Diffraction Pattern of a Laser Beam Obliquely Incident on a Transmission Diffraction Grating

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1 Introduction

A transmission diffraction grating is a material which consists of many parallel slits. In DP Physics class, we investigated how diffraction gratings scatter perpendicularly incident light based on the wavelength of the light. In some spots, the diffracted light constructively interferes, while in others, the light destructively interferes. This causes the light diffracted at certain angles to be much more intense than all the other angles. If we project the diffracted light onto a screen, we can visually observe this phenomenon: several locations on the screen appear as bright dots, while the rest of the screen remains unilluminated. Fig. 1 below illustrates how a diffraction grating with vertically oriented slits diffracts a horizontal laser beam onto a screen.

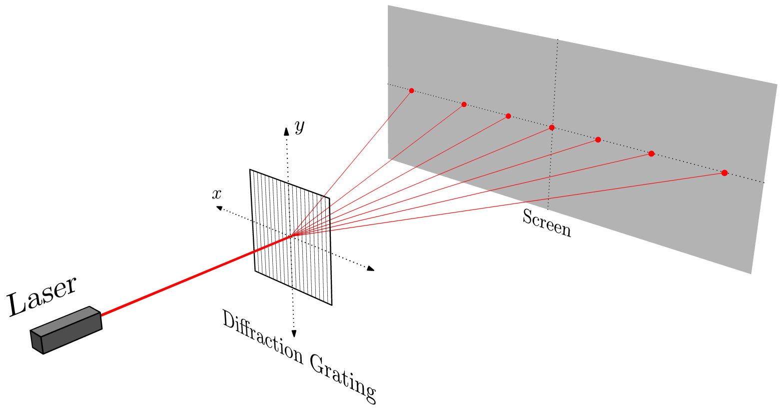


Figure 1: Standard diffraction grating behaviour

Let the laser travel along the z-axis, and let the x- and y- axes be perpendicular to the laser as labeled in Fig. 1, with the origin being at the point the laser intersects the grating. I noticed that if I rotate the diffraction grating about the x-axis as shown below in Fig. 2, the secondary maxima shift upwards and trace a curve along the screen.

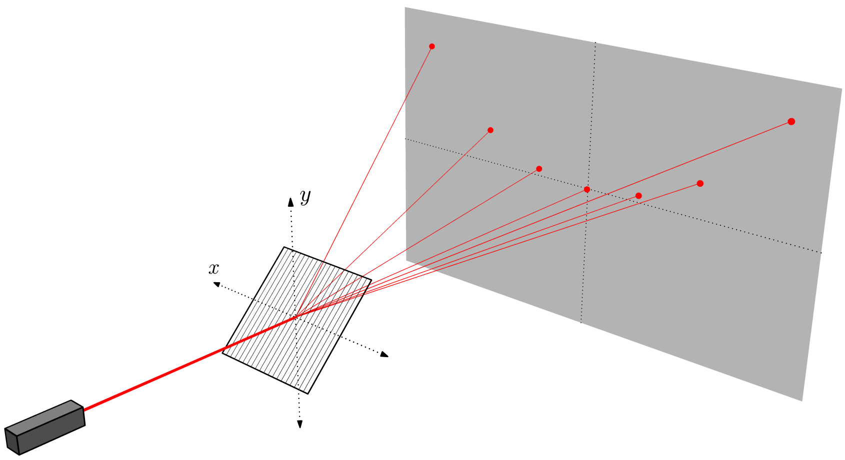


Figure 2: Tilted diffraction grating behaviour

Fig. 3 below shows a side view of Fig. 2 with only the primary and 1st-order maximum.

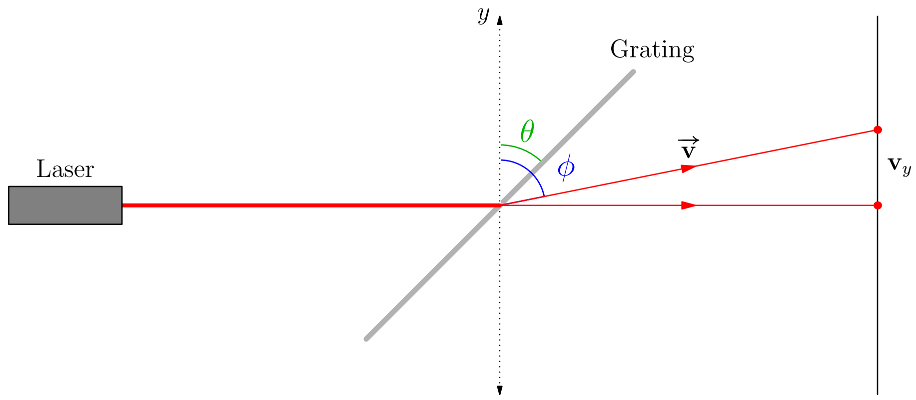


Figure 3: Side view of the tilted diffraction grating

My independent variable is the angle formed by the diffraction grating and the y-axis, θ. I wanted to see how θ affects the amount the 1st-order maximum has been deflected upwards. I chose to use the vertical (y) direction cosine of the 1st-order maxima, β_1, as my dependent variable. This direction cosine is defined as the cosine of the angle formed by $\vec{\mathbf{v}}$ (the vector representing the 1st-order maxima, as labeled above) and the y-axis, or $\cos\phi$, and thus it is unitless. It gives a measure of how "vertical" the vector is – if $\vec{\mathbf{v}}$ is steeper, β_1 is closer to 1, and if $\vec{\mathbf{v}}$ is shallower, β_1 is closer to 0. The y direction cosine is equal to the value $\frac{\mathbf{v}_y}{\|\vec{\mathbf{v}}\|}$, with $\mathbf{v}_y$ representing the y-component of $\vec{\mathbf{v}}$. I chose not to use $\mathbf{v}_y$ by itself as my dependent variable because this value would depend on the distance between the grating and the screen, while the direction cosine gives a better description of the actual direction the light travels in.

I will use a 5mW, 650 nm wavelength laser. Thus, my research question: (can move this focused RQ later, in methodology)

How does the tilt of a transmission diffraction grating, θ (for $0° < \theta < 90°$), affect the vertical direction cosine of the 1st-order maxima, β_1, for a 5mW, 650 nm wavelength laser?

Theoretically, the y direction cosines on each side should be equal, but experimentally there could be some discrepancy due to random error. For my experiment, I will measure the y direction cosine for both of the 1st-order maxima and take the average value.

2 Theory

This phenomenon is known as conical diffraction, because the maxima all lie on a cone whose axis is parallel to the slits of the grating, as shown in Fig. 4 below [2]. Note the screen has been tilted so that it is perpendicular to the axis of the cone to better illustrate this phenomenon.

