ω and 2Ω connected in parallel to a 15 V supply. Also find the effective resistance and total current drawn from the supply.

Data : $R_1 = 3\Omega, R_2 = 5\Omega, R_3 = 2\Omega$; Supply voltage V = 15 volt

Solution :

Effective resistance of parallel combination



$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3} + \frac{1}{5} + \frac{1}{2}$$

$R_p = 0.9677\ \Omega$

Current through R_1, $I_1 = \frac{V}{R_1} = \frac{15}{3} = 5A$

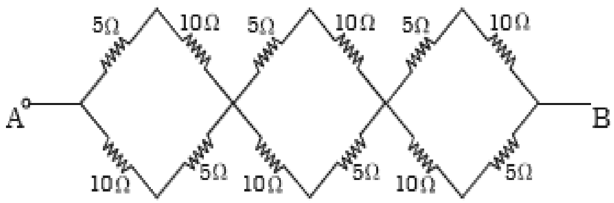
Current through R_2, $I_2 = \dfrac{V}{R_2} = \dfrac{15}{5} = 3A$

Current through R_3, $I_3 = \dfrac{V}{R_3} = \dfrac{15}{2} = 7.5A$

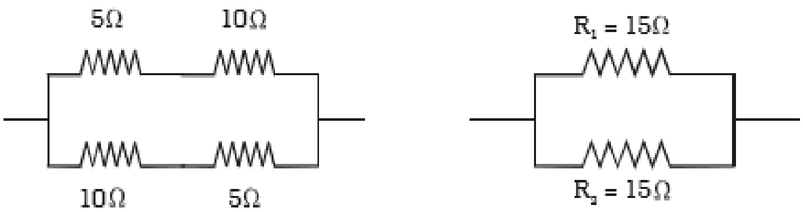
Total current $I = \dfrac{V}{R_p} = \dfrac{15}{0.9677} = 15.5$ A

2.12 In the given network, calculate the effective resistance between points A and B

(i)



Solution : The network has three identical units. The simplified form of one unit is given below :



The equivalent resistance of one unit is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{15} + \frac{1}{15} \text{ or } R_P = 7.5\ \Omega$$

Each unit has a resistance of 7.5 Ω. The total network reduces to

The combined resistance between points A and B is

$R = R' + R' + R'$ $(\because R_s = R_1 + R_2 + R_3)$

$R = 7.5 + 7.5 + 7.5 = 22.5\ \Omega$

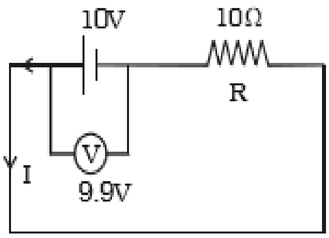
2.13 A 10 Ω resistance is connected in series with a cell of emf 10V. A voltmeter is connected in parallel to a cell, and it reads. 9.9 V. Find internal resistance of the cell.

Data : R = 10 Ω ; E = 10 V ; V = 9.9 V ; r = ?

Solution : $r = \left(\frac{E-V}{V}\right)R$

$= \left(\frac{10-9.9}{9.9}\right)\times 10$

$= 0.101\ \Omega$



Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

2.1 *A charge of 60 C passes through an electric lamp in 2 minutes. Then the current in the lamp is*

(a) 30 A *(b) 1 A* *(c) 0.5 A* *(d) 5 A*

2.2 *The material through which electric charge can flow easily is*

(a) quartz *(b) mica* *(c) germanium* *(d) copper*

2.3 *The current flowing in a conductor is proportional to*

(a) drift velocity

(b) 1/ area of cross section

(c) 1/no of electrons

(d) square of area of cross section.

2.4 *A toaster operating at 240V has a resistance of 120Ω. The power is*

(a) 400 W *(b) 2 W* *(c) 480 W* *(d) 240 W*

2.5 *If the length of a copper wire has a certain resistance R, then on doubling the length its specific resistance*

(a) will be doubled *(b) will become $1/4^{th}$*

(c) will become 4 times *(d) will remain the same.*

2.6 *When two 2Ω resistances are in parallel, the effective resistance is*

(a) 2 Ω *(b) 4 Ω* *(c) 1 Ω* *(d) 0.5 Ω*

2.7 *In the case of insulators, as the temperature decreases, resistivity*

(a) decreases *(b) increases*

(c) remains constant (d) becomes zero

2.8 If the resistance of a coil is 2 Ω at 0ºc and α = 0.004 /ºC, then its resistance at 100º C is

(a) 1.4 Ω (b) 0 Ω (c) 4 Ω (d) 2.8 Ω

2.9 According to Faraday's law of electrolysis, when a current is passed, the mass of ions deposited at the cathode is independent of

(a) current (b) charge (c) time (d) resistance

2.10 When n resistors of equal resistances (R) are connected in series, the effective resistance is

(a) n/R (b) R/n (c) 1/nR (d) nR

2.11 Why is copper wire not suitable for a potentiometer?

2.12 Explain the flow of charges in a metallic conductor.

2.13 Distinguish between drift velocity and mobility. Establish a relation between drift velocity and current.

2.14 State Ohm's law.

2.15 Define resistivity of a material. How are materials classified based on resistivity?

2.16 Write a short note on superconductivity. List some applications of superconductors.

2.17 The colours of a carbon resistor is orange, orange, orange. What is the value of resistor?

2.18 Explain the effective resistance of a series network and parallel network.

2.19 Discuss the variation of resistance with temperature with an expression and a graph.

2.20 Explain the determination of the internal resistance of a cell using voltmeter.

2.21 State and explain Kirchoff's laws for electrical networks.

2.22 Describe an experiment to find unknown resistance and temperature coefficient of resistance using metre bridge?

2.23 Define the term specific resistance. How will you find this using a metre bridge?

2.24 Explain the principle of a potentiometer. How can emf of two cells be compared using potentiometer?

2.25 Distinguish between electric power and electric energy

2.26 State and Explain Faraday's laws of electrolysis. How are the laws verified experimentally?

2.27 Explain the reactions at the electrodes of (i) Daniel cell (ii) Leclanche cell

2.28 Explain the action of the following secondary cell.

(i) lead acid accumulator

2.29 Why automobile batteries have low internal resistance?

Problems

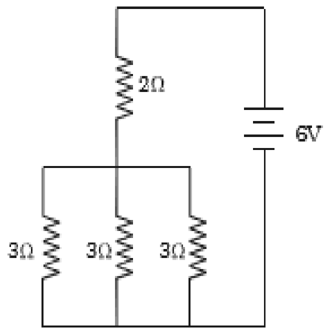
2.30 What is the drift velocity of an electron in a copper conductor having area $10 \times 10^{-6}m^2$, carrying a current of 2 A. Assume that there are 10×10^{28} electrons / m^3.

2.31 How much time 10^{20} electrons will take to flow through a point, so that the current is 200 mA? ($e = 1.6 \times 10^{-19}$ C)

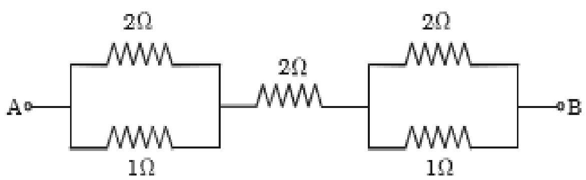
2.32 A manganin wire of length 2m has a diameter of 0.4 mm with a resistance of 70 Ω. Find the resistivity of the material.

2.33 The effective resistances are 10Ω, 2.4Ω when two resistors are connected in series and parallel. What are the resistances of individual resistors?

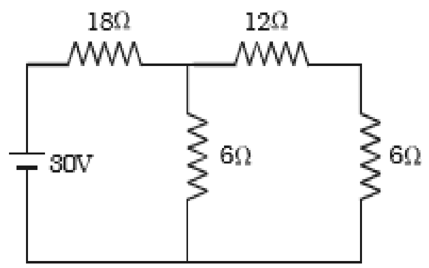
2.34 In the given circuit, what is the total resistance and current supplied by the battery.



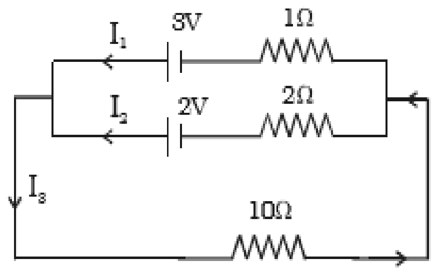
2.35 Find the effective resistance between A and B in the given circuit



2.36 *Find the voltage drop across 18 Ω resistor in the given circuit*



2.37 *Calculate the current I_1, I_2 and I_3 in the given electric circuit.*



2.38 *The resistance of a platinum wire at 0^0 C is 4 Ω. What will be the resistance of the wire at 100°C if the temperature coefficient of resistance of platinum is 0.0038 /^{0} C.*

2.39 *A cell has a potential difference of 6 V in an open circuit, but it falls to 4 V when a current of 2 A is drawn from it. Find the internal resistance of the cell.*

2.40 *In a Wheatstone's bridge, if the galvanometer shows zero deflection, find the unknown resistance. Given P = 1000Ω Q = 10000 Ω and R = 20 Ω*

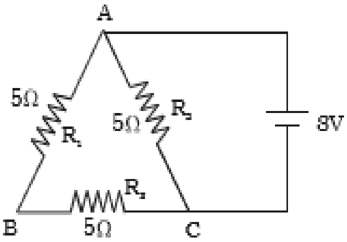
2.41 *An electric iron of resistance 80 Ω is operated at 200 V for two hours. Find the electrical energy consumed.*

2.42 *In a house, electric kettle of 1500 W is used everyday for 45 minutes, to boil water. Find the amount payable per month (30 days) for usage of this, if cost per unit is Rs. 3.25*

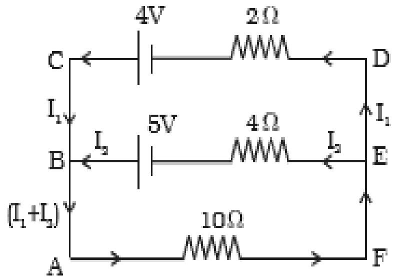
2.43 *A 1.5 V carbon – zinc dry cell is connected across a load of 1000 Ω. Calculate the current and power supplied to it.*

2.44 *In a metre bridge, the balancing length for a 10 Ω resistance in left gap is 51.8 cm. Find the unknown resistance and specific resistance of a wire of length 108 cm and radius 0.2 mm.*

2.45 *Find the electric current flowing through the given circuit connected to a supply of 3 V.*



2.46 *In the given circuit, find the current through each branch of the circuit and the potential drop across the 10 Ω resistor.*



Answers

2.1	*(c)*	**2.2**	*(d)*	**2.3**	*(a)*	**2.4**	*(c)*
2.5	*(d)*	**2.6**	*(c)*	**2.7**	*(b)*	**2.8**	*(d)*
2.9	*(d)*	**2.10**	*(d)*				

2.17	33 kΩ	**2.30**	1.25×10^{-5} m s^{-1}
2.31	80s	**2.32**	4.396 μΩ m
2.33	6 Ω and 4Ω	**2.34**	3 Ω and 2A
2.35	3.33 Ω	**2.36**	24 V
2.37	0.5 A, −0.25 A, 0.25 A	**2.38**	5.52 Ω
2.39	1 Ω	**2.40**	200 Ω
2.41	1 kWh	**2.42**	Rs. 110
2.43	1.5 mA; 2.25 mW	**2.44**	1.082×10^{-6} Ω m
2.45	0.9 A	**2.46**	0.088A, 0.294A, 3.82 V

3. Magnetic Effects of electric current and Magnetism

The concept of electric current, electromotive force having been already discussed in the previous chapter, in this chapter we will discuss the physical consequences of electric current. Living in an electrical and interestingly in an electronic age, we are familiar with many practical applications of electric current, such as bulbs, electroplating, electric fans, electric motors, electric fuse, furnaces etc.

3.1 *Effects of electric current*

There are three main effects of an electric current, viz, Chemical effect of current, Heating effect of current and Magnetic effect of current.

The passage of an electric current through conducting liquid causes chemical reaction and the resulting effect is called chemical effect. Primary cells, Secondary cells and electroplating are applications of chemical effect of current.

In a source of emf, a part of the energy may go into useful work like in an electric motor. The remaining part of the energy is dissipated in the form of heat in the resistors. This is the heating effect of current. Just as current produces thermal energy, thermal energy may also be suitably used to produce an emf. This is thermoelectric effect. This effect is not only a cause but also a consequence of current.

Fig (3.1-3.4) show differentapplications based on heating effect of electric current.

When electric current flows through a wire it behaves like a magnet. This is called magnetic effect of electric current. Motors, generators, transformers, bells, telephones, lifting magnets are some applications of magnetic effect of current.

3.2 *Magnetic effect of electric current*

First time in 1820, Danish Physicist, Oersted discovered magnetic field exists around a current carrying wire. This important physical consequence of current is called magnetic effect of electric current. The direction of magnetic field depends on direction of current. At different points around wire, direction of magnetic field is different.

Oerested's Experiment to Demonstrate the Magnetic Effect of Current

First of all make a simple electrical circuit by joining a long straight wire with a battery and a plug. Now, take a magnetic compass needle and place the straight wire parallel to and over the compass needle. Then switch on the circuit so that current flows through the wire from south to north directions. Now, you will found that the north pole of compass needle gets deflected towards the west.
But, if the direction of current in the wire is reversed then the north pole of the compass needle gets deflected towards the east. The direction of deflection of magnetic needle can be easily determined by SNOW rule. According to SNOW rule, when current flows through a conductor from South to North direction and the wire is held over magnetic needle then the north pole of the magnetic needle deflects towards west.

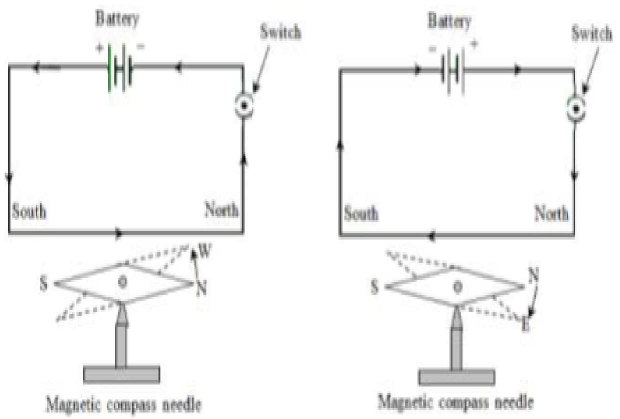


Fig (3.5 & 3.6) represent Oerested's experiment on magnetic effect of current

3.2.1 Magnetic field around a straight conductor carrying current

A smooth cardboard with iron filings spread over it, is fixed in a horizontal plane with the help of a clamp. A straight wire passes through a hole made at the centre of the cardboard (Fig 3.7).

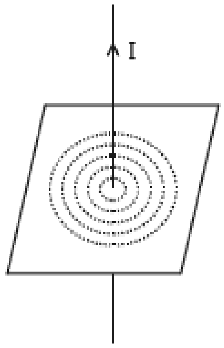


Fig 3.7 Magnetic field around a straight conductor carrying current

A current is passed through the wire by connecting its ends to a battery. When the cardboard is gently tapped, it is found that the iron filings arrange themselves along concentric circles. This clearly shows that magnetic field is developed around a current carrying conductor.

To find the direction of the magnetic field, let us imagine, a straight wire passes through the plane of the paper and perpendicular to it. When a compass needle is placed, it comes to rest in such a way that its axis is always tangential to a circular field around the conductor. When the current is inwards (Fig 3.8a) the direction of the magnetic field around the conductor looks clockwise.

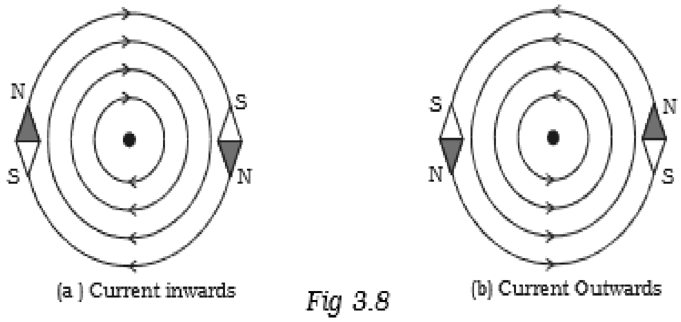


Fig 3.8

When the direction of the current is reversed, that it is outwards, (Fig 3.8b) the direction of the magnetic pole of the compass needle also changes showing the reversal of the direction of the magnetic field. Now, it is anticlockwise around the conductor. This proves that the direction of the magnetic field also depends on the direction of the current in the conductor. This is given by Maxwell's rule.

Maxwells's right hand cork screw rule

If a right handed cork screw is rotated to advance along the direction of the current through a conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.

3.2.2 Magnetic field due to a circular loop carrying current

A cardboard is fixed in a horizontal plane. A circular loop of wire passes through two holes in the cardboard as shown in Fig 3.9. Iron filings are sprinkled over the cardboard. Current is passed through the loop and the card board is gently tapped. It is observed that the iron filings arrange themselves along the resultant magnetic field. The magnetic lines of force are almost circular around the wire where it passes through the cardboard. At the centre of the loop, the line of force is almost straight and perpendicular to the plane of the circular loop.

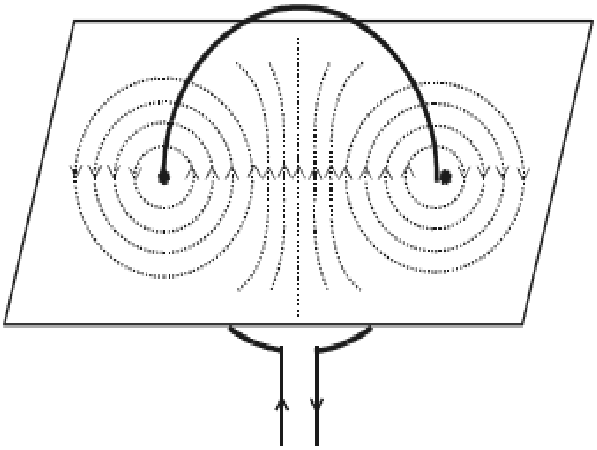


Fig 3.9 Magnetic field due to a circular loop carrying current

3.3 Biot – Savart Law

Biot and Savart conducted many experiments to determine the factors on which the magnetic field due to current in a conductor depends.

The results of the experiments are summarized as Biot-Savart law.

Let us consider a conductor XY carrying a current I (Fig 3.10). AB = dl is a small element of the conductor. P is a point at a distance r from the mid point O of AB. According to Biot and Savart, the magnetic induction dB at P due to the element of length dl is

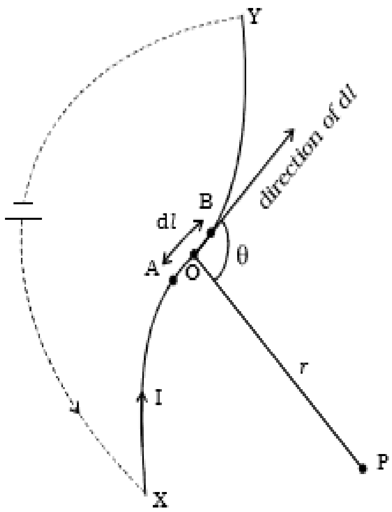


Fig 3.10 Biot - Savart Law

(i) directly proportional to the current (I)

(ii) directly proportional to the length of the element (dl)

(iii) directly proportional to the sine of the angle between dl and the line joining element dl and the point P ($\sin\theta$)

(iv) inversely proportional to the square of the distance of the point from the element ($\frac{1}{r^2}$)

$$\therefore \quad dB \ \alpha \ \frac{I\,dl\sin\theta}{r^2}$$

$$dB = K\frac{I\,dl\sin\theta}{r^2}$$, K is the constant of proportionality

The constant $K = \frac{\mu}{4\pi}$ where μ is the permeability of the medium.

$$dB = \frac{\mu}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$\mu = \mu_r\,\mu_o$ where μ_r is the relative permeability of the medium and μ_0 is the permeability of free space. $\mu_o = 4\pi \times 10^{-7}$ henry/metre. For air $\mu_r = 1$.

So, in air medium $dB = \frac{\mu_o}{4\pi}\cdot\frac{I\,.\,dl\sin\theta}{r^2}$

In vector form, $\overline{dB} = \frac{\mu_o}{4\pi}\frac{\overrightarrow{Idl}\times\overline{r}}{r^3}$ or $\overline{dB} = \frac{\mu_o}{4\pi}\frac{\overline{Idl}\times\hat{r}}{r^2}$

The direction of dB is perpendicular to the plane containing current element Idl and r (i.e plane of the paper) and acts inwards. The unit of magnetic induction is tesla (or) weber m^{-2}.

3.3.1 Magnetic induction due to infinitely long straight conductor carrying current

XY is an infinitely long straight conductor carrying a current I (Fig 3.11). P is a point at a distance a from the conductor. AB is a small element of length dl. θ is the angle between the current element $I\,dl$ and the line joining the element dl and the point P. According to Biot-Savart law, the magnetic induction at the point P due to the current element Idl is

$$dB = \frac{\mu_o}{4\pi}\frac{Idl.\sin\theta}{r^2} \qquad ...(1)$$

AC is drawn perpendicular to BP from A.

$\angle OPA = \phi$, $\angle APB = d\phi$

In Δ ABC, $\sin\theta = \frac{AC}{AB} = \frac{AC}{dl}$

$\therefore$ AC = $dl \sin\theta$ ...(2)

From Δ APC, AC = $rd\phi$ ...(3)

From equations (2) and (3), $rd\phi = dl\sin\theta$ ...(4)

substituting equation (4) in equation (1)

$$dB = \frac{\mu_o}{4\pi}\frac{I\, rd\phi}{r^2} = \frac{\mu_o}{4\pi}\frac{I\, d\phi}{r} \quad ...(5)$$

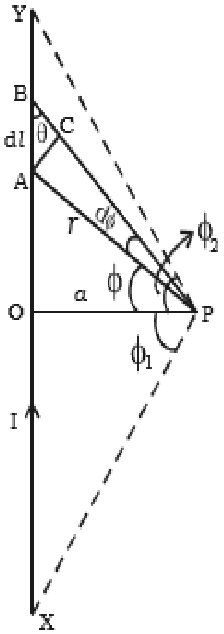


Fig 3.11 Straight conductor

In Δ OPA, $\cos\phi = \frac{a}{r}$

$$\therefore \quad r = \frac{a}{\cos\phi} \quad ...(6)$$

substituting equation (6) in equation (5)

$$dB = \frac{\mu_o}{4\pi}\frac{I}{a}\cos\phi\, d\phi$$

The total magnetic induction at P due to the conductor XY is

$$B = \int_{-\phi_1}^{\phi_2} dB = \int_{-\phi_1}^{\phi_2} \frac{\mu_o I}{4\pi a}\cos\phi\, d\phi$$

$$B = \frac{\mu_o I}{4\pi a}[\sin\phi_1 + \sin\phi_2]$$

For infinitely long conductor, $\phi_1 = \phi_2 = 90^o$

$$\therefore \quad B = \frac{\mu_o I}{2\pi a}$$

If the conductor is placed in a medium of permeability μ,

$$B = \frac{\mu I}{2\pi a}$$

3.3.2 Magnetic induction along the axis of a circular coil carrying current

Let us consider a circular coil of radius 'a' with a current I as shown in Fig 3.12. P is a point along the axis of the coil at a distance x from the centre O of the coil.

AB is an infinitesimally small element of length dl. C is the mid point of AB and CP = r

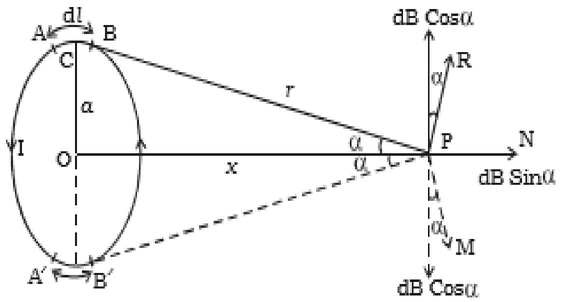


Fig. 3.12 Circular coil

According to Biot – Savart law, the magnetic induction at P due to the element dl is

$$dB = \frac{\mu_o}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$, where θ is the angle between Idl and r

Here, $\theta = 90^o$

$$\therefore \qquad dB = \frac{\mu_o}{4\pi}\frac{I\,dl}{r^2}$$

The direction of dB is perpendicular to the current element Idl and CP. It is therefore along PR perpendicular to CP.

Considering the diametrically opposite element A′B′, the magnitude of dB at P due to this element is the same as that for AB but its direction is along PM. Let the angle between the axis of the coil and the line joining the element (dl) and the point (P) be α.

dB is resolved into two components :- dB sin α along OP and dB cos α perpendicular to OP. dB cos α components due to two opposite elements cancel each other whereas dB sin α components get added up. So, the total magnetic induction at P due to the entire coil is

$$B = \int dB\sin\alpha = \int \frac{\mu_o}{4\pi}\frac{Idl}{r^2}\frac{a}{r} = \frac{\mu_o}{4\pi}\frac{Ia}{r^3}\int dl$$

$$= \frac{\mu_o Ia}{4\pi r^3}2\pi a$$

$$= \frac{\mu_o Ia^2}{2(a^2+x^2)^{\frac{3}{2}}} \qquad (\because r^2 = a^2 + x^2)$$

If the coil contains n turns, the magnetic induction is

$$B = \frac{\mu_o nIa^2}{2(a^2+x^2)^{\frac{3}{2}}}$$

At the centre of the coil, $x = 0$

$$B = \frac{\mu_o nI}{2a}$$

Ex. A circular coil of 200 turns and of radius 20 cm carries a current of 5A. Calculate the magnetic induction at a point along its axis, at a distance three times the radius of the coil from its centre.

Data : n = 200; a = 20cm = 2 × 10^{-1}m; I = 5A; $x = 3a$; B = ?

Solution : $B = \dfrac{\mu_o n I a^2}{2(a^2 + x^2)^{3/2}}$

$$B = \frac{\mu_o n I a^2}{2(a^2 + 9a^2)^{3/2}} = \frac{\mu_o n I a^2}{2(10a^2)^{3/2}} = \frac{\mu_o n I}{a \times 20\sqrt{10}}$$

$$B = \frac{\mu_o n I \sqrt{10}}{a \times 200} = \frac{4\pi \times 10^{-7} \times 200 \times 5 \times \sqrt{10}}{2 \times 10^{-1} \times 200}$$

$$B = 9.9 \times 10^{-5}\ \mathrm{T}$$

3.4 Ampere's Circuital Law

Biot – Savart law expressed in an alternative way is called Ampere's circuital law.

The magnetic induction due to an infinitely long straight current carrying conductor is

$$B = \frac{\mu_o I}{2\pi a}$$

$$B\,(2\pi a) = \mu_o I$$

B $(2\pi a)$ is the product of the magnetic field and the circumference of the circle of radius 'a' on which the magnetic field is constant. If L is the perimeter of the closed curve and I_o is the net current enclosed by the closed curve, then the above equation may be expressed as,

$$BL = \mu_o I_o \qquad ....(1)$$

In a more generalized way, Ampere's circuital law is written as

$$\oint \vec{B}.\vec{dl} = \mu_o I_o \qquad ....(2)$$

The line integral does not depend on the shape of the path or the position of the wire within the magnetic field. If the current in the wire is in the opposite direction, the integral would have the opposite sign. If the closed path does not encircle the wire (if a wire lies outside the path), the line integral of the field of that wire is zero. Although derived for the case of a number of long straight parallel conductors, the law is true for conductors and paths of any shape. Ampere's circuital law is hence defined using equation (1) as follows :

The line integral $\oint \vec{B}.\vec{dl}$ for a closed curve is equal to μ_o times the net current I_o through the area bounded by the curve.

