

CRYSTALLOGRAPHY AND CRYSTAL DEFECTS

PART-A

SHORT QUESTIONS WITH SOLUTIONS

Q1. Define crystal lattice and crystal structure.

Answer :

Crystal Lattice

It is the space arrangement of atoms in a crystal. However, pure crystal contains regularly arranged atoms. This regular or periodic arrangement varies in different directions. So, we imagine points in space about which these atoms are located. Such points are called 'lattice points' and totality of such points is called 'crystal lattice'. Crystal lattice is a three-dimensional array of atoms (points) in which surrounding is similar for every point.

Crystal Structure

A structure formed by associating a basis to every lattice point in a space lattice is known as crystal structure.

Q2. Define primitive cell.

Answer :

The unit cell formed by the primitives a , b and c is called primitive cell. In a primitive cell there is only one lattice point. If there are two or more lattice points, then it is not a primitive cell. In the case of simple cubic crystal lattice, the primitive cell and unit cell are equal. It has only one lattice point in its unit cell. But, most of the unit cells of various crystal lattices contain two or more lattice points and it is not necessary that unit cell should be equal to the primitive cell.

Q3. Define lattice points, space lattices.

Answer :

Lattice Points

In a pure crystal, the points that are located in space about which the atoms are arranged in a regular manner are called lattice points. These points have identical surrounding.

Space Lattice

A group of lattice points form a space lattice. It is also defined as an infinite array of points in space arranged in such a way that it divides space into equal volumes, with no space excluded.

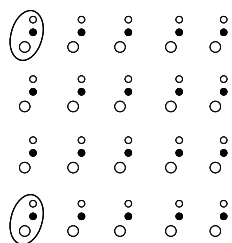
Q4. Define basis.

Answer :

Basis can be defined as the assembly of atoms or molecules similar in composition. It is also called as the pattern. A crystal structure can be generated if we associate basis with lattice.

Crystal structure = Lattice + Basis

The basis or the pattern can be explained by the figure below,



Figure

It can be seen that a basis consists of three different atoms and also it is identical in composition, arrangement and orientation.

Q5. How unit cell is different from primitive cell?

Answer :

Unit cell is the smallest portion of the lattice, during its repetition in all directions gives rise to lattice structure. If the unit cell contains only one atom at the corners then it is called primitive cell or primitive unit cell. It is equivalent of one lattice point. The main difference between unit cell and primitive cell is that unit cell is not restricted to equivalent of one lattice point. Unit cell may not be primitive cell that all primitive cells need not to be unit cell.

Q6. Define Bravais lattice.

Answer :

Depending on the Bravais theory, a total of 14 lattices are enough to illustrate all crystals. These lattices are referred to as Bravais lattices and are categorized into 7 crystal systems on cell parameter basis. The Bravais lattices are classified as,

1. Primitive lattice (*P*)
2. Body centered lattice (*I*)
3. Face centered lattice (*F*)
4. Base centered lattice (*C*).

Q7. Explain the term nearest neighbour distance.

Answer :

The nearest neighbour distance is defined as the distance between the centres of two nearest neighbouring atoms as shown in figure below. This is denoted with '*r*'.

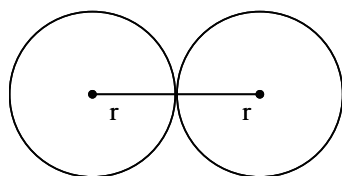


Figure: Distance Between the Centres of Two Atoms

The crystal which is of closely packed has the nearest neighbour of $2r$, where ' r ' is the radius of the atom.

Q8. Define lattice planes.

Answer :

An aggregation of a set of parallel equidistant planes, passing through the lattice points in a crystal system are called as lattice planes. The determination of distance between these lattice planes in various directions helps in finding the crystal structure.

Q9. Define Miller indices and give example.

Answer :

The three smallest possible integers with the same ratio as that of reciprocals of the intercepts of the plane concerned on the three axes are referred to as Miller indices.

Example

Let us consider the Miller indices in a particular case, where the plane cuts the intercepts of 2, 3 and 4 units along the three axes,

- (i) Intercepts are 2, 3, 4.
- (ii) Reciprocals of these are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.
- (iii) L.C.M of the denominators i.e., 2, 3, 4 is 12. Hence, multiplying by 12, we get 6, 4, 3. Thus, the Miller indices of this plane are (6, 4, 3).

Q10. The Miller indices of a set of parallel planes which make equal intercepts on the three axes are

- (a) (121)
- (b) (111)
- (c) (100)
- (d) (101)

Answer :

Jan.-12, Q5(i)

- (b) (111)

Q11. If (3, 2, 6) are Miller indices of a plane the intercepts made by the plane on the three crystallographic axes are,

- (a) (a, b, c)
- (b) (a, 2b, 3c)
- (c) (2a, 3b, c)
- (d) None of the above

Answer :

June-15, Q5

Given that,

Miller indices of a plane = (3 2 6)

Step 1

First step is to take reciprocals of the Miller indices.

Taking reciprocal of the Miller indices,

$$\left(\frac{1}{3}, \frac{1}{2}, \frac{1}{6}\right)$$

Step 2

Next step is to reduce these reciprocals as follows,

The LCM of (3, 2, 6) is equal to 6.

Multiplying each reciprocal by 6,

$$\begin{aligned}\left(\frac{1}{3}, \frac{1}{2}, \frac{1}{6}\right) &= \left(\frac{6}{3}, \frac{6}{2}, \frac{6}{6}\right) \\ &= (2, 3, 1) \\ &= (2a, 3b, c)\end{aligned}$$

Therefore, $(2a \ 3b \ c)$ is the solution where a, b, c are primitives.

Q12. Give the expression for inter-planar separation in terms of Miller indices (h, k, l) for a cubic structure of dimension (a).

Answer :

The expression for inter-planar separation (d) in terms of Miller indices (h, k, l) for a cubic structure of dimension (a) is given by,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Q13. State Bragg's law of X-ray diffraction.

Answer :

Bragg's law of X-ray diffraction specifies the reinforcement of reflected waves occur only at particular values of θ , corresponding to ' λ ' and ' d ' value as,

$$2d \sin \theta = n\lambda$$

Where,

d - Inter-planar spacing

θ - Glancing angle

λ - Wavelength of monochromatic X-rays

n - Any integer = 1, 2, 3

Q14. Write about the defects in crystals and their classification.

Answer :

An ideal crystal is one which does not contain any lattice defects. There are no ideal crystals in nature and they can never be produced by artificial methods. The presence of defect affects the physical, chemical and electromagnetic properties of a crystal. An imperfection or irregularity in the atomic arrangement of a crystal is called a defect. It can be called as a deviation from perfect periodicity or ideal crystalline structure. Factors such as density, stiffness, dielectric capacitivity, specific heats which are explained on the basis of perfect periodicity and are affected due to the defects.

The defects in crystals are classified into three categories. They are,

- Point defects
- Line defects or dislocations
- Surface and volume defects.

Q15. Define point defects.

Answer :

In a crystal, point defects are produced by the absence of atoms or the presence of foreign atoms. The internal energy of a crystal with point defects is high, while it is compared with the perfect crystal. The effects caused by these defects will produce distortion within the crystal structure. The properties of crystals that get affected by the presence of point defects are,

- Electrical resistance
- Few atomic diameters of distortion or strain
- Mechanical strength.

Q16. Explain the types of point defects observed in crystals.

Answer :

May/June-12, Q4

Three types of point defects are observed in crystals,

- Vacancy
- Substitutional and
- Interstitial.

(i) Vacancy

Point defects observed due to missing of atom in the crystal lattice is known as vacancy point defect.

(ii) Substitutional

Replacement of an atom of the host crystal by a foreign atom is known as substitutional point defect.

(iii) Interstitial

Occupation of the void space in the host lattice by a small sized atom, without dislodging the atoms of the host material produces interstitial point defect.

Q17. Give the expression for the equilibrium concentration of vacancies in a solid at a given temperature.

Answer :

The expression for the equilibrium concentration of vacancies in a solid at a given temperature is,

$$m = N \exp \left(\frac{-E_V}{K_B T} \right)$$

Where,

N - Total number of ion pairs

E_v - Amount of energy consumed in moving a cation and anion from the interior to the surface of the crystal

K_B - Boltzmann constant

T - Temperature.

Q18. Give the expression for concentration of Schottky defect in an ionic crystal.

Answer :

The expression for the concentration of Schottky defect in a solid at a given temperature is,

$$m = M \exp \left(\frac{-E_v}{2K_B T} \right)$$

Where,

M - Total number of ion pairs

E_v - Amount of energy consumed in moving a cation and anion from the interior to the surface of the crystal

K_B - Boltzmann constant

T - Temperature.

Q19. Give the expression for concentration of Frenkel defect in an ionic crystal.

Answer :

The expression for the concentration of Frenkel defect in a solid at a given temperature is,

$$m = (MM_i)^{1/2} \exp \left[\frac{-E_v}{2k_B T} \right]$$

Where,

M - Total number of ion pairs

M_i - Number of interstitial sites.

E_v - Amount of energy consumed in moving a cation and anion from the interior to the surface of the crystal.

K_B - Boltzmann constant.

T - Temperature.

PART-B

ESSAY QUESTIONS WITH SOLUTIONS

1.1 CRYSTALLOGRAPHY

1.1.1 Introduction - Types of Crystal Systems - Bravais Lattices - Lattice Planes and Miller Indices (Cubic System) - Inter Planar Spacing (Cubic System)

Q20. Explain the terms,

- (i) **Basis**
- (ii) **Space lattice**
- (iii) **Unit cell.**

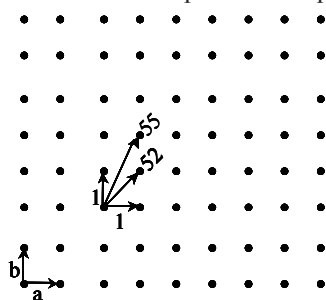
Answer :

- (i) **Basis**

For answer refer Unit-1, Q4.

- (ii) **Space Lattice**

A space lattice is defined as an infinite array of points in three-dimensions in which every point has surroundings identical to that of every other point in the array. For representation on paper consider a two-dimensional square array and points. By repeated translation of the two vectors \vec{a} and \vec{b} on the plane of the paper, we can generate the square array.



Figure

The magnitudes of a and b are equal and can be taken to as unity. The angle between them is 90° . The \vec{a} and \vec{b} are called the fundamental translation vectors, that generate the square array. We will assume that the array can be extended infinitely. If we locate ourselves at any point in the array and look out in a particular direction that lies on the plane of the paper, the scenery is same irrespective of where we consider the immediate surroundings of a point in the array. If we look due north or due east from this point, we see another point at a distance of 1 unit. Along north east, we see the nearest point at a distance of $\sqrt{2}$ units and along north-north east, the nearest point is at a distance of $\sqrt{5}$ units. As this is true for every point in the array, satisfies the definition and it can be called a two-dimensional square lattice.

A space lattice can be defined by referring to a unit cell. The unit cell is the smallest unit which when repeated in space indefinitely, will generate the space lattice. In the example of square lattice, the unit cell is the square obtained by joining four neighbouring lattice points. Since every corner of this square is common to four units cells meeting at that corner, the effective number of lattice points in the unit cell is only one i.e., the unit cell can be visualized and with none at the corners.

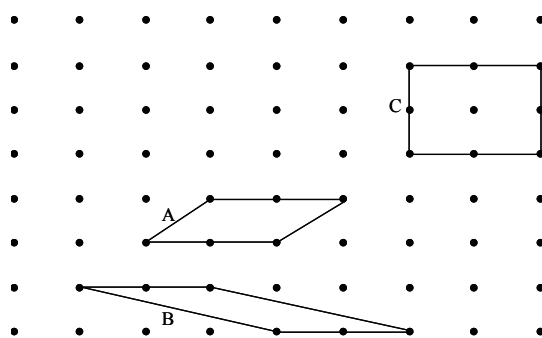
A three dimensional space lattice is generated by repeated translation of three coplanar vectors \vec{a} , \vec{b} and \vec{c} . The translational vector $\vec{T} = n_1 \vec{a} + n_2 \vec{b} + n_3 \vec{c}$ where n_1 , n_2 and n_3 are number of transactions in dimensions a , b and c respectively. There are only fourteen distinguishable ways of arranging points in three dimensional space, such that each arrangement conforms to the definition of a space lattice. These 14 space lattices are known as Bravais lattices. They belong to 7 crystal systems.

A crystal is an arrangement of atoms that repeats itself indefinitely in three-dimensions.

- (iii) **Unit Cell**

The unit cell is the smallest block from which the entire crystal is built up by repetition in three-dimensions, it represents the crystal and it reflects all the properties of crystal.

With reference to a given crystal, there exists a number of possible unit cells. Let us consider the space lattice.



Figure

The point, which must be considered as unit cell i.e., a conventional unit cell. In fact, it is a matter of convenience usually a cell with shortest possible size area $(\bar{a} \times \bar{b})$ in two dimensions and $\bar{a} \cdot \bar{b} \times \bar{c}$ volume in three-dimensions and the sides are chosen as convenient cell. In this respect, a parallelogram A is chosen. This cell contains lattice point only at the corners. Since, a lattice point is shared by four unit cells. We can say that $\frac{1+n}{4}$ of a lattice point is associated with this cell and thus, there is a lattice density of one point associated with this cell. The cell which contains only single lattice point is called primitive cell. Hence, the unit cell is the primitive cell with minimum area.

Q21. Describe in detail, the seven crystal systems with diagrams.

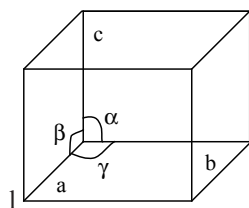
Answer :

Based on the values of lattice parameters, crystal systems are classified into seven categories. The primitives and interfacial angles for seven crystal systems are given in the following table.

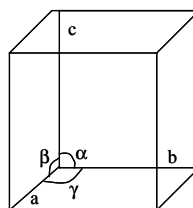
S.No.	Name of Crystal System	Primitives and Interfacial Angles	Examples
1.	Cubic	$a = b = c$ $\alpha = \beta = \gamma = 90^\circ$	Na, Cu ₂ O, Ag, Pb,
2.	Tetragonal	$a = b \neq c$ $\alpha = \beta = \gamma = 90^\circ$	TiO ₂ , SnO ₂ KH ₂ PO ₄
3.	Orthorhombic	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90^\circ$	C ₁₅ H ₂₀ O ₂ PbCO ₃ , KNO ₃ , K ₂ SO ₄
4.	Monoclinic	$a \neq b \neq c$ $\alpha = \gamma = 90^\circ \neq \beta$	CaSO ₄ , 2H ₂ O K ₂ MgSO ₄ , 5H ₂ O
5.	Triclinic	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90^\circ$	K ₂ Cr ₂ O ₇ CuSO ₄ ·5H ₂ O
6.	Rhombohedral (Trigonal)	$a = b = c$ $\alpha = \beta = \gamma \neq 90^\circ$	Calcite, As, Sb, Bi
7.	Hexagonal	$a = b \neq c$ $\alpha = \beta = 90^\circ$ $\gamma = 120^\circ$	Quartz, Cd

Table

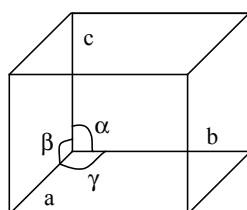
The unit cells of all the crystal systems are shown in the following figures.



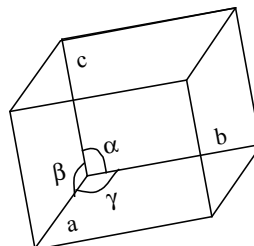
(1) Cubic Crystal



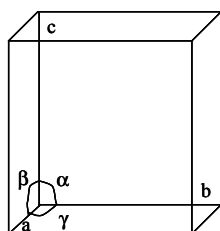
(2) Tetragonal Crystal



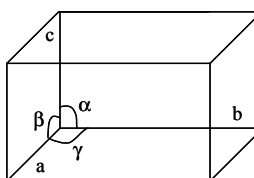
(3) Orthorhombic Crystal



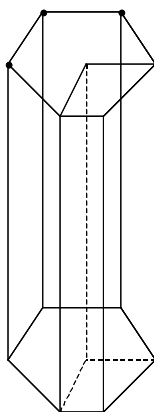
(4) Monoclinic Crystal



(5) Triclinic Crystal



(6) Rhombohedral Crystal



(7) Hexagonal Crystal

Figures

Q22. What are Bravais lattices?

Answer :

Depending on the Bravais theory, a total of 14 lattices are enough to illustrate all crystals. These lattices are referred to as Bravais lattices and are categorized into 7 crystal systems on cell parameters basis. The bravais lattices are classified as,

1. Primitive lattice (*P*)
2. Body centered lattice (*I*)
3. Face centered lattice (*F*)
4. Base centered lattice (*C*)

Table shows the seven crystals system and the corresponding Bravais lattice.

S.No	Crystal Lattices System	Types of Bravais Lattices	Number of Bravais Lengths and Angles	Relation between Lengths and Angles
1	Cubic	P, I, F	3	$a = b = c$ $\alpha = \beta = \gamma = 90^\circ$
2	Tetragonal	P, I	2	$a = b \neq c$ $\alpha = \beta = \gamma = 90^\circ$
3	Orthorhombic	P, I, F, C	4	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90^\circ$
4	Monoclinic	P, C	2	$a \neq b \neq c$ $\alpha = \gamma = 90^\circ \neq \beta$
5	Triclinic	P	1	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma$
6	Rhombohedral	P	1	$a = b = c$ $\alpha = \beta = \gamma \neq 90^\circ < 120^\circ$
7	Hexagonal	P	1	$a = b \neq c$ $\alpha = \beta = 90^\circ$ and $\gamma = 120^\circ$

Table

Q23. What are Miller indices? How are they obtained?

Answer :

Miller Indices

The three smallest possible integers with the same ratio as that of reciprocals of the intercepts of the plane concerned on the three axes are referred to as Miller indices.

The rules for finding Miller indices are as follows,

1. Take any atom as the origin in the crystal and draw coordinate axes from this atom in the directions of the basis vectors.
2. Choose one plane of the set of interest and note its intercepts on the axis \bar{a} , \bar{b} , \bar{c} in terms of lattice constants. The plane must be chosen so that no intercept is at the origin.
3. Take the reciprocals of these intercepts and convert these into the smallest set of integers that can be obtained by multiplying each of the fractions by the same number say L.C.M.

The result is conventionally enclosed in parenthesis ($h\ k\ l$).

Example

Consider the Miller indices in a particular case, where the plane cuts the intercepts of 2, 3 and 4 units along the three axes.

- (i) Intercepts are 2, 3, 4.
- (ii) Reciprocals of these are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$
- (iii) L.C.M of the denominators i.e., 2, 3, 4 is 12. Hence, multiplying by 12, we get 6, 4, 3. Thus, the Miller indices of this plane are (6, 4, 3).

Q24. Show that in a cubic crystal the spacing (d) between consecutive parallel planes of Miller indices (h k l) is given by $d = a (h^2 + k^2 + l^2)^{-1/2}$.

OR

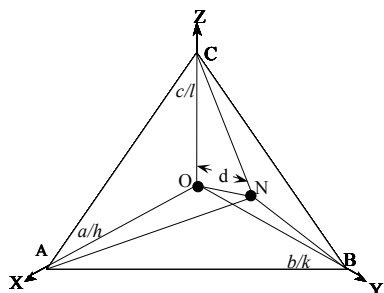
Deduce the expression for the inter-planar separation in terms of Miller indices for a cubic structure.

Answer :

The separation between lattice planes of a cubic, orthorhombic and tetragonal systems of a crystal, in which the edges of the unit cell are perpendicular. The Cartesian geometry is applicable and can be deduced as follows,

Let OA , OB and OC be the orthogonal axes. Let us consider any set of parallel planes referred by Miller indices $(h\ k\ l)$. Consider the plane passing through the origin as the

reference plane. The next plane (x, y, z) makes intercepts $\frac{a}{h}$, $\frac{b}{k}$ and $\frac{c}{l}$ along the three orthogonal axis. Let the line ON be the normal to the plane passing through the origin, the length “ d ” of which represents the inter-planar spacing between two consecutive planes. Let us find the value of “ d ” in terms of a , b and c as d is normal to the plane, from the figure.



Figure

$$d = \left(\frac{a}{h}\right) \cos \alpha = \left(\frac{b}{k}\right) \cos \beta = \left(\frac{c}{l}\right) \cos \gamma$$

Where α , β , and γ are the interfacial angles, which the normal makes with crystallographic axes.

$$\angle NOX = \alpha, \angle NOY = \beta, \angle NOZ = \gamma$$

From cosine theorem,

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

Substituting the values of $\cos \alpha$, $\cos \beta$ and $\cos \gamma$ from the above equation,

$$\frac{d^2}{\left(\frac{a}{h}\right)^2} + \frac{d^2}{\left(\frac{b}{k}\right)^2} + \frac{d^2}{\left(\frac{c}{l}\right)^2} = 1$$

$$d^2 \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2} \right] = 1$$

$$d^2 = \frac{1}{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}$$

$$d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$$

It is to be noted that the formula is applicable only to primitive lattice in cubic, orthorhombic and tetragonal system. For a cubic lattice $a = b = c$ and therefore the above equation becomes,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Q25. Draw the (1, 1, 2), (1, 2, 0) (3, 2, 1) and (0, 1, 0) crystal planes of simple cubic crystal.

Answer :

Given crystal planes of simple cubic crystal are,

$$(h_1, k_1, l_1) = (1, 1, 2)$$

$$(h_2, k_2, l_2) = (1, 2, 0)$$

$$(h_3, k_3, l_3) = (3, 2, 1)$$

$$(h_4, k_4, l_4) = (0, 1, 0)$$

Plane $(h_1, k_1, l_1) = (1, 1, 2)$

For this plane, the intercepts (i.e., reciprocals of h_1, k_1, l_1) are,

$$\frac{1}{h_1}, \frac{1}{k_1}, \frac{1}{l_1} = \frac{1}{1}, \frac{1}{1}, \frac{1}{2} = 1, 1, 0.5$$

Now, a plane with intercepts 1, 1, 0.5 in the direction of x, y, z axis is shown in figure (1).

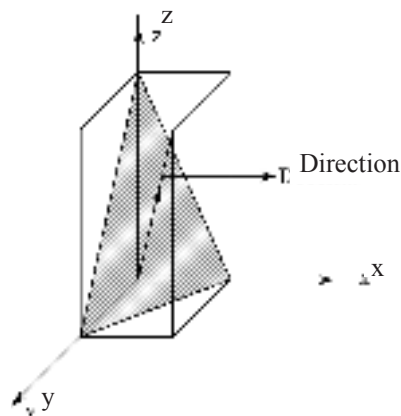


Figure (1)

Plane $(h_2, k_2, l_2) = (1, 2, 0)$

For this plane, the intercepts (i.e., reciprocals of (h_2, k_2, l_2)) are,

$$\frac{1}{h_2}, \frac{1}{k_2}, \frac{1}{l_2} = \frac{1}{1}, \frac{1}{2}, \frac{1}{0} = 1, 0.5, \infty$$

Now, a plane with intercepts 1, 0.5, ∞ in the direction of x, y, z axis is shown in figure (2).

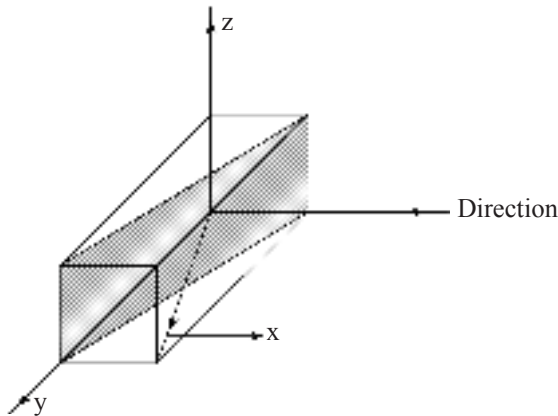


Figure (2)

Plane $(h_3, k_3, l_3) = (3, 2, 1)$

For this plane, the intercepts (i.e., reciprocals of h_3, k_3, l_3) are,

$$\frac{1}{h_3}, \frac{1}{k_3}, \frac{1}{l_3} = \frac{1}{3}, \frac{1}{2}, \frac{1}{1} = 0.333, 0.5, 1$$

Now, a plane with intercepts 0.333, 0.5, 1 in the direction of x, y, z axis is shown in figure (3).

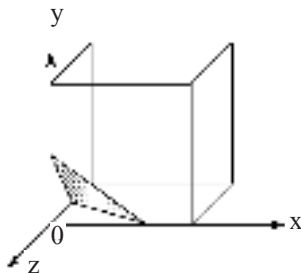


Figure (3)

Plane $(h_4, k_4, l_4) = (0, 1, 0)$

For this plane, the intercepts (i.e., reciprocals of h_4, k_4, l_4) are,

$$\frac{1}{h_4}, \frac{1}{k_4}, \frac{1}{l_4} = \frac{1}{0}, \frac{1}{1}, \frac{1}{0} = \infty, 1, \infty$$

Now, a plane with intercepts $\infty, 1, \infty$ in the direction of x, y, z axis is shown in figure (4).

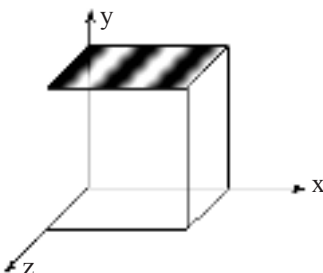


Figure (4)

Q26. The distance between (1 1 0) planes in a body centered cubic structure is 0.203 nm. What is the size of the unit cell? What is the radius of the atom?

Answer :

Given that,

For a plane in body centered cubic structure,

$$(h, k, l) = (1, 1, 0)$$

$$d = 0.203 \text{ nm}$$

Then, the expression for separation between lattice planes in a cubic crystal is given by,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$\Rightarrow a = d \times \sqrt{h^2 + k^2 + l^2}$$

$$= 0.203 \times 10^{-9} \times \sqrt{1^2 + 1^2 + 0^2}$$

$$= 0.203 \times 10^{-9} \times \sqrt{2}$$

$$\Rightarrow a = 0.287 \times 10^{-9} \text{ m}$$

$$\therefore a = 0.287 \times 10^{-9} \text{ m}$$

For a BCC lattice,

$$a = \frac{4r}{\sqrt{3}}$$

$$\Rightarrow r = a \times \frac{\sqrt{3}}{4}$$

$$\Rightarrow r = 0.287 \times 10^{-9} \times \frac{\sqrt{3}}{4} = 0.124 \times 10^{-9} \text{ m}$$

$$\therefore r = 0.124 \times 10^{-9} \text{ m}$$

Q27. What are miller indices? Sketch the miller indices for (1 1 0) (1 2 1) and (0 1 0) planes.

Answer :

June-17, Q16(a)

Miller Indices

For answer refer Unit-1, Q23, Topic: Miller Indices.

Given crystal planes are,

$$(h_1, k_1, l_1) = (1, 1, 0)$$

$$(h_2, k_2, l_2) = (1, 2, 1)$$

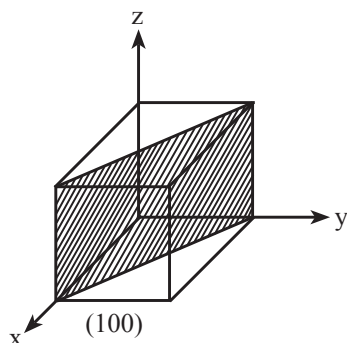
$$(h_3, k_3, l_3) = (0, 1, 0)$$

Plane $(h_1, k_1, l_1) = (1, 1, 0)$

For this plane, the intercepts (i.e., reciprocals of h_1, k_1, l_1) are,

$$\frac{1}{h_1}, \frac{1}{k_1}, \frac{1}{l_1} = \frac{1}{1}, \frac{1}{1}, \frac{1}{0} = 1, 1, \infty$$

Now, a plane with intercepts 1, 1, ∞ in the direction of $x y z$ axis is shown in figure (1).



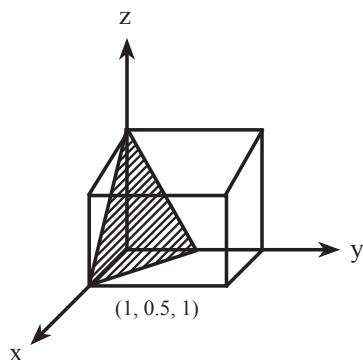
Figure(1)

Plane $(h_2, k_2, l_2) = (1, 2, 1)$

For this plane, the intercepts (i.e., reciprocals of (h_2, k_2, l_2)) are,

$$\frac{1}{h_2}, \frac{1}{k_2}, \frac{1}{l_2} = \frac{1}{1}, \frac{1}{2}, \frac{1}{1} = 1, 0.5, 1$$

Now, a plane with intercepts 1, 0.5, 1 in the direction of $x y z$ axis is shown in figure (2).



Figure(2)

Plane $(h_3, k_3, l_3) = (0, 1, 0)$

For this plane, the intercepts (i.e., reciprocals of (h_3, k_3, l_3)) are,

$$\frac{1}{h_3}, \frac{1}{k_3}, \frac{1}{l_3} = \frac{1}{0}, \frac{1}{1}, \frac{1}{0} = \infty, 1, \infty$$

Now, a plane with intercepts $\infty, 1, \infty$ in the direction of $x y z$ axis is shown in figure (3).

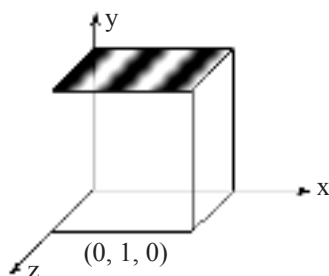


Figure (3)

1.1.2 Bragg's Law - Powder Diffraction Method

Q28. State and explain Bragg's law in X-ray diffraction.
May/June-12, Q6

OR

Obtain an expression for Bragg's law.

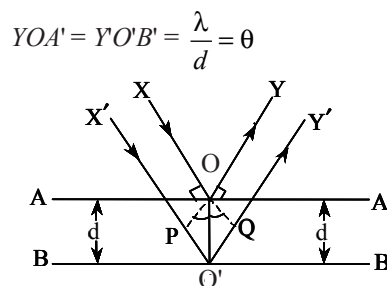
Answer :

Jan.-10, Q15(b)

Bragg's Law

For answer refer Unit-1, Q13.

Consider a set of parallel lattice planes of a crystal in which spacing between the successive planes is d . If a narrow beam of X-rays of wavelength λ be incident on the planes with a glancing angle θ , the reflected beam leaves the plane at an angle θ as shown in figure. Consider the rays reflected at two consecutive planes AA' and BB' . Thus, the rays XO and $X'O'$ are reflected at O and O' respectively along OY and $O'Y'$ such that,



Figure

If OP and $O'Q$ are perpendicular lines drawn from O and $X'O'$ and O' and Y' the path difference between the beams reflected at two consecutive planes will be,

$$\Delta = PO' + O'Q \quad \dots (1)$$

$$\text{From } \triangle OPO', \sin \theta = \frac{O'P}{OO'} = \frac{O'P}{d}$$

$$\Rightarrow O'P = d \sin \theta \quad \dots (2)$$

$$\text{From } \triangle O'QO', \sin \theta = \frac{O'Q}{OO'} = \frac{O'Q}{d}$$

$$\Rightarrow O'Q = d \sin \theta \quad \dots (3)$$

Substituting equations (2) and (3) in equation (1),

Path difference, $\Delta = d \sin \theta + d \sin \theta = 2d \sin \theta$

$$\therefore \Delta = 2d \sin \theta$$

If this path difference is an integral multiple of the wavelength λ , the reflected beam will interfere constructively giving maximum intensity, i.e., for constructive diffraction.

$$2d \sin \theta = n\lambda \quad \dots (4)$$

Where, n is an integer ($n = 1, 2, 3, \dots$)

Equation (4) is known as Bragg's law.

Q29. What is Bragg's law ? Explain the experimental determination of lattice constant by powder diffraction method.

April-16, Q15(a)

OR

Describe the powder diffraction method for the determination of lattice constant of a cubic unit cell.

Answer :

May/June-12, Q14(a)

Bragg's law

For answer refer Unit-1, Q13.

Experimental determination of lattice constant by powder diffraction

The powder method is applicable to finely divided crystalline powder or to a very fine grained polycrystalline specimen. This is also called Debye-Scherrer method. The powder method is more convenient since crystals are not required.

The powdered specimen is kept inside a small capillary tube which would not undergo diffraction by x-rays. An arrow pencil of monochromatic X-rays is diffracted from the powder and recorded by the photographic film as a series of lines of varying curvature as shown in the following figure.

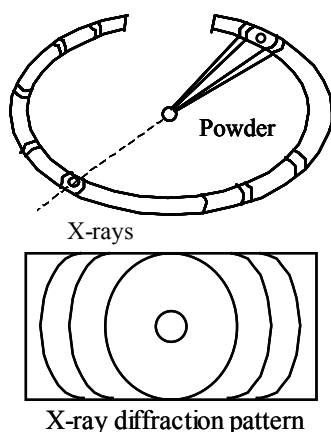


Figure: Powder Method

The full opening angle of the diffraction cone 4θ is determined by measuring the distance S between two corresponding arcs on the powder photograph symmetrically displaced about the exit point of the direct beam. The distance S on the film between two diffraction lines corresponding to a particular plane is related to the Bragg's angle by the equation.

$$4\theta = \frac{S}{R} \text{ radian or } 4\theta = \left(\frac{S}{R} \right) \left(\frac{180}{\pi} \right) \text{ deg}$$

Where, R is the specimen to film distance, usually the radius of the camera housing the film.

Thus, a list of θ values can be obtained from the measured values of S . Since wavelength is known, substituting of θ and λ gives a list of values of spacing d . Each spacing is the distance between neighbouring planes (h, k, l).

So, the reflections can be indexed and the unit cell parameters evaluated from $d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$. For different values of temperature, a can be determined.

Uses

1. This method is used for determination of lattice parameters in crystals of specified structure accurately.
2. This method is also used for identification of elements and compounds.

Q30. The Bragg's angle in the first order for (2, 2, 0) reflection from nickel (FCC) is 38.2° when X-rays of wavelength 1.54 \AA are employed in a diffraction experiment. Determine the lattice parameter of nickel.

Answer :

Given that,

In a diffraction experiment,

Bragg's angle, $\theta = 38.2^\circ$

Wavelength, $\lambda = 1.54 \text{ \AA}$

Miller indices, (h, k, l) = (2, 2, 0)

Order, $n = 1$

From Bragg's law, $2d \sin \theta = n \lambda$... (1)

Where d is the separation between adjacent lattice planes,

Equation (1) can be written as,

$$d = \frac{n\lambda}{2 \sin \theta} \quad \dots (2)$$

Substituting the values of n, λ, θ in equation (2)

$$d = \frac{1.54 \times 10^{-10} \text{ m}}{2 \sin(38.2^\circ)}$$

$$\therefore d = 1.245 \text{ \AA}$$

For face centered cubic nickel, the lattice parameters are $a = b = c$ and $\alpha = \beta = \gamma = 90^\circ$

Separation between adjacent lattice planes in a cubic crystal is given by,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad \dots (3)$$

Where, a - Lattice constant

(h, k, l) - Miller indices.

From equation (3),

$$a = d \sqrt{h^2 + k^2 + l^2}$$

$$\Rightarrow a = 1.54 \times 10^{-10} \left(\sqrt{2^2 + 2^2 + 0^2} \right)$$

$$\therefore a = 4.356 \text{ \AA}$$

Therefore, the lattice parameters of nickel are $a = b = c = 4.356 \text{ \AA}$ and $\alpha = \beta = \gamma = 90^\circ$

Q31. Calculate the inter planar spacing for (231) plane of FCC structure whose atomic radius is 0.175 nm.

Answer :

Given that,

For an FCC structure,

$$\text{Plane} = (231)$$

Atomic radius, $r = 0.175 \text{ nm}$

Inter planar spacing is given by,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Where,

$$\text{Atomic radius, } r = \frac{\sqrt{2}}{4} a$$

$$\Rightarrow a = \frac{4r}{\sqrt{2}}$$

$$\Rightarrow d = \frac{4r}{\sqrt{2} \sqrt{h^2 + k^2 + l^2}}$$

Substituting the corresponding values in above equation,

$$\begin{aligned} d &= \frac{4 \times 0.175 \times 10^{-9}}{\sqrt{2} \sqrt{2^2 + 3^2 + 1^2}} \\ &= \frac{0.7 \times 10^{-9}}{\sqrt{2} \sqrt{14}} \\ &= \frac{0.7 \times 10^{-9}}{2\sqrt{7}} \\ &= 0.132 \times 10^{-9} \end{aligned}$$

$$\therefore d = 0.132 \text{ nm}$$

1.2 CRYSTAL DEFECTS

1.2.1 Classification of Point Defects

Q32. Write a short note on point defects in solids.

Jan.-13, Q13(b)

OR

What are point defects and explain them?

Answer :

Jan.-10, Q15(a)

Point Defects

In a crystal, point defects are produced by the absence of atoms or the presence of foreign atoms. The internal energy of a crystal with point defects is high, while it is compared with the perfect crystal. The effects caused by these defects will produce distortion within the crystal structure. The properties of crystals that get affected by the presence of point defects are,

1. Electrical resistance
2. Few atomic diameters of distortion or strain
3. Mechanical strength.

The various types of point defects are discussed below,

(i) Vacancies

Vacancy in a crystal is referred to as an empty site of an atom position or simply a missing atom as shown in figure (1). These vacancies are produced by either of two following ways,

1. Improper packing during the original crystallization.
2. Increase of electrons due to thermal vibrations.

During the industrial processes like annealing, precipitation, sintering and surface hardening, vacancies are useful in the transportation of atoms through lattice.

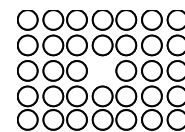


Figure (1)

(ii) Interstitial

Interstitial defect in a crystal is referred to as a defect caused by the occupation of an additional atom in the empty space between the atoms (i.e., void space) as shown in figure (2). Based on the size of interstitial atom, the amount mechanical strain on the surrounding atoms will vary. These atoms might available in the low packing fraction crystals. Some of the parent atoms, which occupy void spaces by applying enormous stress on crystal results in self-interstitial defects.

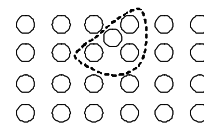


Figure (2)

(iii) Compositional Defects

During the crystallization process, the compositional defects occur by either the following two impurities. They are,

- (a) Unwanted impurities
- (b) Purposely doped impurities.

Compositional defects play a vital role in semiconductors (particularly diodes, transistors etc.). The two types of these defects are,

1. Substitutional impurity, which specifies a foreign atom that replaces a parent atom in the crystal structure as shown in figure (3).

Example

Silicon (extrinsic semiconductor) doped with aluminium or phosphorous.

2. Interstitial impurity which specifies a small sized foreign atom occupy the void space in the crystal without disturbing parent atoms as shown in figure (4).

Example

In the octahedral void same of FCC iron of atomic radius 2.25\AA , carbon atoms of radius 0.777\AA are occupied.

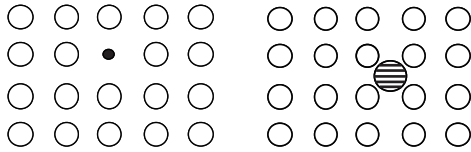


Figure (3)

Figure (4)

(iv) Electronic Defects

In solids, the errors in charge distribution are responsible for electronic defects. The free movement of these defects within the crystal is possible using the external electric field. These defects play a vital role understanding the electrical conductivity phenomenon.

1.2.2 Concentration of Schottky Defects in Metals and Ionic Crystals - Concentration of Frenkel Defects

Q33. Explain Schottky and Frenkel defects with the help of suitable figures.

Answer :

In ionic crystals, the charged ions exhibit two types of point defects namely, Frenkel and Schottky defects.

Schottky Defect

In an ionic crystal, the absence of one cation and one anion from the site is as shown in figure (a). Such a cation vacancy pair is known as Schottky defect.

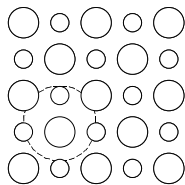


Figure (a): Schottky Defect

This defect is caused by removing one cation and one anion from the interior of the crystal and placing both of them at the surface. Both cations and anions have the same charge. However, the charge neutrality is maintained as every anion such as vacancy exists along with a cation vacancy.

Example: Alkali halides like NaCl, KCl, KBr etc.

Frenkel Defect

The defect caused by a cation which leaves its normal position and moves into an interstitial site in figure (b) is referred to as Frenkel defect. However, there is no change in charge because the cation maintains the same positive charge as an interstitial. Crystals like CaF_2 , AgBr and AgI develop this type of defect.

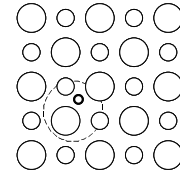


Figure (b): Frenkel Defect

Q34. Obtain the expression for the equilibrium concentration of vacancies in a solid at a given temperature.

Answer :

Generally, the vacancies are available in all type of crystals. The main reasons for occurrence of defects in crystals are,

- Thermal agitation
- Constant creation and removal of vacancies.

The amount of energy necessary to create 'n' number of isolated vacancies can be expressed as,

$$U = nE_v$$

Where,

E_v - Energy consumed in moving a crystal lattice from interior to the surface of the crystal.

The total number of ways to move 'n' number of atoms in a crystal on the surface will be,

$$P = \frac{N!}{(N-n)!n!}$$

The entropy due to formation of n vacancies can be written as,

$$S = K_B \log P$$

$$\Rightarrow S = K_B \log \left[\frac{N!}{(N-n)!n!} \right]$$

But, the expression for amount of free energy is,

$$F = U - TS$$

$$\text{And } U = nE_v$$

Then,

$$\Rightarrow F = nE_v - K_B T \log \left[\frac{N!}{(N-n)!n!} \right]$$

$$F = nE_v - K_B T [\log N! - \log(N-n)! - \log n!]$$

Using Stirling's approximation $\log x! = x \log x - x$, the second terms on R.H.S can be simplified as,

$$F = nE_v - K_B T [N \log N - N - (N-n) \log(N-n) + (N-n) - n \log n + n]$$

$$F = nE_v - K_B T [N \log N - (N-n) \log(N-n) - n \log n + n]$$

At thermal equilibrium, free energy is constant and minimum with respect to n , hence.

$$\begin{aligned} \left[\frac{\delta F}{\delta n} \right]_T &= 0 \\ &= E_V - K_B T \left[0 - (N-n) \frac{-1}{N-n} - (-1) \log(N-n) - n \frac{1}{n} - \log n \right] \\ E_V &= K_B T [1 + \log(N-n) - 1 - \log n] \\ \Rightarrow E_V &= K_B T \log \left[\frac{(N-n)}{n} \right] \\ \frac{N-n}{n} &= \exp \left(\frac{E_V}{K_B T} \right) \\ \Rightarrow n &= (N-n) \exp \left(\frac{-E_V}{K_B T} \right) \end{aligned}$$

If $n \ll N$, n can be neglected on the right hand side. Thus,

$$n = N \exp \left(\frac{-E_V}{K_B T} \right)$$

Q35. Obtain an expression for the concentration of Schottky defects in an ionic crystals.

