Reductions

Tuesday, November 5, 2019 8:33 PM

CLIQUE

IND. SET

HAM

VERTEX COVER

ILP

SAT

⋮

$P \subseteq NP \cap co\text{-}NP$

$L \in P \Rightarrow \forall x$ if $x \in L$ there is a poly size certificate

if $x \notin L$ ——————— .

Reduction

$A \longrightarrow B$

$x \in A?$ —— $y \in B?$

efficiently given access

A can be solved efficiently given access to a solver for B.

Ex. 1. IND. SET $\longrightarrow$ CLIQUE

$G = (V, E)$ $\exists$ IND. SET of size $\geq k$?

$\Updownarrow$

$\overline{G} = (V, \overline{E})$ $\exists$ clique of size $\geq k$?

Ex. 2. CLIQUE $\longrightarrow$ IND. SET.

Time for reduction? polynomial!

Ex. 3

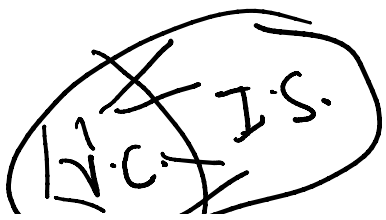
V.C. $\longrightarrow$ I.S.

I.S. $\longrightarrow$ V.C.

$G = (V, E)$ $\exists$ V.C. size $\leq k$?

$\Updownarrow$

$\exists$ ind. set of size $\geq n - k$?



~~I.N.C.~~

Ex.4 CNF SAT $\longrightarrow$ I.L.P.

$(x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_2 \vee \bar{x}_1 \vee x_4 \vee x_5) \wedge \dots$

$1 = T$

$0 = F.$

$$x_1 + x_2 + (1 - x_3) \geq 1$$

$$x_2 + (1 - x_1) + x_4 + x_5 \geq 1$$

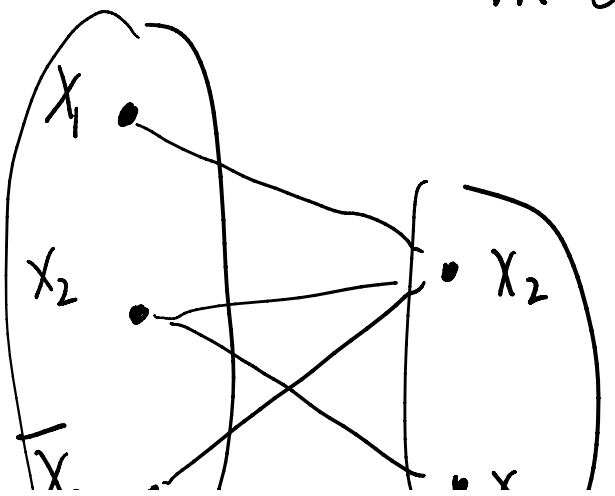
$$\vdots$$

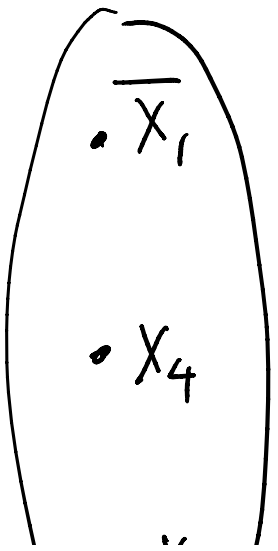
$$0 \leq x_i \leq 1.$$

Ex.5 CNF SAT $\longrightarrow$ CLIQUE

$(x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_2 \vee x_3) \dots (\bar{x}_1 \vee x_4 \vee x_5)$

m clauses.





$(x_i, x_j) \in E$ unless $x_i, \bar{x}_i$ if $x_j \neq \bar{x}_i$ and not corresponding to same clause.

<u>Lemma</u>: $F \in SAT \Leftrightarrow G$ has a clique of size m.

$F \in SAT$: pick one true literal from each clause $\rightarrow$ clique.

G has an m clique. must be one from each clause.

Set all literals in clique to True.

then $x_i, \bar{x}_i$ are not both in clique.