3.4.1 Solenoid

A long closely wound helical coil is called a solenoid. Fig 3.15 shows a section of stretched out solenoid. The magnetic field due to the solenoid is the vector sum of the magnetic fields due to current through individual turns of the solenoid. The magnetic fields associated with each single turn are almost concentric circles and hence tend to cancel between the turns. At the interior mid point, the field is strong and along the axis of the solenoid (i.e) the field is parallel to the axis. For a point such as P, the field due to the upper part of the solenoid turns tend to cancel the field due to the lower part of the solenoid turns, acting in opposite directions. Hence the field outside the solenoid is nearly zero. The direction of the magnetic field due to circular closed loops (solenoid) is given by right hand palm-rule.

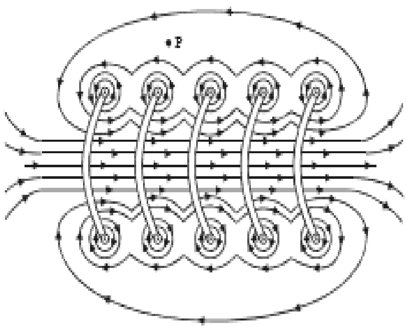


Fig 3.15 Magnetic field due to a current carrying solenoid.

Right hand palm rule

The coil is held in the right hand so that the fingers point in the direction of the current in the windings. The extended thumb, points in the direction of the magnetic field.

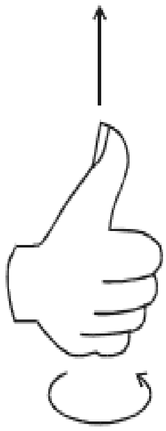


Fig 3.16 Right hand palm rule

3.4.2 Magnetic induction due to a long solenoid carrying current.

Let us consider an infinitely long solenoid having n turns per unit length carrying a current of I. For such an ideal solenoid (whose length is very large compared to its radius), the magnetic field at points outside the solenoid is zero.

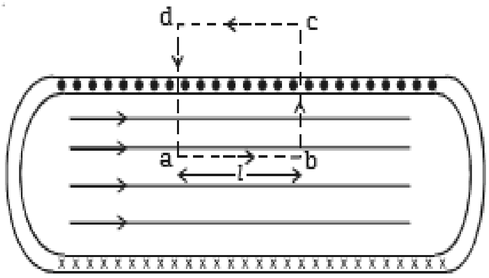


Fig 3.17 Magnetic field due to a long solenoid.

A long solenoid appears like a long cylindrical metal sheet (Fig 3.17). The upper view of dots is like a uniform current sheet coming out of the plane of the paper. The lower row of crosses is like a uniform current sheet going into the plane of the paper. To find the magnetic induction (B) at a point inside the solenoid, let us consider a rectangular Amperean loop $abcd$. The line integral $\oint \vec{B}.\vec{dl}$ for the loop $abcd$ is the sum of four integrals.

$$\therefore \oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl}$$

If l is the length of the loop, the first integral on the right side is Bl. The second and fourth integrals are equal to zero because $\vec{B}$ is at right angles for every element $\vec{dl}$ along the path. The third integral is zero since the magnetic field at points outside the solenoid is zero.

$$\therefore \oint \vec{B}.\vec{dl} = Bl \qquad ...(1)$$

Since the path of integration includes nl turns, the net current enclosed by the closed loop is

$$I_o = Inl \quad ...(2)$$

Ampere's circuital law for a closed loop is

$$\oint \vec{B}.\vec{dl} = \mu_o I_o \quad ...(3)$$

Substituting equations (1) and (2) in equation (3)

$$Bl = \mu_o \, Inl$$

$$\therefore \quad B = \mu_o nI \quad ...(4)$$

The solenoid is commonly used to obtain uniform magnetic field. By inserting a soft iron core inside the solenoid, a large magnetic field is produced

$$B = \mu nI = \mu_o \, \mu_r nI \quad ...(5)$$

when a current carrying solenoid is freely suspended, it comes to rest like a suspended bar magnet pointing along north-south. The magnetic polarity of the current carrying solenoid is given by End rule.

End rule

When looked from one end, if the current through the solenoid is along clockwise direction Fig 3.18a, the nearer end corresponds to south pole and the other end is north pole.

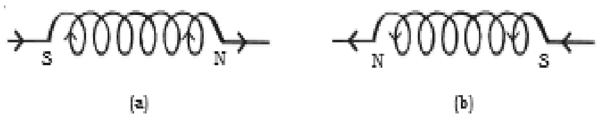


Fig 3.18 End rule

When looked from one end, if the current through the solenoid is along anti-clock wise direction, the nearer end corresponds to north pole and the other end is south pole (Fig 3.18b)

3.5 Magnetic Lorentz force

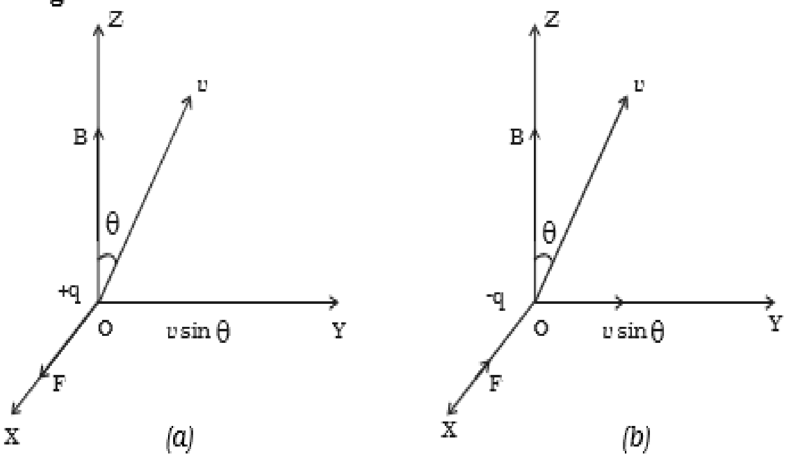


Fig 3.19 Lorentz force

Let us consider a uniform magnetic field of induction B acting along the Z-axis. A particle of charge + q moves with a velocity v in YZ plane making an angle θ with the direction of the field (Fig 3.19a). Under the influence of the field, the particle experiences a force F.

H.A.Lorentz formulated the special features of the force F (Magnetic lorentz force) as under :

(i) the force F on the charge is zero, if the charge is at rest. (i.e) the moving charges alone are affected by the magnetic field.

(ii) the force is zero, if the direction of motion of the charge is either parallel or anti-parallel to the field and the force is maximum, when the charge moves perpendicular to the field.

(iii) the force is proportional to the magnitude of the charge (q)

(iv) the force is proportional to the magnetic induction (B)

(v) the force is proportional to the speed of the charge (v)

(vi) the direction of the force is oppositely directed for charges of opposite sign (Fig 3.19b).

All these results are combined in a single expression as

$$\vec{F} = q\ (\vec{v} \times \vec{B})$$

The magnitude of the force is

$$F = Bqv \sin\theta$$

Since the force always acts perpendicular to the direction of motion of the charge, the force does not do any work.

In the presence of an electric field E and magnetic field B, the total force on a moving charged particle is

$$\vec{F} = q\ [(\vec{v} \times \vec{B}) + \vec{E}]$$

3.5.1 Motion of a charged particle in a uniform magnetic field.

Let us consider a uniform magnetic field of induction B acting along the Z-axis. A particle of charge q and mass m moves in XY plane. At a point P, the velocity of the particle is v. (Fig 3.20)

The magnetic lorentz force on the particle is $\vec{F} = q\ (\vec{v} \times \vec{B})$. Hence $\vec{F}$ acts along PO perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. Since the force acts perpendicular to its velocity, the force does not do any work. So, the magnitude of the velocity remains constant and only its direction changes. The force F acting towards the point O acts as the centripetal force and makes the particle to move along a circular path. At points Q and R, the particle experiences force along QO and RO respectively.

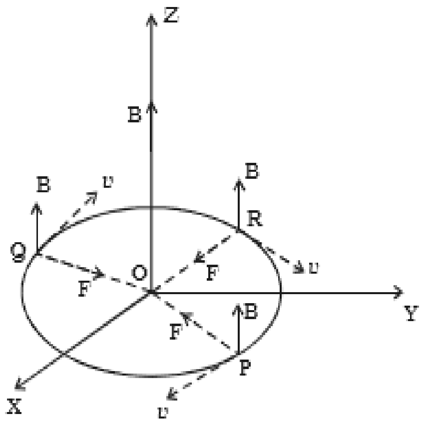


Fig 3.20 Motion of a charged particle

Since $\vec{v}$ and $\vec{B}$ are at right angles to each other

$$F = Bqv \sin 90^0 = Bqv$$

This magnetic lorentz force provides the necessary centripetal force.

$$Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq} \qquad \text{...(1)}$$

It is evident from this equation, that the radius of the circular path is proportional to (i) mass of the particle and (ii) velocity of the particle

From equation (1), $\frac{v}{r} = \frac{Bq}{m}$

$$\omega = \frac{Bq}{m} \qquad \text{...(2)}$$

This equation gives the angular frequency of the particle inside the magnetic field.

Period of rotation of the particle,

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi m}{Bq} \qquad \text{...(3)}$$

From equations (2) and (3), it is evident that the angular frequency and period of rotation of the particle in the magnetic field do not depend upon (i) the velocity of the particle and (ii) radius of the circular path.

3.5.2 Cyclotron

Cyclotron is a device used to accelerate charged particles to high energies. It was devised by Lawrence.

Principle

Cyclotron works on the principle that a charged particle moving normal to a magnetic field experiences magnetic lorentz force due to which the particle moves in a circular path.

Construction

It consists of a hollow metal cylinder divided into two sections D_1 and D_2 called Dees, enclosed in an evacuated chamber (Fig 3.21). The Dees are kept separated and a source of ions is placed at the centre in the gap between the Dees. They are placed between the pole pieces of a strong electromagnet. The magnetic field acts perpendicular to the plane of the Dees. The Dees are connected to a high frequency oscillator.

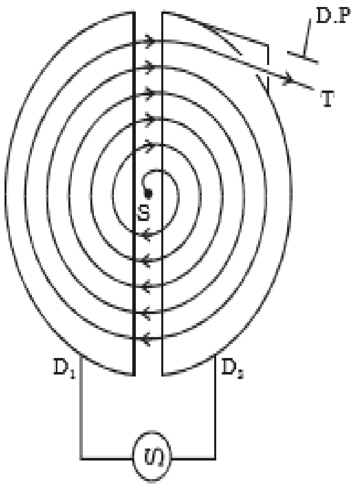


Fig 3.21 Cyclotron

Working

When a positive ion of charge q and mass m is emitted from the source, it is accelerated towards the Dee having a negative potential at that instant of time. Due to the normal magnetic field, the ion experiences magnetic lorentz force and moves in a circular path. By the time the ion arrives at the gap between the Dees, the polarity of the Dees gets reversed. Hence the particle is once again accelerated and moves into the other Dee with a greater velocity along a circle of greater radius. Thus the particle moves in a spiral path of increasing radius and when it comes near the edge, it is taken out with the help of a deflector plate (D.P). The particle with high energy is now allowed to hit the target T.

When the particle moves along a circle of radius r with a velocity v, the magnetic Lorentz force provides the necessary centripetal force.

$$Bq\upsilon = \frac{m\upsilon^2}{r}$$

$$\therefore \quad \frac{\upsilon}{r} = \frac{Bq}{m} = \text{constant} \qquad ...(1)$$

The time taken to describe a semi-circle

$$t = \frac{\pi r}{\upsilon} \qquad ...(2)$$

Substituting equation (1) in (2),

$$t = \frac{\pi m}{Bq} \qquad ...(3)$$

It is clear from equation (3) that the time taken by the ion to describe a semi-circle is independent of

(i) the radius (r) of the path and (ii) the velocity (υ) of the particle

Hence, period of rotation T = 2t

$$\therefore \quad T = \frac{2\pi m}{Bq} = \text{constant} \qquad ...(4)$$

So, in a uniform magnetic field, the ion traverses all the circles in exactly the same time. The frequency of rotation of the particle,

$$\upsilon = \frac{1}{T} = \frac{Bq}{2\pi m} \qquad ...(5)$$

If the high frequency oscillator is adjusted to produce oscillations of frequency as given in equation (5), resonance occurs.

Cyclotron is used to accelerate protons, deutrons and α - particles.

Limitations

(i) Maintaining a uniform magnetic field over a large area of the Dees is difficult.

(ii) At high velocities, relativistic variation of mass of the particle upsets the resonance condition.

(iii) At high frequencies, relativistic variation of mass of the electron is appreciable and hence electrons cannot be accelerated by cyclotron.

3.6 Force on a current carrying conductor placed in a magnetic field.

Let us consider a conductor PQ of length l and area of cross section A. The conductor is placed in a uniform magnetic field of induction B making an angle θ with the field [Fig 3.22]. A current I flows along PQ. Hence, the electrons are drifted along QP with drift velocity v_d. If n is the number of free electrons per unit volume in the conductor, then the current is

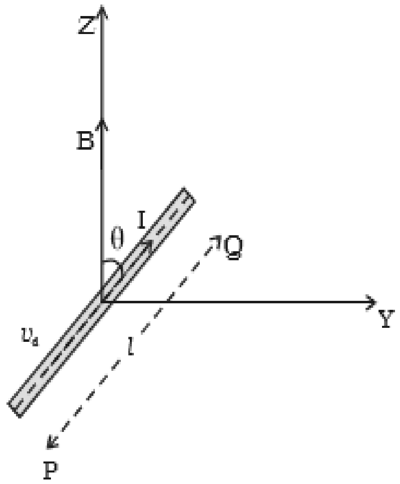


Fig 3.22 Force on a current carrying conductor placed in a magnetic field

$$I = nAv_de$$

Multiplying both sides by the length l of the conductor,

$$\therefore \quad Il = nAv_del.$$

Therefore the current element,

$$I\vec{l} = -nA\vec{v}_del \qquad ...(1)$$

The negative sign in the equation indicates that the direction of current is opposite to the direction of drift velocity of the electrons.

Since the electrons move under the influence of magnetic field, the magnetic lorentz force on a moving electron.

$$\vec{f} = -e\ (\vec{v}_d \times \vec{B}) \qquad ...(2)$$

The negative sign indicates that the charge of the electron is negative.

The number of free electrons in the conductor

$$N = nAl \qquad ...(3)$$

The magnetic lorentz force on all the moving free electrons

$$\vec{F} = N\vec{f}$$

Substituting equations (2) and (3) in the above equation

$$\vec{F} = nAl\ \{ -e\ (\vec{v}_d \times \vec{B}) \}$$

$$\vec{F} = -nAl\ e\ \vec{v}_d \times \vec{B} \qquad ...(4)$$

Substituting equation (1) in equation (4)

$$\vec{F} = I\vec{l} \times \vec{B}$$

This total force on all the moving free electrons is the force on the current carrying conductor placed in the magnetic field.

Magnitude of the force

The magnitude of the force is $F = BIl \sin \theta$

(i) If the conductor is placed along the direction of the magnetic field, $\theta = 0^o$, Therefore force $F = 0$.

(ii) If the conductor is placed perpendicular to the magnetic field, $\theta = 90^o$, $F = BIl$. Therefore the conductor experiences maximum force.

Direction of force

The direction of the force on a current carrying conductor placed in a magnetic field is given by Fleming's Left Hand Rule.

The forefinger, the middle finger and the thumb of the left hand are stretched in mutually perpendicular directions. If the forefinger points in the direction of the magnetic field, the middle finger points in the direction of the current, then the thumb points in the direction of the force on the conductor.

3.6.1 Force between two long parallel current-carrying conductors

AB and CD are two straight very long parallel conductors placed in air at a distance a. They carry currents I_1 and I_2 respectively. (Fig 3.23). The magnetic induction due to current I_1 in AB at a distance a is

$$B_1 = \frac{\mu_o I_1}{2\pi a} \qquad \ldots(1)$$

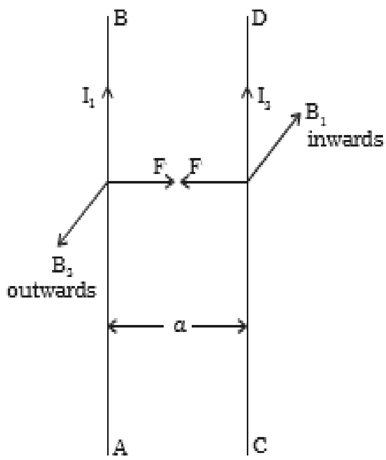


Fig. 3.23 Force between two long parallel current-carrying conductors

This magnetic field acts perpendicular to the plane of the paper and inwards. The conductor CD with current I_2 is situated in this magnetic field. Hence, force on a segment of length l of CD due to magnetic field B_1 is

$$F = B_1 I_2 l$$

substituting equation (1)

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \qquad ...(2)$$

By Fleming's Left Hand Rule, F acts towards left. Similarly, the magnetic induction due to current I_2 flowing in CD at a distance a is

$$B_2 = \frac{\mu_0 I_2}{2\pi a} \qquad ...(3)$$

This magnetic field acts perpendicular to the plane of the paper and outwards. The conductor AB with current I_1, is situated in this field. Hence force on a segment of length l of AB due to magnetic field B_2 is

$$F = B_2 I_1 l$$

substituting equation (3)

$$\therefore \quad F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \qquad ...(4)$$

By Fleming's left hand rule, this force acts towards right. These two forces given in equations (2) and (4) attract each other. Hence, two parallel wires carrying currents in the same direction attract each other and if they carry currents in the opposite direction, repel each other.

Definition of ampere

The force between two parallel wires carrying currents on a segment of length l is

$$F = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

$\therefore$ Force per unit length of the conductor is

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

If $I_1 = I_2 = 1A$, $a = 1m$

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{1 \times 1}{1} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7}\ Nm^{-1}$$

The above conditions lead the following definition of ampere.

Ampere is defined as that constant current which when flowing through two parallel infinitely long straight conductors of negligible cross section and placed in air or vacuum at a distance of one metre apart, experience a force of 2×10^{-7} newton per unit length of the conductor.

3.7 Torque experienced by a current loop in a uniform magnetic field

Let us consider a rectangular loop PQRS of length l and breadth b (Fig 3.24). It carries a current of I along PQRS. The loop is placed in a uniform magnetic field of induction B. Let θ be the angle between the normal to the plane of the loop and the direction of the magnetic field.

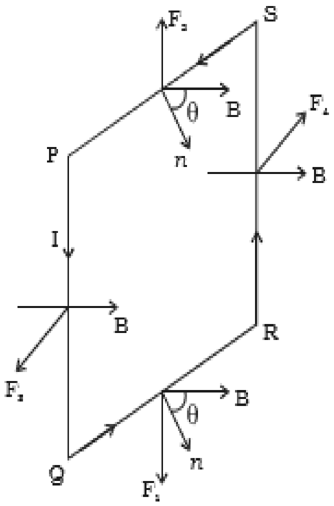


Fig 3.24 Torque on a current loop placed in a magnetic field

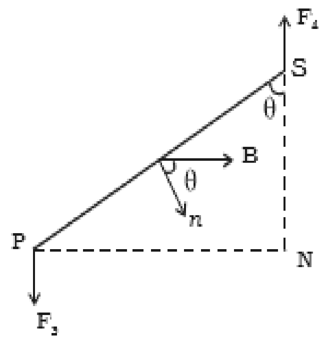


Fig 3.25 Torque

Force on the arm QR, $\vec{F_1} = \overline{I(QR)} \times \vec{B}$

Since the angle between $\overline{I(QR)}$ and $\vec{B}$ is $(90^o - \theta)$,

Magnitude of the force $F_1 = BIb \sin (90^o - \theta)$

ie. $\quad F_1 = BIb \cos \theta$

Force on the arm SP, $\vec{F_2} = \overline{I(SP)} \times \vec{B}$

Since the angle between $\overline{I(SP)}$ and $\vec{B}$ is $(90^o + \theta)$,

Magnitude of the force $F_2 = BIb \cos \theta$

The forces F_1 and F_2 are equal in magnitude, opposite in direction and have the same line of action. Hence their resultant effect on the loop is zero.

Force on the arm PQ, $\vec{F_3} = \overline{I(PQ)} \times \vec{B}$

Since the angle between $\overline{I(PQ)}$ and $\vec{B}$ is 90^o, Magnitude of the force $F_3 = BIl \sin 90^o = BIl$

F_3 acts perpendicular to the plane of the paper and outwards.

Force on the arm RS, $\vec{F_4} = \overline{I(RS)} \times \vec{B}$

Since the angle between $\overline{I(RS)}$ and $\vec{B}$ is 90^o,

Magnitude of the force $F_4 = BIl \sin 90^o = BIl$

F_4 acts perpendicular to the plane of the paper and inwards.

The forces F_3 and F_4 are equal in magnitude, opposite in direction and have different lines of action. So, they constitute a couple.

Hence, Torque $= BIl \times PN = BIl \times PS \times \sin\theta$ (Fig 3.25)

$= BIl \times b \sin\theta = BIA \sin\theta$

If the coil contains n turns, $\tau = nBIA \sin\theta$

So, the torque is maximum when the coil is parallel to the magnetic field and zero when the coil is perpendicular to the magnetic field.

3.7.1 Moving coil galvanometer

Moving coil galvanometer is a device used for measuring the current in a circuit.

Principle

Moving coil galvanometer works on the principle that a current carrying coil placed in a magnetic field experiences a torque.

Construction

It consists of a rectangular coil of a large number of turns of thin insulated copper wire wound over a light metallic frame *(Fig 3.26)*. The coil is suspended between the pole pieces of a horse-shoe magnet by a fine phosphor – bronze strip from a movable torsion head. The lower end of the coil is connected to a hair spring (HS) of phosphor bronze having only a few turns. The other end of the spring is connected to a binding screw. A soft iron cylinder is placed symmetrically inside the coil. The hemispherical magnetic poles produce a radial magnetic field in which the plane of the coil is parallel to the magnetic field in all its positions *(Fig 3.27)*.

A small plane mirror (m) attached to the suspension wire is used along with a lamp and scale arrangement to measure the deflection of the coil.

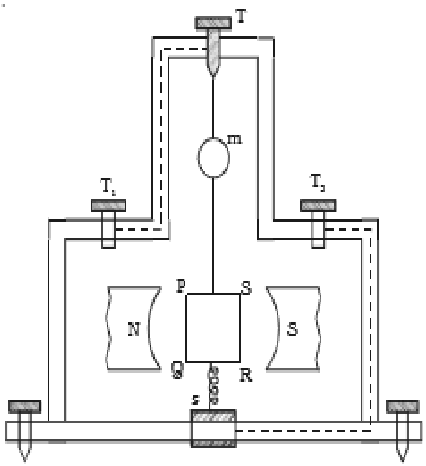


Fig 3.26 Moving coil galvanometer

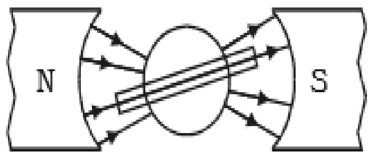


Fig 3.27 Radial magnetic field

Theory

Let PQRS be a single turn of the coil (Fig 3.28). A current I flows through the coil. In a radial magnetic field, the plane of the coil is always parallel to the magnetic field. Hence the sides QR and SP are always parallel to the field. So, they do not experience any force. The sides PQ and RS are always perpendicular to the field.

PQ = RS = l, length of the coil and PS = QR = b, breadth of the coil

Force on PQ, F = BI (PQ) = BIl. According to Fleming's left hand rule, this force is normal to the plane of the coil and acts outwards.

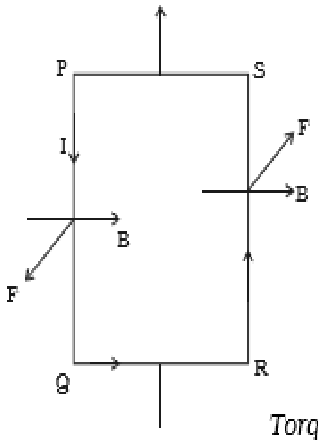
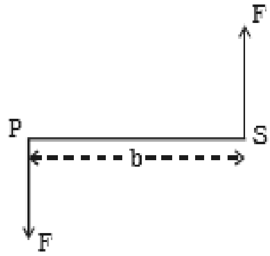


Fig 3.28



Torque on the coil

Fig 3.29

Force on RS, F = BI (RS) = BIl.

This force is normal to the plane of the coil and acts inwards. These two equal, oppositely directed parallel forces having different lines of action constitute a couple and deflect the coil. If there are n turns in the coil,

moment of the deflecting couple $= n\,BIl \times b$ (Fig 3.29)

$= nBIA$

When the coil deflects, the suspension wire is twisted. On account of elasticity, a restoring couple is set up in the wire. This couple is proportional to the twist. If θ is the angular twist, then,

moment of the restoring couple = $C\theta$

where C is the restoring couple per unit twist

At equilibrium, deflecting couple = restoring couple

$$nBIA = C\theta$$

$$\therefore \quad I = \frac{C}{nBA}\theta$$

$I = K\,\theta$ where $K = \dfrac{C}{nBA}$ is the galvanometer constant.

i.e $I \propto \theta$. Since the deflection is directly proportional to the current flowing through the coil, the scale is linear and is calibrated to give directly the value of the current.

3.7.2 Pointer type moving coil galvanometer

The suspended coil galvanometers are very sensitive. They can measure current of the order of 10^{-8} ampere. Hence these galvanometers have to be carefully handled. So, in the laboratory, for experiments like Wheatstone's bridge, where sensitivity is not required, pointer type galvanometers are used. In this type of galvanometer, the coil is pivoted on ball bearings. A lighter aluminium pointer attached to the coil moves over a scale when current is passed. The restoring couple is provided by a spring.

3.7.3 Current sensitivity of a galvanometer.

The current sensitivity of a galvanometer is defined as the deflection produced when unit current passes through the galvanometer. A galvanometer is said to be sensitive if it produces large deflection for a small current.

In a galvanometer, $I = \frac{C}{nBA}\theta$

$\therefore$ Current sensitivity $\frac{\theta}{I} = \frac{nBA}{C}$ ...(1)

The current sensitivity of a galvanometer can be increased by

(i) increasing the number of turns

(ii) increasing the magnetic induction

(iii) increasing the area of the coil

(iv) decreasing the couple per unit twist of the suspension wire. This explains why phosphor-bronze wire is used as the suspension wire which has small couple per unit twist.

3.7.4 *Voltage sensitivity of a galvanometer*

The voltage sensitivity of a galvanometer is defined as the deflection per unit voltage.

$\therefore$ Voltage sensitivity $\frac{\theta}{V} = \frac{\theta}{IG} = \frac{nBA}{CG}$ ...(2)

where G is the galvanometer resistance.

An interesting point to note is that, increasing the current sensitivity does not necessarily, increase the voltage sensitivity. When the number of turns (n) is doubled, current sensitivity is also doubled (equation 1). But increasing the number of turns correspondingly increases the resistance (G). Hence voltage sensitivity remains unchanged.

