hw 7

Oviya Seeniraj

Problem 1

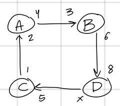
PART A

Each loop must be consistent

x must make A → B → D → C consistent

y must make A → B → E consistent

x:



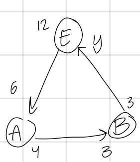
$$M = \begin{array}{c} \begin{array}{cccc} A & B & D & C \end{array} \\ \begin{bmatrix} 4 & -3 & 0 & 0 \\ 0 & 6 & -8 & 0 \\ 0 & 0 & x & -5 \\ -2 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 4 & -3 & 0 & 0 \\ 0 & 6 & -8 & 0 \\ 0 & 0 & x & -5 \\ -2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[r4 = r1 + 2r4]{r4 \to r1} \begin{bmatrix} -2 & 0 & 0 & 1 \\ 0 & 6 & -8 & 0 \\ 0 & 0 & x & -5 \\ 0 & -3 & 0 & 2 \end{bmatrix} \xrightarrow[r_4 = r_4 + 0.5 r_2]{} \begin{bmatrix} -2 & 0 & 0 & 1 \\ 0 & 6 & -8 & 0 \\ 0 & 0 & x & -5 \\ 0 & 0 & -4 & 2 \end{bmatrix} \xrightarrow[r_4 = r_4 + 0.4 r_3]{x = 10} \begin{bmatrix} -2 & 0 & 0 & 1 \\ 0 & 6 & -8 & 0 \\ 0 & 0 & 10 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{rrrrr} & 4 & -3 & 0 & 0 \\ + & -4 & 0 & 0 & 2 \\ \hline & 0 & -3 & 0 & 2 \end{array}$$

For M to have rank n-1, $\boxed{x \text{ must be } 10}$

y:



$$M = \begin{array}{c} \begin{array}{ccc} A & B & E \end{array} \\ \begin{bmatrix} 4 & -3 & 0 \\ 0 & 3 & -y \\ -6 & 0 & 12 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 4 & -3 & 0 \\ 0 & 3 & x \\ -6 & 0 & 12 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 & 0 & 2 \\ 0 & 3 & x \\ 4 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 & 0 & 2 \\ 0 & 3 & -8 \\ 0 & -3 & 8 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 & 0 & 2 \\ 0 & 3 & -8 \\ 0 & 0 & 0 \end{bmatrix}$$

For M to have rank n-1, $\boxed{y \text{ must be } 8}$

Check for overall system:

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} P & A & B & C & D & E & S \end{array} \\
M = & \begin{bmatrix}
1 & -3 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & -3 & 0 & 0 & 0 & 0 \\
0 & 0 & 3 & 0 & 0 & -y & 0 \\
0 & 0 & 6 & 0 & -8 & 0 & 0 \\
0 & -6 & 0 & 0 & 0 & 12 & 0 \\
0 & 0 & 0 & -5 & x & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & -4 \\
0 & 2 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

$V_S(P) - 3V_S(A) = 0$

$4V_S(A) - 3V_S(B) = 0$

$3V_S(B) - yV_S(E) = 0$

$6V_S(B) - 8V_S(A) = 0$

$xV_S(D) - 5V_S(C) = 0$

$2V_S(D) - 4V_S(S) = 0$

$V_S(C) - 2V_S(A) = 0$

System consistent when $\text{rank}(M) = n-1$

when $x=10$, $y=8$, $\text{rank}(M) = 6 = n-1 \rightarrow$ system consistent

Null space $\Rightarrow$ $q = [18 \ \ 6 \ \ 8 \ \ 12 \ \ 6 \ \ 3 \ \ 3]^T$

PART B

Most simplified null space w/ no further common factor: $[18 \ \ 6 \ \ 8 \ \ 12 \ \ 6 \ \ 3 \ \ 3]^T$ so run each letter that many times in minimum length sched:

- $P = 18$
- $A = 6$
- $B = 8$
- $C = 12$
- $D = 6$
- $E = 3$
- $S = 3$

E.g. of min length sched: $\boxed{(18P)(6A)(8B)(3E)(6D)(12C)(3S)}$

Sched needs $\boxed{\text{48 buffers}}$: 36 on $E \rightarrow A$ trans, 12 on $C \rightarrow A$ trans for the $(6A)$ part of the sched.

(May not be a minimum buffer sched but question doesn't require it)

Problem 2

PROBLEM 8

a) $q_A = 3q_B$

$2q_B = 3q_C$

least pos int soln: $\boxed{\begin{array}{l} q_A = 9 \\ q_B = 3 \\ q_C = 2 \end{array}}$

b) AAABAAABCAAABC

A → B buffer size: 3

B → C buffer size: 4

PROBLEM 9

a)

$$M = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & -3 & 0 \\ 1 & 0 & -6 \\ 0 & 2 & -3 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 1 & 0 & -6 \\ 0 & 2 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & 3 & -6 \\ 0 & 2 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -2 \\ 0 & 2 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Rank(M) = 3 = n → $\boxed{\text{No}}$ unbounded execution w/ bounded buffers

b)

$$M = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & -3 & 0 \\ 0 & 2 & -3 \\ 1 & 0 & -n \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 2 & -3 \\ 1 & 0 & -n \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & 2 & -3 \\ 0 & 0 & -n \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 6 & -9 \\ 0 & 6 & -2n \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & 6 & -9 \\ 0 & 0 & 9-2n \end{bmatrix}$$

To have rank(M) ≤ n − 1, $9 - 2n = 0 \rightarrow n = 4.5 \rightarrow$ not integer, so $\boxed{\text{no}}$ unbounded execution w/ bounded buffers

PROBLEM 10

a)

$$\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} M & -1 & 0 \\ N & 0 & -2 \\ 0 & 2 & -1 \\ -2M & 0 & 1 \end{bmatrix} \end{array} \rightarrow \begin{bmatrix} M & -1 & 0 \\ N & 0 & -2 \\ 0 & 2 & -1 \\ 0 & -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} M & -1 & 0 \\ N & 0 & -2 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} M & -1 & 0 \\ N & -4 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} M & -1 & 0 \\ N-4M & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank(M) = n - 1 = 2 so N & M must make N-4M = 0 → $\boxed{N = 4M}$ must be satisfied for consistent model

b) System start: A needs at least 2M tokens: $z_{min} \geq 2M$ to avoid deadlock
System simulation shows that z=2M runs all actors & can return to initial state
∴, $\boxed{z = 2M}$ is the minimum z that avoids deadlock

Min length sched: (A) (MB) (2M C)

c) w, x, y need to be such that C can run 2M times (based on part B)

~~y=4M, x=2M~~ → $\underbrace{y = 4M,\ x = 0,\ w = M}$
lower w + x + y val
that also achieves x = 2M
using w = M

w+x+y is minimized & deadlock-free @ $\boxed{y = 4M,\ x = 0,\ w = M}$

d) Minimum buffer sizes:
$b_w = M$
$b_x = 2$
$b_y = 4M$
$b_z = 2M$

can acheive these sizes w/ the schedule A (M x BCC)

Problem 3

a) As a CE who took 153B beforehand, this class really helped complete my understanding of embedded systems and enjoy my major more. I love how lab-heavy the class is, giving me more real-world experience.

b) Rather than course concepts (all of which were vital & enjoyable), the most important thing I learned from this course was time commitment & backing up my data when engaging with softwares like NoMachine, Xilinx, and Vivado. Getting the hang of them was difficult and taught me that I needed to start early with dedicated chunks of time to really understand & learn from the labs. My favorite lab was probably 3A: it took me the longest, but latency is such a crucial concept I'm glad we got to experiment with.