	MTH 103	Quiz 2 Solutions
Name:	Section 068	9/19/13

1. (4 points) Compute the discriminant of the quadratic polynomial in the following equation and use that information to determine how many **real number** solutions the equation has (you do not need to actually solve the equation).

$$2x^2 + 4x + 2 = 0$$

Discriminant: $\underline{b^2 - 4ac = 4^2 - 4(2)(2) = 16 - 16 = 0}$

Number of real solutions: $\underline{1}$

2. (4 points) I have a TV whose width is one inch less than twice its height. The area of the screen is 465 square inches. What are the dimensions of the TV?

Let w be the width of the TV, and h be the height. Then the information from the story problem gives us that $w = 2h - 1$. Since the area of a rectangle is given by $A = hw$, we obtain the equation

$$465 = h(2h - 1) = 2h^2 - h$$

We can subtract 465 to get this in standard form:

$$0 = 2h^2 - h - 465$$

Now we just plug everything into the quadratic formula to get

$$\begin{aligned} h &= \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-465)}}{2(2)} \\ &= \frac{1 \pm \sqrt{3721}}{4} \\ &= \frac{1 \pm 61}{4} \end{aligned}$$

Obviously one of these two solutions is negative, and we don't want that one. So we take the positive one:

$$h = \frac{1 + 61}{4} = \frac{62}{4} = 15.5$$

We said earlier on that $w = 2h - 1$, so $w = 2(15.5) - 1 = 30$.

Width: <u>30”</u> **Height:** <u>15.5”</u>

3. (3 points) Write the reciprocal of the complex number $3+5i$ in **standard form** ($a+bi$ for real numbers a and b).

$$\begin{aligned}\frac{1}{3+5i} &= \left(\frac{1}{3+5i}\right)\left(\frac{3-5i}{3-5i}\right)\\ &= \frac{3-5i}{3(3)+15i-15i-5i(5i)}\\ &= \frac{3-5i}{9-25i^2}\\ &= \frac{3-5i}{9+25}\\ &= \frac{3-5i}{34}\end{aligned}$$

$\frac{1}{3+5i} = \underline{\frac{3-5i}{34}}$

4. (4 points) Find **all** solutions (real or complex) to the following quadratic equation, and write your solution(s) in **set notation**.

$$x^2 - 4x + 13 = 0$$

Just use the quadratic formula:

$$\begin{aligned}x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}\\ &= \frac{4 \pm \sqrt{16-52}}{2}\\ &= \frac{4 \pm \sqrt{-36}}{2}\\ &= \frac{4 \pm 6i}{2}\\ &= 2 \pm 3i\end{aligned}$$

Solution: $\underline{\{2-3i, 2+3i\}}$

5. (5 points) A 5 foot long ladder is leaning against a wall. Let h be the height at which the ladder touches the wall, and w be the distance from the wall to where the ladder touches the ground. If h is one foot more than w, then what are h and w? (Hint: use the Pythagorean Theorem. If a and b are the lenghts of the legs of a right triangle, and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$).

The hint was to use the Pythagorean Theorem, with w and h being the lengths of the legs of the right triangle, and 5 is the length of the hypotenuse. So we get

$$w^2 + h^2 = 5^2 = 25$$

The problem said that $h = w + 1$, so we can substitute this into the above equation to reduce it to just having one variable:

$$\begin{aligned} w^2 + (w+1)^2 &= 25 \\ w^2 + (w^2 + 2w + 1) &= 25 \\ 2w^2 + 2w + 1 &= 25 \\ 2w^2 + 2w - 24 &= 0 \\ 2\left(w^2 + w - 12\right) &= 0 \\ w^2 + w - 12 &= 0 \\ (w+4)(w-3) &= 0 \end{aligned}$$

So the two solutions to this quadratic equation are $w = 3$ and $w = -4$. But negative lengths don't make sense, so our only sensical answer is $w = 3$. Since $h = w + 1$, we have $h = 4$.

$h = \underline{4'}$ $w = \underline{3'}$