3.7.5 *Conversion of galvanometer into an ammeter*

A galvanometer is a device used to detect the flow of current in an electrical circuit. Eventhough the deflection is directly proportional to the current, the galvanometer scale is not marked in ampere. Being a very sensitive instrument, a large current cannot be passed through the galvanometer, as it may damage the coil. However, a galvanometer is converted into an ammeter by connecting a low resistance in parallel with it. As a result, when large current flows in a circuit, only a small fraction of the current passes through the galvanometer and the remaining larger portion of the current passes through the low

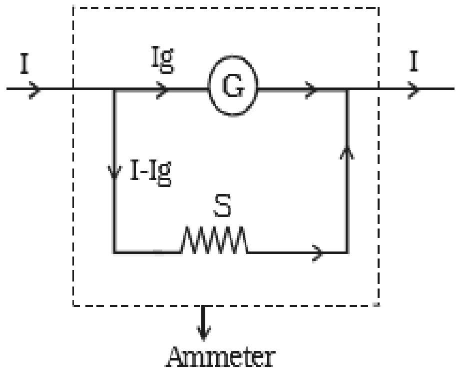


Fig 3.30 Conversion of galvanometer into an ammeter

resistance. The low resistance connected in parallel with the galvanometer is called shunt resistance. The scale is marked in ampere.

The value of shunt resistance depends on the fraction of the total current required to be passed through the galvanometer. Let I_g be the maximum current that can be passed through the galvanometer. The current I_g will give full scale deflection in the galvanometer.

Galvanometer resistance = G

Shunt resistance = S

Current in the circuit = I

$\therefore$ Current through the shunt resistance = $I_s = (I-I_g)$

Since the galvanometer and shunt resistance are parallel, potential is common.

$$\therefore \quad I_g \cdot G = (I - I_g)S$$

$$S = G\,\frac{I_g}{I-I_g} \qquad ...(1)$$

The shunt resistance is very small because I_g is only a fraction of I.

The effective resistance of the ammeter R_a is (G in parallel with S)

$$\frac{1}{R_a} = \frac{1}{G} + \frac{1}{S}$$

$$\therefore \quad R_a = \frac{GS}{G+S}$$

R_a is very low and this explains why an ammeter should be connected in series. When connected in series, the ammeter does not appreciably change the resistance and current in the circuit. Hence an ideal ammeter is one which has zero resistance.

3.7.6 Conversion of galvanometer into a voltmeter

Voltmeter is an instrument used to measure potential difference between the two ends of a current carrying conductor.

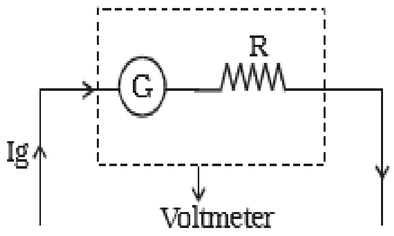


Fig 3.31 Conversion of galvanometer into voltmeter

A galvanometer can be converted into a voltmeter by connecting a high resistance in series with it. The scale is calibrated in volt. The value of the resistance connected in series decides the range of the voltmeter.

Galvanometer resistance = G

The current required to produce full scale deflection in the galvanometer = I_g

Range of voltmeter = V

Resistance to be connected in series = R

Since R is connected in series with the galvanometer, the current through the galvanometer,

$$I_g = \frac{V}{R+G}$$

$$\therefore \quad R = \frac{V}{I_g} - G$$

From the equation the resistance to be connected in series with the galvanometer is calculated.

The effective resistance of the voltmeter is

$$R_v = G + R$$

R_v is very large, and hence a voltmeter is connected in parallel in a circuit as it draws the least current from the circuit. In other words, the resistance of the voltmeter should be very large compared to the resistance across which the voltmeter is connected to measure the potential difference. Otherwise, the voltmeter will draw a large current from the circuit and hence the current through the remaining part of the circuit decreases. In such a case the potential difference measured by the voltmeter is very much less than the actual potential difference. The error is eliminated only when the voltmeter has a high resistance. An ideal voltmeter is one which has infinite resistance.

3.8 Current loop as a magnetic dipole

Ampere found that the distribution of magnetic lines of force around a finite current carrying solenoid is similar to that produced by a bar magnet. This is evident from the fact that a compass needle when moved around these two bodies show similar deflections. After noting the close resemblance between these two, Ampere demonstrated that a simple current loop behaves like a bar magnet and put forward that all the magnetic phenomena is due to circulating electric current. This is Ampere's hypothesis.

The magnetic induction at a point along the axis of a circular coil carrying current is

$$B = \frac{\mu_o n I a^2}{2\,(a^2+x^2)^{\frac{3}{2}}}$$

The direction of this magnetic field is along the axis and is given by right hand rule. For points which are far away from the centre of the coil, $x >> a$, a^2 is small and it is neglected. Hence for such points,

$$B = \frac{\mu_o n I a^2}{2x^3}$$

If we consider a circular loop, n = 1, its area $A = \pi a^2$

$$\therefore \quad B = \frac{\mu_o I A}{2\pi x^3} \quad \text{...(1)}$$

The magnetic induction at a point along the axial line of a short bar magnet is

$$B = \frac{\mu_o}{4\pi} \cdot \frac{2M}{x^3}$$

$$B = \frac{\mu_o}{2\pi} \cdot \frac{M}{x^3} \quad \text{...(2)}$$

Comparing equations (1) and (2), we find that

$$M = IA \quad \text{...(3)}$$

Hence a current loop is equivalent to a magnetic dipole of moment M = IA

The magnetic moment of a current loop is defined as the product of the current and the loop area. Its direction is perpendicular to the plane of the loop.

3.9 The magnetic dipole moment of a revolving electron

According to Neil Bohr's atom model, the negatively charged electron is revolving around a positively charged nucleus in a circular orbit of radius r. The revolving electron in a closed path constitutes an electric current. The motion of the electron in anticlockwise direction produces conventional current in clockwise direction.

Current, $i = \frac{e}{T}$ where T is the period of revolution of the electron.

If v is the orbital velocity of the electron, then

$$T = \frac{2\pi r}{v}$$

$$\therefore \quad i = \frac{ev}{2\pi r}$$

Due to the orbital motion of the electron, there will be orbital magnetic moment μ_l

$\mu_l = iA$, where A is the area of the orbit

$$\mu_l = \frac{ev}{2\pi r} \cdot \pi r^2$$

$$\mu_l = \frac{evr}{2}$$

If m is the mass of the electron

$$\mu_l = \frac{e}{2m}(mvr)$$

mvr is the angular momentum (l) of the electron about the central nucleus.

$$\mu_l = \frac{e}{2m}\, l \qquad \text{... (1)}$$

$\frac{\mu_l}{l} = \frac{e}{2m}$ is called gyromagnetic ratio and is a constant. Its value is 8.8×10^{10} C kg^{-1}. Bohr hypothesised that the angular momentum has only discrete set of values given by the equation.

$l = \frac{nh}{2\pi}$ ...(2) where n is a natural number

and h is the Planck's constant = 6.626×10^{-34} Js.

substituting equation (2) in equation (1)

$$\mu_l = \frac{e}{2m} \cdot \frac{nh}{2\pi} = \frac{neh}{4\pi m}$$

The minimum value of magnetic moment is

$$(\mu_l)_{min} = \frac{eh}{4\pi m},\ n = 1$$

The value of $\frac{eh}{4\pi m}$ is called Bohr magneton

By substituting the values of e, h and m, the value of Bohr magneton is found to be 9.27×10^{-24} Am^2

In addition to the magnetic moment due to its orbital motion, the electron possesses magnetic moment due to its spin. Hence the resultant magnetic moment of an electron is the vector sum of its orbital magnetic moment and its spin magnetic moment.

3.1 A long straight wire carrying current produces a magnetic induction of 4×10^{-6}T at a point, 15 cm from the wire. Calculate the current through the wire.

Data : $B = 4 \times 10^{-6}$T, a=15×10^{-2}m, I=?

Solution : $B = \frac{\mu_o I}{2\pi a}$

$$\therefore I = \frac{B \times 2\pi a}{\mu_o} = \frac{4 \times 10^{-6} \times 2\pi \times 15 \times 10^{-2}}{4\pi \times 10^{-7}}$$

$\therefore$ I = 3A

3.10. Magnetism

The word magnetism is derived from iron ore magnetite (Fe_3O_4), which was found in the island of magnesia in Greece. It is believed that the Chinese had known the property of the magnet even in 2000 B.C. and they used magnetic compass needle for navigation in 1100 AD. But it was Gilbert who laid the foundation for magnetism and had suggested that Earth itself behaves as a giant bar magnet. The field at the surface of the Earth is approximately 10^{-4} T and the field extends upto a height of nearly five times the radius of the Earth.

3.10.1 Earth's magnetic field and magnetic elements

A freely suspended magnetic needle at a point on Earth comes to rest approximately along the geographical north - south direction. This shows that the Earth behaves like a huge magnetic dipole with its magnetic poles near its geographical poles. Since the north pole of the magnetic needle approximately points towards geographic north (N_G) it is appropriate to call the magnetic pole near N_G as the magnetic south pole of Earth S_m. Also, the pole near S_G is the magnetic north pole of the Earth (N_m). (Fig. 3.32)

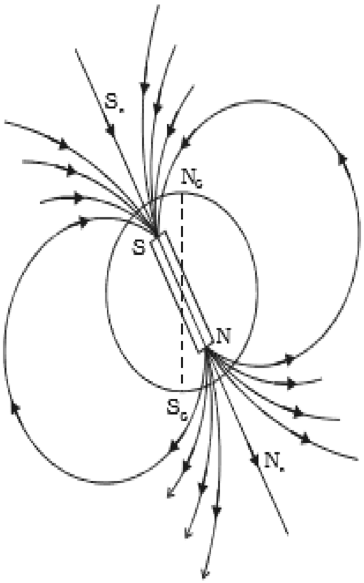


Fig. 3.32 Magnetic field of Earth

The Earth's magnetic field at any point on the Earth can be completely defined in terms of certain quantities called magnetic elements of the Earth, namely

(i) Declination or the magnetic variation θ.

(ii) Dip or inclination δ and

(iii) The horizontal component of the Earth's magnetic field B_h

Causes of the Earth's magnetism

The exact cause of the Earth's magnetism is not known even today. However, some important factors which may be the cause of Earth's magnetism are:

(i) Magnetic masses in the Earth.

(ii) Electric currents in the Earth.

(iii) Electric currents in the upper regions of the atmosphere.

(iv) Radiations from the Sun.

(v) Action of moon etc.

However, it is believed that the Earth's magnetic field is due to the molten charged metallic fluid inside the Earth's surface with a core of radius about 3500 km compared to the Earth's radius of 6400 km.

3.10.2 Bar magnet

The iron ore magnetite which attracts small pieces of iron, cobalt, nickel etc. is a natural magnet. The natural magnets have irregular shape and they are weak. A piece of iron or steel acquires magnetic properties when it is rubbed with a magnet. Such magnets made out of iron or steel are artificial magnets. Artificial magnets can have desired shape and desired strength. If the artificial magnet is in the form of a rectangular or cylindrical bar, it is called a bar magnet.

3.10.3 Basic properties of magnets

(i) When the magnet is dipped in iron filings, they cling to the ends of the magnet. The attraction is maximum at the two ends of the magnet. These ends are called poles of the magnet.

(ii) When a magnet is freely suspended, it always points along north-south direction. The pole pointing towards geographic north is called north pole N and the pole which points towards geographic south is called south pole S.

(iii) Magnetic poles always exist in pairs. (i.e) isolated magnetic pole does not exist.

(iv) The magnetic length of a magnet is always less than its geometric length, because the poles are situated a little inwards from the free ends of the magnet. (But for the purpose of calculation the geometric length is always taken as magnetic length.)

(v) Like poles repel each other and unlike poles attract each other. North pole of a magnet when brought near north pole of another magnet, we can observe repulsion, but when the north pole of one magnet is brought near south pole of another magnet, we observe attraction.

(vi) The force of attraction or repulsion between two magnetic poles is given by Coulomb's inverse square law.

Note : In recent days, the concept of magnetic poles has been completely changed. The origin of magnetism is traced only due to the flow of current. But anyhow, we have retained the conventional idea of magnetic poles in this chapter. Pole strength is denoted by m and its unit is ampere metre.

Magnetic moment

Since any magnet has two poles, it is also called a magnetic dipole.

The magnetic moment of a magnet is defined as the product of the pole strength and the distance between the two poles.

If m is the pole strength of each pole and $2l$ is the distance between the poles, the magnetic moment

$$\vec{M} = m\,(2\vec{l})$$

Magnetic moment is a vector quantity. It is denoted by M. Its unit is A m^2. Its direction is from south pole to north pole.

Magnetic field

Magnetic field is the space in which a magnetic pole experiences a force or it is the space around a magnet in which the influence of the magnet is felt.

Magnetic induction

Magnetic induction is the fundamental character of a magnetic field at a point.

Magnetic induction at a point in a magnetic field is the force experienced by unit north pole placed at that point. It is denoted by B. Its unit is $\frac{N}{Am}$. It is a vector quantity. It is also called as magnetic flux density.

If a magnetic pole of strength m placed at a point in a magnetic field experiences a force F, the magnetic induction at that point is

$$\overline{B} = \frac{\overline{F}}{m}$$

Magnetic lines of force

A magnetic field is better studied by drawing as many number of magnetic lines of force as possible.

A magnetic line of force is a line along which a free isolated north pole would travel when it is placed in the magnetic field.

Properties of magnetic lines of force

(i) Magnetic lines of forces are closed continuous curves, extending through the body of the magnet.

(ii) The direction of line of force is from north pole to south pole outside the magnet while it is from south pole to north pole inside the magnet.

(iii) The tangent to the magnetic line of force at any point gives the direction of magnetic field at that point. (i.e) it gives the direction of magnetic induction ($\vec{B}$) at that point.

(iv) They never intersect each other.

(v) They crowd where the magnetic field is strong and thin out where the field is weak.

Magnetic flux and magnetic flux density

The number of magnetic lines of force passing through an area A is called magnetic flux. It is denoted by ϕ. Its unit is weber. It is a scalar quantity.

The number of magnetic lines of force crossing unit area kept normal to the direction of line of force is magnetic flux density. Its unit is Wb m^{-2} or tesla or N $A^{-1}m^{-1}$.

$\therefore$ Magnetic flux $\phi = \vec{B} \cdot \vec{A}$

Uniform and non-uniform magnetic field

Magnetic field is said to be uniform if the magnetic induction has the same magnitude and the same direction at all

Fig. 3.33 Uniform Magneticfield

the points in the region. It is represented by drawing parallel lines (Fig. 3.33).

An example of uniform magnetic field over a wide area is the Earth's magnetic field.

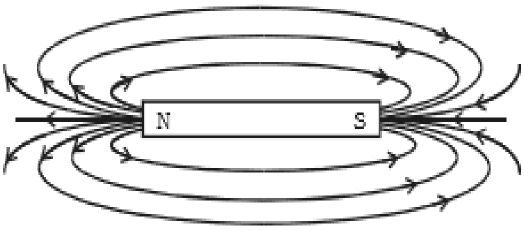


Fig. 3.34 Non-uniform magnetic field

If the magnetic induction varies in magnitude and direction at different points in a region, the magnetic field is said to be non-uniform. The magnetic field due to a bar magnet is non-uniform. It is represented by convergent or divergent lines (Fig. 3.34).

3.11. Force between two magnetic poles

In 1785, Coulomb made use of his torsion balance and discovered the law governing the force between the two magnetic poles.

Coulomb's inverse square law

Coulomb's inverse square law states that the *force of attraction or repulsion between the two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.*

If m_1 and m_2 are the pole strengths of two magnetic poles separated by a distance of d in a medium, then

$$F \propto m_1 m_2 \text{ and } F \propto \frac{1}{d^2}$$

$$\therefore F \propto \frac{m_1 m_2}{d^2}$$

$$F = k\frac{m_1 m_2}{d^2}$$

where k is the constant of proportionality and $k = \frac{\mu}{4\pi}$ where μ is the permeability of the medium.

But $\mu = \mu_o \times \mu_r$

$$\therefore \mu_r = \frac{\mu}{\mu_o}$$

where μ_r - relative permeability of the medium

μ_o - permeability of free space or vacuum.

Let $m_1 = m_2 = 1$

and $d = 1\ m$

$$k = \frac{\mu_o}{4\pi}$$

In free space, $\mu_o = 4\pi \times 10^{-7}\ H\ m^{-1}$

$$\therefore \quad F = \frac{10^{-7} \times m_1 \times m_2}{d^2}$$

$$F = \frac{10^{-7} \times 1 \times 1}{1^2}$$

$$F = 10^{-7}\ N$$

Therefore, *unit pole is defined as that pole which when placed at a distance of 1 metre in free space or air from an equal and similar pole, repels it with a force of 10^{-7} N.*

3.12. Magnetic induction at a point along the axial line due to a magnetic dipole (Bar magnet)

NS is the bar magnet of length $2l$ and of pole strength m. *P* is a point on the axial line at a distance d from its mid point *O* (Fig. 3.35).

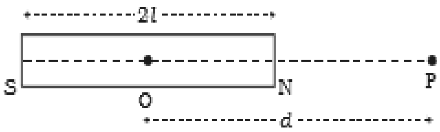


Fig. 3.35 *Magnetic induction along the axial line*

According to inverse square law, $F = \frac{\mu_o}{4\pi}\frac{m_1 m_2}{d^2}$

$\therefore$ Magnetic induction (B_1) at P due to north pole of the magnet,

$$B_1 = \frac{\mu_o}{4\pi}\frac{m}{NP^2} \text{ along NP} \quad \left(\because B = \frac{F}{m}\right)$$

$$= \frac{\mu_o}{4\pi}\frac{m}{(d-l)^2} \text{ along NP}$$

Magnetic induction (B_2) at P due to south pole of the magnet,

$$B_2 = \frac{\mu_o}{4\pi}\frac{m}{SP^2} \text{ along } PS$$

$$B_2 = \frac{\mu_o}{4\pi}\frac{m}{(d+l)^2} \text{ along } PS$$

$\therefore$ Magnetic induction at P due to the bar magnet,

$$B = B_1 - B_2$$

$$B = \frac{\mu_o}{4\pi}\frac{m}{(d-l)^2} - \frac{\mu_o}{4\pi}\frac{m}{(d+l)^2} \quad \text{along NP}$$

$$B = \frac{\mu_o m}{4\pi}\left(\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right)$$

$$B = \frac{\mu_o m}{4\pi}\left(\frac{(d+l)^2-(d-l)^2}{(d^2-l^2)^2}\right)$$

$$B = \frac{\mu_o m}{4\pi}\left(\frac{4ld}{(d^2-l^2)^2}\right)$$

$$B = \frac{\mu_o m}{4\pi}\frac{2l\times 2d}{(d^2-l^2)^2}$$

$$B = \frac{\mu_o}{4\pi}\frac{2Md}{(d^2-l^2)^2}$$

where M = $2ml$ (magnetic dipole moment).

For a short bar magnet, l is very small compared to d, hence l^2 is neglected.

$$\therefore B = \frac{\mu_o}{4\pi}\frac{2M}{d^3}$$

The direction of B is along the axial line away from the north pole.

3.13. Magnetic induction at a point along the equatorial line of a bar magnet

NS is the bar magnet of length $2l$ and pole strength m. P is a point on the equatorial line at a distance d from its mid point O (Fig. 3.36).

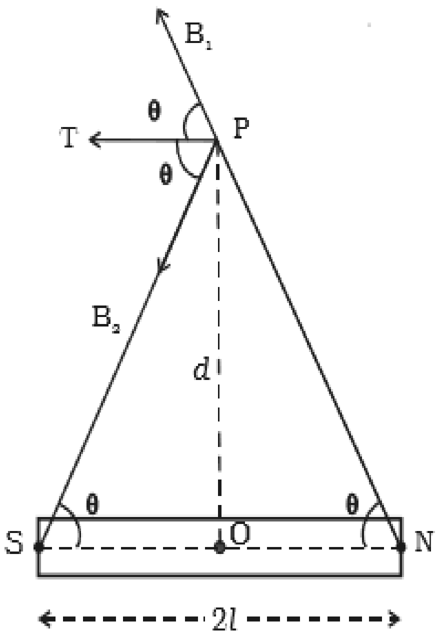


Fig. 3.36 Magnetic induction along the equatorial line

Magnetic induction (B_1) at P due to north pole of the magnet,

$$B_1 = \frac{\mu_o}{4\pi}\frac{m}{NP^2} \text{ along } NP$$

$$= \frac{\mu_o}{4\pi}\frac{m}{(d^2+l^2)} \text{ along NP}$$

$$(\because NP^2 = NO^2 + OP^2)$$

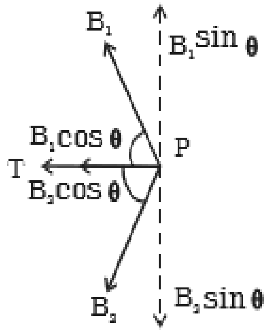


Fig. 3.37 Components of magnetic fields

Magnetic induction (B_2) at P due to south pole of the magnet,

$$B_2 = \frac{\mu_o}{4\pi}\frac{m}{PS^2} \text{ along } PS$$

$$= \frac{\mu_o}{4\pi}\frac{m}{(d^2+l^2)} \text{ along PS}$$

Resolving B_1 and B_2 into their horizontal and vertical components.

Vertical components $B_1 \sin\theta$ and $B_2 \sin\theta$ are equal and opposite and therefore cancel each other (Fig. 3.37).

The horizontal components $B_1 \cos\theta$ and $B_2 \cos\theta$ will get added along PT.

Resultant magnetic induction at P due to the bar magnet is

$B = B_1 \cos\theta + B_2 \cos\theta.$ (along PT)

$$B = \frac{\mu_o}{4\pi}\frac{m}{d^2+l^2}\cdot\frac{l}{\sqrt{d^2+l^2}} + \frac{\mu_o}{4\pi}\frac{m}{(d^2+l^2)}\cdot\frac{l}{\sqrt{d^2+l^2}}$$

$$\left(\because \cos\theta = \frac{SO}{PS} = \frac{NO}{NP}\right)$$

$$= \frac{\mu_o}{4\pi}\frac{2ml}{(d^2+l^2)^{3/2}}$$

$$B = \frac{\mu_o}{4\pi}\frac{M}{(d^2+l^2)^{3/2}}, \quad \text{(where } M = 2ml\text{)}$$

For a short bar magnet, l^2 is neglected.

$$\therefore B = \frac{\mu_o}{4\pi}\frac{M}{d^3}$$

The direction of 'B' is along PT parallel to NS.

3.14. Mapping of magnetic field due to a bar magnet

A bar magnet is placed on a plane sheet of a paper. A compass needle is placed near the north pole of the magnet. The north and south poles of the compass are marked by pencil dots. The compass needle is shifted and placed so that its south pole touches the pencil dot marked for north pole. The process is repeated and a series of dots are obtained. The dots are joined as a smooth curve. This curve is a magnetic line of force. Even though few lines are drawn around a bar magnet the magnetic lines exists in all space around the magnet.

(i) Magnet placed with its north pole facing geographic north

A sheet of paper is fixed on a drawing board. Using a compass needle, the magnetic meridian is drawn on it. A bar magnet is placed on the magnetic meridian such that its north pole points towards geographic north. Using a compass needle, magnetic lines of force are drawn around the magnet. (Fig. 3.38)

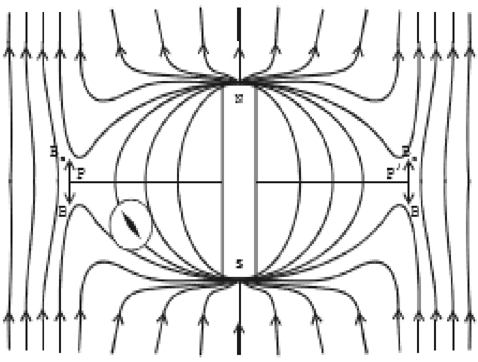


Fig. 3.38 Neutral points - equatorial line

The magnetic lines of force is due to the combined effect of the magnetic field due to the bar magnet and Earth. It is found that when the compass is placed at points P and P ′ along the equatorial line of the magnet, the compass shows no deflection. They are called "neutral points." At these points the magnetic field due to the magnet along its equatorial line (B) is exactly balanced by the horizontal component of the Earth's magnetic field. (B_h)

Hence, *neutral points are defined as the points where the resultant magnetic field due to the magnet and Earth is zero.*

Hence, at neutral points

$$B = B_h$$

$$\frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = B_h$$

(ii) Magnet placed with its south pole facing geographic north

A sheet of paper is fixed on a drawing board. Using a compass needle, the magnetic meridian is drawn on it. A bar magnet is placed on a magnetic meridian such that its south pole facing geographic north. Using a compass needle, the magnetic lines of force are drawn around the magnet as shown in Fig. 3.39.

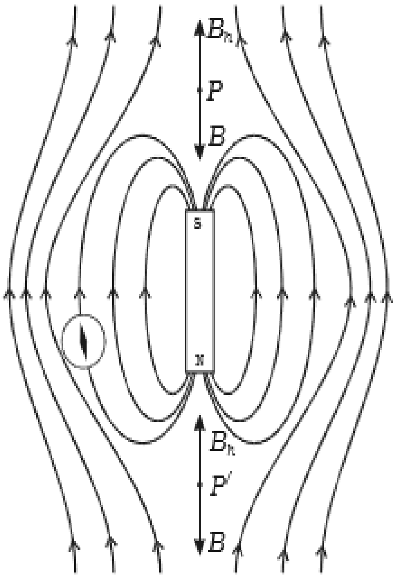


Fig. 3.39 Neutral points - axial line

The magnetic lines of force is due to the combined effect of the magnetic field due to the bar magnet and Earth. It is found that when the compass is placed at points P and P' along the axial line of the magnet, the compass shows no deflection. They are called neutral points. At these points the magnetic field (B) due to the magnet along its axial line is exactly balanced by the horizontal component of the Earth's magnetic field (B_h).

Hence at neutral points, $B = B_h$

$$\therefore \frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2} = B_h$$

3.15. Torque on a bar magnet placed in a uniform magnetic field

Consider a bar magnet NS of length $2l$ and pole strength m placed in a uniform magnetic field of induction B at an angle θ with the direction of the field (Fig. 3.40).

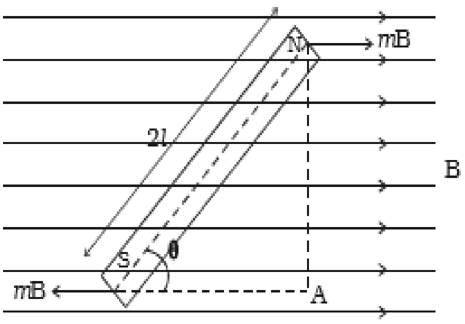


Fig. 3.40 Torque on a bar magnet

Due to the magnetic field B, a force mB acts on the north pole along the direction of the field and a force mB acts on the south pole along the direction opposite to the magnetic field.

These two forces are equal and opposite, hence constitute a couple. The torque τ due to the couple is

τ = one of the forces $\times$ perpendicular distance between them

$$\tau = F \times NA$$

$$= mB \times NA \quad ...(1)$$

$$= mB \times 2l \sin\theta$$

$$\therefore \tau = MB \sin\theta \quad ...(2)$$

Vectorially,

$$\vec{\tau} = \vec{M} \times \vec{B}$$

The direction of τ is perpendicular to the plane containing $\vec{M}$ and $\vec{B}$.

