Market Allocation and Bidder Coalition in Auctions with a Dual-Roled Principal

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Abstract

Motivated by the procurement auctions of COVID-19 critical medical resources, we examine the United States federal government's dual roles as both principal and agent in such auctions. We propose a resource allocation mechanism called Optimal Resource Allocation under Second-price Auction (ORASP), which consists of a first-stage second-price auction and a second-stage market game. In the first-stage auction of an indivisible bundle of items, we examine how the surplus is distributed when collusion behaviors are allowed. A nested knockout mechanism is proposed at this stage, which turns out to achieve the Shapley value. In the second stage, items are traded in a market game and the core of the two-stage game is studied. Since the federal government has an information advantage about players' valuation, our analysis shows that a dual-roled federal government may bid untruthfully to raise the market price, reducing the aggregate utility. Numerical examples are provided using ventilator procurement data in the United States. We show that the proposed two-stage mechanism can outperform popular models such as second-price auctions, centralized allocation, and dynamic auctions in terms of collective payoff (the aggregate utility). Beyond the COVID-19 contexts, our results caution against dual roles in procurement auctions and provide policy guidelines in such pandemics response/management.

Keywords: dual-roled principal, cooperative games, second-price auction, collusion, resource allocation

1 Introduction

The healthcare system in the United States has been under great pressure from COVID-19 due to the limited medical supplies[1], including personal protective equipment (PPE), testing equipment, and ventilation-related products. Ventilators are among the most lacking resources, which is essential for preventing respiratory failure. To purchase more supplies for medical staff, the federal government and states created an auction[2]. The federal government plays dual roles in the allocation of ventilators. Being the principal, the federal government organized the auction, while also participating directly or indirectly. Governor complained that it was like the "eBay" style bidding war for ventilators[3] as 50 states and the Federal Emergency Management Agency (FEMA) were competing, who was expected to centralize procurement and create efficient allocation. This phenomenon is not unique in the allocation of medical resources facing emergence responses. In general, we are interested in a dual-roled principal in resource allocation via auctions. For example, Google profits from selling keywords and receiving bids from advertisers. However, Google also operates other businesses such as cloud computing, smart devices, and autonomous vehicles. The concern arises that Google might hawk its own product option over other advertisers when displaying search results to consumers.

Motivated by the bidding war for ventilators, our objective is to investigate how can such auction mechanisms achieve efficient allocation while avoiding severe competition. In particular, we investigate the strategic behavior of the federal government, which can potentially benefit from its dual roles as well as informational sophistication. In this paper, we model the resource allocation in a cooperative game theoretic manner and propose a two-stage resource allocation mechanism, called Optimal Resource Allocation under Second-price Auction, comprising a first-stage bidding game and a second-stage market game. We investigate how the surplus is distributed in the first-stage bidding game, a second-price auction of an indivisible bundle of goods when collusion behaviors are allowed. At this stage, a nested knockout is adopted, and it achieves the Shapley value. In the second stage, players exchange goods in a market game, aiming to maximize the aggregate utility. Specifically, we consider a dual-roled federal government that participates in the game as both a principal and a player, who is concerned with both the social welfare and its personal interest as

[1] https://www.fda.gov/medical-devices/coronavirus-covid-19-and-medical-devices/medical-device-shortages-during-covid-19-public-health-emergency

[2] https://www.cnbc.com/2020/04/09/why-states-and-the-federal-government-are-bidding-on-ppe.html

[3] https://www.theguardian.com/us-news/2020/mar/31/new-york-andrew-cuomo-coronavirus-ventilators

well. We model the self-interested federal government by a weighting factor λ, which describes the federal government's bias towards self-interest against social welfare.

Our analysis shows that the proposed mechanism always has a nonempty core and thus is a stable way for resource allocation. It also achieves efficient allocation and outperforms several popular mechanisms in a real-world case study. The self-interested and dual-roled federal government in this mechanism has an incentive to bid higher than its true valuation in the first-stage bidding game, which can skew the market price and the allocation in the second-stage market game. This result coincides with the fact that FEMA bade a high price on top of the fifty states in the ventilator auction, and provides insight for dual roles in general auctions. Since our research objective is to understand and improve the resource allocation mechanism from a cooperative game perspective, our model contributes to the literature by considering bidders' heterogeneity and coalition formation in a two-tiered process reflecting the ventilator procurement auction practice. Another distinctive feature of our model is the dual-roled principal in mechanism design, which has not been closely studied before. This paper proceeds as follows. We review the related literature in Section 2. Section 3 introduces the preliminaries of cooperative game theory including basic concepts such as core, the Shapley value, convex game, and market game. In section 4, we formulate our model consisting of the main auction (bidding game) and a resource allocation after bidding. Then we discuss a special scenario where the principal also participates in the game in Section 5. A numerical case study of ventilator procurement in the US is presented in Section 6. We summarize the main results with a discussion in Section 7.

2 Related Works

To capture complex collusive behaviors in the auctions, a cooperative game framework is adopted to analyze the two-stage allocation process and evaluate the welfare distribution. Cooperative games has long roots in Operations Research. Recently, there is a re-emergence of applications of cooperative games from the operations management community. Ryan et al. (2022) model a sourcing network as a cooperative game to investigate its stability, and both myopic and farsighted stability are considered. Fang and Cho (2020) analyze two approaches of firms to audit their supplier for social responsibility, and a cooperative game in partition function form is applied. Liu et al. (2022) consider a problem that firms allocate different demands to different facilities to minimize the cost, and capacity sharing is formulated as a cooperative game. Gopalakrishnan et al. (2020) study an emission responsibility allocation problem in a supply chain and propose an allocation scheme, which is based on the Shapley value. In this paper, we also use the Shapley value for payoff allocation.

Our model consists of a second-price auction in the first stage and a market game for resource allocation in the second stage. Therefore, we briefly review related auction literature. Oren and Williams (1975) discuss competitive bidding and find that the highest bidder usually overvalues the object when the value is uncertain. In our paper, we design a secondary resource allocation to avoid this issue. Applications of auctions can be found in many fields, such as resource allocation, procurement, matching, etc., and we focus on resource allocation problems. Barrera and Garcia (2015) design an iterative auction mechanism focusing on service capacity allocation, which ensures that bidders report their demand truthfully. Chen et al. (2005) propose an auction for procurement in supply networks that minimizes both transportation costs and production costs. They also show that the buyer may prefer auctions with suboptimal total supply chain cost which lower the buyer's payment. Bimpikis et al. (2020) investigate the effect of market thickness in liquidation auctions in business-to-business markets. Feng et al. (2007) study the simultaneous pooled auction for ranked items, and show that a reserve price may provide the seller with higher revenue. Chen et al. (2009) investigate a dynamic setting for performance-based keyword auctions, where bidders may increase their performance level. Chu and Shen (2008) propose two truthful double auction mechanisms for an exchange market, which reduce the aggregate transaction cost. Bhargava et al. (2019) propose an incentive-compatible auction format for a mixed matching problem wherein both exclusive and shared matching exist.

In particular, in line with Graham et al. (1990), we focus on auction analysis when bidders' collusive behaviors are allowed. This work investigates the nested knockout within a bidder coalition, and the expected payment for each player is given, which coincides with the Shapley value. Marshall and Marx (2009) study the vulnerability to bidder collusion in second-price auctions and gives some necessary conditions for achieving the first-best collusive outcome. Since the object auctioned in our model is a bundle of items, our paper is also related to the literature on multi-unit auctions. Bachrach (2010) discusses collusion in VCG auctions. He analyzes the collusion behavior from a cooperative game theoretic perspective and shows that the Shapley value is contained in the core of the game. Decarolis et al. (2020) investigate the impact of collusion on online advertisement auctions including the GRP auction and the VCG auction. Our mechanism can be considered as a special case of collusion in multi-unit auctions, and we present a simple way to realize a core allocation, which can be computationally expensive for collusion in multi-unit auctions. The salient feature in our model is the dual-roled principal, who is concerned with both his own payoff as well as the aggregate utility. There has been little discussion about a dual-roled principal in auction or mechanism design literature in general. A rare example is Chen (2017) where the dynamic auction design for online advertising is studied, and the publisher has a dual role in the sense that he is the system designer as well as a bidder on behalf of some advertisers. Relatedly, Caillaud and Jéhiel (1998) consider the collusion when externalities among bidders exist, and show that the collusion can be imperfect with externalities. So far, however, there has been little discussion about such a dual-roled principal in auction collusion.

Our paper is also related to the studies on resource allocation in (medical) emergence response or other humanitarian operations. Zemach (1970) presents a data-driven model, which describes the total utilization of medical resources, the resource allocation, and the cost of health care. Chen et al. (2001) discuss the coordination mechanisms in a distribution system, and the optimal strategy is proposed to maximize the overall profits in both centralized and decentralized systems. Long et al. (2018) propose a two-stage model of medical resource allocation for emerging epidemics. The first stage forecasts the epidemics and the second stage allocates resources across affected regions. Alban et al. (2021) study the long-term allocation of mobile health units with sigmoidal demands. Westerink-Duijzer et al. (2020) discuss the cooperation problems in vaccinations, and derive the conditions under which beneficial redistribution can be achieved. Moreover, trading for a market price is likely to result in a core allocation, which motivates us to construct our two-stage model.

3 Preliminaries on Cooperative Game Theory

This section briefly introduces some basic concepts of cooperative games, which is the basis of the proposed mechanism. A coalitional (cooperative) game is a game where players can make binding agreements to determine the distribution of payoffs or the choice of strategies (e.g. through contract law) (Harsanyi and Selten, 1988). In a coalitional game, some of the players are willing to form a coalition, a subset of the set of players, to coordinate the decision-making. Cooperative games with transferable utility (TU) will be of interest in this paper, which enables the payoff to be distributed arbitrarily among coalition members.

Let (N, v) be a cooperative TU game, and $N = \{1, 2, 3..., n\}$ is the grand coalition with all the players in it. $S \subseteq N$ is one of the 2^n coalitions. The characteristic function $v(S)$ assigns a value to each coalition S, which is the payoff that the members in S can divide among themselves in any possible way. It is assumed that $v(S)$ is zero if the subset S is empty. One widely used solution concept is *core*, the set of all allocations $\mathbf{x} \in \mathbb{R}^N$ that is not dominated, which means players are willing to stay in the current coalition under core allocations. Formally, the core is a set of payoff allocations satisfying the efficiency condition and the stability condition:

$$\sum_{i \in N} x_i = v(N), \tag{1}$$

$$\sum_{i \in S} x_i \geq v(S), \forall S \subset N. \tag{2}$$

A game (N, v) is convex if the characteristic function v is supermodular, i.e. $v(S \cup T) \geq v(s) + v(T) - v(S \cap T)$ for all $S, T \subseteq N$. The convexity of a game captures the intuition that the incentives for joining a coalition increase with the coalition size. An important conclusion about convex games is that a convex game has a nonempty core (Peleg and Sudhölter, 2007).

Among the the allocation rules, the Shapley Value $\phi(v) = (\phi_1(v), \phi_2(v), ..., \phi_n(v))$ is a common way of allocating the resource. It divides payoffs among members in the coalition based on the following equation:

$$\phi_i(v) = \sum_{S \in N/\{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(S \cup \{i\}) - v(S)). \tag{3}$$

Here $|S|$ is the number of players in coalition S. The Shapley Value captures the "marginal contributions" of player i, averaging over all the possible coalitions. It provides a fair and stable way of payoff distribution and a reasonable expectation of the payoff before the game. The Shapley value is also related to the core in the sense that the Shapley value must be in the core if the game is convex (Peleg and Sudhölter, 2007).

We finish this section by introducing the market game, a class of coalitional TU games (Peleg and Sudhölter, 2007). It is an exchange economy consisting of several key components Osborne and Rubinstein (1994): a finite set of the players(traders) N, a number of trading goods m, the initial endowments vector $y_i \in \mathbb{R}_+^m$ as well as the continuous, non-decreasing and concave utility functions $f_i : \mathbb{R}_+^m \longrightarrow \mathbb{R}_+$ for each player $i \in N$. The so-called input vector $(m_i)_{i \in N}$ describes how many items each player owns. In such a market game with TU $(N, m, (y_i), (f_i))$, which can be considered as a cooperative game with TU (N, g), players are trying to figure out a way of redistributing the total input in order to maximize the total output:

$$g(S) = \max_{(m_i)_{i\in S}} \left\{ \sum_{i\in S} f_i(m_i) : m_i \in \mathbb{R}_+^m \text{ and } \sum_{i\in N} m_i = \sum_{i\in N} y_i \right\} \tag{4}$$

Here $g(S)$ is the maximal collective payoff of the coalition S. The core $\{\psi_i\}$ of the market game can be found with a corresponding market price Θ. The concept of market game plays a central role in the secondary exchange market in our resource allocation mechanism.

