1 Sets

1.1 Definitions

$$
\begin{aligned}
\mathbb{N} &= \{0, 1, 2, \ldots\} && \text{natural numbers} \\
\mathbb{Z} &= \{\ldots, -1, 0, 1, \ldots\} && \text{integers} \\
\mathbb{Q} &= \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\} && \text{rationals} \\
\mathbb{R} &= \text{real numbers} \\
\mathbb{C} &= \{a + bi : a, b \in \mathbb{R}\} && \text{complex numbers}
\end{aligned}
$$

$\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_i \in \mathbb{R}\}$ *n-dimensional space*
$\mathbb{R}^* = \{(x_1, x_2, \ldots) : x_i \in \mathbb{R}\}$ *infinite dimensional space*

1.2 Set Operations

$A \cup B = \{x : x \in A \text{ or } x \in B\}$ *union*
$A \cap B = \{x : x \in A \text{ and } x \in B\}$ *intersection*
$A \setminus B = \{x \in A : x \notin B\}$ *set difference*
$A \triangle B = (A \setminus B) \cup (B \setminus A)$ *symmetric difference*

1.3 Venn Diagrams

[Venn diagrams would be inserted here]

2 Functions

Let $(f_n)_{n \in \mathbb{N}}$ be an indexed family of functions (where $n \in \mathbb{N}$).

$$
\begin{aligned}
\text{Ex (1)} \quad & f : \mathbb{N} \to \mathbb{N} \\
& A_n = \{k, n+1\} \\
& \text{definition}
\end{aligned}
$$

$$
\begin{aligned}
(2) \quad & f : \mathbb{R}^* \to \{x \in \mathbb{R} : x \geq 0\} \\
& A_n = (x_n, \infty)
\end{aligned}
$$

$$
\begin{aligned}
\textstyle\bigcup_{n \in \mathbb{N}} A_n &= \{x : x \in A_n \text{ for some } n \in \mathbb{N}\} \\
\textstyle\bigcap_{n \in \mathbb{N}} A_n &= \{x : x \in A_n \text{ for every } n \in \mathbb{N}\}
\end{aligned}
$$

These are called *union* and *intersection* of a family of sets $(A_n)_{n \in \mathbb{N}}$.

Let A, B be sets ($A \neq B$). A map $f : A \to B$ is called a *function* or *mapping* or *transformation*.

If $A \subseteq B$ and $f : A \to B$, then f is called an *embedding* of A into B if f is injective.

Note: f maps A onto $B \Leftrightarrow f$ is surjective

If $f : A \to B$ is both injective and surjective, then f is called *bijective* (or *one-to-one*).

f^{-1} is called the *inverse* (if it exists) of f.

If $A' \subseteq A$, then the map $f' : A' \to B$ defined by $f'(x) = f(x)$ is called the *restriction* of f to A' (denoted by $f|_{A'}$).

Ex (1) $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto x^3$

(2) $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$, $x \mapsto \frac{1}{x}$

(3) $g : \mathbb{R} \to \mathbb{R}$, $x \mapsto e^x$

(4) $f : \mathbb{C} \to \mathbb{C}$, $z \mapsto z^2$

(5) $f : \mathbb{R}^2 \to \mathbb{R}$, $(x, y) \mapsto x + y$

(6) $D : C^1(\mathbb{R}) \to C(\mathbb{R})$, $f \mapsto f'$ *derivative*