Team Note of Seunghwan Kim, Seoul National University

Seunghwan Kim (overnap)

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Contents

1 Data Structure for RMQ

1.1 Sparse Table

Usage: RMQ l r: `min(lift[l][len], lift[r-(1<<len)+1][len])`
Time Complexity: $\mathcal{O}(N) - \mathcal{O}(1)$

```
int k = ceil(log2(n));
vector<vector<int>> lift(n, vector<int>(k));
for (int i=0; i<n; ++i)
    lift[i][0] = lcp[i];
for (int i=1; i<k; ++i) {
    for (int j=0; j<=n-(1<<i); ++j)
        lift[j][i] = min(lift[j][i-1], lift[j+(1<<(i-1))][i-1]);
}
vector<int> bits(n+1);
for (int i=2; i<=n; ++i) {
    bits[i] = bits[i-1];
    while (1 << bits[i] < i)
        bits[i]++;
    bits[i]--;
}
```

1.2 Persistence Segment Tree

Time Complexity: $\mathcal{O}(\log^2 N)$

```
struct pst {
  struct node {
    dat d = ID;
    array<int, 2> go{};
  };
  int n;
  vector<node> tree;
  vector<int> roots;
  pst(int sz) {
    n = int(ceil(log2(sz)));
    roots.push_back(1);
    tree.resize(1 << (n + 1));
    for (int i = 1; i < (1 << n); ++i) {
      tree[i].go[0] = i * 2;
      tree[i].go[1] = i * 2 + 1;
    }
  }
  int insert(int x, int prev, const dat &value) {
    int curr = tree.size();
    tree.emplace_back();
    roots.push_back(curr);
    vector<int> st;
    for (int i = n - 1; i >= 0; --i) {
      st.push_back(curr);
      const int next = x >> i & 1;
      tree[curr].go[next] = tree.size();
      tree.emplace_back();
      tree[curr].go[!next] = tree[prev].go[!next];
      curr = tree[curr].go[next];
      prev = tree[prev].go[next];
    }
    tree[curr].d = value;
    while (!st.empty()) {
      const int x = st.back();
      st.pop_back();
      tree[x].d = tree[tree[x].go[0]].d + tree[tree[x].go[1]].d;
    }
    return roots.back();
  }
  dat query(int t, int l, int r) {
    function<dat(int, int, int)> q = [&](int x, int s, int e) ->
    dat {
      if (r < s || e < l)
        return ID;
      if (l <= s && e <= r)
```

```
      return tree[x].d;
    const int m = (s + e) / 2;
    return q(tree[x].go[0], s, m) + q(tree[x].go[1], m + 1, e);
  };
  return q(t, 0, (1 << n) - 1);
}
int walk(int t, int l, int k) {
  function<int(int, int, int)> q = [&](int x, int s, int e) ->
  int {
    if (e < l || tree[x].d.v >= k)
      return -1;
    if (s == e)
      return s;
    const int m = (s + e) / 2;
    const int res = q(tree[x].go[0], s, m);
    if (res != -1)
      return res;
    return q(tree[x].go[1], m + 1, e);
  };
  const int ret = q(t, 0, (1 << n) - 1);
  return ret == -1 ? (1 << n) - 1 : ret - 1;
}
};
```

1.3 Segment Tree Beats

Usage: Note the potential function
Time Complexity: $\mathcal{O}(\log^2 N)$

```
void off(int x, int s, int e, int l, int r, int v) {
  push(x, s, e);
  if (e < l || r < s || (tree[x].o & v) == 0)
    return;
  if (l <= s && e <= r && !(v & (tree[x].a^tree[x].o))) {
    tree[x].l -= v & tree[x].o;
    push(x, s, e);
  } else {
    const int m = (s+e) / 2;
    off(x*2, s, m, l, r, v);
    off(x*2+1, m+1, e, l, r, v);
    tree[x] = tree[x*2] + tree[x*2+1];
  }
}
void on(int x, int s, int e, int l, int r, int v) {
  push(x, s, e);
  if (e < l || r < s || (tree[x].a & v) == v)
    return;
  if (l <= s && e <= r && !(v & (tree[x].a^tree[x].o))) {
    tree[x].l += v & ~tree[x].o;
    push(x, s, e);
  } else {
    const int m = (s+e) / 2;
    on(x*2, s, m, l, r, v);
    on(x*2+1, m+1, e, l, r, v);
    tree[x] = tree[x*2] + tree[x*2+1];
  }
}
int sum(int x, int s, int e, int l, int r) {
  push(x, s, e);
  if (e < l || r < s)
    return 0;
  if (l <= s && e <= r)
    return tree[x].x;
  const int m = (s+e) / 2;
  return max(sum(x*2, s, m, l, r), sum(x*2+1, m+1, e, l, r));
}
```

1.4 Fenwick RMQ

Time Complexity: Fast $\mathcal{O}(\log N)$

```
struct fenwick {
  static constexpr pii INF = {1e9 + 7, -(1e9 + 7)};
  vector<pii> tree1, tree2;
  const vector<int> &arr;
  static pii op(pii l, pii r) {
    return {min(l.first, r.first), max(l.second, r.second)};
  }
  fenwick(const vector<int> &a) : arr(a) {
    const int n = a.size();
    tree1.resize(n + 1, INF);
    tree2.resize(n + 1, INF);
```

```cpp
    for (int i = 0; i < n; ++i)
      update(i, a[i]);
  }
  void update(int x, int v) {
    for (int i = x + 1; i < tree1.size(); i += i & -i)
      tree1[i] = op(tree1[i], {v, v});
    for (int i = x + 1; i > 0; i -= i & -i)
      tree2[i] = op(tree2[i], {v, v});
  }
  pii query(int l, int r) {
    pii ret = INF;
    l++, r++;
    int i;
    for (i = r; i - (i & -i) >= l; i -= i & -i)
      ret = op(tree1[i], ret);
    for (i = l; i + (i & -i) <= r; i += i & -i)
      ret = op(tree2[i], ret);
    ret = op({arr[i - 1], arr[i - 1]}, ret);
    return ret;
  }
};
```

