A dynamic discrete choice activity based travel demand model

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During the last decades, many activity-based models have been developed in the literature. However, especially in random utility based models timing decisions are often treated poorly or inconsistently with other choice dimensions. In this paper we show how dynamic discrete choice can be used to overcome this problem. In the proposed model, trip decisions are made sequentially in time, starting at home in the morning and ending at home in the evening. At each decision stage, the utility of an alternative is the sum of the one-stage utility of the action and the expected future utility in the reached state.

The model generates full daily activity schedules with any number of trips that each is a combination of one of 6 activities, 1240 locations and 4 modes. The ability to go from all to all locations makes evaluating the model very time consuming and sampling of alternatives were therefore used for estimation. The model is estimated on travel diaries and simulation results indicates that it is able to reproduce timing decisions, trip lengths and distribution of the number trips within sample.

To explain when people perform different activities, two sets of parameters are used: firstly, the utility of being at home varies depending on the time of day; and secondly, constants determine the utility of arriving to work at specific times. This was enough to also obtain a good distribution of the starting times for free-time activities.

1. Introduction

Travel demand models have during the last decades evolved from highly aggregated trip based models, through tour based models into activity based models that considers the choice of all activities and all transportation for a full day (or longer) at an individual (or household) level. The motivation behind this gradual increase in complexity is the realization that the demand for travel is derived from the demand for activity participation. Accurate predictions of how individuals will react to infrastructure investments and policy changes should therefore ideally take into account to what extent individuals are flexible in their activity participation. There is only a limited amount of time each day and some activities are more or less fixed in both location and time, such as working and picking up or dropping of children at school/daycare. These constraints severely restrict individuals' ability to adapt and not considering them is therefore likely

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to result in unrealistic forecasts. This is especially the case when considering policy changes such as congestion charge that are becoming increasingly important in today's traffic planning.

A natural way to accomplish an interdependent choice of all aspects of a daily travelling would be to directly model the choice of a full daily travel pattern. The problem with this approach is that the number of possible ways to plan a day is immense. One could definitely argue that people do not actually consider all these alternatives, but there are definitely complex aspects of the activity schedule that do influence how people plan their days. For example, when considering what time to leave for work, they are likely to take into consideration when they will get home and so the preferred departure time to work should be derived from a trade-off between time spent at home in the morning versus time spent at home (or on some activity) in the evening.

How to spend the limited time budget when the preferences for time is dependent on the time of day should be the key determinant for when, where and if people choose to conduct different activities in activity based models. Although many activity based models up to date result in full day activity schedules, most fall short in their treatment of time. For a comprehensive overview of activity based models, see e.g., Pinjari and Bhat (2011) or Rasouli and Timmermans (2014). As an extension to the tour-based approach, Bowman and Ben-Akiva (2001) developed a nested-logit structure that treats tours and activities sequentially based on their importance for the individual. The model consists of five nests: 1) the choice of activity pattern, including the number of tours carried out during the day; 2) the choice of time of day for the primary tour and all its trips; 3) the mode and destination for the primary tour; 4) the time of day for the secondary tours; and 5) the mode and destination of the secondary tours. The nested-logit structure ensures that higher decisions, such as the choice of activity pattern, includes the individual specific information about all available tour-combinations that the pattern includes. However, since secondary tours are not conditioned on each other, they are not temporarily consistent. For instance, it is technically possible to end up with daily patterns that take more than a full day to complete. The model has been further developed by combining with a duration and departure time model, which is not integrated into the nested-logit structure (Bradley et al., 2010; Vovsha and Bradley, 2004) and so does not integrate upwards.

There are a number of models in the literature that is related to the approach taken in this paper. First, Habib (2011) presents a discrete-continuous random utility model for weekend travelling. Agents choose mode, destination and activity based on the utility of the combination. Future time is contained in a time-of-day dependent composite good, which is parameterized and estimated. Second, in the Albatross model system, choices are also made sequentially in time (Arentze et al., 2000), and at every time step a heuristic decision rule determines determine the next action so that time-space constraints are fulfilled. One challenge with these approaches is the difficulty in treating value of future time consistently, in the sense that a dynamically consistent model should either directly model the choice of full day-schedules, or the value of future time that individuals consider should be the same as the expectation of utility that they can obtain during the remaining day.

Some other models of travel demand do include the full time spent on different activities over a full day. In particular related to the approach in this paper, we would like to mention utility-based models in which individuals (or households) are searching for a day-path that maximize the sum of the utility gathered during the day. One idea is to simulate day-paths and through a search algorithm approach paths that maximize the sum of the utility gathered over a day (Balmer et al., 2005). The utility of a day-path is once again the sum of the utility for respectively activity or travel episode, and individuals are choosing the alternative with the highest utility. The original model did not include any random or unknown term in the utility function but some kind of randomness was introduced through the search process. This was not enough when including location choice and Horni et al. (2011) found that formulating the choice of day-paths as an MNL gave a realistic distribution of travel times. In AURORA (Joh et al., 2003, 2005), individuals are considering the utility of the full daily travel pattern but are assumed to use search-heuristics to schedule (and reschedule) their day.

Yet another approach is based on mathematical programming. For instance, the shortest path problem can equivalently be expressed as a mixed-integer programming problem. Time-space constraints that define when certain links in the network are available can be included as constraints in the programming problem, which makes this approach tractable. Recker (2001) show how the choice of an activity schedule including mode, activity duration and participation can be solved in this way. The model has been estimated using a genetic algorithm (Recker et al., 2008) and extended to include destination choice (Kang and Recker, 2013).

The frameworks mentioned above are promising in their attempts to model and simulate the choice of daily travel patterns. In this paper, we follow the random utility-based approach where individuals maximize the achieved utility from a path through activity-location-time-space throughout one day. Following this approach, we acknowledge that it is unrealistic that a single individual consider all possible paths during one day. The rationale following this utility-maximization approach is twofold. First, the standard practice in the field is based on random utility maximization, and even more specifically nested logit. The approach taken in this paper closely follows the standard practice, but introduces time explicitly, respecting that time has a direction and that decisions in real-life actually can be made sequentially in time, given the information available at each point in time when a decision is made. Second, individual rationality is to-date a corner stone for welfare economics, enabling the approach to be theoretically and consistently translated into a tool for cost-benefit analysis. Both of these arguments are strong, independently, and the latter argument is difficult to ignore without having a theory for behavioural welfare economics in sight.

Further, although they are based on utility maximization, it is unclear how they should be used for other purposes than prediction. In a discrete choice framework, it is logical to use the expected (maximum) utility from a day-path as input to other models and for cost appraisal (Geurs et al., 2010). The expected utility could also be used to get detailed disaggregated measures of accessibility (Dong et al., 2006; Jonsson et al., 2013). With a dynamically consistent model, such measures of accessibility could be used to see how, e.g., the fact that some activities such as picking up children and going shopping are mandatory influences the accessibility of different work locations.

The number of different daily activity patterns is immense, but the number of possible actions available to an individual at a specific time of the day is relatively easy to define. Dynamic discrete choice theory could therefore present a way of simplifying the activity scheduling problem without making any restrictions on the choice set, as proposed by Karlström (2005). It could also give detailed time-dependent individual accessibility measures, as demonstrated by Jonsson et al. (2013). The basic idea is to model choice of a travel patterns as a sequence of simultaneous choices of activity type, duration, mode of transport and location conditional on the expected future utility given respectively choice. The sequencing of actions and the expected future utility components ensures that both the history and the future is taken into account; makes it easy to include time-space constraints; and allows for rescheduling due to unexpected events. However, the idea has only been implemented in small example cases and no model has been previously estimated.

In this paper we propose an estimation method for a dynamic discrete choice activity based model based on sampling of alternatives that is used to estimate an implementation of the model. Section 2 will present and discuss the modelling framework and specification; section 3 will discuss the estimation method proposed; section 4 the data; section 5 the utility specification; and section 6 estimation result and simulation validations.

