Computational Economics, University of California Irvine,
Instructor: Oliko Vardishvili

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Take Home Exam 1 [1]

(Upload a zip of code & pdf files in Canvas) Note that for the final grade, I will ask you questions during lecture from your own solutions.

Problem 0 Warm up: Elastic Labor Supply

Now we will depart from the Aiyagari model with capital accumulation studied in class by introducing elastic labor supply. Now the preferences take the following form:

$$\frac{\left(c^{\nu}(1-l)^{1-\nu}\right)^{1-\mu}}{1-\mu} \tag{1}$$

Take the following parameters: $\nu = 0.374, \mu = 4$.[2] Use the following parameters: $\beta = 0.96$, $\alpha = 0.36$, $\delta = 0.08$, $\rho = 0.6$, $\sigma_\epsilon^2 = 0.16$, such that $var(log(z_t)) = 0.32$. Besides, set $m = 2.45$ to discretize shocks.[3]

1. Set up the dynamic programming problem and derive the first-order conditions.
2. Compute the steady state of the model using discrete value function iteration.

Problem 1 Steady States

Now we again consider inelastic labor supply. We will depart from the Aiyagari model with capital accumulation studied in class by introducing taxes in two very simple ways.

Use the following parameters: $\gamma = 0.5$, $\beta = 0.96$, $\alpha = 0.36$, $\delta = 0.08$, $\rho = 0.6$, $\sigma_\epsilon^2 = 0.16$, such that $var(z_t) = 0.04$. Besides, set $m = 2.45$ to discretize shocks.

Utility: $u = \frac{c^{1-\gamma}}{1-\gamma}$, production function: $Y = AK^\alpha L^{1-\alpha}$

[1] This take home is prepared for and adopted from Arpad Abraham's lecture series.
[2] Note that the values imply a relative risk aversion is equal to 2, a standard value in macro literature.
[3] $[prob, logs, invdist] = markovappr(\rho, \sigma_\epsilon, 2.45, N)$ in Matlab.

- Reform 1: Introducing a 15% consumption tax. The tax revenues are rebated equally across all agents of the economy.

- Reform 2: Introducing a capital income tax which generates the same revenues as the previous tax schedule (tax on rk, not on the principal). The tax revenues are rebated equally across all agents of the economy again. [Hint: $\tau_k rk = Tr_c$]

1. Define the recursive competitive equilibrium in these cases. Note that you have to add a government in the definition. Derive the Euler equations analytically and compare them with each other and with the basic model (without taxes).

2. Solve using some adjustments of the programs (with 3 solution methods: value function iteration, policy function iteration, and endogenous grid point methods provided in the class) for the steady state level of aggregate capital and the stationary decision rules and distribution of agents for the two tax reforms. [Hint: Note that tax revenues and consequently the tax rebate is a function of aggregate capital (or the interest rate), so you have to make only small modifications.]

3. Plot asset policy, consumption policy, and distributions and compare the smoothness of the functions across these solution methods. Explain which solution method approximates better the policy functions. Why?

4. Check how aggregate capital accumulation changes as a result of the two tax reforms. Provide intuition in terms of insurance and output efficiency.

5. Check how the distribution of agents across consumption levels and asset levels changes due to the two reforms. You may want to use both graphical representation and some statistics such as the Gini coefficient or coefficient of variation.

6. Calculate the welfare effect of these tax reforms. Do it in two ways: (i) use the aggregate social welfare and compare it across the three cases (benchmark and two reforms); (ii) check also who benefits and who loses due to these reforms. You can use the consumption equivalent measures for these welfare comparisons.

7. Comment on what sense, these welfare comparisons across steady states are meaningful or misleading.

8. Find an interesting quantitative question which you can answer using this model and answer it.

Problem 2 Transitional dynamics

Now assume that at t=0 the tax schedule is in the steady state with consumption taxes (which you solved above. i.e., steady-state economy under Reform 1). At the beginning of period 1, the government makes a surprise announcement that it abolishes consumption taxes and switches entirely to taxing a capital income (the second steady state you solved above).

1. Define the recursive competitive equilibrium with transitions.

2. Compute the transition path of the economy using the algorithm provided in handout and the matlab code provided in tutorial. [try T=200]. Plot the transition paths of interest rate, wage, capital and welfare. Comment on the results you obtain.

3. a. Answer the following question: "How much do we need to change consumption of the agent in every state in the stationary equilibrium so that he'd be indifferent between living through the tax reform and living in the pre-reform economy?" Decompose welfare increase due to *increased consumption level* and due to *reduced uncertainty.*

   b. Now discuss which tax system is welfare improving taking into account the transitional dynamics (as opposed to steady state comparisons.). Also discuss which taxes are more distortionary.

4. What fraction of the overall population would support the reform? Compute and plot the measure of consumption equivalent variation.

5. Use your results to analyze which tax schedule is better in terms of efficiency and distribution and why.

6. Find an interesting quantitative question which you can answer using this model and answer it.