1.5 Link/Cut Tree

```cpp
struct Node {
  Node *l, *r, *p;
  bool flip;
  int sz;
  T now, sum, lz;
  Node() {
    l = r = p = nullptr;
    sz = 1;
    flip = false;
    now = sum = lz = 0;
  }
  bool IsLeft() const { return p && this == p->l; }
  bool IsRoot() const { return !p || (this != p->l && this !=
  p->r); }
  friend int GetSize(const Node *x) { return x ? x->sz : 0; }
  friend T GetSum(const Node *x) { return x ? x->sum : 0; }
  void Rotate() {
    p->Push();
    Push();
    if (IsLeft())
      r && (r->p = p), p->l = r, r = p;
    else
      l && (l->p = p), p->r = l, l = p;
    if (!p->IsRoot())
      (p->IsLeft() ? p->p->l : p->p->r) = this;
    auto t = p;
    p = t->p;
    t->p = this;
    t->Update();
    Update();
  }
  void Update() {
    sz = 1 + GetSize(l) + GetSize(r);
    sum = now + GetSum(l) + GetSum(r);
  }
  void Update(const T &val) {
    now = val;
    Update();
  }
  void Push() {
    Update(now + lz);
    if (flip)
      swap(l, r);
    for (auto c : {l, r})
      if (c)
        c->flip ^= flip, c->lz += lz;
    lz = 0;
    flip = false;
  }
};
Node *rt;
Node *Splay(Node *x, Node *g = nullptr) {
  for (g || (rt = x); x->p != g; x->Rotate()) {
    if (!x->p->IsRoot())
      x->p->p->Push();
    x->p->Push();
    x->Push();
    if (x->p->p != g)
```

```cpp
      (x->IsLeft() ^ x->p->IsLeft() ? x : x->p)->Rotate();
  }
  x->Push();
  return x;
}
Node *Kth(int k) {
  for (auto x = rt;; x = x->r) {
    for (; x->Push(), x->l && x->l->sz > k; x = x->l)
      ;
    if (x->l)
      k -= x->l->sz;
    if (!k--)
      return Splay(x);
  }
}
Node *Gather(int s, int e) {
  auto t = Kth(e + 1);
  return Splay(t, Kth(s - 1))->l;
}
Node *Flip(int s, int e) {
  auto x = Gather(s, e);
  x->flip ^= 1;
  return x;
}
Node *Shift(int s, int e, int k) {
  if (k >= 0) { // shift to right
    k %= e - s + 1;
    if (k)
      Flip(s, e), Flip(s, s + k - 1), Flip(s + k, e);
  } else { // shift to left
    k = -k;
    k %= e - s + 1;
    if (k)
      Flip(s, e), Flip(s, e - k), Flip(e - k + 1, e);
  }
  return Gather(s, e);
}
int Idx(Node *x) { return x->l->sz; }
//////////// Link Cut Tree Start ////////////
Node *Splay(Node *x) {
  for (; !x->IsRoot(); x->Rotate()) {
    if (!x->p->IsRoot())
      x->p->p->Push();
    x->p->Push();
    x->Push();
    if (!x->p->IsRoot())
      (x->IsLeft() ^ x->p->IsLeft() ? x : x->p)->Rotate();
  }
  x->Push();
  return x;
}
void Access(Node *x) {
  Splay(x);
  x->r = nullptr;
  x->Update();
  for (auto y = x; x->p; Splay(x))
    y = x->p, Splay(y), y->r = x, y->Update();
}
int GetDepth(Node *x) {
  Access(x);
  x->Push();
  return GetSize(x->l);
}
Node *GetRoot(Node *x) {
  Access(x);
  for (x->Push(); x->l; x->Push())
    x = x->l;
  return Splay(x);
}
Node *GetPar(Node *x) {
  Access(x);
  x->Push();
  if (!x->l)
    return nullptr;
  x = x->l;
  for (x->Push(); x->r; x->Push())
    x = x->r;
  return Splay(x);
}
void Link(Node *p, Node *c) {
  Access(c);
  Access(p);
```

```cpp
  c->l = p;
  p->p = c;
  c->Update();
}
void Cut(Node *c) {
  Access(c);
  c->l->p = nullptr;
  c->l = nullptr;
  c->Update();
}
Node *GetLCA(Node *x, Node *y) {
  Access(x);
  Access(y);
  Splay(x);
  return x->p ? x->p : x;
}
Node *Ancestor(Node *x, int k) {
  k = GetDepth(x) - k;
  assert(k >= 0);
  for (;; x->Push()) {
    int s = GetSize(x->l);
    if (s == k)
      return Access(x), x;
    if (s < k)
      k -= s + 1, x = x->r;
    else
      x = x->l;
  }
}
void MakeRoot(Node *x) {
  Access(x);
  Splay(x);
  x->flip ^= 1;
  x->Push();
}
bool IsConnect(Node *x, Node *y) { return GetRoot(x) ==
GetRoot(y); }
void PathUpdate(Node *x, Node *y, T val) {
  Node *root = GetRoot(x); // original root
  MakeRoot(x);
  Access(y); // make x to root, tie with y
  Splay(x);
  x->lz += val;
  x->Push();
  MakeRoot(root); // Revert
  // edge update without edge vertex...
  Node *lca = GetLCA(x, y);
  Access(lca);
  Splay(lca);
  lca->Push();
  lca->Update(lca->now - val);
}
T VertexQuery(Node *x, Node *y) {
  Node *l = GetLCA(x, y);
  T ret = l->now;
  Access(x);
  Splay(l);
  if (l->r)
    ret = ret + l->r->sum;
  Access(y);
  Splay(l);
  if (l->r)
    ret = ret + l->r->sum;
  return ret;
}
Node *GetQueryResultNode(Node *u, Node *v) {
  if (!IsConnect(u, v))
    return 0;
  MakeRoot(u);
  Access(v);
  auto ret = v->l;
  while (ret->mx != ret->now) {
    if (ret->l && ret->mx == ret->l->mx)
      ret = ret->l;
    else
      ret = ret->r;
  }
  Access(ret);
  return ret;
} // code from justicehui
```

2 Graph & Flow

2.1 Hopcroft-Karp & Kőnig's

Usage: Dinic's variant. Maximum Matching = Minimum Vertex Cover = S - Maximum Independence Set

Time Complexity: $\mathcal{O}(\sqrt{V}E)$

```
while (true) {
  vector<int> level(sz, -1);
  queue<int> q;
  for (int x : l) {
    if (match[x] == -1) {
      level[x] = 0;
      q.push(x);
    }
  }
  while (!q.empty()) {
    const int x = q.front();
    q.pop();
    for (int next : e[x]) {
      if (match[next] != -1 && level[match[next]] == -1) {
        level[match[next]] = level[x] + 1;
        q.push(match[next]);
      }
    }
  }
  if (level.empty() || *max_element(level.begin(), level.end()) ==
-1)
    break;
  function<bool(int)> dfs = [&](int x) {
    for (int next : e[x]) {
      if (match[next] == -1 ||
          (level[match[next]] == level[x] + 1 &&
          dfs(match[next]))) {
        match[next] = x;
        match[x] = next;
        return true;
      }
    }
    return false;
  };
  int total = 0;
  for (int x : l) if (level[x] == 0)  total += dfs(x);
  if (total == 0) break;
  flow += total;
}
set<int> alt; // Konig
function<void(int, bool)> dfs = [&](int x, bool left) {
  if (alt.contains(x)) return;
  alt.insert(x);
  for (int next : e[x]) {
    if ((next != match[x]) && left) dfs(next, false);
    if ((next == match[x]) && !left) dfs(next, true);
  }
};
for (int x : l) if (match[x] == -1) dfs(x, true);
int test = 0;
for (int i : l) {
  if (alt.contains(i)) {
    auto &[y, x] = pos[i];
    s[y][x] = 'C';
  }
}
for (int i : r) {
  if (!alt.contains(i)) {
    auto &[y, x] = pos[i];
    s[y][x] = 'C';
  }
}
```

2.2 Dinic's

Time Complexity: $\mathcal{O}(V^2E)$, $\mathcal{O}(\min(V^{2/3}E, E^{3/2}))$ on unit capacity

```
while (true) {
  vector<int> level(dt, -1);
  queue<int> q;
  level[st] = 0;
  q.push(st);
  while (!q.empty()) {
    const int x = q.front();
    q.pop();
```

```cpp
    for (int nid : eid[x]) {
      const auto &[_, next, cap, flow] = e[nid];
      if (level[next] == -1 && cap - flow > 0) {
        level[next] = level[x] + 1;
        q.push(next);
      }
    }
  }
  if (level[dt] == -1) break;
  vector<int> vis(dt);
  function<int(int, int)> dfs = [&](int x, int total) {
    if (x == dt) return total;
    for (int &i = vis[x]; i < eid[x].size(); ++i) {
      auto &[_, next, cap, flow] = e[eid[x][i]];
      if (level[next] == level[x] + 1 && cap - flow > 0) {
        const int res = dfs(next, min(total, cap - flow));
        if (res > 0) {
          auto &[_next, _x, bcap, bflow] = e[eid[x][i] ^ 1];
          assert(next == _next && x == _x);
          flow += res;
          bflow -= res;
          return res;
        }
      }
    }
    return 0;
  };
  while (true) {
    const int res = dfs(st, 1e9 + 7);
    if (res == 0) break;
    ans += res;
  }
}
```

2.3 Biconnected Component

Time Complexity: $\mathcal{O}(N)$

```cpp
int idx = 0;
vector<int> vis(n, -1);
vector<pii> st;
vector<vector<pii>> bcc;
vector<bool> cut(n); // articulation point
function<int (int, int)> dfs = [&] (int x, int p) {
    int ret = vis[x] = idx++;
    int child = 0;
    for (int next : e[x]) {
        if (next == p)
            continue;
        if (vis[next] < vis[x])
            st.emplace_back(x, next);
        if (vis[next] != -1)
            ret = min(ret, vis[next]);
        else {
            int res = dfs(next, x);
            ret = min(ret, res);
            child++;
            if (vis[x] <= res) {
                if (p != -1)
                    cut[x] = true;
                bcc.emplace_back();
                while (st.back() != pii{x, next}) {
                    bcc.back().push_back(st.back());
                    st.pop_back();
                }
                bcc.back().push_back(st.back());
                st.pop_back();
            } // vis[x] < res to find bridges
        }
    }
    if (p == -1 && child > 1)
        cut[x] = true;
    return ret;
};
```