4 Model Formulation

The proposed mechanism involves a second-price auction and a market game. Before formulating the model, we made some assumptions about the utility functions:

Assumption 1. Let m denote the total number of objects to be sold in the auction.

(1.1) Each utility function $f_i(\cdot)$ is differentiable, strictly concave and increasing;

(1.2) The utility of player i is zero if the player does not receive any items, i.e. $f_i(0) = 0$;

(1.3) The following inequalities hold: $f_1(m) \geq, ..., \geq f_n(m)$.

Now we explain Assumption 1 in detail. The utility function for player i, denoted by $f_i(\cdot)$, is concave and increasing, which captures the diminishing marginal utility in the quantities of items. This assumption reflects a setting where agents are more sensitive towards shortages in allocated resources, while increased quantities induce diminishing gratifications. Specifically, when a player does not receive any item, the corresponding utility is zero. Assumption (1.3) captures the heterogeneity in players' preferences, which can be always weakly ranked without loss of generality. The model comprises a first-stage bidding game and a second-stage market game. In the first stage there is a second-price auction, wherein players form coalitions and distribute collusive gains through a nested knockout. After that, players in the winning coalition trade items in a second-stage market game. Note that an implicit assumption is that the players in the first-stage bidding game are myopic and unaware of the second-stage exchange market; this reflects a setting where the principal is more sophisticated than the agents.

4.1 The First-Stage Bidding Game

The bidding game is based on the second-price (Vickrey) auction. We assume that the utility functions are public information, motivated by the application contexts wherein demand data for emergence response resources (e.g., ventilators) are publicly disclosed. In the second-price auction, there is a reserve price p_0 for all players (bidders). Every player is required to submit a sealed bid. The player who submits the highest price among all the players win the object with the payment equal to the second highest bid. It is known that such a mechanism is incentive-compatible in the sense that player i's payoff winning the item $f_i(m)$ is consistent with his true valuation for the item. Therefore, player i's bidding price is $f_i(m)$ in the first-stage bidding game. Clearly, if all the players participate in the second-price auction alone, only bidder 1 who submits the highest bidding price $f_1(m)$ will win the whole indivisible object in the auction with the payment $f_2(m)$, and other bidders will get nothing. The equilibrium outcome in the first-stage bidding game reflects a wholesale procurement process wherein an indivisible batch of resources (consisting of m items) will be allocated to the agent with the highest demand (or budget).

To secure a non-zero payoff from the first-stage bidding game, players are allowed to form coalitions to coordinate their bidding strategies and split the payoff afterwards accordingly to pre-specified binding agreement. Here we adopt the setting in Graham et al. (1990). Players in a coalition S report their bids to a third-party intermediary and the player who reports the highest bidding price is selected by the coalition as the representative. He will participate in the second-price auction on behalf of other players in the coalition S. Let (N, h) denote the cooperative (coalitional) bidding game, where N is the grand coalition including all players and h corresponds to the characteristic (value) function. The game (N, h) can be described as follows:

$$h(S) = \begin{cases} f_1(m) - \max\limits_{j \in N \setminus S} f_j(m), & 1 \in S, \\ 0, & 1 \notin S. \end{cases} \tag{5}$$

Here the characteristic function $h(S)$ corresponds to the maximum value that coalition S can get from the second-price auction. The set $N \setminus S$ includes players who are members of coalition N but not members of the coalition S. From the characteristic function, a player receives a non-zero payoff if and only if he is in the winning coalition S. Consequently, the set S includes all the players with a valuation higher than the reserve price p_0, which contributes to the collective payoff of the entire coalition. We can also derive an important property of this coalition game from the characteristic function.

Proposition 1. *The bidding game (N, h) is a convex game, i.e.*

$$h(S \cup T) \geq h(s) + h(T) - h(S \cap T). \tag{6}$$

Not surprisingly, the first-stage bidding game implies monotonicity in that the collective payoff from a coalition increases in its size. More importantly, it is known that the Shapley value is in the core of a convex game (Peleg and Sudhölter, 2007), and therefore, this proposition ensures that the allocation introduced below is a core allocation.

The nested knockout. The subsequent problem for the winning coalition S is how to redistribute the collective payoff among all its participants. To solve this problem, here we introduce the nested knockout from Graham et al. (1990), and the nested knockout imputation is shown in Table 1. Suppose there are k players in the winning coalition. Then k coalitions $C_1, C_2, ..., C_k$ will be formed and $C_1 = \{1, 2, ..., k\}$, $C_2 = \{1, 2, 3, .., k-1\}$, ..., $C_i = \{1, ..., k+1-i\}$. The coalitions are nested in the sense that $C_{i+1} \subset C_i$ and we have $C_i \backslash C_{i+1} = \{i\}$.

Knockout	$C_1 \backslash C_2$	$C_2 \backslash C_3$	$C_3 \backslash C_4$	$\cdots$	$C_k \backslash C_{k-1}$
1_{st}	$\frac{f_k(m)-p_0}{k}$	$\frac{f_k(m)-p_0}{n}$	$\frac{f_k(m)-p_0}{k}$	$\frac{f_k(m)-p_0}{k}$	$\frac{f_k(m)-p_0}{k}$
2_{nd}		$\frac{f_{k-1}(m)-f_k(m)}{k-1}$	$\frac{f_{k-1}(m)-f_k(m)}{k-1}$	$\frac{f_{k-1}(m)-f_k(m)}{k-1}$	$\frac{f_{k-1}(m)-f_k(m)}{k-1}$
3_{rd}			$\frac{f_{k-2}(m)-f_{k-1}(m)}{k-2}$	$\frac{f_{k-2}(m)-f_{k-1}(m)}{k-2}$	$\frac{f_{k-2}(m)-f_{k-1}(m)}{k-2}$
$\vdots$				$\vdots$	$\vdots$
Total	$\phi_k = \frac{f_k(m)-p_0}{k}$	$\phi_{k-1} = x_k + \frac{f_{k-1}(m)-f_k(m)}{k-1}$	$\phi_{k-2} = \phi_{k-1} + \frac{f_{k-2}(m)-f_{k-1}(m)}{k-2}$	$\cdots$	$\phi_1 = \phi_2 + \frac{f_1(m)-f_2(m)}{1}$

Table 1: Payoff distribution in the nested knockout structure.

The nested knockout structure generates a series of knockouts based on the second-price auction to decide the payoff for each player in the coalition. In the first knockout, members in coalition C_2 will bid against the member in coalition $C_1 \backslash C_2$. That is, player $\{k\}$ will bid against players $\{1, ..., k-1\}$. Player 1 reporting the highest bidding price in coalition C_2 will be the representative to bid against $\{k\}$. Coalition C_2 will win in the first knockout, and then the second knockout will be carried out. The "surplus" generated by the first knockout is $(f_k(m) - p_0)$, which is equally distributed among all members in coalition C_1, every player receiving $\frac{f_k(m)-p_0}{k}$. In the second knockout, the members in coalition C_2 are considered as the active players, and player k will quit the game. In this turn, the player in $C_2 \backslash C_3 = \{k-1\}$ will compete with coalition C_3. Similarly, on behalf of C_3, player 1 will also win in the second knockout. In this round, the payoff generated is $(f_{k-1}(m) - f_k(m))$. Knockouts would be repeated until player 1 finally wins the items in the k_{th} knockout, wherein the "surplus" is $(f_1(m) - f_2(m))$. Based on Table 1, the nested knockout imputation $(\phi_1(h), \phi_2(h), ..., \phi_k(h))$ for each player can be summarized as follows:

$$\begin{aligned} \phi_k &= \frac{f_k(m) - p_0}{k}, \\ \phi_i &= \phi_{i+1} + \frac{f_i(m) - f_{i+1}(m)}{i}, \ \forall k-1 \geq i \geq 1. \end{aligned} \tag{7}$$

The nested knockout imputation exactly equals the Shapley value of the game (N, h) (Graham et al. (1990)). As we have shown that the bidding game (N, h) is a convex game and the Shapley value must be in the core, the nested knockout can be an efficient way to achieve the core allocation of the bidding game.

4.2 The Second-Stage Market Game

After the bidding game, a bundle of items will be allocated to the winning coalition and all membership players continue the resource re-allocation through a secondary exchange market. Let m_i denote the number of items distributed to player i during the second-stage market game. Considering the heterogeneity of the bidders, each player has a unique utility function with respect to the number of items, $f_1(m_i), f_2(m_i), ..., f_k(m_i)$. The market is also modeled as a cooperative game. We extend the model by considering the market game after bidding, which becomes:

$$V(S) = \begin{cases} g(S) - \max\limits_{j \in N \setminus S} f_j(m), & 1 \in S, \\ 0, & 1 \notin S. \end{cases} \tag{8}$$

Here $V(S)$ corresponds to the value of coalition S of the entire two-stage game. $g(S)$ is the maximal collective payoff of coalition S, which is derived from the following optimization problem:

$$\begin{aligned} \max_{m_i \geq 0} \quad & \sum_{i \in S} f_i(m_i), \\ \text{s.t.} \quad & \sum_{i \in S} m_i = \sum_{i \in S} y_i. \end{aligned} \tag{9}$$

Here $\boldsymbol{y}$ is a $1 \times n$ vector, which corresponds to the allocated quantities of every participant in the winning coalition from the first-stage bidding game, prior to the second-stage market game. We have $y = [m, 0, ..., 0]$, because there is a single representative (with the highest valuation) in the winning coalition who receives the entire bundle of m items. As we will show later in Proposition 3, the grand coalition forms in the market game, i.e., all players join one coalition.

The resource allocation proceeds as a market game. Players trade under a fixed price to better allocate the items. The representative becomes the only seller in the market, and other bidders in the coalition are the buyers. We illustrate how the market price forms in Section 4.4. Combing both stages in the allocation process, the game (N, V) consisting of the bidding game and the market game can be formulated as follows:

$$V(S) = \begin{cases} h(S) + g(S) - f_1(m), & 1 \in S, \\ 0, & 1 \notin S. \end{cases} \tag{10}$$

Note that the corresponding $g(S)$ is zero when $1 \notin S$, because of the loss of coalition S during the auction.

4.3 The Integrated Mechanism

Based on the formulated model above, we now propose an integrated two-stage mechanism. We will show that the entire mechanism is able to implement the optimal resource allocation. We shall call the proposed mechanism the Optimal Resource Allocation under Second-price Auction (ORASP): The whole game includes two parts, a first-stage bidding game and a second-stage market game. In the bidding game, several players will form a coalition. Players report their valuation to a third-party center of the coalition S. The player who reports the highest price among the members of the coalition is selected as the representative who will bid on behalf of the coalition. Other members of the coalitions will not participate in the auction. Under the rule of the second-price auction, an indivisible bundle of items is auctioned. Members in coalition S will receive nothing if the representative does not win the item. Otherwise, the representative wins the item with the payment $\max\limits_{j \in N \setminus S} f_j(m)$, and redistributes the auction payoff via a nested knockout. In the second stage, the game proceeds as a market game with resource allocation issues. The representative is the seller who has all the winning items and other members in coalition S are the buyers. The players are heterogeneous in their demands and utilities for the items. Aiming to maximize the collective payoff, players trade items under a fixed market price.

4.4 Equilibrium Price in the Secondary Market

Here we explain how the market price in the second stage is determined. Under some agreements, players are willing to maximize the collective payoff by trading. In the market game, because of our concavity assumption, there exists a unique price that optimizes the aggregate utility of the coalition. From optimization problem (9), we can derive the optimal market price, which corresponds to the Lagrange multiplier with respect to the constraint in (9), i.e., the shadow price. Let $(m_i^*)_{i \in S}$ be an optimal allocation. Then the following equation shows the optimal market price:

$$\psi_i = f_i(m_i^*) + \Theta(y_i - m_i^*). \tag{11}$$

Here Θ is the Lagrange multiplier of the constraint of optimization problem (9), which is also the market price in this case. If every player's utility is maximized at this price, i.e., their marginal utilities are equal to the market clearing price, which is called the competitive equilibrium, the aggregate utility is also maximized (Osborne and Rubinstein, 1994). It is known that such an equilibrium is individually rational, and the allocation for each player is just the payoff from the competitive equilibrium. We illustrate the two-stage game with the following example.

