

# Team Note of WayInWilderness

jwpassion1, overnap, penguin1234

Compiled on February 23, 2025

## Contents

<b>1 Data Structures</b>	<b>2</b>	<b>5 String</b>	<b>14</b>
1.1 Sparse Table . . . . .	2	5.1 Knuth-Moris-Pratt . . . . .	14
1.2 Persistence Segment Tree . . . . .	2	5.2 Rabin-Karp . . . . .	15
1.3 Segment Tree Beats . . . . .	3	5.3 Manacher . . . . .	15
1.4 Fenwick RMQ . . . . .	3	5.4 Suffix Array and LCP Array . . . . .	15
1.5 Link/Cut Tree . . . . .	4	5.5 Suffix Automaton . . . . .	16
<b>2 Graph &amp; Flow</b>	<b>7</b>	5.6 Aho-Corasick . . . . .	16
2.1 Hopcroft-Karp & König's . . . . .	7	<b>6 DP Optimization</b>	<b>17</b>
2.2 Dinic's . . . . .	8	6.1 Convex Hull Trick w/ Stack . . . . .	17
2.3 Dominator Tree . . . . .	8	6.2 Convex Hull Trick w/ Li-Chao Tree . . . . .	17
2.4 Strongly Connected Component . . . . .	9	6.3 Divide and Conquer Optimization . . . . .	18
2.5 Biconnected Component . . . . .	9	6.4 Monotone Queue Optimization . . . . .	18
2.6 Centroid Decomposition . . . . .	10	6.5 Aliens Trick . . . . .	19
<b>3 Geometry</b>	<b>10</b>	6.6 Knuth Optimization . . . . .	19
3.1 Counter Clockwise . . . . .	10	6.7 Slope Trick . . . . .	19
3.2 Line intersection . . . . .	10	6.8 Sum Over Subsets . . . . .	19
3.3 Graham Scan . . . . .	10	<b>7 Number Theory</b>	<b>19</b>
3.4 Monotone Chain . . . . .	11	7.1 Modular Operator . . . . .	19
3.5 Rotating Calipers . . . . .	11	7.2 Modular Inverse in $\mathcal{O}(N)$ . . . . .	20
3.6 Bulldozer Trick . . . . .	11	7.3 Extended Euclidean . . . . .	20
3.7 Point in Convex Polygon . . . . .	12	7.4 Floor Sum . . . . .	20
<b>4 Fast Fourier Transform</b>	<b>12</b>	7.5 Miller-Rabin . . . . .	20
4.1 Fast Fourier Transform . . . . .	12	7.6 Lucy Hedgehog . . . . .	21
4.2 Number Theoretic Transform and Kitamasa . . . . .	13	7.7 Chinese Remainder Theorem . . . . .	21
4.3 Fast Walsh Hadamard Transform . . . . .	14	7.8 Pollard Rho . . . . .	21
4.4 Fast Walsh Hadamard Transform XOR . . . . .	14		

8	ETC	22
8.1	Gaussian Elimination . . . . .	22
8.2	Useful Stuff . . . . .	22
8.3	Template . . . . .	23
8.4	자주 쓰이는 문제 접근법 . . . . .	23
8.5	DP 최적화 접근 . . . . .	24
8.6	Fast I/O . . . . .	24
8.7	Bitset Add Sub . . . . .	25

## 1 Data Structures

### 1.1 Sparse Table

Usage: RMQ l r: min(lift[l][len], lift[r-(1<<len)+1][len])

Time Complexity:  $\mathcal{O}(N) - \mathcal{O}(1)$

```
int k = ceil(log2(n));
vector<vector<int>> lift(n, vector<int>(k));
for (int i=0; i<n; ++i)
    lift[i][0] = lcp[i];
for (int i=1; i<k; ++i) {
    for (int j=0; j<=n-(1<<i); ++j)
        lift[j][i] = min(lift[j][i-1], lift[j+(1<<(i-1))][i-1]);
}
vector<int> bits(n+1);
for (int i=2; i<=n; ++i) {
    bits[i] = bits[i-1];
    while (1 << bits[i] < i)
        bits[i]++;
    bits[i]--;
}
```

### 1.2 Persistence Segment Tree

Time Complexity:  $\mathcal{O}(\log^2 N)$

```
struct pst {
    struct node {
        int cnt = 0;
        array<int, 2> go{};
    };
};
```

```
};
vector<node> tree;
vector<int> roots;
pst() {
    roots.push_back(1);
    tree.resize(1 << 18);
    for (int i = 1; i < (1 << 17); ++i) {
        tree[i].go[0] = i * 2;
        tree[i].go[1] = i * 2 + 1;
    }
}
int insert(int x, int prev) {
    int curr = tree.size();
    roots.push_back(curr);
    tree.emplace_back();
    for (int i = 16; i >= 0; --i) {
        const int next = (x >> i) & 1;
        tree[curr].go[next] = tree.size();
        tree.emplace_back();
        tree[curr].go[!next] = tree[prev].go[!next];
        tree[curr].cnt = tree[prev].cnt + 1;
        curr = tree[curr].go[next];
        prev = tree[prev].go[next];
    }
    tree[curr].cnt = tree[prev].cnt + 1;
    return roots.back();
}
int query(int u, int v, int lca, int lca_par, int k) {
    int ret = 0;
    for (int i = 16; i >= 0; --i) {
        const int cnt = tree[tree[u].go[0]].cnt +
            tree[tree[v].go[0]].cnt -
            tree[tree[lca].go[0]].cnt -
            tree[tree[lca_par].go[0]].cnt;

        if (cnt >= k) {
            u = tree[u].go[0];
            v = tree[v].go[0];
            lca = tree[lca].go[0];
            lca_par = tree[lca_par].go[0];
        } else {
            ret = i;
            break;
        }
    }
    return ret;
}
```

```

        k -= cnt;
        u = tree[u].go[1];
        v = tree[v].go[1];
        lca = tree[lca].go[1];
        lca_par = tree[lca_par].go[1];
        ret += 1 << i;
    }
}
return ret;
}
};

```

### 1.3 Segment Tree Beats

**Usage:** Note the potential function

**Time Complexity:**  $\mathcal{O}(\log^2 N)$

```

struct seg {
    vector<node> tree;
    void push(int x, int s, int e) {
        tree[x].x += tree[x].l;
        tree[x].o += tree[x].l;
        tree[x].a += tree[x].l;
        if (s != e) {
            tree[x*2].l += tree[x].l;
            tree[x*2+1].l += tree[x].l;
        }
        tree[x].l = 0;
    }
    void init(int x, int s, int e, const vector<int> &a) {
        if (s == e)
            tree[x].x = tree[x].o = tree[x].a = a[s];
        else {
            const int m = (s+e) / 2;
            init(x*2, s, m, a);
            init(x*2+1, m+1, e, a);
            tree[x] = tree[x*2] + tree[x*2+1];
        }
    }
    void off(int x, int s, int e, int l, int r, int v) {
        push(x, s, e);

```

```

        if (e < l || r < s || (tree[x].o & v) == 0)
            return;
        if (l <= s && e <= r && !(v & (tree[x].a ^ tree[x].o))) {
            tree[x].l -= v & tree[x].o;
            push(x, s, e);
        } else {
            const int m = (s+e) / 2;
            off(x*2, s, m, l, r, v);
            off(x*2+1, m+1, e, l, r, v);
            tree[x] = tree[x*2] + tree[x*2+1];
        }
    }
    void on(int x, int s, int e, int l, int r, int v) {
        push(x, s, e);
        if (e < l || r < s || (tree[x].a & v) == v)
            return;
        if (l <= s && e <= r && !(v & (tree[x].a ^ tree[x].o))) {
            tree[x].l += v & ~tree[x].o;
            push(x, s, e);
        } else {
            const int m = (s+e) / 2;
            on(x*2, s, m, l, r, v);
            on(x*2+1, m+1, e, l, r, v);
            tree[x] = tree[x*2] + tree[x*2+1];
        }
    }
    int sum(int x, int s, int e, int l, int r) {
        push(x, s, e);
        if (e < l || r < s)
            return 0;
        if (l <= s && e <= r)
            return tree[x].x;
        const int m = (s+e) / 2;
        return max(sum(x*2, s, m, l, r), sum(x*2+1, m+1, e, l, r));
    }
};

