1. Solve the following inequality:

$$(x+2)(x^2-3x+2) \geq 0$$

2. Solve the rational inequality:

$$\frac{x+6}{x-7} \leq 1$$

3. Solve the equation:

$$\left|x^2 - 1\right| = 1$$

4. Solve the inequality:

$$2|3x - 4| - 5 > 7$$

Solutions

1.

$$(x+2)(x^2-3x+2) \geq 0$$

First, factor x^2-3x+2 into $(x-2)(x-1)$. So the original problem becomes

$$(x+2)(x-2)(x-1) \geq 0$$

The boundary points will be the zeros of this polynomial, which we can immediately see are -2, 1, and 2. So we must partition the number line using these points, and test a point in each resultant interval.

First let's pick a point in $(-\infty, -2)$. I'll use -3.

$$(-3+2)(-3-2)(-3-1) = (-1)(-5)(-4) = -20 < 0$$

Next, let's try a point in $(-2, 1)$. I'll use 0.

$$(0+2)(0-2)(0-1) = (2)(-2)(-1) = 4 > 0$$

Next, let's try a point in $(1, 2)$. I'll use $\frac{3}{2}$.

$$\left(\frac{3}{2}+2\right)\left(\frac{3}{2}-2\right)\left(\frac{3}{2}-1\right) = \left(\frac{7}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = -\frac{7}{8} < 0$$

Finally, let's try a point in $(2, \infty)$. I'll use 3.

$$(3+2)(3-2)(3-1) = (5)(1)(2) = 10 > 0$$

So we see that the intervals on which $(x+2)(x-2)(x-1) > 0$ are $(-2, 1)$ and $(2, \infty)$. So the final solution will at least contain the set $(-2, 1) \cup (2, \infty)$. The only question that remains is whether we want the endpoints. Since we want points where $(x+2)(x-2)(x-1)$ is greater than *or equal to* zero, we also want to include the points at which this expression is *exactly* zero, which are the boundary points. So we want to include all the endpoints.

Solution set: $[-2, 1] \cup [2, \infty)$

2.

$$\frac{x+6}{x-7} \leq 1$$

First we want to get the right-hand side to be zero. So we subtract 1 from both sides:

$$\begin{aligned} \frac{x+6}{x-7} &\leq 1 \\ \frac{x+6}{x-7} - 1 &\leq 0 \\ \frac{x+6}{x-7} - \frac{x-7}{x-7} &\leq 0 \\ \frac{x+6-(x-7)}{x-7} &\leq 0 \\ \frac{x+6-x+7}{x-7} &\leq 0 \\ \frac{13}{x-7} &\leq 0 \qquad (*) \end{aligned}$$

Now it is in a form that we can work with. We get our boundary points by looking at the zeros of the numerator and denominator. There are no zeros of the numerator, since it is just a constant. The only zero of the denominator is 7. So we have only two intervals to test out: $(-\infty, 7)$ and $(7, \infty)$. If we test a point less than 7, we get a negative number, and if we test a point greater than 7, we get a positive number. Since we are testing our points in $(*)$, we want points that give us a negative number. So the solution contains at least the set $(-\infty, 7)$. The last question to ask is whether we want to include 7. But since we got 7 from the denominator, we do not want to include it.

Solution set: $(-\infty, 7)$

3.

$$\left|x^2 - 1\right| = 1$$

Since $|x^2 - 1| = 1$ we know that one of two things can be true: either $x^2 - 1 = 1$ or $x^2 - 1 = -1$. So we just need to solve these two equations to get all possible solutions.

First let's solve $x^2 - 1 = 1$:

$$\begin{aligned} x^2 - 1 &= 1 \\ x^2 &= 2 \\ x &= \pm\sqrt{2} \end{aligned}$$

Now let's solve the second equation:

$$\begin{aligned} x^2 - 1 &= -1 \\ x^2 &= 0 \\ x &= 0 \end{aligned}$$

The solution set of the original equation is then just the union of the solution sets of these two equations we've just solved.

Solution set: $\{0, \pm\sqrt{2}\} = \{0, -\sqrt{2}, \sqrt{2}\}$

4.

$$2|3x - 4| - 5 > 7$$

First let's isolate the absolute value expression:

$$\begin{aligned} 2|3x - 4| - 5 &> 7 \\ 2|3x - 4| &> 12 \\ |3x - 4| &> 6 \end{aligned}$$

This last statement ($|3x - 4| > 6$) is true when one of two things happens: if $3x - 4 > 6$ *or* if $3x - 4 < -6$. So we must solve these two inequalities. Let's solve the first one:

$$\begin{aligned} 3x - 4 &> 6 \\ 3x &> 10 \\ x &> \frac{10}{3} \end{aligned}$$

Now let's solve the second one:

$$\begin{aligned} 3x - 4 &< -6 \\ 3x &< -2 \\ x &< -\frac{2}{3} \end{aligned}$$

The solution set to the original problem is then the union of the solution sets of these two inequalities.

Solution set: $(-\infty, -\frac{2}{3}) \cup (\frac{10}{3}, \infty)$