# 1 Sets

## 1.1 Basic Definitions

$\mathbb{N} = \{0, 1, 2, \dots\}$ - natural numbers
$\mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$ - integers
$\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$ - rationals
$\mathbb{R}$ - real numbers
$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$ - complex numbers
$i^2 = -1$ (imaginary unit)
$\mathbb{R}^n = \{(x_1, \dots, x_n) : x_i \in \mathbb{R}\}$ - $n$-dimensional space
$\mathbb{R}^\infty = \{(x_1, x_2, \dots) : x_i \in \mathbb{R}\}$ - space of infinite sequences of reals

## 1.2 Set Operations

*Union*: $A \cup B = \{x : x \in A \text{ or } x \in B\}$
*Intersection*: $A \cap B = \{x : x \in A \text{ and } x \in B\}$
*Difference*: $A \setminus B = \{x \in A : x \notin B\}$
*Symmetric difference*: $(A \setminus B) \cup (B \setminus A)$

## 1.3 Examples

Let $\{a_n\}_{n \in \mathbb{N}}$ be an indexed family of sets (for $n \in \mathbb{N}$).

$$\bigcup_{n \in \mathbb{N}} A_n = \{x : x \in A_n \text{ for some } n \in \mathbb{N}\}$$
$$\bigcap_{n \in \mathbb{N}} A_n = \{x : x \in A_n \text{ for every } n \in \mathbb{N}\}$$

*Note*: If $\exists n_0 \in \mathbb{N} : A_n = \emptyset$ (empty set), then $\bigcap_{n \in \mathbb{N}} A_n = \emptyset$.

## 1.4 Functions

A *function* is a mapping or transformation.
Let $A, B$ be sets. $A \xrightarrow{f} B$
$f : A \to B$ is a rule that assigns to each $a \in A$ a unique $b \in B$.
$\Delta$ is called the *domain* (by definition) of $f$.
$B$ is called the *codomain* or *target space* of $f$.

## 1.5 Image and Preimage

If $A' \subset A$, then the *image* of $A'$ under $f$ is:

$$f(A') = \{f(x) : x \in A'\}$$

This is called the *restriction* of $f$ to $A'$ (denoted by $f|_{A'}$).
If $B' \subset B$, then the *preimage* of $B'$ under $f$ is:

$$f^{-1}(B') = \{x \in A : f(x) \in B'\}$$

## 1.6 Examples

1. $f : \mathbb{R} \to \mathbb{R},\ x \mapsto x^2$
2. $f : \mathbb{R}^+ \to \mathbb{R},\ x \mapsto \log x$
3. $g : \mathbb{R} \to \mathbb{R},\ x \mapsto e^x$
4. $h : \mathbb{R} \to \mathbb{R},\ x \mapsto \sin x$
5. $f : \mathbb{C} \to \mathbb{C},\ z \mapsto z^2$
6. $D : C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R}),\ f \mapsto f'$ (derivative)