Peer Effects in Linear-in-Means Models with Heterogeneous Interaction Effects*

Magne Mogstad† Alex Torgovitsky‡ Oscar Volpe§

November 13, 2024

Abstract

We study peer effects in linear-in-means models with heterogeneous interaction effects. The classical linear-in-means model imposes strict homogeneity on the interaction effects, yielding testable implications that can easily be examined in data. We relax these restrictions to allow for both positive and negative interaction effects that vary within and across groups. These extensions make the linear-in-means model suited to study a wide range of economic behaviors in addition to peer effects, including joint labor supply decisions within households and strategic interactions among firms. We analyze what can and cannot be learned from frequently used OLS and IV estimands for linear-in-means models under heterogeneous interaction effects. While these estimands do not lead to point identification, they can still be used to draw inferences about key economic quantities. We apply these results to two economic applications: classroom peer effects in Kenyan primary schools and strategic pricing decisions among cocoa traders in Sierra Leone. In each application, we reject homogenous interaction effects. Yet, we still draw meaningful inferences about endogenous interactions and social multipliers while allowing for heterogeneous interaction effects.

I. Introduction

Peer effects models are widely used in economics to study how individuals' actions are shaped by those around them, with applications ranging from education and health to labor markets and beyond. The classical linear-in-means model (Manski, 1993) remains the most commonly used framework for empirically analyzing these interactions.[1] This model typically assumes strict homogeneity in the endogenous interaction effects, requiring that all individuals, within and across peer groups, are positively influenced in exactly the same way

*We thank Lancelot Henry de Frahan for valuable comments. All errors are our own.

†Department of Economics, University of Chicago, Statistics Norway, and NBER.

‡Department of Economics, University of Chicago.

§Department of Economics, University of Chicago.

[1] Applications of the linear-in-means framework include Sacerdote (2001), Guryan et al. (2009), Patacchini & Zenou (2009), Duflo et al. (2011), Dahl et al. (2014), and Casaburi & Reed (2022), among others. Blume et al. (2015) and Boucher et al. (2024) study the microfoundations and economic properties of these models.

by the average outcome of their peers. Identification and estimation is well-studied under this homogeneity assumption, with researchers typically relying on linear OLS and IV estimators to recover parameters of interest (Kline & Tamer, 2020). However, the identification arguments behind these estimators do not readily transfer to settings with heterogeneous interaction effects. Known conditions for point identification under heterogeneous effects, such as those developed by Masten (2017), put strong demands on available instruments and can be difficult to implement.

The goal of our paper is to analyze peer effects in linear-in-means models with heterogeneity in endogenous interaction effects. We consider a setting with two or more groups, where each group g comprises a set of agents, denoted by $\mathcal{N}_g$. Each agent i in group g has an outcome Y_{ig}, which is affected by the outcomes of other agents in the group. This interdependence is characterized by the following system of linear simultaneous equations.

$$Y_{ig} = \alpha_{ig} + \frac{\beta_{ig}}{|\mathcal{N}_g| - 1} \sum_{j \neq i} Y_{jg} + Z'_{ig}\gamma_{ig}, \quad \text{for } i \in \mathcal{N}_g. \tag{1}$$

In these equations, $\{\alpha_{ig}\}_{i \in \mathcal{N}_g}$, $\{\beta_{ig}\}_{i \in \mathcal{N}_g}$, and $\{\gamma_{ig}\}_{i \in \mathcal{N}_g}$ are all unknown structural parameters. Additionally, $\{Z_{ig}\}_{i \in \mathcal{N}_g}$ is a set of observed variables, which could include individual-level shifters, if $Z_{ig} \neq Z_{jg}$ for $i \neq j$, as well as group-level covariates, if $Z_{ig} = Z_{jg}$ for all $i, j \in \mathcal{N}_g$.

In this model, the parameter β_{ig} represents the individual interaction effect, indicating how each agent i in group g is influenced by the average outcome in the rest of the group. Whereas the classical linear-in-means model maintains that β_{ig} is constant across individuals i and groups g, we allow the interaction effects to differ along both these dimensions. Also, unlike previous work, we do not restrict the sign or magnitude of β_{ig}. Therefore, agents may be positively or negatively affected, however intensely, by their peers. The parameters α_{ig} and γ_{ig} specify how the variables Z_{ig} would determine an agent i's outcome Y_{ig} in absence of spillover effects. We allow these terms to vary freely among agents within and across groups. We also do not restrict the size or composition of each group, as characterized by the set $\mathcal{N}_g$.

In Section II, we begin by reviewing the economic quantities commonly studied in models with constant effects, along with the identification strategies used to recover these quantities. We show that the constant effects assumption yields testable implications in the form of overidentification tests, which can be used to assess whether individuals have uniform interaction effects. We apply these tests to data for two economic applications that employ the linear-in-means model with constant effects: peer effects in Kenyan primary schools (Duflo et al., 2011) and competition between cocoa traders in Sierra Leone (Casaburi & Reed, 2022). In both instances, we find evidence against homogeneous interaction effects. In the first application, we conclude that peer effects vary across classrooms. In the second application, we find that traders respond strategically in different ways to their competitors' actions.

Motivated by these findings, we consider, in Section III, the heterogeneous effects model, which allows α_{ig}, β_{ig}, and γ_{ig} to vary freely among agents within and across groups. Under this framework, we derive new expressions for the equilibrium outcomes in terms of the individual interaction effects. We use these expressions to characterize equilibrium behavior in the presence of heterogeneity, demonstrating how different combinations of interaction effects distort group-level outcomes. With heterogeneous effects, the equilibrium impact of an exogenous shock on group-level outcomes depends on which agents in the group are directly exposed to that shock. These equilibrium effects may also differ across groups. To guide and interpret our results, we draw on three economic examples: classroom peer effects, household labor supply decisions, and competition among firms in oligopolies. In each example, we show how the linear-in-means model with heterogeneous effects can be used to analyze the economic questions of interest without placing strong restrictions on individual behavior.

In Sections IV and V, we investigate what features of the model are recovered from OLS and IV estimation under heterogeneous effects. We start by examining a class of OLS estimands that result from regressing the outcomes Y on the exogenous variables Z (or linear combinations of Z). We demonstrate that a correctly specified OLS regression can recover the average equilibrium effects of Z on Y across groups, even in the presence of heterogeneous effects. Additionally, these regressions tell us about social multiplier effects, which quantify how network spillovers distort the impacts of individual shocks on group outcomes (Glaeser et al., 2003). In our framework, OLS does not lead to point identification of social multipliers. Still, we show that OLS can be used to test for the presence of positive (or negative) multiplier effects, providing insight into the role of heterogeneous spillovers in amplifying (or suppressing) the impacts of policies in equilibrium. Furthermore, we demonstrate that OLS allows us to test for positive or negative interaction effects in a variety of empirical settings.

Next, we analyze what economic quantities are recovered from IV estimation. We study a large class of IV estimands that use exclusion restrictions to recover the interaction effects β_{ig}. We show that, with heterogeneous effects, the IV estimand represents a particular weighted average of interaction effects, which places higher weight on groups where aggregate outcomes are more responsive to the instruments. We then derive necessary and sufficient conditions for these weights to be non-negative, which we view as a minimal requirement for the IV estimand to be informative about interaction effects. We also show how the IV estimand compares to an unweighted average of interaction effects. In general, this relationship depends on the signs of the interactions, whether they are positive or negative. We prove that in many common network settings, such as classical peer effects, oligopoly models, and public goods games, the IV estimand will necessarily overstate the average interaction effect.

In the last section, we take the linear-in-means model with heterogeneous interaction effects to our two empirical applications. In the analysis of social interactions in Kenyan primary schools, we find evidence that peer effects are present in many classrooms and are positive for a significant number of students. Our estimate of the upper bound on the average interaction effect suggests that, on average, a 1 point increase in the average test score of one's peers does not directly influence a student's own test score by more than 0.45 points. We also find evidence of significant positive social multiplier effects, suggesting that, in at least some classrooms, factors influencing individual achievement are amplified throughout the class due to social interactions. In the analysis of competition between cocoa traders in Sierra Leone, we find evidence of strategic interactions and imperfect competition in price setting. We estimate an upper bound on the average conduct parameter, indicating the extent to which traders respond strategically to their competitors' pricing decisions. Our estimated bound suggests that increasing competitors' cocoa purchases by 1 pound does not directly reduce a trader's own purchases by more than 0.02 pounds on average. We do not find evidence of social multiplier effects, suggesting that strategic interactions do not materially matter for conclusions about how trader-specific changes in demand or costs affect total market output.

Our paper contributes to two literatures. First, we contribute to a literature on the empirical analysis of social interactions; see Paula (2017) and Kline & Tamer (2020) for recent surveys.[2] Within this literature, there is increasing recognition of the importance of accounting for individual heterogeneity in endogenous interaction effects. While the economic theory is well-studied in these cases (Jackson & Zenou, 2015), there is less work addressing the identification of models with heterogeneous interaction effects. One key exception is Masten (2017), who studies identification for a linear peer effects model with random coefficients.[3] He proves that the marginal distributions of the coefficients are point identified if there is an instrument with continuous variation over a large support. However, he also shows that instruments are insufficient for recovering the full joint distribution of random coefficients. These results raise questions about what can be learned about other economic quantities, such as equilibrium effects and social multipliers, in the presence of heterogeneity. Our paper addresses this question by analyzing how to interpret and learn from OLS and IV estimation in contexts with heterogeneous interaction effects. We view our results as constructive. While point identification might not be achievable, we find that meaningful inferences can still be made from frequently used OLS and IV estimators. Our approach is broadly applicable for a variety of settings where access to a continuous instrument with large support is not feasible.

The second literature to which we contribute is concerned with the interpretation of linear OLS and IV estimands in settings with unobserved heterogeneity in treatment effects. Mogstad & Torgovitsky (2024) give a recent survey of this work. In a seminal paper, Imbens & Angrist (1994) pioneer a framework for interpreting linear IV estimands as weighted averages of local average treatment effects, and Angrist et al. (2000) extend these interpretations to supply and demand models consisting of two simultaneous equations. The system of linear simultaneous equations for peer effects differs in two important ways from the linear supply

[2]See Blume et al. (2011) for more discussion. Also, Sacerdote (2011) surveys the literature on peer effects in education, and Browning et al. (2014) discusses the use of social interaction models for household behavior.

[3]Hurwicz (1950), Kelejian (1974), and Hahn (2001) also examine simultaneous equations with random coefficients. Hurwicz (1950) does not give explicit identification results. In addition, as Masten (2017) points out, Kelejian (1974) and Hahn (2001) conduct analyses that are based on self-contradictory assumptions.

and demand system studied by Angrist et al. (2000). First, the supply and demand system is restricted to a network of two agents: a representative firm and a representative consumer in each market. Second, the supply and demand system focuses on specific interaction effects where the sign is known, i.e., upward-sloping supply and downward-sloping demand. In contrast, the system of linear simultaneous equations we consider does not place restrictions on the signs of the interaction effects, which means that agents' outcomes could be strategic substitutes and/or complements. Therefore, we can apply our model to a wide range of settings that involve substitutabilities and/or complementarities in decision-making, including peer effects, household labor supply decisions, and competition among firms in an oligopoly.

Our paper contributes to this literature by demonstrating how to interpret linear OLS and IV estimands for linear peer effects models with heterogeneous interaction effects. Our analysis finds that many of the existing tools for interpreting these estimands do not easily transfer to peer effects models. For example, with peer groups larger than two, the standard monotonicity conditions for IV to have a causal interpretation (Imbens & Angrist, 1994) place strong restrictions on the peer effects, which are unlikely to apply in many practical settings. We propose alternative, weaker conditions under which IV retains a causal interpretation. We then demonstrate how this causal parameter allows us to learn about economic quantities of interest. Overall, our analysis gives an accessible framework for learning about heterogeneous interaction effects and social multipliers from frequently used linear OLS and IV estimands.

II. The Classical Linear-in-Means Model

In this section, we present the classical linear-in-means model, define economic quantities of interest, and discuss how these quantities can be recovered from the data under the assumption that agents exhibit homogeneous endogenous interaction effects. We then show that this assumption has testable implications, which are rejected in the data for our two applications.

II.A. Model and Assumptions

The classical linear-in-means model assumes that the interaction effects are constant. Specifically, in equation (1), it assumes that $\beta_{ig} = \beta_{jg}$ for any two agents i and j in group g, such that agents are uniformly affected by other members of their group. Additionally, the model assumes that $\beta_{ig} = \beta_i$ for all i and g, such that all groups exhibit identical spillover effects.

While there are many variants of the linear-in-means model, it is common to assume that the interaction effects are positive: $\beta_{ig} \geq 0$ for all agents i and groups g. This restriction leads to uniform strategic complementarities, where all agents conform to the average outcome of their peers. In addition, the literature generally assumes that $|\beta_{ig}| < 1$ for all i and g, which ensures that the spillover effects are small in magnitude. Many papers also maintain that the coefficient γ_{ig} is constant and that the intercept α_{ig} satisfies both $\mathrm{E}(\alpha_{ig}|Z_{ig}, g) = \mathrm{E}(\alpha_{ig})$ and $\mathrm{Cov}(\alpha_{ig}, \alpha_{jg'}|Z_{ig}, Z_{jg'}, g, g') = 0$. We summarize these assumptions below for reference.

Classical Linear-in-Means Assumptions

C.1 *(Homogeneous Interactions within Groups).* $\beta_{ig} = \beta_{jg}$ for any two agents $i, j \in \mathcal{N}_g$.
C.2 *(Homogeneous Interactions across Groups).* $\beta_{ig} = \beta_i$ for all agents i and groups g.
C.3 *(Positive and Bounded Spillovers).* $0 \leq \beta_{ig} < 1$ for all agents i and groups g.
C.4 *(Homogeneous Incidence of Z).* $\gamma_{ig} = \gamma$ for all agents i and groups g.
C.5 *(Mean Independence).* $\mathrm{E}(\alpha_{ig}|Z_{ig}, g) = \mathrm{E}(\alpha_{ig})$ and $\mathrm{Cov}(\alpha_{ig}, \alpha_{jg'}|Z_{ig}, Z_{jg'}, g, g') = 0$.

Assumption C.3 ensures that the system of equations (1) possesses a unique solution. Under the classical linear-in-means assumptions, we derive the following reduced form equations.

$$Y_{ig} = (1 + \beta\delta_g)(\alpha_{ig} + \gamma Z_{ig}) + \delta_g \sum_{j \neq i}(\alpha_{jg} + \gamma Z_{jg}), \quad \text{for } i \in \mathcal{N}_g, \tag{2}$$

where $\delta_g = \frac{\beta}{(1-\beta)(|\mathcal{N}_g|-1+\beta)}$ is a term that tends to zero as the group size $|\mathcal{N}_g|$ tends to infinity.

II.B. Economic Quantities of Interest

Depending on the empirical context, researchers may be interested in learning about a variety of reduced form and structural parameters in the model. In Table 1, we list several economic quantities that are often studied in the classical linear-in-means model. For each of these quantities, we provide definitions and derive expressions in terms of the structural parameters, both for the heterogeneous effects specification (1) and for the constant effects special case. We postpone the analysis of these quantities under heterogeneous effects to Section III.

To ease notation in our derivations, we remove group subscripts in the model. We also define $\mathcal{N} = \{1, \ldots, N\}$, while noting that the number of agents can freely vary across groups. Finally, for expositional purposes, we assume that Z_{ig} is one-dimensional, so that $Z_{ig} \in \mathbb{R}$, although including a vector of shifters/covariates does not meaningfully change our analysis.

The first three quantities in the table are reduced form coefficients, which specify how the exogenous variables Z affect agents' outcomes Y in equilibrium, after accounting for network spillovers. In the classical linear-in-means model, the total effect of Z_i on Y_i is uniform across i, meaning that all agents are affected in the same way by an exogenous shock to their own outcome. Also, the individual spillover effect of Z_i on Y_j is uniform across i and j, implying that these spillovers do not depend on which agent is directly affected by a shock or on which agent is indirectly affected by it. Furthermore, the total effect of Z_i on $\bar{Y}$ is uniform across i, which means that the same shock to any agent's outcome would have an identical impact on the average outcome in the group. These equilibrium effects are also uniform across groups.