```

### 1.4 Fenwick RMQ

**Time Complexity:** Fast  $\mathcal{O}(\log N)$

```

struct fenwick {
    static constexpr pii INF = {1e9 + 7, -(1e9 + 7)};
    vector<pii> tree1, tree2;
    const vector<int> &arr;
    static pii op(pii l, pii r) {
        return {min(l.first, r.first), max(l.second, r.second)};
    }
    fenwick(const vector<int> &a) : arr(a) {
        const int n = a.size();
        tree1.resize(n + 1, INF);
        tree2.resize(n + 1, INF);
        for (int i = 0; i < n; ++i)
            update(i, a[i]);
    }
    void update(int x, int v) {
        for (int i = x + 1; i < tree1.size(); i += i & -i)
            tree1[i] = op(tree1[i], {v, v});
        for (int i = x + 1; i > 0; i -= i & -i)
            tree2[i] = op(tree2[i], {v, v});
    }
    pii query(int l, int r) {
        pii ret = INF;
        l++, r++;
        int i;
        for (i = r; i - (i & -i) >= l; i -= i & -i)
            ret = op(tree1[i], ret);
        for (i = l; i + (i & -i) <= r; i += i & -i)
            ret = op(tree2[i], ret);
        ret = op({arr[i - 1], arr[i - 1]}, ret);
        return ret;
    }
};

```

## 1.5 Link/Cut Tree

```

struct Node {
    Node *l, *r, *p;
    bool flip;
    int sz;
    T now, sum, lz;

```

```

Node() {
    l = r = p = nullptr;
    sz = 1;
    flip = false;
    now = sum = lz = 0;
}
bool IsLeft() const { return p && this == p->l; }
bool IsRoot() const { return !p || (this != p->l && this != p->r); }
}
friend int GetSize(const Node *x) { return x ? x->sz : 0; }
friend T GetSum(const Node *x) { return x ? x->sum : 0; }
void Rotate() {
    p->Push();
    Push();
    if (IsLeft())
        r && (r->p = p), p->l = r, r = p;
    else
        l && (l->p = p), p->r = l, l = p;
    if (!p->IsRoot())
        (p->IsLeft() ? p->p->l : p->p->r) = this;
    auto t = p;
    p = t->p;
    t->p = this;
    t->Update();
    Update();
}
void Update() {
    sz = 1 + GetSize(l) + GetSize(r);
    sum = now + GetSum(l) + GetSum(r);
}
void Update(const T &val) {
    now = val;
    Update();
}
void Push() {
    Update(now + lz);
    if (flip)
        swap(l, r);
    for (auto c : {l, r})
        if (c)

```

```

        c->flip ^= flip, c->lz += lz;
        lz = 0;
        flip = false;
    }
};
Node *rt;
Node *Splay(Node *x, Node *g = nullptr) {
    for (g || (rt = x); x->p != g; x->Rotate()) {
        if (!x->p->IsRoot())
            x->p->p->Push();
        x->p->Push();
        x->Push();
        if (x->p->p != g)
            (x->IsLeft() ^ x->p->IsLeft() ? x : x->p)->Rotate();
    }
    x->Push();
    return x;
}
Node *Kth(int k) {
    for (auto x = rt;; x = x->r) {
        for (; x->Push(), x->l && x->l->sz > k; x = x->l)
            ;
        if (x->l)
            k -= x->l->sz;
        if (!k--)
            return Splay(x);
    }
}
Node *Gather(int s, int e) {
    auto t = Kth(e + 1);
    return Splay(t, Kth(s - 1))->l;
}
Node *Flip(int s, int e) {
    auto x = Gather(s, e);
    x->flip ^= 1;
    return x;
}
Node *Shift(int s, int e, int k) {
    if (k >= 0) { // shift to right
        k %= e - s + 1;

```

```

        if (k)
            Flip(s, e), Flip(s, s + k - 1), Flip(s + k, e);
    } else { // shift to left
        k = -k;
        k %= e - s + 1;
        if (k)
            Flip(s, e), Flip(s, e - k), Flip(e - k + 1, e);
    }
    return Gather(s, e);
}
int Idx(Node *x) { return x->l->sz; }
////////// Link Cut Tree Start //////////
Node *Splay(Node *x) {
    for (; !x->IsRoot(); x->Rotate()) {
        if (!x->p->IsRoot())
            x->p->p->Push();
        x->p->Push();
        x->Push();
        if (!x->p->IsRoot())
            (x->IsLeft() ^ x->p->IsLeft() ? x : x->p)->Rotate();
    }
    x->Push();
    return x;
}
void Access(Node *x) {
    Splay(x);
    x->r = nullptr;
    x->Update();
    for (auto y = x; x->p; Splay(x))
        y = x->p, Splay(y), y->r = x, y->Update();
}
int GetDepth(Node *x) {
    Access(x);
    x->Push();
    return GetSize(x->l);
}
Node *GetRoot(Node *x) {
    Access(x);
    for (x->Push(); x->l; x->Push())
        x = x->l;
}

```

```

    return Splay(x);
}
Node *GetPar(Node *x) {
    Access(x);
    x->Push();
    if (!x->l)
        return nullptr;
    x = x->l;
    for (x->Push(); x->r; x->Push())
        x = x->r;
    return Splay(x);
}
void Link(Node *p, Node *c) {
    Access(c);
    Access(p);
    c->l = p;
    p->p = c;
    c->Update();
}
void Cut(Node *c) {
    Access(c);
    c->l->p = nullptr;
    c->l = nullptr;
    c->Update();
}
Node *GetLCA(Node *x, Node *y) {
    Access(x);
    Access(y);
    Splay(x);
    return x->p ? x->p : x;
}
Node *Ancestor(Node *x, int k) {
    k = GetDepth(x) - k;
    assert(k >= 0);
    for (;;) x->Push() {
        int s = GetSize(x->l);
        if (s == k)
            return Access(x), x;
        if (s < k)
            k -= s + 1, x = x->r;
    }
}

```

```

    else
        x = x->l;
    }
}
void MakeRoot(Node *x) {
    Access(x);
    Splay(x);
    x->flip ^= 1;
    x->Push();
}
bool IsConnect(Node *x, Node *y) { return GetRoot(x) == GetRoot(y); }
void PathUpdate(Node *x, Node *y, T val) {
    Node *root = GetRoot(x); // original root
    MakeRoot(x);
    Access(y); // make x to root, tie with y
    Splay(x);
    x->lz += val;
    x->Push();
    MakeRoot(root); // Revert
    // edge update without edge vertex...
    Node *lca = GetLCA(x, y);
    Access(lca);
    Splay(lca);
    lca->Push();
    lca->Update(lca->now - val);
}
T VertexQuery(Node *x, Node *y) {
    Node *l = GetLCA(x, y);
    T ret = l->now;
    Access(x);
    Splay(l);
    if (l->r)
        ret = ret + l->r->sum;
    Access(y);
    Splay(l);
    if (l->r)
        ret = ret + l->r->sum;
    return ret;
}

```

```

Node *GetQueryResultNode(Node *u, Node *v) {
    if (!IsConnect(u, v))
        return 0;
    MakeRoot(u);
    Access(v);
    auto ret = v->l;
    while (ret->mx != ret->now) {
        if (ret->l && ret->mx == ret->l->mx)
            ret = ret->l;
        else
            ret = ret->r;
    }
    Access(ret);
    return ret;
} // code from justicehui

```

## 2 Graph & Flow

### 2.1 Hopcroft-Karp & König's

**Usage:** Dinic's variant. Maximum Matching = Minimum Vertex Cover = S - Maximum Independence Set

**Time Complexity:**  $\mathcal{O}(\sqrt{VE})$

```

while (true) {
    vector<int> level(sz, -1);
    queue<int> q;
    for (int x : 1) {
        if (match[x] == -1) {
            level[x] = 0;
            q.push(x);
        }
    }
    while (!q.empty()) {
        const int x = q.front();
        q.pop();
        for (int next : e[x]) {
            if (match[next] != -1 && level[match[next]] == -1) {
                level[match[next]] = level[x] + 1;
                q.push(match[next]);
            }
        }
    }
}

```

```

    }
}
}
if (level.empty() || *max_element(level.begin(), level.end()) == -1)
    break;
function<bool(int)> dfs = [&](int x) {
    for (int next : e[x]) {
        if (match[next] == -1 ||
            (level[match[next]] == level[x] + 1 && dfs(match[next])))
        {
            match[next] = x;
            match[x] = next;
            return true;
        }
    }
    return false;
};
int total = 0;
for (int x : 1) if (level[x] == 0) total += dfs(x);
if (total == 0) break;
flow += total;
}
set<int> alt; // Konig
function<void(int, bool)> dfs = [&](int x, bool left) {
    if (alt.contains(x)) return;
    alt.insert(x);
    for (int next : e[x]) {
        if ((next != match[x]) && left) dfs(next, false);
        if ((next == match[x]) && !left) dfs(next, true);
    }
};
for (int x : 1) if (match[x] == -1) dfs(x, true);
int test = 0;
for (int i : 1) {
    if (alt.contains(i)) {
        auto &[y, x] = pos[i];
        s[y][x] = 'C';
    }
}
}

