Multidimensional screening with substitutable attributes*

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Abstract

A principal wants to choose whether to accept or reject an agent. The principal can perform a costly test that measures a combination of the agent's valuable qualities without revealing each quality separately. The agent can present evidence on some qualities but not others. I call the latter qualities talent. Although favorable, evidence can make the principal ascribe the test result to a certain quality, thereby negatively affecting his assessment of the agent's talent. Thus, testing may interfere with the agent's incentives to present evidence. Indeed, when the test is less sensitive to talent than talent is valuable to the principal, a conflict arises between the two evaluation methods: (i) testing and (ii) asking for evidence. The optimal mechanism leads to two types of errors, both favoring high- over low-evidence agents: (i) It accepts without testing some unworthy high-evidence agents, and (ii) it accepts after testing some unworthy medium-evidence agents while rejecting some worthy low-evidence ones.

Keywords: evidence game, signal jamming, manipulation, scoring, testing, under-disclosure, multidimensional screening
JEL classification codes: D82, D83

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1 Introduction

In many environments, a principal (he) needs to evaluate an agent (she) by using a test that measures a combination of the agent's (valuable) qualities—without revealing each quality separately. The agent can present favorable evidence on some of her (verifiable) qualities but has no evidence on other (unverifiable) qualities. Ideally, the principal would like to evaluate the agent in two ways: (i) by testing her and (ii) by having her present evidence. Both the test and evidence provide valuable information to the principal.

In principle, presenting all her evidence to convince the principal of her verifiable qualities is in the agent's best interest. However, by presenting evidence, the agent may affect how the principal interprets the test result. Particularly, evidence on one quality may make the principal ascribe the test result—which measures a combination of the agent's qualities—to that quality, thereby negatively affecting the principal's assessment of the agent's unverifiable qualities. Thus, testing may interfere with the agent's incentives to present evidence.

When does this conflict between the two ways of evaluating the agent arise? When it does, should the principal try to resolve it or should he design incentives that induce the agent to hide evidence? How should he test and evaluate the agent while taking the conflict into account? For example, should he in some cases refrain from testing so as not to interfere with the agent's incentives to present evidence? If so, in which cases? If he mostly values the unverifiable qualities, should he be most skeptical and require a higher test score (i) when the agent presents little evidence (possibly hiding some) or (ii) when she presents a lot of evidence (indicating the test result is to a large extent attributable to the verifiable qualities, which the principal does not value much)? This paper aims to answer such questions.

The conflict can arise in various settings where people are evaluated. A college applicant may downplay her privileged background or how much effort she has exerted to paint her academic performance and standardized test scores as results of her brilliance rather than effort and high-quality education and get admitted by a college that values talent and potential. For example, she can hide her background or how intensively she has studied in the past by (i) overstating the struggles that she has gone through, (ii) not mentioning tutoring or extracurricular activities, (iii) withholding information on her parents' education and professions, or even (iv) hiding her race.[1] A job candidate may hide her privileged background and prior effort to make the employer attribute her achievements and pre-employment test results to talent and hire her. An employee may understate how long she took to complete a task to make the employer attribute her

[1] Indeed, in a 2021 survey, 34% of white Americans admitted to lying about being a racial minority on their college application (see https://www.intelligent.com/34-of-white-college-students-lied-about-their-race-to-improve-chances-of-admission-financial-aid-benefits). 48% of people who lied claimed to be Native American, and 3/4 of those who lied were accepted by the colleges that they lied to.

productivity to ability (i.e., the rate at which her work hours translate into value to the firm) and promote her. This strategy can pay off if promotion decisions rely mostly on the employer's beliefs concerning the employee's ability (because in the higher position, ability is—relative to working long hours—more important than in the current position). A micro theorist on the academic job market may not present some results that she has already derived in order to use them to answer the audience's questions, thereby making the hiring committee attribute her answers to intelligence and ability to think on her feet and hire her.

This way of thinking is so fundamental that children also seem to follow it. Students often eagerly proclaim they have not studied hard for an exam—not only when they have performed poorly but also when they have performed exceptionally well. By stressing their low effort (or even understating it), they may be trying to have their score attributed to their (overstated) brilliance. Effortless perfection (i.e., the need to seem perfect without apparent effort) and hiding one's effort have been documented among university students (Travers et al., 2015; Casale et al., 2016).

Despite how fundamental this way of thinking is, to the best of my knowledge, no previous work has studied the problem of evaluating people when—to affect how a combined signal of their various virtues is interpreted—they can hide evidence that both (i) is in principle favorable to them and (ii) contains useful information for the evaluator. I study the problem in the following setting. In the baseline setting, an agent has a bidimensional type.[2] The first dimension is her *evidence* (e.g., a college or job applicant's socioeconomic background, effort, and training, an employee's effort, a researcher's knowledge) and the second is her *talent* (e.g., a college or job applicant's innate ability, an employee's efficiency or managerial skills, a researcher's ability to think fast).[3] The agent can verifiably disclose any part of her evidence but cannot prove she is not withholding evidence. She cannot unilaterally prove anything about her talent, although she privately observes it.

The value of the agent to the principal is non-decreasing in both her evidence and her talent. The principal ultimately wants to make a binary choice: accept the agent (and receive the value of the agent as payoff) or reject her (and receive payoff 0). He does so by committing to a direct mechanism that, conditional on (i) the evidence presented and (ii) the cheap talk statement made by the agent about her talent, (possibly) tests the agent at a cost and then decides whether to accept her. If performed, the test returns a one-dimensional (deterministic) signal (i.e., the test score) of the agent's bidimensional type. The test score is increasing in both the agent's evidence and talent. The agent

[2] Section 3.5 extends the results to types of any finite dimension.

[3] Although plausibly endogenous in some cases (e.g., when an employer decides whether to promote an employee), I solve the problem for exogenous evidence and then extend the model to allow for endogenous evidence production. Section 5.3 shows that the structure of the optimal mechanism remains qualitatively the same even if evidence is endogenous (i.e., produced by the agent before her interaction with the principal), as long as the principal cannot influence evidence production by committing to a mechanism before the agent produces evidence.

wants to get accepted independently of her type.

If the test score measures exactly what the principal values in an agent (and, thus, the principal wants to accept the agent if and only if her test score is high enough), then the test's usefulness is apparent. But what happens if he values talent to a different degree (relative to evidence) than the test measures talent (relative to evidence)? Or, in economics jargon, what if his marginal rate of substitution between talent and evidence differs from the marginal rate of substitution between the two in the test (i.e., holding fixed the test score)?

The main result concerns the optimal screening mechanism when the test (score) is less sensitive to talent than talent is valuable to the principal. The optimal mechanism features double penalization of low-evidence agents, favoring high-evidence agents in two ways: (i) It accepts some high-evidence agents—including undeserving ones (i.e., who give the principal a negative payoff when accepted)—without testing them but rather only by asking them for a certain threshold level of evidence; and (ii) among agents who do not meet that threshold level of evidence, it accepts (after testing) some undeserving agents with high evidence but low talent while rejecting some deserving agents with high talent but low evidence. Remarkably, this is the structure of the optimal mechanism in the extreme case where the principal *only* values talent (i.e., his payoff for accepting the agent is increasing in talent and constant in evidence).[4] The principal still optimally favors high-evidence agents even though evidence is worthless to him. He does so (i) to save on testing costs by accepting high-evidence agents without testing them and (ii) to accept (after testing) some deserving low-evidence agents by also testing and accepting some undeserving high-evidence agents, who can hide their high evidence to imitate the more talented deserving agents.

I now discuss the results in more detail. A screening mechanism is incentive-compatible if and only if three conditions are satisfied. First, among agents with the same level of evidence, an agent with higher talent should be accepted with (weakly) higher probability (than a less talented one), because she can successfully imitate an agent with (the same amount of evidence but) lower talent, given that the test score is increasing in talent. Second, among agents with the same level of evidence, to accept a talented agent with (strictly) higher probability (than a less talented one), the principal needs to test the talented agent with high enough probability to prevent the less talented one from posing as the more talented agent in the hope that she will be accepted without a test. Third, among agents with the same (potential) test score (i.e., the test score that they will achieve if tested), an agent with higher evidence should be accepted with (weakly) higher probability, because she can hide (part of) her evidence and over-report her talent to imitate an agent with the same test score, lower talent, and higher evidence.

[4]When the principal's payoff for accepting the agent depends only on talent, the test is automatically less sensitive to talent than talent is valuable to the principal.

Given this characterization of incentive-compatible mechanisms, I first study the principal's problem under free testing. If the test is more sensitive to talent than talent is valuable to the principal, testing does not create incentives for the agents to hide evidence, so the principal's problem is easy. By observing the agent's test score *and* evidence—which the agent presents fully—the principal learns the agent's type, thereby achieving the first-best. On the other hand, if the test is less sensitive to talent than talent is valuable to the principal, testing *does* create incentives for the agents to hide evidence. The optimal mechanism makes two types of errors: (i) a Type I error of rejecting some deserving agents with high talent but low evidence to avoid accepting undeserving agents with low talent but high evidence, and (ii) a Type II error of accepting some undeserving agents with high evidence in order to also accept deserving ones with low evidence. The less sensitive the test is to talent, the larger the errors are.

The results capture a stark contrast in the difficulty of hiring different types of employees. When verifiable qualities (which can be proven through hard evidence) are most valuable, the hiring process is easy. On the other hand, when talent—which is assessed by tests that are also sensitive to the candidate's verifiable qualities, which the candidate can hide—is most valuable, the hiring process is flawed, favoring candidates with high-quality training and education at the expense of equally or more valuable candidates with limited training.

Under costly testing, the optimal mechanism is as follows. If the test is less sensitive to talent than talent is valuable to the principal, the principal gives the agent two paths to getting accepted: (i) provide enough evidence to meet a certain threshold or (ii) score high enough in the test without providing evidence.[5] As in the case of free testing, the optimal mechanism makes a Type I and a Type II error. The test score threshold balances these two errors. The evidence threshold captures the trade-off between the benefit of testing (i.e., rejecting some unworthy high-evidence agents) and its cost. As in the case of free testing, if the test is more sensitive to talent than talent is valuable to the principal, testing does not create incentives for the agents to hide evidence. Every agent with at least a certain threshold of evidence is accepted without a test, and among agents who do not meet that threshold, an agent is accepted (after testing) if and only if her value to the principal is high enough to cover the testing cost. High-evidence agents are favored only to the extent that their evidence is high enough for them to be accepted without a test. Among agents who do not have such high levels of evidence, the mechanism accepts every deserving agent—without favoring high- (or low-) evidence agents.

The results have implications for hiring by prestigious employers (or, in general, hiring for highly desirable positions), promotions, and college admissions. Consider, first, hiring by a prestigious employer. Evidence is the candidate's CV quality (e.g., high school quality, undergraduate institution quality and GPA, awards, distinctions, reference letters), and

[5] The first option need not always be provided (e.g., when the testing cost is low enough).

talent is her ability and drive not captured by the evidence. Testing amounts to letting a less prestigious employer hire the candidate—with the option to poach the candidate later at a cost (after observing her performance with that employer). In the optimal mechanism, Ivy-Leaguers are immediately hired by prestigious employers, whereas worthy candidates with less impressive credentials have to go through less prestigious employers to prove their worth before they land a prestigious position. If the candidates' performance in the less prestigious position is less sensitive to talent than talent is valuable in the more prestigious position—an arguably reasonable assumption, worthy candidates with low credentials are at a disadvantage not only in the first stage of hiring by the prestigious employer but also in the poaching stage.

In the context of promotions, evidence can be understood as the employee's effort, and talent can be understood as her efficiency (i.e., the rate at which effort translates into productivity or value to the firm) or managerial skills.[6] Testing amounts to monitoring the employee's productivity. Then, the payoff to the principal from accepting (i.e., promoting) the employee is the difference between her productivity in the new position (if promoted) and her productivity in her current position. The payoff is, as assumed, non-decreasing in effort and efficiency if both effort and efficiency have a (weakly) higher marginal productivity in the higher position. This is indeed the case if the higher position comes with increased responsibilities that allow the employee's effort and talent to have a larger impact. Talent being (relative to effort) more important in the higher position than in the current one is also a natural assumption. Then, the test (i.e., current productivity) is less sensitive to talent than talent is valuable to the employer, which means some hard-working employees are (optimally) promoted—either with or without their productivity monitored—although their promotion destroys firm value. At the same time, some talented but not hard-working employees are not promoted to managerial positions, although their promotion would generate value for the firm.

Lastly, the results have implications for affirmative action in college admissions (i.e., trying to control for applicants' unequal backgrounds). Affirmative action is not very effective if both of the following conditions are satisfied: (i) College applicants can to a large extent hide their privilege and (ii) standardized test scores reflect talent (e.g., relative to socioeconomic background and prior education, training, and test preparation) less than colleges value talent. If both conditions hold, the optimal admissions policy requires roughly the same test score from every applicant for admission—regardless of background. However, if any of the two conditions fails, affirmative action is effective, and we should expect its reversal to significantly reduce diversity in college admissions.

After a discussion of related literature, section 2 presents the model. Section 3 characterizes incentive-compatible mechanisms and then solves the principal's problem.

[6]The employee has chosen effort in a previous stage (see section 5.3 for a discussion of endogenous evidence production) and can show or hide how much effort she has exerted.

Section 4 discusses applications. Section 5 presents extensions of the model. Section 6 concludes. Proofs are gathered in Appendix A.

Related literature. My analysis contributes to the literature of multidimensional screening (see, e.g., Armstrong, 1996; Rochet and Choné, 1998; Rochet and Stole, 2003). Although duality approaches have proven useful in verifying a mechanism's optimality (Rochet and Choné, 1998; Carroll, 2017; Daskalakis et al., 2017; Cai et al., 2019), full characterizations of multidimensional screening problems remain challenging. Partial characterizations have, for example, been obtained (i) for the case where the principal can use costly instruments in screening (Yang, 2022) or (ii) that show when offering menus with specific characteristics are optimal for a multi-product monopolist (Haghpanah and Hartline, 2021; Yang, 2023). I contribute to this literature by proposing a novel multidimensional screening problem and deriving a full characterization under general assumptions. I assume the agent's type admits a full support density but impose no other restrictions on the type distribution. Also, I make no parametric assumptions on the principal's preferences or testing technology; they are only assumed to satisfy a single-crossing condition.[7] My analysis does not rely on ironing procedures (see, e.g., Mussa and Rosen, 1978; Myerson, 1981; Rochet and Choné, 1998) or the duality approach. Instead, I show the principal's problem can be reduced to a maximization problem where the objective is a linear (and thus convex) and continuous functional and the domain is a (convex and compact) space of monotone functions.[8] Bauer's maximum principle then implies an extreme point solves the problem.[9] The proof proceeds using properties of extreme points of spaces of monotone functions. In that sense, my paper is also related to recent papers that characterize extreme points of spaces of monotone functions (see, e.g., Kleiner et al., 2021; Yang and Zentefis, 2024) and then use Bauer's maximum principle.

The paper has links to a few other strands of the literature.

It connects to the literature of persuasion games, where a sender discloses verifiable information (i.e., her type) to a decision-maker to influence his actions.[10] When the sender (e.g., seller) perfectly knows her type and can costlessly and verifiably disclose it to the decision-maker (e.g., buyer), whose payoff is increasing in the type, full unraveling emerges in equilibrium: every sender type (except possibly the lowest one) discloses her quality (Viscusi, 1978; Grossman, 1981; Milgrom, 1981). In my setting, this happens when the principal only values the agent's evidence.

It also has links to the literature on evidence games, where an agent chooses what

[7] They are also assumed to be monotone, but this assumption is not made for tractability. Rather, it captures the main force under study.

[8] For this result, it is important that the principal's (final) choice is binary.

[9] Manelli and Vincent (2007) also use Bauer's maximum principle to study a multi-dimensional screening problem.

[10] See Milgrom (2008) for a review.

part of her verifiable evidence to disclose to the principal without being able to prove whether she has hidden evidence (see, e.g., Shin, 1994; Dziuda, 2011; Hart et al., 2017). This literature several differences to mine. Most importantly, the principal cannot obtain a signal of the agent's type after she provides evidence.[11]

The ability of the principal to perform an experiment after the agent transmits information is, for example, considered in Glazer and Rubinstein (2004), Carroll and Egorov (2019), Bizzotto et al. (2020), Li (2020, 2021), Kattwinkel and Knoepfle (2023). However, in Bizzotto et al. (2020), Li (2020), and Kattwinkel and Knoepfle (2023), the agent's type is one-dimensional. Even in Glazer and Rubinstein (2004), Carroll and Egorov (2019), and Li (2021), where the agent's type is multi-dimensional, testing by the principal produces a signal that does not combine multiple dimensions but rather reveals one of the dimensions. Therefore, the interpretation of the test result is not influenced by the agent's initial disclosure as in my model, where the substitutability between the two dimensions is key.[12]

Nevertheless, the composite signal that the testing generates is not entirely new to the economics literature. It is reminiscent of the signal jamming problem in career concern models (see, e.g., Holmström, 1999). Still, in these models the main force is the agent's incentives to exert effort in order to influence the principal's learning (though costless observation of the agent's productivity) of the agent's talent. Here, I focus on information transmission and testing.[13] I show that if the principal can ask for hard evidence of effort, the signal jamming problem is mitigated if productivity is sensitive enough to talent—compared to the principal's preferences for accepting (e.g., promoting) the agent. However, when productivity is *not* sensitive enough to talent, the signal jamming problem persists even if the principal can ask for evidence of effort. Agents have incentives to withhold evidence, which they should be paid information rents to reveal.

The ability of the agent to influence the informational content of the designer's signal is considered in another strand of the literature. In Perez-Richet and Skreta (2022), Frankel and Kartik (2019, 2022), and Ball (2024), the agent can manipulate the signal at a cost.[14] In my setting, withholding evidence does not manipulate the signal itself but it can affect the informational content of the signal. The amount of evidence that the agent possesses can then be interpreted as the ability of the agent to manipulate the

[11] There are more differences. For example, in Hart et al. (2017) the principal does not always prefer more evidence to less and the agent can choose to withhold damaging evidence. Damaging evidence also exists in Dziuda (2011), where the existence of a behavioral type is central, while in my model all agents are strategic.

[12] Also, in Glazer and Rubinstein (2004) rather than providing verifiable evidence, the agent only sends a cheap talk message. In Carroll and Egorov (2019), the principal has partial commitment power: he can commit on testing but not on action (after testing) decisions. He can commit to an action only to severely punish the agent.

[13] Another difference from career concerns models is that the principal chooses whether to test the agent at a cost. Also, the principal is actually an employer without commitment power.

[14] In Frankel and Kartik (2022), the receiver has no commitment power.

signal. This gaming ability is exogenous and fixed in Perez-Richet and Skreta (2022) but stochastic and privately observed by the agent in Frankel and Kartik (2019, 2022) and Ball (2024), like evidence is in my model. However, in my setting, evidence controls the set of messages that the agent can send rather than how costly it is to send a certain message. Also, unlike in models of costly signal manipulation, there is no pecuniary or direct cost of hiding evidence in my model. Still, there is an *endogenous* cost of hiding evidence given that the principal values evidence—unlike in the other models, where gaming ability is not valuable to the principal—and chooses how to reward it through the mechanism. These differences also set apart my setting from models of costly lying (e.g., Kartik, 2009; Sobel, 2020).

2 A model of multidimensional screening with substitutable attributes

There are an agent (she) and a principal (he). The agent is privately informed of her bidimensional type (e,t), which has a full-support density $f : [0,1]^2 \to \mathbb{R}_{++}$.[15] e is the agent's *evidence*. An agent of type (e,t) can prove to the principal that her e is at least r for any $r \in [0,e]$ by presenting evidence $r \in [0,e]$. If she reveals $r < e$, we say that she hides evidence. However, for no $r \in [0,1)$ can she prove that her e is not higher than r; in other words, she cannot prove that she is not withholding evidence. t is the agent's *talent*, which she cannot unilaterally prove anything about. The principal can test the agent by paying a cost $c \geq 0$.

The testing technology. The test is imperfect and works as follows.[16] Testing the agent amounts to observing a deterministic signal $\sigma(e,t) \in [0,1]$ of the agent's type (e,t). $\sigma : [0,1]^2 \to [0,1]$ is increasing and continuous in e and t. The assumption of a deterministic increasing signal is not uncommon. In fact, it is more general than the assumption that the test reveals one of the dimensions of the agent's type, which is for example made in Glazer and Rubinstein (2004), Carroll and Egorov (2019), and Kattwinkel and Knoepfle (2023).[17]

Payoffs. Ultimately, the principal wants to choose whether to accept or reject the agent. He receives (gross of testing costs) Bernoulli payoff $u(e,t)$ from accepting an agent of type

[15] Section 3.5 extends the results to types of any finite dimension.

[16] Although called a test, this need not be a written test. For example, it can also be the agent's performance in an interview or her productivity as an employee.

[17] That is, if we allow σ to be constant in e or t (but not both). This case is easy to deal with but uninteresting in our setting. If σ is constant in e (and, thus, reveals t exactly), the optimal mechanism is the same as under pro-t biased testing (see section 3). If σ is constant in t (and, thus, reveals e exactly), the test is useless, and there exists an optimal mechanism that only asks for evidence.

(e,t), where $u : [0,1]^2 \to \mathbb{R}$ is non-decreasing and continuous in e and t. If he rejects the agent, he receives payoff normalized to 0. An isocurve of the principal's (gross) payoff is given by $I_u(\overline{u}) := \{(e,t) \in [0,1]^2 : u(e,t) = \overline{u}\}$.[18] Define also the (strict) upper and lower contour sets $I_u^{\uparrow}(\overline{u}) := \{(e,t) \in [0,1]^2 : u(e,t) > \overline{u}\}$ and $I_u^{\downarrow}(\overline{u}) := \{(e,t) \in [0,1]^2 : u(e,t) < \overline{u}\}$, respectively. The agent's Bernoulli payoff is equal to 1 if accepted and 0 if rejected.

Canonical examples. In a linear example, $u(e,t) := \gamma_u e + (1-\gamma_u)t - \underline{q}$, where $\gamma_u \in [0,1]$ measures how much the principal values e versus t, and $\underline{q} \in (0,1)$ measures the threshold quality that the agent needs to have to be of (positive) value to the principal. Similarly, $\sigma(e,t) := \gamma_s e + (1-\gamma_s)t$, where $\gamma_s \in (0,1)$ measures how sensitive the test is to e relative to t. In a Cobb-Douglas specification, $u(e,t) := e^{\gamma_u} t^{1-\gamma_u} - \underline{q}$ and $\sigma(e,t) := e^{\gamma_s} t^{1-\gamma_s}$ with $\gamma_u \in [0,1]$ and $\gamma_s, \underline{q} \in (0,1)$. No parametric assumptions are imposed on u or σ but for simplicity in depiction, all figures use the linear parametrization.

