

18.100A Assignment 12

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Problem 1

(a)

Proof. Suppose $\exists c \in [a, b]$ such that $f(c) > 0$. Since f is continuous, $\exists \delta > 0$ such that if $|x - c| < \delta$, then $|f(x) - f(c)| < \frac{f(c)}{2}$, i.e. $\frac{f(c)}{2} < f(x)$. We compute

$$0 > \int_a^b f \quad (1)$$

$$> \int_a^b \frac{f(c)}{2} \quad (2)$$

$$= \frac{f(c)}{2}(b - a) \quad (3)$$

$$> 0, \quad (4)$$

i.e. $0 > 0$ ($\Rightarrow \Leftarrow$), which is clearly a contradiction.

Therefore $f(x) = 0 \forall x \in [a, b]$. \square

(b)

Proof. Let $E = \int_a^b (u')^2 dx$. Then $E \geq 0$ since $(u')^2 \geq 0$. Using integration by parts, we have

$$E = \int_a^b u' u' dx \quad (5)$$

$$= uu'|_a^b - \int_a^b uu'' dx \quad (6)$$

$$= u'(b)u(b) - u'(a)u(a) - \int_a^b u(Vu) dx \quad (7)$$

$$= - \int_a^b Vu^2 dx. \quad (8)$$

But $V(x) \geq 0$ and $u^2 \geq 0$, so $-(Vu^2) \leq 0$, hence $E \leq 0$. Thus $E = 0$, which must mean that

$$\int_a^b (u')^2 dx = 0, \tag{9}$$

and by part **(a)**, this implies that $(u')^2 = 0 \ \forall x \in [a, b]$. Hence $u'(x) = 0$ for all x , and since $u(a) = 0$, then u remains constant at 0; i.e. $u = 0$ everywhere. \square