

# **Physics for Engineering**



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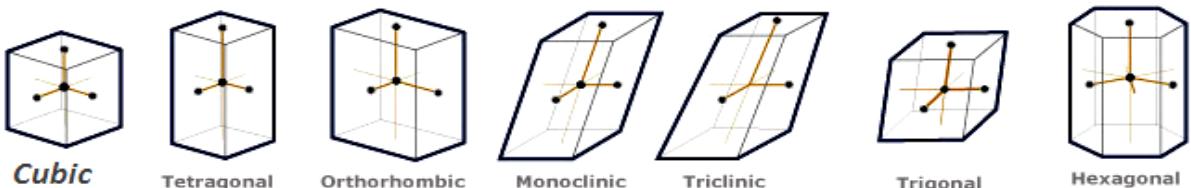
# Unit-I Crystallography

## 1. Write a short note on crystal systems (3M)

On the basis of values of lattice parameters, crystals are classified into seven categories

S.No	Crystal System	Lattice Constants	Crystallography Angles	Examples
1	Cubic	$a=b=c$	$\alpha =\beta = \gamma = 90^\circ$	$P_0$ , Ag, Au, Cu, Pb,
2	Tetragonal	$a=b \neq c$	$\alpha =\beta = \gamma = 90^\circ$	$TiO_2$ , $SnO_2$ , $KH_2PO_4$
3	Rhombic (Orthorhombic)	$a \neq b \neq c$	$\alpha =\beta = \gamma = 90^\circ$	$KNO_3$ , $K_2SO_4$
4	Monoclinic	$a \neq b \neq c$	$\alpha =\beta = \gamma \neq 90^\circ$	Gypsum, $K_2MgSO_4 \cdot 5H_2O$
5	Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$	$K_2Cr_2O_7$ , $CuSO_4 \cdot 5H_2O$
6	Rhombohedral (Trigonal)	$a=b=c$	$\alpha =\beta = \gamma \neq 90^\circ$	As, Sb, Bi
7	Hexagonal	$a=b \neq c$	$\alpha =\beta = 90^\circ, \gamma = 120^\circ$	Quartz, Zn, Cd

The Unit cells of all the crystal systems are shown below



## 2. Explain Bravais lattice (3M)

According to Bravais, there are only fourteen independent ways of arranging points in three-dimensional space such that each arrangement confirms to the definitions of a space lattice. Thus there are fourteen (14) possible types of space lattices among the seven crystal systems and are called Bravais Lattices.

S.NO	CRYSTAL SYSTEM	Bravais Lattices	Name of Bravais Lattice
1	Cubic	3	Simple, Body centered and face centered
2	Tetragonal	2	Simple, Body centered
3	Rhombic	4	Simple, Body Centered, face centered Base Centered
4	Monoclinic	2	Simple Base Centered
5	Triclinic	1	Simple
6	Rhombohedral	1	Simple
7	Hexagonal	1	Simple

### 3. Write as short notes on crystal planes and miller Indices (3M)

Crystallographic planes are specified by 3 indices  $(h k l)$  called **Miller indices**.

- The three indices are not separated by commas and are enclosed in open brackets.
- If any of the indices is negative, a bar is placed in top of that index
- If plane is parallel to that axis, intercept is infinity

Procedure for determining miller indices  $(h k l)$ :

1. Record the intercept values in order  $x, y, z$ .
2. Take reciprocals of the intercept values.
3. Convert the reciprocals into the smallest integers by taking L.C.M of denominators if necessary.

**Example:**

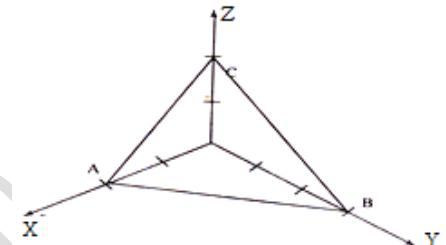
Plane ABC has intercepts of 2 units along X-axis, 3 units along Y-axis and 2 units along Z-axis. find the miller indices for a plane shown in figure.

**Solution:**

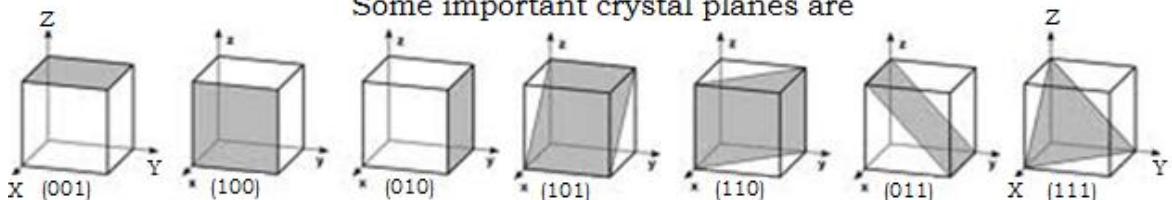
Intercepts are  $(2 \ 3 \ 2)$

Reciprocal of the three intercepts are  $(\frac{1}{2} \ \frac{1}{3} \ \frac{1}{2})$

Convert reciprocals into small integers by multiplying with LCM of denominators i.e., with 6. Then we get miller indices of the plane ABC is  $(3 \ 2 \ 3)$ .



Some important crystal planes are



4.

### 4. Derive interplanar spacing in crystals (2M)

Let  $(h k l)$  be the miller indices of the plane ABC. Let ON=d be a normal to the plane passing through the origin O. Let this ON make angles  $\alpha'$ ,  $\beta'$  and  $\gamma'$  with x, y and z axes respectively. Then  $\cos \alpha' = \frac{d}{OA} = \frac{d}{a/h} = \frac{dh}{a}$

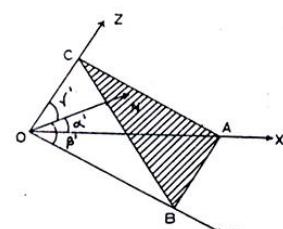
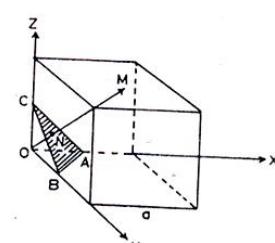
$$\cos \beta' = \frac{d}{OB} = \frac{d}{a/k} = \frac{dk}{a}$$

$$\cos \gamma' = \frac{d}{OC} = \frac{d}{a/l} = \frac{dl}{a}$$

Now  $\cos^2 \alpha' + \cos^2 \beta' + \cos^2 \gamma' = 1$

$$\text{Hence } \left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{a}\right)^2 + \left(\frac{dl}{a}\right)^2 = 1$$

$$(\text{or}) \quad d = ON = \frac{a}{\sqrt{h^2+k^2+l^2}}$$



Let the next plane A'B'C' parallel to ABC be at a distance OM from the origin. Then its intercepts are  $\frac{2a}{h}$ ,  $\frac{2a}{k}$  and  $\frac{2a}{l}$

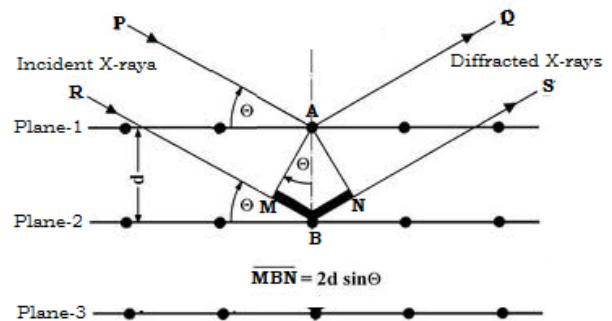
$$\text{Therefore } OM = 2d = \frac{2a}{\sqrt{h^2+k^2+l^2}}$$

Hence the spacing between the adjacent planes= OM-ON=N.M

$$\text{i.e., } d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

## 5. Derive Bragg's Law (3M)

Consider a set of parallel planes of a crystal separated by a distance 'd' apart. Suppose a beam of monochromatic X-rays incident at an angle ' $\theta$ ' on these planes. The beam PA is reflected from atom 'A' in plane-1 whereas beam RB is reflected from atom 'B' in plane-2 as shown in figure. These two reflected rays will be in phase or out of phase with each other depending on their path difference. This path difference can be found by drawing perpendiculars AM and AN. It is obvious that second ray travels an extra distance



$$= MB+BN$$

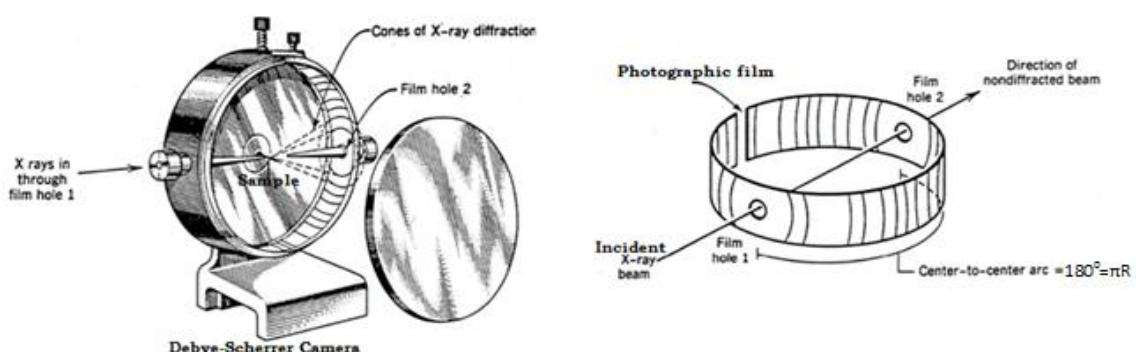
Hence the path difference the two reflected beams

$$= MB+BN=d \sin \theta + d \sin \theta= 2d \sin \theta$$

Bragg's law states that the two reflected beams will be in-phase to each other, if this path difference equals to integral multiple of  $\lambda$

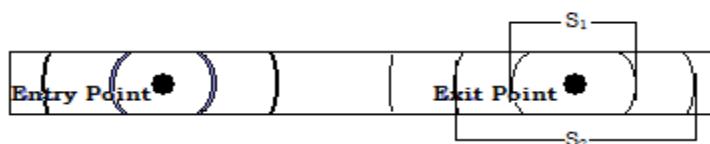
i.e.  $2d \sin \theta = n\lambda$  Where  $n=1,2,3$  for the first order, second order and third order maxima respectively.

## 6. Explain powder diffraction method for determination of lattice constant (5M)



### (Debye-Scherrer method)

The experimental arrangement of powder method is shown in figure. The powdered specimen is kept inside a small capillary tube present at the centre of the camera. A photographic film strip placed along the drum of the camera. A fine pencil beam X-rays is made to fall on the powdered sample. The powder diffracts the x-rays in accordance with Bragg's law to produce cones of diffracted beams. These cones intersect the photographic film. Due to narrow width of the film only pair of arcs of the circle are recorded on the film. The non-diffracted rays leave the camera via the exit port.



The film is removed from the camera, processed and flattened. It shows the diffraction lines and the holes for the incident and transmitted beams. The distance between two successive arcs S is measured and using the relation  $4\theta = \frac{S}{R} \left( \frac{180}{\pi} \right)$ , a list of  $\theta$  values can be obtained. Where 'R' is the radius of the camera. Since the wavelength  $\lambda$  is known, substituting these  $\theta$  values in Bragg's formula, a list of inter-planar spacing d can be calculated. From the ratio of interplanar spacing, the type of the lattice can be identified as well as lattice constant a can be calculated.

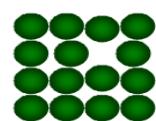
### 7. Write a short note on Classification of defects (5M)

Classification of defects in crystals			
1. Point Defects (0-Dimensional defects)	2. Line Defects (1-Dimensional defects)	3. Surface Defects (2-Dimensional defects)	4. Volume Defects (3-Dimensional defects)
a. Vacancies b. Interstitialcies c. Impurities	a. Edge dislocation b. Screw dislocation	a. Grain Boundaries b. Tilt Boundaries c. Twin Boundaries d. Stacking Faults	a. Cracks b. Voids or Air bubbles

#### 1. Point Defects

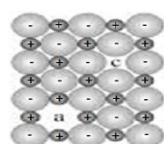
##### (a) Vacancies: Vacancies also called Shottky defects

**Schottky defect:** Ion vacancies are called Shottky defects. A shottky defect is the combination of one cation vacancy and one anion vacancy. A pair of one cation and one anion missing from an ionic crystal(a schottky defect) is shown in figure. These defects are normally generated in equal number of anion and cation vacancies hence electrical neutrality is maintained in the crystal.



##### (b) Interstitialcies: Interstitial defect also called Frenkel defect

**Frenkel defect:** In the case of ionic crystal, an ion displaces from the lattice into an interstitial site is called a Frenkel defect. A Frenkel defect is the combination of one cation vacancy and one cation interstitial defect.



## 2. Line defects

Line defects are called dislocations. These are one-dimensional imperfections in the geometrical sense. The two basic types of dislocations are **a.** edge dislocation and **b.** screw dislocation.

**Edge dislocation:** An edge location is created in the crystal by introducing an extra half plane. The atoms below the edge of the extra plane are squeezed together and are in a state of compression.

**Screw dislocation:** Screw dislocation results from a displacement of the atoms in one part of a crystal relative to the rest of the crystal, forming a spiral ramp around the dislocation line. .

**3. Surface defects** The defects that occur along the surface of the crystal is said to be surface defects. The surface defects are two dimensional defects. The different types of surface defects are

**Grain boundaries:** Grain boundaries are those surface imperfections which Separate crystals of different orientations in a polycrystalline aggregate.

**Tilt boundaries:** Tilt boundary is another type of surface imperfection and it may be regarded as an array of edge dislocations. It is also a class of low angle boundaries.

**Twin boundaries:** Surface imperfections which separate two orientations that are mirror images of one another are called twin boundaries. Twin boundaries occur in pairs, such that the orientation change introduced by one boundary is restored by the other as shown in figure.

**Stacking faults:** A stacking fault is a surface defect that results from the stacking of one atomic plane out of sequence on another, while the lattice on either side of the fault is perfect. The stacking fault is a discrepancy in the packing sequence of the layers of atoms.

