15-150 Assigment 2

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Section S
May 24, 2016

Task 2.1

Theorem 1: For all values `l : (string * int) list`, `zip(unzip l)` $\cong$ `l`.
Proof: By structural induction on `l`.
Case `[]`: To show: `zip(unzip [])` $\cong$ `[]`
Proof:

`zip(unzip [])`	$\cong$	`zip(case [] of [] => ([],[])| ...)`	[step]
	$\cong$	`zip(([],[]))`	[step]
	$\cong$	`case ([],[]) of ([],_) => [] | ...)`	[step]
	$\cong$	`[]`	

By extensional equivalence, `zip(unzip [])` $\cong$ `[]`.
Case `(x1, x2)::xs` for some `x1`, `x2`, `xs`.
Inductive Hypothesis: `zip(unzip xs) => xs`
***Note I:** `unzip xs => (fst unzip xs, snd unzip xs)`
***Note II:** The IH $\implies$ `unzip xs` is a valuable expression
To show: `zip(unzip((x1, x2)::xs)` $\cong$ `(x1, x2)::xs`
Proof:

`zip(unzip (x1, x2)::xs)`	$\cong$	`zip(case (x1, x2) of [] => ([],[]) | ...)`	[step]
	$\cong$	`zip(let val (l1, l2) = unzip xs in...)`	[step]
	$\cong$	`zip(x1::fst(unzip xs), x2::snd(unzip xs))`	[Rule 1, Note I, Note II]
	$\cong$	`case (x1::fst(unzip xs), x2::snd(unzip xs) of...)`	[step]
	$\cong$	`(x1, x2)::zip(fst (unzip xs), snd (unzip xs))`	[step]
	$\cong$	`(x1, x2:)::zip(unzip xs)`	[Rule 2]
	$\cong$	`(x1,x2)::xs`	[IH]

By extensional equivalence, `zip(unzip (x1, x2)::xs)` $\cong$ `(x1, x2)::xs`.
Since the Base Case and the Inductive Step hold, Theorem 1 must be true.

Task 2.2

Theorem 2: For all values `l1 : string list`, `l2 : int list`, `unzip(zip(l1, l2))` $\cong$ `(l1, l2)`
Theorem 2 is false: Proof by counterexample.
Let `l1 = ["hi"]`, `l2 = [1, 2]`. Then,

`unzip(zip(l1, l2))`	$\cong$	`unzip(zip(["hi"], [1, 2]))`	
	$\cong$	`unzip(case (["hi],[1, 2]) of...)`	[step]
	$\cong$	`unzip(("hi",1)::zip([],[2]))`	[step]
	$\cong$	`unzip(("hi",1)::[]`	[step]
	$\cong$	`unzip([("hi",1)])`	[step]
	$\cong$	`case [("hi",1)] of [] => ([],[])| ...`	[step]
	$\cong$	`let val (l1, l2) = unzip [] in ("hi"::l1, 1::l2) end`	[step]
	$\cong$	`("hi"::fst(unzip []), 1::snd(unzip []))`	[Rule 1]
	$\cong$	`("hi"::[], 1::[])`	[step]
	$\cong$	`(["hi"],[1])`	[step]

$\therefore$ `unzip(zip(["hi"], [1, 2, 3]))` $\Longrightarrow$ `(["hi"],[1])` $\ncong$ `(["hi"], [1, 2, 3])`
Thus by counterexample, Theorem 2 is false.

Task 5.2

Given a list `L` with length n, the asymptotic bound for the work of `prefixSum L` is n^2. In other words, $W_{\texttt{prefixSum}}(n)$ is $O(n^2)$, polynomial time.

Task 5.4

Given a list `L` with length n, the asymptotic bound for the work of `prefixSumFast L` is n. In other words, $W_{\texttt{prefixSumFast}}(n)$ is $O(n)$, linear time.