

Linear TES: Type and Effect System

Orpheas van Rooij

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1 Language

```
op ::= + | - | * | / | not | and | or
v ::= () | ℓ (∈ Loc) | b (∈ Bool) | i (∈ Int) | λx. e | ⊙ (∈ op) | (v, v) | cont ℓ N
e ::= v | x | e e | (e, e) | let (x, x) = e in e | if e then e else e
    | do e | shallow-try e with v | v | eff v K
N ::= • | N e | v N | (N, e) | (v, N) | let (x, x) = N in e | if N then e else e | do N
K ::= N | shallow-try K with v | v
```

Figure 1: Syntax of effect values, values, expressions, and evaluation contexts

2 Head Reduction

$(\lambda x. e) v / \sigma$	\rightsquigarrow	$e[v/x] / \sigma$
$\odot v / \sigma$	\rightsquigarrow	v' / σ
		$\llbracket \odot \rrbracket v = v'$
$\odot v_1 v_2 / \sigma$	\rightsquigarrow	v / σ
		$v_1 \llbracket \odot \rrbracket v_2 = v$
$\text{let } (x_1, x_2) = (v_1, v_2) \text{ in } e_3 / \sigma$	\rightsquigarrow	$e_3[v_1/x_1][v_2/x_2] / \sigma$
$\text{if true then } e_1 \text{ else } e_2 / \sigma$	\rightsquigarrow	e_1 / σ
$\text{if false then } e_1 \text{ else } e_2 / \sigma$	\rightsquigarrow	e_2 / σ
$\text{do } v / \sigma$	\rightsquigarrow	$\text{eff } v \bullet / \sigma$
$\text{shallow-try } v \text{ with } h \mid r / \sigma$	\rightsquigarrow	$r v / \sigma$
$\text{shallow-try } (\text{eff } v N) \text{ with } h \mid r / \sigma$	\rightsquigarrow	$h v (\text{cont } \ell N) / \sigma[\ell \mapsto \text{false}]$
		$\ell \notin \text{dom } \sigma$
$(\text{cont } \ell N) v / \sigma[\ell \mapsto \text{false}]$	\rightsquigarrow	$N[v] / \sigma[\ell \mapsto \text{true}]$
$(\text{eff } v_1 N) e_2 / \sigma$	\rightsquigarrow	$\text{eff } v_1 (N e_2) / \sigma$
$v_1 (\text{eff } v_2 N) / \sigma$	\rightsquigarrow	$\text{eff } v_2 (v_1 N) / \sigma$
$(\text{eff } v_1 N, e_2) / \sigma$	\rightsquigarrow	$\text{eff } v_1 (N, e_2) / \sigma$
$(v_1, \text{eff } v_2 N) / \sigma$	\rightsquigarrow	$\text{eff } v_2 (v_1, N) / \sigma$
$\text{let } (x_1, x_2) = (\text{eff } v_1 N) \text{ in } e_2 / \sigma$	\rightsquigarrow	$\text{eff } v_1 (\text{let } (x_1, x_2) = N \text{ in } e_2) / \sigma$
$\text{if } (\text{eff } v N) \text{ then } e \text{ else } e / \sigma$	\rightsquigarrow	$\text{eff } v (\text{if } N \text{ then } e \text{ else } e) / \sigma$
$\text{do } (\text{eff } v N) / \sigma$	\rightsquigarrow	$\text{eff } v (\text{do } N) / \sigma$

Figure 2: The head reduction relation

3 Types

$\tau, \kappa, \iota ::= \text{unit} \mid \text{bool} \mid \text{int} \mid \tau \xrightarrow{\rho} \tau \mid \tau * \tau$
$\rho ::= \langle \rangle \mid \tau \Rightarrow \tau$

Figure 3: Syntax of types, and row signatures

4 Typing Rules

UNIT $\Gamma \vdash () : \rho : \text{unit}$	BOOL $\Gamma \vdash b : \rho : \text{bool}$	INT $\Gamma \vdash i : \rho : \text{int}$	OP $\frac{\vdash_{Op} \odot : \tau \rightarrow \kappa}{\Gamma \vdash \odot : \rho : \tau \xrightarrow{\rho} \kappa}$
VAR $\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \rho : \tau}$	FUN $\frac{\Gamma, x : \tau \vdash e : \rho : \kappa}{\Gamma \vdash \lambda x. e : \langle \rangle : \tau \xrightarrow{\rho} \kappa}$	APP $\frac{\Gamma_1 \vdash e : \rho : \tau \xrightarrow{\rho} \kappa \quad \Gamma_2 \vdash e' : \rho : \tau}{\Gamma_1 ++ \Gamma_2 \vdash e e' : \rho : \kappa}$	
PAIR $\frac{\Gamma_1 \vdash e_1 : \rho : \tau \quad \Gamma_2 \vdash e_2 : \rho : \kappa}{\Gamma_1 ++ \Gamma_2 \vdash (e_1, e_2) : \rho : \tau * \kappa}$	PAIR-ELIMINATION $\frac{\Gamma_1 \vdash e_1 : \rho : \tau * \kappa \quad \Gamma_2, x_1 : \tau, x_2 : \kappa \vdash e_2 : \rho : \iota}{\Gamma_1 ++ \Gamma_2 \vdash \text{let } (x_1, x_2) = e_1 \text{ in } e_2 : \rho : \iota}$		
IF-THEN-ELSE $\frac{\Gamma_1 \vdash e_1 : \rho : \text{bool} \quad \Gamma_2 \vdash e_2 : \rho : \tau \quad \Gamma_2 \vdash e_3 : \rho : \tau}{\Gamma_1 ++ \Gamma_2 \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \rho : \tau}$			
DO $\frac{\rho = (\iota \Rightarrow \kappa) \quad \Gamma \vdash e : \rho : \iota}{\Gamma \vdash \text{do } e : \rho : \kappa}$	SHALLOW-HANDLER $\frac{\Gamma_1 \vdash e : \rho : \tau \quad \rho = (\iota \Rightarrow \kappa) \quad \emptyset \vdash e : \langle \rangle : \iota \multimap (\kappa \xrightarrow{\rho} \tau') \xrightarrow{\rho'} \tau' \quad \Gamma_2 \vdash r : \langle \rangle : \tau \xrightarrow{\rho'} \tau'}{\Gamma_1 ++ \Gamma_2 \vdash \text{shallow-try } e \text{ with } h \mid r : \rho' : \tau'}$		

