On Understanding Data Abstraction

...

Revisited

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Dedicated to P. Wegner

Objects

????

Abstract Data Types

Warnings!

No “Objects Model the Real World”

No Inheritance

No State

No Subtyping!

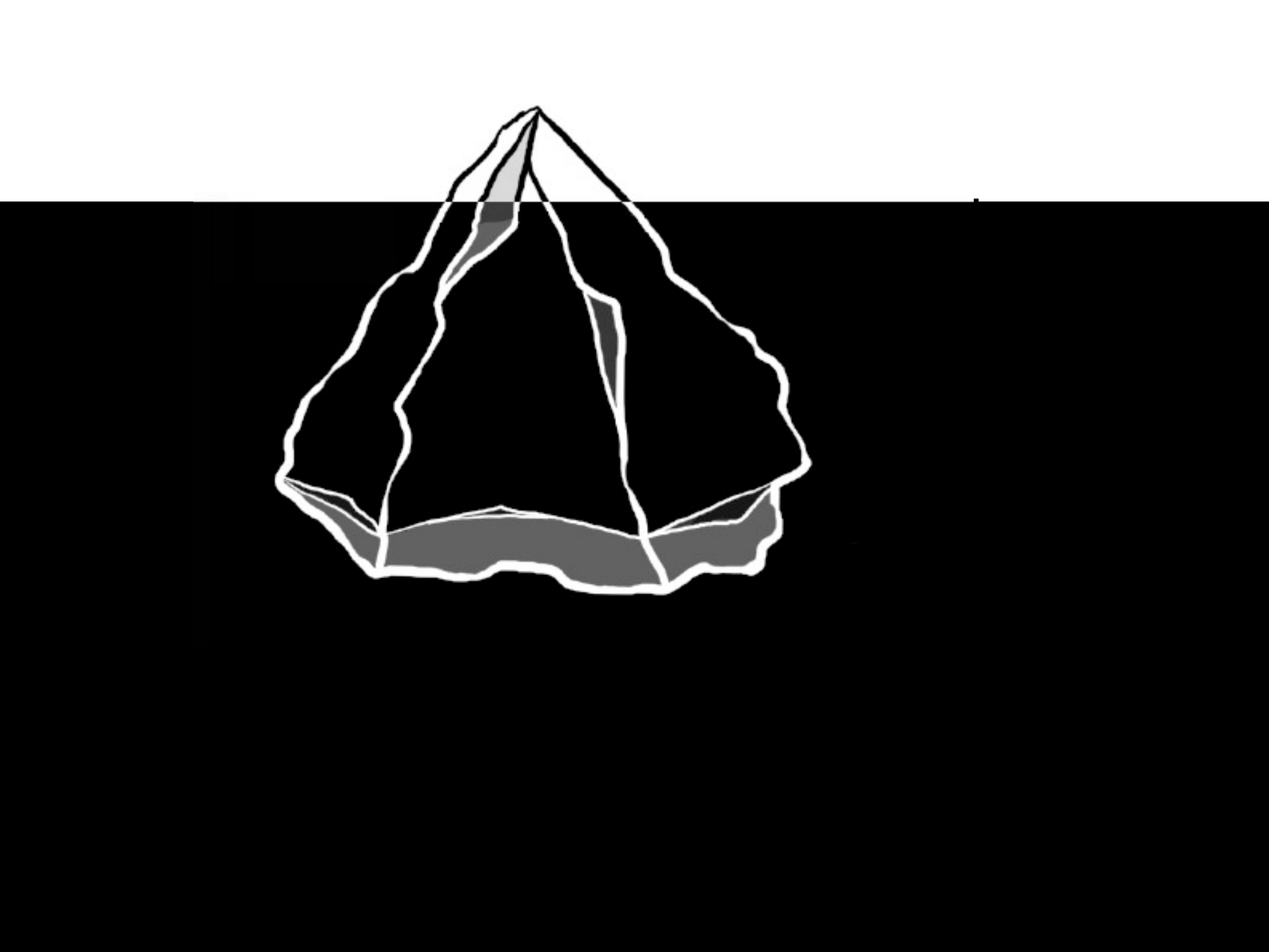
Interfaces as types

Not Essential

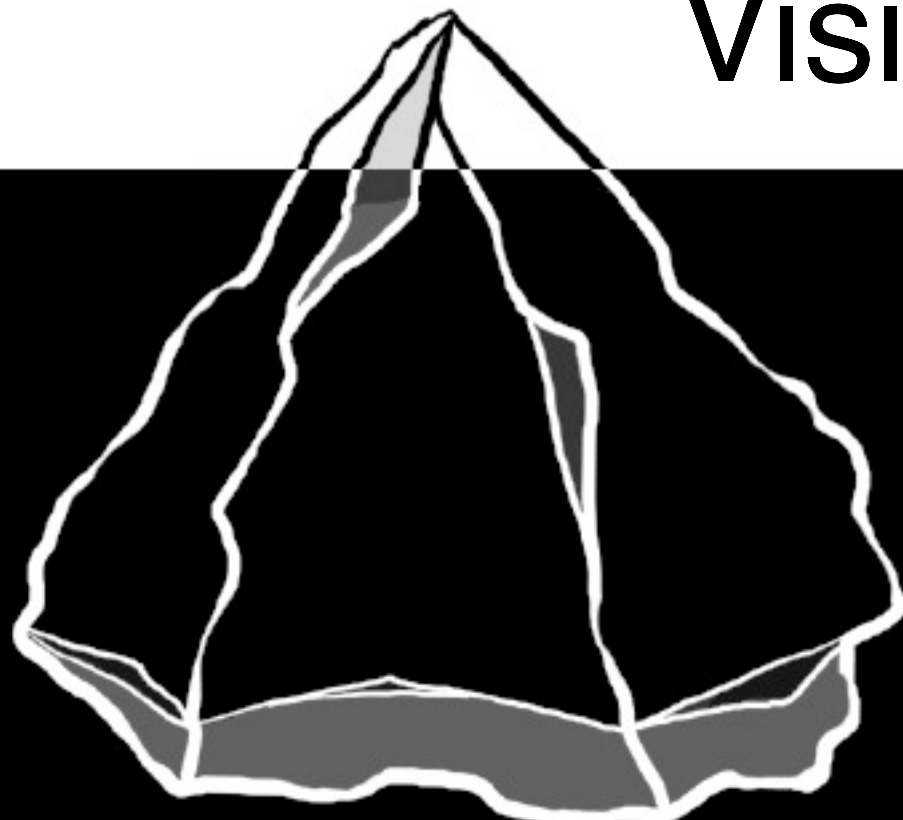
(very nice but not essential)

[discuss
inheritance
later]

Abstraction



Visible



Hidden

Procedural Abstraction

```
bool f(int x) { … }
```

Procedural Abstraction

int → bool

(one kind of)

Type Abstraction

class Set<T>

(one kind of)

Type Abstraction

$\forall T.Set[T]$

Abstract Data Type

```
signature Set
  empty       : Set
  insert      : Set, Int → Set
  isEmpty     : Set → Bool
  contains    : Set, Int → Bool
```

Abstract Data Type

Abstract

signature <u>Set</u>

empty	: Set
insert	: Set, Int → Set
isEmpty	: Set → Bool
contains	: Set, Int → Bool