2. MODEL

When individuals decide what time to leave for work, they take into consideration how it will influence their afternoon. If they leave ten minutes later it means that they get ten minutes less for activities in the afternoon. The difference in how they value afternoon-time versus morning-time will therefore be one factor determining when they leave for work. Even people with flexible working hours seem to prefer to go to work during rush hours, when roads and public transport are heavily congested. Probably they have other time-constraints – they might have to pick up their children from school, have a gym-class or are meeting friends – or time preferences of some other sort – they might value having dinner with their family at 6 p.m. Whatever the reasons, it is clear that the timing of trips in the morning is influenced by their plans for the afternoon, and that the benefits of travelling at less congested times would not outweigh the costs of changing these plans. The decision on when, where and how to travel should therefore be explained by trade-off's on how a limited amount of time should be spent and a correct representation of time is crucial in activity-based travel demand models.

A daily schedule of trips and activities could be represented by a path between states where a state s_t defines the location and time of day t among other things. An action a_t, defining activity, duration and mode of transport, gives a new state s_{t+1}, and a sequence of such actions starting in the morning and ending in the evening is what we will call a day-path. A day-path defines a travel pattern or schedule but further contains information of the sequence of actions and states that have been traversed. The way in which the action a_t in state s_t yields a new state might contain randomness, caused by for example day-to-day variations in travel times. The state might further only be partially visible, where uncertainty might account for unexpected opportunities or needs. For example, a friend might suddenly call or a meeting is delayed, demanding a replanning of the travel schedule.

The environment is thus stochastic. A rational agent in an uncertain environment that starts in a state s would behave according to a policy π, determining the action a_t to take when in state s_t, that maximizes the expected future utility:

$$V(s_0) = \max_{\pi} E_s \left\{ \sum_{t=0}^{T} \beta^t u(s_t, a_t) \middle| s_0 = s \right\} \tag{2.1}$$

for some one-stage utility function $u(s_t, a_t)$ and discount factor β, assuming that the utility is additively separable between decision stages.

Finding the utility maximizing decision rule is a daunting task. Consider, for example, an individual with 10 h of free time during a day. If there are 8 different activities that can be conducted at 100 alternative locations and 4 available modes of transport, and each activity-travel episode is at least 1 h long, there are $(8 \cdot 100 \cdot 4)^{10} \approx 10^{35}$ alternative sequences of actions. The problem is thus immense, and we do not propose that people actually consider all these options. For one thing, a specific individual probably only considers a small set of locations for each activity. However, constructing consistent models for how individuals are constructing their choice sets is a very complex problem, and considering the universal choice set is thus a tractable option. Fortunately, even this immense problem can be solved with dynamic programming.

The value function $V(s)$ in (2.1) can be defined recursively through Bellman's equation as (Rust, 1987):

$$V(s_t) = \max_{a_t} \left\{ u(s_t, a_t) + \beta \int V(s_{t+1}) p(ds_{t+1} | s_t, a_t) \right\} \tag{2.2}$$

where $p(s_{t+1}|s_t, a_t)$ is the probability to reach state s_{t+1} when taking action a_t in state s_t. To make (2.2) computationally tractable, Rust (1987) introduces a number of assumptions. Firstly, the state s_t is divided into (x_t, ϵ_t), where x_t is known to both the econometricians and the individual whereas ϵ_t is unknown for the econometricians but known by the individual for the current period t. The unknown part ϵ_t is further assumed to be Gumbel distributed and i.i.d. over alternatives and time. We will assume that the error term is translated to $\epsilon \sim G(-\gamma, 1)$, where the location is $-\gamma$ to ensure that the mean of ϵ is zero rather than γ. The utility function $u(s_t, a_t)$ is assumed to be additively separable into a known and unknown part: $u(s_t, a_t) = u(x_t, a_t) + \epsilon_t$.

We will make two additional restrictions that give a more tractable specification. Firstly, individuals are not assumed to discount within the day, so $\beta = 1$. As the time frame of decisions is short, we do not see this as a big restriction. We will, secondly, assume that all uncertainty in the state transitions is captured by ϵ_t. This means that, e.g., travel time uncertainty is cannot be taken into account by individuals. Being able to explicitly model travel time uncertainty would definitely be of great value, and we will attempt to overcome this restriction in future work.

With these two additional restrictions (and $\epsilon_t \sim G(-\gamma, 1)$), the expected value of the value function (the *expected value function*) becomes:

$$\begin{aligned} EV(x_t) &= \int V(s_{t+1}) p(ds_{t+1} | s_t, a_t) \\ &= \log \left(\sum_{a \in A(x_t)} e^{u(x_t, a) + EV(x_{t+1})} \right) \end{aligned} \tag{2.3}$$

and the probability that an alternative a is choosen when in state x is given by the well-known MNL-formula:

$$P(a_t|x_t) = \frac{e^{u(a_t, x_t) + EV(x_{t+1})}}{\sum\limits_{a'_t \in A(x_t)} e^{u(a'_t, x_t) + EV(x'_{t+1})}} \tag{2.4}$$

$$= e^{u(a_t, x_t) + EV(x_{t+1}) - EV(x_t)} \tag{2.5}$$

where (2.3) together with (2.4) gives (2.5).

When modelling daily planning, there is logical terminal time T in the end of the day. We will restrict ourselves to a single feasible state x_T in the end of day with $EV(x_T) = 0$. This is not a restriction per se; multiple states in the end of the day could be included by adding a link from all of these states to a common fictive terminal state x_T. With $EV(x_T)$ defined, it is possible to use backward induction to calculate EV in all states using (2.3).

2.1. *Specification*

We restrict the model to working days and people that arrive at work between 6 a.m. and 11 a.m. and return home before 11 p.m. We also omit lunch activities and business trips. Car ownership, work location, working time and whether people have fixed or flexible working schedules is exogenous in the current implementation. Escort trips of children to or from school/daycare are mandatory for individuals that do the trip on the survey day, and drop off/pick up location is exogenous.

Below we will first describe the state space and choice set, and then how time-space constraints can be expressed in terms of restrictions on either the state space or on a state specific choice set.

2.1.1. *States and actions*

A state should include the information needed to determine available actions and utility of these actions. Here a state x consists of:

Time $t \in [5\,\mathrm{am}, 11\,\mathrm{pm}]$: Continuous variable for time of day. A day starts at 5 am and ends at 11 pm

Location $L \in [1, 1240]$: Current location. One of 1240 zones in the region of Stockholm.

Activity A: New activity, end activity, social, recreational, shop, home, work and escorting children are the alternative activity states.

The activity must be included in the state since the individual can choose to continue with the same activity for yet another time-period, but have to travel (possibly within the zone) to change activity. The purpose of the new activity and end activity states will be discussed below.

Errand indicator $E \in [0, 3]$: A state keeping track of the number of finished mandatory activities. The number of mandatory activities varies from 1 to 3 depending on the individual, as will be explained later.

Car dummy $\delta_{car} \in \{true, false\}$: Dummy for car availability. An individual have to travel with car if $\delta_{car} = true$ and if out of home and cannot travel with car if $\delta_{car} = false$.

The set of actions a that are available in a state x for individual n is denoted $A_n(x)$. The universal choice set consists of any combination of activity A, mode M and location L:

Activity A: Activity for new action.

Location L: New location.

Mode: Car, public transport, bike and walk are the modelled modes. When continuing with the same activity, the mode of the action is "no-mode".

When starting a new activity with flexible duration it is initially conducted for one time-step, which we have chosen to be 10 minutes. Depending on the activity and on time-space constraints it can be possible to continue with the same activity for another time step. The action-space is thus discrete and finite. This means that every 10th minute individuals can decide whether to continue with the current activity for another 10 minutes. Travel times are not divisible by these time step lengths, and it therefore makes sense to have a continuous state variable for time. Since there are a finite number of actions in each state there will still only be a finite number of reachable states. It is sometimes argued that activity length should be a continuous variable, as it is included in, e.g., (Habib, 2011), (Pinjari and Bhat, 2010) and (Kang and Recker, 2013), but from a behavioural perspective we think it makes at least as much sense to assume that people are considering whether to, e.g., spend 10, 20 or 30 minutes shopping as to assume that they decide to spend exactly 17.3123 minutes.

