COSE474-2024F Deep Learning HW 2

- **Chapter 7**
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0.1. Installation

[ ] ↳ 숨겨진 셀 2개

7.1. From Fully Connected Layers to Convolutions

To this day, the models that we have discussed so far remain appropriate options when we are dealing with tabular data. By tabular, we mean that the data consist of rows corresponding to examples and columns corresponding to features. With tabular data, we might anticipate that the patterns we seek could involve interactions among the features, but we do not assume any structure *a priori* concerning how the features interact.

Sometimes, we truly lack the knowledge to be able to guide the construction of fancier architectures. In these cases, an MLP may be the best that we can do. However, for high-dimensional perceptual data, such structureless networks can grow unwieldy.

For instance, let's return to our running example of distinguishing cats from dogs. Say that we do a thorough job in data collection, collecting an annotated dataset of one-megapixel photographs. This means that each input to the network has one million dimensions. Even an aggressive reduction to one thousand hidden dimensions would require a fully connected layer characterized by $10^6 \times 10^3 = 10^9$ parameters. Unless we have lots of GPUs, a talent for distributed optimization, and an extraordinary amount of patience, learning the parameters of this network may turn out to be infeasible.

A careful reader might object to this argument on the basis that one megapixel resolution may not be necessary. However, while we might be able to get away with one hundred thousand pixels, our hidden layer of size 1000 grossly underestimates the number of hidden units that it takes to learn good representations of images, so a practical system will still require billions of parameters. Moreover, learning a classifier by fitting so many parameters might require collecting an enormous dataset. And yet today both humans and computers are able to distinguish cats from dogs quite well, seemingly contradicting these intuitions. That is because images exhibit rich structure that can be exploited by humans and machine learning models alike. Convolutional neural networks (CNNs) are one creative way that machine learning has embraced for exploiting some of the known structure in natural images.

7.1.1. Invariance

Imagine that we want to detect an object in an image. It seems reasonable that whatever method we use to recognize objects should not be overly concerned with the precise location of the object in the image. Ideally, our system should exploit this knowledge. Pigs usually do not fly and planes usually do not swim. Nonetheless, we should still recognize a pig were one to appear at the top of the image. We can draw some inspiration here from the children's game "Where's Waldo" (which itself has inspired many real-life imitations, such as that depicted in :numref: `img_waldo`). The game consists of a number of chaotic scenes bursting with activities. Waldo shows up somewhere in each, typically lurking in some unlikely location. The reader's goal is to locate him. Despite his characteristic outfit, this can be surprisingly difficult, due to the large number of distractions. However, *what Waldo looks like* does not depend upon *where Waldo is located*. We could sweep the image with a Waldo detector that could assign a score to each patch, indicating the likelihood that the patch contains Waldo. In fact, many object detection and segmentation algorithms are based on this approach :cite: `Long.Shelhamer.Darrell.2015`. CNNs systematize this idea of *spatial invariance*, exploiting it to learn useful representations with fewer parameters.

We can now make these intuitions more concrete by enumerating a few desiderata to guide our design of a neural network architecture suitable for computer vision:

1. In the earliest layers, our network should respond similarly to the same patch, regardless of where it appears in the image. This principle is called *translation invariance* (or *translation equivariance*).
2. The earliest layers of the network should focus on local regions, without regard for the contents of the image in distant regions. This is the *locality* principle. Eventually, these local representations can be aggregated to make predictions at the whole image level.
3. As we proceed, deeper layers should be able to capture longer-range features of the image, in a way similar to higher level vision in nature.

Let's see how this translates into mathematics.

7.1.2. Constraining the MLP

To start off, we can consider an MLP with two-dimensional images $\mathbf{X}$ as inputs and their immediate hidden representations $\mathbf{H}$ similarly represented as matrices (they are two-dimensional tensors in code), where both $\mathbf{X}$ and $\mathbf{H}$ have the same shape. Let that sink in. We now imagine that not only the inputs but also the hidden representations possess spatial structure.

Let $[\mathbf{X}]_{i,j}$ and $[\mathbf{H}]_{i,j}$ denote the pixel at location (i,j) in the input image and hidden representation, respectively. Consequently, to have each of the hidden units receive input from each of the input pixels, we would switch from using weight matrices (as we did previously in MLPs) to representing our parameters as fourth-order weight tensors W. Suppose that $\mathbf{U}$ contains biases, we could formally express the fully connected layer as

$$\begin{aligned} \left[\mathbf{H}\right]_{i,j} &= [\mathbf{U}]_{i,j} + \sum_k \sum_l [\mathsf{W}]_{i,j,k,l} [\mathbf{X}]_{k,l}\\ &= [\mathbf{U}]_{i,j} + \sum_a \sum_b [\mathsf{V}]_{i,j,a,b} [\mathbf{X}]_{i+a,j+b}.\end{aligned}$$

The switch from W to V is entirely cosmetic for now since there is a one-to-one correspondence between coefficients in both fourth-order tensors. We simply re-index the subscripts (k,l) such that $k = i+a$ and $l = j+b$. In other words, we set $[\mathsf{V}]_{i,j,a,b} = [\mathsf{W}]_{i,j,i+a,j+b}$. The indices a and b run over both positive and negative offsets, covering the entire image. For any given location (i, j) in the hidden representation $[\mathbf{H}]_{i,j}$, we compute its value by summing over pixels in x, centered around (i,j) and weighted by $[\mathsf{V}]_{i,j,a,b}$. Before we carry on, let's consider the total number of parameters required for a *single* layer in this parametrization: a 1000×1000 image (1 megapixel) is mapped to a 1000×1000 hidden representation. This requires 10^{12} parameters, far beyond what computers currently can handle.

7.1.2.1. Translation Invariance

Now let's invoke the first principle established above: translation invariance :cite: `Zhang.ea.1988`. This implies that a shift in the input $\mathbf{X}$ should simply lead to a shift in the hidden representation $\mathbf{H}$. This is only possible if V and $\mathbf{U}$ do not actually depend on (i,j). As such, we have $[\mathsf{V}]_{i,j,a,b} = [\mathbf{V}]_{a,b}$ and $\mathbf{U}$ is a constant, say u. As a result, we can simplify the definition for $\mathbf{H}$:

$$[\mathbf{H}]_{i,j} = u + \sum_a\sum_b [\mathbf{V}]_{a,b} [\mathbf{X}]_{i+a,j+b}.$$

This is a *convolution*! We are effectively weighting pixels at $(i+a, j+b)$ in the vicinity of location (i,j) with coefficients $[\mathbf{V}]_{a,b}$ to obtain the value $[\mathbf{H}]_{i,j}$. Note that $[\mathbf{V}]_{a,b}$ needs many fewer coefficients than $[\mathsf{V}]_{i,j,a,b}$ since it no longer depends on the location within the image. Consequently, the number of parameters required is no longer 10^{12} but a much more reasonable 4×10^6: we still have the dependency on $a, b \in (-1000, 1000)$. In short, we have made significant progress. Time-delay neural networks (TDNNs) are some of the first examples to exploit this idea :cite: `Waibel.Hanazawa.Hinton.ea.1989`.

7.1.2.2. Locality

Now let's invoke the second principle: locality. As motivated above, we believe that we should not have to look very far away from location (i,j) in order to glean relevant information to assess what is going on at $[\mathbf{H}]_{i,j}$. This means that outside some range $|a|> \Delta$ or $|b| > \Delta$, we should set $[\mathbf{V}]_{a,b} = 0$. Equivalently, we can rewrite $[\mathbf{H}]_{i,j}$ as

$$[\mathbf{H}]_{i,j} = u + \sum_{a = -\Delta}^{\Delta} \sum_{b = -\Delta}^{\Delta} [\mathbf{V}]_{a,b} [\mathbf{X}]_{i+a,j+b}.$$

:eqlabel: `eq_conv-layer`

This reduces the number of parameters from 4×10^6 to $4 \Delta^2$, where Δ is typically smaller than 10. As such, we reduced the number of parameters by another four orders of magnitude. Note that :eqref: `eq_conv-layer`, is what is called, in a nutshell, a *convolutional layer*. *Convolutional neural networks* (CNNs) are a special family of neural networks that contain convolutional layers. In the deep learning research community, $\mathbf{V}$ is referred to as a *convolution kernel*, a *filter*, or simply the layer's *weights* that are learnable parameters.

While previously, we might have required billions of parameters to represent just a single layer in an image-processing network, we now typically need just a few hundred, without altering the dimensionality of either the inputs or the hidden representations. The price paid for this drastic reduction in parameters is that our features are now translation invariant and that our layer can only incorporate local information, when determining the value of each hidden activation. All learning depends on imposing inductive bias. When that bias agrees with reality, we get sample-efficient models that generalize well to unseen data. But of course, if those biases do not agree with reality, e.g., if images turned out not to be translation invariant, our models might struggle even to fit our training data.

This dramatic reduction in parameters brings us to our last desideratum, namely that deeper layers should represent larger and more complex aspects of an image. This can be achieved by interleaving nonlinearities and convolutional layers repeatedly.

7.1.3. Convolutions

Let's briefly review why :eqref: `eq_conv-layer` is called a convolution. In mathematics, the *convolution* between two functions :cite: `Rudin.1973`, say $f, g: \mathbb{R}^d \to \mathbb{R}$ is defined as

$$(f * g)(\mathbf{x}) = \int f(\mathbf{z}) g(\mathbf{x}-\mathbf{z}) d\mathbf{z}.$$

That is, we measure the overlap between f and g when one function is "flipped" and shifted by $\mathbf{x}$. Whenever we have discrete objects, the integral turns into a sum. For instance, for vectors from the set of square-summable infinite-dimensional vectors with index running over $\mathbb{Z}$ we obtain the following definition:

$$(f * g)(i) = \sum_a f(a) g(i-a).$$

For two-dimensional tensors, we have a corresponding sum with indices (a, b) for f and $(i-a, j-b)$ for g, respectively:

$$(f * g)(i, j) = \sum_a\sum_b f(a, b) g(i-a, j-b).$$

:eqlabel: `eq_2d-conv-discrete`

This looks similar to :eqref: `eq_conv-layer` , with one major difference. Rather than using $(i+a, j+b)$, we are using the difference instead. Note, though, that this distinction is mostly cosmetic since we can always match the notation between :eqref: `eq_conv-layer` and :eqref: `eq_2d-conv-discrete` . Our original definition in :eqref: `eq_conv-layer` more properly describes a *cross-correlation*. We will come back to this in the following section.

7.1.4. Channels

Returning to our Waldo detector, let's see what this looks like. The convolutional layer picks windows of a given size and weighs intensities according to the filter V, as demonstrated in :numref: `fig_waldo_mask` . We might aim to learn a model so that wherever the "waldoness" is highest, we should find a peak in the hidden layer representations.

There is just one problem with this approach. So far, we blissfully ignored that images consist of three channels: red, green, and blue. In sum, images are not two-dimensional objects but rather third-order tensors, characterized by a height, width, and channel, e.g., with shape $1024 \times 1024 \times 3$ pixels. While the first two of these axes concern spatial relationships, the third can be regarded as assigning a multidimensional representation to each pixel location. We thus index X as $[\mathsf{X}]_{i, j, k}$. The convolutional filter has to adapt accordingly. Instead of $[\mathbf{V}]_{a,b}$, we now have $[\mathsf{V}]_{a,b,c}$.

Moreover, just as our input consists of a third-order tensor, it turns out to be a good idea to similarly formulate our hidden representations as third-order tensors H. In other words, rather than just having a single hidden representation corresponding to each spatial location, we want an entire vector of hidden representations corresponding to each spatial location. We could think of the hidden representations as comprising a number of two-dimensional grids stacked on top of each other. As in the inputs, these are sometimes called *channels*. They are also sometimes called *feature maps*, as each provides a spatialized set of learned features for the subsequent layer. Intuitively, you might imagine that at lower layers that are closer to inputs, some channels could become specialized to recognize edges while others could recognize textures.

To support multiple channels in both inputs (X) and hidden representations (H), we can add a fourth coordinate to V: $[\mathsf{V}]_{a, b, c, d}$. Putting everything together we have:

$$[\mathsf{H}]_{i,j,d} = \sum_{a = -\Delta}^{\Delta} \sum_{b = -\Delta}^{\Delta} \sum_c [\mathsf{V}]_{a, b, c, d} [\mathsf{X}]_{i+a, j+b, c},$$

:eqlabel: `eq_conv-layer-channels`

where d indexes the output channels in the hidden representations H. The subsequent convolutional layer will go on to take a third-order tensor, H, as input. We take :eqref: `eq_conv-layer-channels` , because of its generality, as the definition of a convolutional layer for multiple channels, where V is a kernel or filter of the layer.

There are still many operations that we need to address. For instance, we need to figure out how to combine all the hidden representations to a single output, e.g., whether there is a Waldo *anywhere* in the image. We also need to decide how to compute things efficiently, how to combine multiple layers, appropriate activation functions, and how to make reasonable design choices to yield networks that are effective in practice. We turn to these issues in the remainder of the chapter.

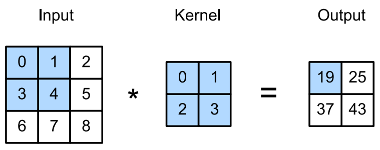
7.2. Convolutions for Images

```
import torch
from torch import nn
from d2l import torch as d2l
```

7.2.1. The Cross-Correlation Operation

Recall that strictly speaking, convolutional layers are a misnomer, since the operations they express are more accurately described as cross-correlations. Based on our descriptions of convolutional layers in :numref: `sec_why-conv` , in such a layer, an input tensor and a kernel tensor are combined to produce an output tensor through a (**cross-correlation operation.**)

Let's ignore channels for now and see how this works with two-dimensional data and hidden representations. In :numref: `fig_correlation`, the input is a two-dimensional tensor with a height of 3 and width of 3. We mark the shape of the tensor as 3×3 or $(3, 3)$. The height and width of the kernel are both 2. The shape of the *kernel window* (or *convolution window*) is given by the height and width of the kernel (here it is 2×2).



:label: `fig_correlation`

In the two-dimensional cross-correlation operation, we begin with the convolution window positioned at the upper-left corner of the input tensor and slide it across the input tensor, both from left to right and top to bottom. When the convolution window slides to a certain position, the input subtensor contained in that window and the kernel tensor are multiplied elementwise and the resulting tensor is summed up yielding a single scalar value. This result gives the value of the output tensor at the corresponding location. Here, the output tensor has a height of 2 and width of 2 and the four elements are derived from the two-dimensional cross-correlation operation:

$$
\begin{aligned}
0\times0+1\times1+3\times2+4\times3=19,\\
1\times0+2\times1+4\times2+5\times3=25,\\
3\times0+4\times1+6\times2+7\times3=37,\\
4\times0+5\times1+7\times2+8\times3=43.
\end{aligned}
$$

Note that along each axis, the output size is slightly smaller than the input size. Because the kernel has width and height greater than 1, we can only properly compute the cross-correlation for locations where the kernel fits wholly within the image, the output size is given by the input size $n_\textrm{h} \times n_\textrm{w}$ minus the size of the convolution kernel $k_\textrm{h} \times k_\textrm{w}$ via

$$(n_\textrm{h}-k_\textrm{h}+1) \times (n_\textrm{w}-k_\textrm{w}+1).$$

This is the case since we need enough space to "shift" the convolution kernel across the image. Later we will see how to keep the size unchanged by padding the image with zeros around its boundary so that there is enough space to shift the kernel. Next, we implement this process in the `corr2d` function, which accepts an input tensor `X` and a kernel tensor `K` and returns an output tensor `Y`.

```
def corr2d(X, K):
    """Compute 2D cross-correlation."""
    h, w = K.shape
    Y = torch.zeros((X.shape[0] - h + 1, X.shape[1] - w + 1))
    for i in range(Y.shape[0]):
        for j in range(Y.shape[1]):
            Y[i, j] = (X[i:i + h, j:j + w] * K).sum()
    return Y
```

```
X = torch.tensor([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]])
K = torch.tensor([[0.0, 1.0], [2.0, 3.0]])
corr2d(X, K)
```

```
tensor([[19., 25.],
        [37., 43.]])
```

7.2.2. Convolutional Layers

```
class Conv2D(nn.Module):
    def __init__(self, kernel_size):
        super().__init__()
        self.weight = nn.Parameter(torch.rand(kernel_size))
        self.bias = nn.Parameter(torch.zeros(1))

    def forward(self, x):
        return corr2d(x, self.weight) + self.bias
```

7.2.3. Object Edge Detection in Images

```
X = torch.ones((6, 8))
X[:, 2:6] = 0
X
```

```
tensor([[1., 1., 0., 0., 0., 0., 1., 1.],
        [1., 1., 0., 0., 0., 0., 1., 1.],
        [1., 1., 0., 0., 0., 0., 1., 1.],
        [1., 1., 0., 0., 0., 0., 1., 1.],
        [1., 1., 0., 0., 0., 0., 1., 1.],
        [1., 1., 0., 0., 0., 0., 1., 1.]])
```

Next, we construct a kernel `K` with a height of 1 and a width of 2. When we perform the cross-correlation operation with the input, if the horizontally adjacent elements are the same, the output is 0. Otherwise, the output is nonzero. Note that this kernel is a special case of a finite difference operator. At location (i, j) it computes $x_{i,j} - x_{(i+1),j}$, i.e., it computes the difference between the values of horizontally adjacent pixels. This is a discrete approximation of the first derivative in the horizontal direction. After all, for a function $f(i, j)$ its derivative $-\partial_i f(i, j) = \lim_{\epsilon \to 0} \frac{f(i,j) - f(i+\epsilon,j)}{\epsilon}$. Let's see how this works in practice.

```
K = torch.tensor([[1.0, -1.0]])
```

We are ready to perform the cross-correlation operation with arguments `X` (our input) and `K` (our kernel). As you can see, [**we detect 1 for the edge from white to black and -1 for the edge from black to white.**] All other outputs take value 0.

```
Y = corr2d(X, K)
Y
```

```
tensor([[ 0.,  1.,  0.,  0.,  0., -1.,  0.],
        [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
        [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
        [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
        [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
        [ 0.,  1.,  0.,  0.,  0., -1.,  0.]])
```

We can now apply the kernel to the transposed image. As expected, it vanishes. The kernel K only detects vertical edges.

```
corr2d(X.t(), K)
```

```
tensor([[0., 0., 0., 0., 0.],
        [0., 0., 0., 0., 0.],
        [0., 0., 0., 0., 0.],
        [0., 0., 0., 0., 0.],
        [0., 0., 0., 0., 0.],
        [0., 0., 0., 0., 0.],
        [0., 0., 0., 0., 0.],
        [0., 0., 0., 0., 0.]])
```

7.2.4. Learning a Kernel

Designing an edge detector by finite differences `[1, -1]` is neat if we know this is precisely what we are looking for. However, as we look at larger kernels, and consider successive layers of convolutions, it might be impossible to specify precisely what each filter should be doing manually.

Now let's see whether we can [**learn the kernel that generated `Y` from `X`**] by looking at the input--output pairs only. We first construct a convolutional layer and initialize its kernel as a random tensor. Next, in each iteration, we will use the squared error to compare `Y` with the output of the convolutional layer. We can then calculate the gradient to update the kernel. For the sake of simplicity, in the following we use the built-in class for two-dimensional convolutional layers and ignore the bias.

```
# Construct a two-dimensional convolutional layer with 1 output channel and a
# kernel of shape (1, 2). For the sake of simplicity, we ignore the bias here
conv2d = nn.LazyConv2d(1, kernel_size=(1, 2), bias=False)

# The two-dimensional convolutional layer uses four-dimensional input and
# output in the format of (example, channel, height, width), where the batch
# size (number of examples in the batch) and the number of channels are both 1
X = X.reshape((1, 1, 6, 8))
Y = Y.reshape((1, 1, 6, 7))
lr = 3e-2  # Learning rate

for i in range(10):
    Y_hat = conv2d(X)
    l = (Y_hat - Y) ** 2
    conv2d.zero_grad()
    l.sum().backward()
    # Update the kernel
    conv2d.weight.data[:] -= lr * conv2d.weight.grad
    if (i + 1) % 2 == 0:
        print(f'epoch {i + 1}, loss {l.sum():.3f}')
```

```
epoch 2, loss 7.803
epoch 4, loss 1.545
epoch 6, loss 0.356
epoch 8, loss 0.099
epoch 10, loss 0.033
```

Note that the error has dropped to a small value after 10 iterations. Now we will take a look at the kernel tensor we learned.

```
conv2d.weight.data.reshape((1, 2))
```

```
tensor([[ 1.0035, -0.9693]])
```

7.2.5. Cross-Correlation and Convolution

Recall our observation from :numref: `sec_why-conv` of the correspondence between the cross-correlation and convolution operations. Here let's continue to consider two-dimensional convolutional layers. What if such layers perform strict convolution operations as defined in :eqref: `eq_2d-conv-discrete` instead of cross-correlations? In order to obtain the output of the strict *convolution* operation, we only need to flip the two-dimensional kernel tensor both horizontally and vertically, and then perform the *cross-correlation* operation with the input tensor.

It is noteworthy that since kernels are learned from data in deep learning, the outputs of convolutional layers remain unaffected no matter such layers perform either the strict convolution operations or the cross-correlation operations.

