1 Sets and Number Systems

1.1 Definitions

$\mathbb{N} = \{0, 1, 2, \ldots\}$: natural numbers
$\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$: integers
$\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\}$: rationals
$\mathbb{R}$: real numbers
$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$: complex numbers
$i^2 = -1$ (imaginary unit)
$\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_i \in \mathbb{R}\}$: *n-dimensional space*
$\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$: positive real numbers

1.2 Set Operations

$A \cup B = \{x : x \in A \text{ or } x \in B\}$: union
$A \cap B = \{x : x \in A \text{ and } x \in B\}$: intersection
$A \setminus B = \{x \in A : x \notin B\}$: *set difference*
$A \oplus B = (A \cup B) \setminus (A \cap B)$: *symmetric difference*

1.3 Intervals

$(a, b) = \{x \in \mathbb{R} : a < x < b\}$
$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$

1.4 Set Builder Notation

$$\bigcup_{n \in \mathbb{N}} A_n = \{x : x \in A_n \text{ for some } n \in \mathbb{N}\}$$
$$\bigcap_{n \in \mathbb{N}} A_n = \{x : x \in A_n \text{ for every } n \in \mathbb{N}\}$$

2 Functions

A function f is a rule that assigns to each element x in a set A exactly one element y in a set B.

2.1 Notation

$f : A \to B$
$x \mapsto f(x)$
f is *surjective* if $\forall y \in B, \exists x \in A : f(x) = y$
f is *injective* if $\forall x_1, x_2 \in A, f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
f is *bijective* if it is both surjective and injective

2.2 Inverse Function

If $f : A \to B$ is bijective, then the inverse function $f^{-1} : B \to A$ exists.

2.3 Composition of Functions

If $f : A \to B$ and $g : B \to C$, then the composition $g \circ f : A \to C$ is defined by $(g \circ f)(x) = g(f(x))$.

2.4 Examples

1. $f : \mathbb{R} \to \mathbb{R}, x \mapsto x^2$ 2. $f : \mathbb{R}^+ \to \mathbb{R}, x \mapsto \log x$ 3. $g : \mathbb{R} \to \mathbb{R}, x \mapsto e^x$ 4. $f : \mathbb{R} \to \mathbb{R}, x \mapsto \sin x$ 5. $f : \mathbb{Z} \to \mathbb{Z}, n \mapsto n^2$