Work and child errands have fixed duration (10 minutes for dropping off children and working hours as observed). The remaining alternatives can be continued for any number of time steps. Most computations come from calculating the log-sums in (2.3) for all states, and as activities and locations are both states and alternatives in each state, the computational time will increase quadratic with both the number of locations and the number of activities. To reduce computational time, the "start-activity" and "end-activity" states are added. The reason for this can be illustrated with an example. In each state, an individual can choose to either continue with the same activity or start a new activity at any location. This gives $1 + N_{act} \cdot N_m \cdot N_{loc}$ alternatives, and calculating EV in a state therefore requires summing up $1 + N_{act} \cdot N_m \cdot N_{loc}$ factors. In each time step, there are approximately $N_{act} \cdot N_{loc}$ states for which this operation is performed. In a "start-activity"-state, the only available alternatives are to start one of the available activities, so there are approximately N_{act} factors that needs to be summed together. Once we have EV in this state, we can divide the choice of a new activity into two steps, firstly the choice of a new location and mode, and secondly the choice of activity. This will not change the choice probability when they are given by (2.4). With this new state, the number of terms reduces to $1 + N_m \cdot N_{loc}$ (where 1 is the alternative to continue with the same activity), and approximately decreases with a factor N_{act}. This comes at the expense of calculating the log-sum of N_{act} in N_{loc} states. When considering a new action, the future utility is independent of the current activity, and it is therefore possible to create an "end-activity" where the sums of all possible "start-activity" states is calculated. Instead of having to sum up $1 + N_{act} \cdot N_m \cdot N_{loc}$ factors in $N_{act} \cdot N_{loc}$ states we sum up the $N_m \cdot N_{loc}$ "start-activity" factors in N_{loc} "end-activity" states, so the computational savings can be significant. This also means that increasing the number of activities only have a minor effect on the computation time.

We currently do not consider any mixed modes, or mode chains, such as taking the bike to the train station. This would definitely be possible conceptually, but would further increase the computation time and has therefore not been included at this stage. Car is only a possible choice if the individual has a car available at home. Further, if a car is used for a trip away from home, all consecutive trips on the same tour must be done using car. This is controlled through the car-dummy δ_{car}. Certainly it happens that individuals use a car for only some trips in a tour. They might leave their car at work and pick it up the next day or leave it for another family member. To correctly include the possibility to leave a car at any location, another state variable would be needed that remembered the location where the car was parked and the state space would become 1000-times larger. Since that kind of behaviour is quite uncommon, we think this restriction on car usage is reasonable.

Each individual is considering all possible locations for each new action. before, locations are both state variables and alternative actions so the computation time increases quadratically with the number of locations. Restricting the choice set of locations for individuals is therefore extremely tempting, but combining an activity scheduling model with a location choice set model in a consistent way seems extremely complex. This curse of dimensionality connected to the number of zones is sometimes solved by sampling a number of zones through some auxiliary model (see e.g., (Liao et al., 2013)), or by approximating the log-sums through importance sampling (similar to how Bradley et al. 2010 does in a nested framework). We want to avoid such approximations if possible. However, if the zones would be refined or increase for other reasons, we would likely have to resolve to some sort of sampling. Rust (1997) shows how randomization can be used to approximate EV in dynamic discrete choice models, and it would be one possible way to decrease computation time.

2.1.2. *Time as a continuous variable.*

Time is modelled as a continuous variable, but the number of states in which EV can be calculated is limited by computation time. It is therefore not possible to exactly calculate the expected value functions in all reachable states. Instead, EV is calculated on a discretized time-grid containing every 10:th minute and linear interpolation is used to approximate the value between these points. The linear interpolation approximation in a state x is denoted $\overline{EV}$ (x). The calculation of EV will therefore be based on approximations of EV in future states. We will therefore never know the exact expected value function in any state, but rather the (1:st order) approximate expected value function $\widetilde{EV}$, given by:

$$\widetilde{EV}(x_t) = \log \left(\sum_{a \in A(x_t)} e^{u(x_t,a)+\overline{EV}(x_{t'})} \right) \tag{2.6}$$

where $t_{k-1} \leq t' \leq t_k$ gives:

$$\overline{EV}(x_{t'}) = \alpha_1 \widetilde{EV}(x_{t_k}) + \alpha_2 \widetilde{EV}(x_{t_{k+1}}) \tag{2.7}$$

where $\alpha_1 = \frac{t_{k+1}-t'}{t_{k+1}-t_k}$ and $\alpha_2 = \frac{t'-t_k}{t_{k+1}-t_k}$.

This approximation makes the order in which EV is calculated important. When backward induction is used to calculate EV, it is updated one time step at a time starting in the end of the day T and moving backward. If an action is less than 10 minutes long, the approximation in (2.7) will be based on $\widetilde{EV}$ in the current time step, causing self-dependence which would require value iteration or the solution to an equation system. The only way this can occur with the current discretization and activity durations is if the new action involves a trip to a "new activity" state. As the only available alternative in a "new activity" state is to start an activity, $\widetilde{EV}$ in these states can be calculated first without risking self dependence. After $\widetilde{EV}$ has been calculated in the "new activity" states, it can be calculated in the remaining states for that time step. Note that for this reason, it is important that the shortest possible activity duration is longer than the duration between time-steps.

2.1.3. *Time-space constraints.*

Time space constraints define when and where an individual can participate in different activities and thereby impose a structure on the day. Time-space constraints can be of the type "I have to be at work by 7 a.m.", and both explicitly determine where an individual will be at 7 a.m. (at work) and implicitly influence where they can be at 6 : 50 a.m. (not more than 10 minutes away from work with available modes of transport). To check that a specific trip is possible, one must look multiple future trips into the future to ensure that all time-space constraints can be satisfied if that specific trip is carried out. Finding feasible activity schedules in a dynamic discrete choice model is trivial since expected value function $EV = -\infty$ in any explicitly or implicitly infeasible state, as by definition there are no actions leading from such a state to another state with $EV \neq -\infty$. Actions that are implicitly infeasible due to time-space constraints will therefore have zero probability.

Some activities are time constrained. Time constraints on when activities can be started or when they must be completed can easily be included by restricting the choice set at times that do not meet these constraints. Location constraints, i.e., constraints specifying where different activities can take place, are treated in the same way.

People can have fixed or flexible working hours. People with fixed working hours must arrive at work when the workday start and leave when the workday ends. People with flexible working hours can choose to arrive between 6 a.m. and 10 a.m., but the length of a working day is still fixed. The individual specifications on working hour type, working length, start and end hours must be provided from elsewhere. Children can be dropped off between $6:30$ a.m. and $12:00$ a.m.. Pick up trips must be completed between 12 a.m. and $6:30$ p.m. All individuals must start and end their days at home. There is no need to restrict the state space in the start of the day. Such restrictions are ensured by the choice of the initial state used when, e.g., simulating day paths.

Picking up and dropping of children at school as well as going to work are considered mandatory activities with fixed location and time constraints. These three activities further have an internal order: dropping of children is done before going to work which must be done before picking up the children again. To model this order of activities, we introduce the errand indicator E. When $E = 0$, only dropping of children is possible. After having finished a drop-off activity, E increases by one and the only available activity in the group is work. Enforcing that all activities are finished during the day is done by restricting E in the end of the day, and time-constraints are treated as above.

More generally, a constraint could impose that some activity or a group of activities must be conducted a number of times N during a day. This can be modeled by introducing an errand indicator state variable, say Q, for each such group of mandatory activities and setting the expected value function to $-\infty$ whenever $Q \neq N$ in the end of the day. Whenever an activity in the group is started, Q is increased by one. If the day is started in a state with $Q = 0$, all feasible activity schedules will do activities in the group exactly N times. Introducing an extra state variable is not without costs. The number of states will increase linearly with N and the number of actions in each state will not decrease substantially, so the computation time will increase almost linearly with N in each basic activity constraint. If there are multiple groups of mandatory activities where the activities in a group i must be conducted N_i times, the number of states will increase with a factor $\prod_i (N_i + 1)$ times.

