18.100A Assignment 4

Octavio Vega

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Problem 1

(a)

Proof. Define the complement of $[a, b]$ via

$$[a, b]^c := \{x \in \mathbb{R} \mid x < a, \ x > b\}. \tag{1}$$

We can write this complement as the union of two sets:

$$[a, b]^c = \{x \in \mathbb{R} \mid x < a\} \cup \{x \in \mathbb{R} \mid x > b\} \tag{2}$$
$$= (-\infty, a) \cup (b, \infty). \tag{3}$$

Both the sets $(-\infty, a)$ and (b, ∞) are open, as proved in assignment 3. We also proved that the union of open sets is open. Thus, $[a, b]^c$ is open.

Therefore, we conclude that $[a, b]$ is closed. $\square$