Université catholique de Louvain

LMECA2830 - Aerospace Dynamics

Homework 1 : Performance and static stability studies

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1 F/A-18 Super Hornet

1.1 Aircraft Polar

For the relation between the drag and lift coefficients, we will assume a relation of the type:

$$C_D = C_{D,0} + kC_L^2 \tag{1}$$

with $C_{D,0}$ the zero-lift drag and $k = \frac{1}{\pi(AR)e}$ where AR is the aspect ratio and e is the Oswald efficiency. Using the values given, we can find the polar curve:

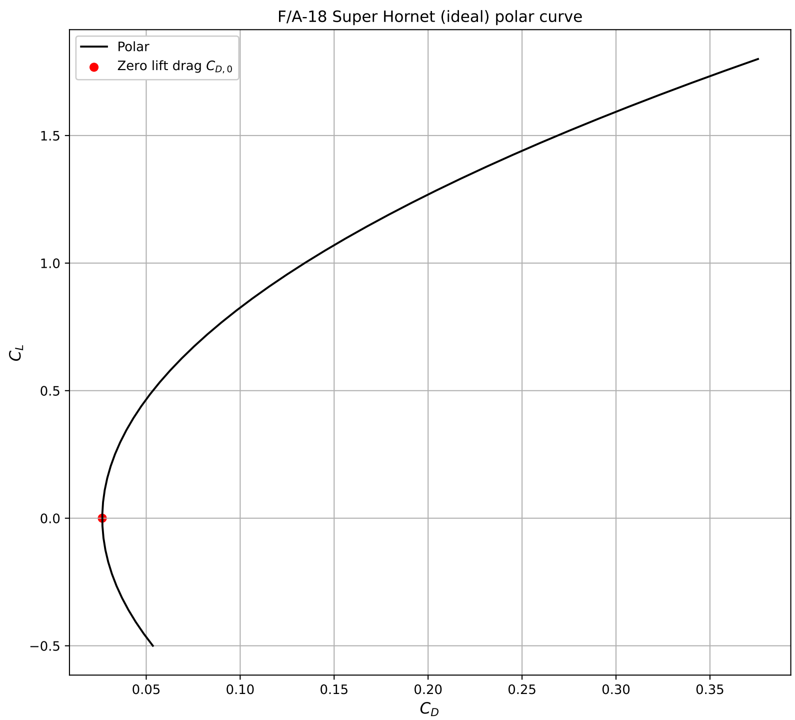

Figure 1: Polar for the Super Hornet

This relation is ideal and in reality, the C_L attains a maximal value.

1.2 First part of the mission

a)

We will try to find a differential equation linking the aircraft range and its weight. By the definition of the $TSFC$, we have:

$$dW = -g\,(TSFC)\,T dt$$

Dividing by W and using the fact that in level flight $\frac{T}{W} = \frac{D}{L} = \frac{C_D}{C_L}$, we can find:

$$\frac{dW}{W} = -g\,(TSFC)\,\frac{C_D}{C_L}dt$$

By definition of the velocity, we have $\frac{dx}{dt} = V = \sqrt{\frac{2W}{\rho S C_L}}$. Replacing this in the equation and simplifying gives:

$$\frac{dW}{\sqrt{W}} = -g\,(TSFC)\,C_D\sqrt{\frac{\rho S}{2C_L}}dx$$

Integrating this equation between the two states 0 and 1, we find:

$$\sqrt{W_1} - \sqrt{W_0} = -\frac{g\,(TSFC)}{2}C_D\sqrt{\frac{\rho S}{2C_L}}\,(x_1 - x_0)$$

$$W_0 = \left(\sqrt{W_1} + \frac{g\,(TSFC)}{2}C_D\sqrt{\frac{\rho S}{2C_L}}\,(x_1 - x_0)\right)^2$$

In our case, W_0 will be the initial weight of the aircraft (empty mass and mass of fuel times g), W_1 will be the empty weight and $(x_1 - x_0)$ will be the distance of the trip so $600km$.

b)