The fourth parameter that we define is the social multiplier effect. For the classical linear-in-means model, Glaeser et al. (2003) define the social multiplier as the ratio of aggregate coefficients to individual coefficients in the reduced form. Specifically, the multiplier equals:

$$M^{\text{constant}} = \frac{\Delta\bar{Y}/\Delta\bar{Z}}{\Delta Y_i/\Delta Z_i} = \frac{\beta + N - 1}{\beta + (1-\beta)(N-1)}. \tag{3}$$

This parameter measures how the equilibrium impact of Z on Y changes at different levels of aggregation. As the size of the group N grows large, the multiplier effect tends to $(1-\beta)^{-1}$.

The next three parameters are the structural coefficients α_i, β_i, and γ_i. Among these quantities, the interaction effect β_i is often a primary target parameter in peer effects models since it measures the amount of social pressure that an individual experiences (e.g., see Sacerdote, 2011). The parameters α_i and γ_i also have an economic significance, as they indicate how the variable Z_i would impact an agent's outcome Y_i without network interference. For some applications, it may be important to distinguish between the direct treatment responses and the indirect effects of treatments arising from social interactions (e.g., see Manski, 1993).

Table 1: Economic Quantities of Interest

Estimand	**Definition**	**Structural Interpretation**	
		Constant Effects	Heterogeneous Effects
<u>Reduced Form Quantities</u>			
Total Individual Effect	$\Delta Y_i/\Delta Z_i$	$\gamma + \frac{\beta^2\gamma}{(1-\beta)(N-1+\beta)}$	$\gamma_i + \frac{\beta_i\gamma_i\left(\frac{1}{N-1}\sum_{j\neq i}\beta_j\nu_{ij}\right)}{(N-1)\det(I-\mathbf{B})}$
Individual Spillover Effect	$\Delta Y_j/\Delta Z_i$	$\frac{\beta\gamma}{(1-\beta)(N-1+\beta)}$	$\frac{\beta_j\gamma_i\nu_{ij}}{(N-1)\det(I-\mathbf{B})}$
Total Effect on the Average	$\Delta\bar{Y}/\Delta Z_i$	$\frac{1}{N}\times\frac{\gamma}{(1-\beta)}$	$\frac{1}{N}\times\frac{\gamma_i\nu_i}{\det(I-\mathbf{B})}$
Individual Social Multiplier	$\frac{\sum_{j=1}^{N}\Delta Y_j/\Delta Z_i}{\Delta Y_i/\Delta Z_i}$	$\frac{\beta+N-1}{\beta+(1-\beta)(N-1)}$	$\frac{\nu_i}{\nu_i-\frac{1}{N-1}\sum_{j\neq i}\beta_j\nu_{ij}}$
Aggregate Social Multiplier	$\frac{\sum_{i=1}^{N}\Delta\bar{Y}/\Delta Z_i}{\frac{1}{N}\sum_{j=1}^{N}\Delta Y_j/\Delta Z_j}$	$\frac{\beta+N-1}{\beta+(1-\beta)(N-1)}$	$\frac{\frac{1}{N}\sum_{i=1}^{N}\nu_i\gamma_i}{\frac{1}{N}\sum_{j=1}^{N}\left(\nu_j-\frac{1}{N-1}\sum_{k\neq j}\beta_k\nu_{jk}\right)\gamma_j}$
<u>Structural Quantities</u>			
No Interference Outcome	$Y_i\|(\bar{Y}_{-i},Z_i)=\mathbf{0}$	α_i	α_i
No Interference Effect	$\Delta Y_i/\Delta Z_i\|\bar{Y}_{-i}$	γ	γ_i
Individual Interaction Effect	$\Delta Y_i/\Delta\bar{Y}_{-i}$	β	β_i
Interaction Effect Correlation	$\operatorname{corr}\left(\Delta Y_i/\Delta\bar{Y}_{-i},\Delta Y_j/\Delta\bar{Y}_{-j}\right)$	0	$\operatorname{corr}(\beta_i,\beta_j)$

Notes. The reduced form effects $\Delta Y_i/\Delta Z_i$, $\Delta Y_i/\Delta Z_i$, and $\Delta Y_i/\Delta Z_i$ are defined by holding $\{Z_j\}_{j\neq i}$ fixed. To ease notation in the last column, we let $\nu_i=\prod_{\ell\neq i}\left(1+\frac{\beta_\ell}{N-1}\right)$ and $\nu_{ij}=\prod_{\ell\notin\{i,j\}}\left(1+\frac{\beta_\ell}{N-1}\right)$ for any i and j.

II.C. Standard Linear Peer Effects Estimands and Testable Implications

We now review the identification strategies typically used to recover reduced form and structural parameters in models with constant effects. In our analysis, we assume that the variables Z_i are individual-level shifters in the model. We require that these shifters $\{Z_i\}_{i=1}^{N}$ are not perfectly collinear, such that $\mathrm{E}(ZZ')$ is nonsingular. In addition, we assume that $\gamma\neq 0$, which ensures that each variable Z_i has a nonzero effect on observed outcomes. Under this assumption, any exogenous, individual-level shifter is a valid instrument in our framework.

Since this model involves simultaneity, instruments play a crucial role in identification. Namely, they generate exclusion restrictions, which are factors that directly affect a subset of the agents in a group, while leaving others unaffected. Examples of exclusions include policy variables that shift an agent's marginal cost of action. Alternatively, an exclusion could be a restriction on the interactions in the network, whereby some agents do not directly influence certain members of their group. In the Appendix, we show how to extend the linear-in-means model to reformulate these restrictions as instruments. In doing so, our analysis speaks to a wide range of identification strategies that use exclusions to recover structural parameters.[4]

II.C.1. Frequently Used OLS Estimands

We begin by analyzing OLS estimands that result from projecting the outcomes Y on group-level averages of $\{Z_i\}_{i=1}^N$. In particular, we consider the estimands $\beta_{Y_i}^{\text{OLS}} = \text{E}(\tilde{Z}_i\tilde{Z}_i')^{-1}\,\text{E}(\tilde{Z}_iY_i)$ and $\beta_{\bar{Y}}^{\text{OLS}} = \text{E}(\tilde{Z}\tilde{Z}')^{-1}\,\text{E}(\tilde{Z}\bar{Y})$, corresponding to regressions of the individual outcome Y_i on the vector $\tilde{Z}_i = (1, Z_i, \bar{Z}_{-i})$ and the average outcome $\bar{Y}$ in the group on the vector $\tilde{Z} = (1, \bar{Z})$.

Under Assumptions C.1-C.5, these estimands recover the total individual effect $\Delta Y_i/\Delta Z_i$, the individual spillover effects $\{\Delta Y_i/\Delta Z_j\}_{j\neq i}$, and the total effect on the average $\Delta\bar{Y}/\Delta Z_i$. To see why, recall that homogeneous effects ensures that $\Delta Y_i/\Delta Z_j$ is constant across $j \neq i$ and $\Delta\bar{Y}/\Delta Z_j$ is constant across $j \in \{1, \ldots, N\}$. So, in the reduced form, Y_i can be expressed as a linear function of Z_i and $\bar{Z}_{-i}$. Similarly, $\bar{Y}$ can be expressed as a linear function of $\bar{Z}$.

One key testable implication of Assumption C.1 is that the reduced form effects $\Delta Y_j/\Delta Z_i$ and $\Delta Y_k/\Delta Z_i$ are equal for any agents $i, j, k \in \mathcal{N}$. The lemma below formalizes this property.

Lemma 1. For any distinct agents $i, j, k \in \mathcal{N}$, $\beta_j = \beta_k$ if and only if $\Delta Y_j/\Delta Z_i = \Delta Y_k/\Delta Z_i$.

The economic intuition behind Lemma 1 is that, if two agents j and k are influenced in the same way by another agent i, then an exogenous shock to agent i's outcome would produce identical spillover effects on agent j and agent k. This property gives a testable restriction for the hypothesis $H_0 : \beta_j = \beta_k$, which posits that the interaction effects are constant among agents in the same group. To test this hypothesis, we evaluate the following OLS estimands:

$$\tilde{\beta}_{Y_j}^{\text{OLS}} = \text{E}(\tilde{Z}_{jk}\tilde{Z}_{jk}')^{-1}\,\text{E}(\tilde{Z}_{jk}Y_j) \quad \text{and} \quad \tilde{\beta}_{Y_k}^{\text{OLS}} = \text{E}(\tilde{Z}_{jk}\tilde{Z}_{jk}')^{-1}\,\text{E}(\tilde{Z}_{jk}Y_k),$$

where $\tilde{Z}_{jk} = (1, Z_j, Z_k, \bar{Z}_{-i,-k})$. If the regression coefficients on $\bar{Z}_{-i,-k}$ differ between the two estimands, then it must be that $\beta_j \neq \beta_k$, and it follows that Assumption C.1 is violated.

II.C.2. Frequently Used IV Estimands

We now shift attention to a large class of IV estimands that use instruments to recover the interaction effect β. This quantity is often the main target parameter in constant effects models, and our framework nests a wide variety of existing approaches that are used to recover it.

We define an IV estimand for β that uses $\tilde{Z}_{-i}$ as the excluded instrument for $\bar{Y}_{-i}$ in an agent i's outcome equation. We allow $\tilde{Z}_{-i}$ to be any monotonic transformation of the vector Z_{-i}. In particular, we define $\tilde{Z}_{-i} = g(Z_{-i})$, where g is a monotone mapping taking values in

[4] See Kline & Tamer (2020) for discussion. Bramoullé et al. (2009) formalize how to use network exclusions—where not all agents interact with one another—for identification of classical linear-in-means models.

the support of Z_{-i}.[5] Our specification encompasses a wide array of IV strategies, including: *(1)* using one instrument individually, *(2)* using multiple instruments jointly, and *(3)* using an increasing transformation of multiple instruments, e.g., a group-level average of $\{Z_j\}_{j\neq i}$. For any realization of z_i in the support of Z_i, we can write down an IV estimand as follows:

$$\beta_i^{\text{IV}}(z_i) = \frac{\text{Cov}(Y_i, \mathbf{L}(\bar{Y}_{-i}|\tilde{Z}_{-i})|Z_i = z_i)}{\text{Cov}(\bar{Y}_{-i}, \mathbf{L}(\bar{Y}_{-i}|\tilde{Z}_{-i})|Z_i = z_i)}, \tag{4}$$

where $\mathbf{L}(\bar{Y}_{-i}|\tilde{Z}_{-i})$ represents the population fitted values from a regression of $\bar{Y}_{-i}$ on $(1, \tilde{Z}_{-i})$.

Under constant effects, the interaction effect β is point-identified from this IV estimand. In fact, even if the interaction effects vary within a group, the estimand will still recover the interaction effect β_i for agent i, provided that the interaction effects are homogeneous across groups. This result is well-established in the literature, and it is reviewed in many econometrics textbooks tracing back to Fisher (1966).

Lemma 2. Under Assumptions C.2-C.5, the IV estimand $\beta_i^{\text{IV}}(z_i)$ recovers the parameter β_i.

This lemma generates a testable implication for Assumptions C.2-C.5. Specifically, under these assumptions, the IV estimand $\beta_i^{\text{IV}}(z_i)$ always recovers the same parameter, regardless of which excluded instruments $\tilde{Z}_{-i}$ are used in the regression. Therefore, we can validate the classical linear-in-means assumptions by conducting an over-identification test. For $N > 2$, there may be multiple valid instruments $\{Z_j\}_{j\neq i}$ for the endogenous variable $\bar{Y}_{-i}$ in an agent i's outcome equation. We can leverage this over-identification to construct two IV estimands $\beta_i^{\text{IV},1}$ and $\beta_i^{\text{IV},2}$ for β_i using two distinct instruments $\tilde{Z}_{-i,1}$ and $\tilde{Z}_{-i,2}$, respectively. We can then empirically assess whether Assumptions C.2-C.5 hold by testing the null $H_0 : \beta_i^{\text{IV},1} = \beta_i^{\text{IV},2}$.

II.D. Empirical Applications of Linear Peer Effects Estimators

We now consider two applications: peer effects in Kenyan primary schools (Duflo et al., 2011) and strategic pricing decisions of cocoa traders in Sierra Leone (Casaburi & Reed, 2022). Both studies use a linear-in-means model with constant effects. Also, in both studies, the model is over-identified, as individual-level shifters affect the outcomes of multiple agents in a group. We leverage this feature to test the assumption of homogeneous interaction effects.[6]

II.D.1. Classroom Peer Effects in Kenya

Our first application comes from Duflo et al. (2011), who study peer effects and the impact of ability tracking in primary schools in Kenya. The study included 121 schools, each assigning students to one of two classrooms. Students in *treatment* schools were assigned to classrooms based on ability, as measured by their baseline test score, while students in *control* schools

[5] Formally, we restrict g to the set of functions $\mathcal{G} = \{g : \text{supp}(Z_{-i}) \to \mathbb{R}^n | g(z'_{-i}) \geq g(z_{-i}) \text{ for } z'_{-i} \geq z_{-i}\}$. For $\tilde{Z}_{-i}$ to be a relevant instrument, we require that g is strictly increases in at least one component of Z_{-i}.

[6] As usual, we also need to maintain the other assumptions of the model for the over-identifying restrictions to be a test of homogeneous interaction effects. Otherwise, it is a test of all the model assumptions C.1-C.5.

were randomly assigned. Following Duflo et al. (2011), we restrict the sample to the control group. This sample is composed of 2,849 students over 61 schools, each split into two rooms.[7]

To measure peer effects in classrooms, Duflo et al. (2011) consider the following model:

$$Y_i = \beta \bar{Y}_{-i} + Z_i' \gamma + \nu_s + \varepsilon_i, \tag{5}$$

where Y_i is the endline test score of a student i, $\bar{Y}_{-i}$ is the average endline test score of i's classmates, Z_i is a vector of controls that includes i's own baseline score, and ν_s is a school fixed effect. The authors use the average baseline score of i's classmates $\bar{Z}_{-i}$ as an instrument for $\bar{Y}_{-i}$. As outcome variables, they consider math, reading, and total endline test scores.[8]

Table 2 presents results from our re-analysis of the data. The first three columns of Panel A show estimates from OLS regressions of Y_i on Z_i and $\bar{Z}_{-i}$ with school fixed effects, the same specification used by Duflo et al. (2011). For a classical linear-in-means model with equal class sizes, this regression recovers the equilibrium effects $\Delta Y_i/\Delta Z_i = \gamma + \frac{\beta^2\gamma}{(1-\beta)(N-1+\beta)}$ and $\Delta Y_i/\Delta Z_j = \frac{\beta\gamma}{(1-\beta)(N-1+\beta)}$, for $j \neq i$. The last three columns of Panel A present estimates from OLS regressions of $\bar{Y}_{-i}$ on Z_i and $\bar{Z}_{-i}$ with school fixed effects. The first three columns of Panel B provide estimates for the main IV specification in Duflo et al. (2011). Under the classical linear-in-means assumptions, these regressions recover the constant peer effect β.

The last three columns of Table 2, Panel B, report estimates from alternate IV specifications that use multiple excluded instruments. In addition to $\bar{Z}_{-i}$, we include four more instrumental variables: *(1)* minimum baseline score of peers, *(2)* maximum baseline score of peers, *(3)* average baseline score among female peers, and *(4)* average baseline score among male peers. Under constant effects, any combination of these instruments lead to the same IV estimand. Under heterogeneous effects, the IV estimands could differ. To test for constant effects in the model, we conduct a Sargan–Hansen test for over-identifying restrictions using all five excluded instruments. This test allows us to assess the validity of over-identifying restrictions using any linear combination of the excluded instruments. We find that this test is rejected at the significance level 0.05, suggesting that peer effects vary across classrooms.