```

```

for (int i : r) {
    if (!alt.contains(i)) {
        auto &[y, x] = pos[i];
        s[y][x] = 'C';
    }
}

```

## 2.2 Dinic's

**Time Complexity:**  $\mathcal{O}(V^2E)$ ,  $\mathcal{O}(\min(V^{2/3}E, E^{3/2}))$  on unit capacity

```

while (true) {
    vector<int> level(dt, -1);
    queue<int> q;
    level[st] = 0;
    q.push(st);
    while (!q.empty()) {
        const int x = q.front();
        q.pop();
        for (int nid : eid[x]) {
            const auto &[_ , next, cap, flow] = e[nid];
            if (level[next] == -1 && cap - flow > 0) {
                level[next] = level[x] + 1;
                q.push(next);
            }
        }
    }
    if (level[dt] == -1) break;
    vector<int> vis(dt);
    function<int(int, int)> dfs = [&](int x, int total) {
        if (x == dt) return total;
        for (int &i = vis[x]; i < eid[x].size(); ++i) {
            auto &[_ , next, cap, flow] = e[eid[x][i]];
            if (level[next] == level[x] + 1 && cap - flow > 0) {
                const int res = dfs(next, min(total, cap - flow));
                if (res > 0) {
                    auto &[_next, _x, bcap, bflow] = e[eid[x][i] ^ 1];
                    assert(next == _next && x == _x);
                    flow += res;
                    bflow -= res;
                    return res;
                }
            }
        }
    };
}

```

```

    }
}
return 0;
};
while (true) {
    const int res = dfs(st, 1e9 + 7);
    if (res == 0) break;
    ans += res;
}
}

```

## 2.3 Dominator Tree

**Time Complexity:**  $\mathcal{O}(N \log N)$

```

vector<int> DominatorTree(const vector<vector<int>> &g, int src){ //
// 0-based
    int n = g.size();
    vector<vector<int>> rg(n), buf(n);
    vector<int> r(n), val(n), idom(n, -1), sdom(n, -1), o, p(n), u(n);
    iota(all(r), 0); iota(all(val), 0);
    for(int i=0; i<n; i++) for(auto j : g[i]) rg[j].push_back(i);
    function<int(int)> find = [&](int v){
        if(v == r[v]) return v;
        int ret = find(r[v]);
        if(sdom[val[v]] > sdom[val[r[v]]]) val[v] = val[r[v]];
        return r[v] = ret;
    };
    function<void(int)> dfs = [&](int v){
        sdom[v] = o.size(); o.push_back(v);
        for(auto i : g[v]) if(sdom[i] == -1) p[i] = v, dfs(i);
    };
    dfs(src); reverse(all(o));
    for(auto &i : o){
        if(sdom[i] == -1) continue;
        for(auto j : rg[i]){
            if(sdom[j] == -1) continue;
            int x = val[find(j), j];
            if(sdom[i] > sdom[x]) sdom[i] = sdom[x];
        }
    }
}

```



```

    buf[o[o.size() - sdom[i] - 1]].push_back(i);
    for(auto j : buf[p[i]]) u[j] = val[find(j), j];
    buf[p[i]].clear();
    r[i] = p[i];
}
reverse(all(o)); idom[src] = src;
for(auto i : o) // WARNING : if different, takes idom
    if(i != src) idom[i] = sdom[i] == sdom[u[i]] ? sdom[i] :
        idom[u[i]];
for(auto i : o) if(i != src) idom[i] = o[idom[i]];
return idom; // unreachable -> ret[i] = -1
}

```

## 2.4 Strongly Connected Component

Time Complexity:  $\mathcal{O}(N)$

```

int idx = 0, scnt = 0;
vector<int> scc(n, -1), vis(n, -1), st;
function<int (int)> dfs = [&] (int x) {
    int ret = vis[x] = idx++;
    st.push_back(x);
    for (int next : e[x]) {
        if (vis[next] == -1)
            ret = min(ret, dfs(next));
        else if (scc[next] == -1)
            ret = min(ret, vis[next]);
    }
    if (ret == vis[x]) {
        while (!st.empty()) {
            const int t = st.back();
            st.pop_back();
            scc[t] = scnt;
            if (t == x)
                break;
        }
        scnt++;
    }
    return ret;
};

```

## 2.5 Biconnected Component

Time Complexity:  $\mathcal{O}(N)$

```

int idx = 0;
vector<int> vis(n, -1);
vector<pii> st;
vector<vector<pii>> bcc;
vector<bool> cut(n); // articulation point
function<int (int, int)> dfs = [&] (int x, int p) {
    int ret = vis[x] = idx++;
    int child = 0;
    for (int next : e[x]) {
        if (next == p)
            continue;
        if (vis[next] < vis[x])
            st.emplace_back(x, next);
        if (vis[next] != -1)
            ret = min(ret, vis[next]);
        else {
            int res = dfs(next, x);
            ret = min(ret, res);
            child++;
            if (vis[x] <= res) {
                if (p != -1)
                    cut[x] = true;
                bcc.emplace_back();
                while (st.back() != pii{x, next}) {
                    bcc.back().push_back(st.back());
                    st.pop_back();
                }
                bcc.back().push_back(st.back());
                st.pop_back();
            } // vis[x] < res to find bridges
        }
    }
    if (p == -1 && child > 1)
        cut[x] = true;
    return ret;
};

```

## 2.6 Centroid Decomposition

**Usage:** `cent[x]` is the parent in centroid tree

**Time Complexity:**  $\mathcal{O}(N \log N)$

```
vector<int> sz(n);
vector<bool> fin(n);
function<int (int, int)> get_size = [&] (int x, int p) {
    sz[x] = 1;
    for (int next : e[x])
        if (!fin[next] && next != p) sz[x] += get_size(next, x);
    return sz[x];
};
function<int (int, int, int)> get_cent = [&] (int x, int p, int all)
{
    for (int next : e[x])
        if (!fin[next] && next != p && sz[next]*2 > all) return
            get_cent(next, x, all);
    return x;
};
vector<int> cent(n, -1);
function<void (int, int)> get_cent_tree = [&] (int x, int p) {
    get_size(x, p);
    x = get_cent(x, p, sz[x]);
    fin[x] = true;
    cent[x] = p;
    function<void (int, int, int, bool)> dfs = [&] (int x, int p,
        int d, bool test) {
        if (test) // update answer
        else // update state
        for (int next : e[x])
            if (!fin[next] && next != p) dfs(next, x, d, test);
    };
    for (int next : e[x]) {
        if (!fin[next]) {
            dfs(next, x, init, true);
            dfs(next, x, init+curr, false);
        }
    }
    for (int next : e[x])
        if (!fin[next] && next != p) get_cent_tree(next, x);
};
```

```
};
get_cent_tree(0, -1);
```

## 3 Geometry

### 3.1 Counter Clockwise

**Usage:** It returns  $\{-1, 0, 1\}$  - the ccw of  $b - a$  and  $c - b$

**Time Complexity:**  $\mathcal{O}(1)$

```
auto ccw = [] (const pii &a, const pii &b, const pii &c) {
    pii x = { b.first - a.first, b.second - a.second };
    pii y = { c.first - b.first, c.second - b.second };
    ll ret = 1LL * x.first * y.second - 1LL * x.second * y.first;
    return ret == 0 ? 0 : (ret > 0 ? 1 : -1);
};
```

### 3.2 Line intersection

**Usage:** Check the intersection of  $(x_1, x_2)$  and  $(y_1, y_2)$ . It requires an additional condition when they are parallel

**Time Complexity:**  $\mathcal{O}(1)$

```
ccw(x1, x2, y1) != ccw(x1, x1, y2) && ccw(y1, y2, x1) != ccw(y1, y2,
x2)
```

### 3.3 Graham Scan

**Time Complexity:**  $\mathcal{O}(N \log N)$

```
struct point {
    int x, y, p, q;
    point() { x = y = p = q = 0; }
    bool operator < (const point& other) {
        if (1LL * other.p * q != 1LL * p * other.q)
            return 1LL * other.p * q < 1LL * p * other.q;
        else if (y != other.y)
            return y < other.y;
        else
            return x < other.x;
    }
};
```

```

};
swap(points[0], *min_element(points.begin(), points.end()));
for (int i=1; i<points.size(); ++i) {
    points[i].p = points[i].x - points[0].x;
    points[i].q = points[i].y - points[0].y;
}
sort(points.begin()+1, points.end());
vector<int> hull;
for (int i=0; i<points.size(); ++i) {
    while (hull.size() >= 2 && ccw(points[hull[hull.size()-2]],
    points[hull.back()], points[i]) < 1)
        hull.pop_back();
    hull.push_back(i);
}