2.2. *One-stage utility functions*

For an individual n, the instantaneous utility $u_n(a|s)$ is the sum of the (dis)utility of traveling $u_{n,m}$ and the utility of participating in an activity $u_{n,p}$. The utility of traveling is dependent on the travel cost, travel time and mode, which in turn will be dependent on time of day, origin and destination. The current state is given by $s = (t, l, p, e, c)$, where t is the time, l the location, p the activity (or purpose), e the errand indicator and c the car dummy; and the action is $a = (p', l', m)$ where p' is the new activity, l' the new location and m the mode of transport. The utility of travelling with mode m for individual n can then be written as $u_{n,m}(a|s) = u_{n,m}(l, l', t)$, as it is dependent on the individual, the origin, destination, time of day and mode. For respectively mode it is specified as:

$$\begin{aligned}
u_{n,\text{car}}(l, l', t) &= c_{\text{car}} + \theta_{\text{car},t} TT_{\text{car}}(l, l', t) + \theta_c C_{\text{car}}(l, l', t) \\
u_{n,\text{PT}}(l, l', t) &= c_{\text{PT}} + \theta_{\text{PT},t} TT_{\text{PT}}(l, l', t) + \theta_c C_{\text{PT}}(l, l', t) \\
u_{n,\text{bike}}(l, l', t) &= c_{\text{bike}} + \theta_{\text{bike},t} TT_{\text{bike}}(l, l', t) \\
u_{n,\text{walk}}(l, l', t) &= c_{\text{walk}} + \theta_{\text{walk},t} TT_{\text{walk}}(l, l', t) + \theta_{\text{same zone}} \delta_{\text{same zone}}
\end{aligned}$$

where $TT_m(l_1, l_2, t)$ and $C_m(l_1, l_2, t)$ denote the travel time and cost with mode m at time t for a trip from origin l_1 to destination l_2 , $\delta_{samezone}$ is a dummy indicating if the trip is done within the same zone and c_m are mode specific constants.

When arriving at the destination l' at time $t' = t + TT_m(l, l', t)$, the new activity p is started and performed for an activity dependent duration Δt_p. Starting the activity gives a time-of-day dependent constant utility $c_p(t)$ and a duration and time-of-day dependent utility $U_{n,p}(t, \Delta t_p)$. Choosing to continue with the same activity for another time step only gives the duration utility $U_{n,p}(t, \Delta t_p)$. Not all activities have time-of-day specific parameters. In order to keep down the number of parameters, the constant utility $c_p(t)$ is only time dependent for the work activity and the duration utility $U_{n,p}(t, \Delta t_p)$ is only time dependent for the home activity. Time-of-day varying parameters are specified on discrete time steps T_k with values θ_{p,T_k} and c_{p,T_k}. The activity specific constant is given by linear interpolation between the closest defined parameters:

$$c_p(t) = \frac{c_{p,T_k}(T_{k+1} - t) + c_{p,T_{k+1}}(t - T_k)}{T_{k+1} - T_k}$$

where $t \in (T_k, T_{k+1})$. For the duration utility we specify the *marginal* utility of activity participation at time t as given by linearly interpolation between the closest parameters, so:

$$u_t(t,p) = \frac{\theta_{p,T_k}(T_{k+1} - T_k) + \theta_{p,T_{k+1}}(t - T_k)}{T_{k+1} - T_k}.$$

The utility of an activity episode of duration Δt_p, when $T_k \leq t$ and $t + \Delta t_p \leq T_{k+1}$, then becomes:

$$U_p(t, \Delta t_p) = \int_t^{t+\Delta t_p} u_t(\tau, p)\, \mathrm{d}\tau = \alpha_{T_k}\theta_{p,T_k} + \alpha_{T_{k+1}}\theta_{p,T_{k+1}} \tag{2.8}$$

where:

$$\alpha_{T_k} = \Delta t_p \frac{T_{k+1} - t - 0.5\Delta t_p}{T_{k+1} - T_k}$$
$$\alpha_{T_{k+1}} = \Delta t_p \frac{t + 0.5\Delta t_p - T_k}{T_{k+1} - T_k}.$$

Observe that $\alpha_{T_k} + \alpha_{T_{k+1}} = \Delta t_p$, so if $\theta_{p,T_k} = \theta_{p,T_{k+1}}$ the duration utility becomes $\Delta t_p \theta_{p,T_{k+1}}$. If $t + \Delta t_p > T_{k+1}$, $u_t(\tau, p)$ in (2.8) becomes a stepwise linear function but is otherwise treated in the same way.

Besides activity specific constants, each location l has size parameters representing the number of available opportunities for each activity at that location. This utility is given by:

$$u_{p,\text{size}}(l) = \theta_{p,\text{size}} \log\left(\sum_{s=1}^{s=S_p} x_{p,l,s} e^{\theta_{p,s}}\right)$$

where S_p is the number of size variables for activity p, and the size variables $x_{p,l,s}$ can be, e.g., the number of employees in a specific sector at location l. Since the model contains activity specific constants, one of the parameters $\theta_{p,s}$ should be fixed for all activities. This also provides an alternative interpretation of the activity specific constants as scales for the size variables $x_{p,l,s}$. A complete list of size variables included for respectively activity is given in table 2.

2.3. *Computation time*

With the current implementation, calculating the value function in all states and thus evaluating the probability of a path once for a single individual takes between $4 - 10$ s. Almost all (98%) of the computation time is consumed by the function calculating the value function in all end-activity states when summing up the alternative trips that can be started. When excluding working hours, an example individual have 11 h of free time left between 5:00AM and 23:00PM. The value function is evaluated on a 10 minute grid, giving 65 grid points in which it will be evaluated. In each of these grid points, there are a total of 4 modes available for a car owner and 1 240 locations that are both states and destinations. This gives a total of $65 \cdot 4 \cdot 1\,240 \cdot 1\,240 \sim 4 \cdot 10^8$ links. For each of these links, one must calculate the travel time (taking 16% of the time), one-stage utilities (23%), obtain the future expected value function and sum this with the utility (41%) and finally perform the exponent $e^{u_i+EV_j}$ for all links (20%). All-in-all, this takes around 4-10 s when using a single core on a Intel(R) Core(TM) i7-6820HQ CPU @ 2.70GHZ. The main program is written in C# but Intel MKL has been used for vector mathematics when applicable and C++ routines has been used for other time consuming parts. As a comparison, simply performing e^x in MATLAB for a vector x with 10^8 elements on the same computer takes 1 s.

As discussed above, the from-all-to-all destinations operation is currently the limiting computational factor. One possible way to speed up the program would be to sample locations. If 100 locations were sampled, the computation time could potentially decrease to 0.05 s/individual.

The program is parallelized using MPI and could potentially benefit from hundreds of cores reducing the computation time to days. A possible future solution to obtain efficient estimates and allow for nested logit or GEV formulations would be to use inefficient estimates obtained using sampling of locations or some alternative approximative or possibly biased estimation technique to find a good enough specification of the model and then finally obtain full-information estimates using a cluster.

