ETE 305

Pulse Code Modulation

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Pulse code modulation

Definition: Pulse Code Modulation(PCM) is essentially analog to digital conversion of a special type where the information contained in the instantaneous samples of an analog signal is represented by digital word in a serial bit streams.

Advantages:

- Relatively expensive digital circuitry are used extensively.
- PCM signals from analog sources may be merged with data signals and transmitted over a high speed digital communication system. This merging is called Time Division Multiplexing.
- In long distance digital telephone system, a clean PCM waveform can be regenerated at the output of each repeater.
- The probability of error for the system output can be reduced by using appropriate coding techniques.

Disadvantages: A much wider bandwidth than that of the corresponding analog signal

PCM (2)

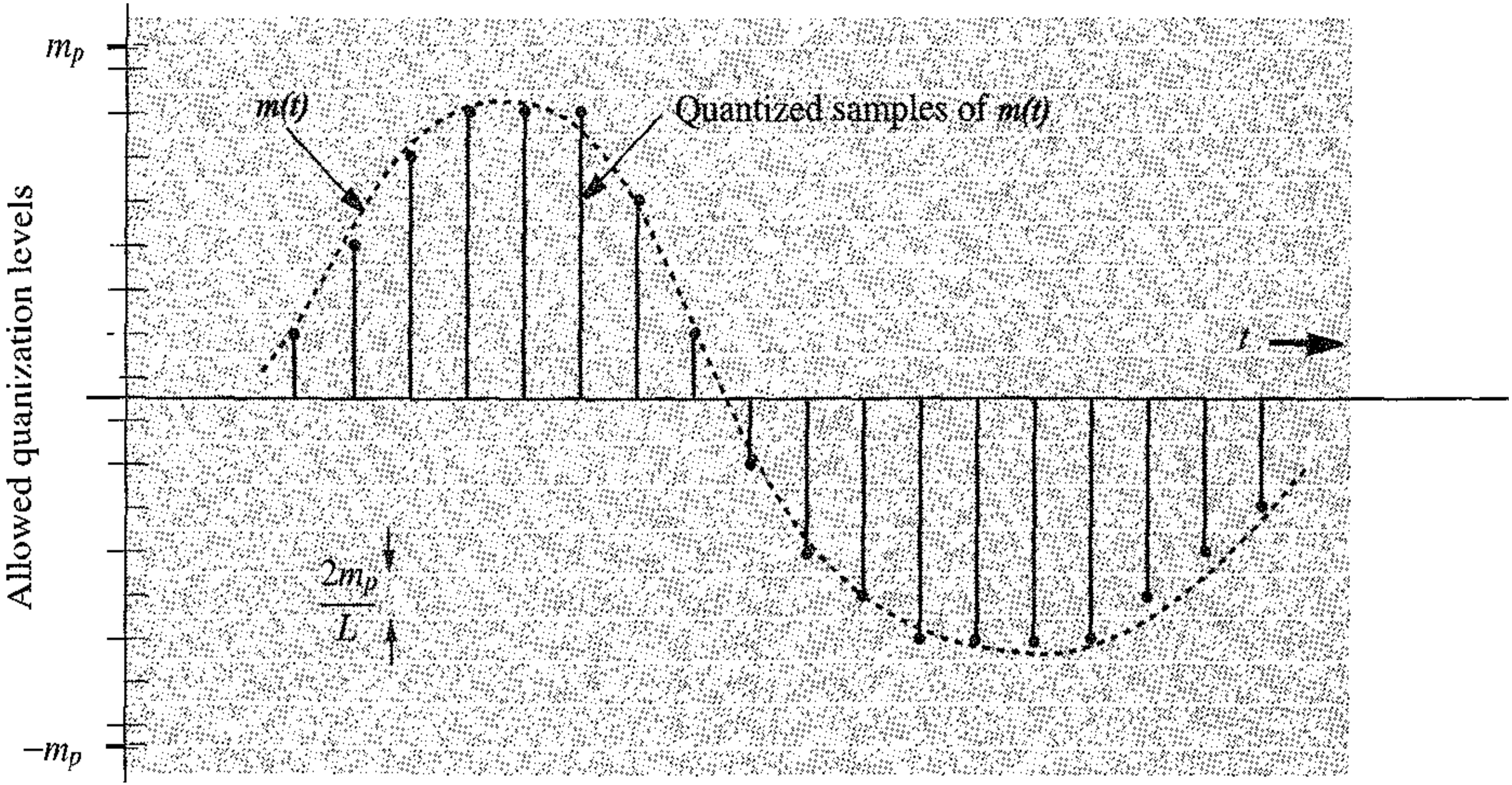


Figure 6.10 Quantization of a sampled analog signal.

Pulse code modulation (3)

- The PCM signal is generated by carrying out 3 basic operations:

  1. Sampling
  2. Quantizing
  3. Encoding.

- Each sample value from the analog signal can be any one of an infinite number of levels. So, instead of using exact sample value W(kTs,) the sample is replaced by the closest allowed value. Therefore error is introduced into the recovered output analog signal called quantizing noise (round-off) error.

- Then for encoding binary code, Gray code or any other code may be used, but Gray code was chosen because it has only one bit change in the quantized level. So, will caused minimal error.