## 4. Volume Defects

Volume defects are 3-dimensional defects. These include pores, cracks, foreign inclusions and other phases. These defects are normally introduced during processing and fabrication steps. All these defects are capable of acting as stress raisers, and thus deleterious to parent metal's mechanical behavior. However, in some cases foreign particles are added purposefully to strengthen the parent material.

## 8. Derive equilibrium Concentration of vacancy defects in metal

Let  $E_v$  = Energy of formation of a vacancy

$N$  = Number of lattice sites

$n$  = number of vacancies

The total number of ways in which  $n$  number of vacancies can be formed is given by

$$W = \frac{N!}{n!(N-n)!}$$

Since disorder increases, the increase in Entropy is given by  $S = k \log W$

$$S = k \log \frac{N!}{n!(N-n)!}$$

$$S = k [\log N! - \log n! - \log(N-n)!]$$

Applying Stirling Approximation i.e.  $\log x! = x \log x - x$  we get

$$S = k [N \log N - N - n \log n + n - (N-n) \log(N-n) + N - n]$$

$$S = k [N \log N - n \log n - (N-n) \log(N-n)]$$

From the law of thermodynamics, the corresponding change in free energy is given by  $F = U - TS$  ( $U = nE_v$ )

$$F = nE_v - Tk [N \log N - n \log n - (N-n) \log(N-n)]$$

At equilibrium, the change in free energy is minimum, i.e.,  $\frac{\partial F}{\partial n} = 0$

$$\begin{aligned} 0 &= E_v - kT \left[ -n \frac{1}{n} - \log n - (N-n) \frac{1}{(N-n)} (-1) - \log(N-n)(-1) \right] \\ 0 &= E_v - kT [-1 - \log n + 1 + \log(N-n)] \\ 0 &= E_v - kT [-\log n + \log(N-n)] \end{aligned}$$

$$E_v = kT [-\log n + \log(N-n)]$$

$$\frac{E_v}{kT} = [-\log n + \log(N-n)]$$

$$\frac{E_v}{kT} = \log \frac{N-n}{n}$$

$$e^{\frac{E_v}{kT}} = \frac{N-n}{n}$$

$$e^{\frac{E_v}{kT}} = \frac{N-n}{n} \quad (\text{Since } n \ll N, n \text{ is neglected}) \therefore n = N e^{-\frac{E_v}{kT}}$$

## 9. Derive equilibrium Concentration of vacancy defects in Ionic crystal

Let in a perfect crystal

$E_s$  = Energy required to create a Schottky defect

$N$  = Number of atoms

$n$  = concentration of Schottky defects

The total number of ways in which  $n$  no. of Schottky defects can be formed given

$$\text{by } W = \left[ \frac{N!}{n!(N-n)!} \right]^2$$

Since disorder increases, the increase in Entropy is given by  $S = k \log W$

$$\begin{aligned} S &= k \log \left[ \frac{N!}{n!(N-n)!} \right]^2 \\ S &= 2k \log \left[ \frac{N!}{n!(N-n)!} \right] \\ S &= 2k [\log N! - \log n! - \log(N-n)!] \end{aligned}$$

Applying Stirling Approximation i.e.  $\log x! = x \log x - x$  we get

$$S = 2k [N \log N - N - n \log n + n - (N-n) \log(N-n) + N - n]$$

$$S = 2k [N \log N - n \log n - (N-n) \log(N-n)]$$

From the law of thermodynamics, the corresponding change in free energy is given by  $F = U - TS$

$$F = nE_s - 2Tk [N \log N - n \log n - (N-n) \log(N-n)]$$

At equilibrium, the change in free energy is minimum, i.e.,  $\frac{\partial F}{\partial n} = 0$

$$0 = E_s - 2kT \left[ -n \frac{1}{n} - \log n - (N-n) \frac{1}{(N-n)} (-1) - \log(N-n)(-1) \right]$$

$$0 = E_s - 2kT [-1 - \log n + 1 + \log(N-n)]$$

$$E_s = 2kT [-\log n + \log(N-n)]$$

$$\frac{E_s}{2kT} = [-\log n + \log(N-n)]$$

$$\frac{E_s}{2kT} = \log \frac{N-n}{n}$$

$$e^{\frac{E_s}{2kT}} = \frac{N-n}{n}$$

$$e^{\frac{E_s}{2kT}} = \frac{N}{n} \quad (\text{Since } n \ll N, n \text{ is neglected})$$

$$\therefore n = N e^{-\frac{E_s}{2kT}}$$

This gives the concentration of Schottky defects in a crystal at temperature T. The number of defects increases exponentially with the temperature.

## 10. Derive equilibrium concentration of Frenkel defect

Let in a perfect crystal

$E_f$  = Energy required to create a Frenkel defect

$N$  = Number of atoms

$N_i$  = Number of interstitial atoms

$n$  = concentration of Frenkel defects

The total number of ways in which Frenkel defects can be formed is given by

$$W = \frac{N!}{n!(N-n)!} \times \frac{N_i!}{n!(N_i-n)!}$$

Since disorder increases, the increase in Entropy is given by

$$S = k \log \left[ \frac{N!}{n!(N-n)!} \times \frac{N_i!}{n!(N_i-n)!} \right]$$

$$S = k [\log N! - \log n! - \log(N-n)! + \log N_i! - \log n! - \log(N_i-n)!]$$

Applying Stirling Approximation i.e.  $\log x! = x \log x - x$  we get

$$\begin{aligned} S &= k [N \log N - N - n \log n + n - (N \\ &\quad - n) \log(N-n) + N - n + N_i \log N_i - N_i - n \log n + n - (N_i \\ &\quad - n) \log(N_i-n) + N_i - n] \end{aligned}$$

$$\begin{aligned} S &= k [N \log N - n \log n - (N \\ &\quad - n) \log(N-n) + N_i \log N_i - n \log n - (N_i - n) \log(N_i - n)] \end{aligned}$$

From the law of thermodynamics, the corresponding change in free energy is given by

$$F = U - TS$$

$$\begin{aligned} F &= nE_f - Tk [N \log N - n \log n - (N \\ &\quad - n) \log(N-n) + N_i \log N_i - n \log n - (N_i - n) \log(N_i - n)] \end{aligned}$$

At equilibrium, the change in free energy is minimum, i.e.,  $\frac{\partial F}{\partial n} = 0$

$$\begin{aligned} 0 &= E_f - kT \left[ -n \frac{1}{n} - \log n - (N - n) \frac{1}{(N - n)} (-1) - \log(N - n)(-1) - n \frac{1}{n} - \log n \right. \\ &\quad \left. - (N_i - n) \frac{1}{(N_i - n)} (-1) - \log(N_i - n)(-1) \right] \end{aligned}$$

$$0 = E_f - kT [-1 - \log n + 1 + \log(N - n) - 1 - \log n + 1 + \log(N_i - n)]$$

$$E_f = kT [-2 \log n + \log(N - n) + \log(N_i - n)]$$

$$\frac{E_f}{kT} = [-\log n^2 + \log(N - n) + \log(N_i - n)]$$

$$\frac{E_f}{kT} = \log \frac{(N - n)(N_i - n)}{n^2}$$

$$e^{\frac{E_f}{kT}} = \frac{(N-n)(N_i-n)}{n^2}$$

$$e^{\frac{E_f}{kT}} = \frac{NN_i}{n^2} \quad (\text{Since } n \ll N \text{ and } n \ll N_i, n \text{ is neglected})$$

$$n^2 = NN_i e^{-\frac{E_f}{kT}}$$

$$\therefore n = \sqrt{NN_i} e^{-\frac{E_f}{2kT}}$$

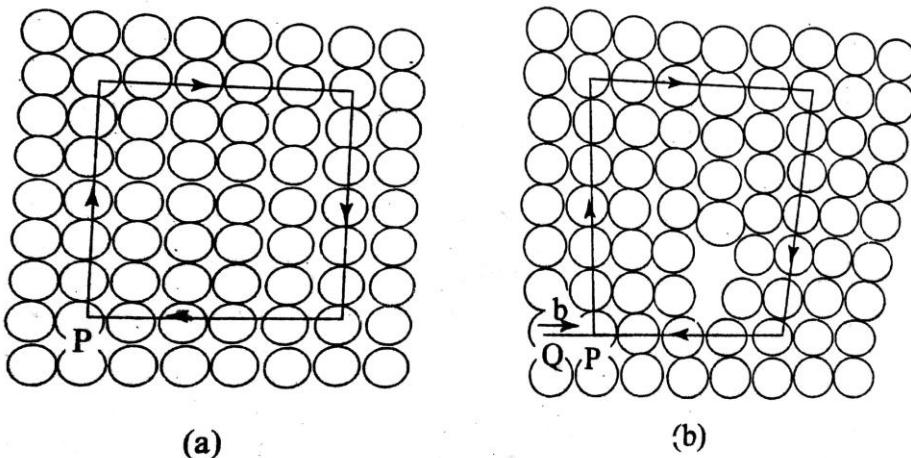
This gives the concentration of Frenkel defects in a crystal at temperature T. The number of defects increases exponentially with the temperature.

## 10. What is Burger's Vector and explain with diagram?

The magnitude and the direction of the displacement are defined by a vector called Burger's vector. To understand the concept of Burger's vector let us draw a Burgers circuit's in two crystals one perfect and another with edge dislocation as shown in the figure (a) and (b) respectively. In figure (a) starting from the point  $P$ , we go up by 6 steps, then move towards right by 5 steps, and move down by 6 steps, and finally move towards left by 5 steps to reach the starting point  $P$ ; the Burger's circuit is closed. When the same operation is performed on the crystal fig(b) we end up at  $Q$  instead of the starting point  $P$ . Now we have to move an extra step  $QP$  to return to  $P$  in order to close the Burger's circuit.

The magnitude and the direction of the step defines the Burger's vector (BV)

$$\text{BV} = \vec{QP} = \mathbf{b}$$

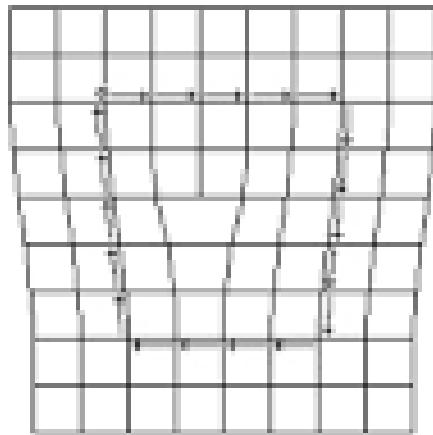


## 11. Write a short note on Edge and Screw Dislocation

### Edge Dislocation

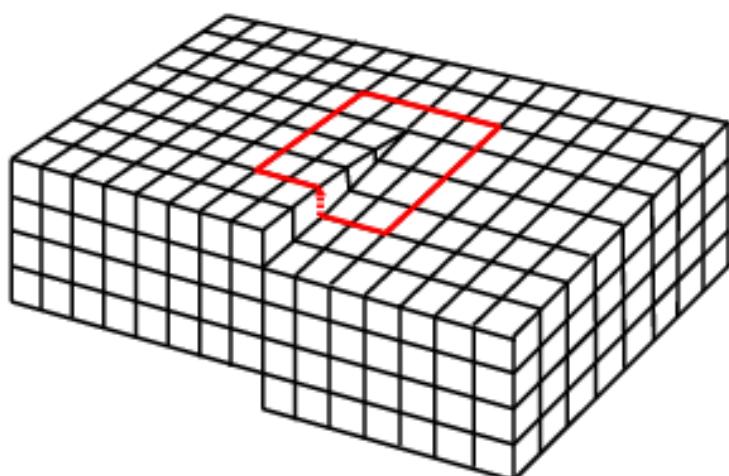
In perfect crystal, atoms are arranged in both vertical and horizontal planes parallel to the side faces. If one of these vertical planes does not extend to the full length, but ends in between, within the crystal it is called edge dislocation.

Because of dislocation, just above the discontinuity atoms are squeezed and are in a state of compression. Just below the discontinuity the atoms are pulled apart and are in a state of tension. The region of maximum imperfection called an edge dislocation. Edge dislocations are symbolically represented by  $\pm$  or T.



### Screw Dislocation

Screw dislocation results from a displacement of the atoms in one part of a crystal relative to the rest part of the crystal, forming a spinal ramp around the dislocation line as illustrated in figure. It shows a method of determining the burgers vector applied to an edge dislocation



# Unit-II Band Theory of Solids & Semiconductors

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## 1. Discuss Classical free electron theory (5m)

**Introduction:** In 1900 Drude proposed electron gas model to explain the electrical conduction in metals. It was extended by Lorentz in 1909 and become known as Drude-Lorentz theory or classical free electron theory.

### Postulates:

1. In a metal, the valence electrons of the atoms are assumed to be completely detached from their ions forming an free **electron gas**.
2. The free electrons are treated equivalent to gas molecules, and thus are assumed to obey the laws of kinetic theory of gases.
3. electron-electron interactions are completely neglected
4. A metal is imagined as a structure of 3-D array of ions in between which, there are freely moving valence electrons confined to the body of the metal.
5. The potential inside the metal is considered constant. Hence, free electrons are moving in a completely uniform potential field due to ions which are fixed in the lattice. But in a particular direction the resultant velocity of the free electron is zero.
6. When we apply the potential difference across the metal, the free electrons slowly drift towards the positive potential even though they continue to collide with each other and positive ions.

### Success:

1. It is used to verify ohm's law
2. It is used to explain the electrical and thermal conductivities of metals
3. It is used to explain the optical properties of metals
4. It is used to explain Ductility and malleability of metals.

### Failures:

1. According to classical free electron theory, resistivity  $\rho \propto \sqrt{T}$  But from experiments  $\rho \propto T$
2. According to classical free electron theory, the paramagnetic susceptibility is inversely proportional to the temperature. But it is observed that it is nearly independent of temperature.
3. With the help of this model, we cannot explain the electrical conductivity of semiconductors and insulators.
4. Ferromagnetism cannot be explained by this theory.
5. Photoelectric effect, Compton Effect and black body radiation cannot be explained by this theory.