To illustrate this, suppose that a convolutional layer performs *cross-correlation* and learns the kernel in :numref: `fig_correlation`, which is here denoted as the matrix $\mathbf{K}$. Assuming that other conditions remain unchanged, when this layer instead performs strict *convolution*, the learned kernel $\mathbf{K}'$ will be the same as $\mathbf{K}$ after $\mathbf{K}'$ is flipped both horizontally and vertically. That is to say, when the convolutional layer performs strict *convolution* for the input in :numref: `fig_correlation` and $\mathbf{K}'$, the same output in :numref: `fig_correlation` (cross-correlation of the input and $\mathbf{K}$) will be obtained.

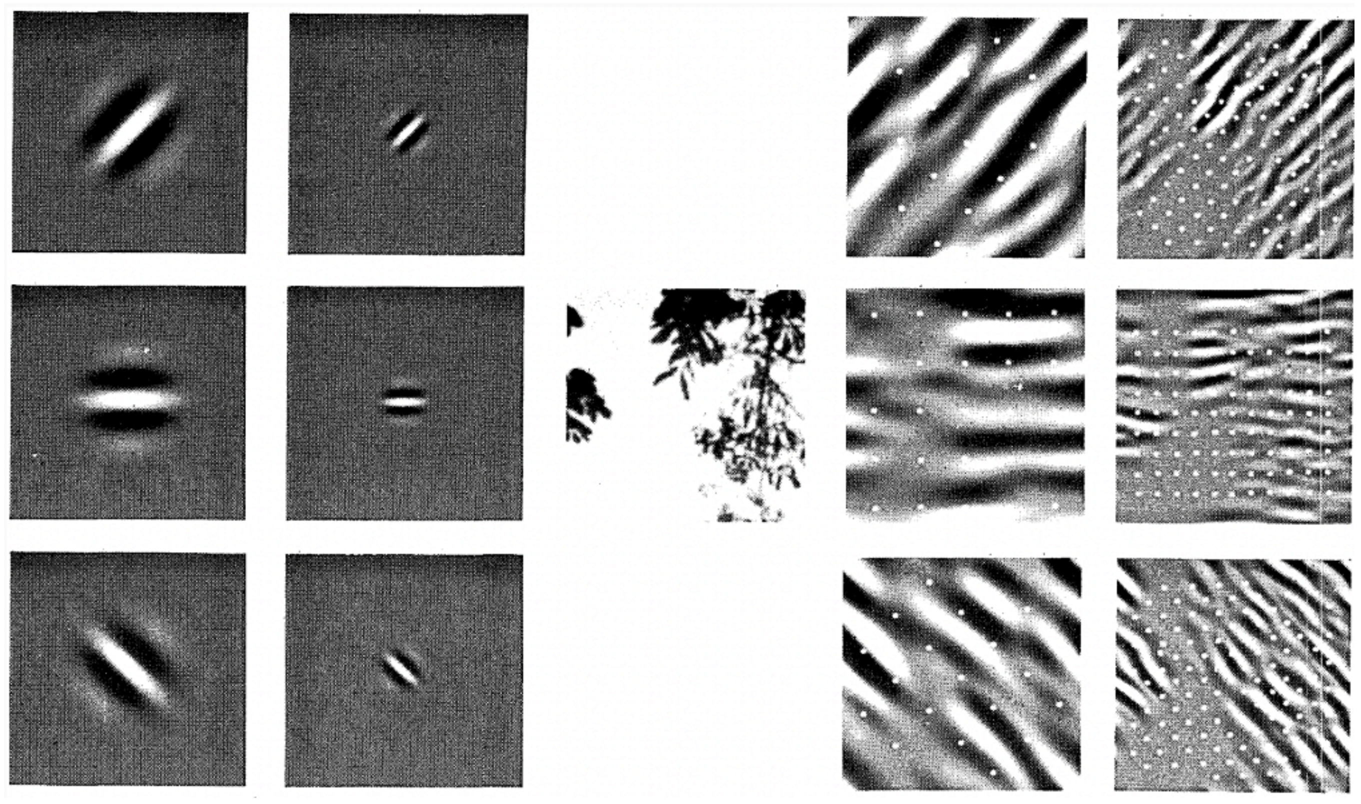
In keeping with standard terminology in deep learning literature, we will continue to refer to the cross-correlation operation as a convolution even though, strictly-speaking, it is slightly different. Furthermore, we use the term *element* to refer to an entry (or component) of any tensor representing a layer representation or a convolution kernel.

7.2.6. Feature Map and Receptive Field

As described in :numref: `subsec_why-conv-channels`, the convolutional layer output in :numref: `fig_correlation` is sometimes called a *feature map*, as it can be regarded as the learned representations (features) in the spatial dimensions (e.g., width and height) to the subsequent layer. In CNNs, for any element x of some layer, its *receptive field* refers to all the elements (from all the previous layers) that may affect the calculation of x during the forward propagation. Note that the receptive field may be larger than the actual size of the input.

Let's continue to use :numref: `fig_correlation` to explain the receptive field. Given the 2×2 convolution kernel, the receptive field of the shaded output element (of value 19) is the four elements in the shaded portion of the input. Now let's denote the 2×2 output as $\mathbf{Y}$ and consider a deeper CNN with an additional 2×2 convolutional layer that takes $\mathbf{Y}$ as its input, outputting a single element z. In this case, the receptive field of z on $\mathbf{Y}$ includes all the four elements of $\mathbf{Y}$, while the receptive field on the input includes all the nine input elements. Thus, when any element in a feature map needs a larger receptive field to detect input features over a broader area, we can build a deeper network.

Receptive fields derive their name from neurophysiology. A series of experiments on a range of animals using different stimuli :cite: `Hubel.Wiesel.1959,Hubel.Wiesel.1962,Hubel.Wiesel.1968` explored the response of what is called the visual cortex on said stimuli. By and large they found that lower levels respond to edges and related shapes. Later on, :citet: `Field.1987` illustrated this effect on natural images with, what can only be called, convolutional kernels. We reprint a key figure in :numref: `field_visual` to illustrate the striking similarities.



An example of coding with six different channels. (Left) Examples of the six types of sensor associated with each channel. (Right) Convolution of the image in (Middle) with the six sensors shown in (Left). The response of the individual sensors is determined by sampling these filtered images at a distance proportional to the size of the sensor (shown with dots). This diagram shows the response of only the even symmetric sensors.](../img/field-visual.png)

As it turns out, this relation even holds for the features computed by deeper layers of networks trained on image classification tasks, as demonstrated in, for example, `Kuzovkin.Vicente.Petton.ea.2018`. Suffice it to say, convolutions have proven to be an incredibly powerful tool for computer vision, both in biology and in code. As such, it is not surprising (in hindsight) that they heralded the recent success in deep learning.

7.3. Padding and Stride

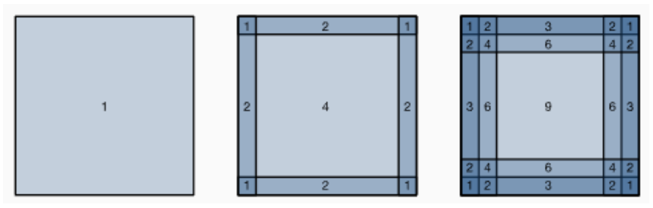
Recall the example of a convolution in :numref: `fig_correlation`. The input had both a height and width of 3 and the convolution kernel had both a height and width of 2, yielding an output representation with dimension 2×2. Assuming that the input shape is $n_\mathrm{h} \times n_\mathrm{w}$ and the convolution kernel shape is $k_\mathrm{h} \times k_\mathrm{w}$, the output shape will be $(n_\mathrm{h}-k_\mathrm{h}+1) \times (n_\mathrm{w}-k_\mathrm{w}+1)$: we can only shift the convolution kernel so far until it runs out of pixels to apply the convolution to.

In the following we will explore a number of techniques, including padding and strided convolutions, that offer more control over the size of the output. As motivation, note that since kernels generally have width and height greater than 1, after applying many successive convolutions, we tend to wind up with outputs that are considerably smaller than our input. If we start with a 240×240 pixel image, ten layers of 5×5 convolutions reduce the image to 200×200 pixels, slicing off 30% of the image and with it obliterating any interesting information on the boundaries of the original image. *Padding* is the most popular tool for handling this issue. In other cases, we may want to reduce the dimensionality drastically, e.g., if we find the original input resolution to be unwieldy. *Strided convolutions* are a popular technique that can help in these instances.

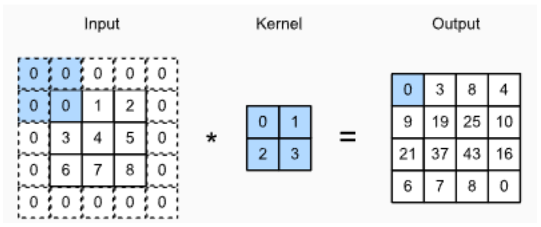
```
import torch
from torch import nn
```

7.3.1. Padding

As described above, one tricky issue when applying convolutional layers is that we tend to lose pixels on the perimeter of our image. Consider :numref: `img_conv_reuse` that depicts the pixel utilization as a function of the convolution kernel size and the position within the image. The pixels in the corners are hardly used at all.



Since we typically use small kernels, for any given convolution we might only lose a few pixels but this can add up as we apply many successive convolutional layers. One straightforward solution to this problem is to add extra pixels of filler around the boundary of our input image, thus increasing the effective size of the image. Typically, we set the values of the extra pixels to zero. In :numref: `img_conv_pad`, we pad a 3×3 input, increasing its size to 5×5. The corresponding output then increases to a 4×4 matrix. The shaded portions are the first output element as well as the input and kernel tensor elements used for the output computation: $0 \times 0 + 0 \times 1 + 0 \times 2 + 0 \times 3 = 0$.



In general, if we add a total of p_h rows of padding (roughly half on top and half on bottom) and a total of p_w columns of padding (roughly half on the left and half on the right), the output shape will be

$$(n_\mathrm{h} - k_\mathrm{h} + p_\mathrm{h} + 1) \times (n_\mathrm{w} - k_\mathrm{w} + p_\mathrm{w} + 1).$$

This means that the height and width of the output will increase by p_h and p_w, respectively.

In many cases, we will want to set $p_\mathrm{h} = k_\mathrm{h} - 1$ and $p_\mathrm{w} = k_\mathrm{w} - 1$ to give the input and output the same height and width. This will make it easier to predict the output shape of each layer when constructing the network. Assuming that k_h is odd here, we will pad $p_\mathrm{h}/2$ rows on both sides of the height. If k_h is even, one possibility is to pad $\lceil p_\mathrm{h}/2\rceil$ rows on the top of the input and $\lfloor p_\mathrm{h}/2\rfloor$ rows on the bottom. We will pad both sides of the width in the same way.

CNNs commonly use convolution kernels with odd height and width values, such as 1, 3, 5, or 7. Choosing odd kernel sizes has the benefit that we can preserve the dimensionality while padding with the same number of rows on top and bottom, and the same number of columns on left and right.

Moreover, this practice of using odd kernels and padding to precisely preserve dimensionality offers a clerical benefit. For any two-dimensional tensor `X`, when the kernel's size is odd and the number of padding rows and columns on all sides are the same, thereby producing an output with the same height and width as the input, we know that the output `Y[i, j]` is calculated by cross-correlation of the input and convolution kernel with the window centered on `X[i, j]`.

In the following example, we create a two-dimensional convolutional layer with a height and width of 3 and (**apply 1 pixel of padding on all sides.**) Given an input with a height and width of 8, we find that the height and width of the output is also 8.

```
# We define a helper function to calculate convolutions. It initializes the
# convolutional layer weights and performs corresponding dimensionality
# elevations and reductions on the input and output
def comp_conv2d(conv2d, X):
    # (1, 1) indicates that batch size and the number of channels are both 1
    X = X.reshape((1, 1) + X.shape)
    Y = conv2d(X)
    # Strip the first two dimensions: examples and channels
    return Y.reshape(Y.shape[2:])

# 1 row and column is padded on either side, so a total of 2 rows or columns
# are added
conv2d = nn.LazyConv2d(1, kernel_size=3, padding=1)
X = torch.rand(size=(8, 8))
comp_conv2d(conv2d, X).shape
```

```
torch.Size([8, 8])
```

When the height and width of the convolution kernel are different, we can make the output and input have the same height and width by setting different padding numbers for height and width.

```
# We use a convolution kernel with height 5 and width 3. The padding on either
# side of the height and width are 2 and 1, respectively
```

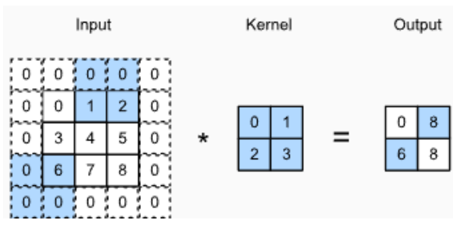
```
conv2d = nn.LazyConv2d(1, kernel_size=(5, 3), padding=(2, 1))
comp_conv2d(conv2d, X).shape
```

```
torch.Size([8, 8])
```

7.3.2. Stride

When computing the cross-correlation, we start with the convolution window at the upper-left corner of the input tensor, and then slide it over all locations both down and to the right. In the previous examples, we defaulted to sliding one element at a time. However, sometimes, either for computational efficiency or because we wish to downsample, we move our window more than one element at a time, skipping the intermediate locations. This is particularly useful if the convolution kernel is large since it captures a large area of the underlying image.

We refer to the number of rows and columns traversed per slide as *stride*. So far, we have used strides of 1, both for height and width. Sometimes, we may want to use a larger stride. :numref: `img_conv_stride` shows a two-dimensional cross-correlation operation with a stride of 3 vertically and 2 horizontally. The shaded portions are the output elements as well as the input and kernel tensor elements used for the output computation: $0 \times 0 + 0 \times 1 + 1 \times 2 + 2 \times 3 = 8, 0 \times 0 + 6 \times 1 + 0 \times 2 + 0 \times 3 = 6$. We can see that when the second element of the first column is generated, the convolution window slides down three rows. The convolution window slides two columns to the right when the second element of the first row is generated. When the convolution window continues to slide two columns to the right on the input, there is no output because the input element cannot fill the window (unless we add another column of padding).



In general, when the stride for the height is s_h and the stride for the width is s_w, the output shape is

$$\lfloor(n_\mathrm{h}-k_\mathrm{h}+p_\mathrm{h}+s_\mathrm{h})/s_\mathrm{h}\rfloor \times \lfloor(n_\mathrm{w}-k_\mathrm{w}+p_\mathrm{w}+s_\mathrm{w})/s_\mathrm{w}\rfloor.$$

If we set $p_\mathrm{h}=k_\mathrm{h}-1$ and $p_\mathrm{w}=k_\mathrm{w}-1$, then the output shape can be simplified to $\lfloor(n_\mathrm{h}+s_\mathrm{h}-1)/s_\mathrm{h}\rfloor \times \lfloor(n_\mathrm{w}+s_\mathrm{w}-1)/s_\mathrm{w}\rfloor$. Going a step further, if the input height and width are divisible by the strides on the height and width, then the output shape will be $(n_\mathrm{h}/s_\mathrm{h}) \times (n_\mathrm{w}/s_\mathrm{w})$.

Below, we [**set the strides on both the height and width to 2**], thus halving the input height and width.

```
conv2d = nn.LazyConv2d(1, kernel_size=3, padding=1, stride=2)
comp_conv2d(conv2d, X).shape
```

```
torch.Size([4, 4])
```

Let's look at a slightly more complicated example.

```
conv2d = nn.LazyConv2d(1, kernel_size=(3, 5), padding=(0, 1), stride=(3, 4))
comp_conv2d(conv2d, X).shape
```

```
torch.Size([2, 2])
```

7.4. Multiple Input and Multiple Output Channels

While we described the multiple channels that comprise each image (e.g., color images have the standard RGB channels to indicate the amount of red, green and blue) and convolutional layers for multiple channels in :numref: `subsec_why-conv-channels`, until now, we simplified all of our numerical examples by working with just a single input and a single output channel. This allowed us to think of our inputs, convolution kernels, and outputs each as two-dimensional tensors.

When we add channels into the mix, our inputs and hidden representations both become three-dimensional tensors. For example, each RGB input image has shape $3 \times h \times w$. We refer to this axis, with a size of 3, as the *channel* dimension. The notion of channels is as old as CNNs themselves: for instance LeNet-5 :cite: `LeCun.Jackel.Bottou.ea.1995` uses them. In this section, we will take a deeper look at convolution kernels with multiple input and multiple output channels.

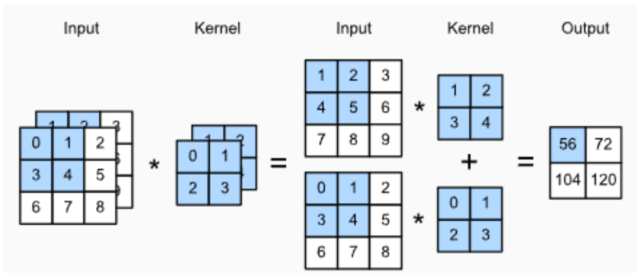
```
import torch
from d2l import torch as d2l
```

7.4.1. Multiple Input Channels

When the input data contains multiple channels, we need to construct a convolution kernel with the same number of input channels as the input data, so that it can perform cross-correlation with the input data. Assuming that the number of channels for the input data is $c_\textrm{i}$, the number of input channels of the convolution kernel also needs to be $c_\textrm{i}$. If our convolution kernel's window shape is $k_\textrm{h}\times k_\textrm{w}$, then, when $c_\textrm{i}=1$, we can think of our convolution kernel as just a two-dimensional tensor of shape $k_\textrm{h}\times k_\textrm{w}$.

However, when $c_\textrm{i}>1$, we need a kernel that contains a tensor of shape $k_\textrm{h}\times k_\textrm{w}$ for *every* input channel. Concatenating these $c_\textrm{i}$ tensors together yields a convolution kernel of shape $c_\textrm{i}\times k_\textrm{h}\times k_\textrm{w}$. Since the input and convolution kernel each have $c_\textrm{i}$ channels, we can perform a cross-correlation operation on the two-dimensional tensor of the input and the two-dimensional tensor of the convolution kernel for each channel, adding the $c_\textrm{i}$ results together (summing over the channels) to yield a two-dimensional tensor. This is the result of a two-dimensional cross-correlation between a multi-channel input and a multi-input-channel convolution kernel.

:numref: `fig_conv_multi_in` provides an example of a two-dimensional cross-correlation with two input channels. The shaded portions are the first output element as well as the input and kernel tensor elements used for the output computation: $(1\times1+2\times2+4\times3+5\times4)+(0\times0+1\times1+3\times2+4\times3)=56$.



To make sure we really understand what is going on here, we can implement cross-correlation operations with multiple input channels ourselves. Notice that all we are doing is performing a cross-correlation operation per channel and then adding up the results.

```
def corr2d_multi_in(X, K):
    # Iterate through the 0th dimension (channel) of K first, then add them up
    return sum(d2l.corr2d(x, k) for x, k in zip(X, K))
```

We can construct the input tensor X and the kernel tensor K corresponding to the values in Fig. 7.4.1 to validate the output of the cross-correlation operation.

```
X = torch.tensor([[[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]],
               [[1.0, 2.0, 3.0], [4.0, 5.0, 6.0], [7.0, 8.0, 9.0]]])
K = torch.tensor([[[0.0, 1.0], [2.0, 3.0]], [[1.0, 2.0], [3.0, 4.0]]])

corr2d_multi_in(X, K)
```

```
tensor([[ 56.,  72.],
        [104., 120.]])
```

7.4.2. Multiple Output Channels

Regardless of the number of input channels, so far we always ended up with one output channel. However, as we discussed in :numref: `subsec_why-conv-channels`, it turns out to be essential to have multiple channels at each layer. In the most popular neural network architectures, we actually increase the channel dimension as we go deeper in the neural network, typically downsampling to trade off spatial resolution for greater *channel depth*. Intuitively, you could think of each channel as responding to a different set of features. The reality is a bit more complicated than this. A naive interpretation would suggest that representations are learned independently per pixel or per channel. Instead, channels are optimized to be jointly useful. This means that rather than mapping a single channel to an edge detector, it may simply mean that some direction in channel space corresponds to detecting edges.

Denote by $c_\textrm{i}$ and $c_\textrm{o}$ the number of input and output channels, respectively, and by $k_\textrm{h}$ and $k_\textrm{w}$ the height and width of the kernel. To get an output with multiple channels, we can create a kernel tensor of shape $c_\textrm{i}\times k_\textrm{h}\times k_\textrm{w}$ for *every* output channel. We concatenate them on the output channel dimension, so that the shape of the convolution kernel is $c_\textrm{o}\times c_\textrm{i}\times k_\textrm{h}\times k_\textrm{w}$. In cross-correlation operations, the result on each output channel is calculated from the convolution kernel corresponding to that output channel and takes input from all channels in the input tensor.