2.4 Heavy-Light Decomposition

Usage: Query with the ETT number and it's root node
Time Complexity: $\mathcal{O}(N) - \mathcal{O}(\log N)$

```cpp
vector<int> par(n), ett(n), rt(n), d(n), sz(n);
function<void (int)> dfs1 = [&] (int x) {
```

```cpp
    sz[x] = 1;
    for (int &next : e[x]) {
        if (next == par[x]) continue;
        d[next] = d[x]+1;
        par[next] = x;
        dfs1(next);
        sz[x] += sz[next];
        if (e[x][0] == par[x] || sz[e[x][0]] < sz[next])
            swap(e[x][0], next);
    }
};
int idx = 1;
function<void (int)> dfs2 = [&] (int x) {
    ett[x] = idx++;
    for (int next : e[x]) {
        if (next == par[x]) continue;
        rt[next] = next == e[x][0] ? rt[x] : next;
        dfs2(next);
    }
};
```

2.5 Centroid Decomposition

Usage: cent[x] is the parent in centroid tree
Time Complexity: $\mathcal{O}(N \log N)$

```cpp
vector<int> sz(n);
vector<bool> fin(n);
function<int (int, int)> get_size = [&] (int x, int p) {
    sz[x] = 1;
    for (int next : e[x])
        if (!fin[next] && next != p) sz[x] += get_size(next, x);
    return sz[x];
};
function<int (int, int, int)> get_cent = [&] (int x, int p, int
all) {
    for (int next : e[x])
        if (!fin[next] && next != p && sz[next]*2 > all) return
        get_cent(next, x, all);
    return x;
};
vector<int> cent(n, -1);
function<void (int, int)> get_cent_tree = [&] (int x, int p) {
    get_size(x, p);
    x = get_cent(x, p, sz[x]);
    fin[x] = true;
    cent[x] = p;
    function<void (int, int, int, bool)> dfs = [&] (int x, int p,
    int d, bool test) {
        if (test) // update answer
        else // update state
        for (int next : e[x])
            if (!fin[next] && next != p) dfs(next, x, d, test);
    };
    for (int next : e[x]) {
        if (!fin[next]) {
            dfs(next, x, init, true);
            dfs(next, x, init+curr, false);
        }
    }
    for (int next : e[x])
        if (!fin[next] && next != p) get_cent_tree(next, x);
};
get_cent_tree(0, -1);
```

3 Geometry

3.1 Line intersection

Usage: Check the intersection of (x_1, x_2) and (y_1, y_2). It requires an additional condition when they are parallel
Time Complexity: $\mathcal{O}(1)$

```cpp
ccw(x1, x2, y1) != ccw(x1, x1, y2) && ccw(y1, y2, x1) != ccw(y1,
y2, x2)
```

3.2 Graham Scan

Time Complexity: $\mathcal{O}(N \log N)$

```cpp
struct point {
    int x, y, p, q;
```

```cpp
    point() { x = y = p = q = 0; }
    bool operator < (const point& other) {
        if (1LL * other.p * q != 1LL * p * other.q)
            return 1LL * other.p * q < 1LL * p * other.q;
        else if (y != other.y)
            return y < other.y;
        else
            return x < other.x;
    }
};
swap(points[0], *min_element(points.begin(), points.end()));
for (int i=1; i<points.size(); ++i) {
    points[i].p = points[i].x - points[0].x;
    points[i].q = points[i].y - points[0].y;
}
sort(points.begin()+1, points.end());
vector<int> hull;
for (int i=0; i<points.size(); ++i) {
    while (hull.size() >= 2 && ccw(points[hull[hull.size()-2]],
    points[hull.back()], points[i]) < 1)
        hull.pop_back();
    hull.push_back(i);
}
```

3.3 Rotating Calipers

Usage: Get the maximum distance of the convex hull
Time Complexity: $\mathcal{O}(N)$

```cpp
auto ccw4 = [&] (point& a1, point& a2, point& b1, point& b2) {
    return 1LL * (a2.x - a1.x) * (b2.y - b1.y) > 1LL * (a2.y -
    a1.y) * (b2.x - b1.x);
};
auto dist = [] (point& a, point& b) {
    return 1LL * (a.x - b.x) * (a.x - b.x) + 1LL * (a.y - b.y) *
    (a.y - b.y);
};
ll maxi = 0;
for (int i=0, j=1; i<hull.size();) {
    maxi = max(maxi, dist(hull[i], hull[j]));
    if (j < hull.size()-1 && ccw4(hull[i], hull[i+1], hull[j],
    hull[j+1]))
        j++;
    else
        i++;
}
```

3.4 Bulldozer Trick

Usage: Traverse the entire sorting state of 2D points
Time Complexity: $\mathcal{O}(N^2 \log N)$

```cpp
struct Line{
  ll i, j, dx, dy; // dx >= 0
  Line(int i, int j, const Point &pi, const Point &pj)
    : i(i), j(j), dx(pj.x-pi.x), dy(pj.y-pi.y) {}
  bool operator < (const Line &l) const {
    return make_tuple(dy*l.dx, i, j) < make_tuple(l.dy*dx, l.i,
    l.j);
  }
  bool operator == (const Line &l) const {
    return dy * l.dx == l.dy * dx;
  }
};
void Solve(){
  sort(A+1, A+N+1); iota(P+1, P+N+1, 1);
  vector<Line> V; V.reserve(N*(N-1)/2);
  for(int i=1; i<=N; i++) for(int j=i+1; j<=N; j++)
  V.emplace_back(i, j, A[i], A[j]);
  sort(V.begin(), V.end());
  for(int i=0, j=0; i<V.size(); i=j){
    while(j < V.size() && V[i] == V[j]) j++;
    for(int k=i; k<j; k++){
      int u = V[k].i, v = V[k].j; // point id, index -> Pos[id]
      swap(Pos[u], Pos[v]); swap(A[Pos[u]], A[Pos[v]]);
      if(Pos[u] > Pos[v]) swap(u, v);
      // @TODO
    }
  }
} // code from justicehui
```

3.5 Point in Convex Polygon

Time Complexity: $\mathcal{O}(\log N)$

```cpp
bool onsegment(pii a, pii b, pii c) {
    return ccw(a, b, c) == 0 && (a - c) * (b - c) <= 0;
}
bool pointinhull(pii* H, int n, pii p, bool strict = true) {
    int a = 1, b = n - 1, r = !strict;
    if (n < 3) return r && onsegment(H[0], H[n - 1], p);
    if (sign(ccw(H[0], H[a], H[b])) > 0) swap(a, b);
    if (sign(ccw(H[0], H[a], p)) >= r || sign(ccw(H[0], H[b], p)
    <= -r))
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (sign(ccw(H[0], H[c], p)) > 0 ? b : a) = c;
    }
    return sign(ccw(H[a], H[b], p)) < r;
}
```

3.6 Line Hull Intersection

Time Complexity: $\mathcal{O}(\log N)$

```cpp
/*
 * lineHull(line, poly) returns a pair describing the intersection
 of a line with the polygon:
 * (-1, -1 if no collision,
 * (i, -1) if touching the corner $i$,
 * (i, i) if along side $(i, i+1)$,
 * (i, j) if crossing sides $(i, i+1)$ and $(j, j+1)$.
 * In the last case, if a corner $i$ is crossed, this is treated
 as happening on side $(i, i+1)$.
 * The points are returned in the same order as the line hits the
 polygon.
 * extrVertex returns the point of a hull with the max projection
 onto a line.
 */
T cross(P p) const { return x*p.y - y*p.x; }
T cross(P a, P b) const { return (a-*this).cross(b-*this); }
P perp() const { return P(-y, x); } // rotates +90 degrees

#define cmp(i,j) sign(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}

#define cmpL(i) sign(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  for (int i = 0; i < 2; i++) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

4 Fast Fourier Transform

4.1 Fast Fourier Transform

Usage: FFT and multiply polynomials
Time Complexity: $\mathcal{O}(N \log N)$