```

### 3.4 Monotone Chain

**Usage:** Get the upper and lower hull of the convex hull

**Time Complexity:**  $\mathcal{O}(N \log N)$

```

pair<vector<pii>, vector<pii>> getConvexHull(vector<pii> pt){
    sort(pt.begin(), pt.end());
    vector<pii> uh, dh;
    int un=0, dn=0; // for easy coding
    for (auto &tmp : pt) {
        while(un >= 2 && ccw(uh[un-2], uh[un-1], tmp))
            uh.pop_back(), --un;
        uh.push_back(tmp); ++un;
    }
    reverse(pt.begin(), pt.end());
    for (auto &tmp : pt) {
        while(dn >= 2 && ccw(dh[dn-2], dh[dn-1], tmp))
            dh.pop_back(), --dn;
        dh.push_back(tmp); ++dn;
    }
    return {uh, dh};
} // ref: https://namnamseo.tistory.com

```

### 3.5 Rotating Calipers

**Usage:** Get the maximum distance of the convex hull

**Time Complexity:**  $\mathcal{O}(N)$

```

auto ccw4 = [&] (point& a1, point& a2, point& b1, point& b2) {
    return 1LL * (a2.x - a1.x) * (b2.y - b1.y) > 1LL * (a2.y - a1.y)
    * (b2.x - b1.x);
};
auto dist = [] (point& a, point& b) {
    return 1LL * (a.x - b.x) * (a.x - b.x) + 1LL * (a.y - b.y) *
    (a.y - b.y);
};
ll maxi = 0;
for (int i=0, j=1; i<hull.size(); i++) {
    maxi = max(maxi, dist(hull[i], hull[j]));
    if (j < hull.size()-1 && ccw4(hull[i], hull[i+1], hull[j],
    hull[j+1]))
        j++;
    else
        i++;
}

```

### 3.6 Bulldozer Trick

**Usage:** Traverse the entire sorting state of 2D points

**Time Complexity:**  $\mathcal{O}(N^2 \log N)$

```

struct Line{
    ll i, j, dx, dy; // dx >= 0
    Line(int i, int j, const Point &pi, const Point &pj)
        : i(i), j(j), dx(pj.x-pi.x), dy(pj.y-pi.y) {}
    bool operator < (const Line &l) const {
        return make_tuple(dy*l.dx, i, j) < make_tuple(l.dy*dx, l.i,
        l.j);
    }
    bool operator == (const Line &l) const {
        return dy * l.dx == l.dy * dx;
    }
};
void Solve(){
    sort(A+1, A+N+1); iota(P+1, P+N+1, 1);
    vector<Line> V; V.reserve(N*(N-1)/2);
}

```

```

for(int i=1; i<=N; i++) for(int j=i+1; j<=N; j++)
V.emplace_back(i, j, A[i], A[j]);
sort(V.begin(), V.end());
for(int i=0, j=0; i<V.size(); i=j){
    while(j < V.size() && V[i] == V[j]) j++;
    for(int k=i; k<j; k++){
        int u = V[k].i, v = V[k].j; // point id, index -> Pos[id]
        swap(Pos[u], Pos[v]); swap(A[Pos[u]], A[Pos[v]]);
        if(Pos[u] > Pos[v]) swap(u, v);
        // @TODO
    }
}
} // code from justicehui

```

### 3.7 Point in Convex Polygon

**Time Complexity:**  $\mathcal{O}(\log N)$

```

bool Check(const vector<Point> &v, const Point &pt){
    if(CCW(v[0], v[1], pt) < 0) return false;
    int l = 1, r = v.size() - 1;
    while(l < r){
        int m = l + r + 1 >> 1;
        if(CCW(v[0], v[m], pt) >= 0) l = m; else r = m - 1;
    }
    if(l == v.size() - 1) return CCW(v[0], v.back(), pt) == 0 && v[0]
    <= pt && pt <= v.back();
    return CCW(v[0], v[l], pt) >= 0 && CCW(v[l], v[l+1], pt) >= 0 &&
    CCW(v[l+1], v[0], pt) >= 0;
}

```

## 4 Fast Fourier Transform

### 4.1 Fast Fourier Transform

**Usage:** FFT and multiply polynomials

**Time Complexity:**  $\mathcal{O}(N \log N)$

```

#include <string>
#pragma GCC optimize("O3")

```

```

#pragma GCC target("avx,avx2,fma")
#include <bits/stdc++.h>
#include <immintrin.h>
#include <smmmintrin.h>
__m256d mult(__m256d a, __m256d b) {
    __m256d c = _mm256_movedup_pd(a);
    __m256d d = _mm256_shuffle_pd(a, a, 15);
    __m256d cb = _mm256_mul_pd(c, b);
    __m256d db = _mm256_mul_pd(d, b);
    __m256d e = _mm256_shuffle_pd(db, db, 5);
    __m256d r = _mm256_addsub_pd(cb, e);
    return r;
}
void fft(int n, __m128d a[], bool invert) {
    for (int i = 1, j = 0; i < n; ++i) {
        int bit = n >> 1;
        for (; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len <= 1) {
        double ang = 2 * 3.14159265358979 / len * (invert ? -1 : 1);
        __m256d wlen;
        wlen[0] = cos(ang), wlen[1] = sin(ang);
        for (int i = 0; i < n; i += len) {
            __m256d w; w[0] = 1; w[1] = 0;
            for (int j = 0; j < len / 2; ++j) {
                w = _mm256_permute2f128_pd(w, w, 0);
                wlen = _mm256_insertf128_pd(wlen, a[i + j + len / 2], 1);
                w = mult(w, wlen);
                __m128d vw = _mm256_extractf128_pd(w, 1);
                __m128d u = a[i + j];
                a[i + j] = _mm_add_pd(u, vw);
                a[i + j + len / 2] = _mm_sub_pd(u, vw);
            }
        }
    }
    if (invert) {
        __m128d inv; inv[0] = inv[1] = 1.0 / n;
        for (int i = 0; i < n; ++i) a[i] = _mm_mul_pd(a[i], inv);
    }
}

```

```

    }
}
vector<int64_t> multiply(vector<int64_t> &v, vector<int64_t> &w) {
    int n = 2;
    while (n < v.size() + w.size()) n <= 1;
    __m128d *fv = new __m128d[n];
    for (int i = 0; i < n; ++i) fv[i][0] = fv[i][1] = 0;
    for (int i = 0; i < v.size(); ++i) fv[i][0] = v[i];
    for (int i = 0; i < w.size(); ++i) fv[i][1] = w[i];
    fft(n, fv, 0); // (a+bi) is stored in FFT
    for (int i = 0; i < n; i += 2) {
        __m256d a;
        a = _mm256_insertf128_pd(a, fv[i], 0);
        a = _mm256_insertf128_pd(a, fv[i + 1], 1);
        a = mult(a, a);
        fv[i] = _mm256_extractf128_pd(a, 0);
        fv[i + 1] = _mm256_extractf128_pd(a, 1);
    }
    fft(n, fv, 1);
    vector<int64_t> ret(n);
    for (int i = 0; i < n; ++i) ret[i] = (int64_t)round(fv[i][1] / 2);
    delete[] fv;
    return ret;
}

```

## 4.2 Number Theoretic Transform and Kitamasa

**Usage:** FFT with integer - to get better accuracy

**Time Complexity:**  $\mathcal{O}(N \log N)$

```

// w is the root of mod e.g. 3/998244353 and 5/1012924417
void ntt(vector<ll> &f, const ll w, const ll mod) {
    const int n = f.size();
    if (n == 1)
        return;
    vector<ll> odd(n/2), even(n/2);
    for (int i=0; i<n; ++i)
        (i&1 ? odd : even)[i/2] = f[i];
    ntt(odd, w*w%mod, mod);
    ntt(even, w*w%mod, mod);
    ll x = 1;

