18.100A Assignment 7

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Problem 1

Proof. Since $\sum_n a_n$ and $\sum_n b_n$ converge absolutely, suppose that $\sum_n |a_n| < M$ and $\sum_n |b_n| < N$. Then

$$\begin{aligned}
\sum_{n=0}^{m} |c_n| &= \sum_{n=0}^{m} \left| \sum_{k=0}^{n} a_k b_{n-k} \right| && (1)\\
&\leq \sum_{n=0}^{m} \sum_{k=0}^{n} |a_k b_{n-k}| && (2)\\
&= |a_0 b_0| + (|a_0 b_1| + |a_1 b_0|) + \cdots + \\
&(|a_0 b_m| + |a_1 b_{m-1}| + \cdots + |a_m b_0|) && (3)\\
&= \sum_{n=0}^{m} |a_n| \sum_{k=0}^{m-n} |b_k| && (4)\\
&< MN. && (5)
\end{aligned}$$

Thus $\sum_n |c_n|$ is bounded above and monotone, so it converges. $\square$