Type
+
Operations

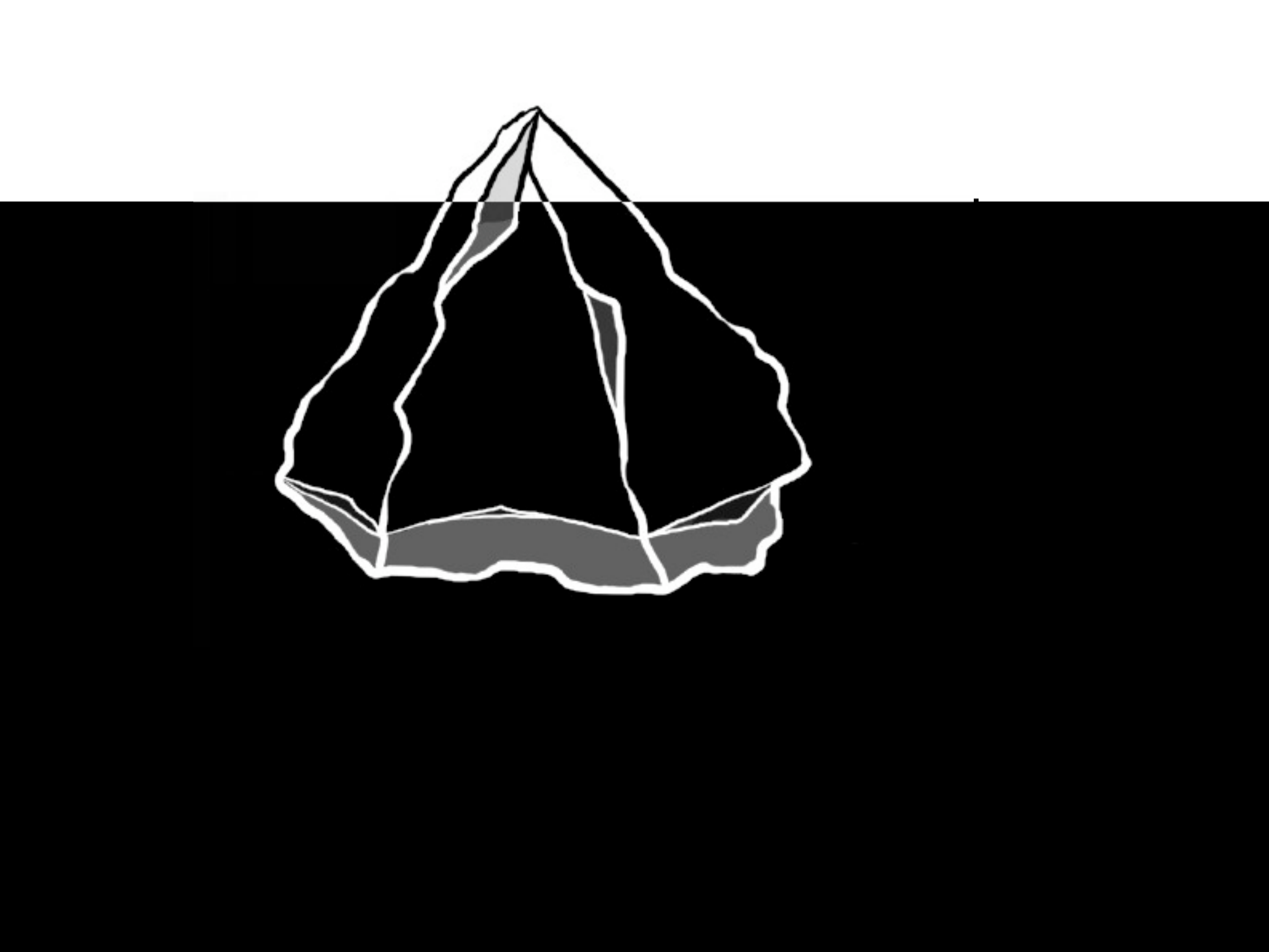
ADT Implementation

abstype Set = List of Int

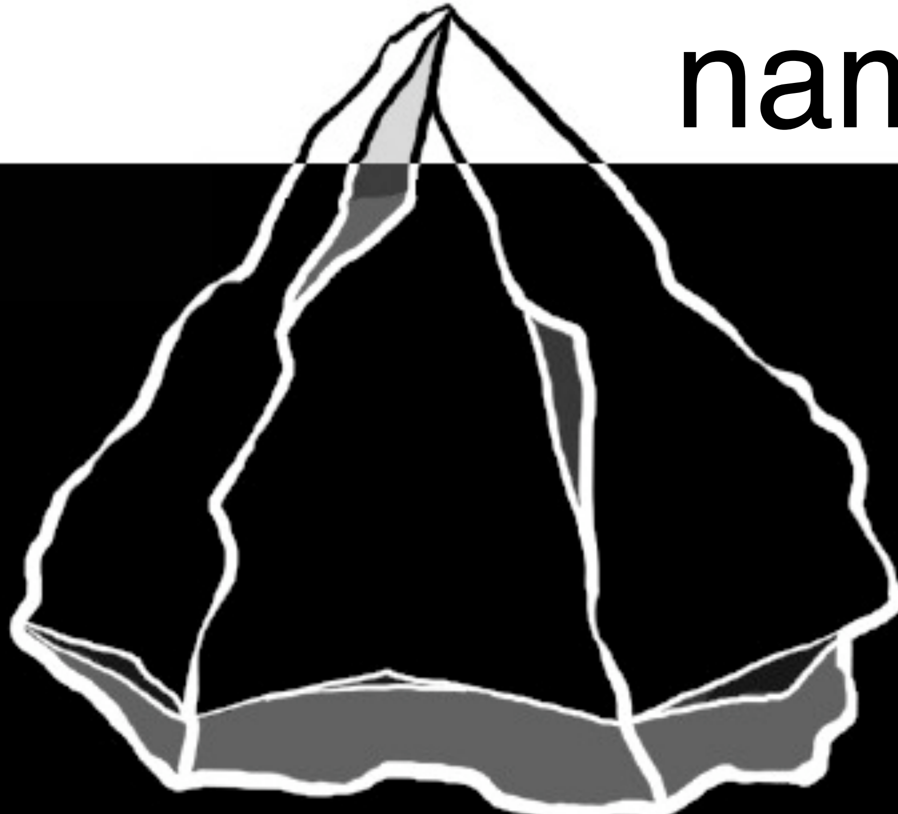
```
  empty             = []
  insert(s, n)      = (n : s)
  isEmpty(s)        = (s == [])
  contains(s, n)    = (n ∈ s)
```

Using ADT values

```
Set x = empty
Set y = insert(x, 3)
Set z = insert(y, 5)
print( contains(z, 2) )
==> false
```



Visible name: Set



Hidden representation: List of Int

```
ISetModule = ∃Set.{
    empty      : Set
    insert     : Set, Int → Set

    isEmpty    : Set → Bool
    contains   : Set, Int → Bool
}
```

Natural!

just like
built-in types

Mathematical
Abstract Algebra

Type Theory

$$\exists x.P$$

(existential types)

Abstract Data Type

=

Data Abstraction

Right?

$$S = \{ 1, 3, 5, 7, 9 \}$$

Another way

$$P(n) = \text{even}(n) \ \& \ 1 \leq n \leq 9$$

$$S = \{ 1, 3, 5, 7, 9 \}$$

$$P(n) = \text{even}(n) \ \& \ 1 \leq n \leq 9$$

Sets as characteristic functions

type Set =

Int → Bool

Empty =

$\lambda n.$ false

$$\text{Insert}(s, m) =$$

$$\lambda n.\ (n{=}m) \vee s(n)$$

Using them is easy

```
Set x = Empty
Set y = Insert(x, 3)
Set z = Insert(y, 5)
print( z(2) )
==> false
```

So What?

Flexibility

set of all
even numbers

Set ADT:
Not Allowed!

or…
break open ADT
& change
representation

set of
even numbers
as a
function?

Even =

$$\lambda n.\ (n \text{ rem } 2 = 0)$$

Even interoperates

```
Set x = Even
Set y = Insert(x, 3)
Set z = Insert(y, 5)
print( z(2) )
==> true
```

Sets-as-functions
are
objects!

No type abstraction

$$\text{type Set} = \text{Int} \rightarrow \text{Bool}$$

multiple
methods?

sure...

```
interface Set {
  contains: Int → Bool
  isEmpty: Bool
}
```

What about
Empty and Insert?