```

```

    for (int i=0; i<n/2; ++i) {
        f[i] = (even[i] + x * odd[i] % mod) % mod;
        f[i+n/2] = (even[i] - x * odd[i] % mod + mod) % mod;
        x = x*w%mod;
    }
}
vector<int> mult(vector<int> f, vector<int> g) {
    int sz;
    for (sz = 1; sz < f.size() + g.size(); sz *= 2);
    vector<int> ret(sz);
    f.resize(sz), g.resize(sz);
    int w = modpow(W, (MOD - 1) / sz, MOD);
    ntt(f, w), ntt(g, w);
    for (int i = 0; i < sz; ++i)
        ret[i] = 1LL * f[i] * g[i] % MOD;
    ntt(ret, modpow(w, MOD - 2, MOD));
    const int szinv = modpow(sz, MOD - 2, MOD);
    for (int i = 0; i < sz; ++i)
        ret[i] = 1LL * ret[i] * szinv % MOD;
    while (!ret.empty() && ret.back() == 0)
        ret.pop_back();
    return ret;
}
vector<int> inv(vector<int> f, const int DMOD) {
    vector<int> ret = {modpow(f[0], MOD - 2, MOD)};
    for (int i = 1; i < DMOD; i *= 2) {
        vector<int> tmp(f.begin(), f.begin() + min((int)f.size(), i * 2));
        tmp = mult(ret, tmp);
        tmp.resize(i * 2);
        for (int &x : tmp) x = (MOD - x) % MOD;
        tmp[0] = (tmp[0] + 2) % MOD;
        ret = mult(ret, tmp);
        ret.resize(i * 2);
    }
    ret.resize(DMOD);
    return ret;
}
vector<int> div(vector<int> a, vector<int> b) {
    if (a.size() < b.size()) return {};

```

```

const int DMOD = a.size() - b.size() + 1;
reverse(a.begin(), a.end());
reverse(b.begin(), b.end());
if (a.size() > DMOD) a.resize(DMOD);
if (b.size() > DMOD) b.resize(DMOD);
b = inv(b, DMOD);
auto res = mult(a, b);
res.resize(DMOD);
reverse(res.begin(), res.end());
while (!res.empty() && res.back() == 0) res.pop_back();
return res;
}

vector<int> mod(vector<int> &a, vector<int> b) {
    auto tmp = mult(div(a, b), b);
    tmp.resize(a.size());
    for (int i = 0; i < a.size(); ++i)
        a[i] = (a[i] - tmp[i] + MOD) % MOD;
    while (!a.empty() && a.back() == 0) a.pop_back();
    return a;
}

vector<int> res = {1}, xn = {0, 1};
while (n) {
    if (n & 1) res = mod(mult(res, xn), c);
    n /= 2;
    xn = mod(mult(xn, xn), c);
}

```

### 4.3 Fast Walsh Hadamard Transform

Usage: XOR convolution

Time Complexity:  $\mathcal{O}(N \log N)$

```

void fwht(vector<ll> &f) {
    const int n = f.size();
    if (n == 1)
        return;
    vector<ll> odd(n/2), even(n/2);
    for (int i=0; i<n; ++i)
        (i&1 ? odd : even)[i/2] = f[i];
    fwht(odd);
    fwht(even);
}

```

```

for (int i=0; i<n/2; ++i) {
    f[i*2] = even[i] + odd[i];
    f[i*2+1] = even[i] - odd[i];
}
}

```

### 4.4 Fast Walsh Hadamard Transform XOR

Usage: XOR between two frequency array

Time Complexity:  $\mathcal{O}(N \log N)$

```

void fwht_xor(vector<ll> &a, bool inv = false) {
    ll n = a.size();
    for (int s = 2, h = 1; s <= n; s <= 1, h <= 1) {
        for (int l = 0; l < n; l += s) {
            for (int i = 0; i < h; i++) {
                ll t = a[l + h + i];
                a[l + h + i] = a[l + i] - t;
                a[l + i] += t;
                if (inv)
                    a[l + h + i] /= 2, a[l + i] /= 2;
            }
        }
    }
}

```

```

vector<ll> a, b, c;
fwht_xor(a);
fwht_xor(b);
for (int i = 0; i < sz; i++)
    c[i] = a[i] * b[i];
fwht_xor(c, true);

```

## 5 String

### 5.1 Knuth-Morris-Pratt

Time Complexity:  $\mathcal{O}(N)$

```

vector<int> fail(m);
for (int i=1, j=0; i<m; ++i) {

```

```

    while (j > 0 && p[i] != p[j]) j = fail[j-1];
    if (p[i] == p[j]) fail[i] = ++j;
}
vector<int> ans;
for (int i=0, j=0; i<n; ++i) {
    while (j > 0 && t[i] != p[j]) j = fail[j-1];
    if (t[i] == p[j]) {
        if (j == m-1) {
            ans.push_back(i-j);
            j = fail[j];
        } else j++;
    }
}

```

## 5.2 Rabin-Karp

**Usage:** The Rabin fingerprint for const-length hashing

**Time Complexity:**  $\mathcal{O}(N)$

```

ull hash, p;
vector<ull> ht;
for (int i=0; i<=l-mid; ++i) {
    if (i == 0) {
        hash = s[0];
        p = 1;
        for (int j=1; j<mid; ++j) {
            hash = hash * pi + s[j];
            p = p * pi; // pi is the prime e.g. 13
        }
    } else
        hash = (hash - p * s[i-1]) * pi + s[i+mid-1];
    ht.push_back(hash);
}

```

## 5.3 Manacher

**Usage:** Longest radius of palindrome substring

**Time Complexity:**  $\mathcal{O}(N)$

```

vector<int> man(m);
int r = 0, p = 0;

```

```

for (int i=0; i<m; ++i) {
    if (i <= r)
        man[i] = min(man[p*2 - i], r - i);
    while (i-man[i] > 0 && i+man[i] < m-1 && v[i-man[i]-1] ==
v[i+man[i]+1])
        man[i]++;
    if (r < i + man[i]) {
        r = i + man[i];
        p = i;
    }
}

```

## 5.4 Suffix Array and LCP Array

**Time Complexity:**  $\mathcal{O}(N \log N) - \mathcal{O}(N)$

```

const int m = max(255, n)+1;
vector<int> sa(n), ord(n*2), nord(n*2);
for (int i=0; i<n; ++i) {
    sa[i] = i;
    ord[i] = s[i];
}
for (int d=1; d<n; d*=2) {
    auto cmp = [&] (int i, int j) {
        if (ord[i] == ord[j])
            return ord[i+d] < ord[j+d];
        return ord[i] < ord[j];
    };
    vector<int> cnt(m), tmp(n);
    for (int i=0; i<n; ++i)
        cnt[ord[i+d]]++;
    for (int i=0; i+1<m; ++i)
        cnt[i+1] += cnt[i];
    for (int i=n-1; i>=0; --i)
        tmp[--cnt[ord[i+d]]] = i;
    fill(cnt.begin(), cnt.end(), 0);
    for (int i=0; i<n; ++i)
        cnt[ord[i]]++;
    for (int i=0; i+1<m; ++i)
        cnt[i+1] += cnt[i];
    for (int i=n-1; i>=0; --i)

```

```

        sa[--cnt[ord[tmp[i]]]] = tmp[i];
nord[sa[0]] = 1;
for (int i=1; i<n; ++i)
    nord[sa[i]] = nord[sa[i-1]] + cmp(sa[i-1], sa[i]);
swap(ord, nord);
}
vector<int> inv(n), lcp(n);
for (int i=0; i<n; ++i)
    inv[sa[i]] = i;
for (int i=0, k=0; i<n; ++i) {
    if (inv[i] == 0)
        continue;
    for (int j=sa[inv[i]-1]; max(i+j)+k<n&&s[i+k]==s[j+k]; ++k);
    lcp[inv[i]] = k ? k-- : 0;
}

```

## 5.5 Suffix Automaton

**Usage:** Suffix link corresponds to suffix tree of  $\text{rev}(S)$

**Time Complexity:**  $\mathcal{O}(N) - \mathcal{O}(N)$  using hashmap or  $\mathcal{O}(1)$  size array

```

struct suffix_automaton {
    struct node {
        int len, slink;
        map<int, int> go;
    };
    int last = 0;
    vector<node> sa = {{0, -1}};
    void insert(int x) {
        sa.emplace_back(sa[last].len + 1, 0);
        int p = last;
        last = sa.size() - 1;
        while (p != -1 && !sa[p].go.contains(x))
            sa[p].go[x] = last, p = sa[p].slink;
        if (p != -1) {
            const int t = sa[p].go[x];
            if (sa[p].len + 1 < sa[t].len) {
                const int q = sa.size();
                sa.push_back(sa[t]);
                sa[q].len = sa[p].len + 1;
                sa[t].slink = q;
            }
        }
    }
};

```

```

        while (p != -1 && sa[p].go[x] == t)
            sa[p].go[x] = q, p = sa[p].slink;
        sa[last].slink = q;
    } else
        sa[last].slink = t;
    }
}
};