If $B = 1$ and $\theta = 90°$

Then from equation (2), $\tau = M$

Hence, *moment of the magnet M is equal to the torque necessary to keep the magnet at right angles to a magnetic field of unit magnetic induction.*

Ex. *A bar magnet of moment 7.5 Am^2 experiences a torque of 1.5 x 10^{-4} Nm, when placed inclined at 30^0 in a uniform magnetic field. Find the magnetic induction of the field.*

Solution: $$B = \frac{\tau}{M\sin\theta}$$

$$B = \frac{1.5x10^{-4}}{7.5x\sin30^0}$$

$$B = \frac{1.5x10^{-4}}{7.5x0.5}$$

$$B = 0.4x10^{-4} = 4x10^{-5}T$$

So, the magnetic induction of the field is $4x10^{-5}$T

3.16. Magnetic properties of materials

The study of magnetic properties of materials assumes significance since these properties decide whether the material is suitable for permanent magnets or electromagnets or cores of transformers etc. Before classifying the materials depending on their magnetic behaviour, the following important terms are defined.

(i) Magnetising field or magnetic intensity

The magnetic field used to magnetise a material is called the magnetising field. It is denoted by H and its unit is A m^{-1}.

(Note : Since the origin of magnetism is linked to the current, the magnetising field is usually defined in terms of ampere turn which is out of our purview here.)

(ii) Magnetic permeability

Magnetic permeability is the ability of the material to allow the passage of magnetic lines of force through it.

Relative permeability μ_r of a material is defined as the ratio of number of magnetic lines of force per unit area B inside the material to the number of lines of force per unit area in vacuum B_o produced by the same magnetising field.

$$\therefore \text{ Relative permeability } \mu_r = \frac{B}{B_o}$$

$$\mu_r = \frac{\mu H}{\mu_o H} = \frac{\mu}{\mu_o}$$

(since μ_r is the ratio of two identical quantities, it has no unit.)

$\therefore$ *The magnetic permeability of the medium $\mu = \mu_o \mu_r$ where μ_o is the permeability of free space.*

Magnetic permeability μ of a medium is also defined as the ratio of magnetic induction B inside the medium to the magnetising field H inside the same medium.

$$\therefore \mu = \frac{B}{H}$$

- - -

(iii) Intensity of magnetisation

Intensity of magnetisation represents the extent to which a material has been magnetised under the influence of magnetising field H.

Intensity of magnetisation of a magnetic material is defined as the magnetic moment per unit volume of the material.

$$I = \frac{M}{V}$$

Its unit is A m^{-1}.

For a specimen of length $2l$, area A and pole strength m,

$$I = \frac{2lm}{2lA}$$

$$\therefore I = \frac{m}{A}$$

Hence, *intensity of magnetisation is also defined as the pole strength per unit area of the cross section of the material.*

(iv) Magnetic induction

When a soft iron bar is placed in a uniform magnetising field H, the magnetic induction inside the specimen B is equal to the sum of the magnetic induction B_o produced in vacuum due to the magnetising field and the magnetic induction B_m due to the induced magnetisation of the specimen.

$$B = B_o + B_m$$

But $\quad B_o = \mu_o H$ and $B_m = \mu_o I$

$$B = \mu_o H + \mu_o I$$

$$\therefore B = \mu_o (H + I)$$

(v) Magnetic susceptibility

Magnetic susceptibility χ_m is a property which determines how easily and how strongly a specimen can be magnetised.

Susceptibility of a magnetic material is defined as the ratio of intensity of magnetisation I induced in the material to the magnetising field H in which the material is placed.

Thus $\chi_m = \frac{I}{H}$

Since I and H are of the same dimensions, χ_m has no unit and is dimensionless.

Relation between χ_m and μ_r

$$\chi_m = \frac{I}{H}$$

$$\therefore I = \chi_m H$$

We know $B = \mu_o (H + I)$

$$B = \mu_o (H + \chi_m H)$$

$$B = \mu_o H (1 + \chi_m)$$

If μ is the permeability, we know that $B = \mu H$.

$$\therefore \mu H = \mu_o H (1 + \chi_m)$$

$$\frac{\mu}{\mu_o} = (1 + \chi_m)$$

$$\therefore \mu_r = 1 + \chi_m$$

3.17 Classification of magnetic materials

On the basis of the behaviour of materials in a magnetising field, the materials are generally classified into three categories namely, (i) Diamagnetic, (ii) Paramagnetic and (iii) Ferromagnetic

(i) Properties of diamagnetic substances

Diamagnetic substances are those in which the net magnetic moment of atoms is zero.

1. The susceptibility has a low negative value. (For example, for bismuth $\chi_m = -0.00017$).

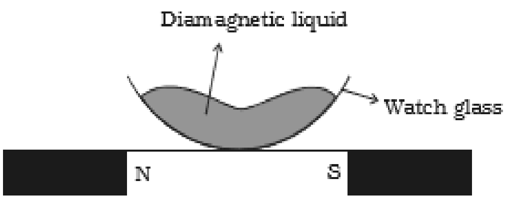


Fig. 3.41 Diamagnetic liquid

2. Susceptibility is independent of temperature.

3. The relative permeability is slightly less than one.

4. When placed in a non uniform magnetic field they have a tendency to move

away from the field. (i.e) from the stronger part to the weaker part of the field. They get magnetised in a direction opposite to the field as shown in the Fig. 3.41.

5. When suspended freely in a uniform magnetic field, they set themselves perpendicular to the direction of the magnetic field (Fig. 3.42).

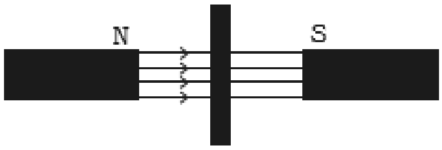


Fig. 3.42 Diamagnetic material perpendicular to the field

Examples : Bi, Sb, Cu, Au, Hg, H_2O, H_2 etc.

(ii) Properties of paramagnetic substances

Paramagnetic substances are those in which each atom or molecule has a net non-zero magnetic moment of its own.

1. Susceptibility has a low positive value.

(For example : χ_m for aluminium is +0.00002).

2. Susceptibiltity is inversely proportional to absolute temperature

(i.e) $\chi_m \alpha \frac{1}{T}$. As the temperature increases susceptibility decreases.

3. The relative permeability is greater than one.

4. When placed in a non uniform magnetic field, they have a tendency to move from weaker part to the stronger part of the field. They get magnetised in the direction of the field as shown in Fig. 3.43.

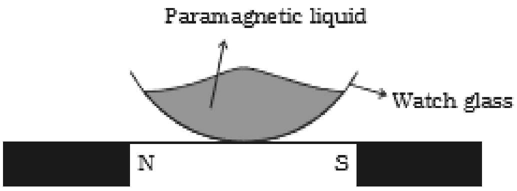


Fig. 3.43 Paramagnetic liquid

5. When suspended freely in a uniform magnetic field, they set themselves parallel to the direction of magnetic field (Fig. 3.44).

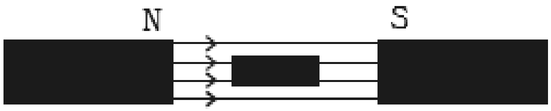


Fig. 3.44 Paramagnetic material parallel to the field

Examples : Al, Pt, Cr, O_2, Mn, $CuSO_4$ etc.

(iii) Properties of ferromagnetic substances

Ferromagnetic substances are those in which each atom or molecule has a strong spontaneous net magnetic moment. These substances exhibit strong paramagnetic properties.

1. The susceptibility and relative permeability are very large.

(For example : μ_r for iron = 200,000)

2. Susceptibility is inversely proportional to the absolute temperature.

(i.e) $\chi_m \alpha \frac{1}{T}$. As the temperature increases the value of susceptibility decreases. *At a particular temperature, ferro magnetics become para magnetics. This transition temperature is called curie temperature.* For example curie temperature of iron is about 1000 K.

3. When suspended freely in uniform magnetic field, they set themselves parallel to the direction of magnetic field.

4. When placed in a non uniform magnetic field, they have a tendency to move from the weaker part to the stronger part of the field. They get strongly magnetised in the direction of the field.

Examples : Fe, Ni, Co and a number of their alloys.

3.18 Hysteresis

Consider an iron bar being magnetised slowly by a magnetising field H whose strength can be changed. It is found that the magnetic induction B inside the material increases with the strength of the magnetising field and then attains a saturated level. This is depicted by the path OP in the Fig. 3.45.

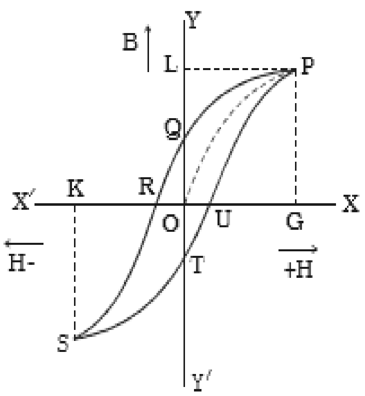


Fig. 3.45 Hysteresis loop

If the magnetising field is now decreased slowly, then magnetic induction also decreases but it does not follow the path PO. Instead, when H = 0, B has non zero value equal to OQ. This implies that some magnetism is left in the specimen. *The value of magnetic induction of a substance, when the magnetising field is reduced to zero, is called remanance or residual magnetic induction of the material.* OQ represents the residual magnetism of the material. Now, if we apply the magnetising field in the reverse direction, the magnetic induction decreases along QR till it becomes zero at R. Thus to reduce the residual magnetism (remanent magnetism) to zero, we have to apply a magnetising field OR in the opposite direction.

The value of the magnetising field H which has to be applied to the magnetic material in the reverse direction so as to reduce its residual magnetism to zero is called its coercivity.

When the strength of the magnetising field H is further increased in the reverse direction, the magnetic induction increases along RS till it acquires saturation at a point S (points P and S are symmetrical). If we now again change the direction of the field, the magnetic induction follows the path STUP. *This closed curve PQRSTUP is called the 'hysteresis loop' and it represents a cycle of magnetisation.* The word 'hysteresis' literally means lagging behind. We have seen that magnetic induction B lags behind the magnetising field H in a cycle of magnetisation. *This phenomenon of lagging of magnetic induction behind the magnetising field is called hysteresis.*

Hysteresis loss

In the process of magnetisation of a ferromagnetic substance through a cycle, there is expenditure of energy. The energy spent in magnetising a specimen is not recoverable and there occurs a loss of energy in the form of heat. This is so because, during a cycle of magnetisation, the molecular magnets in the specimen are oriented and reoriented a number of times. This molecular motion results in the production of heat. It has been found that *loss of heat energy per unit volume of the specimen in each cycle of magnetisation is equal to the area of the hysteresis loop.*

The shape and size of the hysteresis loop is characteristic of each material because of the differences in their retentivity, coercivity, permeability, susceptibility and energy losses etc. By studying hysteresis loops of various materials, one can select suitable materials for different purposes.

3.18.1. *Uses of ferromagnetic materials*

(i) Permanent magnets

The ideal material for making permanent magnets should possess high retentivity (residual magnetism) and high coercivity so that the magnetisation lasts for a longer time. Examples of such substances are steel and alnico (an alloy of Al, Ni and Co).

(ii) Electromagnets

Material used for making an electro-magnet has to undergo cyclic changes. Therefore, the ideal material for making an electromagnet has to be one which has the least hysteresis loss. Moreover, the material should attain high values of magnetic induction B at low values of magnetising field H. Soft iron is preferred for making electromagnets as it has a thin hysteresis loop (Fig. 3.46) [small area, therefore less hysteresis loss] and low retentivity. It attains high values of B at low values of magnetising field H.

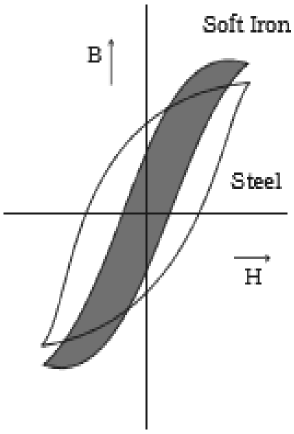


Fig. 3.46 Hysteresis loop for steel and soft iron

(iii) Core of the transformer

A material used for making transformer core and choke is subjected to cyclic changes very rapidly. Also, the material must have a large value of magnetic induction B. Therefore, soft iron that has thin and tall hysteresis loop is preferred. Some alloys with low hysteresis loss are: radio-metals, pern-alloy and mumetal.

(iv) Magnetic tapes and memory store

Magnetisation of a magnet depends not only on the magnetising field but also on the cycle of magnetisation it has undergone. Thus, the value of magnetisation of the specimen is a record of the cycles of magnetisation it has undergone. Therefore, such a system can act as a device for storing memory.

Ferro magnetic materials are used for coating magnetic tapes in a cassette player and for building a memory store in a modern computer. Examples : Ferrites (Fe, Fe_2O, $MnFe_2O_4$ etc.).

3.19. Bar magnet as an equivalent solenoid

1. A solenoid is an arrangement in which a long insulated wire is wound in a closely packed helix.
2. A current carrying coil behaves like a bar magnet, a solenoid also behaves like a bar magnet.
3. In solenoid, the length of the helix is very large as compared to its diameter as shown in fig. 3.47. Each turn of a current carrying solenoid acts approximately as a circular loop.

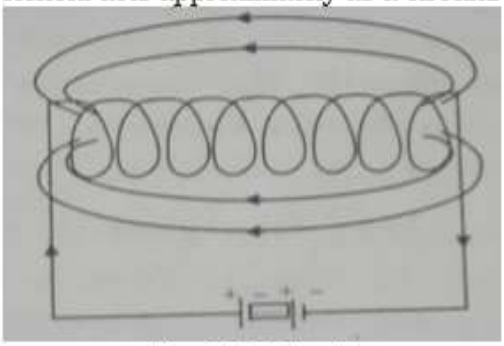


Fig. 3.47 Solenoid

4. The magnetic field lines in case of a solenoid diverge from one end and converge at the other end which is similar to that of a bar magnet. Thus, one end of the solenoid behaves like a north pole and other end behaves like the south pole.
5. Consider a bar magnet of magnetic dipole moment M. The magnitude of the magnetic induction at a point d along the axis of a short magnetic dipole is given by,

$$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

6. If the current through the solenoid is I then at a distance d from the centre of a solenoid of area of cross section A, the magnitude of magnetic induction is given by,

$$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

Where, n = total number of turns.
A = cross sectional area of coil.
M = nIA is the magnetic moment

7. Comparing equations (1) and (2), we came to know, A magnetic dipole and a solenoid produce similar magnetic fields.
8. Hence, there is an equivalence between bar magnet (magnetic dipole) and solenoid

3.20. Magnetic field of the earth (terrestrial magnetism)

A study of earth's magnetic field is called terrestrial magnetism. It is also called geomagnetism.

When a bar magnet is suspended freely, its N-pole points towards the geographic north pole and S-pole towards geographic south pole.

William Gilbert in 1600 showed that the earth behaves as a huge magnet. The surrounding space of the earth possesses a magnetic field. This magnetic field is responsible for the force on a suspended magnet. The magnetic field on the surface of the earth can be approximately represented as if caused by a huge imaginary bar magnet in the interior of the earth. The north end of the imaginary magnet has been assigned a south polarity while the south end has been assigned north polarity. The axis of the earth's magnetic dipole does not coincide with the axis of the rotation of the earth but it is inclined at an angle of nearly 11.5^0 to it. The strength of the earth's magnetic field on the surface of the earth is of the order of 10^{-4} tesla. The strength and d direction of the earth's magnetic field not only varies from place to place, but also varies over a long period of time. The direction and magnitude of the earth's magnetic field at any place is commonly specified in terms of magnetic elements (see fig 3.48):

1. Magnetic field declination or Declination (θ)
2. Angle of dip or Dip (δ)
3. Horizontal component of the earth's magnetic field (B_H)

3.20.1 Various terms with regard to earth's magnetic field.

1. **Geographic Axis:** Geographic axis is defined as a straight line around which the earth spins.
2. **Geographic meridian:** Geographic meridian at a given place is defined as, a vertical plane passing through the geographic north and south poles of the earth.
3. **Geographic equator:** geographic equator is defined as a great imaginary circle on the surface of the earth, in a plane perpendicular to the geographic axis.
4. **Magnetic axis:** Magnetic axis is defined as a straight line passing through the magnetic poles of the earth.
5. **Magnetic Meridian:** Magnetic meridian at a given place is defined as, the vertical plane passing through the magnetic north and south pole of the earth.
6. **Magnetic equator:** Magnetic equator is defined as a great imaginary circle on the surface of the earth, in a plane perpendicular to the magnetic axis.

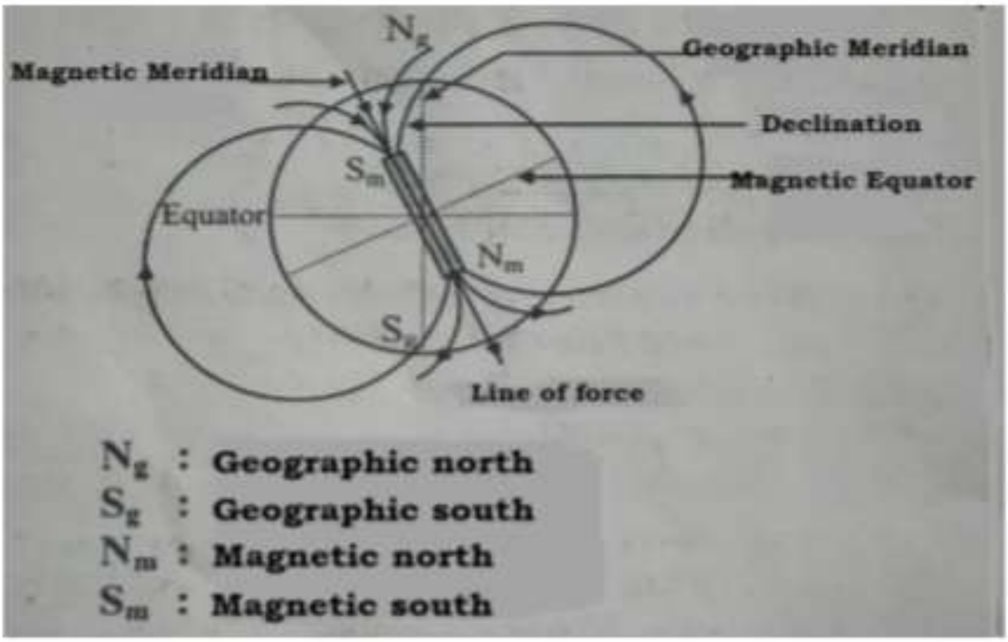


Fig 3.48 Earth's magnetic field

23.20.2. Detail of Magnetic elements

(i) *Declination*

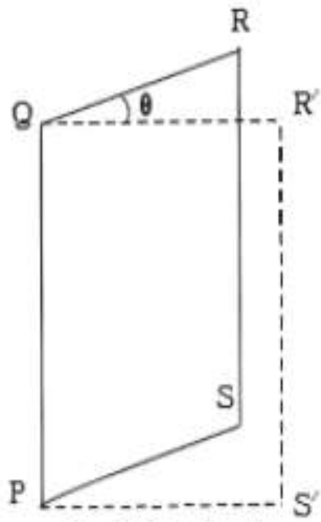


Fig. 3.49 Declination

A vertical plane passing through the axis of a freely suspended magnetic needle is called magnetic meridian and the vertical plane passing through the geographic north - south direction (axis of rotation of Earth) is called geographic meridian (Fig.).

In the Fig. 3.49 the plane $PQRS$ represents the magnetic meridian and the plane $PQR'S'$ represents the geographic meridian.

Declination at a place is defined as the angle between magnetic meridian and the geographic meridian at that place.

Determination of declination

A simple method of determining the geographical meridian at a place is to erect a pole of 1 to 1.5 m height on the ground and a circle is drawn with the pole O as centre and its height as radius as shown in the Fig. 3.50.

Mark a point P_1 on the circle before noon, when the tip of the shadow of the pole just touches the circle.

Again mark a point P_2 when the tip of the shadow touches the circle in the afternoon. The line POQ drawn bisecting the angle P_1OP_2 is the geographical meridian at that place.

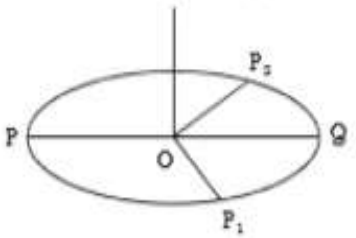


Fig. 3.50 Geographic Meridian

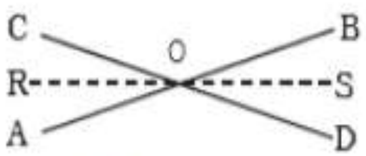


Fig. 3.51 Magnetic meridian

Magnetic meridian is drawn by freely suspending a magnetic needle provided with two pins fixed vertically at its ends.

When the needle is at rest, draw a line AB joining the tips of the two pins. The magnetic needle is reversed upside down. Pins are fixed at the ends of the needle. When the magnet is at rest, draw a line CD joining the tips of pins. O is the point of intersection of AB and CD. The line RS bisecting the angle BOD is the magnetic meridian at that place (Fig. 3.51).

Now the angle between geographic meridian PQ and the magnetic meridian RS is the angle of declination θ (Fig.3.52).

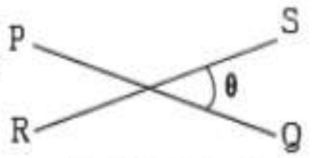


Fig. 3.52 Declination

(ii) *Dip*

Dip is defined as the angle between the direction of Earth's magnetic field and the direction of horizontal component of earths magentic field. It is the angle by which the total Earth's magnetic field dips or comes out of the horizontal plane. It is denoted by δ. The value of dip varies from place to place. It is O° along the equator and 90° at the poles. At Chennai the value of dip is about 9°7'.

At a place the value of dip is measured by an instrument called dip circle.

Dip circle

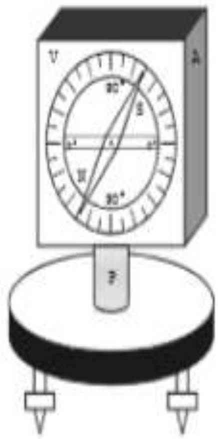


Fig. 3.53 Dip Circle

A magnetic needle NS is pivoted at the centre of a circular vertical scale V by means of a horizontal rod. The needle is free to move over this circular scale. The scale has four segments and each segment is graduated from 0° to 90° such that it reads 0° - 0° along the horizontal and 90° - 90° along the vertical. The needle and the scale are enclosed in a rectangular box A with glass window. The box is mounted on a vertical pillar P on a horizontal base, which is provided with levelling screws. The base has a circular scale graduated from 0° to 360° (FIg.3.53).The box can be rotated about a vertical axis and its position can be read on the circular scale with the help of a vernier (not shown in the figure).

The levelling screws are adjusted such that the base is horizontal and the scale inside the box is vertical. The box is rotated so that the ends of the magnetic needle NS read 90° - 90° on the vertical scale.

The needle, in this position is along the vertical component of the Earth's field. The horizontal component of Earth's field being perpendicular to the plane, does not affect the needle. This shows that the vertical scale and the needle are in a plane at right angles to the magnetic meridian. Now the box is rotated through an angle of 90° with the help of the horizontal circular scale. The magnetic needle comes to rest exactly in the magnetic meridian. The reading of the magnetic needle gives the angle of dip at that place.

(iii) The horizontal component of earth's magnetic field

The earth's magnetic induction 'B' is resolved into two components. The horizontal component of earth's magnetic induction is B_H and vertical component of earth's magnetic induction is B_V as shown in Fig. 3.54.

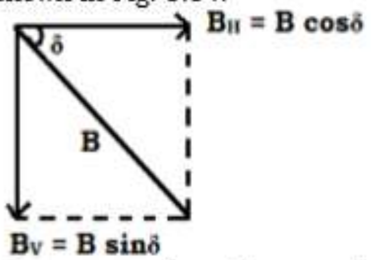


Fig. 3.54 Components of earth's magnetic induction

From the figure

$$B_H = B\,cos\delta \qquad -----(1)$$

$$B_V = B\,sin\delta \qquad -----(2)$$

Divide equation (1) by (2), we have,

$$\frac{B_V}{B_H} = \frac{B\,sin\delta}{B\,cos\delta}$$

$$\frac{B_V}{B_H} = tan\delta \qquad ----- \;(3)$$

Squaring and adding equations (10 and (2). We get,

$$B_H{}^2 + B_V{}^2 = B^2cos^2\theta + B^2sin^2\theta = B^2\,(cos^2\theta + sin^2\theta)$$

$$B_H{}^2 + B_V{}^2 = B^2$$

$$B = \sqrt{B_H{}^2 + B_V{}^2} \qquad ---- \;(4)$$

Thus, magnetic elements describe the magnitude and direction of magnetic field of the earth at a given place.

Ex. *The vertical and horizontal components of the earth's magnetic field at a place are $2 \times 10^{-5}\,T$ and $3.464 \times 10^{-5}\,T$ respectively. Calculate the angle of dip and resultant earth's magnetic field at that place.*

Solution: (i) $tan\delta = \frac{B_V}{B_H}$

$$\delta = tan^{-1}\left(\frac{B_V}{B_H}\right) = tan^{-1}\left(\frac{2 \times 10^{-5}}{3.464 \times 10^{-5}}\right) = tan^{-1}\,(0.577) = 30^0$$

(ii) $B = \sqrt{(3.464 \times 10^{-5})^2 + (2 \times 10^{-5})^2} = (\sqrt{12+4}) \times 10^{-5}$

$$B = 4 \times 10^{-5}\,T$$

3.21. Electromagnets

Electromagnets are artificial magnets prepared by passing an electric current through an insulated wire wound around a piece of soft iron.

Electromagnet consists of a solenoid with soft iron core. Due to current flowing through the coil of the solenoid, a strong magnetic field is produced along the axis of the solenoid. The iron core inside the solenoid produces a strong magnetic field. To make the electromagnet strong, the material of an electromagnet must have high value of saturation magnetization but low retentivity and coercivity. Also the material must have low hysteresis loss so that the heating effect during magnetization and demagnetization would be very less*.

Factors affecting the strength of electromagnets

The factors affecting the strength of an electromagnet are,'

1. Current through the solenoid.
2. Saturation magnetization of the core.
3. Number of turns per unit length of the solenoid.

Uses of electromagnets

1. Electromagnets are generally used to lift ferromagnetic substances like iron part, car or scrap.
2. Electromagnets are used in technology and in relays, circuit breakers, breaking system of the train.
3. High power electromagnets are used in cyclotrons which are used to accelerate charged particles.

** Please go through the topic 3.18.1*

Solved problems

3.1 A solenoid is 2m long and 3 cm in diameter. It has 5 layers of windings of 1000 turns each and carries a current of 5A. Find the magnetic induction at its centre along its axis.

