Université catholique de Louvain

LMECA2830 - Aerospace Dynamics

Homework 2 : Dynamic stability studies

Academic year 2023-2024

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1 Dimensional derivatives

To get the dimensional derivatives, we can use the given adimensional derivatives and multiply them by the correct coefficient to make them dimensional (see course slides). In order to do this, we need the flight speed $u_0 = Mc = M\sqrt{\gamma RT}$. For this, we used $\gamma = 1.4$ and $R = \frac{p}{\rho T}$. Putting these values in a table gives:

	Y	L	N
v	-7.121	-8.397	2.802
p	0	-28.762	-4.328
r	0	9.645	-1.531

In the above table, the column gives the quantity that we are differentiating and the line is the variable by which we are differentiating. For example, Y_v is the entry $(1,1)$ in the table so $Y_v = -7.121$.

2 A matrix

In order to study the lateral stability of the aircraft, we will use the linearized equations of motion for the lateral motions. These equations can be written in matrix form as:

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{Y_v}{m} & \frac{Y_p}{m} & \frac{Y_r}{m} - u_0 & g\cos\theta_0 \\ I'_{zz}L_v - I'_{xz}N_v & I'_{zz}L_p - I'_{xz}N_p & I'_{zz}L_r - I'_{xz}N_r & 0 \\ I'_{xx}N_v - I'_{xz}L_v & I'_{xx}N_p - I'_{xz}L_p & I'_{xx}N_r - I'_{xz}L_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + B \tag{1}$$

$$\dot{\omega} = \boldsymbol{A}\omega + B \tag{2}$$

Where ω is the vector containing the dependant variables and B is a vector that does not depend on the variables in ω and contains terms linked with aircraft control (rudder, ailerons, ...). The stability of this system will be determined only the matrix $\boldsymbol{A}$.

Assuming that we take the aircraft coordinate frame such that $\theta_0 = 0$ and using $I'_k = \frac{I_k}{I_{xx}I_{zz} - I_{xz}^2}$ with $k \in \{xx, zz, xz\}$ and the derivatives found in the previous section, we can get the matrix $\boldsymbol{A}$:

$$\begin{bmatrix} -0.570 & 0.0 & -20.418 & 9.81 \\ -1.459 & -5.005 & 1.681 & 0.0 \\ 0.254 & -0.346 & -0.144 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \end{bmatrix} \tag{3}$$

3 Characteristic polynomial

In order to study the stability of the aircraft, we need to find the eigenvalues of this matrix. To do that, we first need to get the characteristic polynomial and the eigenvalues of the matrix will be the roots of this polynomial. Calculating $det(\boldsymbol{A} - \lambda \boldsymbol{I})$ gives:

$$p(\lambda) = \lambda^4 + 5.719\lambda^3 + 9.420\lambda^2 + 51.417\lambda - 2.114 \tag{4}$$

As we can see one of the coefficient in the polynomial is negative so we know that some of the eigenvalues will have a positive real part and some modes will be unstable.

If we assume that one of the roots will be very small in magnitude compared to the other ones (which is to be expected for the spiral mode in lateral stability analysis), we can keep only the linear term in the polynomial to find:

$$\lambda = \frac{2.114}{51.417} = 0.0411 \tag{5}$$

If we assume that this is the correct value of the one of the roots of (4), then it can be rewritten as:

$$p(\lambda) \approx (\lambda - 0.0411)(\lambda^3 + 5.760\lambda^2 + 9.657\lambda + 51.814) \tag{6}$$

Looking at the second term of the right hand side of (6), which is a polynomial of degree 3, we now see that all of its coefficients are positive and the Routh criterium for polynomials of degree 3 ($5.76 * 9.657 > 51.8145$) is true. We can therefore assume that all of the other roots will have a negative real part and the modes will be stable.

4 Eigenvalues

We can now compute exactly the roots of (4) to find:

$$\lambda_1 = 0.0408$$
$$\lambda_2 = -5.669$$
$$\lambda_{3,4} = -0.0454 \pm 3.023i$$

As we can see, these are in agreement to what was said in the previous section, the root λ_1 is close to the one that we found above and all of the other roots lead to stable modes. We also note that λ_3 and λ_4 are complex conjugates so we will analyse them together.

From these roots, we can compute the time to half/double (depending on the sign of the real part) and for the complex conjugate roots, we can also compute the period and the number of cycles to half. We then find:

	$T_{1/2}$	T	$N_{1/2}$
λ_1	0.122	/	/
λ_2	16.992	/	/
$\lambda_{3,4}$	15.252	2.079	7.338

If we look at these values and compare them to typical values of the lateral stability analysis of an aircraft, we can expect the first mode to be the the spiral mode as it has a long time to double, the second mode to be the rolling convergence as it is heavily damped and the third mode to be the dutch roll as it is oscillatory and lightly damped. As we can see, the unstable mode is the spiral mode but we can notice that the time to double is quite large. This will therefore usually not be a big problem as the pilot will have time to react and correct the instability using the aircraft controls.

5 Eigenvectors

To confirm the claims made in the previous section, we will compute the eigenvectors of the matrix to see which state variables play a part in each mode. We will also compute additional state variables such as ψ the yaw angle and y_0 the lateral displacement of the aircraft. Using $\theta_0 = 0$, we have $\dot{\psi} = \sec\theta_0 r = r$ and $\Delta\dot{y}_0 = u_0 \cos\theta_0 \psi + v = u_0\psi + v$. Since the variables will change like $e^{\lambda t}$, taking the integral with respect to time is equivalent to dividing by λ. If we then make all of the variables adimensional and normalize each mode by one of the variables, we can find:

	λ_1		λ_2		$\lambda_{3,4}$	
	Module	Phase	Module	Phase	Module	Phase
$\beta = \hat{v}$	$1.693 \cdot 10^{-3}$	$0.0°$	0.138	$180.0°$	0.577	$64.97°$
$\hat{p}$	$2.818 \cdot 10^{-4}$	$0.0°$	0.450	$180.0°$	0.240	$-90.86°$
$\hat{r}$	$3.237 \cdot 10^{-3}$	$0.0°$	0.0179	$180.0°$	0.104	$159.30°$
ϕ	0.087	$0.0°$	1.0	$0.0°$	1.0	$0.0°$
ψ	1.0	$0.0°$	0.0397	$0.0°$	0.434	$-109.84°$
$\frac{y_0}{u_0 t^*}$	$1.616 \cdot 10^{3}$	$0.0°$	1.146	$0.0°$	3.262	$140.65°$

By looking at this table, we can confirm that the first mode involves mainly yaw, no side-slip and a little roll so we can say that it corresponds to the spiral mode. The second mode involves mainly roll so it corresponds to the rolling convergence. The third mode has roll and yaw coupled so it is the dutch roll.