Pulse code modulation (4)

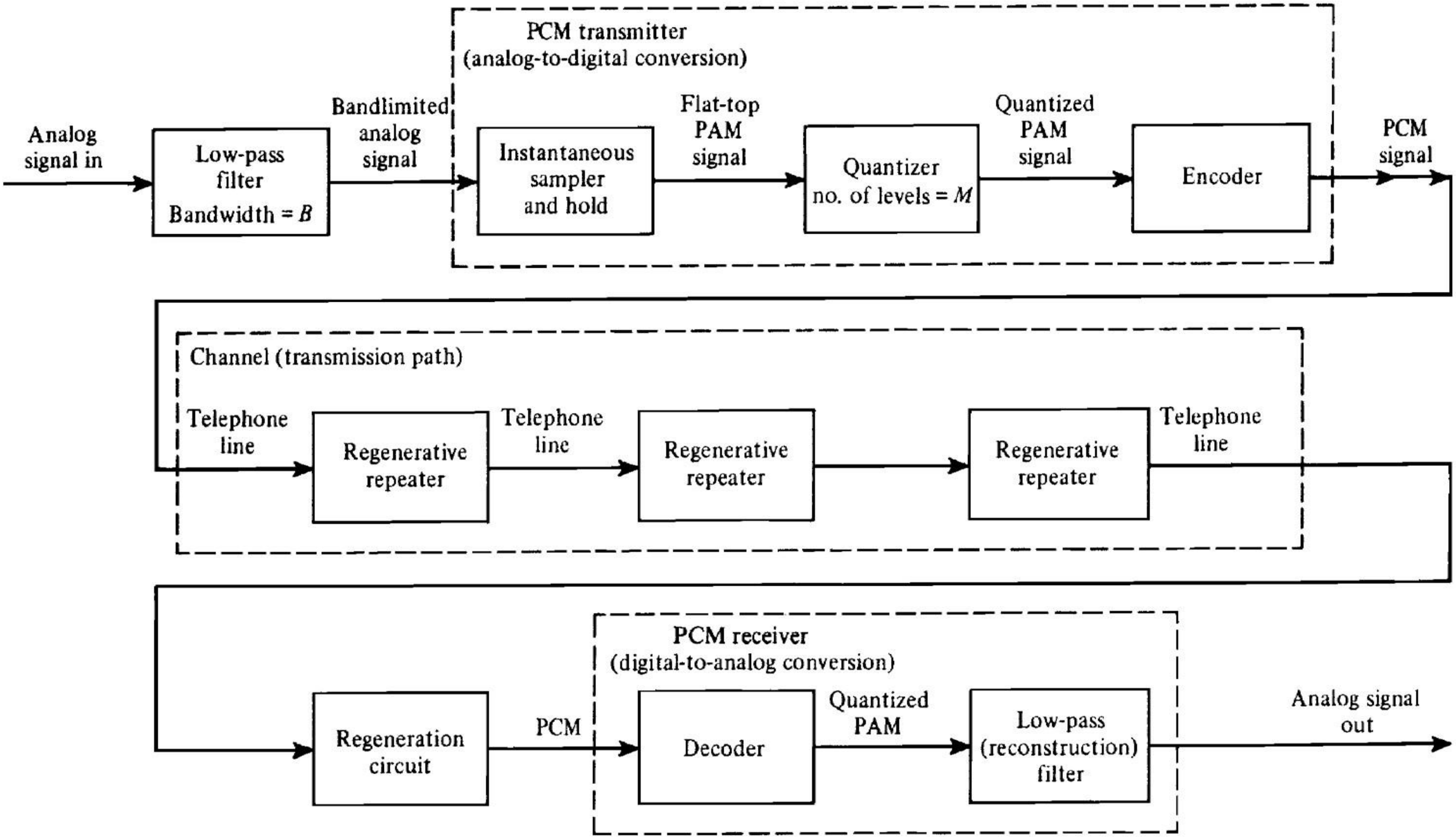


Figure 3-7 PCM trasmission system.

Pulse code modulation (5)

Practical PCM circuits:

- ❑ **ADC:**
  - ❖ Counting or Ramp
  - ❖ Serial or successive appx
  - ❖ Parallel or flash encoders.

- ➢ ICL 7126 CMOS ADC (Counting)
- ➢ ADC 0804 8 bit (Successive)
- ➢ CA 3318 8 bit (Parallel)

✓Serial ↔ Parallel (If necessary) is done by SIO (Serial Input Output Chip) –

- ❖ UART
- ❖ USRT
- ❖ USART

✓At the receiving end the PCM signal is decoded back into an analog signal by using DAC chip.

- ❖ DAC 0808 (National Semiconductor)
- ❖ MC 145503 (Motorola)
- ❖ TCM 300 AC54 (Codec)

  Texas instrument

Pulse code modulation (6)

Bandwidth of PCM :

Audio signal bandwidth = 15 kHz

But above 300 Hz are suppressed.

Then ,Sampled at a rate = 8 kHz. ($3.4\times2=6.8$ kHz Nyquist rate.)

Each Sample is finally quantized into 256 levels (L=256) which requires a group of 8 binary pulses ($2^8=256$).

Thus a telephone signal requires(8×8000)=64000 binary pulses /s.

In case of Compact Disc (CD)

Bandwidth = 15 kHz

Sampling rate = 44.1 kHz.

levels L=65536 to reduce quantizing error.

$B_{PCM} = R = nfs$ [First null bandwidth]

n = Number of bit in PCM word.

The quantizer is said to uniform because all of the steps are of equal size.

Pulse code modulation (7)

Effect of noise:

The analog signal that is recovered at the PCM system is corrupted by noise. Two main effects produce this noise or distortion –

1) Quantizing noise that is caused by the M step quantizer

2) Bit errors in the recovered PCM signal. This bit errors are caused by channel noise.

Pulse code modulation (8)

Quantization noise:

Mean square of the quantization error in PCM:

m(t) = Amplitude of message signal.

$-m_p$ to m_p = Range of quantizer which is spaced into L equal intervals.

The Amplitude of m(t) beyond $\pm\, m_p$ are chopped off.

The quantum levels are separated by $\Delta v = \frac{2m_p}{L}$

Since a sample value is approximated by the midpoint $\hat{m}$ of the subinterval of height Δv.