```
#include <string>
#pragma GCC optimize("O3")
#pragma GCC target("avx,avx2,fma")
#include <bits/stdc++.h>
#include <immintrin.h>
#include <smmintrin.h>
__m256d mult(__m256d a, __m256d b) {
  __m256d c = _mm256_movedup_pd(a);
  __m256d d = _mm256_shuffle_pd(a, a, 15);
  __m256d cb = _mm256_mul_pd(c, b);
  __m256d db = _mm256_mul_pd(d, b);
  __m256d e = _mm256_shuffle_pd(db, db, 5);
  __m256d r = _mm256_addsub_pd(cb, e);
  return r;
}
void fft(int n, __m128d a[], bool invert) {
  for (int i = 1, j = 0; i < n; ++i) {
    int bit = n >> 1;
    for (; j >= bit; bit >>= 1) j -= bit;
    j += bit;
    if (i < j) swap(a[i], a[j]);
  }
  for (int len = 2; len <= n; len <<= 1) {
    double ang = 2 * 3.14159265358979 / len * (invert ? -1 : 1);
    __m256d wlen;
    wlen[0] = cos(ang), wlen[1] = sin(ang);
    for (int i = 0; i < n; i += len) {
      __m256d w; w[0] = 1; w[1] = 0;
      for (int j = 0; j < len / 2; ++j) {
        w = _mm256_permute2f128_pd(w, w, 0);
        wlen = _mm256_insertf128_pd(wlen, a[i + j + len / 2], 1);
        w = mult(w, wlen);
        __m128d vw = _mm256_extractf128_pd(w, 1);
        __m128d u = a[i + j];
        a[i + j] = _mm_add_pd(u, vw);
        a[i + j + len / 2] = _mm_sub_pd(u, vw);
      }
    }
  }
  if (invert) {
    __m128d inv; inv[0] = inv[1] = 1.0 / n;
    for (int i = 0; i < n; ++i) a[i] = _mm_mul_pd(a[i], inv);
  }
}
vector<int64_t> multiply(vector<int64_t> &v, vector<int64_t> &w) {
  int n = 2;
  while (n < v.size() + w.size()) n <<= 1;
  __m128d *fv = new __m128d[n];
  for (int i = 0; i < n; ++i) fv[i][0] = fv[i][1] = 0;
  for (int i = 0; i < v.size(); ++i) fv[i][0] = v[i];
  for (int i = 0; i < w.size(); ++i) fv[i][1] = w[i];
  fft(n, fv, 0); // (a+bi) is stored in FFT
  for (int i = 0; i < n; i += 2) {
    __m256d a;
    a = _mm256_insertf128_pd(a, fv[i], 0);
    a = _mm256_insertf128_pd(a, fv[i + 1], 1);
    a = mult(a, a);
    fv[i] = _mm256_extractf128_pd(a, 0);
    fv[i + 1] = _mm256_extractf128_pd(a, 1);
  }
  fft(n, fv, 1);
  vector<int64_t> ret(n);
  for (int i = 0; i < n; ++i) ret[i] = (int64_t)round(fv[i][1] /
  2);
  delete[] fv;
  return ret;
}
```

4.2 Number Theoretic Transform and Kitamasa

Usage: FFT with integer - to get better accuracy
Time Complexity: $\mathcal{O}(N \log N)$

```
// w is the root of mod e.g. 3/998244353 and 5/1012924417
void ntt(vector<ll> &f, const ll w, const ll mod) {
  const int n = f.size();
```

```
  if (n == 1)
    return;
  vector<ll> odd(n/2), even(n/2);
  for (int i=0; i<n; ++i)
    (i&1 ? odd : even)[i/2] = f[i];
  ntt(odd, w*w%mod, mod);
  ntt(even, w*w%mod, mod);
  ll x = 1;
  for (int i=0; i<n/2; ++i) {
    f[i] = (even[i] + x * odd[i] % mod) % mod;
    f[i+n/2] = (even[i] - x * odd[i] % mod + mod) % mod;
    x = x*w%mod;
  }
}
vector<int> mult(vector<int> f, vector<int> g) {
  int sz;
  for (sz = 1; sz < f.size() + g.size(); sz *= 2);
  vector<int> ret(sz);
  f.resize(sz), g.resize(sz);
  int w = modpow(W, (MOD - 1) / sz, MOD);
  ntt(f, w), ntt(g, w);
  for (int i = 0; i < sz; ++i)
    ret[i] = 1LL * f[i] * g[i] % MOD;
  ntt(ret, modpow(w, MOD - 2, MOD));
  const int szinv = modpow(sz, MOD - 2, MOD);
  for (int i = 0; i < sz; ++i)
    ret[i] = 1LL * ret[i] * szinv % MOD;
  while (!ret.empty() && ret.back() == 0)
    ret.pop_back();
  return ret;
}
vector<int> inv(vector<int> f, const int DMOD) {
  vector<int> ret = {modpow(f[0], MOD - 2, MOD)};
  for (int i = 1; i < DMOD; i *= 2) {
    vector<int> tmp(f.begin(), f.begin() + min((int)f.size(), i *
    2));
    tmp = mult(ret, tmp);
    tmp.resize(i * 2);
    for (int &x : tmp) x = (MOD - x) % MOD;
    tmp[0] = (tmp[0] + 2) % MOD;
    ret = mult(ret, tmp);
    ret.resize(i * 2);
  }
  ret.resize(DMOD);
  return ret;
}
vector<int> div(vector<int> a, vector<int> b) {
  if (a.size() < b.size()) return {};
  const int DMOD = a.size() - b.size() + 1;
  reverse(a.begin(), a.end());
  reverse(b.begin(), b.end());
  if (a.size() > DMOD) a.resize(DMOD);
  if (b.size() > DMOD) b.resize(DMOD);
  b = inv(b, DMOD);
  auto res = mult(a, b);
  res.resize(DMOD);
  reverse(res.begin(), res.end());
  while (!res.empty() && res.back() == 0) res.pop_back();
  return res;
}
vector<int> mod(vector<int> &&a, vector<int> b) {
  auto tmp = mult(div(a, b), b);
  tmp.resize(a.size());
  for (int i = 0; i < a.size(); ++i)
    a[i] = (a[i] - tmp[i] + MOD) % MOD;
  while (!a.empty() && a.back() == 0) a.pop_back();
  return a;
}
vector<int> res = {1}, xn = {0, 1};
while (n) {
  if (n & 1) res = mod(mult(res, xn), c);
  n /= 2;
  xn = mod(mult(xn, xn), c);
}
```

4.3 Fast Walsh Hadamard Transform

Usage: XOR convolution
Time Complexity: $\mathcal{O}(N \log N)$

```
void fwht(vector<ll> &f) {
  const int n = f.size();
```

```
  if (n == 1)
    return;
  vector<ll> odd(n/2), even(n/2);
  for (int i=0; i<n; ++i)
    (i&1 ? odd : even)[i/2] = f[i];
  fwht(odd);
  fwht(even);
  for (int i=0; i<n/2; ++i) {
    f[i*2] = even[i] + odd[i];
    f[i*2+1] = even[i] - odd[i];
  }
}
```

5 String

5.1 Knuth-Moris-Pratt

Time Complexity: $\mathcal{O}(N)$

```
vector<int> fail(m);
for (int i=1, j=0; i<m; ++i) {
    while (j > 0 && p[i] != p[j]) j = fail[j-1];
    if (p[i] == p[j]) fail[i] = ++j;
}
vector<int> ans;
for (int i=0, j=0; i<n; ++i) {
    while (j > 0 && t[i] != p[j]) j = fail[j-1];
    if (t[i] == p[j]) {
        if (j == m-1) {
            ans.push_back(i-j);
            j = fail[j];
        } else j++;
    }
}
```

5.2 Rabin-Karp

Usage: The Rabin fingerprint for const-length hashing
Time Complexity: $\mathcal{O}(N)$

