The Quadratic Formula

Start with the general quadratic equation:

$$ax^2 + bx + c = 0$$

We can complete the square to derive the quadratic formula:

$$\begin{aligned}
x^2 + \frac{b}{a} \cdot x + \frac{c}{a} &= 0 \\
x^2 + \frac{b}{a} \cdot x &= -\frac{c}{a} \\
x^2 + \frac{b}{a} \cdot x + \left(\frac{b}{2a}\right)^2 &= \left(\frac{b}{2a}\right)^2 - \frac{c}{a} \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2}{4a^2} - \frac{c}{a} \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2}{4a^2} - \frac{4ac}{4a^2} \\
x + \frac{b}{2a} &= \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \\
x + \frac{b}{2a} &= \pm\frac{\sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$