We implement a cross-correlation function to [**calculate the output of multiple channels**] as shown below.

```
def corr2d_multi_in_out(X, K):
    # iterate through the 0th dimension of K, and each time, perform
    # cross-correlation operations with input X. All of the results are
    # stacked together
    return torch.stack([corr2d_multi_in(X, k) for k in K], 0)
```

We construct a trivial convolution kernel with three output channels by concatenating the kernel tensor for K with K+1 and K+2.

```
K = torch.stack((K, K + 1, K + 2), 0)
K.shape
```

```
torch.Size([3, 2, 2, 2])
```

Below, we perform cross-correlation operations on the input tensor X with the kernel tensor K. Now the output contains three channels. The result of the first channel is consistent with the result of the previous input tensor X and the multi-input channel, single-output channel kernel.

```
corr2d_multi_in_out(X, K)
```

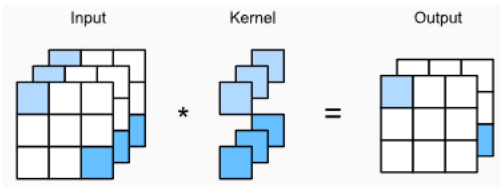
```
tensor([[[ 56.,  72.],
         [104., 120.]],

        [[ 76., 100.],
         [148., 172.]],

        [[ 96., 128.],
         [192., 224.]]])
```

7.4.3 1×1 Convolutional Layer

At first, a [1×1 **convolution**], i.e., $k_\mathrm{h} = k_\mathrm{w} = 1$, does not seem to make much sense. After all, a convolution correlates adjacent pixels. A 1×1 convolution obviously does not. Nonetheless, they are popular operations that are sometimes included in the designs of complex deep networks :cite:`Lin.Chen.Yan.2013,Szegedy.Ioffe.Vanhoucke.ea.2017`. Let's see in some detail what it actually does.

Because the minimum window is used, the 1×1 convolution loses the ability of larger convolutional layers to recognize patterns consisting of interactions among adjacent elements in the height and width dimensions. The only computation of the 1×1 convolution occurs on the channel dimension.

:numref:`fig_conv_1x1` shows the cross-correlation computation using the 1×1 convolution kernel with 3 input channels and 2 output channels. Note that the inputs and outputs have the same height and width. Each element in the output is derived from a linear combination of elements *at the same position* in the input image. You could think of the 1×1 convolutional layer as constituting a fully connected layer applied at every single pixel location to transform the c_i corresponding input values into c_o output values. Because this is still a convolutional layer, the weights are tied across pixel location. Thus the 1×1 convolutional layer requires $c_\mathrm{o} \times c_\mathrm{i}$ weights (plus the bias). Also note that convolutional layers are typically followed by nonlinearities. This ensures that 1×1 convolutions cannot simply be folded into other convolutions.



Let's check whether this works in practice: we implement a 1×1 convolution using a fully connected layer. The only thing is that we need to make some adjustments to the data shape before and after the matrix multiplication.

```
def corr2d_multi_in_out_1x1(X, K):
    c_i, h, w = X.shape
    c_o = K.shape[0]
    X = X.reshape((c_i, h * w))
    K = K.reshape((c_o, c_i))
    # Matrix multiplication in the fully connected layer
    Y = torch.matmul(K, X)
    return Y.reshape((c_o, h, w))
```

When performing 1×1 convolutions, the above function is equivalent to the previously implemented cross-correlation function `corr2d_multi_in_out`. Let's check this with some sample data.

```
X = torch.normal(0, 1, (3, 3, 3))
K = torch.normal(0, 1, (2, 3, 1, 1))
Y1 = corr2d_multi_in_out_1x1(X, K)
```

```
Y2 = corr2d_multi_in_out(X, K)
assert float(torch.abs(Y1 - Y2).sum()) < 1e-6
```

7.5. Pooling

In many cases our ultimate task asks some global question about the image, e.g., does it contain a cat? Consequently, the units of our final layer should be sensitive to the entire input. By gradually aggregating information, yielding coarser and coarser maps, we accomplish this goal of ultimately learning a global representation, while keeping all of the advantages of convolutional layers at the intermediate layers of processing. The deeper we go in the network, the larger the receptive field (relative to the input) to which each hidden node is sensitive. Reducing spatial resolution accelerates this process, since the convolution kernels cover a larger effective area.

Moreover, when detecting lower-level features, such as edges (as discussed in Section 7.2), we often want our representations to be somewhat invariant to translation. For instance, if we take the image X with a sharp delineation between black and white and shift the whole image by one pixel to the right, i.e., Z[i, j] = X[i, j + 1], then the output for the new image Z might be vastly different. The edge will have shifted by one pixel. In reality, objects hardly ever occur exactly at the same place. In fact, even with a tripod and a stationary object, vibration of the camera due to the movement of the shutter might shift everything by a pixel or so (high-end cameras are loaded with special features to address this problem).

This section introduces pooling layers, which serve the dual purposes of mitigating the sensitivity of convolutional layers to location and of spatially downsampling representations.

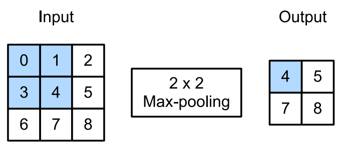
```
import torch
from torch import nn
from d2l import torch as d2l
```

7.5.1. Maximum Pooling and Average Pooling

Like convolutional layers, *pooling* operators consist of a fixed-shape window that is slid over all regions in the input according to its stride, computing a single output for each location traversed by the fixed-shape window (sometimes known as the *pooling window*). However, unlike the cross-correlation computation of the inputs and kernels in the convolutional layer, the pooling layer contains no parameters (there is no *kernel*). Instead, pooling operators are deterministic, typically calculating either the maximum or the average value of the elements in the pooling window. These operations are called *maximum pooling* (*max-pooling* for short) and *average pooling*, respectively.

Average pooling is essentially as old as CNNs. The idea is akin to downsampling an image. Rather than just taking the value of every second (or third) pixel for the lower resolution image, we can average over adjacent pixels to obtain an image with better signal-to-noise ratio since we are combining the information from multiple adjacent pixels. *Max-pooling* was introduced in :citet: `Riesenhuber.Poggio.1999` in the context of cognitive neuroscience to describe how information aggregation might be aggregated hierarchically for the purpose of object recognition; there already was an earlier version in speech recognition :cite: `Yamaguchi.Sakamoto.Akabane.ea.1990`. In almost all cases, max-pooling, as it is also referred to, is preferable to average pooling.

In both cases, as with the cross-correlation operator, we can think of the pooling window as starting from the upper-left of the input tensor and sliding across it from left to right and top to bottom. At each location that the pooling window hits, it computes the maximum or average value of the input subtensor in the window, depending on whether max or average pooling is employed.



The output tensor in :numref: `fig_pooling` has a height of 2 and a width of 2. The four elements are derived from the maximum value in each pooling window:

$$\begin{aligned} \max(0, 1, 3, 4) &= 4,\\ \max(1, 2, 4, 5) &= 5,\\ \max(3, 4, 6, 7) &= 7,\\ \max(4, 5, 7, 8) &= 8. \end{aligned}$$

More generally, we can define a $p \times q$ pooling layer by aggregating over a region of said size. Returning to the problem of edge detection, we use the output of the convolutional layer as input for 2×2 max-pooling. Denote by `X` the input of the convolutional layer input and `Y` the pooling layer output. Regardless of whether or not the values of `X[i, j]`, `X[i, j + 1]`, `X[i+1, j]` and `X[i+1, j + 1]` are different, the pooling layer always outputs `Y[i, j] = 1`. That is to say, using the 2×2 max-pooling layer, we can still detect if the pattern recognized by the convolutional layer moves no more than one element in height or width.

In the code below, we (**implement the forward propagation of the pooling layer**) in the `pool2d` function. This function is similar to the `corr2d` function in :numref: `sec_conv_layer`. However, no kernel is needed, computing the output as either the maximum or the average of each region in the input.

```
def pool2d(X, pool_size, mode='max'):
    p_h, p_w = pool_size
    Y = torch.zeros((X.shape[0] - p_h + 1, X.shape[1] - p_w + 1))
    for i in range(Y.shape[0]):
        for j in range(Y.shape[1]):
            if mode == 'max':
                Y[i, j] = X[i: i + p_h, j: j + p_w].max()
            elif mode == 'avg':
                Y[i, j] = X[i: i + p_h, j: j + p_w].mean()
    return Y
```

We can construct the input tensor X in Fig. 7.5.1 to validate the output of the two-dimensional max-pooling layer.

```
X = torch.tensor([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]])
pool2d(X, (2, 2))
```

```
tensor([[4., 5.],
        [7., 8.]])
```

Also, we can experiment with the average pooling layer.

```
pool2d(X, (2, 2), 'avg')
```

```
tensor([[2., 3.],
        [5., 6.]])
```

7.5.2. Padding and Stride

As with convolutional layers, pooling layers change the output shape. And as before, we can adjust the operation to achieve a desired output shape by padding the input and adjusting the stride. We can demonstrate the use of padding and strides in pooling layers via the built-in two-dimensional max-pooling layer from the deep learning framework. We first construct an input tensor X whose shape has four dimensions, where the number of examples (batch size) and number of channels are both 1.

```
X = torch.arange(16, dtype=torch.float32).reshape((1, 1, 4, 4))
X
```

```
tensor([[[[ 0.,  1.,  2.,  3.],
          [ 4.,  5.,  6.,  7.],
          [ 8.,  9., 10., 11.],
          [12., 13., 14., 15.]]]])
```

Since pooling aggregates information from an area, deep learning frameworks default to matching pooling window sizes and stride. For instance, if we use a pooling window of shape (3, 3) we get a stride shape of (3, 3) by default.

```
pool2d = nn.MaxPool2d(3)
# Pooling has no model parameters, hence it needs no initialization
pool2d(X)
```

```
tensor([[[[10.]]]])
```

Needless to say, the stride and padding can be manually specified to override framework defaults if required.

```
pool2d = nn.MaxPool2d(3, padding=1, stride=2)
pool2d(X)
```

```
tensor([[[[ 5.,  7.],
          [13., 15.]]]])
```

Of course, we can specify an arbitrary rectangular pooling window with arbitrary height and width respectively, as the example below shows.

```
pool2d = nn.MaxPool2d((2, 3), stride=(2, 3), padding=(0, 1))
pool2d(X)
```

```
tensor([[[[ 5.,  7.],
          [13., 15.]]]])
```

7.5.3. Multiple Channels

When processing multi-channel input data, the pooling layer pools each input channel separately, rather than summing the inputs up over channels as in a convolutional layer. This means that the number of output channels for the pooling layer is the same as the number of input channels. Below, we will concatenate tensors X and X + 1 on the channel dimension to construct an input with two channels.

```
X = torch.cat((X, X + 1), 1)
X
```

```
tensor([[[[ 0.,  1.,  2.,  3.],
          [ 4.,  5.,  6.,  7.],
          [ 8.,  9., 10., 11.],
          [12., 13., 14., 15.]],

         [[ 1.,  2.,  3.,  4.],
          [ 5.,  6.,  7.,  8.],
          [ 9., 10., 11., 12.],
          [13., 14., 15., 16.]]]])
```

As we can see, the number of output channels is still two after pooling.

```
pool2d = nn.MaxPool2d(3, padding=1, stride=2)
pool2d(X)
```

```
tensor([[[[ 5.,  7.],
          [13., 15.]],

         [[ 6.,  8.],
          [14., 16.]]]])
```

7.6. Convolutional Neural Networks (LeNet)

We now have all the ingredients required to assemble a fully-functional CNN. In our earlier encounter with image data, we applied a linear model with softmax regression (:numref: `sec_softmax_scratch`) and an MLP (:numref: `sec_mlp-implementation`) to pictures of clothing in the Fashion-MNIST dataset. To make such data amenable we first flattened each image from a 28×28 matrix into a fixed-length 784-dimensional vector, and thereafter processed them in fully connected layers. Now that we have a handle on convolutional layers, we can retain the spatial structure in our images. As an additional benefit of replacing fully connected layers with convolutional layers, we will enjoy more parsimonious models that require far fewer parameters.

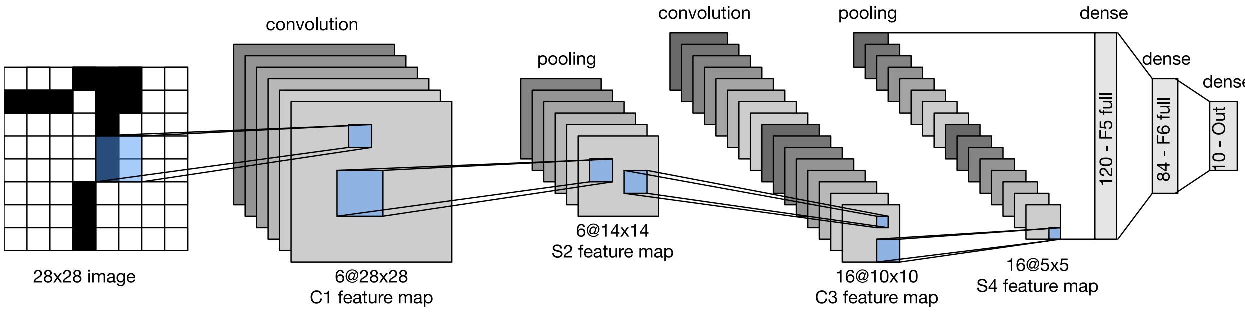
In this section, we will introduce *LeNet*, among the first published CNNs to capture wide attention for its performance on computer vision tasks. The model was introduced by (and named for) Yann LeCun, then a researcher at AT&T Bell Labs, for the purpose of recognizing handwritten digits in images :cite: `LeCun.Bottou.Bengio.ea.1998`. This work represented the culmination of a decade of research developing the technology; LeCun's team published the first study to successfully train CNNs via backpropagation :cite: `LeCun.Boser.Denker.ea.1989`.

At the time LeNet achieved outstanding results matching the performance of support vector machines, then a dominant approach in supervised learning, achieving an error rate of less than 1% per digit. LeNet was eventually adapted to recognize digits for processing deposits in ATM machines. To this day, some ATMs still run the code that Yann LeCun and his colleague Leon Bottou wrote in the 1990s!

```
import torch
from torch import nn
from d2l import torch as d2l
```

7.6.1. LeNet

At a high level, (**LeNet (LeNet-5) consists of two parts: (i) a convolutional encoder consisting of two convolutional layers; and (ii) a dense block consisting of three fully connected layers**). The architecture is summarized in :numref: `img_lenet`.



The basic units in each convolutional block are a convolutional layer, a sigmoid activation function, and a subsequent average pooling operation. Note that while ReLUs and max-pooling work better, they had not yet been discovered. Each convolutional layer uses a 5×5 kernel and a sigmoid activation function. These layers map spatially arranged inputs to a number of two-dimensional feature maps, typically increasing the number of channels. The first convolutional layer has 6 output channels, while the second has 16. Each 2×2 pooling operation (stride 2) reduces dimensionality by a factor of 4 via spatial downsampling. The convolutional block emits an output with shape given by (batch size, number of channel, height, width).

In order to pass output from the convolutional block to the dense block, we must flatten each example in the minibatch. In other words, we take this four-dimensional input and transform it into the two-dimensional input expected by fully connected layers: as a reminder, the two-dimensional representation that we desire uses the first dimension to index examples in the minibatch and the second to give the flat vector representation of each example. LeNet's dense block has three fully connected layers, with 120, 84, and 10 outputs, respectively. Because we are still performing classification, the 10-dimensional output layer corresponds to the number of possible output classes.

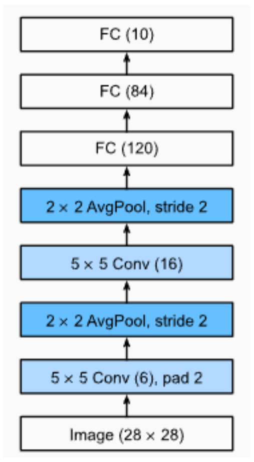
While getting to the point where you truly understand what is going on inside LeNet may have taken a bit of work, we hope that the following code snippet will convince you that implementing such models with modern deep learning frameworks is remarkably simple. We need only to instantiate a `Sequential` block and chain together the appropriate layers, using Xavier initialization as introduced in :numref: `subsec_xavier`.

```
def init_cnn(module):
    """Initialize weights for CNNs."""
    if type(module) == nn.Linear or type(module) == nn.Conv2d:
        nn.init.xavier_uniform_(module.weight)
```

```
class LeNet(d2l.Classifier):
    """The LeNet-5 model."""
    def __init__(self, lr=0.1, num_classes=10):
        super().__init__()
        self.save_hyperparameters()
        self.net = nn.Sequential(
            nn.LazyConv2d(6, kernel_size=5, padding=2), nn.Sigmoid(),
            nn.AvgPool2d(kernel_size=2, stride=2),
            nn.LazyConv2d(16, kernel_size=5), nn.Sigmoid(),
            nn.AvgPool2d(kernel_size=2, stride=2),
            nn.Flatten(),
            nn.LazyLinear(120), nn.Sigmoid(),
            nn.LazyLinear(84), nn.Sigmoid(),
            nn.LazyLinear(num_classes))
```

We have taken some liberty in the reproduction of LeNet insofar as we have replaced the Gaussian activation layer by a softmax layer. This greatly simplifies the implementation, not least due to the fact that the Gaussian decoder is rarely used nowadays. Other than that, this network matches the original LeNet-5 architecture.

Let's see what happens inside the network. By passing a single-channel (black and white) 28×28 image through the network and printing the output shape at each layer, we can [**inspect the model**] to ensure that its operations line up with what we expect from :numref: `img_lenet_vert`.



```
@d2l.add_to_class(d2l.Classifier)
def layer_summary(self, X_shape):
    X = torch.randn(*X_shape)
    for layer in self.net:
        X = layer(X)
        print(layer.__class__.__name__, 'output shape:\t', X.shape)

model = LeNet()
model.layer_summary((1, 1, 28, 28))
```

```
Conv2d output shape:     torch.Size([1, 6, 28, 28])
Sigmoid output shape:    torch.Size([1, 6, 28, 28])
AvgPool2d output shape:  torch.Size([1, 6, 14, 14])
Conv2d output shape:     torch.Size([1, 16, 10, 10])
Sigmoid output shape:    torch.Size([1, 16, 10, 10])
AvgPool2d output shape:  torch.Size([1, 16, 5, 5])
Flatten output shape:    torch.Size([1, 400])
Linear output shape:     torch.Size([1, 120])
Sigmoid output shape:    torch.Size([1, 120])
Linear output shape:     torch.Size([1, 84])
Sigmoid output shape:    torch.Size([1, 84])
Linear output shape:     torch.Size([1, 10])
```

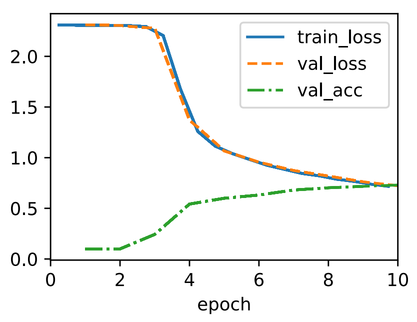
7.6.2. Training

Now that we have implemented the model, let’s run an experiment to see how the LeNet-5 model fares on Fashion-MNIST.

While CNNs have fewer parameters, they can still be more expensive to compute than similarly deep MLPs because each parameter participates in many more multiplications. If you have access to a GPU, this might be a good time to put it into action to speed up training. Note that the d2l.Trainer class takes care of all details. By default, it initializes the model parameters on the available devices. Just as with MLPs, our loss function is cross-entropy, and we minimize it via minibatch stochastic gradient descent.

```
trainer = d2l.Trainer(max_epochs=10, num_gpus=1)
data = d2l.FashionMNIST(batch_size=128)
```

```
model = LeNet(lr=0.1)
model.apply_init([next(iter(data.get_dataloader(True)))[0]], init_cnn)
trainer.fit(model, data)
```



Discussions & Exercises

7.1. Discussion

Invariance: Translation invariance allows the system to detect objects irrespective of their exact location.

Locality: Early layers of CNNs focus on local regions of the image.

Convolution: By using a convolutional operation, the number of parameters can be drastically reduced, making the network more efficient.

Channels: Images are typically represented with multiple channels (e.g., RGB), and CNNs account for this by having multiple input and hidden channels (feature maps).

CNNs balance reducing parameters while maintaining the ability to capture important patterns in data by leveraging inductive biases like translation invariance and locality, leading to practical models that generalize well.

- Importance of Convolutions in Image Processing: The shift from MLPs to CNNs is crucial for handling large-scale perceptual data, especially high-resolution images. Understanding how convolutions reduce parameters and make image processing feasible is central to modern AI applications.
- Translation Invariance: How CNNs handle translation invariance is an important topic to explore further. How does this property make CNNs so effective for tasks like image classification and object detection?
- Channels and Feature Maps: The concept of channels (RGB) and their evolution into feature maps through convolutional layers can be discussed in more depth. How do CNNs learn to detect different features, like edges, textures, and complex objects through multiple layers?
- Efficient Model Design: Further exploration of how CNNs maintain efficiency by using fewer parameters and focusing on locality might be important in areas like hardware optimization and edge computing.

