

18.100A Assignment 6

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Problem 1

(a)

$$\sum_{n=1}^{\infty} \frac{3}{9n+1} = 3 \sum_{n=1}^{\infty} \frac{1}{9n+1} \quad (1)$$

$$= \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n + \frac{1}{9}} \quad (2)$$

$$= \frac{1}{3} \sum_{n=2}^{\infty} \frac{1}{(n-1) + \frac{1}{9}} \quad (3)$$

$$= \frac{1}{3} \sum_{n=2}^{\infty} \frac{1}{n - \frac{8}{9}} \quad (4)$$

$$> \sum_{n=2}^{\infty} \frac{1}{n}. \quad (5)$$

But the Harmonic series, $\sum_n \frac{1}{n}$, diverges.

Therefore, we conclude by comparison that the series $\sum_n \frac{3}{9n+1}$ diverges.

(b)

$$\sum_{n=1}^{\infty} \frac{1}{2n-1} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n - \frac{1}{2}} > \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}. \quad (6)$$

Therefore, by comparison, $\sum_n \frac{1}{2n-1}$ diverges.