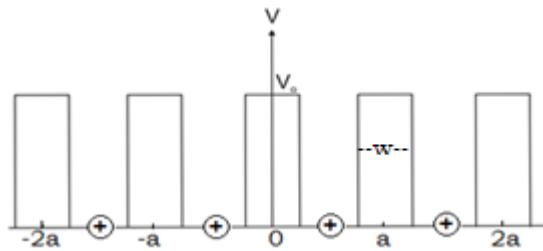
## 2. Explain Kronig- Penney Model (5M)

In 1930, Kronig and Penney proposed a simpler potential in the form of an array of square wells as shown in figure.

The schrodinger wave equation for one dimensional periodic potential field denoted by

$$V(x) \text{ can be written as } \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V(x)]\psi = 0 \\ \text{where } V(x)=V(x+a) \quad \dots(1)$$

The solutions of this equation have the form



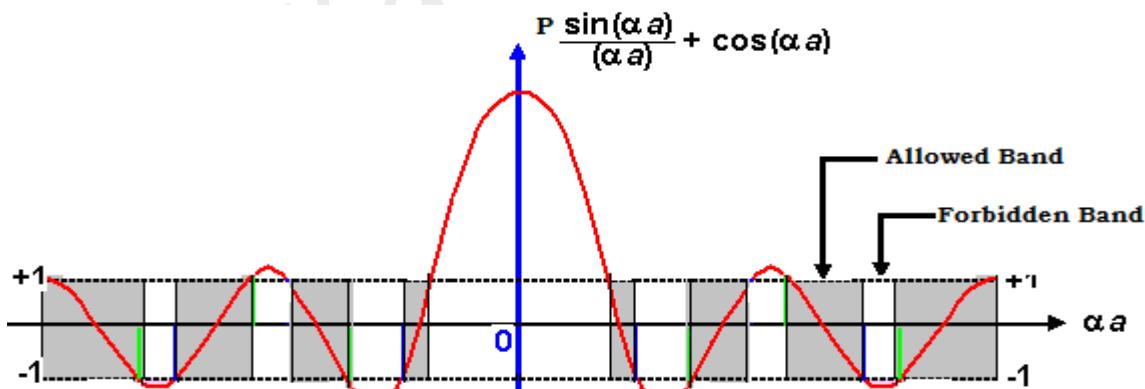
$$\psi(x) = U_k(x)e^{ikx} \text{ where } U_k(x) = U_k(x + a) \quad \dots(2)$$

In order to simplify the computations, an assumption is made regarding the potential barrier shown in figure. As  $V_0$  increases the width of the barrier  $w$  decreases so that the product  $V_0w$  remains constant. It turns out that solutions are possible only for energies given by the relation,

$$\cos ka = \frac{P \sin \alpha a}{\alpha a} + \cos \alpha a \quad \dots(3)$$

Where  $P = \frac{ma}{\hbar^2} V_0 w$  and  $\alpha = \frac{\sqrt{2mE}}{\hbar}$  ( $P$  is referred to as scattering power of the potential barrier)

The left hand side of eq.(3) imposes a limitation on the values that the right side function can have, namely, a maximum value of +1 and a minimum value of -1. Hence only certain range of values of  $\alpha$  are allowed. This means that energy  $E$  is restricted to lie within certain ranges which form the allowed energy bands. This concept is best understood by considering the plot of the right side of the equation against  $\alpha a$  as in figure.



Following interesting conclusions can be drawn from the figure

1. Allowed ranges of  $\alpha a$  which permit a wave mechanical solution to exist are shown by the shadow portions. The motion of electrons in a periodic lattice is characterized by the bands of allowed energy bands separated by forbidden regions.

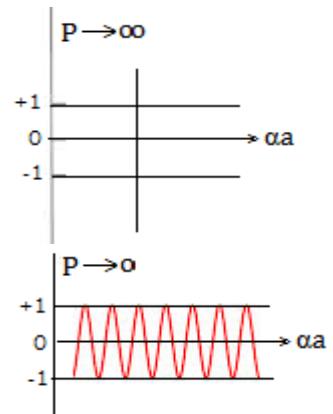
- As the value of  $\alpha a$  increases the width of the allowed energy bands also increases and the width of the forbidden band decreases.
- If the potential barrier strength  $P$  is large, the function described by the right hand side of the equation crosses +1 and -1 region at a steeper angle. Thus the allowed bands become narrower and forbidden bands become wider.
- If  $P \rightarrow \infty$ , the allowed band reduces to one single energy level corresponding to the discrete energy level of an isolated atom.
- In the other extreme case, when  $P \rightarrow 0$ , no energy levels exist. The particle is completely free. Thus by varying  $P$  from 0 to  $\infty$  we find that the completely free electron becomes completely bound.

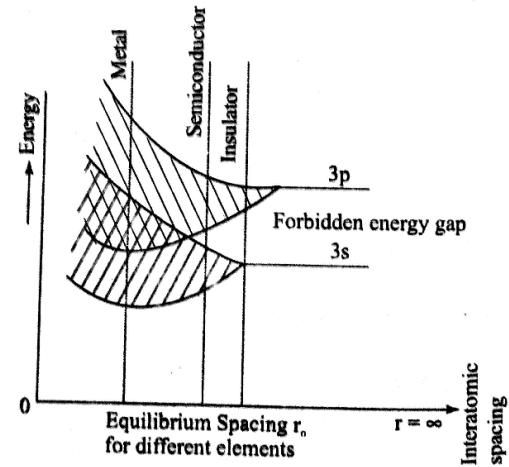
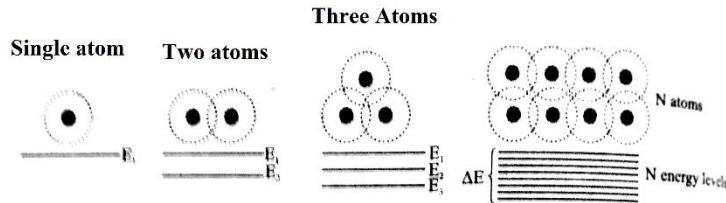
### 3. Discuss Energy band formation in solids (5m)

The solid crystals are formed when the isolated atoms are brought together. Various interactions occur between the neighboring atoms. At a particular interatomic spacing  $d$ , there is a proper balance between forces of attraction to form a crystal. In this process, the changes occur in the electron energy level configurations. In case of a single atom, there is a single energy for an electron orbit. However, when two atoms are brought close to each other, it leads to intermixing of electrons in the valence shell. As a result, the number of permissible energy levels is formed, which is called an energy band. Three bands are important from the conductivity point of view, which are,

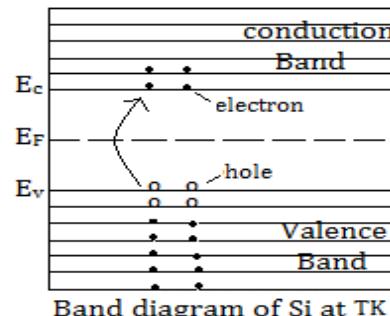
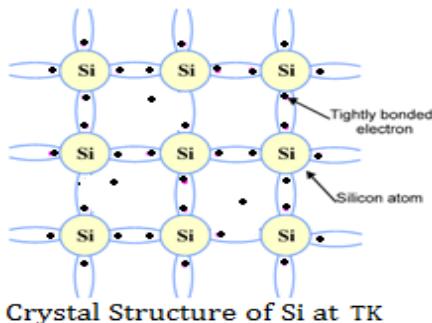
- Valence band
- Conduction band
- Forbidden gap or band

In the normal state, the electrons involved in the covalent bonds in the crystal occupy the valence band and the conduction band is empty. Hence the electrons in the outermost shell are called valence electrons and the outermost shell is called valence shell. At higher temperature, these electrons acquire energy and move to the conduction band as electron is not allowed to occupy any energy state in forbidden gap. These electrons are called free electrons. For any given type of material, the forbidden energy gap may be large, small or nonexistent. The classification of materials as insulators, conductors and semiconductors is mainly dependent on the widths of the forbidden energy gap.





#### 4. Discuss creation of hole and electron Intrinsic semiconductors



Intrinsic semiconductors are those in which impurities are not present and are therefore called pure semiconductors. When a semiconductor is taken at 0K, then it behaves as an insulator and conduction occurs at higher temperature due to thermal excitations of electrons from valence band to the conduction band. The best examples of intrinsic semiconductors are Silicon and Germanium.

At 0K, all the valence electrons of Si atoms are participating in covalent bonding. So at 0K valence band is full and conduction band is empty of electrons. The allowed band of energies above valence band is called conduction band.

As the temperature increases, electrons will come out by breaking their covalent bonds and they move freely inside the crystal. The emitted electrons participate in electrical conduction. As an electron comes out from valence band a vacancy is created in valence band called hole.

Thus at temperature T K, in an intrinsic semiconductor, if 'n' covalent bonds are broken per unit volume of the material, then there will be n electrons in the conduction band and same number of holes in the valence band. Usually the number of electrons present in the conduction band per unit volume of the semiconductor is called **electron concentration (n)**. Similarly the number of holes present in the valence band per unit volume of the semiconductor is called **hole concentration (p)**. Both the free electrons and holes present in the material participate in electrical conduction. The free electrons and holes present per unit volume of the semiconductor is called **carrier concentration (n<sub>i</sub>)**.

## 5. Derive carrier concentration n<sub>i</sub> in Intrinsic semiconductors

In intrinsic semiconductor number of holes is equal to number of electrons

$$n=p=n_i \therefore n_i^2 = np \text{ where } n_i \text{ is called intrinsic carrier concentration}$$

Electron concentration (n) is given by

$$n = 2 \left( \frac{2\pi m_e^* KT}{h^2} \right)^{\frac{3}{2}} \exp \left( \frac{E_F - E_c}{KT} \right) \dots \dots \dots (1)$$

Hole concentration (P) is given by

$$p = 2 \left( \frac{2\pi m_h^* KT}{h^2} \right)^{\frac{3}{2}} \exp \left( \frac{E_v - E_F}{KT} \right) \dots \dots \dots (2)$$

Multiply above two equation i.e. n=p=n<sub>i</sub> ∴ n<sub>i</sub><sup>2</sup> = np

$$\begin{aligned} n_i^2 &= 2 \left( \frac{2\pi m_e^* KT}{h^2} \right)^{\frac{3}{2}} \exp \left( \frac{E_F - E_c}{KT} \right) 2 \left( \frac{2\pi m_h^* KT}{h^2} \right)^{\frac{3}{2}} \exp \left( \frac{E_v - E_F}{KT} \right) \\ &= 4 \left( \frac{2\pi KT}{h^2} \right)^3 (m_e^* m_h^*)^{\frac{3}{2}} \exp \left( \frac{E_F - E_c + E_v - E_F}{KT} \right) \\ &= 4 \left( \frac{2\pi KT}{h^2} \right)^3 (m_e^* m_h^*)^{\frac{3}{2}} \exp \left( \frac{-(E_c - E_v)}{KT} \right) \\ &= 4 \left( \frac{2\pi KT}{h^2} \right)^3 (m_e^* m_h^*)^{\frac{3}{2}} \exp \left( \frac{-E_g}{KT} \right) \\ \therefore n_i &= 2 \left( \frac{2\pi KT}{h^2} \right)^{\frac{3}{2}} (m_e^* m_h^*)^{\frac{3}{4}} \exp \left( \frac{-E_g}{2KT} \right) \end{aligned}$$

## 6. Discuss conductivity of semiconductor

We know that for a semiconducting material, electrical conductivity  $\sigma$  is given by

$$\sigma = (n e \mu_e + n e \mu_h)$$

Where  $\mu_e$  and  $\mu_h$  are the mobility of electron and hole respectively

Hence  $\sigma = n i e (\mu_e + \mu_h)$  ----- (1)

Since  $n = p = n_i$

$$\text{but } n_i = 2 \left( \frac{2\pi kT}{h^2} \right)^{\frac{3}{2}} (m_e^* m_h^*)^{\frac{3}{4}} \exp \left( \frac{-E_g}{2kT} \right)$$

Substitute above value in equation (1)

$$\sigma = 2 \left( \frac{2\pi kT}{h^2} \right)^{\frac{3}{2}} (m_e^* m_h^*)^{\frac{3}{4}} \exp \left( \frac{-E_g}{2kT} \right) e (\mu_e + \mu_h)$$

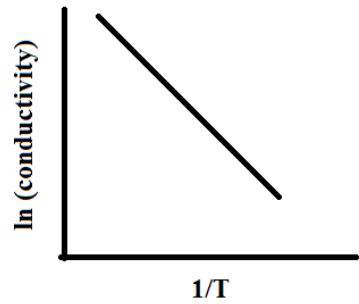
Thus conductivity depend upon the negative exponential of the forbidden energy gap between the valence and conduction band, and on the mobility s of both hole and electron

Hence electrical conductivity of an intrinsic semiconductor can be written as (Take all constants as A)

$$\sigma = A \exp \left( \frac{-E_g}{2kT} \right)$$

Where A is a constant

Taking logarithms on both sides



$$\ln \sigma = \ln A - \left( \frac{E_g}{2kT} \right)$$

A typical graph between  $1/T$  and  $\ln \sigma$  is shown in figure. From the graph we understand that conductivity increases with temperature

## 7. State and derive Hall Effect

When a piece of conductor (metal or semiconductor) carrying current is placed in a transverse magnetic field, an electric field is produced inside the conductor in a direction normal to both the current and the magnetic field. This phenomenon is known as the **Hall effect** and the generated voltage is known as **Hall voltage**.

Let B be the magnetic field applied at right angles to the direction of current flow. If the material is N-type, and 'v' is the velocity of electrons then the electrons experience a force ' $Bev$ ' due to magnetic field at right angles to both flow direction and B. This causes the electron current to be deflected causing a negative charge to accumulate on the face(1) of the slab as shown in

figure. A potential difference is therefore established across the faces(1) and (2) causing a field called Hall electric field ' $E_H$ '.

This field gives rise to a force ' $eE_H$ ' on electrons in the opposite direction.

At equilibrium,  $eE_H = Bev$

$$E_H = Bv \quad \dots \dots (1)$$

If  $J$  is the current density, then  $J = nev \rightarrow v = J/ne$

Therefore eq(1) can be written as  $E_H = \frac{1}{ne} BJ$

$$E_H = R_H BJ$$

Where  $R_H = \frac{1}{ne}$  is the Hall coefficient.