In order to find the optimal lift coefficient, we will try to maximize $\frac{C_L V}{C_D}$ as it will give the best range for a given quantity of fuel. The calculations are then:

$$\frac{d}{dC_L}\left(\frac{C_L V}{C_D}\right) = 0$$

$$\frac{d}{dC_L}\left(\frac{C_L^{1/2}}{C_D}\right) = 0$$

$$\frac{d}{dC_L}\left(\frac{1}{C_{D,0}C_L^{-1/2} + kC_L^{3/2}}\right) = 0$$

This last expression is equivalent to:

$$\frac{d}{dC_L}\left(C_{D,0}C_L^{-1/2} + kC_L^{3/2}\right) = 0$$

$$\Rightarrow -\frac{1}{2}C_{D,0}C_L^{-3/2} + \frac{3}{2}kC_L^{1/2} = 0$$

$$\Rightarrow C_{L,opt} = \sqrt{\frac{C_{D,0}}{3k}}$$

Using this equation, we can get that $\boxed{C_{L,opt} = 0.287}$ and using (1), we have $\boxed{C_{D,opt} = 0.0355}$.

c)

Using the equation for W_0 and the calculated C_L and C_D, we can find that $W_0 = 151871N$. By removing the empty weight of the aircraft then dividing by g, we can find that the fuel mass needed is $\boxed{m_{fuel} = 1616.33kg}$.

d)

Using the formula $V = \sqrt{\frac{2W}{\rho C_L S}}$, we can calculate the initial and final velocities using the initial and final weights. This gives us $\boxed{V_{init} = 166.83m/s = 600.59km/h}$ and $\boxed{V_{final} = 157.88m/s = 568.37km/h}$. Because the aircraft is flying at a constant altitude, we always need to have the lift force that is equal to the weight. Since C_L and ρ stay constant, the only way to change the lift force is to change the velocity. We then have $L \propto V^2$. At the beginning of the flight, the weight of the aircraft is larger than at the end since the fuel has been used and therefore to maintain $L = W$, the velocity will also be larger at the beginning than at the end.

1.3 Second part of the mission

a)

When the pilot is in a level flight upside down, the load factor will be equal to -1 and the pilot will be pushed out of his seat. By doing the inverted pull up, the load factor will increase up to its maximal (positive) value. When pushing on the stick in order to go down, the load factor will decrease to negative values. In an aircraft, the maximal load factor will usually be higher in absolute value than the minimal load factor so the airplane can be pushed to a greater limit when doing the inverted pull-up. The influence on the pilot also plays a role as it is better to be pushed into the seat rather than being pulled out of it.

b)

To draw the flight envelope, we need to compute the critical velocity using $V^* = \sqrt{\frac{2n_{max}W/S}{\rho C_{L,max}}}$ Plugging the values that we have, we find $\boxed{V^* = 202.37m/s = 728.54km/h}$. The flight envelope is then:

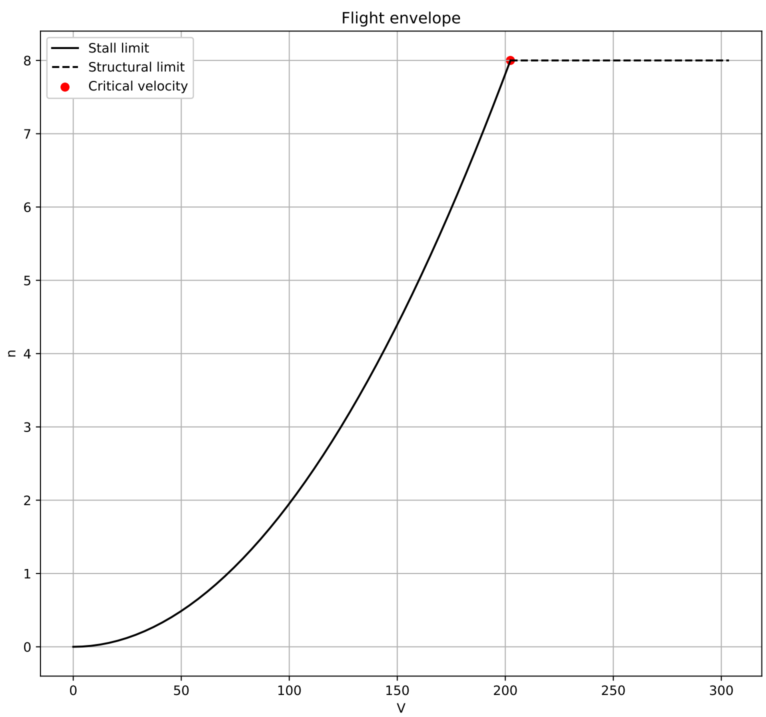

Figure 2: Flight envelope

c)