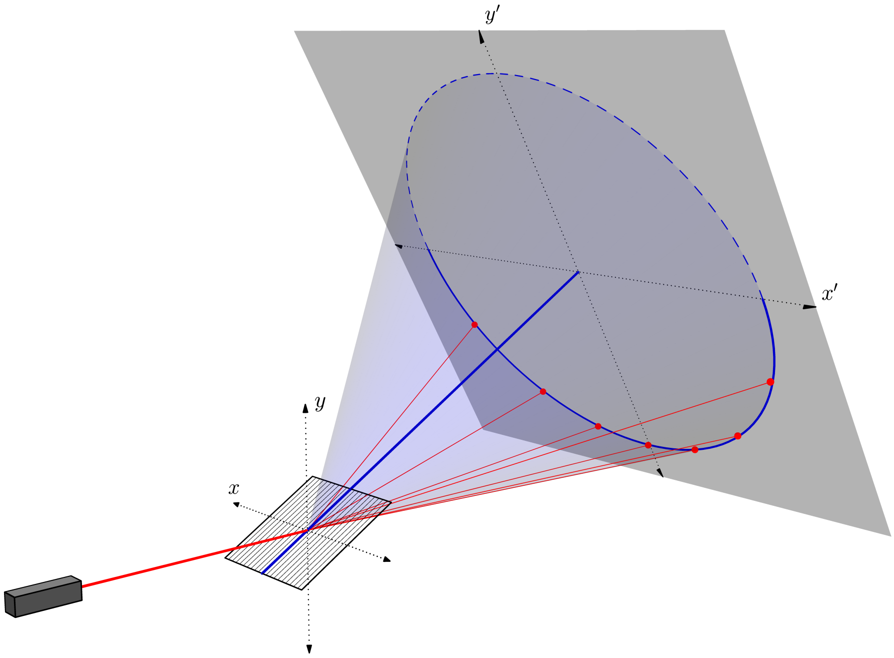


Figure 4: Conical geometry of the diffracted maxima

In my research, I could not find an explanation as to why the diffracted maxima form a cone that didn't use advanced math and physics (such as the Fraunhofer Diffraction Integral), so I fashioned my own argument based on path length difference shown in Appendix D. The main idea is that, for 2 light rays passing through 2 points on the same slit to have no path length difference, the diffracted light must lie on the cone with its axis along the slit.

Returning to Fig. 4, we can see that when the screen is perpendicular to the axis of the cone all the diffracted rays have the same length. If we position the screen such that the length of each diffracted ray is 1, then the x and y direction cosines of the diffracted light vectors are just the absolute x and y coordinates of the point formed on the screen. As we just described, the half-angle of this cone is $\frac{\pi}{2} - \theta$, which implies the radius of the circle on the screen is $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$. Let x' and y' be a new set of coordinate axes on the tilted screen, as shown in Fig. 4. If we first find the x direction cosine (the cosine of the angle formed by the x-axis) of the 1st-order maxima, α_1, we can substitute that for the x'-coordinate of the point formed on the screen. This will allow us to find the y'-coordinate, and finally the y direction cosine β_1.

Although β_1 changes with the tilt of the diffraction grating θ, the value α_1 is actually independent of θ. Fig. 5 on the right depicts 2 parallel rays of light passing through 2 adjacent slits in a grating. When the light is diffracted at a specific angle from the x-axis, the path length difference (the green segment) is equal to the wavelength of the light λ. Thus, the diffracted rays constructively interfere with each other, forming a 1st-order maximum. The blue cones represent all possible directions the diffracted light could travel in such that the two rays have a path length difference of λ. Even when we tilt the diffraction grating, for the diffracted light to constructively interfere it must lie on this blue cone.

Let C represent the half-angle of this cone (the magenta angle), and let d be the distance between adjacent slits. The x direction cosine for any ray along this blue cone can be calculated as

$$\alpha_1 = \cos C = \frac{\lambda}{d}. \tag{1}$$

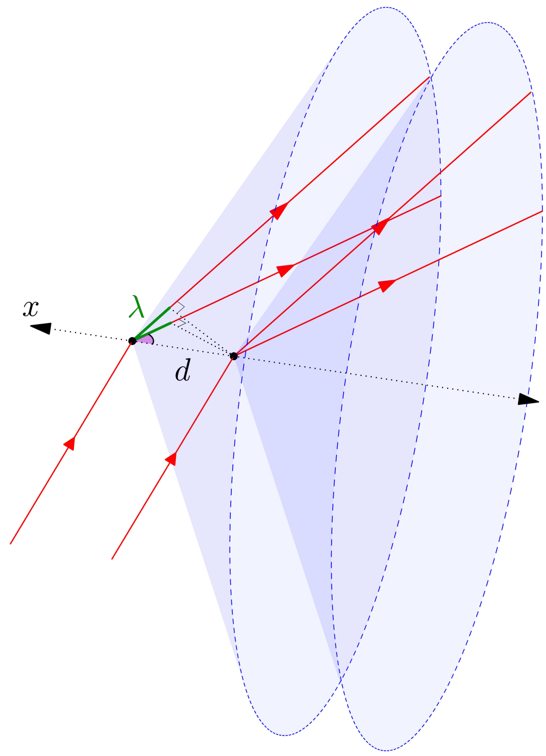


Figure 5: Diffraction in the x direction

Once again returning to Fig. 4, the x direction cosine α_1 is equal to the x'-coordinate of the point formed on the screen by the 1st-order maximum. Fig. 6 below shows the lower half of the circle formed on the screen by the cone along which the diffracted maxima lie, with the red dot being the 1st-order maximum. Recall the radius of this circle was determined to be $\cos\theta$ above.

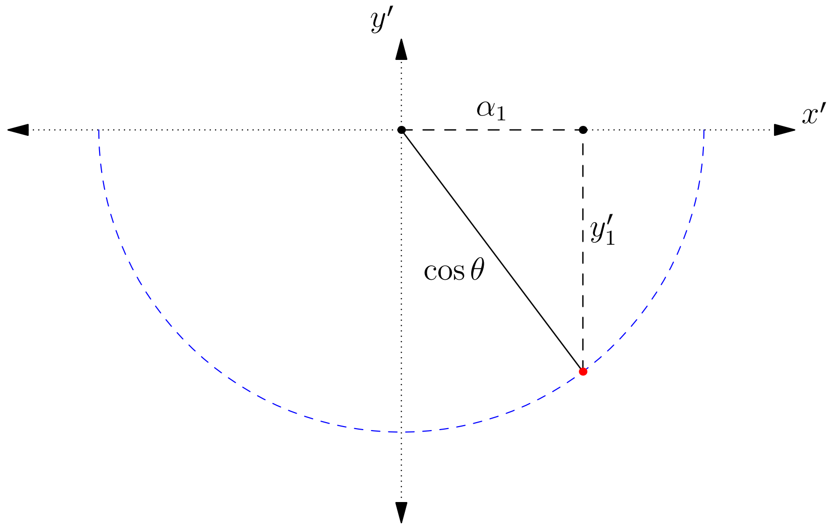


Figure 6: y'-coordinate of 1st-order maxima on the tilted screen

By the pythagorean theorem, we have $y_1' = \sqrt{\cos^2\theta - \alpha_1^2}$. Returning to Fig. 4, we can calculate the difference in height between the center of the circle on the screen and the 1st-order maxima on the screen as $\Delta y = y_1' \cos\left(\frac{\pi}{2} - \theta\right) = y_1' \sin\theta$. The y-coordinate of the center of the circle on the screen is the radius of the circle multiplied by $\cos\left(\frac{\pi}{2} - \theta\right)$ or $\sin\theta$, which is $\cos\theta\sin\theta$. Therefore, the y-coordinate of the 1st-order maxima on the tilted screen is

