15-150 Assigment 8

Jonathan Li
jlli
Section S
June 15, 2016

Task 2.1

If we first attempt to find a recurrence for $W_{\texttt{length}}(n)$, we find that:

$$\begin{aligned} W_{\texttt{length}}(n) &= k_2 + W_{\texttt{length}}(\frac{n}{2}) + W_{\texttt{length}}(\frac{n}{2}) \\ &= k_2 + 2W_{\texttt{length}}(\frac{n}{2}) \end{aligned}$$

We can safely say that, in the recursive case, the calls to `length(L)` and `length(R)` evaluate inputs of size $\frac{n}{2}$ because of our *Balance Invariant:* that the sizes of `L` and `R` differ by at most 1. Thus, if the size of the input shrub to `length` is of size n, the number of inputs in each of `L` and `R` is around $\frac{n}{2}$. The recurrence above holds, and boils down to a Big-Oh bound of $O(n)$ for $W_{\texttt{length}}(n)$[1].

Similarly, since the calls to `length(L)` and `length(R)` are made in parallel, and by the same Balance Invariant, we find that:

$$\begin{aligned} S_{\texttt{length}}(n) &= k_2 + \texttt{max}(S_{\texttt{length}}(\frac{n}{2}), S_{\texttt{length}}(\frac{n}{2})) \\ &= k_2 + S_{length}(\frac{n}{2}) \end{aligned}$$

...which is $O(logn)$.

[1] I referred to the recurrence table given here:
http://www.cs.cmu.edu/ 15150/resources/lectures/06/WorkAnalysis.pdf