(they are classes)

```
class Empty {
  contains(n) { return false;}
  isEmpty()   { return true;}
}
```

```
class Insert(s, m) {
  contains(n) { return (n=m)
                  ∨ s.contains(n) }
  isEmpty() { return false }
}
```

Using Classes

```
Set x = Empty()
Set y = Insert(x, 3)
Set z = Insert(y, 5)
print( z.contains(2) )
==> false
```

An object
is the
set of observations that
can be
made upon it

Including more methods

```
interface Set {
  contains: Int → Bool
  isEmpty: Bool
  insert    : Int → Set
}
```

```
interface Set {
  contains  : Int → Bool
  isEmpty   : Bool
  insert    : Int → Set
}
```

Type
Recursion

```
class Empty {
  contains(n) { return false;}
  isEmpty()   { return true;}
  insert(n)  { return
                    Insert(this, n);}
}
```

```
class Empty {
  contains(n)  { return false;}
  isEmpty()    { return true;}
  insert(n)    { return
                    Insert(this, n);}
}
```

Value Recursion

Using objects

```
Set x = Empty
Set y = x.insert(3)
Set z = y.insert(5)
print( z.contains(2) )
==> false
```

Autognosis

Autognosis

(Self-knowledge)

Autognosis

An object can access other objects only through public interfaces

operations
on
multiple objects?

union
of
two sets

```
class Union(a, b) {
  contains(n) { a.contains(n)
                ∨ b.contains(n); }
  isEmpty()   { a.isEmpty(n)
              ∧ b.isEmpty(n); }
  ...
}
```

```
interface Set {
  contains: Int → Bool
  isEmpty: Bool
  insert       : Int → Set
  union        : Set → Set
}
```

Complex Operation

(binary)

intersection

of

two sets

??

```
class Intersection(a, b) {
  contains(n) { a.contains(n)
              ∧ b.contains(n); }

  isEmpty()   { ? no way! ? }
  ...
}
```

Autognosis: Prevents some operations (complex ops)

Autognosis:
Prevents some
optimizations
(complex ops)

Inspecting two representations & optimizing operations on them are easy with ADTs

Objects are fundamentally different from ADTs

Object Interface
(recursive types)

$$
\begin{aligned}
&\mathrm{Set} = \{\\
&\quad \mathrm{isEmpty} &&: \mathrm{Bool}\\
&\quad \mathrm{contains} &&: \mathrm{Int} \to \mathrm{Bool}\\
&\quad \mathrm{insert} &&: \mathrm{Int} \to \mathrm{Set}\\
&\quad \mathrm{union} &&: \mathrm{Set} \to \mathrm{Set}\\
&\}\\
&\mathrm{Empty} : \mathrm{Set}\\
&\mathrm{Insert} : \mathrm{Set} \to \mathrm{Set}\\
&\mathrm{Union} : \mathrm{Set} \to \mathrm{Set}
\end{aligned}
$$

ADT
(existential types)

$$
\begin{aligned}
&\mathrm{SetImpl} = \exists\, \mathrm{Set}\, .\, \{\\
&\quad \mathrm{empty} &&: \mathrm{Set}\\
&\quad \mathrm{isEmpty} &&: \mathrm{Set} \to \mathrm{Bool}\\
&\quad \mathrm{contains} &&: \mathrm{Set}, \mathrm{Int} \to \mathrm{Bool}\\
&\quad \mathrm{insert} &&: \mathrm{Set}, \mathrm{Int} \to \mathrm{Set}\\
&\quad \mathrm{union} &&: \mathrm{Set}, \mathrm{Set} \to \mathrm{Set}\\
&\}
\end{aligned}
$$

Operations/Observations

	s	
	Empty	Insert(s', m)
isEmpty(s)	true	false
contains(s, n)	false	n=m ∨ contains(s', n)
insert(s, n)	false	Insert(s, n)
union(s, s'')	isEmpty(s'')	Union(s, s'')

ADT Organization

	s	
	Empty	Insert(s', m)
isEmpty(s)	true	false
contains(s, n)	false	n=m $\vee$ contains(s', n)
insert(s, n)	false	Insert(s, n)
union(s, s'')	isEmpty(s'')	Union(s, s'')

OO Organization

	s	
	Empty	Insert(s', m)
isEmpty(s)	true	false
contains(s, n)	false	n=m ∨ contains(s', n)
insert(s, n)	false	Insert(s, n)
union(s, s'')	isEmpty(s'')	Union(s, s'')

Objects are fundamental (too)

Mathematical

functional representation of data

Type Theory

$\mu X.P$

(recursive types)

ADTs <u>require</u>
a
static type system

Objects work well with or without static typing

“Binary” Operations?