$$\begin{aligned}\cos\theta\sin\theta - \Delta y &= \cos\theta\sin\theta - y_1'\sin\theta \\ &= \cos\theta\sin\theta - \sin\theta\sqrt{\cos^2\theta - \alpha_1}.\end{aligned} \tag{2}$$

As explained above, since the distance of the light ray is 1, this value is equal to the y direction cosine. Finally, substituting equation (1) into this result, we get

$$\beta_1 = \sin\theta\cos\theta - \sin\theta\sqrt{\cos^2\theta - \frac{\lambda^2}{d^2}}. \tag{3}$$

The domain of this function only goes up to $\arccos\left(\frac{\lambda}{d}\right)$ (for $\theta < \frac{\pi}{2}$). This is because, when the angle θ is larger than $\arccos\left(\frac{\lambda}{d}\right)$, the inside of the square root in the expression for β_1 becomes negative[1]. Thus, there are no secondary maxima when $\theta > \arccos\left(\frac{\lambda}{d}\right)$.

3 Method

3.1 Setup

I first found a steady flat surface with some space in front of it. I placed a laser on the surface such that it points towards a vertical wall, more than 1m away from the laser (this will be the screen). I collected data by placing masking tape on the wall and drawing dots on the tape.

Next, I attached two lengths of wood to a diffraction grating parallel to the direction of the slits, as shown in Fig. 7 to the right. I rested the pieces of wood on a clamp with an adjustable height so that the grating is in the path of the laser, as depicted in Fig. 7. If the laser beam was too high or low to pass through the grating, I elevated the laser or grating off of the surface using blocks of wood or a similarly steady and flat object.

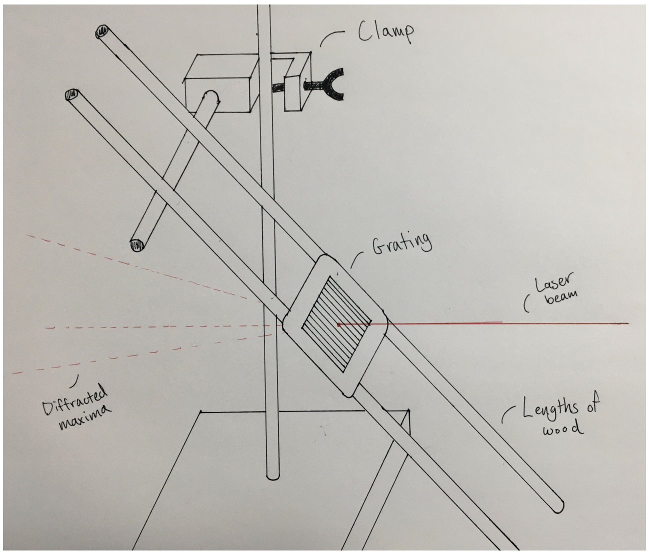
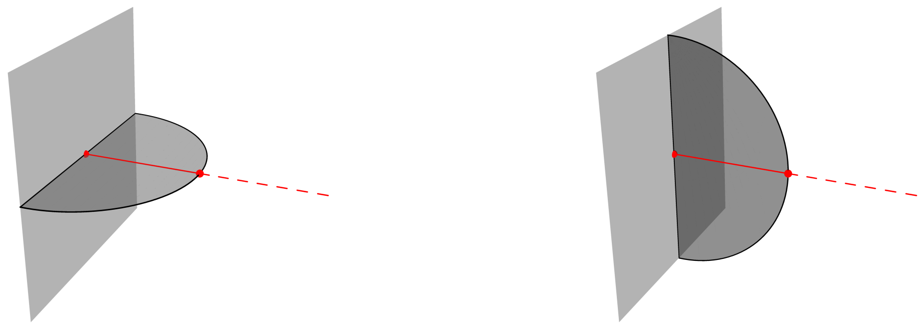


Figure 7: Experimental setup of the tilted diffraction grating

I made sure the laser beam is perpendicular to the screen by the following process. I checked the laser is not rotated about the x- or y-axes. Holding a transparent protractor perpendicular to the screen such that the flat side is parallel to the x-axis and the origin of the protractor lies in the path of the laser, as shown below in Fig. 8a, I confirmed the laser passes through the 90 degree mark. I repeated the process with the flat side of the protractor parallel to the y-axis to check that the laser is not rotated about the x-axis, as illustrated in Fig. 8b.



(a) Rotation about the y-axis

(b) Rotation about the x-axis

Figure 8: One way to verify the laser is perpendicular to the screen

[1]This is not a quirk of the algebra; we can also explain this upper bound using the conical geometry from before. If $\theta > \arccos\left(\frac{\lambda}{d}\right)$, then $\theta = \arccos(k)$ for some value $k < \frac{\lambda}{d}$. Returning to Fig. 6, the radius of the circle is $\cos\theta = k$. However, (1) tells us $\alpha_1 = \frac{\lambda}{d} > k$. This is a contradiction.

3.2 Measurements

I marked the primary maxima and the two 1st-order maxima on the screen and turned off the laser for safety. My measurement process was quite involved, requiring 7 separate measurements per trial.

First, I measured the independent variable – the tilt of the grating (θ) – with a protractor. I measured the angle formed by the bottom edge of one of the wooden beams and the horizontal surface to reduce error. Next, I measured the horizontal distance from the grating to the screen (labelled D in Fig. 9 below). Then, the heights of the primary and 1st-order maxima (h_{-1}, h_0, h_1). Finally, I measured the distance from each 1st-order maxima to the primary maxima (l_{-1}, l_1). Although only the measurements on 1 side are strictly necessary, averaging both sides will give more accurate data.