Example 1. *Consider a 3 - player game with independent utility functions:*

$$\begin{aligned} f_1(m_1) &= \log(m_1 + 1), \\ f_2(m_2) &= \log(10m_2 + 1), \\ f_3(m_3) &= \log(20m_3 + 1). \end{aligned} \tag{12}$$

The number of the items being auctioned m is set to be 60. The bids of three players are $f_1(60) = 4.11$, $f_2(60) = 6.40$, and $f_3(60) = 7.09$. The reserve price p_0 is set as 0.5. Based on the rule of the second-price auction, we have characteristic function $h(S)$, which is shown in Table 2. With

S	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
h(S)	0	0	0.69	0	0.69	2.98	6.59

Table 2: Value of characteristic function $h(S)$.

the efficiency and stability conditions of the core, there are many possible core allocations. For example, $[1, 1, 4.59]$ is a core allocation. Also, the Shapley value of (N, h) is $[1.20, 2.35, 3.04]$, which is also in the core.

Now coalition $\{1, 2, 3\}$ win the 60 items in the first-stage auction. The initial items of the three players are $[0, 0, 60]$ at the beginning of the second-stage market game. Player 3 is the only seller, and player 1, and 2 are buyers. Items will be traded with a fixed market price Θ per unit to maximize the aggregate utility:

$$\begin{aligned} \max \quad & \log(m_1 + 1) + \log(10m_2 + 1) + \log(20m_3 + 1), \\ \text{s.t.} \quad & m_1 + m_2 + m_3 = 60. \end{aligned} \tag{13}$$

Let (m_1^*, m_2^*, m_3^*) be a solution to the above optimization problem, which corresponds to the optimal numbers of items for each player. Then the allocation of this market game is:

$$\begin{aligned} \psi_1 &= \log(m_1^* + 1) + \Theta(0 - m_1^*), \\ \psi_2 &= \log(10m_2^* + 1) + \Theta(0 - m_2^*), \\ \psi_3 &= \log(20m_3^* + 1) + \Theta(60 - m_3^*). \end{aligned} \tag{14}$$

Here Θ equals 0.049 by solving all the equations above. Thus, the three players trade items with the market price 0.049 per unit item. A feasible core allocation of this market game is $[2.06, 4.32, 7.96]$. Combined with the Shapley value of the first-stage bidding game, a core allocation for the entire game is $[3.26, 6.67, 3.91]$.

A special case with two payers. In fact, to achieve optimal resource allocation, the market price need not coincide with the competitive equilibrium price. We provide a simple example of a market game with two players, a seller and a buyer. Here, we use the subscripts b and s to refer to "buyer" and "seller".

Proposition 2. *In the case of a two-player market game, the core is the set of all market allocations corresponding to the optimal resource allocation (m_b^*, m_s^*) with a fixed market price Θ satisfying:*

$$\Theta \in \left[\frac{f_s(m) - f_s(m_s^*)}{m - m_s^*}, \frac{f_b(m_b^*)}{m_b^*} \right]. \tag{15}$$

he seller can earn a certain utility by selling some of the items to the buyer. The market price Θ should be not too low for the seller and not too high for the buyer. Because there are only two players involved in the game, the profit received by the seller is paid by the buyer.

Specifically, we can derive the explicit expression of the market price and the optimal allocation for the 2 players in the market game. Similar to before, we assume the utility functions are $f_1(m_1) = \log(m_1 + 1)$, $f_2(m_2) = \log(10m_2 + 1)$. Let (m_1^*, m_2^*) be the optimal allocation which maximizes the aggregate utility of two players. Let Θ be the slope of a tangent to the utility function at m_1^* and m_2^*, and l be an affine function with slope Θ that $l(m_1^*) = f_1(m_1^*)$ and $l(m_2^*) = f_2(m_2^*)$. Then we can derive explicit expressions of the optimal allocation and the market price:

$$m_1^* = \frac{m}{2} - \frac{9}{20},\ m_2^* = \frac{m}{2} + \frac{9}{20},\ \Theta = \frac{20}{10m + 11} \tag{16}$$

This provides an alternative way to calculate the Θ and m_i^*. Furthermore, if there are three players in the market with the same utility functions as those in Example 1, then the result is:

$$m_1^* = \frac{m}{3} - \frac{37}{60},\ m_2^* = \frac{m}{3} + \frac{17}{60},\ m_3^* = \frac{m+1}{3},\ \Theta = \frac{60}{20m + 23}. \tag{17}$$

One can verify that the same result of Example 1 can be derived with $m = 60$.

4.5 The Core of the Game

Now we discuss the core of the entire two-stage game. It is known that the second-stage market game (N, g) always has a nonempty core (Osborne and Rubinstein, 1994). Let $\{x_i\}_{i \in N}$ be a core allocation of the bidding game and $\{y_i\}_{i\in N}$ be a core allocation of the market game. Then we define ζ_i as

$$\zeta_i = \begin{cases} x_i + y_i - f_1(m), & i = 1, \\ x_i + y_i, & i \neq 1, \end{cases} \tag{18}$$

which turns out to be in the core of the entire game.

Theorem 1. *$\{\zeta_i\}_{i\in N}$ satisfy the efficiency condition $\sum_{i\in N} \zeta_i = V(N)$ and stability condition $\sum_{i\in S} \zeta_i \geq V(S)$. Therefore, $\{\zeta_i\}_{i\in N}$ is in the core of game (N, V).*

Recall that the bidding game (N, h) is a convex game with the Shapley value $\{\phi_i\}_{i\in N}$ in the core, and we know that the competitive payoff $\{\psi_i\}_{i\in N}$ is also in the core of the market game (Osborne and Rubinstein, 1994). Therefore, this theorem ensures that the distribution of the whole game is in the core. A noteworthy fact is that, as we have shown in Section 4.1, the winning coalition in the bidding game can be any coalition that contains all the players whose valuation is higher than p_0. These coalitions are all equivalent in the sense that they lead to the same allocation. As players are myopic, they are indifferent between any two possible winning coalitions. However, different coalitions will result in different outcomes in the market game.

Proposition 3. *Let S, T be two coalitions containing all the players whose valuation is higher than p_0. If $S \subsetneq T$, then, compared with being in coalition S,*

1. *the collective payoff increases, i.e. $g(S) < g(T)$, where $g(S)$ is defined by (9);*
2. *the seller (player 1) will gain a higher utility when in coalition T;*
3. *all the buyers will gain a lower utility when in coalition T.*

Due to the strict concaveness of the utility function, the collective payoff increases in the coalition size. Intuitively, the addition of new players increases the aggregate demand and alleviated the effect of diminished marginal utility. Another important observation is that the seller in the secondary market benefits from a large coalition while the buyers do not. Though buyers prefer to form a small coalition, all players finally join the grand coalition N. Because only the seller has initial inventory, a coalition without the seller (player 1) receives no payoff. Also, the seller will not deviate from the grand coalition, as the seller prefers large coalitions. Thus no subset of players can benefit from deviation, and the grand coalition is formed.

5 A Dual-Roled Principal

The salient feature in the emergence response setting is the dual-roled principal: the federal government who designs the allocation mechanism also participates in the game. In this section, the principal is denoted as the first agent, and we specify that the utility function f_1 is not known to other players. We still assume that the utility functions of other agents are in descending order, i.e. $f_2(m) \geq, ..., \geq f_n(m)$. The utility functions of all players except for player 1 are public knowledge. For ease of exposition, we consider a news-vendor's utility function in this section to model the ventilator procurement problem:

$$f_i(m_i) = \mathbb{E}\min\{m_i, \mu_i D\}, D \sim N(1, \sigma^2). \tag{19}$$

Here μ_i represents the expected demand of an agent (state or federal government), and D is a random variable that captures the uncertainty in demand. Each agent aims to procure sufficient items to meet demand $D_i = \mu_i D$, while the marginal utility of any additional item beyond the demand quantity is zero. We normalize $f_i(0)$ to zero and overstep the issue with negative quantities similar to standard treatment in news-vendor literature (e.g., Li and Zhang (2008)). This function satisfies Assumption 1, and the proof can be found in Appendix.A. Let $f(x) = \mathbb{E}\min\{x, D\}$, then $f_i(m_i) = \mu_i f(m_i/\mu_i)$. In the following discussion, we always assume that $m_i < \mu_i$, which reflects a setting of resource scarcity in general. This gives $q_D^{'}(m_i/\mu_i) > 0$, where q_D is the pdf of D. Without loss of generality, we normalize the reservation price to zero, i.e. $p_0 = 0$. Under our assumption of the function form, a bidding price $f_i(m)$ is associated with a utility function $f_i(m) = \mu_i f(m/\mu_i)$, which is uniquely determined by the expected demand m_i. Thus a bidding price in fact reflects a player's expected demand for items. Intuitively, a player with a high demand has a high valuation for the items, so he also bids a high price. Due to the one-to-one correspondence between bidding price and expected demand, we use "demand" and "bidding price" interchangeably for convenience.

The rule maker is concerned with the aggregate utility and his personal interest as well,

$$\max f_1(m_1) - q_1 + \lambda[\sum_{i=2}^{n} f_i(m_i)] - \lambda \sum_{i=2}^{n} q_i. \tag{20}$$

Here f_i denotes agent i's payoff function, m_i is the quantity of resources allocated to agent i, and q_i is agent i's payment (which can be negative if agent i receives money). Finally, a weighting factor $\lambda \in [0, 1]$ measures the relative preference in the aggregate utility against the principal's own payoff. A lower weight implies that the principal is more self-concerned and vice versa.

5.1 A Myopic Principal

We start with a benchmark wherein the federal government is myopic in the first-stage bidding game and ignores its consequences in the second-stage market game. Notice that when $\lambda = 0$, the principal has no difference with other players, which degenerates to the case we analyzed in Section 4. Therefore, we focus on the case when $\lambda > 0$ in what follows.

The first-stage bidding game. In the bidding game, the federal government (player 1) aims to maximize its weighted objective. Anticipating the truthfully bidding of state governments, the federal government is able to bid any price exactly at its targeted ranking. If the federal government reports the highest bidding price, it becomes the representative of the coalition and the weighted objective is

$$f_1(m) - (1-\lambda)(\sum_{i=2}^{n} s_i). \tag{21}$$

Here s_i is the Shapley value, $s_i = \sum_{j=i}^{n-1} \frac{f_j(m)-f_{j+1}(m)}{j} + \frac{f_n(m)}{n}$. If the federal government reports a bidding price that is the second highest ($f_{n+1}(m) < f_2(m)$), the weighted objective is

$$(1-\lambda)(s_2 - \frac{f_2(m) - f_{n+1}(m)}{2}) + \lambda f_2(m). \tag{22}$$

The first term is the payment from player 2 (the representative) according to the nested knockout rule. Because the payment is increasing with $f_{n+1}(m)$, the federal government will never report a bidding price that is the third highest or even lower. In fact, the federal government will report a bidding price that is slightly lower than $f_2(m)$. For simplicity, we denote this bidding price by $f_2(m)$, i.e. $f_{n+1}(m) = f_2(m)$. Since $s_2 + \sum_{i=2}^{n} s_i = f_2(m)$, subtracting (22) from (21) gives $f_1(m) - f_2(m)$. So for any λ, the federal government will report the highest bidding price if and only if $f_1(m) > f_2(m)$.

Proposition 4. *In the bidding game, the federal government will report the highest bidding price if $f_1(m) > f_2(m)$. Otherwise, the federal government will report $f_2(m)$.*

The federal government reports truthfully only when $f_1(m) \geq f_2(m)$. In other circumstances, the federal government reports a bidding price higher than its true valuation, which increases its contribution to the coalition, in order to gain more payoff.

The second-stage market game. The game proceeds as illustrated in Section 4. The equilibrium allocation of the market game is given by the solution to the following optimization problem:

$$\begin{aligned} \max_{m_i} \quad & \sum_{i=1}^{n} f_i(m_i) \\ \text{s.t.} \quad & \sum_{i=1}^{n} m_i = m. \end{aligned} \tag{23}$$

Recall that $f_i(x) = \mu_i f(x/\mu_i)$ where μ_i is player i's expected demand. The federal government's demand μ_1 is derived from the bidding price in the bidding game. Then the market price θ is $\theta = f^{'}(\frac{m_i}{\mu_i}), \forall i$. The following proposition characterizes the equilibrium allocation and price in the market game.

