

# 18.100A Assignment 3

Octavio Vega

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## Problem 1

*Proof.* Let  $x, y \in \mathbb{R}$ . By the density of  $\mathbb{Q}$ , we have that  $\exists r \in \mathbb{Q}$  such that  $x < r < y$ .

Then  $x + \sqrt{2} < y + \sqrt{2}$ . Then  $\exists r \in \mathbb{Q}$  such that

$$x + \sqrt{2} < r < y + \sqrt{2} \tag{1}$$

$$\implies x < r - \sqrt{2} < y. \tag{2}$$

But since  $r \in \mathbb{Q}$  and  $\sqrt{2} \notin \mathbb{Q}$ , then the number  $i := r - \sqrt{2} \notin \mathbb{Q}$ .

So  $x < i < y$  with  $i \in \mathbb{R} \setminus \mathbb{Q}$ , as desired. □