Data : $l = 2m$, $N = 5 \times 1000$ turns, $I = 5A$, $B = ?$

Solution : $B = \mu_o nI = \mu_o \frac{N}{l}.I$

$$B = \frac{4\pi \times 10^{-7} \times 5000 \times 5}{2}$$

$$B = 1.57 \times 10^{-2}\ T$$

3.2 An α-particle moves with a speed of $5 \times 10^5\ ms^{-1}$ at an angle of 30^o with respect to a magnetic field of induction 10^{-4} T. Find the force on the particle. [α particle has a +ve charge of 2e]

Data : $B = 10^{-4}$ T, $q = 2e$, $v = 5 \times 10^5\ ms^{-1}$, $\theta = 30^0$, $F = ?$

Solution $F = Bqv \sin\theta$

$$= B(2e)\ v \sin 30^o$$

$$= 10^{-4} \times 2 \times 1.6 \times 10^{-19} \times 5 \times 10^5 \times \frac{1}{2}$$

$$F = 8 \times 10^{-18} N$$

3.3 A stream of deutrons is projected with a velocity of $10^4\ ms^{-1}$ in XY – plane. A uniform magnetic field of induction 10^{-8} T acts along the Z-axis. Find the radius of the circular path of the particle. (Mass of deuteron is 3.32×10^{-27} kg and charge of deuteron is 1.6×10^{-19}C)

Data : $v = 10^4\ ms^{-1}$, $B = 10^{-8}$T, $m = 3.32 \times 10^{-27}$ kg

$e = 1.6 \times 10^{-19}$C, $r = ?$

Solution : $Bev = \frac{mv^2}{r}$

$$\therefore r = \frac{mv}{Be} = \frac{3.32 \times 10^{-27} \times 10^4}{10^{-8} \times 1.6 \times 10^{-19}} = 2.08 \times 10^{-1}$$

$$r = 0.208m$$

3.4 A uniform magnetic field of induction 0.5 T acts perpendicular to the plane of the Dees of a cyclotron. Calculate the frequency of the oscillator to accelerate protons. (mass of proton = 1.67×10^{-27} kg)

Data : $B = 0.5$ T, $m_p = 1.67 \times 10^{-27}$ kg, $q = 1.6 \times 10^{-19}$C, $\nu = ?$

Solution: $\nu = \dfrac{Bq}{2\pi m_p}$

$$= \frac{0.5\times1.6\times10^{-19}}{2\times3.14\times1.67\times10^{-27}} = 0.763 \times 10^{7} = 7.63 \times 10^{6} \text{ Hz}$$

$\therefore\ \nu = 7.63$ MHz

3.5 A conductor of length 50 cm carrying a current of 5A is placed perpendicular to a magnetic field of induction 2×10^{-8} T. Find the force on the conductor.

Data : $l = 50$ cm $= 5\times10^{-1}$m, $I = 5$A, $B = 2\times10^{-8}$T; $\theta = 90^o$, $F = ?$

Solution: $F = BIl \sin\theta$

$$= 2 \times 10^{-8} \times 5 \times 5 \times 10^{-1} \times \sin 90^0$$

$\therefore\ F = 5 \times 10^{-8}$ N

3.6 Two parallel wires each of length 5m are placed at a distance of 10 cm apart in air. They carry equal currents along the same direction and experience a mutually attractive force of 3.6×10^{-4} N. Find the current through the conductors.

Data : $I_1 = I_2 = I$,

$l = 5$m, $a = 10^{-1}$ m,

$F = 3.6\times10^{-4}$N, $I = ?$

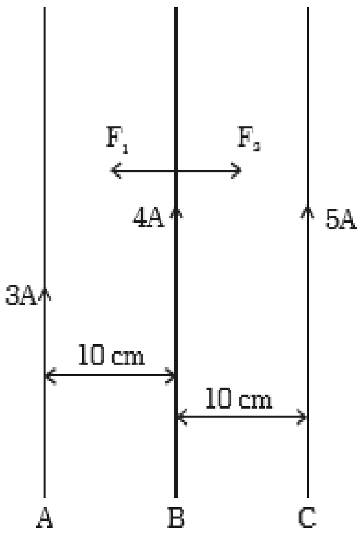
Solution: $F = \dfrac{\mu_o I_1 I_2 l}{2\pi a}$

$$F = \frac{2\times10^{-7} I^2 l}{a}$$

$$\therefore I^2 = \frac{F.a}{2\times10^{-7} l} = \frac{3.6\times10^{-4}\times10^{-1}}{2\times10^{-7}\times5} = 36$$

$\therefore\ I = 6$A

3.7 A, B and C are three parallel conductors each of length 10 m, carrying currents as shown in the figure. Find the magnitude and direction of the resultant force on the conductor B.



Solution : Between the wires A and B, force of attraction exists.

F_1 acts towards left

$$F_1 = \frac{2\times10^{-7} I_1 I_2 l}{a} = \frac{2\times10^{-7}\times3\times4\times10}{10^{-1}}$$

$F_1 = 24 \times 10^{-5}$ N

Between the wires B and C, force of attraction exists

F_2 acts towards right

$$F_2 = \frac{2\times10^{-7} I_1 I_2 l}{a} = \frac{2\times10^{-7}\times4\times5\times10}{10^{-1}}$$

$F_2 = 40 \times 10^{-5}$ N

$F_2 - F_1 = 16 \times 10^{-5}$ N

The wire B is attracted towards C with a net force of 16×10^{-5} N.

3.8 A rectangular coil of area 20 cm × 10 cm with 100 turns of wire is suspended in a radial magnetic field of induction 5×10^{-3} T. If the galvanometer shows an angular deflection of 15^0 for a current of 1mA, find the torsional constant of the suspension wire.

Data : $n = 100$, $A = 20 \text{ cm} \times 10 \text{ cm} = 2 \times 10^{-1} \times 10^{-1}\ \text{m}^2$

$B = 5 \times 10^{-3}$ T, $\theta = 15^0$, $I = 1\text{mA} = 10^{-3}$A, $C = ?$

Solution : $\theta = 15^0 = \frac{\pi}{180}\times15 = \frac{\pi}{12}$ rad

$nBIA = C\theta$

$$\therefore C = \frac{nBIA}{\theta} = \frac{10^2\times5\times10^{-3}\times10^{-3}\times2\times10^{-1}\times10^{-1}}{\left(\frac{\pi}{12}\right)}$$

$C = 3.82 \times 10^{-5}$ N m rad^{-1}

3.9 A moving coil galvanometer of resistance 20 Ω produces full scale deflection for a current of 50 mA. How you will convert the galvanometer into (i) an ammeter of range 20 A and (ii) a voltmeter of range 120 V.

Data : $G = 20\ \Omega$; $I_g = 50 \times 10^{-3}$ A ; I = 20 A, S = ?

V = 120 V, R = ?

Solution : (i) $S = G \cdot \dfrac{I_g}{I - I_g} = \dfrac{20 \times 50 \times 10^{-3}}{20 - 50 \times 10^{-3}} = \dfrac{1}{20 - 0.05}$

$S = 0.05\ \Omega$

A shunt of 0.05 Ω should be connected in parallel

(ii) $R = \dfrac{V}{Ig} - G$

$= \dfrac{120}{50 \times 10^{-3}} - 20 = 2400 - 20 = 2380\ \Omega$

$R = 2380\ \Omega$

A resistance of 2380 Ω should be connected in series with the galvanometer.

3.10 The deflection in a galvanometer falls from 50 divisions to 10 divisions when 12 Ω resistance is connected across the galvanometer. Calculate the galvanometer resistance.

Data : $\theta_1 = 50$ divs, $\theta_g = 10$ divs, $S = 12\Omega$ G = ?

Solution : $I \propto \theta_1$

$I_g \propto \theta_g$

In a parallel circuit potential is common.

$\therefore G.\ I_g = S\ (I - I_g)$

$\therefore G = \dfrac{S\,(I - I_g)}{Ig} = \dfrac{12\,(50 - 10)}{10}$

$\therefore G = 48\ \Omega$

3.11 A short bar magnet is placed with its north pole pointing north. The neutral point is 10 cm away from the centre of the magnet. If $B = 4 \times 10^{-5}$ T, calculate the magnetic moment of the magnet.

Data : *$d = 10 \times 10^{-2}$ m; $B = 4 \times 10^{-5}$ T; $M = ?$*

Solution : *When the north pole of a bar magnet points north, the neutral points will lie on its equatorial line.*

$\therefore$ The field at the neutral point on the equatorial line of a short bar magnet is, $B = \frac{\mu_o}{4\pi}\frac{M}{d^3}$

$\therefore \quad M = B \times d^3 \times 10^7 = 4 \times 10^{-5} (10 \times 10^{-2})^3 \times 10^7$

$M = 0.4 \text{ A m}^2$

3.12 A bar magnet is suspended horizontally by a torsionless wire in magnetic meridian. In order to deflect the magnet through 30^o from the magnetic meridian, the upper end of the wire has to be rotated by 270^o. Now this magnet is replaced by another magnet. In order to deflect the second magnet through the same angle from the magnetic meridian, the upper end of the wire has to be rotated by 180^o. What is the ratio of the magnetic moments of the two bar magnets. (Hint : $\tau = C\theta$)

Solution : *Let C be the deflecting torque per unit twist and M_1 and M_2 be the magnetic moments of the two magnets.*

The deflecting torque is $\tau = C\theta$

The restoring torque is $\tau = MB \sin \theta$

In equilibrium

deflecting torque = restoring torque

For the Magnet – I

$$C\,(270^o - 30^o) = M_1\, B_h \sin \theta \qquad \text{... (1)}$$

For the magnet – II

$$C\,(180^o - 30^o) = M_2\, B_h \sin \theta \qquad \text{... (2)}$$

Dividing (1) by (2)

$$\frac{M_1}{M_2} = \frac{240^o}{150^o} = \frac{8}{5}$$

3.13 A short bar magnet of magnetic moment 5.25×10^{-2} A m^2 is placed with its axis perpendicular to the Earth's field direction. At what distance from the centre of the magnet on (i) its equatorial line and (ii) its axial line, is the resultant field inclined at 45° with the Earth's field. Magnitude of the Earth's field at the place is 0.42×10^{-4} T.

Data :

$M = 5.25 \times 10^{-2}\ A\ m^{-2}$

$\theta = 45^o$

$B_h = 0.42 \times 10^{-4}\ T$

$d = ?$

Solution : *From Tangent Law*

$$\frac{B}{B_h} = Tan\ \theta$$

$$B = B_h \tan\theta = 0.42 \times 10^{-4} \times \tan 45^o$$

$$B = 0.42 \times 10^{-4}\ T$$

(i) *For the point on the equatorial line*

$$B = \frac{\mu_o}{4\pi}\frac{M}{d^3}$$

$$d^3 = \frac{\mu_o}{4\pi}\frac{M}{B}$$

$$d^3 = \frac{4\pi \times 10^{-7} \times 5.25 \times 10^{-2}}{4\pi \times 0.42 \times 10^{-4}}$$

$$= 12.5 \times 10^{-5} = 125 \times 10^{-6}$$

$$\therefore\ d = 5 \times 10^{-2}\ m$$

(ii) *For the point on the axial line*

$$B = \frac{\mu_o}{4\pi}\frac{2M}{d^3} \quad (or) \quad d^3 = \frac{\mu_o}{4\pi}\frac{2M}{B}$$

$$d^3 = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 5.25 \times 10^{-2}}{0.42 \times 10^{-4}}$$

$$d^3 = 250 \times 10^{-6} = 2 \times 125 \times 10^{-6}$$

$d = 2^{1/3} \cdot (5 \times 10^{-2})$

$d = 6.3 \times 10^{-2}$ m.

3.14 A bar magnet of mass 90 g has magnetic moment 3 A m^2. If the intensity of magnetisation of the magnet is 2.7×10^5 A m^{-1}, find the density of the material of the magnet.

Data : $m = 90 \times 10^{-3}$ kg; $M = 3$ A m^2

$I = 2.7 \times 10^5$ A m^{-1}; $\rho = ?$

Solution : Intensity of magnetisation, $I = \frac{M}{V}$

But, volume $V = \frac{m}{\rho}$

$\therefore I = \frac{M\rho}{m}$

$$\rho = \frac{Im}{M} = \frac{2.7 \times 10^5 \times 90 \times 10^{-3}}{3} = 8100$$

$\rho = 8100$ kg m^{-3}

3.15 A magnetising field of 50 A m^{-1} produces a magnetic field of induction 0.024 T in a bar of length 8 cm and area of cross section 1.5 cm^2. Calculate (i) the magnetic permeability (ii) the magnetic susceptibility.

Data : $H = 50$ A m^{-1}, $B = 0.024$ T $= 2.4 \times 10^{-2}$ T,

$2l = 8 \times 10^{-2}$ m, $A = 1.5 \times 10^{-4}$ m^2 $\mu = ?$; $\chi_m = ?$

Solution : Permeability $\mu = \frac{B}{H} = \frac{2.4 \times 10^{-2}}{50} = 4.8 \times 10^{-4} H\ m^{-1}$

Susceptibility, $\chi_m = \mu_r - 1 = \frac{\mu}{\mu_o} - 1$

$$\chi_m = \frac{4.8 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 381.16$$

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

3.1 Which of the following equations represents Biot-savart law?

(a) $dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$ (b) $\vec{dB} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$

(c) $\vec{dB} = \frac{\mu_0}{4\pi}\frac{\overline{Idl}\times\bar{r}}{r^2}$ (d) $\vec{dB} = \frac{\mu_0}{4\pi}\frac{\overline{Idl}\times\bar{r}}{r^3}$

3.2 Magnetic induction due to an infinitely long straight conductor placed in a medium of permeability μ is

(a) $\frac{\mu_0 I}{4\pi a}$ (b) $\frac{\mu_0 I}{2\pi a}$

(c) $\frac{\mu I}{4\pi a}$ (d) $\frac{\mu I}{2\pi a}$

3.3 The period of revolution of a charged particle inside a cyclotron does not depend on

(a) the magnetic induction (b) the charge of the particle

(c) the velocity of the particle (d) the mass of the particle

3.4 The torque on a rectangular coil placed in a uniform magnetic field is large, when

(a) the number of turns is large

(b) the number of turns is less

(c) the plane of the coil is perpendicular to the field

(d) the area of the coil is small

3.5 Phosphor – bronze wire is used for suspension in a moving coil galvanometer, because it has

(a) high conductivity (b) high resistivity

(c) large couple per unit twist (d) small couple per unit twist

3.6 Of the following devices, which has small resistance?

(a) moving coil galvanometer (b) ammeter of range 0 – 1A

(c) ammeter of range 0–10 A (d) voltmeter

3.6 A galvanometer of resistance G Ω is shunted with S Ω .The effective resistance of the combination is R_a. Then, which of the following statements is true?

(a) G is less than S

(b) S is less than R_a but greater than G.

(c) R_a is less than both G and S

(d) S is less than both G and R_a

3.7 An ideal voltmeter has

(a) zero resistance

(b) finite resistance less than G but greater than Zero

(c) resistance greater than G but less than infinity

(d) infinite resistance

3.8 Two magnetic poles kept separated by a distance d in vacuum experience a force of 10 N. The force they would experience when kept inside a medium of relative permeability 2, separated by the same distance is

(a) 20 N (b) 10 N

(c) 5 N (d) 40 N

3.9 The magnetic moment of a magnet is 5 A m^2. If the pole strength is 25 A m, what is the length of the magnet?

(a) 10 cm (b) 20 cm

(c) 25 cm (d) 1.25 cm

3.10 A long magnetic needle of length 2l, magnetic moment M and pole strength m is broken into two at the middle. The magnetic moment and pole strength of each piece will be

(a) M, m (b) $\frac{M}{2}, \frac{m}{2}$

(c) M, $\frac{m}{2}$ (d) $\frac{M}{2}$, m

3.11 *The relative permeability of a specimen is 10001 and magnetising field strength is 2500 A m^{-1}. The intensity of magnetisation is*

(a) 0.5×10^{-7} A m^{-1} (b) 2.5×10^{-7} A m^{-1}

(c) $2.5 \times 1.0^{+7}$ A m^{-1} (d) 2.5×10^{-1} A m^{-1}

3.12 *For which of the following substances, the magnetic susceptibility is independent of temperature?*

(a) diamagnetic

(b) paramagnetic

(c) ferromagnetic

(d) diamagnetic and paramagnetic

3.13 *At curie point, a ferromagnetic material becomes*

(a) non–magnetic (b) diamagnetic

(c) paramagnetic (d) strongly ferromagnetic

3.14 *Electromagnets are made of soft iron because soft iron has*

(a) low susceptibility and low retentivity

(b) high susceptibility and low retentivity

(c) high susceptibility and high retentivity

(d) low permeability and high retentivity

3.15 *State Biot – Savart law*

3.16 *Obtain an expression for the magnetic induction at a point due to an infinitely long straight conductor carrying current.*

3.17 *Deduce the relation for the magnetic induction at a point along the axis of a circular coil carrying current.*

3.18 *What is Ampere's circuital law?*

3.19 *Applying Amperes circuital law, find the magnetic induction due to a straight solenoid.*

3.20 *Define ampere*

3.21 *Deduce an expression for the force on a current carrying conductor placed in a magnetic field.*

3.22 *Explain in detail the principle, construction and the theory of moving coil galvanometer.*

3.23 *Obtain the expressions for the magnetic induction at a point on the (i) axial line and (ii) equatorial line of a bar magnet.*

3.24 *Find the torque experienced by a magnetic needle in a uniform magnetic field.*

Problems

3.25 *Find the magnetic induction at a point, 10 cm from a long straight wire carrying a current of 10A*

3.26 *A circular coil of radius 20 cm has 100 turns wire and it carries a current of 5A. Find the magnetic induction at a point along its axis at a distance of 20 cm from the centre of the coil.*

3.27 *A straight wire of length one metre and of resistance 2 Ω is connected across a battery of emf 12V. The wire is placed normal to a magnetic field of induction 5×10^{-3} T. Find the force on the wire.*

3.28 *A circular coil of 50 turns and radius 25 cm carries a current of 6A. It is suspended in a uniform magnetic field of induction 10^{-3} T. The normal to the plane of the coil makes an angle of 60^0 with the field. Calculate the torque of the coil.*

3.29 *A uniform magnetic field 0.5 T is applied normal to the plane of the Dees of a Cyclotron. Calculate the period of the alternating potential to be applied to the Dees to accelerate deutrons (mass of deuteron = 3.3×10^{-27} kg and its charge = 1.6×10^{-19}C).*

3.30 *A rectangular coil of 500 turns and of area 6×10^{-4} m^2 is suspended inside a radial magnetic field of induction 10^{-4} T by a suspension wire of torsional constant 5×10^{-10} Nm per degree. calculate the current required to produce a deflection of 10^o.*

3.31 *Two straight infinitely long parallel wires carrying equal currents and placed at a distance of 20 cm apart in air experience a mutally attractive force of 4.9×10^{-5} N per unit length of the wire. Calculate the current.*

3.32 *A long solenoid of length 3m has 4000 turns. Find the current through the solenoid if the magnetic field produced at the centre of the solenoid along its axis is 8×10^{-3} T.*

3.33 *A galvanometer has a resistance of 100 Ω. A shunt resistance 1 Ω is connected across it. What part of the total current flows through the galvanometer?*

3.34 *A galvanometer has a resistance of 40 Ω. It shows full scale deflection for a current of 2 mA. How you will convert the galvanometer into a voltmeter of range 0 to 20V?*

3.35 *A galvanometer with 50 divisions on the scale requires a current sensitivity of 0.1 m A/division. The resistance of the galvanometer is 40 Ω. If a shunt resistance 0.1 Ω is connected across it, find the maximum value of the current that can be measured using this ammeter.*

3.36 *The force acting on each pole of a magnet placed in a uniform magnetic induction of 5×10^{-4} T is 6×10^{-3} N. If the length of the magnet is 8 cm, calculate the magnetic moment of the magnet.*

3.37 *Two magnetic poles, one of which is twice stronger than the other, repel one another with a force of 2×10^{-5} N, when kept seperated at a distance of 20 cm in air. Calculate the strength of each pole.*

3.38 *Two like poles of unequal pole strength are placed 1 m apart. If a pole of strength 4 A m is in equilibrium at a distance 0.2 m from one of the poles, calculate the ratio of the pole strengths of the two poles.*

3.39 *A magnet of pole strength 24.6×10^{-2} A m and length 10 cm is placed at 30° with a magnetic field of 0.01 T. Find the torque acting on the magnet.*

3.40 *The magnetic moment of a bar magnet of length 10 cm is 9.8×10^{-1} A m^2. Calculate the magnetic field at a point on its axis at a distance of 20 cm from its midpoint.*

3.41 *Two mutually perpendicular lines are drawn on a table. Two small magnets of magnetic moments 0.108 and 0.192 A m^2 respectively are placed on these lines. If the distance of the point of intersection of these lines is 30 cm and 40 cm respectively from these magnets, find the resultant magnetic field at the point of intersection.*

3.42 *The intensity of magnetisation of an iron bar of mass 72 g, density 7200 kg m^{-3} is 0.72 A m^{-1}. Calculate the magnetic moment.*

3.43 *A magnet of volume 25 cm^3 has a magnetic moment of 12.5×10^{-4} A m^2. Calculate the intensity of magnetisation.*

3.44 *A magnetic intensity of 2×10^3 A/m produces a magnetic induction of 4π Wb/m^2 in a bar of iron. Calculate the relative permeability and susceptibility.*

Answers

3.1 (d) **3.2** (d) **3.3** (c) **3.4** (a) **3.5** (d)

3.6 (c) **3.7** (d) **3.8** (a) **3.9** (b) **3.10** (d)

3.11 (c) **3.12** (a) **3.13** (c) 3.14 (b)

3.25 2×10^{-5} T

3.26 5.55×10^{-4} T

3.27 3×10^{-2} N

3.28 5.1×10^{-2} N m

3.29 2.6×10^{-7} s

3.30 0.166 m A

3.31 7 A

3.32 4.77 A

3.33 1/101

3.34 9960 Ω in series

3.35 2 A

3.36 0.96 A m^2

3.37 2 A m, 4 A m

3.38 1 :16

3.39 1.23×10^{-4} N m

3.40 2.787×10^{-5} T

3.41 10^{-6} T

3.42 7.2×10^{-6} A m^2

3.43 50 A m^{-1}

3.44 5000, 4999

4. Electromagnetic Induction and Alternating Current

In the year 1820, Hans Christian Oersted demonstrated that a current carrying conductor is associated with a magnetic field. Thereafter, attempts were made by many to verify the reverse effect of producing an induced emf by the effect of magnetic field.

4.1 Electromagnetic induction

Michael Faraday demonstrated the reverse effect of Oersted experiment. He explained the possibility of producing emf across the ends of a conductor when the magnetic flux linked with the conductor changes. This was termed as electromagnetic induction. The discovery of this phenomenon brought about a revolution in the field of power generation.

4.1.1 Magnetic flux

The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing a closed area (A) (Fig 4.1). If θ is the angle between the direction of the field and normal to the area, then

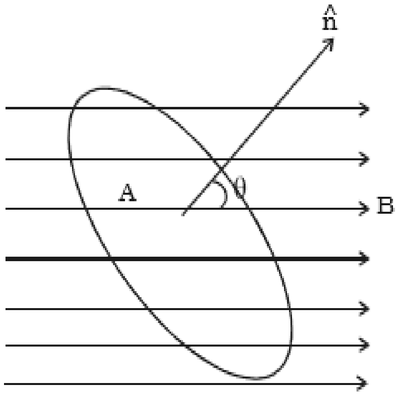


Fig 4.1 Magnetic flux

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA \cos \theta$$

4.1.2 Induced emf and current – Electromagnetic induction.

Whenever there is a change in the magnetic flux linked with a closed circuit an emf is produced. This emf is known as the induced emf and the current that flows in the closed circuit is called induced current. The phenomenon of producing an induced emf due to the changes in the magnetic flux associated with a closed circuit is known as electromagnetic induction.

Faraday discovered the electromagnetic induction by conducting several experiments.

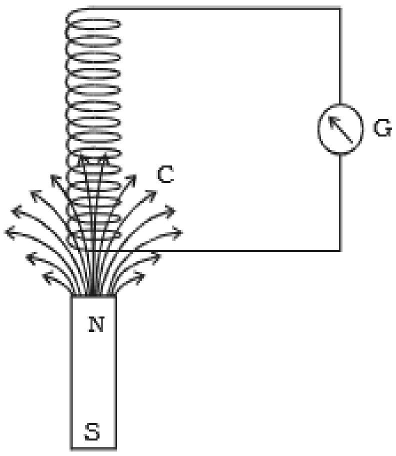


Fig 4.2 Electromagnetic Induction

Fig 4.2 consists of a cylindrical coil C made up of several turns of insulated copper wire connected in series to a sensitive galvanometer G. A strong bar magnet NS with its north pole pointing towards the coil is moved up and down. The following inferences were made by Faraday.

(i) Whenever there is a relative motion between the coil and the magnet, the galvanometer shows deflection indicating the flow of induced current.

(ii) The deflection is momentary. It lasts so long as there is relative motion between the coil and the magnet.

(iii) The direction of the flow of current changes if the magnet is moved towards and withdrawn from it.

(iv) The deflection is more when the magnet is moved faster, and less when the magnet is moved slowly.

(v) However, on reversing the magnet (i.e) south pole pointing towards the coil, same results are obtained, but current flows in the opposite direction.

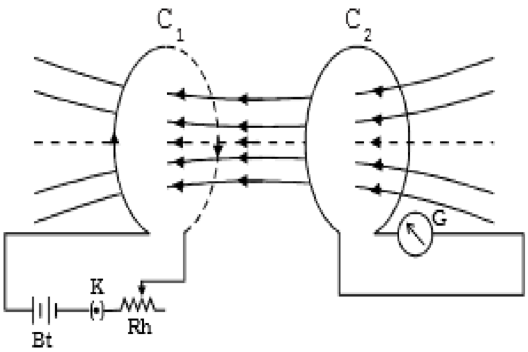


Fig 4.3 Electromagnetic Induction

Faraday demonstrated the electro-magnetic induction by another experiment also.

Fig 4.3 shows two coils C_1 and C_2 placed close to each other.

The coil C_1 is connected to a battery Bt through a key K and a rheostat. Coil C_2 is connected to a sensitive galvanometer G and kept close to C_1. When the key K is pressed, the galvanometer connected with the coil C_2 shows a sudden momentary deflection. This indicates that a current is induced in coil C_2. This is because when the current in C_1 increases from zero to a certain steady value, the magnetic flux linked with the coil C_1 increases. Hence, the magnetic flux linked with the coil C_2 also increases. This causes the deflection in the galvanometer.

On releasing K, the galvanometer shows deflection in the opposite direction. This indicates that a current is again induced in the coil C_2. This is because when the current in C_1 decreases from maximum to zero value, the magnetic flux linked with the coil C_1 decreases. Hence, the magnetic flux linked with the coil C_2 also decreases. This causes the deflection in the galvanometer in the opposite direction.

4.1.3 Faraday's laws of electromagnetic induction

Based on his studies on the phenomenon of electromagnetic induction, Faraday proposed the following two laws.

First law

Whenever the amount of magnetic flux linked with a closed circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

Second law

The magnitude of emf induced in a closed circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

Let ϕ_1 be the magnetic flux linked with the coil initially and ϕ_2 be the magnetic flux linked with the coil after a time t. Then

Rate of change of magnetic flux = $\frac{\phi_2 - \phi_1}{t}$

According to Faraday's second law, the magnitude of induced emf is, $e \propto \frac{\phi_2 - \phi_1}{t}$. If $d\phi$ is the change in magnetic flux in a time dt, then the above equation can be written as $e \propto \frac{d\phi}{dt}$

4.1.4Lenz's law

The Russian scientist H.F. Lenz in 1835 discovered a simple law giving the direction of the induced current produced in a circuit. Lenz's law states that the induced current produced in a circuit always flows in such a direction that it opposes the change or cause that produces it.

If the coil has N number of turns and ϕ is the magnetic flux linked with each turn of the coil then, the total magnetic flux linked with the coil at any time is $N\phi$

$$\therefore \quad e = -\frac{d}{dt}(N\phi) \quad e = -\frac{Nd\phi}{dt} = -\frac{N(\phi_2 - \phi_1)}{t}$$

Lenz's law - a consequence of conservation of energy

Copper coils are wound on a cylindrical cardboard and the two ends of the coil are connected to a sensitive galvanometer. A magnet is moved towards the coil (Fig 4.4). The upper face of the coil acquires north polarity.