II.D.2. Strategic Pricing Decisions in Sierra Leone

Our second application builds on the work of Casaburi & Reed (2022), who examine the strategic behavior of traders who purchase cocoa from farmers in Sierra Leone. During an experiment conducted from October to December 2011, half of the 80 traders in the sample were randomly assigned a subsidy of 150 leones per pound of cocoa sold at village markets. Data on prices and quantities from these transactions was subsequently collected for analysis.

[7] After removing missing data, we retain 2,190 students over 48 schools.

[8] Equation (5) corresponds to (E4) in Duflo et al. (2011). We adapt their notation slightly to align with our framework. We replicate the reduced form and IV estimates presented in Table 4 of the original paper.

Table 2: Classroom Peer Effects—Primary Schools in Kenya

	Own Endline Score			Peers' Mean Endline Score		
	Total (1)	Math (2)	Literature (3)	Total (4)	Math (5)	Literature (6)
Panel A. Reduced Form						
Own Baseline Score	0.507***	0.496***	0.413***	0.007**	0.006*	0.007**
	(0.026)	(0.022)	(0.030)	(0.003)	(0.003)	(0.003)
Peers' Mean Baseline Score	0.345**	0.324**	0.291**	0.788***	0.697***	0.704***
	(0.150)	(0.160)	(0.131)	(0.157)	(0.174)	(0.134)
Observations	2,188	2,188	2,189	2,188	2,188	2,189
	One Instrument Spec.			Multiple Instrument Spec.		
	Total	Math	Literature	Total	Math	Literature
Panel B. Instrumental Variables						
Peers' Mean Endline Score	0.444***	0.469***	0.422***	0.424***	0.488***	0.487***
	(0.117)	(0.124)	(0.120)	(0.094)	(0.103)	(0.117)
First-Stage F-Stat	371.8	371.6	1970	293.4	463.4	590.9
Sargan-Hansen Test[a]				15.12	12.53	12.76
				(0.004)	(0.014)	(0.013)
Observations	2,188	2,188	2,189	2,188	2,188	2,188

Notes. Data comes from Duflo et al. (2011). Following the authors' specifications, we include school fixed effects and controls for gender, age, and being assigned to the contract teacher. Columns (1)-(3) in Panel B use peers' mean baseline score as an excluded instrument. Columns (4)-(6) in Panel B use as excluded instruments: peers' mean baseline score, peers' minimum and maximum baseline scores, and mean baseline scores of male and female peers. Standard errors clustered at the school level.

[a]We report the Sargan-Hansen χ^2_4 test statistic with the corresponding p-value in parentheses below. *p<0.1; **p<0.05; ***p<0.01.

Casaburi & Reed (2022) specify a model of imperfect competition among buyers. Each market consists of N buyers and a unit measure of homogenous producers. The price P_i that a buyer i pays to producers is given by the inverse supply $P_i = \lambda + \kappa Q_i + \theta \sum_{j \neq i} Q_j$, which is micro-founded by assuming there exists a representative producer with a love for variety.[9] A buyer's profit function equals $\Pi_i = Q_i(v + sZ_i - P_i)$, where v represents the wholesale price net of costs and Z_i indicates whether the buyer is randomly assigned a subsidy valued at s.

In equilibrium, the buyers choose their quantities Q_i to maximize profit, while accounting for optimal decisions $\{Q_j\}_{j \neq i}$ of their competitors. The profit-maximizing quantities satisfy

[9]Following footnote 6 in Casaburi & Reed (2022), a producer's profit is: $V(P, Q) = Q_0 + \sum_{i=1}^{N} P_i Q_i - C(Q)$, where $C(Q) = \lambda \sum_{i=1}^{N} Q_i + \frac{1}{2}\kappa \sum_{i=1}^{N} Q_i^2 + \theta \sum_{j \neq i} Q_i Q_j$ is the cost of production, and Q_0 is any unsold output.

the linear-in-means model with constant effects, where the interaction effect β is $\theta(N-1)/2\kappa$.

$$\begin{aligned} Q_i &= \frac{v-\lambda}{2\kappa} - \frac{\theta}{2\kappa}\sum_{j\neq i} Q_j + \frac{s}{2\kappa} Z_i \\ &= \underbrace{\alpha}_{(v-\lambda)/2\kappa} + \underbrace{\frac{\beta}{N-1}}_{-\theta/2\kappa} \sum_{j\neq i} Q_j + \underbrace{\gamma}_{s/2\kappa} Z_i, \quad \text{for } i \in \{1,\dots,N\}. \end{aligned} \tag{6}$$

In this setting, we can interpret $\theta/2\kappa$ as a conduct parameter that measures how a buyer i's demand depends on the total quantity purchased by i's competitors. Under constant effects, the conduct parameter is identified from IV, where the quantity purchased by i's competitors $\sum_{j\neq i} Q_j$ is instrumented by the treatment statuses of i's competitors, denoted by $\{Z_j\}_{j\neq i}$.[10]

Table 3 presents estimates from our re-analysis of the data. The first two columns of Panel A show estimates from OLS regressions of a buyer i's own purchases Q_i on his or her own treatment status Z_i and the total number of treated competitors $\sum_{j\neq i} Z_j$, with and without trader controls. Under constant effects, this regression recovers the total individual effect of the subsidy Z_i on i's own purchases Q_i, along with the individual spillover effect of another trader j receiving a subsidy on i's own purchases. The last two columns of Panel A show OLS estimates from regressing $\sum_{j\neq i} Q_j$ on Z_i and $\sum_{j\neq i} Z_j$, with and without trader controls. The first two columns of Panel B report estimates from IV regressions of Q_i on $\sum_{j\neq i} Q_j$, where the number of treated competitors $\sum_{j\neq i} Z_j$ is used as the excluded instrument. For constant effects, this regression recovers the (negative) conduct parameter $-\theta/2\kappa$.

The last two columns of Table 3, Panel B, present estimates from alternate IV specifications that use multiple instruments. We use these regressions to test whether traders exhibit identical conduct parameters. In addition to $\sum_{j\neq i} Z_j$, we introduce three extra instruments: *(1)* number of treated competitors who have access to a storage facility, *(2)* number of treated competitors older than the median age (37), and *(3)* number of treated competitors with baseline sales above the median (300 lbs of cocoa). Each of these instruments is valid by the same identification arguments used in the original paper. We then conduct a Sargan–Hansen test for over-identifying restrictions using all four excluded instruments. From this exercise, we find strong evidence against the constant effects assumption. This suggests that different traders likely respond strategically in different ways to their competitors' pricing decisions.

[10] Casaburi & Reed (2022) do not run this IV regression since they never explicitly define a market in their empirical analysis. Rather, they rely on additional model assumptions to estimate the market size N while never explicitly assigning traders to markets. To conduct our analysis, however, we need to know which traders belong to which markets. We achieve this objective by defining a market as the interaction between a week and a chiefdom, which represents a small administrative unit in Sierra Leone. In the data, we find that 90% of traders operate in a single chiefdom in a given week and that over 98% of traders make more than half of their sales in the same chiefdom. We leverage this observation to assign traders to chiefdoms.

Table 3: Strategic Interactions—Cocoa Traders in Sierra Leone

	Trader Quantity		Competitors' Total Quantity	
	(1)	(2)	(1)	(2)
Panel A. Reduced Form				
Treatment Trader	416.663***	454.895***	-166.995	-61.516
	(45.733)	(49.594)	(248.156)	(267.626)
Number of Treated Competitors	-10.733***	-7.423**	507.685***	522.394***
	(2.975)	(3.697)	(16.141)	(19.948)
Observations	610	602	610	602
Trader Controls		X		X
	One Instrument Spec.		**Multiple Instrument Spec.**	
	(1)	(2)	(1)	(2)
Panel B. Instrumental Variables				
Competitors' Total Quantity	-0.007	-0.020***	-0.004	-0.018***
	(0.006)	(0.007)	(0.006)	(0.007)
First-Stage F-Stat	23.06	14.15	22.90	14.09
Sargan-Hansen Test[a]			9.82	12.35
			(0.02)	(0.006)
Observations	610	602	610	602
Trader Controls		X		X

Notes. Data comes from Casaburi & Reed (2022). Following the original paper, we include week fixed effects. Trader controls are: baseline pounds of cocoa sold, number of villages where trader operates, baseline share of suppliers receiving credit from trader, age, years working with wholesaler, ownership of a cement or tile floor, mobile phone, and access to a storage facility. Sample sizes differ between (1) and (2) due to missing data about trader controls.

[a]We report a Sargan-Hansen χ^2_3 test statistic with a corresponding p-value in parentheses.
*p<0.1; **p<0.05; ***p<0.01.

III. The Linear-in-Means Model with Heterogeneous Interaction Effects

Motivated by the findings in the previous section, we now relax Assumptions C.1-C.5 to allow agents to exhibit interaction effects of different signs and magnitudes, which vary within and across groups. Going forward, we treat $\alpha_g = [\alpha_{ig}]_{i\in\mathcal{N}_g}$, $\beta_g = [\beta_{ig}]_{i\in\mathcal{N}_g}$, and $\gamma_g = [\gamma'_{ig}]_{i\in\mathcal{N}_g}$ as random vectors that are distributed according to a joint density f. We place no parametric restrictions on this density. In particular, we allow the random coefficients to depend on one another. For example, an agent i's interaction effect β_{ig} could be shaped by the interaction effects of i's peers, as well as by the interaction effects that are realized in the other groups. Moreover, since we permit the coefficients γ_{ig} to be heterogeneous, we allow for the possibility that the incidence of Z_{ig} varies and may even depend on the characteristics of other agents.[11]

To accommodate heterogeneous effects, we replace Assumption C.5 with a new condition.

Assumption I (Independence of Observed Variables)**.** $Z_g \perp (\alpha_g, \beta_g, \gamma_g, \mathcal{N}_g)$.

This assumption is standard in the literature on random coefficients. It guarantees that the distribution of unobservables is statistically independent from the vector of observables Z_g.[12]

III.A. Economic Interpretations of the Model under Heterogeneous Effects

We now illustrate how the linear-in-means model with heterogeneous effects can be derived as the estimating equation in three distinct economic decision problems: peer effects in schools, joint labor supply decisions within households, and strategic interactions between firms in oligopolistic markets. In each of these examples, it is necessary to make strong assumptions about preferences or technology to justify the restrictions of homogeneous interaction effects. Relaxing the assumption of homogeneous effects therefore makes the model better suited for studying economic behavior in these different settings. Throughout the rest of the paper, we will draw on these examples to guide and interpret our analysis.

III.A.1. Peer Effects

Consider a peer group g, where each individual i makes a choice Y_{ig} from an action space $\mathbb{R}$. When making their decisions, individuals seek to conform to (or deviate from) the average behavior of their peers. These social pressures enter directly into the agent's utility function.

$$U_{ig}(Y_{ig}|Z_{ig}, \bar{Y}_{-ig}) = \big(\alpha_{ig} + Z_{ig}'\gamma_{ig}\big)Y_{ig} + \beta_{ig}\bar{Y}_{-ig}Y_i - \frac{1}{2}Y_{ig}^2.$$

The first component of utility captures the non-social determinants of an agent's choice Y_{ig}. The second term contains the peer effect, where the coefficient β_{ig} indicates how agent i is influenced by the average behavior in the peer group g. The third term is a convex cost of action. In equilibrium, agents' optimal decisions $\{Y_{ig}\}_{i\in\mathcal{N}_g}$ satisfy the system of equations (1).

This utility function is commonly used in the education literature to study peer effects; e.g., see Epple & Romano (1998) and Calvó-Armengol et al. (2009).[13] This work tends to rule out the possibility of heterogeneous peer effects by assuming that all students have the same

[11] We allow α_g, β_g, and γ_g to be correlated with the group size and composition, as characterized by the set $\mathcal{N}_g$. This correlation could be economically meaningful. For example, the social pressures that individuals experience might depend on the number or types of peers within the group. Moreover, this correlation ties our hands by preventing us from using group size variation as a source of identification. Both Lee (2007) and Davezies et al. (2009) study how variation in group sizes can be used for identification of peer effects.

[12] If Z_g includes covariates, then we can relax Assumption I to allow for independence of individual-level shifters conditional on covariates: $Z_g^s \perp (\alpha_g, \beta_g, \gamma_g, \mathcal{N}_g)|Z_g^c$, where Z_g^s are shifters and Z_g^c are covariates. In addition, if the set of agents $\mathcal{N}_g$ in a group is observed, then we can relax it by writing $Z_g \perp (\alpha_g, \beta_g, \gamma_g)|\mathcal{N}_g$.

[13] Blume et al. (2015) specify utility as $U_{ig}(Y_{ig}|Z_{ig}, \bar{Y}_{-ig}) = \big(\alpha_{ig} + Z_{ig}'\gamma_{ig}\big)Y_{ig} - \frac{1}{2}\beta_{ig}(Y_{ig} - \bar{Y}_{-ig})^2 - \frac{1}{2}Y_{ig}^2$. This function also rationalizes our framework, where the coefficients α_{ig}, β_{ig} and γ_{ig} are rescaled by $\frac{1}{1+\beta_{ig}}$.

marginal rate of substitution between private and social utility. By relaxing this assumption, we allow students to face different social pressures. It could be that some students deviate from, rather than conform to, their peers; or, it could be that all students conform but some do so more than others. Our extension allows for such nuances in the study of peer effects.

III.A.2. Household Labor Supply

Consider the following non-unitary model of household labor supply (see Donni & Chiappori, 2011). In household g, the members choose how much of their total time T to allocate between labor and leisure. Let h_{ig} be the amount of time that person i chooses to work, and let W_{ig} be the wage. Each member i of household g earns an income, denoted by $Y_{ig} = W_{ig}h_{ig}$.

Members of the household pool their incomes. These incomes are then redistributed so that each member i receives a fraction $\kappa_{ig} \in [0,1]$ to spend on personal consumption. The total value of household consumption, denoted by C_g, cannot exceed total household income. In addition to consuming $\kappa_{ig}C_g$, each member i can also consume non-transferable goods. These goods may come in the form of workplace amenities or social assistance benefits (e.g., healthcare services that only i can access). The value of these goods to person i equals a_{ig}.

Each individual maximizes welfare from leisure and consumption. The returns from each input are marginally decreasing, as captured by the following log-additive utility function.

$$\max_{h_{ig}} \; U_{ig}(h_{ig}|W_{ig}, C_g) = \mu_{ig}\log(T - h_{ig}) + (1-\mu_{ig})\log(a_{ig} + \kappa_{ig}C_g), \quad \text{s.t.} \quad C_g = \sum_{j\in\mathcal{N}_g} W_{jg}h_{jg}.$$

The parameter $\mu_{ig} \in [0,1]$ denotes person i's relative preference for leisure over consumption. As long as each person i spends some time $h_{ig} \in (0,T)$ working, an interior solution exists.

$$\begin{aligned} Y_{ig} &= -\frac{\mu_{ig}a_{ig}}{\kappa_{ig}} - \mu_{ig}\sum_{j\neq i} Y_{jg} + (1-\mu_{ig})TW_{ig} \\ &= \underbrace{\alpha_{ig}}_{-\frac{\mu_{ig}a_{ig}}{\kappa_{ig}}} + \underbrace{\frac{\beta_{ig}}{|\mathcal{N}_g|-1}}_{-\mu_{ig}} \sum_{j\neq i} Y_{jg} + \underbrace{\gamma_{ig}}_{(1-\mu_{ig})T} W_{ig}, \quad \text{for } i \in \mathcal{N}_g. \end{aligned}$$

The equilibrium equations satisfy the linear-in-means framework where the interaction effect β_{ig} equals $-\mu_{ig}(|\mathcal{N}_g|-1)$. The parameter μ_{ig} determines how much a person i's income falls when the rest of the household earns more. This parameter also governs the elasticity of i's earnings with respect to the wage W_{ig}. The variable Z_{ig} can be anything affecting i's wage.

The assumption of constant interaction effects implies that the marginal rate of substitution between consumption and leisure is the same both among the members of a given household and across households. By allowing for heterogeneous interaction effects, we permit the trade-offs to differ along both dimensions. It could be that the labor supply responses vary between primary and secondary earners in a household. These responses could also vary across households based on contextual factors, such as the number of children in the home.