The principal's problem. To decide whether to accept the agent, the principal designs (with commitment) a direct mechanism $M \equiv \langle T,P \rangle$ that specifies (i) the probability $T(e,t) \in [0,1]$ with which the principal will test the agent if she presents evidence e and sends cheap talk message t and (ii) the probability $P(e,t,s)$, which should be non-decreasing in $s \in [0,1]$, with which the principal will accept the agent after the agent has presented evidence e, sent cheap talk message t, and the test has returned result $s \in [0,1]$.[19] If no test is performed, $s = \emptyset$ and the agent is accepted with probability $P(e,t,\emptyset)$. Notice that (e,t) refers to the message sent by the agent. When necessary to avoid confusion, we will denote by (e',t') the agent's message (i.e., evidence e' presented and cheap talk message t' sent) to differentiate it from the agent's type, which in those cases will be denoted by (e,t). Overall, the principal chooses a mechanism $M \equiv \langle T,P \rangle$, where $T : [0,1]^2 \to [0,1]$ and $P : [0,1]^2 \times ([0,1] \cup \{\emptyset\}) \to [0,1]$ with $P(e,t,s)$ non-decreasing in $s \in [0,1]$, and (breaking the agent's indifferences in his favor) an agent response rule $\phi : [0,1]^2 \to [0,1]^2$ to maximize

$$\int_0^1 \int_0^1 \left\{ \underbrace{\begin{bmatrix} T(\phi(e,t))P(\phi(e,t),\sigma(e,t)) \\ +[1-T(\phi(e,t))]P(\phi(e,t),\emptyset) \end{bmatrix}}_{\text{total probability that } (e,t) \text{ is accepted}} u(e,t) - c\underbrace{T(\phi(e,t))}_{\substack{\text{probability} \\ \text{that}(e,t) \\ \text{is tested}}} \right\} f(e,t)\,dt\,de$$

[18] $I_u(\overline{u})$ is assumed to be a curve for any $\overline{u}$. This is the case if, for example, $u(e,t)$ is increasing in e or t.

[19] The condition that $P(e,t,s)$ be non-decreasing in $s \in [0,1]$ can be understood as an incentive-compatibility condition in a model where $\sigma(e,t)$ gives the maximum score that agent type (e,t) can achieve but the agent can intentionally reduce her score.

subject to the agent's incentive compatibility (IC) constraint

$$\phi(e,t) \in \underset{(\widehat{e},\widehat{t})\leq(e,1)}{\arg\max} \underbrace{\left\{T(\widehat{e},\widehat{t})P(\widehat{e},\widehat{t},\sigma(e,t)) + (1-T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t},\emptyset)\right\}}_{\text{total probability that } (e,t) \text{ is accepted if she reports } (\widehat{e},\widehat{t})}.$$

3 Optimal bidimensional screening with substitutable attributes

This section characterizes incentive-compatible (IC) mechanisms and then solves the principal's problem.

3.1 Simplifying the class of mechanisms

Before characterizing IC mechanisms, we show that we can without loss restrict the class of mechanisms that we need to consider.

Truthful mechanisms are without loss. The first simplification comes from the fact that the principal can without loss of optimality restrict attention to truthful mechanisms (i.e., mechanisms that induce truth-telling). To see why, notice that the correspondence $(e,t) \mapsto \{(e',t') \in [0,1]^2 : e' \leq e\}$ (from the type space to the message space) that determines the admissible messages for each agent type (e,t) satisfies the Nested Range Condition (NRC) of Green and Laffont (1986), who show that under this condition, the set of implementable social choice functions coincides with the set of truthfully implementable social choice functions.[20] Therefore, from now on, we restrict attention to truthful mechanisms and define IC mechanisms as follows.

Definition 1. A mechanism $M \equiv \langle T,P\rangle$ is IC if for every $(e,t) \in [0,1]^2$

$$(e,t) \in \underset{(e',t')\in[0,e]\times[0,1]}{\arg\max} \{T(e',t')P(e',t',\sigma(e,t)) + (1-T(e',t'))P(e',t',\emptyset)\}.$$

Pass-or-fail tests are without loss. Next, we can constrain attention to mechanisms with threshold acceptance policies conditional on testing; that is, mechanisms such that

$$P(e,t,s) = \begin{cases} 0 & \text{if } s < \sigma(e,t) \\ P_{at}(e,t) & \text{if } s \geq \sigma(e,t) \end{cases} \tag{1}$$

[20] Essentially, the principal implements a social choice function $g : [0,1]^2 \to [0,1]^2 \times [0,1]^{[0,1]}$, where $g_1(e,t)$ the probability of testing, $g_2(e,t)$ the probability of accepting conditional on not testing, and $g_3(e,t,\cdot)$ a self-map on $[0,1]$ that (conditional on testing) maps the test result s to the probability $g_3(e,t,s)$ of acceptance.

for any (e,t) for some $P_{at} : [0,1]^2 \to [0,1]$, where *at* is a mnemonic for the probability of accepting the agent *after testing* (given that the threshold test score is met). If type (e,t) reports her type truthfully and is tested, she is then accepted with probability $P_{at}(e,t)$. Notice that the threshold is set exactly equal to the test score that a truthfully-reporting agent can achieve. To see why constraining attention to such mechanisms is without loss of optimality, observe that among all mechanisms that (conditional on testing) accept type (e,t) with probability $P_{at}(e,t)$, the one that satisfies equation (1) minimizes incentives of other types to imitate (e,t).[21]

Moreover, agents who meet the test score threshold are accepted with certainty. To see this, notice that the total probability with which agent (e,t) is accepted if she truthfully reports her type is equal to $\Pi(e,t) := (1 - T(e,t))P(e,t,\emptyset) + T(e,t)P_{at}(e,t)$, and define outcome-equivalent mechanisms as follows.

Definition 2. A mechanism $M' \equiv \langle T',P' \rangle$ is outcome-equivalent to a mechanism $M \equiv \langle T,P \rangle$ if for every (e,t), $\Pi(e,t) = \Pi'(e,t)$, where $\Pi(e,t) \equiv (1-T(e,t))P(e,t,\emptyset)+T(e,t)P_{at}(e,t)$ and $\Pi'(e,t) \equiv (1 - T'(e,t))P'(e,t,\emptyset) + T'(e,t)P'_{at}(e,t)$.

Lemma 1 shows that when testing is costly, an agent who is tested and passes the test is accepted with probability 1 in any optimal mechanism. When testing is free, it is still without loss to constrain attention to mechanisms that accept the agent with probability 1 when she passes the test.

Lemma 1. Given any IC mechanism M, there exists an IC mechanism $M' \equiv \langle T',P' \rangle$ with $P'_{at}(e,t) = 1$ for every (e,t) that is outcome-equivalent to M. Also, for $c > 0$, in any optimal mechanism $M \equiv \langle T,P \rangle$, $P_{at}(e,t) = 1$ for any (e,t) such that $T(e,t) > 0$.[22]

The intuition behind this result is as follows. The only reason to test an agent before accepting her—rather than accept her without a test—is to prevent others from imitating her. The total probability with which each agent is accepted is the sum of (i) the probability $(1 - T(e,t))P(e,t,\emptyset)$ of being accepted without getting tested and (ii) the probability $T(e,t)P_{at}(e,t)$ of being accepted after getting tested (and passing the test). Simply put, if the principal pays the cost to test an agent, he may as well assign as large a part as possible of the total probability of accepting her to the case where he accepts her after a test.

Specifically, if agent (e,t) is not accepted with certainty after passing the test (i.e., $P_{at}(e,t) < 1$ and $T(e,t) > 0$), we can (i) increase the probability $P_{at}(e,t)$ with which she is

[21]Namely, accepting the agent with even higher probability for performing above $\sigma(e,t)$ will result in the same probability of accepting type (e,t) after testing her and only give additional incentives to other agents to imitate (e,t). Similarly, there is no reason to accept the agent for test scores lower than $\sigma(e,t)$. Particularly, this argument holds when we compare all mechanisms that test (e,t) with the same probability, and, thus have the same testing costs.

[22]Strictly put, $P_{at}(e,t)$ can be lower than 1 for a zero-measure set of (e,t) with $T(e,t) > 0$.

accepted conditional on getting tested (and passing the test), (ii) decrease the probability $T(e,t)$ with which she is tested, and (iii) decrease (if positive) the probability $P(e,t,\emptyset)$ of accepting her conditional on not testing her, keeping fixed both (a) the probability $(1 - T(e,t))P(e,t,\emptyset)$ of accepting her without testing her and (b) the probability $T(e,t)P_{at}(e,t)$ of accepting her after testing. By doing so, we (i) keep fixed the total probability $\Pi(e,t)$ of accepting (e,t), (ii) do not change the incentives of other types to imitate (e,t), since any agent imitating (e,t) will be accepted with probability $(1 - T(e,t))P(e,t,\emptyset)$ (if she only has at least as much evidence as (e,t) but cannot test as high as her) or $\Pi(e,t)$ (if she can also test as high as (e,t)), and (iii) reduce the probability of testing (e,t), thereby limiting testing costs. Thus, from now on, we constrain attention to mechanisms with $P_{at}(e,t) = 1$ for any (e,t).[23]

Untalented agents do not need to be tested. Lemma 2 shows that we can further simplify the analysis by constraining attention to mechanisms where agents with zero talent are never tested.

Lemma 2. Given any IC mechanism M, there exists an IC mechanism $M' \equiv \langle T',P' \rangle$ with $T'(e,0) = 0$ for every e that is outcome-equivalent to M and has the same (expected) testing cost as M.

Here is the intuition behind this result. The only reason to test an agent before accepting her—rather than accept her without a test—is to prevent others from imitating her. But any agent (e',t') who has sufficient evidence (i.e., $e' \geq e$) to imitate an agent $(e,0)$ with no talent can also score (if tested) at least as high as the untalented agent $(e,0)$, since the test score is increasing in e and t. Therefore, there is no point in testing untalented agents, as doing so does not reduce incentives of others to imitate them.[24]

3.2 Incentive-compatible mechanisms

Given what we have seen, we constrain attention to truthful mechanisms with pass-or-fail tests where untalented agents are never tested. Let $\tau(e,s)$ be implicitly given by $\sigma(e,\tau(e,s)) = s$. $\tau(e,s)$ gives the level of talent that an agent with evidence e should have to achieve test score (exactly) s. $\tau(e,s)$ is well-defined for (e,s) such that $s \in [0,1]$ and $e \in [\underline{e}(s),\overline{e}(s)]$, where $\underline{e}(s) \coloneqq \min\{e \in [0,1] : \sigma(e,1) \geq s\}$ and $\overline{e}(s) \coloneqq \max\{e \in [0,1] : \sigma(e,0) \leq s\}$.[25] Proposition 1 then characterizes IC mechanisms.

[23]For (e,t) with $T(e,t) = 0$, the value of $P_{at}(e,t)$ does not matter, so we can again set $P_{at}(e,t) = 1$ without loss.

[24]Of course, agents with $t = 0$ are a zero-measure set. Thus, even if testing is costly, the principal could optimally test them. However, restricting attention to mechanisms with $T(e,0) = 0$ for every e helps simplify notation.

[25]$\underline{e}(s)$ (resp. $\overline{e}(s)$) is the minimum (resp. maximum) level of evidence that an agent can have while achieving test score (exactly) s. That is, agents with evidence lower than $\underline{e}(s)$ score less than s even if

Proposition 1. A mechanism $M \equiv \langle T,P \rangle$ is IC if and only if

(i) $\Pi(e,t)$ is non-decreasing in t for every $e \in [0,1]$,

(ii) $\Pi(e,\tau(e,s))$ is non-decreasing in e over $e \in [\underline{e}(s),\overline{e}(s)]$ for every $s \in [0,1]$, and

(iii) $(1 - T(e,t))P(e,t,\emptyset) \leq \Pi(e,0)$ for every $(e,t) \in [0,1]^2$,

where $\Pi(e,t) \equiv (1 - T(e,t))P(e,t,\emptyset) + T(e,t)$ is the probability with which agent (e,t) is accepted if she truthfully reports her type.

Figure 1 schematically summarizes IC conditions (i) and (ii) of Proposition 1.

Condition (i) is necessary and sufficient to ensure that an agent (e,t) does not want to reveal her evidence but under-report her talent to imitate agent (e,t') with $t' < t$, pass (e,t')'s test (since the test score is increasing in talent), and get accepted with probability $\Pi(e,t')$.

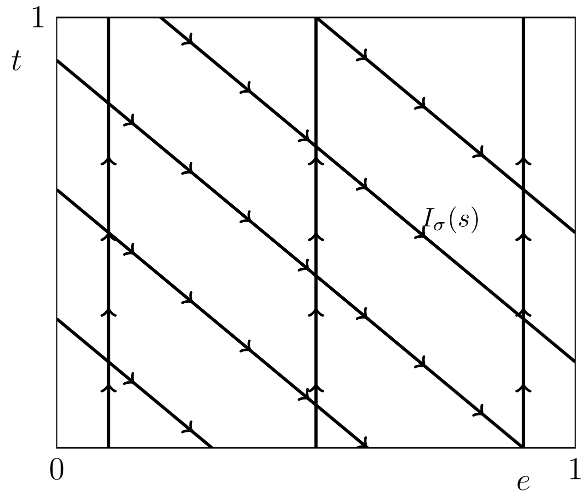
Condition (iii) is necessary and sufficient to ensure that no untalented agent $(e,0)$ has incentives to over-report her talent, imitating an agent (e,t)—whose test score she *cannot* achieve—and possibly getting accepted in case she is not tested. Put differently, among agents with the same level of evidence e, in order to accept talented agents more frequently (than the untalented agent $(e,0)$), the principal needs to test them with high enough probability to prevent agent $(e,0)$ from imitating them. Conditions (i) and (iii) combined also imply that $\Pi(e,t) \geq \Pi(e,0) \geq (1 - T(e,t'))P(e,t',\emptyset)$ for every e,t,t', so no agent has incentives to present all her evidence but overstate her talent.

Last, condition (ii) is necessary and sufficient to ensure that agents do not want to hide some of their evidence in order to overstate their talent, thereby imitating agents whose test score they *can* achieve. Namely, an agent (e,t) does not want to imitate an agent (e',t') with less evidence $e' < e$, more talent $t' > t$, and equal test score $\sigma(e',t') = \sigma(e,t)$ in order to get accepted with probability $\Pi(e',t')$ instead of $\Pi(e,t)$. Notice that for any possible level of evidence $e' < e$ that agent (e,t) may reveal, if it is not profitable for (e,t) to overstate her talent so much that she will fail the test if tested, then because of condition (i), she will want to overstate her talent as much as possible (making sure that she will be able to pass the test), up to the point where $\sigma(e',t') = \sigma(e,t)$.

We have so far seen that conditions (i), (ii), and (iii) are necessary and sufficient for the agent not to have incentives to deviate in any of the following three ways: (a) present all her evidence but under-report her talent, (b) present all her evidence but overstate her talent, or (c) hide some of her evidence and overstate her talent, imitating agents whose test score she *can* achieve. To see why they are necessary and sufficient for IC, it remains to observe that these conditions also rule out the fourth type of deviations

they have talent $t = 1$. Analogously, agents with evidence higher than $\overline{e}(s)$ score more than s even if they have talent $t = 0$.

Figure 1: Directions of (weak) increase in $\Pi(e,t)$ in IC mechanisms



Note: the arrowed lines show the directions in which $\Pi(e,t)$ is non-decreasing in IC mechanisms.

by the agent: hiding evidence and overstating talent to imitate agents whose test score she *cannot* achieve. To see this, notice that conditions (i), (ii), and (iii) combined imply that $\Pi(e,t) \geq \Pi(e,0) \geq \Pi(e',0) \geq (1 - T(e',t'))P(e',t',\emptyset)$ for any $e' < e$, where the second inequality follows from conditions (i) and (ii) combined, ensuring that (e,t) does not want to hide evidence and overstate her talent so much (to a point where $\sigma(e',t') > \sigma(e,t)$) that she fails the test.

Condition (iii) of Proposition 1 is satisfied with equality. Lemma 3 shows that when testing is costly and some talented agents are (optimally) accepted with higher probability than untalented ones with the same level of evidence, the optimal mechanism satisfies condition (iii) of Proposition 1 with equality. Under free testing or when it is not optimal to accept talented agents with higher probability, it is still without loss to constrain attention to mechanisms that satisfy condition (iii) of Proposition 1 with equality.

Lemma 3. Given any IC mechanism $M \equiv \langle T,P \rangle$, there exists an IC mechanism $M' \equiv \langle T',P' \rangle$ with $(1 - T'(e,t))P'(e,t,\emptyset) = \Pi'(e,0)$ for every (e,t) that is outcome-equivalent to M and has at most as high testing costs as M. For $c > 0$, if also $\Pi(e,t) > \Pi(e,0)$ for a positive measure of agent types, then M' has lower testing costs than M.

Here is the intuition behind this result. Take any IC mechanism $M \equiv \langle T,P \rangle$. When $\Pi(e,0) > (1 - T(e,t))P(e,t,\emptyset)$, it means that untalented agent $(e,0)$ strictly prefers to not overstate her talent. This strict preference is due to overtesting of talented agents. Namely, $T(e,t)$ can be reduced and $P(e,t,\emptyset)$ can be increased keeping $\Pi(e,t) \equiv (1-T(e,t))P(e,t,\emptyset) + T(e,t)$ fixed and increasing $(1 - T(e,t))P(e,t,\emptyset)$ maintaining $(1 - T(e,t))P(e,t,\emptyset) \leq \Pi(e,0)$, so that condition (iii) of Proposition 1 is still satisfied. Conditions (i) and (ii) of Proposition

1 are also still satisfied since Π does not change. Then, talented agents are tested with lower but high enough probability to prevent untalented agents from imitating them.

From now on, we constrain attentions to mechanisms with $(1-T(e,t))P(e,t,\emptyset) = \Pi(e,0)$, or equivalently, $\Pi(e,t) = \Pi(e,0) + T(e,t)$, for every (e,t). Given that untalented agents are never tested, condition (iii) being satisfied with equality means that $(1-T(e,0))P(e,0,\emptyset) = \Pi(e,0) = (1-T(e,t))P(e,t,\emptyset)$ for every (e,t); that is, the probability of accepting an agent without a test depends only on presented evidence. The total probability of accepting the agent has two components: (i) a base probability $\Pi(e,0)$ of accepting the agent for her evidence without a test and (ii) an additional probability $T(e,t)$ of accepting the agent for her talent, which (through testing) allows her to differentiate herself from less talented agents with the same level of evidence.

3.3 Optimal screening under free testing

We are now ready to characterize the optimal mechanisms under free testing (i.e., $c = 0$). The principal's objective function is $\int_0^1 \int_0^1 \Pi(e,t)u(e,t)f(e,t)dtde$, which can be written as

$$\int_0^1 \int_{\underline{e}(s)}^{\bar{e}(s)} \Pi(e,\tau(e,s))u(e,\tau(e,s))f(e,\tau(e,s))deds, \tag{2}$$

where instead of integrating over e and t, we integrate over (test score level) s and e. The principal's problem amounts to choosing $\Pi(e,\tau(e,s))$, seen as a function of (e,s) instead of (e,t), non-decreasing in s (condition (i) of Proposition 1) and e (condition (ii) of Proposition 1) to maximize (2), which is linear (and, thus, convex) in Π.[26] Bauer's maximum principle then implies that there exists an extreme Π (i.e., an extreme point of the space of non-decreasing functions from $\{(e,s) \in [0,1]^2 : e \in [\underline{e}(s),\bar{e}(s)]\}$ to $[0,1]$) that maximizes (2). It is an extension of Lemma 2.7 in Börgers (2015) that an extreme Π maps each agent to either 0 or 1.

Lemma 4. Let $c = 0$. There exists an optimal deterministic mechanism (i.e., an optimal mechanism where $\Pi(e,t) \in \{0,1\}$ for all (e,t)).

3.3.1 Testing technology biased in favor of talent

We are now ready to derive the optimal mechanism. Consider first the case where the testing technology is biased in favor of talent in the sense that the test is more sensitive to talent than talent is valuable to the principal.[27]

[26] Condition (iii) of Proposition 1 is immaterial, since testing is free. As implied by Lemma 3, any Π that satisfies conditions (i) and (ii) of Proposition 1 can be implemented with T and P such that $(1-T(e,t))P(e,t,\emptyset) = \Pi(e,0)$ for every (e,t). However, there are many other ways to implement any Π that satisfies conditions (i) and (ii). For example, setting $P(e,t,\emptyset) = 0$ and $T(e,t) = \Pi(e,t)$ for every (e,t) (i.e., nobody is ever accepted without a test) automatically satisfies condition (iii) of Proposition 1.

[27] We define pro-t biased testing for any testing cost c. The optimal mechanism under costly testing is studied in section 3.4.

Definition 3. σ is pro-t biased if for every test score $s \in [0,1]$ there exists e_s such that if $e > e_s$ (resp. $e < e_s$) and $\sigma(e,t) = s$, then $u(e,t) > c$ (resp. $u(e,t) < c$).

This is a single-crossing condition. It says that iso-test-score curves cross the principal's indifference curve "from below." Here is the intuition behind the definition. Because the test is overly sensitive to talent, it is too generous towards those with high talent and low evidence and too strict towards those with low talent and high evidence. Therefore, among all agents with the same test score (if tested), the principal wants to accept those with high but not those with low evidence.

Clearly, if the principal's payoff from accepting the agent is increasing along iso-test-score curves, σ is pro-t biased. This is the case if the principal's MRS of talent for evidence is higher (in absolute value) than the test's MRS of talent for evidence.

Claim 1. If $u(e,\tau(e,s))$ is increasing in e over $e \in [\underline{e}(s),\overline{e}(s)]$ for every $s \in [0,1]$, then σ is pro-t biased (for any c). The condition is satisfied if $\frac{\partial u(e,t)/\partial e}{\partial u(e,t)/\partial t} > \frac{\partial \sigma(e,t)/\partial e}{\partial \sigma(e,t)/\partial t}$ for every (e,t).

Proposition 2 shows that when testing is (i) free and (ii) pro-t biased, the principal can achieve the full information benchmark.

Proposition 2. Let $c = 0$, and assume that σ is pro-t biased. Then, $\Pi(e,t) = \mathbf{I}\left(u(e,t) \geq 0\right)$ is IC, so the principal achieves the full information first-best.[28]

When the principal only values evidence, he can trivially achieve the first best—much like in the case where talent was absent from the model. Namely, accepting every agent with sufficient evidence to be of positive value to the principal is IC, because it does not create incentives for agents to understate their evidence and/or overstate their talent as only evidence is rewarded. Similarly, if the principal also values talent but less strongly than the test score depends on talent, agents do not have incentives to hide evidence in order to overstate their talent when the principal accepts every agent of positive value. Figure 3(a) presents the optimal mechanism under pro-t biased testing.

A way for the principal to implement the first-best Π (and the one that we have restricted attention to given Lemma 3) is by setting $T(e,t) = \mathbf{I}(u(e,t) \geq 0 \wedge u(e,0) < 0)$ and $P(e,t,\emptyset) = \mathbf{I}(u(e,0) \geq 0)$. That is, agents who are not valuable to the principal truthfully report their type and are rejected without getting tested. Agents who are valuable but cannot prove so by presenting evidence e such that $u(e,0) > 0$ (which would prove that even if they have $t = 0$, they are valuable) are tested and then accepted. Finally, agents who can prove that they are valuable by presenting evidence e such that $u(e,0) \geq 0$ do so and are accepted without a test. Clearly, since testing is free, $T(e,t) = \mathbf{I}(u(e,t) \geq 0)$ and $P(e,t,\emptyset) = 0$ for every (e,t) is, for example, also optimal (and differs from the former implementation if there exists e such that $u(e,0) \geq 0$), as is testing every agent and accepting only the valuable ones.

[28] By assumption, $\{(e,t) \in [0,1]^2 : u(e,t) = 0\}$ is a zero-measure set, so $\Pi(e,t) = \mathbf{I}\left(u(e,t) > 0\right)$ and $\Pi(e,t) = \mathbf{I}\left(u(e,t) \geq 0\right)$ are both optimal.