Figure 4: The type system

5 Protocol

<p><i>Protocol</i></p> $ \begin{aligned} \text{Protocol} &\triangleq \text{Val} \rightarrow (\text{Val} \rightarrow \text{iProp}) \rightarrow \text{iProp} \\ !\vec{x}(v) \{P\}. ?\vec{y}(w) \{Q\} &\triangleq \lambda u \Psi. \exists \vec{x}. \ulcorner u = v \urcorner * P * (\forall \vec{y}. Q \multimap \Psi(w)) \\ \perp &\triangleq !x(x) \{\text{False}\}. ?y(y) \{\text{True}\} \end{aligned} $

Figure 5: Definition of a protocol

6 Extended Weakest Precondition

The extended Weakest Precondition that we will use for the semantic typing is an enhancement of the usual weakest precondition that captures safety to incorporate reasoning with effects and effect handlers.

The $\text{ewp } e \langle \Psi \rangle \{ \Phi \}$ specifies that expression e can either call an effect according to protocol Ψ or it evaluates safely such that if it evaluates to a value that value satisfies Φ .

<p><i>Extended weakest precondition</i></p> $ \begin{aligned} \text{ewp } v \langle \Psi \rangle \{ \Phi \} &\triangleq \models \Phi(v) \\ \text{ewp } (\text{eff } v \ N) \langle \Psi \rangle \{ \Phi \} &\triangleq (\uparrow \Psi) v (\lambda w. \triangleright \text{ewp } N[w] \langle \Psi \rangle \{ \Phi \}) \\ \text{ewp } e \langle \Psi \rangle \{ \Phi \} &\triangleq \forall \sigma. S(\sigma) \equiv * \left\{ \begin{array}{l} \exists e', \sigma'. e / \sigma \longrightarrow e' / \sigma' * \\ \forall e', \sigma'. e / \sigma \longrightarrow e' / \sigma' \equiv * \triangleright \models \\ S(\sigma') * \text{ewp } e' \langle \Psi \rangle \{ \Phi \} \end{array} \right. \end{aligned} $ <p><i>Upward closure</i></p> $ (\uparrow \Psi) v \Phi \triangleq \exists \Phi'. \Psi v \Phi' * (\forall w. \Phi'(w) \multimap \Phi(w)) $

Figure 6: Definition of the weakest precondition

7 Semantic Interpretation

Interpretation of types

$$\begin{aligned}
\mathcal{V}[\![\mathbf{unit}]\!](v) &\triangleq \lceil v = () \rceil \\
\mathcal{V}[\![\mathbf{bool}]\!](v) &\triangleq \exists b. \lceil v = \#b \rceil \\
\mathcal{V}[\![\mathbf{int}]\!](v) &\triangleq \exists i. \lceil v = \#i \rceil \\
\mathcal{V}[\![\tau \xrightarrow{\rho} \kappa]\!](v) &\triangleq \forall w. \mathcal{V}[\![\tau]\!](w) \multimap \text{ewp } (v \ w) \ \langle \mathcal{R}[\![\rho]\!] \rangle \{ \mathcal{V}[\![\kappa]\!] \} \\
\mathcal{V}[\![\tau * \kappa]\!](v) &\triangleq \exists v_1 v_2. \lceil v = (v_1, v_2) \rceil * \mathcal{V}[\![\tau]\!](v_1) * \mathcal{V}[\![\kappa]\!](v_2)
\end{aligned}$$

Interpretation of a row

$$\begin{aligned}
\mathcal{R}[\![\langle \rangle]\!] &\triangleq \perp \\
\mathcal{R}[\![\tau \Rightarrow \iota]\!] &\triangleq !x(x) \{ \mathcal{V}[\![\tau]\!](x) \}. ?y(y) \{ \mathcal{V}[\![\iota]\!](y) \}
\end{aligned}$$

Interpretation of typing judgments

$$\begin{aligned}
\Gamma \models e : \rho : \tau &\triangleq \forall vs. \mathcal{G}[\![\Gamma]\!](vs) \multimap \text{ewp } e[vs] \ \langle \mathcal{R}[\![\rho]\!] \rangle \{ \mathcal{V}[\![\tau]\!] \} \\
\mathcal{G}[\![\Gamma]\!](vs) &\triangleq \forall \{x \mapsto \tau\} \subseteq \Gamma. \mathcal{V}[\![\tau]\!](vs(x))
\end{aligned}$$

Figure 7: Interpretation of types, rows, and typing judgments