1 Sets

1.1 Basic Definitions

$\mathbb{N} = \{0, 1, 2, \ldots\}$ *natural numbers*
$\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$ *integers*
$\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$ *rationals*
$\mathbb{R}$ - *real numbers*
$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$ *complex*
$\mathbb{R}^2$ - *2-dimensional real plane*
$\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_i \in \mathbb{R}\}$ *n-dimensional space*
$\mathbb{R}^\infty = \{(x_1, x_2, \ldots) : x_i \in \mathbb{R}\}$ *space of infinite sequences of reals*

1.2 Set Operations

$A \cup B = \{x : x \in A \text{ or } x \in B\}$ *union*
$A \cap B = \{x : x \in A \text{ and } x \in B\}$ *intersection*
$A \setminus B = \{x \in A : x \notin B\}$ *set difference*
$A \triangle B = (A \setminus B) \cup (B \setminus A)$ *symmetric difference*

1.3 Examples

Let $A_n = \{x_1, \ldots, x_n\}$ be an indexed family (where $n \in \mathbb{N}$).

$$\bigcup_{i=1}^{n} A_i = \{x : x \in A_i \text{ for some } i \in \mathbb{N}\}$$
$$\bigcap_{i=1}^{n} A_i = \{x : x \in A_i \text{ for every } i \in \mathbb{N}\}$$

These are generalized to arbitrary indexed families.

2 Functions

A function $f : A \to B$ is a mapping that assigns to each $a \in A$ a unique $b \in B$.
$A_f = \{(a, f(a)) : a \in A\}$ is the *graph* of f.
If $f : A \to B$ is a function, then:

- f is *injective* (1-1) if $\forall a_1, a_2 \in A, f(a_1) = f(a_2) \Rightarrow a_1 = a_2$
- f is *surjective* (onto) if $\forall b \in B, \exists a \in A : f(a) = b$
- f is *bijective* if it is both injective and surjective

If $f : A \to B$ is bijective, then there exists a unique function $f^{-1} : B \to A$ called the *inverse of f* (denoted by $f^{-1} = f_{inv}$).

2.1 Examples of Functions

1. $f : \mathbb{R} \to \mathbb{R}, x \mapsto x^2$
2. $f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto x + y$
3. $g : \mathbb{R} \to \mathbb{R}, x \mapsto e^x$
4. $h : \mathbb{R} \to \mathbb{R}, x \mapsto \sin x$
5. $f : \mathbb{C} \to \mathbb{C}, z \mapsto z^2 + 1$