III.A.3. Firm Oligopoly

Lastly, consider a model of oligopolistic competition where firms face heterogeneous, convex cost curves. Following Bresnahan (1981) and Perry (1982), we consider a general framework that nests the theories of Bertrand and Cournot oligopoly. This approach assumes that firms form conjectures about other firms' decisions that are consistent with equilibrium outcomes.

Each market g contains multiple firms i, each producing output Y_{ig}. The price that clears the market is given by an inverse demand: $P_g = a_g - b_g \sum_{i \in \mathcal{N}_g} Y_{ig}$, where a_g and b_g can vary across markets g. A firm's production costs are given by $c_{ig}(Y_{ig}) = (\lambda_{ig0} + Z'_{ig}\lambda_{ig1})Y_{ig} + \frac{1}{2}\delta_{ig}Y_{ig}^2$, where λ_{ig0}, λ_{ig1}, and δ_{ig} can vary both across firms i and across markets g. Assume that the vector Z_{ig} contains observable cost-shifters that directly influence the firm's productivity.

We suppose that every firm i has some reference output Y_{ig}^0, which is common knowledge in the market. The firm conjectures that increasing its own output Y_{ig} relative to Y_{ig}^0 causes the other firms to adjust their total output by θ_{ig}, believing that $\sum_{j \neq i} Y_{jg}$ equals $\sum_{j \neq i} Y_{jg}^0 + \theta_{ig}(Y_{ig} - Y_{ig}^0)$. Given these conjectures, each firm i in market g maximizes its profit by solving:

$$\max_{Y_{ig}} \; \Pi_{ig}(Y_{ig} | Z_{ig}, \{Y_{jg}^0\}_{j \neq i}) = P_g Y_{ig} - c_{ig}(Y_{ig}), \quad \text{s.t.} \quad \begin{aligned} P_g &= a_g - b_g \sum_{i \in \mathcal{N}_g} Y_{ig} \\ \sum_{j \neq i} Y_{jg} &= \sum_{j \neq i} Y_{jg}^0 + \theta_{ig}(Y_{ig} - Y_{ig}^0) \\ c_{ig}(Y_{ig}) &= (\lambda_{ig0} + Z'_{ig}\lambda_{ig1})Y_{ig} + \frac{1}{2}\delta_{ig}Y_{ig}^2. \end{aligned}$$

In equilibrium, the output Y_{ig} equals the reference output Y_{ig}^0. So, an equilibrium is given by:

$$\begin{aligned} Y_{ig} &= \frac{1}{\delta_{ig} + b_g(2 + \theta_{ig})} \left[a_g - \lambda_{ig0} - b_g \sum_{j \neq i} Y_{jg} - Z'_{ig}\lambda_{ig1} \right] \\ &= \underbrace{\alpha_{ig}}_{\frac{a_g - \lambda_{ig0}}{\delta_{ig} + b_g(2+\theta_{ig})}} + \underbrace{\frac{\beta_{ig}}{|\mathcal{N}_g| - 1}}_{-\frac{b_g}{\delta_{ig} + b_g(2+\theta_{ig})}} \sum_{j \neq i} Y_{jg} + Z'_{ig} \underbrace{\gamma_{ig}}_{-\frac{\lambda_{ig1}}{\delta_{ig} + b_g(2+\theta_{ig})}}, \quad \text{for } i \in \mathcal{N}_g. \end{aligned}$$

In this model, θ_{ig} is the conjectural variation, which measures firm i's perceived influence in market g. Three special cases are particularly notable. First, if $\theta_{ig} = 0$ for all i, then the model corresponds to Cournot oligopoly. In this case, firms do not internalize the effect of their own output decisions on the behavior of other firms. Second, if $\theta_{ig} = -1$ for all i, then the model is one of Bertrand competition. Here, firms expect that their actions have no effect on total market output. Third, if $\theta_{ig} = |\mathcal{N}_g| - 1$ for all i, then the market is monopolistic. In this setting, each firm acts as if it fully controls the market, which leads to perfect collusion.

Given this range of possibilities, it seems natural to permit θ_{ig} to be between -1 and $|\mathcal{N}_g|-1$.

An equilibrium in this model produces the linear-in-means model as an estimating equation. The interaction effect β_{ig} equals $-\frac{b_g(|\mathcal{N}_g|-1)}{\delta_{ig}+b_g(2+\theta_{ig})}$, which represents a firm-specific conduct parameter as defined by Weyl & Fabinger (2013). This quantity measures how a firm i's output responds to the output of other firms in the market. Note that β_{ig} depends on the elasticity of consumer demand b_g, the slope δ_{ig} of the marginal cost curve, and the conjectural variation θ_{ig}. So, the constant effects assumption implies: *(1)* consumer demand is equally elastic in all markets, *(2)* firms' marginal costs have the same curvature, and *(3)* all firms have the same beliefs about market competition. The linear-in-means model with heterogeneous interaction effects allows each of these factors to vary within and across markets.

III.B. Characterization of an Equilibrium

To analyze the equilibrium behavior of the linear-in-means model with heterogeneous interaction effects, we first derive the necessary and sufficient conditions for there to be a unique solution to the system of equations (1). The condition that we derive will significantly relax Assumption C.3. Specifically, rather than placing bounds on the signs and magnitudes of the endogenous interaction effects, our condition only rules out a single equality constraint.

Assumption II (Unique Solution). $\sum_{i\in\mathcal{N}_g}(1-\beta_{ig})\prod_{j\in\mathcal{N}_g\setminus i}(|\mathcal{N}_g|-1+\beta_{jg})\neq 0$ for any group g.

Assumption II is a rank condition. It ensures that $I-\mathbf{B}_g$ is invertible, where I denotes the identity matrix and $\mathbf{B}_g$ is the adjacency matrix specifying the interaction effects in group g:

$$\mathbf{B}_g = \frac{1}{|\mathcal{N}_g|-1}\begin{bmatrix} 0 & \beta_{1g} & \cdots & \beta_{1g} \\ \beta_{2g} & 0 & \cdots & \beta_{2g} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{|\mathcal{N}_g|g} & \beta_{|\mathcal{N}_g|g} & \cdots & 0 \end{bmatrix}. \tag{7}$$

This assumption rules out cases where the outcome equations (1) correspond to parallel lines. If these lines are parallel to each other, then they either never intersect or they overlap. In the first case, the model has no solution. In the second case, it has infinitely-many solutions.[14] By eliminating these two cases, Assumption II ensures that the equilibrium is well-defined.

We now present a closed form representation of the equilibrium, showing how the outcomes $\{Y_{ig}\}_{i\in\mathcal{N}_g}$ depend on the variables $\{Z_{ig}\}_{i\in\mathcal{N}_g}$ after accounting for spillover effects. In general, spillovers have the potential amplify or suppress the impacts of $\{Z_{ig}\}_{i\in\mathcal{N}_g}$ on agents' outcomes. These distortions are driven by the interaction effects $\{\beta_{ig}\}_{i\in\mathcal{N}_g}$, which can be positive or negative in our framework. Moreover, as we allow the interaction effects to vary

[14] Tamer (2003) discusses issues of *incoherency* and *incompleteness* of simultaneous equation models. When a model is incoherent, it has no solution. When a model is incomplete, it has multiple solutions. In our setting, nonintersecting lines makes the model incoherent, and overlapping lines makes the model incomplete. In the Appendix, we provide a graphical illustration of these two cases, discussing why they are both problematic.

among agents, the nature of these distortions becomes more complex as the group size $|\mathcal{N}_g|$ grows larger. The following proposition gives a general characterization of the equilibrium.

Proposition 1. A unique solution to system (1) exists if and only if Assumption II holds. In equilibrium, the outcomes $\{Y_{ig}\}_{i\in\mathcal{N}_g}$ in group g satisfy $Y_{ig} = \alpha_{ig} + \beta_{ig}\bar{Y}_{g,-i} + Z'_{ig}\gamma_{ig}$, where:

$$\bar{Y}_g = \frac{\sum_{j\in\mathcal{N}_g}\left[\prod_{\ell\in\mathcal{N}_g\setminus j}\left(1+\frac{\beta_{\ell g}}{|\mathcal{N}_g|-1}\right)\right]\times(\alpha_{jg}+Z'_{jg}\gamma_{jg})}{|\mathcal{N}_g|\times\det(I-\mathbf{B}_g)}, \quad \text{and:}$$

$$\bar{Y}_{g,-i} = \frac{\sum_{j\in\mathcal{N}_g}\nu_{ijg}\times\left[\frac{\beta_{jg}}{|\mathcal{N}_g|-1}(\alpha_{ig}+Z'_{ig}\gamma_{ig})+(\alpha_{jg}+Z'_{jg}\gamma_{jg})\right]}{(|\mathcal{N}_g|-1)\times\det(I-\mathbf{B}_g)}, \quad \text{for } i\in\mathcal{N}_g.$$

Here, we define $\nu_{ijg}=1$ for $|\mathcal{N}_g|=2$ and $\nu_{ijg}=\prod_{\ell\in\mathcal{N}_g\setminus\{i,j\}}\left(1+\frac{\beta_{\ell g}}{|\mathcal{N}_g|-1}\right)$ for $|\mathcal{N}_g|>2$. The determinant of $I-\mathbf{B}_g$ also has a closed-form expression, which is provided in the Appendix.

While prior work derives similar formulas for two- or three-agent special cases (e.g., Masten, 2017), our equilibrium formulas apply to groups of any size. Given this generality, our analysis extends to a wide range of settings with varying group size and composition, such as peer effects in Kenyan primary schools and competition among traders in Sierra Leone.

Remark 1. Moment Determinacy.

Although Assumption II rules out models with parallel lines, it does not eliminate models with *nearly* parallel lines, in which $\det(I-\mathbf{B}_g)$ is close to zero with high probability. This distinction becomes important when we consider mean-based identification strategies, since the moments of the reduced form coefficients may not exist if $\det(I-\mathbf{B}_g)$ is very close to zero. For the reduced form moments to be well-defined, we need a slightly stronger assumption.

One sufficient condition for moment determinacy is that the vector of outcomes Y_g has a bounded support. Moreover, as Masten (2017) shows, the reduced form moments can exist even when Y_g takes full support if the tails of the outcome distributions are sufficiently thin. By reformulating Assumption A6 in Masten (2017) for our framework, we arrive at the following sufficient condition, which is expressed as a restriction on the structural parameters.[15]

Assumptions III (Sufficient Conditions for Moment Determinacy)**.**
III.1. $\mathrm{P}\left(\left|\sum_{i\in\mathcal{N}_g}(1-\beta_{ig})\prod_{j\in\mathcal{N}_g\setminus i}(|\mathcal{N}_g|-1+\beta_{jg})\right|\geq\tau\right)=1$ for some scalar $\tau>0$.
III.2. The marginal distributions of $\{\alpha_{ig}\}_{i,g}$ and $\{\gamma_{ig}\}_{i,g}$ have subexponential tails.

Remark 2. Preservation of Order.

Although not necessary for identification, it is often helpful for interpreting economic quantities if the structural coefficients $\{\gamma_{ig}\}_{i,g}$ have the same signs (respectively) as the reduced

[15]For more discussion, as well as necessary conditions for moment determinacy, we refer to Masten (2017).

form effects $\{\Delta Y_{ig}/\Delta Z_{ig}\}_{i,g}$. That is, if Z_{ig} has a positive direct effect on the outcome Y_{ig}, when does Z_{ig} have a positive effect on Y_{ig} in equilibrium? Consider the following condition.

Assumption IV (Bounded Interactions)**.** $1 - |\mathcal{N}_g| < \beta_{ig} < 1$ for all agents i and groups g.

By ensuring that γ_{ig} and $\Delta Y_{ig}/\Delta Z_{ig}$ share the same sign, Assumption IV rules out equilibrium behaviors that might seem illogical. For example, in a peer effects model, a student's achievement would not fall when the marginal utility of effort rises. In a household labor supply model, a person's income would not decrease after receiving a raise. In an oligopoly model, a firm's output would not fall as a consequence of becoming more productive.[16]

III.C. Definition and Interpretation of Economic Quantities under Heterogeneous Effects

We now define and interpret the economic quantities in Table 1 under heterogeneous effects. As before, we ease notation by removing group subscripts and treating Z_i as one-dimensional.

Total Individual Effect

The first quantity that we reexamine is the total effect of Z_i on Y_i in equilibrium. By Proposition 1, we can decompose this quantity to distinguish between direct and indirect effects of Z_i.

$$\frac{\Delta Y_i}{\Delta Z_i} = \gamma_i + \underbrace{\beta_i \frac{\Delta \bar{Y}_{-i}}{\Delta Z_i}}_{\text{Indirect Effect}}, \quad \text{where} \quad \frac{\Delta \bar{Y}_{-i}}{\Delta Z_i} = \frac{\sum_{j \neq i} \left[\beta_j \prod_{\ell \notin \{i,j\}} \left(1 + \frac{\beta_\ell}{N-1}\right)\right]}{(N-1)^2 \times \det(I - \mathbf{B})} \times \gamma_i. \tag{8}$$

The indirect effect accounts for network distortions. It depends on the cycles in the network, which specify how an agent's behavior is reflected back onto itself via interactions with others. This feedback loop may either reinforce or undermine the direct effect of the variable Z_i.

To interpret the indirect effect of Z_i on Y_i, we first need to determine how Z_i affects $\bar{Y}_{-i}$. If Assumption IV holds, then the sign of $\Delta \bar{Y}_{-i}/\Delta Z_i$ is determined by the product of γ_i and:

$$\psi_i = \frac{1}{N-1} \sum_{j \neq i} \left[\beta_j \prod_{\ell \notin \{i,j\}} \left(1 + \frac{\beta_\ell}{N-1}\right)\right]. \tag{9}$$

The term ψ_i specifies how agent i's action Y_i affects the average outcome $\bar{Y}_{-i}$ in the rest of the group. If $\psi_i > 0$, e.g., in a peer effects model with positive social interactions, then increasing agent i's action will raise the average outcome of others. If $\psi_i < 0$, e.g., in a household labor supply model or a model of oligopolistic competition, then increasing Y_i will decrease $\bar{Y}_{-i}$.

[16] Assumption IV is a special case of Assumption II. Thus, it also ensures that there is a unique equilibrium. In addition, it implies that $\det(I - \mathbf{B}_g) > 0$ with probability 1 (see the Appendix for a proof). In a household labor supply model, Assumption IV holds if all people value consumption: $\mu_{ig} \neq 1$ for all i. In an oligopoly model, it rules out Bertrand competition for firms with constant marginal costs: $(\theta_{ig}, \delta_{ig}) \neq (-1, 0)$ for all i. Such models do not possess an interior solution, since firms would always seek to undercut one another until they are all left with zero profit. This phenomenon is known as the Bertrand paradox (Edgeworth, 1925).

To understand when network spillovers would amplify or suppress the impact Z_i on Y_i, we must examine the product of β_i and ψ_i. This product represents an indirect interaction effect that the agent has with herself, as understood by evaluating all the cycles in the network that start and end with agent i. If $\beta_i \times \psi_i$ is positive, e.g., in a peer effects model with positive spillovers, a household labor supply model, and an oligopoly model, then agent i's behavior is self-reinforcing. In this case, the interaction effects would magnify the impact of an exogenous shock: $|\Delta Y_i/\Delta Z_i| > |\gamma_i|$. Conversely, if $\beta_i \times \psi_i$ is negative, then agent i's actions are self-undermining, which would suppress the impact of a shock: $|\Delta Y_i/\Delta Z_i| < |\gamma_i|$.

Individual Spillover Effect

The second parameter we study is the spillover effect of Z_i on Y_j. In a peer effects model, this measures how student j is indirectly affected by factors that alter student i's achievement. For household labor supply, it captures how j's income is affected by the wage of family member i. For an oligopoly, it specifies how the output at firm j responds to the productivity at another firm i. By Proposition 1, we write down the spillover effect of Z_i on Y_j as follows:

$$\frac{\Delta Y_j}{\Delta Z_i} = \frac{\beta_j \prod_{\ell \notin \{i,j\}} \left(1 + \frac{\beta_\ell}{N-1}\right)}{(N-1) \times \det(I - \mathbf{B})} \times \gamma_i. \tag{10}$$

Under Assumption IV, this effect always has the same sign as $\beta_j \times \gamma_i$. So, when β_j is positive, a positive shock to agent i's outcome has a positive spillover effect on agent j's outcome. Conversely, when β_j is negative, a positive shock to Y_i has a negative spillover effect on Y_j.

