1 Sets

1.1 Definitions

$N = \{0, 1, 2, \ldots\}$ natural numbers
$Z = \{\ldots, -1, 0, 1, \ldots\}$ integers
$Q = \{\frac{a}{b} : a, b \in Z, b \neq 0\}$ rationals
$R =$ real numbers
$C = \{a + bi : a, b \in R\}$ complex numbers
$i = \sqrt{-1}$ imaginary unit
$R^n = \{(x_1, \ldots, x_n) : x_i \in R\}$ n-dimensional space
$R^\infty = \{(x_1, x_2, \ldots) : x_i \in R\}$ space of infinite sequences of reals

1.2 Operations

Union: $A \cup B = \{x : x \in A \text{ or } x \in B\}$
Intersection: $A \cap B = \{x : x \in A \text{ and } x \in B\}$
Difference: $A \setminus B = \{x \in A : x \notin B\}$
Symmetric difference: $(A \cup B) \setminus (A \cap B)$

2 Functions

Let $(A_n)_{n \in N}$ be an indexed family of sets (where $n \in N$).

$$\bigcup_{n \in N} A_n = \{x : x \in A_n \text{ for some } n \in N\}$$
$$\bigcap_{n \in N} A_n = \{x : x \in A_n \text{ for every } n \in N\}$$

These are called *union* and *intersection* of an indexed family.

Definition: $A_1 \to A_2$ is the set of all functions mapping from A_1 to A_2.

If $A \xrightarrow{f} B$, then f is an *assignment* of $V \in B$ to every $a \in A$, denoted by $f(a)$.

Note: If $\forall a \in A$, there exists exactly one $b \in B$ such that $(a, b) \in f$, then f is called a *function* (by definition).

f is called the *domain* (by definition) of f.

B is called the *codomain* or *target space* of f.

Definition: If $A' \subseteq A$, then the *image* of A' under f (denoted by $f(A')$) is called the *restriction* of f to A'.

2.1 Examples

1. $f : R \to R, x \mapsto x^2$
2. $f : R[X] \to R, p \mapsto p(0)$
3. $g : R \to R, x \mapsto \frac{1}{x}$
4. $f : V \to V^*$
5. $f : V \xrightarrow{\sim} V^{**}$
6. $D : C^\infty(R) \to C^\infty(R), f \mapsto f'$