For n-type semiconductors, Hall coefficient  $R_H = \frac{-E_H}{BJ} = \frac{-1}{ne}$  where  $n$  is the density of electrons

For p-type semiconductors, Hall coefficient  $R_H = \frac{E_H}{BJ} = \frac{1}{pe}$  where  $p$  is the density of holes.

If 'w' is the width of sample across which Hall voltage  $V_H$  is measured  $E_H = \frac{V_H}{w}$

$$\text{Hence, } R_H = \frac{E_H}{BJ} = \frac{V_H}{wBJ} \Rightarrow V_H = R_H BJ w$$

If 't' is the thickness of the sample, then its cross-section is  $wt$  and current density  $J = \frac{I}{A} = \frac{I}{wt}$

$$\text{Hence } V_H = R_H B \frac{I}{wt} w = R_H B \frac{I}{t} \Rightarrow R_H = \frac{V_H t}{BI}$$

' $V_H$ ' will be opposite for N-type and P-type semiconductors.

### Applications of Hall Effect

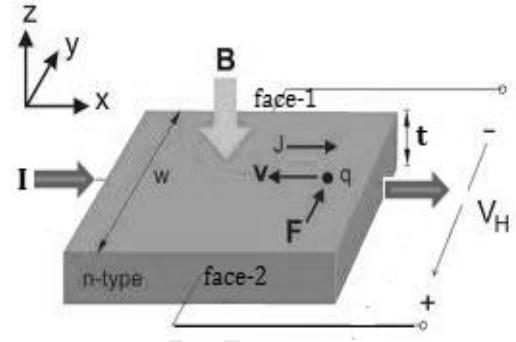
- i) Determination of type of semiconductor

For an N-type semiconductor, the hall coefficient is negative whereas for a P-type it positive. Thus from the direction of Hall voltage developed we can find out the type of semiconductor

- ii) Calculation of carrier concentration

Once the Hall coefficient  $R_H$  is measured, the carrier concentration  $n$  can be obtained from

$$n = \frac{1}{eR_H}$$



iii) Determination of mobility

We know that conductivity  $\sigma_n = ne\mu_e \Rightarrow \mu_e = \frac{\sigma_n}{ne} = -\sigma_n R_H$

iv) Measurement of magnetic flux density

Using a semiconductor sample of known Hall coefficient, the magnetic flux density can be deduced from the formula  $R_H = \frac{V_{Ht}}{BI} \Rightarrow B = \frac{V_{Ht}}{IR_H}$

## 8. What is Thermistor ? Discuss its different types

We know that the resistance of a conductor increases with the increase in temperature whereas in semiconductor resistance decreases with the increase in temperature. It is further found that the variation in the resistance is more in semiconductors than in conductors. **The device whose resistance is very sensitive to temperature variation is called thermistor.**

Thermistors can be classified into two types:

- 1) Positive temperature coefficient (**PTC**) thermistor: -resistance increase with increase in temperature.
- 2) Negative temperature coefficient (**NTC**) thermistor: -resistance decrease with increase in temperature.

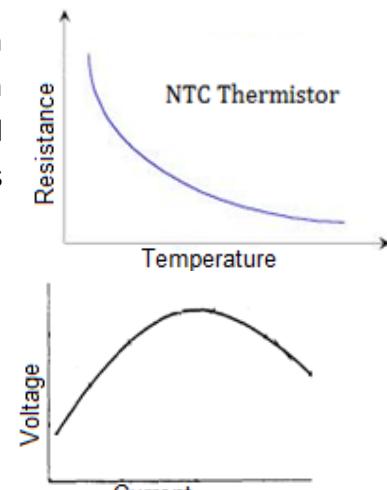
NTC thermistors are normally very small in size, but are complex in construction and material composition. They come in variety of sizes and shapes like beads, washers, rods etc.

The resistance of a thermistor as a function of temperature is given by  $R=A \exp(E/kT)$

Where A is constant and E is the activation energy for the conduction process. The resistance of a thermistor falls off exponentially with temperature as shown in figure(1). It is observed that at low potential difference, a thermistor obeys ohm's law. At high currents it exhibits negative resistance due to self heating as shown in figure.

If we plot a graph between current and time it can be observed that there is a time delay to reach the maximum current at a particular temperature as shown in fig. These characteristics help in introducing a time delay in an electronic circuit.

**Uses:** NTC Thermistor can be used as temperature sensors, temperature controllers, pressure meters and flow meters.



## 9. Explain I-V characteristics of PN junction diode.

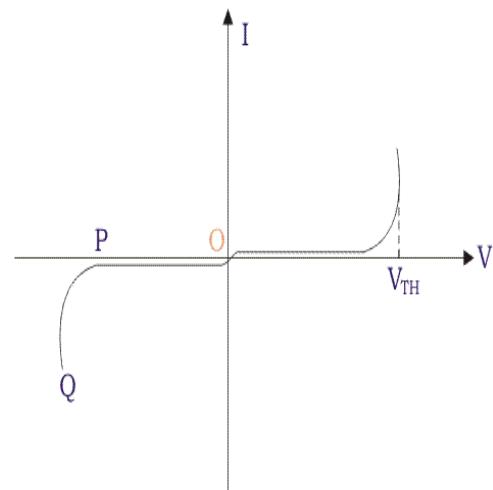
**P-N junction diode** is the most fundamental and the simplest electronics device. When, one side is made p-type by doping with n-type material, a p-n junction diode is formed. This is a two terminal device. It appeared in 1950's. **P-N junction** can be step graded or linearly graded. In step graded the concentration of dopants both, in n - side and in p - side are constant up to the junction. But in linearly graded junction, the doping concentration varies almost linearly with the distance from the junction.

When the **P-N diode** is in unbiased condition that is no voltage is applied across it, electrons will diffuse through the junction to p-side and holes will diffuse through the junction to n-side and they combine with each other. Thus the acceptor atom near the p-side and donor atom near n-side are left unutilized. An electron field is generated by these uncovered charges. This opposes further diffusion of carriers. So, no movement of region is known as space charge or depletion region. If, we apply forwards bias to the **p-n junction diode**. That means if positive side of the battery is connected to the p – side, then the depletion regions width decreases and carriers flow across the junction. If the bias is reversed the depletion width increases and no charge can flow across the junction.

Let's a voltage  $V$  is applied across a **p-n junction** and total current  $I$ , flows through the junction.

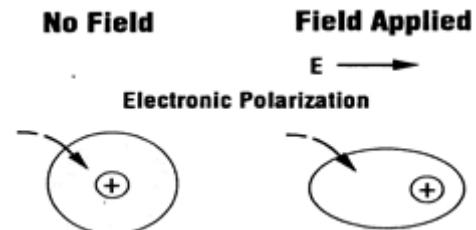
It is given as.  $I = I_s [exp(eV/\eta K_B T) - 1]$  Here,  $I_s$  = reverse saturation current  $e$  = charge of electron  $\eta$  = emission coefficient  $K_B$  = Boltzmann constant  $T$  = temperature

The current voltage characteristics plot is given below. When,  $V$  is positive the junction is forward biased and when  $V$  is negative, the junction is reversing biased. When  $V$  is negative and less than  $V_{TH}$ , the current is very small. But when  $V$  exceeds  $V_{TH}$ , the current suddenly becomes very high. The voltage  $V_{TH}$  is known as threshold or cut in voltage. For Silicon [diode](#)  $V_{TH} = 0.6$  V. At a reverse voltage corresponding to the point P, there is abrupt increment in reverse current. The PQ portion of the characteristics is known as breakdown region.

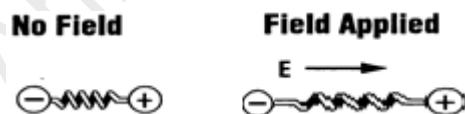


**Unit-II****DIELECTRICS MATERIALS****1. Explain different types of Dielectric polarization****Electronic Polarization**

The electronic polarization is due to the displacement of the positively charged nucleus and the negative electrons of an atom in opposite directions, on application of an electric field. On applying a field, the electron cloud around the nucleus readily shifts towards the positive end of the field. As the nucleus and the center of the electron cloud are separated by a certain distance, dipole moment is created within each atom. The extent of this shift is proportional to the field strength. This type of polarization is independent of temperature.

**Ionic Polarization**

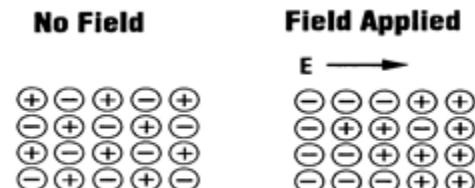
The ionic polarization is due to the displacement of cations and anions in opposite directions. This occurs in ionic solids. This polarization is independent of temperature. It does not occur in covalent crystals.

**Orientation Polarization**

When an external field is applied to polar dielectrics, they tend to align themselves in the direction of external applied field. The polarization due to such alignment is called Orientational polarization. The Orientational polarization is strongly temperature dependent. With increase of temperature, the thermal energy tends to randomize the alignment. An expression for Orientational polarization can be obtained by following the same procedure adopted in Langevin theory of paramagnetism. Here, instead of magnetic field, electric field is used.

**Fig: Orientational Polarization****Space- charge Polarization**

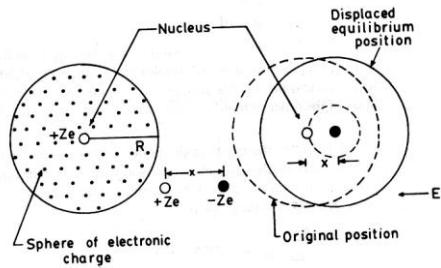
Space charge polarization occurs due to accumulation of charges at the electrodes or at the interfaces in multiphase dielectrics. As shown in figure, the ions diffuse over appreciable distance in response to the applied field. This gives rise to redistribution of charges in dielectric medium. The space charge polarization is not an important factor in most common dielectrics. It is found in ferrites and semiconductors.

**Fig: Space Charge Polarization**

## 2. Derive expression for Electronic polarization

The displacement of the positively charged nucleus and the (negative) electrons of an atom in opposite directions, on application of an electric field, result in electronic polarization

From above diagram nucleus of charge  $Ze$  is surrounded by an electronic cloud  $-Ze$  distributed in a sphere of radius  $R$ . The charge density  $\rho$  is given by



$$\rho = \frac{-Ze}{4/3\pi R^3} = \frac{-3}{4} \left[ \frac{Ze}{\pi R^3} \right] \quad (1)$$

When an external field of intensity  $E$  is applied, the nucleus and the electrons experience Lorentz forces of magnitude  $ZeE$  in opposite directions. Hence the nucleus and electron cloud are pulled apart. When they are separated a coulomb force develops between them, which tend to oppose the displacement. When these forces are equal and opposite, equilibrium is reached. Let  $x$  be the displacement. Since nucleus is much heavier than the electron cloud it is assumed that only the electron cloud is displaced when external field is applied.

$$\text{Lorentz force} = -ZeE$$

$$\text{Coulomb force} = \frac{1}{4\pi\epsilon_0} Ze \times \left[ \frac{\text{Charge enclosed in the sphere of radius } x}{x^2} \right]$$

The charge enclosed in the sphere after applying the field is given by = density X Area

$$\begin{aligned} &= \frac{4}{3}\pi x^3 \rho \text{ (substituted from equation 1)} \\ &= \frac{4}{3}\pi x^3 \left[ \frac{-3}{4} \left[ \frac{Ze}{\pi R^3} \right] \right] \\ &= -\frac{Zex^3}{R^3} \end{aligned}$$

Hence Coulomb force is  $\frac{Ze}{4\pi\epsilon_0 x^2} \left[ \frac{-Zex^3}{R^3} \right] = -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3}$

Under equilibrium conditions, these two forces are equal and opposite. Therefore,  $ZeE =$

$$\frac{1}{4\pi\epsilon_0} \frac{Z^2 e^2 x}{R^3}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Zex}{R^3} \quad \text{or} \quad x = \frac{4\pi\epsilon_0 R^3 E}{Ze}$$

Thus the displacement of the electron cloud is proportional to the applied field. Thus the two electric charges  $+Ze$  and  $-Ze$  are separated by a distance  $x$  under the action of the applied field thus constituting induced dipoles. Induced dipole moment is given by  $\mu_e =$

$$\text{either charge} \times \text{distance between charges} = Ze x = Ze \frac{4\pi\epsilon_0 R^3 E}{Ze} = 4\pi\epsilon_0 R^3 E \quad \text{---- (3)}$$

But from the definition of electronic Polarizability  $\mu_e = \alpha_e E \quad \text{---- (4)}$

Comparing (3) and (4) we get  $\alpha_e = 4\pi\epsilon_0 R^3$

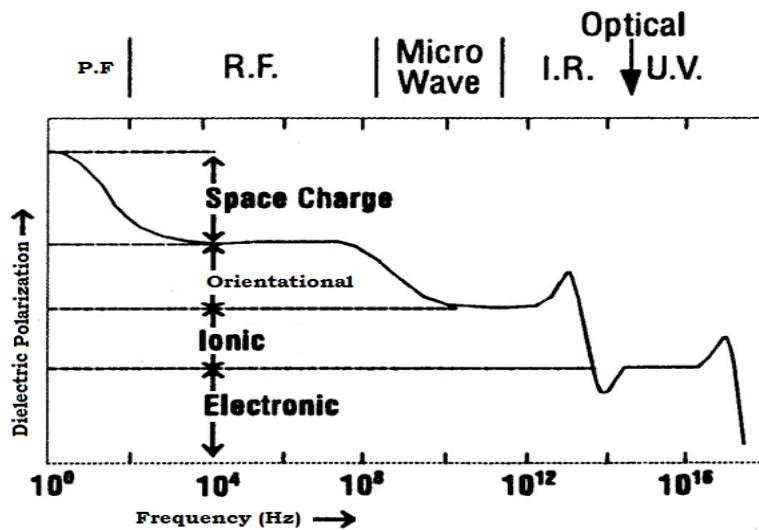
Thus electronic Polarizability is directly proportional to the volume of the atom.