7.1 Exercises & My Own Experiments

```
import tensorflow as tf
from tensorflow.keras import datasets, layers, models
import matplotlib.pyplot as plt

# MNIST 데이터셋을 불러오고 전처리합니다.
(train_images, train_labels), (test_images, test_labels) = datasets.mnist.load_data()

# 데이터를 CNN에 맞게 (28, 28, 1) 모양으로 reshape하고 normalize(0~1 범위로 변경)합니다.
train_images = train_images.reshape((train_images.shape[0], 28, 28, 1)).astype('float32') / 255
test_images = test_images.reshape((test_images.shape[0], 28, 28, 1)).astype('float32') / 255

# 각 이미지의 라벨을 one-hot encoding 합니다.
train_labels = tf.keras.utils.to_categorical(train_labels)
test_labels = tf.keras.utils.to_categorical(test_labels)



model = models.Sequential()

# 첫 번째 합성곱 레이어: 필터 수 32, 커널 크기 (3, 3), 활성화 함수 ReLU
model.add(layers.Conv2D(32, (3, 3), activation='relu', input_shape=(28, 28, 1)))
model.add(layers.MaxPooling2D((2, 2)))

# 두 번째 합성곱 레이어: 필터 수 64, 커널 크기 (3, 3), 활성화 함수 ReLU
model.add(layers.Conv2D(64, (3, 3), activation='relu'))
model.add(layers.MaxPooling2D((2, 2)))

# 세 번째 합성곱 레이어: 필터 수 64, 커널 크기 (3, 3), 활성화 함수 ReLU
model.add(layers.Conv2D(64, (3, 3), activation='relu'))

# 완전 연결 층을 위해 1차원으로 펼치기
model.add(layers.Flatten())

# 완전 연결 층 (Dense Layer): 뉴런 수 64, 활성화 함수 ReLU
model.add(layers.Dense(64, activation='relu'))

# 출력 레이어: 뉴런 수 10 (0~9 숫자 클래스), 활성화 함수 softmax
model.add(layers.Dense(10, activation='softmax'))

# 모델 요약 출력
model.summary()
```

```
Model: "sequential_1"
┏━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━┳━━━━━━━━━━━━━━━━━━━━━━━━━━━━━┳━━━━━━━━━━━━━━━━┓
┃ Layer (type)                      ┃ Output Shape                ┃        Param # ┃
┡━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━╇━━━━━━━━━━━━━━━━━━━━━━━━━━━━━╇━━━━━━━━━━━━━━━━┩
│ conv2d_2 (Conv2D)                 │ (None, 26, 26, 32)          │            320 │
├───────────────────────────────────┼─────────────────────────────┼────────────────┤
│ max_pooling2d_2 (MaxPooling2D)    │ (None, 13, 13, 32)          │              0 │
├───────────────────────────────────┼─────────────────────────────┼────────────────┤
│ conv2d_3 (Conv2D)                 │ (None, 11, 11, 64)          │         18,496 │
├───────────────────────────────────┼─────────────────────────────┼────────────────┤
│ max_pooling2d_3 (MaxPooling2D)    │ (None, 5, 5, 64)            │              0 │
├───────────────────────────────────┼─────────────────────────────┼────────────────┤
│ conv2d_4 (Conv2D)                 │ (None, 3, 3, 64)            │         36,928 │
├───────────────────────────────────┼─────────────────────────────┼────────────────┤
│ flatten_1 (Flatten)               │ (None, 576)                 │              0 │
├───────────────────────────────────┼─────────────────────────────┼────────────────┤
│ dense_2 (Dense)                   │ (None, 64)                  │         36,928 │
├───────────────────────────────────┼─────────────────────────────┼────────────────┤
│ dense_3 (Dense)                   │ (None, 10)                  │            650 │
└───────────────────────────────────┴─────────────────────────────┴────────────────┘
 Total params: 93,322 (364.54 KB)
 Trainable params: 93,322 (364.54 KB)
 Non-trainable params: 0 (0.00 B)
```

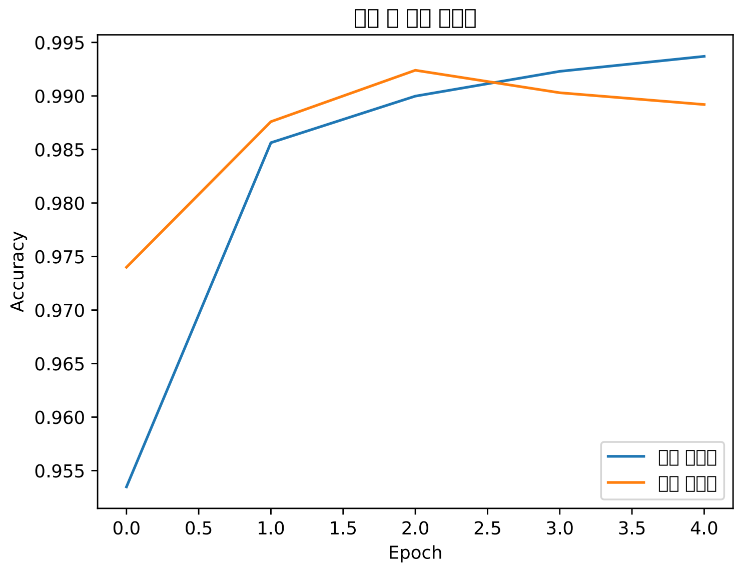
```
# 모델 컴파일
model.compile(optimizer='adam',
              loss='categorical_crossentropy',
              metrics=['accuracy'])

# 모델 훈련
history = model.fit(train_images, train_labels, epochs=5,
                    validation_data=(test_images, test_labels))
```

```
Epoch 1/5
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 67s 34ms/step - accuracy: 0.8878 - loss: 0.3543 - val_accuracy: 0.9740 - val_loss: 0.0847
Epoch 2/5
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 94s 41ms/step - accuracy: 0.9850 - loss: 0.0487 - val_accuracy: 0.9876 - val_loss: 0.0378
Epoch 3/5
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 72s 35ms/step - accuracy: 0.9898 - loss: 0.0324 - val_accuracy: 0.9924 - val_loss: 0.0259
Epoch 4/5
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 83s 36ms/step - accuracy: 0.9926 - loss: 0.0228 - val_accuracy: 0.9903 - val_loss: 0.0321
Epoch 5/5
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 82s 36ms/step - accuracy: 0.9938 - loss: 0.0198 - val_accuracy: 0.9892 - val_loss: 0.0388
```

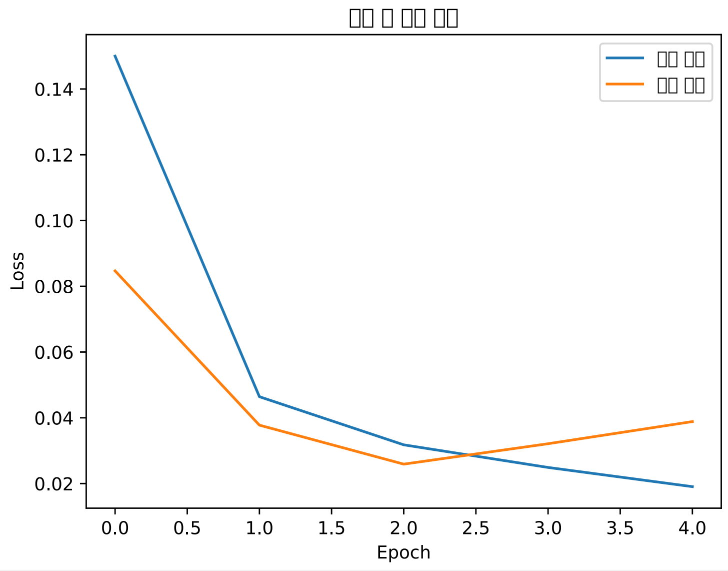
```
# 테스트 데이터로 모델 평가
test_loss, test_acc = model.evaluate(test_images, test_labels, verbose=2)
print(f"테스트 정확도: {test_acc}")

# 훈련 및 검증 과정에서의 정확도와 손실 그래프 시각화
plt.plot(history.history['accuracy'], label='훈련 정확도')
plt.plot(history.history['val_accuracy'], label='검증 정확도')
plt.title('훈련 및 검증 정확도')
plt.xlabel('Epoch')
plt.ylabel('Accuracy')
plt.legend(loc='lower right')
plt.show()

plt.plot(history.history['loss'], label='훈련 손실')
plt.plot(history.history['val_loss'], label='검증 손실')
plt.title('훈련 및 검증 손실')
plt.xlabel('Epoch')
plt.ylabel('Loss')
```

```
    plt.legend(loc='upper right')
    plt.show()
```

```
313/313 - 3s - 9ms/step - accuracy: 0.9892 - loss: 0.0388
테스트 정확도: 0.9891999959945679
/usr/local/lib/python3.10/dist-packages/IPython/core/pylabtools.py:151: UserWarning: Glyph 54984 (\N{HANGUL SYLLABLE HUN}) missing from current
  fig.canvas.print_figure(bytes_io, **kw)
/usr/local/lib/python3.10/dist-packages/IPython/core/pylabtools.py:151: UserWarning: Glyph 47144 (\N{HANGUL SYLLABLE RYEON}) missing from curren
  fig.canvas.print_figure(bytes_io, **kw)
/usr/local/lib/python3.10/dist-packages/IPython/core/pylabtools.py:151: UserWarning: Glyph 48143 (\N{HANGUL SYLLABLE MIC}) missing from current
  fig.canvas.print_figure(bytes_io, **kw)
/usr/local/lib/python3.10/dist-packages/IPython/core/pylabtools.py:151: UserWarning: Glyph 44160 (\N{HANGUL SYLLABLE GEOM}) missing from current
  fig.canvas.print_figure(bytes_io, **kw)
/usr/local/lib/python3.10/dist-packages/IPython/core/pylabtools.py:151: UserWarning: Glyph 51613 (\N{HANGUL SYLLABLE JEUNG}) missing from curren
  fig.canvas.print_figure(bytes_io, **kw)
/usr/local/lib/python3.10/dist-packages/IPython/core/pylabtools.py:151: UserWarning: Glyph 51221 (\N{HANGUL SYLLABLE JEONG}) missing from curren
  fig.canvas.print_figure(bytes_io, **kw)
/usr/local/lib/python3.10/dist-packages/IPython/core/pylabtools.py:151: UserWarning: Glyph 54869 (\N{HANGUL SYLLABLE HWAG}) missing from current
  fig.canvas.print_figure(bytes_io, **kw)
/usr/local/lib/python3.10/dist-packages/IPython/core/pylabtools.py:151: UserWarning: Glyph 46020 (\N{HANGUL SYLLABLE DO}) missing from current f
  fig.canvas.print_figure(bytes_io, **kw)
```



```
/usr/local/lib/python3.10/dist-packages/IPython/core/pylabtools.py:151: UserWarning: Glyph 49552 (\N{HANGUL SYLLABLE SON}) missing from current
  fig.canvas.print_figure(bytes_io, **kw)
/usr/local/lib/python3.10/dist-packages/IPython/core/pylabtools.py:151: UserWarning: Glyph 49892 (\N{HANGUL SYLLABLE SIL}) missing from current
  fig.canvas.print_figure(bytes_io, **kw)
```



7.2. Discussion

Edge Detection:

- Concept: Detects edges in images by identifying changes in pixel intensity.
- Kernel: A simple kernel like [1, -1] detects horizontal edges by subtracting adjacent pixels. This works by calculating the difference between neighboring pixels, outputting non-zero values where intensity changes (edges).

- Horizontal and Vertical Edge Detection: Applying the kernel detects horizontal edges, but applying it to the transposed image does not detect vertical edges. Specific kernels are needed for different edge orientations.
- Practical Use: Edge detection is a basic step in image processing and can be used in object recognition, feature extraction, and computer vision tasks.

Learning Kernels:

- Automatic Kernel Learning: Instead of manually defining filters like [1, -1], CNNs learn optimal kernels through training, adjusting the kernel weights to minimize loss during backpropagation.
- Example: In edge detection, the learned kernel might resemble [1, -1], but as we increase the kernel size and layers, the network can detect more complex features automatically.

Receptive Field:

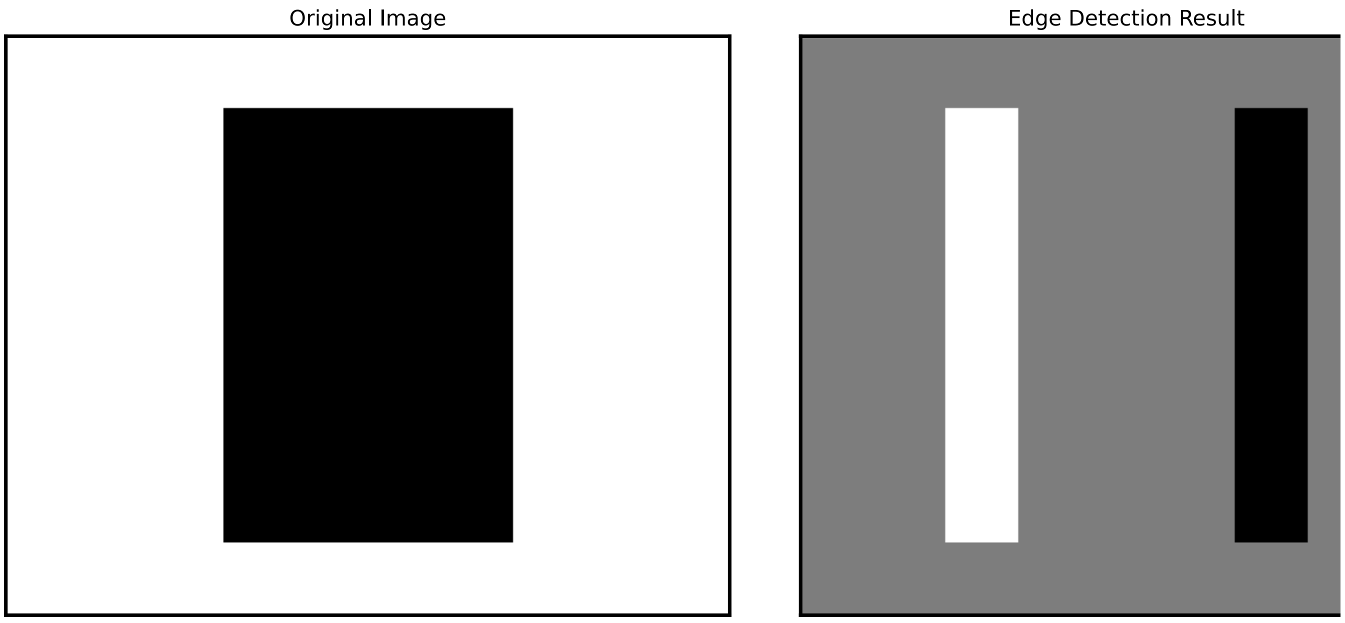
- Definition: The receptive field refers to the portion of the input image that contributes to a single output value in a convolutional layer.
- Importance: As the network deepens, the receptive field increases, allowing the network to detect broader and more global features of the input image.

7.2. Exercises & My Own Experiments

```
import torch
import matplotlib.pyplot as plt
import numpy as np

def corr2d(X, K):
    """Compute 2D cross-correlation."""
    h, w = K.shape
    Y = torch.zeros((X.shape[0] - h + 1, X.shape[1] - w + 1))
    for i in range(Y.shape[0]):
        for j in range(Y.shape[1]):
            Y[i, j] = (X[i:i + h, j:j + w] * K).sum()
    return Y

# Create input tensor with padding
X = torch.ones((8, 10))
X[1:-1, 3:-3] = 0  # The main image content is now 6x4
print("Input image:")
print(X)

# Create kernel for edge detection
K = torch.tensor([[1.0, -1.0]])

# Perform edge detection
Y = corr2d(X, K)
print("WnEdge detection result:")
print(Y)

# Visualize results
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12, 5))

# Function to add a border to the plot
def add_border(ax, image):
    ax.imshow(image, cmap='gray')
    ax.set_xticks([])
    ax.set_yticks([])
    for spine in ax.spines.values():
        spine.set_visible(True)
        spine.set_color('black')
        spine.set_linewidth(2)

add_border(ax1, X.numpy())
ax1.set_title('Original Image')

add_border(ax2, Y.numpy())
ax2.set_title('Edge Detection Result')

plt.tight_layout()
plt.show()

# Learning a kernel
# Convert X and Y to the shape expected by nn.Conv2d
X = X.reshape((1, 1, 8, 10))
Y = Y.reshape((1, 1, 8, 9))

# Create a convolutional layer
conv2d = torch.nn.Conv2d(1, 1, kernel_size=(1, 2), bias=False)
```

```
# Training loop
lr = 3e-2
for i in range(10):
    Y_hat = conv2d(X)
    l = (Y_hat - Y) ** 2
    l.sum().backward()
    # Update the kernel
    conv2d.weight.data[:] -= lr * conv2d.weight.grad
    conv2d.weight.grad.zero_()
    if (i + 1) % 2 == 0:
        print(f'epoch {i + 1}, loss {l.sum():.3f}')

print("WnLearned kernel:")
print(conv2d.weight.data.reshape((1, 2)))
```

```
Input image:
tensor([[1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
        [1., 1., 1., 0., 0., 0., 0., 1., 1., 1.],
        [1., 1., 1., 0., 0., 0., 0., 1., 1., 1.],
        [1., 1., 1., 0., 0., 0., 0., 1., 1., 1.],
        [1., 1., 1., 0., 0., 0., 0., 1., 1., 1.],
        [1., 1., 1., 0., 0., 0., 0., 1., 1., 1.],
        [1., 1., 1., 0., 0., 0., 0., 1., 1., 1.],
        [1., 1., 1., 1., 1., 1., 1., 1., 1., 1.]])

Edge detection result:
tensor([[ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  1.,  0.,  0.,  0., -1.,  0.,  0.],
        [ 0.,  0.,  1.,  0.,  0.,  0., -1.,  0.,  0.],
        [ 0.,  0.,  1.,  0.,  0.,  0., -1.,  0.,  0.],
        [ 0.,  0.,  1.,  0.,  0.,  0., -1.,  0.,  0.],
        [ 0.,  0.,  1.,  0.,  0.,  0., -1.,  0.,  0.],
        [ 0.,  0.,  1.,  0.,  0.,  0., -1.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.]])
```



```
epoch 2, loss 2.079
epoch 4, loss 20.956
epoch 6, loss 7727.283
epoch 8, loss 2896238.000
epoch 10, loss 1085537408.000

Learned kernel:
tensor([[-10804.3555, -10806.3428]])
```

7.3. Discussion

Padding helps preserve the spatial dimensions of the input by adding extra pixels (usually zero) around the boundary. Without padding, the image shrinks after every convolution layer, especially when using larger kernels. Padding ensures that the output height and width are the same as the input, particularly when using odd-sized kernels, which are common in practice.

Stride refers to the step size of the convolution window as it slides over the input. A larger stride reduces the output size because fewer positions are sampled. This is useful for downsampling the input, either for computational efficiency or when reducing the resolution of the input data. Setting stride and padding appropriately helps control the size of the feature map while keeping important features intact.