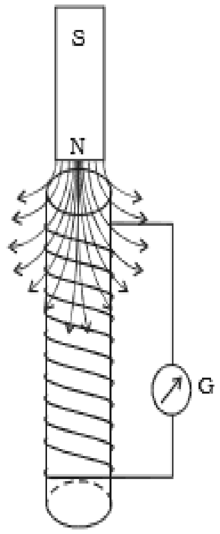


Fig 4.4 Lenz's law

Consequently work has to be done to move the magnet further against the force of repulsion. When we withdraw the magnet away from the coil, its upper face acquires south polarity. Now the workdone is against the force of attraction. When the magnet is moved, the number of magnetic lines of force linking the coil changes, which causes an induced current to flow through the coil. The direction of the induced current, according to Lenz's law is always to oppose the motion of the magnet. The workdone in moving the magnet is converted into electrical energy. This energy is dissipated as heat energy in the coil. If on the contrary, the direction of the current were to help the motion of the magnet, it would start moving faster increasing the change of magnetic flux linking the coil. This results in the increase of induced current. Hence kinetic energy and electrical energy would be produced without any external work being done, but this is impossible. Therefore, the induced current always flows in such a direction to oppose the cause. Thus it is proved that Lenz's law is the consequence of conservation of energy.

4.1.5 Fleming's right hand rule

The forefinger, the middle finger and the thumb of the right hand are held in the three mutually perpendicular directions. If the forefinger points along the direction of the magnetic field and the thumb is along the direction of motion of the conductor, then the middle finger points in the direction of the induced current. This rule is also called generator rule.

4.2. Self Induction

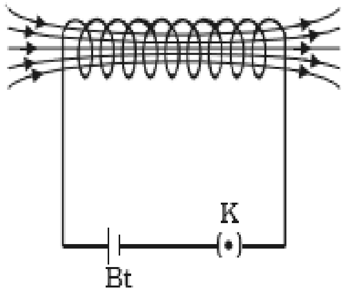


Fig 4.5 Self Induction

The property of a coil which enables to produce an opposing induced emf in it when the current in the coil changes is called self induction.

A coil is connected in series with a battery and a key (K) (Fig. 4.5). On pressing the key, the current through the coil increases to a maximum value and correspondingly the magnetic flux linked with the coil also increases. An induced current flows through the coil which according to Lenz's law opposes the further growth of current in the coil.

On releasing the key, the current through the coil decreases to a zero value and the magnetic flux linked with the coil also decreases. According to Lenz's law, the induced current will oppose the decay of current in the coil.

4.2.1 Coefficient of self induction

When a current I flows through a coil, the magnetic flux (ϕ) linked with the coil is proportional to the current.

$\phi \propto I$ or $\phi = LI$

where L is a constant of proportionality and is called coefficient of self induction or self inductance.

If I = 1A, $\phi = L \times 1$, then $L = \phi$ Therefore, coefficient of self induction of a coil is numerically equal to the magnetic flux linked with a coil when unit current flows through it. According to laws of electromagnetic induction.

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(LI) \quad \text{or} \quad e = -L\frac{dI}{dt}$$

If $\frac{dI}{dt} = 1\ A\ s^{-1}$, then $L = -e$

The coefficient of self induction of a coil is numerically equal to the opposing emf induced in the coil when the rate of change of current through the coil is unity. The unit of self inductance is henry (H).

One henry is defined as the self-inductance of a coil in which a change in current of one ampere per second produces an opposing emf of one volt.

4.2.2 Self inductance of a long solenoid

Let us consider a solenoid of N turns with length l and area of cross section A. It carries a current I. If B is the magnetic field at any point inside the solenoid, then

Magnetic flux per turn = B × area of each turn

But, $B = \frac{\mu_o NI}{l}$

Magnetic flux per turn $= \frac{\mu_o NIA}{l}$

Hence, the total magnetic flux (ϕ) linked with the solenoid is given by the product of flux through each turn and the total number of turns.

$$\phi = \frac{\mu_o NIA}{l} \times N$$

i.e $$\phi = \frac{\mu_o N^2 IA}{l} \quad ...(1)$$

If L is the coefficient of self induction of the solenoid, then

$$\phi = LI \quad ...(2)$$

From equations (1) and (2)

$$LI = \frac{\mu_o N^2 IA}{l}$$

$$\therefore \quad L = \frac{\mu_o N^2 A}{l}$$

If the core is filled with a magnetic material of permeability μ,

then, $L = \frac{\mu N^2 A}{l}$

4.2.3 Energy associated with an inductor

Whenever current flows through a coil, the self–inductance opposes the growth of the current. Hence, some work has to be done by external agencies in establishing the current. If e is the induced emf then,

$$e = -L\frac{dI}{dt}$$

The small amount of work dw done in a time interval dt is

$$dw = e.I\,dt$$

$$= -L\frac{dI}{dt}\,I.dt$$

The total work done when the current increases from 0 to maximum value (I_0) is

$$w = \int dw = \int_0^{I_0} -L\,I\,dI$$

This work done is stored as magnetic potential energy in the coil.

∴ Energy stored in the coil

$$= -L\int_0^{I_0} I dI = -\frac{1}{2}L\,I_0^2$$

Negative sign is consequence of Lenz's Law. Hence, quantitatively, the energy stored in an inductor is $\frac{1}{2}L\,I_0^2$

4.2.4 Mutual induction

Whenever there is a change in the magnetic flux linked with a coil, there is also a change of flux linked with the neighbouring coil, producing an induced emf in the second coil. This phenomenon of producing an induced emf in a coil due to the change in current in the other coil is known as mutual induction.

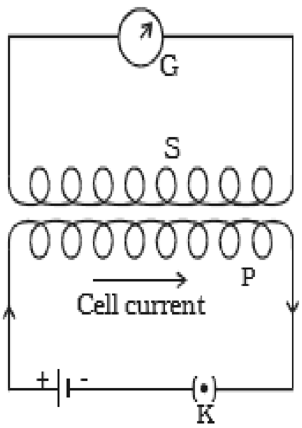


Fig 4.6 Mutual induction

P and S are two coils placed close to each other (Fig. 4.6). P is connected to a battery through a key K. S is connected to a galvanometer G. On pressing K, current in P starts increasing from zero to a maximum value. As the flow of current increases, the magnetic flux linked with P increases. Therefore, magnetic flux linked with S also increases producing an induced emf in S. Now, the galvanometer shows the deflection. According to Lenz's law the induced current in S would oppose the increase in current in P by flowing in a direction opposite to the current in P, thus delaying the growth of current to the maximum value. When the key 'K' is released, current starts decreasing from maximum to zero value, consequently magnetic flux linked with P decreases. Therefore magnetic flux linked with S also decreases and hence, an emf is induced in S. According to Lenz's law, the induced current in S flows in such a direction so as to oppose the decrease in current in P thus prolonging the decay of current.

4.2.5 Coefficient of mutual induction

I_P is the current in coil P and ϕ_S is the magnetic flux linked with coil S due to the current in coil P.

$\therefore \quad \phi_S \propto I_P \quad$ or $\quad \phi_S = M I_P$

where M is a constant of proportionality and is called the coefficient of mutual induction or mutual inductance between the two coils.

If I_P = 1A, then, $M = \phi_S$

Thus, coefficient of mutual induction of two coils is numerically equal to the magnetic flux linked with one coil when unit current flows through the neighbouring coil. If e_S is the induced emf in the coil (S) at any instant of time, then from the laws of electromagnetic induction,

$$e_S = -\frac{d\phi_s}{dt} = -\frac{d}{dt}(MI_P) = -M\frac{dI_P}{dt}$$

$$\therefore M = -\frac{e_s}{\left(\frac{dI_P}{dt}\right)}$$

If $\frac{dI_P}{dt} = 1 \text{ A s}^{-1}$, then, $M = -e_S$

Thus, the coefficient of mutual induction of two coils is numerically equal to the emf induced in one coil when the rate of change of current through the other coil is unity. The unit of coefficient of mutual induction is henry.

One henry is defined as the coefficient of mutual induction between a pair of coils when a change of current of one ampere per second in one coil produces an induced emf of one volt in the other coil.

The coefficient of mutual induction between a pair of coils depends on the following factors

(i) Size and shape of the coils, number of turns and permeability of material on which the coils are wound.

(ii) proximity of the coils

Two coils P and S have their axes perpendicular to each other (Fig. 4.7a). When a current is passed through coil P, the magnetic flux linked with S is small and hence, the coefficient of mutual induction between the two coils is small.

The two coils are placed in such a way that they have a common axis (Fig. 4.7b). When current is passed through the coil P the magnetic flux linked with coil S is large and hence, the coefficient of mutual induction between the two coils is large.

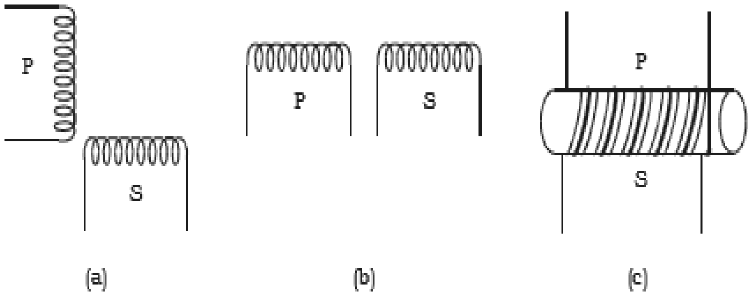


Fig 4.7 Mutual induction

If the two coils are wound on a soft iron core (Fig 4.7c) the mutual induction is very large.

4.2.6 Mutual induction of two long solenoids.

S_1 and S_2 are two long solenoids each of length l. The solenoid S_2 is wound closely over the solenoid S_1 (Fig 4.8).

N_1 and N_2 are the number of turns in the solenoids S_1 and S_2 respectively. Both the solenoids are considered to have the same area of cross section A as they are closely wound together. I_1 is the current flowing through the solenoid S_1. The magnetic field B_1 produced at any point inside the solenoid S_1 due to the current I_1 is

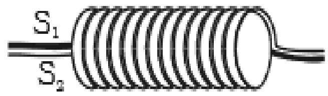


Fig 4.8 Mutual induction between two long solenoids

$$B_1 = \mu_o \frac{N_1}{l} I_1 \qquad ...(1)$$

The magnetic flux linked with each turn of S_2 is equal to B_1A.

Total magnetic flux linked with solenoid S_2 having N_2 turns is

$$\phi_2 = B_1AN_2$$

Substituting for B_1 from equation (1)

$$\phi_2 = \left(\mu_o \frac{N_1}{l} I_1\right) A \ N_2$$

$$\phi_2 = \frac{\mu_o N_1 N_2 A I_1}{l} \qquad ...(2)$$

But $\phi_2 = MI_1 \qquad ...(3)$

where M is the coefficient of mutual induction between S_1 and S_2.

From equations (2) and (3)

$$MI_1 = \frac{\mu_o N_1 N_2 A I_1}{l}$$

$$M = \frac{\mu_o N_1 N_2 A}{l}$$

If the core is filled with a magnetic material of permeability μ,

$$M = \frac{\mu N_1 N_2 A}{l}$$

4.3 Methods of producing induced emf

We know that the induced emf is given by the expression

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA \cos \theta)$$

Hence, the induced emf can be produced by changing

(i) the magnetic induction (B)

(ii) area enclosed by the coil (A) and

(iii) the orientation of the coil (θ) with respect to the magnetic field.

4.3.1 Emf induced by changing the magnetic induction.

The magnetic induction can be changed by moving a magnet either towards or away from a coil and thus an induced emf is produced in the coil.

The magnetic induction can also be changed in one coil by changing the current in the neighbouring coil thus producing an induced emf.

$$\therefore \qquad e = -NA \cos \theta \left(\frac{dB}{dt}\right)$$

4.3.2 Emf induced by changing the area enclosed by the coil

PQRS is a conductor bent in the shape as shown in the Fig 4.9. L_1M_1 is a sliding conductor of length l resting on the arms PQ and RS. A uniform magnetic field 'B' acts perpendicular to the plane of the conductor. The closed area of the conductor is L_1QRM_1. When L_1M_1 is moved through a distance dx in time dt, the new area is L_2QRM_2. Due to the change in area $L_2L_1M_1M_2$, there is a change in the flux linked with the conductor. Therefore, an induced emf is produced.

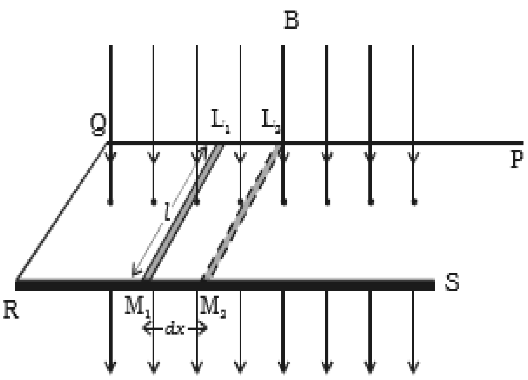


Fig 4.9 Emf induced by changing the area

Change in area dA = Area $L_2L_1M_1M_2$

$\therefore \quad dA = l\,dx$

Change in the magnetic flux, $d\phi = B.dA = Bl\,dx$

But $\quad e = -\dfrac{d\phi}{dt}$

$\therefore \quad e = -\dfrac{Bldx}{dt} = -Blv$

where v is the velocity with which the sliding conductor is moved.

4.3.3 Emf induced by changing the orientation of the coil

PQRS is a rectangular coil of N turns and area A placed in a uniform magnetic field B (Fig 4.10). The coil is rotated with an angular velocity ω in the clockwise direction about an axis perpendicular to the direction of the magnetic field. Suppose, initially the coil is in vertical position, so that the angle between normal to the plane of the coil and magnetic field is zero. After a time t, let θ (=ωt) be the angle through which the coil is rotated. If ϕ is the flux linked with the coil at this instant, then

$\phi = NBA \cos \theta$

The induced emf is,

$$e = -\frac{d\phi}{dt} = -NBA\frac{d}{dt}\cos(\omega t)$$

$$\therefore\ e = NBA\omega \sin \omega t \quad ...(1)$$

The maximum value of the induced emf is, $E_0 = NAB\omega$

Hence, the induced emf can be represented as $e = E_0 \sin \omega t$

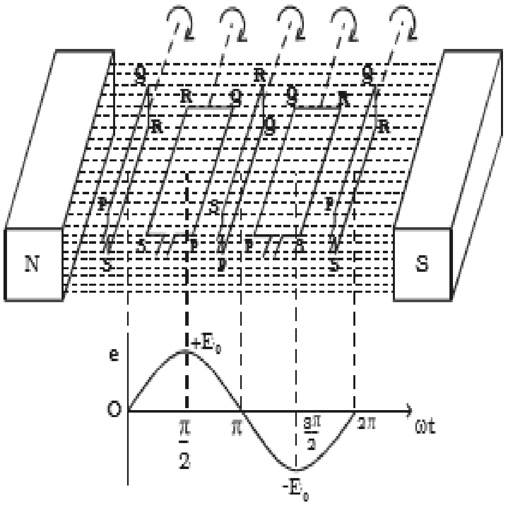


Fig 4.10 Induced emf by changing the orientation of the coil

The induced emf e varies sinusoidally with time t and the frequency being ν cycles per second $\left(\nu = \frac{\omega}{2\pi}\right)$.

(i) When $\omega t = 0$, the plane of the coil is perpendicular to the field B and hence $e = 0$.

(ii) When $\omega t = \pi/2$, the plane of the coil is parallel to B and hence $e = E_0$

(iii) When $\omega t = \pi$, the plane of the coil is at right angle to B and hence $e = 0$.

(iv) When $\omega t = 3\pi/2$, the plane of the coil is again parallel to B and the induced emf is $e = -E_0$.

(v) When $\omega t = 2\pi$, the plane of the coil is again perpendicular to B and hence $e = 0$.

If the ends of the coil are connected to an external circuit through a resistance R, current flows through the circuit, which is also sinusoidal in nature.

4.4 AC generator (Dynamo) – Single phase

The ac generator is a device used for converting mechanical energy into electrical energy. The generator was originally designed by a Yugoslav scientist Nikola Tesla.

Principle

It is based on the principle of electromagnetic induction, according to which an emf is induced in a coil when it is rotated in a uniform magnetic field.

Essential parts of an AC generator

(i) Armature

Armature is a rectangular coil consisting of a large number of loops or turns of insulated copper wire wound over a laminated soft iron core or ring. The soft iron core not only increases the magnetic flux but also serves as a support for the coil

(ii) Field magnets

The necessary magnetic field is provided by permanent magnets in the case of low power dynamos. For high power dynamos, field is provided by electro magnet. Armature rotates between the magnetic poles such that the axis of rotation is perpendicular to the magnetic field.

(iii) Slip rings

The ends of the armature coil are connected to two hollow metallic rings R_1 and R_2 called slip rings. These rings are fixed to a shaft, to which the armature is also fixed. When the shaft rotates, the slip rings along with the armature also rotate.

(iv) Brushes

B_1 and B_2 are two flexible metallic plates or carbon brushes. They provide contact with the slip rings by keeping themselves pressed against the ring. They are used to pass on the current from the armature to the external power line through the slip rings.

Working

Whenever, there is a change in orientation of the coil, the magnetic flux linked with the coil changes, producing an induced emf in the coil. The direction of the induced current is given by Fleming's right hand rule.

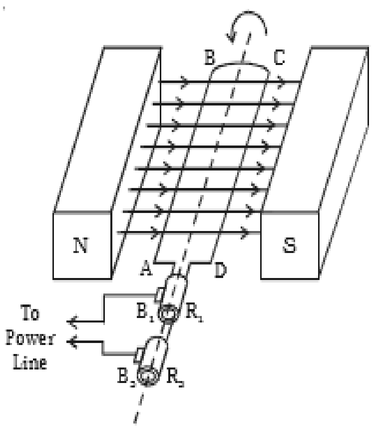


Fig 4.11 AC dynamo

Suppose the armature ABCD is initially in the vertical position. It is rotated in the anticlockwise direction. The side AB of the coil moves downwards and the side DC moves upwards (Fig. 4.11). Then according to Flemings right hand rule the current induced in arm AB flows from B to A and in CD it flows from D to C. Thus the current flows along DCBA in the coil. In the external circuit the current flows from B_1 to B_2.

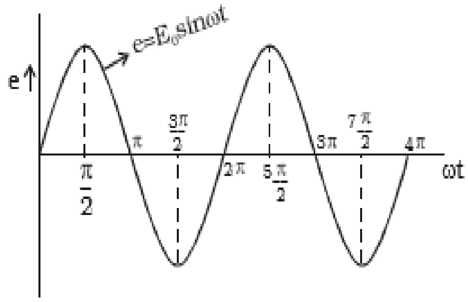


Fig 4.12 emf varies sinusoidally

On further rotation, the arm AB of the coil moves upwards and DC moves downwards. Now the current in the coil flows along ABCD. In the external circuit the current flows from B_2 to B_1. As the rotation of the coil continues, the induced current in the external circuit keeps changing its direction for every half a rotation of the coil. Hence the induced current is alternating in nature (Fig 4.12). As the armature completes ν rotations in one second, alternating current of frequency ν cycles per second is produced. The induced emf at any instant is given by $e = E_o \sin \omega t$

The peak value of the emf, $E_o = NBA\omega$

where N is the number of turns of the coil,

A is the area enclosed by the coil,

B is the magnetic field and

ω is the angular velocity of the coil

4.4.1 AC generator (Alternator) – Three phase

A single phase a.c. generator or alternator has only one armature winding. If a number of armature windings are used in the alternator it is known as polyphase alternator. It produces voltage waves equal to the number of windings or phases. Thus a polyphase system consists of a numerous windings which are placed on the same axis but displaced from one another by equal angle which depends on the number of phases. Three phase alternators are widely preferred for transmitting large amount of power with less cost and high efficiency.

Generation of three phase emf

In a three – phase a.c. generator three coils are fastened rigidly together and displaced from each other by 120^o. It is made to rotate about a fixed axis in a uniform magnetic field. Each coil is provided with a separate set of slip rings and brushes.

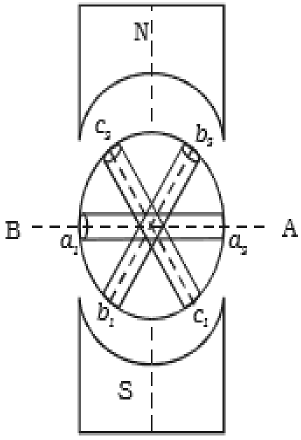


Fig 4.13a Section of 3 phase ac generator

An emf is induced in each of the coils with a phase difference of 120^o. Three coils $a_1 a_2$, $b_1 b_2$ and $c_1 c_2$ are mounted on the same axis but displaced from each other by 120^o, and the coils rotate in the anticlockwise direction in a magnetic field (Fig 4.13a).

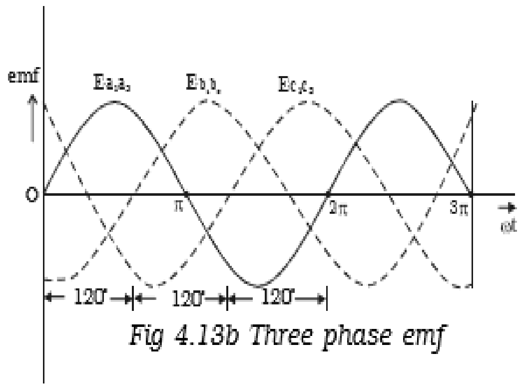


Fig 4.13b Three phase emf

When the coil a_1a_2 is in position AB, emf induced in this coil is zero and starts increasing in the positive direction. At the same instant the coil b_1b_2 is 120^o behind coil $a_1 a_2$, so that emf induced in this coil is approaching its maximum negative value and the coil $c_1 c_2$ is 240^o behind the coil $a_1 a_2$, so the emf induced in this coil has passed its positive maximum value and is decreasing. Thus the emfs induced in all the three coils are equal in magnitude and of same frequency. The emfs induced in the three coils are ;

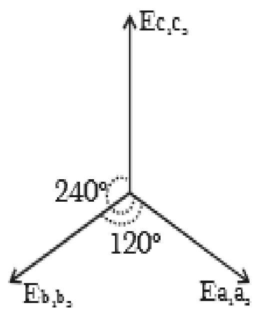


Fig 4.13c Angular displacement between the armature

$$e_{a_1 a_2} = E_o \sin \omega t$$
$$e_{b_1 b_2} = E_o \sin (\omega t - 2\pi/3)$$
$$e_{c_1 c_2} = E_o \sin (\omega t - 4\pi/3)$$

The emfs induced and phase difference in the three coils $a_1 a_2$, $b_1 b_2$ and $c_1 c_2$ are shown in Fig 4.13b & Fig 4.13c.

4.5 Eddy currents

Foucault in the year 1895 observed that when a mass of metal moves in a magnetic field or when the magnetic field through a stationary mass of metal is altered, induced current is produced in the metal. This induced current flows in the metal in the form of closed loops resembling 'eddies' or whirl pool. Hence this current is called eddy current. The direction of the eddy current is given by Lenz's law.

When a conductor in the form of a disc or a metallic plate as shown in Fig 4.14, swings between the poles of a magnet, eddy currents are set up inside the plate. This current acts in a direction so as to oppose the motion of the conductor with a strong retarding force, that the conductor almost comes to rest. If the metallic plate with holes drilled in it is made to swing inside the magnetic field, the effect of eddy current is greatly reduced consequently the plate swings freely inside the field. Eddy current can be minimised by using thin laminated sheets instead of solid metal.

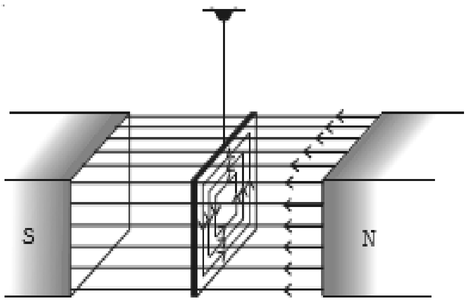


Fig 4.14 Eddy current

Applications of Eddy current

(i) Dead beat galvanometer

When current is passed through a galvanometer, the coil oscillates about its mean position before it comes to rest. To bring the coil to rest immediately, the coil is wound on a metallic frame. Now, when the coil oscillates, eddy currents are set up in the metallic frame, which opposes further oscillations of the coil. This inturn enables the coil to attain its equilibrium position almost instantly. Since the oscillations of the coil die out instantaneously, the galvanometer is called dead beat galvanometer.

(ii) Induction furnace

In an induction furnace, high temperature is produced by generating eddy currents. The material to be melted is placed in a varying magnetic field of high frequency. Hence a strong eddy current is developed inside the metal. Due to the heating effect of the current, the metal melts.

(iii) Induction motors

Eddy currents are produced in a metallic cylinder called rotor, when it is placed in a rotating magnetic field. The eddy current initially tries to decrease the relative motion between the cylinder and the rotating magnetic field. As the magnetic field continues to rotate, the metallic cylinder is set into rotation. These motors are used in fans.

(iv) Electro magnetic brakes

A metallic drum is coupled to the wheels of a train. The drum rotates along with the wheel when the train is in motion.When the brake is applied, a strong magnetic field is developed and hence, eddy currents are produced in the drum which oppose the motion of the drum. Hence, the train comes to rest.

(v) Speedometer

In a speedometer, a magnet rotates according to the speed of the vehicle. The magnet rotates inside an aluminium cylinder (drum) which is held in position with the help of hair springs. Eddy currents are produced in the drum due to the rotation of the magnet and it opposes the motion of the rotating magnet. The drum inturn experiences a torque and gets deflected through a certain angle depending on the speed of the vehicle. A pointer attached to the drum moves over a calibrated scale which indicates the speed of the vehicle.

4.6 Transformer

Transformer is an electrical device used for converting low alternating voltage into high alternating voltage and vice versa. It transfers electric power from one circuit to another. The transformer is based on the principle of electromagnetic induction.

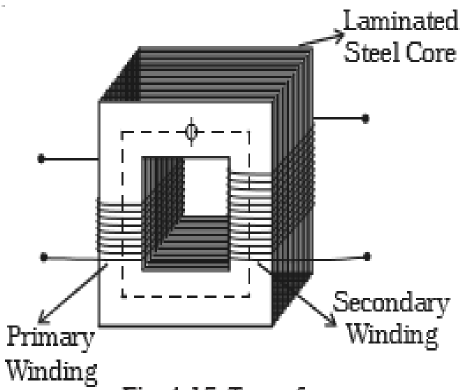


Fig 4.15 Transformer

A transformer consists of primary and secondary coils insulated from each other, wound on a soft iron core (Fig 4.15). To minimise eddy currents a laminated iron core is used. The a.c. input is applied across the primary coil. The continuously varying current in the primary coil produces a varying magnetic flux in the primary coil, which in turn produces a varying magnetic flux in the secondary. Hence, an induced emf is produced across the secondary.

Let E_P and E_S be the induced emf in the primary and secondary coils and N_P and N_S be the number of turns in the primary and secondary coils respectively. Since same flux links with the primary and secondary, the emf induced per turn of the two coils must be the same

(i.e) $$\frac{E_P}{N_P} = \frac{E_s}{N_s}$$

or $$\frac{E_s}{E_p} = \frac{N_s}{N_p} \qquad ...(1)$$

For an ideal transformer, input power = output power

$$E_p I_p = E_s I_s$$

where I_p and I_s are currents in the primary and secondary coils.

(i.e.) $$\frac{E_s}{E_P} = \frac{I_P}{I_s} \qquad ...(2)$$

From equations (1) and (2)

$$\frac{E_s}{E_P} = \frac{N_s}{N_p} = \frac{I_P}{I_S} = k$$

where k is called transformer ratio.