```

## 5.6 Aho-Corasick

**Time Complexity:**  $\mathcal{O}(N + \sum M)$

```

struct trie {
    array<trie *, 3> go;
    trie *fail;
    int output, idx;
    trie() {
        fill(go.begin(), go.end(), nullptr);
        fail = nullptr;
        output = idx = 0;
    }
    ~trie() {
        for (auto &x : go)
            delete x;
    }
    void insert(const string &input, int i) {
        if (i == input.size())
            output++;
        else {
            const int x = input[i] - 'A';
            if (!go[x])
                go[x] = new trie();
            go[x]->insert(input, i+1);
        }
    }
};

queue<trie*> q; // make fail links; requires root->insert before
root->fail = root;
q.push(root);
while (!q.empty()) {

```



```

trie *curr = q.front();
q.pop();
for (int i=0; i<26; ++i) {
    trie *next = curr->go[i];
    if (!next)
        continue;
    if (curr == root)
        next->fail = root;
    else {
        trie *dest = curr->fail;
        while (dest != root && !dest->go[i])
            dest = dest->fail;
        if (dest->go[i])
            dest = dest->go[i];
        next->fail = dest;
    }
    if (next->fail->output)
        next->output = true;
    q.push(next);
}
}
trie *curr = root; // start query
bool found = false;
for (char c : s) {
    c -= 'a';
    while (curr != root && !curr->go[c])
        curr = curr->fail;
    if (curr->go[c])
        curr = curr->go[c];
    if (curr->output) {
        found = true;
        break;
    }
}
}

```

## 6 DP Optimization

### 6.1 Convex Hull Trick w/ Stack

Usage:  $dp[i] = \min(dp[j] + b[j] * a[i]), b[j] \geq b[j+1]$

**Time Complexity:**  $\mathcal{O}(N \log N) - \mathcal{O}(N)$  where  $a[i] \leq a[i+1]$

```

struct lin {
    ll a, b;
    double s;
    ll f(ll x) { return a*x + b; }
};

inline double cross(const lin &x, const lin &y) {
    return 1.0 * (x.b - y.b) / (y.a - x.a);
}

vector<ll> dp(n);
vector<lin> st;
for (int i=1; i<n; ++i) {
    lin curr = { b[i-1], dp[i-1], 0 };
    while (!st.empty()) {
        curr.s = cross(st.back(), curr);
        if (st.back().s < curr.s)
            break;
        st.pop_back();
    }
    st.push_back(curr);
    int x = -1;
    for (int y = st.size(); y > 0; y /= 2) {
        while (x+y < st.size() && st[x+y].s < a[i])
            x += y;
    }
    dp[i] = s[x].f(a[i]);
}
while (x+1 < st.size() && st[x+1].s < a[i]) ++x; // O(N) case

```

### 6.2 Convex Hull Trick w/ Li-Chao Tree

Usage: `update(l, r, 0, { a, b })`

**Time Complexity:**  $\mathcal{O}(N \log N)$

```

static constexpr ll INF = 2e18;
struct lin {
    ll a, b;
    ll f(ll x) { return a*x + b; }
};
struct lichao {

```

```

struct node {
    int l, r;
    lin line;
};
vector<node> tree;
void init() { tree.push_back({-1, -1, { 0, -INF }}); }
void update(ll s, ll e, int n, const lin &line) {
    lin hi = tree[n].line;
    lin lo = line;
    if (hi.f(s) < lo.f(s))
        swap(lo, hi);
    if (hi.f(e) >= lo.f(e)) {
        tree[n].line = hi;
        return;
    }
    const ll m = s + e >> 1;
    if (hi.f(m) > lo.f(m)) {
        tree[n].line = hi;
        if (tree[n].r == -1) {
            tree[n].r = tree.size();
            tree.push_back({-1, -1, { 0, -INF }});
        }
        update(m+1, e, tree[n].r, lo);
    } else {
        tree[n].line = lo;
        if (tree[n].l == -1) {
            tree[n].l = tree.size();
            tree.push_back({-1, -1, { 0, -INF }});
        }
        update(s, m, tree[n].l, hi);
    }
}
ll query(ll s, ll e, int n, ll x) {
    if (n == -1)
        return -INF;
    const ll m = s + e >> 1;
    if (x <= m)
        return max(tree[n].line.f(x), query(s, m, tree[n].l, x));
    else
        return max(tree[n].line.f(x), query(m+1, e, tree[n].r, x));
}

```

```

}
};

```

### 6.3 Divide and Conquer Optimization

Usage:  $dp[t][i] = \min(dp[t-1][j] + c[j][i])$ ,  $c$  is Monge

Time Complexity:  $\mathcal{O}(KN \log N)$

```

vector<vector<ll>> dp(n, vector<ll>(t));
function<void (int, int, int, int, int)> dnc = [&] (int l, int r,
int s, int e, int u) {
    if (l > r)
        return;
    const int mid = (l + r) / 2;
    int opt;
    for (int i=s; i<=min(e, mid); ++i) {
        ll x = sum[i][mid] + C;
        if (i && u)
            x += dp[i-1][u-1];
        if (x >= dp[mid][u]) {
            dp[mid][u] = x;
            opt = i;
        }
    }
    dnc(l, mid-1, s, opt, u);
    dnc(mid+1, r, opt, e, u);
};
for (int i=0; i<t; ++i)
    dnc(0, n-1, 0, n-1, i);

```

### 6.4 Monotone Queue Optimization

Usage:  $dp[i] = \min(dp[j] + c[j][i])$ ,  $c$  is Monge, find cross

Time Complexity:  $\mathcal{O}(N \log N)$

```

auto cross = [&] (ll p, ll q) {
    ll lo = min(p, q) - 1, hi = n + 1;
    while (lo + 1 < hi) {
        const ll mid = (lo + hi) / 2;
        if (f(p, mid) < f(q, mid)) lo = mid;
        else hi = mid;
    }
}

```

```

    }
    return hi;
};
deque<pll> st;
for (int i = 1; i <= n; ++i) {
    pll curr{i - 1, 0};
    while (!st.empty() &&
           (curr.second = cross(st.back().first, i - 1)) <=
            st.back().second)
        st.pop_back();
    st.push_back(curr);
    while (st.size() > 1 && st[1].second <= i) st.pop_front();
    dp[i] = f(st[0].first, i);
}

```

## 6.5 Aliens Trick

**Usage:**  $dp[t][i] = \min(dp[t-1][j] + c[j+1][i])$ ,  $c$  is Monge, find lambda w/ half bs

**Time Complexity:**  $\mathcal{O}(N \log N)$

```

ll lo = 0, hi = 1e15;
while (lo + 1 < hi) {
    const ll mid = (lo + hi) / 2;
    auto [dp, cnt] = dec(mid); // the best DP[N][K] and its K value
    if (cnt < k) hi = mid;
    else lo = mid;
}
cout << (dec(lo).first - lo * k) / 2;

```

## 6.6 Knuth Optimization

**Usage:**  $dp[i] = \min(dp[i][k] + dp[k][j]) + c[i][j]$ , Monge, Monotonic

**Time Complexity:**  $\mathcal{O}(N^2)$

```

vector<vector<int>> dp(n, vector<int>(n)), opt(n, vector<int>(n));
for (int i=0; i<n; ++i)
    opt[i][i] = i;
for (int j=1; j<n; ++j) {
    for (int s=0; s<n-j; ++s) {
        int e = s+j;

```

```

        dp[s][e] = 1e9+7;
        for (int o=opt[s][e-1]; o<min(opt[s+1][e]+1, e); ++o) {
            if (dp[s][e] > dp[s][o] + dp[o+1][e]) {
                dp[s][e] = dp[s][o] + dp[o+1][e];
                opt[s][e] = o;
            }
        }
        dp[s][e] += sum[e+1] - sum[s];
    }
}

```

## 6.7 Slope Trick

**Usage:** Use priority queue, convex condition

**Time Complexity:**  $\mathcal{O}(N \log N)$

```

pq.push(A[0]);
for (int i=1; i<N; ++i) {
    pq.push(A[i] - i);
    pq.push(A[i] - i);
    pq.pop();
    A[i] = pq.top();
}

```

## 6.8 Sum Over Subsets

**Usage:**  $dp[mask] = \sum(A[i])$ ,  $i$  is in mask

**Time Complexity:**  $\mathcal{O}(N 2^N)$

```

for (int i=0; i<(1<<n); i++)
    f[i] = a[i];
for (int j=0; j<n; j++)
    for(int i=0; i<(1<<n); i++)
        if (i & (1<<j)) f[i] += f[i ^ (1<<j)];

