18.100A Assignment 9

Octavio Vega

May 22, 2023

Problem 1

Proof. We have that $\forall x \in \mathbb{R}$, $|\arctan(x)| < \frac{\pi}{2}$, i.e.

$$\arctan(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \tag{1}$$

which is an open set. So $\forall |y| < \frac{\pi}{2}$, $\exists \epsilon > 0$ such that $(y-\epsilon, y+\epsilon) \subset (-\frac{\pi}{2}, \frac{\pi}{2})$. Thus for every such y, we can always find a $y_0 < y$ and $y_1 > y$ inside this open set. This means that there is no x_1 such that $\arctan(x_1) \geq \arctan(x)$ nor an x_0 such that $\arctan(x_0) \leq \arctan(x)$ $\forall x$.

Hence $f(x) = \arctan(x)$ does not achieve an absolute minimum or maximum. $\square$

Problem 2

Proof. Let $x, y \in (c, \infty)$. Choose $L = \frac{1}{c^2}$. Then

$$\begin{align} |f(y) - f(x)| &= \left|\frac{1}{y} - \frac{1}{x}\right| \tag{2} \\ &= \frac{|x-y|}{xy} \tag{3} \\ &< \frac{|x-y|}{c^2} \tag{4} \\ &= L|x-y|. \tag{5} \end{align}$$

Therefore $f(x) = \frac{1}{x}$ is Lipschitz continuous. $\square$