3.3.2 Testing technology biased in favor of evidence

Consider now the case where the testing technology is biased in favor of evidence in the sense that the test is more sensitive to evidence than evidence is valuable to the principal.[29]

Definition 4. σ is pro-e biased if for every test score $s \in [0,1]$ there exists e_s such that if $e < e_s$ (resp. $e > e_s$) and $\sigma(e,t) = s$, then $u(e,t) > c$ (resp. $u(e,t) < c$).

This is again a single-crossing condition. It says that iso-test-score curves cross the principal's indifference curve "from above." Here is the intuition behind the definition. Because the test is overly sensitive to evidence, it is too generous towards those with high evidence and low talent and too strict towards those with low evidence and high talent. Therefore, among all agents with the same test score (if tested), the principal wants to accept those with low but not those with high evidence.

Clearly, if the principal's payoff from accepting the agent is decreasing along iso-test-score curves, σ is pro-e biased. This is the case if the principal's MRS of talent for evidence is lower (in absolute value) than the test's MRS of talent for evidence.

Claim 2. If $u(e,\tau(e,s))$ is decreasing in e over $e \in [\underline{e}(s),\overline{e}(s)]$ for every $s \in [0,1]$, then σ is pro-e biased (for any c). The condition is satisfied if $\frac{\partial u(e,t)/\partial e}{\partial u(e,t)/\partial t} < \frac{\partial \sigma(e,t)/\partial e}{\partial \sigma(e,t)/\partial t}$ for every (e,t).

It is easy to see that the first-best is no longer achievable even if testing is free (i.e., $c = 0$).[30] Indeed, Figure 2 shows that accepting (almost) every agent with $u(e,t) > 0$ and rejecting (almost) every agent with $u(e,t) < 0$ is not IC, as it creates incentives for agents with $u(e,t) < 0$ to hide evidence and imitate more talented and valuable agents.

But what *can* actually be achieved when the test is less sensitive to talent than talent is valuable to the principal? Proposition 3 describes the optimal mechanism when testing is (i) free and (ii) pro-e biased.

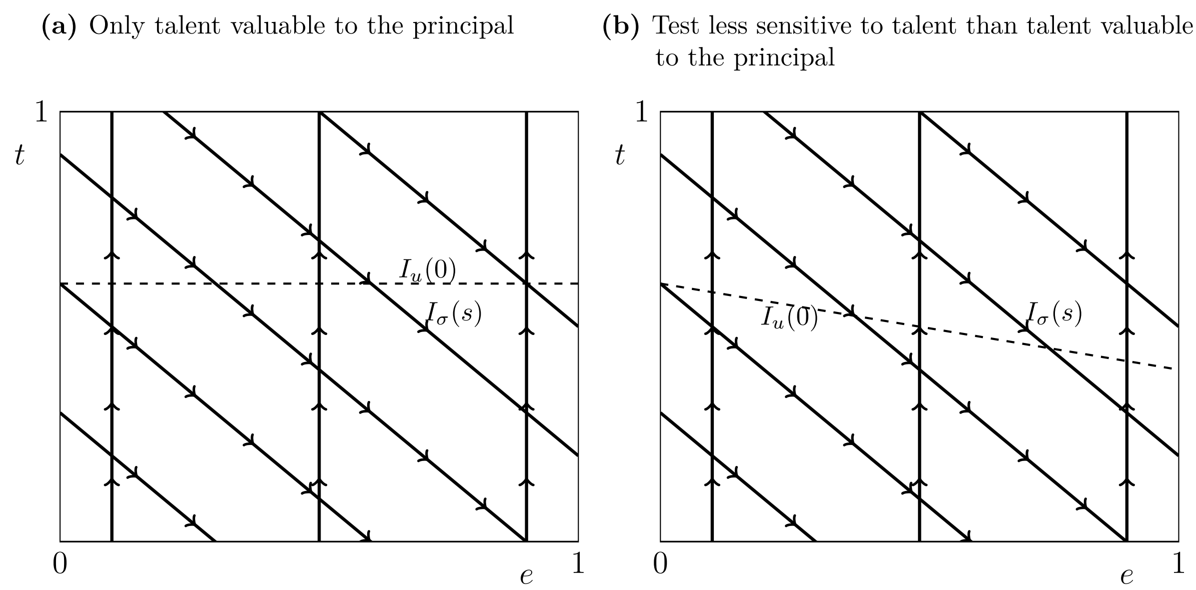
Proposition 3. Let $c = 0$, and assume that σ is pro-e biased. Then, there exists an optimal mechanism with $\Pi(e,t) = \mathbf{I}(\sigma(e,t) \geq s^*)$.

Proposition 3 shows that in the optimal mechanism, agent (e,t) is accepted if and only if $\sigma(e,t) \geq s^*$. An optimal way to implement this Π (and the one that we have restricted attention to given Lemma 3) is by setting $T(e,t) = \mathbf{I}(\sigma(e,t) \geq s^* \wedge e \leq \overline{e}(s^*))$ and $P(e,t,\emptyset) = \mathbf{I}(e > \overline{e}(s^*))$. That is, agents who cannot achieve test score (at least) s^* truthfully report their type and are rejected without getting tested. Agents who can achieve that test score and cannot prove this by presenting evidence $e > \overline{e}(s^*)$ (which

[29] We define pro-e biased testing for any testing cost c. The optimal mechanism under costly testing is studied in section 3.4.

[30] σ being pro-e biased is not necessary for this conclusion. The conclusion still applies as long as the conditions in definition 4 is satisfied for a positive measure of $s \in [0,1]$.

Figure 2: *Not* achieving the first-best: testing technology biased in favor of evidence



Note: the arrowed lines represent the directions of (weak) increase in $\Pi(e,t)$ in any IC mechanism. The dashed lines represent the principal's indifference curve $I_u(0)$.

would prove that even if they have $t = 0$, they can achieve test score s^*) are tested and then accepted. Finally, agents who can prove that they can meet the test score threshold by presenting evidence $e \geq \overline{e}(s^*)$ do so and are accepted without a test.[31]

Finding the optimal mechanism is remarkably simple. It amounts to solving a one-dimensional optimization problem on a closed interval with a continuous objective function. The principal needs to find $s^* \in \arg\max_{\widetilde{s}\in[0,1]} v(\widetilde{s})$, where

$$v(\widetilde{s}) := \int_{\widetilde{s}}^{1} \int_{\underline{e}(s)}^{\overline{e}(s)} u(e,\tau(e,s))f(e,\tau(e,s))deds$$

is continuous in $\widetilde{s}$.[32] When $s^* \in (0,1)$, it solves $\int_{\underline{e}(s^*)}^{\overline{e}(s^*)} u(e,\tau(e,s^*))f(e,\tau(e,s^*))de = 0$. The principal effectively chooses a threshold test score s^* and accepts every agent who can achieve this score. In choosing this threshold, he balances the Type I (i.e., rejecting agents in $I_u^{\uparrow}(0)$) and Type II (i.e., accepting agents in $I_u^{\downarrow}(0)$) errors. This trade-off can be seen in Figure 3(b).

Here is a sketch of the proof of Proposition 3. Because σ is pro-e biased, for any two types of zero value to the principal $(e,t),(e',t') \in I_u(0)$ with $e' > e$, $\sigma(e',t') \geq \sigma(e,t)$. But then, if $\sigma(e',t') \geq \sigma(e,t)$ and $e' > e$, IC requires $\Pi(e',t') \geq \Pi(e,t)$. In other words, $\Pi(e,t)$ has to be non-decreasing as e increases along the $I_u(0)$ curve. Therefore, in any

[31] Clearly, since testing is free, $T(e,t) = \mathbf{I}(\sigma(e,t) \geq s^*)$ and $P(e,t,\emptyset) = 0$ for every (e,t) is, for example, also optimal (and differs from the former implementation if $\overline{e}(s^*) < 1$), as is testing every agent and accepting only those that pass the test score threshold s^*.

[32] The principal's problem reduces to this because all mechanisms with $\Pi(e,t) = \mathbf{I}(\sigma(e,t) \geq s^*)$ and appropriate T are IC.

deterministic IC mechanism, there exists a threshold type on the $I_u(0)$ curve such that agents on the $I_u(0)$ curve with more (resp. less) evidence than the threshold type are accepted (resp. rejected). Next, observe that IC requires that $\Pi(e,t)$ be non-decreasing along iso-test-score curves. Thus, having fixed $\Pi(e,t)$ along the $I_u(0)$ curve, keeping $\Pi(e,t)$ constant along iso-test-score curves maximizes the principal's payoff. That is because, on the part of an iso-test-score curve that lies below (resp. above) $I_u(0)$, the principal wants to make $\Pi(e,t)$ as low (resp. high) as possible but is constrained to set $\Pi(e,t)$ at least (resp. most) equal to its value on the curve $I_u(0)$ for that specific test score level. Condition (i) of Proposition 1 is automatically satisfied.

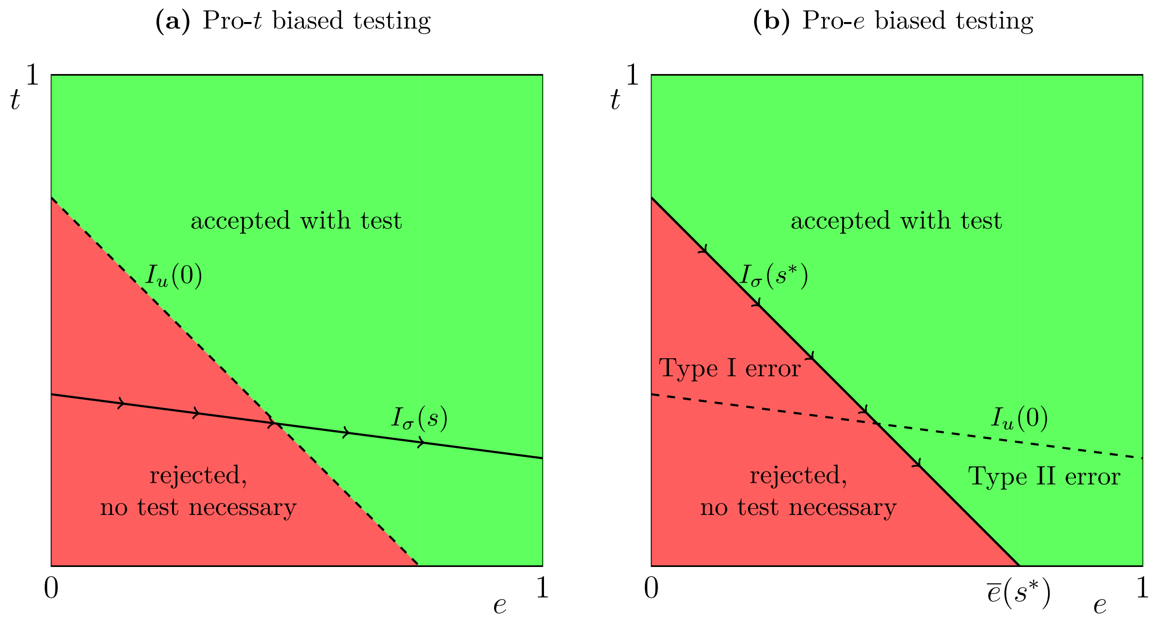
Discussion. When seen against the results under pro-t biased testing (see Proposition 2), Proposition 3 reveals a stark contrast in the difficulty of, for example, hiring different types of employees. When skills and knowledge that can be proven through hard evidence are most valuable, the hiring process is easy. On the other hand, when talent—which is assessed by tests and interviews that are also sensitive to the candidate's training and knowledge—is most valuable, the hiring process is flawed, favoring some unworthy candidates with advanced training at the expense of agents with limited training who are, however, more valuable to the firm.

The revealed difference in the difficulty of hiring talented versus well-trained employees can be partly the reason behind the fact that firm survival rates increase with firm age and size (Evans, 1987; Dunne and Hughes, 1994; Farinas and Moreno, 2000; Agarwal and Gort, 2002; Bartelsman et al., 2005). To the extent that start-ups firms often face new challenges without the established procedures or clearly defined roles of older and larger firms, the success of a start-up will depend crucially on the ability of its employees to adapt and learn new tasks fast (i.e., $u(e,t)$ is very sensitive to t). On the other hand, the continued success of an established firm—where each employee's tasks are more clearly and narrowly defined—will depend (relatively) more on employee training, knowledge, and expertise (i.e., $u(e,t)$ is relatively more sensitive to e). Thus, hiring should be harder in start-ups than in established firms.

3.4 Optimal screening under costly testing

We now allow for a positive testing cost $c > 0$. The principal now needs compare the benefit of testing to its cost. The benefit of testing is that it increases accuracy: it allows the principal to accept talented agents with higher probability that untalented ones. The principal's objective function is $\int_0^1 \int_0^1 [\Pi(e,t)u(e,t) - cT(e,t)]\, f(e,t)dtde$. By Lemma 3, condition (iii) of Proposition 1 is satisfied with equality by the optimal mechanism, so in the objective function we can substitute $T(e,t) = \Pi(e,t) - \Pi(e,0)$. Then, the objective

Figure 3: The optimal mechanism under free testing



Note: the dashed line represents the principal's indifference curve $I_u(0)$; the arrowed line represents an iso-test-score curve (at an arbitrary level s in the left panel and at level s^* in the right panel). The green (resp. red) area denotes the set of agents who are accepted (resp. rejected) in the optimal mechanism. The Type I error corresponds to the part of the red area that lies above the dashed line. The Type II error corresponds to the part of the green area that lies below the dashed line.

function reads

$$\int_0^1 \int_{\underline{e}(s)}^{\bar{e}(s)} \left[\Pi(e,\tau(e,s))(u(e,\tau(e,s)) - c) + c\Pi(e,0)\right] f(e,\tau(e,s))deds, \tag{3}$$

which is again linear in Π, so by Bauer's maximum principle, there exists an extreme Π—among all Π that are non-decreasing in s and e—that solves the principal's problem.

Lemma 5. There exists an optimal deterministic mechanism.

3.4.1 Testing technology biased in favor of talent

Proposition 4 characterizes the optimal mechanism under pro-t biased testing, generalizing Proposition 2 by allowing for possibly costly testing (i.e., $c \geq 0$).

Proposition 4. If σ is pro-t biased, then there exists an optimal mechanism with $\Pi(e,t) = \mathbf{I}(u(e,t) \geq c \text{ or } e \geq e^*)$ and $T(e,t) = \mathbf{I}(u(e,t) \geq c \text{ and } e < e^*)$ for some $e^* \in [0,1]$.

The principal's problem amounts to choosing a threshold level of evidence $e^* \in$

$\arg\max_{\tilde{e}\in[0,1]} v(\tilde{e})$,[33] where

$$v(\tilde{e}) := \int_0^1 \int_0^{\tilde{e}} (u(e,t) - c)\mathbf{I}(u(e,t) \geq c) f(e,t) de dt + \int_0^1 \int_{\tilde{e}}^1 u(e,t) f(e,t) de dt.$$

Every agent with evidence $e \geq e^*$ evidence is accepted without a test, while agents with evidence $e < e^*$ are tested and accepted if their value $u(e,t)$ to the principal is higher than the cost c of testing. The remaining agents are rejected without getting tested. Figure 4(a) presents the structure of the optimal mechanism.

When $e^* \in (0,1)$, the first-order condition is

$$v'(e^*) = \int_0^1 (u(e^*,t) - c)\mathbf{I}(u(e^*,t) \geq c) f(e^*,t) dt - \int_0^1 u(e^*,t) f(e^*,t) dt = 0,$$

or equivalently

$$v'(e^*) = \overbrace{-\int_0^1 u(e^*,t)\mathbf{I}(u(e^*,t) \leq 0) f(e^*,t) dt}^{>0\text{: gain from rejection of unworthy agents (ii)}} - \overbrace{\int_0^1 u(e^*,t)\mathbf{I}(0 < u(e^*,t) < c) f(e^*,t) dt}^{>0\text{: loss from rejection of worthy agents (iii)}}$$
$$- \underbrace{c\int_0^1 \mathbf{I}(u(e^*,t) \geq c) f(e^*,t) dt}_{>0\text{: loss from increase in testing costs (i)}} = 0.$$

An increase in the threshold e^* would lead to: (i) increased testing costs by making additional agents who lie above $I_u(c)$ get tested before being accepted (who were accepted without a test before the increase in e^*), (ii) the rejection without a test of additional agents who lie below $I_u(0)$ (who were accepted without a test before the increase in e^*), but also (iii) the rejection without a test of additional agents who lie below $I_u(c)$ but above $I_u(0)$ (who were accepted without a test before the increase in e^*). Channels (i) and (iii) negatively affect the principal's payoff, while channel (ii) tends to increase his payoff. In choosing the optimal threshold e^*, the principal trades off testing costs (i.e., effect (i)) with the increase in accuracy (i.e., the net effect of (ii) and (iii)).

Comparative statics. We now briefly discuss some comparative statics. For simplicity, assume that $e^* \in (0,1)$ is unique with the second-order condition of the principal's problem satisfied strictly and that some agents are optimally tested.[34] First, an increase in c causes the (combined) magnitude of channels (i) and (iii) to increase without affecting the magnitude of channel (ii).[35] Thus, e^* is decreasing in c; the more costly testing is, the

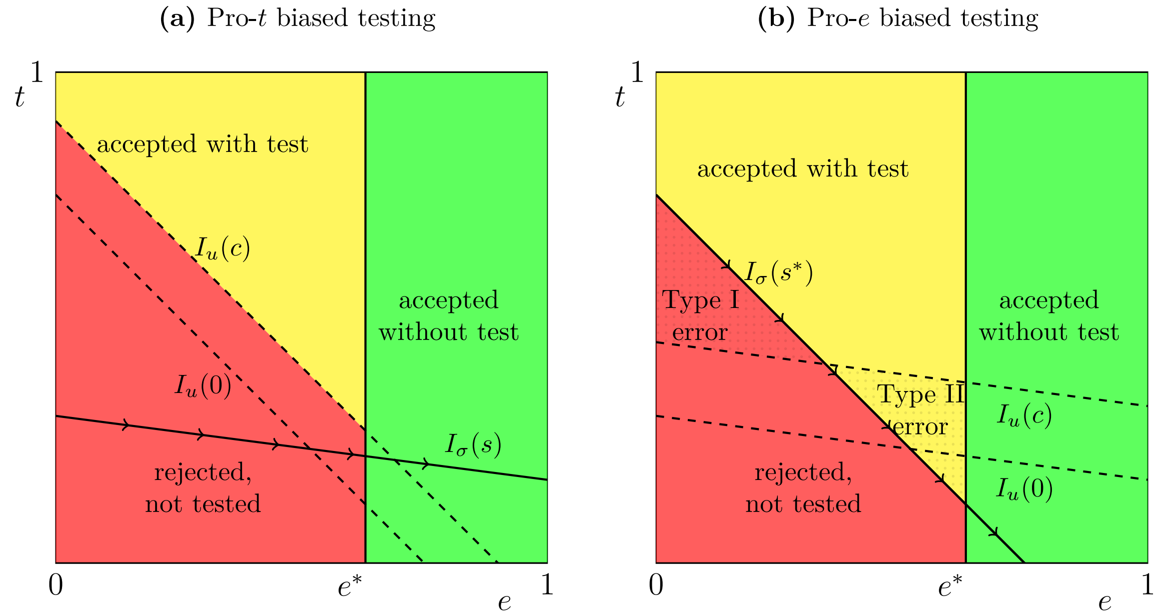
[33] The principal's problem reduces to this because all mechanisms with $\Pi(e,t) = \mathbf{I}(u(e,t) \geq c \text{ or } e \geq e^*)$ and $T(e,t) = \mathbf{I}(u(e,t) \geq c \text{ and } e < e^*)$ for some $e^* \in [0,1]$ are IC.

[34] Namely, $e^* > 0$ and $u(e,t) > c$ for a positive measure of agents with $e < e^*$. This rules out the case $u(e,t) = e - \underline{q}$, where the principal only cares about evidence, in which case he does not test.

[35] In more detail, the partial derivative of $-c\int_0^1 \mathbf{I}(u(e^*,t) \geq c) f(e^*,t) dt$ with respect to c is $-\int_0^1 \mathbf{I}(u(e^*,t) \geq c) f(e^*,t) dt + cf(e^*,t')$ where t' is such that $u(e^*,t') = c$. The partial derivative of

more high-evidence agents are accepted without a test. Particularly, $v'(e)$ is decreasing in c with $\partial v'(e)/\partial c = -\int_0^1 \mathbf{I}(u(e^*,t) \geq c)f(e^*,t)dt < 0$, and by the Implicit Function Theorem $de^*/dc = -\partial v'(e)/\partial c|_{e=e^*}/v''(e^*) < 0$. Second, the principal's optimal payoff is decreasing in c. Third, since the principal's objective function is independent of the testing technology σ, the optimal mechanism and payoff are the same under any two pro-t biased testing technologies with the same testing cost c.

Figure 4: The optimal mechanism under costly testing



Note: the dashed line represents the principal's indifference curve $I_u(c)$: the principal is indifferent between accepting with test and rejecting without test agents on that curve. The arrowed line represents an iso-test-score curve (at an arbitrary level s in the left panel and at level s^* in the right panel). The green area denotes the set of agents who are accepted without getting tested in the optimal mechanism. The red one denotes the set of agents who are rejected without getting tested in the optimal mechanism. The yellow area denotes the set of agents who are accepted after getting tested in the optimal mechanism.

3.4.2 Testing technology biased in favor of evidence

Proposition 5 characterizes the optimal mechanism under pro-e biased testing, generalizing Proposition 3 by allowing for possibly costly testing (i.e., $c \geq 0$).

Proposition 5. If σ is pro-e biased, then there exists an optimal mechanism with $\Pi(e,t) = \mathbf{I}(\sigma(e,t) \geq s^* \text{ or } e \geq e^*)$ and $T(e,t) = \mathbf{I}(\sigma(e,t) \geq s^* \text{ and } e < e^*)$ for some $(e^*,s^*) \in [0,1]^2$.

$\int_0^1 u(e^*,t)\mathbf{I}(0 < u(e^*,t) < c)f(e^*,t)dt$ with respect to c is $u(e^*,t')f(e^*,t') = cf(e^*,t') > 0$, which cancels out with the corresponding term in the derivative of $-c\int_0^1 \mathbf{I}(u(e^*,t) \geq c)f(e^*,t)dt$.

Finding an optimal mechanism is again remarkably simple. The principal's problem amounts to choosing threshold test score and evidence levels $(e^*,s^*) \in \arg\max_{(\widetilde{e},\widetilde{s})\in[0,1]^2} v(\widetilde{e},\widetilde{s})$, where

$$v(\widetilde{e},\widetilde{s}) := \int_{\widetilde{s}}^{1} \int_{\underline{e}(s)}^{\max\{\min\{\overline{e}(s),\widetilde{e}\},\underline{e}(s)\}} (\widetilde{u}(e,s) - c)\, \widetilde{f}(e,s) deds + \int_0^1 \int_{\max\{\underline{e}(s),\widetilde{e}\}}^{\max\{\overline{e}(s),\widetilde{e}\}} \widetilde{u}(e,s)\widetilde{f}(e,s)deds,$$

and $\widetilde{u}(e,s) := u(e,\tau(e,s))$ and $\widetilde{f}(e,s) := f(e,\tau(e,s))$.[36] Every agent with evidence $e \geq e^*$ is accepted without a test, while agents with evidence $e < e^*$ are tested and accepted if their (potential) test score is at least $\sigma(e,t) \geq s^*$. The remaining agents are rejected without getting tested. Figure 4(b) presents the structure of the optimal mechanism under pro-e biased testing.

