Université catholique de Louvain

LMECA2550 - Aircraft propulsion systems

Homework 1 : Blade Element Momentum theory

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Otis Van Kerm : 35561900

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1 Introduction

The goal of this homework was to implement the blade element momentum theory. This method allows us to calculate the thrust and power of a propeller or a wind turbine. To solve the non-linear equations, we will use an under-relaxation method. In the code I used a coefficient $\omega = 0.7$ as it gives a fast convergence while staying stable.

In both cases (propeller and wind turbine), the data describing the problem are:

- Diameter $D = 2R$
- Hub diameter $D_h = 2R_h$
- Number of blades B
- Chord $c(r)$
- Pitch angle $\beta(r)$
- Profile polars $C_l(\alpha)$ and $C_d(\alpha)$

When talking about a propeller, the relevant coefficients are:

- Advance ratio $J = \frac{u_0}{nD}$
- Thrust coefficient $k_T = \frac{T}{\rho n^2 D^4}$
- Torque coefficient $k_Q = \frac{Q}{\rho n^2 D^5}$
- Power coefficient $k_P = \frac{P}{\rho n^3 D^5}$
- Efficiency $\eta_P = J\frac{k_T}{k_P}$

When talking about a wind turbine, we have:

- Tip speed ratio $\lambda = \frac{\Omega R}{u_0}$
- Thrust coefficient $\frac{T}{\frac{1}{2}\rho u_0^2 \pi R^2}$
- Power coefficient $\frac{P}{\frac{1}{2}\rho u_0^3 \pi R^2}$

In order to calculate these coefficients, we will have to calculate the induction factors a and a'. Their definitions can be found in the course notes. The formula to calculate them depends on the configuration. For a propeller, we have:

$$a = \frac{\sigma c_N (1+a)}{2(1-\cos(2\phi))}$$
$$a' = \frac{\sigma c_T (1-a')}{2\sin(2\phi)}$$

Where $\sigma(r) = \frac{c(r)B}{2\pi r}$ and $\phi(r) = \beta(r) - \alpha(r) = \arctan\left(J\frac{R}{\pi r}\frac{(1+a)}{(1-a')}\right)$. The two coefficients c_N and c_T are found with:

$$c_N = C_l\cos(\phi) - C_d\sin(\phi)$$
$$c_T = C_l\sin(\phi) + C_d\cos(\phi)$$

For the wind turbine, the induction factors are found with:

$$a = \frac{\sigma c_N (1-a)}{2(1-\cos(2\phi))}$$
$$a' = \frac{\sigma c_T (1+a')}{2\sin(2\phi)}$$

Where $\sigma(r) = \frac{c(r)B}{2\pi r}$ and $\phi(r) = \beta(r) + \alpha(r) = \arctan\left(\frac{R}{\lambda r}\frac{(1-a)}{(1+a')}\right)$. The two coefficients c_N and c_T are found with:

$$c_N = C_l \cos(\phi) + C_d \sin(\phi)$$
$$c_T = C_l \sin(\phi) - C_d \cos(\phi)$$

2 Verification

Using the data given, we can verify the solver against known results. Here is the comparison for a two bladed propeller with a flat plate as the profile:

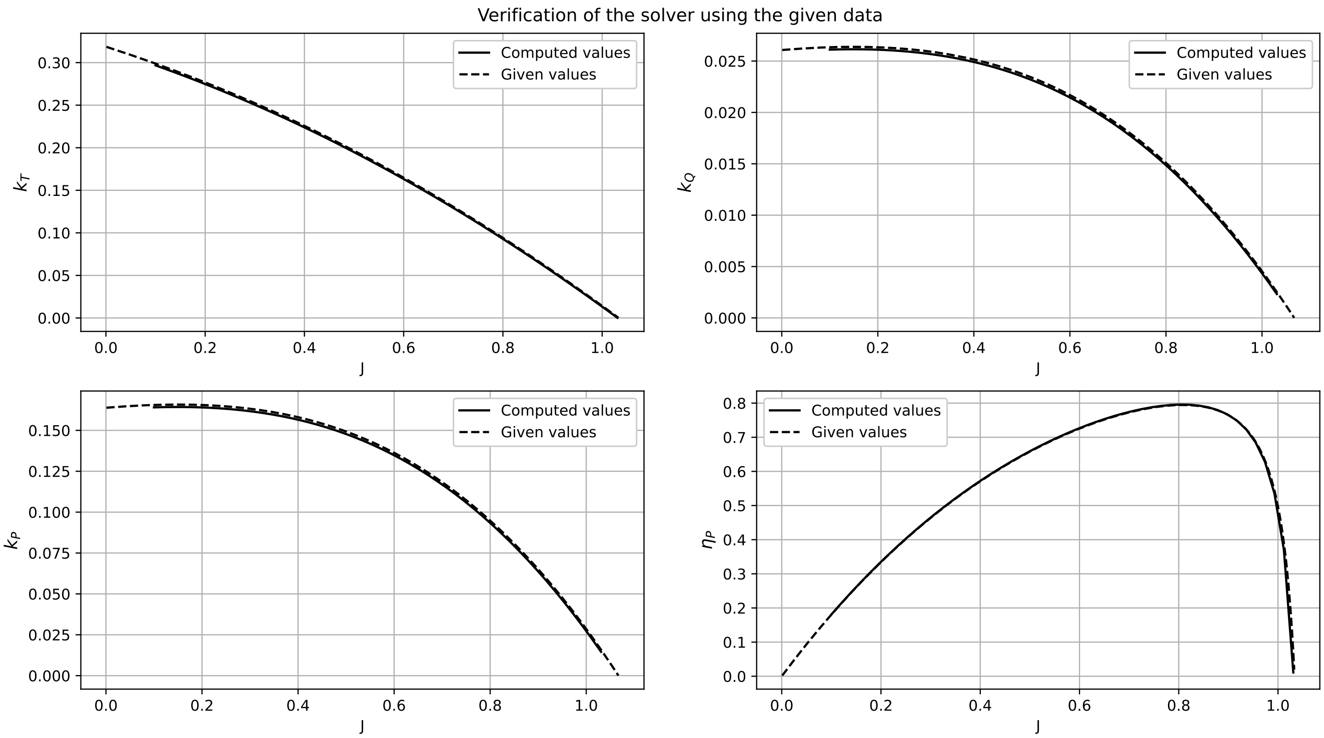

Figure 1: Verification of the solver

Since we are talking about a propeller, we must have positive thrust so the plots have been cut at the value of J where $k_T = 0$. Because the solving time was much longer when J was close to 0, I decided to start the graphs at $J = 0.1$.

As we can see on the above plots, the two lines are close so we are confident that our solver is correct. Adding more elements to the blade would improve the accuracy at the cost of computation time. Increasing the tolerance for the under-relaxed solver would also improve the accuracy of the results. In the next computations, we have used 50 elements and a tolerance of 10^{-8}.

3 P-51D propeller

For the P-51D propeller, we will also change the pitch setting of the blades so we'll add an offset to β to impose its value at $r = 0.75R$. Here is what the curves of the different coefficients look like for different pitch settings between $10°$ and $60°$:

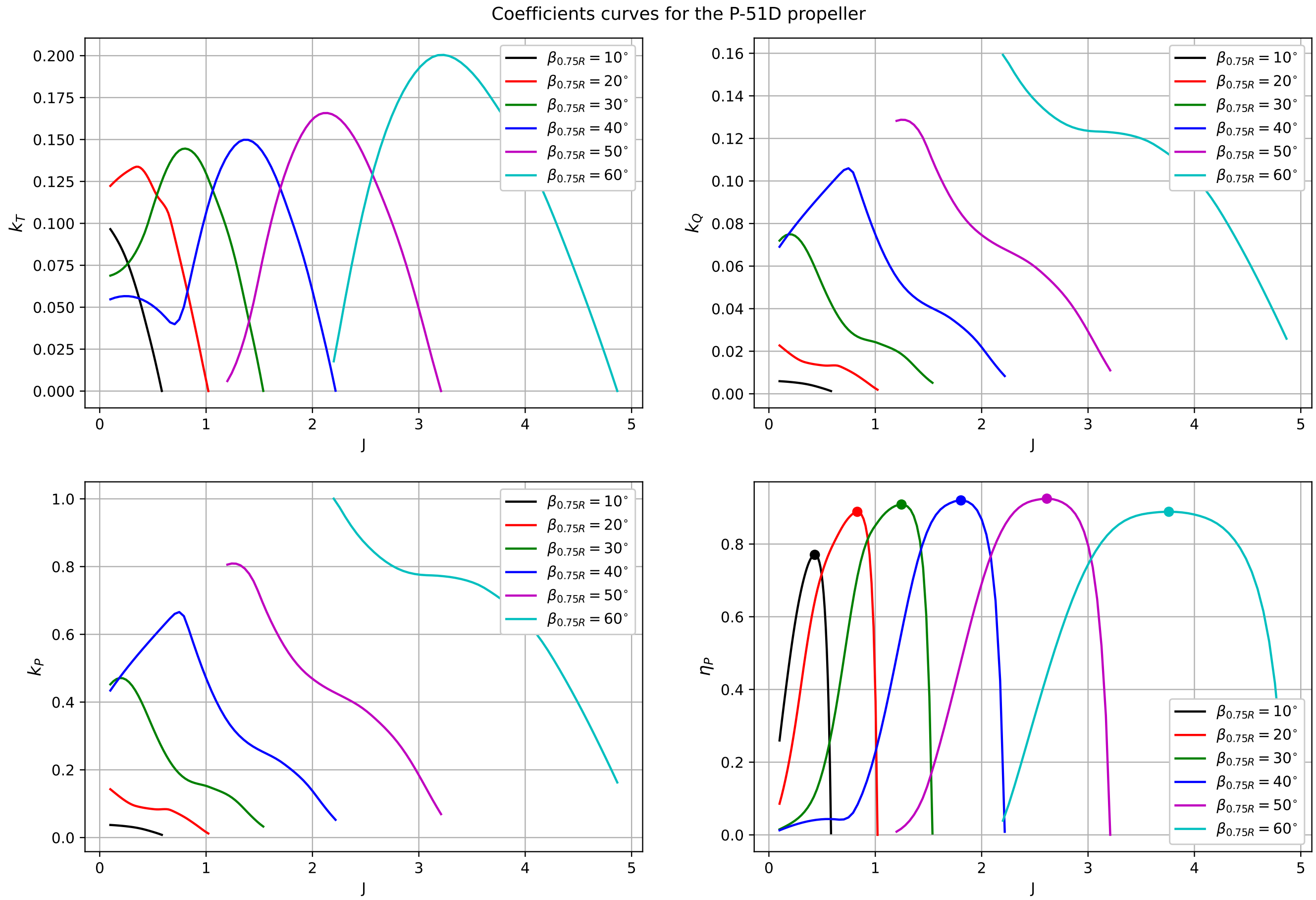

Figure 2: Coefficients for the P-51D propeller for different pitch settings

We can note a few things by looking at these plots. Firstly, as the pitch setting increases, we tend to get a higher maximal torque coefficient but it happens at different values of J. Secondly, the maximum propuslive efficiencies that we can get stay almost constant for every pitch setting but also happen at different values of J. Even if the maximum efficiency stays almost constant, it is smaller when $\beta_{0.75R} = 10°$ and we can see that the overall best efficiency happens between $\beta_{0.75R} = 40°$ and $\beta_{0.75R} = 50°$. Thirdly, we can see very different operating ranges (with $k_T > 0$) depending on the pitch setting.

One thing that appeared during the solving process was that for the pitch settings $\beta_{0.75R} = 50°$ and $\beta_{0.75R} = 60°$, the thrust coefficient was positive on two distinct intervals. The first interval, with the smallest values of J was also smaller in size and the second interval, with larger values of J was the longest one in length. Of the two, only the largest one, the one with larger values of J was drawn as all of the coefficients were very small in the first interval. If we are in conditions with a value of J in the first interval, an aircraft will use one of the other pitch settings in order to have better thrust and power.

Depending on the flight condition, we will have to use different pitch settings. If we are at a standstill, with $u_0 = 0$ so $J = 0$, we see that to get the most thrust, we want to have a pitch setting between $10°$ and $20°$. If we are in cruise conditions, we want to maximize the range so get the best possible efficiency. This can be done by varying the RPM in order to vary the value of J to get the best efficiency. If we cannot change the engine RPM, we could change the pitch setting to get the best possible efficiency for a fixed value of J. We must however be careful that changing either the RPM or the pitch setting will also change the thrust coefficient which would in turn change the speed so change the value of J.

