1 Set Theory

1.1 Sets and Notations

- $\mathbb{N} = \{0, 1, 2, \ldots\}$ - natural numbers
- $\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$ - integers
- $\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\}$ - rationals
- $\mathbb{R}$ - real numbers
- $\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$ - complex numbers

1.2 Set Operations

- $\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_i \in \mathbb{R}\}$ - n-dimensional space
- $\mathbb{R}^\infty = \{(x_1, x_2, \ldots) : x_i \in \mathbb{R}\}$ - space of infinite sequences of reals

1.3 Set Relations

- $A \cup B = \{x : x \in A \text{ or } x \in B\}$ - union
- $A \cap B = \{x : x \in A \text{ and } x \in B\}$ - intersection
- $A \setminus B = \{x \in A : x \notin B\}$ - difference
- $A \triangle B = (A \setminus B) \cup (B \setminus A)$ - symmetric difference

1.4 Properties

Let $\{A_\alpha\}_{\alpha \in I}$ be an indexed family of sets. Then:

1. $I \subseteq \mathbb{N}$
2. $A_\alpha = \{x, y, z\}$
3. $I = \mathbb{R}^+$, $A_\alpha = [0, \alpha]$

Define:

$$\bigcup_{\alpha \in I} A_\alpha = \{x : x \in A_\alpha \text{ for some } \alpha \in I\}$$
$$\bigcap_{\alpha \in I} A_\alpha = \{x : x \in A_\alpha \text{ for every } \alpha \in I\}$$

1.5 Functions

A *function* is a mapping or transformation.

Let A, B be sets. $f : A \to B$ means:

- f assigns to each $a \in A$ a unique $b \in B$
- $a \mapsto b$ or $f(a) = b$

f is *surjective* if $\forall b \in B, \exists a \in A$ such that $f(a) = b$.
f is *injective* if $a_1 \neq a_2 \implies f(a_1) \neq f(a_2)$.
f is *bijective* if it is both surjective and injective.

1.6 Inverse Function

If $f : A \to B$ is bijective, then the *inverse function* of f (denoted by $f^{-1} : B \to A$) is defined by $f^{-1}(b) = a$.

Examples:

1. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$
2. $f : \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{0\}$, $f(x) = \frac{1}{x}$
3. $g : \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$
4. $f : \mathbb{R} \to [0, \infty)$, $x \mapsto |x|$
5. $f : \mathbb{R} \to (-1, 1)$, $x \mapsto \tanh(x)$