Proposition 5. *The quantity of items allocated to player i is $m_i = \mu_i \bar{m}$, and the equilibrium market price is $f^{'}(\bar{m})$, where $\bar{m}$ is the quantity of items averaged over demands $\bar{m} = m/(\sum_{i=1}^{n} \mu_i)$.*

In the market game, the aggregate utility is maximized, and the optimal quantity of items allocated to player i is in proportion to player i's demand. Recall that the competitive equilibrium is also achieved in the market game, and thus the market price corresponds to the marginal utility of any player. Note that if the federal government reports untruthfully in the bidding game, the allocation in the market game will be inefficient. In this case, compared to the optimal allocation, the federal government receives more items as the reported demand is higher than its true demand, while the quantity of items allocated to state governments is lower.

5.2 A Strategic Principal

As the principal in the two-stage integrated mechanism, it is natural to consider a more sophisticated federal government that strategically bids in the first stage in anticipation of the secondary market. Similar to the myopic benchmark, we start by analyzing the federal government's bidding strategy in the first-stage. If the federal government reports the highest bidding price, it becomes the representative of the coalition and will be the only seller in the market game. If the federal government reports a bidding price lower than $f_2(m)$, then the federal government will receive payment from the representative (player 2) instead of holding all the items, and consequently, player 2 will hold all the items, who becomes the seller in the market game. Meanwhile, from Proposition 5, the quantity of items allocated to player i is in proportion to player i's demand, which is reflected by his bidding price. Anticipating different allocation results in the secondary market, the federal government will use backward induction and determine its optimal bidding strategy in the first stage. To simplify the notation, we analyze the demand μ_{n+1} corresponding to the federal government's bidding price instead of the bidding price $f_{n+1}(m) = \mu_{n+1} f(m/\mu_{n+1})$. Additionally, we focus on the case where the federal government's true demand μ_1 is no less than μ_3, i.e., the federal government has at least the second highest demand. Other cases, for example, the federal government has the third highest demand, can be analyzed analogously.

Overbidding strategies. When the federal government overbids, i.e. reports the highest bidding price in the bidding game, it becomes the seller in the market game. Adding up the weighted objective of the two stages, we have

$$f_1(m) - (1-\lambda)(\sum_{i=2}^{n} s_i) - f_1(m) + AU^s. \tag{24}$$

Here AU^s is the weighted aggregate utility in the market game. Recall that μ_i is the expected demand of player i and μ_{n+1} is the demand corresponding to the bidding price $f_{n+1}(m)$ reported by the federal government. $\bar{m} = m/(\sum_{i=2}^{n+1})$ is the quantity of items averaged over players' demands and $\theta = f^{'}(\bar{m})$ is the equilibrium market price.

$$AU^s = f_1(\mu_{n+1}\bar{m}) + \lambda[\sum_{i=2}^{n} f_i(\mu_i \bar{m})] + (1-\lambda)\sum_{i=2}^{n} \theta \mu_i \bar{m}. \tag{25}$$

The first two terms correspond to utility from items and the last term is the profit from selling items. Since as long as $\mu_{n+1} > \mu_i, i \geq 2$, reporting different bidding prices does not influence the outcome of the bidding game, the optimal μ_{n+1} in this case maximizes the outcome of the market game, which is the optimal solution to the following problem:

$$\begin{aligned} \max_{\mu_{n+1}} \quad & AU^s = f_1(\mu_{n+1}\bar{m}) + \lambda[\sum_{i=2}^{n} f_i(\mu_i \bar{m})] + (1-\lambda)\sum_{i=2}^{n} \theta \mu_i \bar{m} \\ \text{s.t.} \quad & \mu_{n+1} \geq \mu_2. \end{aligned} \tag{26}$$

The constraint ensures that the federal government is the seller. The following proposition compares the demand μ_{n+1} corresponding to the federal government's bidding price and the true demand μ_1, and the result is also illustrated in Figure 1.

Proposition 6. *When the federal government overbids in the first stage, the weighted aggregate utility is* (24)*. Then the demand μ_{n+1} corresponding to the federal government's optimal bidding price satisfies the following properties.*

- *μ_{n+1} increases as λ decreases.*
- *When $\mu_1 \geq \mu_2$, the optimal demand $\mu_{n+1} \geq \mu_1$, and equality holds if and only if $\lambda = 1$.*
- *When $\mu_1 < \mu_2$, the optimal demand $\mu_{n+1} = \mu_2$ if $\lambda > \lambda^s$, and $\mu_{n+1} > \mu_2$ if $\lambda \leq \lambda^s$, where λ^s is a function of $\mu_1, \mu_2, ..., \mu_n$.*

The first statement indicates that the demand μ_{n+1} corresponding to the federal government's bidding price increases when the federal government is more self-interested, and the bidding price in the first stage increases accordingly. Increasing μ_{n+1} benefits a self-interested federal government in two aspects. First, as μ_{n+1} becomes higher, the aggregate demand of all players increases, which raises the equilibrium market price θ. Being the only seller in the market, the federal government earns more (the third term in (26)) because of the high market price. Second, the federal government also receives a higher quantity of items (the first term in (26)), because the market allocation is in proportion to a player's demand, as mentioned in Proposition 5. Though state governments receive a lower quantity of items (the second term in (26)) when μ_{n+1} increases, the self-interested federal government is less concerned about state governments' utility as λ decreases.

The second and third statements follow from the first statement. When $\lambda = 1$, the federal government is concerned with social welfare. It is optimal for the federal government to bid truthfully since the market allocation achieves the maximal aggregate utility, and the federal government reports a price $f_{n+1}(m)$ as close to its valuation $f_1(m)$ as possible, i.e. the corresponding demand μ_{n+1} is close to μ_1. Note that in the overbidding strategy, μ_{n+1} should be greater than μ_2 to ensure that the government is a seller in the secondary market. Thus when $\lambda = 1$, the demand corresponding to the government's bidding price $\mu_{n+1} = \mu_1$ if $\mu_1 > \mu_2$ and $\mu_{n+1} = \mu_2$ if $\mu_1 \leq \mu_2$. When λ decreases, the bidding price $f_{n+1}(m)$ and the corresponding demand μ_{n+1} increase, which explains the second and third statements.

Underbidding strategy. When the federal government underbids, i.e. reports a bidding price lower than the highest one in the bidding game, the federal government becomes a buyer in the second-stage market game. Since we have assumed that $\mu_1 \geq \mu_3$, here we consider the case where the federal government reports the second highest bidding price. The federal government maximizes the following function in the two-stage game,

$$(1-\lambda)(s_2 - \frac{f_2(m) - f_{n+1}(m)}{2}) + \lambda f_2(m) - \lambda f_2(m) + AU^b. \tag{27}$$

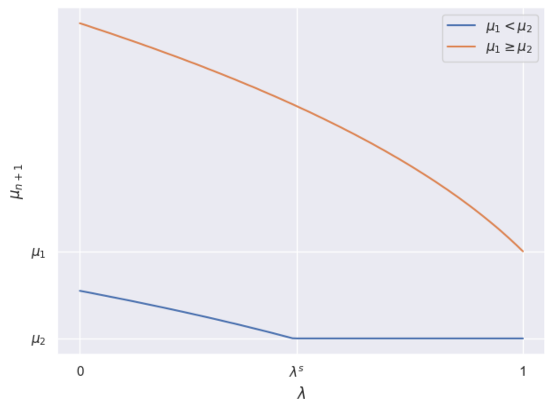

Figure 1: The demand μ_{n+1} corresponding to the bidding price when the federal government is the seller.

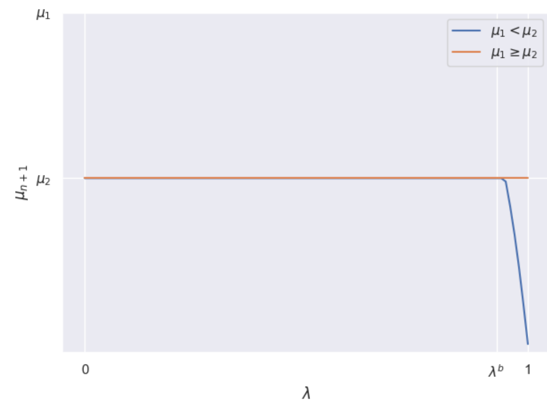

Figure 2: The demand μ_{n+1} corresponding to the bidding price when the federal government is a buyer.

AU^b is the weighted aggregate utility from the market game,

$$AU^b = f_1(\mu_{n+1}\bar{m}) + \lambda[\sum_{i=2}^{n} f_i(\mu_i\bar{m})] - (1-\lambda)\theta\mu_{n+1}\bar{m}. \tag{28}$$

Here $\theta = f^{'}(\bar{m})$ is the market price. The first two terms correspond to utility from items and the last term is the payment the federal government pays for the items. The optimal μ_{n+1} in this case is the optimal solution to the following problem,

$$\begin{aligned} \max_{\mu_{n+1}} \quad & \frac{1-\lambda}{2} f_{n+1}(m) + f_1(\mu_{n+1}\bar{m}) + \lambda[\sum_{i=2}^{n} f_i(\mu_i\bar{m})] - (1-\lambda)\theta\mu_{n+1}\bar{m} \\ \text{s.t.} \quad & \mu_3 \leq \mu_{n+1} \leq \mu_2. \end{aligned} \tag{29}$$

The constraint ensures that the federal government is a buyer and reports the second-highest bidding price. The following proposition compares the demand corresponding to the federal government's bidding price μ_{n+1} and its true demand μ_1, and the result is also illustrated in Figure 2.

Proposition 7. *When the federal government underbids in the first stage, the weighted aggregate utility is* (27)*. Then the demand μ_{n+1} corresponding to the federal government's optimal bidding price satisfies the following properties.*

- *When $\mu_1 \geq \mu_2$, the optimal demand $\mu_{n+1} = \mu_2$ for all $0 \leq \lambda \leq 1$.*
- *When $\mu_1 < \mu_2$, the optimal demand $\mu_{n+1} < \mu_2$ if $\lambda > \lambda^b$, and $\mu_{n+1} = \mu_2$ if $\lambda \leq \lambda^b$, where λ^b is a function of $\mu_1, \mu_2, ..., \mu_n$. Specifically, $\mu_{n+1} = \mu_1$ when $\lambda = 1$.*

Similar to the explanation for Proposition 6, when $\lambda = 1$, bidding truthfully is optimal since the market allocation achieves the maximal aggregate utility. So the demand corresponding to the federal government's bidding price μ_{n+1} is close to μ_1. Specifically, $\mu_{n+1} = \mu_2$ if its true demand $\mu_1 \geq \mu_2$, and $\mu_{n+1} = \mu_1$ if $\mu_{n+1} \leq \mu_2$. As λ decreases, the federal government has no incentive to decrease the bidding price, so $\mu_{n+1} = \mu_2$ for all λ if $\mu_1 \geq \mu_2$ and $\mu_{n+1} = \mu_2$ if $\lambda \leq \lambda^b, \mu_1 < \mu_2$. The reasons are as follows. The federal government benefits from increasing bidding price in two ways. The payoff from the bidding game (the first term in (29)) increases in its bidding price according to the nested knockout allocation rule. Also, because the market allocation is in proportion to a player's demand, as mentioned in Proposition 5, the quantity of items received by the federal government in the market game (the second term in (29)) increases. Though increasing $\mu - n + 1$ leads to a decrease in the quantity of items received by state governments, the self-interested federal government is less concerned about state governments' utility when λ decreases. A low demand μ_{n+1} corresponding to a low bidding price can decrease the aggregate demand, and thus lower the market price, reducing the procurement cost (the fourth term in (29)). However, this cannot compensate for the loss caused by a low Shapley value.