When $\underline{e}(s^*) < e^*$ (i.e., some agents are accepted after being tested) and $e^*,s^* \in (0,1)$,[37] the first-order conditions are

$$v_1(e^*,s^*) = \overbrace{-\int_0^{s^*} \widetilde{u}(e^*,s)\widetilde{f}(e^*,s)ds}^{\substack{>0\text{: gain from rejection}\\ \text{of unworthy agents (i)}}} - \overbrace{c\int_{s^*}^{1} \widetilde{f}(e^*,s)ds}^{\substack{>0\text{: loss from increase}\\ \text{in testing costs (ii)}}} = 0,$$

$$v_2(e^*,s^*) = \underbrace{-\int_{\underline{e}(s^*)}^{e^*} \min\{\widetilde{u}(e,s^*) - c,0\}\widetilde{f}(e,s^*)de}_{>0\text{: gain from decrease in Type II error}} - \underbrace{\int_{\underline{e}(s^*)}^{e^*} \max\{\widetilde{u}(e,s^*) - c,0\}\widetilde{f}(e,s^*)de}_{>0\text{: loss from increase in Type I error}} = 0,$$

given that optimality requires $e^* \leq \overline{e}(s^*)$.[38] The principal chooses s^* considering the trade-off between Type I and Type II errors—conditional on the fact that only agents with evidence $e \geq e^*$ are accepted without a test; for agents with evidence $e < e^*$, the principal chooses between accepting after testing and rejecting without testing. The Type I error is due to the fact that the principal rejects without testing some agents whom he would prefer to accept after testing. The Type II error is due to the fact that the principal tests and accepts some agents whom he would prefer to reject without testing. An increase in the threshold e^* would lead to: (i) the rejection of additional agents who lie below $I_u(0)$ (who were accepted without a test before the increase in e^*) and (ii) increased testing costs by making additional agents who lie above $I_\sigma(s^*)$ get tested before being accepted (who were accepted without a test before the increase in e^*). Channel (ii) negatively affects the principal's payoff, while channel (i) tends to increase his payoff. In choosing the optimal threshold e^*, the principal trades off testing costs (i.e., effect (ii)) with the increase in accuracy (i.e., effect (i)).

[36]The principal's problem reduces to this because all mechanisms with $\Pi(e,t) = \mathbf{I}(\sigma(e,t) \geq s^*$ or $e \geq e^*)$ and $T(e,t) = \mathbf{I}(\sigma(e,t) \geq s^*$ and $e < e^*)$ for some $(e^*,s^*) \in [0,1]^2$ are IC.

[37]Notice that $e^* \leq \overline{e}(s^*)$ (for if $e^* > \overline{e}(s^*)$ and $c > 0$, reducing e^* would increase $v(e^*,s^*)$). For $c = 0$, $e^* = 1$ without loss.

[38]If $e^* > \overline{e}(s^*)$, decreasing e^* to make it equal to $\overline{e}(s^*)$ would decrease testing costs without changing the set of agents who are accepted, thereby increasing the principal's payoff.

Comparative statics. We now briefly discuss some comparative statics. For simplicity, assume that $s^*, e^* \in (0,1)$ are unique with the second-order condition of the principal's problem satisfied strictly and that some agents are optimally tested. Denote by $J(e^*,s^*)$ the Jacobian matrix of the first derivatives evaluated at (e^*,s^*), which is by assumption negative definite. Particularly, $v_{11}(e^*,s^*), v_{22}(e^*,s^*) < 0$ and $\det(J(e^*,s^*)) > 0$. Also, $v_{12}(e^*,s^*) = v_{21}(e^*,s^*) = -\left(\tilde{u}(e^*,s^*) - c\right)\tilde{f}(e^*,s^*) > 0$. First, the total derivatives of $e^*$and s^* with respect to c are:

$$\frac{de^*}{dc} \propto \overbrace{-v_{1c}(e^*,s^*)v_{22}(e^*,s^*)}^{\substack{\text{<0: direct effect of } c \text{ on} \\ e^* \text{ due to increase in} \\ \text{marginal testing costs}}} + \overbrace{v_{2c}(e^*,s^*)v_{12}(e^*,s^*)}^{\substack{\text{>0: indirect effect of } c \\ \text{on } e^* \text{ through direct} \\ \text{effect of } c \text{ on } s^*}},$$
$$\frac{ds^*}{dc} \propto \underbrace{-v_{2c}(e^*,s^*)v_{11}(e^*,s^*)}_{\substack{\text{>0: direct effect of } c \text{ on} \\ s^* \text{ due to increase in} \\ \text{marginal testing costs}}} + \underbrace{v_{1c}(e^*,s^*)v_{21}(e^*,s^*)}_{\substack{\text{<0: indirect effect of } c \\ \text{on } s^* \text{ through direct} \\ \text{effect of } c \text{ on } e^*}},$$

where $v_{1c}(e^*,s^*) = -\int_{s^*}^{1}\tilde{f}(e^*,s)ds < 0$ and $v_{2c}(e^*,s^*) = \int_{\underline{e}(s^*)}^{e^*}\tilde{f}(e,s^*)de > 0$ are the partial derivatives of v_1 and v_2 with respect to c. An increase in the (marginal) cost c of testing tends to directly cause (i) e^* to decrease by magnifying the testing cost savings associated with a decrease in e^* and (ii) s^* to increase by magnifying the testing cost savings associated with an increase in e^*.[39] However, an increase in s^* tends to cause e^* to increase by reducing the marginal increase in testing costs associated with an increase in e^*. Conversely, an increase in e^* tends to cause s^* to increase by increasing the marginal (with respect to s^*) Type II error. Therefore, although an increase in c tends to directly cause e^* to fall and s^* to rise, the interaction between e^* and s^* works in the opposite direction making the net effect ambiguous. Still, we know that if s^* decreases in response to an increase in c, e^* should also decrease and—the contrapositive—if e^* increases in response to an increase in c, s^* should also increase. Second, the principal's optimal payoff is decreasing in c. Third, the optimal payoff is higher under less pro-e biased testing technologies. Namely, take any two pro-e biased testing technologies σ' and σ. If all iso-test-score curves of σ cross the iso-test-score curves of σ' from above (i.e., σ is less pro-e biased than σ'), the principal's optimal payoff is higher under σ' than under σ.[40] Fourth, the principal's payoff should tend to increase with the correlation between evidence and talent. A strong (positive) correlation between e and t means that there are not many agents with high (resp. low) talent and low (resp. high) evidence, which implies

[39]Put differently, an increase in c can be seen to increase the marginal (with respect to s^*) Type II error and decrease the marginal Type II error, thereby tending to make s^* increase to equalize the magnitudes to the two errors.

[40]Comparative statics of s^* and e^* with respect to σ would have little value, since the optimal test score thresholds (which are determined simultaneously with the optinal evidence thresholds) under different testing technologies are not comparable, as they can only be interpreted with respect to their corresponding testing technologies.

that both Type I and Type II errors are small. As e and t become perfectly (positively) correlated, the principal achieves the first-best just by asking for evidence—regardless of his preferences and the testing technology.

Implementation of the optimal mechanism. We have so far restricted (without loss) attention to truth-telling mechanisms. However, the optimal mechanism under pro-e biased testing can be implemented in the following simple way. The principal gives the agent two paths to getting accepted: (i) provide evidence e^* and you will be accepted without a test or (ii) take a test without providing any evidence, score at least s^*, and you will be accepted. The first option is not always provided (e.g., when testing is free, the first option is not necessary in the optimal mechanism). Asking for evidence is useful to the principal (as long as $e^* < 1$). Last, notice that a similarly simple implementation of the optimal mechanism under pro-t biased testing is not possible. In that case, the principal needs to ask for evidence also from agents who are tested.[41]

3.5 $(m+n)$-dimensional screening with substitutable attributes

We now generalize the results allowing for multiple dimensions of evidence and talent. Let the agent's type be $(e_1,e_2,\ldots,e_m,t_1,t_2,\ldots,t_n)$ with full-support density $f : [0,1]^{m+n} \to \mathbb{R}_{++}$. $(e_1,e_2,\ldots,e_m)$ are different dimensions of evidence and $(t_1,t_2,\ldots,t_m)$ are different dimensions of talent. The agent can present any combination of evidence $\boldsymbol{r} \in [\boldsymbol{0},\boldsymbol{e}]$. The testing technology $\sigma : [0,1]^{m+n} \to [0,1]$ is continuous and increasing. $u(\boldsymbol{e},\boldsymbol{t})$ is continuous and non-decreasing. It follows by the same arguments as before that (i) truthful mechanisms, (ii) pass-or-fail tests, and (iii) $T(\boldsymbol{e},\boldsymbol{0}) = 0$ for every $\boldsymbol{e}$ are still without loss.

Lemma 6 makes the following additional observation: among agents with the same evidence and potential test score, IC mechanisms cannot screen for different dimensions of talent. That $\Pi(\boldsymbol{e},\boldsymbol{t}) = \Pi(\boldsymbol{e},\boldsymbol{t}')$ for every $\boldsymbol{e},\boldsymbol{t},\boldsymbol{t}'$ with $\sigma(\boldsymbol{e},\boldsymbol{t}) = \sigma(\boldsymbol{e},\boldsymbol{t}')$ is necessary to ensure that no agent has incentives to present all her evidence but misreport her talent to imitate an agent with the same test score.

Lemma 6. If a mechanism $M \equiv \langle T,P\rangle$ is IC, then $\Pi(\boldsymbol{e},\boldsymbol{t}) = \Pi(\boldsymbol{e},\boldsymbol{t}')$ for every $\boldsymbol{e},\boldsymbol{t},\boldsymbol{t}'$ such that $\sigma(\boldsymbol{e},\boldsymbol{t}) = \sigma(\boldsymbol{e},\boldsymbol{t}')$.

Therefore, we restrict attention to mechanisms with $\Pi(\boldsymbol{e},\boldsymbol{t}) = \Pi(\boldsymbol{e},\boldsymbol{t}')$ for every $\boldsymbol{e},\boldsymbol{t},\boldsymbol{t}'$ such that $\sigma(\boldsymbol{e},\boldsymbol{t}) = \sigma(\boldsymbol{e},\boldsymbol{t}')$. Lemma 7 shows that we can further restrict attention to mechanisms that treat agents with the same evidence and potential test score exactly the same way with respect to testing and acceptance probabilities conditional on test results.

[41] These observations on the implementation of optimal mechanisms also imply that under free testing (i.e., $c = 0$), if the principal (optimally) asks for evidence—which he does not need to do under pro-e biased testing, then he most likely values evidence (i.e., $u(e,t)$ is increasing in e).

Lemma 7. Given any IC mechanism M, there exists an IC mechanism $M' \equiv \langle T', P' \rangle$ with $T'(\boldsymbol{e},\boldsymbol{t}) = T'(\boldsymbol{e},\boldsymbol{t}')$ and $P'(\boldsymbol{e},\boldsymbol{t},\emptyset) = P'(\boldsymbol{e},\boldsymbol{t}',\emptyset)$ for every $\boldsymbol{e},\boldsymbol{t},\boldsymbol{t}'$ such that $\sigma(\boldsymbol{e},\boldsymbol{t}) = \sigma(\boldsymbol{e},\boldsymbol{t}')$ that is outcome-equivalent to M. Also, for $c > 0$, in any optimal mechanism $M \equiv \langle T,P \rangle$, $T(\boldsymbol{e},\boldsymbol{t}) = T(\boldsymbol{e},\boldsymbol{t}')$ for almost every $\boldsymbol{e},\boldsymbol{t},\boldsymbol{t}'$ such that $\sigma(\boldsymbol{e},\boldsymbol{t}) = \sigma(\boldsymbol{e},\boldsymbol{t}')$.

The intuition behind this result goes as follows. The only reason to test an agent before accepting her—rather than accept her without a test—is to prevent others from imitating her. Take any agent $(\boldsymbol{e},\boldsymbol{t})$ who contemplates which of the agents in the set $X(\boldsymbol{e}',s) := \{(\boldsymbol{e}',\boldsymbol{t}') : \sigma(\boldsymbol{e}',\boldsymbol{t}') = s\}$, where $\boldsymbol{e}' \leq \boldsymbol{e}$, to imitate. By Lemma 6, Π is the same for every agent in $X(\boldsymbol{e},s)$, so if $\sigma(\boldsymbol{e},\boldsymbol{t}) \geq s$, then agent $(\boldsymbol{e},\boldsymbol{t})$'s payoff from imitating an agent in $X(\boldsymbol{e}',s)$ does not depend on which particular agent she chooses to imitate. If, on the other hand, $\sigma(\boldsymbol{e},\boldsymbol{t}) < s$, agent $(\boldsymbol{e},\boldsymbol{t})$'s payoff from imitating an agent $(\boldsymbol{e}',\boldsymbol{t}') \in X(\boldsymbol{e}',s)$ is increasing (resp. decreasing) in $P(\boldsymbol{e}',\boldsymbol{t}',\emptyset)$ (resp. $T(\boldsymbol{e}',\boldsymbol{t}')$). Thus, the principal can decrease $T(\boldsymbol{e}',\boldsymbol{t}')$ and increase $P(\boldsymbol{e}',\boldsymbol{t}',\emptyset)$ for every agent $(\boldsymbol{e}',\boldsymbol{t}') \in X(\boldsymbol{e}',s)$ with $T(\boldsymbol{e}',\boldsymbol{t}') > \inf_{(\boldsymbol{e}'',\boldsymbol{t}'') \in X(\boldsymbol{e}',s)} T(\boldsymbol{e}'',\boldsymbol{t}'')$ (and thus, $P(\boldsymbol{e}'',\boldsymbol{t}',\emptyset) < \sup_{(\boldsymbol{e}'',\boldsymbol{t}'') \in X(\boldsymbol{e}',s)} P(\boldsymbol{e}',\boldsymbol{t}'',\emptyset)$) keeping Π fixed. There is no point in testing (or accepting without testing) two agents with the same evidence and potential test score with different probabilities, as doing so does not reduce incentives of others to imitate them and makes the principal incur higher than necessary testing costs.

Thus, we can restrict attention to mechanisms with $\Pi(\boldsymbol{e},\boldsymbol{t}) = \Pi(\boldsymbol{e},\boldsymbol{t}')$, $T(\boldsymbol{e},\boldsymbol{t}) = T(\boldsymbol{e},\boldsymbol{t}')$, $P'(\boldsymbol{e},\boldsymbol{t},\emptyset) = P'(\boldsymbol{e},\boldsymbol{t}',\emptyset)$, and $P(\boldsymbol{e},\boldsymbol{t},s) = P(\boldsymbol{e},\boldsymbol{t}',s)$ for every $\boldsymbol{e},\boldsymbol{t},\boldsymbol{t}',s$ such that $\sigma(\boldsymbol{e},\boldsymbol{t}) = \sigma(\boldsymbol{e},\boldsymbol{t}')$.[42] In other words, the principal can constrain attention to mechanisms that ask agents only for evidence and a statement about the test score they claim they can achieve (rather than a whole profile of talent dimensions). The principal designs a mechanism $M \equiv \langle T,P \rangle$, where $T : [0,1]^{m+1} \to [0,1]$ and $P : [0,1]^{m+1} \times ([0,1] \cup \{\emptyset\}) \to [0,1]$. Proposition 6 extends the IC characterization of Proposition 1 to the case of $(m+n)$-dimensional screening.

Proposition 6. A mechanism $M \equiv \langle T,P \rangle$ is IC if and only if

(i) $\Pi(\boldsymbol{e},s)$ is non-decreasing in s over $s \in [\sigma(\boldsymbol{e},\boldsymbol{0}),\sigma(\boldsymbol{e},\boldsymbol{1})]$ for every $\boldsymbol{e} \in [0,1]^m$,

(ii) $\Pi(\boldsymbol{e},s)$ is non-decreasing in $\boldsymbol{e}$ over $\boldsymbol{e} \in \{\boldsymbol{e} \in [0,1]^m : \sigma(\boldsymbol{e},\boldsymbol{0}) \leq s \leq \sigma(\boldsymbol{e},\boldsymbol{1})\}$ for every $s \in [0,1]$, and

(iii) $(1 - T(\boldsymbol{e},s))P(\boldsymbol{e},s,\emptyset) \leq \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\boldsymbol{0}))$ for every $(\boldsymbol{e},s) \in [0,1]^{m+1}$,

where $\Pi(\boldsymbol{e},s) := (1 - T(\boldsymbol{e},s))P(\boldsymbol{e},s,\emptyset) + T(\boldsymbol{e},s)$ is the probability with which agent an agent is accepted if she truthfully reports her evidence $\boldsymbol{e}$ and test score s.

[42] That $P(\boldsymbol{e},\boldsymbol{t},s) = P(\boldsymbol{e},\boldsymbol{t}',s)$ for every $s \in [0,1]$ when $\sigma(\boldsymbol{e},\boldsymbol{t}) = \sigma(\boldsymbol{e},\boldsymbol{t}')$ follows already from restricting attention to pass-or-fail tests.

The intuition behind the conditions is analogous to the one behind the conditions of Proposition 1. Notice that there are no IC condition on the comparison between the values of T, P, or Π for agent types $(\boldsymbol{e},s)$ and $(\boldsymbol{e}',s')$ such that $\boldsymbol{e}$ and $\boldsymbol{e}'$ are incomparable (i.e., $\boldsymbol{e} \ngeq \boldsymbol{e}'$ and $\boldsymbol{e} \nleq \boldsymbol{e}'$). That is, because neither agent type has the evidence to imitate the other.

Lemma 8 extends Lemma 3 to show that when testing is costly and some talented agents are (optimally) accepted with higher probability than untalented ones with the same evidence, then the optimal mechanism satisfies condition (iii) of Proposition 6 with equality. Under free testing or when it is not optimal to accept talented agents with higher probability, it is still without loss to constrain attention to mechanisms that satisfy condition (iii) of Proposition 6 with equality.

Lemma 8. Given any IC mechanism $M \equiv \langle T,P \rangle$, there exists an IC mechanism $M' \equiv \langle T',P' \rangle$ with $(1 - T'(\boldsymbol{e},s))P'(\boldsymbol{e},s,\emptyset) = \Pi'(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0}))$ for every $(\boldsymbol{e},s)$, $s \in [\sigma(\boldsymbol{e},\mathbf{0}),\sigma(\boldsymbol{e},\mathbf{1})]$ that is outcome-equivalent to M and has at most as high testing costs as M. For $c > 0$, if also $\Pi(\boldsymbol{e},s) > \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0}))$ for a positive measure of $(\boldsymbol{e},s)$'s, then M' has lower testing costs than M.

Define $\widetilde{f}(\boldsymbol{e},s) := \int_{\boldsymbol{t}\in[0,1]^n} \mathbf{I}(\sigma(\boldsymbol{e},\boldsymbol{t}) = s) f(\boldsymbol{e},\boldsymbol{t}) d\boldsymbol{t}$, the probability density of agents with evidence $\boldsymbol{e}$ and potential test score s, and $\widetilde{u}(\boldsymbol{e},s) := \mathbb{E}_{\mathbf{t}}[u(\boldsymbol{e},\boldsymbol{t})|\sigma(\boldsymbol{e},\boldsymbol{t}) = s] = \int_{\mathbf{t}\in[0,1]^n} u(\mathbf{e},\mathbf{t})\mathbf{I}(\sigma(\mathbf{e},\mathbf{t}) = s) f(\mathbf{e},\mathbf{t}) d\mathbf{t}/\widetilde{f}(\boldsymbol{e},s)$, the principal's expected payoff from accepting all agents with evidence $\boldsymbol{e}$ and potential test score s.[43] Assume that $\widetilde{u}(\boldsymbol{e},s)$ is non-decreasing in s. The principal's objective function is $\int_0^1 \cdots \int_0^1 \int_{\sigma(\boldsymbol{e},\mathbf{0})}^{\sigma(\boldsymbol{e},\mathbf{1})} [\Pi(\boldsymbol{e},s)\widetilde{u}(\boldsymbol{e},s) - cT(\boldsymbol{e},s)] \widetilde{f}(\boldsymbol{e},s) ds\, de_1 \cdots de_m$. By Lemma 8, condition (iii) of Proposition 6 is satisfied with equality by the optimal mechanism, so in the objective function we can substitute $T(\boldsymbol{e},s) = \Pi(\boldsymbol{e},s) - \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0}))$. Then, the objective function reads

$$\int_0^1 \cdots \int_0^1 \int_{\sigma(\boldsymbol{e},\mathbf{0})}^{\sigma(\boldsymbol{e},\mathbf{1})} [\Pi(\boldsymbol{e},s)(\widetilde{u}(\boldsymbol{e},s) - c) + c\Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0}))] \widetilde{f}(\boldsymbol{e},s) ds de_1 \cdots de_m, \tag{4}$$

which is linear in Π, so by Bauer's maximum principle, there exists an extreme Π—among all Π that are non-decreasing in s and $\boldsymbol{e}$—that solves the principal's problem.

Lemma 9. There exists an optimal deterministic mechanism.

3.5.1 Testing technology biased in favor of talent

The definition of pro-t biased testing is extended to the case of $(m + n)$-dimensional screening as follows.

[43] For $(\boldsymbol{e},s)$ such that $s = \sigma(\boldsymbol{e},\mathbf{0})$, $\widetilde{u}(\boldsymbol{e},s) \equiv u(\boldsymbol{e},\mathbf{0}))$. For simplicity, the following Regularity Assumption is made: $\widetilde{u}(\boldsymbol{e},s)$ is increasing in s.

Definition 5. σ is pro-t biased if for every $\boldsymbol{e},\boldsymbol{e}' \in [0,1]^m$ and every test score $s \in [\max\{\sigma(\boldsymbol{e},\mathbf{0}),\sigma(\boldsymbol{e}',\mathbf{0})\}, \min\{\sigma(\boldsymbol{e},\mathbf{1}),\sigma(\boldsymbol{e}',\mathbf{1})\}]$, if $\tilde{u}(\boldsymbol{e},s) \geq c \geq \tilde{u}(\boldsymbol{e}',s)$ with at least one inequality holding strictly, then $\boldsymbol{e}' \not\geq \boldsymbol{e}$.

Generalizing Proposition 4, Proposition 7 derives the optimal mechanism under pro-t biased testing.

Proposition 7. If σ is pro-t biased, then there exists an optimal mechanism with $\Pi(\boldsymbol{e},s) = \mathbf{I}(\tilde{u}(\boldsymbol{e},s) > c \text{ or } \boldsymbol{e} \in E^*)$ and $T(\boldsymbol{e},s) = \mathbf{I}(\tilde{u}(\boldsymbol{e},s) > c \text{ and } \boldsymbol{e} \notin E^*)$ for some upper set E^* of $[0,1]^m$ (i.e., $E^* \subseteq [0,1]^m$ such that for any $\boldsymbol{e} \in E^*$ and $\boldsymbol{e}' \in [0,1]^m$, if $\boldsymbol{e}' \geq \boldsymbol{e}$, then $\boldsymbol{e}' \in E^*$).