Stack, Socket, Window, Service, DOM, Enterprise Data, ...

Objects are <u>very</u> higher-order

(functions passed as data and returned as results)

Verification

ADTs: construction

Objects: observation

ADTs: induction

Objects: coinduction complicated by: callbacks, state

Objects are designed to be as difficult as possible to verify

Simulation

One object can
simulate another!
(identity is bad)

Java

What is a type?

Declare variables

Classify values

Class as type

=> representation

Class as type

=> ADT

Interfaces as type

=> behavior
   pure objects

Harmful!

instanceof *Class*
(*Class*) exp
Class x;

Object-Oriented subset of Java: class name used <u>only</u> after “new”

It's not an accident that "int" is an ADT in Java

Smalltalk

```
class True
  ifTrue: a ifFalse: b
      ^a

class False
  ifTrue: a ifFalse: b
      ^b
```

True =

$\lambda a . \lambda b .$

a

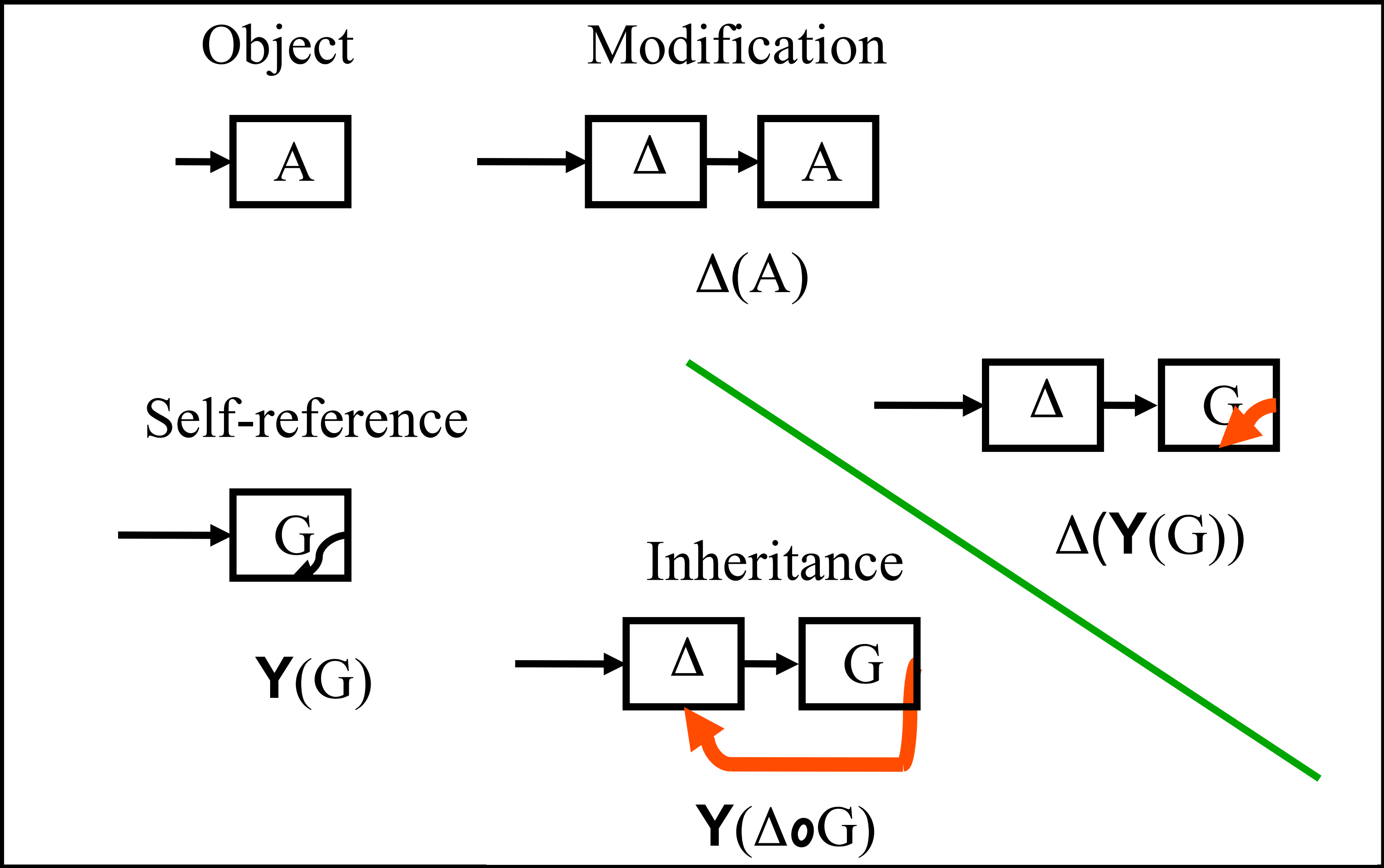
False =

$\lambda a . \lambda b .$

b

Inheritance
(in one slide)

Inheritance



History

1 User-Defined Types and Procedural Data Structures as Complementary Approaches to Data Abstraction

John C. Reynolds

Abstract

User-defined types (or modes) and procedural (or functional) data structures are complementary methods for data abstraction, each providing a capability lacked by the other. With user-defined types, all information about the representation of a particular kind of data is centralized in a type definition and hidden from the rest of the program. With procedural data structures, each part of the program which creates data can specify its own representation, independently of any representations used elsewhere for the same kind of data. However, this decentralization of the description of data is achieved at the cost of prohibiting primitive operations from accessing the representations of more than one data item. The contrast between these approaches is illustrated by a simple example.

Introduction

User-defined types and procedural data structures have both been proposed as methods for data abstraction, i.e., for limiting and segregating the portion of a program that depends upon the representation used for some kind of data. In this paper we suggest, by means of a simple example, that these methods are complementary, each providing a capability lacked by the other.