When the aircraft is upside down at the top of the circular curve, the vertical equilibrium equation is:

$$L + W = \frac{W}{g}\frac{V^2}{R}$$
$$L = W\left(\frac{V^2}{gR} - 1\right)$$

Introducing the load factor $n = L/W$, we get:

$$n = \left(\frac{V^2}{gR} - 1\right) \tag{2}$$

d)

From equation (2), we have that $R = \frac{V^2}{g(n+1)}$. Using the expression for V^*, we can get a formula for the smallest turning radius:

$$R_{min} = \frac{2n_{max}W/S}{\rho C_{L,max}(n_{max}+1)g}$$

This gives us $\boxed{R_{min} = 463.86m}$. If we were doing a simple pull up maneuver, the minimal turning radius would be:

$$R_{min} = \frac{2n_{max}W/S}{\rho C_{L,max}(n_{max}-1)g}$$

Which gives $\boxed{R_{min} = 596.40m}$. As we can see, we are able to get a much smaller turning radius when doing the inverted pull-up rather that a right side up pull-up.

2 Static stability, trim and stability augmentation

2.1 Static margin

Since we do not have any information about the body or the propulsion system of the aircraft, we will only consider the wing and the tail in the calculations. The static margin $h_n - h$ can be found by finding the neutral point h_n with:

$$h_n = \frac{\overline{V_H} C_{L_T,\alpha}}{C_{L,\alpha}} + h_{nW}$$

In this equation, $\overline{V_H}$ can be found using:

$$\begin{aligned}\overline{V_H} &= V_H + (h - h_{nW})\frac{S_T}{S}\\ &= \left(\frac{l_T}{\bar{c}} + h - h_{nW}\right)\frac{S_T}{S}\\ &= (h_{nT} + h - h_{nW})\frac{S_T}{S}\end{aligned}$$

The tail lift coefficient derivative needs to incorporate the downwash. We therefore have $C_{L_T,\alpha} = C_{L_T,\alpha_t}\left(1 - \frac{\partial\epsilon}{\partial\alpha_w}\right)$. The term $C_{L,\alpha}$ is the total lift coefficient derivative with respect to the angle of attack. It includes the tail and wing contributions, it is then $C_{L,\alpha} = C_{L_W,\alpha_w} + \frac{S_T}{S}C_{L_T,\alpha_t}\left(1 - \frac{\partial\epsilon}{\partial\alpha_w}\right)$. Using the values given and plugging them into these equation gives us the static margin: $\boxed{h_n - h = 0.148 = 14.8\%}$. It is greater than 5% so we can say that our aircraft is stable with some room.

2.2 New static margin

In order to find the new static margin, we can reuse the above formulas by replacing S_T with $0.8S_T$ to account for the 20% reduction in the tail surface area. This then gives us a static margin $\boxed{h_n - h = 0.120 = 12.0\%}$.

2.3 Stability augmentation system

In order to find the coefficient K, we will find the linearized equation for C_m then impose that $C_{m,\alpha} = 0$ at the original neutral point h_n. The general expression for C_m is:

$$C_m = C_{m_{AC_W}} + C_L \left(h - h_{nW}\right) - \overline{V_H} C_{L_T}$$

The $\overline{V_H}$ term is found using the same formula as before. We can look at the tail lift coefficient:

$$\begin{aligned}
C_{L_T} &= C_{L_T,\alpha_t}\alpha_t + C_{L_{t,\delta_e}}\delta_e \\
&= C_{L_T,\alpha_t}\left(\alpha_w - \epsilon - i_T\right) + C_{L_{t,\delta_e}} K\alpha \\
&= C_{L_T,\alpha_t}\left(\alpha_w - \epsilon_0 - \frac{\partial \epsilon}{\partial \alpha_w}\alpha_w - i_T\right) + C_{L_{t,\delta_e}} K\alpha \\
&= C_{L_T,\alpha_t}\left(\alpha_w\left(1 - \frac{\partial \epsilon}{\partial \alpha_w}\right) - \epsilon_0 - i_T\right) + C_{L_{t,\delta_e}} K\alpha
\end{aligned}$$

