1 Set Theory

1.1 Basic Sets

$$\begin{aligned}
\mathbb{N} &= \{0, 1, 2, \ldots\} && \textit{natural numbers} \\
\mathbb{Z} &= \{\ldots, -1, 0, 1, \ldots\} && \textit{integers} \\
\mathbb{Q} &= \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\} && \textit{rationals} \\
\mathbb{R} &= \textit{real numbers} \\
\mathbb{C} &= \{a + bi : a, b \in \mathbb{R}\} && \textit{complex numbers}
\end{aligned}$$

1.2 Set Operations

- $\mathbb{R}^* = \{x \in \mathbb{R} : x \neq 0\}$ *(non-zero reals)*
- $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$ *(positive reals)*

1.3 Set Relations

$$\begin{aligned}
A \cup B &= \{x : x \in A \text{ or } x \in B\} \\
A \cap B &= \{x : x \in A \text{ and } x \in B\}
\end{aligned}$$

Venn diagrams are provided for visual representation.

$$\begin{aligned}
A \setminus B &= \{x \in A : x \notin B\} \\
A \triangle B &= (A \cup B) \setminus (A \cap B)
\end{aligned}$$

2 Functions

Let $(a_n)_{n \in \mathbb{N}}$ be an infinite sequence (where $n \in \mathbb{N}$).

$$\begin{aligned}
\text{Ex 1:} \quad I &: \mathbb{N} \to \mathbb{N} \\
A_n &= \{n, n+1\}
\end{aligned}$$

Definition

$$\begin{aligned}
\text{Ex 2:} \quad I &: \mathbb{R}^* \to \{x \in \mathbb{R} : x > 0\} \\
A_x &= \{x, -x\}
\end{aligned}$$

Domain and codomain

$$\begin{aligned}
\bigcup_{n \in \mathbb{N}} A_n &= \{n : n \in \mathbb{N} \text{ for some } n \in \mathbb{N}\} \\
\bigcap_{n \in \mathbb{N}} A_n &= \{n : n \in A_n \text{ for every } n \in \mathbb{N}\}
\end{aligned}$$

These are *countable* operations.
Any $f : A \to B$ is a subset (not $\in$) of $A \times B$ satisfying:

- $\forall a \in A, \exists b \in B : (a, b) \in f$
- If $(a, b_1) \in f$ and $(a, b_2) \in f$, then $b_1 = b_2$

Note: If $\forall x \in X \exists A : x \in A$, then $X \subseteq A$.
f is called the *graph* of the function (by definition).
If $A' \subseteq A$, then $f|_{A'} : A' \to B$ is called the *restriction* of f to A'.

3 Function Properties

If $A' \subseteq A$, then the *image* of A' under $f : A \to B$ is:

$$f(A') = \{f(a) : a \in A'\}$$

This is called the *restriction* of f to A' (denoted by $f|_{A'}$).

1. $f : \mathbb{R} \to \mathbb{R}$
2. $f : \{x\} \to \{y\}$
3. $f : A \setminus \{x\} \to B \setminus \{y\}$
4. $g : B \to C$
5. $f : \frac{1}{x} \mapsto \frac{1}{x^2}$
6. $D : f(x) = 2f'(x)$