```
ull hash, p;
vector<ull> ht;
for (int i=0; i<=l-mid; ++i) {
    if (i == 0) {
        hash = s[0];
        p = 1;
        for (int j=1; j<mid; ++j) {
            hash = hash * pi + s[j];
            p = p * pi; // pi is the prime e.g. 13
        }
    } else
        hash = (hash - p * s[i-1]) * pi + s[i+mid-1];
    ht.push_back(hash);
}
```

5.3 Manacher

Usage: Longest radius of palindrome substring
Time Complexity: $\mathcal{O}(N)$

```
vector<int> man(m);
int r = 0, p = 0;
for (int i=0; i<m; ++i) {
    if (i <= r)
        man[i] = min(man[p*2 - i], r - i);
    while (i-man[i] > 0 && i+man[i] < m-1 && v[i-man[i]-1] == v[i+man[i]+1])
        man[i]++;
    if (r < i + man[i]) {
        r = i + man[i];
        p = i;
    }
}
```

5.4 Suffix Array and LCP Array

Time Complexity: $\mathcal{O}(N \log N) - \mathcal{O}(N)$

```
const int m = max(255, n)+1;
vector<int> sa(n), ord(n*2), nord(n*2);
for (int i=0; i<n; ++i) {
    sa[i] = i;
```

```
    ord[i] = s[i];
}
for (int d=1; d<n; d*=2) {
    auto cmp = [&] (int i, int j) {
        if (ord[i] == ord[j])
            return ord[i+d] < ord[j+d];
        return ord[i] < ord[j];
    };
    vector<int> cnt(m), tmp(n);
    for (int i=0; i<n; ++i)
        cnt[ord[i+d]]++;
    for (int i=0; i+1<m; ++i)
        cnt[i+1] += cnt[i];
    for (int i=n-1; i>=0; --i)
        tmp[--cnt[ord[i+d]]] = i;
    fill(cnt.begin(), cnt.end(), 0);
    for (int i=0; i<n; ++i)
        cnt[ord[i]]++;
    for (int i=0; i+1<m; ++i)
        cnt[i+1] += cnt[i];
    for (int i=n-1; i>=0; --i)
        sa[--cnt[ord[tmp[i]]]] = tmp[i];
    nord[sa[0]] = 1;
    for (int i=1; i<n; ++i)
        nord[sa[i]] = nord[sa[i-1]] + cmp(sa[i-1], sa[i]);
    swap(ord, nord);
}
vector<int> inv(n), lcp(n);
for (int i=0; i<n; ++i)
    inv[sa[i]] = i;
for (int i=0, k=0; i<n; ++i) {
    if (inv[i] == 0)
        continue;
    for (int j=sa[inv[i]-1]; max(i,j)+k<n&&s[i+k]==s[j+k]; ++k);
    lcp[inv[i]] = k ? k-- : 0;
}
```

5.5 Suffix Automaton

Usage: Suffix link corresponds to suffix tree of rev(S)
Time Complexity: $\mathcal{O}(N) - \mathcal{O}(N)$ using hashmap or $\mathcal{O}(1)$ size array

```
struct suffix_automaton {
  struct node {
    int len, slink;
    map<int, int> go;
  };
  int last = 0;
  vector<node> sa = {{0, -1}};
  void insert(int x) {
    sa.emplace_back(sa[last].len + 1, 0);
    int p = last;
    last = sa.size() - 1;
    while (p != -1 && !sa[p].go.contains(x))
      sa[p].go[x] = last, p = sa[p].slink;
    if (p != -1) {
      const int t = sa[p].go[x];
      if (sa[p].len + 1 < sa[t].len) {
        const int q = sa.size();
        sa.push_back(sa[t]);
        sa[q].len = sa[p].len + 1;
        sa[t].slink = q;
        while (p != -1 && sa[p].go[x] == t)
          sa[p].go[x] = q, p = sa[p].slink;
        sa[last].slink = q;
      } else
        sa[last].slink = t;
    }
  }
};
```

5.6 Aho-Corasick

Time Complexity: $\mathcal{O}(N + \sum M)$

```
struct trie {
  array<trie *, 3> go;
  trie *fail;
  int output, idx;
  trie() {
    fill(go.begin(), go.end(), nullptr);
    fail = nullptr;
    output = idx = 0;
```

```cpp
  }
  ~trie() {
    for (auto &x : go)
      delete x;
  }
  void insert(const string &input, int i) {
    if (i == input.size())
      output++;
    else {
      const int x = input[i] - 'A';
      if (!go[x])
        go[x] = new trie();
      go[x]->insert(input, i+1);
    }
  }
};
queue<trie*> q; // make fail links; requires root->insert before
root->fail = root;
q.push(root);
while (!q.empty()) {
    trie *curr = q.front();
    q.pop();
    for (int i=0; i<26; ++i) {
        trie *next = curr->go[i];
        if (!next)
            continue;
        if (curr == root)
            next->fail = root;
        else {
            trie *dest = curr->fail;
            while (dest != root && !dest->go[i])
                dest = dest->fail;
            if (dest->go[i])
                dest = dest->go[i];
            next->fail = dest;
        }
        if (next->fail->output)
            next->output = true;
        q.push(next);
    }
}
trie *curr = root; // start query
bool found = false;
for (char c : s) {
    c -= 'a';
    while (curr != root && !curr->go[c])
        curr = curr->fail;
    if (curr->go[c])
        curr = curr->go[c];
    if (curr->output) {
        found = true;
        break;
    }
}
```

6 DP Optimization

6.1 Convex Hull Trick w/ Stack

Usage: `dp[i] = min(dp[j] + b[j] * a[i]), b[j] >= b[j+1]`
Time Complexity: $\mathcal{O}(N \log N) - \mathcal{O}(N)$ where `a[i] <= a[i+1]`

```cpp
struct lin {
  ll a, b;
  double s;
  ll f(ll x) { return a*x + b; }
};
inline double cross(const lin &x, const lin &y) {
  return 1.0 * (x.b - y.b) / (y.a - x.a);
}
vector<ll> dp(n);
vector<lin> st;
for (int i=1; i<n; ++i) {
    lin curr = { b[i-1], dp[i-1], 0 };
    while (!st.empty()) {
        curr.s = cross(st.back(), curr);
        if (st.back().s < curr.s)
            break;
        st.pop_back();
    }
    st.push_back(curr);
```

```
    int x = -1;
    for (int y = st.size(); y > 0; y /= 2) {
        while (x+y < st.size() && st[x+y].s < a[i])
            x += y;
    }
    dp[i] = s[x].f(a[i]);
}
while (x+1 < st.size() && st[x+1].s < a[i]) ++x; // O(N) case
```

6.2 Convex Hull Trick w/ Li-Chao Tree

Usage: `update(l, r, 0, { a, b })`
Time Complexity: $\mathcal{O}(N \log N)$

```
static constexpr ll INF = 2e18;
struct lin {
  ll a, b;
  ll f(ll x) { return a*x + b; }
};
struct lichao {
  struct node {
    int l, r;
    lin line;
  };
  vector<node> tree;
  void init() { tree.push_back({-1, -1, { 0, -INF }}); }
  void update(ll s, ll e, int n, const lin &line) {
    lin hi = tree[n].line;
    lin lo = line;
    if (hi.f(s) < lo.f(s))
      swap(lo, hi);
    if (hi.f(e) >= lo.f(e)) {
      tree[n].line = hi;
      return;
    }
    const ll m = s + e >> 1;
    if (hi.f(m) > lo.f(m)) {
      tree[n].line = hi;
      if (tree[n].r == -1) {
        tree[n].r = tree.size();
        tree.push_back({-1, -1, { 0, -INF }});
      }
      update(m+1, e, tree[n].r, lo);
    } else {
      tree[n].line = lo;
      if (tree[n].l == -1) {
        tree[n].l = tree.size();
        tree.push_back({-1, -1, { 0, -INF }});
      }
      update(s, m, tree[n].l, hi);
    }
  }
  ll query(ll s, ll e, int n, ll x) {
    if (n == -1)
      return -INF;
    const ll m = s + e >> 1;
    if (x <= m)
      return max(tree[n].line.f(x), query(s, m, tree[n].l, x));
    else
      return max(tree[n].line.f(x), query(m+1, e, tree[n].r, x));
  }
};
```

6.3 Divide and Conquer Optimization

Usage: `dp[t][i] = min(dp[t-1][j] + c[j][i]), c is Monge`
Time Complexity: $\mathcal{O}(KN \log N)$