Answer :

[Dec.-17, Q11(b) | Jan.-12, Q13(b)]

In an ionic crystal, charge neutrality is maintained by the formation of equal number of anions and cations. Suppose ' m ' is the number of cation-anion pairs present in an ionic crystal at thermal equilibrium temperature ' T ' and ' M ' is the total number of pairs of ions. Therefore, the number of ways in which M -pairs of vacancies are created is given by,

$$P = \left[\frac{M!}{(M-m)! m!} \right]^2 \quad \dots (1)$$

The formation of vacancies in a crystal results in crystal disorders and change in free energy (F). These disorders are measured in terms of entropy (S). Therefore, according to thermodynamics change in entropy due to formation of vacancies in the crystal is,

$$S = k_B \log P \quad \dots (2)$$

Where, k_B - Boltzmann constant

Substituting equation (1) in equation (2),

$$S = k_B \log \left[\frac{M!}{(M-m)! m!} \right]^2 \quad \dots (3)$$

Similarly, change in free energy due to the formation of vacancies is given as,

$$F = U - TS \quad \dots (4)$$

Where, U - Internal energy of the crystal at temperature T .

$$\text{But, } U = mE_p \quad \dots (5)$$

Where, E_p - Amount of energy consumed in moving a cation and anion from the interior to the surface of the crystal.

Substituting equations (3) and (5) in equation (4),

$$\begin{aligned} \Rightarrow F &= mE_p - k_B T \log \left[\frac{M!}{(M-m)! m!} \right]^2 \\ \therefore F &= mE_p - 2 k_B T [\log M! - \log(M-m)! - \log m!] \quad \dots (6) \end{aligned}$$

Using Stirling's approximation, equation (5) can be simplified as,

$$\log x! = x \log x - x$$

Therefore, equation (5) can be written as,

$$F = mE_p - 2k_B T [[M \log M - M] - [(M - m) \log (M - m) - (M - m)] - [m \log m - m]]$$

$$\Rightarrow F = mE_p - 2k_B T [M \log M - (M - m) \log (M - m) - m \log m] \quad \dots (7)$$

As the ionic crystal is in thermal equilibrium condition the free energy F will be minimum with respect to all the changes in ' m '. Hence,

$$\left[\frac{\partial F}{\partial m} \right]_T = 0$$

$$\Rightarrow E_p - 2k_B T \left[0 - \left\{ -(M - m) \cdot \frac{1}{M - m} + \log(M - m)(-1) \right\} - \left\{ m \cdot \frac{1}{m} + \log m \cdot 1 \right\} \right] = 0$$

$$\Rightarrow E_p - 2k_B T [1 + \log(M - m) - 1 - \log m] = 0$$

$$\Rightarrow E_p - 2k_B T [\log(M - m) - \log m] = 0$$

$$\Rightarrow E_p - 2k_B T \log \left[\frac{M - m}{m} \right] = 0$$

$$\Rightarrow \log \left[\frac{M - m}{m} \right] = \left[\frac{E_p}{2k_B T} \right] \quad \dots (8)$$

$$\Rightarrow \frac{M - m}{m} = \exp \left[\frac{E_p}{2k_B T} \right]$$

Assume that right hand side term of above expression is much greater than '1'. Thus,

$$\frac{M}{m} \simeq \exp \left[\frac{E_p}{2k_B T} \right]$$

$$\therefore m \simeq M \exp \left[\frac{E_p}{2k_B T} \right]$$

Q36. Derive an expression for the concentration of Frenkel defect in an ionic crystal.

June-17, Q11(a)

OR

What is Frenkel defect? Derive an expression for the concentration of Frenkel defects in a crystal.

June-14, Q14

OR

Derive an expression for the equilibrium concentration of Frenkel defects in crystals.

Answer :

Jan.-13, Q17(b)

The vacancy created by transfer of an atom, from a regular array of atoms, to an interstitial location is called Frenkel defect.

In an ionic crystal, charge neutrality is maintained by the formation of equal number of anions and cations. Suppose ' m ' is the number of cation-anion pairs (or) number of Frenkel defects present in an ionic crystal at thermal equilibrium temperature ' T ' and ' M ' is the total number of pairs of ions. Therefore, the number of ways in which M -pairs of vacancies are created is given by,

$$P = \left[\frac{M!}{(M-m)! m!} \right] \left[\frac{M_i!}{(M_i-m)! m!} \right] \quad \dots (1)$$

Where,

M_i - Number of interstitial sites.

The formation of vacancies in a crystal results in crystal disorders and change in free energy (F). These disorders are measured in terms of entropy (S). Therefore, according to thermodynamics change in entropy due to formation of vacancies in the crystal is,

$$S = k_B \log P$$

Where, k_B - Boltzmann constant. ... (2)

Substituting equation (1) in equation (2),

$$S = k_B \log \left\{ \left[\frac{M!}{(M-m)! m!} \right] \left[\frac{M_i!}{(M_i-m)! m!} \right] \right\} \quad \dots (3)$$

Similarly, change in free energy due to the formation of vacancies is given as,

$$F = U - T S \quad \dots (4)$$

But, $U = U - m E_p$... (5)

Where, E_p is Amount of energy consumed in moving a cation and anion from the interior to the surface of the crystal.

Substituting equations (3) and (5) in equation (4),

$$\Rightarrow F = m E_p - k_B T \log \left\{ \left[\frac{M!}{(M-m)! m!} \right] \left[\frac{M_i!}{(M_i-m)! m!} \right] \right\}$$

$$\therefore F = m E_p - k_B T [\log M! + \log M_i! - [\log (M-m)! + \log (M_i-m)! + 2 \log m!]] \quad \dots (6)$$

Using Stirling's approximation, equation (5) can be simplified i.e.,

$$\log x! = x \log x - x$$

Therefore, equation (5) becomes,

$$\therefore F = m E_p - k_B T \{ [M \log M - M] + [M_i \log M_i - M_i] - [(M-m) \log (M-m) - (M-m)] - [(M_i-m) \log (M_i-m) - (M_i-m)] - 2 [m \log m - m] \}$$

$$= m E_p - k_B T \{ M \log M + M_i \log M_i - (M-m) \log (M-m) - (M_i-m) \log (M_i-m) - 2m \log m \} \quad \dots (7)$$

As the ionic crystal is in thermal equilibrium condition, the free energy F will be minimum with respect to all the changes in 'm'. Hence,

$$\left[\frac{\partial F}{\partial m} \right]_T = 0$$

$$\Rightarrow E_p - k_B T = \{ 0 + 0 - [(M-m) \cdot \frac{1}{M-m} + \log (M-m) \cdot (-1)] - [(M_i-m) \cdot \frac{1}{M_i-m} + \log (M_i-m) \cdot (-1)] - 2[m \cdot \frac{1}{m} + \log m \cdot 1] \}$$

$$\Rightarrow E_p - k_B T [1 + \log (M-m) + 1 + \log (M_i-m) - 2 - 2 \log m] = 0$$

$$\Rightarrow E_p - k_B T [\log (M-m) (M_i-m) - \log m^2] = 0$$

$$\Rightarrow E_p - k_B T \log \left[\frac{(M-m)(M_i-m)}{m^2} \right] = 0$$

$$\Rightarrow \frac{E_p}{k_B T} = \log \left[\frac{(M-m)(M_i-m)}{m^2} \right] \quad \dots (8)$$

$$\Rightarrow \frac{(M-m)(M_i-m)}{m^2} = \exp \left[\frac{E_p}{k_B T} \right]$$

In the case of smaller Frenkel defects (i.e., $M \gg m$ and $M_i \gg m$), the above expression reduces to,

$$\frac{MM_i}{m^2} = \exp \left[\frac{E_p}{k_B T} \right]$$

$$\Rightarrow \frac{m^2}{MM_i} = \exp \left[\frac{-E_p}{k_B T} \right]$$

$$\Rightarrow m^2 = MM_i \exp \left[\frac{-E_p}{k_B T} \right]$$

$$\therefore m = (MM_i)^{1/2} \exp \left[\frac{-E_p}{2k_B T} \right]$$

Q37. The average energy required to create a Schottky defect in a crystal is 1 eV. Calculate the ratio of number of Schottky defects at 50°C and 250°C.

Answer :

Given that,

Energy, $E = 1 \text{ eV}$

The ratio of number of Schottky defects is given as,

$$\frac{n}{N} = e^{-E/kT} \quad \dots (1)$$

At, $t = 50^\circ\text{C}$

$$T = 273 + 50 = 323 \text{ K}$$

$$\Rightarrow \frac{n}{N} = e^{-1/1.38 \times 10^{-23} \times 323}$$

$$\therefore \frac{n}{N} = 0$$

At, $t = 250^\circ\text{C}$

$$T = 250 + 273$$

$$= 523 \text{ K}$$

$$\Rightarrow \frac{n}{N} = e^{-1/1.38 \times 10^{-23} \times 523}$$

$$\therefore \frac{n}{N} = 0$$

Q38. If the average energy required to create a Frenkel defect in an ionic crystal is 1.4 eV, calculate the ratio of number of Frenkel defects at 20°C and 300°C.

Answer :

Given that,

Average energy = 1.4 eV

Temperature, $t_1 = 20^\circ\text{C}$

Temperature, $t_2 = 300^\circ\text{C}$

Number of ways of forming Frenkel defects is given as,

$$n = (NN_i)^{1/2} \exp \left(\frac{-E_f}{2kT_i} \right)$$

At $t_1 = 20^\circ\text{C}$,

$$T_1 = 273 + 20$$

$$= 293 \text{ K}$$

At $t_2 = 300^\circ\text{C}$,

$$T_2 = 273 + 300$$

$$= 573 \text{ K}$$

Then,

$$n_{T_1} = (NN_i)^{1/2} \exp \left(\frac{-1.4}{586 \text{ K}} \right)$$

$$n_{T_2} = (NN_i)^{1/2} \exp \left(\frac{-1.4}{1146 \text{ K}} \right)$$

Therefore,

$$\frac{n_{T_1}}{n_{T_2}} = \exp \left(\frac{-1.4}{K} \right) \left(\frac{1}{586} - \frac{1}{1146} \right)$$

$$= \exp \left(\frac{-1.4}{8.625 \times 10^{-5}} \right) \left(\frac{1146 - 586}{(586)(1146)} \right)$$

$$= \left(\frac{-1.4 \times 560}{8.625 \times 671556 \times 10^{-5}} \right)$$

$$\left(\frac{n_{T_2}}{n_{T_1}} \right) = \frac{784}{57.921}$$

$$\ln \left(\frac{n_{T_2}}{n_{T_1}} \right) = 13.535$$

$$\left(\frac{n_{T_2}}{n_{T_1}} \right) = 7.56 \times 10^5$$

1.2.3 Line Defects - Screw and Edge Dislocations - Burger's Vector

Q39. Write a notes on line defects of crystals.

OR

Describe line imperfections in a crystal with Burger's vectors.

Answer :

Line Defect

The line defect also referred to as dislocation is defined as a defect that occurs due to the misalignment of atoms in a crystal.

Classification of Line Defects

The line defects are classified into three types. They are,

- (i) Edge dislocation
- (ii) Screw dislocation
- (iii) Mixed dislocation.

(i) Edge Dislocation

In a perfect crystal, when an extra layer of atom is inserted (either from top or bottom) and gets terminated at the middle, then such defect is referred to as edge dislocation. It usually occurs along the edge of extra atoms in a row, hence the name edge dislocation.

If the insertion of atomic layers is from the top then the dislocation is represented as ' \perp ' and if the insertion is from the bottom, then the dislocation is represented as ' ∇ '.

Figure (1) depicts the edge dislocation in a crystal.

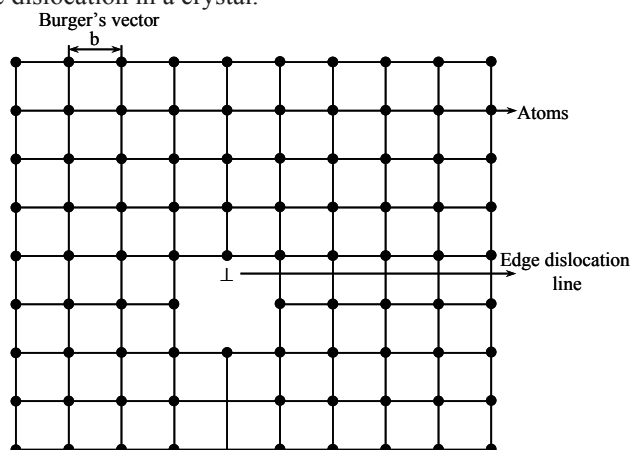
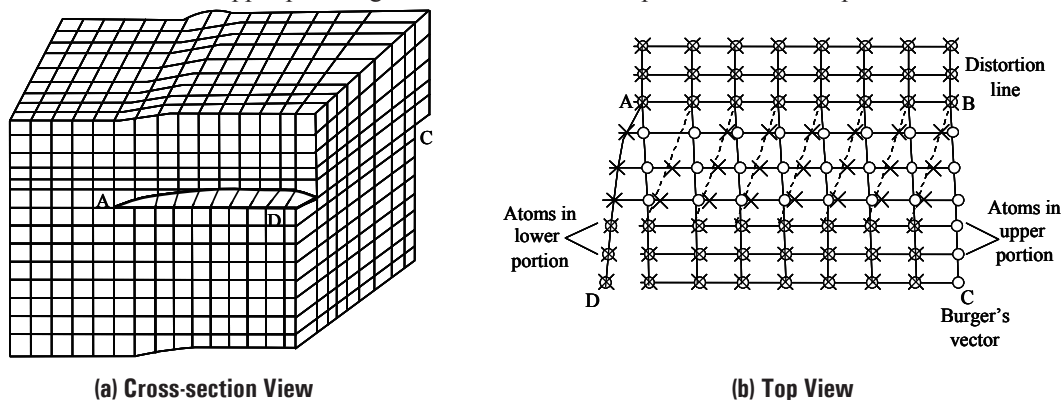


Figure (1): Edge Dislocation

The perpendicular line that passes through the termination point in the extra layer of atom is known as edge dislocation line. The atoms present above this line occurs in a compressed manner and those present below are pulled away.

(ii) Screw Dislocation

The displacement of atoms through one atomic distance by the upper portion of crystal by applied shearing stress is referred as screw dislocation. The upper portion gets dislocated more compared to the lower portion as shown in figure (2).



(a) Cross-section View

(b) Top View

Figure (2): Screw Dislocation

Here, the line AB is the dislocation line that is parallel to Burger's vector and dislocation line are parallel to each other, then screw dislocation is represented as \curvearrowright and if Burger's vector and dislocation line are perpendicular to each other, then screw dislocation is represented as \curvearrowleft .

(iii) **Mixed Dislocation**

The crystals in which both edge and screw dislocations occurs simultaneously is referred to as mixed dislocation and is depicted in figure (3). Here line AB is the dislocation line. The Burger vector is neither parallel nor perpendicular to the dislocation line.

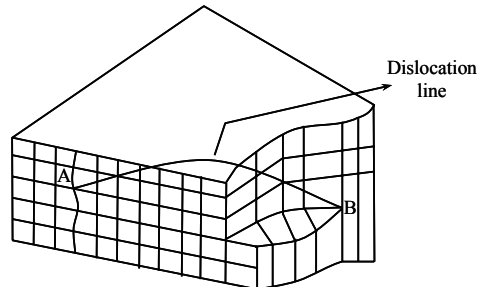


Figure (3): Mixed Dislocation

Q40. Distinguish between edge and screw dislocations.

Answer :

Differences between Edge and Screw Dislocation

Edge Dislocation		Screw Dislocation	
1.	The Burger's vector lies perpendicular to the dislocation line.	1.	The Burger's vector lies parallel to the dislocation line.
2.	Edge dislocation occurs due to slight variation in the placement of adjacent part of growing crystal.	2.	Screw dislocation occurs due to twist in stacking sequence of atoms, during crystallization process.
3.	It contains shear, tensile and compressive stresses.	3.	It contains only shear stress.
4.	It consist of an incomplete plane which lies above (or) below the slip plane.	4.	It consist of lattice planes which scroll around the lattice planes identical to left hand (or) right hand screw.
5.	It has a specific glide plane.	5.	It doesn't have a unique glide plane.
6.	Along an edge of the lattice, the region of disturbances increases.	6.	In case of screw dislocation, the region of disturbances increases in two distinct planes (which are perpendicular to each other).

Q41. What is Burger's vector? In what direction do the Burger's vector lie with respect to,

- (i) An edge dislocation
- (ii) Screw dislocation.

Answer :

Burger's Vector

Burger's vector of a dislocation is an essential property of a dislocation as it helps in describing the entire dislocation. The method of finding the Burger's vector of an edge dislocation is shown in figure.

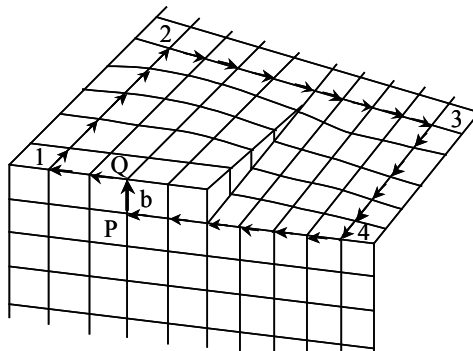


Figure: Burger's Circuit and Vector of a Screw Dislocation

Determining Burger's vector in a positive direction and find a vector which closes the loop. The loop so formed is called a Burger's circuit. The loop needs to close by itself and if it does not include any dislocation then it fails to do so by an amount called Burger's vector (b). As the loop fails to get closed along itself, a Burger's vector ' b ' (\overrightarrow{PQ}) can be represented in the figure which points in a direction parallel to the dislocation.

Significance of Burger's Vectors

The Burger vector of a dislocation is an important property of a dislocation because if the Burger vector and the orientation of the dislocation line are known, the dislocation is completely described. This indicates how much and in what direction the lattice above the slip plane appears to have been shifted with respect to the lattice below the slip plane.

(i) Edge Dislocation

The Burger's vector lies perpendicular to the dislocation line.

(ii) Screw Dislocation

The Burger's vector lies parallel to the dislocation line.

Q42. Describe the line defects and surface defects.

Answer :

Dec.-17, Q16(a)

Line Defects

For answer refer Unit-1, Q39.

Surface Defects

The imperfections present in the regions of atoms causing disturbances in the atomic diameter are called surface defects. Surface defects are classified as follows :

- (a) Grain Boundary
- (b) Tilt Boundary
- (c) Twin Boundaries
- (d) Stacking Faults

Grain Boundary

An individual crystal present in a polycrystalline solid is known as grain. Two grains are separated by a region in a crystal, wherein the atoms are arranged in a systematic order. The misalignment of atoms in the region between the two grains is known as Grain boundary.

At the grain boundaries atoms are loosely arranged which gives less efficient packing of atoms. Hence, atoms in the grain boundary possess higher energy than those inside the grains.

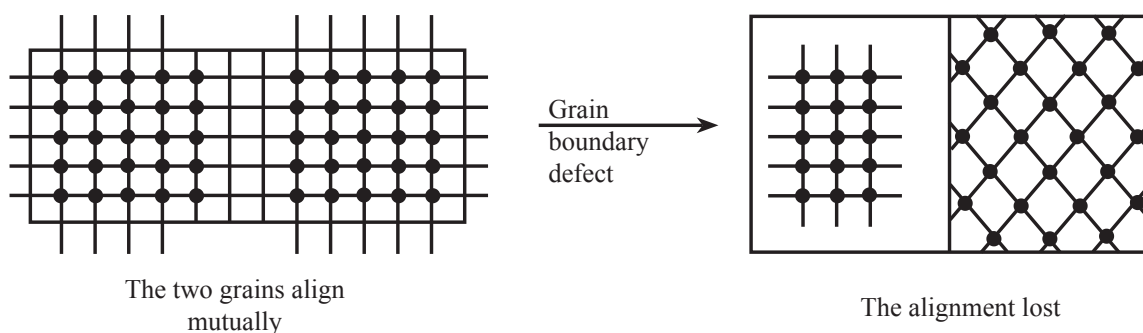


Figure (i): Grain Boundary Defect

Tilt Boundary

This defect is an advancement of grain boundary. Tilt is produced when the orientation between two neighbouring grains is less than 10° . It is an array of dislocations. It is also known as low angle grain boundary. The grain boundary obtained when the orientation between two grains is greater than $10-15^\circ$ is called high angle grain boundary.

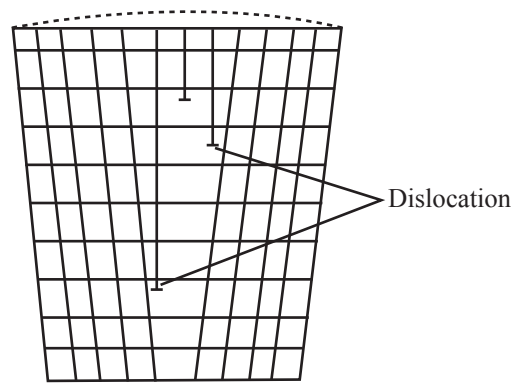


Figure (ii): Tilt Boundary Imperfection

Twin Boundaries

The surface defect in which two orientations are separated by a region and one orientation is the mirror image of the other is known as Twin boundary. The region between these two orientations is called twinned plane (or) region.

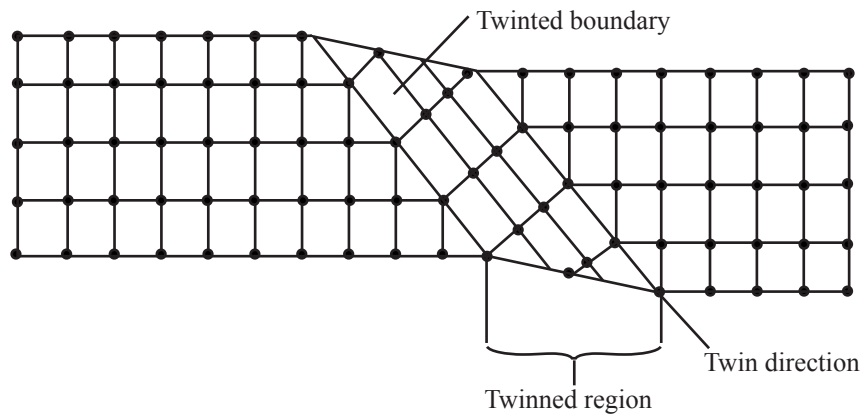


Figure (iii): Twin Boundaries Imperfection

Stacking Faults

When the stacking sequence of atomic planes is broken by a dislocation, it results in stacking fault. In this type of defect, an atomic plane will be missing from the normal sequence. This type of defect is generally observed in FCC crystal structure.

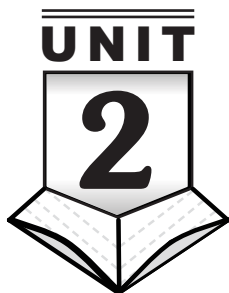
Example : In FCC crystal, the normal sequence of stacking atomic planes is,

.....ABC ABC ABC.....

If A is missing from the above sequence then the atomic sequence thus obtained is,

.....ABC BC ABC.....

This type of surface defect is known as stacking fault.



BAND THEORY OF SOLIDS AND SEMICONDUCTORS, DIELECTRIC MATERIALS

PART-A SHORT QUESTIONS WITH SOLUTIONS

Q1. Write the assumptions involved in classical free electron model.

Answer :

The assumptions involved in classical free electron theory of metals are,

- (i) The valence electrons in atoms of metals are free to move. These electrons move in the spaces between the ions within metal similar to gaseous molecules and are called free electron gas. These electrons participate in electrical conduction and hence are called conduction electrons.
- (ii) There is no interaction between conduction electrons.
- (iii) The interaction between free electrons and ion cores is negligible
- (iv) The free electrons form uniform electric field in the metal.

Q2. What are the drawbacks of the classical theory of free electrons in metals?

Answer :

The drawbacks of classical theory of free electrons in metals are,

1. The value of specific heat given by classical free electron theory ($\approx 4.5 R$) is not same as that of experimental value ($\approx 3 R$), where 'R' is called the universal gas constant.
2. This theory will not allow us to describe the electrical conductivity of semiconductors and insulators.
3. The electrical and thermal conductivities of metal at low temperatures are not constant, which violates the classical free electron theory at any temperature.
4. The classical free electron theory value of paramagnetic susceptibility will vary from experimental value. However, the concept of ferromagnetism is not explained by this theory.
5. The phenomenons like photoelectric effect, compton effect and the black body radiation are not able to explained by this theory.

Q3. Give the expression for electrical conductivity of a metal on the basis of classical free electron theory.

Answer :

The expression for electrical conductivity of a metal on the basis of classical free electron theory is given by,

$$\sigma = \frac{ne^2\tau}{m} = \frac{nev_d}{E} = ne\mu$$

Where,

n - Number of electrons per unit volume

e - Charge of electron

τ - Average relaxation time

m - Mass of electron

v_d - Drift velocity

E - Electric field intensity

μ - Mobility of electrons.

Q4. The relaxation time of electrons in a conductor at 300 K is equal to 1.99×10^{-14} sec. Determine the electrical conductivity of that conductor. [Concentration of electrons = 8.5×10^{28} per m^3]

Answer :

Given that,

For a conductor,

Relaxation time, $\tau = 1.99 \times 10^{-14}$ sec

Concentration of electrons, $n = 8.5 \times 10^{28}/m^3$

Temperature, $T = 300$ K

Then, the expression for electrical conductivity of a conductor is given by,

$$\begin{aligned}\sigma &= \frac{ne^2\tau}{m} \\ &= \frac{8.5 \times 10^{28} \times (1.602 \times 10^{-19})^2 \times 1.99 \times 10^{-14}}{9.1 \times 10^{-31}} \\ &= \frac{43.41 \times 10^{-24}}{9.1 \times 10^{-31}} \\ &= 4.76 \times 10^7 \text{ ohm}^{-1} \text{ m}^{-1} \\ \therefore \sigma &= 4.76 \times 10^7 \text{ ohm}^{-1} \text{ m}^{-1}\end{aligned}$$

Q5. For a metal having 6.5×10^{28} conduction electrons per m^3 , calculate relaxation time of electrons, if the metal has the resistivity 1.43×10^{-8} ohm-m (Mass of electron = 9.1×10^{-31} kg).

Answer :

Given that,

For a metal,

Concentration of electrons, $n = 6.5 \times 10^{28}/m^3$

Resistivity, $\rho = 1.43 \times 10^{-8}$

Mass of electron, $m = 9.1 \times 10^{-31}$ kg

Then, the expression for relaxation time ' τ ' is given as,

$$\tau = \frac{m}{ne^2\rho} \quad \dots (1)$$

Substituting the values of m , n , e and ρ in equation (1),

$$\begin{aligned}\tau &= \frac{9.1 \times 10^{-31}}{6.5 \times 10^{28} \times (1.602 \times 10^{-19})^2 \times 1.43 \times 10^{-8}} \\ &= 38.15 \times 10^{-15} \text{ sec} \\ \therefore \tau &= 38.15 \times 10^{-15} \text{ sec}\end{aligned}$$

Q6. Write the conclusions given by Kronig-Penney model.

Answer :

The conclusions drawn from Kronig-Penney model are,

- The energy spectrum of the electron consists of a number of allowed energy band separated by forbidden region.
- The width of the allowed energy band increases with increasing value of energy.
- The allowed bands are narrowest for low value of energy and become broader as energy increase.

- The width of a particular allowed band decreases with increasing binding energy of the electron and when binding energy of electron becomes zero, then we have free electron model and energy spectrum is continuous.

Q7. Calculate the root mean square velocity of the electron at 27°C on the basis of classical free electron theory (Note: $K = 8.6 \times 10^{-5}$ eV/k, $m = 9.1 \times 10^{-31}$ Kg).

Answer :

Dec.-17, Q2

Given that,

For a basis of classical free electron theory,

Temperature, $T = 27^\circ\text{C} = 27 + 273 = 300\text{K}$

Mass, $m = 9.1 \times 10^{-31}$ kg

Boltzmann constant, $k = 8.6 \times 10^{-5}$ eV/k

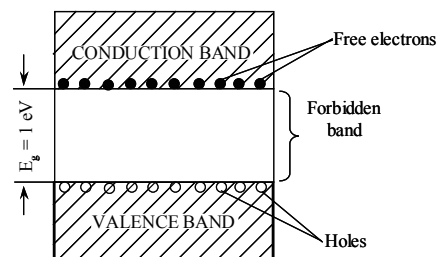
The expression for root mean square velocity of an electron of mass ' m ' is given by,

$$\begin{aligned}V_{rms} &= \sqrt{\frac{3kT}{m}} \\ &= \sqrt{\frac{3 \times 8.6 \times 10^{-5} \times 1.602 \times 10^{-19} \times 300}{9.1 \times 10^{-31}}} \\ &= \sqrt{\frac{123.99 \times 10^{-22}}{9.1 \times 10^{-31}}} \\ &= \sqrt{13.6 \times 10^9} \\ &= 116727.3 \\ &= 1.167 \times 10^5 \\ \therefore V_{rms} &= 1.167 \times 10^5 \text{ m/s.}\end{aligned}$$

Q8. Define semiconductor.

Answer :

A semiconductor is a substance for which width of the forbidden energy region is relatively small (i.e., ~ 1 eV). The conductivity of semiconductor is much greater than that of an insulator because, the number of free electrons in a semiconductor lies in the range of 10^7 to 10^{28} . Figure below shows the energy band structure of a semiconductor.



Figure

Q9. Define intrinsic semiconductor.

Answer :

Intrinsic Semiconductor

This type of semiconductor is made out of the semiconductor material in its extreme pure form.

Example

Ge and Si which have forbidden energy gaps of 0.72 eV and 1.1 eV respectively.

When an electric field is applied to an intrinsic semiconductor at a temperature greater than 0 K, conductor's electrons move to the anode and also the holes in the valence band move to the cathode.

An intrinsic semiconductor may be defined as in which the number of conduction electrons are equal to the number of holes.

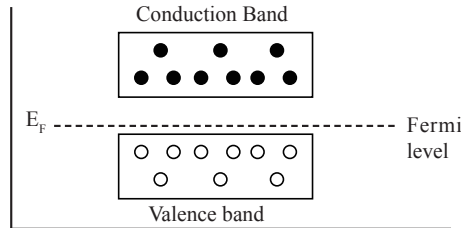


Figure: Energy Band Diagram of an Intrinsic Semiconductor at Room Temperature

Q10. For an intrinsic semiconductor having band gap $E_g = 0.7$ eV, calculate the density of holes and electrons at room temperature (27°C). Given, $K = 1.38 \times 10^{-23}$ J/K and $h = 6.62 \times 10^{-34}$ J.

Answer :

April-16, Q7

Given that,

For an intrinsic semiconductor,

Band gap, $E_g = 0.7$ eV

Room temperature, $T = 27^\circ\text{C}$

Boltzmann's constant, $K = 1.38 \times 10^{-23}$ J/K

Planck's constant, $h = 6.62 \times 10^{-34}$ J

Then, the expression of hole density in an intrinsic semiconductor is given by,

$$\begin{aligned}
 p &= \frac{1}{4} \left[\frac{2m_h K T}{\pi h^2} \right]^{\frac{3}{2}} \exp \left[\frac{-E_g}{K T} \right] \\
 &= \frac{1}{4} \left[\frac{2 \times 1.66 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}{\pi \times (6.62 \times 10^{-34})^2} \right]^{\frac{3}{2}} \exp \left[\frac{-0.7 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300} \right] \\
 &= \frac{1}{4} \times (9.983 \times 10^{18})^{\frac{3}{2}} \times 1.782 \times 10^{-12} \\
 &= 14.052 \times 10^{15} \text{ m}^{-3} \\
 \therefore p &= 14.052 \times 10^{15} \text{ m}^{-3}
 \end{aligned}$$

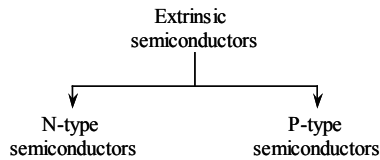
And the expression of electron density in an intrinsic semiconductor is given by,

$$\begin{aligned}
 n &= \frac{1}{4} \left[\frac{2m_e K T}{\pi h^2} \right]^{\frac{3}{2}} \exp \left[\frac{-E_g}{K T} \right] \\
 &= \frac{1}{4} \left[\frac{2 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{\pi \times (6.62 \times 10^{-34})^2} \right]^{\frac{3}{2}} \exp \left[\frac{-0.7 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300} \right] \\
 &= \frac{1}{4} \times (54.727 \times 10^{14})^{\frac{3}{2}} \times 1.782 \times 10^{-12} \\
 &= \frac{1}{4} \times 404.858 \times 10^{21} \times 1.782 \times 10^{-12} \\
 &= 1.804 \times 10^{11} \text{ m}^{-3} \\
 \therefore n &= 1.804 \times 10^{11} \text{ m}^{-3}
 \end{aligned}$$

Q11. Give the classification of extrinsic semiconductors.

Answer :

Depending on the type of doping material, extrinsic semiconductors are classified as follows,



(i) N-type Semiconductors

N-type semiconductors are obtained by adding pentavalent impurities like Phosphorous, Arsenic (other pentavalent elements) etc., to pure (or intrinsic) semiconductors. The electrons are the majority carrier in an N-type semiconductor. They are also called as donor type semiconductor, because they are capable of donating an electron.

(ii) P-type Semiconductors

When impurities like Aluminium, Boron and other trivalent elements are added to a pure semiconductor a P-type semiconductor is formed. In this type of semiconductor, large number of holes are present and thus this type is always ready to accept electrons. Hence, it is also known as acceptor type semiconductor.

Q12. Given the expression for carrier concentration of an intrinsic semiconductor.

Answer :

The expression for carrier concentration of an intrinsic semiconductor is given by,

$$n_i^2 = AT^3 \exp [-E_g/k_B T]$$

Where,

$$A - \text{Constant value} = 4 \left[\frac{2\pi k_B}{h^2} \right]^3 (m_e m_h)^{3/2}$$

T - Ambient temperature

E_g - Energy gap of the semiconductor $= E_c - E_v$

E_c - Conduction band energy level

E_v - Valance band energy level

k_B - Boltzman constant.

Q13. Mobilities of electrons and holes in a sample of intrinsic Ge at 300 K are $0.36 \text{ m}^2 \text{ V}^{-1} \text{ S}^{-1}$ and $0.17 \text{ m}^2 \text{ V}^{-1} \text{ S}^{-1}$ respectively. If the resistivity of the specimen is $2.12 \Omega \text{ m}$, compute the intrinsic concentration of carriers for Ge.

Where $m_e^* = 0.5 m_0$ and $m_h^* = 0.37 m_0$

Answer :

Jan.-12, Q6

Given that,

For a Ge semiconductor,

Operating temperature, $T = 300 \text{ K}$

Mobility of electrons, $\mu_e = 0.36 \text{ m}^2 \text{ V}^{-1} \text{ S}^{-1}$

Mobility of holes, $\mu_p = 0.17 \text{ m}^2 \text{ V}^{-1} \text{ S}^{-1}$

Resistivity of specimen, $\rho = 2.12 \Omega \text{ m}$

Mass of electron, $m_e^* = 0.5 m_0$

Mass of hole, $m_h^* = 0.37 m_0$

Then, the expression for intrinsic carrier concentration of a semiconducting material is,

$$\begin{aligned} n_i &= \frac{1}{\rho e(\mu_p + \mu_e)} \quad \left[\because \sigma = \frac{1}{\rho} \right] \\ &= \frac{1}{2.12 \times 1.6 \times 10^{-19} \times (0.36 + 0.17)} \\ &= 5.562 \times 10^{18} \text{ m}^{-3} \\ \therefore n_i &= 5.562 \times 10^{18} \text{ m}^{-3} \end{aligned}$$

Q14. Give the expressions for carrier concentration of N-type and P-type semiconductors.

Answer :

The expression for carrier concentration of an N-type semiconductor is given by,

$$N_D = N_c \exp \left[\frac{E_F - E_c}{k_B T} \right]$$

Where,

N_c - Number of carriers in conduction band

E_F - Fermi level energy

E_c - Conduction band energy level

T - Ambient temperature

k_B - Boltzmann constant.

And the expression for carrier concentration of a P-type semiconductor is given by,

$$N_A = N_v \exp \left[\frac{E_v - E_F}{k_B T} \right]$$

Where, N_v - Number of carriers in valence band

E_v - Valence band energy level.

Q15. Give the expression for diode current.

Answer :

The expression for diode current is given by,

$$I = I_0 \left[e^{(V/\eta V_T)} - 1 \right]$$

Where,

I_0 - Diode reverse saturation current

V - Externally applied voltage to diode

η - Constant = 1 for Ge = 2 for Si

V_T - Thermal voltage $= \frac{T}{11600}$

T - Ambient temperature.

Q16. Draw the V-I characteristics of diode in forward bias and reverse bias condition.

Answer :

V-I Characteristics of Diode in Forward Bias

Figure (1) shows the V-I characteristic of the PN junction diode in forward bias.

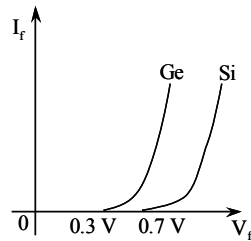


Figure (1): V-I Characteristics of Junction Diode

V-I Characteristics of Diode in Reverse Bias

Figure (2) shows the V-I characteristics of PN junction diode in reverse bias.

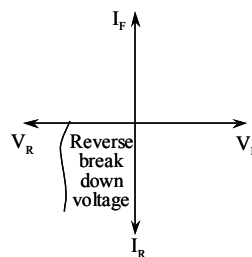


Figure (2): V-I Characteristics of Reverse Bias

Q17. What is Hall effect?

Answer :

When a current carrying semiconductor bar is kept in a transverse magnetic field, an electric field is produced within the semiconductor. The direction of generated electric field is perpendicular to both the current and applied magnetic field. This process is referred to as Hall effect. Due to the generated electric field, a voltage is developed across the surfaces of semiconductor bar known as Hall voltage.

Q18. Mention the uses of thermistor.

Answer :

Dec.-17, Q5

Uses of Thermistor

- (i) It is used as temperature sensors.
- (ii) It is widely used in electronics.
- (iii) It uses sensors to regulate cold, heat and voltage.
- (iv) It is used for controlling volume, time delays and circuit protection.

Q19. What is a dielectric?

Answer :

When a non-conducting material is characterized by charge storage (as the main function) in an electric field, then the material is termed as "Dielectric". Any insulator can be considered as a dielectric.

Q20. Define electronic polarization.

Answer :

Electronic polarization is the phenomenon of separating negatively charged electron shell from a positively charged nucleus, in the presence of electric field.

Electronic polarization can be obtained from the expression of dipole moment as,

$$\mu_e = 4\pi\epsilon_0 R^3 E$$

or

$$\mu_e = \alpha_e E$$

Where,

$\alpha_e = 4\pi\epsilon_0 R^3$ is a proportionality constant called electronic polarizability.

Q21. Define ionic polarization.

Answer :

A polarization which is caused by relative displacements between positive and negative ions in ionic crystals (i.e., NaCl) is called as ionic polarization.

Ionic polarization causes ferroelectric transition as well as dipolar polarization. The transition which is caused by the order of the directional orientations of permanent dipoles along a particular direction is called order-disorder phase transition. The transition which is caused by ionic polarizations in crystals is called displacive phase transition

Q22. Define orientation (or) dipolar polarization.

Answer :

The phenomenon in which presence of electric field produces alignment of polar substances in the direction of applied field is referred to as orientational or dipolar polarization. It is denoted by P_0 and is given as,

$$P_0 = N \vec{\mu}$$

$$\therefore P_0 = N\alpha_0 E$$

Where,

α_0 - Orientational polarizability.

Q23. What is dielectric constant?

Answer :

It is defined as the ratio of permittivity of medium (ϵ) to the permittivity of free space (ϵ_0). It is denoted by ϵ_r i.e.,

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

Since, it is the ratio of permittivity, it has no units. It is used to determine the dielectric characteristics of a material.

Q24. Define ferroelectricity.

Answer :

The property of the materials exhibiting the spontaneous electric polarization for below curie temperature is defined as ferro electricity. The spontaneous electric polarization is the polarization of material exhibited without application of any external field.

The materials exhibiting the property of ferroelectricity are called ferro-electric materials.

Q25. Explain the concept of spontaneous polarization in ferroelectrics.

Answer :

May/June-12, Q7

Ferroelectric materials comprises of tiny parts that are polarized in different directions in the absence of an electric field. This is because, the center of gravity of positive and negative charges do not coincide, the substance has an electric dipole moment and is said to be spontaneously polarized.

PART-B

ESSAY QUESTIONS WITH SOLUTIONS

2.1 BAND THEORY OF SOLIDS

2.1.1 Classical Free Electron Theory (Qualitative), Kronig - Penney Model (Qualitative Treatment) - Energy Band Formation in Solids

Q26. Discuss the free electron theory of metals.

Answer :

June-11, Q13(a)

This theory was developed by Drude and Lorentz in the year 1900. It states that, the metals which contain free electrons must obey the laws of classical mechanics.

The classical free electron theory depends on certain postulates as mentioned below,

1. A metal consists of atoms which have a nucleus around which the electrons revolve.
2. The valence electrons in the atoms can move over the entire volume of the metal. Electron gas is a combination of all such valence electrons in the atoms.
3. These free electrons move randomly in all directions, which results in a collision with positive ions fixed to the lattice or other free electrons. These collisions are elastic, as there is no energy loss.
4. The motion of free electrons obey the laws of classical kinetic theory of gases.
5. The metal electron velocities obey the classical. Maxwell-Boltzmann distribution of velocities.
6. Due to the fixed ions in the lattice, the movement of free electrons will be in a uniform potential field.
7. In an electric field, the free electrons in the metal accelerate in the direction opposite to the direction of electric field.

Q27. Describe the qualitatively theory of Kronig-Penney model and what are the conclusions can be drawn from it.

Dec.-17, Q11(a)

OR

Explain, how Kronig-Penney model of solids lead to energy band formation?

Answer :

June-11, Q13(b)

The potential energy of an electron according to the Kronig-Penney model is assumed to be in the form of a square well as shown in figure (1).