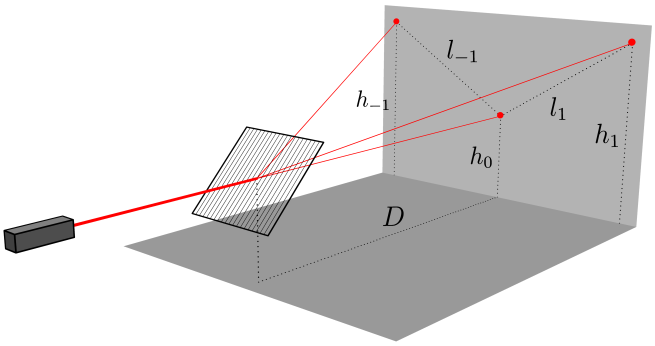


Figure 9: Measurements

My dependent variable, the y direction cosine of the 1st-order maxima, can be calculated by taking the y-components of the two maxima, $h_1 - h_0$ and $h_{-1} - h_0$, and dividing by the total distance the light travels. This total distance can be calculated by using the pythagorean theorem on the right triangle with legs l_1 and D (using l_{-1} for the other maxima). Thus, we have the two equations

$$\beta_1 = \frac{h_1 - h_0}{\sqrt{l_1^2 + D^2}} \tag{4}$$

$$\beta_{-1} = \frac{h_{-1} - h_0}{\sqrt{l_{-1}^2 + D^2}} \tag{5}$$

As mentioned in the introduction, these two values should theoretically be equal, but in reality they could differ by some small amount due to random error caused by some geometrical asymmetry in the setup. Therefore, the dependent variable should be the average of β_1 and β_{-1} for each trial.

I repeated this process with a series of different tilt angles, starting at an angle close to 0 and going up to the point where the diffracted maxima do not consistently appear, which was around 44 degrees. After completing measurements for at least 5 different tilt angles (θ) across this range, I repeated two more trials for each value of θ (I did not conduct trials of the same tilt angle consecutively, as there is likely some amount of random error in the setup). Since I had extra time after collecting data for 5 angles, I conducted trials of more tilt angles to get more data.

3.3 Safety Considerations

The laser I used was a 5 mW laser. The IEC (International Electrotechnical Commission) advises not to actively look directly into this class of laser, but that there is a low risk of injury for fully attentive users with a normal blink reflex [3]. As part of the laser beam will also reflect off of the transmission grating, I made sure nobody is behind the setup when the laser is on, as the reflected light beam could shine into their eyes. As mentioned in the methodology, I turned off the laser when it is not needed to minimize the risk of looking into the laser. There are no other significant safety concerns for my experiment.

4 Results

Appendix B is a table with the raw values of the 7 variables I measured. I used an uncertainty of $\pm 3°$ for the angle of the diffraction grating because the grating material was slightly warped as shown in Fig. 10 below. Depending on where the laser passed through the grating, the tilt of the grating could differ by a significant amount, which I estimated to be around 3°. Next, the secondary maxima often didn't form fully resolved points on the screen, but streaks and blobs of light, as depicted in Fig. 11 below. Whenever the exact location of the maxima was unclear, I assumed the maxima lay on the brightest, most central point of the shape. I estimated the absolute uncertainty in choosing this location for each measurement, recorded in Table 2.

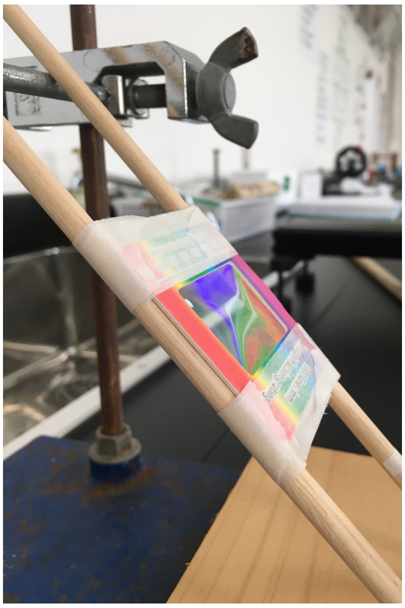


Figure 10: Warped grating

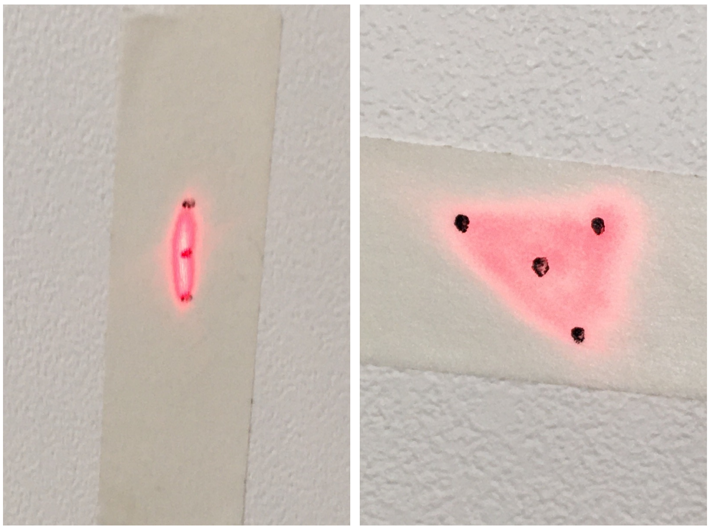


Figure 11: Shapes formed by diffracted maxima

The y-direction cosines for the 1st-order maxima are given by equations (4) and (5). Table 1 shows the average y direction cosine, $\frac{\beta_1+\beta_{-1}}{2}$, for each trial, as well as the average y direction cosine across all three trials and the statistical uncertainty.

I calculated the measurement uncertainties for each value using the rules for propagating uncertainties – adding absolute uncertainties when adding or subtracting values, adding relative uncertainties when multiplying or dividing, and multiplying the relative uncertainty by the exponent when raising values to a power. The full uncertainty calculations are shown in Appendix C. Note that these uncertainties are just the propagated measurement uncertainties for the 7 measured variables, and do not account for uncertainties in the setup (such as the uncertainty in the angle of the incident laser beam). As the statistical uncertainty is equal or greater than the measurement uncertainty in every case, the statistical uncertainty values should be used instead of the measurement uncertainty.