Maximum quantization error ,

$$q = m - \hat{m}$$
$$= \pm \frac{\Delta v}{2}$$

Pulse code modulation (9)

Assuming that the error is equally likely to lie anywhere in the range

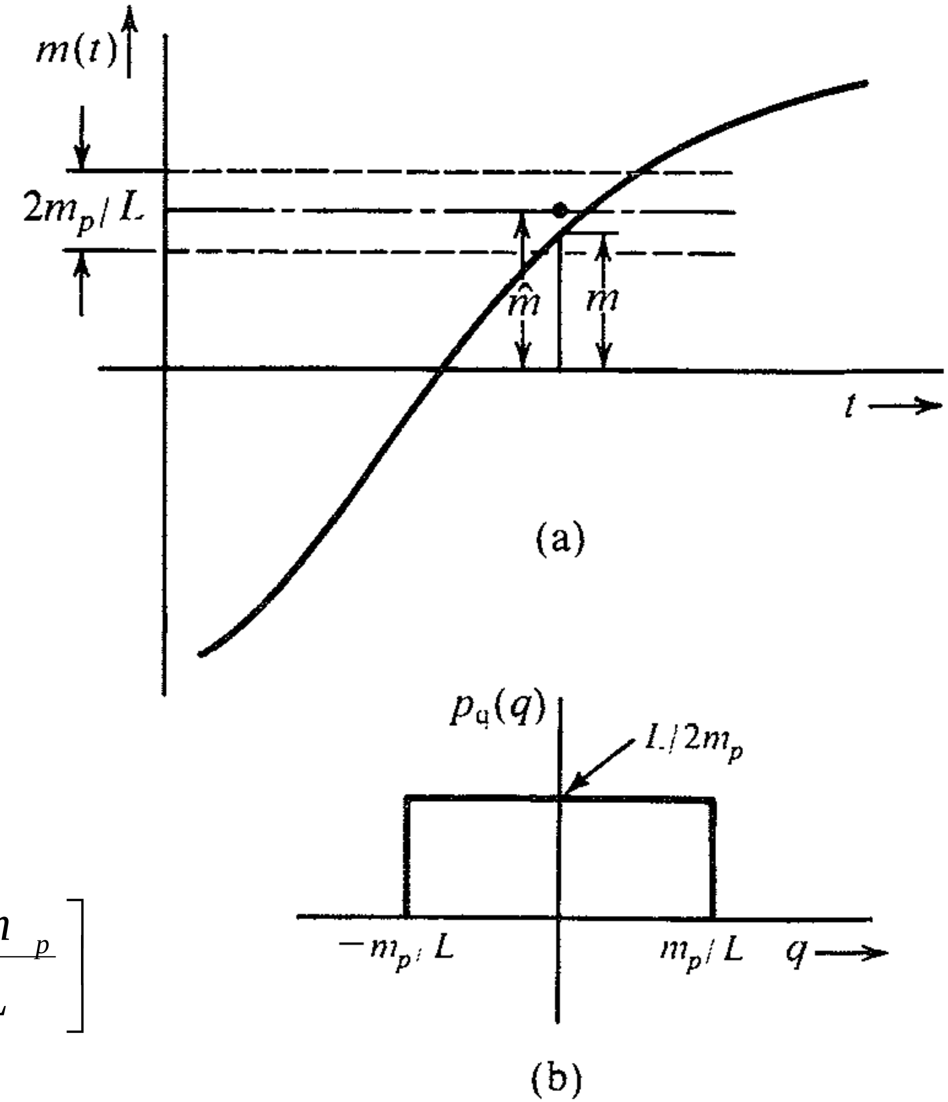
$$\left(-\frac{\Delta v}{2}, \frac{\Delta v}{2}\right)$$

the main square quantizing error,

$$\tilde{q}^2 = \frac{1}{\Delta v}\int_{-\frac{\Delta v}{2}}^{\frac{\Delta v}{2}} q^2\, dq$$

$$= \frac{1}{\Delta v}\left.\frac{q^3}{3}\right|_{-\frac{\Delta v}{2}}^{\frac{\Delta v}{2}}$$

$$= \frac{\Delta v^2}{12} = \frac{m_p^2}{3L^2} \quad \left[\because \Delta v = \frac{2m_p}{L}\right]$$



Qunatization error in PCM & its PDF

Pulse code modulation (10)

For sampled pulse train, (if sampled by 2B Hz) –

Quantization noise, $\boxed{N_q = \widetilde{q^2(t)} = \dfrac{m_p^2}{3L^2}}$

Same, since , $\tilde{q}^2(t)$= Average of total number of pulse

Assuming that the pulse detection error at the receiver is negligible, the reconstructed signal $\hat{m}(t)$ at the receiver output is

$$\hat{m}(t) = m(t) + q(t)$$

PCM (11)

SNR: The power of the message signal m(t) is $\widetilde{m^2}(t)$

Then
$$\boxed{\begin{aligned} S_o &= \widetilde{m^2(t)} \\ N_o = N_q &= \frac{m_p^2}{3L^2} \end{aligned}}$$

So,
$$\boxed{\frac{S_o}{N_o} = 3L^2 \frac{\widetilde{m^2(t)}}{m_p^2}}$$

This means SNR is a linear function of $\widetilde{m^2}(t)$

which varies –

- From talker to talker.
- Depends on the length of the circuit.
- Even for the same talker, when speaks softly.

In these situation (in fact most of the time) the signal will deteriorate markedly.

The root of this difficulty lies in the fact that the quantizing steps are uniform. Thus the problem can be solved by using smaller steps for smaller amplitudes (nonuniform quantizing).

PCM (12)

Transmissionbandwidth: For a binary PCM, a distinct group of n binary digits (bits) are assigned to each of the L quantization levels.

$$L = 2^n \qquad \text{or} \qquad n = \log_2 L$$

Because a signal m(t) band limited to B Hz requires a minimum of 2B samples/sec, we require a total of 2nB bits/sec (bps) , for PCM.