- With padding:

$$\text{Output shape} = (n_h - k_h + p_h + 1) \times (n_w - k_w + p_w + 1)$$

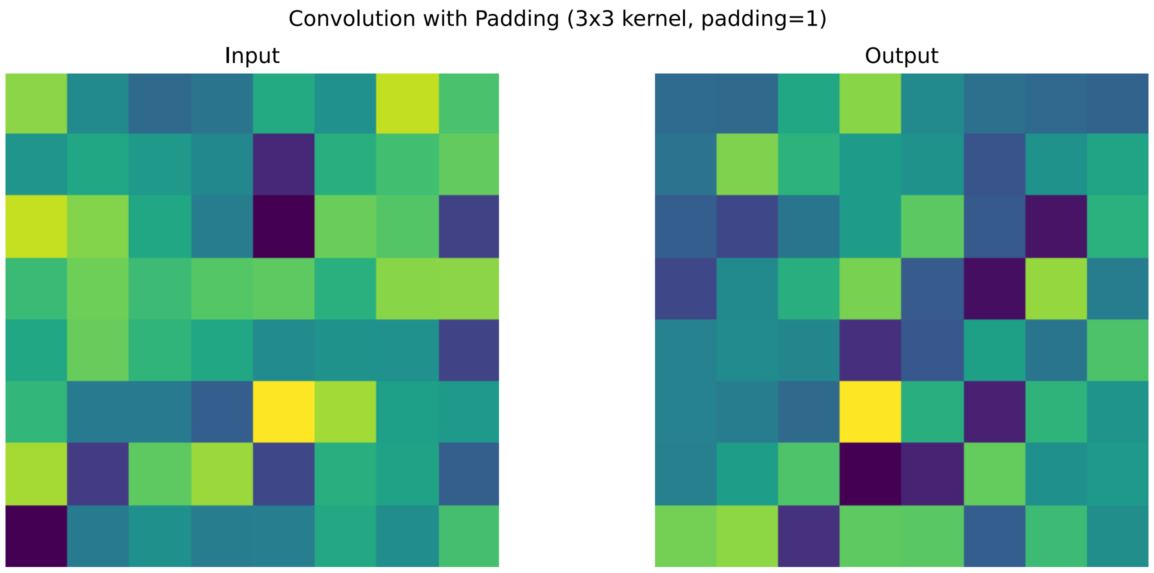
- With stride:

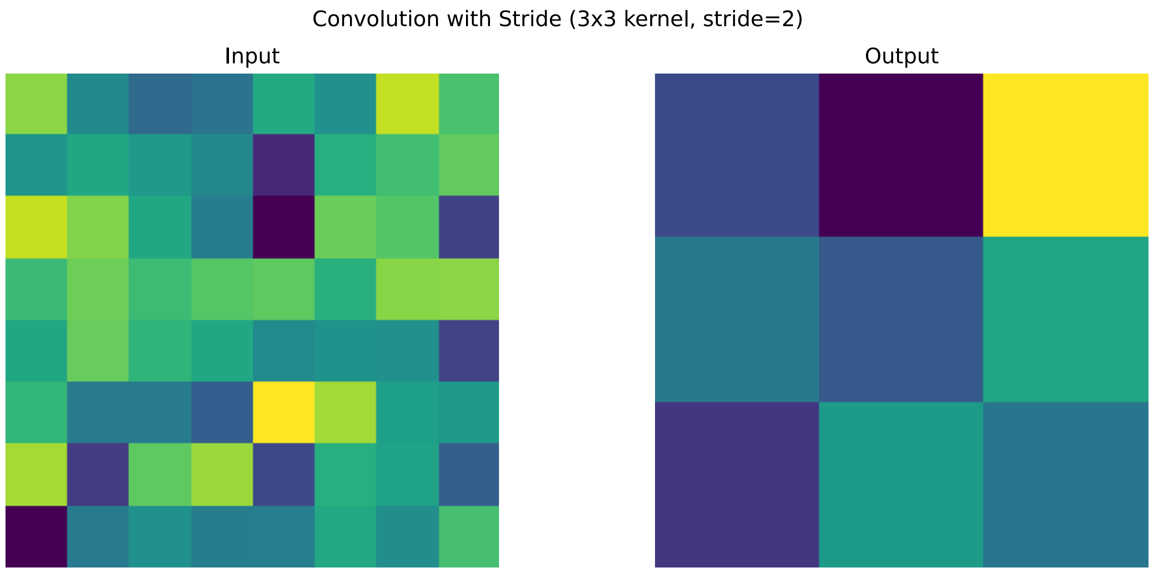
$$\text{Output shape} = \left\lfloor \frac{n_h - k_h + p_h + s_h}{s_h} \right\rfloor \times \left\lfloor \frac{n_w - k_w + p_w + s_w}{s_w} \right\rfloor$$

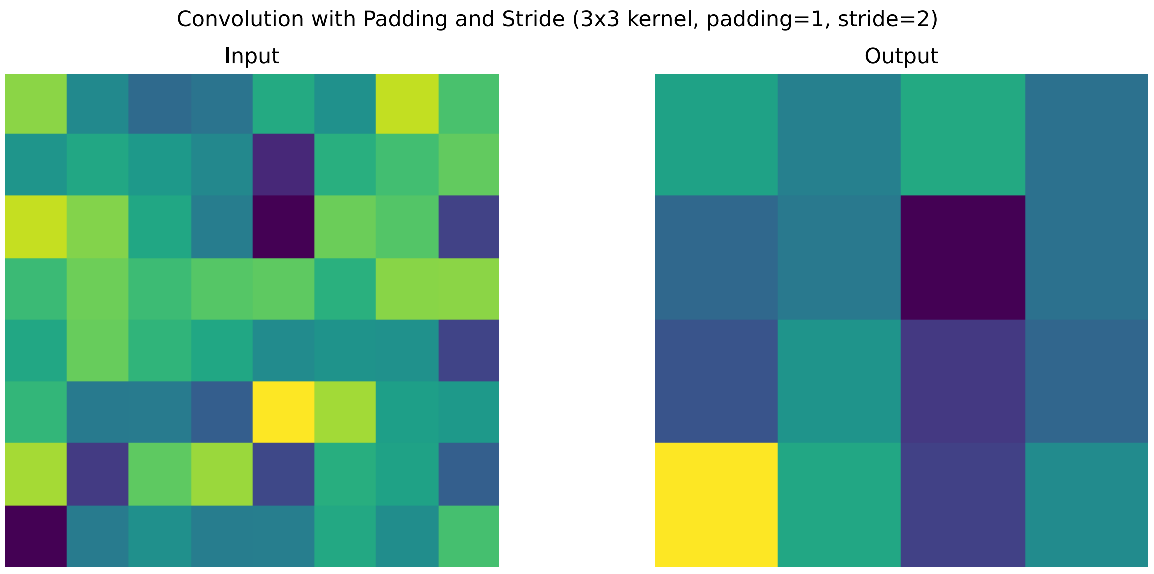
1. **Padding and Stride in Convolutions:** How they directly influence output dimensions and are essential in designing CNN architectures, particularly for applications in image processing, where preserving the size of the feature map or reducing it in a controlled way is important.
2. **Effect of Kernel Size on Output:** Larger kernels can quickly reduce the size of the image representation, potentially losing important edge information, which emphasizes the need for padding. Conversely, strided convolutions allow for downsampling, but they reduce the resolution of the output feature maps.
3. **Balancing Computation and Output Size:** A significant challenge is maintaining a balance between computational efficiency and retaining useful information. Using larger strides or kernels can speed up computations but risks losing important spatial information.

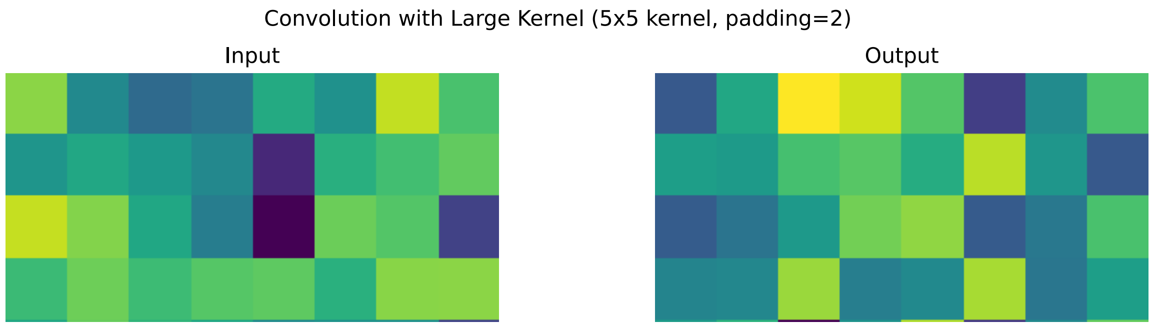
7.3. Exercises & My Own Experiments

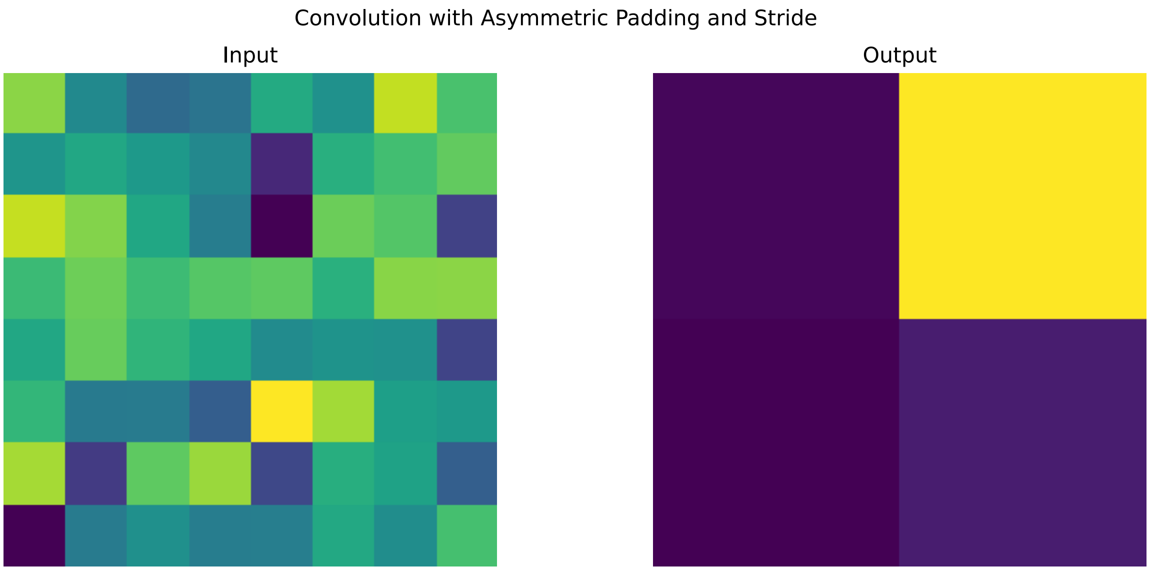
```
import torch
import torch.nn as nn
import matplotlib.pyplot as plt

def visualize_conv2d(input_tensor, conv_layer, title):
    # Apply convolution
    output = conv_layer(input_tensor)

    # Create a figure with two subplots
    fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12, 5))

    # Plot input
    ax1.imshow(input_tensor.squeeze().detach().numpy(), cmap='viridis')
    ax1.set_title('Input')
    ax1.axis('off')

    # Plot output
    ax2.imshow(output.squeeze().detach().numpy(), cmap='viridis')
    ax2.set_title('Output')
    ax2.axis('off')

    plt.suptitle(title)
    plt.show()

# Create a sample input tensor
input_tensor = torch.randn(1, 1, 8, 8)

# Experiment 1: Padding
conv_pad = nn.Conv2d(1, 1, kernel_size=3, padding=1)
visualize_conv2d(input_tensor, conv_pad, "Convolution with Padding (3x3 kernel, padding=1)")

# Experiment 2: Stride
conv_stride = nn.Conv2d(1, 1, kernel_size=3, stride=2)
visualize_conv2d(input_tensor, conv_stride, "Convolution with Stride (3x3 kernel, stride=2)")

# Experiment 3: Combination of Padding and Stride
conv_pad_stride = nn.Conv2d(1, 1, kernel_size=3, padding=1, stride=2)
visualize_conv2d(input_tensor, conv_pad_stride, "Convolution with Padding and Stride (3x3 kernel, padding=1, stride=2)")

# Experiment 4: Different kernel sizes
conv_large_kernel = nn.Conv2d(1, 1, kernel_size=5, padding=2)
visualize_conv2d(input_tensor, conv_large_kernel, "Convolution with Large Kernel (5x5 kernel, padding=2)")

# Experiment 5: Asymmetric padding and stride
conv_asymmetric = nn.Conv2d(1, 1, kernel_size=(3, 5), padding=(0, 1), stride=(3, 4))
visualize_conv2d(input_tensor, conv_asymmetric, "Convolution with Asymmetric Padding and Stride")

print("Experiment complete. Check the visualizations to understand the effects of padding and stride.")
```


Convolution with Padding (3x3 kernel, padding=1)
Input
Output


Convolution with Stride (3x3 kernel, stride=2)
Input
Output


Convolution with Padding and Stride (3x3 kernel, padding=1, stride=2)
Input
Output


Convolution with Large Kernel (5x5 kernel, padding=2)
Input
Output



```
Experiment complete. Check the visualizations to understand the effects of padding and stride.
```

7.4. Discussion

1. **Importance of Multi-Channel Input/Output in CNNs:** Multi-channel input and output are essential for handling the complexity of modern neural networks, particularly in image processing tasks where each channel can represent different features or aspects of the image.
2. **Cross-Correlation with Multiple Channels:** The process of cross-correlation for multi-channel data is a key operation that enables deep learning models to extract richer feature representations from input data.
3. **Significance of 1×1 Convolutions:** Despite its simplicity, the 1×1 convolution plays a critical role in reducing the number of parameters and computational complexity in deep networks. It is also useful for manipulating the depth of feature maps.

7.4. Exercises & My Own Experiments

```
import torch
import torch.nn.functional as F

def corr2d_multi_in(X, K):
    return sum(F.conv2d(x.unsqueeze(0).unsqueeze(0), k.unsqueeze(0).unsqueeze(0)) for x, k in zip(X, K))

def corr2d_multi_in_out(X, K):
    return torch.stack([corr2d_multi_in(X, k) for k in K])

# 1. Multi-input channel convolution
print("1. Multi-input channel convolution")
X = torch.tensor([[[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]],
                  [[1.0, 2.0, 3.0], [4.0, 5.0, 6.0], [7.0, 8.0, 9.0]]])
K = torch.tensor([[[0.0, 1.0], [2.0, 3.0]],
                  [[1.0, 2.0], [3.0, 4.0]]])

print("Input shape:", X.shape)
print("Kernel shape:", K.shape)
Y = corr2d_multi_in(X, K)
print("Output:")
print(Y)
print("Output shape:", Y.shape)

# 2. Multi-input, multi-output channel convolution
print("Wn2. Multi-input, multi-output channel convolution")
K = torch.stack((K, K + 1, K + 2), 0)
print("New kernel shape:", K.shape)
Y = corr2d_multi_in_out(X, K)
```

```
print("Output:")
print(Y)
print("Output shape:", Y.shape)

# 3. 1x1 Convolution
print("\n3. 1x1 Convolution")
def corr2d_multi_in_out_1x1(X, K):
    c_i, h, w = X.shape
    c_o = K.shape[0]
    X = X.reshape((c_i, h * w))
    K = K.reshape((c_o, c_i))
    Y = torch.matmul(K, X)
    return Y.reshape((c_o, h, w))

X = torch.normal(0, 1, (3, 3, 3))
K = torch.normal(0, 1, (2, 3, 1, 1))

Y1 = corr2d_multi_in_out_1x1(X, K)
Y2 = corr2d_multi_in_out(X, K)

print("1x1 Convolution output shape:", Y1.shape)
print("Regular convolution output shape:", Y2.shape)
print("Difference between outputs:", float(torch.abs(Y1 - Y2).sum()))

# 4. Using PyTorch's built-in Conv2d
print("\n4. Using PyTorch's built-in Conv2d")
conv2d = torch.nn.Conv2d(in_channels=3, out_channels=2, kernel_size=1)
Y3 = conv2d(X.unsqueeze(0)).squeeze(0)
print("PyTorch Conv2d output shape:", Y3.shape)
print("Difference between PyTorch and our implementation:",
      float(torch.abs(Y3 - Y1).sum()))
```

```
1. Multi-input channel convolution
Input shape: torch.Size([2, 3, 3])
Kernel shape: torch.Size([2, 2, 2])
Output:
tensor([[[[ 56.,  72.],
          [104., 120.]]]])
Output shape: torch.Size([1, 1, 2, 2])

2. Multi-input, multi-output channel convolution
New kernel shape: torch.Size([3, 2, 2, 2])
Output:
tensor([[[[[ 56.,  72.],
           [104., 120.]]]],



        [[[[ 76., 100.],
           [148., 172.]]]],



        [[[[ 96., 128.],
           [192., 224.]]]]])
Output shape: torch.Size([3, 1, 1, 2, 2])

3. 1x1 Convolution
1x1 Convolution output shape: torch.Size([2, 3, 3])
Regular convolution output shape: torch.Size([2, 1, 1, 3, 3])
Difference between outputs: 20.021825790405273

4. Using PyTorch's built-in Conv2d
PyTorch Conv2d output shape: torch.Size([2, 3, 3])
Difference between PyTorch and our implementation: 14.509613990783691
```

7.5. Discussion

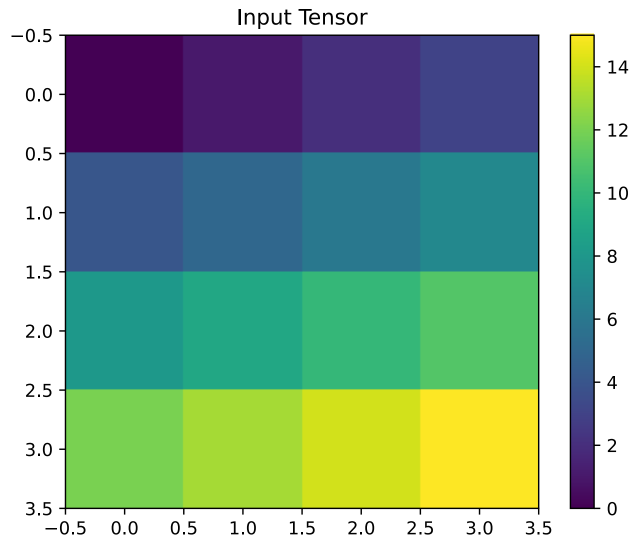
1. **Max-Pooling vs. Average Pooling:** Max-pooling selects the maximum value within a window, while average pooling takes the average. Max-pooling is generally preferred for better performance in tasks like object recognition.
2. **Pooling Window:** The window is slid over the input tensor, and at each position, either the max or average value is computed. A common choice is a 2x2 window.
3. **Padding and Stride:** Just like in convolutional layers, padding and stride can be used in pooling layers to control the output shape. These can either be set to default values or customized as needed.
4. **Handling Multiple Channels:** For multi-channel inputs, each channel is pooled independently, which means the number of output channels remains the same as the input.

- Convolutional Neural Networks (CNNs): CNN architecture and its essential layers, including convolution and pooling layers, are central to understanding how computers recognize and process images.
- Importance of Pooling: Pooling layers reduce spatial resolution, making the network less sensitive to slight movements or transformations of the input image. This is crucial for tasks like object detection, where objects rarely appear in exactly the same place.
- Feature Translation Invariance: This concept ensures that the network can recognize objects even if they are shifted slightly, which is vital for making models robust to different inputs.

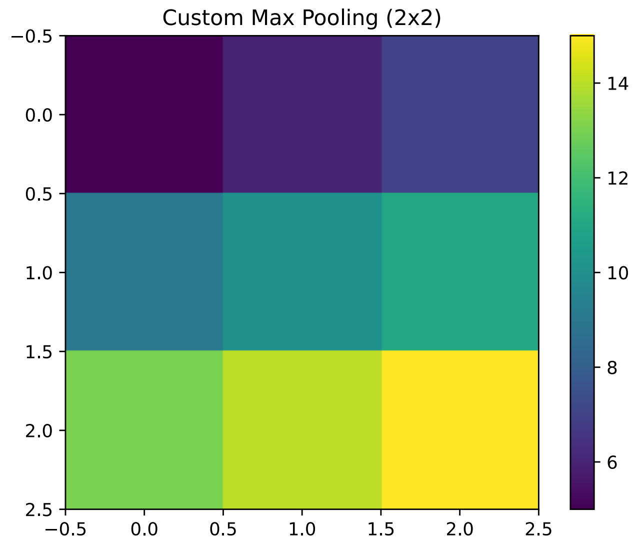
7.5. Exercises & My Own Experiments

```
import torch
import torch.nn as nn
import matplotlib.pyplot as plt

def custom_pool2d(X, pool_size, mode='max'):
    p_h, p_w = pool_size
    Y = torch.zeros((X.shape[0] - p_h + 1, X.shape[1] - p_w + 1))
    for i in range(Y.shape[0]):
        for j in range(Y.shape[1]):
            if mode == 'max':
                Y[i, j] = X[i: i + p_h, j: j + p_w].max()
            elif mode == 'avg':
                Y[i, j] = X[i: i + p_h, j: j + p_w].mean()
    return Y

def visualize_tensor(tensor, title):
    plt.imshow(tensor.numpy(), cmap='viridis')
    plt.colorbar()
    plt.title(title)
    plt.show()

# Create input tensor
X = torch.tensor([[0.0, 1.0, 2.0, 3.0],
                  [4.0, 5.0, 6.0, 7.0],
                  [8.0, 9.0, 10.0, 11.0],
                  [12.0, 13.0, 14.0, 15.0]])

print("Input tensor:")
print(X)
visualize_tensor(X, "Input Tensor")

# Custom max pooling
custom_max_pooled = custom_pool2d(X, (2, 2), 'max')
print("₩nCustom max pooling result (2x2):")
print(custom_max_pooled)
visualize_tensor(custom_max_pooled, "Custom Max Pooling (2x2)")

# Custom average pooling
custom_avg_pooled = custom_pool2d(X, (2, 2), 'avg')
print("₩nCustom average pooling result (2x2):")
print(custom_avg_pooled)
visualize_tensor(custom_avg_pooled, "Custom Average Pooling (2x2)")

# PyTorch built-in max pooling
max_pool = nn.MaxPool2d(2)
torch_max_pooled = max_pool(X.unsqueeze(0).unsqueeze(0)).squeeze()
print("₩nPyTorch built-in max pooling result (2x2):")
print(torch_max_pooled)
visualize_tensor(torch_max_pooled, "PyTorch Max Pooling (2x2)")

# PyTorch built-in average pooling
avg_pool = nn.AvgPool2d(2)
torch_avg_pooled = avg_pool(X.unsqueeze(0).unsqueeze(0)).squeeze()
print("₩nPyTorch built-in average pooling result (2x2):")
print(torch_avg_pooled)
visualize_tensor(torch_avg_pooled, "PyTorch Average Pooling (2x2)")

# Test with different pooling window size
custom_max_pooled_3x3 = custom_pool2d(X, (3, 3), 'max')
print("₩nCustom max pooling result (3x3):")
print(custom_max_pooled_3x3)
visualize_tensor(custom_max_pooled_3x3, "Custom Max Pooling (3x3)")

# PyTorch pooling with padding and stride
max_pool_padded = nn.MaxPool2d(2, padding=1, stride=1)
torch_max_pooled_padded = max_pool_padded(X.unsqueeze(0).unsqueeze(0)).squeeze()
print("₩nPyTorch built-in max pooling result (padding=1, stride=1):")
print(torch_max_pooled_padded)
visualize_tensor(torch_max_pooled_padded, "PyTorch Max Pooling (padding=1, stride=1)")
```