```
vector<vector<ll>> dp(n, vector<ll>(t));
function<void (int, int, int, int, int)> dnc = [&] (int l, int r,
int s, int e, int u) {
    if (l > r)
        return;
    const int mid = (l + r) / 2;
    int opt;
    for (int i=s; i<=min(e, mid); ++i) {
        ll x = sum[i][mid] + C;
        if (i && u)
            x += dp[i-1][u-1];
        if (x >= dp[mid][u]) {
            dp[mid][u] = x;
            opt = i;
```

```
        }
    }
    dnc(l, mid-1, s, opt, u);
    dnc(mid+1, r, opt, e, u);
};
for (int i=0; i<t; ++i)
    dnc(0, n-1, 0, n-1, i);
```

6.4 Monotone Queue Optimization

Usage: `dp[i] = min(dp[j] + c[j][i]), c is Monge, find cross`
Time Complexity: $\mathcal{O}(N \log N)$

```
auto cross = [&](ll p, ll q) {
  ll lo = min(p, q) - 1, hi = n + 1;
  while (lo + 1 < hi) {
    const ll mid = (lo + hi) / 2;
    if (f(p, mid) < f(q, mid)) lo = mid;
    else hi = mid;
  }
  return hi;
};
deque<pll> st;
for (int i = 1; i ≤ n; ++i) {
  pll curr{i - 1, 0};
  while (!st.empty() &&
          (curr.second = cross(st.back().first, i - 1)) ≤
          st.back().second)
    st.pop_back();
  st.push_back(curr);
  while (st.size() > 1 && st[1].second ≤ i) st.pop_front();
  dp[i] = f(st[0].first, i);
}
```

6.5 Aliens Trick

Usage: `dp[t][i] = min(dp[t-1][j] + c[j+1][i]), c is Monge, find lambda w/ half bs`
Time Complexity: $\mathcal{O}(N \log N)$

```
  ll lo = 0, hi = 1e15;
  while (lo + 1 < hi) {
    const ll mid = (lo + hi) / 2;
    auto [dp, cnt] = dec(mid); // the best DP[N][K] and its K
    value
    if (cnt < k) hi = mid;
    else lo = mid;
  }
  cout << (dec(lo).first - lo * k) / 2;
```

6.6 Knuth Optimization

Usage: `dp[i] = min(dp[i][k] + dp[k][j]) + c[i][j]`, Monge, Monotonic
Time Complexity: $\mathcal{O}(N^2)$

```
vector<vector<int>> dp(n, vector<int>(n)), opt(n, vector<int>(n));
for (int i=0; i<n; ++i)
    opt[i][i] = i;
for (int j=1; j<n; ++j) {
    for (int s=0; s<n-j; ++s) {
        int e = s+j;
        dp[s][e] = 1e9+7;
        for (int o=opt[s][e-1]; o<min(opt[s+1][e]+1, e); ++o) {
            if (dp[s][e] > dp[s][o] + dp[o+1][e]) {
                dp[s][e] = dp[s][o] + dp[o+1][e];
                opt[s][e] = o;
            }
        }
        dp[s][e] += sum[e+1] - sum[s];
    }
}
```

6.7 Slope Trick

Usage: Use priority queue, convex condition
Time Complexity: $\mathcal{O}(N \log N)$

```
pq.push(A[0]);
for (int i=1; i<N; ++i) {
    pq.push(A[i] - i);
    pq.push(A[i] - i);
    pq.pop();
```

```
    A[i] = pq.top();
}
```

6.8 Sum Over Subsets

Usage: `dp[mask] = sum(A[i]), i is in mask`
Time Complexity: $\mathcal{O}(N2^N)$

```
for (int i=0; i<(1<<n); i++)
    f[i] = a[i];
for (int j=0; j<n; j++)
    for(int i=0; i<(1<<n); i++)
      if (i & (1<<j)) f[i] += f[i ^ (1<<j)];
```

7 Number Theory

7.1 Modular Operator

Usage: For Fermat's little theorem and Pollard rho
Time Complexity: $\mathcal{O}(\log N)$

```
using ull = unsigned long long;
ull modmul(ull a, ull b, ull n) { return ((unsigned __int128)a *
b) % n; }
ull modmul(ull a, ull b, ull n) { // if __int128 isn't available
   if (b == 0) return 0;
   if (b == 1) return a;
   ull t = modmul(a, b/2, n);
   t = (t+t)%n;
   if (b % 2) t = (t+a)%n;
   return t;
}
ull modpow(ull a, ull d, ull n) {
    if (d == 0) return 1;
    ull r = modpow(a, d/2, n);
    r = modmul(r, r, n);
    if (d % 2) r = modmul(r, a, n);
    return r;
}
ull gcd(ull a, ull b) { return b ? gcd(b, a%b) : a; }
```

7.2 Modular Inverse in $\mathcal{O}(N)$

Usage: Get inverse of factorial
Time Complexity: $\mathcal{O}(N) - \mathcal{O}(1)$

```
const int mod = 1e9+7;
vector<int> fact(n+1), inv(n+1), factinv(n+1);
fact[0] = fact[1] = inv[1] = factinv[0] = factinv[1] = 1;
for (int i=2; i<=n; ++i) {
    fact[i] = 1LL * fact[i-1] * i % mod;
    inv[i] = mod - 1LL * mod/i * inv[mod%i] % mod;
    factinv[i] = 1LL * factinv[i-1] * inv[i] % mod;
}
```

7.3 Extended Euclidean

Usage: get a and b as arguments and return the solution (x, y) of equation $ax + by = \gcd(a, b)$.
Time Complexity: $\mathcal{O}(\log a + \log b)$

```
pair<ll, ll> extGCD(ll a,ll b){
    if (b != 0) {
        auto tmp = extGCD(b, a % b);
        return {tmp.second, tmp.first - (a / b) * tmp.second};
    } else return {1ll, 0ll};
}
```

7.4 Floor Sum

Usage: sum of $\lfloor (ax + b)/c \rfloor$ where $x \in [0, n]$
Time Complexity: $\mathcal{O}(\log N)$

```
ll floor_sum(ll a, ll b, ll c, ll n) {
  ll ans = 0;
  if (a < 0) {
    ans -= (n * (n + 1) / 2) * ((a % c + c - a) / c);
    a = a % c + c;
  }
  if (b < 0) {
    ans -= (n + 1) * ((b % c + c - b) / c);
    b = b % c + c;
  }
```

```
  if (a == 0) return ans + b / c * (n + 1);
  if (a >= c or b >= c)
    return ans + (n * (n + 1) / 2) * (a / c) + (n + 1) * (b / c) +
           floor_sum(a % c, b % c, c, n);
  ll m = (a * n + b) / c;
  return ans + m * n - floor_sum(c, c - b - 1, a, m - 1);
}
```

7.5 Miller-Rabin

Usage: Fast prime test for big integers
Time Complexity: $\mathcal{O}(k \log N)$

```
bool is_prime(ull n) {
    const ull as[7] = {2, 325, 9375, 28178, 450775, 9780504,
    1795265022};
    // const ull as[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,
    37}; // easier to remember
    auto miller_rabin = [] (ull n, ull a) {
        ull d = n-1, temp;
        while (d % 2 == 0) {
            d /= 2;
            temp = modpow(a, d, n);
            if (temp == n-1)
                return true;
        }
        return temp == 1;
    };
    for (ull a : as) {
        if (a >= n)
            break;
        if (!miller_rabin(n, a))
            return false;
    }
    return true;
}
```

7.6 Lucy_Hedgehog

Usage: Fast prime DP; runs within 4 secs where $N = 10^{12}$
Time Complexity: $\mathcal{O}(N^{3/4})$

