18.100A Assignment 6

Octavio Vega

March 6, 2023

Problem 1

(a)

$$\sum_{n=1}^{\infty} \frac{3}{9n+1} = 3\sum_{n=1}^{\infty} \frac{1}{9n+1} \tag{1}$$

$$= \frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{n+\frac{1}{9}} \tag{2}$$

$$= \frac{1}{3}\sum_{n=2}^{\infty} \frac{1}{(n-1)+\frac{1}{9}} \tag{3}$$

$$= \frac{1}{3}\sum_{n=2}^{\infty} \frac{1}{n-\frac{8}{9}} \tag{4}$$

$$> \sum_{n=2}^{\infty} \frac{1}{n}. \tag{5}$$

But the Harmonic series, $\sum_n \frac{1}{n}$, diverges.

Therefore, we conclude by comparison that the series $\sum_n \frac{3}{9n+1}$ diverges.