Optimal bidding strategies. Now we compare the outcome of the two strategies and investigate the equilibrium behavior of the federal government. Denote the demand corresponding to the federal government's optimal bidding price in the overbidding (underbidding) strategy by μ^s (μ^b). If $\mu_1 < \mu_2$, $\lambda \leq \lambda^b$ or $\mu_1 \geq \mu_2$, then $\mu^b = \mu_2$. To see the difference between two weighted aggregate utility (24) and (27), we substitute $\mu_{n+1} = \mu_2$ into (27) and rearrange it to derive the following equation.

$$\begin{aligned} & AU^b(\mu_2) - AU^s(\mu^s) + (1-\lambda) f_2(m) \\ = \quad & \underbrace{AU^s(\mu_2) - AU^s(\mu^s)}_{\text{difference in allocation efficiency}} \quad + \underbrace{(1-\lambda)\left[f_2(m) - \theta(\mu_2)m\right]}_{\text{difference in profit}}. \end{aligned} \tag{30}$$

Here $\theta(\mu_2)$ is the market price when the federal government reports μ_2. Since μ^s is the maximum point of AU^s when $\mu_{n+1} \geq \mu_2$, we have $AU^s(\mu_2) \leq AU^s(\mu^s)$. The following lemma shows that the second term is also negative.

Lemma 1. *$f_2(m) - \theta(\mu_2)m < 0$.*

Therefore, overbidding is a better strategy than underbidding in this case. If $\lambda = 1$, we have shown that the federal government reports truthfully because the market allocation already maximizes the aggregate utility, so the federal government overbids if $\mu_1 \geq \mu_2$ and underbids if

$\mu_1 < \mu_2$. Because of the continuity of utility functions, when λ is close to 1 ($\lambda > \hat{\lambda}$ for some $\hat{\lambda}$), the federal government still remains the choice. The optimal bidding strategies are summarized in the following proposition.

Proposition 8. *If the federal government fully anticipates the second-stage market game in advance, its strategy can be described as follows.*

- *When $\mu_1 \geq \mu_2$, the federal government overbids in the first stage, reporting the highest bidding price, and becomes the seller in the second stage.*
- *When $\mu_1 < \mu_2$, $\lambda \leq \lambda^b$, the federal government overbids in the first stage, reporting the highest bidding price, and becomes the seller in the second stage.*
- *When $\mu_1 < \mu_2$, $\lambda > \hat{\lambda}$, the federal government underbids in the first stage, reporting a bidding price lower than the highest one, and becomes a buyer in the second stage.*

When the federal government has the highest demand, i.e. $\mu_1 \geq \mu_2$, it incurs a loss in two ways for him to underbid in the first stage and procure in the secondary market (in contrast to overbid). First, as the federal government lowers its bidding price, the efficiency in recourse allocation overall is suboptimal, i.e. $AU_1(\mu_2) < AU_1(\mu^*)$. Moreover, Lemma 1 indicates that the profit loss in the market game is greater than the payoff from the bidding game. Thus the federal government prefers on the sell side, because it monopolizes the secondary market and may raise the market price by bidding strategically, which explains the first statement. The explanation for the second statement is similar. When the federal government has the second highest demand, i.e. $\mu_1 \leq \mu_2$, and the weighting factor λ is low enough, the federal government is less concerned with state governments' utility. Then the federal government treats state governments as they have lower demands (compared with their true demands), so its optimal strategy is the same with the first case (when the federal government has the highest demand). When the federal government has the second highest demand, i.e. $\mu_1 \leq \mu_2$, and the weighting factor λ is high enough, the federal government focuses on allocation efficiency rather than monetary profit, and the bidding price is close to the true valuation, thus underbidding. As we have discussed before, a change in the federal government's bidding price will distort the market allocation. Case $\lambda^b \leq \lambda \leq \hat{\lambda}$ is analytically intractable, but our numerical result in Figure 3 shows that the federal government underbids if and only if $\lambda > \hat{\lambda}$.

So far, we focus on the federal government's perceived payoff, weighted by a factor λ. Finally, we derive how the aggregate utility (instead of the weighted one) is affected by the self-interested federal government.

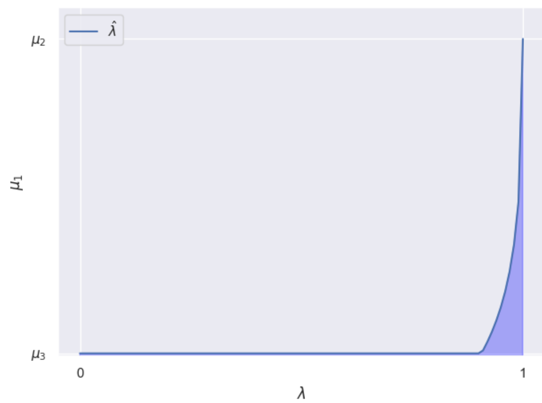

Figure 3: For (μ_1, λ) in the blue area, the federal government underbids.

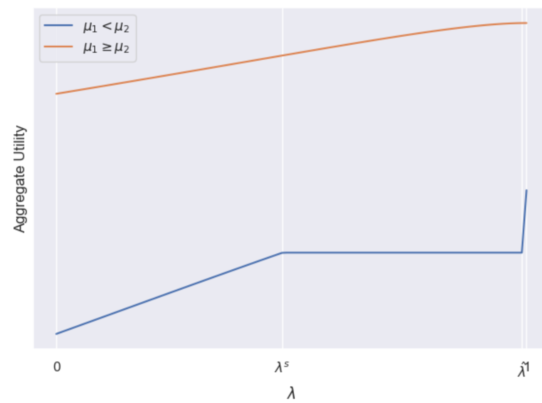

Figure 4: The aggregate utility increases with λ.

Corollary 1. *If the federal government overbids, the aggregate utility increases with* λ.

From our numerical result, this conclusion can also be applied to the case when the federal government underbids, which is shown in the third segment of the blue line in Figure 4 ($\hat{\lambda} \leq \lambda \leq 1$). We summarize the intuition behind the results in this section as follows. A more self-interested federal government acts as it has a higher demand than its true demand, resulting in a higher bidding price. This skews the market price and the allocation, so the aggregate utility decreases. Our result describes the behavior of a self-interested and dual-roled principal in such an allocation mechanism, which helps explain the high bidding price of the dual-roled federal government.

6 Case Study-Ventilator Bidding in the US

In this section, we apply our model to a case study focusing on the ventilator procurement issue in the US. Due to the serious situation of COVID-19, 50 states and 1 distinct are bidding against each other for ventilators. We use the ventilator demand data from Institute for Health Metrics and Evaluation in 2020 (IHME COVID-19 health service utilization forecasting team, 2020), and the data is shown in Appendix.B.

We first consider the minimal winning coalition discussed in Section 4.5. There are 4 states whose valuation is higher than $p_0 = 75000$, and the winning coalition is {Michigan, New Jersey, New York, Texas}. Table 3 shows the allocation of the two subgames and the whole game. Differentiating $f_i(m_i)$ with respect to m_i gives the market price $\theta = 49.93$ per ventilator. The aggregate utility is

62242. When the coalition size increases, as mentioned in Proposition 3, the aggregate utility also rises. In Figure 5, we show how the aggregate utility changes as the coalition becomes larger. Each time the player with the highest demand outside the coalition is added to the coalition. That is, in this figure, a coalition with size k includes player $\{1, 2, ..., k\}$.

No.	State	The bidding game	The market game	The whole game
1	New York	41296	1864	43159
2	Michigan	10574	75	10649
3	Texas	4765	65	4830
4	New Jersey	3472	61	3533
5	California	20	50	70

Table 3: The allocation of the game. The aggregate utility is 62242.

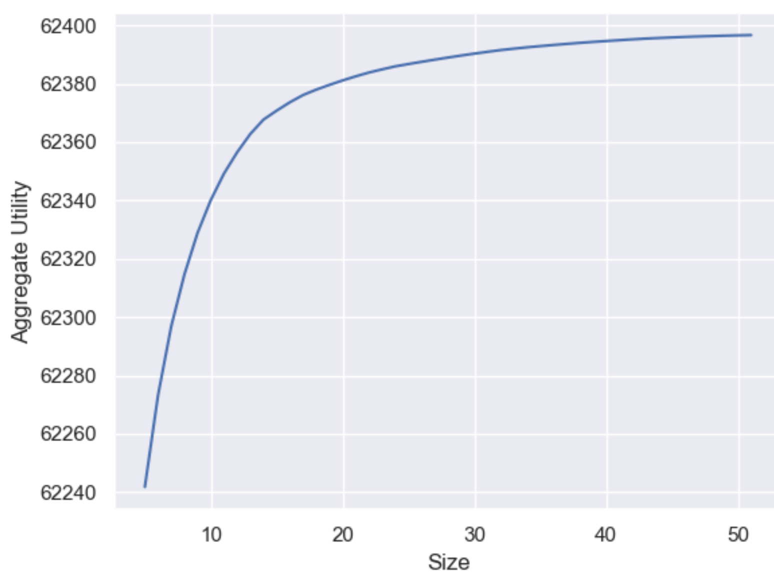

Figure 5: The aggregate utility and the coalition size.

There are two parameters in this numerical case, the number of ventilators m and the standard deviation of demand distribution σ. Figure 6 shows how the number of active bidders and the aggregate utility change with m. Each player who has a valuation $f_i(m)$ higher than the reserve price $p_0 m$ is an active bidder and is involved in the winning coalition, receiving a positive payoff in the nested knockout. As the number of ventilators rises, fewer bidders remain active. The aggregate utility increases with m almost linearly, because the utility function has a linear growth at the beginning. Though hard to see, the curve corresponding to the aggregate utility is still concave.

Figure 7 presents how market price and the aggregate utility change with σ. The two curves show similar trends. Since market price is the marginal value of a ventilator, under a fixed number of ventilators, a high market price indicates a high valuation of ventilators and thus a high aggregate utility. The standard deviation σ of the demand distribution affects the uncertainty of demand, which is reflected by the concaveness of the utility functions. When $\sigma = 0$, the demand is deterministic, and the utility function will be a piece-wise linear function $f(m) = \min\{m, D\}$. As σ increases, the expected value of a ventilator decreases because of the high uncertainty. Therefore, the market price and the aggregate utility both decrease.

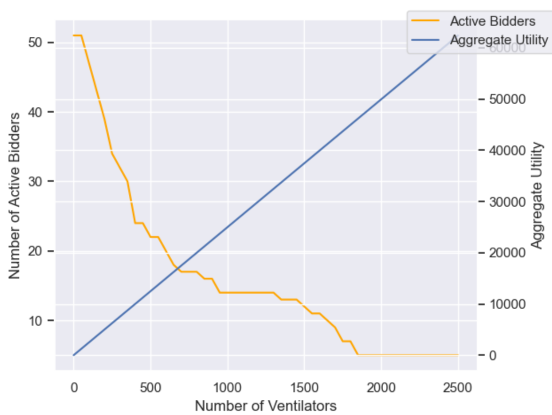

Figure 6: Effect of m.

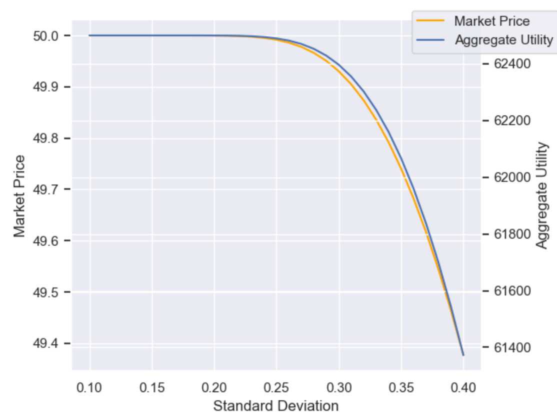

Figure 7: Effect of standard deviation.

To give a clear view of the advantage of our mechanism, we now compare ORASP and some other popular mechanisms. We still use the data in Table 4. The comparison result is concluded in Figure 8, and the allocation details for these mechanisms are listed in Appendix.B.

The second-price auction. 2500 ventilators are auctioned as an indivisible bundle under the rule of second-price auction. Among 50 states and 1 distinct, player 1 who reports the highest price $f_1(m)$, the New York state, wins m ventilators with the payment of $f_2(m)$. In this game, only player 1 wins the ventilators and generates a corresponding utility $f_{51}(m)$. Other 49 states and 1 distinct get nothing and their utilities are zero. Thus, the aggregate utility is $f_{51}(m) - f_{50}(m) = 30722$.

The centralized allocation. The Federal government purchases m ventilators at the price of $p = 45$ and allocates them to maximize the group welfare:

$$\begin{aligned} \max \quad & \sum_{i=1}^{51} f_i(m_i) \\ \text{s.t.} \quad & \sum_{i=1}^{51} m_i = m, \\ & m_i \geq 0, 1 \leq i \leq 51. \end{aligned} \tag{31}$$

All states and distinct get a number of ventilators corresponding to their demands. From Proposition 5, we know that this is equivalent to proportional allocation. The total payoff of the centralized allocation is $\max \sum_{i=1}^{51} f_i(mi) - 45m = 12397$.

