18.100A Assignment 4

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Problem 1

(a)

Proof. Define the complement of $[a, b]$ via

$$[a, b]^c := \{x \in \mathbb{R} \mid x < a,\ x > b\}. \tag{1}$$

We can write this complement as the union of two sets:

$$\begin{aligned} [a, b]^c &= \{x \in \mathbb{R} \mid x < a\} \cup \{x \in \mathbb{R} \mid x > b\} & (2) \\ &= (-\infty, a) \cup (b, \infty). & (3) \end{aligned}$$

Both the sets $(-\infty, a)$ and (b, ∞) are open, as proved in assignment 3. We also proved that the union of open sets is open. Thus, $[a, b]^c$ is open.

Therefore, we conclude that $[a, b]$ is closed. □

(b)

Proof. Consider the complement of the integers in the real numbers, $\mathbb{Z}^c = \mathbb{R} \backslash \mathbb{Z}$. We may write this complement as a union of open sets, where each of the open sets represents the set of numbers between (but not including) consecutive integers:

$$\mathbb{Z}^c = \bigcup_{n \in \mathbb{Z}} (n, n+1). \tag{4}$$

Since the sets being unioned are all open, then so is the union, i.e. $\mathbb{Z}^c$ is open.

Thus, $\mathbb{Z}$ is closed. □