

# 18.100A Midterm

Octavio Vega

March 20, 2023

## Problem 1

(a)

*Proof.* Let  $x \in f^{-1}(C \cap D)$ . Then

$$\implies f(x) \in C \cap D \quad (1)$$

$$\implies f(x) \in C \text{ and } f(x) \in D \quad (2)$$

$$\implies x \in f^{-1}(C) \text{ and } x \in f^{-1}(D) \quad (3)$$

$$\implies x \in f^{-1}(C) \cap f^{-1}(D). \quad (4)$$

Thus,

$$f^{-1}(C \cap D) \subseteq f^{-1}(C) \cap f^{-1}(D). \quad (5)$$

Now let  $x \in f^{-1}(C) \cap f^{-1}(D)$ . Then

$$\implies f(x) \in C \text{ and } f(x) \in D \quad (6)$$

$$\implies f(x) \in C \cap D \quad (7)$$

$$\implies x \in f^{-1}(C \cap D). \quad (8)$$

Thus,

$$f^{-1}(C) \cap f^{-1}(D) \subseteq f^{-1}(C \cap D). \quad (9)$$

Therefore by equations (5) and (9),  $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$ .  $\square$