Clearly, if $c = 0$, $E^* = \emptyset$ without loss. If $c > 0$, $E^* \supseteq \{\boldsymbol{e} \in [0,1]^m : \tilde{u}(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) > c\}$. This has to be true, because among the agents who are accepted, an agent should be tested only if this will prevent others from imitating her. Any agent who has enough evidence to imitate an agent $(\boldsymbol{e},\mathbf{0})$ with $\tilde{u}(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) > c$ and get accepted can also achieve a test score at least as high as $(\boldsymbol{e},\mathbf{0})$ can. Therefore, $(\boldsymbol{e},\mathbf{0})$ with $\tilde{u}(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) > c$ should not be tested.

3.5.2 Testing technology biased in favor of evidence

The definition of pro-e biased testing is extended to the case of $(m+n)$-dimensional screening as follows.

Definition 6. σ is pro-e biased if for every $\boldsymbol{e},\boldsymbol{e}' \in [0,1]^m$ and every test score $s \in [\max\{\sigma(\boldsymbol{e},\mathbf{0}),\sigma(\boldsymbol{e}',\mathbf{0})\}, \min\{\sigma(\boldsymbol{e},\mathbf{1}),\sigma(\boldsymbol{e}',\mathbf{1})\}]$, if $\tilde{u}(\boldsymbol{e},s) \geq c \geq \tilde{u}(\boldsymbol{e}',s)$ with at least one inequality holding strictly, then $\boldsymbol{e}' \geq \boldsymbol{e}$.

Generalizing Proposition 5, Proposition 8 derives the optimal mechanism under pro-e biased testing.

Proposition 8. If σ is pro-e biased, then there exists an optimal mechanism with $\Pi(\boldsymbol{e},s) = \mathbf{I}(s \geq s^* \text{ or } \boldsymbol{e} \in E^*)$ and $T(\boldsymbol{e},s) = \mathbf{I}(s \geq s^* \text{ and } \boldsymbol{e} \notin E^*)$ for some $s^* \in [0,1]$ and some upper set E^* of $[0,1]^m$.

If $c > 0$, then E^* is a maximal upper set of $E^* \supseteq \{\boldsymbol{e} \in [0,1]^m : \tilde{u}(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) > s^*\}$, which again has to be true, because among the agents who are accepted, an agent should be tested only if this will prevent others from imitating her.

4 Applications

In this section, I use the model to discuss hiring by prestigious employers, promotion decisions, college admissions, and academic job market hiring.

4.1 Hiring by prestigious employers

A job candidate's evidence e is her CV quality (e.g., high school quality, undergraduate institution quality and GPA, awards, distinctions, reference letters). t is her ability and drive not captured by e. A prestigious employer wants to decide whether to hire the candidate. Testing amounts to letting some other employer hire the candidate. Testing is costly because if the employer wants to then hire the candidate, he will have to poach her at a cost.

In the optimal mechanism, Ivy-Leaguers with high credentials get immediately hired by prestigious employers thanks to their evidence. On the other hand, talented candidates with education from lower-ranked institutions and lower grades have to go through less prestigious employers to prove their worth before they land a prestigious position. Also, if the candidates' performance in the less prestigious position is less sensitive to talent than talent is valuable in the more prestigious position—a natural assumption, then worthy candidates with low credentials are at a disadvantage also in the poaching stage.

4.2 Promotions

An employee of efficiency t has exerted effort e. $\sigma(e,t)$ is the employee's productivity, increasing in e and in t. The employee can provide evidence or not on e by, for example, working at the office or from home. Testing (by the employer/manager) amounts to verifying the employee's productivity $\sigma(e,t)$. The value to the principal of the agent who is not promoted (i.e., continues to work in her current position) is $\sigma(e,t)$. His value of the agent if promoted is $\tilde{u}(e,t)$. Then, his problem is equivalent to the one in section 2 with $u(e,t) := \tilde{u}(e,t) - \sigma(e,t)$, as long as the difference $\tilde{u}(e,t) - \sigma(e,t)$ is non-decreasing in both e and t.[44] This condition on the difference can be interpreted to say that both effort and talent have a (weakly) higher marginal return in the higher position, which comes with increased responsibilities that allow the employee's talent and effort to have a larger impact.

Under differentiability and given Claims 1 and 2, the test is pro-t (resp. pro-e) biased if for every (e,t), $\partial u(e,t)/\partial e/(\partial u(e,t)/\partial t)$ is higher (resp. lower) than $\partial\sigma(e,t)/\partial e/(\partial\sigma(e,t)/\partial t)$, or equivalently,

$$\frac{\partial\tilde{u}(e,t)/\partial e}{\partial\tilde{u}(e,t)/\partial t} \overset{(\text{resp. } <)}{>} \frac{\partial\sigma(e,t)/\partial e}{\partial\sigma(e,t)/\partial t},$$

that is, if the marginal rate of substitution of effort for talent is higher (resp. lower) in the production function of the new position than of the current one.

[44] $u(e,t)$ could also be defined as $u(e,t) := \tilde{u}(e,t) - \sigma(e,t) - \underline{q}$, where $\underline{q}$ is the threshold productivity differential for the promotion to be beneficial to the firm (e.g., $\underline{q}$ could be the productivity differential of another employee who could be promoted instead).

4.3 College admissions and standardized testing

A college applicant's evidence e is her high school quality, grades, private tutoring received, awards, and extracurricular activities. t is her "natural" ability or drive that is not captured by e. The college wants to decide whether to admit the applicant or not. Testing amounts to requiring the applicant to take the standardized test.[45]

In the optimal mechanism, if the standardized test is not sensitive enough to talent, students can withhold evidence, which makes admission decisions imperfect at the expense of students with low evidence (e.g., those with limited access to quality education, tutoring, extracurricular activities, and opportunities to participate in competitions). Particularly, if colleges want diversity and only value talent (trying to control for the applicants' unequal backgrounds), the above problem is necessarily present under standardized testing to the extent that applicants can pretend to be from a more modest background than they actually are. Students from a privileged background have an advantage over equally good—or even somewhat better—students from a more modest background.

Even if universities do not only value talent in applicants (but instead value the total ability of the candidate, part of which is due to nurture), students from advantaged backgrounds are still favored over equally able students from disadvantaged backgrounds when the test is not sensitive enough to talent (compared to the total ability that colleges care about).

4.4 Academic job market talks

An academic job market candidate's research topic is comprised of a "mass" $b > 1$ of (uncountably infinitely many) problems.[46] $e \in [0,1]$ is the candidate's knowledge, the mass of problems which she has found answers to. t is her ability to think on her feet. More concretely, it is the probability with which she finds an answer on the spot to a problem that she has not already solved. After the candidate presents answers to a mass $e' \in [0,e]$ of problems and makes a claim about t, the hiring committee may test her. Testing amounts to posing to the candidate countably infinitely many problems randomly sampled from the mass of problems that the candidate has not already disclosed answers to.[47] Thus, if she presents answers to mass $e' \in [0,e]$ of problems and is tested, she will

[45] In this setting, the college does not condition the requirement to take a test on the candidate's report. However, when the college requires a test score, the optimal mechanism takes the same form as in the case of $c = 0$.

[46] The analysis can apply to presentations more generally (e.g., by a start-up founder to a venture capital firm).

[47] This can be understood as there being a set of problems with cardinality equal to the cardinality of $\mathbb{R}$. There is no interdependence among the problems (e.g., the agent having the answer to a problem x carries no information with regard to whether she also has the answer to a problem y). Also, the agent is equally likely to have or find an answer to any of the problems. Thus, there is no need to identify problems with an index.

answer proportion

$$p(e,t,r) := \frac{e - e' + (b - e)t}{b - e'}$$

of the problems posed to her. This is the sum of (i) the proportion $(e - e')/(b - e')$ of problems sampled from the set of problems that the candidate already has answers to (but has not disclosed them) and (ii) the proportion $(b - e)/(b - e')$ of problems sampled from the set of problems that the candidate does not already have answers to multiplied by the proportion t to which the candidate will find answers on the spot. $u(e,t)$ is the hiring committee's surplus from hiring the candidate (compared to the committee's outside option). Observing e' and $p(e,t,e')$ is equivalent to observing e' and $\sigma(e,t) := e + (b - e)t$, so the committee's problem is equivalent to the problem that we have studied.

Different testing technologies can be interpreted as different values of b. An increase in the mass b of the universe of problems makes it less likely that the candidate will be asked a question that she already has an answer to (but has not presented), thereby making the test more sensitive to talent and increasing the principal's payoff.

5 Extensions and robustness

This section first discusses optimal screening under alternative evidence structures. Then, it studies two extensions of the model: (i) one where the principal has to pay a cost *before* the agent reports her type in order to design the test, which he will then (after the agent reports her type) choose whether to administer at an additional cost and (ii) the case where evidence is not exogenous but rather endogenously produced by the agent before she interacts with the principal.

5.1 Optimal screening under alternative evidence structures

I study optimal screening under three alternative scenarios: (i) The agent cannot hide evidence, (ii) the agent can present evidence also on talent, or (iii) the agent cannot present evidence (on either dimension of her type).[48]

5.1.1 Optimal screening when the agent cannot hide evidence

Assume that e is observed by the principal. Then, given that the test is at least somewhat sensitive to t, testing reveals t and, thus, the agent's type completely. The principal's problem is decoupled: he can solve it for each e separately.[49] It is easy to see that the

[48] The case where the agent can present evidence on t but not on e is a relabeling of the main model.

[49] If the agent's type is (e_p,e,t) distributed over $[0,1]^3$, where e_p is the publicly observed part of evidence and e is the part that can be hidden, the principal can solve the problem for each e_p separately. The optimal mechanism is a collection mechanisms like the one described in section 3: one mechanism for

optimal mechanism is described by Proposition 9.

Proposition 9. Assume that the agent cannot hide evidence. In the optimal mechanism, for every level of evidence $e \in [0,1]$, if

(i) $u_{\text{accept}}(e) > \max\{u_{\text{test}}(e), 0\}$, then every agent with evidence e is accepted without a test,

(ii) $0 > \max\{u_{\text{accept}}(e), u_{\text{test}}(e)\}$, then every agent with evidence e is rejected without getting tested,

(iii) $u_{\text{test}}(e) \geq \max\{u_{\text{accept}}(e), 0\}$, then an agent with evidence e is tested and accepted if $u(e,t) \geq c$; otherwise, she is rejected without getting tested,

where $u_{\text{accept}}(e) \coloneqq \int_0^1 u(e,t)f(t)dt$ and $u_{\text{test}}(e) \coloneqq \int_0^1 (u(e,t) - c)\mathbf{I}(u(e,t) \geq c)f(t)dt$.

This implies that under pro-e (resp. pro-t biased testing), if two agents (e_1,t_1) and (e_2,t_2), $e_2 > e_1$, both need to be tested (based on their level of evidence) to get accepted, then the test score threshold that (e_1,t_1) needs to meet is lower (resp. higher) than the test score threshold that (e_2,t_2) needs to meet.[50] For example, if the principal only values talent (i.e., $u(e,t) = t - \underline{q}$ for some $\underline{q} \in (0,1)$), agents with less evidence do not need to test as high (as those with more evidence) to get accepted. This is in stark contrast with the the optimal mechanism where agents can hide evidence, in which case every agent faces the same test score cutoff.

This analysis implies the following for college admissions. If (i) college applicants can to a large extent hide their privilege and (ii) standardized tests reflect talent less than colleges value talent, then every applicant will have to achieve roughly the same test score to get admitted, and affirmative action (i.e., trying to control for unequal backgrounds, measured by e) will not be very effective in admitting a diverse class of talented students. If any of the two condition fails, affirmative action is effective. Particularly, if (i) college applicants *cannot* hide evidence of privilege and (ii) standardized tests reflect talent less than colleges value talent, then applicants from disadvantaged backgrounds will face lower test score cutoffs, and affirmative action is effective. If standardized tests are sensitive enough to talent (compared to college preferences), then testing does not create incentives for applicants to hide evidence of privilege (even if they can do so), and affirmative action is effective regardless of whether college applicants can hide evidence of privilege or not. Thus, if condition (i) or (ii) fails, a reversal of affirmative action would have significant effects on diversity in college admissions.

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each value of e_p. In the case of pro-e biased testing, for example, the optimal evidence $e^*(e_p)$ and test score cutoffs $s^*(e_p)$ depend on the observable part of evidence e_p.

[50] To see this, notice that under pro-e (resp. pro-t biased testing), if $u(e_1,t_1) = u(e_2,t_2) = c$ and $e_2 > e_1$, then $\sigma(e_1,t_1) < \sigma(e_1,t_1)$ (resp. $\sigma(e_1,t_1) > \sigma(e_1,t_1)$).

5.1.2 Optimal screening when the agent can present evidence also on talent

Consider the case where—apart from e—t also has an evidence structure. That is, agent (e,t) can report any $(e',t') \leq (e,t)$ but not $e' > e$ or $t' > t$. Then, the principal can achieve the full information first-best, inducing—without testing—every agent to present all her evidence on both e and t. The conclusion is the same if t is observed (at no cost) by the principal and e is evidence.

The comparison between this and the main model emphasizes (i) the difference in peoples' incentives to present evidence that is in principle (i.e., absent testing) favorable to them and (ii) how these incentives shape the principal's problem of evaluating them. The existence of an agent characteristic that is valuable to the principal but which (i) the agent cannot provide evidence on and (ii) the principal can only imperfectly test using a test that is overly (compared to the principal's preferences) sensitive to other (valuable) agent characteristics creates incentives for the agent to understate those other characteristics that she can actually provide favorable evidence on. This problem vanishes (i) if the agent can provide evidence on every characteristic (or if those that she cannot provide evidence on are observed by the principal) or (ii) if the test is sensitive only to the characteristic that the agent cannot provide evidence on.

These results are consistent with the finding that hiding one's effort is particularly prevalent among younger individuals. Effortless perfection (i.e., the need to seem perfect without apparent effort) and hiding one's effort have been documented among university students (Travers et al., 2015; Casale et al., 2016).The psychology literature has emphasized personality traits that may lie behind this finding. Namely, hiding effort has been identified as a unique expression of perfectionistic self-presentation (Flett et al., 2016). My model hints towards an alternative (or complementary) interpretation of this finding. If as an individual progresses in her career, her talent is revealed during all the evaluation stages that she goes through, then individuals that are further along in their career paths should have reduced incentives to hide their hard work.

5.1.3 Optimal screening when the agent cannot present evidence

Consider the case where the agent can present evidence on neither e nor t. That is, agent (e,t) can report any $(e',t') \in [0,1]^2$. We can still restrict attention to truthful mechanisms with pass-or-fail tests. Proposition 10 characterizes IC mechanisms.

Proposition 10. Assume that the agent cannot present evidence. A mechanism $M \equiv \langle T,P \rangle$ is IC if and only if

(i) $\Pi(e,t)$ is non-decreasing in t for every e,

(ii) $\Pi(e, \tau(e,s))$ is constant in e over $e \in [\underline{e}(s),\overline{e}(s)]$ for every $s \in [0,1]$, and

(iii) $(1 - T(e,t))P(e,t,\emptyset) \leq \Pi(0,0)$ for every (e,t),

where $\Pi(e,t) \equiv (1 - T(e,t))P(e,t,\emptyset) + T(e,t)$.

Condition (i) is identical to the one in Proposition 10, where e is evidence. Condition (iii) is stronger (when combined with the other two conditions) than the corresponding condition (iii) of Proposition 10. It ensures that the *least* talented agent with the *least* evidence does not have incentives to over-report her talent and/or evidence to imitate an agent (e,t) whose test score she *cannot* achieve.[51] Put differently, in order to accept some agents with higher probability (than agent (0,0)), the principal needs to test those agents with high enough probability to prevent agent (0,0) from imitating them to get accepted in case she is not tested. The condition is stricter than the one in Proposition 10 because now agents can also imitate agents with higher e to get accepted in the case that they are not tested. Thus, that agents cannot present evidence on e enhances the need to test.

Last, condition (ii) makes sure that agents do not want to under- or over-report their e to imitate agents whose test score they *can* achieve. Namely, an agent (e,t) does not want to imitate an agent (e',t') with evidence $e' > e$ (resp. $e' < e$), talent $t' < t$ (resp. $t' > t$), and equal test score $\sigma(e',t') = \sigma(e,t)$ in order to get accepted with probability $\Pi(e',t')$ instead of $\Pi(e,t)$. Notice that for any possible level of evidence e' that agent (e,t) may reveal, because of condition (i), she will want to report her talent to be as high as possible (making sure that she will be able to pass the test), up to the point where $\sigma(e',t') = \sigma(e,t)$. The condition is stricter than the one in Proposition 10 because now agents can not only understate but also overstate e. This nullifies the advantage that agents with high e have (relative to agents with the same test score but lower e) when they can present evidence.

The probability of getting accepted without a test is the same for everyone. Lemma 10 shows that when testing is costly and some agents are (optimally) accepted with higher probability than other ones, the optimal mechanism satisfies condition (iii) of Proposition 1 with equality. Under free testing or when it is not optimal to accept some agents with higher probability, it is still without loss to constrain attention to mechanisms that satisfy condition (iii) of Proposition 10 with equality.

Lemma 10. Assume that the agent cannot present evidence. Given any IC mechanism $M \equiv \langle T,P \rangle$, there exists an IC mechanism $M' \equiv \langle T',P' \rangle$ with $(1 - T'(e,t))P'(e,t,\emptyset) = \Pi'(0,0)$ for every (e,t) that is outcome-equivalent to M and has at most as high testing costs as M. For $c > 0$, if also $\Pi(e,t) > \Pi(0,0)$ for a positive measure of agent types, then M' has lower testing costs than M.

[51]Combined with conditions (i) and (ii), this also means that no other agent has incentives to imitate an agent whose test score she cannot achieve.

By Lemma 10 $\Pi(e,t) = \Pi(0,0) + T(e,t)$. Thus, the principal's objective function, $\int_0^1 \int_0^1 [\Pi(e,t)u(e,t) - cT(e,t)]\, f(e,t)dtde$, can be written as

$$\int_0^1 \int_{\underline{e}(s)}^{\overline{e}(s)} [\Pi(e,\tau(e,s))(u(e,\tau(e,s)) - c) + c\Pi(0,0)]\, f(e,\tau(e,s))deds, \tag{5}$$

which is linear in Π, so by Bauer's maximum principle, there exists an extreme Π (among $\Pi(e, \tau(e,s))$ that are constant in e and non-decreasing in s) that solves the principal's problem. Proposition 11 describes that extreme optimal mechanism.

Proposition 11. Assume that the agent cannot present evidence. There exists an optimal mechanism with $\Pi(e,t) = \mathbf{I}(\sigma(e,t) \geq s^*)$ and $T(e,t) = \Pi(e,t) - \Pi(0,0)$ for some $s^* \in (0,1)$. That is, either

(i) $s^* = 0$, and every agent is accepted without a test or

(ii) $s^* > 0$, and each agent (e,t) is (a) accepted after getting tested if $\sigma(e,t) \geq s^*$ or (b) rejected without getting tested if $\sigma(e,t) < s^*$.

The inability of agents to present evidence on one of their attributes limits the set of IC mechanisms, thereby decreasing—in most cases—the principal's optimal payoff. Assume for simplicity that the optimal mechanism is unique. When the testing technology is pro-e biased, if some—but not all–agents are optimally accepted without a test when e is actually evidence (i.e., the optimal evidence threshold for acceptance without a test lies strictly between 0 and 1), then the principal's payoff is lower if evidence is not available to the agents. When the testing technology is pro-t biased, if not all agents are optimally accepted without a test when e is actually evidence, then the principal's payoff is lower if evidence is not available to the agents. Particularly, the principal now has to choose s^* trading-off Type I and Type II errors even when σ is pro-t biased. Pro-t biased tests are not inherently better than pro-e biased tests when the agents cannot present evidence on e. Regardless of whether it is pro-t or -e biased, the more closely the test aligns with the principal's preferences, the higher the principal's optimal payoff is.

The comparison between the baseline model and the case where the agent cannot present evidence implies the following about the "signal jamming" problem that arises in career concern models (see, e.g., Holmström, 1999). Under—as in career concerns models—free monitoring of the employee's productivity,[52] if the employer can ask for hard evidence of effort, then the signal jamming problem is mitigated if productivity is sensitive enough to talent—compared to the employer's preferences for accepting (e.g., promoting) the employee. However, when productivity is *not* sensitive enough to talent, the signal jamming problem persists even if the employer can ask for evidence of effort. Agents have incentives to withhold evidence, which they should be paid information rents to reveal.

[52]The argument still holds as long as monitoring is not too costly. If it is, then no monitoring occurs, so there can be no signal jamming either.

5.2 Costly test design

Treating the testing technology σ as exogenous is reasonable in several applications. For example, in hiring by prestigious employers (section 4.1), the employee's production function in the less prestigious position is not chosen by the prestigious employer. In promotion decisions (section 4.2), the employee's production function in the current position depends on her current job description and responsibilities, which should mostly reflect the firm's regular operating needs rather than support the employer's promotion decisions.

However, in other cases (e.g., hiring decisions where testing amounts to actual tests and interviews), the principal may be able to choose the testing technology. How does his problem change in that case? Let there be a cost $C(\sigma)$ that the principal needs to pay before the interaction with the agent, so that she can use testing technology σ during the interaction with the agent. Indeed, it is reasonable that the principal needs to design a test (if she designs a test at all) *before* the interaction with the agent due to time constraints and the complexity of designing a test. During the interaction with the agent, the principal can only choose whether to administer the test at cost c. Then, the principal's problem can be solved in two steps: (i) finding the optimal mechanism for each possible testing technology $\sigma \in \Sigma$, and then (ii) choosing the optimal testing technology $\sigma^* \in \Sigma$ form the set Σ of conceivable testing technologies. The solution to the first step is the one we have already described.[53]

If tests that are more sensitive to talent are more expensive to devise, our results imply that as long as the test is under-sensitive (compared to the principal's preferences) to talent, there are gains from increasing its sensitivity to it, which the principal will have to compare to the cost of making the test more sensitive to talent. The principal will want to make the tests at most as sensitive to talent as his preferences are, since tests that are overly sensitive to talent are as effective as those that are exactly aligned with the principal's preferences.[54]

However, when agents cannot present evidence on any of their attributes, the principal can always gain from finely calibrating the test's sensitivity (to the agent's attributes) to align it with his preferences. Regardless of whether it is pro-t or -e biased, the more closely the test aligns with the principal's preferences, the higher the principal's payoff is.

[53] That is, assuming that Σ contains only pro-t and pro-e biased testing technologies (and possibly a testing technology that exactly coincides with the principal's preferences). Also, it is easy to see that there are no gains from designing multiple tests to the extent that all tests have the same administration cost c.

[54] Still, if there is uncertainty over a test's properties, in a robust approach, the principal will not need to worry about making the test overly sensitive to talent.

5.3 Endogenous evidence production

If the agent produces evidence before the interaction with the principal, in some applications, the principal may be able to affect the agent's evidence production by committing to a mechanism *before* the agent produces evidence. Indeed, in promotion decisions (section 4.2), the employer may use the prospect of promotion to incentivize the employee to exert effort.[55] Treating evidence as exogenous is more in line with other applications. For instance, in hiring decisions (section 4.1), a single employer has little labor market power to affect the candidate's (effort to obtain) credentials. Similarly, in college admissions (section 4.3), a single college cannot affect how hard high school students study.