4 P-51D propeller as a wind turbine

Using the geometric data from the P-51D propeller, we can turn it into a wind turbine. From this, we can find the thrust and power coefficients for different values of the pitch setting to get:

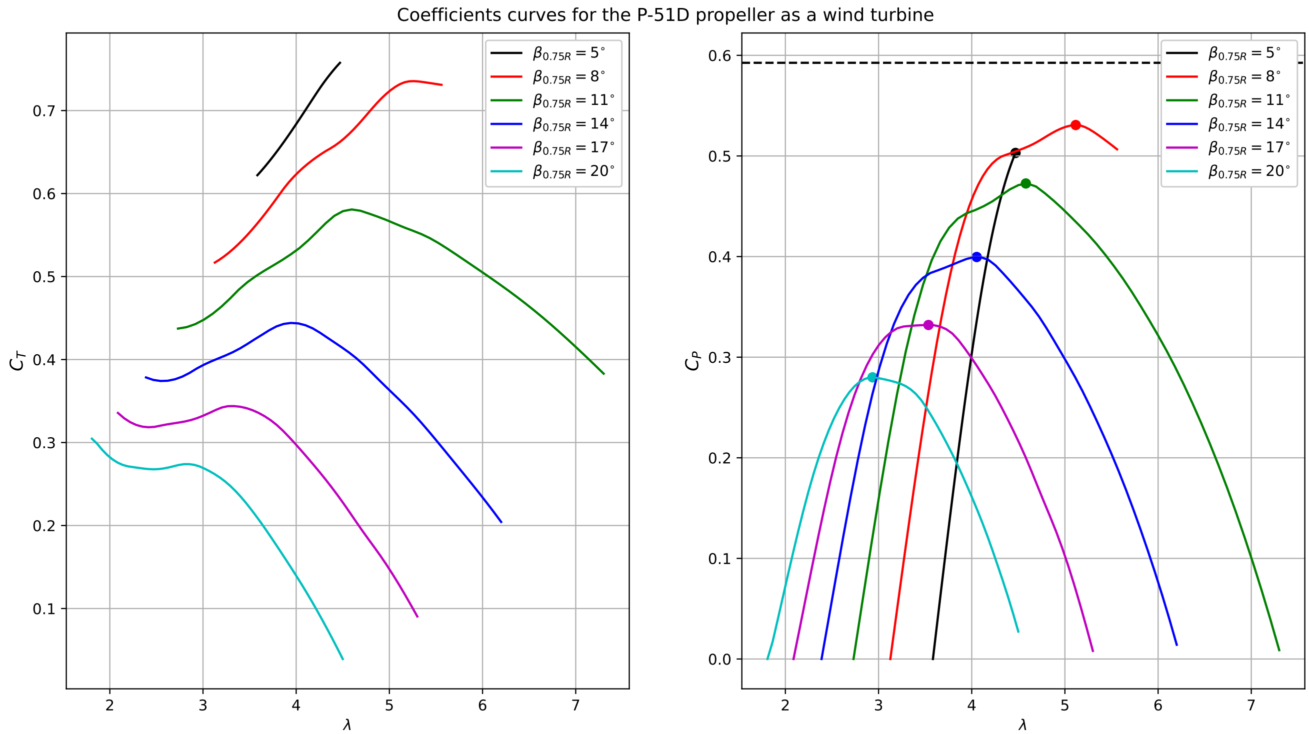

Figure 3: Plots for the P-51D propeller as a wind turbine

On the plot on the right, the dashed line represents the maximal value of Betz optimum case so with $C_{p,max} = \frac{16}{27}$. As we can see, every configuration that we have induces a lower maximal power coefficient than the Betz optimum case. From these plots, we can also see that we get better power and lift coefficients as the $\beta_{0.75R}$ decreases. We should note that for the curves with $\beta_{0.75R} = 5°$ and $\beta_{0.75R} = 10°$, we have to cut them off on the right because they correspond to situations where we have $a > 0.5$. This is one of the limits of the BEM theory. If we could extend our theory for higher values of λ for the these two cases, we might be able to get even closer to the Betz optimum.

To find the optimal configuration, we will do the computations for different values of $\beta_{0.75R}$ and find the maximal C_P. Computing the maximal power coefficents for $\beta_{0.75R} \in [5°, 6°, 7°, 8°, 9°]$, we can find that the optimal configuration happens for $\beta_{0.75R} = 8°$. At this value, we have $C_{P,opt} = 0.531$ and $\lambda_{opt} = 5.116$.