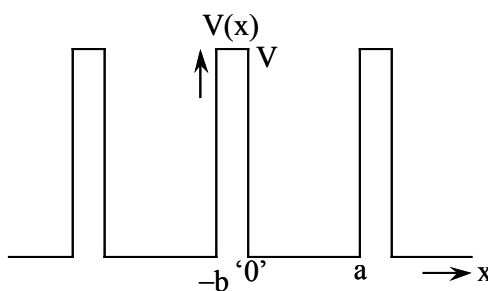


Figure (1): Kronig-Penney Model

Potential near an atom is supposed to be represented by each potential well. If the period of the potential is $(a + b)$ then,

$$U = 0 \text{ for } 0 < x < a$$

$$U = V \text{ for } -b < x < 0$$

Therefore, the Schrodinger equation for the above two regions is given as,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E \psi = 0 \text{ for } 0 \leq x \leq a \quad \dots (1)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0 \text{ for } -b \leq x \leq 0 \quad \dots (2)$$

To evaluate the boundary conditions, we request two more real quantities which can be defined assuming that energy of the electron (E) is less than the potential of electron (V). Therefore, the two defined real quantities are, α and β , and are defined as,

$$\alpha^2 = \frac{2mE}{h^2} \text{ and } \beta^2 = \frac{2m(V-E)}{h^2}$$

From this two definitions equations (1) and (2) can be written as,

$$\frac{\partial^2 \psi}{\partial x^2} + \alpha^2 \psi = 0 \quad \text{for } 0 \leq x \leq a \quad \dots (3)$$

$$\text{And } \frac{\partial^2 \psi}{\partial x^2} - \beta^2 \psi = 0 \quad \text{for } -b \leq x \leq 0 \quad \dots (4)$$

Solving the above second order differential equations,

$$U_1 = We^{i(\alpha-k)x} + Xe^{i(\alpha+k)x} \text{ for } 0 < x < a \quad \dots (5)$$

$$U_2 = Ye^{(\beta-ik)x} + De^{-(\beta+ik)x} \text{ for } b < x < 0 \quad \dots (6)$$

Where W, X, Y and Z are constants, which are selected in such a way that they satisfy the below condition,

$$U_1(0) = U_2(0)$$

$$U_1(a) = U_2(-b)$$

$$\left(\frac{dU_1}{dx} \right)_{x=0} = \left(\frac{dU_2}{dx} \right)_{x=0}$$

$$\left(\frac{dU_1}{dx} \right)_{x=a} = \left(\frac{dU_2}{dx} \right)_{x=-b}$$

Because of the boundary conditions, a new condition arises i.e.,

$$\cos k(a+b) = \frac{\beta^2 - \alpha^2}{2\alpha\beta} \cdot \sinh\beta b \cdot \sin\alpha a + \cosh\beta b \cdot \cos\alpha a \quad \dots (7)$$

Kronig and Penney also assumed that potential barrier will change to delta functions containing V which tends to infinity when ' b ' approaches zero. But in reality on contrary to their assumption, the product of V and b i.e., V_b remains finite. As a result, equation (7) changes to,

$$\left(\frac{mb}{h^2\alpha} \right) \sin\alpha a + \cos\alpha a + \cos ka$$

or

$$\cos ka = \left(\frac{P}{\alpha a} \right) \sin\alpha a + \cos\alpha a \quad \dots (8)$$

Where,

$$P = \frac{mba}{h^2}$$

Figure (2) shows a plot of the left and right side of equation (8). As the left side of the equation (8) is $\cos ka$, the value of functions of left side of the equation should fall only between -1 and $+1$. The energy spectrum region between the range -1 to $+1$ is called 'allowed energy band' and the region outside the range is known as '1' forbidden energy band,

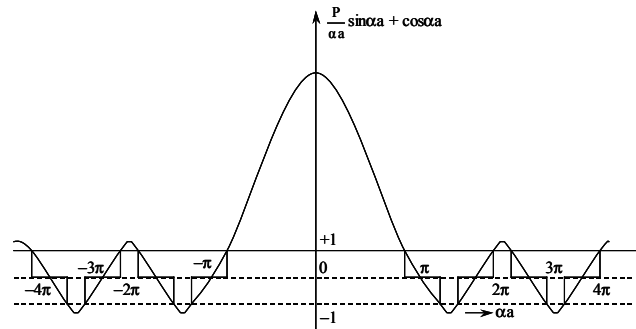


Figure (2): Left Hand Side of the Equation Plotted against αa

From the above figure, it is clear that some values for the left side of the equation (8) exceed more than 1 and fall in the forbidden region. Hence, it is clear that energy spectrum of electron contains numerous allowed energy bands separated by forbidden energy bands.

Q28. Discuss the band theory of solids and explain the formation of bands.

Answer :

Band Theory of Solids

In case of solids, the atoms are arranged in a systematic space lattice and hence the atom is greatly influenced by neighbouring atoms. The closeness of atoms results in the intermixing of electrons of neighbouring atoms. For the valence electrons in the outermost shells which are not strongly bonded by nucleus. Due to intermixing, the number of permissible energy levels increase or there are significant changes in the energy levels. Hence, in case of solids, instead of single energy levels associated with the single atom, there will be bands of energy levels. A set of such closely packed energy levels is called an energy band. Now, the bands of energy levels are referred to the entire solid as a whole and not to the single individual atom. Here it should be remembered that individual energies within the band are so close together that for many purposes, the energy bands may be considered to be continuous.

The concept of energy bands can easily be understood with the help of figure (1).

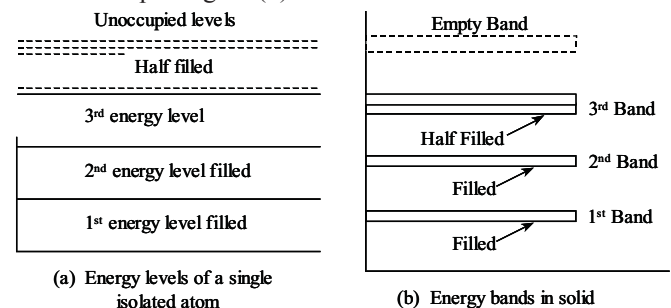


Figure (1): Energy Bands of Silicon

Figure 1(a) shows the energy levels of a single isolated atom of silicon. Here an electron can have only single energy corresponding to the orbit in which it exists.

Each silicon atom has 14 electrons, two of them will occupy the K shell, 8 occupy the L shell and 4 electrons occupies the M-shell. However, two electrons of M shell are in sub-shell, 3s and two are in sub-shell 3p, i.e., the 3p sub-shell is half-filled because it can accommodate a total of 6 electrons and 3d sub-shell is left unoccupied.

Consider a silicon atom in the solid (containing 200 atoms). It is obvious that there would be 200 K-shells differing in energy by extremely small amounts. The reason is that no two electrons in this shell see exactly the same charge environment. The 200 K-shells, differing in extremely small amounts of energy are so close as to merge into one energy band. This is called first band as shown in figure 1(b). Similarly, second orbit electrons from second energy band and so on.

Valence Band, Conduction Band and Forbidden Energy Gap

Generally, the atoms of a solid are arranged in a regular repeated geometric pattern and the electrons of the atoms rotate around their nuclei in certain permitted energy levels. The electrons in the inner shells are strongly bounded to their nuclei while the electrons in the outer most shells are not strongly bonded to their nuclei. The electrons in the outermost shell are called valence electrons. The band formed by a series of energy levels containing the valence electrons is known as valence band. The valence band may be defined as a band which is occupied by the valence electrons or a band having highest occupied band energy.

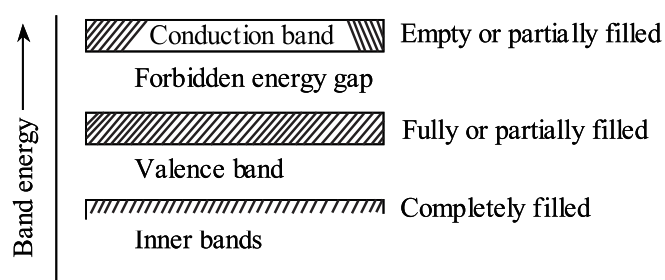


Figure (2): Valence Band and Conduction Band

The valence band is shown in figure (2). The valence band may be partially or completely filled depending upon the nature of the crystal. This band can never be empty.

In certain materials (metals), the valence electrons are loosely attached to the nucleus. Even at ordinary temperature, some of valence electrons leave the valence band. These are called free electrons. These are responsible for the conduction of current in a conductor and hence also called as conduction

electrons. The band occupied by these electrons is called conduction band as shown in figure (2). This band is next to the valence band. The conduction band may be empty or partially filled. In conduction band, the electrons can move freely. When a substance has empty conduction band, the current conduction is not possible. Generally, insulators have empty conduction band.

The separation between conduction band and valence band is known as forbidden energy gap. There is no allowed energy state in this gap and hence no electron can stay in the forbidden energy gap. It should be remembered that greater is the energy gap, more tightly the valence electrons are bound to the nucleus. In order to push an electron from valence band to conduction band external energy is required which is equal to the forbidden energy gap.

Formation of Bands

This can be explained with the help of energy level spectrums of an isolated atoms. When atoms of a matter are at far distances, as in the case of gases, the electron clouds of atoms will not overlap. Therefore, mutual interaction between neighbouring atoms can be neglected. As such, the energy level's structure of such matter is almost similar to that of an isolated atom.

However, when atoms are brought much closer, as in the case of solids, electron clouds of neighbouring atoms overlap on each other. Due to this (interaction), the original energy levels of individual atoms get mix-up and rearrange themselves such that they form a closely spaced continuum of levels. Each energy band of this state consists of as many levels as the number of atoms in the solid. This new energy level structure represents the spectrum of whole solid.

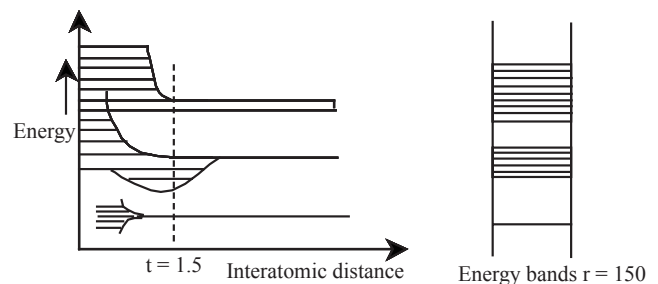


Figure (3)

Figure depicts the energy band formation in solids. Energy spectrum is discrete at large values of interatomic distance r . But as the value of r decreases, energy levels of individual atoms mix-up forming energy bands. At a particular distance $r = r_0$, each energy band is separated by a finite energy gap.

Q29. Classify the solids into conductors, insulators and semiconductors on the basis of band theory of solids.

Answer :

Jan.-13, Q5

Based on band theory of solids, materials are classified into insulators, semiconductors and conductors are described as follows,

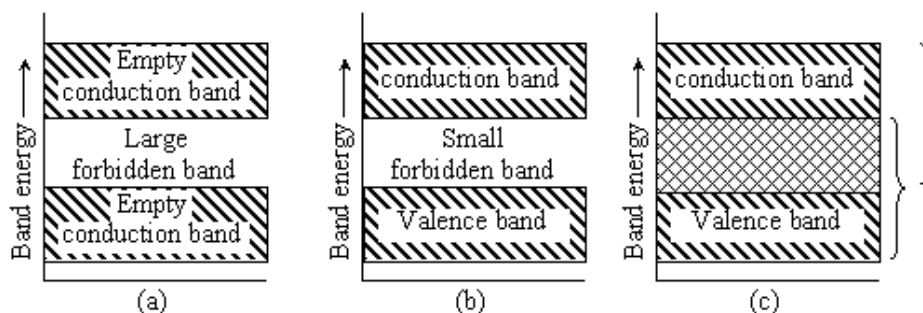


Figure (1): Insulator (b) Semiconductor (c) Conductor

Insulator

A material which is having extremely poor electrical conductivity is known as an insulator. Figure (2) shows the energy band structure of an insulator. In this, a large separation is present between filled valence region and vacant conduction band. Thus, it is not possible to carry the particle from filled region to vacant band due to the small energy applied by external field. Hence the electron cannot acquire sufficient energy from externally applied energy, which makes conduction just impossible. For example, diamond is an insulator with band energy gap of 6 eV.

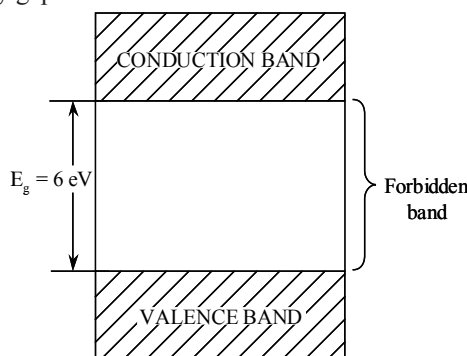


Figure (2)

Semiconductor

A semiconductor is a substance for which width of the forbidden energy region is relatively small (i.e., ~ 1 eV). The conductivity of semiconductor is much greater than that of an insulator because, the number of free electrons in a semiconductor lies in the range of 10^7 to 10^{28} . Figure (3) shows the energy band structure of a semiconductor. In this, the conductivity of the material is increased by the following two ways,

1. By increasing temperature (These are called as intrinsic semiconductors).
2. By adding impurity atoms into the crystal (These are called as extrinsic semiconductors).

For example, germanium and silicon are semiconductors with energy gap values of 0.785 eV and 1.21 eV respectively.

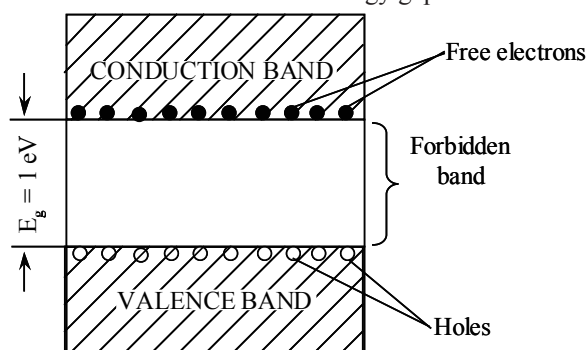


Figure (3)

Conductor

A material in which conduction is only due to the electrons is known as conductor. Due to the absence of forbidden energy region in conductors, the valence band merges into the conduction band, which is empty. Figure (4) shows the energy band structure of a conductor. The electrons in the low energy state may acquire additional energy and move into higher energy states under the influence of an applied electric field.

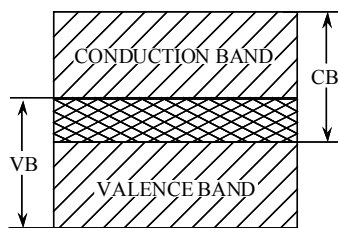


Figure (4)

Q30. Based on energy bands, distinguish between conductors, semiconductors and insulators.

Answer :

June-14, Q15(a)

Conductors		Semiconductors		Insulators	
1.	In conductors, forbidden energy gap is not present.	1.	Forbidden energy gap is small so that, considerable number of electrons cross it.	1.	Forbidden energy gap is very large so that very few electrons cross it at room temperature.
2.	Energy gap is zero.	2.	Energy gap is very less and is about 1 eV.	2.	Energy gap varies from 5 eV to 10 eV.
3.	Effective number of electrons is very large.	3.	Effective number of electrons is non-zero but less than in metals.	3.	Effective number of electrons is zero.
4.	In conductors there is high electrical conductivity.	4.	In these, electrical conductivity is much smaller than in metals.	4.	In these, there is poor electrical conductivity.
5.	Conductors have positive temperature coefficient of resistivity.	5.	Semiconductors have negative temperature coefficient of resistivity.	5.	Insulators have negative temperature coefficient of resistivity.
6.	In conductors, conduction is by the electrons.	6.	In semiconductors, conduction is by holes and electrons.	6.	In insulators, there is no conduction.

2.2 SEMICONDUCTORS

2.2.1 Intrinsic and Extrinsic Semiconductors

Q31. Briefly explain about the classification of semiconductors with suitable examples.

Answer :

Semiconductors can be classified into two groups. They are as follows,

1. Intrinsic (or) pure semiconductors
2. Extrinsic (or) impure semiconductors.

1. Intrinsic Semiconductors

For answer refer Unit-2, Q9.

2. Extrinsic Semiconductors

The intrinsic semiconductors to which some suitable impurity (or) doping agent has been added in extremely small amount are called extrinsic semiconductors.

Case I

The doping agents are pentavalent atoms having five valence electrons in their outer most orbit (or) trivalent atoms having three valence electrons in the outer most orbit. Pentavalent doping atom is known as donor atom because it contributes one electron to the conduction band of pure germanium.

Case II

Trivalent atom is called as acceptor atom because it accepts one electron from the germanium atom.

Finally, depending on the type of doping material used, extrinsic semiconductors can be further divided into two groups. They are as follows,

- (a) *n*-type semiconductors
- (b) *p*-type semiconductors.

Q32. Distinguish between intrinsic and extrinsic semiconductors.**Answer :**

June-14, Q6

The differences between intrinsic and extrinsic semiconductors is mentioned below:

Intrinsic Semiconductors		Extrinsic Semiconductors	
1.	Intrinsic semiconductor is a pure form of semiconductor.	1.	Extrinsic semiconductor is an impure form of semiconductor.
2.	The electrical conductivity of intrinsic semiconductor is relatively low.	2.	Extrinsic semiconductor has higher electrical conductivity.
3.	Number of holes is equal to number of electrons in intrinsic semiconductor.	3.	Number of holes is not equal to number of electrons in extrinsic semiconductor.
4.	In intrinsic semiconductors, conductivity is increased by increasing the temperature.	4.	In extrinsic semiconductors, conductivity is increase by adding small amount of impurities to intrinsic semiconductors through doping process.
5.	Example: Group-IV elements such as crystals of pure silicon and Germanium.	5.	Example: Crystals of germanium and silicon doped with impurities such as As(Arsenic), Sb(Antimony) and P(Phosphorus).

Q33. Explain the need for doping and discuss different types.**Answer :**

The process of adding impurities to the crystal of an intrinsic (pure) semiconductors is called doping. Generally, at room temperature semiconductors do not conduct electricity and act as insulators. So, some amount of impurities must be added to them to imbalance the covalent bonds of crystal structure to make them conduct. Therefore, the impurity added is called 'Dopant' and the doped semiconductor material is called 'extrinsic semiconductor'. The conductivity of intrinsic semiconductors increases due to doping and in practice, extrinsic semiconductors are preferred for making various electronic devices such as diodes, transistors, etc.

Based on the type of impurities used, the extrinsic semiconductors are classified as follows,

1. N-type semiconductor
2. P-type semiconductor.

1. N-type Semiconductors

N-type semiconductors are obtained by adding pentavalent impurities like Phosphorous, Arsenic (other pentavalent elements) etc., to pure (or intrinsic) semiconductors. The electrons are the majority carrier in an N-type semiconductor. They are also called as donor type semiconductor, because they are capable of donating an electron.

The concentration of impurity atoms added to the pure semiconductor is very small, such that one pentavalent impurity atom is surrounded by 4 atoms of Ge (or) Si (pure semiconductor). The 5 valence electrons of impure atoms are shared by 4 Ge (or) Si to form covalent bonds. The extra 5th electron of the impure atom can be dislodged by supplying small range of energies (0.01 eV to 0.05 eV). Hence, free electron exists for every impurity atom. Thus, the number of electrons are high in this type of semiconductor.

Figure (1) shows the covalent bond between a pentavalent impurity atom Ar and Ge.

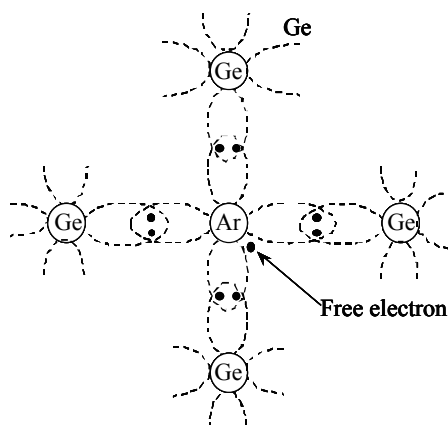


Figure (1): Covalent Bonds between Ar and Ge

2. P-type Semiconductor

When impurities like Aluminium, Boron and other trivalent elements are added to a pure semiconductor a P-type semiconductor is formed. In this type of semiconductor, large number of holes are present and thus this type is always ready to accept electrons. Hence, it is also known as acceptor type semiconductor.

The impurities like Aluminium, Boron and other trivalent atoms have three valence electrons and same as in case of N-type semiconductor, each impure atom is surrounded by four Ge (or) Si atoms.

The three valence electrons form covalent bonds with three Ge(or) Si atoms and the fourth Ge(or) Si atom has no electron to form a bond with the impure atom. So, when sufficient energy is supplied, the electron deficient bond gets the electron from the nearby bond, this way the electrons moves in one direction which is nothing but movement of hole in the reverse direction to that of electrons. In this way, hole moves and generates hole current.

Figure (2) shows the covalent bond between a trivalent impurity Al and Ge.

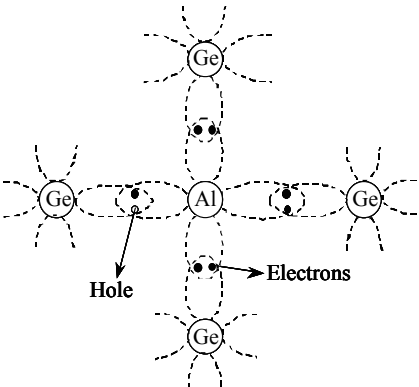


Figure (2): Covalent Bond between Aluminium (Al) and Germanium (Ge)

Q34. Write the differences between n-type and p-type semiconductor materials.

Answer :

n-type Semiconductor		p-type Semiconductor	
1.	n-type semiconductors are obtained by adding pentavalent impurities (like phosphorous, Arsenic) to pure semiconductors.	1.	p-type semiconductors are obtained by adding trivalent impurities (like Aluminium, Boron) to pure semiconductors.
2.	These are also known as DONOR type semi-conductors, because they are capable of donating electrons.	2.	These are also known as ACCEPTOR type semi-conductors, because they are ready to accept electrons.
3.	Electrons are the majority charge carriers.	3.	Holes are the majority charge carriers.
4.	It consists of 5 valence electrons.	4.	It consists of 3 valence electrons.
5.	Electrons move and generate electron current.	5.	Holes move and generate hole current.

2.2.2 Concept of a Hole - Carrier Concentration and Conductivity in Intrinsic Semiconductors

Q35. Explain concept of hole.

Answer :

June-17, Q5

Concept of Hole

Hole specifies the absence of an electron. The properties of hole are same as that of electron except that they possess a positive charge. These are present in the valence band alone. The direction of motion of holes is in the direction of applied electric field due to its positive charge. Other properties of hole are mentioned below.

- 1. Energy of hole is same as that of electron but opposite in magnitude i.e.,

$$E_h = - E_e$$

- 2. Effective mass of hole at the top of the valence band is positive.
- 3. The expression for current of a hole is given by,

$$J_h = neV.$$

Q36. Deduce an expression for carrier concentration in intrinsic semiconductor.

June-17, Q13(b)

OR

Derive an expression for the carrier concentration of an intrinsic semiconductor.

Answer :

In a semiconductor, two types of free charge carriers exist i.e., electrons in the conduction band and holes in the valence band. In general, it is taken that the conduction band is extended from E_C to $+\infty$ and the valence band is extended from ∞ to E_V .

(i) Computation of Electron Density

Let ' n ' be the number of electrons per unit volume of a homogeneously doped semiconductor crystal in equilibrium. If the conduction band extends from E_C to ∞ , then the electron density can be written as,

$$n = \int g_e(E) p_e(E) dE \quad \dots (1)$$

$$\text{But, } g_e(E) = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} (E - E_C)^{1/2} \text{ and}$$

$$p_e(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{K_B T} \right)}}$$

Hence,

$$n = \int_{E_C}^{\infty} \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} \frac{(E - E_C)^{1/2}}{1 + \exp\left(\frac{E - E_F}{K_B T} \right)} \frac{(E - E_C)^{1/2}}{1 + e^{\left(\frac{E - E_F}{K_B T} \right)}} dE$$

In this integral, the energy E being considered is always greater than E_C . If temperature T is $E_C - E_F \gg 5K_B T$ then the expression reduces to,

$$n = \int_{E_C}^{\infty} \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} \frac{(E - E_C)^{1/2}}{e^{\left(\frac{E - E_F}{K_B T} \right)}} dE \quad \dots (2)$$

$$\text{Substituting } x = \frac{E - E_C}{K_B T}, dx = \frac{1}{K_B T} dE,$$

$$dE = K_B T dx \text{ and } (E - E_C)^{1/2} = (K_B T)^{1/2} (x)^{1/2}$$

$$\text{And } e^{\left(\frac{E - E_F}{K_B T} \right)} = e^{\left(\frac{E - E_C}{K_B T} \right)} e^{\left(\frac{E_C - E_F}{K_B T} \right)} = e^x e^{\left(\frac{E_C - E_F}{K_B T} \right)}$$

Substituting the above values and changing the limits in equation (2),

$$n = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} \int_0^{\infty} \frac{(K_B T)^{1/2} x^{1/2} (K_B T) dx}{e^x e^{\left(\frac{E_C - E_F}{K_B T} \right)}}$$

$$n = \frac{1}{2\pi^2} \left(\frac{2m_e K_B T}{\hbar^2} \right)^{3/2} e^{\left[\frac{-(E_C - E_F)}{K_B T} \right]} \int_0^{\infty} x^{1/2} e^{-x} dx \quad \dots (3)$$

From standard mathematical tables,

$$\int_0^{\infty} x^{1/2} e^{-x} dx = \frac{\sqrt{\pi}}{2} \quad \dots (4)$$

Substituting equation (4) in equation (3),

$$n = \frac{1}{2\pi^2} \left(\frac{2m_e K_B T}{\hbar^2} \right)^{3/2} e^{\left[\frac{-(E_C - E_F)}{K_B T} \right]} \frac{\sqrt{\pi}}{2}$$

$$n = \frac{1}{4\pi^2} \left(\frac{2m_e K_B T}{\hbar^2} \right)^{3/2} \sqrt{\pi} e^{\left[\frac{-(E_C - E_F)}{K_B T} \right]}$$

$$\text{i.e., } n = \frac{1}{4} \left(\frac{2m_e K_B T}{\hbar^2 \pi} \right)^{3/2} e^{\left[\frac{-(E_C - E_F)}{K_B T} \right]}$$

$$n = \frac{1}{4} \left[\frac{2m_e K_B T}{\hbar^2 \pi} \right]^{3/2} e^{\left[\frac{-(E_C - E_F)}{K_B T} \right]} \left[\because \hbar = \frac{h}{2\pi} \right]$$

$$n = \frac{8}{4} \left(\frac{2\pi m_e K_B T}{h^2} \right)^{3/2} e^{\left[\frac{-(E_C - E_F)}{K_B T} \right]}$$

$$n = \left(\frac{2\pi m_e K_B T}{h^2} \right)^{3/2} e^{\left(\frac{E_F - E_C}{K_B T} \right)}$$

$$n = N_C e^{\left(\frac{E_F - E_C}{K_B T} \right)} \quad \dots (5)$$

Where,

$$N_C = 2 \left(\frac{2\pi m_e K_B T}{h^2} \right)^{3/2} \quad \dots (6)$$

Which is called the effective electron density in the conduction band.

(ii) Computation of Hole Density

Let p be the number of holes per unit volume of a homogeneously doped semiconductor crystal in equilibrium. If the valence band extend from $-\infty$ to E_V , then the hole density, p in a homogeneous doped semiconductor can be written as,

$$p = \int_{-\infty}^{E_V} g_h(E) p_h(E) dE \quad \dots (7)$$

$$\text{But, } g_h(E) = \frac{1}{2\pi^2} \left(\frac{2m_h}{\hbar^2} \right)^{3/2} (E_V - E)^{1/2}$$

$$\text{And } p_h(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{K_B T} \right)}}$$

Hence,

$$p = \int_{-\infty}^{E_V} \frac{1}{2\pi^2} \left(\frac{2m_h}{\hbar^2} \right)^{3/2} \frac{(E_V - E)^{1/2}}{1 + e^{\left(\frac{E - E_F}{K_B T} \right)}} dE$$

In general, the energy E being considered always lesser than E_V . If the temperature is such that $E_{Fi} - E_V \approx 5K_B T$.

$$p = \int_{\infty}^{E_V} \frac{1}{2\pi^2} \left(\frac{2m_h}{\hbar^2} \right)^{3/2} e^{\left[\frac{E_V - E_{Fi}}{K_B T} \right]} (E_V - E)^{1/2} dE \quad \dots (8)$$

$$\text{Sub } \frac{E_V - E}{K_B T} = x, dx = \frac{-dE}{K_B T}, dE = -K_B T dx,$$

$$\text{And } (E_V - E)^{1/2} = (K_B T)^{1/2} x^{1/2},$$

$$\text{And } e^{\left[\frac{(E_V - E_{Fi})}{K_B T} \right]} = e^{\left[\frac{-(E_V - E_{Fi})}{K_B T} \right]} e^{\left[\frac{(E_V - E_{Fi})}{K_B T} \right]} \\ = e^{-x} e^{\left(\frac{(E_V - E_{Fi})}{K_B T} \right)}$$

Substituting the above values and changing the limits in equation (8),

$$p = \frac{1}{2\pi^2} \left(\frac{2m_h}{\hbar^2} \right)^{3/2} \int_0^{\infty} - (x)^{1/2} (K_B T)^{3/2} e^{-x} dx e^{\left(\frac{E_V - E_{Fi}}{K_B T} \right)} \\ p = \frac{1}{2\pi^2} \left(\frac{2m_h K_B T}{\hbar^2} \right)^{3/2} e^{\left(\frac{E_V - E_{Fi}}{K_B T} \right)} \int_0^{\infty} x^{1/2} e^{-x} dx \quad \dots (9)$$

From standard integrals,

$$\int_0^{\infty} x^{1/2} e^{-x} dx = \sqrt{\pi}/2 \quad \dots (10)$$

Substituting equation (10) in equation (11),

$$p = 2 \left(\frac{2\pi m_h K_B T}{\hbar^2} \right)^{3/2} e^{\left(\frac{E_V - E_{Fi}}{K_B T} \right)} \quad \dots (11)$$

$$p = N_V e^{\left(\frac{E_V - E_{Fi}}{K_B T} \right)}$$

Where,

$$N_V = 2 \left(\frac{2\pi m_h K_B T}{\hbar^2} \right)^{3/2} \quad \dots (12)$$

Which is known as effective hole density in the valence band.

From mass-action law,

$$np = n_i^2$$

$$\Rightarrow np = 4 \left[\frac{2\pi m_e K_B T}{\hbar^2} \right]^{3/2} \left[\frac{2\pi m_h K_B T}{\hbar^2} \right]^{3/2} e^{\left(\frac{E_i - E_C}{K_B T} \right)} e^{\left(\frac{E_V - E_i}{K_B T} \right)} \\ = n_i^2$$

$$\Rightarrow n_i^2 = 4 \left(\frac{2\pi K_B}{\hbar^2} \right)^3 (m_e m_h)^{3/2} T^3 e^{\left(\frac{E_V - E_C}{K_B T} \right)} \quad [\because n_i = p_i]$$

$$\therefore n_i^2 = AT^3 e^{\left(-E_g/K_B T \right)}$$

Where, $A = 4 \left(\frac{2\pi K_B}{\hbar^2} \right)^3 (m_e m_h)^{3/2}$ is a constant value

of semiconductor and $E_g = E_C - E_V$ is the energy gap of the semiconductor.

It is to be noted that A and E_g are constants for a given semiconductor and n_i is called the intrinsic concentration which indicates thermally generated electrons and holes. It is a strong function of temperature T .

Q37. Explain the nature of conductivity in intrinsic semiconductors.

Answer :

Jan.-13, Q6

The conductivity of semiconductor lies between insulators and conductors. At 0°K temperature they behave as insulators. When temperature is raised their conductivity increases, that is, conductivity is directly proportional to temperature.

$$\sigma \propto T$$

When temperature increases, large number of charge carriers are generated due to breaking of covalent bonds, this charge carries increases the conductivity.

Derivation for Conductivity in Intrinsic Semiconductor

In a semiconductor (Intrinsic (or) extrinsic), the conduction is possible due to negative charged electron and positive charged hole. If the electric field is applied to the semiconductor, the motion of electrons is towards positive terminal and holes towards the negative terminal, but the direction of current is same for both. Hence, the total current density is obtained by the addition of electron and hole current density.

$$\text{i.e., } J = J_p + J_n \quad \dots (1)$$

Where,

J_p - Hole current density, and is given by $Pq\mu_p E$

J_n - Electron current density and is given by $nq\mu_n E$

Substituting J_p and J_n values in (1),

$$J = (n\mu_n + p\mu_p) qE \quad \dots (2)$$

But the current density (J) is given as SE

$$\text{i.e., } J = \sigma E \quad \dots (3)$$

Comparing equations (2) and (3),

$$\sigma = (n\mu_n + p\mu_p) q \quad \dots (4)$$

Where,

σ - Conductivity of semiconductor

p - Concentration of holes

n - Concentration of electrons

μ_p - Mobility of holes

μ_n - Mobility of electrons.

For intrinsic semiconductor, number of holes is equal to number of electrons i.e.,

$$n = p = n_i \quad \dots (5)$$

Substituting equation (5) in equation (4),

$$\sigma = (n_i \mu_n + n_i \mu_p) q$$

$$\Rightarrow \sigma = n_i (\mu_n + \mu_p) q$$

$$\therefore \text{Conductivity of intrinsic semiconductor } (\sigma_i) = n_i (\mu_n + \mu_p) q.$$

2.2.3 Formation of P-N Junction Diode and its I-V Characteristics

Q38. What do you understand by depletion region at PN junction? What is the effect of forward and reverse biasing of PN junction on the depletion region? Explain with necessary diagrams.

Answer :

Depletion Region

In a PN junction P-type consists of holes and N-type consists of electrons. The PN junction with P-type on left side and N-type on right side is shown in figure (1).

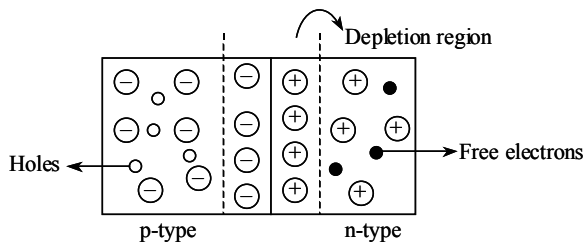


Figure (1)

Due to diffusion, the large number of holes from left side diffuse to right side and similarly, the large number of electrons from right side diffuse to left side.

Due to this displacement, P-side losses holes and forms a negative electric field to the left side of junction and N-side losses electrons and forms a positive electric field to the right side of the junction. Because of this large movement of holes and electrons, a barrier potential is developed across the junction. Finally, the holes will combine with free electrons and gets disappear leaving negative potential at P-side near the junction. Similarly, the free electrons will combine with holes and gets disappear leaving positive potential at N-side near the junction. This region at the junction is known as depletion region.

The thickness of depletion region is the order of few microns.

Where,

$$1 \text{ micron} = 10^{-4} \text{ cm.}$$

Forward Bias

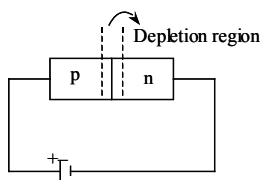


Figure (2)

In forward bias, the thickness of depletion layer is very thin because p-type is connected to positive terminal and the n-type is connected to the negative terminal. This causes the holes and electrons to move freely across the junction, hence resulting in a large current.

Reverse Bias

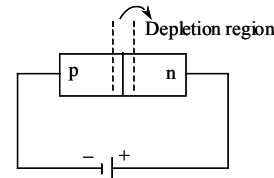


Figure (3)

In reverse bias, as the p-type is connected to the negative terminal and n-type is connected to the positive terminal, the force of attraction takes place, so the holes from p-side and the electrons from n-side moves away from the junction, thus increasing the width of depletion region. This results in a very little current, almost equal to zero. Therefore, in reverse bias the thickness of the depletion layer is large.

Q39. Explain the volt-ampere characteristics of PN diode.

Answer :

PN Junction under Forward Bias

Figure (1) shows the V-I characteristic of the PN junction diode.

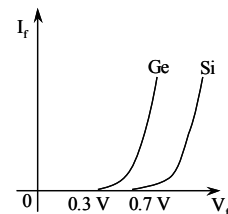


Figure (1): V-I Characteristics of Junction Diode

The forward current I_f will be zero because the forward voltage is increased to $V_f < V_0$, by the potential barrier, it will now prevents the holes for P-region and electrons from N-region to flow across the depletion layer but in opposite direction.

If $V_f > V_0$ the barrier will completely disappear and by this the holes will cross the P-region and enters to N-region and the electrons will enter into the P-region. By this large current will be flowing in the circuit. The threshold voltage for germanium and silicon is 0.3 V and 0.7 V.

PN Junction under Reverse Bias

Figure (2) shows the V-I characteristics of PN junction diode in reverse bias. The covalent bond which was formed by the electrons in the semiconductor atom now they will absorb the energy from light and heat to cause breaking of some bonds. By this electrons and holes form a pair continuously. In this reverse bias the holes in the P-region and electrons in N-region will be attracted towards the supply terminals. By this there will be some current produced which is called reverse current. It is also called as reverse saturation current.

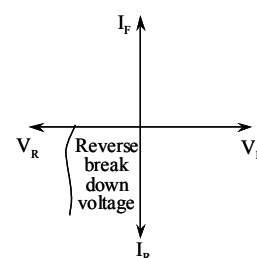


Figure (2): V-I Characteristics of Reverse Bias

This current depends upon the temperature. If this reverse current goes on increasing at a point we get a condition called avalanche of free electrons at this state the junction will be completely broken this voltage is called breakdown voltage V_{BD} .

2.2.4 Thermistor and its Characteristics - Hall Effect and Its Characteristics

Q40. Write a short note on thermistor.

Answer :

Thermistor is a temperature sensitive resistor, that is, the resistance varies with temperature in a pre-designed fashion. Thermistors are widely known as temperature sensors. These have high temperature coefficients. Depending upon the temperature coefficient the thermistors can be classified as,

1. Positive temperature coefficient thermistor and
2. Negative temperature coefficient thermistor.

Positive temperature coefficient thermistor is the one whose resistance increases with increase in temperature and negative temperature coefficient thermistor is the one whose resistance decreases with increase in temperature.

The relationship between resistance and temperature obeys exponential form of law as,

$$R = R_0 e^{B/T}$$

Where,

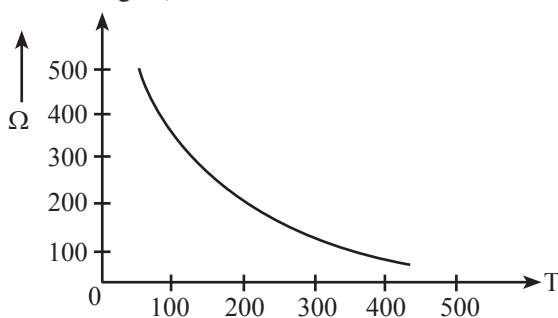
R - Resistance of thermistor at a temperature ' T '

R_0 - Resistance of thermistor at absolute zero temperature ' T '

B - Constant

T - Temperature

The resistance temperature characteristics of thermistor is shown in figure,



Figure

Q41. State and explain Hall effect.

Answer :

Dec.-17, Q17(b)

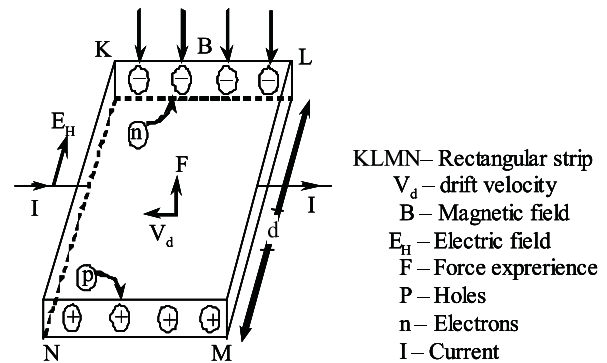
Hall Effect

When a current carrying semiconductor bar is kept in a transverse magnetic field, an electric field is produced within the semiconductor. The direction of generated electric field is perpendicular to both the current and applied magnetic field.

This process is referred as Hall effect. Due to the generated electric field, a voltage is developed across the surfaces of semiconductor bar known as Hall voltage.

Explanation

Figure shows a rectangular strip of semiconductor carrying current along the positive x direction.



Figure

The majority charge carriers of the unknown type of semiconductor continue to conduct the current as long as there is no external field.

Whenever magnetic field B is applied perpendicularly to the direction of current the majority charge carriers experience a force. It is assumed that the majority charge carriers in the semiconductor (shown in the diagram) are electrons. These electrons are deflected with a force F and move towards the KL face of the rectangular strip. The minority carriers (i.e., holes) also divert towards the MN face of the semiconductor strip.

On account of this charge gradient across KL and MN , an electric field is setup in the rectangular strip.

The mathematical expression for equilibrium condition of the electric and magnetic force is given as,

$$e E_H = e V_d B \quad \dots (1)$$

$$\Rightarrow \frac{V_H}{d} = V_d B \quad \left(\because E = \frac{V}{d} \right)$$

$$\Rightarrow V_H = V_d B$$

Where, V_H - Hall voltage

$$\Rightarrow V_H = \frac{J}{ne} B d \quad [\because \text{Current density, } J = ne V_d] \dots (2)$$

Where, n -type of charge carrier (i.e., electrons). Thus, by determining Hall voltage and n , the type of the semiconductor can be determined.

Q42. What is Hall effect? Deduce the expression for Hall coefficient in case of a semiconductor.

Answer :

May/June-12, Q14(b)

Hall Effect

For answer refer Unit-2, Q17.

Expression for Hall Coefficient

Hall Coefficient of p-type Semiconductor

When a *p*-type semiconductor bar is placed in a transverse magnetic field, an electric field is established in the direction normal to both magnetic field and current '*I*'. Figure(i) shows the *p*-type semiconductor bar, which is aligned with the coordinate system.

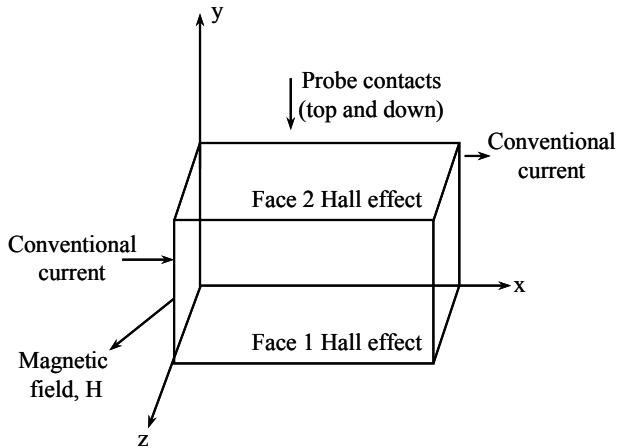


Figure (i)

The magnetic force F_L exerted by the holes on application of magnetic field '*B*' is given by,

$$F_L = eBv_d \quad \dots (1)$$

Where,

e - Charge of electron or hole

v_d - Average drift velocity of holes.