Our characterization of the optimal mechanism then still applies—even if evidence is endogenous, as long as the principal cannot influence evidence production by committing ex ante to a mechanism. Let the agent's talent t follow a distribution with density g and support [0,1]. Taking as given the principal's mechanism, summarized by evidence and test score thresholds (e^*,s^*), the agent exerts costly effort $x \in \mathbb{R}_+$ to produce evidence.[56] Exerting effort x has cost $C_t(x)$, non-decreasing in x. Evidence is distributed, conditional on x, according to density function $h_x(e)$ with support [0,1]. Denote by $x^*(t)$ the equilibrium level of effort by type t. An equilibrium is a fixed point (x^*,e^*,s^*) where $x^* : [0,1] \to \mathbb{R}_+$ is a best-response to (e^*,s^*) and (e^*,s^*) is a best-response to x^* (i.e., the thresholds (e^*,s^*) that solve the principal's problem when the agent's type has density $f(e,t) = g(t)h_{x^*(t)}(e)$). (x^*,e^*,s^*) can be interpreted as a symmetric equilibrium where each of multiple "effort-taking" principals chooses thresholds (e^*,s^*).

While a detailed analysis of endogenous evidence production is beyond the scope of this paper, the following observation shows the importance of the fact that the optimal mechanism has been characterized under minimal assumptions on the agent's type distribution (i.e., that it admits a full-support density). In equilibrium, by exerting effort x, agent t will earn expected payoff $\int_{\min\{e^*,\varepsilon(t,s^*)\}}^1 h_x(e)de - C_t(x)$, where $\varepsilon(t,s)$ is implicitly given by $\sigma(\varepsilon(t,s),t) = s$. If, for example, $c = 0$, $e^* = 1$ and so $x^*(t) = 0$ for every $t \geq \tau(0,s^*)$ or $t \leq \tau(1,s^*)$. That is, agents so talented that they are accepted even without evidence and agents so untalented that they are rejected even if they present evidence $e = 1$ do not exert effort. More generally, the agent's incentives to exert effort—and, thus, effort itself—will often be non-monotone in t.[57] Thus, evidence and talent may be stochastically dependent in complicated ways.

[55] Still, if promotions are not the main motive for the employee to exert effort (e.g., a bonus could be the main motive), effort can still be taken as approximately exogenous. For example, if the employee can obtain a higher position by changing employers, the prospect of promotion in the current company may not significantly affect her effort.

[56] Notice that the optimal mechanism can always be summarized by these two thresholds. Under pro-t biased testing, there is only an evidence threshold.

[57] For example, let $x \in [0,1]$ with $C_t(x) := \xi(t)x^2/2$, where $\xi(t) > 0$ is decreasing in t, $u(e,t) := \gamma_u e + (1-\gamma_u)t - \underline{q}$, $\sigma(e,t) := \gamma_s e + (1-\gamma_s)t$, where $1 > \gamma_s > \gamma_u$, and $H_x(e) := \int_0^e h_x(y)dy = -2xe(1-e) + e(2-e)$.

6 Conclusion

This paper has proposed a model of multidimensional screening, where an agent (she) with two attributes—evidence and talent—presents evidence (i.e., verifiably discloses possibly part of her evidence) and is (possibly) tested at a cost by the principal (he), who then decides whether to accept or reject the agent. The agent cannot unilaterally prove anything about her talent. The test delivers a signal (i.e., the test score)—increasing in both evidence and talent—of the agent's type and the principal (weakly) values both evidence and talent in an agent. If the principal is going to test the agent, then the agent may have incentives to hide evidence—although the principal values evidence—to influence how the principal interprets the test result. Particularly, she may want to hide evidence to make the principal attribute the test result to talent, thereby overestimating her talent.

This problem arises when the test (score) is less sensitive to talent than talent is valuable to the principal. In that case, the optimal mechanism features two types of inefficiencies, both of which favor high-evidence agents over low-evidence ones: (i) It accepts some undeserving agents without testing them but rather only by asking them to present a certain level of evidence, and (ii) even among agents who cannot meet that evidence threshold, it accepts (after testing) some undeserving agents with high evidence but low talent, while it rejects some deserving agents with high talent but low evidence. Remarkably, this is the structure of the optimal mechanism even when the principal *only* values talent. The principal still optimally rewards evidence even though it is worthless to him.

The results indicate how less worthy individuals with high credentials or effort to show are favored—by an optimal and objective evaluation mechanism—over more worthy ones, who have however lower credentials (or effort to show). Ivy-Leaguers are immediately hired by prestigious employers, while those from more modest backgrounds have to go through less prestigious employers to prove their worth before landing a prestigious position. Even controlling for the fact that they need to first take a less prestigious position, they may still be at a disadvantage when trying to transition to a more prestigious one. Hard-working employees with mediocre managerial skills are promoted to managerial positions over less hard-working ones who would, however, make better managers.

Then,

$$x^*(t) = \begin{cases} 0 & \text{if } t \leq \dfrac{s^* - \gamma_s}{1-\gamma_s} \\ \dfrac{2[s^* - (1-\gamma_s)t][\gamma_s - s^* + (1-\gamma_s)t]}{\gamma_s^2 \xi(t)} & \text{if } t \in \left(\dfrac{s^* - \gamma_s}{1-\gamma_s}, \dfrac{s^*}{1-\gamma_s}\right) \\ 0 & \text{if } t \geq \dfrac{s^*}{1-\gamma_s}. \end{cases}$$

Last, in college admissions, high school students from privileged backgrounds have an advantage over equally good (or even better) students from modest backgrounds—even if colleges value diversity and try to control for the applicants' unequal backgrounds but their evaluation mechanisms (e.g., standardized tests) are sensitive to the applicant's prior training. Affirmative action (i.e., trying to control for college applicants' unequal backgrounds) has limited effectiveness if two conditions are satisfied: (i) Applicants have considerable room to hide evidence of privilege, and (ii) standardized test scores reflect talent less than colleges value talent. If any of the two condition fails, then affirmative action is effective, and we should expect its reversal to have significant effects on diversity in college admissions.

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A Proofs

Proof of Lemma 1 Take an IC mechanism $M \equiv \langle T,P\rangle$. Construct the mechanism $M' \equiv \langle T',P'\rangle$ with (i) $P'_{at}(e,t) = 1$, (ii) $T'(e,t) = T(e,t)P_{at}(e,t) \leq T(e,t)$, and (iii) $P'(e,t,\emptyset) = (1-T(e,t))P(e,t,\emptyset)/(1-T'(e,t))$ for any (e,t).[58]

We have then that (a) $T'(e,t)P'_{at}(e,t) = T(e,t)P_{at}(e,t)$, (b) $(1-T'(e,t))P'(e,t,\emptyset) = (1-T(e,t))P(e,t,\emptyset)$ and (c) $\Pi'(e,t) = \Pi(e,t)$ for any (e,t). (a)-(c) combined imply that the problem of every agent type under M' is the same as it was under M, so M' is also IC. (c) means that M' is outcome-equivalent to M.

Last, to see why the second part is true, notice that for $c > 0$, M' saves on testing costs compared to M if there exists (a positive measure of types) (e,t) with $T(e,t) > 0$ and $P_{at}(e,t) < 1$. **Q.E.D.**

Proof of Lemma 2 Let $M \equiv \langle T,P\rangle$ be an IC mechanism. Then, construct the mechanism $M' := \langle T',P'\rangle$ with (i) $T'(e,0) = 0$ for every e and $T'(e,t) = T(e,t)$ for every (e,t) with $t > 0$, (ii) $P'(e,t,\emptyset) = P(e,t,\emptyset) + (T(e,t) - T'(e,t))\,(1 - P(e,t,\emptyset))$ for every (e,t), and (iii)

$$P'(e,t,s) := \begin{cases} 0 & \text{if } s < \sigma(e,t) \\ P_{at}(e,t) & \text{if } s \geq \sigma(e,t) \end{cases}$$

for every (e,t) and $s \in [0,1]$.

If every type reports truthfully, M' accepts each agent type with the same probability that M does, so it remains to show that M' is IC.

By IC of M we have that for every (e,t)

$$(e,t) \in \underset{(\widehat{e},\widehat{t})\leq(e,1)}{\arg\max} \left\{(1 - T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t},\emptyset) + T(\widehat{e},\widehat{t})\mathbf{I}\left(\sigma(e,t) \geq \sigma(e',t')\right)\right\}. \tag{6}$$

[58] In $P'(e,t,\emptyset)$, if $T'(e,t) = 1$, cancel $(1 - T(e,t))$ in the numerator and $(1 - T'(e,t))$ in the denominator.

By construction, we have that $T(\widehat{e},\widehat{t}) = T'(\widehat{e},\widehat{t})$ and $P(\widehat{e},\widehat{t},\emptyset) = P'(\widehat{e},\widehat{t},\emptyset)$ for every $(\widehat{e},\widehat{t})$ with $\widehat{t} > 0$, so no type (e,t) has incentives to imitate any type $(\widehat{e},\widehat{t})$ with $\widehat{t} > 0$ under mechanism M'. Also, for any (e,t) and any $(\widehat{e},\widehat{t})$ with $\widehat{t} = 0$ and $\widehat{e} \leq e$, $\sigma(e,t) \geq \sigma(\widehat{e},\widehat{t})$, which means that the payoff of (e,t) from reporting $(\widehat{e},\widehat{t})$ with $\widehat{t} = 0$ is equal to

$$\begin{aligned}(1 - T'(\widehat{e},\widehat{t}))P'(\widehat{e},\widehat{t},\emptyset) + T'(\widehat{e},\widehat{t})\mathbf{I}\left(\sigma(e,t) \geq \sigma(e',t')\right) = \\ (1-0)P'(\widehat{e},\widehat{t},\emptyset) = P(\widehat{e},\widehat{t},\emptyset) + \left(T(\widehat{e},\widehat{t}) - T'(\widehat{e},\widehat{t})\right)\left(1 - P(\widehat{e},\widehat{t},\emptyset)\right) = \\ (1 - T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t},\emptyset) + T(\widehat{e},\widehat{t}) = (1 - T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t},\emptyset) + T(\widehat{e},\widehat{t})\mathbf{I}\left(\sigma(e,t) \geq \sigma(e',t')\right) \\ \leq (1 - T(e,t))P(e,t,\emptyset) + T(e,t),\end{aligned}$$

where the inequality follows from (6). Thus, no type (e,t) has incentives to imitate any type $(\widehat{e},\widehat{t})$ with $\widehat{t} = 0$ under mechanism M'. We conclude that M' is IC. **Q.E.D.**

Proof of Proposition 1 Denote the total probability with which type (e,t) is accepted if she reports $(\widehat{e},\widehat{t})$ (with $\widehat{e} \leq e$) by

$$\widetilde{P}(\widehat{e},\widehat{t};e,t) := (1 - T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t},\emptyset) + T(\widehat{e},\widehat{t})\mathbf{I}\left(\sigma(e,t) \geq \sigma(\widehat{e},\widehat{t})\right).$$

Also, define condition (iii') (a strengthening of condition (iii)) to say that $(1 - T(e,t))P(e,t,\emptyset) \leq \Pi(e',0)$ for every e,t,e' with $e \leq e'$.

Step 1: I first show that condition (i) is necessary for IC by showing the contrapositive. Assume that for some e,t_1,t_2 with $t_2 > t_1$, $\Pi(e,t_2) < \Pi(e,t_1)$. Then, IC of type (e,t_2) is violated, since $\widetilde{P}(e,t_1;e,t_2) = \Pi(e,t_1) > \Pi(e,t_2)$, that is, (e,t_2) can imitate (e,t_1) to (reach (e,t_1)'s test score threshold and) get accepted with higher probability that she would if she truthfully reported her type.

Step 2: I now show that condition (iii') is necessary for IC by showing the contrapositive.[59] Assume that for some e,e',t with $e' \geq e$, $(1 - T(e,t))P(e,t,\emptyset) > \Pi(e',0)$. Then, IC of type $(e',0)$ is violated, since $\widetilde{P}(e,t;e',0) \geq (1 - T(e,t))P(e,t,\emptyset) > \Pi(e',0)$, that is, $(e',0)$ can imitate (e,t) to get accepted with higher probability that she would if she truthfully reported her type (even if she cannot achieve (e,t)'s test score).

Step 3: I now show that provided that (i) and (iii') are satisfied, $\Pi(r,\tau(r,\sigma(e,t)))$ being non-decreasing in r over $r \in [\underline{e}(\sigma(e,t)),e]$ for every (e,t) is necessary and sufficient for IC.

IC of type (e,t) is satisfied if and only if

$$\max_{(\widehat{e},\widehat{t}) \leq (e,1)} \left[(1 - T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t};\emptyset) + T(\widehat{e},\widehat{t})\mathbf{I}\left(\sigma(e,t) \geq \sigma(\widehat{e},\widehat{t})\right)\right] = \Pi(e,t). \tag{7}$$

Assume that conditions (i) and (iii') are satisfied. Then, $\Pi(e,t) \geq \Pi(e,0) \geq (1 -$

[59]That $P(e,0,\emptyset) = \Pi(e,0)$ follows from $T(e,0) = 0$.

$T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t},\emptyset)$ for any $(\widehat{e},\widehat{t})$ with $\widehat{e} \leq e$. Therefore, (7) is equivalent to

$$\max_{(\widehat{e},\widehat{t}) \in \{(x,y) \in [0,1]^2 : x \leq e \text{ and } \sigma(e,t) \geq \sigma(x,y)\}} \Big[(1 - T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t};\emptyset) + T(\widehat{e},\widehat{t})\Big] = \Pi(e,t). \tag{8}$$

Given that $\Pi(e,t)$ is non-decreasing in t (condition (i)), (8) can equivalently be written as

$$\max_{r \in [\underline{e}(\sigma(e,t)),e]} \{[1 - T(r, \tau(r,\sigma(e,t)))]P(r,\tau(r,\sigma(e,t)),\emptyset) + T(r,\tau(r,\sigma(e,t)))\} = \Pi(e,t)$$

or equivalently,

$$e \in \operatorname*{arg\,max}_{r \in [\underline{e}(\sigma(e,t)),e]} \Pi(r, \tau(r,\sigma(e,t))). \tag{9}$$

Thus, IC is satisfied for every type if and only if for every (e,t), (9) is satisfied. This is true if and only if $\Pi(r, \tau(r,\sigma(e,t)))$ is non-decreasing in r for $r \in [\underline{e}(\sigma(e,t)),e]$ for every (e,t).

That the latter is sufficient for (9) to hold for every (e,t) is immediate. I show necessity by showing the contrapositive. Assume that for some (e,t), $\Pi(r, \tau(r,\sigma(e,t)))$ is *not* non-decreasing in r for $r \in [\underline{e}(\sigma(e,t)),e]$. That is, for some (e,t) there exist r_1, r_2 with $\underline{e}(\sigma(e,t)) \leq r_1 < r_2 \leq e$ such that $\Pi(r_2, \tau(r_2,\sigma(e,t))) < \Pi(r_1, \tau(r_1,\sigma(e,t)))$. Then,

$$r_2 \notin \operatorname*{arg\,max}_{x \in [\underline{e}(\sigma(e,t)),r_2]} \Pi(x, \tau(x,\sigma(e,t))).$$

Namely, IC of type $(r_2, \tau(r_2,\sigma(e,t)))$ is violated, as she prefers to imitate type $(r_1, \tau(r_1,\sigma(e,t)))$.

Step 4: It is easy to see that $\Pi(r, \tau(r,\sigma(e,t)))$ being non-decreasing in r over $r \in [\underline{e}(\sigma(e,t)),e]$ for every (e,t) is equivalent to condition (ii).

Step 5: Finally, notice that provided that conditions (i) and (ii) hold, conditions (iii) and (iii') are equivalent. That (iii') implies (iii) is immediate. We will show that the opposite direction also holds. Assume that conditions (i), (ii), and (iii) hold. Then, for any e,e',t with $e' \geq e$

$$\Pi(e',0) \geq \Pi(e, \tau(e,\sigma(e',0))) \geq \Pi(e,0) \geq (1 - T(e,t))P(e,t,\emptyset),$$

where the first inequality follows from condition (ii),[60] the second from condition (i), and the third from condition (iii). **Q.E.D.**

Proof of Lemma 3 Take any IC mechanism $M \equiv \langle T,P \rangle$. Condition (iii) of Proposition 1 says that $\Pi(e,0) \geq (1 - T(e,t))P(e,t,\emptyset)$ for any (e,t). Then, construct the mechanism

[60] The first inequality assumes that $e \geq \underline{e}(\sigma(e',0))$. If this is not the case, using conditions (i) and (ii) iteratively, we can still show that $\Pi(e',0) \geq \Pi(e,0)$.

$M' \coloneqq \langle T', P' \rangle$ with[61]

$$\begin{aligned} T'(e,t) &\coloneqq \Pi(e,t) - \Pi(e,0) = (1 - T(e,t))P(e,t,\emptyset) + T(e,t) - \Pi(e,0) \\ &\leq \Pi(e,0) + T(e,t) - \Pi(e,0) = T(e,t), \quad \text{and} \\ P'(e,t,\emptyset) &\coloneqq \frac{\Pi(e,0)}{1 - \Pi(e,t) + \Pi(e,0)} \geq \frac{(1 - T(e,t))P(e,t,\emptyset)}{1 - \Pi(e,t) + (1 - T(e,t))P(e,t,\emptyset)} = P(e,t,\emptyset) \end{aligned}$$

for every (e,t), where the inequalities follow from $\Pi(e,0) \geq (1 - T(e,t))P(e,t,\emptyset)$.

By construction we have that $\Pi'(e,t) = \Pi(e,t)$ for every (e,t), so M' satisfies conditions (i) and (ii) of Proposition 1. By construction, we also have that for every (e,t)

$$\Pi'(e,0) = \Pi(e,0) = (1 - T'(e,t))P'(e,t,\emptyset),$$

so M' also satisfies condition (iii) of Proposition 1. Therefore, M' is IC.

Last, to see why the second part is true, notice that for $c > 0$, M' saves on testing costs compared to M if there exists (a positive measure of) (e,t) with $P(e,t,\emptyset)(1 - T(e,t)) < \Pi(e,0)$, since $T'(e,t) < T(e,t)$ for such (e,t). **Q.E.D.**

Proof of Lemmata 4 and 5 I prove the more general Lemma 5. It is useful to look at the principal's choice as a function $\Pi(e,\tau(e,s))$ of (e,s). The objective function (3) is continuous and linear (and, thus, convex) in Π. By Bauer's maximum principle, it follows that there exists a maximizing function $(e,s) \to \Pi(e,\tau(e,s))$ that is an extreme point of the set of non-decreasing functions from $\{(e,s) \in [0,1]^2 : e \in [\underline{e}(s),\overline{e}(s)]\}$ to $[0,1]$.[62] Last, a function $(e,s) \to \Pi(e,\tau(e,s))$ is an extreme point of that set if and only if $\Pi(e,\tau(e,s)) \in \{0,1\}$ for all (e,s) is its domain.[63] The proof of this part is analogous to the one of Lemma 2.7 in Börgers (2015).

If direction: consider any non-decreasing (in e and s) Π with $\Pi(e,\tau(e,s)) \in \{0,1\}$ for all (e,s), and take any function $g : \{(e,s) \in [0,1]^2 : e \in [\underline{e}(s),\overline{e}(s)]\} \to \mathbb{R}$ such that $g(e^*,s^*) \neq 0$ for some (e^*,s^*). If $g(e^*,s^*) > 0$ and $\Pi(e^*,\tau(e^*,s^*)) = 0$, then $\Pi(e^*,\tau(e^*,s^*)) - g(e^*,s^*) < 0$. If $g(e^*,s^*) > 0$ and $\Pi(e^*,\tau(e^*,s^*)) = 1$, then $\Pi(e^*,\tau(e^*,s^*)) + g(e^*,s^*) > 1$. Similarly, if $g(e^*,s^*) < 0$ and $\Pi(e^*,\tau(e^*,s^*)) = 0$, then $\Pi(e^*,\tau(e^*,s^*)) + g(e^*,s^*) < 0$. If $g(e^*,s^*) < 0$ and $\Pi(e^*,\tau(e^*,s^*)) = 1$, then $\Pi(e^*,\tau(e^*,s^*)) - g(e^*,s^*) > 1$. Thus, Π is an extreme point.

Only if direction: now consider any non-decreasing (in e and s) Π with $\Pi(e^*,\tau(e^*,s^*)) \notin \{0,1\}$ for some (e^*,s^*). Construct function g as follows. $g(e,s) \coloneqq \Pi(e,\tau(e,s))$ for every (e,s) such that $\Pi(e,\tau(e,s)) \leq 1/2$ and $g(e,s) \coloneqq 1 - \Pi(e,\tau(e,s))$ for every (e,s) such that $\Pi(e,\tau(e,s)) > 1/2$. $g(e^*,s^*) \in (0,1)$, so $g \neq 0$. Consider the function $(e,s) \mapsto \Pi(e,\tau(e,s)) + g(e,s)$. Take any e_1,e_2,s with $e_2 \geq e_1$ and observe that if $\Pi(e_2,\tau(e_2,s)) > 1/2$,

[61] For (e,t) such that $\Pi(e,t) = 1$ and $\Pi(e,0) = 0$, set $P'(e,t,\emptyset) = 0$.

[62] Observe that this set of functions is convex and compact in the norm topology.

[63] More precisely, this should hold for almost all (e,s) is its domain (see Börgers, 2015).

then

$$\Pi(e_2,\tau(e_2,s)) + g(e_2,s) = 1 \geq \Pi(e_1,\tau(e_1,s)) + g(e_1,s)$$

since by construction $\Pi(e,\tau(e,s)) + g(e,s) \leq 1$ for every (e,s), while if $\Pi(e_2,\tau(e_2,s)) \leq 1/2$, then (since Π is non-decreasing) $\Pi(e_1,\tau(e_1,s)) \leq \Pi(e_2,\tau(e_2,s)) \leq 1/2$, and so

$$\Pi(e_2,\tau(e_2,s)) + g(e_2,s) = 2\Pi(e_2,\tau(e_2,s)) \geq 2\Pi(e_1,\tau(e_1,s)) = \Pi(e_1,\tau(e_1,s)) + g(e_1,s).$$

Similarly, it can be seen that $\Pi(e,\tau(e,s_2)) + g(e,s_2) \geq \Pi(e,\tau(e,s_1)) + g(e,s_1)$ for any s_1,s_2,e with $s_2 \geq s_1$. Also $\Pi(e,\tau(e,s)) + g(e,s) \in [0,1]$ for every (e,s). Therefore, the function $(e,s) \mapsto \Pi(e,\tau(e,s)) + g(e,s)$ lies in the set of non-decreasing functions from $\{(e,s) \in [0,1]^2 : e \in [\underline{e}(s),\overline{e}(s)]\}$ to $[0,1]$. Similarly, it can be seen that the function $(e,s) \mapsto \Pi(e,\tau(e,s)) - g(e,s)$ lies in the set of non-decreasing functions from $\{(e,s) \in [0,1]^2 : e \in [\underline{e}(s),\overline{e}(s)]\}$ to $[0,1]$. Thus, Π is not an extreme point. **Q.E.D.**

Proof of Claims 1 and 2 The first part is immediate. The total derivative of $u(e,\tau(e,s))$ with respect to e is equal to

$$\begin{aligned}\frac{du(e,\tau(e,s))}{de} &= \frac{\partial u(e,\tau(e,s))}{\partial e} + \frac{\partial \tau(e,s)}{\partial e}\left.\frac{\partial u(e,t)}{\partial t}\right|_{t=\tau(e,s)} \\ &= \frac{\partial u(e,t)}{\partial e} + \frac{\partial \sigma(e,t)/\partial e}{\partial \sigma(e,t)/\partial t}\left.\frac{\partial u(e,t)}{\partial t}\right|_{t=\tau(e,s)},\end{aligned}$$

and the second part follows. **Q.E.D.**

Proof of Proposition 2 We need to show that $\Pi(e,t) = \mathbf{I}\,(u(e,t) > 0)$ satisfies conditions (i) and (ii) of Proposition 1.

