

18.100A Assignment 5

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Problem 1

Proof. We have that

$$L = \lim_{n \rightarrow \infty} \frac{|x_{n+1} - x|}{|x_n - x|} < 1. \quad (1)$$

Then for every $\epsilon > 0$, $\exists N \in \mathbb{N}$ such that $\forall n \geq N$,

$$\left| \frac{x_{n+1} - x}{x_n - x} \right| - 1 < \epsilon. \quad (2)$$

Rearranging gives

$$|x_{n+1} - x| < (1 + \epsilon)|x_n - x|. \quad (3)$$

By taking ϵ to be arbitrarily small, we have, $\forall n \geq N$,

$$\xrightarrow{\epsilon \rightarrow 0} |x_{n+1} - x| < |x_n - x|. \quad (4)$$

Define $y_n := |x_n - x|$. Then $\forall n \geq N$,

$$0 \leq y_{n+1} < y_n, \quad (5)$$

so $\{y_n\}_n$ is a decreasing sequence bounded below by 0. Hence,

$$\implies y_n \rightarrow 0 \quad (6)$$

$$\implies |x_n - x| \rightarrow 0 \quad (7)$$

$$\implies x_n \rightarrow x. \quad (8)$$

Therefore, $\{x_n\}_n$ converges to x . \square