Due to the electric field generated, holes experience an electric force ' F_E ' and is given by,

$$F_E = eE \quad \dots (2)$$

At equilibrium condition, the electric force ' F_E ' balances the magnetic force ' F_L ' i.e.,

$$\begin{aligned} F_E &= F_L \\ \Rightarrow eE &= eBv_d \\ \Rightarrow E &= Bv_d \quad \dots (3) \end{aligned}$$

If the width of the *p*-type semiconductor bar is '*W*', then the electric field intensity can be written as,

$$E = \frac{V_H}{W} \quad \dots (4)$$

From equations (3) and (4),

$$Bv_d = \frac{V_H}{W} \quad \dots (5)$$

For a *p*-type semiconductor, surface '1' will be more positive compare to surface '2'. Then, the expression for current density of holes is given by,

$$J = \frac{I}{A} = pev_d \quad \dots (6)$$

Substituting the value of ' V_d ' from equation (6) in equation (5),

$$\begin{aligned} B \left(\frac{J}{pe} \right) &= \frac{V_H}{W} \\ V_H &= \frac{BJW}{pe} \quad \dots (7) \end{aligned}$$

But, the Hall coefficient is defined as,

$$R_H = \frac{E}{JB} = \frac{V_H}{WJB}$$

From equation (7),

$$\begin{aligned} R_H &= \frac{I}{A} \\ &= \frac{1}{\rho} \quad [\because \rho = pe] \\ \therefore R_H &= \frac{1}{pe} = \frac{1}{\rho} \end{aligned}$$

Hall Coefficient of n-type Semiconductor

When a *n*-type semiconductor bar is placed in a transverse magnetic field, an electric field is established in the direction normal to both magnetic field and current '*I*'. Figure (ii) shows the *n*-type semiconductor bar, which is aligned with the coordinate system.

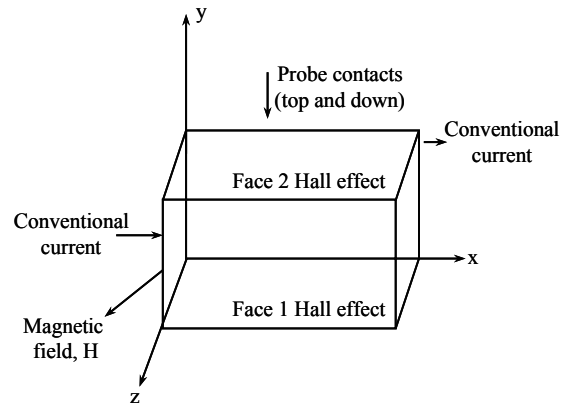


Figure (ii)

The magnetic force F_L exerted by the electrons on application of magnetic field '*B*' is given by,

$$F_L = eBv_d \quad \dots (1)$$

Where,

e - Charge of electron or hole

v_d - Average drift velocity of electrons.

Due to the electric field generated, electrons experience an electric force ' F_E ' and is given by,

$$F_E = eE \quad \dots (2)$$

At equilibrium condition, the electric force ' F_E ' balances the magnetic force ' F_L ' i.e.,

$$\begin{aligned} F_E &= F_L \\ \Rightarrow eE &= eBv_d \\ \Rightarrow E &= Bv_d \quad \dots (3) \end{aligned}$$

If the width of the n -type semiconductor bar is ' W ', then the electric field intensity can be written as,

$$E = \frac{V_H}{W} \quad \dots (4)$$

From equations (3) and (4),

$$BV_d = \frac{V_H}{W} \quad \dots (5)$$

For a n -type semiconductor, surface '1' will be more negative compare to surface '2'. Then, the expression for current density of electrons is given by,

$$J = \frac{I}{A} = -nev_d \quad \dots (6)$$

Here, '- ve' sign indicates that the charge of electrons is negative.

Substituting the value of ' V_d ' from equation (6) in equation (5),

$$B \left(\frac{-J}{ne} \right) = \frac{V_H}{W}$$

$$V_H = \frac{-BJW}{ne} \quad \dots (7)$$

But, the Hall coefficient is defined as,

$$R_H = \frac{E}{JB} = \frac{V_H}{WJB}$$

From equation (7),

$$R_H = \frac{-1}{ne} = \frac{1}{\rho} \quad [\because \rho = -ne]$$

$$\therefore R_H = \frac{-1}{ne} = \frac{1}{\rho}$$

Q43. What is Hall effect and derive the expression for the Hall coefficient, carrier concentration and mobility?

Answer :

June-14, Q15(b)

Hall Effect and Expression for Hall Coefficient

For answer refer Unit-2, Q42.

Carrier Concentration

For p-type semiconductor,

$$\text{Hall coefficient, } R_H = \frac{1}{pe} = \frac{1}{\rho}$$

$$\therefore p = \frac{1}{R_H e} = \frac{\rho}{e}$$

Where,

p - Hole concentration

For n-type semiconductor,

$$\text{Hall coefficient, } R_H = \frac{1}{ne} = \frac{1}{\rho}$$

$$\therefore n = \frac{1}{R_H e} = \frac{\rho}{e}$$

Where,

n - Electron concentration

Therefore,

$$p = n = \frac{\rho}{e}$$

Mobility

For p-type semiconductor,

$$\text{Conductivity, } \sigma = pe\mu_p$$

Where,

μ_p - Hole mobility

$$\therefore \mu_p = \frac{\sigma}{pe} = \sigma R_H$$

For n-type semiconductor,

$$\text{Conductivity, } \sigma = ne\mu_n$$

Where,

μ_n - Electron mobility

$$\therefore \mu_n = \frac{\sigma}{ne} = \sigma R_H$$

Therefore,

$$\mu_p = \mu_n = \sigma R_H$$

Q44. Explain the applications of Hall-effect.

Answer :

Applications of Hall-effect

1. Determination of Type of Semiconductor

The sign of the Hall coefficient depends on the type of semiconductor placed in the transverse magnetic field. Hall coefficient is negative for an n -type semiconductor and positive for a p -type semiconductor. Therefore, from the sign of the Hall coefficient we can determine whether a given semiconductor is n or p -type.

2. Calculation of Carrier Concentration

Concentration of the carrier inside the semiconductor bar placed in the magnetic field can be determined using the expression.

$$n = \frac{1}{e} R_H = \frac{1}{e} \frac{V_H b}{I_x B}$$

Where,

B - Magnetic flux density in Wb/m²

I_x - Current flowing through the semiconductor bar

' b ' - Width of the bar and

' V_H ' - Hall voltage.

The Hall voltage ' V_H ' is measured by placing two probes at the centers of the top and bottom surfaces of the semiconductor bar.

3. Determination of Mobility

Assuming that the flow of current in the n -type semiconductor bar placed in the magnetic field is only due to majority carriers, that is, electrons, we have,

$$\sigma = nq\mu_n$$

$$\Rightarrow \mu_n = \frac{\sigma}{ne} = \sigma R_H$$

$$\text{Since, } R_H = \frac{V_H b}{I_x B}$$

$$\therefore \mu_n = \sigma \left(\frac{V_H b}{I_x B} \right)$$

Using σ , mobility μ_n can be determined.

Similarly, we can determine μ_p by placing a p -type semiconductor bar in the magnetic field.

4. Hall Effect Multiplier

If the magnetic field ' B ' (in which the semiconductor bar is placed) is produced by an air-core coil carrying a current ' I' ' through it, then B is proportional to I' . The Hall voltage which is a product of B and I , (I is the current flowing through the bar) will now be a product of I' and I . This forms a basis of the multiplier.

2.3 DIELECTRIC MATERIALS

2.3.1 Dielectrics - Types of Polarizations - Electronic, Ionic - Orientational and Space, Charge Polarizations - Expression for Electronics Polarizability

Q45. What are dielectric materials? How are they classified based on permanent dipole moment?

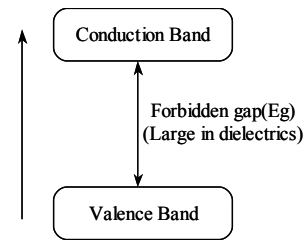
Answer :

Dielectric Materials

When a non-conducting material is characterized by charge storage (as the main function) in an electric field, then the material is termed as "Dielectric". Any insulator can be considered as a dielectric.

A dielectric has a very large forbidden gap(E_g) that enables to excite electrons from full valence band to empty conduction band which is impossible under normal conditions. Therefore, the resistivity of an ideal dielectric is infinitely higher value and in practice, they conduct to a very negligible range of resistivities (10^{10} to 10^{20} ohm-m).

Dielectrics are characterized by its properties such as dielectric constant, dielectric loss, dielectric strength and resistivity. Dielectric constant and dielectric loss are strongly dependent on frequency variation.



Figure

Dielectric materials can be classified into two types based on permanent or induced dipole moment. They are,

- (i) Non-polar dielectrics
- (ii) Polar dielectrics.

(i) Non-polar Dielectrics

The dielectrics which undergoes polarization only in the presence of electric field are referred to as non-polar dielectrics. There is no permanent dipole moment in the absence of electric field.

Consider a dielectric slab, which is subjected to an electric field by placing between the plates of a charged condenser. Due to the electric field applied, the positive charges in the dielectric move towards the negative plate of the condenser, while the negative charges are pulled by the positive and negative charges are separated resulting in dielectric polarization.

The behaviour of non-polar dielectrics in electric field is as illustrated in figure (1).

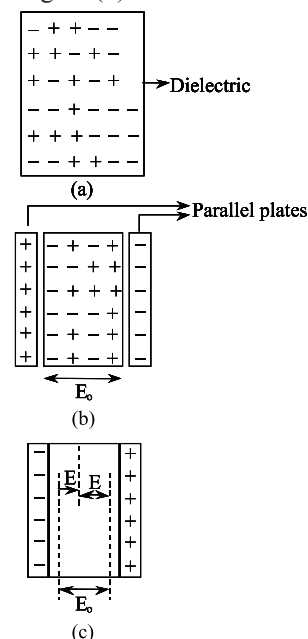


Figure (2): Non-polar Dielectrics in Electric Field

Figure 2(a) shows a non-polar dielectric in the absence of electric field having random distribution of positive and negative charges. The application of electric field (E_0) to this dielectric results in accumulation of charges at the surface as

shown in figures 2(b) and 2(c). The electric field (E') setup by the induced surface charges opposes the external electric field (E_0) as such the resultant field (E) is,

$$E = E_0 - E'$$

Thus, the resultant field is obtained as the vector sum of the fields E_0 and E' .

(ii) Polar Dielectrics

Polar dielectrics have permanent dipole moments with their random orientations as shown in figure.

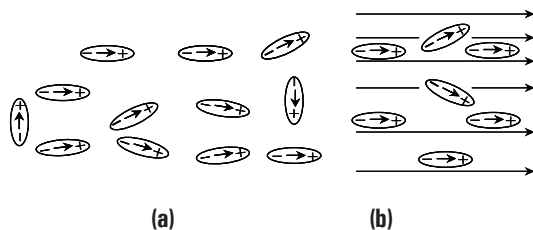


Figure (3): Polar Dielectrics in an Electric Field

In polar dielectric permanent dipole moment existed even in the absence of electric field. In the presence of an electric field, the partial alignment of dipoles takes place as shown in figure 2(b). As the molecules are always in thermal agitation, the alignment will never be perfect. The alignment increases with the increase of electric or with the decrease of temperature.

Q46. Explain the space charge polarization.

Dec.-17, Q10

(Refer Only Space Charged Polarization)

OR

Explain different types of polarization mechanism in dielectrics.

June-17, Q13(a)

OR

Explain the origin of different kinds of Polarization in dielectric materials.

Answer :

Polarization in dielectric materials are mainly classified into three types, based on the displacement of charges in the electric field. The different kinds of polarization in dielectric materials are,

- (i) Electronic polarization
- (ii) Ionic polarization and
- (iii) Orientation or dipolar polarization
- (iv) Space charged Polarization

(i) Electronic Polarization

For answer refer Unit-2, Q20.

(ii) Ionic Polarization

For answer refer Unit-2, Q21.

(iii) Orientation or Dipolar Polarization

For answer refer Unit-2, Q22.

(iv) Space charged Polarization

A polarization which is produced by the separation of positive and negative charges in an electrolyte for an applied electric field is called as space charged polarization. This polarization is negligibly small.

Consider a container with electrolyte solution and two electrodes. In this, the positive and negative ions are in stable state i.e., not separated.

When an electric field is produced by connecting the electrodes to the battery, the positive charges gets accumulated on the negative electrode and negative charges on positive electrode. This migrations or separation produces an induced polarization due to the existence of dipole moment as shown in figures (a) and (b).

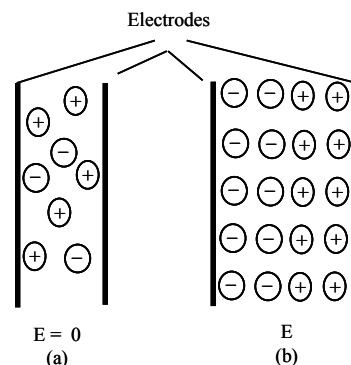


Figure : Space Charge Polarization

Q47. What are dielectrics? Explain various electrical polarization mechanism.

Answer :

Jan.-12, Q14(a)

Dielectrics

For answer refer Unit-2, Q19.

Types of Electrical Polarization Mechanisms

For answer refer Unit-2, Q46.

Q48. Derive the expression for electronic polarizability in case of dielectrics.

Answer :

May/June-12, Q15(a)

Electronic Polarization

For answer refer Unit-2, Q20.

Derivation of Electronic Polarizability

Consider a sphere of volume $\frac{4}{3}\pi R^3$ as shown in figure (i) below,

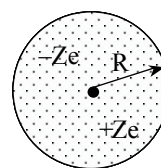


Figure (i)



Figure (ii)

Here, the nucleus charge ' Ze ' is surrounded by electron charges $-Ze$. The electron charge density of electron is given by,

$$\rho_e = \frac{-Ze}{\frac{4}{3}\pi R^3} \quad \dots (1)$$

Where, R - Radius of the sphere.

When the electric field is applied on the sphere, the nucleus and electrons experience the Lorentz forces of magnitude $|ZeE|$ in opposite directions. As a result Ze and $-Ze$ get separated forming a circle of radius ' r ' as shown in figure (ii).

The electron charge within the sphere of radius ' r ' is given by,

$$q_n = \rho_e \frac{4}{3}\pi r^3 \quad \dots (2)$$

From equation (1),

$$\rho_e \frac{4}{3}\pi = \frac{-Ze}{R^3} \quad \dots (3)$$

Where,

R - Outer radius of sphere

Substituting equation (3) in equation (2),

$$q_n = -Ze \left(\frac{r}{R} \right)^3$$

The coulomb force developed between the separated charges is given by,

$$\begin{aligned} F_c &= \frac{1}{4\pi\epsilon_0} \left[\frac{\text{Charge enclosed by the sphere of radius 'r'}}{r^2} \right] \times Ze \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{-Ze \left(\frac{r}{R} \right)^3}{r^2} \right] \times Ze \\ &= \frac{-(Ze)^2 r}{4\pi\epsilon_0 R^3} \end{aligned}$$

Where,

ϵ_0 - Permittivity

At equilibrium state, Lorentz force = - Coulomb force

$$\text{i.e., } ZeE = \frac{(Ze)^2 r}{4\pi\epsilon_0 R^3}$$

$$\Rightarrow E = \frac{Ze r}{4\pi\epsilon_0 R^3}$$

$$\Rightarrow r = \frac{4\pi\epsilon_0 R^3}{Ze} E$$

The separated charges in a sphere form an electric dipole. Then, the induced dipole moment is given by,

$$\begin{aligned} \mu &= Ze r \\ &= Ze \left[\frac{4\pi\epsilon_0 R^3}{Ze} E \right] \\ &= 4\pi\epsilon_0 R^3 E \quad \dots (4) \end{aligned}$$

The dipole moment is also defined as,

$$\mu = \alpha_e E \quad \dots (5)$$

Comparing equations (4) and (5),

$$\alpha_e = 4\pi\epsilon_0 R^3$$

\therefore Electronic polarizability, $\alpha_e = 4\pi\epsilon_0 R^3$

2.3.2 Frequency and Temperature Dependence of Dielectric Polarizations - Determination of Dielectric Constant by Capacitance Bridge Method

Q49. Discuss the frequency dependence of various polarization processes in dielectric materials.

Answer :

April-16, Q16(a)

Frequency Dependence of Polarization

Polarization process occurs as a function of time under the application of alternating electric field. Different types of polarizations have different effects on them as the frequency electronic polarization occurs at all frequencies.

When the frequency of the applied voltage is high in the optical range, ionic polarization occurs which is slower than electronic polarization. Ionic polarization occurs only upto infrared frequencies because ions in the dielectric do not respond to the electric field (due to inertia).

Oriental or dipolar polarization is slower than ionic polarization and does not respond beyond microwave frequency. It occurs only at lower frequencies (50-60 Hertz).

Thus beyond microwave frequency, the total polarizability is due to ionic and electronic processes and beyond infrared frequencies, the total polarizability is electronic polarizability.

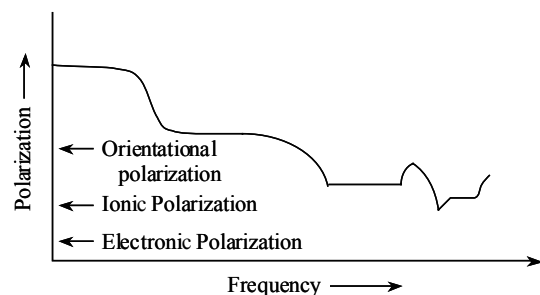


Figure :Frequency Dependence of Various Polarization Processes

Q50. Outline the effect of temperature on dielectric polarization.

Answer :

June-17, Q6

Temperature Dependence of Polarization

Among three types of polarization only orientational polarization depends strongly on temperature. Ionic and

electronic polarizations are independent of temperature. Orientational polarization decreases with increase in temperature for poly-atomic gases and liquids.

Normal temperature opposes the permanent dipoles to align in the direction of field. But higher temperatures ease the movement of ions and molecule so that polarization processes which are not possible at higher frequencies and lower temperature occur at lower frequencies and higher temperature.

Q51. Describe the experimental determination of dielectric constant of dielectric by Schering method.

Dec.-17, Q13(b)

OR

Explain the experimental method of Schering bridge for determination of dielectric constant.

Jan.-13, Q15(a)

OR

What is dielectric constant? Explain the method of Schering bridge for determination of dielectric constant.

Answer :

Dielectric Constant

For answer refer Unit-2, Q23.

Schering Bridge Method

The schematic diagram of a Schering bridge used for measuring dielectric constant is shown in figure as,

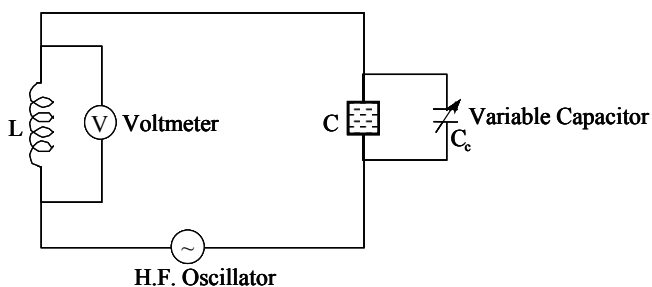


Figure: Schematic Diagram of a Schering Bridge

It consists of a resonant circuit made up of L and C as shown in figure. Further, it uncompasses a calibrated variable capacitor (C_c) connected in parallel with C . The voltage across the inductor is evaluated using a voltmeter and frequency of the resonant circuit is varied using a high frequency oscillator.

The dielectric constant of any material is evaluated by comparing the capacitance of an empty capacitor with the capacitance of the same capacitor filled with the material

If C_0 is the capacity of empty capacitor and C_s is the capacity of capacitor filled with dielectric constant ' ϵ_r ', then

$$\therefore \epsilon_r = \frac{C_s}{C_0}, \text{ at resonance}$$

Working of the Bridge

- Initially, the space between the two plates of capacitor ' C ' is kept empty and the variable capacitor ' C_c ' is adjusted to obtain the resonant frequency. Let C_0 be the capacitance at which resonance is obtained.

If ' A ' is the area of the capacitor ' C ' and ' d ' is the distance of separation between the plates, then the capacitance of the capacitor ' C ' is given by,

$$C_0 = \frac{\epsilon_0 A}{d} \quad \dots (1)$$

- Now the empty space between the two plates of capacitor ' C ' is filled with material whose dielectric constant is to be measured. The variable capacitor ' C_c ' is adjusted to obtain the resonant frequency. Let C_s be the capacitance at which resonance is obtained.

Since ' A ' and ' d ' of the capacitor ' C ' remains same the capacitance ' C_s ' is given by,

$$C_s = \frac{\epsilon_r \epsilon_0 A}{d}$$

Where, ... (2)

ϵ_r - Dielectric constant of the material between the plates.

Dividing equation (2) with equation (1),

$$\frac{C_s}{C_0} = \frac{\frac{\epsilon_r \epsilon_0 A}{d}}{\frac{\epsilon_0 A}{d}}$$

$$\therefore \epsilon_r = \frac{C_s}{C_0}$$

Thus, the dielectric constant is evaluated.

2.3.3 Ferroelectricity - Barium Titanate - Applications of Ferroelectrics

Q52. What is ferroelectricity? Give the characteristics of ferroelectric materials.

Answer :

Ferroelectricity

For answer refer Unit-2, Q24.

Characteristics of Ferroelectric Materials

- Ferroelectric materials exhibit spontaneous polarization for below particular temperature.
- The polarization can be reversed on the application of external electric field.
- Ferroelectric materials lose their Ferroelectricity property at temperature greater than Curie temperature.
- All the ferroelectric materials exhibit piezoelectric effect.
- They generate hysteresis curve.

Q53. Explain the spontaneous polarization in barium titanate.

April-16, Q16(b)

OR

Draw the crystal structure of Barium titanate above 393 K and explain how its structure and polarization changes with decreasing temperature.

Jan.-12, Q7

OR

Explain the phenomenon of ferroelectricity and discuss how dielectric constant of Barium Titanate changes as its temperature is decreased?

June-11, Q14(a)

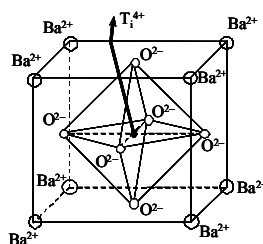
OR

Explain the ferro electricity in Barium titanate.

Answer :

Barium titanate (BaTiO_3) is a dielectric material which show spontaneous polarization below Curie temperature. This spontaneous polarization is due to asymmetric ionic displacements in the crystal structure, when the temperature is reduced below curie temperature. This phenomenon gives rise to ferroelectricity.

Above curie temperature Barium titanate has cubic structure as shown in figure. Barium ions (Ba^{2+}) occupy the corners of the cube, oxygen ions (O^{2-}) are at the centre of faces of cube forming an octahedron and finally titanium ion (Ti^{4+}) is at the centre of octahedron as shown in figure.



Figure

Since, the centre of the negative charges coincide with centre of positive charges the net dipole moment is zero. When temperature is reduced below curie temperature (T_c), the barium and titanium ions are displaced upward by 2.8% and oxygen atoms are displaced downward by 1%. Now, the centre of positive charges is different from that of negative charges. This favours the spontaneous polarization. The spontaneous polarization may lie along any one of the cube edges thus giving a total of 6 possible directions. The material expands in a direction of polarization and contracts in a direction perpendicular to polarization. As a result the unit cell changes from cubic to tetragonal structure. This spontaneous polarization in barium titanate results in ferroelectricity

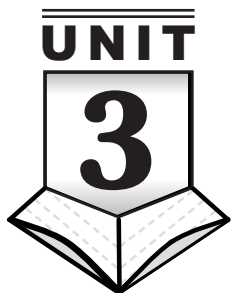
Q54. Write few applications of ferroelectrics.

Answer :

June-11, Q14(b)

Ferroelectric materials have many applications in many electronic and electro-optic devices some of them are,

1. The FRAMs made from ferroelectric materials make use of ferroelectricity phenomena to store data.
2. The high dielectric permittivity of ferroelectric materials such as BaTiO_3 , PMN and PZT is useful for capacitor applications.
3. The ferroelectric materials are convenient in growing single crystals and also the material geometry is convenient for device design.
4. In the fabrication of Surface Acoustic Wave (SAW) devices, ferroelectric materials are very much useful in depositing inter-digital electrodes on the surface.



WAVE MECHANICS, ELECTROMAGNETIC THEORY AND ELECTROMAGNETIC WAVES

PART-A SHORT QUESTIONS WITH SOLUTIONS

Q1. What are matter waves?

Answer :

Waves that are related with the moving material particle such as electrons, protons etc., are referred to as matter waves. The expression for the wavelength of matter waves is given by de-Broglie equation as,

$$\lambda = \frac{h}{m\lambda}$$

Q2. An electron is moving under a potential field of 15 kV. Calculate the wavelength of the electron waves.

Answer :

Given that,

For an electron,

Potential field, $V = 15 \text{ kV}$

The expression for wavelength of electron wave is given by,

$$\lambda = \frac{1.227}{\sqrt{V}} \text{ nm}$$

Substituting the corresponding values in above equation,

$$\lambda = \frac{1.227}{\sqrt{15 \times 10^3}} = \frac{1.227}{\sqrt{15000}}$$

$$\Rightarrow \lambda = \frac{1.227}{122.47} = 0.01 \text{ nm}$$

$$\therefore \lambda = 0.01 \text{ nm}$$

Therefore, wavelength of electron wave is 0.01 nm.

Q3. Mention the ideas which prompted De Broglie to propose his concept of matter waves.

Answer :

The ideas that prompted De Broglie to propose his concept of matter waves are as follows,

- Like radiation, wave and particle, matter should also exhibit dual nature.
- Optical phenomena like photoelectric effect related to radiation were explained by attributing certain particle characteristics to radiation. Similarly, to completely understand the behaviour of matter, certain wave aspects should also be attributed to material particles.
- Electrons present in the stable orbits should be attributed with wave characteristics i.e., while finding the path of electrons, they should be treated as a wave.

Q4. Define wave function.**Answer :**

In the context of matter waves, a quantity that changes periodically is referred to as wave function. It is denoted with $\psi(x, t)$, which should be finite, continuous and single valued for all 'x' and 't'.

Q5. List the properties of wave function.

May/June-18, Q15(b)

OR**Discuss the properties of wave function.****Answer :**

June-11, Q12(a)

Properties of wave function are,

1. Wave function is a continuous function.
2. It includes all the measurable information regarding particle, but cannot be measured using any physical instrument.
3. With the help of Schrodinger's equation, wave function permits energy calculations.
4. Wave function establishes the probability distribution in three dimensions.
5. It allows the determination of effective average value of a given variable.

Q6. Give the Schrodinger's time dependent wave equation.**Answer :**

The Schrodinger's time dependent wave equation is given by,

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{-\hbar^2}{2m} \left[\frac{\partial^2 \Psi}{\partial x^2} \right] + V \cdot \Psi \quad (\text{One-dimensional})$$

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{-\hbar^2}{2m} \left[\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} \right] + V \cdot \Psi \quad (\text{Three-dimensional})$$

Where,

Ψ - Wave function

$$\hbar = \frac{h}{2\pi}$$

h - Planck's constant

m - Mass of electron

V - Field potential.

Q7. Give the Schrodinger's time independent wave equation.**Answer :**

The Schrodinger's time independent wave equation is given by,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0 \quad (\text{One-dimensional})$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0 \quad (\text{Three-dimensional})$$

Where,

ψ - Wave function

m - Mass of electron

h - Planck's constant

E - Total energy

V - Field potential.

Q8. Give the expression for energy of an electron confined to a potential box of width 'x'.**Answer :**

The expression for energy of an electron confined to a potential box of width 'x' is given by,

$$E = \frac{\hbar^2}{8m} \left[\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right]$$

Where,

$$\hbar = \frac{h}{2\pi}$$

h - Planck's constant

n_x, n_y, n_z - Set of three positive numbers

a, b, c - Constants.

Q9. A quantum particle confined to one-dimensional box of width 'a' is known to be in its first excited state. Determine the probability of the particle in the central half.**Answer :**

The wave function of the particle in first excited state ($n = 1$) is,

$$\Psi_1 = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$$

The probability of finding the particle in the central half of the box $x = \frac{L}{2}$ is,

$$P = \Psi_1^2 = \frac{2}{L} \left[\sin \frac{\pi(L/2)}{L} \right]^2 = \frac{2}{L} \sin^2 \frac{\pi}{2} = \frac{2}{L}$$

But, given width of the box = a .

i.e., Here, $L = a$

$$\therefore P = \frac{2}{a}$$

Q10. The energy of an electron contained to move in a one-dimensional box of length 4.0 Å is 9.664 × 10⁻¹⁷ J. Find out the order of excited state.**Answer :**

Given that,

For an electron moving in a one-dimensional box,

Width of the potential box, $a = 4 \text{ Å}$

Energy of the electron, $E = 9.664 \times 10^{-17} \text{ J}$.

Then, the expression for energy of an electron confined to move in a one-dimensional box is given by,

$$\begin{aligned}
 E &= \frac{n^2 h^2}{8 m a^2} \\
 \Rightarrow n^2 &= \frac{8 E m a^2}{h^2} \\
 &= \frac{8 \times 9.664 \times 10^{-17} \times 9.1 \times 10^{-31} \times (4 \times 10^{-10})^2}{(6.625 \times 10^{-34})^2} \\
 [\because h &= 6.625 \times 10^{-34} \text{ JS and } m = 9.1 \times 10^{-31} \text{ kg}] \\
 &= 256.47 \\
 \Rightarrow n &= \sqrt{256.47} = 16.01 \approx 16 \\
 \therefore n &= 16
 \end{aligned}$$

Q11. Complete the de-Broglie wavelength of a proton whose kinetic energy is equal to the rest energy of an electron ($m_e = 9.1 \times 10^{-31} \text{ kg}$, m of proton $= 1.67 \times 10^{-27} \text{ kg}$). (Note: Compute is misprinted as complete and electron is misprinted as election)

Answer : June/July-17, Q9

Given that,
Kinetic energy of proton = rest mass energy of electron
de-Broglie wave length is given by,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \quad (\because p = \sqrt{2mK})$$

Kinetic energy of proton, $k = m_e c^2$

$$\begin{aligned}
 \therefore \lambda &= \frac{h}{\sqrt{2m(m_e c^2)}} \\
 &= \frac{h}{\sqrt{2m(m_e c^2)}} \\
 &= \frac{h}{\sqrt{2} c \sqrt{m m_e}} \\
 &= \frac{h}{\sqrt{2} c \times (m_e)} \\
 \lambda &= \frac{6.63 \times 10^{-34}}{1.414 \times (3 \times 10^8) \times 9.1 \times 10^{-31} \times 42.8} \\
 \Rightarrow \lambda &= 4 \times 10^{-14} \text{ m.} \\
 \therefore \lambda &= 0.0004 \text{ \AA} .
 \end{aligned}$$

Q12. Define Poynting theorem.

Answer : June/July-17, Q10

According to poynting theorem, the total power ' W ' flowing out of a closed surface is equal to the closed surface integral of poynting vector ' \vec{p} '.

$$\text{i.e., } W = \oint_S \vec{p} \cdot d\vec{s}$$

Q13. State and explain the concept of "Displacement current".

Answer :

Dec.-16, Q10

Displacement current is defined as the current at any given point, passing through a unit surface area when the surface is normal to the direction of displacement current. It is denoted by J_D .

Displacement current results when the current flows through the insulating medium.

Displacement current density is given by,

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}.$$

Displacement current is greater when compared to conduction current in a dielectric medium.

Example: Current flowing through conductors and resistors.

Q14. State Coulomb's law of magnetism.

Answer :

Coulomb's law states that, "The force between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them" i.e.,

$$\begin{aligned}
 F &\propto m_1 m_2 \\
 \Rightarrow F &\propto \frac{1}{R^2} \\
 \therefore F &= \frac{m_1 m_2}{4\pi\mu_0 R^2} \text{ Newtons}
 \end{aligned}$$

Where, m_1 and m_2 are the magnetic poles

R is the distance between the poles.

PART-B

ESSAY QUESTIONS WITH SOLUTIONS

3.1 WAVE MECHANICS

3.1.1 Matter Waves - de-Broglie Wavelength - Properties of Wave Function

Q15. Explain in detail the properties of matter waves.

Answer :

Matter waves are proposed by Louis Victor De Broglie in the year 1923 and these waves are associated with material particle and exhibit wave nature. Hence, these waves are also called as De Broglie waves. As per this hypothesis matter has dual nature (i.e., wave and particle).

Properties of Matter Waves

1. A matter wave is not a physical phenomenon. These are pilot waves in the sense that their only function is to pilot or guide the material particles. Hence, they are also treated as probability waves.
2. Matter waves do not possess electromagnetic wave nature.
3. Matter waves are associated only with moving particles.
4. Velocity and wavelength of particle are inversely proportional to each other i.e., smaller the velocity of the particle, greater is the wavelength associated with it.
5. Mass and wavelength of particle are inversely proportional to each other i.e., lighter the particle, greater is the wavelength of the matter wave.
6. The velocity of light is always less than the velocity of matter wave.

Q16. Derive an expression for the De Broglie wavelength of an electron.

Answer :

According to Planck and Einstein theories, the energy of a photon whose frequency ν is expressed as,

$$E = h\nu \quad \dots (1)$$

Where,

h - Planck's constant.

According to Einstein's mass energy relation,

$$E = mc^2 \quad \dots (2)$$

Where,

m - Mass of a photon

c - Velocity of a photon.

From equations (1) and (2),

$$h\nu = mc^2$$

$$\Rightarrow \frac{hc}{\lambda} = mc^2 \quad \left[\because \nu = \frac{c}{\lambda} \right]$$

$$\Rightarrow \lambda = \frac{h}{mc}$$

"∴ Momentum of a photon, $p = mc$

$$\therefore \lambda = \frac{h}{p}$$

In the same way, according to De Broglie hypothesis, if an electron of charge e , mass m , is moving with a velocity v in the presence of potential V , then the wavelength associated with that electron can be expressed as,

$$\lambda = \frac{h}{mV} = \frac{h}{p} \quad \dots (3)$$

Equation (3) is called the De Broglie's equation.

And the energy of the electron in terms of potential can be expressed as $E = eV$ and the kinetic energy of electron is given as,

$$\begin{aligned} E &= \frac{1}{2}mv^2 \\ \Rightarrow eV &= \frac{1}{2}mv^2 \\ \Rightarrow meV &= \frac{1}{2}m^2v^2 \\ \Rightarrow 2meV &= m^2v^2 \quad [\because p = mv] \\ \Rightarrow 2meV &= p^2 \\ \Rightarrow p &= \sqrt{2meV} \end{aligned} \quad \dots (4)$$

Substituting equation (4) in equation (3),

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Q17. State and explain de Broglie's concept of matter waves and derive the expression for wavelength associated with an electron.

Answer : June/July-17, Q17(b)

For answer refer Unit-3, Q15 and Q16.

3.1.2 Physical Significance - Schrodinger Time Dependent and Time Independent Wave Equations

Q18. Define wave function and explain the physical significance of wave function.

OR

Explain the physical significance of wave function " Ψ ".

Answer : [Dec.-16, Q9 | Jan.-16, Q3]

Wave Function

For answer refer Unit-3, Q4.

Physical Significance of Wave Function

Even though the wave function does not have any direct physical significance, its indirect impact on particular parameters associated with the matter waves is discussed below.

Consider a potential function $V(x)$ is assumed to be independent of time, and m is the mass of the particle. The wave function $\Psi(x, t)$ is used to describe the behaviour of the system and mathematically $\Psi(x, t)$ can be a complex quantity. H and E are called Hamiltonian and energy operators respectively.

The probability density of x is expressed as,

$$P = |\Psi|^2 = \Psi\Psi^*$$

Where,

Ψ^* - Complex conjugate of Ψ .

Any wave function $\Psi(x, t)$ can be represented as,

$$\Psi(x, t) = \Psi(x) \cdot \phi(t)$$

Where,

$\Psi(x)$ - Independent of time

$\phi(t)$ - Independent of x .

The probability density can be expressed as,

$$(|\Psi(x, t)|)^2 = |\Psi(x)|^2 \text{ and is independent of time.}$$

Since the function $|\Psi(x)|^2$ represents the probability density function, then for a single particle,

$$\rho = \int_{-\infty}^{+\infty} |\Psi(x)|^2 dx = 1$$

In a volume of ' V ',

$$\rho = \iiint \Psi^2 dx dy dz$$

The above relation is called normalized condition. The wave function that satisfies the relation is called a normalized wave function.

Q19. Derive an expression for Schrodinger's time independent wave equations.

May/June-18, Q15(a)

OR

Derive the expression for Schrodinger's time independent wave equation.

Dec.-16, Q15(a)

OR

Derive the Schrodinger's time-independent wave equation.

Answer : Jan.-16, Q12(b)

Schrodinger Time Independent Wave Equation

Schrodinger's wave equation is a basic principle, which cannot be derived from other basic principles of science. One can arrive at the equation starting with De Broglie's idea of matter waves. According to De Broglie theory, a particle of mass m moving with velocity v has a wavelength (λ) associated with it and is expressed as,

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \dots (1)$$

Where,

p - Momentum of the particle.

In classical physics, we deal with the wave equation. For a wave moving along the x -direction, the displacement y is given by,

$$y = A \sin \frac{2\pi}{\lambda} x \quad \dots (2)$$

Where,

A - Amplitude

x - Position coordinate

λ - Wavelength.

An equation similar to the one given above should also represent the De Broglie wave associated with a moving particle. By analogy, we may write,

$$\psi = A \sin \frac{2\pi x}{\lambda} \quad \dots (3)$$

Periodic changes in ψ are responsible for the wave nature of a moving particle.

Differentiating equation (3) twice with respect to ' x ',

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2} A \sin \frac{2\pi x}{\lambda} = -\frac{4\pi^2}{\lambda^2} \psi$$

$$\frac{1}{\lambda^2} = \frac{1}{4\pi^2\psi} \frac{d^2\psi}{dx^2} \quad \dots (4)$$

Then, the kinetic energy of a moving particle is given by,

$$K.E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\text{From equation (1), } p = \left(\frac{h}{\lambda}\right)$$

Hence,

$$K.E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

If the particle is moving in a field of potential ' V ' its total energy,

$$E = K.E + P.E = \frac{p^2}{2m} + V$$

$$\Rightarrow \frac{p^2}{2m} = E - V \quad \dots (5)$$

Combining equations (4) and (5),

$$-\frac{h^2}{8\pi^2m} \cdot \frac{1}{\psi} \frac{\partial^2\psi}{\partial x^2} = E - V$$

$$\Rightarrow \frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2m}{h^2} (E - V)\psi = 0$$

This is the Schrodinger's time independent wave equation in one dimension. In three dimensions, the equation takes the form as follows,

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{8\pi^2m}{h^2} (E - V)\psi = 0$$

$$\left(\because \frac{h}{2\pi} = \hbar \right)$$

The simplified form of 3-dimensional Schrodinger's time independent wave equation is expressed as,

$$\therefore \nabla^2\psi + 2m \frac{(E - V)}{\hbar^2} \psi = 0$$

Q20. Obtain Schrodinger's time dependent and time independent wave equations for matter waves.

Answer :

June-10, Q13(b)

Schrodinger's Time Dependent Wave Equation

Schrodinger's wave equation is the fundamental equation of quantum mechanics. It was proposed by Erwin Schrodinger in 1926.

Consider a particle travelling in the positive x -direction. Then, the wave function $\psi(x, t)$, associated with this particle is given by,

$$\psi = A.e^{-i\omega[t - \frac{x}{c}]} \quad \dots (1)$$

Where,

ψ - Wave function

A - Amplitude of the wave

ω - Angular frequency (rad/sec)

c - Speed of light (m/sec).

Since the particle which represents a photon, is the smallest unit of light and therefore travels with the speed of light and is given by,

$$c = \nu\lambda \quad \dots (2)$$

Where,

ν - Frequency

λ - Wavelength

And,

$$\omega = 2\pi\nu \quad \dots (3)$$

Substituting the values of c and ω from equations (2) and (3) in equation (1),

$$\psi = A.e^{-i(2\pi\nu)[t - \frac{x}{\nu\lambda}]}$$

$$= A.e^{-i[2\pi\nu t - (\frac{2\pi\nu}{\nu\lambda})x]}$$

$$= A.e^{-i[2\pi\nu t - (\frac{2\pi}{\lambda})x]} \quad \dots (4)$$

From the Planck's relation or the Planck-Einstein equation is given by,

$$E = h\nu \quad \dots (5)$$

Where,

E - Energy of photon

h - Planck's constant

$$(6.626 \times 10^{-34} \text{ J.s or } 4.135 \times 10^{-15} \text{ eV.s})$$

But, the reduced Planck's constant or the Dirac constant is given by,

$$\hbar (h - \text{bar}) = \frac{h}{2\pi}$$

$$\Rightarrow h = 2\pi \hbar \quad \dots (6)$$

Substituting $h = 2\pi \hbar$ in equation (5),

$$E = (2\pi \hbar) \cdot \nu$$

$$\Rightarrow \nu = \frac{E}{2\pi \hbar} \quad \dots (7)$$

Also,

$$\lambda = \frac{h}{P}$$

Where, P - Momentum of particle.

$$\Rightarrow \lambda = \frac{2\pi \hbar}{P} \quad [\because \text{From equation (6)}] \quad \dots (8)$$

Now,

Substituting the value of ν and λ from equations (7) and (8) in equation (4),

$$\psi = A e^{-i \left[\left\{ 2\pi \times \frac{E}{2\pi \hbar} \right\} t - \left\{ \frac{2\pi}{\frac{2\pi \hbar}{P}} \right\} x \right]}$$

$$= A e^{-i \left[\left(\frac{E}{\hbar} \right) t - \left(\frac{P}{\hbar} \right) x \right]}$$

$$= A e^{\frac{-i}{\hbar} [Et - Px]} \quad \dots (9)$$

Equation (9) represents the mathematical description of the wave equivalent of a free particle whose total energy and momentum is E and P respectively in the $+x$ -direction.

Differentiating equation (9) partially with respect to ' t ',

$$\frac{\partial \psi}{\partial t} = A e^{\frac{-i}{\hbar} [Et - Px]} \left[\frac{-iE}{\hbar} \right]$$

$$\Rightarrow \frac{\partial \psi}{\partial t} = \psi \left[\frac{-iE}{\hbar} \right] \quad \left[\because \psi = A e^{\frac{-i}{\hbar} [Et - Px]} \right]$$

$$\Rightarrow \frac{\partial \psi}{\partial t} = \psi \left(\frac{i}{\hbar} \right) \left(\frac{iE}{\hbar} \right)$$

$$\Rightarrow \frac{\partial \psi}{\partial t} = \psi \left(\frac{E}{i\hbar} \right) (-i^2)$$

$$\Rightarrow \frac{\partial \psi}{\partial t} = \frac{\psi E}{i\hbar} \quad [\because i^2 = -1]$$

$$\Rightarrow E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \dots (10)$$

Differentiating equation (9) twice, partially with respect to ' x ',

$$\frac{\partial \psi}{\partial x} = A e^{\frac{-i}{\hbar} [Et - Px]} \left[\frac{Pi}{\hbar} \right]$$

$$\frac{\partial^2 \psi}{\partial x^2} = A e^{\frac{-i}{\hbar} [Et - Px]} \left[\frac{Pi}{\hbar} \right] \left[\frac{Pi}{\hbar} \right]$$

$$\frac{\partial^2 \psi}{\partial x^2} = \psi \left[\frac{Pi}{\hbar} \right]^2$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{-P^2 \psi}{\hbar^2}$$

$$\Rightarrow P^2 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \quad \dots (11)$$

The total energy of the particle is given by,

E = Kinetic energy ($K.E$) + Potential energy

$$= \frac{1}{2} m v^2 + V = \frac{1}{2m} (mv)^2 + V$$

$$= \frac{1}{2m} (P)^2 + V \quad \dots (12)$$

$[\because \text{Momentum, } P = \text{Mass}(m) \times \text{Velocity}(V)]$

Multiplying equation (12) by ψ on both sides,

$$E\psi = \frac{P^2 \psi}{2m} + V \cdot \psi \quad \dots (13)$$

Substituting the values of $E\psi$ and $P^2 \psi$ from equations (10) and (11) in equation (13),

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m} \left[-\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \right] + V \cdot \psi$$

$$\Rightarrow i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \left[\frac{\partial^2 \psi}{\partial x^2} \right] + V \cdot \psi \quad \dots (14)$$

Thus, equation (14) represents one dimensional Schrodinger time dependent wave equation.