3. ESTIMATION

Consider an individual n who has been observed to choose a day-path starting from state x_0 consisting of a sequence of decisions $\mathbf{a} = (a_0, ..., a_{T-1})$ and thus traversing the states $\mathbf{x} = (x_1, ..., x_T)$. The likelihood for this sequence of decisions is, according to (2.5) given by:

$$\begin{aligned} P(\mathbf{a}|x_0) &= \prod_{i=0}^{T} P(a_i|x_i) \\ &= \prod_{i=0}^{T} e^{u(a_i,x_i)+EV(x_{i+1})-EV(x_i)} \end{aligned} \tag{3.1}$$

The standard method for estimating dynamic discrete choice models is the Nested Fixed Point Method (NFXP) (Rust, 1988). This involves first calculating EV and its gradients in each state of the network and then directly use (3.1) for estimation. Although possible to apply on the proposed model, the method would be extremely time consuming given the discussion on computation time in Section 2.3. If estimation would rely on calculating the value function and gradient of the value function in each state in each iteration, the computation time would likely be $0.4 \cdot N_{\text{variables}} \cdot t$ per observation per iteration (as the gradient must be calculated for each variable, requiring at least the additional sum of the gradient of $u + EV$ for each variable). With 10 s/observation to calculate EV, 70 variables, 3 300 observations and 100 iterations before convergence, this would require $10 \cdot 0.4 \cdot 70 \cdot 3\,300 \cdot 100\,\text{s} \approx 1\,000$ days to estimate using a single core.

Methods used to speed up estimation of dynamic discrete choice models, e.g., the method proposed in Aguirregabiria and Mira (2002), reduces the burden of value iterations when calculating the value function. As no iteration is needed to evaluate the value function in this model, this would likely not help. Some approximative method is therefore needed.

3.1. *Sampling of alternatives*

In the context of route choice modeling, Fosgerau et al. (2013a) showed that an MNL over routes in a directed network can be expressed by (2.5) and estimated using the NFXP. From (3.1), observe that:

$$\begin{aligned} P(\mathbf{a}|x_0) &= \prod_{i=0}^{T} e^{u(a_i,x_i)+EV(x_{i+1})-EV(x_i)} \\ &= e^{u(\mathbf{a},x_0)+EV(x_{T+1})-EV(x_0)} \end{aligned} \tag{3.2}$$

where $u(\mathbf{a}, x_0) = \sum_{i=0}^{T-1} u(a_i, x_i)$. Since $EV(x_T)$ and $EV(x_0)$ are the same for all alternative action sequences starting in x_0, the probability for each alternative is proportional to $e^{u(\mathbf{a},x_0)}$. If $\mathscr{A}(x_0)$ is the set of action sequences that, starting from x_0, satisfies all space-time constraints, then:

$$P(\mathbf{a}|x_0) = \frac{e^{u(\mathbf{a},x_0)}}{\sum\limits_{\mathbf{a}' \in \mathscr{A}(x_0)} e^{u(\mathbf{a}',x_0)}}. \tag{3.3}$$

Recursive Logit (RL) models has recently been developed extending the MNL case discussed in Fosgerau et al. (2013a) to cover Nested Logit (Mai et al., 2015), MEV (Mai, 2016) and Mixed Logit specifications (Mai et al., 2016). They have also been applied in a number of scenarios, e.g., in Zimmermann et al. (2017) application to route choice for bikes, possibly with the largest network so far in a RL model. The number of links in these models are between 7 000-40 000. The model presented here is thus around 2 000 times bigger, so although the models are very similar the estimation techniques used for RL-models are to feasible here.

Here, the equivalence between a RL-model in (3.2) and an MNL over routes in (3.3) is utilized to allow estimation using sampling of alternatives. It is a general property of MNL models that estimating over a subset of alternatives gives consistent estimates if a correction term is added to the utility function (McFadden, 1978). The estimates are, however, not efficient and how the choice sets are constructed will determine the efficiency loss. Since the number of alternatives is immense, is is important to use a smart way of sampling alternatives that somewhat resembles the model in order to obtain good estimates. As it is trivial to simulate alternative once the value function has been evaluated, a choice set can be constructed using the described model with an initial guess of the parameters.

Estimation using sampling of alternatives involves sampling a choice set $\tilde{\mathbb{C}}_n \subset \mathbb{C}_n$ and estimating using the conditional choice probability $P_n(\mathbf{a}_n|\tilde{\mathbb{C}}_n)$ instead of the $P_n(\mathbf{a}_n|\mathbb{C}_n)$. A maximum likelihood estimation on a choice set $\tilde{\mathbb{C}}$ gives consistent estimates if the correction term $\log(\bar{q}_n(\tilde{\mathbb{C}}_n|j))$ is added to each alternative and $\bar{q}_n(\tilde{\mathbb{C}}_n|j)$ satisfies the positive conditioning property, i.e., that if $j \in \tilde{\mathbb{C}}_n$ and $\bar{q}_n(\tilde{\mathbb{C}}_n|i) > 0$ for some i, then $\bar{q}_n(\tilde{\mathbb{C}}_n|j) > 0$. This holds if $\tilde{\mathbb{C}}_n$ is sampled from the universal choice set $\mathbb{C}_n$ and all alternatives in $\mathbb{C}_n$ have a non-zero probability of being sampled.

Let N observations form the set of observations $\mathscr{O}_N$. The log-likelihood function for $\mathscr{O}_N$ based on the conditional likelihoods becomes:

$$\bar{\mathscr{L}\mathscr{L}}(\mathscr{O}_N;\theta) = \sum_{n=1}^{N} \log\Big(\frac{e^{u(\mathbf{a}_n)+\log(q_n(\tilde{\mathbb{C}}_n|j))}}{\sum\limits_{\mathbf{a}^*\in\tilde{\mathbb{C}}_n} e^{u(\mathbf{a}^*)+q_n(\tilde{\mathbb{C}}_n|\mathbf{a}^*)}}\Big) \tag{3.4}$$

If all alternatives in $\mathbb{C}_n$ have equal probability of being sampled to the choice set $\tilde{\mathbb{C}}$, the correction term $q_n(\tilde{\mathbb{C}}_n|j)$ will also be the same for all alternatives and therefore cancel out from the likelihood function. However, if some other sampling protocol is used the probability must be calculated. The sampling protocol use here is the same that Frejinger et al. (2009) used to estimate an MNL model over the choice of routes in a traffic network. The sampling protocol consists of drawing R alternatives with replacement from the choice set $\mathbb{C}_n$ consisting of J_n alternatives, and then adding the observed choice to the choice set. The outcome of such a protocol is $(k_{n1}, k_{n2}, \ldots, k_{nJ})$ where k_{nj} is the number of times alternative j appears in the choice set, so that $\sum_{j=1}^{J} k_{nj} = R+1$, since the observed alternative j is added once extra to the choice set. Let $q_n(i)$ denote the probability that alternative $j \in \mathbb{C}_n$ is sampled. The correction term can then be derived to: $q_n(\tilde{\mathbb{C}}_n|j) = K_{\tilde{\mathbb{C}}_n}\frac{k_{nj}}{q_n(j)}$. The constant $K_{\tilde{\mathbb{C}}_n}$ will cancel out from the likelihood function to give:

$$\bar{\mathscr{L}\mathscr{L}}(\mathscr{O}_N;\theta) = \sum_{n=1}^{N} \log\left(\frac{e^{u(\mathbf{a}_n)+\log(\frac{k_{n\mathbf{a}_n}}{q_n(\mathbf{a}_n)})}}{\sum_{\mathbf{a}^*\in\mathbb{C}_n} e^{u(\mathbf{a}^*)+\log(\frac{k_{n\mathbf{a}_n^*}}{q_n(\mathbf{a}_n^*)})}}\right) \tag{3.5}$$

As previously mentioned, a choice set is sampled using (2.4) with a single set of parameters. They were derived by starting from a simple specification of the model, only involving time and cost parameters, and then manually their values until travel times, mode choices and activity episodes were in line with the observations.

3.2. *Correlation between alternatives*

It is common practice in route choice modelling to add a size attribute to each link to take correlation among paths that overlap into account, e.g., using Path-Size Logit (Ben-Akiva and Bierlaire, 1999). For their link-based route choice model, Fosgerau et al. (2013b) obtains a size coefficient by calculating EV in each link using some pre-specified parameters and adding that to the link-utility. In the activity-scheduling model presented here, it is not as easy to define the overlapping of paths, as the network is dynamic. If two paths are identical besides that the start time for all activities in one path is 10 min after the start time in the other path, there can be practically no overlapping as defined by the Path-Size Logit although the two paths would be very similar. How to address this issue in a dynamic network and in the activity-scheduling framework is therefore an open question.