Condition (i): Since $\Pi(e,t) \in \{0,1\}$ for every (e,t), it suffices to show that for any (e,t), if $\Pi(e,t) = 1$, then $\Pi(e,t') = 1$ for every $t' \geq t$. Indeed, we have that for any (e,t)

$$\Pi(e,t) = 1 \implies u(e,t) > 0 \implies u(e,t') > 0 \text{ for every } t' \geq t,$$

where the second implication follows since $u(e,t)$ is non-decreasing in t.

Condition (ii): Similarly, it suffices to show that for any (r,s), if $\Pi(r,\tau(r,s)) = 1$, then $\Pi(r',\tau(r',s)) = 1$ for every $r' \in [r,\overline{e}(s)]$. Indeed, we have that for any (r,s), $\Pi(r,\tau(r,s)) = 1$ implies that $u(r,\tau(r,s)) > 0$, which in turn implies that $u(r',\tau(r',s)) > 0$ for every $r' \in [r,\overline{e}(s)]$.

To see why the last part follows, assume instead that $u(r',\tau(r',s)) \leq 0$ for some $r' \in [r,\overline{e}(s)]$. Particularly, it must be $r' > r$. Since σ is pro-t biased, there exists e_s such that if $e > e_s$ (resp. $e \leq e_s$) and $\sigma(e,t) = s$, then $u(e,t) > 0$ (resp. $u(e,t) \leq 0$). We have that $u(r',\tau(r',s)) \leq 0$, so σ being pro-t biased implies that $r' \leq e_s$. But $r' > r$, so $r < e_s$, and since $\sigma(r,\tau(r,s)) = s$, σ being pro-t biased implies that $u(r,\tau(r,s)) \leq 0$, a contradiction. **Q.E.D.**

Proof of Proposition 3 *Step 1:* In definition 4 of pro-e biased testing, for s such that $u(e,t) > c = 0$ (resp. $u(e,t) \leq 0$) for every $(e,t) \in I_\sigma(s)$, e_s is not uniquely defined. In that case, for s such that $u(e,t) > 0$ (resp. $u(e,t) \leq 0$) for every $(e,t) \in I_\sigma(s)$, set $e_s = \overline{e}(s)$ (resp. $e_s = \underline{e}(s)$). We will show that (under pro-e biased testing) e_s is non-decreasing in s. Take any $\underline{s},\overline{s} \in [0,1]$ with $\overline{s} > \underline{s}$, and define $S \coloneqq (e_{\overline{s}},e_{\underline{s}}) \cap [\underline{e}(\overline{s}), \overline{e}(\overline{s})] \cap [\underline{e}(\underline{s}), \overline{e}(\underline{s})]$.

Step 1, case 1: If $S = \emptyset$, then $e_{\underline{s}} \leq e_{\overline{s}}$. To see this, consider the following two subcases.

Step 1, case 1(a): if $\underline{e}(\overline{s}) \geq \overline{e}(\underline{s})$, then $e_{\underline{s}} \leq \overline{e}(\underline{s}) \leq \underline{e}(\overline{s}) \leq e_{\overline{s}}$, so $e_{\underline{s}} \leq e_{\overline{s}}$, a contradiction.

Step 1, case 1(b): if $\underline{e}(\overline{s}) < \overline{e}(\underline{s})$, then $S = (e_{\overline{s}},e_{\underline{s}}) \cap [\underline{e}(\overline{s}), \overline{e}(\underline{s})]$. Since $S = \emptyset$, either $\underline{e}(\overline{s}) \geq e_{\underline{s}}$ or $\overline{e}(\underline{s}) \leq e_{\overline{s}}$. If $\underline{e}(\overline{s}) \geq e_{\underline{s}}$, then $e_{\underline{s}} \leq \underline{e}(\overline{s}) \leq e_{\overline{s}}$, so $e_{\underline{s}} \leq e_{\overline{s}}$, a contradiction. Similarly, if $\overline{e}(\underline{s}) \leq e_{\overline{s}}$, then $e_{\underline{s}} \leq \overline{e}(\underline{s}) \leq e_{\overline{s}}$, so $e_{\underline{s}} \leq e_{\overline{s}}$, a contradiction.

Step 1, case 2: We now prove by contradiction that if $S \neq \emptyset$, then $e_{\underline{s}} \leq e_{\overline{s}}$. To this end, assume that $S \neq \emptyset$ and $e_{\underline{s}} > e_{\overline{s}}$. Given that $S \neq \emptyset$, we can take some $e^* \in S$. Since $e^* \in [\underline{e}(\underline{s}), \overline{e}(\underline{s})]$ and σ is continuous, there exists $t^* \in [0,1]$ such that $\sigma(e^*,t^*) = \underline{s}$. Since σ is pro-e biased and $e^* < e_{\underline{s}}$, it follows that $u(e^*,t^*) > 0$. Similarly, since σ is pro-e biased, $e^* > e_{\overline{s}}$, and $e^* \in [\underline{e}(\overline{s}), \overline{e}(\overline{s})]$, there exists $t^{**} \in [0,1]$ such that $\sigma(e^*,t^{**}) = \overline{s}$ and $u(e^*,t^{**}) \leq 0$. Also, because $\overline{s} > \underline{s}$ and $\sigma(e,t)$ is increasing in t, $t^{**} > t^*$. Overall, we have $t^{**} > t^*$ and $u(e^*,t^*) > 0 \geq u(e^*,t^{**})$, a contradiction to $u(e,t)$ being non-decreasing in t.

Step 2: Given e_s, define also t_s implicitly given by $\sigma(e_s,t_s) = s$. We have then that for every test score $s \in [0,1]$, (e_s,t_s) is the "threshold" agent who lies on the iso-test-score curve $I_\sigma(s)$. That is, any other agent (e,t) on that iso-test-score curve with $e < e_s$ (resp. $e > e_s$) gives—if accepted—a positive (resp. negative) payoff to the principal.

We divide the problem of finding an optimal IC mechanism in three parts. First, we fix an arbitrary "partial" IC mechanism $s \mapsto \Pi(e_s,t_s)$ for every $s \in [0,1]$. Then, we complete that partial IC mechanism (i.e., we assign a value to $\Pi(e,t)$ for every (e,t) for which $\Pi(e,t)$ has no been assigned a value in the first step), so that the complete mechanism is IC and optimal given the fixed partial mechanism. Finally, we find an optimal partial mechanism.

Step 3: Fix the value of $\Pi(e_s,t_s)$ for every $s \in [0,1]$ such that these values are part of some IC mechanism.[64] Given that e_s is non-decreasing in s, by Proposition 1, the values of $\Pi(e_s,t_s)$ are part of some IC mechanism if and only if $\Pi(e_s,t_s)$ is non-decreasing in s. Therefore, by Proposition 5, there exists an optimal mechanism with $\Pi(e_s,t_s) = \mathbf{I}(s \geq \underline{s})$ for some $\underline{s} \in [0,1]$.

Step 4: It follows then that for IC to be satisfied by the complete mechanism, it must be that (i) $\Pi(e,t) = 1$ for every (e,t) such that $e > e_s$ and $\sigma(e,t) = s$ for some $s \geq \underline{s}$ and

[64]That is, fix the value of $\Pi(e_s,t_s)$ for every $s \in [0,1]$ to be such that there exists IC $\Pi : [0,1]^2 \to [0,1]$ that agrees with the values of $\Pi(e_s,t_s)$ for every $s \in [0,1]$.

(ii) $\Pi(e,t) = 0$ for every (e,t) such that $e < e_s$ and $\sigma(e,t) = s$ for some $s < \underline{s}$. Also, since (e_s,t_s) is the "threshold" agent, the principal wants to make $\Pi(e,t)$ as high (resp. low) as possible for every (e,t) such that $e < e_s$ (resp. $e > e_s$). Thus, given the IC constraint, it is optimal to set (i) $\Pi(e,t) = 1$ for every (e,t) such that $e < e_s$ and $\sigma(e,t) = s$ for some $s \geq \underline{s}$ and (ii) $\Pi(e,t) = 0$ for every (e,t) such that $e > e_s$ and $\sigma(e,t) = s$ for some $s < \underline{s}$. **Q.E.D.**

Proof of Proposition 4 By IC conditions (i) and (ii) of Proposition 1, any IC mechanism has $\Pi(e,0)$ non-decreasing in e. Thus, given Lemma 5, there exists an optimal mechanism with $\Pi(e,0) = \mathbf{I}(e \geq e^*)$ for some $e^* \in [0,1]$. The objective function (3) then becomes

$$\int_0^1 \int_{\min\{\underline{e}(s),e^*\}}^{\min\{\overline{e}(s),e^*\}} \left[\Pi(e,\tau(e,s))(u(e,\tau(e,s)) - c)\right] f(e,\tau(e,s))deds \\ + \int_0^1 \int_{e^*}^1 u(e,t)f(e,t)dedt.$$

The mechanism affects the second term only through e^*. Given e^*, setting $\Pi(e,t) = \mathbf{I}\left(u(e,t) > c \text{ or } e \geq e^*\right)$ maximizes the first term and—given that σ is pro-t biased—makes the mechanism IC, since it satisfies conditions (i) and (ii) of Proposition 1. $T(e,t) = \mathbf{I}(u(e,t) \geq c \text{ and } e < e^*)$ is backed out from Lemma 5. **Q.E.D.**

Proof of Proposition 5 By IC conditions (i) and (ii) of Proposition 1, any IC mechanism has $\Pi(e,0)$ non-decreasing in e. Thus, given Lemma 5, there exists an optimal mechanism with $\Pi(e,0) = \mathbf{I}(e \geq e^*)$ for some $e^* \in [0,1]$. The objective function (3) then becomes

$$\int_0^1 \int_{\min\{\underline{e}(s),e^*\}}^{\min\{\overline{e}(s),e^*\}} \left[\Pi(e,\tau(e,s))(u(e,\tau(e,s)) - c)\right] f(e,\tau(e,s))deds \\ + \int_0^1 \int_{e^*}^1 u(e,t)f(e,t)dedt.$$

The mechanism affects the second term only through e^*. Given e^*, maximizing the first term is equivalent to the problem studied by Proposition 3 with the principal's payoff function given by $u(e,t) - c$. Thus, for $e < e^*$, $\Pi(e,t) = \mathbf{I}(\sigma(e,t) \geq s^*$ for some $s^* \in [0,1]$ maximizes the first term (under the IC conditions, when the problem is restricted to $(e,t) < (e^*,1)$). The complete mechanism then has $\Pi(e,t) = \mathbf{I}(\sigma(e,t) \geq s^* \text{ or } e \geq e^*)$, which satisfies conditions (i) and (ii) of Proposition 1. $T(e,t) = \mathbf{I}(\sigma(e,t) \geq s^* \text{ and } e < e^*)$ is backed out from Lemma 5. **Q.E.D.**

Proof of Lemma 6 Take any two agents $(\boldsymbol{e},\boldsymbol{t})$ and $(\boldsymbol{e},\boldsymbol{t}')$ with $\sigma(\boldsymbol{e},\boldsymbol{t}) = \sigma(\boldsymbol{e},\boldsymbol{t}')$. $(\boldsymbol{e},\boldsymbol{t})$'s IC requires $\Pi(\boldsymbol{e},\boldsymbol{t}) \geq \Pi(\boldsymbol{e},\boldsymbol{t}')$. $(\boldsymbol{e},\boldsymbol{t}')$'s IC requires $\Pi(\boldsymbol{e},\boldsymbol{t}') \geq \Pi(\boldsymbol{e},\boldsymbol{t})$. **Q.E.D.**

Proof of Lemma 7 Take any IC mechanism M. Construct the mechanism $M' \equiv \langle T', P' \rangle$ with[65]

$$T'(\boldsymbol{e},\boldsymbol{t}) \coloneqq \inf_{\boldsymbol{t}' \text{ s.t. } \sigma(\boldsymbol{e},\boldsymbol{t}')=\sigma(\boldsymbol{e},\boldsymbol{t})} T(\boldsymbol{e},\boldsymbol{t}) \leq T(\boldsymbol{e},\boldsymbol{t}), \quad \text{and}$$
$$P'(\boldsymbol{e},\boldsymbol{t},\emptyset) \coloneqq \frac{\Pi(\boldsymbol{e},\boldsymbol{t}) - T'(\boldsymbol{e},\boldsymbol{t})}{1 - T'(\boldsymbol{e},\boldsymbol{t})} \geq \frac{\Pi(\boldsymbol{e},\boldsymbol{t}) - T(\boldsymbol{e},\boldsymbol{t})}{1 - T(\boldsymbol{e},\boldsymbol{t})} = P(\boldsymbol{e},\boldsymbol{t},\emptyset)$$

for every $(\boldsymbol{e},\boldsymbol{t})$. Then, $\Pi'(\boldsymbol{e},\boldsymbol{t}) = (1 - T'(\boldsymbol{e},\boldsymbol{t}))P'(\boldsymbol{e},\boldsymbol{t},\emptyset) + T'(\boldsymbol{e},\boldsymbol{t}) = \Pi(\boldsymbol{e},\boldsymbol{t})$ for every $(\boldsymbol{e},\boldsymbol{t})$, where the second equality follows by construction of M'. Thus, M' is outcome-equivalent to M. Given that M is IC, outcome-equivalence implies that under M', no agent has incentives to imitate an agent whose test score she can achieve.

It remains to show that under mechanism M', no agent has incentives to imitate an agent whose test score she cannot achieve. Take any agent $(\boldsymbol{e},\boldsymbol{t})$, evidence $\boldsymbol{e}' \leq \boldsymbol{e}$, and talent $\boldsymbol{t}'$. It holds that

$$\begin{aligned}
\Pi'(\boldsymbol{e},\boldsymbol{t}) = \Pi(\boldsymbol{e},\boldsymbol{t}) &\geq \sup_{\tilde{\boldsymbol{t}} \text{ s.t. } \sigma(\boldsymbol{e}',\tilde{\boldsymbol{t}})=\sigma(\boldsymbol{e}',\boldsymbol{t}')} \left\{(1 - T(\boldsymbol{e}',\tilde{\boldsymbol{t}}))P(\boldsymbol{e}',\tilde{\boldsymbol{t}},\emptyset)\right\} \\
&= \sup_{\tilde{\boldsymbol{t}} \text{ s.t. } \sigma(\boldsymbol{e}',\tilde{\boldsymbol{t}})=\sigma(\boldsymbol{e}',\boldsymbol{t}')} \left\{\Pi(\boldsymbol{e}',\tilde{\boldsymbol{t}}) - T(\boldsymbol{e}',\tilde{\boldsymbol{t}})\right\} \\
&= \Pi(\boldsymbol{e}',\boldsymbol{t}') + \sup_{\tilde{\boldsymbol{t}} \text{ s.t. } \sigma(\boldsymbol{e}',\tilde{\boldsymbol{t}})=\sigma(\boldsymbol{e}',\boldsymbol{t}')} \left\{-T(\boldsymbol{e}',\tilde{\boldsymbol{t}})\right\} \\
&= \Pi(\boldsymbol{e}',\boldsymbol{t}') - \inf_{\tilde{\boldsymbol{t}} \text{ s.t. } \sigma(\boldsymbol{e}',\tilde{\boldsymbol{t}})=\sigma(\boldsymbol{e}',\boldsymbol{t}')} T(\boldsymbol{e}',\tilde{\boldsymbol{t}}) \\
&= \Pi'(\boldsymbol{e}',\boldsymbol{t}') - T'(\boldsymbol{e}',\boldsymbol{t}') = (1 - T'(\boldsymbol{e}',\boldsymbol{t}'))P'(\boldsymbol{e}',\boldsymbol{t}',\emptyset),
\end{aligned}$$

where (i) the first equality follows by construction of M', (ii) the inequality by IC of M, (iii) the second equality by definition of Π, (iv) the third equality by Lemma 6 and IC of M, which together imply that $\Pi(\boldsymbol{e}',\tilde{\boldsymbol{t}}) = \Pi(\boldsymbol{e}',\boldsymbol{t}')$ for every $\tilde{\boldsymbol{t}}$ such that $\sigma(\boldsymbol{e}',\tilde{\boldsymbol{t}}) = s$, (v) the fifth inequality by construction of M', and the final equality by definition of Π'. We have thus shown that for any agent $(\boldsymbol{e},\boldsymbol{t})$, $\Pi'(\boldsymbol{e},\boldsymbol{t}) \geq (1 - T'(\boldsymbol{e}',\boldsymbol{t}'))P'(\boldsymbol{e}',\boldsymbol{t}',\emptyset)$ for every $(\boldsymbol{e}',\boldsymbol{t}') \leq (\boldsymbol{e},\boldsymbol{1})$, so under mechanism M', no agent has incentives to imitate an agent whose test score she cannot achieve.

For $c > 0$, M' also minimizes testing costs. **Q.E.D.**

Proof of Proposition 6 The proof proceeds like the proof of Proposition 1 and is, thus, omitted. **Q.E.D.**

Proof of Lemma 8 The proof proceeds like the proof of Lemma 3.

Take any IC mechanism $M \equiv \langle T,P \rangle$. Condition (iii) of Proposition 6 says that $\Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\boldsymbol{0})) \geq (1 - T(\boldsymbol{e},s))P(\boldsymbol{e},s,\emptyset)$ for any $(\boldsymbol{e},s)$. Then, construct the mechanism

[65] For $\boldsymbol{e}$ such that $\inf_{\boldsymbol{t}' \text{ s.t. } \sigma(\boldsymbol{e},\boldsymbol{t}')=\sigma(\boldsymbol{e},\boldsymbol{t})} T(\boldsymbol{e},\boldsymbol{t}) = 1$, set $P'(\boldsymbol{e},\boldsymbol{t},\emptyset) = P(\boldsymbol{e},\boldsymbol{t},\emptyset)$.

$M' := \langle T',P' \rangle$ with[66]

$$\begin{aligned} T'(\boldsymbol{e},s) &:= \Pi(\boldsymbol{e},s) - \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) = (1 - T(\boldsymbol{e},s))P(\boldsymbol{e},s,\emptyset) + T(\boldsymbol{e},s) - \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) \\ &\leq \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) + T(\boldsymbol{e},s) - \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) = T(\boldsymbol{e},s), \quad \text{and} \\ P'(\boldsymbol{e},s,\emptyset) &:= \frac{\Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0}))}{1 - \Pi(\boldsymbol{e},s) + \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0}))} \geq \frac{(1 - T(\boldsymbol{e},s))P(\boldsymbol{e},s,\emptyset)}{1 - \Pi(\boldsymbol{e},s) + (1 - T(\boldsymbol{e},s))P(\boldsymbol{e},s,\emptyset)} = P(\boldsymbol{e},s,\emptyset) \end{aligned}$$

for every $(\boldsymbol{e},s)$, where the inequalities follow from $\Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) \geq (1 - T(\boldsymbol{e},s))P(\boldsymbol{e},s,\emptyset)$.

By construction we have that $\Pi'(\boldsymbol{e},s) = \Pi(\boldsymbol{e},s)$ for every $(\boldsymbol{e},s)$, so M' satisfies conditions (i) and (ii) of Proposition 1. By construction, we also have that for every $(\boldsymbol{e},s)$

$$\Pi'(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) = \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) = (1 - T'(\boldsymbol{e},s))P'(\boldsymbol{e},s,\emptyset),$$

so M' also satisfies condition (iii) of Proposition 1. Therefore, M' is IC.

Last, to see why the second part is true, notice that for $c > 0$, M' saves on testing costs compared to M if there exists (a positive measure of) $(\boldsymbol{e},s)$ with $P(\boldsymbol{e},s,\emptyset)(1 - T(\boldsymbol{e},s)) < \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0}))$, since $T'(\boldsymbol{e},s) < T(\boldsymbol{e},s)$ for such $(\boldsymbol{e},s)$. **Q.E.D.**

Proof of Proposition 7 Let $M \equiv \langle T,P \rangle$ be an optimal deterministic mechanism with $\Pi(\boldsymbol{e},s) \equiv (1 - T(\boldsymbol{e},s))P(\boldsymbol{e},s,\emptyset) + T(\boldsymbol{e},s)$. Define $E^* := \{\boldsymbol{e} \in [0,1]^m : \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) = 1\}$ (so $\Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) = 0$ for every $\boldsymbol{e} \notin E^*$). Given that M is IC, conditions (i) and (ii) of Proposition 6 combined imply that E^* is an upper set of $[0,1]^m$. To see this, take any $\boldsymbol{e} \in E^*$ and any $\boldsymbol{e}' \in [0,1]^m$. If $\boldsymbol{e}' \geq \boldsymbol{e}$, then $\Pi(\boldsymbol{e}',\sigma(\boldsymbol{e}',\mathbf{0})) \geq \Pi(\boldsymbol{e},\sigma(\boldsymbol{e}',\mathbf{0})) \geq \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) = 1$, so $\Pi(\boldsymbol{e}',\sigma(\boldsymbol{e}',\mathbf{0})) = 1$ and thus $\boldsymbol{e}' \in E^*$. The first inequality follows from condition (ii) and $\boldsymbol{e}' \geq \boldsymbol{e}$. The second inequality follows from condition (i), $\boldsymbol{e}' \geq \boldsymbol{e}$, and σ being increasing.[67]

Also, condition (i) of Proposition 6 implies that $\Pi(\boldsymbol{e},s) = 1$ for every $\boldsymbol{e} \in E^*$ and every $s \in [0,1]$. Then, the principal's objective function (4) can be written as

$$\int_{\boldsymbol{e} \in E^*} \int_{\sigma(\boldsymbol{e},\mathbf{0})}^{\sigma(\boldsymbol{e},\mathbf{1})} \widetilde{u}(\boldsymbol{e},s)\widetilde{f}(\boldsymbol{e},s)ds d\boldsymbol{e} + \int_{\boldsymbol{e} \notin E^*} \int_{\sigma(\boldsymbol{e},\mathbf{0})}^{\sigma(\boldsymbol{e},\mathbf{1})} [\Pi(\boldsymbol{e},s)(\widetilde{u}(\boldsymbol{e},s) - c)]\, \widetilde{f}(\boldsymbol{e},s)ds d\boldsymbol{e}.$$

The first term depends on the mechanism M only through E^*. The second term depends on the mechanism M only through the values of Π for $\boldsymbol{e} \notin E^*$. Setting $\Pi(\boldsymbol{e},s) = \mathbf{I}(\widetilde{u}(\boldsymbol{e},s) > c)$ for every $\boldsymbol{e} \notin E^*$ maximizes the second term. It is also IC.

To show this, we first prove that $\Pi(\boldsymbol{e},s) = \mathbf{I}(\widetilde{u}(\boldsymbol{e},s) > c \text{ or } \boldsymbol{e} \in E^*)$ satisfies condition (i) of Proposition 6. Take any $\boldsymbol{e},s,s'$ with $s' > s$. It suffices to show that $\Pi(\boldsymbol{e},s') = 0$ implies $\Pi(\boldsymbol{e},s) = 0$. If $\Pi(\boldsymbol{e},s') = 0$, then $\widetilde{u}(\boldsymbol{e},s') \leq c$ and $\boldsymbol{e} \notin E^*$. Since $\widetilde{u}(\boldsymbol{e},s)$ is non-decreasing in s, $\widetilde{u}(\boldsymbol{e},s) \leq \widetilde{u}(\boldsymbol{e},s') \leq c$. Therefore, $\Pi(\boldsymbol{e},s) = 0$.