Since a unit bandwidth (1 Hz) can transmit a maximum of 2 pieces of information per second.

We require a minimum channel bandwidth ,

$$B_T = nB \text{ Hz}$$

PCM (13)

Example 6.2:

m(t), B= 3kHz, $f_s = 33\frac{1}{3}\%$ higher than Nyquistrate

Max acceptable quantization error = 0.5% of m_p

a) Min bandwidth of channel = ?

b) If 24 signals are multiplexed by TDM, Min transmission bandwidth = ?

Soln: B= 3kHz

Nyquist, $R_N = 2\times3000 = 6000$ Hz.

$$Actual\ ,\ R_A = 6000\times\left(1\frac{1}{3}\right) = 8000\,Hz$$

Max quantization error = $\pm\dfrac{\Delta v}{2}$

$$\frac{\Delta v}{2} = \frac{m_p}{L} = \frac{0.5}{100}m_p$$

$$\Longrightarrow L = 200$$

For binary coding, L must be a power of 2. Hence, the next higher value of L that is a power of 2 is $L = 256$.

$n = \log_2 256 = 8$ bits per sample.

total of $C = 8 \times 8000 = 64,000$ bit/s.

Because we can transmit up to 2 bit/s per hertz of bandwidth,

a) Min transmission bandwidth,

$$\boxed{B_T = C/2 = 32 \text{ kHz.}}$$

b) If multiplexed,

$$C_M = 24 \times 64,000 = 1.536 \text{ Mbit/s}$$

$$\boxed{\therefore B_T = \frac{1.536}{2} = 0.768\,MHz}$$

PCM (14)

Exponential increase of the output SNR:

$$\frac{S_o}{N_o} = 3L^2 \frac{\tilde{m}^2(t)}{m_p^2}[Uniform]$$

$$= \frac{3L^2}{[\ln(1+\mu)]^2}[Nonuniform / Compressed]$$

See,
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Putting $L^2 = 2^{2n}$

$$\frac{S_o}{N_o} = c(2)^{2n}$$

$$= c\,2^{\frac{2B_T}{B}} \quad [B_T = nB \;Hz]$$

$$where, c = 3\frac{\tilde{m}^2(t)}{m_p^2}$$

$$= \frac{3}{[\ln(1+\mu)]^2}$$

i.e. SNR increases exponentially with the channel bandwidth B_T. (A small increases in BW yields a large benefit in SNR).

PCM (15)

$$\left(\frac{S_o}{N_o}\right)_{\text{dB}} = 10\log_{10}\left(\frac{S_o}{N_o}\right)$$

$$= 10\log_{10}[c(2)^{2n}]$$

$$= 10\log_{10} c + 2n\log_{10} 2$$

$$= (\alpha + 6n) \quad \text{dB} \quad \text{where } \alpha = 10\ \log_{10} c.$$

This shows that increasing n by 1, quadruples the output SNR (6 db increase).

Thus, if n↑ from 8 to 9,
SNR ↑ by 4 times, but
B_T ↑ from 32 to 36 kHz (For previous example)
(An increase of only 12.5%)

PCM (16)

Mean square error caused by channel noise in PCM:

at the receiver due

Consider some of the pulses are incorrectly detected to channel noise .

$\tilde{m} = Decoded\ sample value\ at\ the\ receiver$

$\hat{m} = Quantized\ sample value\ at\ the\ receiver$

$\therefore Error, \varepsilon = \hat{m} - \tilde{m}$, A random variable(Rv)

The value of Ɛ depends on the position of the incorrectly detected pulses.
The error in the ith digit –

$$\varepsilon_i = \left(2^{-i}\right)F, \qquad F = Full\ scale = 2m_p$$

$$= \left(2^{-i}\right)\left(2m_p\right) \qquad i = 1, 2, \ldots\ldots n$$

$$\overline{\varepsilon^2} = \sum_{i=1}^{n} \varepsilon_i^2 P_\varepsilon(\varepsilon_i)$$

Say, L=16,n=4
Transmitted code = 1101 = 13
Received = 0101 = 5
Error = 8

Received = 1001 = 9
Error = 4

$\varepsilon_i = \left(2^{-i}\right)(16)$

PCM (17)

Since the probability of error for any one bit is the same as that of any other that is P_e .

$$\overline{\varepsilon^2} = P_e \sum_{i=1}^{n} \varepsilon_i^2$$

$$= P_e \sum_{i=1}^{n} 4m_p^2 (2^{-2i})$$

$$= 4m_p^2 P_e \sum_{i=1}^{n} 2^{-2i}$$

This summation is a geometric progression with a common ratio $r = 2^{-2}$, with the first term $a_1 = 2^{-2}$ and the last term $a_n = 2^{-2n}$.

$$\overline{\varepsilon^2} = 4m_p^2 P_e \left[\frac{(2^{-2})2^{-2n} - 2^{-2}}{2^{-2} - 1} \right]$$

$$= \frac{4m_p^2 P_e (2^{2n} - 1)}{3(2^{2n})}$$

$$\boxed{\sum_{m=M}^{N} r^m = \frac{r^{N+1} - r^M}{r - 1} \quad where, r = 1}$$

Its probabilities are symmetrical about $\varepsilon=0$, Hence, $\varepsilon = 0$

$$\sigma_\varepsilon^2 = \overline{\varepsilon^2} = \frac{4m_p^2 P_e(2^{2n} - 1)}{3(2^{2n})}$$

PCM (19)