In trip-generation models, it is common to have nests for mode choice, location choice and activity choice, as in, e.g., Bowman and Ben-Akiva (2001). It would be possible to introduce different scales for the error term when solving (2.3) and obtaining choice probabilities in (2.4) where the scale (which is one here) would be state dependent. This is done in the Nested Recursive Logit model described in Mai et al. (2015). However, the probability of a path would then not reduce to (3.2) and sampling of alternative sequences would not be possible to use for estimation. Guevara and Ben-Akiva (2013b) recently showed that Multivariate Extreme Value (MEV) models such as Nested Logit can be estimated using sampling of alternatives. A Nested RL-model is however not the same as an MEV model, so the transferability of the result is uncertain. If nests are introduced within the network, as in Mai et al. (2015), it would further require that the value function was approximated in all states, so the computational benefit might not be enough. An alternative would be to introduce nests over paths, for example nesting alternatives that include specific activities or modes. This would move the model in the direction of Bowman and Ben-Akiva (2001).

The computation time is however likely to grow linearly with the number of nests. Given that the model already is time demanding, creating nests for all combinations of modes and activities would not be computationally feasible.

Another issue is the correlation in preferences over time. Individuals' variances in preferences for, e.g., mode or activities are likely to be consistent over time and therefore to some extent be the same throughout the day. Including nests on a trip level would not capture this correlation. A possible solution would be to introduce mixed parameters for, e.g., activities and modes, that would be the same for each individual for the full day. Our estimation approach is based on sampling of alternatives and recent research by Guevara and Ben-Akiva (2013a) shows that the same method gives consistent estimates for mixed logit models. This has been explored in an extension of the work presented here in (Zimmermann et al., 2018).

Finally, it is worth noting that the expected value function in (2.4) might pick up some of the correlation in the unobservable ϵ that is usually captured by introducing nests in a trip or tour based model. Since a trip with walk, public transport and bike all share the same state, except for the arrival time, EV will be correlated for the three alternatives.

3.3. *Data*

We have estimated the model using the Stockholm travel survey from 2004, where individuals report a full day travel diary. Estimating using full day-paths puts a high demand on the reported diaries, since the information for all trips in an observation must be correct in order for it to be usable. Further, travel times as reported in the diaries are rarely the same as the data we have on travel times or the travel times we calculate for the same origin-destination, and sometimes the discrepancy is huge. This is always a problem, since the observed travel times only are available for the observed origin-destination pair with the observed mode. One common way of dealing with this is to use the calculated travel times rather than reported travel times, and this is how we choose to proceed. A static traffic assignment model with a nested-logit based travel demand model gives the travel times and costs used for peak and off-peak periods. It is worth noting that when considering day-paths changing the travel time of an observed trip will influence the starting time and/or duration of all remaining actions in the same day. Travel cost with car is calculated as 1.4 kr/km, travel times with bike is calculated assuming a speed of 15 km/h and walk travel times assuming a speed of 4 km/h.

We have so far restricted the model to individuals that go to work on a weekday. This leaves us with 5200 observations with sufficient information. Out of these, 3300 behave in a way that fits the model. The $\sim$ 2000 observations that are removed at this stage behave in ways that the model cannot handle, for example by ending the work day with a business trip, and therefore not ending the work day at their work location; having longer than $2h$ breaks in the middle of their work day; working late (later than 8pm) or starting early (earlier than 6am); leaving the car somewhere or not returning home.

We have demanded that car should be used for either all or no trips on a tour. As passengers and drivers has been treated in the same way, this means that observations with passengers are likely to be removed. Besides individuals reporting that they were passengers, 3% of the observations included a trip of this kind. It is hard to say what is happening here. It is possible that the individual is being dropped off or is being picked up but is driving the car to/from the activity. It is also possible that individuals leave their car for a later day. Parameterizing this behaviour without knowing if the car will be picked up on a later occasion or if someone else is driving it back, and thereby not knowing the attributes associated with such a choice, is likely make the model responds incorrectly to changes in traffic conditions. We have therefore chosen to exclude these observations.

3.4. *Estimation Results*

Table 1 gives estimation result for all parameters except the size-parameters, which are given in 2. Most parameters are significant and have the expected sign. Cost is negative and spending time on activities is preferred to spending time on travelling. Home time is valued higher early in the morning and late in the evening. It is preferable to spend time on freetime activity than to be at home between 1-4PM and no significant difference between 5-9PM. Since not all time parameters can be identified, $\theta_{\text{Rec Time}}$ is fixed, and the linear-in-time parameters can only be compared against each other. Although the choice of parameter to fix does not affect the theoretical properties of the estimates, it can impact the obtained standad devivation. When $\theta_{\text{PT Time}}$ was fixed rather than $\theta_{\text{Rec Time}}$, the standard deviation of all time parameters was close to 0.006, rather than varying between 0.001 and 0.005. Travel time parameters are significantly smaller than activity duration parameters, so participating in an activity is preferred to travelling. Since time parameters can only be interpreted in relation to each other, it is not possible to directly calculate the value of time. A travel time saving with car that gives one minute extra at home between 6 p.m. would be valued $(\theta_{\text{Car Time}} - \theta_{\text{5-6PM Time}})/\theta_{\text{Cost}} = 7.3\,\text{SEK/minute}$.

The time-specific constants for work hours seem quite large in comparison to the time parameters, but this is mainly due to the scaling of the parameters. When comparing two alternative sequence, one that arrives at work at 6 a.m. and arrive back home at 4 p.m. and one that arrives at work at 7 a.m. and back home at 5 p.m., the difference in utility per minute at work from arriving at the different times will be $(\theta_{\text{Work ASC 6AM}} - \theta_{\text{Work ASC 7AM}})/60 = -0.007$. This is greater than the difference between $\theta_{\text{Home 6PM}}$ and $\theta_{\text{Home 7PM}}$ but smaller than the difference between $\theta_{\text{Home 5PM}}$ and $\theta_{\text{Home 4PM}}$. The difference in the valuation of time spent at home at different times of the day and the difference in the valuation of time spent at work at different times of the day will therefore both have a big importance when determining departure time to work.

Interpreting the constants for mode and activities is not entirely straightforward. Firstly, they are all normalized by fixing $\theta_{\text{Home ASC}}$ to zero. Further, the scale of the size parameters is arbitrary and obtained by fixing one of the size parameters to zero.

As estimation is performed using sampling of alternatives efficient estimates are not obtained. Since the number of alternatives of the universal choice set so immense, it would be possible that the obtained estimates were very inefficient or biased. When validating the estimation on simulated data with known parameters and sampling with parameters far from their true values, the estimation result was often very poor. It is also possible that the approximations used to calculate $\widetilde{EV}$ would cause a bias when the model is used to simulate day-paths, as estimation does not take this approximation into account. To check if either of these factors influence the final result, 1000 day-paths per individual was simulated and their aggregated attributes was compared to the observed data. The resulting differences can be found in Table 3. The simulated data deviates from the observed data by $0.1 - 1.7\%$, but relative difference over one percent is only observed for attributes small quantities (the number of other-activities are underpredicted by 0.002 times/day, meaning 2%). In absolute terms it translates to errors of 0.05 min/day in travel time, 0.14 SEK/day in travel cost and 0.005 trips/day per mode. We think that this remaining error is negligible in any practical applications.

4. SIMULATION RESULT

To test how well the model manages to produce realistic behavior we simulated daily activity schedules for the observed individuals and compared some characteristics of the simulated data with the real data. Since we have parameters for travel time, travel cost, activity time, number of trips, etc., many quantities should be the same as in data (if we did not use sampling of alternatives). We are therefore focusing on quantities that we do not directly estimate but that are outcomes of the estimated parameters. Here we report result on activity timing, distribution of trips and tours, and trip length distribution.

