Advanced Macro II, Yale University,
Oliko Vardishvili

Spring 2022
Take Home Exam 2

(upload a zip of code & pdf files in Canvas)

Problem 3 Overlapping Generations Model

We will depart from the benchmark OLG model studied in the tutorial by incorporating government sector into the model as well as elastic labor supply.

Each period a continuum of agents is born and live for $J = 12$ periods. Agents retire at $J_r = 9$ periods and die deterministically at age $J = 12$. Newly born agents hold no initial assets, $a_1 = 0$, and they have equal propability to draw high or low fixed effect as well as idiosyncratic productivity.

The instantaneous utility function of an agent is given by:

$$u(c, l) = \frac{(c^{\nu}(1-l)^{1-\nu})^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}, \tag{1}$$

where $\gamma = 0.5$ and $\nu = 0.335$.

The government collects consumption, $\tau_c = 0.073$, capital, $\tau_r = 0.1$ and labor income, $\tau_w = 1$, taxes and uses the revenue to finance government expenditure $\frac{G}{Y} = 0.19$, where $Y = \Omega K^{\alpha} L^{1-\alpha}$ is the aggregate output, with $\alpha = 0.36$ and $\Omega = 1$. K and L denote respectively aggregate capital and effective labor in the economy.

The pension system operates on a pay-as-you-go basis, i.e., it collects contributions from working age generations and directly redistributes them to current retirees: $\tau_p = \frac{\kappa w \bar{L} m}{wL}$, where m denotes the mass of the retired households and $\bar{L}$ is the average effective labor. In the pension system, the replacement rate $\kappa_t = 0.35$ is exogenously given while the contribution rate t_p adjusts in order to balance the budget.

1. Define the recursive competitive equilibrium in this case. Derive the Euler equation taking into account elastic labor supply. Solve the new general equilibrium numerically, where the government balances its budget by labor income tax rate, τ_w. [You need to incorporate the government sector into the model, who collects labor income, capital income and consumption taxes and pays government expenditure G and pensions to the retired households. Use the benchmark code from the tutorial].

2. Report the following macroeconomic variables for this simulation: Aggregate consumption to GDP ratio, capital to GDP ratio, effective labor to GDP ratio, wage, interest rate, welfare (consumption equivalence terms). Plot life cycle profile of consumption, assets, working hours, and earnings.)

3. Now do the following simulations: (1) the government closes the budget with consumption taxes, setting other taxes to zero. (2) the government closes the budget with income taxes (taxes on labor income as well as capital income, common rate) setting other taxes to zero (3) the government eliminates pay as you go system, by setting $\kappa = 0$, keep everything else as in the benchmark simulation. (4) Decrease fertility $n_p = 0$. Report all macroeconomic variables as you did in the benchmark case. Provide intuition in terms of insurance and output efficiency.

4. Check how the distribution of agents across consumption levels and asset levels changes due to the two reforms. (You may want to plot life cycle profile of consumption, assets.) Comment under each reform which generations win and which generations lose. Explain the intuition.

5. Find an interesting quantitative question which you can answer using this model and answer it.

	n_p	κ	τ_w	τ_r	τ_c	C/Y	K/Y	L	w	r	Y	W
(0)												
(1)												
(2)												
(3)												
(4)												