18.100A Assignment 9

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Problem 1

Proof. We have that $\forall x \in \mathbb{R}$, $|\arctan(x)| < \frac{\pi}{2}$, i.e.

$$\arctan(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \tag{1}$$

which is an open set. So $\forall |y| < \frac{\pi}{2}$, $\exists \epsilon > 0$ such that $(y - \epsilon, y + \epsilon) \subset (-\frac{\pi}{2}, \frac{\pi}{2})$. Thus for every such y, we can always find a $y_0 < y$ and $y_1 > y$ inside this open set. This means that there is no x_1 such that $\arctan(x_1) \geq \arctan(x)$ nor an x_0 such that $\arctan(x_0) \leq \arctan(x)$ $\forall x$.

Hence $f(x) = \arctan(x)$ does not achieve an absolute minimum or maximum. □