### 3. Explain frequency and temperature dependence of dielectric polarization

#### Frequency dependence of the dielectric Polarizations:

On application of an alternating field across the material, the polarization process occurs as a function of time. Electronic polarization is extremely rapid and is complete at any instant of time even when the frequency of the voltage is very high in the optical range. Thus it occurs at all frequencies. But ionic polarization is slower and the ions do not respond at all when the voltage is corresponding to optical frequencies. That is, the electric field here is changing too rapidly for the ions to reorient themselves in response to the field. Therefore, ionic polarization does not occur at optical frequencies where as it occurs at infrared frequencies. The Orientation polarization is slower than the ionic polarization and occurs only at microwave frequencies which are smaller than the infrared frequencies. Space-charge polarization is slowest process and occurs at power frequencies (50-60 per sec).

Thus at low frequencies, the value of total polarization is very high and at high frequencies (optical frequencies) the value of total polarization is very small.



Above  $10^{15}$  Hz, None of the polarization mechanisms are able to switch rapidly enough to remain in step with the field. The material no longer possesses the ability to polarize, and the dielectric constant drops to 1, the same as that of vacuum.

#### Temperature dependence of the dielectric Polarizations:

Usually Orientation polarization depends strongly on temperature whereas the electronic and the ionic polarizabilities are practically independent of temperatures for normal temperatures. Normal temperatures oppose the permanent dipoles to align in the field direction. But higher temperatures facilitate the movement of ions and molecules so that a given polarization process which is not possible at higher frequencies at normal temperature can occur at higher frequencies at high temperatures.

#### 4. Determine dielectric constant by Schering bridge method

**Principle:** It is based on the measurement of the capacitance of the capacitor without the dielectric ( $C_1$ ) and capacitance of the capacitor with dielectric material ( $C_1'$  )

A circuit diagram of a general Schering bridge is shown in Figure.

$C_1$ =capacitor whose capacitance is to be measured.

$R_1$ = a series resistance representing the loss in the capacitor  $C_1$

$C_2$ =Standard capacitor

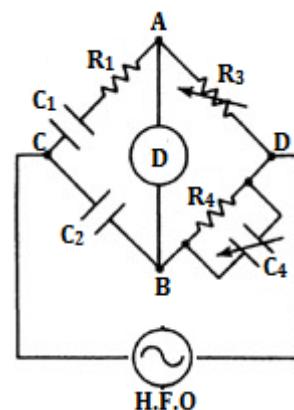
$R_3$ =Non-inductive variable resistance

$R_4$ =Non-inductive Resistance

$C_4$ =Variable capacitor

D= A.C. Null detector

HFO=High Frequency Oscillator



- (i) First without inserting dielectric inside  $C_1$ , the bridge is balanced by varying  $C_4$  and  $R_3$ . When current flowing through the detector becomes zero,

$$\frac{\frac{1}{j\omega C_1} + R_1}{R_3} = \frac{\frac{1}{j\omega C_2}}{\frac{R_4}{1 + j\omega C_4 R_4}}$$

$$\left[ (R_1 + \frac{1}{j\omega C_1}) R_4 \right] = \frac{R_3(1 + j\omega C_4 R_4)}{j\omega C_2}$$

$$R_1 R_4 + \frac{R_4}{j\omega C_1} = \frac{R_3}{j\omega C_2} + \frac{R_3 R_4 C_4}{C_2}$$

Equating the real and imaginary parts, we get

$$R_1 = \frac{R_3 C_4}{C_2} \quad \text{and} \quad C_1 = \frac{R_4 C_2}{R_3}$$

Since  $R_4$  and  $C_2$  are fixed, the dial of  $R_3$  is calibrated to read the capacitance  $C_1$  directly.

- (ii) The dielectric specimen should be in the size of  $C_1$  (in its area) and inserted between the plates  $C_1$ . Now once again the bridge is balanced. Now the dial reading in  $R_3$  will give the value of new capacitance  $C_1'$

Then the dielectric constant can be calculated using the formula  $\epsilon_r = \frac{C}{C_0} = \frac{C_1'}{C_1}$

## 5. Discuss ferroelectricity in dielectrics

Ferroelectricity refers to the creation of enormous value of induced dipole moment in a weak electric field as well as existence of electric polarization even in the absence of applied electric field. Ferroelectricity is a result of dielectric hysteresis.

When a ferroelectric material is subjected to electric field, the polarization  $P$  versus electric field  $E$  describes a closed loop called as hysteresis loop as shown figure. Let initially, the material has polarization equal to zero. When the electric field is applied, the polarization increases along the path OA. After the point A, the polarization will not increase even if  $E$  is increased. This polarization is called saturation polarization  $P_s$ .

When  $E$  is reduced gradually such that  $E=0$ , the curve traces the path AB. It is observed that certain amount of polarization called remnant polarization  $P_r$  is still present. This is the polarization that exists in the absence of external electric field. Therefore, this is called spontaneous polarization.

In order to reduce the polarization to zero, a field in reverse direction must be applied. This is denoted by  $E_c$  called as coercive field. Further increase of field brings the material to saturation state. When the field is brought to zero and increased in the positive direction, a curve FGA is traced. Now, the hysteresis cycle is completed. The area bounded within the curve represents the loss of electrical energy per cycle.

## 6. Write a short note on Barium titanate

The structural changes in Barium titanate ( $\text{BaTiO}_3$ ) crystal due to lattice variation give rise to Ferroelectricity. Above Curie temperature (approx.  $120^\circ\text{C}$ ),  $\text{BaTiO}_3$  has a cubic crystal structure with Barium ions are at the corners the titanium ions exactly at the body center, and Oxygen ions are at the face centers. At those temperatures, there is no spontaneous dipole moment.

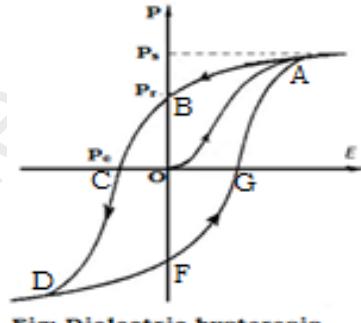


Fig: Dielectric hysteresis in Ferroelectric Materials

When the crystal is cooled below 120°C, one axis (axis C) stretches and the other axes shrink and turn into a tetragonal crystal structure. In this case, polarization happens as a result of the unit shift of axially elongated Ti+ ion crystal. This polarization occurs without applying an external electric field or pressure, and is known as "spontaneous polarization." This characteristic is called "ferroelectricity". The displacements of titanium ions create electric dipoles and all the dipoles of the adjacent unit cells get aligned in the same c-direction. Similarly, at 5°C spontaneous polarization direction corresponds to the face diagonal direction and at -80°C the direction corresponds to a body diagonal. At those temperatures there is an enormous value for dielectric constant.

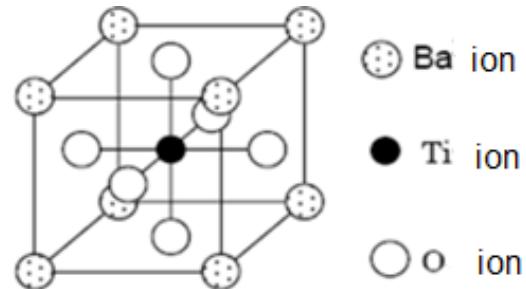
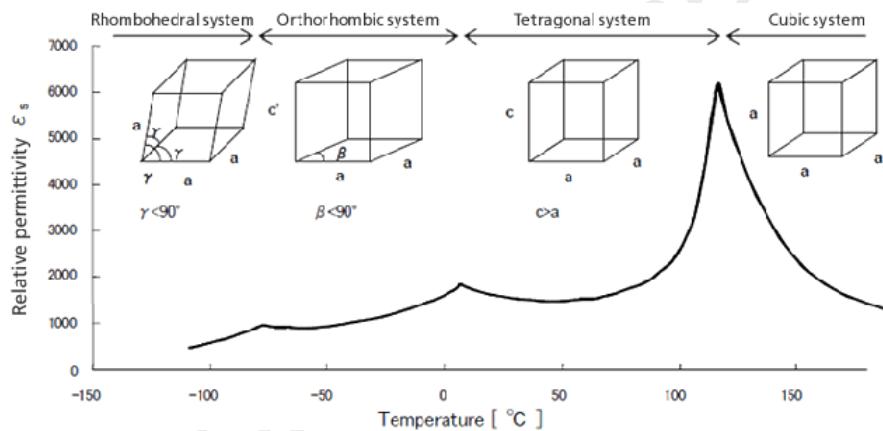


Fig: Unit cell of BaTiO<sub>3</sub> above 120°C



## 7. Write Applications of ferroelectric materials

- Capacitors
- Non-volatile memory
- Piezoelectrics for ultrasound imaging and actuators
- Electro-optic materials for data storage applications
- Thermistors
- Switches known as transchargers or transpolarizers
- Oscillators and filters
- Light deflectors, modulators and displays

### **QUESTIONS FROM PREVIOUS EXAMINATIONS**

1. Explain the phenomenon of Ferro-electricity and discuss how dielectric constant of Barium titanate changes as its temperature is decreased. [June 2011, 7 marks]
2. Write few applications of ferroelectrics. [June 2011, 3 marks]
3. What are dielectrics? Explain various electrical polarization mechanisms. [Jan 2012, 6 marks]
4. Draw the crystal structure of Barium titanate above 393 K and explain how its structure and polarization changes with decreasing temperature. [Jan 2012, 3 marks]
5. Derive the expression for electronic Polarizability in case of dielectrics. [June 2012, 5 marks]
6. Explain the concept of spontaneous polarization in Ferro-electrics. [June 2012, 3 marks]
7. Explain the experimental method of Schering Bridge for determination of dielectric constant. [Jan 2013, 5 marks][April/May-2007,5marks]
8. What are dielectrics? Explain electronic polarization in detail. [Dec-2007& 2008, 3marks]

## Unit-III      WAVE MECHANICS

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### 1. Derive De-Broglie's equation

According to De-Broglie every moving particle is associated with wave. The dual Nature of light possessing both wave and particle properties was explained by combining Planck's expression for the energy of photon  $E = h\nu$  with Einstein's mass energy relation  $E = mc^2$  to give

$$mc^2 = h\nu \quad \text{Or} \quad \nu = \frac{mc^2}{h}$$

$$\frac{c}{\lambda} = \frac{mc^2}{h} \quad (\nu = \frac{c}{\lambda})$$

$$\lambda = \frac{h}{mc} = \frac{h}{p}, \text{ where } \lambda \text{ is the wavelength of photon}$$

$$\text{If particle moves with velocity 'v' then } \lambda = \frac{h}{mv}$$

From above equation if particle velocity is less, then wavelength of wave is more .It was by analogy with this equation associating momentum with a photon that de Broglie expressed the concept of matter wave, according to which a material particle of mass 'm' moving with a velocity 'v' should have an associated wavelength ' $\lambda$ ' , called the de-Broglie wavelength .

### 2. Write difference forms of De-Broglie's Wavelength

(1) De-Broglie's wave equation is given by  $\lambda = \frac{h}{mv}$  ---- (1)

If kinetic Energy of particle is  $E = \frac{1}{2}mv^2$  or  $\sqrt{2Em} = mv$  ---- (2)

substitute Eq (2) in Eq (1)

$$\lambda = \frac{h}{\sqrt{2Em}}$$

(Where E is kinetic Energy)

(2) The wavelength associated with a moving particle is independent of any charge associate with it. If the velocity 'v' is given to an electron by accelerating it through a potential difference 'V' then the work done on the electron is  $eV$ . This work done is converted into the kinetic energy of the electron, then

$$eV = \frac{1}{2}mv^2 \quad \text{Or} \quad v = \left[ \frac{2eV}{m} \right]^{\frac{1}{2}}$$

$$\sqrt{2meV} = mv$$

Substituting this value in the De Broglie equation we have  $\lambda = \frac{h}{\sqrt{2meV}}$

Charge of electron is  $1.6 \times 10^{-19}$  C/s and Mass of electron is  $9.1 \times 10^{-31}$  Kg substitute in above equations

$$\lambda = \frac{12.27}{\sqrt{V}} A^0$$

### **3. What are the properties of de Broglie's waves or Matter Waves?**

- (1) Lighter is the particle, greater is the wavelength associated with it.
- (2) Smaller is the velocity of the particle greater is the wavelength associated with it.
- (3) When  $v = 0$  then  $\lambda = \infty$  i.e., wave becomes indeterminate and if  $v = \infty$  then  $\lambda = 0$  This shows that matter waves are generated by the motion of particles. These waves are produced whether the particles are charged particles or they are uncharged. This fact reveals that these waves are not electromagnetic waves but they are a new kind of waves
- (4) The velocity of matter waves depends on the velocity of material particle i.e. it is not a constant while the velocity of electromagnetic wave is constant.
- (5) The wave nature of matter introduces an uncertainty in the location of the position of the particle because a wave cannot be said exactly at this point or exactly at that point. However, where the wave is large there is good chance of finding the particle while, where the wave is small there is very small chance of finding the particle.

### **4. Derive Schrödinger time independent equation**

Let us consider a group of waves associated with a moving particle. Let  $\psi$  represent the displacement of these waves at any time 't'. Let us consider that the wave motion be represented by classical wave equation.

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} \quad \text{--- (1)}$$

Where  $v$  is the velocity. The solution of the above equation is given by

$$\psi = \psi_o e^{-i\omega t} \quad \text{--- (2)}$$

Differentiate above equation with respect to 't'

$$\frac{\partial \psi}{\partial t} = \psi_o (-i\omega) e^{-i\omega t} \quad \text{And} \quad \frac{\partial^2 \psi}{\partial t^2} = \psi_o (-i\omega)^2 e^{-i\omega t} = -\omega^2 \psi$$

Substitute above value in equation (1) then  $\nabla^2 \psi = \frac{1}{v^2} (-\omega^2 \psi)$

$$\nabla^2\psi + \frac{\omega^2}{v^2}\psi = 0 \quad \text{But } \frac{\omega^2}{v^2} = \frac{(2\pi v)^2}{(v\lambda)^2} = \frac{4\pi^2}{\lambda^2}$$

The wave equation is given by  $\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0$

We can substitute the wavelength of the wave accompanying the particle in terms of the particle like property i.e.  $\lambda = \frac{h}{mv}$  Then

$$\nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0$$

If  $E$  and  $V$  are the total energy and the potential energy of the particle, respectively, then

$$\frac{1}{2}mv^2 = E - V \quad \text{or} \quad mv = \sqrt{2m(E - V)}$$

The wave equation is given by  $\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$  or  $\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0}$

This equation is known as Schrödinger time independent wave equation.

