1 Set Theory

1.1 Basic Definitions

$$
\begin{aligned}
\mathbb{N} &= \{0, 1, 2, \ldots\} && \textit{natural numbers} \\
\mathbb{Z} &= \{\ldots, -1, 0, 1, \ldots\} && \textit{integers} \\
\mathbb{Q} &= \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\} && \textit{rationals} \\
\mathbb{R} &= \textit{real numbers} \\
\mathbb{C} &= \{a + bi : a, b \in \mathbb{R}\} && \textit{complex numbers}
\end{aligned}
$$

1.2 Set Operations

- $\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_i \in \mathbb{R}\}$ *n-dimensional space*
- $\mathbb{R}^\infty = \{(x_1, x_2, \ldots) : x_i \in \mathbb{R}\}$ *space of infinite sequences of reals*

1.3 Set Relations

$$
\begin{aligned}
A \subset B &: A \textit{ is a subset of } B \\
A \cap B &: \textit{intersection} \\
A \cup B &: \textit{union}
\end{aligned}
$$

[Venn diagrams for $A \cap B$ and $A \cup B$]

$$
\begin{aligned}
A \setminus B &= \{x \in A : x \notin B\} \\
A \triangle B &= (A \cup B) \setminus (A \cap B) \\
&= \textit{symmetric difference}
\end{aligned}
$$

2 Intervals

Let $a, b \in \mathbb{R}$ with $a < b$. Then:

$$
\begin{aligned}
(a, b) &= \{x \in \mathbb{R} : a < x < b\} \\
[a, b] &= \{x \in \mathbb{R} : a \leq x \leq b\}
\end{aligned}
$$

3 Functions

$$
\begin{aligned}
f : X \to Y &= \{(x, y) : x \in X, y \in Y\} \\
A_f &= \{x \in X : (x, y) \in f \text{ for some } y \in Y\} \\
&= \textit{domain} \\
\bigcup_{x \in X} f(x) &= \{y : y = f(x) \text{ for some } x \in X\} \\
&= \textit{range}
\end{aligned}
$$

These are generally:

- A_f = domain = preimage of f
- $f(A)$ = image of A under f

A map $f : A \to B$ is called:

- *injective* if $\forall x_1, x_2 \in A$, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
- *surjective* if $\forall y \in B$, $\exists x \in A$ such that $f(x) = y$
- *bijective* if f is both injective and surjective

Note: f injective $\Leftrightarrow$ f^{-1} exists
f^{-1} is called the inverse of f.
Δ *is called the diagonal* (by definition) of X.

4 Composition of Functions

If $f : A \to B$ and $g : B \to C$, then the composition of f and g (denoted by $g \circ f$) is:

$$g \circ f : A \to C$$

This is called the composition of f and g.

4.1 Examples

1. $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto \sin x$
2. $f : \mathbb{R}[X] \to \mathbb{R}$, $p \mapsto p(0)$
3. $g : \mathbb{R} \to \mathbb{R}$, $x \mapsto e^x$
4. $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto \frac{1}{x}$
5. $f : V \to V^*$, $v \mapsto (v, -)$