[66] For $(\boldsymbol{e},s)$ such that $\Pi(\boldsymbol{e},s) = 1$ and $\Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) = 0$, set $P'(\boldsymbol{e},s,\emptyset) = 0$.

[67] If $\sigma(\boldsymbol{e}',\mathbf{0}) > \sigma(\boldsymbol{e},\mathbf{1})$, then $\Pi(\boldsymbol{e},\sigma(\boldsymbol{e}',\mathbf{0}))$ is not well-defined (since there is no agent with evidence $\boldsymbol{e}$ and potential test score $\sigma(\boldsymbol{e}',\mathbf{0})$) but the inequalities still follow if we use conditions (i) and (ii) iteratively.

It remains to show that $\Pi(\boldsymbol{e},s) = \mathbf{I}(\tilde{u}(\boldsymbol{e},s) > c \text{ or } \boldsymbol{e} \in E^*)$ satisfies condition (ii) of Proposition 6. Take any $\boldsymbol{e},\boldsymbol{e}',s$ with $\boldsymbol{e}' \geq \boldsymbol{e}$. We need to show that $\Pi(\boldsymbol{e}',s) = 0$ implies $\Pi(\boldsymbol{e},s) = 0$. If $\Pi(\boldsymbol{e}',s) = 0$, then $\tilde{u}(\boldsymbol{e}',s) \leq c$ and $\boldsymbol{e}' \notin E^*$. It follows then that $\boldsymbol{e} \notin E^*$, since E^* is an upper set of $[0,1]^m$, $\boldsymbol{e}' \geq \boldsymbol{e}$, and $\boldsymbol{e}' \notin E^*$. It remains to show that $\tilde{u}(\boldsymbol{e},s) \leq c$. We will show this by contradiction. Assume that $\tilde{u}(\boldsymbol{e},s) > c$. Then, we have that $\tilde{u}(\boldsymbol{e},s) > c \geq \tilde{u}(\boldsymbol{e}',s)$, which, given that σ is pro-t biased, implies that $\boldsymbol{e}' \ngeq \boldsymbol{e}$, a contradiction. **Q.E.D.**

Proof of Proposition 8 Let $M \equiv \langle T,P \rangle$ be an optimal deterministic mechanism with $\Pi(\boldsymbol{e},s) \equiv (1 - T(\boldsymbol{e},s))P(\boldsymbol{e},s,\emptyset) + T(\boldsymbol{e},s)$. Define $E^* := \{\boldsymbol{e} \in [0,1]^m : \Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) = 1\}$ (so $\Pi(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{0})) = 0$ for every $\boldsymbol{e} \notin E^*$). Given that M is IC, conditions (i) and (ii) of Proposition 6 combined imply that E^* is an upper set of $[0,1]^m$.

Also, condition (i) of Proposition 6 implies that $\Pi(\boldsymbol{e},s) = 1$ for every $\boldsymbol{e} \in E^*$ and every $s \in [0,1]$. Then, the principal's objective function (4) can be written as

$$\int_{\boldsymbol{e}\in E^*} \int_{\sigma(\boldsymbol{e},\mathbf{0})}^{\sigma(\boldsymbol{e},\mathbf{1})} \tilde{u}(\boldsymbol{e},s)\tilde{f}(\boldsymbol{e},s)ds d\boldsymbol{e} + \int_{\boldsymbol{e}\notin E^*} \int_{\sigma(\boldsymbol{e},\mathbf{0})}^{\sigma(\boldsymbol{e},\mathbf{1})} \left[\Pi(\boldsymbol{e},s)(\tilde{u}(\boldsymbol{e},s) - c)\right] \tilde{f}(\boldsymbol{e},s)ds d\boldsymbol{e}.$$

The first term depends on the mechanism M only through E^*. The second term depends on the mechanism M only through the values of Π for $\boldsymbol{e} \notin E^*$.

Take any $(\boldsymbol{e},s),(\boldsymbol{e}',s') \in I_{\tilde{u}}(c) \setminus E^*$ with $s \neq s'$. That $(\boldsymbol{e},s),(\boldsymbol{e}',s') \in I_{\tilde{u}}(c)$ means that $\tilde{u}(\boldsymbol{e},s) = \tilde{u}(\boldsymbol{e}',s') = c$. First, we show that if $\boldsymbol{e}' \ngeq \boldsymbol{e}$, then $s' < s$. Let $\boldsymbol{e}' \ngeq \boldsymbol{e}$:

Case 1: if $s' \in [\sigma(\boldsymbol{e},\mathbf{0}),\sigma(\boldsymbol{e},\mathbf{1})]$, then σ being pro-e biased implies that $\tilde{u}(\boldsymbol{e},s') \leq c$. To see this, notice that if instead $\tilde{u}(\boldsymbol{e},s') > c$, then we would have $\tilde{u}(\boldsymbol{e},s') > c = \tilde{u}(\boldsymbol{e}',s')$, so pro-$e$ biased testing would imply that $\boldsymbol{e}' \geq \boldsymbol{e}$, a contradiction. We then have that $\tilde{u}(\boldsymbol{e},s') \leq c = \tilde{u}(\boldsymbol{e},s)$. Particularly, $\tilde{u}(\boldsymbol{e},s') < c = \tilde{u}(\boldsymbol{e},s)$, because $\tilde{u}(\boldsymbol{e},s') = \tilde{u}(\boldsymbol{e},s) = \tilde{u}(\boldsymbol{e}',s') = c$ is not possible by the Regularity Assumption. Given that $\tilde{u}(\boldsymbol{e},s)$ is non-decreasing in s, $\tilde{u}(\boldsymbol{e},s') < c = \tilde{u}(\boldsymbol{e},s)$ implies that $s' < s$.

Case 2: if $s' < \sigma(\boldsymbol{e},\mathbf{0})$, then since $s \in [\sigma(\boldsymbol{e},\mathbf{0}),\sigma(\boldsymbol{e},\mathbf{1})]$, it follows that $s' < s$.

Case 3a: if $s' > \sigma(\boldsymbol{e},\mathbf{1})$ and $\sigma(\boldsymbol{e},\mathbf{1}) \in [\sigma(\boldsymbol{e}',\mathbf{0}),\sigma(\boldsymbol{e}',\mathbf{1})]$, then because $\sigma(\boldsymbol{e},\mathbf{1}) \geq s$ and $\tilde{u}(\boldsymbol{e},s)$ is non-decreasing in s, it follows that $\tilde{u}(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{1})) \geq \tilde{u}(\boldsymbol{e},s) = c$. Thus, we have $\tilde{u}(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{1})) \geq c = \tilde{u}(\boldsymbol{e}',s') \geq \tilde{u}(\boldsymbol{e}',\sigma(\boldsymbol{e},\mathbf{1}))$ with at least one inequality holding strictly (for otherwise $\tilde{u}(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{1})) = \tilde{u}(\boldsymbol{e}',\sigma(\boldsymbol{e},\mathbf{1})) = \tilde{u}(\boldsymbol{e}',s') = c$ with $s' \neq \sigma(\boldsymbol{e},\mathbf{1})$ and $\boldsymbol{e} \neq \boldsymbol{e}'$, which is not possible by the Regularity Assumption), so pro-e biased testing implies that $\boldsymbol{e}' \geq \boldsymbol{e}$, a contradiction. Therefore, Case 3a is impossible.

Case 3b: if $s' > \sigma(\boldsymbol{e},\mathbf{1})$ and $\sigma(\boldsymbol{e},\mathbf{1}) < \sigma(\boldsymbol{e}',\mathbf{0})$, then by continuity and monotonicity of σ and because $\sigma(\boldsymbol{e},\mathbf{1}) \in [\sigma(\mathbf{0},\mathbf{0}),\sigma(\boldsymbol{e}',\mathbf{0}))$ there exists $\boldsymbol{e}'' \leq \boldsymbol{e}'$ such that $\sigma(\boldsymbol{e}'',\mathbf{0}) = \sigma(\boldsymbol{e},\mathbf{1})$. We have then that

$$\tilde{u}(\boldsymbol{e},\sigma(\boldsymbol{e},\mathbf{1})) \geq \tilde{u}(\boldsymbol{e},s) = c = \tilde{u}(\boldsymbol{e}',s') \geq \min_{\boldsymbol{t}:\sigma(\boldsymbol{e}',\boldsymbol{t})=s'} u(\boldsymbol{e}',\boldsymbol{t})$$

$$= u(\boldsymbol{e}', \underset{\boldsymbol{t}:\sigma(\boldsymbol{e}',\boldsymbol{t})=s'}{\arg\min}\ u(\boldsymbol{e}',\boldsymbol{t})) \geq u(\boldsymbol{e}'', \underset{\boldsymbol{t}:\sigma(\boldsymbol{e}',\boldsymbol{t})=s'}{\arg\min}\ u(\boldsymbol{e}',\boldsymbol{t})) \geq u(\boldsymbol{e}'', \mathbf{0})$$
$$= \mathbb{E}_{\mathbf{t}}\left[u(\boldsymbol{e}'',\boldsymbol{t})|\sigma(\boldsymbol{e}'',\boldsymbol{t}) = \sigma(\boldsymbol{e}'',\mathbf{0})\right] \equiv \widetilde{u}(\boldsymbol{e}'',\sigma(\boldsymbol{e}'',\mathbf{0})) = \widetilde{u}(\boldsymbol{e}'',\sigma(\boldsymbol{e},\mathbf{1}))$$

with at least one inequality holding strictly. The first line follows because $\sigma(\boldsymbol{e},\mathbf{1}) \geq s$, $\widetilde{u}(\boldsymbol{e},s)$ is non-decreasing in s, $(\boldsymbol{e},s),(\boldsymbol{e}',s') \in I_{\widetilde{u}}(c)$, and $\widetilde{u}(\boldsymbol{e}',s') \equiv \mathbb{E}_{\mathbf{t}}[u(\boldsymbol{e}',\boldsymbol{t})|\sigma(\boldsymbol{e}',\boldsymbol{t}) = s'] \geq \min_{\boldsymbol{t}:\sigma(\boldsymbol{e}',\boldsymbol{t})=s'} u(\boldsymbol{e}',\boldsymbol{t})$. The second line follows because $\boldsymbol{e}' \geq \boldsymbol{e}''$, $\arg\min_{\boldsymbol{t}:\sigma(\boldsymbol{e}',\boldsymbol{t})=s'} u(\boldsymbol{e}',\boldsymbol{t}) \geq \mathbf{0}$, and u is non-decreasing. The third line follows because, given that σ is increasing, the only value of $\boldsymbol{t}$ that makes $\sigma(\boldsymbol{e}'',\boldsymbol{t}) = \sigma(\boldsymbol{e}'',\mathbf{0})$ is $\boldsymbol{t} = \mathbf{0}$; also, $\sigma(\boldsymbol{e}'',\mathbf{0}) = \sigma(\boldsymbol{e},\mathbf{1})$.

Case 3c: if $s' > \sigma(\boldsymbol{e},\mathbf{1})$ and $\sigma(\boldsymbol{e},\mathbf{1}) > \sigma(\boldsymbol{e}',\mathbf{1})$, then we arrive at a contradiction since $s' > \sigma(\boldsymbol{e}',\mathbf{1})$ is not possible. Thus, Case 3c is impossible.

We have thus shown that for any $(\boldsymbol{e},s),(\boldsymbol{e}',s') \in I_{\widetilde{u}}(c) \setminus E^*$ with $s \neq s'$, if $\boldsymbol{e}' \not\geq \boldsymbol{e}$, then $s' < s$. This is equivalent to its contrapositive: for any $(\boldsymbol{e},s),(\boldsymbol{e}',s') \in I_{\widetilde{u}}(c) \setminus E^*$, if $s' > s$, then $\boldsymbol{e}' \geq \boldsymbol{e}$. Therefore, by conditions (i) and (ii) of Proposition 6, there exists $s^* \in [0,1]$ such that for any $(\boldsymbol{e},s) \in I_{\widetilde{u}}(c) \setminus E^*$, $\Pi(\boldsymbol{e},s) = \mathbf{I}(s \geq s^*)$.

It remains to find the values for $(\boldsymbol{e},s) \notin I_{\widetilde{u}}(c) \cup E^*$. Take any $(\boldsymbol{e},s)$ in $I_{\widetilde{u}}(c) \cup E^*$.

Case 1: If $\widetilde{u}(\boldsymbol{e}^*,s) = c$ for some $\boldsymbol{e}^*$ such that $s \in [\sigma(\boldsymbol{e}^*,\mathbf{0}),\sigma(\boldsymbol{e}^*,\mathbf{1})]$, then

Case 1a: if $\widetilde{u}(\boldsymbol{e},s) < c$ and $s \geq s^*$, then $\widetilde{u}(\boldsymbol{e}^*,s) = c > \widetilde{u}(\boldsymbol{e},s)$, so because σ is pro-e biased, $\boldsymbol{e} \geq \boldsymbol{e}^*$, and thus IC condition (ii) of Proposition 6 requires that $\Pi(\boldsymbol{e},s) \geq \Pi(\boldsymbol{e}^*,s) = 1$, which implies $\Pi(\boldsymbol{e},s) = 1$.

Case 1b: If $\widetilde{u}(\boldsymbol{e}',s) > c$ and $s < s^*$, then $\widetilde{u}(\boldsymbol{e},s) > c = \widetilde{u}(\boldsymbol{e}^*,s)$, so because σ is pro-e biased, $\boldsymbol{e}^* \geq \boldsymbol{e}$, and thus IC condition (ii) of Proposition 6 requires that $\Pi(\boldsymbol{e},s) \leq \Pi(\boldsymbol{e}^*,s) = 0$, which implies $\Pi(\boldsymbol{e},s) = 0$.

Case 1c: If $\widetilde{u}(\boldsymbol{e},s) < c$ and $s < s^*$, then set $\Pi(\boldsymbol{e},s) = 0$, which is what the principal would ideally want to do with $(\boldsymbol{e},s)$ if he was not constrained by IC.

Case 1d: If $\widetilde{u}(\boldsymbol{e},s) > c$ and $s \geq s^*$, then set $\Pi(\boldsymbol{e},s) = 1$, which is what the principal would ideally want to do with $(\boldsymbol{e},s)$ if he was not constrained by IC.

Case 2: If $\widetilde{u}(\boldsymbol{e}',s) < c$ for every $\boldsymbol{e}'$ such that $s \in [\sigma(\boldsymbol{e}',\mathbf{0}),\sigma(\boldsymbol{e}',\mathbf{1})]$, then it is easy to see that $s < s^*$. Set $\Pi(\boldsymbol{e},s) = 0$, which is what the principal would ideally want to do with $(\boldsymbol{e},s)$ if he was not constrained by IC.

Case 3: If $\widetilde{u}(\boldsymbol{e}',s) > c$ for every $\boldsymbol{e}'$ such that $s \in [\sigma(\boldsymbol{e}',\mathbf{0}),\sigma(\boldsymbol{e}',\mathbf{1})]$, then it is easy to see that $s \geq s^*$. Set $\Pi(\boldsymbol{e},s) = 1$, which is what the principal would ideally want to do with $(\boldsymbol{e},s)$ if he was not constrained by IC.

Putting all the above cases together, we get that for $(\boldsymbol{e},s) \notin I_{\widetilde{u}}(c) \cup E^*$, $\Pi(\boldsymbol{e},s) = \mathbf{I}(s \geq s^*)$. Combining this with the fact that for any $(\boldsymbol{e},s) \in I_{\widetilde{u}}(c) \setminus E^*$, $\Pi(\boldsymbol{e},s) = \mathbf{I}(s \geq s^*)$ and given the definition of E^*, we get that for any $(\boldsymbol{e},s)$ such that $s \in [\sigma(\boldsymbol{e},\mathbf{0}),\sigma(\boldsymbol{e},\mathbf{1})]$, $\Pi(\boldsymbol{e},s) = \mathbf{I}(s \geq s^* \text{ or } \boldsymbol{e} \in E^*)$. To conclude the proof, notice that Π satisfies conditions (i) and (ii) of Proposition 6, and is thus IC. Therefore, by solving a relaxed problem when ignoring the IC constraints in cases 1c, 1d, 2, and 3, we have also solved the original problem. **Q.E.D.**

Proof of Proposition 9 Trivial, and, thus, omitted.

Proof of Proposition 10 Denote the total probability with which type (e,t) is accepted if she reports $(\widehat{e},\widehat{t})$ (with $\widehat{e} \leq e$) by

$$\widetilde{P}(\widehat{e},\widehat{t};e,t) \coloneqq (1-T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t},\emptyset) + T(\widehat{e},\widehat{t})\mathbf{I}\left(\sigma(e,t) \geq \sigma(\widehat{e},\widehat{t})\right).$$

Also, define condition (iii') (a strengthening of condition (iii)) to say that $(1 - T(e,t))P(e,t,\emptyset) \leq \Pi(e',0)$ for every e,t,e'.

Step 1: I first show that condition (i) is necessary for IC by showing the contrapositive. Assume that for some e,t_1,t_2 with $t_2 > t_1$, $\Pi(e,t_2) < \Pi(e,t_1)$. Then, IC of type (e,t_2) is violated, since $\widetilde{P}(e,t_1;e,t_2) = \Pi(e,t_1) > \Pi(e,t_2)$.

Step 2: I now show that condition (iii') is necessary for IC by showing the contrapositive. Assume that for some e,e',t, $(1-T(e,t))P(e,t,\emptyset) > \Pi(e',0)$. Then, IC of type $(e',0)$ is violated, since $\widetilde{P}(e,t;e',0) \geq (1-T(e,t))P(e,t,\emptyset) > \Pi(e',0)$.

Step 3: I now show that provided that (i) and (iii') are satisfied, $\Pi(r,\tau(r,\sigma(e,t)))$ being constant in r over $r \in [\underline{e}(\sigma(e,t)),e]$ for every (e,t) is necessary and sufficient for IC.

IC of type (e,t) is satisfied if and only if

$$\max_{(\widehat{e},\widehat{t})\leq(1,1)} \left[(1-T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t};\emptyset) + T(\widehat{e},\widehat{t})\mathbf{I}\left(\sigma(e,t) \geq \sigma(\widehat{e},\widehat{t})\right)\right] = \Pi(e,t). \tag{10}$$

Assume that conditions (i) and (iii') are satisfied. Then, $\Pi(e,t) \geq \Pi(e,0) \geq (1 - T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t},\emptyset)$ for any $(\widehat{e},\widehat{t})$. Therefore, (10) is equivalent to

$$\max_{(\widehat{e},\widehat{t})\in\{(x,y)\in[0,1]^2:\sigma(e,t)\geq\sigma(x,y)\}} \left[(1-T(\widehat{e},\widehat{t}))P(\widehat{e},\widehat{t};\emptyset) + T(\widehat{e},\widehat{t})\right] = \Pi(e,t). \tag{11}$$

Given that $\Pi(e,t)$ is non-decreasing in t (condition (i)), (11) can equivalently be written as

$$\max_{r\in[\underline{e}(\sigma(e,t)),1]} \{[1-T(r,\tau(r,\sigma(e,t)))]P(r,\tau(r,\sigma(e,t)),\emptyset) + T(r,\tau(r,\sigma(e,t)))\} = \Pi(e,t)$$

or equivalently,

$$e \in \underset{r\in[\underline{e}(\sigma(e,t)),\overline{e}(\sigma(e,t))]}{\arg\max} \Pi(r,\tau(r,\sigma(e,t))). \tag{12}$$

Thus, IC is satisfied for every type if and only if for every (e,t), (12) is satisfied. This is true if and only if $\Pi(r,\tau(r,\sigma(e,t)))$ is constant in r for $r \in [\underline{e}(\sigma(e,t)),\overline{e}(\sigma(e,t))]$ for every (e,t).

That the latter is sufficient for (12) to hold for every (e,t) is immediate. I show necessity by showing the contrapositive. Assume that for some (e,t), $\Pi(r,\tau(r,\sigma(e,t)))$ is *not* constant in r for $r \in [\underline{e}(\sigma(e,t)),1]$. That is, for some (e,t) there exist r_1, r_2 with $\underline{e}(\sigma(e,t)) \leq r_1 < r_2 \leq \overline{e}(\sigma(e,t))$ such that $\Pi(r_2,\tau(r_2,\sigma(e,t))) \neq \Pi(r_1,\tau(r_1,\sigma(e,t)))$. If $\Pi(r_2,\tau(r_2,\sigma(e,t))) < \Pi(r_1,\tau(r_1,\sigma(e,t)))$, IC of type $(r_2,\tau(r_2,\sigma(e,t)))$ is violated, as she prefers to imitate type $(r_1,\tau(r_1,\sigma(e,t)))$. If, instead, $\Pi(r_2,\tau(r_2,\sigma(e,t))) > \Pi(r_1,\tau(r_1,\sigma(e,t)))$, IC of type $(r_1,\tau(r_1,\sigma(e,t)))$ is violated, as she prefers to imitate type $(r_2,\tau(r_2,\sigma(e,t)))$.

Step 4: It is easy to see that $\Pi(r,\tau(r,\sigma(e,t)))$ being constant in r over $r \in [\underline{e}(\sigma(e,t)),\overline{e}(\sigma(e,t))]$ for every (e,t) is equivalent to condition (ii).

Step 5: Finally, notice that provided that conditions (i) and (ii) hold, conditions (iii) and (iii') are equivalent. **Q.E.D.**

Proof of Lemma 10 Take any IC mechanism $M \equiv \langle T,P\rangle$. Condition (iii) of Proposition 1 says that $\Pi(0,0) \geq (1-T(e,t))P(e,t,\emptyset)$ for any (e,t). Then, construct the mechanism $M' \coloneqq \langle T',P'\rangle$ with[68]

$$\begin{aligned} T'(e,t) &\coloneqq \Pi(e,t) - \Pi(0,0) = (1-T(e,t))P(e,t,\emptyset) + T(e,t) - \Pi(0,0) \\ &\leq \Pi(0,0) + T(e,t) - \Pi(0,0) = T(e,t), \quad \text{and} \\ P'(e,t,\emptyset) &\coloneqq \frac{\Pi(0,0)}{1-\Pi(e,t)+\Pi(0,0)} \geq \frac{(1-T(e,t))P(e,t,\emptyset)}{1-\Pi(e,t)+(1-T(e,t))P(e,t,\emptyset)} = P(e,t,\emptyset) \end{aligned}$$

for every (e,t), where the inequalities follow from $\Pi(0,0) \geq (1-T(e,t))P(e,t,\emptyset)$.

By construction we have that $\Pi'(e,t) = \Pi(e,t)$ for every (e,t), so M' satisfies conditions (i) and (ii) of Proposition 10. By construction, we also have that for every (e,t)

$$\Pi'(0,0) = \Pi(0,0) = (1-T'(e,t))P'(e,t,\emptyset),$$

so M' also satisfies condition (iii) of Proposition 10. Therefore, M' is IC.

Last, to see why the second part is true, notice that for $c > 0$, M' saves on testing costs compared to M if there exists (a positive measure of) (e,t) with $P(e,t,\emptyset)(1-T(e,t)) < \Pi(0,0)$, since $T'(e,t) < T(e,t)$ for such (e,t). **Q.E.D.**

[68] If $\Pi(0,0) = 0$, then for (e,t) such that $\Pi(e,t) = 1$, set $P'(e,t,\emptyset) = 0$.