```

# 7 Number Theory

## 7.1 Modular Operator

**Usage:** For Fermat's little theorem and Pollard rho

**Time Complexity:**  $\mathcal{O}(\log N)$

```

using ull = unsigned long long;
ull modmul(ull a, ull b, ull n) { return ((unsigned __int128)a * b)
% n; }
ull modmul(ull a, ull b, ull n) { // if __int128 isn't available
    if (b == 0) return 0;
    if (b == 1) return a;
    ull t = modmul(a, b/2, n);
    t = (t+t)%n;
    if (b % 2) t = (t+a)%n;
    return t;
}
ull modpow(ull a, ull d, ull n) {
    if (d == 0) return 1;
    ull r = modpow(a, d/2, n);
    r = modmul(r, r, n);
    if (d % 2) r = modmul(r, a, n);
    return r;
}
ull gcd(ull a, ull b) { return b ? gcd(b, a%b) : a; }

```

## 7.2 Modular Inverse in $\mathcal{O}(N)$

**Usage:** Get inverse of factorial

**Time Complexity:**  $\mathcal{O}(N) - \mathcal{O}(1)$

```

const int mod = 1e9+7;
vector<int> fact(n+1), inv(n+1), factinv(n+1);
fact[0] = fact[1] = inv[1] = factinv[0] = factinv[1] = 1;
for (int i=2; i<=n; ++i) {
    fact[i] = 1LL * fact[i-1] * i % mod;
    inv[i] = mod - 1LL * mod/i * inv[mod%i] % mod;
    factinv[i] = 1LL * factinv[i-1] * inv[i] % mod;
}

```

## 7.3 Extended Euclidean

**Usage:** get a and b as arguments and return the solution  $(x, y)$  of equation  $ax + by = \gcd(a, b)$ .

**Time Complexity:**  $\mathcal{O}(\log a + \log b)$

```

pair<ll, ll> extGCD(ll a, ll b){
    if (b != 0) {

```

```

        auto tmp = extGCD(b, a % b);
        return {tmp.second, tmp.first - (a / b) * tmp.second};
    } else return {1ll, 0ll};
}

```

## 7.4 Floor Sum

**Usage:** sum of  $\lfloor (ax + b)/c \rfloor$  where  $x \in [0, n]$

**Time Complexity:**  $\mathcal{O}(\log N)$

```

ll floor_sum(ll a, ll b, ll c, ll n) {
    ll ans = 0;
    if (a < 0) {
        ans -= (n * (n + 1) / 2) * ((a % c + c - a) / c);
        a = a % c + c;
    }
    if (b < 0) {
        ans -= (n + 1) * ((b % c + c - b) / c);
        b = b % c + c;
    }
    if (a == 0) return ans + b / c * (n + 1);
    if (a >= c || b >= c)
        return ans + (n * (n + 1) / 2) * (a / c) + (n + 1) * (b / c) +
            floor_sum(a % c, b % c, c, n);
    ll m = (a * n + b) / c;
    return ans + m * n - floor_sum(c, c - b - 1, a, m - 1);
}

```

## 7.5 Miller-Rabin

**Usage:** Fast prime test for big integers

**Time Complexity:**  $\mathcal{O}(k \log N)$

```

bool is_prime(ull n) {
    const ull as[7] = {2, 325, 9375, 28178, 450775, 9780504,
        1795265022};
    // const ull as[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,
        37}; // easier to remember
    auto miller_rabin = [] (ull n, ull a) {
        ull d = n-1, temp;
        while (d % 2 == 0) {

```

```

        d /= 2;
        temp = modpow(a, d, n);
        if (temp == n-1)
            return true;
    }
    return temp == 1;
};
for (ull a : as) {
    if (a >= n)
        break;
    if (!miller_rabin(n, a))
        return false;
}
return true;
}

```

## 7.6 Lucy\_Hedgehog

**Usage:** Fast prime DP; runs within 4 secs where  $N = 10^{12}$

**Time Complexity:**  $\mathcal{O}(N^{3/4})$

```

struct lucy_hedgehog {
    ll n, sq;
    vector<int> sieve, psum;
    vector<ll> a, b, d;
    ll f(ll x) {
        if (x <= sq) return a[x];
        else return b[n / x];
    };
    lucy_hedgehog(ll _n) {
        n = _n, sq = sqrt(n);
        sieve.resize(sq + 1, 1);
        psum.resize(sq + 1);
        sieve[0] = sieve[1] = false;
        for (ll i = 4; i <= sq; i += 2) sieve[i] = false;
        for (ll i = 3; i <= sq; i += 2) {
            if (!sieve[i]) continue;
            for (ll j = i * i; j <= sq; j += i) sieve[j] = false;
        }
        for (int i = 2; i <= sq; ++i) psum[i] = psum[i - 1] + sieve[i];
        a.resize(sq + 1), d = b = a;
    }
};

```

```

for (int i = 1; i <= sq; ++i) {
    d[i] = n / i;    // bottleneck is division
    a[i] = i - 1;    // dp[i]
    b[i] = d[i] - 1; // dp[n/i]
}
for (ll i = 2; i <= sq; ++i) {
    if (!sieve[i]) continue;
    for (ll j = 1; j <= sq && d[j] >= i * i; ++j)
        b[j] = b[j] - (f(d[j] / i) - psum[i - 1]);
    for (int j = sq; j >= i * i; --j)
        a[j] = a[j] - (f(j / i) - psum[i - 1]);
}
};

```

## 7.7 Chinese Remainder Theorem

**Usage:** Solution for the system of linear congruence

**Time Complexity:**  $\mathcal{O}(\log N)$

```

w1 = modpow(mod2, mod1-2, mod1);
w2 = modpow(mod1, mod2-2, mod2);
ll ans = ((__int128)mod2 * w1 * f1[i] + (__int128)mod1 * w2 * f2[i])
% (mod1*mod2);

```

## 7.8 Pollard Rho

**Usage:** Factoring large numbers fast

**Time Complexity:**  $\mathcal{O}(N^{1/4})$

```

void pollard_rho(ull n, vector<ull> &factors) {
    if (n == 1)
        return;
    if (n % 2 == 0) {
        factors.push_back(2);
        pollard_rho(n/2, factors);
        return;
    }
    if (is_prime(n)) {
        factors.push_back(n);
        return;
    }
}

```

```

}
ull x, y, c = 1, g = 1;
auto f = [&] (ull x) { return (modmul(x, x, n) + c) % n; };
y = x = 2;
while (g == 1 || g == n) {
    if (g == n) {
        c = rand() % 123;
        y = x = rand() % (n-2) + 2;
    }
    x = f(x);
    y = f(f(y));
    g = gcd(n, y>x ? y-x : x-y);
}
pollard_rho(g, factors);
pollard_rho(n / g, factors);
}

```

## 8 ETC

### 8.1 Gaussian Elimination

**Time Complexity:**  $\mathcal{O}(\log N)$

```

struct basis {
    const static int n = 30; // log2(1e9)
    array<int, n> data{};
    void insert(int x) {
        for (int i=0; i<n; ++i)
            if (data[i] && (x >> (n-1-i) & 1)) x ^= data[i];
        int y;
        for (y=0; y<n; ++y)
            if (!data[y] && (x >> (n-1-y) & 1)) break;
        if (y < n) {
            for (int i=0; i<n; ++i)
                if (data[i] >> (n-1-y) & 1) data[i] ^= x;
            data[y] = x;
        }
    }
}
basis operator+(const basis &other) {
    basis ret{};

```

```

    for (int x : data) ret.insert(x);
    for (int x : other.data) ret.insert(x);
    return ret;
}
};

```

### 8.2 Useful Stuff

- Catalan Number  
 $1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900$   
 $C_n = \text{binomial}(n * 2, n) / (n + 1);$   
 - 길이가  $2n$ 인 올바른 괄호 수식의 수  
 -  $n + 1$ 개의 리프를 가진 풀 바이너리 트리의 수  
 -  $n + 2$ 각형을  $n$ 개의 삼각형으로 나누는 방법의 수
- Burnside's Lemma  
 경우의 수를 세는데, 특정 transform operation(회전, 반사, ..) 해서 같은 경우들은 하나로 친다. 전체 경우의 수는? 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다 (단, “아무것도 하지 않는다” 라는 operation도 있어야 함!)  
 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)
- 알고리즘 게임
  - Nim Game의 해법 : 각 더미의 돌의 개수를 모두 XOR했을 때 0 이 아니면 첫번째, 0 이면 두번째 플레이어가 승리.
  - Grundy Number : 어떤 상황의 Grundy Number는, 가능한 다음 상황들의 Grundy Number를 모두 모은 다음, 그 집합에 포함 되지 않는 가장 작은 수가 현재 state의 Grundy Number가 된다. 만약 다음 state가 독립된 여러개의 state 들로 나뉠 경우, 각각의 state의 Grundy Number의 XOR 합을 생각한다.
  - Subtraction Game : 한 번에  $k$  개까지의 돌만 가져갈 수 있는 경우, 각 더미의 돌의 개수를  $k + 1$ 로 나눈 나머지를 XOR 합하여 판단한다.
  - Index-k Nim : 한 번에 최대  $k$ 개의 더미를 골라 각각의 더미에서 아무렇게나 돌을 제거할 수 있을 때, 각 binary digit에 대하여 합을  $k + 1$ 로 나눈 나머지를 계산한다. 만약 이 나머지가 모든 digit에 대하여 0이라면 두번째, 하나라도 0이 아니라면 첫번째 플레이어가 승리.
- Pick's Theorem  
 격자점으로 구성된 simple polygon이 주어짐.  $I$ 는 polygon 내부의 격자점 수,  $B$ 는 polygon 선분 위 격자점 수,  $A$ 는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다.  $A = I + B/2 - 1$
- 가장 가까운 두 점 : 분할정복으로 가까운 6개의 점만 확인

