

# 18.100A Assignment 6

Octavio Vega

March 6, 2023

## Problem 1

(a)

$$\sum_{n=1}^{\infty} \frac{3}{9n+1} = 3 \sum_{n=1}^{\infty} \frac{1}{9n+1} \quad (1)$$

$$= \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n + \frac{1}{9}} \quad (2)$$

$$= \frac{1}{3} \sum_{n=2}^{\infty} \frac{1}{(n-1) + \frac{1}{9}} \quad (3)$$

$$= \frac{1}{3} \sum_{n=2}^{\infty} \frac{1}{n - \frac{8}{9}} \quad (4)$$

$$> \sum_{n=2}^{\infty} \frac{1}{n}. \quad (5)$$

But the Harmonic series,  $\sum_n \frac{1}{n}$ , diverges.

Therefore, we conclude by comparison that the series  $\sum_n \frac{3}{9n+1}$  diverges.

(b)

$$\sum_{n=1}^{\infty} \frac{1}{2n-1} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n - \frac{1}{2}} > \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}. \quad (6)$$

Therefore, by comparison,  $\sum_n \frac{1}{2n-1}$  diverges.

(c)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = \sum_{k=1}^{\infty} \frac{1}{(2k)^2} - \sum_{k=1}^{\infty} \frac{1}{(2k+1)^2}, \quad (7)$$

which is the difference of two convergent series.

Therefore,  $\sum_n \frac{(-1)^n}{n^2}$  converges.

(d)

We can express the series  $\sum_n \frac{1}{n(n+1)}$  as a telescoping sum:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \frac{1}{n} - \sum_{n=1}^{\infty} \frac{1}{n+1} \quad (8)$$

$$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots \quad (9)$$

$$= 1. \quad (10)$$

Therefore,  $\sum_n \frac{1}{n(n+1)}$  converges to 1.