## 5. Derive Schrödinger time dependent equation

Let us consider a group of waves associated with a moving particle. Let  $\psi$  represent the displacement of these waves at any time 't'. Let us consider that the wave motion be represented by classical wave equation.

$$\nabla^2\psi = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} \quad \text{--- (1)}$$

Where ' $v$ ' is the velocity. The solution of the above equation is given by

$$\psi = \psi_o e^{-i\omega t} \quad \text{--- (2)}$$

Differentiate above equation with respect to 't' then  $\frac{\partial \psi}{\partial t} = \psi_o (-i\omega) e^{-i\omega t}$

But  $\omega = 2\pi v$  substitute in above equation  $\frac{\partial \psi}{\partial t} = \psi_o (-i2\pi v) e^{-i\omega t} = -2\pi \frac{E}{h} \psi$   $(E = \nu h)$

$$\text{Or} \quad \frac{\partial \psi}{\partial t} = -\frac{iE}{\hbar} \psi$$

Multiplying both sides with ' $i$ '  $i \frac{\partial \psi}{\partial t} \hbar = E \psi$

Substituting this value in time independent wave equation i.e.  $\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(i\hbar\frac{\partial\psi}{\partial t} - V\psi) = 0$$

$$\frac{\hbar^2}{2m}\nabla^2\psi + i\hbar\frac{\partial\psi}{\partial t} - V\psi = 0 \quad \text{Or} \quad i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

Above equation is called as Schrödinger time dependent wave equation. The above equation

can be written as  $E\psi = H\psi$  where  $i\frac{\partial\psi}{\partial t}$   $\hbar = E\psi$  and  $H = -\frac{\hbar^2}{2m}\nabla^2 + V$  as a Hamiltonian Operator.

## 6. Explain Physical significance of wave function ( $\psi$ )

The wave function associated with a physical system contains all relevant information about the system and its future behavior and thus describes it completely. It is natural to assume that the wave function be large where the particle is most likely to be and small elsewhere.

If  $\psi$  is the amplitude of matter waves at any point in space, then the particle density at that point may be taken as proportional to  $\psi^2$ . Thus  $\psi$  is a measure of particle density. When this is multiplied by the charge of the particle, the charge density is obtained. In this way,  $\psi$  is a measure of charge density.

According to Max Born  $\psi \psi^* = \psi^2$  gives the probability of finding the particle in the state.  $\psi$  is a measure of probability density

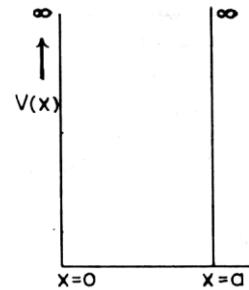
The function  $\psi(\mathbf{r}, t)$  is sometimes called probability amplitude of the particle at position  $\mathbf{r}$  at time  $t$ . The total probability of finding the particle in the region is of course, unity, i.e. the particle is certainly to be found somewhere in space  $\iiint \psi^2 dV = 1$

### Limitation of $\psi$

1.  $\psi$  must be finite for all values of  $x, y, z$  of the region
2.  $\psi$  must be single valued i.e. for each set of values of  $x, y, z$  i.e.  $\psi$  must be have one value only
3.  $\psi$  must be continuous in all region except where potential energy is infinite
4.  $\psi$  is analytical i.e. it possesses continuous first order derivative
5.  $\psi$  Vanishes at the boundaries.

## 7. Apply Schrodinger time independent wave equation to particle in Potential Box

Let us consider a square potential well with infinitely high sides, as indicated below fig... If particle in potential well the potential energy is zero. If particle is moving then potential energy increases.



According to Schrödinger time independent wave equation is given by

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

Potential energy is zero if particle 'x' lies in between **0 to a** i.e.  $V(x) = 0$  for  $0 < x < a$

Boundary condition for wave function is given by

$$\psi = 0 \text{ If particle at } a \text{ i.e. } (\psi)_{x=a} = 0 \text{ And } \psi = 0 \text{ at } 0 \text{ i.e. } (\psi)_{x=0} = 0$$

If potential energy is equal to zero then S. E equation becomes  $\nabla^2\psi + \frac{2m}{\hbar^2}(E)\psi = 0$

$$\text{Or } \nabla^2\psi + \alpha^2\psi = 0 \quad \dots\dots (1) \quad (\alpha^2 = \frac{2mE}{\hbar^2})$$

The solution of above differential equation is  $\psi = A\sin \alpha x + B\cos \alpha x \quad \dots\dots (1)$

Applying the boundary conditions

$$\psi = 0 \text{ at } x = a \text{ and } \psi = 0 \text{ at } x = 0$$

$$\text{i.e } A\sin \alpha a + B\cos \alpha a = 0 \quad \dots\dots (2)$$

$$B\cos \alpha a = 0 \quad \dots\dots (3)$$

If we substitute equation 3 in equation 2 then we have  $A\sin \alpha a = 0$

This means either  $A=0$  or  $\sin \alpha a = 0$

$$\sin \alpha a = 0 \text{ or } \alpha a = \pi, 2\pi, \dots = n\pi$$

$$\text{Where } n=0, 1, 2, 3 \dots \text{ and } \alpha = \frac{n\pi}{a}$$

If we substitute above condition in equation, we have

$$\psi = A \sin \frac{n\pi x}{a} \quad \dots\dots (4)$$

$$\text{And } \alpha^2 = \frac{2mE}{\hbar^2} \text{ or } E = \frac{\alpha^2 \hbar^2}{2m} \text{ or } E_n = \frac{n^2 \hbar^2}{8ma^2}$$

The integer ' $n$ ' introduce above is called a quantum number. The  $E$  values are called energy levels. The particle that is described by the wave function with a certain  $n$  values is said to be in quantum state ' $n$ '. For  $n=1$  the state is called ground state. For 2, 3, ---etc. are known as excited states.

The general form of wavefunction may be written as

$$\psi_n = A \sin \frac{n\pi x}{a} \quad (n = 1, 2, 3, \dots)$$

For A and B values we should normalize above function, normalize condition is

$$\int_0^a \psi^2 dV = 1$$

$$\int_0^a A^2 \sin^2 \frac{n\pi x}{a} dx = 1$$

$$\int_0^a \frac{A^2}{2} (1 - \cos \frac{2n\pi x}{a}) dx = 1$$

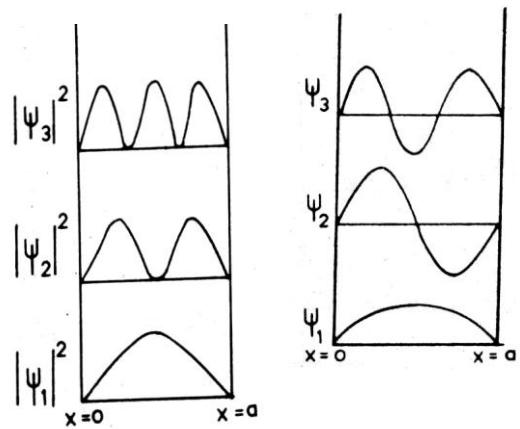
$$[\frac{A}{2}]^2 \left[ x - \frac{a}{2\pi n} \sin \frac{2\pi n x}{a} \right] = 1$$

$$A^2 = \frac{2}{a} \Rightarrow A = \sqrt{\frac{2}{a}}$$

Hence the normalized wavefunctions will have the form

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

The energy level and wave function as shown in the fig.... The Probability of finding the particles more at maximum amplitude.



## University Questions

1. Discuss the properties of wave function?
2. What is the de-Broglie's hypothesis? Obtain the De-broglie's wavelength of matter wave
3. Apply Schrödinger's Wave equation to a particle in Infinite Square well potential and obtain wave function and energy values?
4. Write the properties of matter waves and show that matter waves can travel with speed greater than speed of light in vacuum.
5. Obtain Schrödinger's time independent wave equation?
6. Find the K.E and velocity of Photon associated with de-Brogli's wavelength of  $0.2865\text{A}^0$   

$$h = 6.6 \times 10^{-34} \text{ Js}, m_p = 1.67 \times 10^{-27} \text{ Kg}$$
7. Calculate the wavelength associated with a neutron energy  $0.025\text{eV}$  ( $m_n=1.67 \times 10^{-27}\text{Kg}$ )
8. Compute the de-Broglie wavelength of a proton whose kinetic energy is equal to the rest energy of an electron ( $m_e = 9.1 \times 10^{-31}\text{kg}$ ,  $m_p=1.67 \times 10^{-27} \text{ kg}$ )
9. Calculate the energy of an electron wavelength of  $3 \times 10^{-2} \text{ m}$ . Given  $h = 6.62 \times 10^{-34} \text{ Js}$
10. Compute the De-Broglie wavelength of a proton whose kinetic energy is equal to the rest energy of an electron ( $m_e = 9.1 \times 10^{-31}\text{Kg}$ ,  $m_p=1.67 \times 10^{-27}\text{Kg}$ ) (May06, Dec2004) (2M)
11. Find the first excited state energy of an electron moving along X-axis confined in a box of side length  $10^{-10}\text{m}$  (july02) (2M)
12. Calculate the De-Broglie wavelength of an electron which is accelerated by a potential of  $100\text{V}$   

$$h = 6.6 \times 10^{-34} \text{ Js}, m = 9.1 \times 10^{-31} \text{ Kg}$$
13. Find the energy and momentum of the neutron whose De-Broglie wavelength is  $1.5\text{A}^0$
14. Calculate the de Broglie's wavelength of an electron subjected to a potential difference of  $12.5 \text{ KV}$ .
15. Calculate the energy of an electron wave of wavelength  $3 \times 10^{-2}\text{m}$
16. The electron is confined to a box of length  $10^{-8} \text{ m}$ . Calculate the minimum uncertainty in its velocity.
17. Determine the de-Broglie wavelength of an electron, having kinetic energy of  $1\text{eV}$
18. Compute the energy difference between the first and second quantum state for an electron in one dimension's material having cube side  $1\text{m}$ .

**1. Write the basic equations of Electricity and Magnetism**

$$1. \oint E.dS = \frac{q}{\epsilon_0} \quad \dots\dots(1)$$

This is **Gauss's law of electrostatics** which states that the electric flux through a closed surface is equal to the net charge enclosed by the surface divided by the permittivity constant  $\epsilon_0$

$$2. \oint B.dS = 0 \quad \dots\dots(2)$$

This is **Gauss's law of magnetism**. This states that the magnetic flux through a closed surface is zero.

$$3. \oint E.dl = -\frac{d\phi_B}{dt} \quad \dots\dots(3)$$

This is **Faraday's law** of Electromagnetic induction.

This law states that an electric field is produced by changing magnetic field.

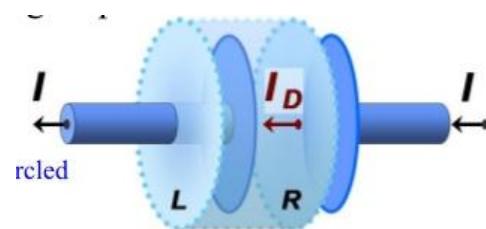
$$4. \oint B.dl = \mu_0 i \quad \dots\dots(4)$$

This is **Ampere's law** for magnetic field due to steady current. This law states that the amount of work done in carrying a unit magnetic pole one around a closed arbitrary path linked with the current is  $\mu_0$  times the current  $i$ .

**2. Derive the expression Displacement Current.**

According to Basic equation (4) i.e. Ampere's law  $\oint B.dl = \mu_0 i$

Maxwell's suggested that above equation ' $i$ ' is not total current. He suggested that something must be added in ' $i$ ' of above equations. In order to know this something Maxwell's postulated that similar to the electric field due to changing magnetic field, there would be a magnetic field due to changing electric field. Thus a changing electric field is equivalent to a current which flows as long as the electric field is changing and produces the same magnetic effect as an ordinary conduction current. This is known as displacement current.



And Ampere's law is valid only for steady state phenomena and not for changing fields  
Let us consider Parallel capacitor for changing field.