The total lift coefficient is defined as:

$$\begin{aligned}
C_L &= C_{L_W} + \frac{S_T}{S} C_{L_T} \\
&= C_{L_W,\alpha_w}\alpha_w + \frac{S_T}{S}\left(C_{L_T,\alpha_t}\left(\alpha_w\left(1 - \frac{\partial \epsilon}{\partial \alpha_w}\right) - \epsilon_0 - i_T\right) + C_{L_{t,\delta_e}} K\alpha\right) \\
&= \left(C_{L_W,\alpha_w} + \frac{S_T}{S} C_{L_T,\alpha_t}\left(1 - \frac{\partial \epsilon}{\partial \alpha_w}\right)\right)\alpha_w - \frac{S_T}{S} C_{L_T,\alpha_t}\left(\epsilon_0 + i_T\right) + \frac{S_T}{S} C_{L_{t,\delta_e}} K\alpha \\
&= \left(C_{L,\alpha} + \frac{S_T}{S} C_{L_{t,\delta_e}} K\right)\alpha
\end{aligned}$$

With $C_{L,\alpha} = C_{L_W,\alpha_w} + \frac{S_T}{S} C_{L_T,\alpha_t} \left(1 - \frac{\partial \epsilon}{\partial \alpha_w}\right)$ and $\alpha = \alpha_w - \frac{S_T}{S} \frac{C_{L_T,\alpha_t}}{C_{L,\alpha}} (\epsilon_0 + i_T)$. We therefore find $\alpha_w = \alpha + \frac{S_T}{S} \frac{C_{L_T,\alpha_t}}{C_{L,\alpha}} (\epsilon_0 + i_T)$. Replacing these expressions into the equation for C_m gives:

$$\begin{aligned}
C_m =& C_{m_{AC_W}} + C_L (h - h_{nW}) - \overline{V_H} C_{L_T} \\
=& C_{m_{AC_W}} \\
&+ (h - h_{nW}) \left(C_{L,\alpha} + \frac{S_T}{S} C_{L_{t,\delta_e}} K \right) \alpha \\
&- \overline{V_H} \left(C_{L_T,\alpha_t} \left(\alpha_w \left(1 - \frac{\partial \epsilon}{\partial \alpha_w} \right) - \epsilon_0 - i_T \right) + C_{L_{t,\delta_e}} K \alpha \right) \\
=& C_{m_{AC_W}} \\
&+ \left[(h - h_{nW}) \left(C_{L,\alpha} + \frac{S_T}{S} C_{L_{t,\delta_e}} K \right) - \overline{V_H} C_{L_{t,\delta_e}} K \right] \alpha \\
&- \overline{V_H} C_{L_T,\alpha_t} \left(1 - \frac{\partial \epsilon}{\partial \alpha_w} \right) \left(\alpha + \frac{S_T}{S} \frac{C_{L_T,\alpha_t}}{C_{L,\alpha}} (\epsilon_0 + i_T) \right) + \overline{V_H} C_{L_T,\alpha_t} (\epsilon_0 + i_T) \\
=& C_{m_{AC_W}} + \overline{V_H} C_{L_T,\alpha_t} (\epsilon_0 + i_T) \left(1 - \frac{S_T}{S} \frac{C_{L_T,\alpha_t}}{C_{L,\alpha}} \left(1 - \frac{\partial \epsilon}{\partial \alpha_w} \right) \right) \\
&+ \left[(h - h_{nW}) \left(C_{L,\alpha} + \frac{S_T}{S} C_{L_{t,\delta_e}} K \right) - \overline{V_H} C_{L_{t,\delta_e}} K - \overline{V_H} C_{L_T,\alpha_t} \left(1 - \frac{\partial \epsilon}{\partial \alpha_w} \right) \right] \alpha \\
=& C_{m0} + C_{m,\alpha} \alpha
\end{aligned}$$

Isolating K in the expression for $C_{m,\alpha}$ when $C_{m,\alpha} = 0$ gives:

$$K = \frac{(h - h_{n_W}) C_{L,\alpha} - \overline{V_H} C_{L_T,\alpha_t} \left(1 - \frac{\partial \epsilon}{\partial \alpha_w}\right)}{\overline{V_H} C_{L_{t,\delta_e}} - \frac{S_T}{S} C_{L_{t,\delta_e}} (h - h_{n_W})}$$

Replacing h with the value of the position of neutral point h_n in the original design, we find $\boxed{K = 0.213}$.