```
struct lucy_hedgehog {
  ll n, sq;
  vector<int> sieve, psum;
  vector<ll> a, b, d;
  ll f(ll x) {
    if (x <= sq) return a[x];
    else return b[n / x];
  };
  lucy_hedgehog(ll _n) {
    n = _n, sq = sqrt(n);
    sieve.resize(sq + 1, 1);
    psum.resize(sq + 1);
    sieve[0] = sieve[1] = false;
    for (ll i = 4; i <= sq; i += 2) sieve[i] = false;
    for (ll i = 3; i <= sq; i += 2) {
      if (!sieve[i]) continue;
      for (ll j = i * i; j <= sq; j += i) sieve[j] = false;
    }
    for (int i = 2; i <= sq; ++i) psum[i] = psum[i - 1] +
    sieve[i];
    a.resize(sq + 1), d = b = a;
    for (int i = 1; i <= sq; ++i) {
      d[i] = n / i;    // bottleneck is division
      a[i] = i - 1;    // dp[i]
      b[i] = d[i] - 1; // dp[n/i]
    }
    for (ll i = 2; i <= sq; ++i) {
      if (!sieve[i]) continue;
      for (ll j = 1; j <= sq && d[j] >= i * i; ++j)
        b[j] = b[j] - (f(d[j] / i) - psum[i - 1]);
      for (int j = sq; j >= i * i; --j)
        a[j] = a[j] - (f(j / i) - psum[i - 1]);
    }
  }
};
```

7.7 Chinese Remainder Theorem

Usage: Solution for the system of linear congruence
Time Complexity: $\mathcal{O}(\log N)$

```
w1 = modpow(mod2, mod1-2, mod1);
w2 = modpow(mod1, mod2-2, mod2);
ll ans = ((__int128)mod2 * w1 * f1[i] + (__int128)mod1 * w2 *
f2[i]) % (mod1*mod2);
```

7.8 Pollard Rho

Usage: Factoring large numbers fast

Time Complexity: $\mathcal{O}(N^{1/4})$

```
void pollard_rho(ull n, vector<ull> &factors) {
    if (n == 1)
        return;
    if (n % 2 == 0) {
        factors.push_back(2);
        pollard_rho(n/2, factors);
        return;
    }
    if (is_prime(n)) {
        factors.push_back(n);
        return;
    }
    ull x, y, c = 1, g = 1;
    auto f = [&] (ull x) { return (modmul(x, x, n) + c) % n; };
    y = x = 2;
    while (g == 1 || g == n) {
        if (g == n) {
            c = rand() % 123;
            y = x = rand() % (n-2) + 2;
        }
        x = f(x);
        y = f(f(y));
        g = gcd(n, y>x ? y-x : x-y);
    }
    pollard_rho(g, factors);
    pollard_rho(n / g, factors);
}
```

8 ETC

8.1 Gaussian Elimination on $\mathbb{Z}_2^n$

Time Complexity: $\mathcal{O}(Nd^2/64)$

```
struct basis {
  const static int n = 30; // log2(1e9)
  array<int, n> data{};
  void insert(int x) {
    for (int i=0; i<n; ++i)
      if (data[i] && (x >> (n-1-i) & 1)) x ^= data[i];
    int y;
    for (y=0; y<n; ++y)
      if (!data[y] && (x >> (n-1-y) & 1)) break;
    if (y < n) {
      for (int i=0; i<n; ++i)
        if (data[i] >> (n-1-y) & 1) data[i] ^= x;
      data[y] = x;
    }
  }
  basis operator+(const basis &other) {
    basis ret{};
    for (int x : data) ret.insert(x);
    for (int x : other.data) ret.insert(x);
    return ret;
  }
};
```

8.2 Gaussian Elimination on $\mathbb{Z}_p^n$

Usage: Kirchhoff's, LGV, etc.
Time Complexity: $\mathcal{O}(N^3)$

```
int det(vector<vector<int>> a, const int mod) {
  const int n = a.size();
  assert(a[0].size() == n);
  int ret = 1;
  for (int j = 0; j < n; ++j) {
    int p = j;
    while (p < n && a[p][j] == 0) p++;
    if (p == n) return 0;
```

```
    if (p > j) {
      ret = mod - ret;
      swap(a[j], a[p]);
    }
    ret = 1LL * ret * a[j][j] % mod;
    const int inv = modpow(a[j][j], mod - 2, mod);
    for (int k = j; k < n; ++k)
      a[j][k] = 1LL * a[j][k] * inv % mod;
    for (int i = j + 1; i < n; ++i) {
      for (int k = n - 1; k >= 0; --k)
        a[i][k] = (a[i][k] - 1LL * a[i][j] * a[j][k] % mod + mod)
        % mod;
    }
  }
  return ret;
}
```

8.3 Useful Stuff

- Catalan Number
  1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012,742900
  $C_n = binomial(n*2,n)/(n+1);$
  - 길이가 2n인 올바른 괄호 수식의 수
  - n + 1개의 리프를 가진 풀 바이너리 트리의 수
  - n + 2각형을 n개의 삼각형으로 나누는 방법의 수

- Burnside' s Lemma
  경우의 수를 세는데, 특정 transform operation(회전, 반사, ..) 해서 같은 경우들은 하나로 친다. 전체 경우의 수는? 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다 (단, "아무것도 하지 않는다" 라는 operation도 있어야 함!) 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)

- 알고리즘 게임
  - Nim Game의 해법 : 각 더미의 돌의 개수를 모두 XOR했을 때 0 이 아니면 첫번째, 0 이면 두번째 플레이어가 승리.
  - Grundy Number : 어떤 상황의 Grundy Number는, 가능한 다음 상황들의 Grundy Number를 모두 모은 다음, 그 집합에 포함 되지 않는 가장 작은 수가 현재 state의 Grundy Number가 된다. 만약 다음 state가 독립된 여러개의 state들로 나뉠 경우, 각각의 state의 Grundy Number의 XOR 합을 생각한다.
  - Subtraction Game : 한 번에 k 개까지의 돌만 가져갈 수 있는 경우, 각 더미의 돌의 개수를 k + 1로 나눈 나머지를 XOR 합하여 판단한다.
  - Index-k Nim : 한 번에 최대 k개의 더미를 골라 각각의 더미에서 아무렇게나 돌을 제거할 수 있을 때, 각 binary digit에 대하여 합을 k + 1로 나눈 나머지를 계산한다. 만약 이 나머지가 모든 digit에 대하여 0이라면 두번째, 하나라도 0이 아니라면 첫번째 플레이어가 승리.

- Pick' s Theorem
  격자점으로 구성된 simple polygon이 주어짐. I 는 polygon 내부의 격자점 수, B는 polygon 선분 위 격자점 수, A는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다. $A = I + B/2 - 1$

- 가장 가까운 두 점 : 분할정복으로 가까운 6개의 점만 확인

- 홀의 결혼 정리 : 이분그래프(L-R)에서, 모든 L을 매칭하는 필요충분 조건 = L에서 임의의 부분집합 S를 골랐을 때, 반드시 (S의 크기) <= (S와 연결되어있는 모든 R의 크기)이다.

- 소수 : 10 007 , 10 009 , 10 111 , 31 567 , 70 001 , 1 000 003 , 1 000 033 , 4 000 037 , 99 999 989 , 999 999 937 , 1 000 000 007 , 1 000 000 009 , 9 999 999 967 , 99 999 999 977

- 소수 개수 : (1e5 이하 : 9592), (1e7 이하 : 664 579) , (1e9 이하 : 50 847 534)

- 10^{15} 이하의 정수 범위의 나눗셈 한번은 오차가 없다.

- N의 약수의 개수 = $O(N^{1/3})$, N의 약수의 합 = $O(NloglogN)$

- $\phi(mn) = \phi(m)\phi(n), \phi(pr^n) = pr^n - pr^{n-1}, a^{\phi(n)} \equiv 1 \pmod{n}$ if coprime

- Euler characteristic : v - e + f (면, 외부 포함) = 1 + c (컴포넌트)

- Euler's phi $\phi(n) = n\prod_{p|n}\left(1 - \frac{1}{p}\right)$

- Lucas' Theorem $\binom{m}{n} = \prod \binom{m_i}{n_i} \pmod{p}$ m_i, n_i는 p^i의 계수

- 스케줄링에서 데드라인이 빠른 걸 쓰는게 이득. 늦은 스케줄이 안들어갈 때 가장 시간 소모가 큰 스케줄 1개를 제거하면 이득.

8.4 Template