Three factors determine the timing of activities: firstly, time constraints on working hours and child-errand times; secondly, preferences for when to arrive at work in the morning; and thirdly, preferences on when to be home. From figure 1 it seems as if these determinants are enough to predict when people go to work and when they get home. For child-errands, the start and end time for school is likely to play an important role, and this has not been included in the model.

Table 1: Parameter estimates obtained using sampling of alternatives. $\theta_{\text{Rec. Time}}$ is fixed and normalize all the time-parameters. $\theta_{\text{Work ASC 8AM}}$ is fixed and normalize the work constants. There is no constant for arriving home ($\theta_{\text{Home ASC}}$ is fixed to zero), and this normalize mode and activity constants can be identified.

Parameter		Estimate	Rob. t-test
Car	Time	-0.084	-12
	ASC	-2.7	-17
PT	Time	-0.038	-2.9
	ASC	-3.7	-21
	Total Wait	0.0041	0.25
Walk	Time	-0.051	-13
	same zone	-0.53	-2.5
	ASC	-1.7	-9.4
Bike	Time	-0.057	-11
	ASC	-4.2	-22
Cost		-0.012	-4.1
Home	6AM Time	0.041	4.5
	7AM Time	0.043	6
	8AM Time	0.02	3
	9-10AM Time	0.015	1.5
	1-4PM Time	-0.011	-4.8
	5-6PM Time	0.0036	1.8
	7-8PM Time	0.0024	1.2
	9PM Time	0.019	5.8
Work	ASC 6AM	1.1	1.4
	ASC 7AM	0.68	1.7
	ASC 8AM	0	Fixed
	ASC 9AM	-1.3	-4.1
	ASC 10AM	-5.1	-7.1
Social	Time	0.00067	0.23
	ASC	-9.2	-21
	LSM Size	0.43	1.7
Shop	Time	-0.021	-7.2
	Small ASC	-6.6	-22
	LSM Size	0.51	4.7
Other	Time	-0.0086	-3.2
	ASC	-7.3	-21
	LSM Size	0.34	2.8
Rec.	Time	0	Fixed
	ASC	-7.7	-24
	LSM Size	0.083	0.98

Table 2: Estimates of size-parameters. Observe that as they enter the utility as e^{γ}, the t-test cannot be used to determine their significance. Population has been fixed to zero for Rec, Social and Shop wheras 'No employed OE' was fixed for Other.

	Parameter	Estimate
Rec.	Population	0
	No Employed Rest.	5.8
Social	Population	0
Shop	Population	0
	No Employed Shop	3.4
Other	No employed OE	0
	No Employed Rest.	3

Start times for free time activities are quite well reproduced by the model, but slightly shifted towards later hours. This could also be due to the lack of opening hours in the model, or because only home-time utility is time-of-day dependent. Overall the model is able to reproduce the timing of activities very well.

The distribution for number of trips and tours in a day is determined by a vast number of factors, but the total number should be correct as tour and trip constants are estimated. The tour constant is equal to the home constant, as each additional tour will include an additional trip home. A specific trip constant would not be possible to identify given the constants we already have for modes and activities, but these constants will together ensure that the number of trips is correct. However, we do not have any constants governing the distribution of the number of trips per day, number of tours per day or number of trips per tour. The overall structure of the model will therefore determine these distributions, and seems to be enough to give good predictions, as figure 2 show.

The length of trips by mode will mainly be determined by the utility of time and money for respectively mode and by network characteristics. This gives a good distribution of travel times, as can be seen in figure 3. Since a large share of the trips are made to and from work, where the location is fixed and the trip is mandatory, the model is guaranteed to reproduce a large share of the trips well. However, the observed mode for the trip to work will only be the chosen mode in some of the simulated observations, and these restrictions will not give the distribution directly.

The "same zone" dummy parameter $\theta_{\text{same zone}}$ for walk is negative, which seems contra-intuitive as one would expect walk to be the preferred mode of transport for shorter distances. From figure 3 it is clear that for same-zone trips (trips with

Table 3: Average simulated and observed statistics. For each individual, 1000 alternatives are sampled. That the difference is very small indicates that the linear approximation of $\widetilde{EV}$ works well and that enough alternatives are sampled to the choice set.

Attribute	Observed	Simulated	Obs-Sim	% difference
Home 6AM Time	87.03	87.00	0.024	-0.028%
Home 7AM Time	45.21	45.15	0.069	-0.152%
Home 8AM Time	19.61	19.55	0.069	-0.354%
Home 9AM Time	3.65	3.62	0.026	-0.726%
Home 1PM Time	11.39	11.49	-0.097	0.844%
Home 5PM Time	58.58	58.62	-0.038	0.065%
Home 7PM Time	102.37	102.41	-0.039	0.038%
Home 9PM Time	146.86	146.73	0.130	-0.089%
Car Time	18.69	18.74	-0.049	0.263%
Car ASC	0.99	0.99	-0.003	0.287%
PT Time	29.96	29.97	-0.004	0.014%
PT ASC	1.06	1.06	-0.001	0.077%
PT Total Wait	22.49	22.50	-0.008	0.034%
Walk Time	9.60	9.63	-0.030	0.315%
Walk same zone	0.06	0.07	-0.001	1.645%
Walk ASC	0.35	0.36	-0.002	0.672%
Bike Time	5.28	5.25	0.033	-0.623%
Bike ASC	0.24	0.24	-0.001	0.501%
Cost	49.13	49.27	-0.140	0.285%
Work ASC 6AM	0.01	0.01	-0.000	1.658%
Work ASC 7AM	0.05	0.06	-0.000	0.900%
Work ASC 9AM	0.15	0.15	0.001	-0.867%
Work ASC 10AM	0.02	0.02	-0.000	0.164%
Shop Time	7.20	7.25	-0.057	0.785%
Social Time	2.82	2.77	0.047	-1.690%
Social ASC	0.03	0.02	0.001	-2.287%
Rec. ASC	0.12	0.12	-0.000	0.197%
Other Time	5.29	5.41	-0.120	2.220%
Other ASC	0.09	0.09	-0.002	2.590%
Shop Small ASC	0.19	0.19	-0.002	1.312%

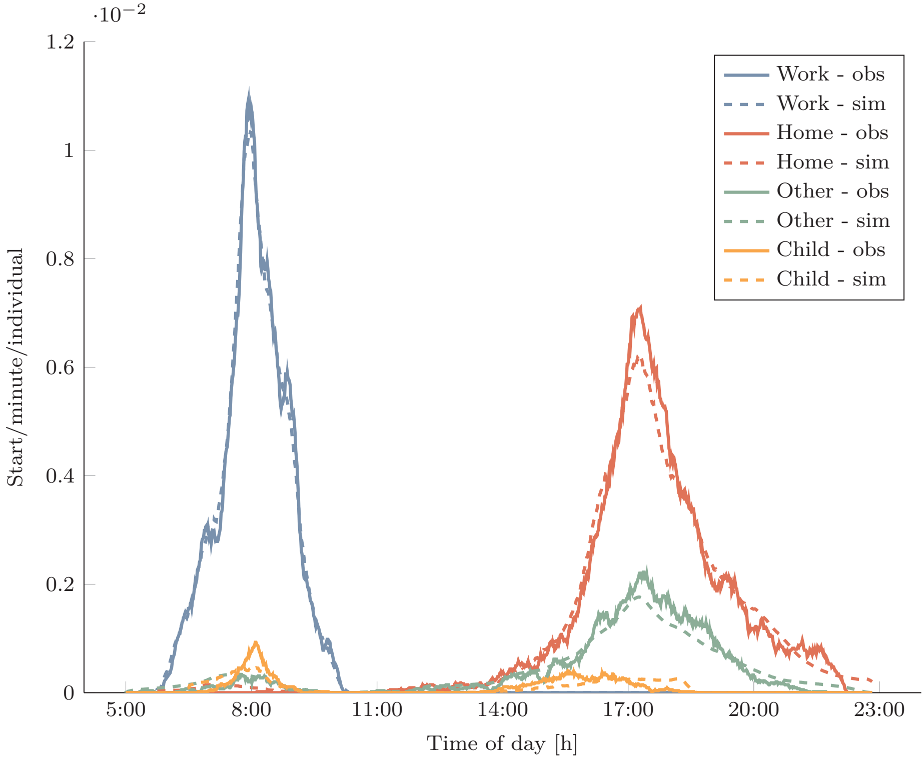

Figure 1: Time of day when each group of activities is started for simulated (solid) and real data (dashed). The plot has been obtained by first constructing a histogram with bin-size of one minute, and then average each time step over the closest 20 minutes.

zero travel time) walk is still the preferred mode of transport. The reason for this is that the mode specific constant C_{walk} is the largest of the constants, even after adding $\delta_{\text{same zone}}$, and when the travel time is small it will therefore have the highest utility.