A three dimensional Schrodinger time dependent wave equation is expressed as,

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right] + V \cdot \psi$$

$$\Rightarrow i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{-\hbar^2}{2m} \nabla^2 + v \right] \psi$$

Where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Thus, the three dimensional Schrodinger time dependent wave equation is represented as,

$$i\hbar \frac{d\psi(r,t)}{dt} = \left[\frac{-\hbar^2}{2m} \nabla^2 + v \right] \psi(r,t)$$

Schrodinger Time Independent Wave Equation

For answer refer Unit-3, Q19.

3.1.3 Particle in a 1-D Box

Q21. Deduce the expression for energy of an electron confined to a potential box of width 'x'.

OR

Derive the expression for wave function of particle in an infinite square well potential.

Answer :

June/July-17, Q15(a)

Particle in a Potential Box

Consider m be the mass of the particle in a one-dimensional box of length L (infinite height).

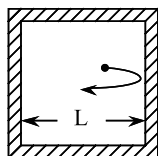


Figure (1): One-dimensional Box

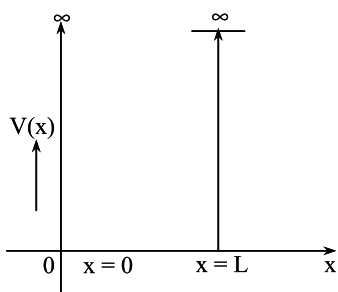


Figure (2): Graphical Representation of Figure (1)

Case 1

Assuming that the Potential Energy (P.E) everywhere within the line (long axis) is constant and zero.

Potential Energy (P.E) for a box is given by,

$$\begin{aligned} V(x) &= 0 \text{ for } x \leq 0; x \geq L \\ &= \infty \text{ for } x \leq L \end{aligned} \quad \dots (1)$$

The wave function $\psi_n(x)$ of the electron in the region $0 < x < L$ is given as follows,

According to the Schrodinger's wave equation, $V = 0$

$$\frac{d^2\psi_n}{dx^2} + \frac{2m}{\hbar^2} E_n \psi_n = 0 \quad \left(\because \hbar = \frac{h}{2\pi} \right) \quad \dots (2)$$

Here,

E_n – Total energy of the electron in the state n .

The solution for $\psi_n(x)$ becomes as,

$$\psi_n(x) = Ae^{ikx} + Be^{-ikx} \quad \dots (3)$$

Where,

$$k^2 = \left(\frac{2m}{\hbar^2} \right) E_n \quad \dots (4)$$

A and B = Arbitrary constants

Case 2

Now the electron is constrained by infinitely high potential barriers.

$$\text{At } x = 0 \text{ and } L = 0$$

$$\psi_n(0) = 0 \text{ and } \psi_n(L) = 0$$

The above equation appears to be physically reasonable, otherwise the product of $V(x) \psi_n(x)$ becomes infinite.

Case 3

If $V(x)\psi_n(x) \rightarrow \infty$, then the kinetic energy (E_n) must be also infinite (if $\psi_n(x)$ be a continuous functions).

According to Schrodinger's wave equation, the first boundary condition $A = -B$, leaving only $\sin kx$ solutions.

Second boundary condition,

$$\sin kL = 0 \text{ or} \quad \dots (5)$$

$$\Rightarrow k = \frac{n\pi}{L} \text{ with } n = 1, 2, 3, \dots$$

Thus both the conditions are satisfied if we choose,

$$\psi_n(x) = A \sin\left(\frac{n\pi}{L}x\right) \quad \dots (6)$$

$$\text{Here, } A = \sqrt{\left(\frac{2}{L}\right)}$$

Therefore, normalized wave functions are,

$$\psi_n(x) = \sqrt{\left(\frac{2}{L}\right)} \sin \frac{n\pi x}{L}$$

Allowed energy Eigen values are defined by substituting equation (5) into equation (4), we get,

$$E_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{L} \right)^2 = \frac{h^2}{2m} \left(\frac{n^2}{2L} \right)^2 \quad \left[\because \hbar = \frac{h}{2\pi} \right]$$

Integral values of n are allowed for wave functions $\psi_n(x)$ and also energy values E_n where n is called the quantum number, whose variation is shown in figure (3).

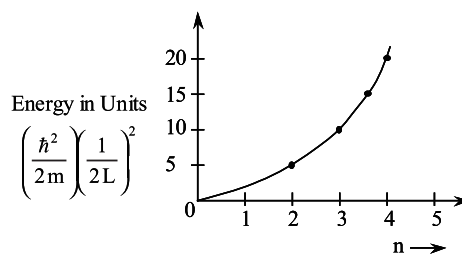


Figure (3)

Q22. Apply the Schrodinger's wave equation to a particle in an infinite square well potential and derive the expression for energy level of the particle.

June-15, Q12(b)

OR

Apply the Schrodinger's wave equation to a particle in an infinite square well potential and obtain the expression for energy of particle.

Dec.-16, Q17(a)

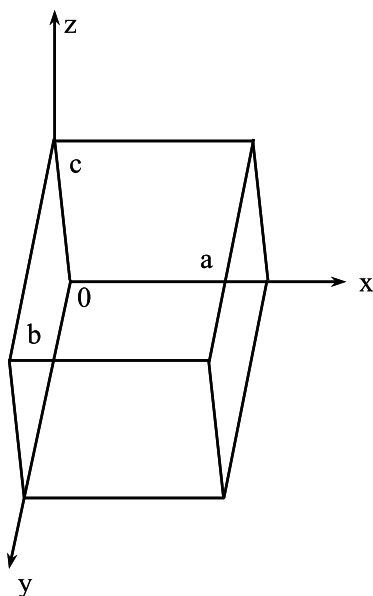
OR

Obtain Schrodinger's wave equation for a particle in a square well potential of infinite depth and discuss energy levels.

Answer :

April-16, Q14

Potential barrier acts a 3D crystal for electrons and confines them in it without allowing them to escape. This 3D crystal is called potential well. Consider an electron of mass 'm' confined in a 3D potential well across xyz plane with origin at one corner as shown in figure.



Figure

Time independent Schrodinger's wave equation for a particle is,

$$\Delta^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0,$$

Where, ψ - Wave function.

Potential energy of an electron inside an potential well is zero and reaches infinity at the boundaries and outside the well, i.e., $V = 0$.

$$\text{Therefore, } \Delta^2\psi + \frac{2mE}{\hbar^2}\psi = 0 \text{ (or)}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad \dots (1)$$

Since ψ is a function of x, y and z , it can be written as,

$$\psi(x, y, z) = X(x) \cdot Y(y) \cdot Z(z) \quad \dots (2)$$

X, Y and Z are functions of x, y and z only respectively.

Hence, the above equation can be simplified further as,

$$\psi = XYZ$$

Differentiating the above equation with respect to x ,

$$\frac{\partial\psi}{\partial x} = YZ \frac{dX}{dx}$$

Differentiating above equation with respect to x ,

$$\frac{\partial^2\psi}{\partial x^2} = YZ \frac{d^2X}{dx^2} \quad \dots (3)$$

Similarly,

$$\frac{\partial^2\psi}{\partial y^2} = ZX \frac{d^2Y}{dy^2} \quad \dots (4)$$

$$\frac{\partial^2\psi}{\partial z^2} = XY \frac{d^2Z}{dz^2} \quad \dots (5)$$

Substituting equations (3), (4) and (5) in equation (1),

$$YZ \frac{d^2X}{dx^2} + XZ \frac{d^2Y}{dy^2} + XY \frac{d^2Z}{dz^2} = -\left[\frac{2mE}{\hbar^2}\right]XYZ$$

Dividing the above equation by XYZ on both the sides,

$$\begin{aligned} \frac{1}{X} \frac{d^2X}{dx^2} + \frac{1}{Y} \frac{d^2Y}{dy^2} + \frac{1}{Z} \frac{d^2Z}{dz^2} &= -\frac{2mE}{\hbar^2} \\ \Rightarrow \frac{1}{X} \frac{d^2X}{dx^2} + \frac{1}{Y} \frac{d^2Y}{dy^2} + XY \frac{d^2Z}{dz^2} &= -K^2 \end{aligned}$$

Where,

$$\begin{aligned} K^2 &= K_x^2 + K_y^2 + K_z^2 \\ &= \frac{2mE}{\hbar^2} \quad \dots (6) \end{aligned}$$

The general solution of the above equation is,

$$\psi(x) = A_x \cos K_x x + B_x \sin K_x x \quad \dots (7)$$

Where, A and B are constants.

Applying boundary condition $\psi(x) = 0$,

- (i) At $x = 0$, is the chances of finding electrons at the boundary of crystal is 0.

Therefore, the general solution becomes,

$$X(x) = B_x \sin K_x x \quad \dots (8)$$

- (ii) At $x = a$, $X = 0 = B_x \sin K_x a$

$$\text{Since, } B_x \neq 0$$

$$\sin K_x a = 0$$

$$\therefore K_x a = \frac{n_x \pi}{a}$$

Where, $n_x = 1, 2, 3, \dots, n_x \neq 0$.

Substituting the above value of K_x in equation (7),

$$X = B_x \sin \frac{n_x \pi x}{a} \quad \dots (9)$$

Normalizing the above equation between $x=0$ and $x=a$.

$$\int_0^a |x|^2 dx = B_x^2 \int_0^a \sin^2 \left[\frac{n_x \pi x}{a} \right] dx = 1$$

Solving the above equation B_x ,

$$\frac{B_x^2 a}{2} = 1$$

$$\therefore B_x = \sqrt{\frac{2}{a}}$$

Substituting the value of B_x in equation (8),

$$X = \sqrt{\frac{2}{a}} \sin \frac{n_x \pi x}{a}$$

$$\text{Similarly, } Y = \sqrt{\frac{2}{b}} \sin \frac{n_y \pi y}{b} \text{ and } Z = \sqrt{\frac{2}{c}} \sin \frac{n_z \pi z}{c}$$

Substituting the above values of X , Y and Z in equation (2),

$$\psi = \sqrt{\frac{2}{a}} \sin \frac{n_x \pi x}{a} \cdot \sqrt{\frac{2}{b}} \sin \frac{n_y \pi y}{b} \cdot \sqrt{\frac{2}{c}} \sin \frac{n_z \pi z}{c}$$

$$\psi_x(x, y, z) = \frac{2\sqrt{2}}{\sqrt{abc}} \cdot \sin \frac{n_x \pi x}{a} \cdot \sin \frac{n_y \pi y}{b} \cdot \sin \frac{n_z \pi z}{c}$$

As a result, equation (6) changes to,

$$E = \frac{h^2}{2m} (K_x^2 + K_y^2 + K_z^2)$$

$$E = \frac{h^2}{2m} \left[\frac{n_x^2 \pi^2}{a^2} + \frac{n_y^2 \pi^2}{b^2} + \frac{n_z^2 \pi^2}{c^2} \right]$$

$$E = \frac{h^2 \pi^2}{2m} \left[\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right]$$

Since $n_x \neq n_y \neq n_z$, E becomes,

$$E_{n_x}, E_{n_y} \text{ and } E_{n_z} = \left(\frac{h^2}{8m} \right) \left[\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right]$$

Where, $n_x = n_y = n_z = 1, 2, 3$.

The above equation represents Eigen values or energy levels of the particle.

3.2 ELECTROMAGNETIC THEORY

3.2.1 Basic Laws of Electricity and Magnetism - Maxwell's Equations in Integral and Differential Forms - Conduction and Displacement Current - Relation between D, E and P

Q23. State and explain ohm's law.

OR

State Ohm's law and write its formula along with limitations.

Answer :

Ohm's Law

According to Ohm's law, the electric current flowing through the conductor at constant physical conditions (i.e., temperature and dimensions) is directly proportional to the potential difference between its two ends.

If ' I ' is the current flowing in the conductor and ' V ' is the potential difference between its ends, then according to Ohm's law,

$$I \propto V$$

$$\therefore I = \frac{V}{R}$$

Where, R – Electrical resistance.

(or)

$$R = \frac{V}{I}$$

At constant temperature, if the applied voltage is increased, the resistance of the conductor remains constant.

Limitations of Ohm's Law

The following are the limitations of ohm's law,

- Ohm's law is not applicable at very high and very low temperatures.
- It can be applied only when physical conditions like temperature and dimensions are constant.
- Ohm's law is applicable to ohmic conductors only. It does not work for non-ohmic substances i.e., diodes, filaments, etc.
- Ohm's law is just an empirical formula.

Q24. State and explain Kirchhoff's law.

Answer :

Kirchhoff's Law

Kirchhoff's laws are related to the steady current flowing through the network of conductors. There are two Kirchhoff's laws, namely,

- Kirchhoff's first law (or) current law.
 - Kirchhoff's second law (or) voltage law.
- (i) Kirchhoff's First Law (or) Current Law**

According to Kirchhoff first law, at any given junction in a circuit, the sum of currents flowing into a junction is equal to the sum of currents flowing out of the junction.

For example, let i_1 , i_2 , i_3 and i_4 are four currents flowing through a circuit as shown in figure (i).

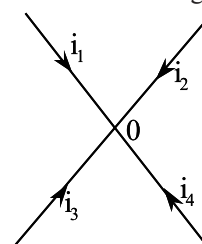


Figure (i)

According to Kirchhoff's first law,

$$i_1 + i_2 = i_3 + i_4$$

It can also be stated as, the algebraic sum of currents in a circuit is zero.

$$\text{i.e., } \sum i = 0 \text{ (or) } i_1 + i_2 - i_3 - i_4 = 0$$

- (ii) **Kirchhoff's Second Law or Voltage Law:** According to Kirchhoff's second law, in any closed circuit or network, the algebraic sum of voltage (product of current and resistance) of all the branches of circuit is equal to zero.

$$\text{i.e., } \sum V = 0$$

Consider a closed loop circuit as shown in figure (ii).

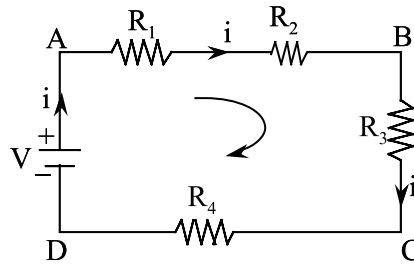


Figure (ii)

Let R_1 , R_2 , R_3 and R_4 represent the four resistances, i be the current flowing around the loop and V be the voltage across the circuit.

Applying Kirchhoff's second law to the loop $ABCD$,

$$iR_1 + iR_2 + iR_3 + iR_4 - V = 0$$

$$\Rightarrow i(R_1 + R_2 + R_3 + R_4) = V$$

$$\therefore i = \frac{V}{R_1 + R_2 + R_3 + R_4}$$

Q25. When two resistances of $5\ \Omega$ and $20\ \Omega$ are connected in parallel across $240\ \text{V}$ supply, calculate the total current and current through each resistance.

Answer :

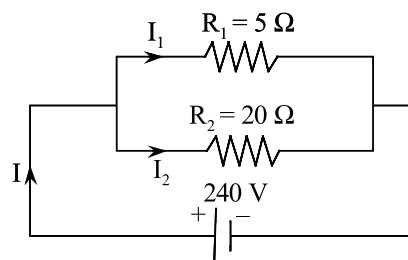
Given that,

Voltage, $V = 240\ \text{V}$

Resistance, $R_1 = 5\ \Omega$

Resistance, $R_2 = 20\ \Omega$

The circuit drawn according to given data is shown in figure.



Figure

Equivalent or total resistance,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 20}{5 + 20} = \frac{100}{25} = 4\ \Omega$$

Total current,

$$I = \frac{V}{R} = \frac{240}{4} = 60 \text{ A}$$

In parallel circuits the voltage across each resistance is same and is equal to supply voltage.

$$\text{i.e., } V_1 = V_2 = V = 240 \text{ V}$$

The current through 5 Ω resistor,

$$I_1 = \frac{V}{R_1} = \frac{240}{5} = 48 \text{ A}$$

The current through 20 Ω resistor,

$$I_2 = \frac{V}{R_2} = \frac{240}{20} = 12 \text{ A}$$

Q26. What is the difference between conduction current and displacement current?

Answer :

Differences Between Conduction Current and Displacement Current

Displacement Current		Conduction Current	
1.	It is defined as the displacement current at given point, passing through a unit surface area when the surface is normal to the direction of displacement current. It is denoted by J_d .	1.	It is defined as the convection current at a given point, passing through a unit surface area normal to the direction of current. It is denoted by J_c .
2.	Displacement current results when the current flows through the insulating medium.	2.	Conduction current results in the conductors in the presence of electric field.
3.	Displacement current density is given by, $\bar{J}_d = \frac{\partial \bar{D}}{\partial t}.$	3.	Conduction current density is given by, $\bar{J}_c = \frac{I_c}{A} = \sigma \bar{E}.$
4.	Displacement current is greater when compared to to conduction current in a dielectric medium.	4.	Conduction current is greater when compared to displacement current in a conductor.
5.	Example: Current flowing through conductors and resistors.	5.	Example: Current flowing through capacitor and all imperfect conductors.

Q27. Explain Maxwell's equations in integral and differential forms.

Dec.-16, Q15(b)

OR

Write down Maxwell's equation in integral form and convert them into differential form.

Answer :

Maxwell's Equation in Integral Form

1. Maxwell's first equation is,

$$\oint D \cdot ds = \int_V \rho_v dv$$

2. Maxwell's second equation is,

$$\oint B \cdot ds = 0$$

3. Maxwell's third equation is,

$$\oint E \cdot dl = - \frac{\partial}{\partial t} \int_s B \cdot ds$$

4. Maxwell's fourth equation is,

$$\oint_1 E \cdot dl = \oint_s \left(J + \frac{\partial D}{\partial t} \right) ds$$

Conversion of Integral Form of Maxwell's Equation to Differential Form

1. Consider Maxwell's first equation or Gauss's law in electric field

$$\text{i.e.,} \quad \oint \vec{D} \cdot d\vec{s} = \int_v \rho dv$$

Applying divergence theorem to L.H.S,

$$\int_v \nabla \cdot \vec{D} dv = \int_v \rho dv$$

Comparing L.H.S and R.H.S,

$$\nabla \cdot \vec{D} = \rho$$

$$\therefore \nabla \cdot \vec{D} = \rho \quad \dots (1)$$

Equation (1) represents the differential form of Maxwell's first equation.

2. Consider Maxwell's second equation or Gauss's law for magnetic field

$$\text{i.e.,} \quad \oint_1 \vec{H} \cdot d\vec{l} \quad \dots (2)$$

Applying divergence theorem to equation (2),

$$\int_v \nabla \cdot \vec{B} dv = 0$$

Since, the volume is arbitrary, the integral is zero,

$$\text{i.e.,} \quad \nabla \cdot \vec{B} = 0$$

$$\therefore \nabla \cdot \vec{B} = 0 \quad \dots (3)$$

Equation (3) represents the differential form of Maxwell's second equation.

3. Consider Maxwell's third equation,

$$\oint_l \vec{H} \cdot d\vec{l} = - \int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \dots (4)$$

Applying Stoke's theorem to L.H.S, of equation (4),

$$\oint_l \vec{H} \cdot d\vec{l} = \int_s (\nabla \times \vec{H}) \cdot d\vec{s} \quad \dots (5)$$

Substituting equation (5) in equation (4),

$$\int_s (\nabla \times \vec{E}) \cdot d\vec{s} = - \int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Comparing L.H.S and R.H.S,

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

$$\therefore \nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \quad \dots (6)$$

Equation (6) represents the differential form of Maxwell's third equation.

4. Consider Maxwell's fourth equation,

$$\oint_l H \cdot dl = \oint_s \left(J + \frac{\partial D}{\partial t} \right) \cdot ds \quad \dots (7)$$

Applying Stoke's theorem to L.H.S of equation (7),

$$\oint_l H \cdot dl = \int_s (\nabla \times H) \cdot ds \quad \dots (8)$$

Substituting equation (8) in equation (7),

$$\int_s (\nabla \times H) \cdot ds = \int_s \left(J + \frac{\partial D}{\partial t} \right) \cdot ds$$

Comparing L.H.S and R.H.S,

$$(\nabla \times H) = J + \frac{\partial D}{\partial t}$$

$$\therefore \nabla \times H = J + \frac{\partial D}{\partial t} \quad \dots (9)$$

Equation (9) represents the differential form of Maxwell's fourth equation.

Q28. Write the Maxwell's equations in integral and differential forms.

Answer :

June-10, Q15(a)

	Differential Form	Integral Form	Word Statement Form
1.	$\nabla \times H = J + \frac{\partial D}{\partial t}$ <p>Where, 'H' is magnetic field density, 'J' is conduction current density, 'D' is displacement current.</p>	$\oint_l H \cdot dl = \oint_s \left(J + \frac{\partial D}{\partial t} \right) \cdot ds$	The magnetomotive force around a closed path is equal to the surface integral of displacement current density and conduction current density over the surface bounded by closed path.
2.	$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_l H \cdot dl = - \int_s \frac{\partial B}{\partial t} \cdot ds$	The electromagnetic force around a closed path is equal to the negative of surface integral of rate of change of magnetic flux density over the surface bounded by closed path.
3.	$\nabla \cdot D = \rho_v$	$\int_s D \cdot ds = \int_{vol} \rho_v \cdot dv$	The total electric flux passing through closed surface is equal to the total charge in the volume enclosed by closed surface.
4.	$\nabla \cdot B = 0$	$\oint B \cdot ds = 0$	The total magnetic flux emerging from closed surface is zero.

Q29. State Gauss law and apply it to dielectrics. Define electric intensity [E], electric polarization [P] and electric displacement [D], deduce the relation among them.

Answer :

Gauss's Law

Statement

According to Gauss's law, "the surface integral of electric field vector (\vec{E}) over a surface is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by that surface".

$$\text{Mathematically, } \oint \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} q$$

(or)

$$\begin{aligned}\epsilon_0 \oint \vec{E} \cdot d\vec{s} &= q \\ \text{(or)} \\ \Rightarrow \oint \epsilon_0 \vec{E} \cdot d\vec{s} &= q \\ \text{(or)} \\ \oint \vec{D} \cdot d\vec{s} &= q\end{aligned}$$

Where, D is electric displacement.

Application of Gauss's Law to Dielectrics

Consider a parallel plate capacitor without dielectric as represented in figure (i).

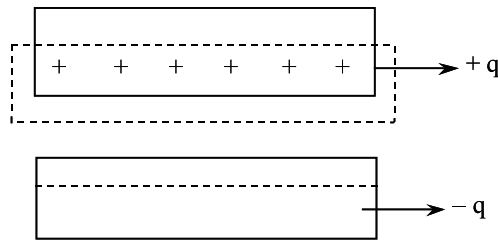


Figure (i): Parallel Plate Capacitor Without Dielectric

If E_0 is the electric field between the plates, then according to Gauss's law,

$$\begin{aligned}\epsilon \oint \vec{E}_0 \cdot d\vec{s} &= q \\ \Rightarrow \epsilon_0 E_0 S &= q \\ \therefore E_0 &= \frac{q}{\epsilon_0 S}\end{aligned} \quad \dots (1)$$

Consider parallel plate capacitors with dielectric material induced between them as shown in figure (ii).

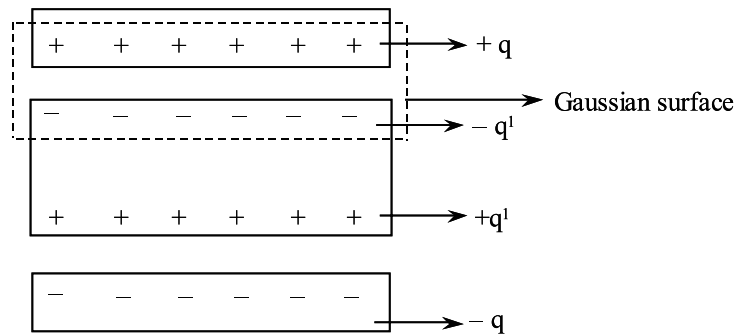


Figure (ii): Parallel Plate Capacitors With Dielectric

From figure (ii), it is observed that the net charge enclosed by Gaussian surface is, $q - q'$

Where, q' represents the induced charge in the dielectric due to polarization.

Therefore, according to Gauss's law,

$$\oint \vec{E} \cdot d\vec{s} = [q - q'] \quad \dots (2)$$

$$\Rightarrow \epsilon_0 E S = q - q'$$

$$\Rightarrow E = \frac{q - q'}{\epsilon_0 S}$$

$$\Rightarrow E = \frac{q}{\epsilon_0 S} - \frac{q'}{\epsilon_0 S} \quad \dots (3)$$

The relative permittivity, k is expressed as,

$$k = \frac{E_0}{E}$$

$$\Rightarrow E = \frac{E_0}{k} \quad \dots (4)$$

Substituting equation (1) in equation (4),

$$E = \frac{q}{k \epsilon_0 S} \quad \dots (5)$$

Substituting equation (5) in equation (3),

$$\frac{q}{k \epsilon_0 S} = \frac{q}{\epsilon_0 S} - \frac{q'}{\epsilon_0 S}$$

$$\frac{q}{\epsilon_0 S k} = \frac{1}{\epsilon_0 S} (q - q')$$

$$\Rightarrow \frac{q}{k} = q - q' \quad \dots (6)$$

$$\Rightarrow \frac{q}{k} - q = -q'$$

$$\Rightarrow q \left(\frac{1}{k} - 1 \right) = -q'$$

$$\Rightarrow q' = q \left(1 - \frac{1}{k} \right)$$

Therefore, the induced charge (q') is less than the free charge (q).

Substituting equation (6) in equation (2),

$$\epsilon_0 \oint \vec{E} \cdot d\vec{s} = \frac{q}{k}$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0 k}$$

$$\Rightarrow \oint (\epsilon_0 k \vec{E}) \cdot d\vec{s} = q$$

$$\Rightarrow \oint \vec{D} \cdot d\vec{s} = q \quad (\because \vec{D} = \epsilon_0 k \vec{E})$$

$$\therefore \vec{D} \cdot d\vec{s} = q$$

Hence proved.

Electric Intensity [E]

Electric field intensity is defined as the ratio of electric force (F) and unit positive charge (Q).

$$\text{i.e., } \vec{E} = \frac{\vec{F}}{Q} \text{ N/C or V/m}$$

(or)

Alternatively, it is also defined as the negative gradient of a potential due to a charge.

$$\text{i.e., } \vec{E} = -\nabla V \text{ V/m}$$

Electric Polarization [P]

Electric polarization is defined as the average electric dipole moment per unit volume of the dielectric. It is represented by the polarization vector \vec{P} and is given as,

$$\vec{P} = N \vec{\mu}$$

Where,

N - Number of molecules per unit volume

$\vec{\mu}$ - Average dipole moment per molecule.

Electric Displacement [D]

Electric displacement is also known as electric flux density. It can be defined as “the electric flux per unit differential area”. This is designated by ‘ \vec{D} ’, and is given by the expression,

$$\vec{D} = \frac{d\psi}{ds} \vec{a}_n \text{ C/m}^2$$

Where, $d\psi$ is electric flux crossing the differential area ds in normal direction.

The units of electric flux density are coulomb/m².

Relation Between \vec{D} , \vec{E} and \vec{P}

Consider parallel-plate capacitors having electric polarization of a dielectric slab as shown in figure (iii).

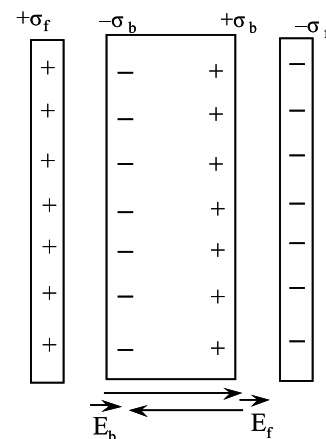


Figure (iii)

Let E_f represents the electric field strength of free charge, then,

$$E_f = \frac{\sigma_f}{\epsilon_0} \quad \dots (1)$$

Where,

σ_f - Surface charge density of free charge

ϵ_0 - Permittivity of free space.

Similarly, let E_b represents the electric field strength of bound charge, then,

$$E_b = \frac{\sigma_b}{\epsilon_0} \quad \dots (2)$$

Where, σ_b - Surface charge density of bound charge.

The net electric field strength is,

$$E = E_f - E_b \quad \dots (3)$$

Substituting equations (1) and (2) in equation (3),

$$E = \frac{\sigma_f}{\epsilon_0} - \frac{\sigma_b}{\epsilon_0}$$

$$\Rightarrow E = \frac{1}{\epsilon_0}(\sigma_f - \sigma_b)$$

$$\Rightarrow E\epsilon_0 = \sigma_f - \sigma_b \quad \dots (4)$$

Generally, the magnitude of polarization vector (\vec{P}) represents the surface charge density of bound charges.

$$\text{i.e., } P = \sigma_b \quad \dots (5)$$

And the magnitudes of electric displacement vector (\vec{D}) represent the surface charge density of free charges.

$$\text{i.e., } D = \sigma_f \quad \dots (6)$$

Substituting equations (5) and (6) in equation (4),

$$E\epsilon_0 = D - P$$

$$D = \epsilon_0 E + P$$

Therefore, the relation between E , P and D is given as,

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

3.3 ELECTROMAGNETIC WAVE

3.3.1 Equation of Plane Wave in Free Space

Q30. Explain transverse wave nature of EM wave.

Answer :

Consider a uniform plane wave, propagating in n -direction. When wave travels in x -direction, the electric and magnetic fields depend on x -direction only. In other words, these are independent of y and z directions.

An electromagnetic wave propagating along the x -direction is as shown in figure below,

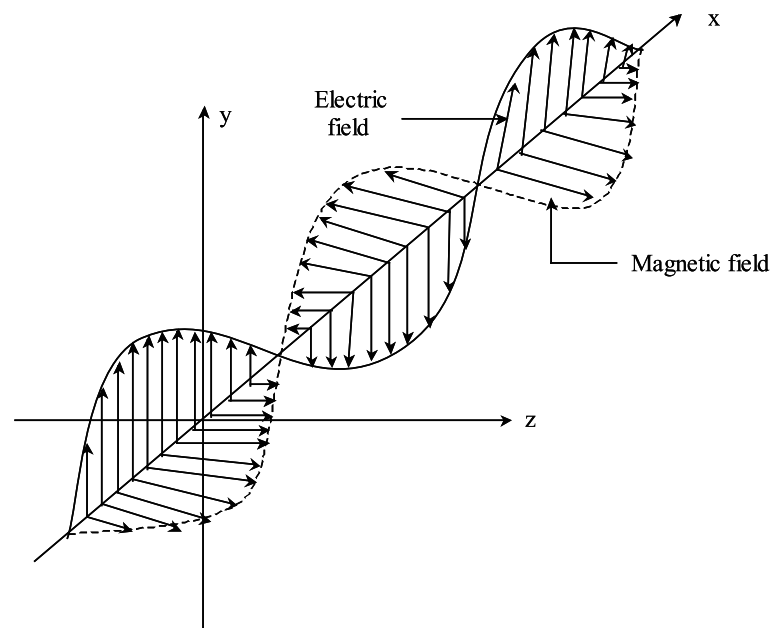


Figure: Electromagnetic Wave

It can be observed from the above figure that, the field remains constant along the y and z -axes and the electric and magnetic fields are perpendicular to each other.

For free space, the equation of plane wave is given as,

$$\nabla^2 E = \mu_0 \epsilon_0 \ddot{E}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \left[\because \nabla^2 E = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} \text{ and } \ddot{E} = \frac{\partial^2 E}{\partial t^2} \right] \quad \dots (1)$$

From the definition of uniform plane wave,

$$E \neq f(y)$$

$$E \neq f(z)$$

$$\frac{\partial^2 E}{\partial y^2} = 0, \frac{\partial^2 E}{\partial z^2} = 0 \quad \dots (2)$$

Substituting equation (2) in equation (1),

$$0 + 0 + \frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \dots (3)$$

Equation (3) can be written in terms of directions (x, y, z) ,

$$\text{i.e., } \frac{\partial^2 E_x}{\partial x^2} a_x + \frac{\partial^2 E_y}{\partial x^2} a_y + \frac{\partial^2 E_z}{\partial x^2} a_z = \mu_0 \epsilon_0 \left[\frac{\partial^2 E_x}{\partial t^2} a_x + \frac{\partial^2 E_y}{\partial t^2} a_y + \frac{\partial^2 E_z}{\partial t^2} a_z \right] \quad \dots (4)$$

Equating the respective components on both sides of equation (4),

$$\frac{\partial^2 E_x}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \quad \dots (5)$$

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_z}{\partial t^2}$$

From Maxwell's equation of free space,

$$\nabla \cdot D = 0$$

$$\Rightarrow \nabla \cdot \epsilon_0 E = 0 \quad [\because D = \epsilon_0 E]$$

$$\Rightarrow \nabla \cdot E = 0$$

$$\therefore \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\frac{\partial E_x}{\partial x} + 0 + 0 = 0 \quad \left[\because \frac{\partial E_y}{\partial y} = 0, \frac{\partial E_z}{\partial z} = 0 \right]$$

$$\frac{\partial E_x}{\partial x} = 0 \quad \dots (6)$$

Substituting equation (6) in equation (5),

$$\mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} = 0$$

$$\frac{\partial^2 E_x}{\partial t^2} = 0$$

$$\Rightarrow E_x = 0$$

Similarly, $H_x = 0$

$$\therefore E_x = 0, H_x = 0 \quad \dots (7)$$

It can be concluded from equation (7) that in a uniform plane wave, there are no electric and magnetic field components in x direction. Uniform plane wave has components of E and H only in direction perpendicular to the direction of propagation. Thus, it indicates that EM-waves in free space are transverse in nature.

Q31. Obtain Wave Equation for Electromagnetic Waves?

Answer :

Wave Equation for Electromagnetic Waves

In conductors, the net flow of charge is zero. Thus, for a uniform conducting medium charge density is zero.

$$\text{i.e., } \nabla \cdot \bar{D} = \rho_v = 0$$

Where, ρ_v - Charge density

$$\text{And, } \nabla \cdot \epsilon \bar{E} = 0$$

$$\Rightarrow \nabla \cdot \bar{E} = 0 \quad \dots (1)$$

From Maxwell's third equation for time varying fields,

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad \dots (2)$$

Applying curl on both sides of equation (2),

$$\begin{aligned} \nabla \times (\nabla \times \bar{E}) &= \frac{-\partial(\nabla \times \bar{B})}{\partial t} \\ \nabla \times (\nabla \times \bar{E}) &= -\mu \frac{\partial(\nabla \times \bar{H})}{\partial t} \quad [\because \bar{B} = \mu \bar{H}] \end{aligned} \quad \dots (3)$$

From Maxwell's fourth equation for time varying fields,

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} + \bar{J} \quad \dots (4)$$

Substituting equation (4) in equation (3),

$$\begin{aligned} \nabla \times (\nabla \times \bar{E}) &= -\mu \frac{\partial}{\partial t} \left(\frac{\partial \bar{D}}{\partial t} + \bar{J} \right) \\ \Rightarrow -\nabla^2 \bar{E} + \nabla(\nabla \cdot \bar{E}) &= -\mu \left(\frac{\partial(\bar{E})}{\partial t} + \sigma \bar{E} \right) \\ &\left[\because \nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}, \right. \\ &\quad \left. D = \epsilon E \text{ and } J = \sigma E \right] \end{aligned}$$

$$\Rightarrow \nabla^2 \bar{E} - \nabla(\nabla \cdot \bar{E}) = \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2} + \mu \sigma \frac{\partial \bar{E}}{\partial t}$$

$$\Rightarrow \nabla^2 \bar{E} - \nabla(0) = \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2} + \mu \sigma \frac{\partial \bar{E}}{\partial t} \quad [\because \text{From equation (1)}]$$

$$\therefore \nabla^2 \bar{E} = \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2} + \mu \sigma \frac{\partial \bar{E}}{\partial t} \quad \dots (5)$$

Applying curl on both sides of equation (4),

$$\begin{aligned} \nabla \times \nabla \times \bar{H} &= \frac{\partial(\nabla \times \bar{D})}{\partial t} + \nabla \times \bar{J} \\ \Rightarrow \nabla \times \nabla \times \bar{H} &= \frac{\partial(\nabla \times \epsilon \bar{E})}{\partial t} + (\nabla \times \sigma \bar{E}) \\ &[\because D = \epsilon \bar{E}, J = \sigma \bar{E}] \end{aligned}$$

$$\begin{aligned} \Rightarrow \nabla \times (\nabla \times \bar{H}) &= \epsilon \frac{\partial(\nabla \times \bar{E})}{\partial t} + \sigma (\nabla \times \bar{E}) \\ \nabla \times (\nabla \times \bar{H}) &= -\epsilon \frac{\partial^2 \bar{B}}{\partial t^2} - \sigma \frac{\partial \bar{B}}{\partial t} \\ &[\because \text{From equation (2)}] \end{aligned}$$

$$\begin{aligned} \Rightarrow \nabla \times (\nabla \times \bar{H}) &= -\mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2} - \mu \sigma \frac{\partial \bar{H}}{\partial t} \\ \therefore \nabla^2 \bar{H} - \nabla(\nabla \cdot \bar{H}) &= \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2} - \mu \sigma \frac{\partial \bar{H}}{\partial t} \quad \dots (6) \\ &[\because \nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}] \end{aligned}$$

From Maxwell's second equation,

$$\begin{aligned} \nabla \cdot \bar{B} &= 0 \\ \Rightarrow \nabla \cdot \mu \bar{H} &= 0 \\ \Rightarrow \nabla \cdot \bar{H} &= 0 \quad \dots (7) \end{aligned}$$

Substituting equation (7) in equation (6),

$$\begin{aligned} \nabla^2 \bar{H} - \nabla(0) &= \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2} + \mu \sigma \frac{\partial \bar{H}}{\partial t} \\ \therefore \nabla^2 \bar{H} &= \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2} + \mu \sigma \frac{\partial \bar{H}}{\partial t} \quad \dots (8) \end{aligned}$$

The equations of conducting medium are,

$$\begin{aligned} \nabla^2 \bar{E} &= \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2} + \mu \sigma \frac{\partial \bar{E}}{\partial t} \\ \nabla^2 \bar{H} &= \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2} + \mu \sigma \frac{\partial \bar{H}}{\partial t} \end{aligned}$$

In free space, sources such as charges or current does not exist. Thus, it acts as a perfect dielectric with zero conductivity and charge density. The wave equations for free space can be obtained by substituting $J = 0$ and $\sigma = 0$ in equations of conducting media,

$$\text{i.e., } \nabla^2 \bar{E} = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} + 0$$

$$\nabla^2 \bar{E} = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\nabla^2 \bar{H} = \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2} + 0$$

$$\nabla^2 \bar{H} = \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}$$

The electromagnetic wave equations for free space are,

$$\nabla^2 \bar{E} = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} \quad \dots (9)$$

$$\nabla^2 \bar{H} = \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2} \quad \dots (10)$$

Generally, $\mu = \mu_0 \mu_r$

$$\epsilon = \epsilon_0 \epsilon_r$$

For free space, $\mu = \mu_0$ and $\epsilon = \epsilon_0$ since, $\mu_r = 1$ and $\epsilon_r = 1$.

Thus, the equation of EM waves for free space are given as,

$$\nabla^2 \bar{E} = \mu_0 \epsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\nabla^2 \bar{H} = \mu_0 \epsilon_0 \frac{\partial^2 \bar{H}}{\partial t^2}$$

3.3.2 Poynting Theorem

Q32. State and explain poynting theorem.

Answer :

[May/June-18, Q17(b) | June/July-17, Q15(b)]

Proof

The relationship between the energy transfer (power) and the electric and magnetic field vectors can be obtained using Maxwell's equations.

From Maxwell's equations,

$$\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t} \quad \dots (1)$$

$$\text{And, } \nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} + \bar{J} = \epsilon \frac{\partial \bar{E}}{\partial t} + \bar{J} \quad \dots (2)$$

Consider a vector identity given as,

$$\nabla(\bar{A} \times \bar{B}) = \bar{B} \cdot (\nabla \times \bar{A}) - \bar{A}(\nabla \times \bar{B}) \quad \dots (3)$$

Applying the above identity for electric and magnetic field vectors \bar{E} and \bar{H} ,

$$\nabla \cdot (\bar{E} \times \bar{H}) = \bar{H} \cdot (\nabla \times \bar{E}) \quad \dots (4)$$

Substituting equations (1) and (2) in equation (3),

$$\nabla \cdot (\bar{E} \times \bar{H}) = \bar{H} \cdot \left(-\mu \frac{\partial \bar{H}}{\partial t} \right) - \bar{E} \cdot \left(\epsilon \frac{\partial \bar{E}}{\partial t} + \bar{J} \right)$$

$$\Rightarrow \nabla \cdot (\bar{E} \times \bar{H}) = -\mu \bar{H} \cdot \frac{\partial \bar{H}}{\partial t} - \epsilon \bar{E} \cdot \frac{\partial \bar{E}}{\partial t} - \bar{E} \cdot \bar{J} \quad \dots (5)$$

Consider another vector identity given as,

$$\frac{\partial(\vec{A} \cdot \vec{B})}{\partial t} = \vec{A} \cdot \frac{\partial \vec{B}}{\partial t} + \vec{B} \cdot \frac{\partial \vec{A}}{\partial t}$$

If $\vec{B} = \vec{A}$,

$$\frac{\partial \vec{A}^2}{\partial t} = 2 \vec{A} \cdot \frac{\partial \vec{A}}{\partial t}$$

$$\therefore \vec{A} \cdot \frac{\partial \vec{A}}{\partial t} = \frac{1}{2} \frac{\partial \vec{A}^2}{\partial t} \quad \dots (6)$$

Applying the above identity for \vec{E} and \vec{H} fields,

$$\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{2} \frac{\partial \vec{E}^2}{\partial t} \quad \dots (7)$$

$$\text{And, } \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = \frac{1}{2} \frac{\partial \vec{H}^2}{\partial t} \quad \dots (8)$$

Substituting equations (7) and (8) in equation (5),

$$\nabla(\vec{E} \times \vec{H}) = \frac{-\mu \partial \vec{H}^2}{2 \partial t} - \frac{\epsilon \partial (\vec{E})^2}{2 \partial t} - \vec{E} \cdot \vec{J} \quad \dots (9)$$

Equation (9) gives the relationship between E and H for a single point. In order to obtain such relationship for all points in space, consider its volume integral as,

$$\int_v \nabla(\vec{E} \times \vec{H}) dv = \int_v \left(\frac{-\mu \partial \vec{H}^2}{2 \partial t} - \frac{\epsilon \partial \vec{E}^2}{2 \partial t} - \vec{E} \cdot \vec{J} \right) dv \quad \dots (10)$$

From Divergence theorem,

$$\oint (\vec{E} \times \vec{H}) \cdot d\vec{s} = \int_v \nabla \cdot (\vec{E} \times \vec{H}) dv \quad \dots (11)$$

Substituting equation (11) in equation (10) and interchanging integral and $\frac{\partial}{\partial t}$,

$$\oint_s (\vec{E} \times \vec{H}) \cdot d\vec{S} = -\frac{\partial}{\partial t} \int_v \left[\frac{1}{2} \epsilon \vec{E}^2 + \frac{1}{2} \mu \vec{H}^2 \right] dv - \int_v \vec{E} \cdot \vec{J} dv \quad \dots (12)$$

Where,

$$\oint_s (\vec{E} \times \vec{H}) \cdot d\vec{S} \text{ - Total power leaving the volume,}$$

$$\frac{\partial}{\partial t} \int_v \left[\frac{1}{2} \epsilon \vec{E}^2 + \frac{1}{2} \mu \vec{H}^2 \right] dv \text{ - Rate of decrease in stored energy in electric and magnetic fields.}$$

$$\int_v \vec{E} \cdot \vec{J} dv \text{ - Ohmic power dissipated.}$$

From law of conservation of energy, the rate of decrease in electromagnetic energy is equal to the total output power coming out of the volume. Thus, the total outward power is given as,

$$W = \oint_s (\vec{E} \times \vec{H}) \cdot d\vec{S}$$

$$\therefore W = \oint_s \vec{P} \cdot d\vec{S} \quad \dots (13)$$

Where,

$$\vec{P} = \vec{E} \times \vec{H} \text{ (Poynting vector)}$$

Hence proved.

