1 Sets and Definitions

1.1 Basic Sets

$\mathbb{N} = \{0, 1, 2, \ldots\}$ - natural numbers
$\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$ - integers
$\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$ - rationals
$\mathbb{R}$ - real numbers
$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$ - complex numbers

1.2 Set Operations

$\cup$ - union
$\cap$ - intersection
$\setminus$ - set difference
$\mathbb{R}^n = \{(x_1, \ldots, x_n) : x_i \in \mathbb{R}\}$ - n-dimensional space
$\mathbb{R}^\infty = \{(x_1, x_2, \ldots) : x_i \in \mathbb{R}\}$ - infinite-dimensional space of real sequences

1.3 Set Relations

$A \subset B$ - subset
$A \cap B = \{x : x \in A \text{ or } x \in B\}$ - union
$A \cap B = \{x : x \in A \text{ and } x \in B\}$ - intersection

$$A \setminus B = \{x \in A : x \notin B\}$$

$A \triangle B = (A \setminus B) \cup (B \setminus A)$ - symmetric difference

1.4 Examples

Let $(a_n)_{n \in \mathbb{N}}$ be an infinite sequence (where $a_n \in \mathbb{R}$).

1. $I = \mathbb{N}$
2. $A_n = [n, \infty)$
3. $I = \mathbb{R}^+$, $A_x = (x, \infty)$

$$\bigcup_{x \in I} A_x = \{y : y \in A_x \text{ for some } x \in I\}$$
$$\bigcap_{x \in I} A_x = \{y : y \in A_x \text{ for every } x \in I\}$$

These are *uncountable* unions/intersections.

1.5 Functions and Mappings

A map $f : A \to B$ is a rule that assigns to each $a \in A$ a unique $b \in B$.
f is *surjective* (or *onto*) if $\forall b \in B, \exists a \in A$ such that $f(a) = b$.
f is *injective* (or *one-to-one*) if $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$.
f is *bijective* if it is both surjective and injective.

1.6 Inverse Functions

If $f : A \to B$ is bijective, then the inverse function $f^{-1} : B \to A$ exists.

$(f^{-1} \circ f)(x) = x$ for all $x \in A$.

This is called the *restriction* of f to A'.

1.7 Examples

1. $f : \mathbb{R} \to \mathbb{R}, x \mapsto x^3$
2. $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}, x \mapsto \frac{1}{x}$
3. $g : \mathbb{R} \to \mathbb{R}, x \mapsto e^x$
4. $f : \mathbb{R} \to (0, \infty), x \mapsto e^x$
5. $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}, x \mapsto \tan x$
6. $D : C(\mathbb{R}) \to C(\mathbb{R}), f \mapsto f'$ - derivative