The dynamic auction. In the dynamic auction, there will be several rounds of second-price auctions. A small number of ventilators will be auctioned in each round. For example, if in each round only one ventilator is auctioned, in the $1st$ round, the bidding prices are $f_1(1)-f_1(0), f_2(1)-f_2(0), ..., f_{51}(1) - f_{51}(0)$. Player 1 will win one ventilator with the payment $f_2(1) - f_2(0)$ according to the second-price auction rule. Thus, the utility of player 1 is $(f_1(1)) - f_1(0) - (f_2(1) - f_2(0))$. Then, in the $2nd$ round, the bidding prices are $f_1(2) - f_1(1), f_2(1) - f_2(0), ..., f_{51}(1) - f_{51}(0)$. The player reporting the highest price will win one ventilator this time. This process will be repeated until all m ventilators are auctioned. From numerical simulations, we derive that the group welfare is 0.05.

Our mechanism performs well as it achieves both fairness and efficiency. In the second-price auction, the state bidding the highest gets all the ventilators, and some states with low ventilator demands are not even active bidders. In contrast, ORASP can distribute ventilators to all the states, which avoids fairness concerns. Also, ORASP eliminates intense competition and provides a robust way to attain a low price. In the dynamic auction, the surplus in each round, which is the difference between the highest bidding price and the second highest bidding price, is almost zero. Thus the outcome of the dynamic auction is not satisfactory, although it achieves an efficient allocation. In centralized allocation, the government may not procure at such a low price because of the market uncertainty and the competition in the market.

Figure 8: Comparison of different mechanisms. "SPA" for second-price auction, "CA" for centralized allocation, and "DA" for dynamic auction.

7 Conclusion

Motivated by the procurement auctions of COVID-19 critical medical resources, we examine the United States federal government's dual roles as both principal and agent in such auctions. We propose a two-stage resource allocation model, i.e., Optimal Resource Allocation under Second-price Auction (ORASP), which consists of a second-price auction in the first stage and a followed market game in the second stage. Under a cooperative game theoretic framework, we explicitly allow bidders' coalition behaviors and consider resource (re-)allocation across (within) coalitions. In the first-stage auction of an indivisible bundle of items, we examine how the surplus is distributed when collusion behaviors are allowed. A nested knockout mechanism is proposed at this stage, which turns out to achieve the Shapley value. At the second stage, items are traded in a market game and the core of the two-stage game is studied. By applying our model to a ventilator procurement competition in the United States, we find that the collusion behaviors eliminate the competition and significantly reduce the procurement cost. Also, the market game effectively redistributes the ventilators which achieves the maximum aggregate utility. We compare the proposed model ORASP with other benchmarks, including the second-price auction, the centralized allocation and the dynamic auction. Numerical results show that ORASP outperforms all other mechanisms in terms of the aggregate welfare, which implies that ORASP can be a promising mechanism for procurement and resource allocation.

We examine the federal government's role as both principal and agent. We propose a weighted aggregate utility framework to capture the federal government's dual roles; the weighting factor λ measures the bias towards self-interest against social welfare. In addition, we consider that the federal government is sophisticated and fully anticipates the secondary market while the other agents are myopic. The principal may overbid or underbid in the first-stage bidding game, and could choose to appear as either a seller or a buyer in the second-stage market game accordingly. Our analysis shows that only when the federal government focuses on the aggregate utility, i.e. $\lambda > \hat{\lambda}$, and its demand for items is low, will it underbid. In this case, the main purpose of the federal government is to realize an efficient allocation in the market, so its bidding price is close to its true valuation. In other cases, the federal government overbids in order to gain profit and increase the quantity of received items. Moreover, as the weighting factor λ decreases, which corresponds to a more self-interested principal, the federal government's bidding price increases. A higher bidding price indicates a higher demand of the federal government and increases the aggregate demand of all the players, which aggravates the scarcity of the items. This raises the equilibrium market price, and benefits the principal who is on the sell side.

References

Alban, A., P. Blaettchen, H. de Vries, and L. Van Wassenhove (2021, 10). Resource allocation with sigmoidal demands: Mobile healthcare units and service adoption. *Manufacturing & Service Operations Management*.

Bachrach, Y. (2010). Honor among thieves: collusion in multi-unit auctions. In *AAMAS*.

Barrera, J. and A. Garcia (2015). Auction design for the efficient allocation of service capacity under congestion. *Operations Research 63*(1), 151–165.

Bhargava, H., G. Csapo, and R. Müller (2019, 10). On optimal auctions for mixing exclusive and shared matching in platforms. *Management Science 66*(6), 2653–2676.

Bimpikis, K., W. J. Elmaghraby, K. Moon, and W. Zhang (2020). Managing market thickness in online business-to-business markets. *Management Science 66*(12), 5783–5822.

Caillaud, B. and P. Jéhiel (1998). Collusion in auctions with externalities. *The RAND Journal of Economics 29*, 680–702.

Chen, F., A. Federgruen, and Y.-S. Zheng (2001). Coordination mechanisms for a distribution system with one supplier and multiple retailers. *Management science 47*(5), 693–708.

Chen, J., J. Feng, and A. B. Whinston (2009). Keyword auctions, unit-price contracts, and the role of commitment. *Production and Operations Management 19*(3), 305–321.

Chen, R. R., R. O. Roundy, R. Q. Zhang, and G. Janakiraman (2005). Efficient auction mechanisms for supply chain procurement. *Management Science 51*(3), 467–482.

Chen, Y.-J. (2017). Optimal dynamic auctions for display advertising. *Operations Research 65*(4), 897–913.

Chu, L. Y. and Z.-J. M. Shen (2008). Truthful double auction mechanisms. *Operations research 56*(1), 102–120.

Decarolis, F., M. Goldmanis, and A. Penta (2020). Marketing agencies and collusive bidding in online ad auctions. *Management Science 66*(10), 4433–4454.

Fang, X. and S.-H. Cho (2020). Cooperative approaches to managing social responsibility in a market with externalities. *Manufacturing & Service Operations Management 22*(6), 1215–1233.

Feng, J., Z.-J. M. Shen, and R. L. Zhan (2007). Ranked items auctions and online advertisement. *Production and Operations Management 16*(4), 510–522.

Gopalakrishnan, S., D. Granot, F. Granot, G. Sosic, and H. Cui (2020, 10). Incentives and emission responsibility allocation in supply chains. *Management Science 67*.

Graham, D. A., R. C. Marshall, and J.-F. Richard (1990). Differential payments within a bidder coalition and the shapley value. *The American Economic Review 80*, 493–510.

Harsanyi, J. C. and R. Selten (1988). A general theory of equilibrium selection in games. *Mit Press Books 1*(5).

IHME COVID-19 health service utilization forecasting team, C. J. M. (2020). Forecasting covid-19 impact on hospital bed-days, icu-days, ventilator-days and deaths by us state in the next 4 months. *medRxiv*.

Li, L. and H. Zhang (2008). Confidentiality and information sharing in supply chain coordination. *Management Science 54*(8), 1467–1481.

Liu, H., Y. Yu, S. Benjaafar, and H. Wang (2022). Price-directed cost sharing and demand allocation among service providers with multiple demand sources and multiple facilities. *Manufacturing & Service Operations Management 24*(1), 647–663.

Long, E. F., E. Nohdurft, and S. Spinler (2018). Spatial resource allocation for emerging epidemics: A comparison of greedy, myopic, and dynamic policies. *Manufacturing & Service Operations Management 20*, 181–198.

Marshall, R. C. and L. M. Marx (2009). The vulnerability of auctions to bidder collusion. *The Quarterly Journal of Economics 124*(2), 883–910.

Oren, M. E. and A. C. Williams (1975). On competitive bidding. *Operations Research 23*(6), 1072–1079.

Osborne, M. J. and A. Rubinstein (1994). *A course in game theory.* MIT press.

Peleg, B. and P. Sudhölter (2007). *Introduction to the theory of cooperative games*, Volume 34. Springer Science & Business Media.

Ryan, J., L. Shao, and D. Sun (2022, 01). Contracting mechanisms for stable sourcing networks. *Manufacturing & Service Operations Management 24*(5), 2558–2576.

Westerink-Duijzer, L. E., L. L. Schlicher, and M. M. Musegaas (2020). Core allocations for cooperation problems in vaccination. *Production and Operations Management 29*(7), 1720–1737.

Zemach, R. (1970). A model of health-service utilization and resource allocation. *Operations Research 18*(6), 1071–1086.

Appendix.A

In this appendix, we provide detailed proofs of the main results.

Proof of Proposition 1. To prove (N, h) is convex, we need to show that $\forall S, T \subseteq N, h(S \cup T) \geq h(s) + h(T) - h(S \cap T)$. Considering any two subsets $S, T \subseteq N$, the corresponding value functions are as follows:

$$h(S) = \begin{cases} f_n(m) - \underset{j \in N \setminus S}{\text{Max}} f_j(m) & n \in S \\ 0 & n \notin S \end{cases}$$

$$h(T) = \begin{cases} f_n(m) - \underset{j \in N \setminus T}{\text{Max}} f_j(m) & n \in T \\ 0 & n \notin T \end{cases}$$

For the above functions, there are 4 possible cases:

1. $n \notin S$ and $n \notin T$;
2. $n \in T$ and $n \notin S$;
3. $n \notin T$ and $n \in S$;
4. $n \in S$ and $n \in T$.

We will discuss the 4 cases respectively:

(1) $n \notin S$ and $n \notin T$: Under this condition, we have $n \notin (S \cup T)$ and $n \notin (S \cap T)$:

$$h(S) = h(T) = h(S \cup T) = h(S \cap T) = 0.$$

(2) $n \in T$ and $n \notin S$: Under this condition, we have $n \notin (S \cap T)$ and,

$$h(S) = h(S \cap T) = 0.$$

Also, from the property of the value function, it is easy to show that

$$\underset{j \in N \setminus (S \cup T)}{\text{Max}} f_j(m) \leq \underset{j \in N \setminus T}{\text{Max}} f_j(m).$$

Thus,

$$h(S \cup T) = f_n(m) - \operatorname*{Max}_{j \in N \backslash (S \cup T)} f_j(m) \geq f_n(m) - \operatorname*{Max}_{j \in N \backslash T} f_j(m) = h(T).$$

(3) $n \notin T$ and $n \in S$: Similar to (2), one can verify that $h(S \cup T) \geq h(s) + h(T) - h(S \cap T)$ holds for case (3).