Q33. A uniform plane wave in free space is having $E = 200 \exp [-j(0.1)z] \cdot \mathbf{a}_x$ V/mt. Find the instantaneous value of poynting vector at $t = 0$, $z = 1$ mt.

Answer :

Given that,

For a uniform plane wave in free space,

$$E = 200 \exp (-j(0.1)z) \mathbf{a}_x \text{ V/mt}$$

The expression for instantaneous value of poynting vector is,

$$P = E \times H^*$$

From the relation between electric field intensity and magnetic field intensity in a uniform plane wave in free space,

$$\frac{E}{H} = \eta = 377 \Omega$$

$$\Rightarrow H = \frac{E}{\eta}$$

$$= \frac{200 e^{-j(0.1)z} \mathbf{a}_x}{377}$$

$$\Rightarrow H^* = \frac{200}{377} e^{j(0.1)z} \mathbf{a}_y$$

Where, H^* – Conjugate of H

$$\therefore P = E \times H^*$$

$$= (200 e^{-j(0.1)z} \mathbf{a}_x) \times \left(\frac{200}{377} e^{j(0.1)z} \mathbf{a}_y \right)$$

$$= \frac{200 \times 200}{377} \cdot [e^{-j(0.1)z} \mathbf{a}_x] \times [e^{+j(0.1)z} \mathbf{a}_y]$$

$$= 106.1 [e^{-j(0.1)z + j(0.1)z} (\mathbf{a}_x \times \mathbf{a}_y)]$$

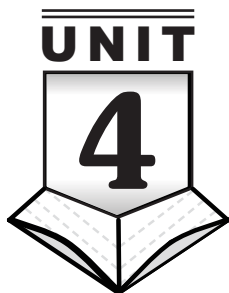
$$= 106.1 (e^0 \cdot \mathbf{a}_z)$$

$$\therefore P = 106.1 \mathbf{a}_z$$

At $t = 0$ and $z = 1$,

$$P = 106.1$$

$$\therefore P = 106.1$$



MAGNETIC MATERIALS AND SUPERCONDUCTIVITY

PART-A SHORT QUESTIONS WITH SOLUTIONS

Q1. Define magnetization.

Answer :

The process or procedure that is followed to convert a non-magnetic material into magnetic material, is called magnetization. The magnetic moment per unit volume of a magnetic material is called its intensity of magnetization (M).

Q2. Define diamagnetic materials.

Answer :

The materials that get feebly repelled by magnetic fields are called diamagnetic materials and the phenomenon is known as diamagnetism. The repulsion is due to the fact that these materials are magnetized weakly in the direction opposite to the applied field. Hence the value of susceptibility, χ is small and negative.

Magnetization of these materials is due to the magnetic dipoles that are induced temporarily by the applied field. When the external field is removed, the induced dipoles disappear. That means there are no permanent dipole magnetic materials. Also diamagnetism does not depend upon temperature.

Examples: Glass, wood, etc.

Q3. Define paramagnetic materials.

Answer :

The substances that get feebly attracted by magnetic field are called 'paramagnetic materials' and the phenomenon is known as, 'paramagnetism'. Internally, these materials are weakly magnetized in the direction of the applied magnetic field. That means susceptibility, χ of these materials is small and positive.

Examples: Aluminium, magnesium etc.

Q4. Define ferromagnetic materials.

Answer :

The materials that are strongly attracted to magnetic fields and magnets, and get permanently magnetized after the removal of external magnetic field are called ferromagnetic materials. Examples of ferromagnetic materials are iron and nickel.

Q5. Define ferromagnetism and list its properties.

Answer :

Ferromagnetism

The process in which electrically uncharged materials strongly attract other materials is defined as ferromagnetism.

Properties

1. Ferromagnetic materials possess enormous permanent dipole moment or magnetic moment. In each atom the induced magnetic moment produced by the external magnetic field is very large and it increases the magnetic induction present in the metal.
2. Susceptibility is always positive and large and it depends upon temperature in a complex manner. This behaviour is as shown in figure (a).

$$\chi = \frac{C}{T - \theta} \text{ (Only in paramagnetic region, i.e., } T > \theta \text{)}$$

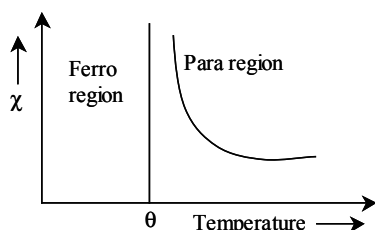


Figure (a)

3. All spins or magnetic moments are orderly oriented as shown in figure (b).

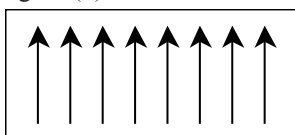


Figure (b)

4. When the temperature of the material is greater than its curie temperature then it gets converted into paramagnetic material.
5. **Examples:** Fe, Ni, Co, Fe_2O_3 , MnO, Zn-ferrite, Ni-ferrite etc.

Q6 What is hysteresis curve?

Answer :

A ferromagnetic specimen acquires magnetization when we apply an external field. This process is known as magnetization. The magnetization of the material does not vary linearly with the magnetizing field. Rather it shows a non-linear behaviour and lags behind the applied field as depicted in the figure below.

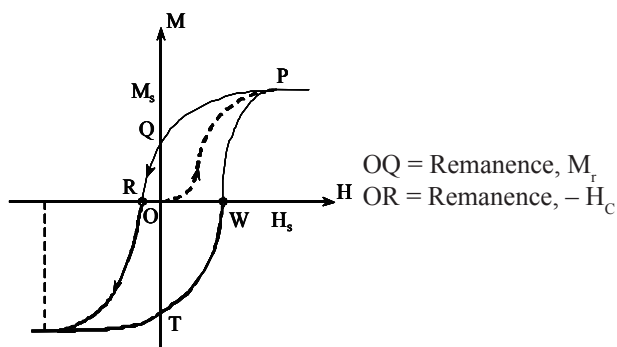


Figure: Hysteresis Curve

This curve is called hysteresis curve.

Q7. What are soft magnetic materials?

Answer :

The materials that are easy to both magnetize and demagnetize by even a small external field are called soft magnetic materials.

These materials are very soft and can be easily moulded into any desired shape. The ferrous-nickel alloy, iron-silicon alloy, aluminium-silicon-iron alloy are few examples of soft magnetic materials.

Q8. What are hard magnetic materials?

Answer :

The materials that are very difficult to both magnetize and demagnetize are called hard magnetic materials. The resistance acting on the domain wall of hard magnetic materials is very high. Due to this, domain walls cannot move easily and require very high magnetic field to get magnetize and demagnetize. Platinum-cobalt alloy, tungsten steel alloy are two most widely used hard magnetic materials.

Q9. Write the applications of ferrites.

Answer :

June-14, Q16(b)

1. Ferrites are used to produce low frequency ultrasonic waves by magnetostriction principle. Further these are used in the electromechanical transducers.
2. Ferrite rods are used in the radio receivers (particularly in medium wave coil) to increase the sensitivity and selectivity of receiver.
3. Ferrites like nickel and zinc are used as cores in audio and T.V. transformers.
4. Based on non-linear tensor permeability property, ferrites can also be used in devices for power limiting and harmonic generation.
5. Ferrites are also used in digital computers and data processing circuits. Generally ferrites with rectangular hysteresis loops are used as magnetic storage generation.

Q10. Define superconductors.

Answer :

The instant disappearance of electrical resistance for below particular temperature is called superconductivity. And, the materials which exhibit superconducting property are called superconductors.

Q11. List the critical parameters of superconductors.

Answer :

The following are the parameters of superconductors,

- (i) Effect of isotopes on superconductors
- (ii) Effect of entropy on superconductors
- (iii) Internal conductivity
- (iv) Superconductivity occurrence
- (v) Zero resistance
- (vi) Magnetic flux density
- (vii) Effect of magnetic flux density
- (viii) Effect of magnetic field.

Q12. Explain the isotopic effect in super conductors.

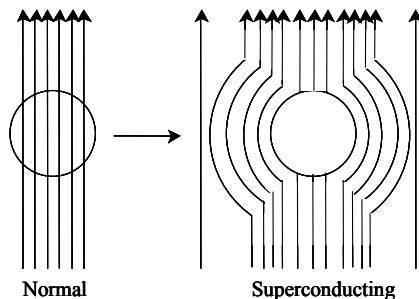
Answer :

June-11, Q8

Isotopes of same metal exhibit different critical temperatures to enter into super conductivity. This effect is known as isotopic effect.

Q13. Define Meissner effect and give its applications.**Answer :****Meissner Effect**

In the presence of weak magnetic field below the critical temperature, T_c the magnetic flux lines produced by the specimen expels out from its region as shown in figure below. This effect is known as Meissner effect.

**Figure****Applications**

1. Meissner effect is used to test the superconducting property of the specimen. (i.e., it proves whether the specimen is superconductor or not).
2. It exhibits levitation effect, the phenomenon in which a larger superconductor repels the smaller magnet and makes to float in the air. This effect is used in the functioning of maglev trains.

Q14. Define the terms critical temperature and critical current.**Answer :****Critical Temperature**

The temperature below which the superconductor will be in superconducting state and above which the superconductor will be in normal state is called as critical temperature.

Critical Current

The maximum current that a wire can carry with zero resistance is referred to as critical current. For a long straight wire, the expression for critical current is given by,

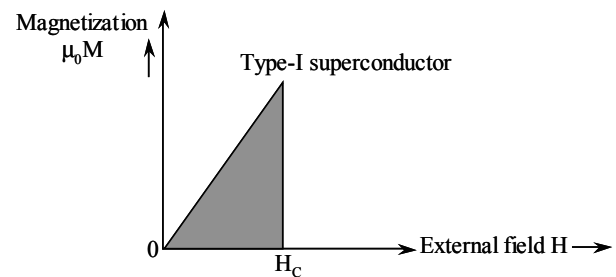
$$I_c = \frac{2\pi a B_c}{\mu_0}$$

Where,

 B_c - Critical magnetic field a - Radius of the wire μ_0 - Permeability of free space.**Q15. Define Type-I and Type-II superconductors.****Answer :****Type-I Superconductors**

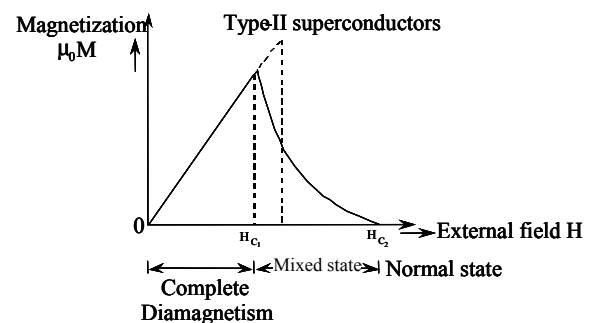
The superconductors which suddenly change to normal state (as shown in figure (i)) at critical value of magnetic field, H_c when placed in a magnetic field are called type-I superconductors.

The magnetization curve for type-I superconductor is shown in figure (i).

**Figure (i)****Type-II Superconductors**

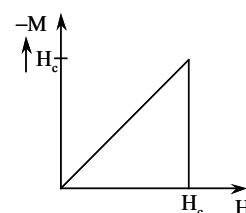
The superconductors which gradually change to normal state (as shown in figure (ii)) at critical value of magnetic field, H_c when placed in a magnetic field are called type-II superconductors.

The magnetic curve for Type-II superconductor is shown in figure (ii),

**Figure (ii)****Q16. Give the properties of Type-I superconductors.****Answer :**

Properties of type-I superconductors,

- (i) These are mostly developed from pure specimens.
- (ii) The transition between normal and superconducting states is sharp.
- (iii) There is only one value of critical magnetic field H_c for these superconductors.
- (iv) Type-I superconductors exhibit complete Meissner effect.
- (v) The values of H_c are too low to have any useful magnetic applications of these superconductors.
Typical values $H_c \approx 1$ kilo Gauss
- (vi) Magnetization curve of Type-I superconductor is shown in figure.

**Figure**

Q17. Define BCS theory.**Answer :**

According to this theory, a slight interaction of conduction electrons with the lattice vibrations (phonons) gives rise to superconductivity. Such an electron-phonon interaction is responsible for the formation of bounded electron pairs called cooper pairs. These electron pairs exhibit opposite momenta and spin.

Q18. Classify superconductors based on transition temperature.**Answer :**

Depending on transition temperature (T_c), superconductors are classified into two types. They are,

- (i) Low T_c superconductors
- (ii) High T_c superconductors.

(i) Low T_c Superconductors

The materials which transform into superconductors at temperature below 24 K are called low T_c superconductors.

(ii) High T_c Superconductors

The materials which transform into superconductors at temperature beyond 27 K are called high T_c superconductors.

Q19. Define the terms (i) retentivity (ii) coercivity.**Answer :****Dec.-17, Q4****(i) Retentivity**

When a ferromagnetic specimen is taken through a cycle of magnetization (i.e., through hysteresis loop) and then the external field is removed then the magnetization, M of the specimen will not become zero. Rather the material will retain some amount of magnetization. This magnetization is called, 'Residual Magnetization' or 'Retentivity'. The value of retentivity varies from material to material.

(ii) Coercivity

Coercivity of a ferromagnetic material is defined as the magnetic field value that is needed to decrease the residual magnetism to zero. It is denoted as H_c . The units of coercivity are Oersted or Ampere/meter (A/m).

Coercivity is used to determine the ferromagnetic material resistance in order to become demagnetized. (B-H) Analyzer is employed in-order to measure coercivity. The materials that have high coercivity are called as hard ferromagnetic materials. The materials having low coercivity are called as soft ferromagnetic materials.

Q20. Explain the effect of magnetic field on a superconductor.**Answer :****June-10, Q10**

The magnetic field set up eddy currents, which will cancel the magnetic field inside the superconductor. As superconductors have no resistivity, the eddy currents do not decay, hence magnetic field cannot penetrate.

Hence, a magnetic field applied to a superconductor cannot penetrate into it.

Q21. The superconducting transition temperature of a metal is 7.26 K. The critical field at 0 K is 64×10^3 A/m. Calculate the critical field at 5 K.**Answer :****[June-17, Q4 | Jan.-13, Q8]**

Given that,

Transition temperature, $T_c = 7.26$ K

Critical field at 0 K, $H_0 = 64 \times 10^3$ A/m

Temperature, $T = 5$ K

The expression for critical field of a super conductor is given by,

$$H_c = H_0 \left[1 - \frac{T^2}{T_c^2} \right]$$

$$= 64 \times 10^3 \left[1 - \frac{(5)^2}{(7.26)^2} \right]$$

$$\therefore H_c = 33.643 \times 10^3 \text{ A/m}$$

PART-B

ESSAY QUESTIONS WITH SOLUTIONS

4.1 MAGNETIC MATERIALS

4.1.1 Classification of Magnetic Materials - Dia, Para, Ferro, Antiferro and Ferrimagnetic Materials

Q22. Give the classification of magnetic materials and also list their corresponding properties.

Answer :

Classification of Magnetic Materials

Depending on the atomic dipoles and interaction between them, the magnetic materials are classified as,

1. Diamagnetic materials
2. Paramagnetic materials
3. Ferromagnetic materials.

1. Diamagnetic Materials

For answer refer Unit-4, Q2.

Properties

Properties of diamagnetic materials are as follows,

1. Diamagnetic materials are repelled by the applied magnetic field.
2. The induced magnetic moment of diamagnetic materials is opposite to the direction of external magnetic field.
3. Diamagnetic materials do not have permanent magnetic dipoles. As a result, magnetic effects are very small.
4. The susceptibility of these materials is small and negative.
5. The susceptibility of these materials is independent of temperature and magnetic field strength.

Examples: Bismuth, niobium, alkali earths etc.

2. Paramagnetic Materials

For answer refer Unit-4, Q3.

Properties

The following are the properties of paramagnetic materials,

- (i) Paramagnetic materials are feebly (weakly) attracted by magnetic fields.
- (ii) When paramagnetic materials are placed in magnetic field, they move from weak magnetic field area to strong magnetic field area.
- (iii) Paramagnetic materials obey Curie law.
- (iv) The susceptibility of paramagnetic materials is very small.
- (v) Paramagnetism depends on temperature.
- (vi) The net dipole moment of paramagnetic atoms is due to their orbital and spin magnetic moments.
- (vii) The permeability of paramagnetic materials is 1.
- (viii) In absence of a magnetic field, the magnetic dipoles of paramagnetic materials align in random directions.

3. Ferromagnetic Materials

For answer refer Unit-4, Q4.

Properties

For answer refer Unit-4, Q5, Topic: Properties.

Q23. Give the applications of anti-ferromagnetic and ferri-magnetic materials.**Answer :****Applications of Antiferro Magnetic Materials**

- (i) Antiferro magnetic materials are used in magnetic thin film devices like magnetic hard disk and sensors.
- (ii) They are used to stabilize the magnetization of nano sized particles at room temperature.
- (iii) They are also used in the modification of switching behaviour of ferromagnetic film.

Applications of Ferrimagnetic Materials

The ferri-magnetic materials are used in,

- (i) The generation of low frequency ultrasonic waves with the help of magnetostriction principle.
- (ii) Electrochemical series.
- (iii) Radio receivers to increase the selectivity and sensitivity of the receiver.
- (iv) Audio and T.V transformers.
- (v) The devices to harmonic generation and power limitation.
- (vi) Digital computers and data processing circuits.

Q24. Distinguish between ferro, antiferro and ferri magnetic materials.**Answer :**

Jan.-13, Q7

Ferromagnetic Materials	Antiferro Magnetic Materials	Ferri Magnetic Materials
1. These materials attract lines of force very strongly.	1. These are special cases of Ferro magnetic materials on the basis of spin alignment.	1. These are special case of antiferro magnetic materials on the basis of amplitude of the spin.
2. Possess very large positive susceptibility, $\chi = \frac{C}{T - \theta}$ T - Absolute temperature θ - Curie temperature	2. Initially the susceptibility increases slightly as the temperature increases and beyond Neal temperature the susceptibility decreases, $\chi = \frac{C}{T + \theta}$ When $T > T_N$	2. Susceptibility is very large and positive if $T > T_N$, $\chi = \frac{C}{T + \theta}$
3. All spins or magnetic moments are orderly oriented $\uparrow \uparrow \uparrow \uparrow \uparrow$	3. The spin alignment is antiparallel to each other and the opposite alignment of adjacent magnetic moments in a solid is generated by an exchange interaction when $T > T_N$. $\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$.	3. It is composed of two sets of different transition metal ions having different values of magnetic moment with antiparallel $\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$.
4. Examples Iron (Fe), Nickel(Ni) Cobalt (Co).	4. Examples 1. Manganese oxide (MnO) 2. Chromium oxide (Cr_2O_3) and 3. Salts of transition metals.	4. Example All ferrites and garnets.

4.1.2 Weiss Molecular Field Theory of Ferromagnetism

Q25. Discuss the Weiss theory of ferromagnetism.

June-14, Q16(a)

OR

Explain Weiss molecular field theory of ferromagnetic material.

Answer :

May/June-12, Q15(b)

P. Weiss in 1907 proposed a theory to explain 'Ferromagnetism'. According to this theory,

1. In ferromagnetic materials, there is internal molecular field ' H_i ' and it sets up spontaneous magnetization.
2. This field magnetizes each domain and which has a definite magnetic moment.
3. The internal field is proportional to magnetization (M) of the material.
4. This magnetization is called spontaneous magnetization.

$$\therefore \text{Internal field, } H_i = \gamma M \quad \dots (1)$$

Where ' γ ' is called Weiss molecular field coefficient and determines the interactions between dipoles.

The effective magnetic field on atomic dipole is

$$H_e = H + H_i \quad \dots (2)$$

Where ' H ' is the applied magnetic field.

Consider a model in which there are ' N ' spin per cubic meter. Each spin will produce a magnetic moment of 1 Bohr magneton, which may lie in same or opposite direction to the applied field.

The total magnetization will be,

$$M = N \mu_B \tanh \left[\left(\frac{\mu_0 \mu_B}{KT} \right) (H + \gamma M) \right] \quad \dots (3)$$

At a very high temperature, the internal field in ferromagnetic material is insufficient to maintain the alignment of magnetic dipole. The behaviour of ferromagnetic material will become same as paramagnetic except for the case in which the applied field on the dipole is given by equation(2). And also,

at high temperature the term $\left(\frac{\mu_0 \mu_B}{KT} \right) (H + \gamma M)$ will be less than unity.

So, $\tanh x \approx x$ (for $x < 1$)

$$\therefore M = \left(N \mu_0 \frac{\mu_B^2}{KT} \right) (H + \gamma M)$$

$$M \left[1 - \left(N \mu_0 \frac{\mu_B^2 \gamma}{KT} \right) \right] = \left(\frac{N \mu_0 \mu_B^2 H}{KT} \right)$$

Susceptibility,

$$x = \frac{M}{H} = \frac{\left(\frac{N \mu_0 \mu_B^2}{K} \right)}{\left[T - \left(\frac{N \mu_0 \mu_B^2 \gamma}{KT} \right) \right]}$$

$$\text{Let } C = \frac{N \mu_0 \mu_B^2}{K}; \theta = \gamma C$$

$$\therefore \chi = \frac{C}{T - \theta}$$

Q26. Explain Weiss molecular field theory of ferromagnetism and obtain the Curie Weiss law.

Answer :

Jan.-13, Q16(a)

Weiss Molecular Field Theory

According to the Weiss molecular field theory, there exists an internal magnetic field H_p that possesses a ferromagnetic property and it is proportional to the magnetization I , which is called as spontaneous magnetization.

$$\text{i.e., } H_i = \lambda I$$

Curie Weiss Law

The phenomenon of exhibiting the magnetization even after removing the magnetic field is known as ferromagnetism.

$$H_i = \lambda I \quad \dots (1)$$

Where,

H_i - Internal molecular field

λ - Weiss coefficient

I - Spontaneous magnetization.

The net effective field is given by,

$$H_e = H + H_i$$

$$\therefore H_e = H + \lambda I \quad \dots (2)$$

According to Langevin theory,

$$I = \frac{\mu_0 \mu^2 N H_e}{3kT}$$

$$\Rightarrow I = \frac{\mu_0 \mu^2 N}{3kT} (H + \lambda I) \quad [\because H_e = H + \lambda I]$$

$$\Rightarrow I = \frac{\mu_0 \mu^2 N H}{3kT} + \frac{\mu_0 \mu^2 N \lambda I}{3kT}$$

$$\Rightarrow I - \frac{\mu_0 \mu^2 N \lambda I}{3kT} = \frac{\mu_0 \mu^2 N H}{3kT}$$

$$\Rightarrow I \left[1 - \frac{\mu_0 \mu^2 N \lambda}{3kT} \right] = \frac{\mu_0 \mu^2 N H}{3kT}$$

Substituting,

$$C = \frac{\mu_0 \mu^2 N}{3k} \text{ and } \theta = C\lambda$$

$$\Rightarrow I \left[1 - \frac{\theta}{T} \right] = \frac{CH}{T} \quad \dots (3)$$

$$\text{Susceptibility, } \chi = \frac{I}{H}$$

From equation (3),

$$\frac{I}{H} = \frac{C}{T \left[1 - \frac{\theta}{T} \right]}$$

$$\therefore \chi = \frac{C}{T - \theta}$$

The above equation is known as Curie Weiss law and 'θ' is Curie temperature.

4.1.3 Magnetic Domains – Hysteresis Curve-Soft and Hard Magnetic Materials

Q27. What are magnetic domains? Using that, how will you explain the formation of hysteresis in ferromagnetic materials?

Answer :

Magnetic Domains

A specimen (Ferromagnetic material) is divided into a number of small regions called domains, in such a way that all of them are spontaneously magnetized but their cumulative effect will give bulk magnetism depending on that temperature if it is below the Curie temperature and also on the external magnetic field.

Hysteresis Based on Domain Theory

Above Curie temperature their moments neutralize completely with one another giving rise to demagnetized state.

P. Weiss explained hysteresis effect that occurs in ferromagnetic materials. According to him, a ferromagnetic material is made of small regions called domain. All the domains in the material are magnetized, as shown in figure (1).

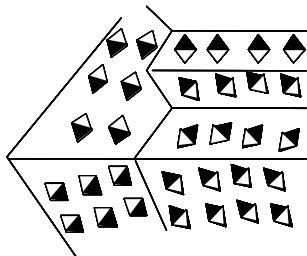


Figure (1): Domains in a Demagnetized Ferromagnetic Solid

It can be seen in the above figure that domains are randomly oriented and because of this the material has zero magnetization. In order to magnetized the material, all the domains must point in the same direction. There are two ways to high domains in the same direction. The first is to allow the

rotation of individual domain magnetization. The second way to magnetized the domain is by using the domains that already point in the direction of magnetic field, as shown in figure (2).

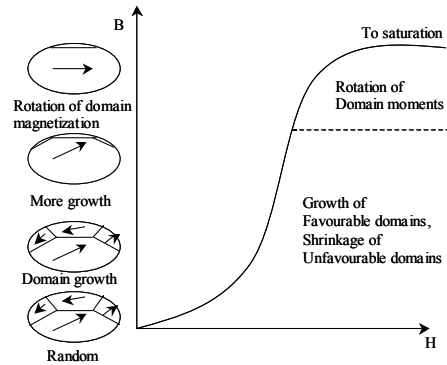


Figure (2)

When an external magnetic field is applied, all the domains point in the same direction and magnetic induction increases very rapidly. After sometime, magnetic induction reaches its saturation limit and stops increasing. Now, when the external magnetic field is removed, the material remains magnetized. This is because only few domains, revert back to their original random direction, while most of the domains still high in the direction of magnetic field. However, if a reverse field is applied, the domain structure may change and once again produce zero magnetic induction.

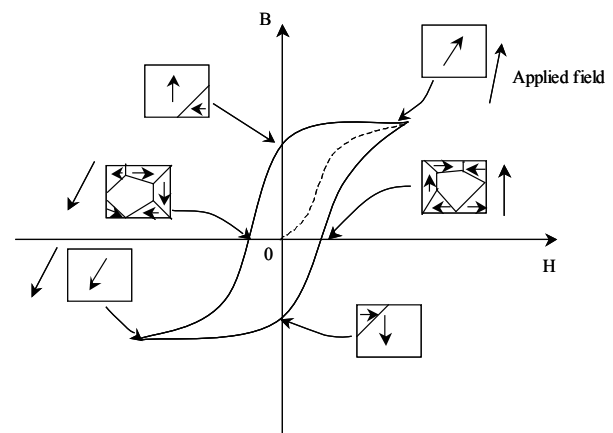


Figure (3): Summary of Domain Microstructures During the Course of a Ferromagnetic Hysteresis Loop

Q28. Draw the nature of magnetic dipole moments and variation of susceptibility with temperature graphs in ferro-ferri and anti-ferromagnetic materials.

Answer :

June-11, Q7

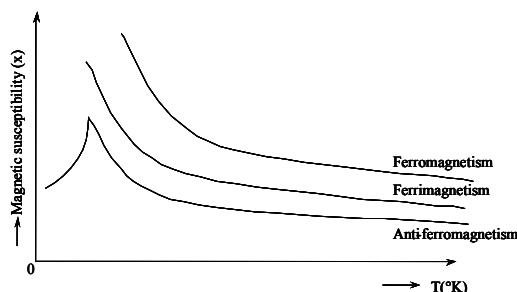
Origin of Magnetic Moment

The magnetic moment in a material originates from the orbital motion and spinning motion of electrons in an atom.

When a magnetic moment is obtained through the motion of electrons in various orbits of an atom, then it is called 'orbital magnetic moment', whose magnitude is always small.

In an atom, generally every two electrons will form a pair such that, they have opposite spins. Thus, the resultant spin magnetic moment, is zero. But in magnetic materials like iron, cobalt, nickel etc., there are unpaired electrons in the 3d orbital. These unpaired electrons spin resulting in magnetic moment interact with the adjacent atom's unpaired electrons in the 3d orbital. These unpaired electrons spin to align in a parallel manner resulting in enormous spin magnetic moment. Thus, these unpaired electron spins are responsible for ferro and paramagnetic behaviour of materials. The value of spin magnetic moment is very large, when we compare it with orbital magnetic moment.

The variation of susceptibility with temperature graphs in ferro-ferri and anti-ferromagnetic materials is shown in below figure.



Figure

Q29. Draw and explain B-H curve for a ferro-magnetic material placed in a magnetic field.

OR

Explain hysteresis of a ferromagnetic material.

Answer :

A ferromagnetic specimen acquires magnetization when we apply an external field. This process is known as 'magnetization'. The magnetization of the material does not vary linearly with the magnetizing field. Rather it shows a non-linear behaviour and lags behind the applied field as depicted in figure. This curve is called Hysteresis Loop. Thus, hysteresis loop of a ferromagnetic specimen, is a curve showing the change in magnetic induction in it, when the intensity of the applied field is varied from H_s to $-H_s$ and back again, where H_s is the saturation field.

1. If the applied field H is increased gradually, starting with an unmagnetized ($M = 0$) bar, the corresponding value of M increases slowly at first and then increases rapidly as H becomes larger and eventually reaches the point P . M remains constant for any further raise in H and the substance is said to be saturated. This value of M called as saturation magnetization is denoted by M_s .

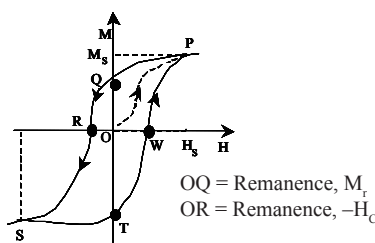


Figure: Hysteresis Loop

2. When H is decreased back gradually, M does not retrace the original path OP . Rather it follows a new curve, PQ as shown in figure. When $H = 0$, some magnetization (OQ) is still retained. This value of M is called as residual magnetization (M_r) or retentivity.
3. The decrease in M continues even when H is increased the in reverse direction and becomes zero at a value of $H = -OR$, where the field OR is called as, 'coercive force of the material'. Further increase in H in the same (i.e., $-H$) direction, results in the saturation of M (now, $-M_s$) at point, S .
4. Now if the field H is decreased ($-H \rightarrow O$) M begins to decrease from ($-M_s$). But this decrease of M does not retrace the old path RS rather it proceeds along STW . Magnetization M will increase in H in the positive direction, and will reach the saturation value, M_s finally.

Q30. What is hysteresis? Draw a hysteresis loop for ferromagnetic material and explain the various important points on it.

Answer :

Hysteresis

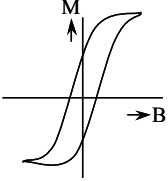
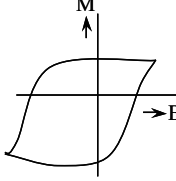
For answer refer Unit-4, Q6.

B-H Curve

For answer refer Unit-4, Q29.

Q31. Distinguish between hard and soft magnetic materials.**Answer :**

[Dec.-17, Q16(b) | June-17, Q3 | Jan-12, Q14(b)]

Soft Magnetic Materials (SMM)		Hard Magnetic Materials (HMM)	
1.	Soft magnetic materials are those that can be easily magnetized and demagnetized.	1.	Hard magnetic materials are those that are difficult to magnetize and demagnetize.
2.	These materials have low coercive fields.	2.	These materials have high coercive fields.
3.	The hysteresis loop for these materials will be thin and long as shown in figure.	3.	The hysteresis loop for these materials is wide as shown in figure.
			
	Figure: Hysteresis Loop for Soft Magnetic Materials		Figure: Hysteresis Loop for Hard Magnetic Materials
4.	Hysteresis loss is less.	4.	Hysteresis loss is large.
5.	Magnetic permeability and magnetic susceptibility are high.	5.	Magnetic permeability and magnetic susceptibility are low.
6.	These are used in the preparation of magnetic core materials.	6.	These are used in the preparation of permanent magnets.
7.	SMM find applications in transformers, electric motors, magnetic amplifiers, magnetic switching circuits etc.	7.	HMM find applications in meters, microphones, magnetic detectors, magnetic separators etc.
8.	Examples of these type of materials are, (i) Pure iron. (ii) Alloys of iron-silicon. (iii) Iron-cobalt. (iv) Iron-nickel.	8.	Examples of these type of materials are, (i) Alnico [Alloy of Al, Ni, Co, Cu and Fe] (ii) Tungsten steel alloy. (iii) Platinum-cobalt alloy. (iv) Invar, etc.

4.1.4 Ferrites - Applications of Ferrites**Q32. What are ferrites? Explain the magnetic properties of ferrites and mention their industrial applications.****Answer :****Ferrites**

The magnetic moments of sublattices in anti-ferromagnetic are equal in magnitudes and opposite in direction, once the net magnetization is a small value. But in ferromagnetic materials a net magnetization is observed even in the absence of applied field because of unequal and antiparallel magnetic moments of sublattice. Ferrites (or) ferrimagnetic materials exhibit a spontaneous magnetic moment below a certain temperature called Curie temperature just as ferromagnetic materials.

The general formula of ferrites may be written in simple form $M_e^{+2} Fe_2^{+2} O_4$.

Where, M_e^{+2} represents divalent metallic ions such as Fe^{+2} , Co^{+2} , Mn^{+2} , Cd^{+2} , Mg^{+2} etc.

Properties

- These are metal oxides, but not metals.
- These are possessing high dielectric constants and varying from ten to few hundreds.
- These materials are having high resistivity values and varying from 10^3 to 10^8 ohm-m.
- These materials exhibit spontaneous magnetization like ferroelectric materials.
- These materials exhibit hysteresis property.
- Ferrites are classified into two types,
 - Normal spinel ferrites
 - Inverse spinel ferrites.

Applications

For answer refer Unit-4, Q9.

4.2 SUPERCONDUCTIVITY

4.2.1 Introduction - General Properties of Superconductors - Meissner Effect - Type-I and Type-II Superconductors

Q33. What is superconductivity? Explain the general properties of superconductors.

Answer :

Superconductivity

The instant disappearance of electrical resistance for below particular temperature is called superconductivity. And, the materials which exhibit superconducting property are called superconductors.

Properties of Superconductors

The following are the properties of superconductors,

1. Superconductors exhibit zero electrical resistance.
2. The current established in a superconductor loop, preserves for a long time and flows without attenuation.
3. Below the critical temperature, superconducting state of the material changes to normal state.
4. Superconductors exhibit perfect diamagnetic properties and hence do not allow magnetic field to pass through them.
5. For the applied magnetic field greater than critical magnetic field, superconducting property of the material disappears.
6. Entropy of the conductors reduce linearly till critical temperature and incredibly below critical temperature.

Q34. Write a note on Meissner effect relating to superconductivity.

OR

What is Meissner effect? Explain.

Answer :

Meissner Effect

For answer refer Unit-4, Q13, Topic: Meissner Effect.

The magnetic induction inside the specimen is expressed as,

$$\begin{aligned} B &= \mu_0(H + M) \quad \dots (1) \\ &= \mu_0(1 + M/H) H \\ &= \mu_0(1 + \chi) H \end{aligned}$$

Where,

H - Magnetic field applied

M - Magnetization

χ - Material susceptibility = $\frac{M}{H}$

For $T < T_c$, $B = 0$

Then, equation (1) becomes,

$$\mu_0(H + M) = 0$$

$$\Rightarrow H + M = 0$$

$$\Rightarrow H = -M$$

Therefore, the material susceptibility, $\chi = \frac{M}{H} = -1$.

Hence, the specimen acts as diamagnet. The state of specimen in which external magnetic field becomes '0' due to magnetization is called perfect diamagnetism.

The perfect diamagnetism and zero resistivity are the two important properties of superconductors.

Applications of Meissner Effect

For answer refer Unit-4, Q13, Topic: Applications.

Q35. Explain the critical magnetic field of a superconductor as a function of temperature.

Answer :

Critical magnetic field is the magnetic field ' B_c ' at which superconductor undergoes transition from superconducting state to normal state. At temperature below critical temperature (T_c), in the absence of any magnetic field, the materials are in superconducting state.

$$\text{At, } T = T_c$$

$$B_c = 0$$

At temperatures below T_c , B_c increases the dependence of the critical field upon the temperature which is expressed as,

$$B_c(T) = B_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

Where,

$B_c(T)$ – Critical field.

$B_c(0)$ and T_c – Constants

The characteristics of the material are as shown in figure.

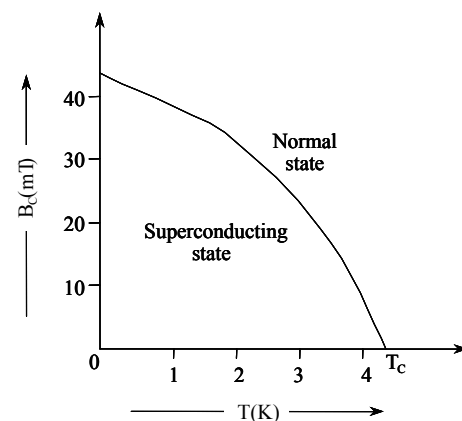


Figure: Characteristics of $B_c(0)$ and T_c

Q36. Define the terms critical temperature, critical magnetic field and critical current.

Answer :

Jan.-12, Q8

Critical Temperature

For answer refer Unit-4, Q14, Topic: Critical Temperature.

Critical Magnetic Field

For answer refer Unit-4, Q35.

Critical Current

For answer refer Unit-4, Q14, Topic: Critical Current.

Q37. Explain about type-I and type-II superconductors.**Answer :**

Based on the magnetic response, superconductors are classified into two types. They are,

1. Type-I superconductors
2. Type-II superconductors.

1. Type-I Superconductors

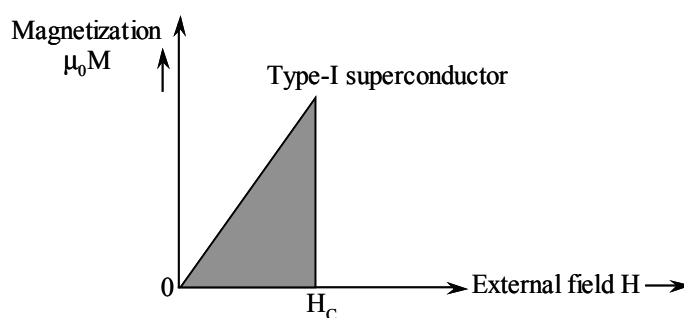
The superconductors which suddenly change to normal state (as shown in figure (i)) at critical value of magnetic field, H_C when placed in a magnetic field are called type-I superconductors.

Type-I super conductors easily obey Meissner effect.

Consider a Type-I super conductor placed in an external magnetic field. As the magnetic field increases, the material remain as diamagnetic. When the magnetic field reaches to a critical value ' H_C ', the conduction becomes normal and hence magnetic flux starts penetrating.

Examples: Aluminium and Zinc.

The magnetization curve for type-I superconductor is shown in figure (i).

**Figure (i)****2. Type-II Superconductors**

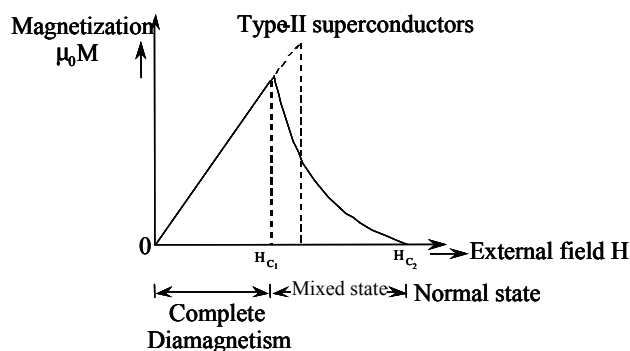
The superconductors which gradually changes to normal state (as shown in figure (ii)) at critical value of magnetic field, H_C when placed in a magnetic field are called type-II superconductors.

Type-II super conductors obey meissner effect only to some extent.

Consider a Type-II Super conductor placed in an external magnetic field. As the magnetic field increases the material remain as diamagnetic when the magnetic field reaches to a lower critical value ' H_{C1} ', the Type-II super conductor gradually starts losing its super conductivity when the magnetic field reaches to a higher critical value ' H_{C2} ' is known as intermediate or vortex state. After H_{C2} , the superconductor behaves as a conductor.

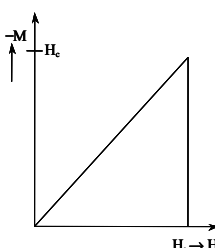
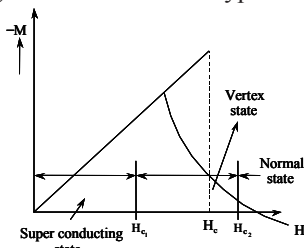
Examples: NbN and BaBi_3 .

The magnetic curve for Type-II superconductor is shown in figure (ii),

**Figure (ii)**

Q38. Distinguish between Type-I and Type-II superconductors.**Answer :**

[Dec.-17, Q17(a) | May/June-12, Q8]

Type-I Superconductors	Type-II Superconductors
<ol style="list-style-type: none"> Type-I superconductors are mostly developed from pure specimens. The transition between normal and superconducting states is sharp. There is only one value of critical magnetic field H_c. Exhibit complete Meissner effect. The values of H_c are too low to have any useful magnetic applications. No intermediate state exists. These are referred to as soft superconductors. There is a sudden loss in magnetization property. Magnetization curve of Type-II superconductors is,  <p style="text-align: center;">Figure</p> <ol style="list-style-type: none"> Examples of Type-I superconductors are lead, tin, mercury. 	<ol style="list-style-type: none"> Type-II superconductors are developed from alloys, compounds, ceramics, transition plates etc. The transition between normal and superconducting states is not sharp. Two critical values of magnetic field H_{c1} and H_{c2} are identified. Exhibit complete Meissner effect in the range $0 < H < H_{c1}$. The mixed state (i.e., $H_{c1} < H < H_{c2}$) is more useful for magnetic applications. Intermediate state exists. These are referred to as hard superconductors. There is gradual loss in magnetization property. Magnetization curve of Type-II superconductors is,  <p style="text-align: center;">Figure</p> <ol style="list-style-type: none"> Examples of Type-II superconductors are Nb - Sn, Nb-Ti, Nb-Zr, and Va-Ga.