Total mean square error in PCM:

m = Signal sample

$\hat{m} = Quantized\ sample$

$\tilde{m} = Detected\ \ sample\ at\ receiver$

$Quantization\ error, q = m - \hat{m}$

$Detection\ error, \varepsilon = \hat{m} - \tilde{m}$

$Total\ error, \hat{m} - \tilde{m} = q + \varepsilon$ where, both q and Ɛ are 0 mean Rv

$$\overline{(\mathrm{m} - \tilde{\mathrm{m}})^2} = \overline{(\mathrm{q} + \epsilon)^2} = \overline{\mathrm{q}^2} + \overline{\epsilon^2}$$ [q and Ɛ are independent]

$$e^2(t) = \frac{1}{3}\left(\frac{m_p}{L}\right)^2 + \frac{4m_p^2 P_\epsilon (2^{2n} - 1)}{3(2^{2n})}$$

Also, because $L = 2^n$,

$$e^2(t) = \frac{{m_p}^2}{3(2^{2n})}[1 + 4P_\epsilon (2^{2n} - 1)]$$

PCM (20)

SNR: $S_o = \overline{m^2}$ $N_o = \overline{e^2(t)}$

$$\frac{S_o}{N_o} = \frac{3(2^{2n})}{1 + 4P_e(2^{2n} - 1)}\left(\frac{\overline{m^2}}{m_p^2}\right)$$

So, by knowing as P_ε (P_e) we can find S_o/N_o for PCM.

P_e = Error probability in detection

$$= Q\left(\left|\frac{A_p}{\sigma_n}\right|\right)$$

A_p = Peak pulse amplitude

σ_n^2 = Channel noise power.

For optimum (matched) filter , $\boxed{\left(\frac{A_p}{\sigma_n}\right)_{\max} = \sqrt{\frac{2E_p}{\mathcal{N}}}}$

Then, $\boxed{(P_e)_{\min} = Q\left(\sqrt{\frac{2E_p}{\mathcal{N}}}\right)}$

E_p = Energy of pulse

PSD of channel noise (white) = N/2

PCM (21)

For B Hz signal, 2B sample and $2B_n$ bit/s
So, received signal power,

$$S_i = 2BnE_p$$

Then,

$$P_e = Q\left(\sqrt{\frac{S_i}{n\mathcal{N}B}}\right)$$

$$= Q\left(\sqrt{\frac{\gamma}{n}}\right)$$

ϒ = Signal power.

$$\frac{S_o}{N_o} = \frac{3(2^{2n})}{1+4(2^{2n}-1)Q\left(\sqrt{\gamma/n}\right)}\left(\frac{\overline{m^2}}{m_p^2}\right)$$

PCM (22)

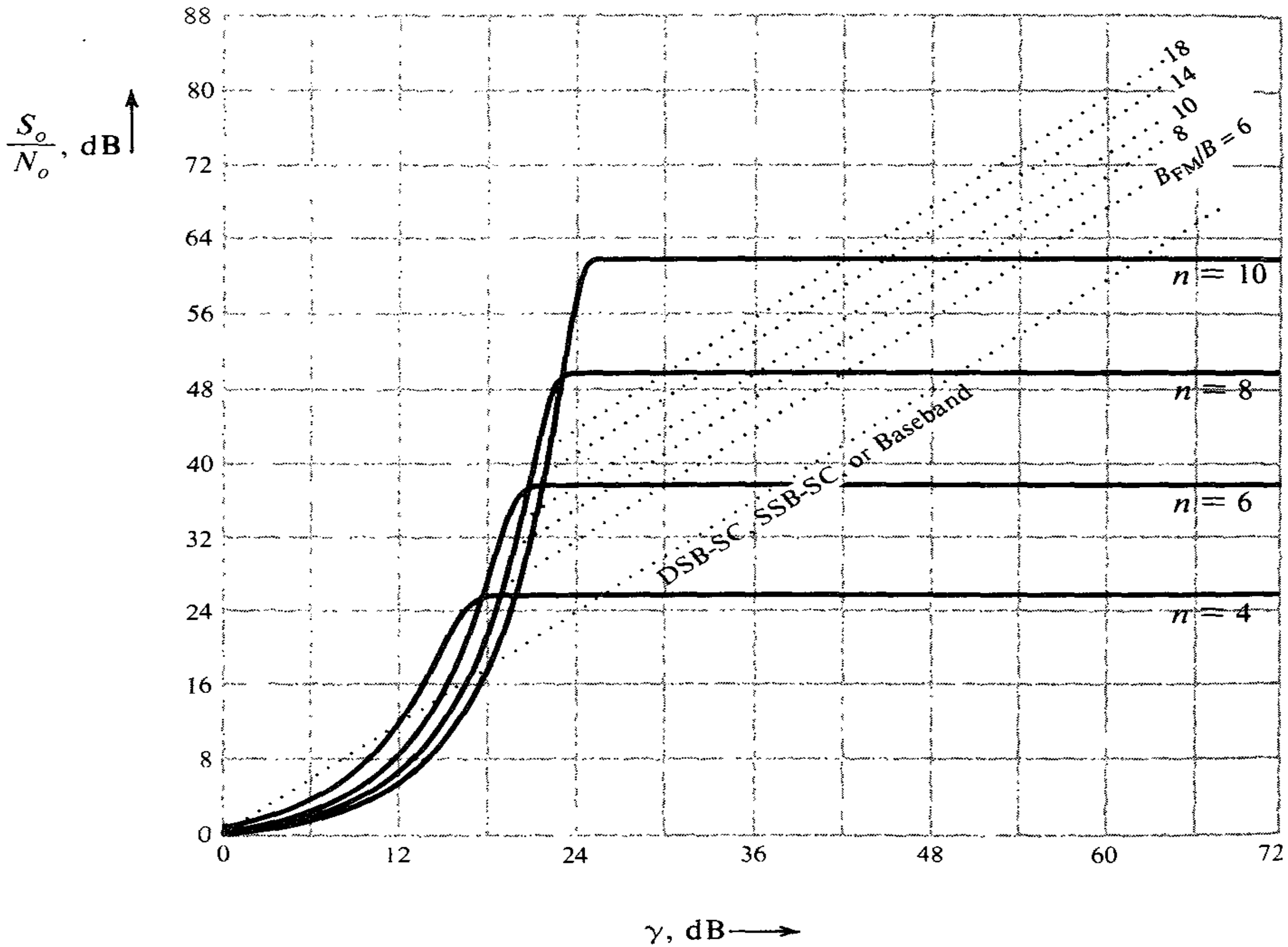

Figure 12.14 Performance of PCM.