Total Effect on the Average

Next, we reinterpret the equilibrium effect of the variable Z_i on the average group outcome $\bar{Y}$.

$$\frac{\Delta \bar{Y}}{\Delta Z_i} = \frac{\prod_{\ell \neq i} \left(1 + \frac{\beta_\ell}{N-1}\right)}{N \times \det(I - \mathbf{B})} \times \gamma_i. \tag{11}$$

Under Assumption IV, this quantity always has the same sign as the coefficient γ_i. Therefore, in a peer effects model, a policy that improves one student's performance always increases the average level of achievement in the group. In a household labor supply model, a wage boost for one individual always raises the total income of the household. In an oligopoly model, improving one firm's productivity always increases the overall output in the market.

Social Multiplier Effects

We now reexamine the social multiplier under heterogeneous effects. We can use this quantity to measure how externalities distort the effect of exogenous shocks on outcomes. Much of the literature on social multipliers (e.g., Goldin & Katz, 2002; Glaeser et al., 2003; Becker & Murphy, 2003) assumes that the interaction effects are positive: $\beta_i \geq 0$ for all agents i. Under this assumption, network spillovers always amplify the impact of a policy shock on group outcomes. However, this pattern need not hold in settings with negative interaction effects. In such cases, network spillovers have a potential to suppress the impact of a policy.

When the endogenous interaction effects are heterogeneous, Glaeser et al.'s (2003) social multiplier is not well-defined because $\bar{Z}$ could affect $\bar{Y}$ in different ways depending on which of the variables $\{Z_i\}_{i=1}^N$ is changed. In other words, the total effect of an exogenous shock on group outcomes depends on which agent(s) in the group are directly exposed to that shock. For heterogeneous effects, we can define an individual-specific social multiplier for an agent i.

$$M_{(i)}^{\text{heterog.}} = \frac{\sum_{j=1}^N \Delta Y_j / \Delta Z_i}{\Delta Y_i / \Delta Z_i} = \frac{1}{1 - \frac{1}{N-1} \sum_{j \neq i} \frac{\beta_j}{1+\beta_j/(N-1)}}. \tag{12}$$

This quantity is defined as the ratio of the total effect of Z_i on $\sum_{j=1}^N Y_j$ to the individual effect of Z_i on Y_i. It generalizes the original definition of the social multiplier by accommodating heterogeneous effects. In a constant effects model, $M_{(i)}^{\text{heterog.}}$ reduces to M^{constant} for every i. Additionally, as the size of the group N becomes large, $M_{(i)}^{\text{heterog.}}$ tends to $(1 - \frac{1}{N-1} \sum_{j \neq i} \beta_j)^{-1}$.

The notion of an individual-specific social multiplier is particularly intuitive when group members assume different roles. In the household labor supply example, $M_{(i)}^{\text{heterog.}}$ measures how an exogenous change in person i's wage would affect total household income relative to i's individual income Y_i. If there is only one primary earner in the household, then it is likely that these multipliers differ across household members i. For example, in a two-person household, $M_{(i)}^{\text{heterog.}}$ equals $1 - \mu_j$, which captures the second household member j's trade-off between consumption and leisure. If member j places high value on leisure (so μ_j is large), then j is more willing to work less when i earns more. In this case, the multiplier $M_{(i)}^{\text{heterog.}}$ is small since the total impact of raising i's wage on total household income would be heavily offset by a reduction in j's labor supply. Alternatively, if j places high value on consumption, then his/her labor supply is less responsive to i's wage, and the multiplier $M_{(i)}^{\text{heterog.}}$ is large.[17]

Averaging across agents, we can construct an aggregate social multiplier effect $M^{\text{heterog.}}$, equal to $\frac{1}{N} \sum_{i=1}^N M_{(i)}^{\text{heterog.}}$. Alternatively, we can take $M^{\text{heterog.}}$ as the ratio of average effects:

$$M^{\text{heterog.}} = \frac{\sum_{i=1}^N \Delta \bar{Y} / \Delta Z_i}{\frac{1}{N} \sum_{j=1}^N \Delta Y_j / \Delta Z_j} = \frac{\frac{1}{N} \sum_{j=1}^N \left(1 + \frac{\beta_j}{N-1}\right)^{-1} \gamma_j}{\frac{1}{N} \sum_{j=1}^N \left(1 + \frac{\beta_j}{N-1}\right)^{-1} \left(1 - \frac{1}{N-1} \sum_{k \neq j} \frac{\beta_k}{1+\beta_k/(N-1)}\right) \gamma_j}. \tag{13}$$

While they have slightly different interpretations, both versions of the aggregate social multiplier reduce to the original definition under constant effects. Throughout the rest of the paper, we take the expression in (13) as our definition of the aggregate social multiplier. If γ_i is constant across agents i, then $M^{\text{heterog.}}$ tends to $\left(1 - \frac{1}{N} \sum_{i=1}^N \beta_i\right)^{-1}$ as N grows large.

[17] Note that the multiplier effects are always less than one in this example, since strategic substitutability suppresses the impact of exogenous wage shocks on total household income, which ensures that $M_{(i)}^{\text{heterog.}} < 1$.

Structural Coefficients and Higher Moments

The next three parameters are α_i, β_i, and γ_i. Of particular interest to us is the interaction effect β_i. In a peer effects model, this term measures how much social pressure someone faces. In a model of household behavior, it specifies how one's income depends on the earnings of others. In an oligopoly model, it measures the degree of strategic interaction between firms.

Lastly, we also seek to learn about the correlation structure between the different interaction effects in a network, i.e., $\operatorname{corr}(\beta_i, \beta_j)$ for different agents i and j. These parameters are novel in the literature, and they can help us to learn about the formation of network ties. For example, if my peer feels a strong pressure to conform to the group, am I also likely to feel that pressure? Do people in the same family share similar preferences over leisure and consumption? The heterogeneous effects framework is well suited to address these questions.

IV. Analysis of OLS and IV Estimands under Heterogeneous Effects

We now analyze what can and cannot be learned from frequently used OLS and IV estimands for linear-in-means models under heterogeneous effects. We show that, while these estimands do not lead to point identification, they still carry information about key economic quantities.

As in Section II, we ease notation by setting $\mathcal{N} = \{1, \ldots, N\}$, omitting group subscripts, and treating Z_{ig} as one-dimensional. Also, as before, we assume that Z_{ig} is an individual-level shifter, and that the shifters are not perfectly collinear. To ensure instrument relevance, we assume that $\gamma_{ig} \neq 0$ for every i and g, so that Z_{ig} has a nonzero effect on observed outcomes.[18]

IV.A. Empirical Analysis with OLS Estimands

We first analyze the OLS estimands $\beta^{\text{OLS}}_{Y_i}$ and $\beta^{\text{OLS}}_{\bar{Y}}$, which are defined in Section II.C.1. To interpret these estimands for heterogeneous effects, first suppose that β_i and γ_i are heterogeneous across groups but homogeneous within groups. In this case, $\beta^{\text{OLS}}_{Y_i}$ and $\beta^{\text{OLS}}_{\bar{Y}}$ recover average equilibrium effects $\mathrm{E}(\Delta Y_i/\Delta Z_i)$, $\{\mathrm{E}(\Delta Y_i/\Delta Z_j)\}_{j\neq i}$, and $\{\mathrm{E}(\Delta \bar{Y}/\Delta Z_j)\}_j$ across groups.

These quantities provide insight into equilibrium behavior. In a peer effects setting, they capture average equilibrium effects (across classrooms) of policies that impact student performance. In a household labor supply context, they represent average equilibrium effects (across families) of wage shocks on household incomes. In the oligopoly example, they quantify average equilibrium effects (across markets) of firm-specific cost shocks on firm output.

If β_i and γ_i are also heterogeneous within groups, then these OLS estimands no longer recover average equilibrium effects. To see why, note that within-group heterogeneity causes $\Delta Y_i/\Delta Z_j$ to differ across agents $j \neq i$ and $\Delta \bar{Y}/\Delta Z_j$ to differ across agents j. Therefore, any OLS regression that includes averages of Z, while excluding $\{Z_j\}_{j=1}^N$ as individual regressors,

[18] This assumption can be relaxed to allow for instruments that affect the outcomes for a subset of agents.

suffers from omitted variable bias.[19] This bias arises even with constant effects across groups. Indeed, as long as β_i and γ_i differ among agents in a group, these regressions are misspecified.

For the regressions to be correctly specified in the presence of within-group heterogeneity, it is important to include the shifters $\{Z_j\}_{j=1}^N$ separately as individual regressors. Specifically, we define the OLS estimands $\beta_{Y_i}^{\text{OLS}} = \mathrm{E}(\tilde{Z}\tilde{Z}')^{-1}\,\mathrm{E}(\tilde{Z}Y_i)$ and $\beta_{\bar{Y}}^{\text{OLS}} = \mathrm{E}(\tilde{Z}\tilde{Z}')^{-1}\,\mathrm{E}(\tilde{Z}\bar{Y})$, which correspond to regressions of Y_i and $\bar{Y}$, respectively, on the entire vector $\tilde{Z} = (1, Z')'$. Even with heterogeneous effects within and across groups, these regressions recover an average of the equilibrium effects $\mathrm{E}(\Delta Y_i/\Delta Z_i)$, $\{\mathrm{E}(\Delta Y_i/\Delta Z_j)\}_{j\neq i}$, and $\{\mathrm{E}(\Delta \bar{Y}/\Delta Z_j)\}_j$ across groups.

Proposition 2. In a linear-in-means model with heterogeneous effects, the economic quantities $\mathrm{E}(\Delta Y_i/\Delta Z_i)$, $\{\mathrm{E}(\Delta Y_i/\Delta Z_j)\}_{j\neq i}$, and $\{\mathrm{E}(\Delta \bar{Y}/\Delta Z_j)\}_j$ are recovered from OLS estimands:

$$\beta_{Y_i}^{\text{OLS}} = \mathrm{E}(\tilde{Z}\tilde{Z}')^{-1}\,\mathrm{E}(\tilde{Z}Y_i) \quad \text{and} \quad \beta_{\bar{Y}}^{\text{OLS}} = \mathrm{E}(\tilde{Z}\tilde{Z}')^{-1}\,\mathrm{E}(\tilde{Z}\bar{Y}), \quad \text{where} \quad \tilde{Z} = (1, Z')'.$$

OLS Estimands for Social Multiplier Effects

For constant effects models, the social multiplier $M^{\text{constant}} = (\Delta\bar{Y}/\Delta\bar{Z})/(\Delta Y_i/\Delta Z_i)$ is point identified from OLS estimands. Specifically, $\Delta\bar{Y}/\Delta\bar{Z}$ is recovered from regressing $\bar{Y}$ on $(1, \bar{Z})$ and $\Delta Y_i/\Delta Z_i$ is recovered from regressing Y_i on $(1, Z_i, \bar{Z}_{-i})$. However, in the linear-in-means model with heterogeneous effects, the social multiplier is no longer point identified from OLS.

To understand why OLS estimands do not recover social multipliers under heterogeneous effects, first recall that M^{constant} is not well-defined in the case of within-group heterogeneity. Instead, we define individual-specific multipliers $M_{(i)}^{\text{heterog.}}$ and aggregate multipliers $M^{\text{heterog.}}$, which are better suited for settings where agents in a group face different interaction effects. If the interactions are constant across groups, then $M_{(i)}^{\text{heterog.}}$ and $M^{\text{heterog.}}$ are both identified from correctly specified OLS regressions, following the previous discussion. However, if the interaction effects vary across groups, then these regressions instead recover the estimands:

$$M_{(i)}^{\text{OLS}} = \frac{\sum_{j=1}^N \mathrm{E}(\Delta Y_j/\Delta Z_i)}{\mathrm{E}(\Delta Y_i/\Delta Z_i)} \quad \text{and} \quad M^{\text{OLS}} = \frac{\sum_{i=1}^N \mathrm{E}(\Delta\bar{Y}/\Delta Z_i)}{\frac{1}{N}\sum_{j=1}^N \mathrm{E}(\Delta Y_j/\Delta Z_j)}.$$

These estimands represent ratios of average equilibrium effects across groups. Yet, they do not correspond to the economic quantities of interest in Table 1. As we show in Section V, we may still be able to use OLS to place informative bounds on the social multiplier effects.

IV.B. Empirical Analysis with IV Estimands

We now reexamine the IV estimand, which is defined in equation (4). Under heterogeneous effects, IV does not lead to point identification of β_i. This negative result motivates our subsequent analysis, examining: When is the IV estimand informative about interaction effects?

[19] If $\{Z_j\}_{j=1}^N$ are all uncorrelated, then the coefficient on Z_i in a regression of Y_i on $(1, Z_i, \bar{Z}_{-i})$ would still recover the average total individual effect $\mathrm{E}(\Delta Y_i/\Delta Z_i)$. Yet, the other coefficients are biased by construction.

We establish conditions under which the IV estimand $\beta_i^{\mathrm{IV}}(z_i)$ will be a positively-weighted average of β_i, which is a minimal requirement for it to be informative about social interaction effects. A standard condition for this property, which is widely used in the treatment effects literature, is proposed by Imbens & Angrist (1994). It requires that the endogenous variable $\bar{Y}_{-i}$ is affected uniformly by any change in the instrument $\tilde{Z}_{-i}$. If we take $\tilde{Z}_{-i}$ to be Z_{-i} (or if $\tilde{Z}_{-i}$ is a one-to-one function of Z_{-i}) then this condition has the following characterization.

Assumption IAM *(Imbens-Angrist Monotonicity)***.** For any vectors (z_{-i}, z_i) and (z'_{-i}, z_i) in the support of Z, either $\mathrm{P}\left(\bar{Y}_{-i}(z_{-i}, z_i) \geq \bar{Y}_{-i}(z'_{-i}, z_i)\right) = 1$ or $\mathrm{P}\left(\bar{Y}_{-i}(z_{-i}, z_i) \leq \bar{Y}_{-i}(z'_{-i}, z_i)\right) = 1$.

We argue that this condition is plausible in settings where the interactions take place between two agents, but we demonstrate that it is unlikely to hold with groups of three or more agents.

Pairs of Agents ($N = 2$)

We consider a special case of the model where the interactions take place between two agents.

$$Y_1 = \alpha_1 + \beta_1 Y_2 + \gamma_1 Z_1 \tag{14}$$

$$Y_2 = \alpha_2 + \beta_2 Y_1 + \gamma_2 Z_2. \tag{15}$$

This special case allows us to study peer effects between pairs of students, joint labor supply decisions in two-person households, and the strategic interactions among firms in duopolies.

For any $j \neq i$, the IV estimand equals $\beta_i^{IV}(z_i) = \mathrm{Cov}(Y_i, Z_j | Z_i = z_i) / \mathrm{Cov}(Y_j, Z_j | Z_i = z_i)$. This estimand can be expressed as a weighted average of all the potential realizations of β_i.

$$\beta_i^{IV}(z_i) = \int_{\mathrm{supp}(\beta_i)} b_i \times \omega(b_i) db_i, \quad \text{where} \quad \omega(b_i) = \frac{\mathrm{E}(\Delta Y_j / \Delta Z_j | \beta_i = b_i) f_{\beta_i}(b_i)}{\mathrm{E}(\Delta Y_j / \Delta Z_j)}. \tag{16}$$

Observe that larger weights $\omega(b_i)$ are placed on values of β_i in groups where the outcome Y_j is more responsive to the instrument Z_j. For the weights to be non-negative, we can impose IAM monotonicity, which requires that Y_j is uniformly affected in the same direction by Z_j. This condition holds if and only if the coefficient γ_j retains the same sign across all networks:

$$\mathrm{P}(\gamma_j \geq 0) = 1 \text{ or } \mathrm{P}(\gamma_j \leq 0) = 1. \tag{17}$$

This condition does not impose restrictions on the interaction effects (β_1, β_2) in the model.[20]

Example (Peer Effects). Consider a model of peer effects with two students: i and j. Let Z_j indicate whether student j receives a scholarship, and assume that this scholarship always raises student achievement, such that $\mathrm{P}(\gamma_j \geq 0) = 1$. In this case, IV recovers the average peer effect β_i in groups where student j's achievement is most impacted by the scholarship.

