# 18.100A Midterm

Octavio Vega

March 20, 2023

## Problem 1

### (a)

*Proof.* Let $x \in f^{-1}(C \cap D)$. Then

$$\implies f(x) \in C \cap D \tag{1}$$
$$\implies f(x) \in C \text{ and } f(x) \in D \tag{2}$$
$$\implies x \in f^{-1}(C) \text{ and } x \in f^{-1}(D) \tag{3}$$
$$\implies x \in f^{-1}(C) \cap f^{-1}(D). \tag{4}$$

Thus,

$$f^{-1}(C \cap D) \subseteq f^{-1}(C) \cap f^{-1}(D). \tag{5}$$

Now let $x \in f^{-1}(C) \cap f^{-1}(D)$. Then

$$\implies f(x) \in C \text{ and } f(x) \in D \tag{6}$$
$$\implies f(x) \in C \cap D \tag{7}$$
$$\implies x \in f^{-1}(C \cap D). \tag{8}$$

Thus,

$$f^{-1}(C) \cap f^{-1}(D) \subseteq f^{-1}(C \cap D). \tag{9}$$

Therefore by equations (5) and (9), $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$. □

### (b)

**Claim**: If $E \subset \mathbb{R}$ is countable, then the complement $\mathbb{R} \backslash E$ is always uncountable.

*Proof.* (By contradiction). Suppose $E^c$ is countable. Then $E \cup E^c$ is countable as well, since it is the union of two countable sets. But $E \cup E^c = \mathbb{R}$, which is uncountable. ($\Rightarrow\Leftarrow$). □