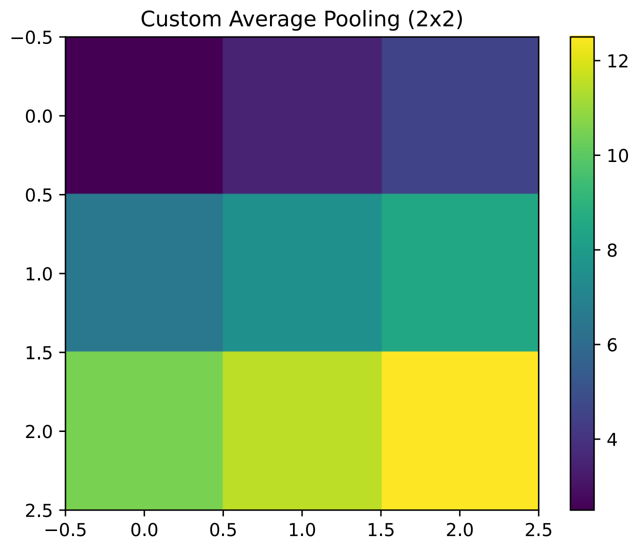
```
Input tensor:
tensor([[ 0.,  1.,  2.,  3.],
        [ 4.,  5.,  6.,  7.],
        [ 8.,  9., 10., 11.],
        [12., 13., 14., 15.]])
```



```
Custom max pooling result (2x2):
tensor([[ 5.,  6.,  7.],
        [ 9., 10., 11.],
        [13., 14., 15.]])
```

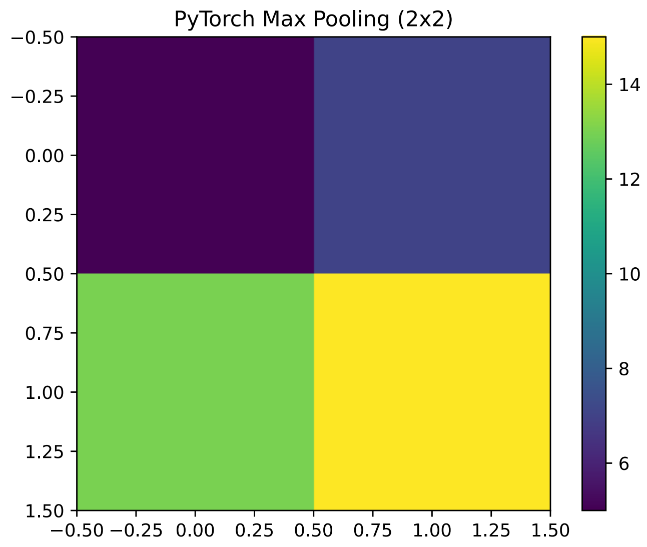


```
Custom average pooling result (2x2):
tensor([[ 2.5000,  3.5000,  4.5000],
        [ 6.5000,  7.5000,  8.5000],
        [10.5000, 11.5000, 12.5000]])
```

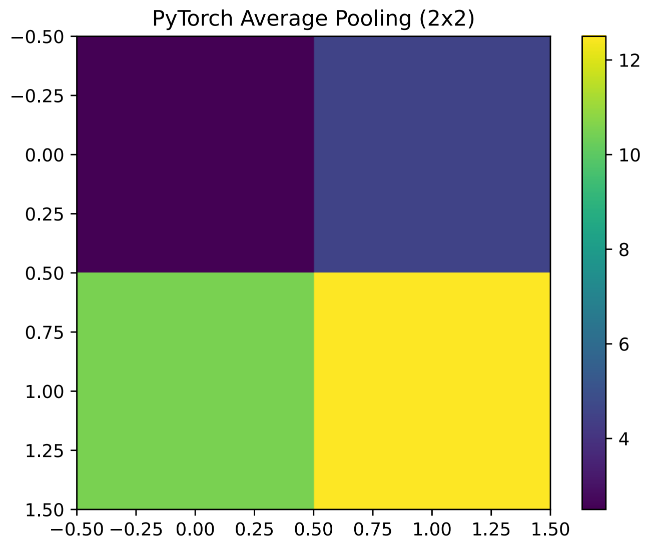


PyTorch built-in max pooling result (2x2):

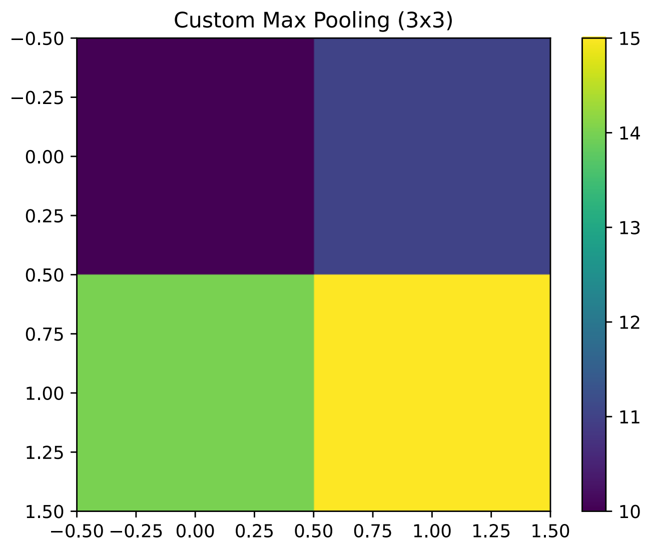
```
tensor([[ 5.,  7.],
        [13., 15.]])
```



```
PyTorch built-in average pooling result (2x2):
tensor([[ 2.5000,  4.5000],
        [10.5000, 12.5000]])
```

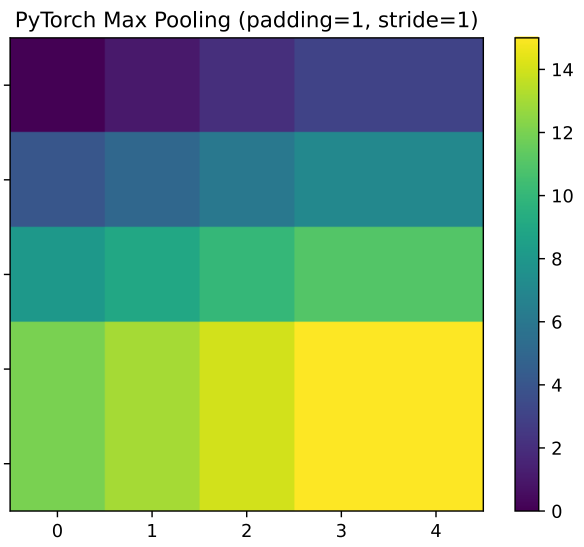


```
Custom max pooling result (3x3):
tensor([[10., 11.],
        [14., 15.]])
```



```
PyTorch built-in max pooling result (padding=1, stride=1):
tensor([[ 0.,  1.,  2.,  3.,  3.],
        [ 4.,  5.,  6.,  7.,  7.],
        [ 8.,  9., 10., 11., 11.],
        [12., 13., 14., 15., 15.],
        [12., 13., 14., 15., 15.]])
```

```
 [12., 13., 14., 15., 15.]])
```


PyTorch Max Pooling (padding=1, stride=1)
0
1
2
3
4
0
1
2
3
4
0
2
4
6
8
10
12
14

7.6. Discussion

LeNet is one of the earliest and most influential Convolutional Neural Networks (CNNs), designed by Yann LeCun in the 1990s for recognizing handwritten digits, achieving less than 1% error. The architecture consists of two main parts:

1. Convolutional Encoder:

- Two convolutional layers, each followed by a sigmoid activation function and average pooling.
- First convolutional layer: 6 channels with a 5 × 5 5×5 kernel.
- Second convolutional layer: 16 channels with a 5 × 5 5×5 kernel.
- Average pooling with a 2 × 2 2×2 window, reducing dimensionality by a factor of 4.

2. Dense Block:

- Three fully connected layers with 120, 84, and 10 outputs (for classification into 10 digit classes).
- The output is passed through a softmax function for classification.

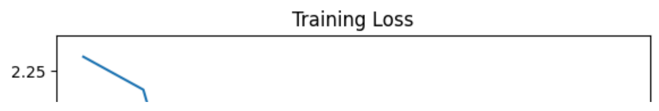
The model can be easily implemented using modern deep learning frameworks such as PyTorch. The experiment on Fashion-MNIST dataset with LeNet demonstrates how convolutional layers retain spatial structure in images, reducing the number of parameters compared to fully connected layers.

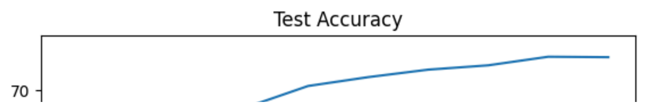
7.6. Exercises & My Own Experiments

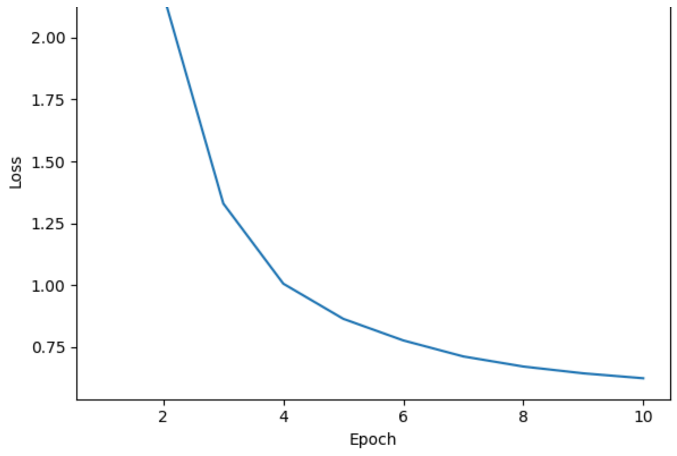
```python
import torch
from torch import nn
from torchvision import datasets, transforms
import torchvision
from torch.utils.data import DataLoader
import matplotlib.pyplot as plt
import numpy as np

# LeNet 모델 정의
class LeNet(nn.Module):
    def __init__(self, num_classes=10):
        super().__init__()
        self.net = nn.Sequential(
            nn.Conv2d(1, 6, kernel_size=5, padding=2), nn.Sigmoid(),
            nn.AvgPool2d(kernel_size=2, stride=2),
            nn.Conv2d(6, 16, kernel_size=5), nn.Sigmoid(),
            nn.AvgPool2d(kernel_size=2, stride=2),
            nn.Flatten(),
            nn.Linear(16 * 5 * 5, 120), nn.Sigmoid(),
            nn.Linear(120, 84), nn.Sigmoid(),
            nn.Linear(84, num_classes)
        )

    def forward(self, x):
        return self.net(x)

# CNN 가중치 초기화
def init_cnn(module):
    if isinstance(module, (nn.Linear, nn.Conv2d)):
        nn.init.xavier_uniform_(module.weight)

# Fashion-MNIST 데이터셋 로드 및 전처리
transform = transforms.Compose([
    transforms.ToTensor(),
    transforms.Normalize((0.5,), (0.5,))
])

train_dataset = datasets.FashionMNIST(root='./data', train=True, download=True, transform=transform)
test_dataset = datasets.FashionMNIST(root='./data', train=False, download=True, transform=transform)

train_loader = DataLoader(train_dataset, batch_size=128, shuffle=True)
test_loader = DataLoader(test_dataset, batch_size=128, shuffle=False)

# 클래스 이름 정의
classes = ['T-shirt/top', 'Trouser', 'Pullover', 'Dress', 'Coat',
           'Sandal', 'Shirt', 'Sneaker', 'Bag', 'Ankle boot']

# 모델, 손실 함수, 옵티마이저 초기화
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
model = LeNet().to(device)
```

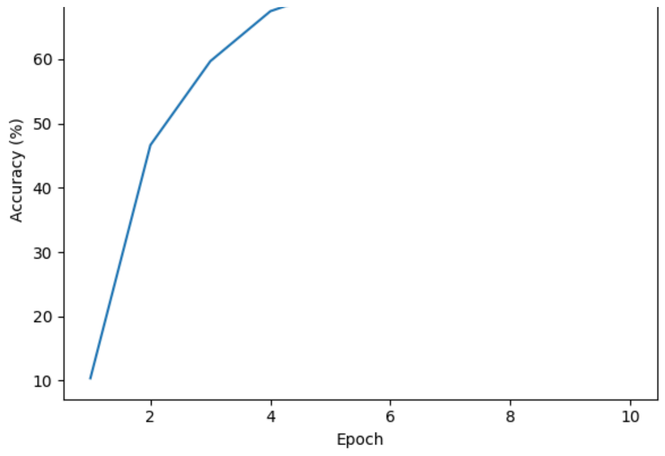
```python
model.apply(init_cnn)
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(model.parameters(), lr=0.1)

# 훈련 결과를 저장할 리스트
train_losses = []
test_accuracies = []

# 훈련 루프
num_epochs = 10
for epoch in range(num_epochs):
    model.train()
    epoch_loss = 0
    for batch_idx, (data, target) in enumerate(train_loader):
        data, target = data.to(device), target.to(device)
        optimizer.zero_grad()
        output = model(data)
        loss = criterion(output, target)
        loss.backward()
        optimizer.step()

        epoch_loss += loss.item()

        if batch_idx % 100 == 0:
            print(f'Epoch {epoch+1}/{num_epochs}, Batch {batch_idx}/{len(train_loader)}, Loss: {loss.item():.4f}')

    avg_loss = epoch_loss / len(train_loader)
    train_losses.append(avg_loss)

    # 테스트 세트에서 평가
    model.eval()
    correct = 0
    total = 0
    with torch.no_grad():
        for data, target in test_loader:
            data, target = data.to(device), target.to(device)
            outputs = model(data)
            _, predicted = torch.max(outputs.data, 1)
            total += target.size(0)
            correct += (predicted == target).sum().item()

    accuracy = 100 * correct / total
    test_accuracies.append(accuracy)
    print(f'Epoch {epoch+1}/{num_epochs}, Test Accuracy: {accuracy:.2f}%')

print('Training finished!')

# 훈련 손실과 테스트 정확도 그래프
plt.figure(figsize=(12, 5))
plt.subplot(1, 2, 1)
plt.plot(range(1, num_epochs+1), train_losses)
plt.title('Training Loss')
plt.xlabel('Epoch')
plt.ylabel('Loss')

plt.subplot(1, 2, 2)
plt.plot(range(1, num_epochs+1), test_accuracies)
plt.title('Test Accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy (%)')

plt.tight_layout()
plt.show()

# 이미지 표시 함수
def imshow(img):
    img = img / 2 + 0.5     # unnormalize
    npimg = img.numpy()
    plt.imshow(np.transpose(npimg, (1, 2, 0)))

# 테스트 세트에서 랜덤 이미지 선택
dataiter = iter(test_loader)
images, labels = next(dataiter)

# 선택된 이미지에 대한 예측
outputs = model(images.to(device))
_, predicted = torch.max(outputs, 1)

# 이미지와 예측 결과 시각화
plt.figure(figsize=(12, 8))
imshow(torchvision.utils.make_grid(images[:16]))
plt.title('Some test images')
plt.axis('off')
plt.show()

print('Ground Truth: ', ' '.join('%5s' % classes[labels[j]] for j in range(16)))
print('Predicted: ', ' '.join('%5s' % classes[predicted[j]] for j in range(16)))
```

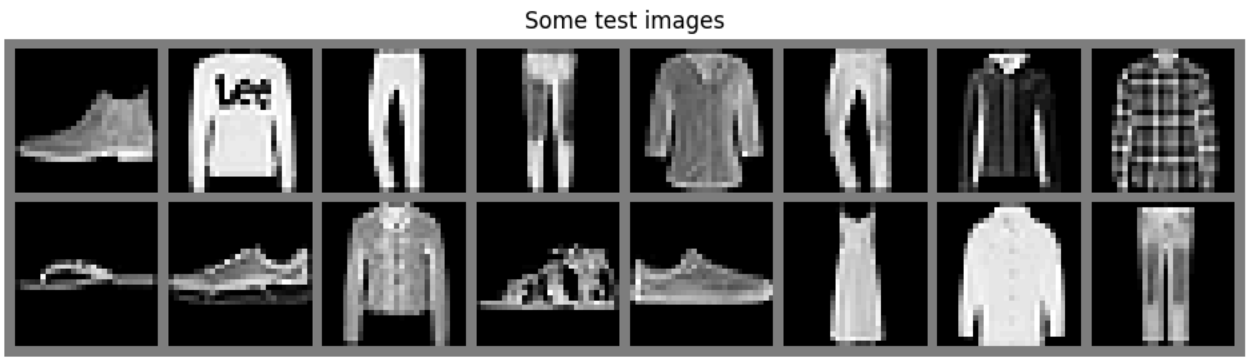
```
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-images-idx3-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-images-idx3-ubyte.gz to ./data/FashionMNIST/raw/train-images-idx3-ul
100%|██████████| 26421880/26421880 [00:03<00:00, 6611086.42it/s]
Extracting ./data/FashionMNIST/raw/train-images-idx3-ubyte.gz to ./data/FashionMNIST/raw

Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-labels-idx1-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-labels-idx1-ubyte.gz to ./data/FashionMNIST/raw/train-labels-idx1-ul
100%|██████████| 29515/29515 [00:00<00:00, 200689.77it/s]
Extracting ./data/FashionMNIST/raw/train-labels-idx1-ubyte.gz to ./data/FashionMNIST/raw

Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-images-idx3-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-images-idx3-ubyte.gz to ./data/FashionMNIST/raw/t10k-images-idx3-uby
100%|██████████| 4422102/4422102 [00:03<00:00, 1194673.62it/s]
Extracting ./data/FashionMNIST/raw/t10k-images-idx3-ubyte.gz to ./data/FashionMNIST/raw

Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-labels-idx1-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-labels-idx1-ubyte.gz to ./data/FashionMNIST/raw/t10k-labels-idx1-uby
100%|██████████| 5148/5148 [00:00<00:00, 6399607.88it/s]
Extracting ./data/FashionMNIST/raw/t10k-labels-idx1-ubyte.gz to ./data/FashionMNIST/raw

Epoch 1/10, Batch 0/469, Loss: 2.4812
Epoch 1/10, Batch 100/469, Loss: 2.3065
Epoch 1/10, Batch 200/469, Loss: 2.3043
Epoch 1/10, Batch 300/469, Loss: 2.3084
Epoch 1/10, Batch 400/469, Loss: 2.3002
Epoch 1/10, Test Accuracy: 10.36%
Epoch 2/10, Batch 0/469, Loss: 2.2941
Epoch 2/10, Batch 100/469, Loss: 2.2917
Epoch 2/10, Batch 200/469, Loss: 2.2839
Epoch 2/10, Batch 300/469, Loss: 2.2084
Epoch 2/10, Batch 400/469, Loss: 1.9980
Epoch 2/10, Test Accuracy: 46.64%
Epoch 3/10, Batch 0/469, Loss: 1.7173
Epoch 3/10, Batch 100/469, Loss: 1.4210
Epoch 3/10, Batch 200/469, Loss: 1.3336
Epoch 3/10, Batch 300/469, Loss: 1.2387
Epoch 3/10, Batch 400/469, Loss: 1.1262
Epoch 3/10, Test Accuracy: 59.71%
Epoch 4/10, Batch 0/469, Loss: 1.0961
Epoch 4/10, Batch 100/469, Loss: 0.9862
Epoch 4/10, Batch 200/469, Loss: 0.9844
Epoch 4/10, Batch 300/469, Loss: 0.9465
Epoch 4/10, Batch 400/469, Loss: 0.9206
Epoch 4/10, Test Accuracy: 67.48%
Epoch 5/10, Batch 0/469, Loss: 0.9333
Epoch 5/10, Batch 100/469, Loss: 0.9238
Epoch 5/10, Batch 200/469, Loss: 0.7317
Epoch 5/10, Batch 300/469, Loss: 0.8737
Epoch 5/10, Batch 400/469, Loss: 0.9831
Epoch 5/10, Test Accuracy: 70.69%
Epoch 6/10, Batch 0/469, Loss: 0.7847
Epoch 6/10, Batch 100/469, Loss: 0.9941
Epoch 6/10, Batch 200/469, Loss: 0.8042
Epoch 6/10, Batch 300/469, Loss: 0.8167
Epoch 6/10, Batch 400/469, Loss: 0.6735
Epoch 6/10, Test Accuracy: 72.06%
Epoch 7/10, Batch 0/469, Loss: 0.7283
Epoch 7/10, Batch 100/469, Loss: 0.6713
Epoch 7/10, Batch 200/469, Loss: 0.6758
Epoch 7/10, Batch 300/469, Loss: 0.6978
Epoch 7/10, Batch 400/469, Loss: 0.8028
Epoch 7/10, Test Accuracy: 73.22%
Epoch 8/10, Batch 0/469, Loss: 0.6930
Epoch 8/10, Batch 100/469, Loss: 0.8399
Epoch 8/10, Batch 200/469, Loss: 0.6570
Epoch 8/10, Batch 300/469, Loss: 0.6557
Epoch 8/10, Batch 400/469, Loss: 0.6941
Epoch 8/10, Test Accuracy: 73.89%
Epoch 9/10, Batch 0/469, Loss: 0.5506
Epoch 9/10, Batch 100/469, Loss: 0.6535
Epoch 9/10, Batch 200/469, Loss: 0.5710
Epoch 9/10, Batch 300/469, Loss: 0.7259
Epoch 9/10, Batch 400/469, Loss: 0.5746
Epoch 9/10, Test Accuracy: 75.22%
Epoch 10/10, Batch 0/469, Loss: 0.6155
Epoch 10/10, Batch 100/469, Loss: 0.5979
Epoch 10/10, Batch 200/469, Loss: 0.5378
Epoch 10/10, Batch 300/469, Loss: 0.6681
Epoch 10/10, Batch 400/469, Loss: 0.6537
Epoch 10/10, Test Accuracy: 75.15%
Training finished!
```











```
Ground Truth:  Ankle boot Pullover Trouser Trouser Shirt Trouser  Coat Shirt Sandal Sneaker  Coat Sandal Sneaker Dress  Coat Trouser
Predicted:  Ankle boot Pullover Trouser Trouser Pullover Trouser Pullover Pullover Sandal Sneaker  Coat Sandal Sandal Dress  Coat Trouser
```

코딩을 시작하거나 AI로 코드를 생성하세요.