Electric field is given by

$$E = \frac{Q}{\epsilon_0 A} \quad (\text{Q is the charge and A is the Area between the plates})$$

For changing electric fields differentiating above equation

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon_0 A} \frac{\partial Q}{\partial t} = \frac{1}{\epsilon_0 A} i$$

$$i = \epsilon_0 A \frac{\partial \mathbf{E}}{\partial t} \quad \left( \frac{\partial Q}{\partial t} = i \right)$$

Displacement Current is given by  $i_d = \epsilon_0 A \frac{\partial \mathbf{E}}{\partial t}$

Now Modified ampere's law is given by  $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (i + i_d)$

### 3. Derive Maxwell differential equations

#### Derivations

$$1. \oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0} \quad (\text{Gauss' law for electricity}) ----- (1)$$

If  $\rho$  be the charge density and  $dV$  be the small volume then charge density

$$\rho = \frac{\text{charge}}{\text{Volume}} = \frac{q}{dV}$$

$$q = \rho \times dV$$

$$\text{For total charge } q = \int_V \rho dV \quad \text{Substitute 'q' value in eq.1}$$

$$\therefore \oint \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon_0} \int_V \rho dV$$

$$\text{or } \oint \epsilon_0 \mathbf{E} \cdot d\mathbf{S} = \int_V \rho dV$$

$$\text{i.e., } \oint \mathbf{D} \cdot d\mathbf{S} = \int_V \rho dV \quad (\because \epsilon_0 \mathbf{E} = \mathbf{D})$$

According to Gauss divergence theorem

$$\oint \mathbf{A} \cdot d\mathbf{S} = \int_V (\vec{\nabla} \cdot \mathbf{A}) dV$$

$$\text{Hence } \oint \mathbf{D} \cdot d\mathbf{S} = \int_V (\vec{\nabla} \cdot \mathbf{D}) dV$$

$$\text{so, } \int_V (\vec{\nabla} \cdot \mathbf{D}) dV = \int_V \rho dV$$

$$\therefore \vec{\nabla} \cdot \mathbf{D} = \rho$$

$$\text{or } \vec{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\text{or } \text{div.E} = \frac{\rho}{\epsilon_0} \quad .....(\text{a})$$

$$\text{or } \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\epsilon_0}$$

$$(2) \oint \mathbf{B} \cdot d\mathbf{S} = 0 \quad (\text{Gauss' law for electricity})$$

Transforming the surface integral into volume integral, we get

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_V \vec{\nabla} \cdot \mathbf{B} \, dV$$

$$\int_V \vec{\nabla} \cdot \mathbf{B} \, dV = 0$$

As the volume is arbitrary, the integral must be zero

$$\therefore \vec{\nabla} \cdot \mathbf{B} = 0 \quad \dots\dots (b)$$

$$\text{or } \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0.$$

$$(3) \oint E \cdot dI = - \frac{d\phi_B}{dt}$$

Magnetic Flux density is given by

$$B = \frac{\text{Flux}}{\text{Area}(dS)}$$

$$\text{Flux} = B \times \text{Area}(dS)$$

$$\text{Total flux}(\phi_B) = \oint_S B \cdot d\mathbf{S}$$

Substitute  $\phi_B$  in above equation then  $\int E \cdot dI = - \oint_S \frac{dB}{dt} \, dS$

Applying Stoke's theorem

$$\oint E \cdot dI = \int_S (\vec{\nabla} \times \mathbf{E}) \cdot d\mathbf{S}$$

$$\therefore \int_S (\vec{\nabla} \times \mathbf{E}) \cdot d\mathbf{S} = - \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

As the equation is true is for all surfaces, we have

$$\vec{\nabla} \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

$$\text{or } \text{curl } \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} \dots\dots (c)$$

$$(4) \oint \mathbf{B} \cdot dI = \mu_0 I \quad (\text{Ampere's law}) \text{ where } I = i + i_d$$

Let us consider Current density  $j = \frac{\text{Current}}{\text{Area}}$

$$\text{Total Current } I = \oint_S J \cdot dS$$

Substitute above value in Ampere's Law

$$\oint B \cdot dI = \mu_0 \oint_S J \cdot dS$$

Applying Stoke's theorem

$$\begin{aligned} \oint B \cdot dI &= \int_s (\vec{\nabla} \times B) \cdot dS \\ \therefore \int_s (\vec{\nabla} \times B) \cdot dS &= \mu_0 \int_S j \cdot dS \\ \text{or} \quad \vec{\nabla} \times B &= \mu_0 j \end{aligned}$$

$$\begin{aligned} \text{But } j &= \frac{\text{Total Current}}{A} = \frac{i + i_d}{A} = \frac{i + \epsilon_o A \frac{\partial E}{\partial t}}{A} = j + \epsilon_o \frac{\partial E}{\partial t} \\ \vec{\nabla} \times B &= \mu_0 (j + \epsilon_o \frac{\partial E}{\partial t}) \end{aligned}$$

#### 4. Write a short note on uniform plane wave (Transverse nature)

A uniform plane wave is a particular case of wave equation for which the electric field is independent of y and z and is a function of x and t only. Such a wave is called uniform plane wave.

Consider the case of electromagnetic wave in which the components of vectors E and B vary with one coordinate only (say  $x$ ) and also with time  $t$ , i.e.

$$\begin{aligned} E &= E(x, t) \quad \text{and} \quad B = B(x, t) \\ \text{But} \quad \vec{\nabla} \cdot E &= 0 \quad \text{or} \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial x} + \frac{\partial E_z}{\partial x} = 0 \\ \therefore \quad \frac{\partial E_x}{\partial x} &= 0 \quad \text{or} \quad E_x = \text{constant} \quad \dots\dots(1) \\ \text{Further} \quad \vec{\nabla} \cdot B &= 0 \quad \text{or} \quad \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial x} + \frac{\partial B_z}{\partial x} = 0 \\ \therefore \quad \frac{\partial B_x}{\partial x} &= 0 \quad \text{or} \quad B_x = \text{constant} \quad \dots\dots(2) \end{aligned}$$

Equations. (1) and (2) are obtained on the fact that the derivative of E and B with respect Y and Z are zero.

Futher  $\text{curl } E = \frac{\partial B}{\partial t}$

$$\therefore \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -\frac{\partial}{\partial t} [iB_x + jB_y + kB_z]$$

Now  $i \left[ \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right] = -i \frac{\partial B_x}{\partial t} = 0 \quad \dots\dots(3)$

$$\therefore \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0$$

From eq.(3)  $\frac{\partial B_x}{\partial t} = 0 \quad \text{or} \quad B_x = \text{constant} \quad \dots\dots(4)$

Similarly, taking curl B, we can show that E = constant.

Hence, we conclude that E and B are constants as regards to time and space. So these components are static components and hence no part of wave motion. Thus,

$$E = jE_y + kE_z$$

$$B = jB_y + kB_z$$

As vectors E and B do not contain any x-component, and hence I-direction being the direction of propagation of the wave. Further both these vectors are perpendicular to the direction of propagation. **Hence, Maxwell electromagnetic waves are purely transverse in nature.**

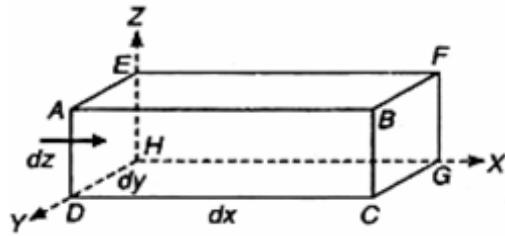
## 5. State and Derive Poynting vector

One important characteristic of electromagnetic waves is that they transport energy from one point to another point. **The amount of field energy passing through unit area of the surface perpendicular to the direction of propagation of energy is called as Poynting vector.** This is denoted by P. For example in a plane electromagnetic wave; E and B are perpendicular to each other and also to the direction of wave propagation. They involve stored energy. So P has a magnitude  $EB \sin 90^\circ = EB$  and points in the direction of wave propagation. The units of P will be Joule/m<sup>2</sup> x sec or Watt/m<sup>2</sup>.

$$P = \frac{1}{\mu_0} (E \times B) \quad \text{or} \quad (E \times H)$$

### Derivation of Expression.

In order to derive the expression for Poynting vector, consider an elementary volume in the form of a rectangular parallelepiped of sides  $dr$ ,  $dy$  and  $dz$  as shown in. The volume of parallelepiped is  $dx dy dz$ . Suppose the electromagnetic energy



is propagated along the X-axis. Now the area perpendicular to the direction of propagation of energy is  $dy \, dz$ . Let the electromagnetic energy in this volume is  $U$ . Then the rate of change of energy is

$$\therefore \frac{\partial U}{\partial t} = - \oint_S P \cdot dS \quad \dots\dots(1)$$

Negative sign is used to show that energy is entering in the volume. So

$$\oint_S P \cdot dS = - \frac{\partial U}{\partial t} \quad \dots\dots(2)$$

We know that:

(1) The energy density per unit volume in electric field E is given by

$$u_E = \frac{1}{2} \epsilon_0 E^2$$

(2) The energy density per unit volume in magnetic field is given by

$$u_B = \frac{1}{2} \mu_0 H^2$$

$$\text{But } \mathbf{B} = \mu_0 \mathbf{H} \quad \text{and } \mathbf{H} = \frac{\mathbf{B}}{\mu_0}$$

$$u_B = \frac{1}{2\mu_0} B^2$$

Total energy

$$\therefore U = u_E + u_B \\ U = \left( \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \right)$$

The rate of decrease of energy in volume  $dV$  is given by

$$-\frac{\partial}{\partial t} \left( \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \right) dV$$

Rate of decrease of energy for volume  $V$

$$\begin{aligned} -\frac{\partial U}{\partial t} &= -\frac{\partial}{\partial t} \int_V \left( \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \right) dV \\ &= -\frac{\partial}{\partial t} \int_V \left( \frac{1}{2} \epsilon_0 2E \frac{\partial E}{\partial t} + \frac{1}{2\mu_0} 2B \frac{\partial B}{\partial t} \right) dV \\ &= \int_V - \left[ \epsilon_0 E \left( \frac{\partial E}{\partial t} \right) + \frac{B}{\mu_0} \left( \frac{\partial B}{\partial t} \right) \right] dV \end{aligned} \quad \dots\dots(3)$$

From Maxwell's equations

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t})$$

(∴ Waves are propagating in non-conducting medium)

$$= \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\therefore \frac{\partial \vec{E}}{\partial t} = \frac{\vec{\nabla} \times \vec{B}}{\mu_0 \epsilon_0} \quad \dots\dots(4)$$

Further  $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$  .....(5)

Substituting the values from eqs. (4) and (5) in eq. (3), we get

$$-\frac{\partial U}{\partial t} = \int_V - \left[ \epsilon_0 E \left( \frac{\vec{\nabla} \times \vec{B}}{\epsilon_0} \right) - \mu_0 B \left( \frac{\vec{\nabla} \times \vec{B}}{\epsilon_0 \mu_0} \right) \right] dV \quad \dots\dots(6)$$

Eq.(6) can be written as

$$\begin{aligned} \frac{\partial U}{\partial t} &= \int_V - \left[ E \cdot (\vec{\nabla} \times \vec{B}) - B \cdot (\vec{\nabla} \times \vec{H}) \right] dV \\ &= \int_V \vec{\nabla} \cdot (E \times H) dV \\ &\quad [ \because \vec{\nabla} \cdot (A \times B) = B \cdot (\vec{\nabla} \times A) - A \cdot (\vec{\nabla} \times B) ] \end{aligned}$$

Using Gauss theorem of divergence, the volume integral can be expressed in terms of surface integral. Thus,

$$= \int_V (E \times H) \cdot n dS \quad \dots\dots(7)$$

Where  $n$  is the unit vector normal to the surface. Comparing eq. (3) with eq. (2), we get

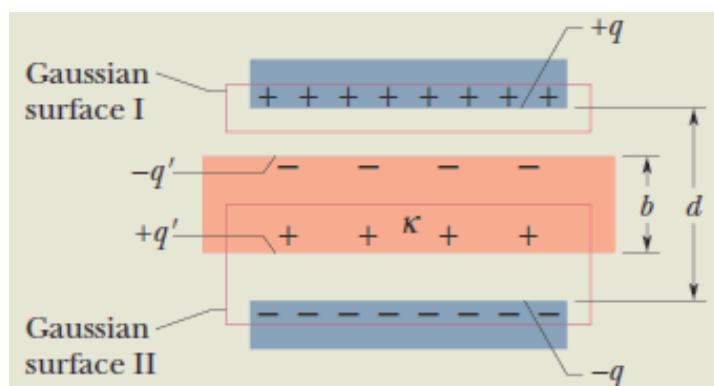
$$\oint_S P \cdot dS = \oint_S (E \times H) \cdot dS$$

Or  $P = (E \times H)$  .....(8)

In magnitude  $P = EH$

This vector shows that energy flow takes place in a direction perpendicular to the plane containing  $E$  and  $H$  or  $B$ . Hence,  $E$  and  $H$  are the instantaneous values.

## 6. Derive the relation between $D$ , $E$ and $P$



Let us consider a parallel plate condenser filled with a dielectric constant 'k'.

When no dielectric present, then Gauss's law is given by

$$\oint E_o \cdot dS = \frac{q}{\epsilon_0}$$

$$E_o \int ds = E_o A = \frac{q}{\epsilon_0} \quad (\text{Where } E_o \text{ is the Electric field without dielectric})$$

$$E_o = \left( \frac{q}{\epsilon_0 A} \right) \dots \dots \dots (1)$$

When dielectric is placed between the plates of the condenser (see fig) the net charge within the Gaussian surface is  $q - q^1$ . Where  $q^1$  is the induced surface charge. Let  $E$  be the resultant field within the dielectric. Then by Gauss's law

$$\begin{aligned} \oint E \cdot dS &= \frac{q - q^1}{\epsilon_0} \\ E \int ds &= EA = \frac{q - q^1}{\epsilon_0} \\ E &= \left( \frac{q - q^1}{\epsilon_0 A} \right) \dots \dots \dots (2) \end{aligned}$$

But dielectric constant  $k = E_0 / E$  and  $E = E_0 / K$

$$\text{Therefore } E = \left( \frac{q}{k \epsilon_0 A} \right) \quad (\text{from equation 1})$$

Substitute  $E$  value in equation 2

$$\left( \frac{q}{k \epsilon_0 A} \right) = \left( \frac{q - q^1}{\epsilon_0 A} \right) = \frac{q}{\epsilon_0 A} - \frac{q^1}{\epsilon_0 A}$$

$$\frac{q}{\epsilon_0 A} = \frac{q}{k \epsilon_0 A} + \frac{q^1}{\epsilon_0 A}$$

$$\text{or} \quad \frac{q}{A} = \epsilon_0 \left( \frac{q}{k \epsilon_0 A} \right) + \frac{q^1}{A}$$

$$\text{We know that } \frac{q}{k \epsilon_0 A} = E \quad \text{and} \quad \frac{q^1}{A} = P$$

$$\therefore \frac{q}{A} = \epsilon_0 E + P$$

We put the ratio  $\left( \frac{q}{A} \right)$  as  $D$ . Hence,

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

# Unit-IV

# Superconductivity

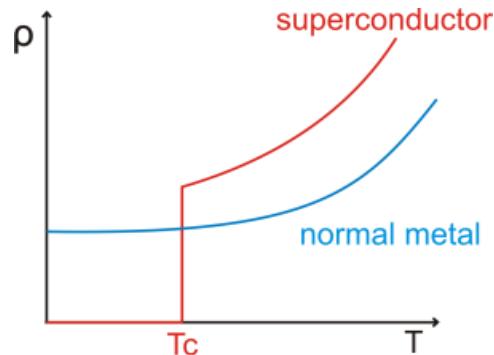
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## 1. Explain the phenomenon of superconductivity (5M)

Before the discovery of superconductivity, it was thought that the electrical resistance of a conductor becomes zero only at absolute zero. But, it is found that, in some materials the electrical resistance becomes zero, when they are cooled to very low temperatures.