```
// precision
cout.precision(16);
cout << fixed;
// gcc bit operator
```

```cpp
__builtin_popcount(bits); // popcountll for ll
__builtin_clz(bits);      // left
__builtin_ctz(bits);      // right
// random number generator
random_device rd;
mt19937 mt(rd()); // or use chrono
uniform_int_distribution<> half(0, 1);
cout << half(mt);
// 128MB = int * 33,554,432
struct custom_hash {
  static uint64_t splitmix64(uint64_t x) {
    // http://xorshift.di.unimi.it/splitmix64.c
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
  size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM =
        chrono::steady_clock::now().time_since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
  }
};
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename K, typename V, typename Comp = less<K>>
using ordered_map =
    tree<K, V, Comp, rb_tree_tag,
    tree_order_statistics_node_update>;
template <typename K, typename Comp = less<K>> // less_equal (MS)
using ordered_set = ordered_map<K, null_type, Comp>;
gp_hash_table<key, int, custom_hash> table;
regex re("^first.[0-9a-z]?*+{n}{n,m}");
regex_match(s, re)
// debug macros
template <class T, class U>
ostream &operator<<(ostream &out, const pair<T, U> &v) {
  out << "(" << v.first << ',' << v.second << ")";
  return out;
}
template <class... Ts>
ostream &operator<<(ostream &out, const tuple<Ts...> &v) {
  out << "(";
  [&]<size_t... Is>(index_sequence<Is...>) {
    ((out << (Is == 0 ? "" : ",") << get<Is>(v)), ...);
  }(index_sequence_for<Ts...>{});
  out << ")";
  return out;
}
template <ranges::range T>
  requires(!is_convertible_v<T, std::string>)
ostream &operator<<(ostream &out, const T &v) {
  out << '[';
  bool first = true;
  for (const auto &x : v) {
    if (!first) out << ", ";
    out << x;
    first = false;
  }
  out << ']';
  return out;
}
#ifndef ONLINE_JUDGE
#define debug(x) cout << "[Debug] " << #x << " = " << x << '\n'
#define dout cout
#else
#define debug(x) void(0)
#define dout if (false) cout
#endif
// CLion CMakeLists.txt
cmake_minimum_required(VERSION 3.30)
project(ps)
set(CMAKE_CXX_STANDARD 20)
include_directories(.)
MATH(EXPR stack_size "1024 * 1024 * 1024")
set(CMAKE_CXX_FLAGS "${CMAKE_CXX_FLAGS} -Wextra -Wall
-fsanitize=address -fsanitize=undefined")
set(CMAKE_EXE_LINKER_FLAGS "-Wl,--stack,${stack_size}")
add_definitions(-DLOCAL)
add_executable(ps main.cpp)
```

8.5 자주 쓰이는 문제 접근법

- 비슷한 문제를 풀어본 적이 있던가?
- 단순한 방법에서 시작할 수 있을까? (brute force)
- 내가 문제를 푸는 과정을 수식화할 수 있을까? (예제를 직접 해결해보면서)
- 문제를 단순화할 수 없을까?
- 그림으로 그려볼 수 있을까?
- 수식으로 표현할 수 있을까?
- 문제를 분해할 수 있을까?
- 뒤에서부터 생각해서 문제를 풀 수 있을까?
- 순서를 강제할 수 있을까?
- 특정 형태의 답만을 고려할 수 있을까? (정규화)
- 특수 조건을 꼭 활용
- 여사건으로 생각하기
- 게임이론 - 거울 전략 혹은 mex DP 연계
- 겁먹지 말고 경우 나누어 생각
- 해법에서 역순으로 가능한가?
- 딱 맞는 시간복잡도에 집착하지 말자
- 문제에 의미있는 작은 상수 이용
- 스몰투라지, 트라이, 해싱, 루트질 같은 트릭 생각
- 너무 추상화하기보단 풀려야 하는 방식으로 생각하기
- 잘못된 방법으로 파고들지 말고 버리자
- 제발 터널 비전에 빠지지 말자
- 헬프 콜은 적극적으로
- 혼자 멘탈 나가지 않기

8.6 DP 최적화 접근

- C[i, j] = A[i] * B[j] 이고 A, B가 단조증가, 단조감소이면 Monge
- l..r의 값들의 sum이나 min은 Monge
- 식 정리해서 일차(CHT) 혹은 비슷한(MQ) 함수를 발견, 구현 힘들면 Li-Chao
- $a <= b <= c <= d$에서 $A[a, c] + A[b, d] <= A[a, d] + A[b, c]$
- Monge 성질을 보이기 어려우면 N^2 나이브 짜서 opt의 단조성을 확인하고 찍맞
- 식이 간단하거나 변수가 독립적이면 DP 테이블을 세그 위에 올려서 해결
- 침착하게 점화식부터 세우고 Monge인지 판별
- Monge에 집착하지 말고 단조성이나 볼록성만 보여도 됨

8.7 Fast I/O

```cpp
#pragma GCC optimize("O3")
#pragma GCC optimize("Ofast")
#pragma GCC optimize("unroll-loops")

inline int readChar();
template<class T = int> inline T readInt();
template<class T> inline void writeInt(T x, char end = 0);
inline void writeChar(int x);
inline void writeWord(const char *s);
static const int buf_size = 1 << 18;
inline int getChar(){
    #ifndef LOCAL
    static char buf[buf_size];
    static int len = 0, pos = 0;
    if(pos == len) pos = 0, len = fread(buf, 1, buf_size, stdin);
    if(pos == len) return -1;
    return buf[pos++];
    #endif
}
inline int readChar(){
    #ifndef LOCAL
    int c = getChar();
    while(c <= 32) c = getChar();
    return c;
```

```
    #else
    char c; cin >> c; return c;
    #endif
}
template <class T>
inline T readInt(){
    #ifndef LOCAL
    int s = 1, c = readChar();
    T x = 0;
    if(c == '-') s = -1, c = getChar();
    while('0' <= c && c <= '9') x = x * 10 + c - '0', c =
    getChar();
    return s == 1 ? x : -x;
    #else
    T x; cin >> x; return x;
    #endif
}
static int write_pos = 0;
static char write_buf[buf_size];
inline void writeChar(int x){
    if(write_pos == buf_size) fwrite(write_buf, 1, buf_size,
    stdout), write_pos = 0;
    write_buf[write_pos++] = x;
}
template <class T>
inline void writeInt(T x, char end){
    if(x < 0) writeChar('-'), x = -x;
    char s[24]; int n = 0;
    while(x || !n) s[n++] = '0' + x % 10, x /= 10;
    while(n--) writeChar(s[n]);
    if(end) writeChar(end);
}
inline void writeWord(const char *s){
    while(*s) writeChar(*s++);
}
struct Flusher{
    ~Flusher(){ if(write_pos) fwrite(write_buf, 1, write_pos,
    stdout), write_pos = 0; }
}flusher;
```

8.8 Bitset Add Sub

```
#define private public
#include <bitset>
#undef private
#include <x86intrin.h>

template <size_t _Nw>
void _M_do_sub(_Base_bitset<_Nw> &A, const _Base_bitset<_Nw> &B) {
  for (int i = 0, c = 0; i < _Nw; i++)
    c = _subborrow_u64(c, A._M_w[i], B._M_w[i],
                       (unsigned long long *)&A._M_w[i]);
}
template <size_t _Nb>
inline bitset<_Nb> operator-(const bitset<_Nb> &A, const
bitset<_Nb> &B) {
  bitset<_Nb> C(A);
  return C -= B;
}

template <size_t _Nw>
void _M_do_add(_Base_bitset<_Nw> &A, const _Base_bitset<_Nw> &B) {
  for (int i = 0, c = 0; i < _Nw; i++)
    c = _addcarry_u64(c, A._M_w[i], B._M_w[i],
                      (unsigned long long *)&A._M_w[i]);
}
```