



**Figure S1.**

Finite-horizon consistency check: optimal dephasing  $\kappa^*$  at fixed  $\gamma$  as the horizon  $T$  increases. The maximizing  $\kappa$  for  $P_{\text{success}}(T)$  converges toward the spectral-gap optimum for sufficiently large  $T$ , consistent with the Liouvillian-gap diagnostic.