(for step up transformer k > 1 and

for step down transformer k < 1)

In a step up transformer $E_s > E_p$ implying that $I_s < I_p$. Thus a step up transformer increases the voltage by decreasing the current, which is in accordance with the law of conservation of energy. Similarly a step down transformer decreases the voltage by increasing the current.

Efficiency of a transformer

Efficiency of a transformer is defined as the ratio of output power to the input power.

$$\eta = \frac{output\ \ power}{input\ \ power} = \frac{E_s I_s}{E_p I_p}$$

The efficiency η = 1 (ie. 100%), only for an ideal transformer where there is no power loss. But practically there are numerous factors leading to energy loss in a transformer and hence the efficiency is always less than one.

Energy losses in a transformer

(1) Hysteresis loss

The repeated magnetisation and demagnetisation of the iron core caused by the alternating input current, produces loss in energy called hysterisis loss. This loss can be minimised by using a core with a material having the least hysterisis loss. Alloys like mumetal and silicon steel are used to reduce hysterisis loss.

(2) Copper loss

The current flowing through the primary and secondary windings lead to Joule heating effect. Hence some energy is lost in the form of heat. Thick wires with considerably low resistance are used to minimise this loss.

(3) Eddy current loss (Iron loss)

The varying magnetic flux produces eddy current in the core. This leads to the wastage of energy in the form of heat. This loss is minimised by using a laminated core made of stelloy, an alloy of steel.

(4) Flux loss

The flux produced in the primary coil is not completely linked with the secondary coil due to leakage. This results in the loss of energy. This loss can be minimised by using a shell type core.

In addition to the above losses, due to the vibration of the core, sound is produced, which causes a loss in the energy.

4.6.1 Long distance power transmission

The electric power generated in a power station situated in a remote place is transmitted to different regions for domestic and industrial use. For long distance transmission, power lines are made of conducting material like aluminium. There is always some power loss associated with these lines.

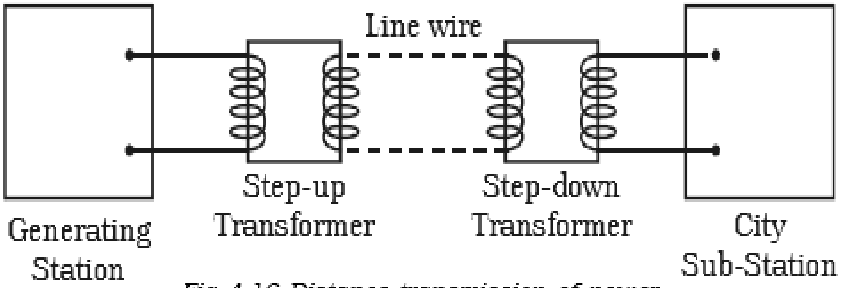


Fig 4.16 Distance transmission of power

If I is the current through the wire and R the resistance, a considerable amount of electric power I^2R is dissipated as heat. Hence, the power at the receiving end will be much lesser than the actual power generated. However, by transmitting the electrical energy at a higher voltage, the power loss can be controlled as is evident from the following two cases.

Case (i) A power of 11,000 W is transmitted at 220 V.

Power P = VI

$$\therefore \quad I = \frac{P}{V} = \frac{11{,}000}{220} = 50A$$

If R is the resistance of line wires,

Power loss = $I^2R = 50^2R = 2500(R)$ watts

Case (ii) 11,000 W power is transmitted at 22,000 V

$$\therefore \quad I = \frac{P}{V} = \frac{11{,}000}{22{,}000} = 0.5 \text{ A}$$

Power loss = $I^2R = (0.5)^2 R = 0.25(R)$ watts

Hence it is evident that if power is trasmitted at a higher voltage the loss of energy in the form of heat can be considerably reduced.

For transmitting electric power at 11,000 W at 220 V the current capacity of line wires has to be 50 A and if transmission is done at 22,000 V, it is only 0.5 A. Thus, for carrying larger current (50A) thick wires have to be used. This increases the cost of transmission. To support these thick wires, stronger poles have to be erected which further adds on to the cost. On the other hand if transmission is done at high voltages, the wires required are of lower current carrying capacity. So thicker wires can be replaced by thin wires, thus reducing the cost of transmission considerably.

For example, 400MW power produced at 15,000 V in the power station at Neyveli, is stepped up by a step-up transformer to 230,000 V before transmission. The power is then transmitted through the transmission lines which forms a part of the grid. The grid connects different parts of the country. Outside the city, the power is stepped down to 110,000 V by a step-down transformer. Again the power is stepped down to 11,000 V by a transformer. Before distribution to the user, the power is stepped down to 230 V or 440 V depending upon the need of the user.

4.7 Alternating current

As we have seen earlier a rotating coil in a magnetic field, induces an alternating emf and hence an alternating current. Since the emf induced in the coil varies in magnitude and direction periodically, it is called an alternating emf. The significance of an alternating emf is that it can be changed to lower or higher voltages conveniently and efficiently using a transformer. Also the frequency of the induced emf can be altered by changing the speed of the coil. This enables us to utilize the whole range of electromagnetic spectrum for one purpose or the other. For example domestic power in India is supplied at a frequency of 50 Hz. For transmission of audio and video signals, the required frequency range of radio waves is between 100 KHz and 100 MHz. Thus owing to its wide applicability most of the countries in the world use alternating current.

4.7.1 Measurement of AC

Since alternating current varies continuously with time, its average value over one complete cycle is zero. Hence its effect is measured by rms value of a.c.

RMS value of a.c.

The rms value of alternating current is defined as that value of the steady current, which when passed through a resistor for a given time, will generate the same amount of heat as generated by an alternating current when passed through the same resistor for the same time.

The rms value is also called effective value of an a.c. and is denoted by I_{rms} or I_{eff}.

when an alter-nating current $i = I_0 \sin \omega t$ flows through a resistor of resistance R, the amount of heat produced in the resistor in a small time dt is

$$dH = i^2 R\, dt$$

The total amount of heat produced in the resistance in one complete cycle is

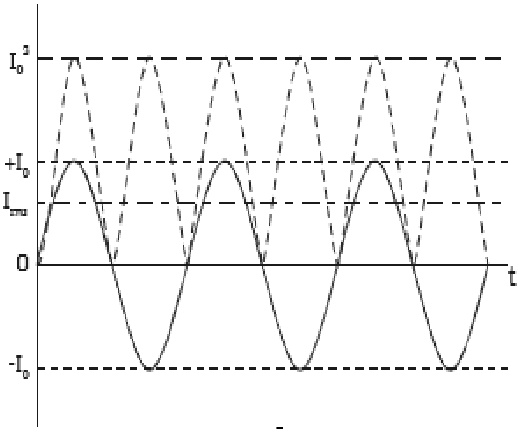


Fig 4.17 Variation I, I^2 and I_{rms} with time

$$H = \int_0^T i^2\, R\, dt = \int_0^T (I_o^2 \sin^2 \omega t) R\, dt$$

$$= I_0^2 R \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt = \frac{I_o^2 R}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t.dt\right]$$

$$= \frac{I_o^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{I_o^2 R}{2}\left[T - \frac{\sin 4\pi}{2\omega}\right] \qquad \left\{\because T = \frac{2\pi}{\omega}\right\}$$

$$H = \frac{I_o^2 RT}{2}$$

But this heat is also equal to the heat produced by rms value of AC in the same resistor (R) and in the same time (T),

(i.e) $H = I^2_{rms}\, RT$

$$\therefore \quad I^2_{rms}\, RT = \frac{I_o^2 RT}{2}$$

$$I_{rms} = \frac{I_o}{\sqrt{2}} = 0.707\, I_0$$

Similarly, it can be calculated that

$$E_{rms} = \frac{E_o}{\sqrt{2}}.$$

Thus, the rms value of an a.c is 0.707 times the peak value of the a.c. In other words it is 70.7 % of the peak value.

4.7.2 AC Circuit with resistor

Let an alternating source of emf be connected across a resistor of resistance R.

The instantaneous value of the applied emf is

$e = E_0 \sin \omega t$ ...(1)

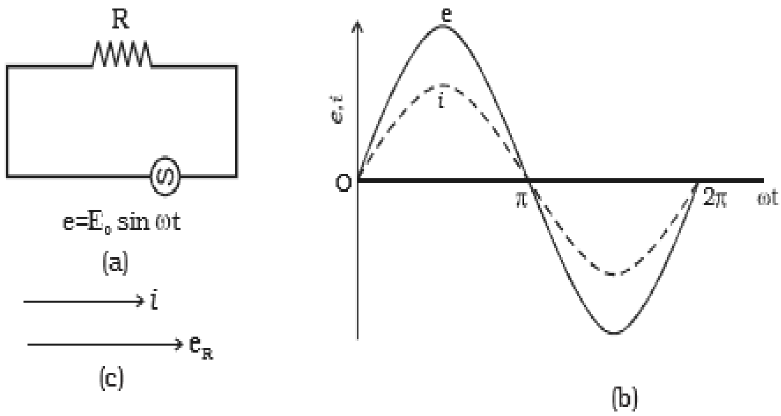


Fig 4.18 a.c. circuit with a resistor

If i is the current through the circuit at the instant t, the potential drop across R is, $e = i R$

Potential drop must be equal to the applied emf.

Hence, $iR = E_0 \sin \omega t$

$i = \frac{E_0}{R} \sin \omega t$; $i = I_0 \sin \omega t$ ...(2)

where $I_0 = \frac{E_0}{R}$, is the peak value of a.c in the circuit. Equation (2) gives the instantaneous value of current in the circuit containing R. From the expressions of voltage and current given by equations (1) and (2) it is evident that in a resistive circuit, the applied voltage and current are in phase with each other (Fig 4.18b).

Fig 4.18c is the phasor diagram representing the phase relationship between the current and the voltage.

4.7.3 AC Circuit with an inductor

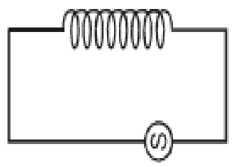
Let an alternating source of emf be applied to a pure inductor of inductance L. The inductor has a negligible resistance and is wound on a laminated iron core. Due to an alternating emf that is applied to the inductive coil, a self induced emf is generated which opposes the applied voltage. (eg) Choke coil.

The instantaneous value of applied emf is given by

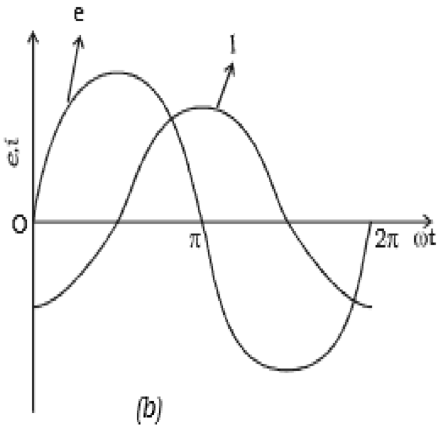
$e = E_0 \sin \omega t \qquad ...(1)$

Induced emf $e' = -L \cdot \frac{di}{dt}$

where L is the self inductance of the coil. In an ideal inductor circuit induced emf is equal and opposite to the applied voltage.



(a)



(b)

Fig 4.19 Pure inductive circuit

Therefore $e = -e'$

$$E_0 \sin \omega t = -\left(-L\frac{di}{dt}\right)$$

$$\therefore \quad E_0 \sin \omega t = L\frac{di}{dt}$$

$$di = \frac{E_o}{L} \sin \omega t \, dt$$

Integrating both the sides

$$i = \frac{E_o}{L}\int \sin \omega t \, dt$$

$$= \frac{E_o}{L}\left[-\frac{\cos \omega t}{\omega}\right] = -\frac{E_o \cos \omega t}{\omega L}$$

$$i = \frac{E_o}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = I_0 \cdot \sin\left(\omega t - \frac{\pi}{2}\right) \quad ...(2)$$

where $I_0 = \frac{E_o}{\omega L}$. Here, ωL is the resistance offered by the coil. It is called inductive reactance. Its unit is ohm .

From equations (1) and (2) it is clear that in an a.c. circuit containing a pure inductor the current i lags behind the voltage e by the phase angle of π/2. Conversely the voltage across L leads the current by the phase angle of π/2. This fact is presented graphically in Fig 4.19b.

Fig 4.19c represents the phasor diagram of a.c. circuit containing only L.

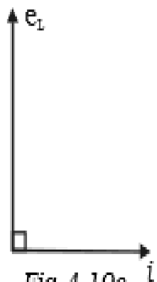


Fig 4.19c
Phasor diagram

Inductive reactance

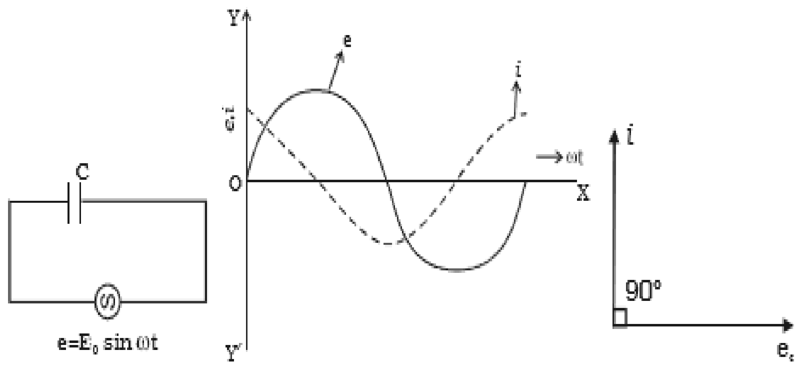
$X_L = \omega L = 2\pi\nu L$, where ν is the frequency of the a.c. supply

For d.c. $\nu = 0$; $\therefore X_L = 0$

Thus a pure inductor offers zero resistance to d.c. But in an a.c. circuit the reactance of the coil increases with increase in frequency.

4.7.4 AC Circuit with a capacitor

An alternating source of emf is connected across a capacitor of capacitance C (Fig 4.20a). It is charged first in one direction and then in the other direction.



(a) (b) (c)

Fig 4.20 Capacitive circuit

The instantaneous value of the applied emf is given by

$$e = E_0 \sin \omega t \qquad ...(1)$$

At any instant the potential difference across the capacitor will be equal to the applied emf

$\therefore e = q/C$, where q is the charge in the capacitor

But $$i = \frac{dq}{dt} = \frac{d}{dt}(Ce)$$

$$i = \frac{d}{dt}(C E_0 \sin \omega t) = \omega CE_0 . \cos \omega t$$

$$i = \frac{E_0}{(1/\omega C)}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad ...(2)$$

where $I_o = \dfrac{E_o}{(1/\omega C)}$

$\dfrac{1}{\omega C} = X_C$ is the resistance offered by the capacitor. It is called capacitive reactance. Its unit is ohm .

From equations (1) and (2), it follows that in an a.c. circuit with a capacitor, the current leads the voltage by a phase angle of $\pi/2$. In otherwords the emf lags behind the current by a phase angle of $\pi/2$. This is represented graphically in Fig 4.20b.

Fig 4.20c represents the phasor diagram of a.c. circuit containing only C.

$$\therefore \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$$

where ν is the frequency of the a.c. supply. In a d.c. circuit $\nu = 0$

$$\therefore \quad X_C = \infty$$

Thus a capacitor offers infinite resistance to d.c. For an a.c. the capacitive reactance varies inversely as the frequency of a.c. and also inversely as the capacitance of the capacitor.

4.7.5 Resistor, inductor and capacitor in series

Let an alternating source of emf e be connected to a series combination of a resistor of resistance R, inductor of inductance L and a capacitor of capacitance C (Fig 4.21a).

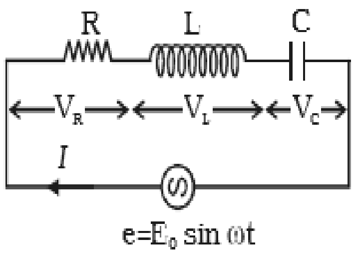
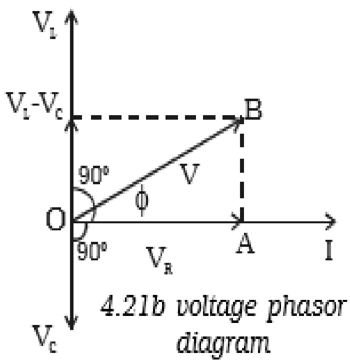


Fig 4.21a RLC sereis circuit



4.21b voltage phasor diagram

Let the current flowing through the circuit be I.

The voltage drop across the resistor is, $V_R = I R$ (This is in phase with I)

The voltage across the inductor coil is $V_L = I\,X_L$

(V_L leads I by $\pi/2$)

The voltage across the capacitor is, $V_C = IX_C$

(V_C lags behind I by $\pi/2$)

The voltages across the different components are represented in the voltage phasor diagram (Fig. 4.21b).

V_L and V_C are 180^0 out of phase with each other and the resultant of V_L and V_C is $(V_L - V_C)$, assuming the circuit to be predominantly inductive. The applied voltage 'V' equals the vector sum of V_R, V_L and V_C.

$$OB^2 = OA^2 + AB^2 \;;$$

$$V^2 = V_R^{\,2} + (V_L - V_C)^2$$

$$V = \sqrt{V_R^{\,2} + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 - (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\frac{V}{I} = Z = \sqrt{R^2 + (X_L - X_C)^2}$$

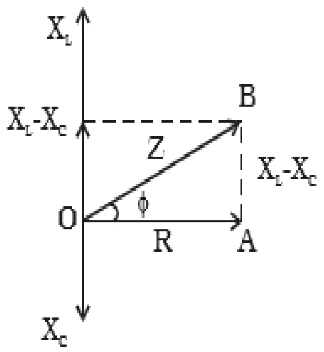


Fig 4.22 Impedance diagram

The expression $\sqrt{R^2 + (X_L - X_C)^2}$ is the net effective opposition offered by the combination of resistor, inductor and capacitor known as the impedance of the circuit and is represented by Z. Its unit is ohm. The values are represented in the impedance diagram (Fig 4.22).

Phase angle ϕ between the voltage and current is given by

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I\,X_L - I\,X_C}{IR}$$

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{\text{net reactance}}{\text{resistance}}$$

$$\therefore \qquad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$\therefore$ $I_0 \sin(\omega t \pm \phi)$ is the instantaneous current flowing in the circuit.

Series resonance or voltage resonance in RLC circuit

The value of current at any instant in a series RLC circuit is given by

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + (\omega_L - \frac{1}{\omega_C})^2}}$$

At a particular value of the angular frequency, the inductive reactance and the capacitive reactance will be equal to each other (i.e.)

$\omega L = \frac{1}{\omega C}$, so that the impedance becomes minimum and it is given by $Z = R$

i.e. I is in phase with V

The particular frequency ν_o at which the impedance of the circuit becomes minimum and therefore the current becomes maximum is called Resonant frequency of the circuit. Such a circuit which admits maximum current is called series resonant circuit or acceptor circuit. Thus the maximum current through the circuit at resonance is

$$I_o = \frac{V}{R}$$

Maximum current flows through the circuit, since the impedance of the circuit is merely equal to the ohmic resistance of the circuit. i.e $Z = R$

$$\omega L = \frac{1}{\omega C}$$

$$\omega = 2\pi \nu_o = \frac{1}{\sqrt{LC}}$$

$$\nu_o = \frac{1}{2\pi\sqrt{LC}}$$

Acceptor circuit

The series resonant circuit is often called an 'acceptor' circuit. By offering minimum impedance to current at the resonant frequency it is able to select or accept most readily this particular frequency among many frequencies.

In radio receivers the resonant frequency of the circuit is tuned to the frequency of the signal desired to be detected. This is usually done by varying the capacitance of a capacitor.

Q-factor

The selectivity or sharpness of a resonant circuit is measured by the quality factor or Q factor. In other words it refers to the sharpness of tuning at resonance.

The Q factor of a series resonant circuit is defined as the ratio of the voltage across a coil or capacitor to the applied voltage.

$$Q = \frac{\textit{voltage across L or C}}{\textit{applied voltage}} \qquad ...(1)$$

$$\text{Voltage across L} = I\,\omega_0 L \qquad ...(2)$$

where ω_0 is the angular frequency of the a.c. at resonance.

The applied voltage at resonance is the potential drop across R, because the potential drop across L is equal to the drop across C and they are 180° out of phase. Therefore they cancel out and only potential drop across R will exist.

$$\text{Applied Voltage} = IR \qquad ...(3)$$

Substituting equations (2) and (3) in equation (1)

$$Q = \frac{I\omega_o L}{IR} = \frac{\omega_o L}{R}$$

$$Q = \frac{1}{\sqrt{LC}}\frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \left\{\because \omega_o = \frac{1}{\sqrt{LC}}\right\}$$

Q is just a number having values between 10 to 100 for normal frequencies. Circuit with high Q values would respond to a very narrow frequency range and vice versa. Thus a circuit with a high Q value is sharply tuned while one with a low Q has a flat resonance. Q-factor can be increased by having a coil of large inductance but of small ohmic resistance.

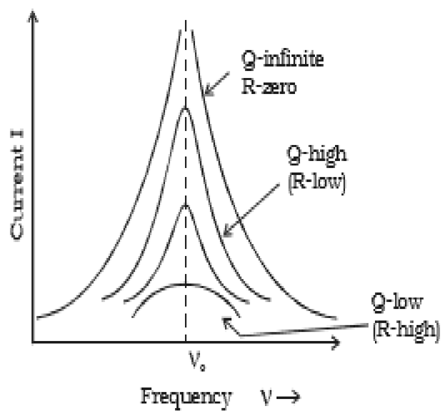


Fig 4.23 variation of current with frequency

Current frequency curve is quite flat for large values of resistance and becomes more sharp as the value of resistance decreases. The curve shown in Fig 4.23 is also called the frequency response curve.

4.7.6 Power in an ac circuit

In an a.c circuit the current and emf vary continuously with time. Therefore power at a given instant of time is calculated and then its mean is taken over a complete cycle. Thus, we define instantaneous power of an a.c. circuit as the product of the instantaneous emf and the instantaneous current flowing through it.

The instantaneous value of emf and current is given by

$$e = E_o \sin \omega t$$

$$i = I_o \sin (\omega t + \phi)$$

where ϕ is the phase difference between the emf and current in an a.c circuit

The average power consumed over one complete cycle is

$$P_{av} = \frac{\int_0^T ie\ dt}{\int_0^T dt} = \frac{\int_0^T [I_o \sin(\omega t + \phi) E_o \sin \omega t]\, dt}{T}.$$

On simplification, we obtain

$$P_{av} = \frac{E_o I_o}{2} \cos \phi$$

$$P_{av} = \frac{E_o}{\sqrt{2}} \cdot \frac{I_o}{\sqrt{2}} . \cos \phi = E_{rms}\ I_{rms}\ \cos \phi$$

P_{av} = apparent power × power factor

where Apparent power = $E_{rms}\ I_{rms}$ and power factor = $\cos \phi$

The average power of an ac circuit is also called the true power of the circuit.

Choke coil

A choke coil is an inductance coil of very small resistance used for controlling current in an a.c. circuit. If a resistance is used to control current, there is wastage of power due to Joule heating effect in the resistance. On the other hand there is no dissipation of power when a current flows through a pure inductor.

Construction

It consists of a large number of turns of insulated copper wire wound over a soft iron core. A laminated core is used to minimise eddy current loss (Fig. 4.24).

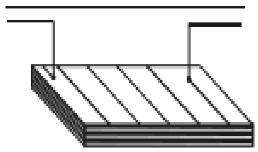


Fig 4.24 Choke coil

Working

The inductive reactance offered by the coil is given by

$X_L = \omega L$

In the case of an ideal inductor the current lags behind the emf by a phase angle $\frac{\pi}{2}$.

$\therefore$ The average power consumed by the choke coil over a complete cycle is

$$P_{av} = E_{rms}\, I_{rms} \cos \pi/2 = 0$$

However in practice, a choke coil of inductance L possesses a small resistance r. Hence it may be treated as a series combination of an inductor and small resistance r. In that case the average power consumed by the choke coil over a complete cycle is

$$P_{av} = E_{rms}\, I_{rms} \cos \phi$$

$$P_{av} = E_{rms}\, I_{rms} \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \qquad ...(1)$$

where $\frac{r}{\sqrt{r^2 + \omega^2 L^2}}$ is the power factor. From equation (1) the value of average power dissipated works out to be much smaller than the power loss I^2R in a resistance R.

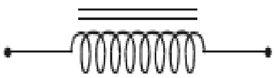


Fig.4.24a A.F Choke

Fig.4.24b R.F. Choke

Chokes used in low frequency a.c. circuit have an iron core so that the inductance may be high. These chokes are known as audio – frequency (A.F) chokes. For radio frequencies, air chokes are used since a low inductance is sufficient. These are called radio frequency (R. F) or high frequency (H.F) chokes and are used in wireless receiver circuits (Fig. 4.24a and Fig. 4.24b).

Choke coils can be commonly seen in fluorescent tubes which work on alternating currents.

Solved problems

4.1 Magnetic field through a coil having 200 turns and cross sectional area 0.04 m^2 changes from 0.1 wb m^{-2} to 0.04 wb m^{-2} in 0.02 s Find the induced emf.

Data : N = 200, A = 0.04 m^2, B_1 = 0.1 wb m^{-2},
B_2 = 0.04 wb m^{-2}, t = 0.02 s, e = ?

Solution : $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(\phi)$

$$e = -\frac{d}{dt}(NBA) = -NA\,.\,\frac{dB}{dt} = -NA.\,\frac{(B_2 - B_1)}{dt}$$

$$e = -200 \times 4 \times 10^{-2}\ \frac{(0.04 - 0.1)}{0.02}$$

$$e = 24\ V$$

4.2 An aircraft having a wingspan of 20.48 m flies due north at a speed of 40 ms^{-1}. If the vertical component of earth's magnetic field at the place is 2×10^{-5} T, Calculate the emf induced between the ends of the wings.

Data : l = 20.48 m; v = 40 ms^{-1}; B = 2×10^{-5}T; e = ?

Solution : $e = -B\,l\,v$

$$= -2 \times 10^{-5} \times 20.48 \times 40$$

$$e = -0.0164\ \text{volt}$$

4.3 A solenoid of length 1 m and 0.05 m diameter has 500 turns. If a current of 2A passes through the coil, calculate (i) the coefficient of self induction of the coil and (ii) the magnetic flux linked with a the coil.

Data : l = 1 m; d = 0.05 m; r = 0.025 m; N = 500 ; I = 2A ;
(i) L = ? (ii) ϕ = ?

Solution : (i) $L = \frac{\mu_o N^2 A}{l} = \frac{\mu_o N^2 \pi r^2}{l}$

$$= \frac{4\pi \times 10^{-7} \times (5 \times 10^2)^2 \times 3.14(0.025)^2}{1} = 0.616 \times 10^{-3}$$

$\therefore$ L = 0.616 mH

(ii) Magnetic flux ϕ = LI

$$= 0.616 \times 10^{-3} \times 2 = 1.232 \times 10^{-3}$$

$$\phi = 1.232\ \text{milli weber}$$

4.4 Calculate the mutual inductance between two coils when a current of 4 A changing to 8 A in 0.5 s in one coil, induces an emf of 50 mV in the other coil.

