# 18.100A Assignment 6

Octavio Vega

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## Problem 1

### (a)

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{3}{9n+1} &= 3\sum_{n=1}^{\infty} \frac{1}{9n+1} &(1)\\ &= \frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{n+\frac{1}{9}} &(2)\\ &= \frac{1}{3}\sum_{n=2}^{\infty} \frac{1}{(n-1)+\frac{1}{9}} &(3)\\ &= \frac{1}{3}\sum_{n=2}^{\infty} \frac{1}{n-\frac{8}{9}} &(4)\\ &> \sum_{n=2}^{\infty} \frac{1}{n}. &(5)\end{aligned}$$

But the Harmonic series, $\sum_n \frac{1}{n}$, diverges.

Therefore, we conclude by comparison that the series $\sum_n \frac{3}{9n+1}$ diverges.

### (b)

$$\sum_{n=1}^{\infty} \frac{1}{2n-1} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n-\frac{1}{2}} > \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n}. \tag{6}$$

Therefore, by comparison, $\sum_n \frac{1}{2n-1}$ diverges.

### (c)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = \sum_{k=1}^{\infty} \frac{1}{(2k)^2} - \sum_{k=1}^{\infty} \frac{1}{(2k+1)^2}, \tag{7}$$

which is the difference of two convergent series.

Therefore, $\sum_n \frac{(-1)^n}{n^2}$ converges.