Example (Household Labor Supply). Suppose that each household has two members, i

[20] An alternative sufficient condition is: $\gamma_j \perp (\beta_1, \beta_2)$. However, this condition does not extend to $N > 2$.

and j, and let Z_j be a policy that increases person j's wage. Then, IV measures the average second earner effect μ_i in households where j's income is particularly affected by the policy.

Example (Duopoly). Consider a duopoly, and let Z_j be a technological shock that always raises the productivity of firm j, i.e., $\mathrm{P}(\lambda_{j1} \leq 0) = 1$. In this case, IV would estimate the average conduct parameter for firm i in the markets where j is most responsive to the shock.

Groups of Three Agents ($N = 3$)

For peer groups of more than two agents, the IAM assumption is more restrictive with respect to the interaction effects. To unpack these restrictions, we will examine the three-agent case.

$$Y_1 = \alpha_1 + \beta_1\Big(\frac{Y_2 + Y_3}{2}\Big) + \gamma_1 Z_1 \tag{18}$$

$$Y_2 = \alpha_2 + \beta_2\Big(\frac{Y_1 + Y_3}{2}\Big) + \gamma_2 Z_2 \tag{19}$$

$$Y_3 = \alpha_3 + \beta_3\Big(\frac{Y_1 + Y_2}{2}\Big) + \gamma_3 Z_2. \tag{20}$$

For distinct agents $i, j, k \in \{1, 2, 3\}$, the endogenous variable in agent i's outcome equation is $\bar{Y}_{-i} = \frac{1}{2}(Y_j + Y_k)$. A researcher can use either Z_j or Z_k as a valid instrument for $\bar{Y}_{-i}$. In this example, we focus on an IV strategy that uses both instruments jointly, i.e., $\tilde{Z}_{-i} = (Z_j, Z_k)$. As in the two-agent case, we can interpret $\beta_i^{\mathrm{IV}}(z_i)$ as a weighted average of interaction effects, where larger weights are given to values of β_i in groups where $\bar{Y}_{-i}$ is more affected by Z_{-i}.

If we impose IAM, then $\beta_i^{IV}(z_i)$ will be a positively-weighted average of β_i's. However, as shown in Figure 1, IAM places strong conditions on the reduced form effects $\Delta\bar{Y}_{-i}/\Delta Z_j$ and $\Delta\bar{Y}_{-i}/\Delta Z_k$.[21] For binary instruments, it requires that these effects have the same signs in all networks and that one of these effects is always larger in magnitude than the other one. For continuous instruments, it requires that the ratio of $\Delta\bar{Y}_{-i}/\Delta Z_j$ to $\Delta\bar{Y}_{-i}/\Delta Z_k$ is constant.

The restrictions on the reduced form also impose restrictions on the interaction effects.

Lemma 3. When $N = 3$ and (Z_j, Z_k) are binary, Assumption IAM holds if and only if:

(*i*) $\mathrm{P}(\gamma_\ell \geq 0) = 1$ or $\mathrm{P}(\gamma_\ell \leq 0) = 1$, for $\ell \in \{j, k\}$.

(*ii*) $\mathrm{P}\left(\frac{1+\frac{1}{2}\beta_j}{1+\frac{1}{2}\beta_k} \geq \left|\frac{\gamma_j}{\gamma_k}\right|\right) = 1$ or $\mathrm{P}\left(\frac{1+\frac{1}{2}\beta_j}{1+\frac{1}{2}\beta_k} \leq \left|\frac{\gamma_j}{\gamma_k}\right|\right) = 1$.

Example (Peer Effects). Suppose that Z_j and Z_k are binary variables indicating whether students j and k, respectively, receive a scholarship. For simplicity, assume that γ_j and γ_k are uniform within and across peer groups. Then, IAM requires that one student always has a larger interaction effect than the other student: either $\mathrm{P}(\beta_j \geq \beta_k) = 1$ or $\mathrm{P}(\beta_j \leq \beta_k) = 1$.[22]

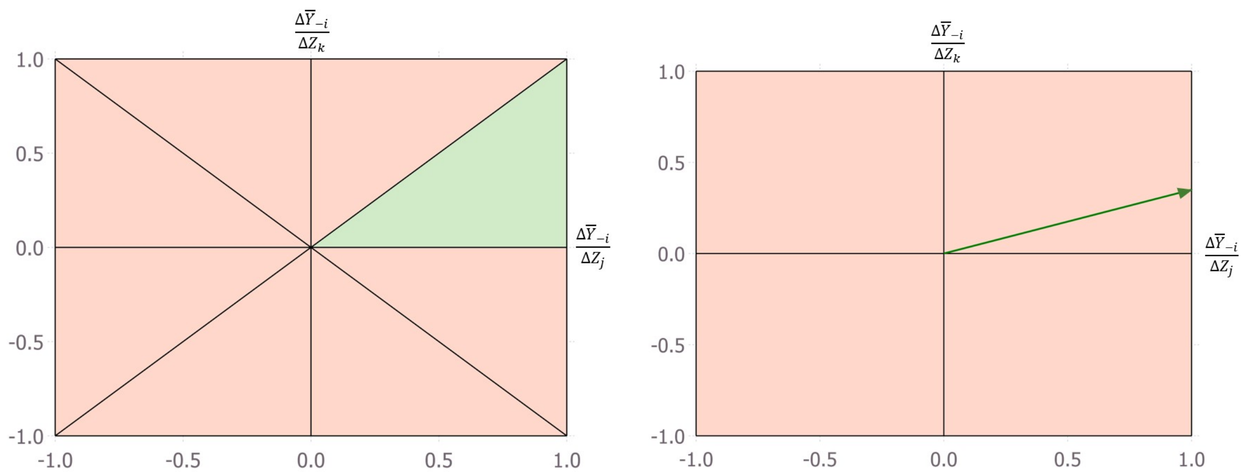
[21] Specifically, the IAM assumption imposes a total order on a vector space, requiring that the relation $\succeq$, where $z_{-i} \succeq z'_{-i}$ if and only if $\mathrm{P}\left(\bar{Y}_{-i}(z_{-i}, z_i) \geq \bar{Y}_{-i}(z'_{-i}, z_i)\right) = 1$, is a total order on the support of Z_{-i}.

[22] If the indices j and k are chosen arbitrarily, then one could overcome this restriction by defining j to be the member of the peer group who experiences the most social pressure. However, if j and k take on specific

Example (Household Labor Supply). Suppose that Z_j and Z_k are binary factors influencing the wages of household members j and k, respectively. In this case, IAM requires that one person always values leisure more than the other: either $\mathrm{P}(\mu_j \geq \mu_k) = 1$ or $\mathrm{P}(\mu_j \leq \mu_k) = 1$.

Example (Oligopoly). Suppose that Z_j and Z_k are binary productivity shocks to firms j and k, respectively. If the coefficients λ_{j1} and λ_{k1} are constant within and across markets, then IAM implies that $\delta_j + b\theta_j$ is always greater than (or always less than) $\delta_k + b\theta_k$. To interpret this statement, recall that δ_j and δ_k are the slopes of firms' marginal cost curves, and $b\theta_j$ and $b\theta_k$ are the (conjectured) indirect effects of firms' actions on the market price. Unless the indices j and k are chosen to satisfy this restriction, it is hard to justify in practice.

Figure 1. Illustration of IAM Conditions for Two Instruments



Case 1. Binary Instruments (Z_j, Z_k) *Case 2.* Continuous Instruments (Z_j, Z_k)

Notes. These plots display feasible regions of the vector $(\Delta \bar{Y}_{-i}/\Delta Z_j, \Delta \bar{Y}_{-i}/\Delta Z_k)$ under Assumption IAM.

If the instruments Z_j and Z_k are continuous, then IAM imposes even stronger restrictions.

Lemma 4. When $N = 3$ and (Z_j, Z_k) are continuous, Assumption IAM holds if and only if:

(i) $\mathrm{P}(\gamma_\ell \geq 0) = 1$ or $\mathrm{P}(\gamma_\ell \leq 0) = 1$, for $\ell \in \{j, k\}$.

(ii) $\mathrm{P}\left(\frac{1+\frac{1}{2}\beta_j}{1+\frac{1}{2}\beta_k} \geq \left|\frac{\gamma_j}{\gamma_k}\right|\right) = 1$ or $\mathrm{P}\left(\frac{1+\frac{1}{2}\beta_j}{1+\frac{1}{2}\beta_k} \leq \left|\frac{\gamma_j}{\gamma_k}\right|\right) = 1$.

Examples. For the peer effects example where $\gamma_j = \gamma_k$, Assumption IAM requires that β_j is a deterministic linear function of β_k, such that $\beta_j = 2(a-1) + a\beta_k$ for $a \in \mathbb{R}$. For a household labor supply model where the wages (W_j, W_k) are used as instruments, this assumption requires that household member j and k's preferences over leisure and consumption are deterministic functions of one another, where $\frac{2-\mu_j}{1-\mu_j} = a \times \frac{2-\mu_k}{1-\mu_k}$. Finally, for an oligopoly model with $\lambda_{j1} = \lambda_{k1}$, it implies that $(\delta_j + b\theta_j) = a \times (\delta_k + b\theta_k) + 1.5b(a-1)$. We are not aware

roles, such as "teacher and student" or "parent and child", then this relabeling approach will not be feasible.

of any meaningful justification for these restrictions. So, for any model with heterogeneous effects and two continuous instruments, IAM would be particularly difficult to rationalize.[23]

Alternative Conditions for Positive Weights

In order to overcome the economic restrictions implied by Imbens-Angrist monotonicity, we propose an alternative assumption, which is sufficient for the IV estimand to be a positively-weighted average of interaction effects. Specifically, under a testable condition on the instrument correlation structure, we can relax IAM by imposing a weaker form of monotonicity.

Assumption PM (Partial Monotonicity)**.** For any $j \neq i$ and any (z_j, z_{-j}) and (z_j', z_{-j}) in the support of Z, either $\mathrm{P}\left(\bar{Y}_{-i}(z_j, z_{-j}) \geq \bar{Y}_{-i}(z_j', z_{-j})\right) = 1$ or $\mathrm{P}\left(\bar{Y}_{-i}(z_j, z_{-j}) \leq \bar{Y}_{-i}(z_j', z_{-j})\right) = 1$.

This form of monotonicity is studied by Mogstad et al. (2021) as an alternative to the Imbens-Angrist condition. It requires that monotonicity holds separately for each instrument instead of for the entire instrument vector. If there is only one instrument, then both assumptions are the same. If there are multiple instruments, then PM is weaker than IAM.

To see what PM implies about the structural parameters, consider the following lemma.

Lemma 5. Assumption PM holds if and only if $\mathrm{P}(\gamma_j \geq 0) = 1$ or $\mathrm{P}(\gamma_j \leq 0) = 1$ for all $j \neq i$.

This result is perhaps surprising given the complex nature of the model. It reveals that PM imposes no restrictions on the interaction effects. Instead, it only requires that the random coefficient γ_j on each instrument Z_j, where $j \neq i$, retains the same sign across all groups.

We now introduce a testable condition that restricts the correlation structure of Z. This condition places a bound the covariances of the instruments $\{Z_j\}_{j \neq i}$ in relation to the average reduced form effects $\{\mathrm{E}(\Delta \bar{Y}_{-i}/\Delta Z_j)\}_{j \neq i}$, which are point identified from OLS regressions.

Assumption NNW *(No Negative Weights)***.** Fix some $z_i \in \mathrm{supp}(Z_i)$. For any $j, k \in \mathcal{N} \setminus i$:

$$\mathrm{Cov}(Z_j, Z_k | z_i) \notin \left(- \sum_{\ell \notin \{i,j\}} \frac{\mathrm{E}(\Delta \bar{Y}_{-i}/\Delta Z_\ell)}{\mathrm{E}(\Delta \bar{Y}_{-i}/\Delta Z_j)} \mathrm{Cov}(Z_\ell, Z_k | z_i), - \sum_{\ell \notin \{i,k\}} \frac{\mathrm{E}(\Delta \bar{Y}_{-i}/\Delta Z_\ell)}{\mathrm{E}(\Delta \bar{Y}_{-i}/\Delta Z_k)} \mathrm{Cov}(Z_\ell, Z_j | z_i) \right).$$

This assumption holds if all the instruments Z_{-i} are uncorrelated. Also, if the components of γ_{-i} share the same sign, then it holds when no two instruments are negatively correlated.

Lemma 6. Assumption NNW is satisfied if either: *(1)* $\mathrm{Cov}(Z_j, Z_k | z_i) = 0$ for all $j, k \in \mathcal{N} \setminus i$ or if *(2)* both $\mathrm{Cov}(Z_j, Z_k | z_i) \geq 0$ for all $j, k \in \mathcal{N} \setminus i$ and $\mathrm{P}(\gamma_{-i} \geq 0) = 1$ or $\mathrm{P}(\gamma_{-i} \leq 0) = 1$.

Note that NNW can be tested empirically as the terms in this condition are identified in the data. So, one can assess whether this restriction holds without making economic arguments.

[23] Even if the instrument $\tilde{Z}_{-i}$ is a non-invertible function of (Z_j, Z_k), Assumption IAM is often still highly restrictive. For example, in the case where $\tilde{Z}_{-i}$ is a linear combination of Z_j and Z_k, the restrictions implied by Lemmas 3 and 4 are similar, if not unchanged. Moreover, even if we were to use only one instrument, setting $\tilde{Z}_{-i} = Z_j$, the restrictions on the interaction effects do not go away unless Z_j and Z_k are uncorrelated.

Suppose that we use a combination of the variables in Z_{-i} as our excluded instrument. Then PM and NNW ensure that the estimand $\beta_i^{\text{IV}}(z_i)$ is a positively-weighted average of β_i's.

Proposition 3. Choose $\tilde{Z}_{-i} \subseteq Z_{-i}$, and suppose that Assumptions PM and NNW both hold. Then the IV estimand is a positively-weighted average of instrument-specific IV estimands:

$$\beta_i^{\text{IV}}(z_i) = \sum_{j \neq i} \omega_j \times \frac{\text{Cov}(Y_i, Z_j | z_i)}{\text{Cov}(\bar{Y}_{-i}, Z_j | z_i)}, \quad \text{where: } \sum_{j \neq i} w_j = 1 \text{ and } w_j \geq 0, \ \forall j \neq i.$$

Additionally, the IV estimand represents a positively-weighted average of interaction effects:

$$\beta_i^{\text{IV}}(z_i) = \int_{\text{supp}(\beta_i)} \beta_i \times \omega(\beta_i | z_i) d\beta_i, \quad \text{where: } \int \omega(\beta_i | z_i) d\beta_i = 1 \text{ and } \omega(\beta_i | z_i) \geq 0, \ \forall \beta_i.$$

From this proposition, we also derive a corollary that applies for any type of instrument $\tilde{Z}_{-i}$.

Corollary 1. For any choice of $\tilde{Z}_{-i}$, the IV estimand is a positively-weighted average of β_i if:

(i) $\text{P}(\gamma_{-i} \geq 0) = 1$ or $\text{P}(\gamma_{-i} \leq 0) = 1$.

(ii) $\text{corr}(Z_j, Z_k | z_i) \geq 0$, for any $j, k \neq i$.

To interpret these results, we now reconsider the special case of the model with three agents.

