Computational Economics
University of California Irvine
Oliko Vardishvili

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Take Home Exam 2

(Send a zip of code & pdf files to me). Note that for the final grade, I will ask you questions during lecture from your own solutions.

Problem 1 Overlapping Generations Model

We will depart from the benchmark OLG model studied in the class by incorporating government sector into the model.

Each period a continuum of agents is born and live for $J = 12$ periods. Population grows at a constant rate $n_p = 1.01^5 - 1$[1]. Agents retire at $J_r = 9$ periods and die deterministically at age $J = 13$. Newly born agents hold no initial assets, $a_1 = 0$, and they have equal propability to draw high or low fixed effect as well as idiosyncratic productivity.

The instantaneous utility function of an agent is given by:

$$u(c,l) = \frac{\left(c^{\nu}(1-l)^{1-\nu}\right)^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}, \tag{1}$$

where $\gamma = 0.5$ and $\nu = 0.335$.

The government collects consumption, $\tau_c = 0.073$, capital, $\tau_r = 0.1$ and labor income, $\tau_w = 0.1$,[2] taxes and uses the revenue to finance government expenditure $G = 0.19Y$, where $Y = \Omega K^{\alpha} L^{1-\alpha}$ is the aggregate output, with $\alpha = 0.36$ and $\Omega = 1$. K and L denote respectively aggregate capital and effective labor in the economy. $\beta = 0.98, \delta = 0.3498$, survival probabilities equal to 1, except age 12 and 13, where $\psi_{12} = 0.96$ and $\psi_{13} = 0$.[3]

The pension system operates on a pay-as-you-go basis, i.e., it collects contributions from working age generations and directly redistributes them to current retirees: $\tau_p = \dfrac{\kappa w \bar{L} m}{wL}$, where m denotes the mass of the retired households and $\bar{L}$ is the average effective labor. In the pension system, the replacement rate $\kappa_t = 0.35$ is exogenously given while the contribution rate t_p adjusts in order to balance the budget.

Productivity Process:
Age deterministic profile: $\epsilon_j = 1.0000, 1.3527, 1.6952, 1.8279, 1.9606, 1.9692, 1.9392, 1.9007$ for age $j = 1, 2, ..., 8$.
Standard deviation of the fixed effect: $\sigma_\theta = 0.242$.
Idiosyncratic AR(1) process: $\eta_j = \rho\eta_{j-1} + \epsilon_j$ with $\sigma_\epsilon = 0.022$ $\rho = 0.5$. Use Tauchen to discretize this process.[4]

1. Define the recursive competitive equilibrium in this case. Derive the Euler equation taking into account elastic labor supply. Then, solve the new general equilibrium numerically, where the government balances its budget by labor income tax rate, τ_w. [Hint: You need to incorporate the

[1] Note that it is converted into a five year growth rate
[2] Note that in some experiments we use this rate to clear the government budget.
[3] Make sure that the aggregate goods market clears by accounting for the accidental bequests properly.
[4] For the speed, discretized the idiosyncratic productivity shocks only with 3 nodes.

government sector into the model, which collects labor income, capital income, and consumption taxes and pays government expenditure G and pensions to the retired households. Assume that the government collects accidental bequests in the economy.]

2. Report the following macroeconomic variables for this simulation: Aggregate consumption to GDP ratio, capital to GDP ratio, effective labor to GDP ratio, wage, interest rate, and welfare (in terms of consumption equivalence). Plot life cycle profile of consumption, assets, working hours, and earnings.) Calculate the variance of log labor earnings (age-specific and aggregate), and decompose it into variances of log hours and log efficiency units, h_j. Compare it to the data moments.[5]

3. Now do the following simulations one by one: (1) the government closes the budget with consumption taxes, setting other taxes to zero. (2) the government closes the budget with income taxes (taxes on labor income as well as capital income, common rate), setting other taxes to zero (3) the government eliminates pay as you go system by setting $\kappa = 0$, keeping everything else as in the benchmark simulation. (4) Decrease fertility $n_p = 0$. Report all macroeconomic variables as you did in the benchmark case. Discuss the intuition why we observe an increase/decrease in macroaggregates/welfare under each reform compared to the benchmark economy and to each other. Provide intuition in terms of insurance and output efficiency.

4. Check how the distribution of agents across consumption levels and asset levels changes due to the two reforms. (You may want to plot the life cycle profile of consumption and assets. You may also use Gini coefficients for labor earnings, consumption and wealth.) Comment under each reform which generations win and which generations lose. Explain the intuition.

5. Find an interesting quantitative question which you can answer using this model and answer it.

Table 1: You may use this structure to summarize part of the results across the experiments.

	n_p	κ	τ_w	τ_r	τ_c	C/Y	K/Y	L	w	r	Y	W	Gini earnings	Gini wealth	Gini consumption
(0)															
(1)															
(2)															
(3)															
(4)															

[5] of variances of age 25 (≈ 0.3), age 60 (≈ 0.9), and a linear increase between them.