Table 1: Processed data

θ ($\pm 3°$)	Trial 1 β_1	Trial 2 β_1	Trial 3 β_1	Avg. β_1	Stat. Unc.
2	0.017 ± 0.002	0.009 ± 0.003	0.012 ± 0.003	0.013 ± 0.003	± 0.004
7	0.042 ± 0.002	0.035 ± 0.003	0.039 ± 0.003	0.038 ± 0.003	± 0.003
14	0.071 ± 0.003	0.052 ± 0.003	0.048 ± 0.003	0.057 ± 0.003	± 0.01
23	0.155 ± 0.004	0.081 ± 0.003	0.001 ± 0.005	0.079 ± 0.005	± 0.08
32	0.163 ± 0.004	0.131 ± 0.005	0.139 ± 0.005	0.144 ± 0.004	± 0.02
40	0.196 ± 0.004	0.234 ± 0.006	0.214 ± 0.004	0.215 ± 0.005	± 0.02
44	0.315 ± 0.005	0.281 ± 0.005	0.226 ± 0.003	0.274 ± 0.004	± 0.04

The highest value of θ for which I could reliably get both 1st-order maxima to form was around 44 degrees. For my setup, I had the values $d = 10^{-6}$ m and $\lambda = 6.5 \times 10^{-7}$ m (calculated from labels on the grating and the laser), which means the theoretical limit for θ was $\arccos\left(\frac{6.5\times10^{-7}}{10^{-6}}\right) = \arccos(0.65) \approx 49$ degrees. As this is reasonably close to 44 degrees, I felt satisfied with the range of values for θ I recorded.

Fig. 12 shows a plot of the average β_1 data versus the tilt angle θ, including error bars. The figure also shows the plot of equation (3), the theoretical relationship between θ and β_1, with $d = 10^{-6}$ m and $\lambda = 6.5 \times 10^{-7}$ m and thus $\frac{\lambda^2}{d^2} = 0.4225$. Note one of the horizontal uncertainties actually extends into the negative x region of the coordinate plane, because the bend of the grating could have caused the grating to be tilted slightly the other direction in certain spots for small tilt angles.

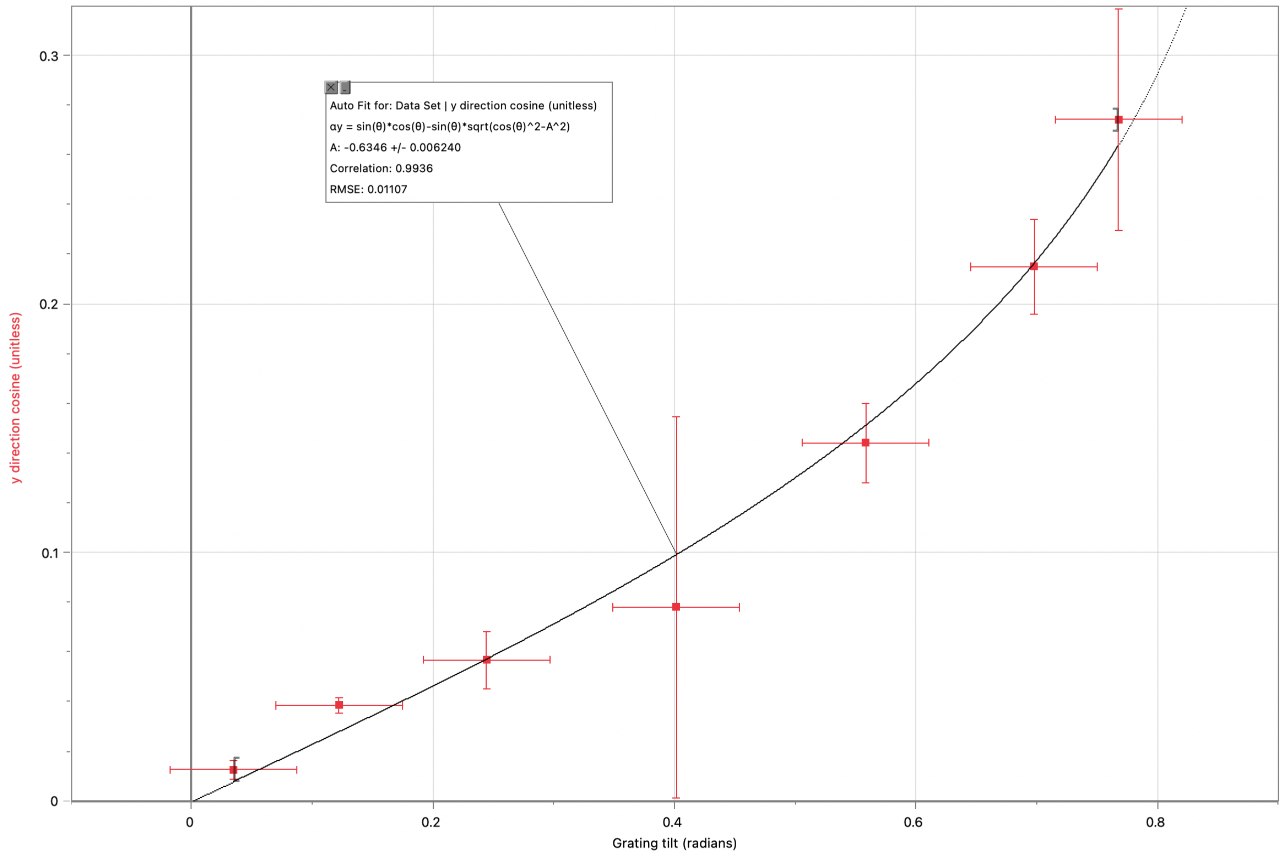


Figure 12: Plot of β_1 vs. θ with theoretical curve $\beta_1 = \sin\theta\cos\theta - \sin\theta\sqrt{\cos^2\theta - 0.4225}$

My data supports the theoretical relationship I derived, as the curve passes through all the error bars. The data point near 0.4 radians seems to have a much larger statistical uncertainty than all the other data points. Due to the specifics of my setup (see Fig. 7), at this angle one of the 1st-order maxima was often obscured by the clamp stand, and I was forced to pass the laser close to the edge of the grating. This could have influenced my data, as the grating was more warped near the edges (see Fig. 10). Thus, I decided to disregard this data point.

I could not linearize this graph because the theoretical relation was too complicated. Instead, I modeled the data points with the function

$$f(x) = \sin x \cos x - \sin x\sqrt{\cos^2 x - A^2}$$

containing a single parameter A. The value of A should theoretically equal the value $\frac{\lambda}{d} = 0.65$. Using a computer regression tool, the value of A that best fits my data is 0.637, which is only 2% off the theoretical value. The purple curve in Fig. 13 below is the graph of $f(x)$ using this value of A. I also found the minimum and maximum values of A such that $f(x)$ passes through all the error bars, depicted by the black curves in Fig. 13.

