

18.100A Assignment 1

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Problem 1

(a)

Proof. We will show that each set is a subset of the other to prove equality. Let $S = A \cap (B \cup C)$ and $T = (A \cup B) \cap (A \cup C)$.

Let $x \in S$. Then $x \in A$ and $x \in B \cup C$,

$$\implies x \in A \text{ and } x \in B, \text{ or } x \in A \text{ and } x \in C$$

$$\implies x \in A \cap B \text{ or } x \in A \cap C$$

$$\implies x \in (A \cap B) \cup (A \cap C) = T.$$

Thus $x \in S \implies x \in T$, so $S \subseteq T$. Now let $x \in T$. Then $x \in (A \cap B) \cup (A \cap C)$,

$$\implies x \in A \cap B \text{ or } x \in A \cap C$$

$$\implies x \in A, \text{ and } x \in B \text{ or } C$$

$$\implies x \in A \cap (B \cup C) = S.$$

Thus $x \in T \implies x \in S$, so $T \subseteq S$, which means $S = T$.

Hence, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

□

(b)

Proof. We proceed as in (a). Let $S = A \cup (B \cap C)$ and $T = (A \cup B) \cap (A \cup C)$.

First let $x \in S$. Then $x \in A$ or $x \in B \cap C$,

$$\implies x \in A \text{ or } x \in B, \text{ and } x \in A \text{ or } x \in C$$

$$\implies x \in (A \cup B) \cap (A \cup C) = T.$$

Thus $x \in S \implies x \in T$, so $S \subseteq T$. Now let $x \in T$. Then $x \in A \cup B$ and $x \in A \cup C$. If $x \in A$, then the requirement is satisfied immediately, regardless

of whether x is in B or C . Otherwise, if $x \notin A$, then $x \in B$ and $x \in C$ must be true. So $x \in A$, or $x \in B$ and $x \in C$

$$\implies x \in A \cup (B \cap C) = S.$$

Thus $x \in T \implies x \in S$, so $T \subseteq S$, which means $S = T$.

Hence, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. □

Problem 2

Proof. (By induction).

The inductive hypothesis $P(n)$ is that, for $n \in \mathbb{N}$, $n < 2^n$.

(Base case): $1 < 2^1$, so $P(1)$ is true.

(Inductive step): Assume $P(n)$ is true for $n = m$, i.e. that $m < 2^m$ holds for $m \in \mathbb{N}$. Then

$$2^{m+1} = 2 \cdot 2^m > 2m = m + m > m + 1, \quad \text{since } m > 1 \implies 2^{m+1} > m + 1. \quad (1)$$

So $P(m) \implies P(m + 1)$, so $P(n)$ is true for all $n \in \mathbb{N}$.

Thus, $2^n > n \forall n \in \mathbb{N}$. □