(4) $n \in S$ and $n \in T$: Under this condition, we have $n \in (S \cap T) \subseteq S, T \subseteq (S \cup T)$ and,

$$0 \leq \operatorname*{Max}_{j \in N \backslash (S \cup T)} f_j(m) \leq \operatorname*{Max}_{j \in N \backslash T} f_j(m), \operatorname*{Max}_{j \in N \backslash S} f_j(m) \leq \operatorname*{Max}_{j \in N \backslash (S \cap T)} f_j(m).$$

Thus,

$$\operatorname*{Max}_{j \in N \backslash (S \cap T)} f_j(m) = \operatorname{Max}[\operatorname*{Max}_{j \in N \backslash T} f_j(m), \operatorname*{Max}_{j \in N \backslash S} f_j(m)],$$

$$\operatorname*{Max}_{j \in N \backslash (S \cup T)} f_j(m) = \operatorname{Min}[\operatorname*{Max}_{j \in N \backslash T} f_j(m), \operatorname*{Max}_{j \in N \backslash S} f_j(m)],$$

and

$$\operatorname*{Max}_{j \in N \backslash (S \cap T)} f_j(m) + \operatorname*{Max}_{j \in N \backslash (S \cup T)} f_j(m) = \operatorname*{Max}_{j \in N \backslash T} f_j(m) + \operatorname*{Max}_{j \in N \backslash S} f_j(m).$$

Therefore, the following relationship holds for case (4):

$$h(S \cup T) = h(s) + h(T) - h(S \cap T).$$

In conclusion, we have proved that $\forall S, T \subseteq N, h(S \cup T) \geq h(s) + h(T) - h(S \cap T)$, which is a condition stronger than subadditivity of cooperative game and also implies the convexity of the bidding game (N, h). ■

Proof of Theorem 1. Since $\{\psi_i\}_{i \in N}$ and $\{\phi_i\}_{i \in N}$ are core allocations of (N, g) and (N, h), respectively. They satisfy

$$\sum_{i \in N} \phi_i = h(N), \ \sum_{i \in N} \psi_i = g(N),$$

and

$$\sum_{i \in S} \phi_i \geq h(S), \ \sum_{i \in S} \psi_i \geq g(S), \ \forall S \subseteq N.$$

Efficiency:

$$\sum_{i\in N\backslash n}(\phi_i+\psi_i)+\phi_n+\psi_n-f_n(m)=h(N)+g(N)-f_n(m)=V(N).$$

Stability:

If $n\in S$,

$$\sum_{i\in S}(\phi_i+\psi_i)\geq h(S)+g(S)-f_n(m)=V(S).$$

If $n\notin S$,

$$\sum_{i\in S}(\phi_i+\psi_i)\geq 0.$$

Thus, $\{\zeta_i\}_{i\in N}$ satisfy the condition of efficiency and stability:

$$\sum_{i\in N}\zeta_i=V(N),\ \sum_{i\in S}\zeta_i\geq V(S),$$

and is in the core of game (N,V). ■

Proof of Proposition 2. Let m_i^* be the optimal resource allocation for coalition S and y_i be the initial items player i have after the bidding game. m is the total number of items in the market game. The utility function for the buyer and the seller are $f_b(\cdot)$ and $f_s(\cdot)$, respectively. We have

$$m_b^*+m_s^*=y_b+y_s=m$$

First, we prove that all core allocations are market allocations in the case of two players. Let (ψ_b,ψ_s) be a core of the game (N,g), and we have:

$$\begin{aligned}\psi_b+\psi_s=&f_b(m_b^*)+f_s(m_s^*)=g(N),\\ \psi_b=&f_b(m_b^*)+\Theta_b(y_b-m_b^*),\\ \psi_s=&f_s(m_s^*)+\Theta_s(y_s-m_s^*).\end{aligned}$$

From the three equations above, we know that

$$\Theta_b=\frac{f_b(m_b^*)-\psi_b}{m_b^*}=\frac{\psi_s-f_s(m_s^*)}{m-m_s^*}=\Theta_s,$$

So $\Theta_b = \Theta_s = \Theta$, which indicates Θ is the market price. Thus, the core allocation (ψ_b, ψ_s) is a market allocation.

Second, we prove that the market allocation is in the core if market price Θ is in the aforementioned interval. If (ψ_b, ψ_s) is a core allocation, it must satisfy the stability condition:

$$\psi_b \geq g(\{b\}) \Leftrightarrow \psi_b = f_b(m_b^*) + \Theta(y_b - m_b^*) \geq 0,$$
$$\psi_s \geq g(\{s\}) \Leftrightarrow \psi_s = f_s(m_s^*) + \Theta(y_s - m_s^*) \geq f_s(m).$$

So

$$\frac{f_s(m) - f_s(m_s^*)}{m - m_s^*} \leq \Theta \leq \frac{f_b(m_b^*)}{m_b^*}$$

Thus, (ψ_b, ψ_s) is in the core if and only if the market price Θ is in the aforementioned interval.

Third, we prove that the interval is not empty. Since the allocation (f_b^*, f_s^*) is optimal, we have:

$$0 + f_s(m) \leq f_b(m_b^*) + f_s(m_s^*)$$

Hence,

$$\frac{f_b(m_b^*)}{m_b^*} = \frac{f_b(m_b^*)}{m - m_s^*} \geq \frac{f_s(m) - f_s(m_s^*)}{m - m_s^*},$$

and the interval is not empty. ■

Proof of Proposition 3. First we assume that $T \backslash S = \{l\}$. We know that $g(S)$ is the maximal value of the following optimization problem,

$$\begin{aligned} \max_{m_i \geq 0} \quad & \sum_{i \in S} f_i(m_i), \\ \text{s.t.} \quad & \sum_{i \in S} m_i = m. \end{aligned}$$

It is easy to verify that the optimal solution m_i^s satisfies $f_i'(m_i^s) = \theta_s,\ \forall i \in S$ for some θ^s. Similarly, $g(T)$ is the maximal value of the following optimization problem,

$$\begin{aligned} \max_{m_i \geq 0} \quad & f_l(m_l) + \sum_{i \in S} f_i(m_i), \\ \text{s.t.} \quad & m_l + \sum_{i \in S} m_i = m, \end{aligned}$$

and the optimal solution m_i^t satisfies $f_i^{'}(m_i^t) = \theta^t,\ \forall i \in T$. Since $m_l^t > 0$, $m_i^s > m_i^t$, $\forall i \in S$. From the strict concaveness of f_i,

$$\begin{aligned} g(T) - g(S) =& \int_0^{m_l^t} f_l^{'}(x)dx - \sum_{i\in S} \int_{m_i^t}^{m_i^s} f_i^{'}(x)dx \\ >& m_l^t \theta^t - \sum_{i\in S}(m_i^s - m_i^t)\theta^t = 0. \end{aligned}$$

By induction, $\forall S \subsetneq T$, $g(T) > g(S)$. From the strict concaveness of f_i, $\theta^t > \theta^s$. In coalition S, the seller (player 1) gets $f_1(m_1^s) + (m - m_1^s)\theta^s$, while in coalition T he gets $f_1(m_1^t) + (m - m_1^t)\theta^t$. Subtracting these two utility gives

$$\begin{aligned} & \int_{m_1^t}^{m_1^s} f_1^{'}(x)dx + (m_1^t - m_1^s)\theta^t + (m - m_1^s)(\theta^s - \theta^t) \\ <& (m - m_1^s)(\theta^s - \theta^t) \\ <& 0. \end{aligned}$$

So the seller benefits more from coalition T. In coalition S, a buyer (player i) gets $f_i(m_i^s) - m_i^s\theta^s$, while in coalition T he gets $f_i(m_i^t) - m_i^t\theta^t$. Subtracting these two utility gives

$$\begin{aligned} & \int_{m_i^t}^{m_i^s} f_i^{'}(x)dx + (m_i^t - m_i^s)\theta^t + m_1^s(\theta^t - \theta^s) \\ =& m_1^s(\theta^t - \theta^s) - \int_{m_i^t}^{m_i^s} \theta^t - f_i^{'}(x)dx \\ >& 0. \end{aligned}$$

So a buyer benefits less from coalition T. ■

Proof: Utility function $f_i(m_i) = \mathbf{E}[\min\{m_i, \mu_i D\}]$ is concave increasing. It suffices to prove that for any non-negative random variable D, $f(m) = \mathbf{E}[\min(m, D)]$ is concave increasing. Let q be the probability density function of D, then we have

$$f(m) = \int_0^\infty \min\{x, m\}q(x)dx = \int_0^m xq(x)dx + m\int_m^\infty q(x)dx.$$

The first order and second order derivatives w.r.t. m are as follows:

$$f^{'}(m) = \int_m^\infty q(x)dx - mq(m) + mq(m) = \int_m^\infty q(x)dx,\ f^{''}(m) = -q(m). \tag{32}$$

Therefore, $f_i(m_i) = \mathbf{E}[\min\{m_i, \mu_i D\}]$ is concave increasing. ■

Proof of Proposition 6. The problem for player 1 is

$$\max_{\mu_{n+1}} \quad AU^s = \mu_1 f(\frac{\mu_{n+1}}{\mu_1}\bar{m}) + \lambda\sum_{i=2}^{n}\mu_i f(\bar{m}) + (1-\lambda)\sum_{i=2}^{n} f^{'}(\bar{m})\mu_i\bar{m}.$$

$$\text{s.t.} \quad \mu_{n+1} \geq \mu_2.$$

Differentiating w.r.t. μ_{n+1} gives

$$\begin{aligned}\frac{\partial AU^s}{\partial \mu_{n+1}} =& f^{'}(\frac{\mu_{n+1}}{\mu_1}\bar{m})\frac{\sum_{i=2}^{n}\mu_i m}{(\sum_{i=2}^{n+1}\mu_i)^2} - \lambda\sum_{i=2}^{n}\mu_i f^{'}(\bar{m})\frac{m}{(\sum_{i=2}^{n+1}\mu_i)^2} \\ &- (1-\lambda)\sum_{i=2}^{n} f^{''}(\bar{m})\frac{m}{(\sum_{i=2}^{n+1}\mu_i)^2}\mu_i\bar{m} - (1-\lambda)\sum_{i=2}^{n} f^{'}(\bar{m})\mu_i\frac{m}{(\sum_{i=2}^{n+1}\mu_i)^2} \\ =& \frac{\sum_{i=2}^{n}\mu_i\bar{m}}{\sum_{i=2}^{n+1}\mu_i}\left[f^{'}(\frac{\mu_{n+1}}{\mu_1}\bar{m}) - f^{'}(\bar{m}) - (1-\lambda)\bar{m}f^{''}(\bar{m})\right].\end{aligned}$$

Let

$$g(\mu_{n+1}) = f^{'}(\frac{\mu_{n+1}}{\mu_1}\bar{m}) - f^{'}(\bar{m}) - (1-\lambda)\bar{m}f^{''}(\bar{m}).$$

From our assumption, $\bar{m} < 1$, so $f^{'''}(\bar{m}) = -q^{'}(\bar{m}) < 0$, where q is the pdf of a $N(1,\sigma)$ variable. Since $f^{''} < 0, f^{'''}(\bar{m}) < 0$,

$$g^{'}(\mu_{n+1}) = \frac{\bar{m}}{\sum_{i=2}^{n+1}\mu_i}\left[f^{''}(\frac{\mu_{n+1}}{\mu_1}\bar{m})\frac{\sum_{i=2}^{n}\mu_i}{\mu_1} + (2-\lambda)f^{''}(\bar{m}) + (1-\lambda)\bar{m}f^{'''}(\bar{m})\right] < 0.$$

Since $\frac{\partial g}{\partial \lambda} < 0$, when λ decreases, the optimal μ_{n+1} increases.

Case 1: In this case, $\mu_1 \geq \mu_2$. If $\lambda = 1$, $\mu_{n+1} = \mu_1$ is the optimal solution as the derivative is zero. If $\lambda < 1$, then we have

$$\frac{\partial AU^s}{\partial \mu_{n+1}}(\mu_1) = \frac{\sum_{i=2}^{n}\mu_i\bar{m}}{\sum_{i=2}^{n+1}\mu_i}\left[-(1-\lambda)\bar{m}f^{''}(\bar{m})\right] > 0,$$

so $\mu_{n+1} > \mu_1$.