2.4 Trimmed and untrimmed curves

To plot the trimmed C_L vs α curve for the original aircraft, we'll use the relation:

$$C_{L_{trim}} = \frac{C_{L,\alpha} \alpha_{trim} \left(\frac{S_T}{S} (h_n - h_{nW}) - \overline{V_H} \right) - \frac{S_T}{S} C_{m00}}{\frac{S_T}{S} (h - h_{nW}) - \overline{V_H}}$$

In this equation, we have $C_{m00} = C_{m_{AC_W}} + \overline{V_H} C_{L_T,\alpha} (\epsilon_0 + i_T) \left(1 - \frac{S_T}{S} \frac{C_{L_T,\alpha_t}}{C_{L,\alpha}} \left(1 - \frac{\partial \epsilon}{\partial \alpha_w}\right)\right)$. Using the given values, we can then plot the trimmed curve:

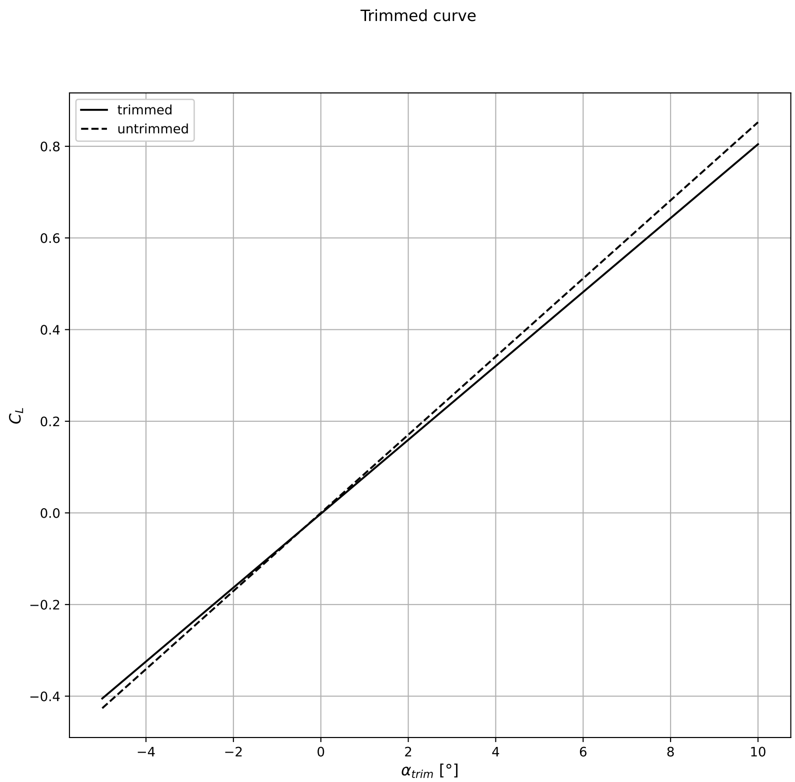

Figure 3: Trimmed curve

As we can see from the graph, we have a lower slope for the trimmed aircraft compared to the untrimmed curve. The new curve also does not go through the point $(0, 0)$.

2.5 Configuration for maximum range

We can use the same formulas as in the first exercise to get the optimal lift coefficient. With this optimal lift coefficient, we can then find the corresponding angle of attack with the trimmed curve using:

$$\alpha = \frac{C_{L,opt}\left[\frac{S_T}{S}(h - h_{nW}) - \overline{V_H}\right] + \frac{S_T}{S}C_{m00}}{C_{L,\alpha}\left(\frac{S_T}{S}(h_n - h_{nW}) - \overline{V_H}\right)}$$

The elevator deflection is then:

$$\delta_e = -\frac{C_{m00} + (h - h_n)C_{L,opt}}{C_{L_{t,\delta_e}}\left(\frac{S_T}{S}(h_n - h_{nW}) - \overline{V_H}\right)}$$

Plugging these values into the equations gives us $\boxed{\alpha = 4.15°}$ for the angle of attack and $\boxed{\delta_e = -3.88°}$ for the aileron deflection. As we can see, it is negative, meaning that the aileron will be deflected up. The camber of the wing will make the aircraft pitch down so this is why we need a negative aileron deflection to counteract this camber effect. To find the equivalent airspeed, we can reuse the formula from above $V = \sqrt{\frac{2W}{\rho S C_{L,opt}}}$. We then have $\boxed{V = 109.65m/s = 394.75km/h}$.