18.100A Final

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Problem 1

We complete the following **negations**:

(i)

Let $S \subset \mathbb{R}$. A function $f : S \to \mathbb{R}$ is **not continuous** at $c \in S$ if $\exists \epsilon_0 > 0$ such that $\forall \delta > 0$ if $|x - c| < \delta$, $|f(x) - f(c)| \geq \epsilon_0$.

(ii)

Let $S \subset \mathbb{R}$. A function $f : S \to \mathbb{R}$ is **not uniformly continuous** on S if $\exists x_0 \in S$ such that $\forall \delta > 0 \ \exists \epsilon_0 > 0$ such that if $|x_0 - x| < \delta$, then $|f(x) - f(x_0)| \geq \epsilon_0$.

(iii)

Let $S \subset \mathbb{R}$. A sequence of functions $f_n : S \to \mathbb{R}$ **does not converge uniformly** to $f : S \to \mathbb{R}$ if $\exists \epsilon_0 > 0$ such that $\forall M \in \mathbb{N} \ \exists n \geq M$ and $x \in S$ such that $|f_n(x) - f(x)| \geq \epsilon_0$.

Problem 2