Case 2: In this case, $\mu_1 < \mu_2$. If $\lambda = 1$,

$$\frac{\partial AU^s}{\partial \mu_{n+1}}(\mu_2) = \frac{\sum_{i=2}^{n}\mu_i\bar{m}}{\sum_{i=2}^{n+1}\mu_i}\left[f^{'}(\frac{\mu_2}{\mu_1}\bar{m}) - f^{'}(\bar{m})\right] < 0,$$

so $\mu_{n+1} = \mu_2$ is the optimal solution. We have shown above that the optimal μ_{n+1} increases as λ decreases. Therefore there exists λ^s s.t. $\frac{\partial AU^s}{\partial \mu_{n+1}}(\mu_2) = 0$. When $\lambda \geq \lambda^s$, $\mu_{n+1} = \mu_2$. When $\lambda < \lambda^s$, $\mu_{n+1} > \mu_2$. When $\lambda = 0$,

$$g(\mu_{n+1}) \geq \bar{m}f^{''}(\bar{m}) - f^{'}(\bar{m}) > 0.$$

This comes from the strict concaveness of f, and thus $\lambda^s > 0$. ■

Proof of Proposition 7. The problem for player 1 is

$$\max_{\mu_{n+1}} \quad U = \frac{1-\lambda}{2}\mu_{n+1} f(\frac{m}{\mu_{n+1}}) + \mu_1 f(\frac{\mu_{n+1}}{\mu_1}\bar{m}) + \lambda \sum_{i=2}^{n} \mu_i f(\bar{m}) - (1-\lambda) f^{'}(\bar{m})\mu_{n+1}\bar{m}$$

$$\text{s.t.} \quad \mu_3 \le \mu_{n+1} \le \mu_2.$$

Differentiating w.r.t. μ_{n+1} gives

$$\begin{aligned}\frac{\partial U}{\partial \mu_{n+1}} =& \frac{1-\lambda}{2}\left[f(\frac{m}{\mu_{n+1}}) - \frac{m}{\mu_{n+1}} f^{'}(\frac{m}{\mu_{n+1}})\right] + \frac{\sum_{i=2}^{n}\mu_i \bar{m}}{\sum_{i=2}^{n+1}\mu_i}\left[f^{'}(\frac{\mu_{n+1}}{\mu_1}\bar{m}) - f^{'}(\bar{m})\right] \\ &+ (1-\lambda) f^{''}(\bar{m}) \frac{\mu_{n+1}\bar{m}^2}{\sum_{i=2}^{n+1}\mu_i}.\end{aligned}$$

Let

$$g(\mu_{n+1}) = \frac{1}{2}\left(f(\frac{m}{\mu_{n+1}}) - \frac{m}{\mu_{n+1}} f^{'}(\frac{m}{\mu_{n+1}})\right) + f^{''}(\bar{m}) \frac{\mu_{n+1}\bar{m}^2}{\sum_{i=2}^{n+1}\mu_i}.$$

From (32), we know that

$$g(\mu_{n+1}) = \frac{1}{2}\int_{-\infty}^{\frac{m}{\mu_{n+1}}} x q(x) dx - \frac{\mu_{n+1}}{\sum_{i=2}^{n+1}\mu_i}\bar{m}^2 q(\bar{m}).$$

Because

$$\mu_{n+1} \le \frac{\mu_{n+1} + \mu_2}{2} \le \frac{1}{2}\sum_{i=2}^{n+1}\mu_i,$$

we have

$$g(\mu_{n+1}) \ge \frac{1}{2}\left[\int_{-\infty}^{2\bar{m}} x q(x) dx - \bar{m}^2 q(\bar{m})\right] > 0.$$

Case 1: In this case, $\mu_1 \ge \mu_2$. Since $\forall \mu_3 \le \mu_{n+1} \le \mu_2$,

$$\frac{\partial U}{\partial \mu_{n+1}} = (1-\lambda) g(\mu_{n+1}) + \frac{\sum_{i=2}^{n}\mu_i \bar{m}}{\sum_{i=2}^{n+1}\mu_i}\left[f^{'}(\frac{\mu_{n+1}}{\mu_1}\bar{m}) - f^{'}(\bar{m})\right] > 0,$$

the optimal $\mu_{n+1} = \mu_2$.

Case 2: In this case, $\mu_1 < \mu_2$. So when $\lambda = 1$,

$$\frac{\partial U}{\partial \mu_{n+1}}(\mu_2) = \frac{\sum_{i=2}^{n}\mu_i \bar{m}}{\sum_{i=2}^{n+1}\mu_i}\left[f^{'}(\frac{\mu_2}{\mu_1}\bar{m}) - f^{'}(\bar{m})\right] < 0,$$

and the optimal $\mu_{n+1} < \mu_2$. In fact, $\mu_{n+1} = \mu_1$ is optimal when $\lambda = 1$. As λ decreases, $\frac{\partial U}{\partial \mu_{n+1}}(\mu_2)$ increases. Therefore, there exists λ^b s.t. $\frac{\partial U}{\partial \mu_{n+1}}(\mu_2) = 0$ when $\lambda = \lambda^b$. When $\lambda > \lambda^b$, $\mu_{n+1} < \mu_2$. When $\lambda \le \lambda^b$, $\mu_{n+1} = \mu_2$. Note that λ^b may be less than 0 and thus $\mu_{n+1} < \mu_2, \forall 0 \le \lambda \le 1$. ■

Proof of Lemma 1. Let $t = \mu_2 + \sum_{i=2}^{n} \mu_i$, and

$$g(m) = \mu_2 f(\frac{m}{\mu_2}) - m f^{'}(\frac{m}{t}).$$

Then

$$\begin{aligned} g^{'}(m) =& f^{'}(\frac{m}{\mu_2}) - f^{'}(\frac{m}{t}) - \frac{m}{t} f^{''}(\frac{m}{t}) \\ =& \int_{m/t}^{m/\mu_2} f^{''}(x)dx - \frac{m}{t} f^{''}(\frac{m}{t}). \end{aligned}$$

Recall that $f^{'} > 0$ is decreasing and $f^{''} < 0$ is decreasing. Since $2\mu_2 \leq t = \mu_2 + \sum_{i=2}^{n} \mu_i$, we have $m/\mu_2 \geq 2m/t$.

$$g^{'}(m) \leq \int_{m/t}^{2m/t} f^{''}(x)dx - \frac{m}{t} f^{''}(\frac{m}{t}) < 0.$$

Combining this with $g(0) = 0$, we have $g(m) < 0$. ■

Proof of Corollary 1. The true aggregate utility is AU_1 with $\lambda = 1$. From the proof of Proposition 7 we know that AU_1 is decreasing w.r.t. μ_{n+1} for any λ when $\mu_{n+1} \geq \mu_1$. Also, the optimal reported demand $\mu^*(\lambda)$ decreases w.r.t λ. Therefore, the true aggregate utility is increasing w.r.t. λ. ■

Appendix.B

The data in Section 6. We adopt the same function form in Section 5, and the standard deviation is set to be 0.3,

$$f_i(m_i) = c\mathbb{E}\min\{m_i, \mu_i D\}, D \sim N(1, 0.3^2). \tag{33}$$

The only parameter for each player is the expected demand μ_i. The demand is estimated from other data such as the number of ICU beds used. We use the ventilator demand data from Institute for Health Metrics and Evaluation in 2020 (IHME COVID-19 health service utilization forecasting team, 2020), and the data is shown in Table 4. For simplicity, we assume that there is no initial inventory of ventilators. The marginal value of a ventilator that is put into use is assumed to be a constant c for each state. From the news [4], "Price of ventilators had skyrocketed from 25,000 dollars each to 45,000 dollars, as 50 states and the federal government all bid against each other for the vital oxygen device", the marginal value of a ventilator is higher than 45 thousand dollars. We assume that the marginal value is $c = 50$ (thousand dollars), and the reserve price is $p_0 = 25$ (thousand dollars) per ventilator. The number of ventilators to be allocated is $m = 2500$.

State	Demand	State	Demand	State	Demand
Alabama	307	Kentucky	95	North Dakota	42
Alaska	44	Louisiana	801	Ohio	746
Arizona	455	Maine	92	Oklahoma	244
Arkansas	184	Maryland	186	Oregon	120
California	1252	Massachusetts	867	Pennsylvania	829
Colorado	185	Michigan	1876	Rhode Island	65
Connecticut	292	Minnesota	336	South Carolina	139
Delaware	61	Mississippi	181	South Dakota	51
District of Columbia	34	Missouri	901	Tennessee	423
Florida	454	Montana	66	Texas	1612
Georgia	868	Nebraska	111	Utah	165
Hawaii	90	Nevada	304	Vermont	175
Idaho	101	New Hampshire	87	Virginia	280
Illinois	751	New Jersey	1531	Washington	238
Indiana	907	New Mexico	134	West Virginia	118
Iowa	190	New York	4305	Wisconsin	110
Kansas	172	North Carolina	661	Wyoming	37

Table 4: The expected demand for ventilators of each state.

[4] https://www.thedailybeast.com/new-york-gov-cuomo-says-price-of-ventilators-has-skyrocketed-to-dollar45000-amid-coronavirus-pandemic

ORASP. The following table shows the outcome of the ORASP. The "Utility" column stands for the utility from ventilators, and the "Cost" column stands for the cost for purchasing ventilators.

No.	State	Number of Ventilators	Utility	Cost
1	New York	444	22181.633	11100.000
2	Michigan	193	9642.030	4825.000
3	Texas	166	8293.140	4150.000
4	New Jersey	158	7893.461	3950.000
5	California	129	6444.667	3225.000
6	Indiana	93	4646.168	2325.000
7	Missouri	93	4646.151	2325.000
8	Georgia	89	4446.333	2225.000
9	Massachusetts	89	4446.330	2225.000
10	Pennsylvania	85	4246.498	2125.000
11	Louisiana	83	4146.555	2075.000
12	Illinois	77	3846.828	1925.000
13	Ohio	77	3846.813	1925.000
14	North Carolina	68	3397.192	1700.000
15	Arizona	47	2348.054	1175.000
16	Florida	47	2348.051	1175.000
17	Tennessee	44	2198.169	1100.000
18	Minnesota	35	1748.542	875.000
19	Alabama	32	1598.666	800.000
20	Nevada	31	1548.728	775.000
21	Connecticut	30	1498.762	750.000
22	Virginia	29	1448.797	725.000
23	Oklahoma	25	1248.970	625.000
24	Washington	25	1248.953	625.000
25	Iowa	20	999.161	500.000
26	Maryland	19	949.219	475.000
27	Colorado	19	949.216	475.000
28	Arkansas	19	949.213	475.000

29	Mississippi	19	949.204	475.000
30	Vermont	18	899.257	450.000
31	Kansas	18	899.248	450.000
32	Utah	17	849.297	425.000
33	South Carolina	14	699.430	350.000
34	New Mexico	14	699.416	350.000
35	Oregon	12	599.514	300.000
36	West Virginia	12	599.508	300.000
37	Nebraska	11	549.557	275.000
38	Wisconsin	11	549.554	275.000
39	Idaho	10	499.598	250.000
40	Kentucky	10	499.580	250.000
41	Maine	9	449.641	225.000
42	Hawaii	9	449.635	225.000
43	New Hampshire	9	449.627	225.000
44	Montana	7	349.705	175.000
45	Rhode Island	7	349.701	175.000
46	Delaware	6	299.760	150.000
47	South Dakota	5	249.800	125.000
48	Alaska	5	249.778	125.000
49	North Dakota	4	199.843	100.000
50	Wyoming	4	199.829	100.000
51	District of Columbia	3	149.888	75.000

Dynamic auction. The following table shows the outcome of the dynamic auction. The "Utility" column stands for the utility from ventilators, and the "Cost" column stands for the cost for purchasing ventilators.

No.	State	Number of Ventilators	Utility	Cost
1	New York	444	22181.633	22181.624
2	Michigan	193	9642.030	9642.026

3	Texas	166	8293.140	8293.137
4	New Jersey	158	7893.461	7893.458
5	California	129	6444.667	6444.665
6	Indiana	93	4646.168	4646.166
7	Missouri	93	4646.151	4646.149
8	Georgia	89	4446.333	4446.332
9	Massachusetts	89	4446.330	4446.328
10	Pennsylvania	85	4246.498	4246.496
11	Louisiana	83	4146.555	4146.554
12	Illinois	77	3846.828	3846.826
13	Ohio	77	3846.813	3846.812
14	North Carolina	68	3397.192	3397.191
15	Arizona	47	2348.054	2348.053
16	Florida	47	2348.051	2348.050
17	Tennessee	44	2198.169	2198.168
18	Minnesota	35	1748.542	1748.541
19	Alabama	32	1598.666	1598.666
20	Nevada	31	1548.728	1548.727
21	Connecticut	30	1498.762	1498.762
22	Virginia	29	1448.797	1448.796
23	Oklahoma	25	1248.970	1248.970
24	Washington	25	1248.953	1248.952
25	Iowa	20	999.161	999.160
26	Maryland	19	949.219	949.219
27	Colorado	19	949.216	949.216
28	Arkansas	19	949.213	949.213
29	Mississippi	19	949.204	949.204
30	Vermont	18	899.257	899.256
31	Kansas	18	899.248	899.247
32	Utah	17	849.297	849.297

33	South Carolina	14	699.430	699.430
34	New Mexico	14	699.416	699.415
35	Oregon	12	599.514	599.514
36	West Virginia	12	599.508	599.508
37	Nebraska	11	549.557	549.557
38	Wisconsin	11	549.554	549.554
39	Idaho	10	499.598	499.597
40	Kentucky	10	499.580	499.580
41	Maine	9	449.641	449.641
42	Hawaii	9	449.635	449.635
43	New Hampshire	9	449.627	449.627
44	Montana	7	349.705	349.705
45	Rhode Island	7	349.701	349.701
46	Delaware	6	299.760	299.759
47	South Dakota	5	249.800	249.800
48	Alaska	5	249.778	249.778
49	North Dakota	4	199.843	199.843
50	Wyoming	4	199.829	199.829
51	District of Columbia	3	149.888	149.888