Data : $I_1 = 4A$; $I_2 = 8A$; $dt = 0.5s$;
$e = 50 \text{ mV} = 50 \times 10^{-3}V$, $M = ?$

Solution : $e = -M \cdot \dfrac{dI}{dt}$

$$\therefore \quad M = -\frac{e}{\left(\frac{dI}{dt}\right)} = -\frac{e}{\left(\frac{I_2 - I_1}{dt}\right)} = -\frac{50 \times 10^{-3}}{\left(\frac{8-4}{0.5}\right)} = -6.25 \times 10^{-3}$$

$\therefore \quad M = 6.25 \text{ mH}$

4.5 An a.c. generator consists of a coil of 10,000 turns and of area 100 cm^2. The coil rotates at an angular speed of 140 rpm in a uniform magnetic field of 3.6×10^{-2} T. Find the maximum value of the emf induced.

Data : $N = 10{,}000 \quad A = 10^2 \text{ cm}^2 = 10^{-2} \text{ m}^2$,

$\nu = 140 \text{ rpm} = \dfrac{140}{60} \text{ rps}, \quad B = 3.6 \times 10^{-2}T \quad E_o = ?$

Solution : $E_o = NAB\omega = NAB\ 2\pi\nu$

$$= 10^4 \times 10^{-2} \times 3.6 \times 10^{-2} \times 2\ \pi \times \frac{7}{3}$$

$E_o = 52.75 \text{ V}$

4.6 Write the equation of a 25 cycle current sine wave having rms value of 30 A.

Data : $\nu = 25$ Hz, $\quad I_{rms} = 30$ A

Solution : $i = I_o \sin \omega t$

$= I_{rms} \sqrt{2} \sin 2\pi\nu t$

$i = 30 \sqrt{2} \sin 2\pi \times 25\ t$

$i = 42.42 \sin 157\ t$

4.7 A capacitor of capacitance 2 μF is in an a.c. circuit of frequency 1000 Hz. If the rms value of the applied emf is 10 V, find the effective current flowing in the circuit.

Data : $C = 2\mu F$, $\nu = 1000$ Hz, $E_{eff} = 10V$

Solution : $X_c = \frac{1}{C\omega} = \frac{1}{C \times 2\pi\upsilon}$

$$X_c = \frac{1}{2 \times 10^{-6} \times 2\pi \times 10^3} = 79.6\ \Omega$$

$$I_{rms} = \frac{E_{eff}}{X_c} = \frac{10}{79.6}$$

$\therefore$ $I_{rms} = 0.126$ A

4.8 A coil is connected across 250 V, 50 Hz power supply and it draws a current of 2.5 A and consumes power of 400 W. Find the self inductance and power factor.

Data : $E_{rms} = 250$ V. $\nu = 50$ Hz; $I_{rms} = 2.5$A; $P = 400$ W; $L = ?$, $\cos\phi = ?$

Solution : Power $P = E_{rms}\, I_{rms} \cos\phi$

$$\therefore \cos\phi = \frac{P}{E_{rms}\ I_{rms}}$$

$$= \frac{400}{250 \times 2.5}$$

$$\cos\phi = 0.64$$

Impedance $Z = \frac{E_{rms}}{I_{rms}} = \frac{250}{2.5} = 100\ \Omega$

From the phasor diagram

$$\sin\phi = \frac{X_L}{Z}$$

$$\therefore \quad X_L = Z \,.\, \sin\phi = Z\sqrt{(1 - \cos^2\phi)}$$

$$= 100\ \sqrt{[1 - (0.64)^2]}$$

$$\therefore \quad X_L = 76.8\ \Omega$$

But $X_L = L\,\omega = L\ 2\ \pi\nu$

$$\therefore \quad L = \frac{X_L}{2\pi\upsilon} = \frac{76.8}{2\pi \times 50}$$

$$\therefore \quad L = 0.244\ H$$

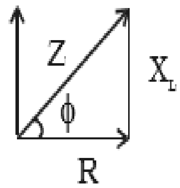
4.9 A bulb connected to 50 V, DC consumes 20 w power. Then the bulb is connected to a capacitor in an a.c. power supply of 250 V, 50 Hz. Find the value of the capacitor required so that the bulb draws the same amount of current.

Data : P = 20 W; V = 50 V; ν = 50 Hz; C= ?

Solution : P = VI

$$\therefore\ I = \frac{P}{V} = \frac{20}{50} = 0.4\ A$$

$$\therefore\ \text{Resistance, } R = \frac{V}{I} = \frac{50}{0.4} = 125\ \Omega$$



$$\text{The impedence, } Z = \frac{V}{I} = \frac{250}{0.4} = 625\Omega$$

$$\therefore\ Z = \sqrt{R^2 + \left(\frac{1}{\omega c}\right)^2} = \sqrt{R^2 + \left(\frac{1}{2\pi\nu C}\right)^2}$$

$$Z^2 = R^2 + \frac{1}{4\pi^2\nu^2C^2}$$

$$C = \frac{1}{2\pi\nu\sqrt{Z^2 - R^2}}$$

$$= \frac{1}{2\pi \times 50\sqrt{(625)^2 - (125)^2}} = \frac{1}{2\pi \times 50 \times 612.37}$$

$$C = 5.198\ \mu F$$

4.10 An AC voltage represented by e = 310 sin 314 t is connected in series to a 24 Ω resistor, 0.1 H inductor and a 25 μF capacitor. Find the value of the peak voltage, rms voltage, frequency, reactance of the circuit, impedance of the circuit and phase angle of the current.

Data : R = 24 Ω, L = 0.1 H, C = 25×10^{-6}F

Solution : e = 310 sin 314 t ... (1)

and e = E_o sin ωt ... (2)

comparing equations (1) & (2)

$$E_o = 310\ V$$

$$E_{rms} = \frac{E_o}{\sqrt{2}} = \frac{310}{\sqrt{2}} = 219.2\ V$$

$\omega t = 314\ t$

$2\pi\nu = 314$

$\nu = \dfrac{314}{2 \times 3.14} = 50\ Hz$

Reactance $= X_L - X_C = L\,\omega - \dfrac{1}{C\omega} = L.2\pi\nu - \dfrac{1}{C.2\pi\upsilon}$

$= 0.1 \times 2\ \pi \times 50 - \dfrac{1}{25 \times 10^{-6} \times 2\pi \times 50}$

$= 31.4 - 127.4 = -96\ \Omega$

$X_L - X_C = -96\ \Omega$

$\therefore \quad X_C - X_L = 96\ \Omega$

$Z = \sqrt{R^2 + (X_C - X_L)^2}$

$= \sqrt{24^2 + 96^2}$

$= \sqrt{576 + 9216}$

$= 98.9\ \Omega$

$\tan\phi = \dfrac{X_C - X_L}{R}$

$= \left(\dfrac{127.4 - 31.4}{24}\right)$

$\tan\phi = \dfrac{96}{24} = 4$

$\phi = 76^o$

Predominance of capacitive reactance signify that current leads the emf by 76^o

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

4.1 Electromagnetic induction is not used in

(a) transformer (b) room heater

(c) AC generator (d) choke coil

4.2 A coil of area of cross section 0.5 m^2 with 10 turns is in a plane which is pendendicular to an uniform magnetic field of 0.2 Wb/m^2. The flux though the coil is

(a) 100 Wb (b) 10 Wb

(c) 1 Wb (d) zero

4.3 Lenz's law is in accordance with the law of

(a) conservation of charges (b) conservation of flux

(c) conservation of momentum (d) conservation of energy

4.4 The self–inductance of a straight conductor is

(a) zero (b) infinity

(c) very large (d) very small

4.5 The unit henry can also be written as

(a) Vs A^{-1} (b) Wb A^{-1}

(c) Ω s (d) all

4.6 An emf of 12 V is induced when the current in the coil changes at the rate of 40 A S^{-1}. The coefficient of self induction of the coil is

(a) 0.3 H (b) 0.003 H

(c) 30 H (d) 4.8 H

4.7 A DC of 5A produces the same heating effect as an AC of

(a) 50 A rms current (b) 5 A peak current

(c) 5A rms current (d) none of these

4.8 Transformer works on

(a) AC only (b) DC only

(c) both AC and DC (d) AC more effectively than DC

4.9 The part of the AC generator that passes the current from the coil to the external circuit is

(a) field magnet (b) split rings

(c) slip rings (d) brushes

4.10 In an AC circuit the applied emf $e = E_o \sin(\omega t + {}^{\pi}/_{2})$ leads the current $I = I_o \sin(\omega t - {}^{\pi}/_{2})$ by

(a) $\pi/2$ (b) $\pi/4$

(c) π (d) 0

4.11 Which of the following cannot be stepped up in a transformer?

(a) input current (b) input voltage

(c) input power (d) all

4.12 The power loss is less in transmission lines when

(a) voltage is less but current is more

(b) both voltage and current are more

(c) voltage is more but current is less

(d) both voltage and current are less

4.13 Which of the following devices does not allow d.c. to pass through?

(a) resistor (b) capacitor

(c) inductor (d) all the above

4.14 In an ac circuit

(a) the average value of current is zero.

(b) the average value of square of current is zero.

(c) the average power dissipation is zero.

(d) the rms current is $\sqrt{2}$ time of peak current.

4.15 What is electromagnetic induction?

4.16 State Faraday's laws of electromagnetic induction.

4.17 Define self-inductance. Give its unit

4.18 Define the unit of self-inductance.

4.19 Define coefficient of mutual induction.

4.20 Give the practical application of self-induction.

4.21 State Fleming's right hand rule.

4.22 Define rms value of a.c.

4.23 State the methods of producing induced emf.

4.24 What is a poly phase AC generator?

4.25 What is inductive reactance?

4.26 Define alternating current and give its expression.

4.27 What is capacitive reactance?

4.28 Mention the difference between a step up and step down transformer.

4.29 What is resonant frequency in LCR circuit?

4.30 Define power factor.

4.31 Why a d.c ammeter cannot read a.c?

4.32 Obtain an expression for the rms value of a.c.

4.33 Define quality factor.

4.34 A capacitor blocks d.c but allows a.c. Explain.

4.35 What happens to the value of current in RLC series circuit, if frequency of the source is increased?

4.36 State Lenz's law and illustrate through an experiment. Explain how it is in accordance with the law of conservation of energy.

4.37 Differentiate between self-inductance and mutual inductance.

4.38 Obtain an expression for the self-inductance of a long solenoid.

4.39 Explain the mutual induction between two long solenoids. Obtain an expression for the mutual inductance.

4.40 Explain how an emf can be induced by changing the area enclosed by the coil.

4.41 Discuss with theory the method of inducing emf in a coil by changing its orientation with respect to the direction of the magnetic field.

4.42 What are eddy currents? Give their applications. How are they minimised?

4.43 Explain how power can be transmitted efficiently to long distance.

4.44 Obtain an expression for the current flowing in a circuit containing resistance only to which alternating emf is applied. Find the phase relationship between voltage and current.

4.22 *Define rms value of a.c.*

4.23 *State the methods of producing induced emf.*

4.24 *What is a poly phase AC generator?*

4.25 *What is inductive reactance?*

4.26 *Define alternating current and give its expression.*

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4.44 *Obtain an expression for the current flowing in a circuit containing resistance only to which alternating emf is applied. Find the phase relationship between voltage and current.*

4.57 An iron cylinder 5cm in diameter and 100cm long is wound with 3000 turns in a single layer. The second layer of 100 turns of much finer wire is wound over the first layer near its centre. Calculate the mutual inductance between the coils (relative permeability of the core = 500).

4.58 A student connects a long air core coil of manganin wire to a 100V DC source and records a current of 1.5A. When the same coil is connected across 100V, 50 Hz a.c. source, the current reduces to 1 A. Calculate the value of reactance and inductance of the coil.

4.59 An emf $e = 100 \sin 200\,\pi t$ is connected to a circuit containing a capacitance of 0.1μF and resistance of 500 Ω in series. Find the power factor of the circuit.

4.60 The primary of a transformer has 400 turns while the secondary has 2000 turns. If the power output from the secondary at 1100 V is 12.1 KW, calculate the primary voltage. If the resistance of primary is 0.2 Ω and that of secondary is 2 Ω and the efficiency of the transformer is 90% calculate

(i) heat loss in the primary coil

(ii) heat loss in the secondary coil

4.61 A resistance of 50 Ω, an inductance of 0.5 H and a capacitance of 5 μF are connected in series with an a.c. supply of $e = 311 \sin(314t)$. Find (i) frequency of a.c. supply (ii) maximum voltage (iii) inductive reactance (iv) capacitive reactance (v) impedance.

4.62 A radio can tune over the frequency range of a portion of broadcast band (800 KHz to 1200 KHz). If its LC circuit has an effective inductance of 200 μ H, what must be the range of its variable capacitance?

4.63 A transformer has an efficiency of 80%. It is connected to a power input of at 4 KW and 100 V. If the secondary voltage is 240 V. Calculate the primary and secondary currents.

4.64 An electric lamp which works at 80 volt and 10 A D.C. is connected to 100 V, 50 Hz alternating current. Calculate the inductance of the choke required so that the bulb draws the same current of 10 A.

Answers

4.1 (b)	**4.2** (c)	**4.3** (d)	**4.4** (a)	**4.5** (d)
4.6 (a)	**4.7** (c)	**4.8** (a)	**4.9** (d)	**4.10** (c)
4.11 (c)	**4.12**(c)	**4.13** (b)	**4.14** (a)	

4.54 0.8 V and 4 mA **4.55** 1 mV

4.56 0.52575 joule **4.57** 0.37 H

4.58 74.54 Ω and 0.237 H **4.59** 0.0314

4.60 220V, (i) 747 W (ii) 242 W

4.61 (i) 50 Hz (ii) 311 V (iii) 157 Ω (iv) 636.9 Ω (v) 482.5 Ω

4.62 87.9 pF to 198 pF

4.63 40 A, 13.3 A

4.64 0.019 H

5. Electromagnetic Waves

According to Faraday's electromagnetic induction, a changing magnetic field at a point with time produces an electric field at that point. On the other hand, Maxwell in 1865, pointed out that there is a symmetry in nature (i.e.) changing electric field with time at a point produces a magnetic field at that point. It means that a change in one field with time (either electric or magnetic) produces another field. This idea led Maxwell to conclude that the variation in electric and magnetic fields perpendicular to each other produces electromagnetic disturbances in space. These disturbances have the properties of a wave and propagate through space without any material medium. These waves are called electromagnetic waves.

5.1 Concept of displacement current

Across inductive coil a varying magnetic field gives rise to varying electric field.

However, in a circuit consisting of a capacitor, current flows only when capacitor is either charging or discharging. Electric field between plates of the capacitor is given by $E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

During charging or discharging of a capacitor, the charge varies, thus electric field between the plates of the capacitor varies. According to Maxwell, to maintain continuity of current in a circuit containing capacitor, there has to be a current which flows from one plate to another, this current is called displacement current.

Displacement current

An imaginary current between plates of a capacitor from one plate to another while charging or discharging of the capacitor is called displacement current. Like conduction current (I_c) through a conductor displacement current (I_d) through a capacitor is also a source of magnetic field. Thus, a magnetic field also exists between plates of a capacitor when capacitor charged or discharged.

It means inside a capacitor a varying electric field gives rise to magnetic field.

Considering the concepts of an inductive coil and a capacitor, the varying electric and magnetic fields regenerate each other. Thus, when varying electric and magnetic fields regenerate each other, they propagate in space as a signal and this signal is called electromagnetic wave.

Need of displacement current concept

By the concept of displacement current laws of electromagnetism become symmetric. Across inductive coil varying magnetic field gives rise to varying electric field and inside capacitor varying electric field gives rise to varying magnetic field.

By the concept of displacement current, the continuity of current in a circuit containing a capacitor is maintained. Conduction current flows in conducting wires and displacement current flows between plates of the capacitor.

By the concept of displacement current, Maxwell modified Ampere circuital law (ACL). The modified Ampere circuital law is given by

$$\oint \vec{B}.\vec{dl} = \mu_0(I_c + I_d)$$

Device	Conduction current	Displacement current	Modified ACL
Conductor/ conducting wire	I_c	0	$\oint \vec{B}.\vec{dl} = \mu_0 I_c$
Capacitor	0	I_d	$\oint \vec{B}.\vec{dl} = \mu_0 I_d$

*($I_c = I_d$)

Magnitude/Expression of displacement current

We know flux linked with any area A inside plate of capacitor is

$$\phi = EA = \frac{\sigma}{\varepsilon_0}A = \frac{\left(Q/_A\right)}{\varepsilon_0}A = \frac{Q}{A\varepsilon_0}A$$

$$\phi = \frac{Q}{\varepsilon_0}$$

Displacement current flows only when electric flux between a capacitor varies. Thus, we need to differentiate above equation both sides w.r.t time so as to acquire displacement current

$$\frac{d\phi}{dt} = \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\,\frac{dQ}{dt} = \frac{1}{\varepsilon_0}\,I_d$$

$$\frac{d\phi}{dt} = \frac{1}{\varepsilon_0}\,I_d$$

$$I_d = \varepsilon_0\,\frac{d\phi}{dt}$$

Modified ACL, thus, can be written as, $\oint \vec{B}.\vec{dl} = \mu_0(I_c + I_d)$

$$\oint \vec{B}.\vec{dl} = \mu_0(I_c + \varepsilon_0\,\frac{d\phi}{dt})$$

5.1.1 Electromagnetic waves

According to Maxwell, an accelerated charge is a source of electromagnetic radiation.

In an electromagnetic wave, electric and magnetic field vectors are at right angles to each other and both are at right angles to the direction of propagation. They possess the wave character and propagate through free space without any material medium. These waves are transverse in nature.

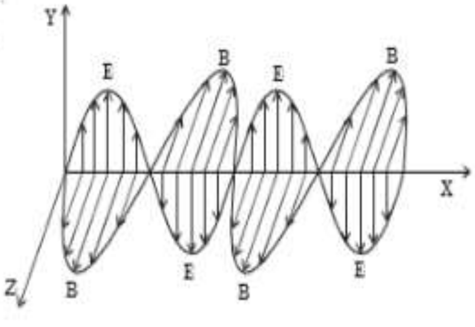

Fig 5.1 Electromagnetic waves.

Fig 5.1 shows the variation of electric field $\vec{E}$ along Y direction and magnetic field $\vec{B}$ along Z direction and wave propagation in + X direction.

5.1.2 Characteristics of electromagnetic waves

(i) Electromagnetic waves are produced by accelerated charges.

(ii) They do not require any material medium for propagation.

(iii) In an electromagnetic wave, the electric $(\vec{E})$ and magnetic $(\vec{B})$ field vectors are at right angles to each other and to the direction of propagation. Hence electromagnetic waves are transverse in nature.

(iv) Variation of maxima and minima in both $\vec{E}$ and $\vec{B}$ occur simultaneously.

(v) They travel in vacuum or free space with a velocity 3×10^8 m s^{-1} given by the relation $C = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$.

(μ_o – permeability of free space and ε_o - permittivity of free space)

(vi) The energy in an electromagnetic wave is equally divided between electric and magnetic field vectors.

(vii) The electromagnetic waves being chargeless, are not deflected by electric and magnetic fields.

5.1.3 Hertz experiment

The existence of electromagnetic waves was confirmed experimentally by Hertz in 1888. This experiment is based on the fact that an oscillating electric charge radiates electromagnetic waves. The energy of these waves is due to the kinetic energy of the oscillating charge.

The experimental arrangement is as shown in Fig 5.2. It consists of two metal plates A and B placed at a distance of 60 cm from each other. The metal plates are connected to two polished metal spheres S_1 and S_2 by means of thick copper wires. Using an induction coil a high potential difference is applied across the small gap between the spheres.

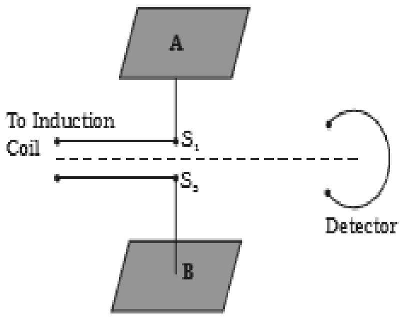

Fig 5.2 Hertz experiment

Due to high potential difference across S_1 and S_2, the air in the small gap between the spheres gets ionized and provides a path for the discharge of the plates. A spark is produced between

S_1 and S_2 and electromagnetic waves of high frequency are radiated. Hertz was able to produce electromagnetic waves of frequency about 5×10^7 Hz.

Here the plates A and B act as a capacitor having small capacitance value C and the connecting wires provide low inductance L. The high frequency oscillation of charges between the plates is given by $\nu = \dfrac{1}{2\pi\sqrt{LC}}$

5.1.4 Electromagnetic Spectrum

After the demonstration of electromagnetic waves by Hertz, electromagnetic waves in different regions of wavelength were produced by different ways of excitation.

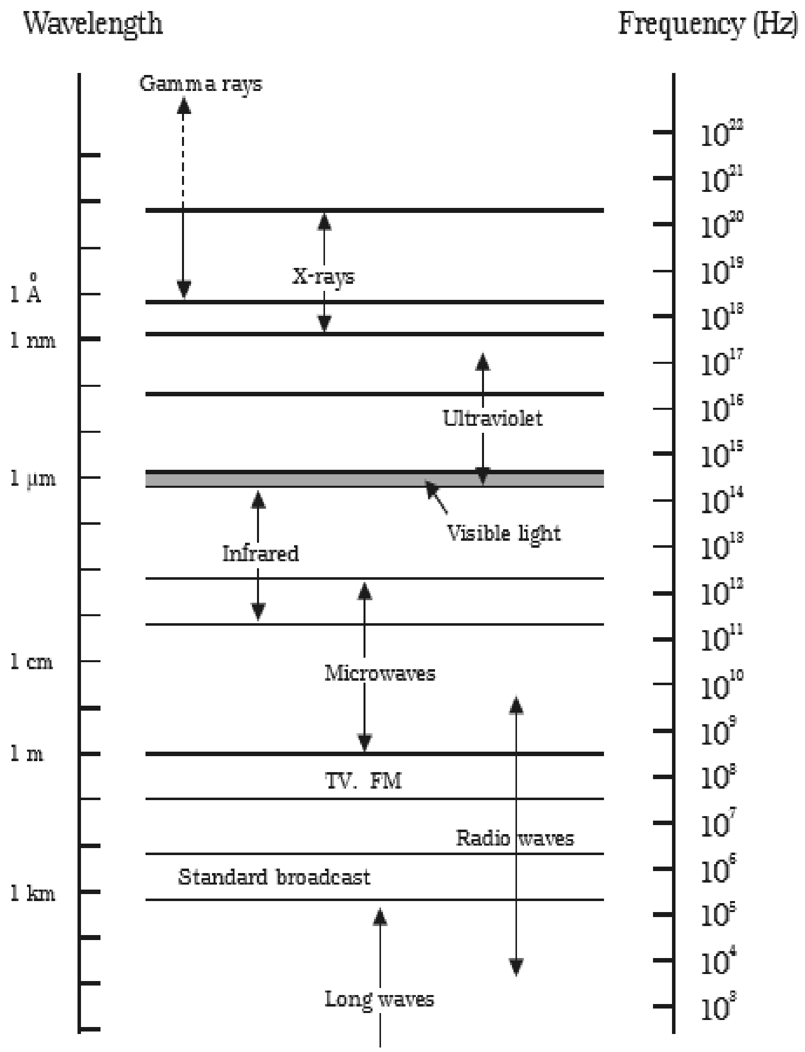

Fig 5.3 Electromagnetic spectrum

The orderly distribution of electromagnetic waves according to their wavelength or frequency is called the electromagnetic spectrum.

Electromagnetic spectrum covers a wide range of wavelengths (or) frequencies. The whole electromagnetic spectrum has been classified into different parts and sub parts, in order of increasing wavelength and type of excitation. All electromagnetic waves travel with the velocity of light. The physical properties of electromagnetic waves are determined by their wavelength and not by their method of excitation. The overlapping in certain parts of the spectrum shows that the particular wave can be produced by different methods.

Table 5.1 shows various regions of electromagnetic spectrum with source, wavelength and frequency ranges of different electromagnetic waves.

Table 5.1

(NOT FOR EXAMINATION)

Sl.No.	Name	Source	Wavelength range (m)	Frequency range (Hz)
1.	γ – rays	Radioactive nuclei, nuclear reactions	10^{-14} – 10^{-10}	3×10^{22} – 3×10^{18}
2.	x – rays	High energy electrons suddenly stopped by a metal target	1×10^{-10}–3×10^{-8}	3×10^{18} – 1×10^{16}
3.	Ultra–violet (UV)	Atoms and molecules in an electrical discharge	6×10^{-10}–4×10^{-7}	5×10^{17} – 8×10^{14}
4.	Visible light	incandescent solids Fluorescent lamps	4×10^{-7} – 8×10^{-7}	8×10^{14} – 4×10^{14}
5.	Infra–red (IR)	molecules of hot bodies	8×10^{-7} – 3×10^{-5}	4×10^{14} – 1×10^{13}
6.	Microwaves	Electronic device (Vacuum tube)	10^{-3} – 0.3	3×10^{11} – 1×10^{9}
7.	Radio frequency waves	charges accelerated through conducting wires	10–10^{4}	3×10^{7} – 3×10^{4}

- - -

5.1.5 Uses of electromagnetic spectrum

The following are some of the uses of electromagnetic waves.

1. Radio waves : These waves are used in radio and television communication systems. AM band is from 530 kHz to 1710 kHz. Higher frequencies upto 54 MHz are used for short waves bands.

Television waves range from 54 MHz to 890 MHz. FM band is from 88 MHz to 108 MHz. Cellular phones use radio waves in ultra high frequency (UHF) band.

2. Microwaves : Due to their short wavelengths, they are used in radar communication system. Microwave ovens are an interesting domestic application of these waves.

3. Infra red waves :

(i) Infrared lamps are used in physiotherapy.

(ii) Infrared photographs are used in weather forecasting.

(iii) As infrared radiations are not absorbed by air, thick fog, mist etc, they are used to take photograph of long distance objects.

(iv) Infra red absorption spectrum is used to study the molecular structure.

4. Visible light : Visible light emitted or reflected from objects around us provides information about the world. The wavelength range of visible light is 4000 Å to 8000 Å.

5. Ultra–violet radiations

(i) They are used to destroy the bacteria and for sterilizing surgical instruments.

(ii) These radiations are used in detection of forged documents, finger prints in forensic laboratories.

(iii) They are used to preserve the food items.

(iv) They help to find the structure of atoms.

6. X rays :

(i) X rays are used as a diagonistic tool in medicine.

(ii) It is used to study the crystal structure in solids.

7. γ–rays : Study of γ rays gives useful information about the nuclear structure and it is used for treatment of cancer.

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

5.1 In an electromagnetic wave

(a) power is equally transferred along the electric and magnetic fields

(b) power is transmitted in a direction perpendicular to both the fields

(c) power is transmitted along electric field

(d) power is transmitted along magnetic field

5.2 Electromagnetic waves are

(a) transverse

(b) longitudinal

(c) may be longitudinal or transverse

(d) neither longitudinal nor transverse

5.3 Refractive index of glass is 1.5. Time taken for light to pass through a glass plate of thickness 10 cm is

(a) 2×10^{-8} s (b) 2×10^{-10} s

(c) 5×10^{-8} s (d) 5×10^{-10} s

5.4 In an electromagnetic wave the phase difference between electric field $\vec{E}$ and magnetic field $\vec{B}$ is

(a) $\pi/4$ (b) $\pi/2$

(c) π (d) zero

5.5 What are electromagnetic waves?

5.6 Mention the characteristics of electromagnetic waves.

5.7 Give the source and uses of electromagnetic waves.

Answers

5.1 *(b)* **5.2** *(a)* **5.3** *(d)* **5.4** *(d)*