4.2.2 BCS Theory (Qualitative) - Introduction to High T_c Superconductors (in Brief) - Applications of Superconductors

Q39. Explain BCS theory of superconductors.**Answer :**

BCS theory is the foundation for quantum theory of superconductivity. It was proposed by three American physicists named Bardeen, Cooper and Schrieffer in the year 1957. Hence the name BCS theory.

According to this theory, a slight interaction of conduction electrons with the lattice vibrations (phonons) gives rise to superconductivity. Such an electron-phonon interaction is responsible for the formation of bounded electron pairs called cooper pairs. These electron pairs exhibit opposite momenta and spin.

When an electron is made to pass through a lattice of positive ions, it tends to attract other positive ions adjacent to it and dislocates them, creating a wave of increased positive charge density as shown in figure.

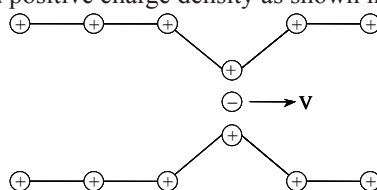


Figure: Electron Travelling through a Lattice of Positive Ions Showing Area of Increased Positive Charge Density

This wave travels along the material in the form of a vibrational wave inside the lattice and is termed as phonons. When another electron passes through the same region of increased positive charge density, a Coulomb interaction takes place between the two electrons with phonons as their mediators. This results in electron-lattice-electron interaction which is attractive and at low temperatures it may exceed the Coulomb force of repulsion between them. Thus, Cooper pairs are formed.

BCS theory also succeeded in explaining the existence of energy gap in superconductors. As the electrons in Cooper pairs have opposite spin and momenta, the overall spin of a cooper pair is zero. Such Cooper pairs collectively form a system of bosons.

This means that any number of cooper pairs can exist within same quantum state and with same energy. At ground state of superconduction with $T = 0$, all the cooper pairs lie in the same state. The emission or absorption of energy by electrons in the superconducting state is possible only by unbinding the cooper pairs. The energy which breaks up a cooper pair is known as superconducting energy gap ' E_g '. At absolute zero, the energy gap is given as,

$$E_g = 3.5 kT_c$$

Where,

K - Boltzmann's constant

The energy gap decreases with increase in temperature and gradually becomes zero at $T = T_0$.

Where,

T_c - Critical temperature.

This theory also proved the existence of Meissner effect and coherence length.

Q40. Describe the preparation of High T_c superconductor.

OR

Explain in detail about high temperature superconductors.

Answer :

High Temperature Superconductors

The materials which transform into superconductors at temperature beyond 24 K are called high temperature superconductors.

Various Materials used in High Superconductors

Initially, before 1980's, metallic alloy Nb_3Ge was used to achieve high superconductivity but due to its low transition temperature i.e., 23.2 K, it is not accomplished.

In the year 1986, Bednorz and Muller used ceramic materials for high superconductivity. They discovered that mixing of metallic oxide i.e., lanthanum-barium-copper ($La_1Ba_2Cu_3O_7$) produces high superconductive material at 30 K. But oxide components have deficiency of oxygen ions. By controlling this oxygen ions deficiency, high superconductivity can be achieved using oxides in the temperature range 30-40 K.

In 1987 Chu and co-workers used another materials yttrium and prepared $YBa_2Cu_3O_7$ with transition temperature of 95K. This material is very economical because it uses very less expensive liquid nitrogen coolant. Thus, it is widely used for preparation of high T_c superconductor. But, due to the deficiency of oxygen in oxide, high T_c superconductor exhibits a defective perovskite structure with three perovskite cubic unit cells stacked on top of each other. Thus, it is necessary to maintain proper oxygen level.

There exist other compound systems such as, Bi-Sr-Cu-O (or) Ba-Ca-Cu-O which is used for high T_c superconductors. For examples, the compounds $Bi_2CaSr_2Cu_2O_{8+x}$ and $Bi_2Ca_2Sr_2Cu_3O_{10+x}$ exhibits superconductivity and has the transition temperature 85 K and 110 K respectively.

In 1993, by using mercury based copper oxide $HgBa_2Ca_2Cu_3O_{1+x}$ material a high T_c superconductor with transition temperature of 133 K is prepared.

All these materials discussed above are categorized into four major families. They are,

- (i) LBCO (mixed oxide of lanthanum, barium and copper).
- (ii) YBCO (mixed oxides of yttrium, barium and copper).
- (iii) BSCCO (mixed oxides of bismuth, strontium, calcium and copper).
- (iv) TBCCO (mixed oxides of thallium, barium, calcium and copper).

Q41. Discuss the properties of high T_c superconductors.

Answer :

The following are the properties of high T_c superconductors,

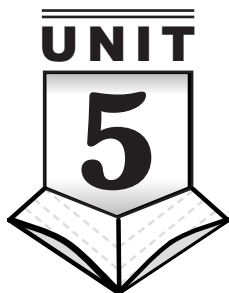
1. The high T_c superconductors are very delicate.
2. Since, the charge carriers in high T_c superconductors are holes, it has positive "Hall coefficient".
3. The materials of these superconductors are highly anisotropic in their normal state.
4. The materials of high T_c superconductors possess highly anisotropic magnetic properties.
5. The effect of pressure is not uniform on all superconducting materials (i.e., different materials has different pressure).
6. The materials of high T_c superconductors has negligible isotope effect.
7. The performance of these superconductors cannot be described using BCS theory.

Q42. Give the applications of superconductors.

Answer :

The following are some of the important applications of superconductors,

1. Superconductors are used as catalysts in industrial processes.
2. They are used in electrical circuits, due to their high speed of operation and efficiency.
3. They can build an extremely fast and large scale computer of compact size with less than 0.5 W power consumption.
4. Superconducting magnets are used in scientific tests and research equipments to generalise high fields with low power consumption. They separate tumour cell from healthy cell by using high gradient magnetic separation method.
Superconducting magnets drive levitated train at a speed of 500 cm/h with minimum energy usage.
5. Due to their small size, superconductors are used to maintain plasma in magneto-hydrodynamic power generators.
6. They are used in magnetic resonance imaging (in the field of medicine) as a diagnostic tool to detect the abnormalities in body tissues and organs.
7. Superconducting cables are used to transmit electric power over long distances without any resistive losses.



LASERS AND FIBER OPTICS

PART-A SHORT QUESTIONS WITH SOLUTIONS

Q1. What is a laser?

Answer :

Laser is an acronym for “Light Amplification by Stimulated Emission of Radiation”. Laser devices are used for generation of light along with the amplification of light. However, these are being artificial light sources, whose functioning is completely different for traditional light sources. These devices are very much suitable for radio and microwave transmitters and generates a highly directional coherent monochromatic polarized light beam.

Q2. What are the characteristics of laser?

Answer :

April-16, Q3

Characteristics of Lasers

The characteristics of lasers which distinguishes them from ordinary light are,

1. Coherence

This characteristic specifies that each and every wave of light beam travel exactly with each other along time and space (i.e., all are in phase). In a laser beam, each part is related to other in phase. These phase transformations are carried over sufficient longer times such that the interference phenomenon can be observed photographically, which is possible with the help of coherence property.

2. Monochromaticity

This characteristic specifies that the light beam has one single colour or wavelength. The laser light is highly monochromatic, which has origin in stimulated emission from one group of atomic energy levels. For the lasers producing more than one colour or wavelength, the generated light is an extremely pure signal with narrow spectral range.

3. Collimation

The characteristic specifies that each and every wave of light travel in parallel over long distances with minimum spread. During the propagation of a laser light, less angular spreading are produced and which in turn occupies less incident area. For instance, consider a laser beam of diameter 10 cm will spread only upto 5 km area on the moon.

Q3. Define spontaneous emission.

Answer :

The process of transition (i.e., decay) of an atom from higher energy level to the lower energy level by emitting a photon of energy ‘ $h\nu$ ’ without any external stimulus is referred to as spontaneous emission. Generally, the atoms present in the excited or higher energy state will be able to stay for only short time intervals (i.e., $\approx 10^{-8}$ sec) and revert back to the lower energy level.

Q4. Define stimulated emission.**Answer :**

The process of transition (i.e., decay) of an atom from higher level to the lower energy level by emitting a photon of energy ' $h\nu$ ' with an external radiation, which makes the transition to lower energy level well before the transition occurred by spontaneous emission is referred to as stimulated emission. This process is necessary in the existence of equilibrium between matter and radiation.

Q5. Give the relation between Einstein coefficients.**Answer :**

The relation between Einstein coefficients, which are occurred during absorption, spontaneous emission and stimulated emission is given by,

$$B_{12} = B_{21} = \frac{c^3}{8\pi h \nu^3 \mu^3} A_{21}$$

Where,

 c - Velocity of light h - Planck's constant ν - Frequency of radiation μ - Permeability of the medium B_{12} - Einstein coefficient for induced absorption B_{21} - Einstein coefficient for induced absorption A_{21} - Einstein coefficient for spontaneous emission.**Q6. The relation between Einstein's coefficients A and B is**

(a) $\frac{8\pi h \nu^3}{c^3}$

(b) $\frac{8\pi^2 h^2 \nu^3}{c^3}$

(c) $\left(\frac{2\pi h \nu}{c}\right)^3$

(d) $\frac{8\pi h c}{A}$

Answer :

April-16, Q10

The relation between Einstein's coefficients A and B is $\frac{8\pi h \nu^3}{c^3}$.

Q7. What is population inversion.**Answer :**

Population inversion is a non equilibrium condition of the material where the number of atoms in higher energy level is greater than the atoms in lower energy level.

Q8. In a Ruby laser, total number of Cr^{+3} ions is 2.8×10^{19} . If the laser emits radiation of wavelength 7000\AA , calculate the energy of the laser pulse.**Answer :**

June-15, Q4

Given that,

In a Ruby laser,

Chromium ions, $n = 2.8 \times 10^{19}$ Wavelength, $\lambda = 7000\text{\AA}$

The expression for the energy of the laser pulse is given as,

ΔE = Energy of one photon \times Total number of photons in the laser pulse

$$\Rightarrow \Delta E = E \times n \quad \dots (1)$$

The expression for energy of one photon is given as,

$$E = \frac{hc}{\lambda} \quad \dots (2)$$

Where,

h - Planck's constant (6.625×10^{-34} J/sec)

c - Speed of light (3×10^8 m/s)

Substituting the corresponding values in equation (2),

$$E = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{7000 \times 10^{-10}} \text{ J}$$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{7 \times 10^{-7} \times 1.6 \times 10^{-19}} \text{ eV} \quad [\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

$$\therefore \Delta E = 1.77 \text{ eV}$$

Substituting the values of ΔE and n in equation (1),

$$E_r = 1.77 \times 2.8 \times 10^{19} = 4.956 \times 10^{19}$$

$$\therefore E_r = 4.9 \times 10^{19} \text{ eV}$$

Q9. What is an optical fiber?

Answer :

An optical fiber is a transparent dielectric wave guide that transmits optical data from one point to other point. It contains two cylindrical shells namely core (inner shell with refractive index n_1) and cladding (other shell with refractive index n_2), always $n_1 > n_2$. It works on the principle of total internal reflection.

Q10. Define critical angle.

Answer :

Critical angle can be defined as an angle of incidence in denser medium for which the angle of refraction in the rarer medium is 90° . It is denoted by C .

Q11. Define total internal reflection and give the conditions to produce total internal reflection in optical fiber.

Answer :

Total Internal Reflection

Total Internal Reflection (TIR) is an optical phenomenon that occurs when a ray of light strikes a medium boundary at an angle larger than the critical angle with respect to the normal to the surface.

If the refractive index is lower on the other side of the boundary, no light can pass through and all of light is reflected. The critical angle is the angle of incidence above which the total internal reflection occurs as shown in figure.

Conditions

The conditions to achieve total internal reflection are given as follows,

- The refractive angle of the core should be greater than that of cladding.
- The incidence angle must be greater than that of the critical angle and it should lie within the acceptance cone of the fiber.

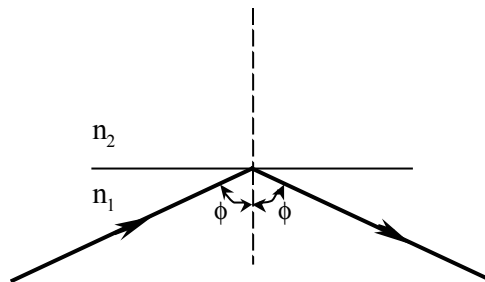


Figure : Total Internal Reflection where $\phi > \phi_c$

Q12. Define “Acceptance Angle”.

May/June-18, Q5

*(Refer Only Acceptance Angle)***OR****Define the terms “Acceptance Angle” and “Numerical Aperture (NA)” of an optical fibre.****Answer :**

June-15, Q3

Acceptance Angle

Acceptance angle of the fiber ' α_m ' is defined as the maximum value of the angle of incidence at the entrance end of the fiber at which the angle of the incidence at the core-cladding interface is equal to critical angle of the core medium.

$$\therefore \sin \alpha_m = (n_1^2 - n_2^2)^{\frac{1}{2}}$$

Numerical Aperture (NA)

Numerical aperture of the fiber is the light collecting efficiency of the fiber and is the measure of the amount of the light rays which can be accepted by the fiber. Mathematically it is defined by the following equation,

$$\therefore NA = (n_1^2 - n_2^2)^{\frac{1}{2}}$$

Q13. The refractive indices of core and cladding for a step index fibre are 1.62 and 1.52, respectively. Find the numerical aperture and acceptance angle.**Answer :**

April - 16, Q5

Given that,

For a step index fibre,

The refractive index of core, $n_1 = 1.62$ The refractive index of cladding, $n_2 = 1.52$

The expression for numerical aperture is given by,

$$\begin{aligned} NA &= \sqrt{n_1^2 - n_2^2} \\ \Rightarrow NA &= \sqrt{(1.62)^2 - (1.52)^2} \\ &= \sqrt{0.314} \\ &= 0.560 \\ \therefore NA &= 0.560 \end{aligned}$$

And acceptance angle (θ_0) is given by,

$$\begin{aligned} \theta_0 &= \sin^{-1} (NA) \\ &= \sin^{-1} (0.560) \\ \Rightarrow \theta_0 &= 34.056 \\ \therefore \theta_0 &= 34.056 \end{aligned}$$

Q14. Calculate the numerical aperture, acceptance angle and critical angle of the fibre from the following data $n_1 = 1.50$ and $n_2 = 1.45$.**Answer :**

Jan.-16, Q4

Given that,

For a fibre,

$$n_1 = 1.50$$

$$n_2 = 1.45$$

The expression for numerical aperture is given by,

$$\begin{aligned} NA &= \sqrt{n_1^2 - n_2^2} \\ &= \sqrt{(1.50)^2 - (1.45)^2} \\ &= \sqrt{0.207} \\ \therefore NA &= 0.455 \end{aligned}$$

The expression for acceptance angle (θ_0) is given by,

$$\theta_0 = \sin^{-1}(NA) = \sin^{-1}(0.455)$$

$$\therefore \theta_0 = 27.065$$

The expression for critical angle (θ_c) is given by,

$$\begin{aligned}\theta_c &= \sin^{-1}\left(\frac{n_2}{n_1}\right) \\ &= \sin^{-1}\left(\frac{1.45}{1.50}\right) \\ &= \sin^{-1}(0.967) \\ &= 75.240\end{aligned}$$

$$\therefore \theta_c = 75.240$$

Q15. Calculate the numerical aperture and acceptance angle for an optical fiber with core and cladding refractive indices being 1.48 and 1.45 respectively.

Answer :

Given that,

For an optical fiber,

Core refractive index, $n_1 = 1.48$

Cladding refractive index, $n_2 = 1.45$

The expression for numerical aperture is given by,

$$\begin{aligned}NA &= \sqrt{n_1^2 - n_2^2} \\ &= \sqrt{(1.48)^2 - (1.45)^2} \\ &= 0.296\end{aligned}$$

$$\therefore NA = 0.296$$

The expression for acceptance angle is given by,

$$\begin{aligned}\theta_0 &= \sin^{-1}[NA] \\ &= \sin^{-1}(0.296) \\ &= 17.2^\circ\end{aligned}$$

$$\therefore \theta_0 = 17.2^\circ$$

Q16. Explain different types of optical fibres.

Answer :

There are two types of optical fibers. They are step index fibre and graded fiber.

Step Index Fiber

The step index fiber consists of a core and cladding. The core is made of pure glass and its diameter is in the range of 6 to 250 μm . It is surrounded by cladding made of either co-axial glass or plastic. The refractive index of cladding is lower than the refractive index of core. Thus, the region between core and cladding acts as a mirror that reflect the signals back into the fiber and stops them from escaping. Hence, the signals are able to travel long distance in optical fiber through internal reflections.

Graded Index Fiber

In graded index fiber, the refractive index of each glass layer of core has a different refractive index. The glass layer nearest to the center of the fiber has highest refractive index and the proceeding glass layers have decreasing refractive index. This arrangement of core ensures that, light doesn't escape the fiber and instead moves forward with the help of refraction and total internal reflection.

Q17. Mention few applications of optical fiber.

Dec.-17, Q3

OR

Mention any four applications of an optical fiber.

Answer :

Dec.-16, Q5

The following are the applications of optical fiber,

- (i) Optical fibre is used to simultaneously transfer large amount of telecommunication data from one place to other with less regenerators.
- (ii) It is also used in sensors.
- (iii) Optical fibre is used as illuminators i.e., to carry light from one place to other.
- (iv) It is also used in medical field to see internal body parts.

PART-B

ESSAY QUESTIONS WITH SOLUTIONS

5.1 LASERS

5.1.1 Characteristics of Lasers - Spontaneous and Stimulated Emission of Radiation - Einstein's Coefficients

Q18. What is a laser? Explain the characteristics of lasers.

Answer :

Laser

Laser is an acronym for “Light Amplification by Stimulated Emission of Radiation”. Laser devices are used for generation of light along with the amplification of light. However, these are being artificial light sources, whose functioning is completely different for traditional light sources. These devices are very much suitable for radio and microwave transmitters and generates a highly directional coherent monochromatic polarized light beam.

Characteristics of Lasers

For answer refer Unit-5, Q2.

Q19. Explain with a neat diagram,

- (i) **Spontaneous emission and**
- (ii) **Stimulated emission of radiation.**

Answer :

(i) Spontaneous Emission

The process of transition (i.e., decay) of an atom from higher energy level to the lower energy level by emitting a photon of energy ' $h\nu$ ' without any external stimulus is referred to as spontaneous emission. Normally, the atoms presents in the excited or higher energy state will be able to stay for only short time intervals (i.e., $\approx 10^{-8}$ sec) and revert back to the lower energy level.

Let us consider that an atom is available at higher energy state ' E_2 '. After a short time interval, the atom in level ' E_2 ' will return to the lower energy state ' E_1 ' by releasing a photon of energy, $h\nu = E_2 - E_1$ without any application of external stimulus as shown in figure (1).

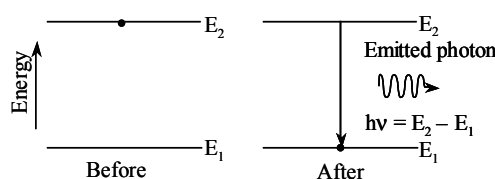
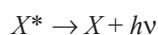


Figure (1)

Mathematically, the spontaneous emission process is expressed as,



Where,

X^* - Atom in higher energy state ' E_2 '

X - Atom in lower energy state ' E_1 '.

And the probability of occurrence of spontaneous transition rely only on the characteristics of energy states E_1 and E_2 i.e.,

$$(P_{21})_{\text{spontaneous}} = A_{21} \quad \dots (1)$$

Where,

A_{21} - Einstein coefficient for spontaneous emission.

While, the reciprocal of Einstein coefficient for spontaneous emission $\left(\text{i.e., } \frac{1}{A_{21}}\right)$ represents the lifetime of higher state against spontaneous decay to the lower state.

Then, the number of spontaneous transitions occurred in a material during the time Δt is obtained as,

$$N_{\text{spontaneous}} = N_2 (P_{21})_{\text{spontaneous}} \Delta t$$

From equation (1),

$$\therefore N_{\text{spontaneous}} = A_{21} N_2 \Delta t$$

Where,

N_2 - Number of atoms in higher energy state ' E_2 '.

In conventional light sources the spontaneous emission process plays a dominant role.

(ii) Stimulated Emission of Radiation

The process of transition (i.e., decay) of an atom from higher level to the lower energy level by emitting a photon of energy ' $h\nu$ ' with an external radiation, which makes the transition to lower energy level well before the transition occurred by spontaneous emission is referred to as stimulated emission. This process is necessary in the existence of equilibrium between matter and radiation.

Let us consider that an atom is available at the higher energy state ' E_2 '. An external radiation of ($h\nu = E_2 - E_1$) is made to fall on the atom in level ' E_2 ' which brings the excited atom to the lower energy level ' E_1 ' well before the transition occurred by spontaneous emission as shown in figure (2). However, there is no effect of externally induced photon on the photon emitted by excited atom.

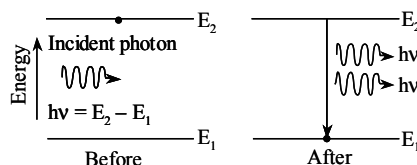
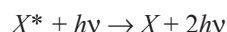


Figure (2)

Mathematically, the stimulated emission process is expressed as,



Where,

X^* - Atom in higher energy state ' E_2 '

X - Atom in lower energy state ' E_1 '.

And the probability of occurrence of stimulated transition varies in direct proportion with photon density $\rho(\nu)$ as,

$$(P_{21})_{\text{stimulated}} \propto \rho(\nu)$$

$$(P_{21})_{\text{stimulated}} = B_{21} \rho(\nu)$$

Where,

B_{21} - Einstein coefficient for stimulated emission.

Then, the number of stimulated transitions occurred in a material during the time Δt is obtained as,

$$N_{\text{stimulated}} = N_2 (P_{21})_{\text{stimulated}} \Delta t$$

From equation (2),

$$N_{\text{stimulated}} = B_{21} N_2 \rho(\nu) \Delta t$$

OR

Distinguish between spontaneous emission and stimulated emission.

Answer :

June-10, Q12(a)

Differences between Spontaneous and Stimulated Emission

Spontaneous Emission		Stimulated Emission	
1.	Spontaneous emission does not require any external radiation.	1.	Stimulated emission is induced by an external photon or radiation.
2.	It is a random and probabilistic process.	2.	It is not a random process.
3.	In spontaneous emission, there is no phase relation between the photons emitted from different atoms.	3.	In stimulated emission, the incident and emitted photons will be in phase.
4.	In this process, non coherent radiations are occurred.	4.	In this process, coherent radiations are occurred.
5.	Population inversion is not necessary.	5.	Population inversion is required.
6.	The direction of emitted photons may be distinct.	6.	The direction of emitted photons will be same as that of incident photon.
7.	Spontaneous emission favours LED principle.	7.	Stimulated emission favours laser action.
8.	The number of spontaneous transitions occurred in a material during the time, Δt given by, $N_{\text{spontaneous}} = A_{21} N_2 \Delta t$ Where, A_{21} - Einstein coefficient for spontaneous emission. N_2 - Number of atoms in higher energy state.	8.	The number of stimulated transitions occurred in a material during the time, Δt is given by, $N_{\text{stimulated}} = B_{21} N_2 \rho(\nu) \Delta t$ Where, B_{21} - Einstein coefficient for stimulated emission. N_2 - Number of atoms in higher energy state. $\rho(\nu)$ - Photon density.

Q21. Explain about Einstein coefficients.**Answer :****Einstein Coefficients**

The coefficients considered during the three process (known as absorption, spontaneous emission and stimulated emission) by the travel of photons through medium are known as Einstein coefficients. They are,

1. Einstein Coefficient for Induced Absorption (B_{12})

It is a proportionality constant and defined as the ratio of probability of occurrence of an absorption transition, P_{12} to the photon density, $\rho(\nu)$ i.e.,

$$B_{12} = \frac{P_{12}}{\rho(\nu)}$$

This coefficient specifies the transition induced by an external photon from lower energy level E_1 to the upper energy level E_2 . It represents the constant characteristic of the atom as well as properties of lower and upper energy states.

2. Einstein Coefficient for Spontaneous Emission (A_{21})

It is a constant and defined as the probability of occurrence of a spontaneous of transition (P_{21})_{spontaneous}.

$$A_{21} = (P_{21})_{\text{spontaneous}}$$

This constant is also known as radiative rate, which also specifies a constant characteristic of the atom. The reciprocal of Einstein coefficient for spontaneous emission is referred as the lifetime of the higher state against spontaneous decay to the lower state. The constant A_{21} represents the transition induced without the help of external photon from higher energy level E_2 to lower energy level E_1 .

3. Einstein Coefficient for Stimulated Emission (B_{21})

It is a probability constant defined as the ratio of probability of occurrence of a stimulated transition, P_{21} to the photon density, $\rho(\nu)$ i.e.,

$$B_{21} = \frac{P_{21}}{\rho(\nu)}$$

This coefficient specifies the transition induced by an external photon from upper energy level E_2 to the lower energy level E_1 . It also represents the constant characteristic of the atom as well as properties of lower and upper energy states.

Q22. Obtain the relation between Einstein coefficients.**Answer :**

Consider a group of atoms in thermal equilibrium state at temperature (T) with radiation of frequency (ν) and photon energy density $\rho(\nu)$.

Then, the number of atoms absorbing photons per second per unit volume are,

$$n_a = N_1 B_{12} \rho(\nu) \quad \dots (1)$$

Where,

N_1 - Population density at level E_1

B_{12} - Einstein coefficient for induced absorption

$\rho(\nu)$ - Photon density.

And the number of atoms emitting photons per second per unit volume are,

$$n_e = A_{21} N_2 + B_{21} N_2 \rho(\nu) \quad \dots (2)$$

Where,

N_2 - Population density at level E_2

A_{21} - Einstein coefficient for spontaneous emission

B_{21} - Einstein coefficient for stimulated emission.

But, the number of induced transitions from E_1 to E_2 must be same as the number of induced transitions from E_2 to E_1 at the thermal equilibrium condition.

Hence,

$$n_a = n_e$$

$$\Rightarrow N_1 B_{12} \rho(\nu) = N_2 A_{21} + B_{21} N_2 \rho(\nu)$$

$$\Rightarrow N_1 B_{12} \rho(\nu) - B_{21} N_2 \rho(\nu) = N_2 A_{21}$$

$$\Rightarrow [N_1 B_{12} - N_2 B_{21}] \rho(\nu) = N_2 A_{21}$$

$$\Rightarrow \rho(\nu) = \frac{N_2 A_{21}}{[N_1 B_{12} - N_2 B_{21}]}$$

$$\Rightarrow \rho(\nu) = \frac{N_2 A_{21}}{N_2 B_{21} \left[\frac{N_1 B_{12}}{N_2 B_{21}} - 1 \right]}$$

$$\Rightarrow \rho(\nu) = \frac{A_{21}}{B_{21} \left[\frac{N_1 B_{12}}{N_2 B_{21}} - 1 \right]} \quad \dots (3)$$

Since the probability of occurrence of stimulated absorption transition is same as the probability of occurrence of stimulated emission i.e.,

$$P_{12} = (P_{21})_{\text{stimulated}}$$

$$\Rightarrow B_{12} \rho(\nu) = B_{21} \rho(\nu)$$

$$\therefore B_{12} = B_{21} \quad \dots (4)$$

From equations (3) and (4),

$$\rho(\nu) = \frac{A_{21}}{B_{21} \left[\frac{N_1}{N_2} - 1 \right]} \quad \dots (5)$$

At thermal equilibrium, the expressions for number of atoms N_1 and N_2 in energy states E_1 and E_2 can be written based on the Boltzmann distribution law as,

$$N_1 = N_0 e^{-E_1/kT} \quad \dots (6)$$

$$N_2 = N_0 e^{-E_2/kT} \quad \dots (7)$$

From equations (6) and (7),

$$\frac{N_1}{N_2} = e^{\left[\frac{E_2}{kT} - \frac{E_1}{kT}\right]}$$

$$\Rightarrow \frac{N_1}{N_2} = e^{\left[\frac{E_2 - E_1}{kT}\right]}$$

$$\Rightarrow \frac{N_1}{N_2} = e^{\left(\frac{h\nu}{kT}\right)} \quad [\because E_2 - E_1 = h\nu]$$

Substituting $\frac{N_1}{N_2}$ value in equation (5),

$$\rho(\nu) = \frac{A_{21}}{B_{21}[e^{h\nu/kT} - 1]} \quad \dots (8)$$

In order to maintain the thermal equilibrium condition, the above radiation must be equivalent to black body radiation and uniform with Planck's radiation law. From Planck's law,

$$\rho(\nu) = \left[\frac{8\pi h\nu^3 \mu^3}{C^3} \right] \left[\frac{1}{e^{h\nu/kT} - 1} \right] \quad \dots (9)$$

Comparing equations (8) and (9),

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3 \mu^3}{C^3}$$

$$\Rightarrow A_{21} = \frac{8\pi h\nu^3 \mu^3}{C^3} B_{21}$$

From equation (4),

$$A_{21} = \frac{8\pi h\nu^3 \mu^3}{C^3} B_{21} = \frac{8\pi h\nu^3 \mu^3}{C^3} B_{12}$$

The above equality shows the relationship between different Einstein coefficients.

5.1.2 Population Inversion - Ruby Laser - Helium - Neon Laser - Semiconductor Laser - Applications of Lasers

Q23. What do you understand by population inversion? How it is achieved?

Answer :

Population Inversion

Generally, the population ratio of a material is decided by Boltzmann factor under thermal equilibrium condition as,

$$\frac{N_2}{N_1} = \exp \left[\frac{-(E_2 - E_1)}{kT} \right]$$

Where,

E_1 - Lower energy level

E_2 - Higher energy level

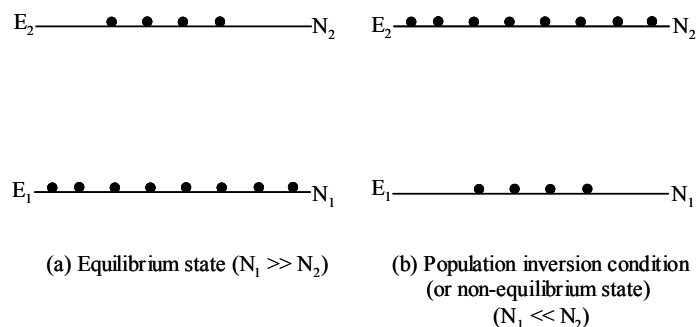
N_1 - Population at level E_1

N_2 - Population at level E_2

k - Boltzmann constant

T - Temperature.

From the above equation it is clear that the number of atoms at the excited level ' E_2 ' are very less compared to that of at ground level ' E_1 ' (i.e. $N_1 \approx 10^{30} N_2$). The state of the material in which population at lower energy level is more compared to that of population at higher energy level is referred to as equilibrium state or normal state as shown in figure below.



Figure

Thermal equilibrium condition, $N_1 \gg N_2$.

While, the state of material in which the number of atoms in higher energy level are very much higher than the number of atoms in lower energy level is referred as the population inversion condition. Thus, under this condition, $N_1 \ll N_2$. This state is also referred to as inverted state, because of the interchange of population at the levels E_1 and E_2 . This population inversion condition exists for only shorter period, which makes the state as non-equilibrium.

Normally, the pumping techniques are used to attain the population inversion by transferring the atoms from lower energy level to upper energy level in large quantity.

Q24. What is population inversion relating to laser action? Explain.

Answer :

Population inversion is a method of performing laser action. In this method, excited state acquires more number of atoms than ground state (i.e., upper state consists of atoms greater than the atoms present in lower state). Because of this, the stimulated emission rate is greater than the absorption rate. The electron energy level structure of a material (i.e., going to act as a laser medium) should be designed in such a way that, the upper layer should have longer spontaneous life span than the lower layer, which should be capable of deteriorating itself to the ground state quickly. There should also be a method of populating the upper level based on the preferences.

States that have a longer life time are referred to as metastable states. These states arise when the condition required to decay the lower excited level to ground state is not satisfied in order to move the upper level into the excited state an input of energy into the medium is required. The process of moving the energy input into the medium is known as pumping which can be performed in different ways based on the nature of the material.

The following are the three important components of any laser. They are,

1. Amplifying Medium

This component consists of set of atoms, molecules or ions which usually act as an amplifier for light waves. Population inversion is required for amplifying the light waves that are attenuated when they pass through the collection of atoms. When the waves that are in the state of population inversion pass through the collection of atoms, induce more emission and are amplified.

2. Pump

Pump acts as an energy source that maintains the medium, whenever the light waves are in population inversion state.

3. Optical Resonator

This component contains pair of mirrors opposite to each other, which are used for providing optical feedback to the amplifier. This is done so that, the optical resonator can act as a radiation source.

Q25. Describe the experimental method of Ruby laser with necessary energy level diagram.

July-17, Q12(b)

OR

Describe the construction and working of Ruby laser.

Jan.-13, Q16(b)

OR

Describe the working of a Ruby laser with suitable energy level diagram.

May/June-12, Q12(a)

OR

Explain the construction and working of Ruby-laser.

June-11, Q16(a)

OR**Draw a neat diagram to represent the components of a Ruby laser and explain its operation.****Answer :**

June-10, Q12(b)

Ruby Laser

Ruby (Al_2O_3) laser uses a crystalline substance as the active material. The different parts are depicted in the figure.

Description

It consists of mainly three parts. They are,

1. An Active Working Material

A rod of ruby crystal in the form of cylindrical rod 3 to 5 mm in diameter and 5 to 30 cm long.

2. A Resonant Cavity

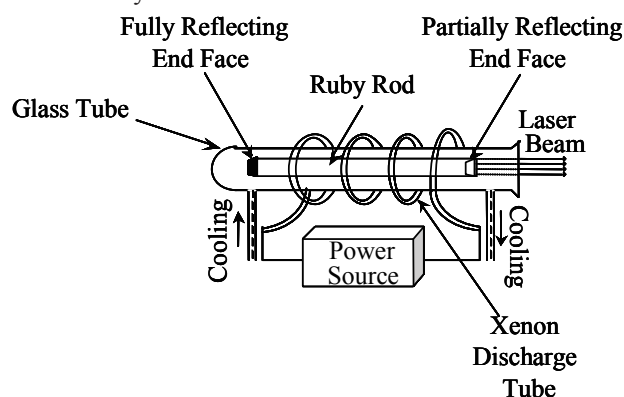
Made of fully reflecting plate at the left of ruby crystal and a partially reflecting plate at the right of ruby crystal. Both the plates are optically plane and exactly parallel to each other.

3. Exciting Systems

A helical xenon flash tube with power supply source.

Working

Ruby ($\text{Al}_2\text{O}_3, \text{Cr}_2\text{O}_3$) is a crystal of aluminium oxide Al_2O_3 in which some aluminium atoms are replaced by chromium atoms (Cr_2O_3). The active material in the ruby are chromium ions Cr^{3+} .

**Figure (1)**

When ruby crystal contains about 0.5% of chromium, its colour is pink. In a ruby laser, pink rod of 4 cm length and 0.5 cm in diameter is generally used. The end faces of the rod are made strictly parallel, ground to a high degree polish. The end faces are then silvered in such a way that one end face becomes fully reflecting while the other end partially reflecting. Sometimes separate pieces are attached at the end faces. The ruby rod is surrounded by a helical xenon tube that lasts several milliseconds and the tube consumes several thousand joules of energy. Only a part of this energy is used in pumping the Cr^{+3} ions while the rest heats up the apparatus. For this purpose, a cooling arrangement is used.

The energy level diagram illustrating the operating principle of a ruby laser is shown in the figure below, which represents the energy levels of chromium ion. In normal state, the chromium ion is in lower levels. When the ruby crystal is irradiated with light of xenon flash, the chromium atoms are excited and pass to upper level where the light absorption band is 5600 \AA . The absorption bandwidth of this level is about 800 \AA . A few excited chromium atoms return to ground level 1 and other to level 2. The transitions $3 \rightarrow 2$ are non-radiative i.e., the chromium atoms give part of their energy to crystal lattice in the form of heat.

Here the transition probabilities between different states are such that most of the excited atoms in state 3 return to state 1 but those which go to state 2 have a little chance to return to state 1 spontaneously. After few milliseconds, the level 2 becomes more populated than level 1 and hence the desired population inversion is achieved. The state of inverted population is not a stable one. The probability of spontaneous transitions at any moment is very high.

So, the spontaneous transitions may cause an induced transition which produces a photon. This photon knocks out a second photon from a neighboring atom. The atom from which the first photon was emitted is now brought to ground level. The two photons will knockout two more photons and their total number will be four and so on. Now the stimulated radiation along the axis starts dominating multiple reflections.

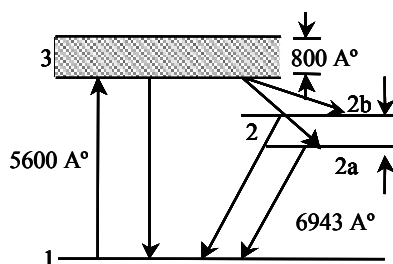


Figure (2)

Main Points

1. Spontaneous component of emission is still not negligible and also the coherent radiation along the axis starts dominating due to multiple reflection.
2. When the pumping radiation falls on the active material the atoms start reaching the excited state till the population inversion is reached.
3. Most of the atoms are initially in unexcited state.
4. The coherent axial photons start becoming further coherent with each other.
5. There is a phase agreement of waves. Almost the entire emission becomes coherent.

Photon beam parallel to the axis of the crystal grows in strength and some of it bursts through the partial reflector and serves as the output laser beam. A stage is reached where the population inversion caused by one flash of xenon tube is used up. The laser beam then ceases till the next flash of xenon tube repeats the process. Thus, the ruby laser is a pulsed laser, of course, continuous wave lasers are also in use.

The output beams have principle wavelengths of 6943\AA equal to about 4.32×10^{14} c/s, which is in the visible spectrum. The duration of output flash is about 300 microseconds. It is very intense of about 10,000 watt. The high power intensity is due to coherence of output beam because the photons moving parallel to the axis have space, time and directional coherence.

Q26. Describe the construction and working of He-Ne gas laser with energy level diagrams.

June-15, Q11(b)

OR

Explain the construction and working of the He-Ne laser.

Answer :

April/May-07, Q12(a)

Helium-Neon Laser

Helium-Neon laser is a gaseous laser system and is used to produce a continuous laser. This laser is highly directional, monochromatic, coherent and stable. But the output power is moderate when it is compared with the solid-state laser. It is very useful in making holograms and interferometric experiments.

Source of energy : R.F oscillator

Active medium : Helium and Neon gas mixture

Optical cavity : Arrangement of reflector

Construction

The experimental arrangement is shown in figure(1). The He-Ne laser consists of quartz tube with a diameter of about 15 cm and 80 cm length which has three electrodes to connect R.F oscillator. This tube is filled with a mixture of Neon (Ne) under a pressure of 0.1 mm of Hg and Helium (He) under a pressure of 1 mm of Hg. There is a majority of helium atoms and minority of neon atoms. At one end of the tube a perfect reflector is arranged while on the other end is a partial reflector. The active material is excited due to energy discharge through the gas mixture by means of radio frequency generator with a frequency of several MHz.

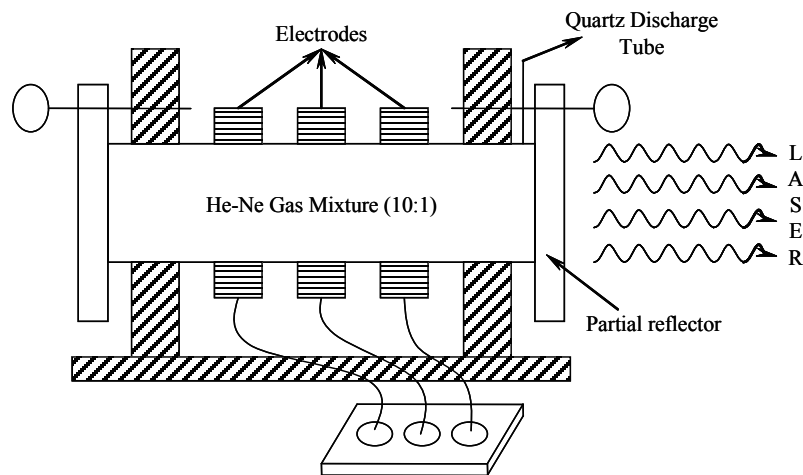


Figure (1)

Working Principle

The schematic energy diagram of He-Ne gas mixture is shown in figure (2). In helium atom three active energy levels are present, they are named as He_1 , He_2 and He_3 , whereas in Neon (Ne) atom six active energy levels are present. They are named as Ne_1 , Ne_2 , Ne_3 , Ne_4 , Ne_5 and Ne_6 . Here, it should be noted at Ne_4 and He_3 levels, when electric discharge is passed through the He-Ne gas mixture then the electrons are accelerated towards the positive electrode. During their passage they collide with helium and neon atoms but only helium atoms are suitable to excite to the upper states labeled He_2 and He_3 . These are metastable states in Helium atoms. Thus, the atoms remain in these levels for a sufficiently long time.

Now these atoms interact with neon atoms which are in the ground state. As the energy exchange continues between He and Ne atoms, the population of neon atoms in the excited states Ne_4 and Ne_6 increase more and more and at a stage population inversion will be achieved in the metastable states Ne_4 and Ne_6 .

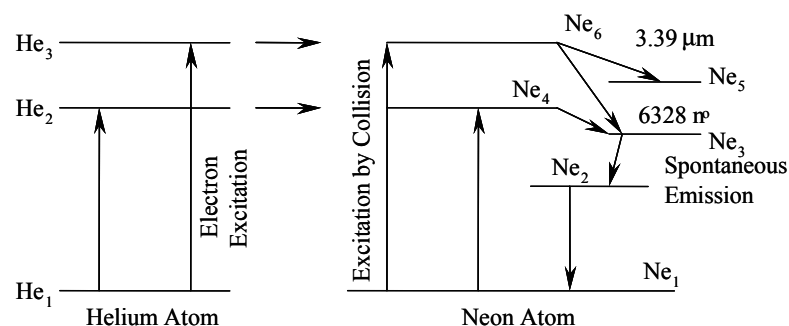


Figure (2)

After Achieving Population Inversion

- Few neon atoms de-excite from Ne_6 to Ne_5 . During this transition, electromagnetic radiation of wavelength of 3390\AA will be emitted.
- Many other neon atoms de-excite from Ne_6 to Ne_3 . During this transition, electromagnetic radiation of wavelength 6328\AA will be emitted. This is the important and major wavelength in this laser system.
- In continuation, atoms in the excited state Ne_4 will de-excite to Ne_3 where an electromagnetic radiation of wavelength 1150\AA is emitted.
- After reaching all the neon atoms to Ne_3 spontaneously those will be de-excited to Ne_2 . During this transition electromagnetic radiation of wavelength 6000\AA is emitted.
- Finally, neon atoms take non-radiative transitions by making collisions with walls of the tube from Ne_2 to Ne_1 .