Groups of Three Agents ($N = 3$)

When there are three agents $i, j, k \in \{1, 2, 3\}$, Assumption PM requires that γ_j and γ_k retain the same signs across all peer groups, and Assumption NNW simplifies in the following way:

$$\text{Cov}(Z_j, Z_k | z_i) \notin \left(-\frac{\text{E}(\Delta \bar{Y}_{-i} / \Delta Z_j)}{\text{E}(\Delta \bar{Y}_{-i} / \Delta Z_k)} \text{Var}(Z_j | z_i), -\frac{\text{E}(\Delta \bar{Y}_{-i} / \Delta Z_k)}{\text{E}(\Delta \bar{Y}_{-i} / \Delta Z_j)} \text{Var}(Z_k | z_i) \right). \tag{21}$$

Example (Peer Effects). First, consider a peer effects model where Z_j and Z_k are factors that raise the achievement of students j and k, respectively. If these factors are not negatively correlated, then the IV estimand $\beta_i^{\text{IV}}(z_i)$ is a causal parameter. It measures the average peer effect β_i in groups where the mean performance of students j and k is most affected by $\tilde{Z}_{-i}$.

Example (Household Labor Supply). Suppose that Z_j and Z_k are the wages of household members j and k, respectively. If these wages are not negatively correlated, then $\beta_i^{\text{IV}}(z_i)$ represents the average value of μ_i in households where the earnings of j and k are most improved by $\tilde{Z}_{-i}$, i.e., where j and k are least inclined to reduce their labor when wages rise.

Example (Oligopoly). Suppose that Z_j and Z_k are positive productivity shocks that are experienced by firms j and k, respectively. As long as these two shocks are not negatively correlated, the parameter $\beta_i^{\text{IV}}(z_i)$ is causal. It measures the average conduct parameter of firm i in markets where the mean output of firms j and k is most responsive to the instrument $\tilde{Z}_{-i}$.

Conditional IV Estimation Using One Instrument

In cases where Assumptions NNW and PM fail to hold, an alternative IV specification may still recover a positively-weighted average of interaction effects. Consider the IV estimand:

$$\beta_i^{IV}(z_{-j}) = \frac{\mathrm{Cov}(Y_i, Z_j | Z_{-j} = z_{-j})}{\mathrm{Cov}(\bar{Y}_{-i}, Z_j | Z_{-j} = z_{-j})} \tag{22}$$

This estimand uses only one instrument Z_j, while controlling for all other instruments Z_{-j}. To interpret this estimand as a positively-weighted average of the interaction effects, we only require that $\Delta\bar{Y}_{-i}/\Delta Z_j$ has the same sign across all networks. This monotonicity condition imposes the same parametric restriction as in the $N = 2$ case. In particular, $\beta_i^{IV}(z_{-j})$ equals a positively-weighted average of β_i-values if and only if $\mathrm{P}(\gamma_j \geq 0) = 1$ or $\mathrm{P}(\gamma_j \leq 0) = 1$.[24] Higher weights are put on β_i-values in groups where $\bar{Y}_{-i}$ is more affected by the instrument Z_j.

V. Learning about Peer Effects and Multipliers under Heterogeneous Effects

In this section, we show how to use OLS and IV regressions to learn about endogenous interaction effects and social multipliers in the linear-in-means model with heterogeneous effects.

V.A. Using IV to Bound Average Peer Effects

First, we show how the IV estimand compares to an unweighted average of interaction effects. The following proposition demonstrates that this relationship is governed by the parameter ψ_i, which is defined in equation (9). This parameter has an important economic interpretation: it determines how an agent i's outcome Y_i affects the average outcome $\bar{Y}_{-i}$ of i's peers.

Proposition 4. Let $\beta_i^{\mathrm{IV}}(z_i)$ be a positively-weighted average of β_i and $\mathrm{E}(\beta_i|\beta_{-i}, \gamma_{-i}) = \mathrm{E}(\beta_i)$.

(*i*) If $\psi_i > 0$ with probability 1, then $\beta_i^{\mathrm{IV}}(z_i) > \mathrm{E}(\beta_i)$.

(*ii*) If $\psi_i < 0$ with probability 1, then $\beta_i^{\mathrm{IV}}(z_i) < \mathrm{E}(\beta_i)$.

There are notable examples where the sign of ψ_i can be easily determined. Specifically, under Assumption IV, $\psi_i > 0$ when $\beta_j > 0$ for all $j \neq i$ and $\psi_i < 0$ when $\beta_j < 0$ for all $j \neq i$.[25]

Examples. Suppose that all the interaction effects share the same sign. Then the IV estimand overstates the magnitude of $\mathrm{E}(\beta_i)$ for any agent i. Namely, for a peer effects model with positive social interactions, IV would overestimate the average peer effect. For a household labor supply model, it would overestimate the average added earner effect. Finally, for an oligopoly model, it would overestimate the average conduct parameter in the market.

Remark. While these examples may suggest that IV generally overestimates the magnitude of $\mathrm{E}(\beta_i)$, there are also notable exceptions. For example, consider a peer effects model

[24] Averaging over Z_{-j}, we can also define the following IV estimand $\beta_i^{IV} = \int \beta_i^{IV}(z_{-j}) f_{Z_{-j}}(z_{-j}) dz_{-j}$.

[25] In the Appendix, we show how this result extends to cases where β_i and β_{-i} are statistically dependent.

where $\beta_i < 0$ and $\beta_j > 0$ for every $j \neq i$. In this setting, everyone seeks to conform to the average action in the group, except for person i, who wishes to deviate. Since ψ_i is less than zero in this case, Proposition 4 shows that $\beta_i^{\mathrm{IV}}(z_i)$ would understate the magnitude of $\mathrm{E}(\beta_i)$.

Pairs of Agents $(N = 2)$

For two-agent groups, we draw comparisons to the mean with the following decomposition.

$$\beta_i^{IV}(z_i) = \mathrm{E}(\beta_i) + \frac{\mathrm{Cov}[\beta_i, \gamma_j/(1-\beta_1\beta_2)]}{\mathrm{E}[\gamma_j/(1-\beta_1\beta_2)]}. \tag{23}$$

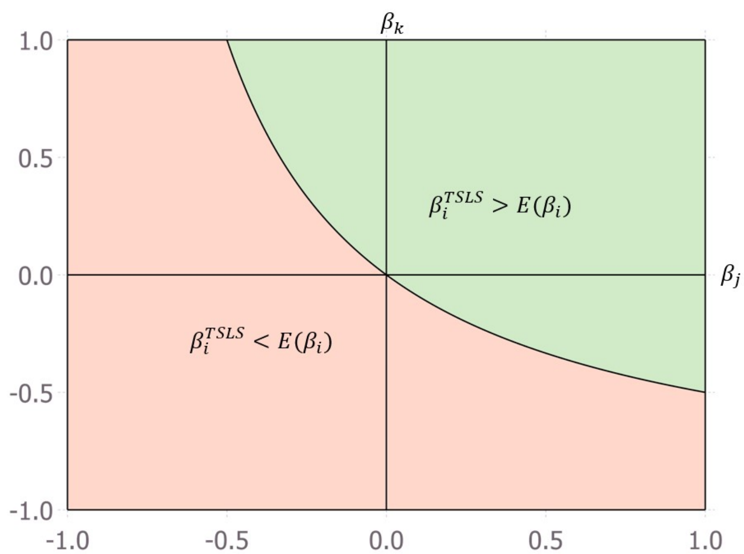
If β_i is mean independent of (β_j, γ_j), then the relationship between $\beta_i^{IV}(z_i)$ and $\mathrm{E}(\beta_i)$ is fully governed by agent j's interaction effect β_j. In particular, *(i)* and *(ii)* in Proposition 4 become:

(i) If $\beta_j > 0$ with probability 1, then $\beta_i^{IV}(z_i) > \mathrm{E}(\beta_i)$.

(ii) If $\beta_j < 0$ with probability 1, then $\beta_i^{IV}(z_i) < \mathrm{E}(\beta_i)$.

One implication of Proposition 4 is that, if β_1 and β_2 have the same sign within and across groups, then IV necessarily overstates the magnitudes of $\mathrm{E}(\beta_1)$ and $\mathrm{E}(\beta_2)$. Alternatively, if β_1 and β_2 always have opposite signs, then IV understates the magnitudes of $\mathrm{E}(\beta_1)$ and $\mathrm{E}(\beta_2)$.

Figure 2. Cases Where $\beta_i^{IV}(z_i) > \mathrm{E}(\beta_i)$ for Three-Agent Groups



Notes. This figure depicts values of (β_j, β_k) where the IV estimand overstates the average interaction effect.

Groups of Three Agents $(N = 3)$

Suppose that each group contains three agents. Then, *(i)* and *(ii)* in Theorem 2 reduce to:

(i) If $\beta_j + \beta_k + \beta_j\beta_k > 0$ with probability 1, then $\beta_i^{\mathrm{IV}}(z_i) > \mathrm{E}(\beta_i)$.

(ii) If $\beta_j + \beta_k + \beta_j\beta_k < 0$ with probability 1, then $\beta_i^{\mathrm{IV}}(z_i) < \mathrm{E}(\beta_i)$.

In Figure 2, we plot the settings where the sum $\beta_j + \beta_k + \beta_j\beta_k$ is positive. If β_j and β_k share the same sign, then the relationship between $\beta_i^{\text{IV}}(z_i)$ and $\mathrm{E}(\beta_i)$ is unambiguous. Alternatively, if these interaction effects have different signs, then it is harder to compare $\beta_i^{\text{IV}}(z_i)$ with $\mathrm{E}(\beta_i)$.

V.B. Using OLS to Test for Peer Effects and Multipliers

We now demonstrate how to use OLS regressions to test for the presence of social multipliers and endogenous interaction effects, as well as to learn about the signs and magnitudes of these interaction effects under heterogeneous effects. Our tests will utilize the average equilibrium quantities $\{\mathrm{E}(\Delta Y_j/\Delta Z_i)\}_{i,j}$, $\{\mathrm{E}(\Delta \bar{Y}/\Delta Z_i)\}_i$, and $\{\mathrm{E}(\Delta \bar{Y}_{-i}/\Delta Z_i)\}_i$, all of which are are point identified from correctly specified OLS regressions, following the discussion in Section IV.A.

Before presenting our tests, we first establish the following proposition, which shows how endogenous interaction effects and social multipliers relate to various reduced form quantities.

Proposition 5. Suppose $\gamma_i > 0$. Then, under Assumptions I-IV, the following results hold:

(a) $M_{(i)}^{\text{heterog.}} - 1$ has the same sign as $\Delta \bar{Y}_{-i}/\Delta Z_i$.

(b) $M^{\text{heterog.}} - 1$ has the same sign as $\sum_{i=1}^{N} \Delta \bar{Y}_{-i}/\Delta Z_i$.

(c) β_j has the same sign as $\Delta Y_j/\Delta Z_i$.

(d) If $\beta_j, \beta_k \geq 0$ or $\beta_j, \beta_k \leq 0$, then $\beta_j - \beta_k$ has the same sign as $\Delta Y_j/\Delta Z_i - \Delta Y_k/\Delta Z_i$.

We will draw on the results presented in Proposition 5 throughout our subsequent analysis.

Testing for Social Multipliers

We begin by showing how to use OLS estimands to draw inference about individual-specific social multipliers $M_{(i)}^{\text{heterog.}}$ and aggregate social multipliers $M^{\text{heterog.}}$, which are both defined in Table 1. If these multipliers are greater (less) than one, then it would suggest that spillover effects amplify (suppress) the impact of individual shocks on the average outcome in a group.

To learn about the social multipliers, we analyze the equilibrium effects $Y_{-i}/\Delta Z_i$, which represent spillover effects of Z_i on agent i's peers. By Proposition 5, we can assess whether social multipliers are greater (less) than one by evaluating the signs of these reduced form quantities. Although we are unable to compute $\{\Delta \bar{Y}_{-i}/\Delta Z_i\}_{i=1}^{J}$ within every group, we can estimate the average reduced form effects $\{\mathrm{E}(\Delta \bar{Y}_{-i}/\Delta Z_i)\}_{i=1}^{J}$ using OLS regressions. With these estimates, we can test whether social multipliers are greater than or less than one for a subset of groups in the population, providing insight into the role of network spillovers. For example, a rejection of the null $H_0 : \mathrm{E}(\Delta \bar{Y}_{-i}/\Delta Z_i) \leq 0$ implies that $\mathrm{P}(M_{(i)}^{\text{heterog.}} > 1) > 0$.

Testing for Positive Interaction Effects

Next, we show how to test for positive (or negative) interaction effects among agents in the population. Recall that positive interaction effects indicate strategic complementarity, which is consistent with classical peer effects, but is inconsistent with household labor supply and oligopoly. In contrast, negative interaction effects indicate strategic substitutability, which is consistent with household labor supply and oligopoly, but not with classical peer effects.

By Proposition 5, the sign of the interaction effect β_j can be inferred from the individual spillover effect $\Delta Y_j/\Delta Z_i$, provided the sign of γ_i is known. Specifically, for two agents i and j where $\gamma_i > 0$, the interaction effect β_j of agent i always shares the same sign as $\Delta Y_j/\Delta Z_i$.

By this property, we can construct a test for the existence of positive interaction effects from OLS regressions. In particular, if we assume that $\mathrm{P}(\gamma_j \geq 0) = 1$, then we can assess whether $\beta_i > 0$ with positive probability by testing the null hypothesis $H_0 : \mathrm{E}(\Delta Y_i/\Delta Z_j) \leq 0$.

In some cases, it may not be feasible to regress the outcomes Y_i on the entire vector Z. Moreover, if the interaction effects are heterogeneous within groups, then using an alternative regression based on averages of $\{Z_j\}_j$ introduces omitted variable bias. This bias confounds our ability to recover the average individual spillover effects $\mathrm{E}(\Delta Y_i/\Delta Z_j)$, which prevents us from conducting the tests outlined above. Fortunately, we can still test for the presence of endogenous interaction effects even when running a correctly specified regression is infeasible.

Lemma 7. Define $\beta^{\mathrm{OLS}}_{Y_i,\bar{Z}_{-i}}$ to be the coefficient on $\bar{Z}_{-i}$ in an OLS regression of Y_i on $(1, Z_i, \bar{Z}_{-i})$. If this estimand is nonzero, then the interaction effect β_i is nonzero with positive probability.

Lemma 7 provides a way to test for endogenous interaction effects, even in the presence of heterogeneous effects, using an OLS regression of Y_i on $(1, Z_i, \bar{Z}_{-i})$. However, it is important to note that this regression does not allow us to determine the sign of the interaction effects.

Testing for the Relative Strengths of Interaction Effects

We can also use OLS to test for the relative strengths of interaction effects. Specifically, for two distinct agents j and k in the group, we may want to empirically assess whether $\beta_j \geq \beta_k$. For example, do female or male students face more social pressure? Do husbands or wives exhibit higher second earner effects? What types of firms have larger conduct parameters?

To conduct this test, we draw on Proposition 5. If β_j and β_k share the same sign and if $\gamma_i > 0$, then difference between agents' interaction effects, $\beta_j - \beta_k$, always has the same sign as the difference in individual spillover effects, $\Delta Y_j/\Delta Z_i - \Delta Y_k/\Delta Z_i$ for any third agent $i \notin \{j, k\}$. Under a monotonicity assumption, $\mathrm{P}(\gamma_i \geq 0) = 1$, we can assess whether $\beta_i > \beta_j$ with positive probability by testing the null hypothesis $H_0 : \mathrm{E}(\Delta Y_i/\Delta Z_k) \leq \mathrm{E}(\Delta Y_j/\Delta Z_k)$.

Testing for Bounded Spillovers

Using OLS regressions, we can also test Assumption IV, which states that $\beta_i \in (1-N, 1)$ for every agent i. One consequence of this assumption is that $\Delta\bar{Y}/\Delta Z_i$ has the same sign as γ_i. Using this property, we can test $\mathrm{P}(1-N < \beta_i < 1) = 1$ through the null $H_0 : \mathrm{E}(\Delta\bar{Y}/\Delta Z_i) > 0$ as long as we maintain a monotonicity assumption that $\mathrm{P}(\gamma_i \geq 0) = 1$. Rejecting this test means that the spillovers are unbounded, which suggests that the model is likely misspecified.