COSE474-2024F Deep Learning HW 2

- **Chapter 8**
- 2021170964 박경빈

0.1. Installation

[ ] ↳ *숨겨진 셀 2개*

8.2. Networks Using Blocks (VGG)

While AlexNet offered empirical evidence that deep CNNs can achieve good results, it did not provide a general template to guide subsequent researchers in designing new networks. In the following sections, we will introduce several heuristic concepts commonly used to design deep networks.

Progress in this field mirrors that of VLSI (very large scale integration) in chip design where engineers moved from placing transistors to logical elements to logic blocks :cite: `Mead.1980`. Similarly, the design of neural network architectures has grown progressively more abstract, with researchers moving from thinking in terms of individual neurons to whole layers, and now to blocks, repeating patterns of layers. A decade later, this has now progressed to researchers using entire trained models to repurpose them for different, albeit related, tasks. Such large pretrained models are typically called *foundation models* :cite: `bommasani2021opportunities`.

Back to network design. The idea of using blocks first emerged from the Visual Geometry Group (VGG) at Oxford University, in their eponymously-named *VGG* network :cite: `Simonyan.Zisserman.2014`. It is easy to implement these repeated structures in code with any modern deep learning framework by using loops and subroutines.

```
import torch
from torch import nn
from d2l import torch as d2l
```

8.2.1. VGG Blocks

The basic building block of CNNs is a sequence of the following: (i) a convolutional layer with padding to maintain the resolution, (ii) a nonlinearity such as a ReLU, (iii) a pooling layer such as max-pooling to reduce the resolution. One of the problems with this approach is that the spatial resolution decreases quite rapidly. In particular, this imposes a hard limit of $\log_2 d$ convolutional layers on the network before all dimensions (d) are used up. For instance, in the case of ImageNet, it would be impossible to have more than 8 convolutional layers in this way.

The key idea of :citet: `Simonyan.Zisserman.2014` was to use *multiple* convolutions in between downsampling via max-pooling in the form of a block. They were primarily interested in whether deep or wide networks perform better. For instance, the successive application of two 3×3 convolutions touches the same pixels as a single 5×5 convolution does. At the same time, the latter uses approximately as many parameters ($25 \cdot c^2$) as three 3×3 convolutions do ($3 \cdot 9 \cdot c^2$). In a rather detailed analysis they showed that deep and narrow networks significantly outperform their shallow counterparts. This set deep learning on a quest for ever deeper networks with over 100 layers for typical applications. Stacking 3×3 convolutions has become a gold standard in later deep networks (a design decision only to be revisited recently by :citet: `liu2022convnet`). Consequently, fast implementations for small convolutions have become a staple on GPUs :cite: `lavin2016fast`.

Back to VGG: a VGG block consists of a *sequence* of convolutions with 3×3 kernels with padding of 1 (keeping height and width) followed by a 2×2 max-pooling layer with stride of 2 (halving height and width after each block). In the code below, we define a function called `vgg_block` to implement one VGG block.

The function below takes two arguments, corresponding to the number of convolutional layers `num_convs` and the number of output channels `num_channels`.

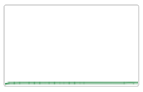
리소스 ✕

Colab Pro 구독 중입니다. 자세히 알아보기
사용 가능: 컴퓨팅 단위 271.67개
사용률: 약 21.18/시간
활성 세션이 2개 있습니다.

세션 관리

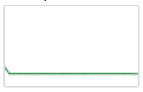
Python 3 Google Compute Engine 백엔드 (GPU)
오후 10:18부터 오후 11:13까지 리소스 표시

시스템 RAM
3.1 / 83.5 GB



GPU RAM
2.8 / 40.0 GB

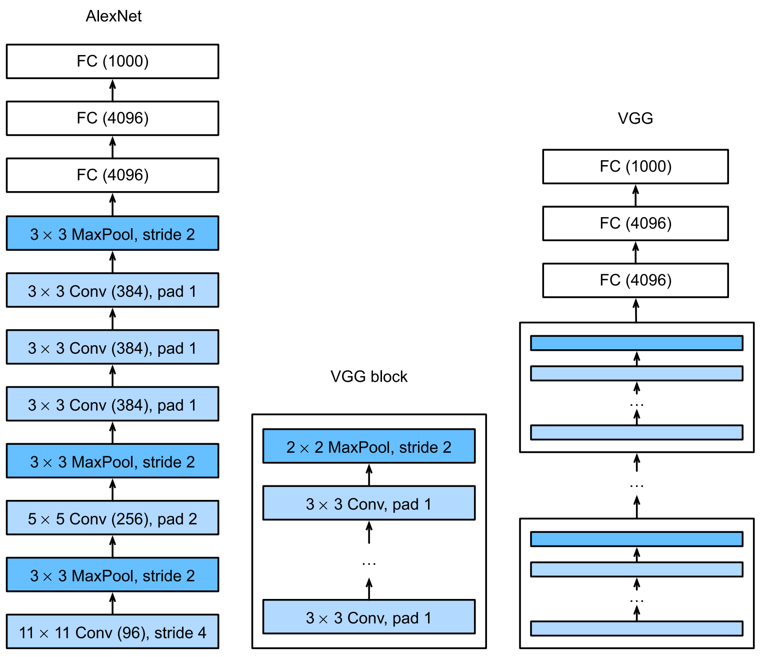
디스크
36.6 / 235.7 GB



```
def vgg_block(num_convs, out_channels):
    layers = []
    for _ in range(num_convs):
        layers.append(nn.LazyConv2d(out_channels, kernel_size=3, padding=1))
        layers.append(nn.ReLU())
    layers.append(nn.MaxPool2d(kernel_size=2,stride=2))
    return nn.Sequential(*layers)
```

8.2.2. VGG Network

Like AlexNet and LeNet, the VGG Network can be partitioned into two parts: the first consisting mostly of convolutional and pooling layers and the second consisting of fully connected layers that are identical to those in AlexNet. The key difference is that the convolutional layers are grouped in nonlinear transformations that leave the dimensonality unchanged, followed by a resolution-reduction step.



The convolutional part of the network connects several VGG blocks from :numref: `fig_vgg` (also defined in the `vgg_block` function) in succession. This grouping of convolutions is a pattern that has remained almost unchanged over the past decade, although the specific choice of operations has undergone considerable modifications. The variable `arch` consists of a list of tuples (one per block), where each contains two values: the number of convolutional layers and the number of output channels, which are precisely the arguments required to call the `vgg_block` function. As such, VGG defines a *family* of networks rather than just a specific manifestation. To build a specific network we simply iterate over `arch` to compose the blocks.

```
class VGG(d2l.Classifier):
    def __init__(self, arch, lr=0.1, num_classes=10):
        super().__init__()
        self.save_hyperparameters()
        conv_blks = []
        for (num_convs, out_channels) in arch:
            conv_blks.append(vgg_block(num_convs, out_channels))
        self.net = nn.Sequential(
            *conv_blks, nn.Flatten(),
            nn.LazyLinear(4096), nn.ReLU(), nn.Dropout(0.5),
            nn.LazyLinear(4096), nn.ReLU(), nn.Dropout(0.5),
            nn.LazyLinear(num_classes))
        self.net.apply(d2l.init_cnn)
```

The original VGG network had five convolutional blocks, among which the first two have one convolutional layer each and the latter three contain two convolutional layers each. The first block has 64 output channels and each subsequent block doubles the number of output channels, until that number reaches 512. Since this network uses eight convolutional layers and three fully connected layers, it is often called VGG-11.

```
VGG(arch=((1, 64), (1, 128), (2, 256), (2, 512), (2, 512))).layer_summary(
    (1, 1, 224, 224))
```

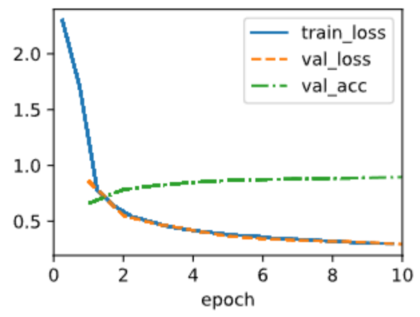
```
Sequential output shape:        torch.Size([1, 64, 112, 112])
Sequential output shape:        torch.Size([1, 128, 56, 56])
Sequential output shape:        torch.Size([1, 256, 28, 28])
Sequential output shape:        torch.Size([1, 512, 14, 14])
Sequential output shape:        torch.Size([1, 512, 7, 7])
Flatten output shape:    torch.Size([1, 25088])
Linear output shape:     torch.Size([1, 4096])
ReLU output shape:       torch.Size([1, 4096])
Dropout output shape:    torch.Size([1, 4096])
Linear output shape:     torch.Size([1, 4096])
ReLU output shape:       torch.Size([1, 4096])
Dropout output shape:    torch.Size([1, 4096])
Linear output shape:     torch.Size([1, 10])
```

As you can see, we halve height and width at each block, finally reaching a height and width of 7 before flattening the representations for processing by the fully connected part of the network. Simonyan and Zisserman (2014) described several other variants of VGG. In fact, it has become the norm to propose families of networks with different speed–accuracy trade-off when introducing a new architecture.

8.2.3. Training

Since VGG-11 is computationally more demanding than AlexNet we construct a network with a smaller number of channels. This is more than sufficient for training on Fashion-MNIST. The model training process is similar to that of AlexNet in Section 8.1. Again observe the close match between validation and training loss, suggesting only a small amount of overfitting.

```
model = VGG(arch=((1, 16), (1, 32), (2, 64), (2, 128), (2, 128)), lr=0.01)
trainer = d2l.Trainer(max_epochs=10, num_gpus=1)
data = d2l.FashionMNIST(batch_size=128, resize=(224, 224))
model.apply_init([next(iter(data.get_dataloader(True)))[0]], d2l.init_cnn)
trainer.fit(model, data)
```



8.6. Residual Networks (ResNet) and ResNeXt

As we design ever deeper networks it becomes imperative to understand how adding layers can increase the complexity and expressiveness of the network. Even more important is the ability to design networks where adding layers makes networks strictly more expressive rather than just different. To make some progress we need a bit of mathematics.

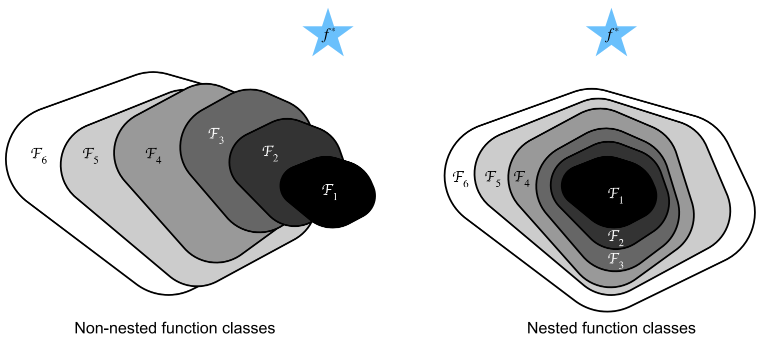
```
import torch
from torch import nn
from torch.nn import functional as F
from d2l import torch as d2l
```

8.6.1. Function Classes

Consider $\mathcal{F}$, the class of functions that a specific network architecture (together with learning rates and other hyperparameter settings) can reach. That is, for all $f \in \mathcal{F}$ there exists some set of parameters (e.g., weights and biases) that can be obtained through training on a suitable dataset. Let's assume that f^* is the "truth" function that we really would like to find. If it is in $\mathcal{F}$, we are in good shape but typically we will not be quite so lucky. Instead, we will try to find some $f^*_{\mathcal{F}}$ which is our best bet within $\mathcal{F}$. For instance, given a dataset with features $\mathbf{X}$ and labels $\mathbf{y}$, we might try finding it by solving the following optimization problem:

$$f^*_\mathcal{F} \stackrel{\textrm{def}}{=} \mathop{\mathrm{argmin}}_f L(\mathbf{X}, \mathbf{y}, f) \textrm{ subject to } f \in \mathcal{F}.$$

We know that regularization :cite: `tikhonov1977solutions,morozov2012methods` may control complexity of $\mathcal{F}$ and achieve consistency, so a larger size of training data generally leads to better $f^*_\mathcal{F}$. It is only reasonable to assume that if we design a different and more powerful architecture $\mathcal{F}'$ we should arrive at a better outcome. In other words, we would expect that $f^*_{\mathcal{F}'}$ is "better" than $f^*_{\mathcal{F}}$. However, if $\mathcal{F} \not\subseteq \mathcal{F}'$ there is no guarantee that this should even happen. In fact, $f^*_{\mathcal{F}'}$ might well be worse. As illustrated by :numref: `fig_functionclasses`, for non-nested function classes, a larger function class does not always move closer to the "truth" function f^*. For instance, on the left of :numref: `fig_functionclasses`, though $\mathcal{F}_3$ is closer to f^* than $\mathcal{F}_1$, $\mathcal{F}_6$ moves away and there is no guarantee that further increasing the complexity can reduce the distance from f^*. With nested function classes where $\mathcal{F}_1 \subseteq \cdots \subseteq \mathcal{F}_6$ on the right of :numref: `fig_functionclasses`, we can avoid the aforementioned issue from the non-nested function classes.

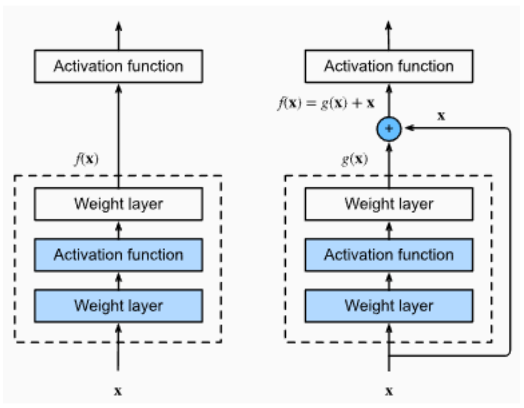


Thus, only if larger function classes contain the smaller ones are we guaranteed that increasing them strictly increases the expressive power of the network. For deep neural networks, if we can train the newly-added layer into an identity function $f(\mathbf{x}) = \mathbf{x}$, the new model will be as effective as the original model. As the new model may get a better solution to fit the training dataset, the added layer might make it easier to reduce training errors.

This is the question that :citet: `He.Zhang.Ren.ea.2016` considered when working on very deep computer vision models. At the heart of their proposed *residual network* (*ResNet*) is the idea that every additional layer should more easily contain the identity function as one of its elements. These considerations are rather profound but they led to a surprisingly simple solution, a *residual block*. With it, ResNet won the ImageNet Large Scale Visual Recognition Challenge in 2015. The design had a profound influence on how to build deep neural networks. For instance, residual blocks have been added to recurrent networks :cite: `prakash2016neural,kim2017residual`. Likewise, Transformers :cite: `Vaswani.Shazeer.Parmar.ea.2017` use them to stack many layers of networks efficiently. It is also used in graph neural networks :cite: `Kipf.Welling.2016` and, as a basic concept, it has been used extensively in computer vision :cite: `Redmon.Farhadi.2018,Ren.He.Girshick.ea.2015`. Note that residual networks are predated by highway networks :cite: `srivastava2015highway` that share some of the motivation, albeit without the elegant parametrization around the identity function.

8.6.2. Residual Blocks

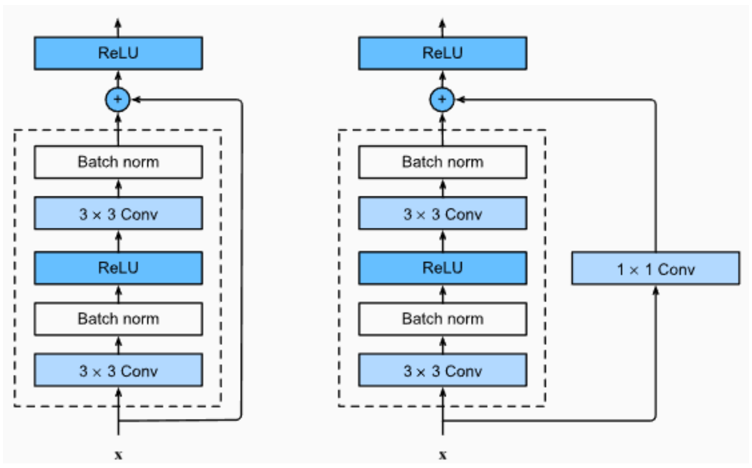
Let's focus on a local part of a neural network, as depicted in :numref: `fig_residual_block`. Denote the input by $\mathbf{x}$. We assume that $f(\mathbf{x})$, the desired underlying mapping we want to obtain by learning, is to be used as input to the activation function on the top. On the left, the portion within the dotted-line box must directly learn $f(\mathbf{x})$. On the right, the portion within the dotted-line box needs to learn the *residual mapping* $g(\mathbf{x}) = f(\mathbf{x}) - \mathbf{x}$, which is how the residual block derives its name. If the identity mapping $f(\mathbf{x}) = \mathbf{x}$ is the desired underlying mapping, the residual mapping amounts to $g(\mathbf{x}) = 0$ and it is thus easier to learn: we only need to push the weights and biases of the upper weight layer (e.g., fully connected layer and convolutional layer) within the dotted-line box to zero. The right figure illustrates the *residual block* of ResNet, where the solid line carrying the layer input $\mathbf{x}$ to the addition operator is called a *residual connection* (or *shortcut connection*). With residual blocks, inputs can forward propagate faster through the residual connections across layers. In fact, the residual block can be thought of as a special case of the multi-branch Inception block: it has two branches one of which is the identity mapping.



ResNet has VGG's full 3×3 convolutional layer design. The residual block has two 3×3 convolutional layers with the same number of output channels. Each convolutional layer is followed by a batch normalization layer and a ReLU activation function. Then, we skip these two convolution operations and add the input directly before the final ReLU activation function. This kind of design requires that the output of the two convolutional layers has to be of the same shape as the input, so that they can be added together. If we want to change the number of channels, we need to introduce an additional 1×1 convolutional layer to transform the input into the desired shape for the addition operation. Let's have a look at the code below.

```
class Residual(nn.Module):
    """The Residual block of ResNet models."""
    def __init__(self, num_channels, use_1x1conv=False, strides=1):
        super().__init__()
        self.conv1 = nn.LazyConv2d(num_channels, kernel_size=3, padding=1,
                                   stride=strides)
        self.conv2 = nn.LazyConv2d(num_channels, kernel_size=3, padding=1)
        if use_1x1conv:
            self.conv3 = nn.LazyConv2d(num_channels, kernel_size=1,
                                       stride=strides)
        else:
            self.conv3 = None
        self.bn1 = nn.LazyBatchNorm2d()
        self.bn2 = nn.LazyBatchNorm2d()

    def forward(self, X):
        Y = F.relu(self.bn1(self.conv1(X)))
        Y = self.bn2(self.conv2(Y))
        if self.conv3:
            X = self.conv3(X)
        Y += X
        return F.relu(Y)
```

This code generates two types of networks: one where we add the input to the output before applying the ReLU nonlinearity whenever use_1x1conv=False; and one where we adjust channels and resolution by means of a convolution before adding. Fig. 8.6.3 illustrates this.