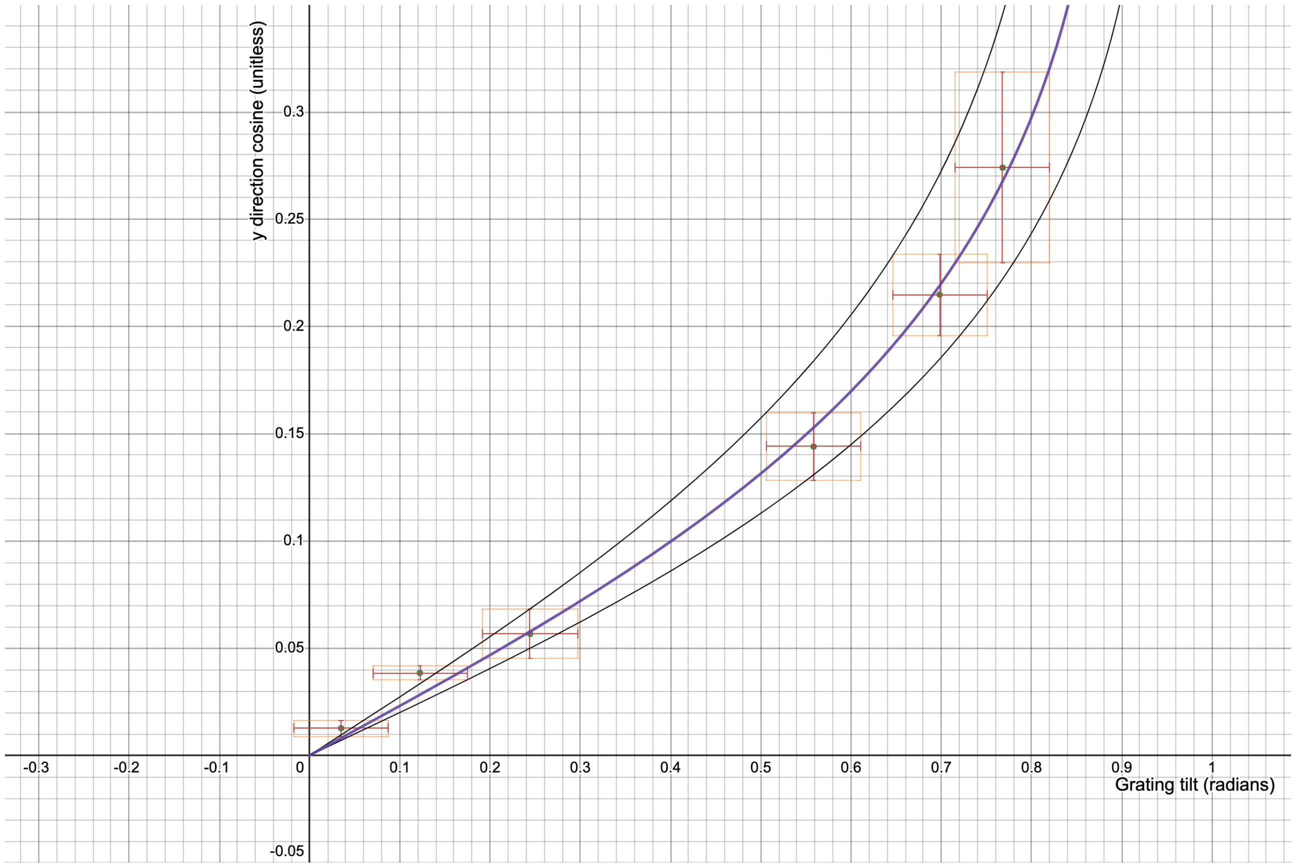


Figure 13: Regression, Minimum, and Maximum curves

The minimum and maximum values of A are 0.598 and 0.684, respectively. This means the average value of A is 0.641 with a statistical uncertainty of 0.043. When rounded to the appropriate number of significant figures, we find that $A = 0.64 \pm 0.04$. This range includes the theoretical value of 0.65, so this again supports my hypothesis that the tilt angle of the grating and the y direction cosine are related by equation (3).

Even excluding the data point near 0.4 radians, my data is not very precise, as each data point has relatively large error bars. The relative uncertainties range from 7% to 150% for θ, and 8% to 31% for β_1. Especially for smaller values of θ, these error bars are large. The predicted value of $\frac{\lambda}{d}$ from the minimum and maximum curves has a lower, but still significant relative uncertainty of 7%. This imprecision was a result of the difficulty of replicating the exact same setup across trials, as well as the warped grating surface. However, my data is accurate, as the theoretical curve passes through all the error bars. There is no evidence in my data to suggest a systematic error.

5 Evaluation

My method was reasonably effective at limiting random error for the equipment that I had available, but there are a number of ways it could be improved. The main sources of random error were likely the exact angle of the grating and the initial angle of the laser beam. Ideally, the grating should not be rotated about the y-axis at all, but there was likely some sideways tilt due to imperfections in the grating surface and the lengths of wood attached to the grating. I taped the wood tightly to the frame of the grating (without bending it), as depicted in Fig. 10, but there could have been some wiggle room for the cylindrical lengths. Fig. 14 demonstrates how the side tilt of the grating impacts the x-direction cosine, which in turn would impact the y-direction cosine. The diagram shows a unit sphere around the point the laser passes through the grating, so that the x-component (the cyan length) of the 1st-order diffracted light vector is its x direction cosine. The diagram is from a different perspective than the other diagrams in this paper – the grating lies in the horizontal plane (the plane with the full circle), and we are looking straight down the lines of the grating (I did not include the grating in the diagram as it was too cluttered). Since the grating has a slight sideways tilt, the x-axis is not perpendicular to the lines of the grating. We can also see the light has a sideways tilt; the primary maxima does not intersect the sphere in the center (marked by the black vertical dotted line) but slightly to the right.

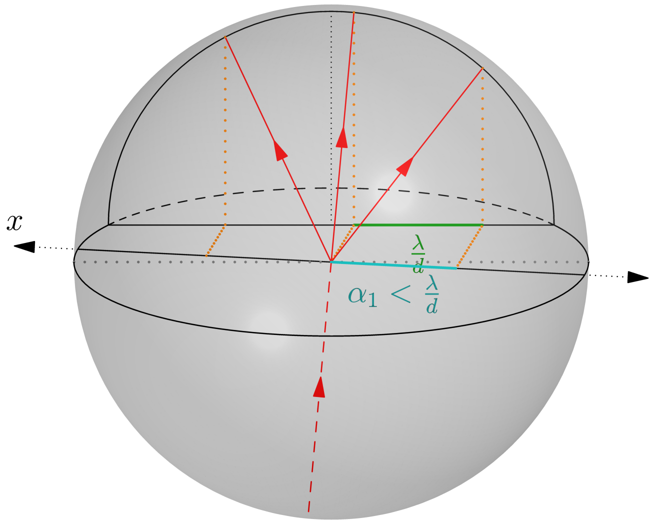


Figure 14: Difference in α_1 caused by side tilt of grating

The sideways (not the x direction, but the direction perpendicular to the slits of the grating, shown by the dotted diameter of the full circle) direction cosine of the diffracted light always has a difference of $\frac{\lambda}{d}$ from the direction cosine of the primary maxima, as labeled. This means the x direction cosine of the 1st-order maxima, α_1, is less than $\frac{\lambda}{d}$. From equation (2), we can see that a decrease in α_1 results in an decrease in β_1, so the measured y direction cosines would be smaller than the theoretical values if the grating is tilted sideways.