Q27. What is population inversion? Discuss with suitable diagram the principle, construction and working of He - Ne laser.

Answer :

For answer refer Unit-5, Q7, Q26.

April-16, Q12(a)

Q28. Explain the construction and working of a semiconductor diode laser.

Answer :

Semiconductor Laser

In this laser, a GaAs semiconductor forward biased diode is used for obtaining a coherent electromagnetic radiation.

Construction

The typical structure of commonly used semiconductor laser diode is shown in figure (1),

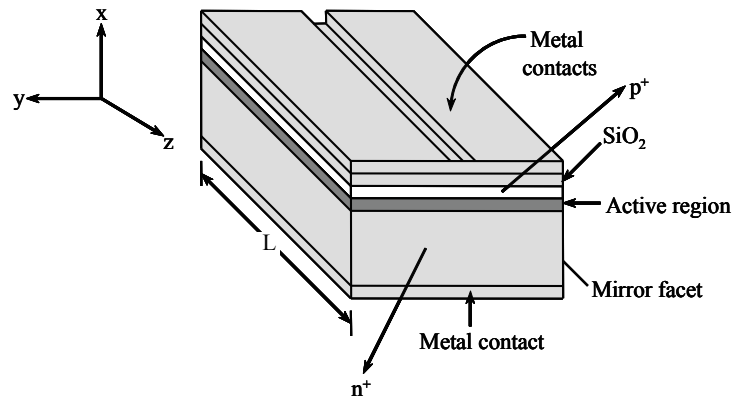


Figure (1)

In figure (1), the layers at PN-junction are positioned in such a way that an active region is formed. This region is responsible for generation of photons using the recombination process. In order to supply an external voltage to the laser diode, a metal layer is available on the top and bottom layers. However, the crystalline semiconductor sides are sliced to function as mirrors at the end of the optical cavity. With these semiconductor diode lasers, laser beams of wavelengths about 7000 \AA to 30000 \AA can be generated. The width of stripe in semiconductor diode laser will range from 2 to 65 mm.

Working

On application of external voltage to the metal contacts available above and below the semiconductor layers, the current is passed through the forward biased PN-junction diode. Due to this, the holes present in P-region and electrons present in N-region will move to the respective opposite region. By the recombination process (i.e., electrons and holes recombination) in the active region, photons are emitted due to the transition of electrons from conduction band to valence band. Because of this phenomenon, a stimulated radiation pattern is obtained from a very thin region close to the junctions as shown in figure (2).

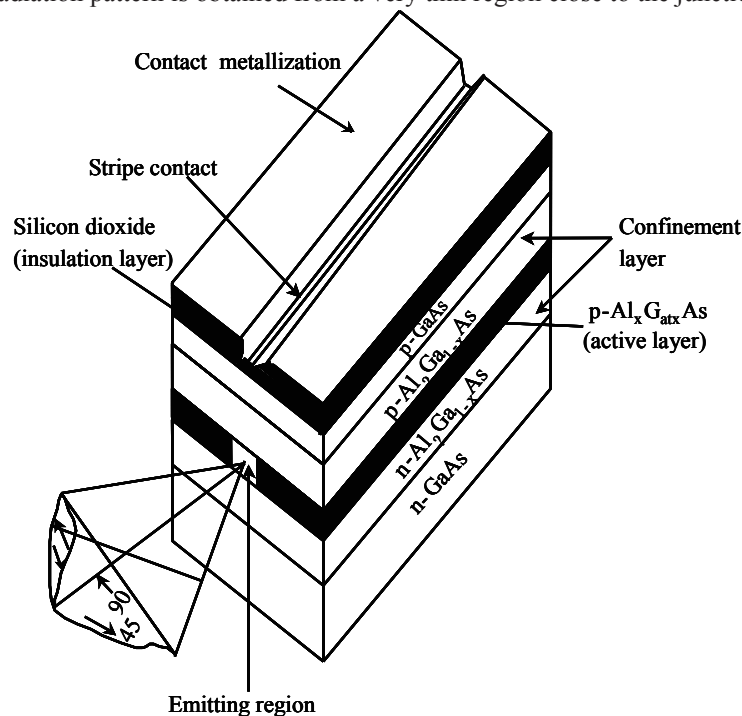


Figure (2): The Structure of Stripe Geometry Laser

In this process, the strength of radiation pattern obtained can be improved by raising the current and reducing the junction thickness.

Q29. List some of the application of laser.

May/June-18, Q4

OR

Describe applications of lasers.

April-16, Q12(b)

OR

Mention some important applications of lasers.**Answer :****Applications of Laser**

The characteristics of lasers like directionality, negligible divergence, high intensity, high degree of coherence and high monochromaticity makes them to have wide range of applications. They are,

1. The narrow bandwidth of lasers will allow them to be used in microwave communication.
2. The higher angular speed of the laser beam has become a mean of communication between earth and moon or other satellites. The distance between earth and moon has been measured with the use of lasers.
3. With the help of lasers, the storage capacity for information in computers is greatly improved due to narrowness of bandwidth. The IBM corporation is trying to transmit an entire memory bank from one computer to another by using a laser beam.
4. The lasers have wide industrial application i.e., they can make holes to diamonds and hard steels easily.
5. They have wide medical applications i.e., they have been used successfully in the treatment of detached retinas.
6. The lasers are also used for military purposes as a war-weapon.
7. They have wide chemical applications. They can initiate or hasten certain chemical reactions which could not be possible in the absence of suitable photons. They can be used for investigating the structure of molecules. Raman spectroscopy is one in which lasers have made so much impact that a separate branch named as Laser Raman spectroscopy has grown rapidly.
8. Laser beam is highly intense. Hence, it can be utilized in applications such as welding, cutting and ablation of materials. The laser beam is particularly suited for welding of fine wires, contacts in miniature assemblies etc.
9. Lasers can also be used for treatment of dental decay, the destruction malignant tumors and the treatment of skin diseases.

5.2 FIBER OPTICS**5.2.1 Introduction - Propagation of Light Through an Optical Fibre - Acceptance Angle - Numerical Aperture (NA)****Q30. Explain the working principle of optical fiber.**

Dec.-17, Q12(b)

OR

Explain about construction and working of optical fibres.**Answer :****Optical Fibre**

An optical fibre is a waveguide or transmission medium which carries optical signal with almost zero loss.

Principle

Optical fibre works on the principle of total internal reflection.

The structure of an optical fibre is divided into three parts. They are.

- (i) The core
- (ii) The cladding and
- (iii) The coating or buffer. Figure (1) shows the structure of an optical fibre.

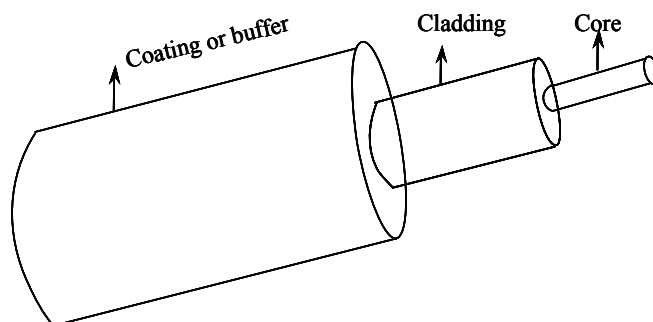


Figure (1) Structure of an Optical Fibre

Core

The core is an cylindrical shape rod made with glass or plastic and has very small radius. It is covered by a layer called cladding light propagates along the core even in the absence of cladding.

Cladding

Cladding is the another layer surrounded by buffer and it is made up of glass or plastic. The material used for making cladding must have refractive index less than the refractive index of core, that can allow total internal reflection to occur easily inside the core.

Coating or Buffer

Buffer is a protecting jacket use for extra protection and can protect the fibre from physical damage. It is made up of plastic. It prevents the wearing away due to friction and scattering losses caused by micro bends of fibre. Figure (2) Shows the Total Internal Reflection along the Optical Fibre.

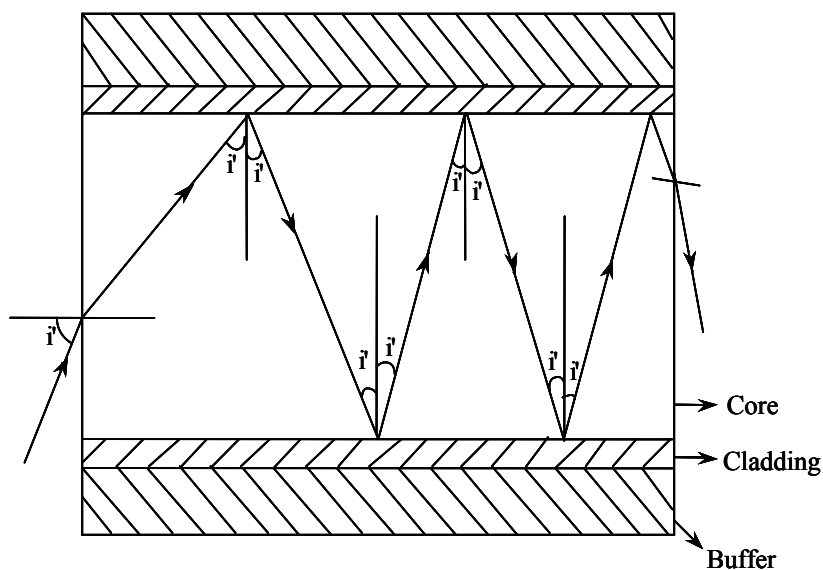


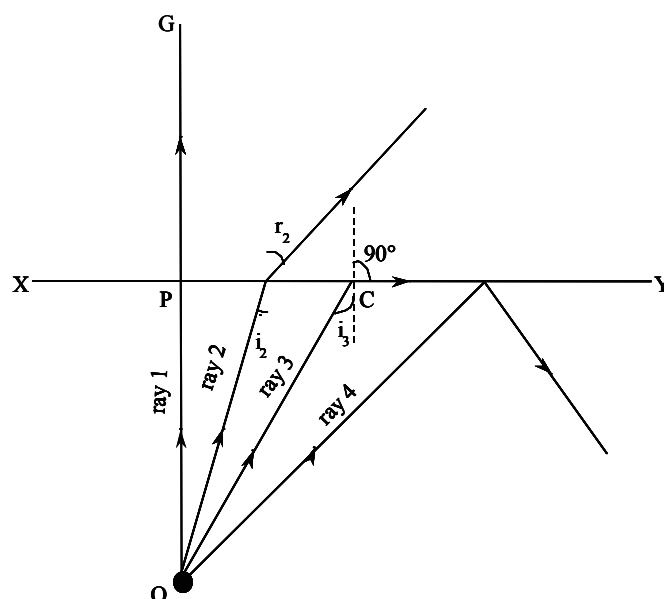
Figure (2) Total Internal reflection in Optical Fibre

A light ray enters into one end of optical fibre, it makes an angle of incidence i at air-core interface and refracts into the core. At core-cladding interface, the angle of incidence is greater than critical angle so the ray reflects in the core and this phenomenon is called total internal reflection. Thus, the ray propagates along the length of fibre by following total internal reflection and goes out from the other end of fibre.

Q31. Explain the concept of critical angle and total internal reflection.

Answer :

Critical angle can be defined as an angle of incidence in denser medium for which the angle of refraction in the rarer medium is 90° . It is denoted by C .



Figure

Let 'O' be the light producing the object in the denser medium having refractive index ' μ '. xy be the separating surface between denser and rarer medium as shown in figure. The ray 1 in the denser medium makes an angle of incidence $i_1 = 0^\circ$ so the ray does not bend and goes along the path 'PG'. The ray 2 crosses the separating surface and makes angle of incidence i_2 in the denser medium and bends in the rarer medium by making angle of refraction r_2 (here $i_1 < r_2$). As angle of incidence increases the angle of refraction also increases. The 'ray 3' in the denser medium makes an angle of incidence i_3 and its angle of refraction in rarer medium becomes 90° and the ray continues to travel along separating surface xy . For ray 4 the angle of incidence is greater than angle of refraction, so the ray reflects back in the same medium.

$$\text{Here } r_3 = 90^\circ = C$$

Where, C is critical angle

Factors

- ❖ Critical angle depends on refractive index (μ) of the same medium
- ❖ Critical angle is inversely proportional to refractive index. As μ increases, C decreases.

Total Internal Reflection

When the light ray travels from denser medium to a rarer medium for which the angle of incidence is greater than the critical angle and the ray reflects back in the denser medium instead of refracting in the rarer medium. This phenomenon is known as total internal reflection.

Conditions for Total Internal Reflection

1. The light ray must pass only from denser to rarer medium.
2. The angle of incidence (i) should have greater value when compared to critical angle (C).

Q32. What is numerical aperture? Derive an expression for numerical aperture.

May/June-18, Q17(a)

OR

Define acceptance angle and numerical aperture. How they are related to the refractive indices of the core and cladding.

Dec.-16, Q13(a)

OR

Define the acceptance angle and numerical aperture. Obtain an expression for the numerical aperture of an optical fiber.

Answer :

(i) Acceptance Angle

Acceptance angle of the fiber ' α_m ' is defined as the maximum value of the angle of incidence at the entrance end of the fiber at which the angle of the incidence at the core-cladding interface is equal to critical angle of the core medium.

$$\therefore \sin \alpha_m = (n_1^2 - n_2^2)^{\frac{1}{2}}$$

Thus,

- (i) Acceptance angle of the fiber is the maximum angle of incidence up to which a light ray can enter into the fiber and still be totally internally reflected.
- (ii) A case of light incident of the entrance end of the fiber will be guided through the fiber, provided the semi-vertical angle of the cone is less than or equal to α_m .

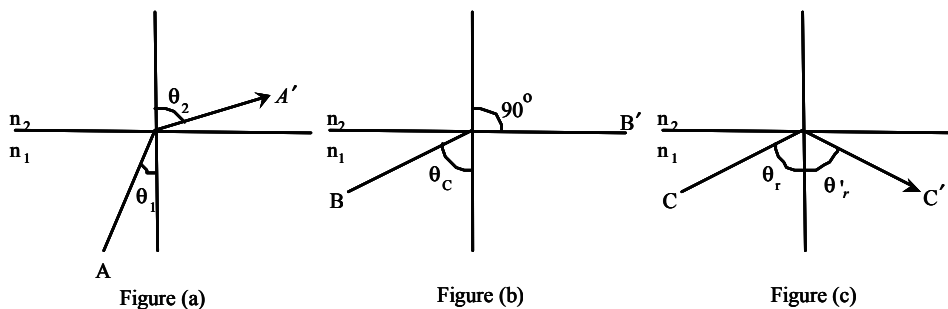
(ii) Numerical Aperture (NA)

Numerical aperture of the fiber is the light collecting efficiency of the fiber and is the measure of the amount of the light rays which can be accepted by the fiber. Mathematically it is defined by the following equation,

$$\therefore NA = (n_1^2 - n_2^2)^{\frac{1}{2}}$$

Expression for Numerical Aperture and Acceptance Angle

Refraction of an electromagnetic wave at the surface of separation of two media is determined by Snell's law and is illustrated in figure (a). A ray of light which is an electromagnetic wave is passing from a medium of higher refractive index n_1 into a medium of lower refractive index n_2 .



(a) Ray AA' is refracted according to Snell's law of refraction $n_1 \sin \theta_1 = n_2 \sin \theta_2$.

(b) Ray BB' is the critical ray $n_1 \sin \theta_c = n_2$.

(c) Ray CC' is totally reflected at the interface $\theta_r = \theta'_r$.

For $0 < \theta_1 < \theta_c$ and $0 < \theta_2 < \frac{\pi}{2}$

$$\frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2}$$

... (1)

Where, θ_1 and θ_2 are the angles of incidence and refraction respectively.

When the angle of refraction θ_2 is equal to $\frac{\pi}{2}$ as shown in figure (b), the angle of incidence $\theta_1 = \theta_c$ and the angle θ_c is known as the “circle angle”. Thus,

$$n_1 \sin \theta_c = n_2 \sin \theta_2 = n_2 \sin \left(\frac{\pi}{2} \right) = n_2 \quad \dots (2)$$

$$\Rightarrow \sin \theta_c = \left(\frac{n_2}{n_1} \right) \quad \dots (3)$$

When the angle of incidence θ_1 is greater than the critical angle θ_c , the total internal reflection of the light takes place with no loss at the boundary as shown in figure (c). Let us consider a cylindrical glass fiber consisting of an inner core of refractive index n_1 and the outer cladding of refractive index n_2 where n_1 is greater than n_2 . The end face of the fiber is cut at right angle to the fiber axis. Figure (d) illustrates that a ray of light enters into end face from air outside with refractive index n_a . The ray will propagate unattenuated along the fiber by means of multiple internal reflections provided the angle of incidence at the core-cladding interface θ_1 is greater than the critical angle θ_c . This requires that the angle of obliqueness to the fiber axis $\phi = \left(\frac{\pi}{2} - \theta_1 \right)$

be less than $\phi_m = \left(\frac{\pi}{2} - \theta_c \right)$ and the angle of incidence ‘ α ’ of the incoming ray into the end face of the fiber will be less than a certain angle α_m . In order to calculate α_m and ϕ_m , it is assumed that the refractive index n_a of air is equal to 1 and Snell’s law is applied.

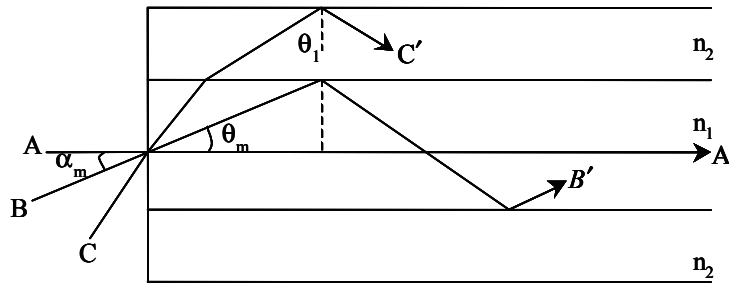


Figure (d): Propagation of a Light Wave in an Optical Fiber

Propagation of a light wave in an optical fiber AA' is an axial ray BB' is the critical ray for total internal reflection at $n_1 - n_2$ interface, CC' is too oblique for reflection at the interface and passes into the cladding by refraction.

$$n_a \sin \alpha_m = n_1 \sin \phi_m \quad \dots (4)$$

Substituting, $n_a = 1$ and $\phi_m = \left(\frac{\pi}{2} - \theta_c \right)$ in equation (4),

$$\sin \alpha_m = n_1 \sin \left(\frac{\pi}{2} - \theta_c \right) = n_1 \cos \theta_c$$

From equation (3),

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\Rightarrow \cos \theta_c = (1 - \sin^2 \theta_c)^{1/2} = \left[1 - \frac{n_2^2}{n_1^2} \right]^{1/2} \quad \dots (5)$$

Substituting the value of $\cos \theta_c$ in equation (4),

$$\begin{aligned} \sin \phi_m &= \cos \theta_c \\ &= \left[1 - \frac{n_2^2}{n_1^2} \right]^{1/2} \\ n_a \sin \alpha_m &= n_1 \sin \phi_m \\ &= n_1 \cos \theta_c \\ &= \frac{n_1 [n_1^2 - n_2^2]^{1/2}}{n_1} \end{aligned} \quad \dots (6)$$

For air, $n_a = 1$

$$\sin \alpha_m = (n_1^2 - n_2^2)^{1/2}$$

Where, α_m = Acceptance angle

$$\sin \alpha_m = [(n_1 + n_2)(n_1 - n_2)]^{1/2} \quad \dots (7)$$

If, $n_1 - n_2 = \Delta n$, $n_1 + n_2 = 2n$, then

$$\sin \alpha_m = (2n \Delta n)^{1/2} \quad \dots (8)$$

The greater value of α_m , the greater is the portion of the incident light on the end face which can be collected by the fiber and propagation takes place by total internal reflection. By analogy, the term used to define the light gathering power of microscope objectives ($n_a \sin \alpha_m$) is known as the Numerical Aperture (NA) of the fiber. Thus, $n_a = 1$.

$$\text{Numerical Aperture, } NA = \sin \alpha_m = (2n \Delta n)^{1/2} \quad \dots (9)$$

$$\begin{aligned} \text{Acceptance angle, } \alpha_m &= \sin^{-1}(2n \Delta n)^{1/2} \\ &= \sin^{-1} NA \end{aligned} \quad \dots (10)$$

5.2.2 Types of Optical Fibres and Refractive Index Profiles - Fibre Drawing Process (Double Crucible Method) - Losses in Optical Fibers - Application of Optical Fibers

Q33. Explain different types of optical fibre and how they effect the pulse dispersion.

June-15, Q16(b)

OR

Describe the classification of optical fibres.

Answer :

Jan.-12, Q16(a)

Different Types of Fibers

There are two different types of fibers that are classified based on their refractive index profiles of the core,

1. Step index fiber
2. Graded index fiber.

1. Step Index Fibers

Step index fiber has a cylindrical symmetric central core with a uniform and constant refractive index N_1 . This core is clad by a semiconductor material of less refractive index N_2 .

Refractive Index Profile of Step Index Fiber

The construction of a step index fiber is as shown in the figure (1).

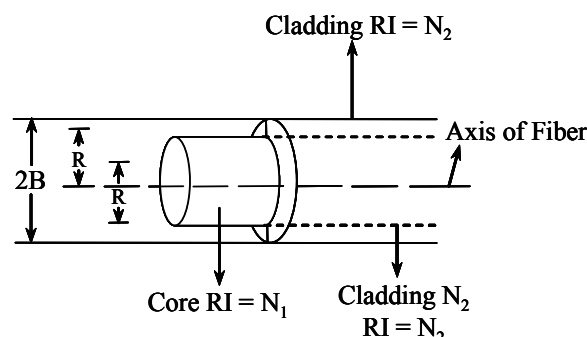


Figure (1): Step Index Fiber

A light falling on one end of the core will propagate to the receiving end in different paths called modes. The wider the diameter of the core, the larger the number of modes. The core has the maximum refractive index maintained through out the cross-section of the core.

If N_1 is the refractive index of the core and N_2 is the refractive index of the cladding, the profile of refractive index in SI fiber is described as $N(R)$.

For $R < A$, $N(R) = N_1$ in the core

For $R > A$, $N(R) = N_2$ in the cladding

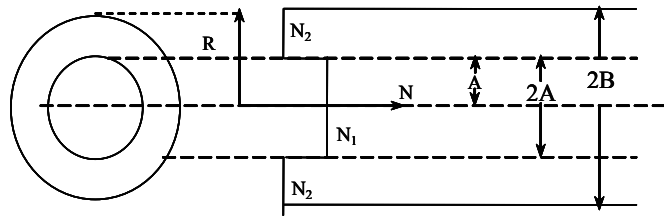


Figure (2): Profile of Refractive Index of a Step-wise Fiber

As shown in the figure (2) above, at $R = A$, the refractive index profile $N(R)$ is a step reduction from N_1 of the core to N_2 of the cladding.

If the reduction is say δ a fraction of N_1 .

$$N_1 - N_2 = \delta N_1$$

$$\Rightarrow N_2 = N_1(1 - \delta)$$

$$\Rightarrow \delta = \frac{N_1 - N_2}{N_1}$$

Propagation Delay of Step Index Fiber

If the angle of incidence is greater than the critical angle, then the ray that passes from core to cladding undergoes Total Internal Reflection (TIR). Because of the cylindrical structure, the ray suffers from TIR even at the lower interface.

The energy of light ray and its information in the form of signal propagates with a group velocity of v_g .

$\therefore v_g$ of fiber with length L

$$\Rightarrow \tau = \frac{L}{v_g}$$

Where, τ is the dispersion, which is the difference between the longest and shortest group delays.

$$\text{Longest group delay, } v_{g1} = \frac{c}{N_1}$$

$$\text{Shortest group delay, } v_{g2} = \frac{c}{N_2}$$

$$\begin{aligned} \therefore \text{Displacement, } \Delta\tau &= \frac{L}{v_{g1}} - \frac{L}{v_{g2}} \\ &= \frac{L(N_1 - N_2)}{c} \end{aligned}$$

Depending on the number of modes propagation, the step index fibers are further classified into two types. They are,

(i) Single Mode Step Index Fiber

In single mode, the pulse dispersion effect is minimized and accepts only the lowest order mode which makes it to be used in long distance systems. These kind of fibres are known as single mode step index fibres as shown in figure 3(a).

(ii) Multi Mode Step Index Fiber

The multimode step index fiber is shown in figure 3(b).

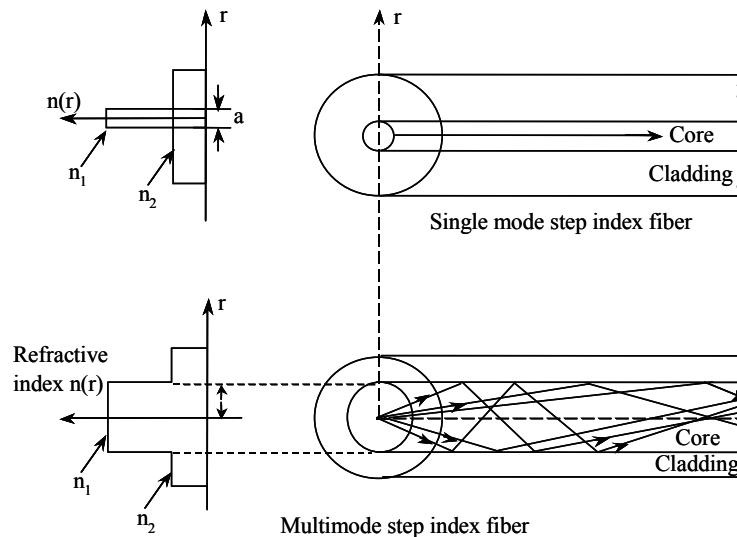


Figure (3): Step Index Fibers

The core diameter of multimode step index fiber is 50 μm or greater due to which, the propagation of many modes within the fiber core is possible.

The figure (4) shows a multimode signal which consists of 3 modes i.e., lower order mode, middle order mode and higher order mode. As shown in the figure, lower order mode travels along the axis and middle order mode reflects twice at the interface. Whereas, higher order mode reflects many times across the fiber, which results pulse broadening and dispersion. Because of these drawbacks multimode step index fiber is used rarely in telecommunication.

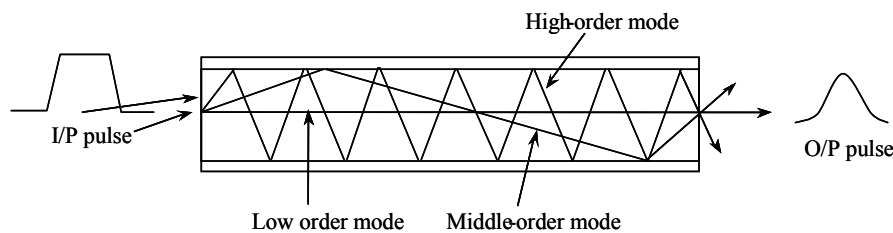


Figure (4): Configuration of Multimode Step Index Fiber

2. Graded Index Fiber

In graded index fiber the refractive index decreases gradually as a ray passes from the core to the cladding. Due to which, the refractive index is maximum at the centre and minimum at the surface of the core. As refractive index changes gradually, the direction of the ray changes regularly so that the ray of larger length spend maximum of the propagation time in the medium having minimum refractive index. This results in the faster propagation of the ray from core to cladding.

The diameter of the core ranges from 50 to 200 μm and outer diameter of the cladding ranges from 100 to 250 μm as shown in figure (5).

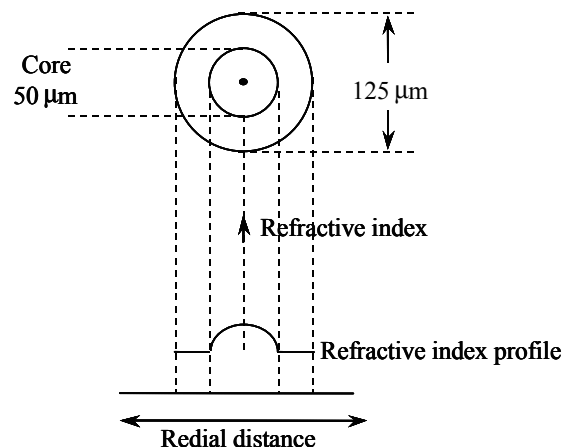


Figure (5): Fiber Cross-sectional View

Refractive Index Profile of Graded Index Fiber

If N_1 is the refractive index of the core and N_2 is the refractive index of the cladding, then the profile of refractive index in graded index is described as $N(R)$.

$$N(R) = N_1 \sqrt{1 - 2(R/A)^2 \Delta} \quad \text{for } R < A$$

$$= N_2 \quad \text{for } R > A$$

Here, ' R ' denotes the distance from the centre of the fiber. ' A ' signifies the core radius,

$$\Delta = \frac{(N_1 - N_2)}{N_1}$$

α is the grading parameter.

The optimal value for α is given by,

$$\alpha_{\text{opt}} = 2(1 - \Delta)$$

Propagation Delay of Graded Index Fiber

The dispersion for graded index fiber is obtained as,

$$\alpha = \Delta \tau_{\text{min}}$$

$$= \frac{L_{N_1} \Delta^2}{8c}$$

$$\text{Where, } \Delta \tau = \frac{L(N_1 - N_2)(\alpha - \alpha_{\text{opt}})}{c(\alpha + 2)}$$

Which represent the propagation delay of graded index fiber.

Q34. What is refractive index of an optical fiber?

Answer :

Refractive Index

Refractive index of an optical fiber determines the optical characteristics of any material. The refractive index of a medium is defined as the ratio of velocity of light in a vacuum to the velocity of light in a medium. It is denoted by ' μ ' or ' n ' and is given by,

$$\mu \text{ or } n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} \quad \dots (1)$$

When a light ray is incident on the interface between two dielectrics of differing refractive indices, part of the ray is reflected into the first medium and the rest is bent or refracted in the second medium. If ' n_1 ' is the refractive index of the first medium and ' n_2 ' is the refractive index of the second medium, the refraction takes place when $n_1 > n_2$ as shown in the figure below.

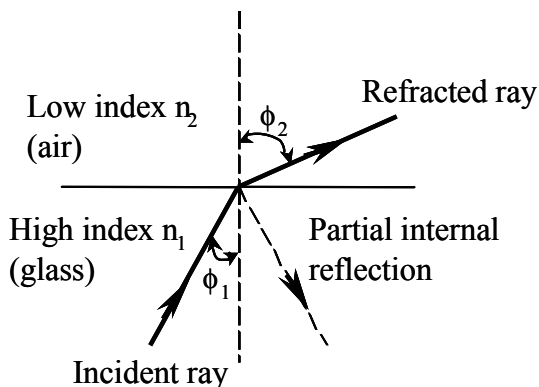


Figure (a): Refraction

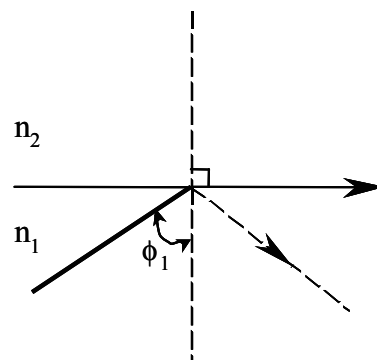


Figure (b): The Limiting Case of Refraction
Showing the Critical Ray at an Angle ϕ_1

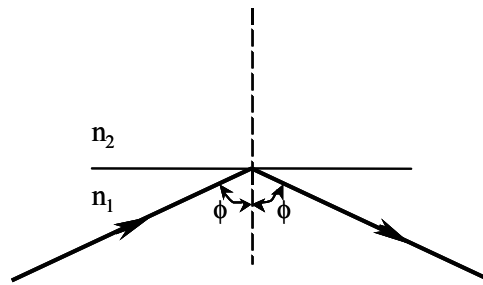


Figure (c): Total Internal Reflection where $\phi > \phi_c$

The bending or refraction of the light ray at the interface is due to the difference in the speed of light in two materials that have different refractive indices.

It is observed from figure (a) that ϕ_1 is the angle of incidence, ϕ_1 is the angle between incident light and normal at the surface of the interface. ϕ_2 is the angle of refraction i.e. it is an angle between refracted light and normal drawn at surface of interface in the lower refractive index medium. The relationship at the interface is known as “Snell’s law” of refraction. It gives the relationship between the angle of incidence and angle of refraction. According to Snell’s law,

$$n_1 \sin \phi_1 = n_2 \sin \phi_2$$

$$\therefore \frac{\sin \phi_1}{\sin \phi_2} = \frac{n_2}{n_1}$$

Q35. Distinguish between multimode step index fiber and single mode step index fiber.

Answer :

Multimode Step Index Fiber		Single Mode or Monomode Step Index Fiber	
1.	Many trajectories of light propagate in the fiber.	1.	Only one trajectory or one mode of light ray is allowed to propagate to the receiving end in single mode SI fiber.
2.	They have a larger diameter of core.	2.	The diameter of single mode SI fiber is small to allow only one mode to propagate.
3.	The diameter of core range upto 50 μm .	3.	The core diameter is lesser than 10 μm .
4.	There is a large difference between core refractive index and cladding refractive index.	4.	There is a small difference between the core refractive index and cladding refractive index.
5.	The strength of signal during propagation weakens or attenuates more in multimode step index fiber. Hence, it is used for short distance communication.	5.	It is used for long distance communications.
6.	Fabrication cost is very less.	6.	Fabrication cost is very high.

Q36. Distinguish between light propagation in,

(i) Step index optical fiber and

(ii) Graded index optical fiber.

OR

Explain the differences between a step index fiber and graded index fiber.

Answer :

Step Index Fiber		Graded Index Fiber	
1.	The core has the same refractive index at all points.	1.	The core has a concentrically varying refractive index, which forms a parabola.
2.	The core diameter is almost double the diameter of a multimode GI fiber.	2.	The core diameter is half to the diameter of GI fiber for multimode.
3.	Propagation of signals is by meridional rays.	3.	Propagation of signals is by helical rays.
4.	Signals distortion is high due to internodal dispersion.	4.	Signal distortion is low due to the absence of internodal dispersion.
5.	Bandwidth is low.	5.	Bandwidth is high.
6.	Attenuation of signals is high.	6.	Attenuation of signal is low.
7.	Numerical aperture is large.	7.	Numeric aperture is small.

Q37. Explain the double crucible method for fibre drawing.

July-17, Q13(a)

OR

Explain the production of optical fibre by Double crucible method and mention any two applications of optical fibres.

Jan.-16, Q16(b)

OR

Explain the fibre drawing process (double crucible) with neat diagram.

Answer :

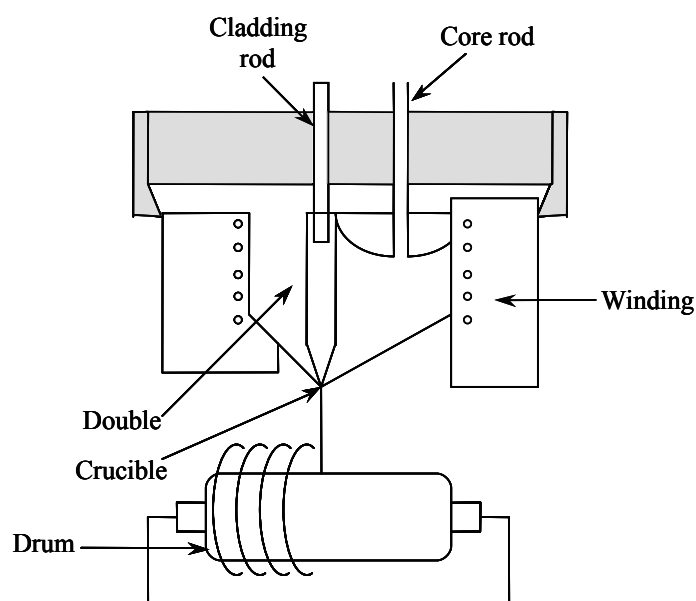
Dec.-09/Jan.-10, Q12(c)

Fiber Drawing Process (Double Crucible Method)

There are two methods of fabricating fibers. They are,

1. Double crucible method
2. Rod-in-tube method.

The double crucible method is shown in figure.

**Figure: Double Crucible Method**

The two high purity glass rods, with different refractive indices are prepared with conventional techniques.

Rods are slowly heated in a the crucibles to melt, and fill up the crucibles with glass-melt. The height of melt must be constant.

The crucible temperature is maintained between 850°C to 1100°C depending upon the choice of glass compositions.

At base, the crucibles down to concentric nozzles, glasses are pulled to a fibre.

Here pull speed is given by the rotating drum on to which the fibre is wound.

By using this method, a fiber is continuously fabricated. But according to the principle, fibres of particular length can be manufactured. Hence, the double crucible method is improved, (to obtain required lengths of fiber by the British post office and by Nippon glass company in Japan).

This method is used to fabricate desired graded index fibers.

Applications of Optical Fibres

For answer refer Unit-5, Q17.

Q38. Explain the losses in optical fibers.

Answer :

There are different types of losses that occur in optical fibres during data transmission from source to destination. They are,

Attenuation

Attenuation is a phenomena of loss of optical power when a light travels through the fibre. It mainly occurs during absorption, scattering and bending losses.

Signal attenuation is also defined as the ratio of input power to the output power of an optical fibre.

The signal attenuation in terms of length is given by,

$$\text{Attenuation} = \frac{10}{L} \log_{10} \left(\frac{P_{in}}{P_{out}} \right) \text{dB/km}$$

Causes for Signal Attenuation

Following are the causes for signal attenuation,

- (i) Composition of optical fiber
- (ii) Purity of material
- (iii) The design of parameters for the fiber to provide the path for the wave
- (iv) Terminal connections
- (v) Splicing to make terminals.

Effects Causing Signal Attenuation

Following are the basic effects causing weakening of signals due to the causes mentioned above,

- (a) Absorption losses
- (b) Scattering losses
- (c) Bending losses.

The attenuation loss is due to absorption and scattering.

(i) Absorption

When an optical power is converted into other form of energy (or heat), attenuation occurs which results in absorption. It occurs from the following factors.

(a) Imperfections in the Atomic Structure

Due to the imperfections in the structure of atoms, missing molecules or oxygen defects occur causing absorption.

(b) Intrinsic Absorption

When the loss of energy occurs due to electron absorption, material absorption and Rayleigh absorption, then intrinsic absorption mechanism is prone to occur.

(c) Extrinsic Absorption

When the loss of energy is due to impurity absorption, then extrinsic absorption occurs.

Scattering Losses

A light beam propagating through the fiber core at critical angle or less will change its direction after hitting on an obstacle in the core region. The obstacle can be any particle that may have diffused inside the core at the time of manufacturing. When the light beam hits the particle it gets scattered and due to this total internal reflection is not achieved hence, the beam is lost through the cladding.

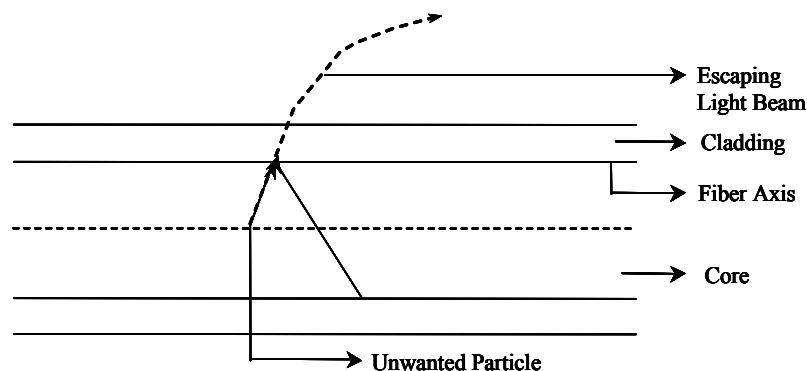


Figure (1): Scattering Loss

Bending Losses

Bending losses occur due to the radiative losses when an optical fiber is bent.

The following are the types of bending losses. They are,

- (i) Fiber bending losses or radiative losses
- (ii) Macrobending or large bending losses
- (iii) Microbending.

(i) Fiber Bending Losses or Radiative Losses

Optical fibers experience radiation losses at the bends or curves on their paths. Radiation losses are predominant when the fiber undergoes a bend of finite radius of curvature. This is due to energy in the evanescent field exceeds the velocity of light at the bends and the guiding mechanism is not applicable. Hence, light energy is radiated from the fiber. The part of the mode outside the bend travels faster than the inside, so that a wave front perpendicular to the direction of propagation is maintained. Hence part of the mode in the cladding needs to travel faster than the velocity of light in that medium. So the energy associated with this part of the mode is lost through radiation. The radiation attenuation coefficient which represents the losses can be expressed as,

$$\alpha_r = C_1 e^{(-C_2 R)} \quad \dots (1)$$

Where,

R - The radius of curvature of the fiber bend

C_1, C_2 - Constants that are independent of ' R '.

(ii) Macrobending or Large Bending Losses

Macrobending losses occur due to the presence of a small imperfection in the fibre cable. The improper cabling leads to high radius bends. External forces also causes small bendings which result in loss of light radiation.

(iii) Microbending

Macro bending losses occur due to small discontinuities or imperfections in the fibre. They also occur due to uneven coating, improper cabling and application of external forces. This increases attenuation as low-order modes couple with high-order modes.

Q39. Write the applications of fiber optics in medicine, communication and sensors.

Answer :

Fiber Optics Applications in Medicine

The applications of optical fibers in the field of medical research are,

1. The optical fibers are used to design wide range of applications like illumination, image and laser signal delivery.
2. Optical fibers will also play a crucial role in traditional applications such as X-ray imaging, light therapy surgical microscopy etc.
3. These are also useful in endoscope applications such as gastroscope (used in stomach inspection), bronchoscope (used to observe the upper passages of lungs), orthoscope (used to observe the small joint spaces), couldoscope (used to examine the female pelvic organs), peritoneoscope (used to examine the abdominal cavity, liver lower parts and gall bladder).
4. In ophthalmology, fibers are used to guide the laser light to attach the separated retina and to correct the flaws in the vision.
5. Optical fibers provide high degree of customization and speciality development in the field of medical research.

Fiber Optic Applications in Communications

The applications of optical fibers in the field of communication are,

1. The properties of optical fibers like flexibility and multiplexing makes them used as a medium for telecommunication and networking.
2. Due to the less attenuation of light through optical fibers, they can be preferred for long distance communication.
3. In the case of short distance communication, the optical fibers occupy very less space in cable ducts when compared to electrical cables.
4. Optical fibers are even immune to electrical interference, which avoids the cross talk among the signals in different cables.

Fiber Optic Applications in Sensors

The applications of optical fibers in the field of sensing are,

1. With the help of an optical fiber a connection between a non-fiber optic sensor and a measurement system is established.
2. By varying the properties of light in the fiber like intensity, phase, polarization and wavelength or transit time the strain, temperature and pressure using fiber optic sensors can be measured.