PCM (23)

Two interesting features are–

1) The threshold for small ϒ, large error, unacceptable meaningless.
2) The saturation, for large ϒ (signal power)

For large ϒ , $P_e = 0$

$$\frac{S_o}{N_o} = 3L^2 \left(\frac{\overline{m^2}}{m_p^2} \right)$$

$$= 3(2^{2n}) \left(\frac{\overline{m^2}}{m_p^2} \right)$$

$$\left(\frac{S_o}{N_o} \right)_{\text{dB}} = 10 \left[\log 3 + 2n \log 2 + \log \left(\frac{\overline{m^2}}{m_p^2} \right) \right]$$

$$= \alpha + 6n$$

In the saturation region,

$$\alpha = 4.77 + 10 \log_{10} (\overline{m^2} / m_p^2).$$

PCM (24)

Assignment:

Problem 12.4-3 $\left\{ \begin{array}{l} \text{B. P. Lathi} \\ \text{Page-576} \end{array} \right.$

Hints:

$S_n(\omega) = NB$ [Beginning of chap 12]

$$P_e = Q\left(\sqrt{\frac{S_i}{n\mathcal{N}B}}\right)$$

$$= Q\left(\sqrt{\frac{\gamma}{n}}\right)$$

$$Q(x) \simeq \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} \quad \text{for } x \gg 1$$ [Table 10.2]

PCM (25)

Quantization noise :

Quantization noise can be categorized into 4 types –

1. Overload noise
2. Random noise.
3. Granular noise.
4. Hunting noise

PCM (26)

1. Overload noise: The level of the analog waveform at the input of the PCM encoder needs to be set so that its peak level does not exceed the design peak of V volts. If the peak input does exceed V, then the recovered analog waveform at the output of the PCM system will have flat-tops near the peak values. This produces Overload noise.

Example: This type of distortion can be heard on a PCM telephone system when they are high levels such as dial tones, busy signals or off-hook signals.

2. Random noise: Random noise is produced by the Random quantization errors in the PCM system under normal operating conditions when the input level is properly set. It has a white hissing sound.

PCM (27)

3. Granular noise: If the input level is reduced further to a relatively small value with respect to the design level, the error values are not equally likely from sample to sample, and the noise has a harsh sound resembling gravel being poured into a barrel. This is called **granular noise.**

- This noise can be randomized (noise power decreased) by increasing the number of quantization levels and PCM bit rate.
- Alternatively , it can be reduced by using a nonuniform quantizer as μ-law or A-law quantizer.

PCM (28)

4. Hunting noise: It can occur when the input analog waveform is nearly constant , including when there is no signal (i. e. 0 level). For these conditions, the sample values at the quantizer output can between 2 adjacent quantization levels, causing an undesired sinusoidal type tone of frequency fs/2 at the output PCM system.

- It can be reduced by filtering out the tone or by designing the quantizer so that there is no vertical step at the **"constant"** value of the input.
- For no signal case, it is also called **idle channel noise** which can be reduced by using a horizontal step at the origin of the quantizer output-input characteristics instead of a vertical step .

PCM (29)

❖SNR is an indication of the quality of the received signal. It can vary widely depending on –

- ➢Talker
- ➢Length of circuit

❖ it will be low most of the time

These problem can be solved by using smaller steps for smaller amplitudes (nonuniform quantizing), as shown in Fig 6.11a .

✓The same result is obtained by first compressing signal samples and then using a uniform quantization. The input output characteristics of a compressor are shown in Fig 6.11b .

PCM (30)

Nonuniform quantizing:

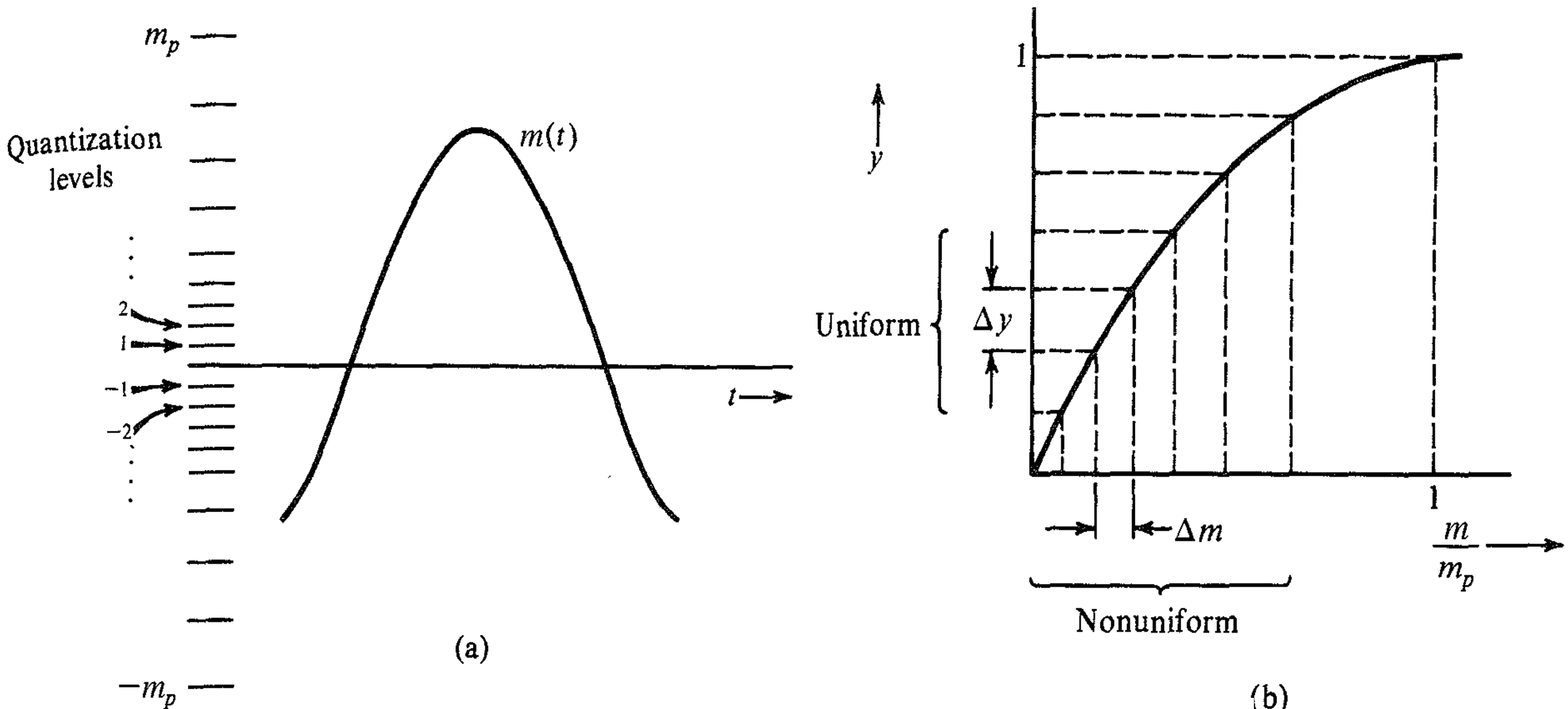

Figure 6.11 Nonuniform quantization.

PCM (31)

Compression laws: 2 Compression laws have been accepted as desirable standards by the CCITT –

1) **μ-law:** used in North America and Japan.

The μ-law (for positive amplitudes) is given by

$$\boxed{y = \frac{1}{\ln(1+\mu)} \ln\left(1 + \frac{\mu m}{m_p}\right) \qquad 0 \le \frac{m}{m_p} \le 1}$$

2) **A-law:** used in Europe and the rest of the world and international routes

The A-law (for positive amplitudes) is

$$\boxed{y = \begin{cases} \dfrac{A}{1+\ln A}\left(\dfrac{m}{m_p}\right) & 0 \le \dfrac{m}{m_p} \le \dfrac{1}{A} \\[2ex] \dfrac{1}{1+\ln A}\left(1 + \ln \dfrac{Am}{m_p}\right) & \dfrac{1}{A} \le \dfrac{m}{m_p} \le 1 \end{cases}}$$

PCM (32)

Both the law curves have odd symmetry about the vertical axis.