To better limit this error, I could have used a better diffraction grating which was not warped. The frame of my grating was also made of a material similar to construction paper, so using a grating with a more rigid frame could have kept the angle of the grating more consistent. My method of using the wooden lengths to prop up the grating measure its tilt worked well, but using rectangular prisms instead of cylinders could have been even more precise, as rectangular lengths could fit flush against the frame of the grating. I should have made sure the lengths were identical in length, so that when they are placed on a flat surface at an angle (Fig. 7), they are perfectly symmetrical and do not induce any sideways tilt. Finally, I could have made sure the arm of the clamp which held up these lengths was parallel to the screen and perpendicular to the laser, such that the lengths and thus the grating were not tilted sideways.

The other major source of random error was the initial angle of the laser beam. I did implement some measures to limit this error as much as possible, as explained in the methodology. However, the laser beam was sometimes difficult to see on the protractor, and thus I could only confirm that the laser was within $\pm 1°$ of being perfectly perpendicular to the screen in both the x- and y- directions. This error was extremely difficult to propagate because there are a plethora of issues which arise with my methodology once we drop the assumption that the laser is perfectly perpendicular to the screen. For example, using pythagorean theorem with l_1 and D no longer works if the laser is not at a right angle to the screen. I couldn't propagate the error caused by the angle of the laser, or even say which angles would cause a positive or negative change in the measured y direction cosine. Instead, I let this error be described by the statistical uncertainty measured across the three trials I conducted for each value of θ. Ideally, I would have conducted more trials to get a better estimate of the uncertainty for each data point, but I determined that gathering more data points was more valuable than increasing my precision for answering my research question.

One way I could have limited the error caused by the tilt of the laser would have been to use a laser stand instead of just placing it on top of wooden blocks, so the angle of the laser would be steadier and easier to manipulate. Alternatively, I could have fixed the grating in place relative to the screen and moved the laser around to measure different angles of incidence on the grating. Then, the angle of the laser could be determined by the position of the primary maxima on the screen, which could be more accurate in measuring the angle of incidence. However, this methodology has other sources of random error, such as the uncertainty in the position and angle of the grating relative to the screen. It is also more difficult to hold the laser steady at an arbitrary angle high off the ground. It is challenging to predict which method would produce more accurate results beforehand, so I used the method which was easier to set up.

References

[1] Salvatore Ganci. Fourier diffraction through a tilted slit. *European Journal of Physics*, 2(3):158–160, nov 1981.

[2] James E. Harvey and Cynthia L. Vernold. Description of diffraction grating behavior in direction cosine space. *Applied Optics*, 37(34):8158–8159, dec 1998.

[3] International Electrotechnical Commission. *Safety of laser products - Part 1: Equipment classification and requirements*, 3.0 edition, may 2014.

Appendices

A Phase shift of light rays passing through the same slit

As mentioned in footnote 1, the green segments in Fig. 15 do not have to be equal as stated in the paper, but could differ by a multiple of the wavelength, $m\lambda$, and still constructively interfere. However, for *all* the light passing through the slit to constructively interfere, the diffracted angle must be independent of s, so that *any* two rays passing through the slit will be in phase. As shown in the paper, the top green segment has a length of $s \sin\theta$ and the bottom green segment has a length of $s \cos\gamma$. Therefore, if the path length of the two rays is not equal, we can write

$$s \sin\theta = s \cos\gamma + m\lambda$$

for some nonzero integer m. Rearranging for γ, we get

$$\gamma = \arccos\left(\sin\theta - \frac{m\lambda}{s}\right).$$

We can see that γ depends on the length s, so all the light will not constructively interfere. Thus, the two paths cannot differ by a multiple of λ, but instead must be equal.

B Raw Data Table

Table 2: Raw Data

θ ($\pm 3°$)	D (cm)	h_{-1} (cm)	h_0 (± 0.1 cm)	h_1 (cm)	l_{-1} (cm)	l_1 (cm)
2	65.1 ± 0.1	13.6 ± 0.1	12.4	14.1 ± 0.1	54.2 ± 0.1	58.3 ± 0.1
7	65.7 ± 0.1	15.4 ± 0.1	12.4	16.6 ± 0.1	57.2 ± 0.1	55.5 ± 0.1
14	66.8 ± 0.2	17.1 ± 0.2	10.7	16.8 ± 0.1	60.9 ± 0.2	55.2 ± 0.1
23	62.6 ± 0.2	26.4 ± 0.2	14.3	28.3 ± 0.2	55.9 ± 0.2	57.1 ± 0.2
32	63.8 ± 0.2	24.7 ± 0.2	12.3	27.9 ± 0.1	57.7 ± 0.2	58.0 ± 0.3
40	66.5 ± 0.4	26.7 ± 0.2	10.8	30.5 ± 0.2	60.8 ± 0.2	62.6 ± 0.2
44	68.0 ± 0.4	38.4 ± 0.2	10.7	45.0 ± 0.3	66.7 ± 0.2	74.6 ± 0.2
2	61.2 ± 0.1	12.9 ± 0.1	12.3	13.2 ± 0.1	50.6 ± 0.1	54.7 ± 0.1
7	61.5 ± 0.1	14.7 ± 0.1	12.4	15.8 ± 0.1	52.6 ± 0.1	53.7 ± 0.1
14	62.3 ± 0.2	16.0 ± 0.1	12.5	17.5 ± 0.1	55.7 ± 0.2	51.8 ± 0.2
23	62.6 ± 0.1	16.7 ± 0.3	11.0	18.7 ± 0.3	53.6 ± 0.3	55.1 ± 0.2
32	63.7 ± 0.2	22.0 ± 0.1	12.5	25.3 ± 0.1	55.6 ± 0.1	57.8 ± 0.2
40	66.9 ± 0.4	33.8 ± 0.3	10.8	31.3 ± 0.3	66.6 ± 0.4	62.0 ± 0.2
44	66.9 ± 0.4	34.7 ± 0.3	12.4	44.1 ± 0.3	65.8 ± 0.2	71.4 ± 0.2
2	60.9 ± 0.1	13.3 ± 0.1	12.4	13.4 ± 0.1	51.1 ± 0.1	53.6 ± 0.1
7	61.7 ± 0.1	15.9 ± 0.1	12.7	15.8 ± 0.1	55.0 ± 0.1	51.1 ± 0.1
14	61.7 ± 0.1	16.3 ± 0.1	11.6	14.8 ± 0.1	60.2 ± 0.1	48.9 ± 0.1
23	62.5 ± 0.2	11.6 ± 0.2	13.7	16.0 ± 0.3	54.0 ± 0.2	55.5 ± 0.1
32	63.9 ± 0.2	22.9 ± 0.3	12.5	26.0 ± 0.3	55.7 ± 0.3	59.1 ± 0.2
40	65.4 ± 0.3	28.0 ± 0.2	12.5	35.4 ± 0.2	60.9 ± 0.2	61.4 ± 0.2
44	66.8 ± 0.4	28.8 ± 0.1	10.7	34.4 ± 0.1	61.0 ± 0.1	66.0 ± 0.1

C Propagation of measurement uncertainties

The formula for the direction cosine was

$$\beta_1 = \frac{h_1 - h_0}{\sqrt{l_1^2 + D^2}}.$$

Let Δx represent the absolute uncertainty for the variable x. The relative uncertainty of x is then $\frac{\Delta(x)}{x}$.