The parameters for travel time are almost the same for car, walk and bike but significantly smaller for PT. Since the travel time is longer for bike and walk, they will be less common for longer trips. PT has the smallest time coefficient and trips with longer travel time are therefore more common with public transport. The lower speed of bike and walk in comparison with the motorized modes will make them less common for longer trips, as can be seen in figure 3. The lower alternative specific constants, and the fact that more locations will become available with the same travel time with PT and car also make bike and walk less common.

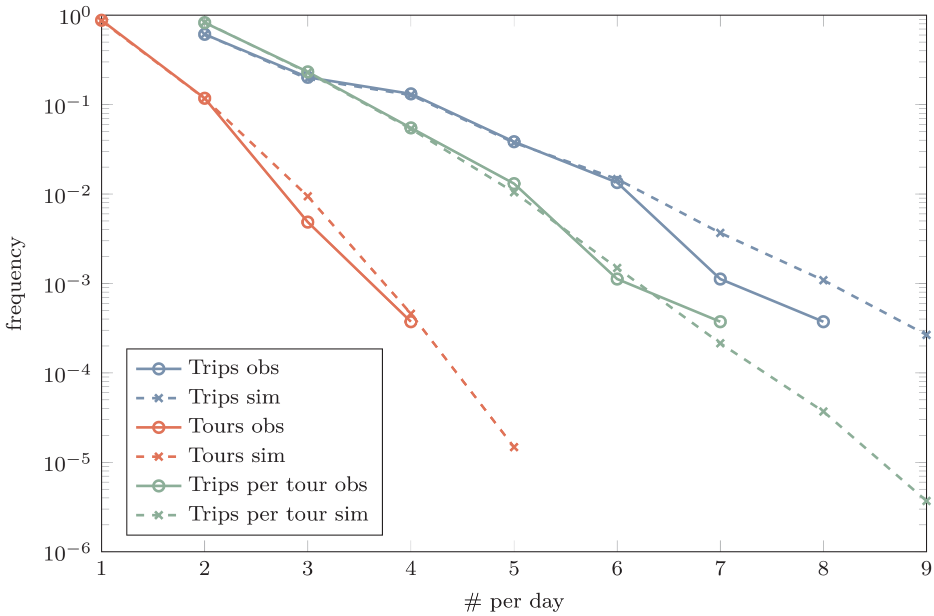

Figure 2: Distribution of number of trips, tours and trips per tour for simulated (dashed) and real data (solid).

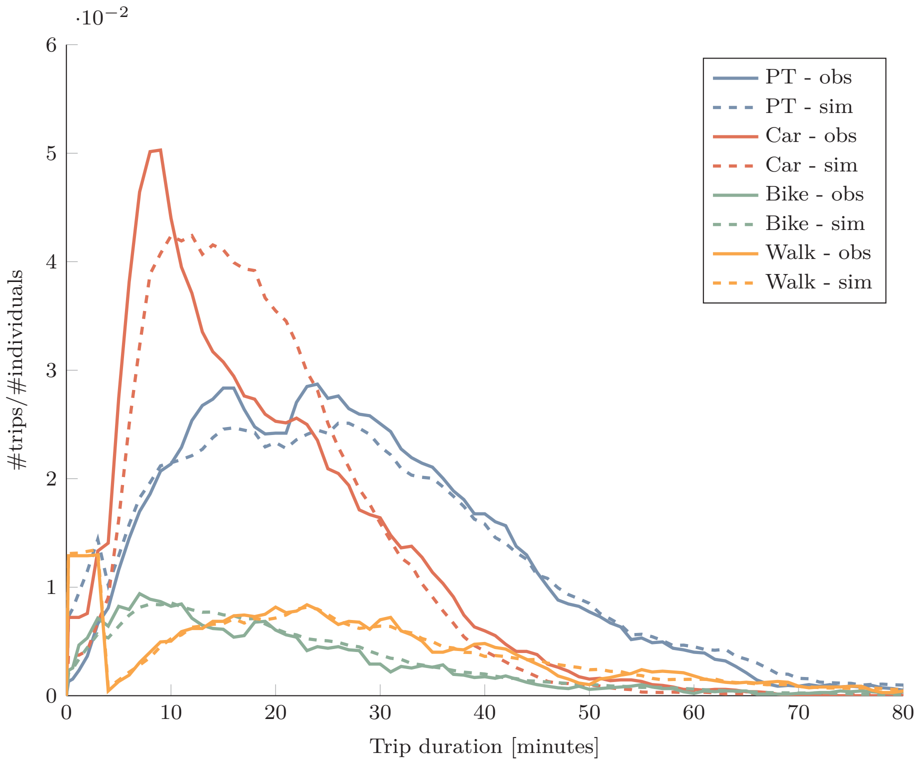

Figure 3: Distribution of trip lengths of respectively mode for observed choices (solid) and simulated (dashed). The plot has been obtained by first constructing a histogram with bin-size of one minute, and then average each time step over the closest 5 minutes.

5. CONCLUSIONS

During the last decades, many activity-based models have been developed in the literature. However, there is still a lack of random-utility based models for which time integrates consistently in all choice dimensions. A natural approach would be to introduce time explicitly in the models, respecting that time has a direction and that it is possible to make decisions sequentially taking into account the available information at that time. It is also natural to respect that people are not completely myopic, but are capable of forward–looking, for instance taking into account the consequences for afternoon activity opportunities when deciding whether to take the car to work in the morning.

The challenge with such a natural extension of the existing state-of-practice modelling framework is the immense combinatorial problem of, at least technically and consistently, considering all possible combinations of activity-location pattern throughout one single day, let alone combinations of days. In this paper, we formulate a dynamic discrete choice model which overcomes this curse of dimensionality using dynamic programming. In the framework, time is respected in the above-mentioned aspects making it dynamically consistent. We also demonstrate that it is indeed possible to estimate the model. Estimation is the main purpose and the main achievement of this paper. The proposed and thus estimated model is also validated in-sample.

There are a number of immediate extensions of this model that can and will be explored in further research. First, it is natural to extend the model to multiple days. Some activities can be postponed to later days, and there is interaction between activity patterns during consecutive days. For instance, shopping for food is an activity in which a planning horizon of more than one day is very relevant to consider.

Another limitation of the model proposed in this paper is the IID assumption between daily activity schedules. In a sequential decision context, it may be important to consider fixed effects, in particular recognizing that the same individual is making the decisions throughout one day. A natural extension of this model is therefore to consider a mixed panel logit model, where preferences for, e.g., cost, time, modes and activities are heterogeneous between individuals but constant throughout the day for a single individual.

The main challenge addressed in this paper was consistent estimation. This has hindered the use of such models in the past. Another very important aspect is to operationalize the model in implementation. For instance, when using the proposed model in the context of (or in conjunction with) a Dynamic Traffic Assignment (DTA) model, it will be necessary to repeatedly simulate travel schedules for millions of individuals. The naive approach would be to first calculate the value function in all states for each individual, but with the current specifications this would be prohibitively time consuming. Methods to speed up this simulation must therefore be developed.

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