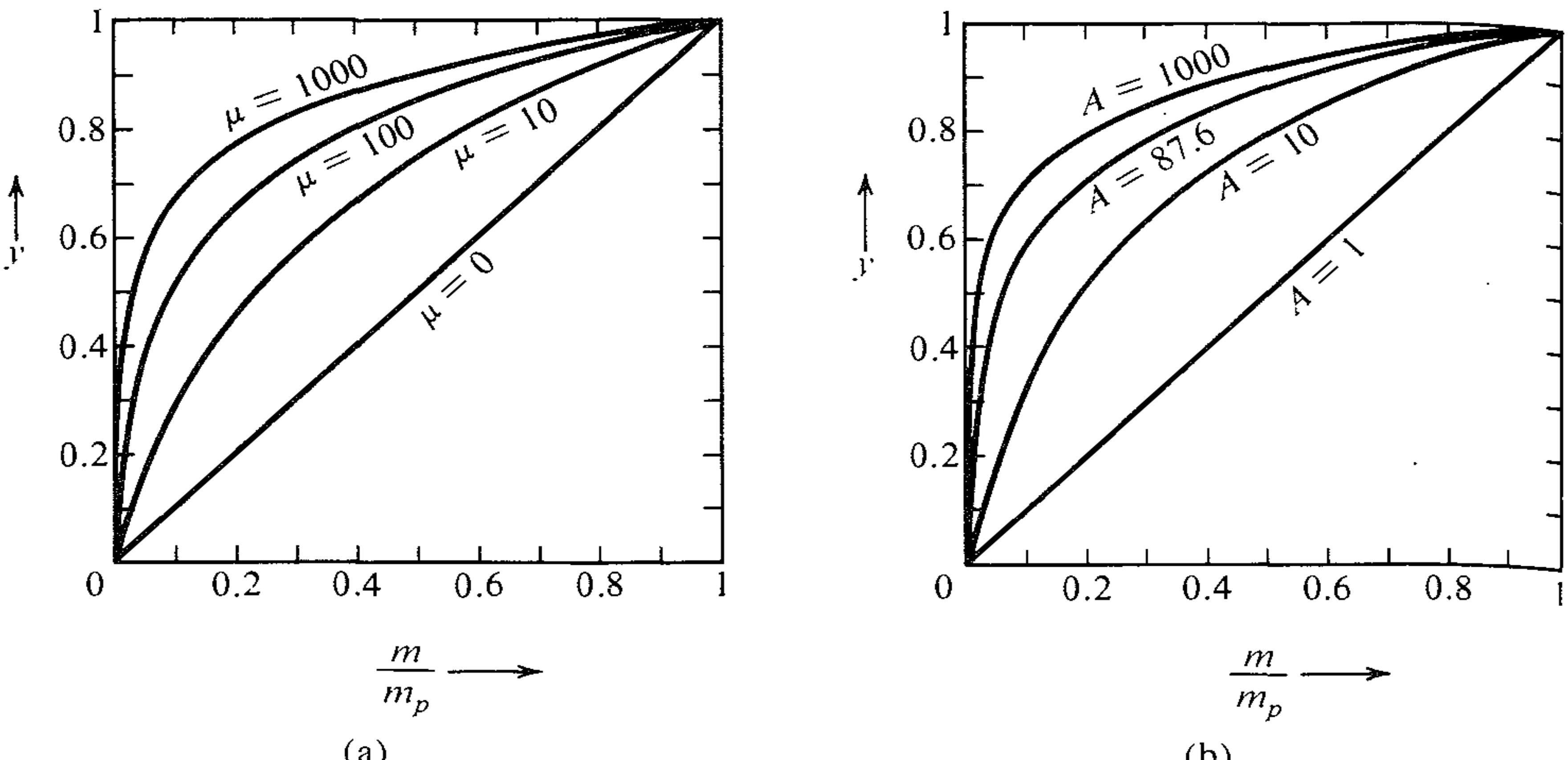

Figure 6.12 (a) μ-law characteristic. (b) A-law characteristic.

PCM (33)

Compandor:

The compressed samples must be restored to their original values at the receiver by using an expander with a characteristic complementary to that of the compressor. The compressor and the expander together are called the **compandor**. Generally speaking, compression of a signal increases its bandwidth. But in PCM, we are compressing not the signal $m(t)$ but its samples. Because the number of samples does not change, the problem of bandwidth increase does not arise here.

when a μ-law compandor is used, the output SNR is

$$\frac{S_o}{N_o} \simeq \frac{3L^2}{[\ln(1+\mu)]^2} \qquad \mu^2 \gg \frac{m_p^2}{\widetilde{m^2(t)}}$$

PCM (34)

The output SNR for the cases of $\mu = 255$ and $\mu = 0$ (uniform quantization) as a function of $\widetilde{m^2}$ (the message signal power) is shown in Fig. 6.13

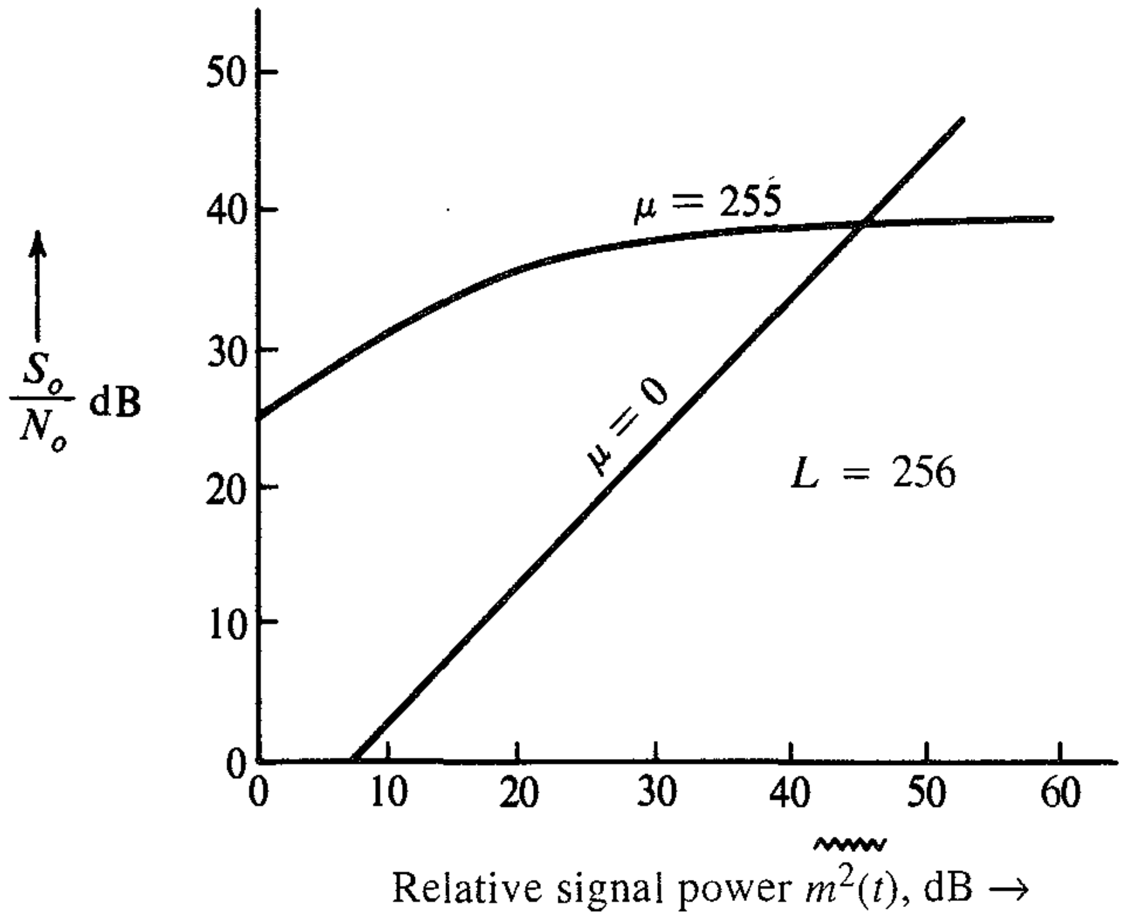

Figure 6.13 Signal-to-quantization-noise ratio in PCM with and without compression.

PCM (35)

Digital signaling: The voltage (or current) waveforms for digital signals can be expressed as an orthogonal series with a finite number of terms, N such as

$$\boxed{w(t) = \sum_{k=1}^{N} w_k \varphi_k(t),\ 0 < t < T_0}$$

Where,

w_k = The digital data
ϕ_k (t), k= 1, 2, ….N are N orthogonal functions that give the waveform its waveshape.
N = Number of dimensions required to describe the waveform.
w(t() = A PCM word or any message of the M message digital source.