The idea of user-defined types has been developed by Morris [1, 2], Liskov and Zilles [3], Fischer and Fischer [4], and Wulf [5], and has its roots in earlier work by Hoare and Dahl [6]. In this approach, each particular conceptual kind of data is called a type, and for each type used in a program, the program is divided into two parts: a type definition and an "outer" or "abstract" program. The type definition specifies the representation to be used for the data type and a set of primitive operations (and perhaps constants), each defined in terms of the representation. The choice of representation is hidden from the outer program by requiring all manipulations of the data type in the outer program to be expressed in terms of the primitive operations. The heart of the matter is that any consistent change in the data representation can be effected by altering the type definition without changing the outer program.

Various notions of procedural (or functional) data structures have been developed by Reynolds [7], Landin [8], and Balzer [9]. In this approach, the abstract form of

First appeared in Stephen A. Schuman, editor, *New Directions in Algorithmic Languages*, 1975, IFIP Working Group 2.1 on Algol, INRIA, pages 157–168. Reprinted in David Gries, editor, *Programming Methodology, A Collection of Papers by Members of IFIP WG 2.3*, 1978, Springer-Verlag, pages 309–317.

User-defined types
and
procedural data
structures
as
complementary
approaches to
data abstraction

by J. C. Reynolds

New Advances in
Algorithmic Languages
INRIA, 1975

Abstract data types

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~~User-defined types~~
and
~~procedural data structures~~ objects
as
complementary
approaches to
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by J. C. Reynolds

New Advances in
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“[an object with two methods] is more a tour de force than a specimen of clear programming.”

- J. Reynolds

Extensibility Problem
(aka Expression Problem)

1975 Discovered by J. Reynolds
1990 Elaborated by W. Cook
1998 Renamed by P. Wadler
2005 Solved by M. Odersky (?)
2025 Widely understood (?)

Summary

It <u>is</u> possible to do Object-Oriented programming in Java

Lambda-calculus was the first object-oriented language (1941)

Data Abstraction

ADT   Objects