- 홀의 결혼 정리 : 이분그래프(L-R)에서, 모든 L을 매칭하는 필요충분 조건 = L에서 임의의 부분집합 S를 골랐을 때, 반드시 (S의 크기)  $\leq$  (S와 연결되어있는 모든 R의 크기)이다.
- 소수 : 10 007 , 10 009 , 10 111 , 31 567 , 70 001 , 1 000 003 , 1 000 033 , 4 000 037 , 99 999 989 , 999 999 937 , 1 000 000 007 , 1 000 000 009 , 9 999 999 967 , 99 999 999 977
- 소수 개수 : (1e5 이하 : 9592), (1e7 이하 : 664 579) , (1e9 이하 : 50 847 534)
- $10^{15}$  이하의 정수 범위의 나눗셈 한번은 오차가 없다.
- N의 약수의 개수 =  $O(N^{1/3})$ , N의 약수의 합 =  $O(N \log \log N)$
- $\phi(mn) = \phi(m)\phi(n)$ ,  $\phi(pr^n) = pr^n - pr^{n-1}$ ,  $a^{\phi(n)} \equiv 1 \pmod n$  if coprime
- Euler characteristic : v - e + f (면, 외부 포함) = 1 + c (컴포넌트)
- Euler's phi  $\phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$
- Lucas' Theorem  $\binom{m}{n} = \prod \binom{m_i}{n_i} \pmod p$   $m_i, n_i$ 는  $p^i$ 의 계수
- 스케줄링에서 데드라인이 빠른 걸 쓰는게 이득. 늦은 스케줄이 안들어갈 때 가장 시간 소모가 큰 스케줄 1개를 제거하면 이득.

### 8.3 Template

```
// precision
cout.precision(16);
cout << fixed;
// gcc bit operator
__builtin_popcount(bits); // popcountll for ll
__builtin_clz(bits);      // left
__builtin_ctz(bits);      // right
// random number generator
random_device rd;
mt19937 mt(rd()); // or use chrono
uniform_int_distribution<> half(0, 1);
cout << half(mt);
// 128MB = int * 33,554,432
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
```

```
x += 0x9e3779b97f4a7c15;
x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
return x ^ (x >> 31);
}
size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM =
        chrono::steady_clock::now().time_since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
}
};
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename K, typename V, typename Comp = less<K>>
using ordered_map =
    tree<K, V, Comp, rb_tree_tag,
    tree_order_statistics_node_update>;
template <typename K, typename Comp = less<K>> // less_equal (MS)
using ordered_set = ordered_map<K, null_type, Comp>;
const int RANDOM =
    chrono::high_resolution_clock::now().time_since_epoch().count();
struct chash {int operator()(int x) const {return x^RANDOM;}};
gp_hash_table<key, int, chash> table;
regex re("^first.[0-9a-z]?*{n}{n,m}");
regex_match(s, re)
```

### 8.4 자주 쓰이는 문제 접근법

- 비슷한 문제를 풀어본 적이 있던가?
- 단순한 방법에서 시작할 수 있을까? (brute force)
- 내가 문제를 푸는 과정을 수식화할 수 있을까? (예제를 직접 해결해보면서)
- 문제를 단순화할 수 있을까?
- 그림으로 그려볼 수 있을까?
- 수식으로 표현할 수 있을까?
- 문제를 분해할 수 있을까?

- 뒤에서부터 생각해서 문제를 풀 수 있을까?
- 순서를 강제할 수 있을까?
- 특정 형태의 답만을 고려할 수 있을까? (정규화)
- 특수 조건을 꼭 활용
- 여사건으로 생각하기
- 게임이론 - 거울 전략 혹은 DP 연계
- 겁먹지 말고 경우 나누어 생각
- 해법에서 역순으로 가능한가?
- 딱 맞는 시간복잡도에 집착하지 말자
- 문제에 의미있는 작은 상수 이용
- 스몰투라지, 트라이, 해싱, 루트질 같은 트릭 생각
- 잘못된 방법으로 파고들지 말고 버리자

## 8.5 DP 최적화 접근

- $C[i, j] = A[i] * B[j]$ 이고 A, B가 단조증가, 단조감소이면 Monge
- l..r의 값들의 sum이나 min은 Monge
- 식 정리해서 일차(CHT) 혹은 비슷한(MQ) 함수를 발견, 구현 힘들면 Li-Chao
- $a \leq b \leq c \leq d$ 에서  $A[a, c] + A[b, d] \leq A[a, d] + A[b, c]$
- Monge 성질을 보이기 어려우면  $N^2$  나이브 짜서 opt의 단조성을 확인하고 짝맞
- 식이 간단하거나 변수가 독립적이면 DP 테이블을 세그 위에 올려서 해결
- 짐작하게 점화식부터 세우고 Monge인지 판별
- Monge에 집착하지 말고 단조성이나 볼록성만 보여도 됨

## 8.6 Fast I/O

```
#pragma GCC optimize("O3")
#pragma GCC optimize("Ofast")
#pragma GCC optimize("unroll-loops")

inline int readChar();
template<class T = int> inline T readInt();
template<class T> inline void writeInt(T x, char end = 0);
inline void writeChar(int x);
inline void writeWord(const char *s);
static const int buf_size = 1 << 18;
inline int getChar(){
    #ifndef LOCAL
        static char buf[buf_size];
        static int len = 0, pos = 0;
        if(pos == len) pos = 0, len = fread(buf, 1, buf_size, stdin);
        if(pos == len) return -1;
        return buf[pos++];
    #endif
}

inline int readChar(){
    #ifndef LOCAL
        int c = getChar();
        while(c <= 32) c = getChar();
        return c;
    #else
        char c; cin >> c; return c;
    #endif
}

template <class T>
inline T readInt(){
    #ifndef LOCAL
        int s = 1, c = readChar();
        T x = 0;
        if(c == '-') s = -1, c = getChar();
        while('0' <= c && c <= '9') x = x * 10 + c - '0', c = getChar();
        return s == 1 ? x : -x;
    #else
        T x; cin >> x; return x;
    #endif
}
```



```

    #endif
}
static int write_pos = 0;
static char write_buf[buf_size];
inline void writeChar(int x){
    if(write_pos == buf_size) fwrite(write_buf, 1, buf_size,
    stdout), write_pos = 0;
    write_buf[write_pos++] = x;
}
template <class T>
inline void writeInt(T x, char end){
    if(x < 0) writeChar('-'), x = -x;
    char s[24]; int n = 0;
    while(x || !n) s[n++] = '0' + x % 10, x /= 10;
    while(n--) writeChar(s[n]);
    if(end) writeChar(end);
}
inline void writeWord(const char *s){
    while(*s) writeChar(*s++);
}
struct Flusher{
    ~Flusher(){ if(write_pos) fwrite(write_buf, 1, write_pos,
    stdout), write_pos = 0; }
}flusher;

```

## 8.7 Bitset Add Sub

```

#define private public
#include <bitset>
#undef private
#include <x86intrin.h>

template <size_t _Nw>
void _M_do_sub(_Base_bitset<_Nw> &A, const _Base_bitset<_Nw> &B) {
    for (int i = 0, c = 0; i < _Nw; i++)
        c = _subborrow_u64(c, A._M_w[i], B._M_w[i],
                            (unsigned long long *)&A._M_w[i]);
}
template <size_t _Nb>

```

```

inline bitset<_Nb> operator-(const bitset<_Nb> &A, const bitset<_Nb>
&B) {
    bitset<_Nb> C(A);
    return C -= B;
}

template <size_t _Nw>
void _M_do_add(_Base_bitset<_Nw> &A, const _Base_bitset<_Nw> &B) {
    for (int i = 0, c = 0; i < _Nw; i++)
        c = _addcarry_u64(c, A._M_w[i], B._M_w[i],
                            (unsigned long long *)&A._M_w[i]);
}

```