VI. Reanalyzing the Empirical Applications under Heterogeneous Effects

We now apply our results to the two applications, analyzing peer effects in Kenyan primary schools and competition among cocoa traders in Sierra Leone under heterogeneous effects.

VI.A. Classroom Peer Effects in Kenya

We reanalyze the peer effects application (Duflo et al., 2011) under the linear-in-means model with heterogeneous effects. To do so, we make two observations about the empirical setting. First, it is plausible that $\mathrm{P}(\gamma_i \geq 0) = 1$ for all i, given that students' baseline test scores are likely to have a positive impact on their endline test scores. This suggests that Assumption PM is satisfied in this setting. Second, under the experimental protocols, students' baseline scores $\{Z_j\}_{j=1}^{N}$ are uncorrelated with one another after controlling for the school that students attend. Therefore, it is also plausible that Assumption NNW is satisfied in this setting.

Analyzing OLS Estimates

In the first three columns of Table 2, Panel A, we provide estimates from an OLS regression of Y_i on $(1, Z_i, \bar{Z}_{-i})$ with school fixed effects. Under heterogeneous effects, the coefficient on Z_i in these regressions represents the average total individual effect $\mathrm{E}(\Delta Y_i/\Delta Z_i)$ of students' own baseline test scores Z_i on their own endline test scores Y_i. We estimate that, on average, scoring 1 point higher on the baseline test leads students to score 0.5 points higher on the endline test. This estimate accounts for spillover effects that take place within the classroom.

In this OLS regression, the coefficient on peers' mean baseline score $\bar{Z}_{-i}$ is estimated to be positive and significant. By Lemma 7, this result allows us to infer that peer effects do exist in at least some of the classrooms in the study. Note that, under heterogeneous effects, we cannot use this estimate to infer the sign of these peer effects, even while they do exist.[26]

Analyzing IV Estimates

In the first three columns of Panel B, we present estimates from the main IV specification in Duflo et al. (2011). We estimate a significant, positive IV coefficient of approximately 0.45. Under homogeneous effects, this estimate corresponds to the constant peer effect β. However, in the presence of heterogeneous effects, it gives a weighted average of β_i across students.

Since Assumptions PM and NNW are both plausible in this setting, Proposition 3 tells us that the IV estimand is a causal parameter, representing a positively-weighted average of peer effects among students.[27] Moreover, if we also assume that no students exhibit negative peer effects, i.e., if $\mathrm{P}(\beta_i \geq 0) = 1$ for all i, then Proposition 4 tells us that this IV estimand

[26] In this application, it is infeasible to regress the outcomes Y on the entire vector $\tilde{Z} = (1, Z')'$ as it requires labeling each student i in a way that is consistent across classrooms. This task may be straightforward in certain applications, e.g., when studying labor supply in two-person households where there is always one primary earner. However, it is impractical in other cases where the number and composition of agents in a group varies. Also, when N is large, there could be more parameters to estimate than there are observations.

[27] Specifically, this IV estimand places larger weights on students for which $\bar{Y}_{-i}$ is more responsive to $\bar{Z}_{-i}$.

represents an upper bound on the average peer effect $\mathrm{E}(\beta_i)$ among students. We estimate that, on average, a 1 point increase in the average test score of one's peers does not directly influence a student's own test score by more than 0.45 points. As this upper bound is large, this would also suggest that peer effects may have a significant influence on student behavior.

Testing for Social Multipliers

In the peer effects application, we find evidence that social multipliers are greater than one. The last three columns of Table 2, Panel A, present estimates from OLS regressions of peers' mean endline score $\bar{Y}_{-i}$ on own baseline score Z_i and peers' mean baseline score $\bar{Z}_{-i}$. The coefficient on Z_i in these regressions corresponds to $\mathrm{E}(\Delta\bar{Y}_{-i}/\Delta Z_i)$, representing the average equilibrium effect of Z_i on $\bar{Y}_{-i}$ in the population.[28] We find that this coefficient is positive and statistically significant at the 5% level. This finding would indicate that factors influencing individual student achievement are amplified within a classroom through social interactions.

VI.B. Strategic Pricing Decisions in Sierra Leone

We now reanalyze competition among cocoa traders in Sierra Leone (Casaburi & Reed, 2022) under the linear-in-means model with heterogeneous effects. In order to interpret the results, we make two observations. First, in this application the coefficient γ_i is proportional to the subsidy s, which is uniform across traders within and across markets. So, partial monotonicity (PM) is plausible in this case. Second, the experimental design ensures that treatment statuses $\{Z_j\}_{j=1}^N$ are mutually uncorrelated. Thus, Assumption NNW is expected to apply.

Analyzing OLS Estimates

In the first two columns of Table 3, Panel A, we provide estimates from OLS regressions of Q_i on $(1, Z_i, \sum_{j\neq i} Z_j)$ with and without trader controls. Under heterogeneous effects, the coefficient on Z_i in these regressions represents the average total individual effect $\mathrm{E}(\Delta Q_i/\Delta Z_i)$ of receiving a subsidy on a trader's purchases. We estimate that, on average, a subsidy leads a trader to buy about 400 more pounds cocoa from farmers, after accounting for spillovers.

In this OLS regression, the coefficient on the number of treated competitors $\sum_{j\neq i} Z_j$ is estimated to be negative and significant. By Lemma 7, this result allows us to conclude that the conduct parameters $\theta_i/2\kappa_i$ are nonzero with positive probability. Therefore, at least some traders exhibit strategic interactions, which implies that markets are imperfectly competitive.

Analyzing IV Estimates

In the first two columns in Panel B, we provide estimates from the IV regressions of Q_i on $\sum_{j\neq i} Q_j$, instrumented by $\sum_{j\neq i} Z_j$. After including trader controls, we estimate a significant, negative IV estimand of -0.02. Under homogeneous effects, this estimand would correspond

[28]Under heterogeneous effects within classrooms, this regression is misspecified, as shown in Section IV.A. Nevertheless, our interpretation of the estimand is unchanged as long as students' baseline scores $\{Z_j\}_{j=1}^N$ are mutually uncorrelated after controlling for schools. This condition is valid under the experimental protocols.

to the constant conduct parameter $\theta/2\kappa$ exhibited by traders. However, under heterogeneous interaction effects, it represents a weighted average of conduct parameters among traders.

Since Assumptions PM and NNW are plausible in this environment, we conclude from Proposition 3 that the IV estimand is a causal parameter, representing a positively-weighted average of conduct parameters.[29] Moreover, as the conduct parameters $\theta_i/2\kappa_i$ are positive by construction, the IV estimand gives an upper bound on the average conduct parameter $\mathrm{E}(\theta_i/2\kappa_i)$ among traders. We find that, on average, raising a competitors' cocoa purchases by 1 pound does not directly reduce a trader's own purchases by more than 0.02 pounds. This upper bound is small, implying that strategic interactions are limited in this setting.

Testing for Social Multipliers

In this application, we find no strong evidence of multiplier effects. The last two columns in Table 3, Panel A, show OLS estimates from regressing competitors' total quantity $\sum_{j\neq i} Q_j$ on own treatment status Z_i and total number of treated competitors $\sum_{j\neq i} Z_j$. The coefficient on Z_i corresponds to $\mathrm{E}\left(\Delta\left(\sum_{j\neq i} Q_j\right)/\Delta Z_i\right)$, which measures the average equilibrium effect of one trader i's treatment status on the total quantity of his or her competitors.[30] This coefficient estimate is small and statistically insignificant, indicating that there is no social multiplier in this setting. We therefore conclude that the strategic interactions have little to no material impact on how changes in traders' demand or costs affect overall market output.

VII. Conclusion

We analyzed a general class of linear simultaneous equations models where agents are influenced by the average outcome of their peers. Our framework nests the classical linear-in-means model (Manski, 1993). Moreover, we extended the model to allow for both positive and negative interaction effects that differ within and across groups. We showed that the assumption of uniform interaction effects significantly limits the scope of economic behavior, making the model unsuitable for many real-world applications. By allowing for heterogeneous effects, we demonstrated that the model can be applied more broadly to study a wide range of network settings, such as joint labor supply decisions within households and strategic interactions between firms. Using the heterogeneous effects framework, we examined what insights are gained from linear peer effects estimators. We found that linear OLS and IV regressions can be used to draw informative inferences about endogenous interaction effects and social multipliers, even while these methods do not yield point identification. We applied our results to two applications from Duflo et al. (2011) and Casaburi & Reed (2022).

[29] Larger weights placed on traders whose competitors' purchases are more responsive to receiving subsidies.

[30] As in the first application, this interpretation requires that $\{Z_j\}_{j=1}^N$ are uncorrelated with one another. This condition is ensured by the experimental protocols, as a trader's treatment status is randomly assigned.

References

Angrist, J. D. (2014): "The Perils of Peer Effects," *Labour Economics*, 30, 98–108.

Angrist, J. D., K. Graddy, and G. W. Imbens (2000): "The Interpretation of Instrumental Variables Estimators in Simultaneous Equations Models with an Application to the Demand for Fish," *The Review of Economic Studies*, 67, 499–527.

Becker, G. S., and K. M. Murphy (2003): *Social Economics*, Cambridge, MA and London, England: Harvard University Press.

Blandhol, C., J. Bonney, M. Mogstad, and A. Torgovitsky (2022): "When is TSLS Actually LATE?" Working Paper 29709, National Bureau of Economic Research.

Blume, L. E., W. A. Brock, S. N. Durlauf, and Y. M. Ioannides (2011): "Identification of Social Interactions," Volume 1 of Handbook of Social Economics: North-Holland, 853–964.

Blume, L. E., W. A. Brock, S. N. Durlauf, and R. Jayaraman (2015): "Linear Social Interactions Models," *Journal of Political Economy*, 123, 444–496.

Boucher, V., M. Rendall, P. Ushchev, and Y. Zenou (2024): "Toward a General Theory of Peer Effects," *Econometrica*, 92, 543–565.

Bramoullé, Y., H. Djebbari, and B. Fortin (2009): "Identification of peer effects through social networks," *Journal of Econometrics*, 150, 41–55.

Bramoullé, Y., R. Kranton, and M. D'Amours (2014): "Strategic Interaction and Networks," *The American Economic Review*, 104, 898–930.

Bresnahan, T. F. (1981): "Duopoly Models with Consistent Conjectures," *The American Economic Review*, 71, 934–945.

Browning, M., P.-A. Chiappori, and Y. Weiss (2014): *Economics of the Family*, Cambridge Surveys of Economic Literature: Cambridge University Press.

Calvó-Armengol, A., E. Patacchini, and Y. Zenou (2009): "Peer Effects and Social Networks in Education," *The Review of Economic Studies*, 76, 1239–1267.

Casaburi, L., and T. Reed (2022): "Using Individual-Level Randomized Treatment to Learn about Market Structure," *American Economic Journal: Applied Economics*, 14, 58–90.

Case, A. (1991): "Spatial Patterns in Household Demand," *Econometrica*, 59, 953–65.

Dahl, G. B., K. V. Løken, and M. Mogstad (2014): "Peer Effects in Program Participation," *American Economic Review*, 104, 2049–74.

Davezies, L., X. D'Haultfoeuille, and D. Fougère (2009): "Identification of Peer Effects Using Group Size Variation," *The Econometrics Journal*, 12, 397–413.

Donni, O., and P.-A. Chiappori (2011): *Household Economic Behaviors*, Chap. Nonunitary Models of Household Behavior: A Survey of the Literature, 1–40: Springer New York.

Duflo, E., P. Dupas, and M. Kremer (2011): "Peer Effects, Teacher Incentives, and the Impact of Tracking: Evidence from a Randomized Evaluation in Kenya," *American Economic Review*, 101, 1739–74.

Edgeworth, F. (1925): *Papers Relating to Political Economy, I.*

Epple, D., and R. Romano (1998): "Competition between Private and Public Schools, Vouchers, and Peer-Group Effects," *American Economic Review*, 88, 33–62.

Fisher, F. M. (1966): *The Identification Problem in Econometrics*: McGraw-Hill.

Glaeser, E. L., J. A. Scheinkman, and B. I. Sacerdote (2003): "The Social Multiplier," *Journal of the European Economic Association*, 1, 345–353.

Goldin, C., and L. F. Katz (2002): "The Power of the Pill: Oral Contraceptives and Women's Career and Marriage Decisions," *Journal of Political Economy*, 110, 730–770.

Guryan, J., K. Kroft, and M. J. Notowidigdo (2009): "Peer Effects in the Workplace: Evidence from Random Groupings in Professional Golf Tournaments," *American Economic Journal: Applied Economics*, 1, 34–68.

Hahn, J. (2001): "Consistent Estimation of the Random Structural Coefficient Distribution from the Linear Simultaneous Equations System," *Economics Letters*, 73, 227–231.

Heckman, J. J. (1978): "Dummy Endogenous Variables in a Simultaneous Equation System," *Econometrica*, 46, 931–959.

Hopenhayn, H., and E. Prescott (1992): "Stochastic Monotonicity and Stationary Distributions for Dynamic Economies," *Econometrica*, 60, 1387–406.

Hurwicz, L. (1950): "Systems with Nonadditive Disturbances," in *Cowles 10* ed. by Koopmans, T. C. Volume 10 of Cowles Commission Monographs, 410–418.

Imbens, G. W., and J. D. Angrist (1994): "Identification and Estimation of Local Average Treatment Effects," *Econometrica*, 62, 467–475.

Jackson, M. O., and Y. Zenou (2015): "Games on Networks," Volume 4 of Handbook of Game Theory with Economic Applications: Elsevier, 95–163.

Kelejian, H. H. (1974): "Random Parameters in a Simultaneous Equation Framework: Identification and Estimation," *Econometrica*, 42, 517–527.

Kline, B., and E. Tamer (2020): "Econometric Analysis of Models with Social Interactions," in *The Econometric Analysis of Network Data* ed. by Graham, B., and de Paula, A.: Academic Press, 149–181.

Lee, L. (2007): "Identification and Estimation of Econometric Models with Group Interactions, Contextual Factors and Fixed Effects," *Journal of Econometrics*, 140, 333–374.

Manski, C. F. (1993): "Identification of Endogenous Social Effects: The Reflection Problem," *The Review of Economic Studies*, 60, 531–542.

——— (2013): "Identification of Treatment Response with Social Interactions," *The Econometrics Journal*, 16, S1–S23.

Masten, M. A. (2017): "Random Coefficients on Endogenous Variables in Simultaneous Equations Models," *The Review of Economic Studies*, 85, 1193–1250.

Mogstad, M., A. Santos, and A. Torgovitsky (2018): "Using Instrumental Variables for Inference about Policy Relevant Treatment Parameters," *Econometrica*, 86, 1589–1619.

Mogstad, M., and A. Torgovitsky (2024): "Instrumental Variables with Unobserved Heterogeneity in Treatment Effects," NBER Working Paper 11.

Mogstad, M., A. Torgovitsky, and C. R. Walters (2021): "The Causal Interpretation of Two-Stage Least Squares with Multiple Instrumental Variables," *American Economic Review*, 111, 3663–98.

Patacchini, E., and Y. Zenou (2009): "Juvenile Delinquency and Conformism," *The Journal of Law, Economics, and Organization*, 28, 1–31.

Paula, A. d. (2017): *Econometrics of Network Models* Volume 1 of Econometric Society Monographs, 268–323: Cambridge University Press.

Perry, M. K. (1982): "Oligopoly and Consistent Conjectural Variations," *The Bell Journal of Economics*, 13, 197–205.

Sacerdote, B. (2001): "Peer Effects with Random Assignment: Results for Dartmouth Roommates*," *The Quarterly Journal of Economics*, 116, 681–704.

——— (2011): "Chapter 4 - Peer Effects in Education: How Might They Work, How Big Are They and How Much Do We Know Thus Far?" Volume 3 of Handbook of the Economics of Education: Elsevier, 249–277.

Tamer, E. (2003): "Incomplete Simultaneous Discrete Response Model with Multiple Equilibria," *The Review of Economic Studies*, 70, 147–165.

Weyl, E. G., and M. Fabinger (2013): "Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition," *Journal of Political Economy*, 121, 528–583.