Now let's look at **a situation where the input and output are of the same shape**, where 1×1 convolution is not needed.

```
blk = Residual(3)
X = torch.randn(4, 3, 6, 6)
blk(X).shape
```

```
torch.Size([4, 3, 6, 6])
```

We also have the option to **halve the output height and width while increasing the number of output channels**. In this case we use 1×1 convolutions via `use_1x1conv=True`. This comes in handy at the beginning of each ResNet block to reduce the spatial dimensionality via `strides=2`.

```
blk = Residual(6, use_1x1conv=True, strides=2)
blk(X).shape
```

```
torch.Size([4, 6, 3, 3])
```

8.6.3. ResNet Model

The first two layers of ResNet are the same as those of the GoogLeNet we described before: the 7×7 convolutional layer with 64 output channels and a stride of 2 is followed by the 3×3 max-pooling layer with a stride of 2. The difference is the batch normalization layer added after each convolutional layer in ResNet.

```
class ResNet(d2l.Classifier):
    def b1(self):
        return nn.Sequential(
            nn.LazyConv2d(64, kernel_size=7, stride=2, padding=3),
            nn.LazyBatchNorm2d(), nn.ReLU(),
            nn.MaxPool2d(kernel_size=3, stride=2, padding=1))
```

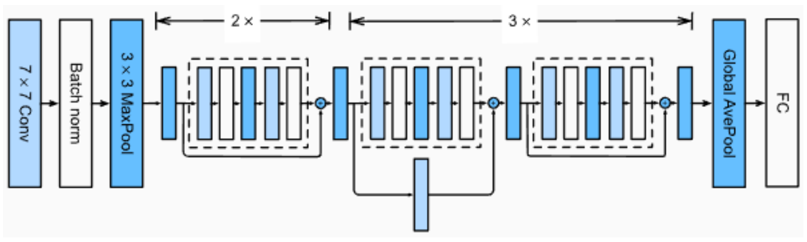
GoogLeNet uses four modules made up of Inception blocks. However, ResNet uses four modules made up of residual blocks, each of which uses several residual blocks with the same number of output channels. The number of channels in the first module is the same as the number of input channels. Since a max-pooling layer with a stride of 2 has already been used, it is not necessary to reduce the height and width. In the first residual block for each of the subsequent modules, the number of channels is doubled compared with that of the previous module, and the height and width are halved.

```
@d2l.add_to_class(ResNet)
def block(self, num_residuals, num_channels, first_block=False):
    blk = []
    for i in range(num_residuals):
        if i == 0 and not first_block:
            blk.append(Residual(num_channels, use_1x1conv=True, strides=2))
        else:
            blk.append(Residual(num_channels))
    return nn.Sequential(*blk)
```

Then, we add all the modules to ResNet. Here, two residual blocks are used for each module. Lastly, just like GoogLeNet, we add a global average pooling layer, followed by the fully connected layer output.

```
@d2l.add_to_class(ResNet)
def __init__(self, arch, lr=0.1, num_classes=10):
    super(ResNet, self).__init__()
    self.save_hyperparameters()
    self.net = nn.Sequential(self.b1())
    for i, b in enumerate(arch):
        self.net.add_module(f'b{i+2}', self.block(*b, first_block=(i==0)))
    self.net.add_module('last', nn.Sequential(
        nn.AdaptiveAvgPool2d((1, 1)), nn.Flatten(),
        nn.LazyLinear(num_classes)))
    self.net.apply(d2l.init_cnn)
```

There are four convolutional layers in each module (excluding the 1×1 convolutional layer). Together with the first 7×7 convolutional layer and the final fully connected layer, there are 18 layers in total. Therefore, this model is commonly known as ResNet-18. By configuring different numbers of channels and residual blocks in the module, we can create different ResNet models, such as the deeper 152-layer ResNet-152. Although the main architecture of ResNet is similar to that of GoogLeNet, ResNet's structure is simpler and easier to modify. All these factors have resulted in the rapid and widespread use of ResNet. :numref: `fig_resnet18` depicts the full ResNet-18.



Before training ResNet, let's [**observe how the input shape changes across different modules in ResNet**]. As in all the previous architectures, the resolution decreases while the number of channels increases up until the point where a global average pooling layer aggregates all features.

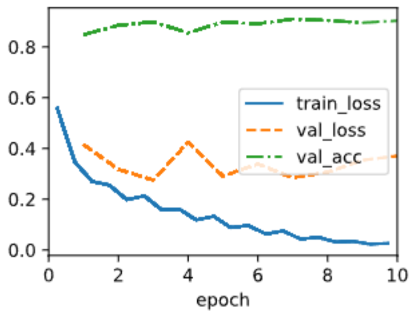
```
class ResNet18(ResNet):
    def __init__(self, lr=0.1, num_classes=10):
        super().__init__(((2, 64), (2, 128), (2, 256), (2, 512)),
                       lr, num_classes)

ResNet18().layer_summary((1, 1, 96, 96))
```

```
Sequential output shape:        torch.Size([1, 64, 24, 24])
Sequential output shape:        torch.Size([1, 64, 24, 24])
Sequential output shape:        torch.Size([1, 128, 12, 12])
Sequential output shape:        torch.Size([1, 256, 6, 6])
Sequential output shape:        torch.Size([1, 512, 3, 3])
Sequential output shape:        torch.Size([1, 10])
```

8.6.4. Training

We train ResNet on the Fashion-MNIST dataset, just like before. ResNet is quite a powerful and flexible architecture. The plot capturing training and validation loss illustrates a significant gap between both graphs, with the training loss being considerably lower. For a network of this flexibility, more training data would offer distinct benefit in closing the gap and improving accuracy.

```
model = ResNet18(lr=0.01)
trainer = d2l.Trainer(max_epochs=10, num_gpus=1)
data = d2l.FashionMNIST(batch_size=128, resize=(96, 96))
model.apply_init([next(iter(data.get_dataloader(True)))[0]], d2l.init_cnn)
trainer.fit(model, data)
```



Discussions & Exercises

8.2. Discussion

The VGG network, developed by the Visual Geometry Group (VGG) at Oxford, introduced the concept of blocks in deep neural networks. While early networks like AlexNet designed layers individually, VGG utilized blocks of layers, repeating patterns that could be reused. These blocks consist of:

1. Multiple convolutional layers with 3×3 kernels, which maintain spatial resolution through padding.

2. ReLU activation functions after each convolution to introduce nonlinearity.
3. Max-pooling layers that downsample the feature maps, reducing the spatial resolution.

The advantage of stacking smaller convolutional filters is that they allow for deeper networks without drastically increasing the number of parameters. For example, two 3×3 convolutions cover the same receptive field as one 5×5 convolution but use fewer parameters, allowing for more complex feature extraction with greater depth.

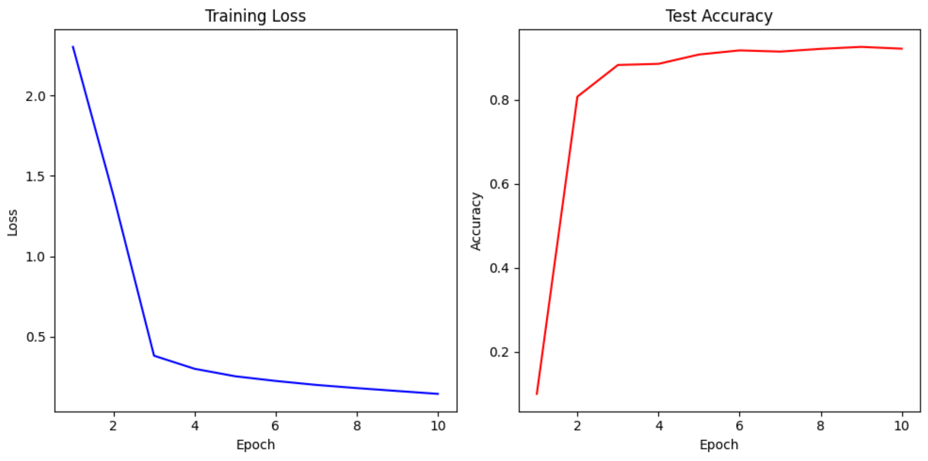
In VGG networks, these blocks are stacked in increasing depth, with channels doubling after each block, allowing for better performance on complex tasks. The final layers are fully connected layers that process the features extracted by the convolutional blocks.

8.2. Exercises & My Own Experiments

```
import torch
import torch.nn as nn
import torch.optim as optim
from torchvision import datasets, transforms
from torch.utils.data import DataLoader
import matplotlib.pyplot as plt

# VGG Block 정의
def vgg_block(num_convs, in_channels, out_channels):
    layers = []
    for _ in range(num_convs):
        layers.append(nn.Conv2d(in_channels, out_channels, kernel_size=3, padding=1))
        layers.append(nn.ReLU())
        in_channels = out_channels
    layers.append(nn.MaxPool2d(kernel_size=2, stride=2))
    return nn.Sequential(*layers)

# VGG 네트워크 정의
class VGG(nn.Module):
    def __init__(self, arch, num_classes=10):
        super(VGG, self).__init__()
        self.features = self._make_layers(arch)
        self.classifier = nn.Sequential(
            nn.Linear(128 * 7 * 7, 4096),  # 128은 마지막 VGG 블록의 출력 채널 수
            nn.ReLU(),
            nn.Dropout(0.5),
            nn.Linear(4096, 4096),
            nn.ReLU(),
            nn.Dropout(0.5),
            nn.Linear(4096, num_classes)
        )

    def forward(self, x):
        x = self.features(x)
        x = x.view(x.size(0), -1)
        x = self.classifier(x)
        return x

    def _make_layers(self, arch):
        layers = []
        in_channels = 1  # Fashion-MNIST는 그레이스케일 이미지
        for x in arch:
            out_channels = x[1]
            layers += [vgg_block(x[0], in_channels, out_channels)]
            in_channels = out_channels
        return nn.Sequential(*layers)

# 데이터 로딩 및 전처리
transform = transforms.Compose([
    transforms.Resize((224, 224)),
    transforms.ToTensor(),
    transforms.Normalize((0.5,), (0.5,))
])

train_dataset = datasets.FashionMNIST(root='./data', train=True, download=True, transform=transform)
test_dataset = datasets.FashionMNIST(root='./data', train=False, download=True, transform=transform)

train_loader = DataLoader(train_dataset, batch_size=64, shuffle=True)
test_loader = DataLoader(test_dataset, batch_size=64, shuffle=False)

# 모델, 손실 함수, 옵티마이저 정의
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
model = VGG(arch=((1, 16), (1, 32), (2, 64), (2, 128), (2, 128))).to(device)
```

```
criterion = nn.CrossEntropyLoss()
optimizer = optim.SGD(model.parameters(), lr=0.01, momentum=0.9)

# 학습 함수
def train(model, train_loader, criterion, optimizer, device):
    model.train()
    running_loss = 0.0
    for inputs, labels in train_loader:
        inputs, labels = inputs.to(device), labels.to(device)
        optimizer.zero_grad()
        outputs = model(inputs)
        loss = criterion(outputs, labels)
        loss.backward()
        optimizer.step()
        running_loss += loss.item()
    return running_loss / len(train_loader)

# 평가 함수
def evaluate(model, test_loader, device):
    model.eval()
    correct = 0
    total = 0
    with torch.no_grad():
        for inputs, labels in test_loader:
            inputs, labels = inputs.to(device), labels.to(device)
            outputs = model(inputs)
            _, predicted = outputs.max(1)
            total += labels.size(0)
            correct += predicted.eq(labels).sum().item()
    return correct / total

# 학습 및 평가
num_epochs = 10
train_losses = []
test_accuracies = []

for epoch in range(num_epochs):
    train_loss = train(model, train_loader, criterion, optimizer, device)
    test_accuracy = evaluate(model, test_loader, device)
    train_losses.append(train_loss)
    test_accuracies.append(test_accuracy)
    print(f"Epoch {epoch+1}/{num_epochs}, Loss: {train_loss:.4f}, Test Accuracy: {test_accuracy:.4f}")

print("Training finished!")

# 결과 시각화
plt.figure(figsize=(10, 5))
plt.subplot(1, 2, 1)
plt.plot(range(1, num_epochs + 1), train_losses, 'b-')
plt.title('Training Loss')
plt.xlabel('Epoch')
plt.ylabel('Loss')

plt.subplot(1, 2, 2)
plt.plot(range(1, num_epochs + 1), test_accuracies, 'r-')
plt.title('Test Accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy')

plt.tight_layout()
plt.show()
```

```
Epoch 1/10, Loss: 2.3031, Test Accuracy: 0.1000
Epoch 2/10, Loss: 1.3758, Test Accuracy: 0.8077
Epoch 3/10, Loss: 0.3810, Test Accuracy: 0.8832
Epoch 4/10, Loss: 0.2998, Test Accuracy: 0.8859
Epoch 5/10, Loss: 0.2533, Test Accuracy: 0.9081
Epoch 6/10, Loss: 0.2242, Test Accuracy: 0.9179
Epoch 7/10, Loss: 0.1994, Test Accuracy: 0.9150
Epoch 8/10, Loss: 0.1795, Test Accuracy: 0.9216
Epoch 9/10, Loss: 0.1615, Test Accuracy: 0.9263
Epoch 10/10, Loss: 0.1438, Test Accuracy: 0.9220
Training finished!
```



8.6. Discussion

Residual Networks (ResNet) introduced a key idea in deep learning: adding layers should increase network expressiveness without hurting performance, which can happen if models become harder to train. The core of ResNet is the residual block, where the network learns the residual mapping instead of the original function directly. This means that, instead of learning f(x), ResNet learns g(x) = f(x) - x. When the desired function is close to the identity, this becomes easier, allowing the network to propagate gradients better and train deeper layers.

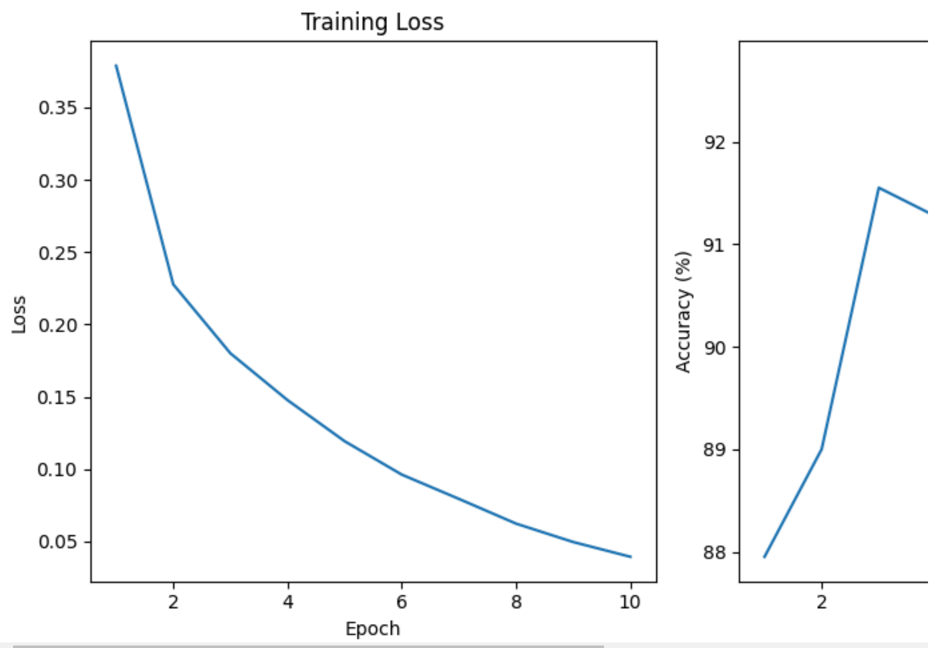
ResNet uses **skip connections** (shortcut connections) where inputs skip a few layers and are directly added to the output, ensuring that layers can easily learn identity functions. This idea is pivotal in stabilizing the training of very deep networks and has been highly influential in architectures such as Transformers and Graph Neural Networks (GNNs).

1. **Residual Block:** Learns residual functions instead of the full function, with skip connections helping to stabilize training.
2. **Shortcut connections:** Enables fast gradient flow through deep networks.
3. **Applications:** Transformers, Graph Neural Networks, and computer vision.

8.6. Exercises & My Own Experiments

```
import torch
import torch.nn as nn
import torch.nn.functional as F
from torchvision import datasets, transforms
from torch.utils.data import DataLoader
import matplotlib.pyplot as plt

# Define the Residual block
class Residual(nn.Module):
    def __init__(self, in_channels, out_channels, use_1x1conv=False, stride=1):
        super(Residual, self).__init__()
        self.conv1 = nn.Conv2d(in_channels, out_channels, kernel_size=3, padding=1, stride=stride)
        self.conv2 = nn.Conv2d(out_channels, out_channels, kernel_size=3, padding=1)
        self.bn1 = nn.BatchNorm2d(out_channels)
        self.bn2 = nn.BatchNorm2d(out_channels)
        if use_1x1conv:
            self.conv3 = nn.Conv2d(in_channels, out_channels, kernel_size=1, stride=stride)
        else:
            self.conv3 = None

    def forward(self, x):
        y = F.relu(self.bn1(self.conv1(x)))
        y = self.bn2(self.conv2(y))
        if self.conv3:
            x = self.conv3(x)
        return F.relu(y + x)

# Define the ResNet18 model
class ResNet18(nn.Module):
    def __init__(self, num_classes=10):
        super(ResNet18, self).__init__()
        self.b1 = nn.Sequential(
            nn.Conv2d(1, 64, kernel_size=7, stride=2, padding=3),
            nn.BatchNorm2d(64), nn.ReLU(),
            nn.MaxPool2d(kernel_size=3, stride=2, padding=1)
        )
        self.b2 = self.block(64, 64, 2, first_block=True)
        self.b3 = self.block(64, 128, 2)
        self.b4 = self.block(128, 256, 2)
        self.b5 = self.block(256, 512, 2)
        self.fc = nn.Linear(512, num_classes)

    def block(self, in_channels, out_channels, num_residuals, first_block=False):
        blk = []
        for i in range(num_residuals):
            if i == 0 and not first_block:
                blk.append(Residual(in_channels, out_channels, use_1x1conv=True, stride=2))
            else:
                blk.append(Residual(out_channels, out_channels))
        return nn.Sequential(*blk)

    def forward(self, x):
        x = self.b1(x)
        x = self.b2(x)
        x = self.b3(x)
        x = self.b4(x)
        x = self.b5(x)
        x = F.adaptive_avg_pool2d(x, (1, 1))
        x = x.view(x.size(0), -1)
        return self.fc(x)

# Set up data loaders
transform = transforms.Compose([
    transforms.Resize((96, 96)),
    transforms.ToTensor(),
    transforms.Normalize((0.5,), (0.5,))
])

train_dataset = datasets.FashionMNIST(root='./data', train=True, download=True, transform=transform)
test_dataset = datasets.FashionMNIST(root='./data', train=False, download=True, transform=transform)

train_loader = DataLoader(train_dataset, batch_size=128, shuffle=True)
test_loader = DataLoader(test_dataset, batch_size=128, shuffle=False)

# Initialize the model, loss function, and optimizer
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
model = ResNet18().to(device)
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(model.parameters(), lr=0.01, momentum=0.9)

# Training loop
num_epochs = 10
train_losses = []
test_accuracies = []

for epoch in range(num_epochs):
    model.train()
    running_loss = 0.0
    for i, (images, labels) in enumerate(train_loader):
        images, labels = images.to(device), labels.to(device)
        
        optimizer.zero_grad()
        outputs = model(images)
        loss = criterion(outputs, labels)
        loss.backward()
        optimizer.step()
        
        running_loss += loss.item()
        
    train_loss = running_loss / len(train_loader)
    train_losses.append(train_loss)
    
    # Evaluate on test set
    model.eval()
    correct = 0
    total = 0
    with torch.no_grad():
        for images, labels in test_loader:
            images, labels = images.to(device), labels.to(device)
            outputs = model(images)
            _, predicted = torch.max(outputs.data, 1)
            total += labels.size(0)
            correct += (predicted == labels).sum().item()
    
    accuracy = 100 * correct / total
    test_accuracies.append(accuracy)
    
    print(f'Epoch [{epoch+1}/{num_epochs}], Train Loss: {train_loss:.4f}, Test Accuracy: {accuracy:.2f}%')

# Plot results
plt.figure(figsize=(10, 5))
plt.subplot(1, 2, 1)
plt.plot(range(1, num_epochs+1), train_losses)
plt.title('Training Loss')
plt.xlabel('Epoch')
plt.ylabel('Loss')

plt.subplot(1, 2, 2)
plt.plot(range(1, num_epochs+1), test_accuracies)
plt.title('Test Accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy (%)')

plt.tight_layout()
plt.show()
```

```
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-images-idx3-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-images-idx3-ubyte.gz to
100%|██████████| 26421880/26421880 [00:03<00:00, 7782215.66it/s]
Extracting ./data/FashionMNIST/raw/train-images-idx3-ubyte.gz to ./data/FashionMNIST/raw

Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-labels-idx1-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-labels-idx1-ubyte.gz to
100%|██████████| 29515/29515 [00:00<00:00, 201876.79it/s]
Extracting ./data/FashionMNIST/raw/train-labels-idx1-ubyte.gz to ./data/FashionMNIST/raw

Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-images-idx3-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-images-idx3-ubyte.gz to
100%|██████████| 4422102/4422102 [00:01<00:00, 3771301.18it/s]
Extracting ./data/FashionMNIST/raw/t10k-images-idx3-ubyte.gz to ./data/FashionMNIST/raw

Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-labels-idx1-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-labels-idx1-ubyte.gz to
100%|██████████| 5148/5148 [00:00<00:00, 6929485.56it/s]
Extracting ./data/FashionMNIST/raw/t10k-labels-idx1-ubyte.gz to ./data/FashionMNIST/raw

Epoch [1/10], Train Loss: 0.3788, Test Accuracy: 87.95%
Epoch [2/10], Train Loss: 0.2277, Test Accuracy: 89.00%
Epoch [3/10], Train Loss: 0.1800, Test Accuracy: 91.55%
Epoch [4/10], Train Loss: 0.1477, Test Accuracy: 91.27%
Epoch [5/10], Train Loss: 0.1192, Test Accuracy: 92.38%
Epoch [6/10], Train Loss: 0.0962, Test Accuracy: 92.22%
Epoch [7/10], Train Loss: 0.0794, Test Accuracy: 92.43%
Epoch [8/10], Train Loss: 0.0623, Test Accuracy: 91.99%
Epoch [9/10], Train Loss: 0.0496, Test Accuracy: 91.92%
Epoch [10/10], Train Loss: 0.0394, Test Accuracy: 92.74%
```



코딩을 시작하거나 AI로 코드를 생성하세요.