The numerator of the expression for β_1 has an absolute uncertainty of $\Delta h_1 + \Delta h_0$, which means it has a relative uncertainty of

$$\frac{\Delta h_1 + \Delta h_0}{h_1 - h_0}.$$

Next, the denominator. The relative uncertainty of l_1^2 is calculated by doubling the relative uncertainty of l_1, which is $2\frac{\Delta l_1}{l_1}$. Multiplying by l_1^2 gives the absolute uncertainty, $2l_1 \cdot \Delta l_1$. Similarly, the absolute uncertainty of D^2 is $2D \cdot \Delta D$. The absolute uncertainty of $l_1^2 + D^2$ is thus the sum $2l_1 \cdot \Delta l_1 + 2D \cdot \Delta D$, implying that the relative uncertainty of $l_1^2 + D^2$ is

$$\frac{2l_1 \cdot \Delta l_1 + 2D \cdot \Delta D}{l_1^2 + D^2}.$$

Since taking the square root is the same as raising to the power of $\frac{1}{2}$, the relative uncertainty of the denominator is half this value:

$$\frac{l_1 \cdot \Delta l_1 + D \cdot \Delta D}{l_1^2 + D^2}.$$

Finally, the relative uncertainty of the whole expression is equal to the sum of the relative uncertainties of the numerator and the denominator, which is

$$\frac{\Delta h_1 + \Delta h_0}{h_1 - h_0} + \frac{l_1 \cdot \Delta l_1 + D \cdot \Delta D}{l_1^2 + D^2}.$$

The absolute uncertainty is thus given by

$$\Delta\beta_1 = \beta_1 \left(\frac{\Delta h_1 + \Delta h_0}{h_1 - h_0} + \frac{l_1 \cdot \Delta l_1 + D \cdot \Delta D}{l_1^2 + D^2} \right).$$

Using this formula in google sheets, I was able to calculate the measurement uncertainty of β_1 for every trial. When taking the average of β_{-1} and β_1, I used the same rules to get an uncertainty of

$$\frac{\Delta\beta_{-1} + \Delta\beta_1}{2}.$$

D Explanation of the Conical Geometry of the Diffracted Maxima Using Path Length Difference

Ganci (1981) [1] mentions that light passing through a single slit also diffracts in a conical pattern, so I started by focusing on one slit in the grating. Fig. 15 shows two light rays passing through the same slit in the grating, and two possible paths each light ray could follow after passing through the slit. The cyan segment represents the path length difference of the two rays before passing through the slit, and the green segment is the path length difference after passing through the slit.

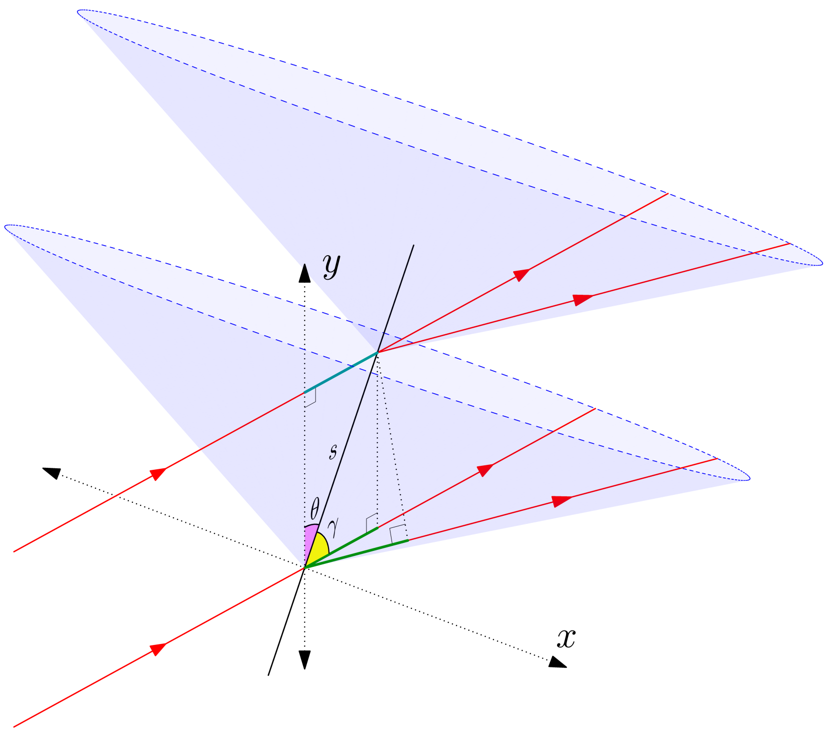


Figure 15: Two rays passing through the same slit

As long as the green segment is equal to the cyan segment[2], the two light rays will be in phase and constructively interfere. We can imagine that if the rays pass straight through the slit without diffracting, then the cyan length and the green length will be equal. Intuitively, rotating the outbound light rays about the slit (forming the blue cones) will keep the length of the green segment the same. More rigorously, let γ represent the angle between the outbound rays and the slit, and let s be the (very small) distance between the points the two rays pass through the slit as labeled above. Also let θ represent the tilt angle of the slit, as before. The green segment of the lower ray is then $s\cos\gamma$, and the cyan segment of the upper ray is $s\sin\theta$. Since these must be equal, we have $s\cos\gamma = s\sin\theta$. Dividing by s yields $\cos\gamma = \sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$. Therefore,

$$\gamma = \frac{\pi}{2} - \theta.$$

This constraint forms the blue cones in the figure, representing all directions the diffracted rays can travel such that they constructively interfere. In the far field, since s is very small, these cones become the single cone along which the diffracted maxima lie.

[2]The lengths of the cyan and green segments can also differ by a multiple of the wavelength, $m\lambda$, and still constructively interfere. However, we can show that it is impossible for the rays to constructively interfere *for all values of* s when this is the case (Appendix A).