For example, the electrical resistance of pure mercury suddenly drops to zero, when it is cooled below 4.2 kelvin.

This phenomenon was first discovered by H.K.Onnes in 1911. This *phenomenon of losing the resistivity absolutely, when cooled to sufficiently low temperature is called superconductivity*. The material which shows this behavior is called **superconductor**. The temperature at which a normal conductor loses its resistivity ( $\rho$ ) and becomes a superconductor is known as **transition or critical temperature  $T_c$** .



## 2. Write general properties of superconductors (3M)

1. The transition temperature is different for different substances.
2. For a chemically pure and structurally perfect specimen, the superconducting transition is very sharp. For chemically impure and structurally imperfect, the transition range is broad.
3. Superconductivity is found to occur in metallic elements in which the number of valence electrons lies between 2 and 8.
4. Superconducting elements, in general lie in the inner columns of the periodic table.
5. Transition metals having odd number of valence electrons are favorable to exhibit superconductivity while metals having even number of valence electrons are unfavorable.
6. Materials having high normal resistivity exhibit superconductivity.
7. Ferromagnetic and antiferromagnetic materials are not superconductors.
8. In superconducting state materials exhibit Meissner effect.
9. The current in a superconducting ring persists for a very long time.
10. In superconducting state there are small changes in thermal conductivity and the volume of the material.

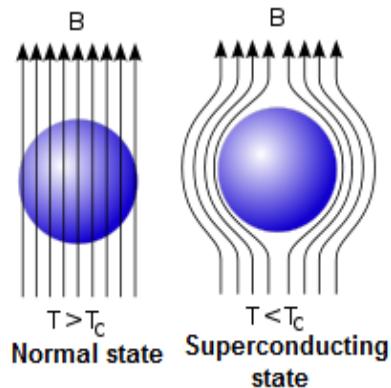
## 3. What is Meissner effect (3M)

When a superconducting material is placed in a weak magnetic field of flux density  $B$ , the magnetic lines of force penetrates through the material as shown in figure(a). When the material is cooled below its transition temperature ( $T \leq T_c$ ), the magnetic lines of forces are expelled as shown in figure(b). We know that a diamagnetic material has the tendency to expel the magnetic lines of forces. Since the superconductor also expels the magnetic lines of forces it behaves as a

*perfect diamagnet*. This behavior was first observed by Meissner and hence called as Meissner effect.

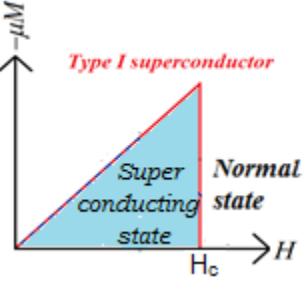
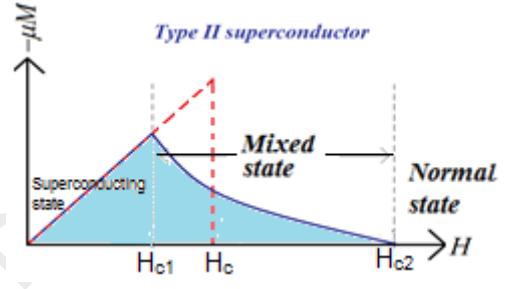
When the strength of the magnetic field applied reaches a critical value  $H_c$  the superconductivity disappears. The dependence of the critical field upon the temperature is given by

$$H_c = H_0 \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$



#### 4. What are Type-I and Type-II superconductors

Based on magnetic behavior, the superconductors are classified into following two categories:

Type-I superconductors	Type-II superconductors
 <p><i>Type I superconductor</i></p>	 <p><i>Type II superconductor</i></p>
The Type-I superconductor becomes a normal conductor abruptly at critical magnetic field $H_c$ .	Type-II superconductor loses its superconducting property gradually, due to increase in magnetic field.
There is only one critical magnetic field $H_c$	There are two critical fields i.e., Lower critical field $H_{c1}$ and upper critical field $H_{c2}$
Type-I superconductors exhibit complete Meissner effect	Type-II superconductors show complete Meissner effect below $H_{c1}$ and allow the flux to penetrate the superconductor between $H_{c1}$ and $H_{c2}$ . Between $H_{c1}$ and $H_{c2}$ , the material shows a region of mixed state.
Above critical field $H_c$ , the superconductor becomes normal conductor	Between $H_{c1}$ and $H_{c2}$ , the superconductor exist in a mixed state called vortex state and above $H_{c2}$ , it comes in normal state.
These are known as soft superconductors	These are known as hard superconductors

The critical field $H_c$ is relatively low	The value of $H_{c2}$ is very large.
Examples: Al, Zn, Ga, etc.	Examples: Lead-Indium alloy, etc.

## 5. Discuss BCS theory of superconductivity

In 1957, Bardeen, Cooper and Schrieffer gave a theory to explain the phenomenon of superconductivity, which is known as BCS theory.

1. The BCS theory is based upon the formation of Cooper pairs, which is purely a quantum mechanical concept. During the flow of current in a superconductor, when an electron comes near a positive ion core of the lattice, it experiences an attractive force because of the opposite charge polarity between electron and the ion core. The ion core will be displaced from its position due to this interaction, which is called lattice distortion. Now, an electron which comes near that place will also interact with the distorted lattice, which tends to reduce the energy of the electron. This process is looked upon as equivalent to interaction between the two electrons via the lattice.
2. The lattice vibrations are quantized in terms of what are called phonons. Thus the process is called "electron-lattice-electron interaction via the phonon field."
3. Cooper pair is a bound pair of electrons formed by the interaction between the electrons with opposite spin and momenta in a phonon field.
4. When the electrons flow in the form of Cooper pairs in materials, they do not encounter any scattering and the resistance factor vanishes or, in other words, conductivity becomes infinity which is named as superconductivity. This is the essence of BCS theory

## 6. Write a short note on High temperature superconductors

In 1986, a real breakthrough in superconductivity took place. Bednorz and Muller discovered metallic, oxygen deficient copper oxide compounds of the Ba-La-Cu-O system with transition temperature of about 35 K. Chu and co-workers formed Y-Ba-Cu-O system with transition temperature of 90 K. Any superconductor with a transition temperature above 10K is in general called high  $T_c$  superconductor. The transition temperature above 10K is in general called high  $T_c$  superconductor. The high  $T_c$  superconductors developed so far belong to five chemical systems having the following general formulae.

1.  $BaPb_{1-x}Bi_xO_3$
2.  $La_{2-x}M_xCuO_{4-x}$  ( $M = Ba, Sr$ )
3.  $Ba_2MCu_3O_{7-x}$ . ( $M = Y$  or rare earth metals such as Gd, Eu, etc.)
4.  $Ba_{2-x}La_{1+x}Cu_3O_\delta$
5.  $Bi_2CaSr_2Cu_2O_\delta$

It is difficult to state the mechanism of high T superconductivity. It has been realized that the role of oxygen is essential for high  $T_c$  oxide superconductors. The unit cell of high T superconductors has complicated structure. If the cell contains one atom of rare earth metal, two barium atoms, three copper atoms and seven oxygen atoms, then such compounds are named 1-2-3 superconductors.

## 7. Explain the applications of superconductors

Superconductors are used in the following applications:

- Maglev (magnetic levitation) trains. These work because a superconductor repels a magnetic field so a magnet will float above a superconductor – this virtually eliminates the friction between the train and the track. However, there are safety concerns about the strong magnetic fields used as these could be a risk to human health.
- Large hadron collider or particle accelerator. This use of superconductors was developed at the Rutherford Appleton Laboratory in Oxfordshire, UK in the 1960s. The latest and biggest large hadron collider is currently being built in Switzerland by a coalition of scientific organizations from several countries. Superconductors are used to make extremely powerful electromagnets to accelerate charged particles very fast (to near the speed of light).
- SQUIDs (Superconducting Quantum Interference Devices) are used to detect even the weakest magnetic field. They are used in mine detection equipment to help in the removal of land mines.
- The USA is developing “E-bombs”. These are devices that make use of strong, superconductor derived magnetic fields to create a fast, high-intensity electromagnetic pulse that can disable an enemy’s electronic equipment. These devices were first used in wartime in March 2003 when USA forces attacked an Iraqi broadcast facility. They can release two billion watts of energy at once. The following uses of superconductors are under development:
  - Making electricity generation more efficient
  - Very fast computing. Other impacts of superconductors on technology will depend on either finding superconductors that work at far higher temperatures than those known at present, or finding cheaper ways of achieving the very cold temperatures currently needed to make them work.

### NUMERICAL PROBLEMS

1. Superconducting tin has a critical temperature of 3.7K at zero magnetic field and a critical field of 0.0306 Tesla at 0K. find the critical field at 2K.

Solution:

$$\text{Critical Field } H_c(T) = H_c(0) \left[ 1 - \frac{T^2}{T_c^2} \right] = 0.0306 \left[ 1 - \frac{2^2}{3.7^2} \right] = 0.02166 \text{ Tesla}$$

2. Calculate the critical current for a wire of lead having a diameter of 1mm at 4.2K. Critical temperature for leads is 7.18K and  $H_c(0) = 6.4 \times 10^4 \text{ A/m}$

Solution:

$$\text{Critical Field } H_c(T) = H_c(0) \left[ 1 - \frac{T^2}{T_c^2} \right] = 6.4 \times 10^4 \left[ 1 - \frac{4.2^2}{7.18^2} \right] = 4.2 \times 10^4 \text{ A/m}$$

$$\text{Critical Current } I_c = 2\pi r H_c = 2 \times 3.14 \times 0.5 \times 10^{-3} \times 4.2 \times 10^4 = 134.26 \text{ A}$$

3. The critical temperature for a metal with isotopic mass 199.5 is 4.185K. Calculate the isotopic mass if the critical temperature falls to 4.133K.

Solution:

We know that isotopic effect is given by  $T_c M^\alpha = \text{constant}$

Hence,

$$T_{c1} M_1^\alpha = T_{c2} M_2^\alpha$$

$$\therefore M_2^\alpha = \frac{M_1^\alpha}{T_{c2}} T_{c1} = \frac{199.5^{0.5}}{4.133} \times 4.185 = 204.536$$

4. The critical temperature for Hg with isotopic mass 199.5 is 4.184K. Calculate the critical temperature when its isotopic mass changes to 203.4K.

Solution:

We know that isotopic effect is given by  $T_c M^\alpha = \text{constant}$

Hence,

$$T_{c1} M_1^\alpha = T_{c2} M_2^\alpha$$

$$\therefore T_{c2} = \frac{M_1^\alpha}{M_2^\alpha} T_{c1} = \frac{199.5^{0.5}}{203.4^{0.5}} \times 4.184 = 4.1435 \text{ K}$$

### Questions from previous papers

1. Write a note on Type I and Type II superconductors. [June 2011, 4 marks]
2. Write a note on High temperature superconductors. [Jan 2012, 5 marks]
3. Distinguish between type I and type II superconductors [June 2012, 2 marks]
4. What is superconductivity? Explain the general properties of superconductors. [Jan 2013, 5 marks]
5. What are type II superconductors? Explain their importance [June-2013, 3marks]
6. What is superconductivity? Describe the preparation of high  $T_c$  superconductors [June-2013, 4marks]
7. The superconducting transition temperature of a metal is 7.26K. The critical field at 0 K is  $64 \times 10^3 \text{ A/m}$ . Calculate the critical field at 5K. [Jan 2013, 3 marks]

**Unit-IV****Magnetic Materials****1. Discuss classification of magnetic materials (5M)**

**Diamagnetic materials:** The materials which when placed in a magnetic field acquire feeble magnetism opposite to the direction of the magnetic field are known as diamagnetic materials.

Bismuth (Bi), Zinc (Zn), Copper (Cu), Silver (Ag), Gold(Au), Salt(NaCl), Water(H<sub>2</sub>O), Mercury(Hg), Hydrogen(H<sub>2</sub>) are the examples of diamagnetic materials.

Properties:

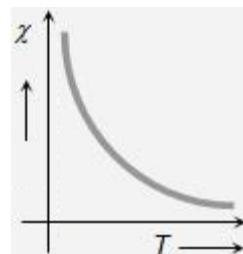
1. When a bar of diamagnetic material is placed in a magnetic field, it tries to repel the lines of force away from it as shown in figure. This shows the magnetic induction B in it is less than the applied field H. So the permeability ( $\mu = \frac{B}{H}$ ) is less than one.
2. When a bar of diamagnetic material is suspended freely between two magnetic poles, the axis of the bar becomes perpendicular to magnetic field.
3. The magnetic susceptibility  $\chi$  is negative and is independent of temperature.

**Paramagnetic materials:** The materials which when placed in a magnetic field, acquire feeble magnetism in the direction of magnetic field are known as paramagnetic materials.

Examples: Aluminium (Al), Platinum (Pt), Manganese (Mn), copper chloride (CuCl<sub>2</sub>), Oxygen (O<sub>2</sub>)

Properties:

1. When a bar of paramagnetic material is placed in a magnetic field, it tries to attract the lines of force towards it as shown in figure. This shows the magnetic induction B in it is greater than the applied field H. so, the permeability is greater than one.
2. When a bar of paramagnetic material is suspended freely between two magnetic poles, its axis becomes parallel to magnetic field as shown in figure. Moreover, the poles produced at the ends of the bar are opposite to nearer magnetic poles.
3. The molecules of paramagnetic material possess permanent magnetic dipole moments. In the absence of external magnetic field, the dipoles are randomly oriented. Hence, the net magnetic moment is zero. In the presence of magnetic field, the dipoles tend to align along the field direction. This leads to magnetization in the direction of field. However, this tendency is opposed by thermal agitation. Therefore, only partial alignment takes place.
4. The paramagnetic susceptibility is positive and depends greatly on temperature as per Curie's law  $\chi \propto \frac{1}{T}$ , where C is curie constant.



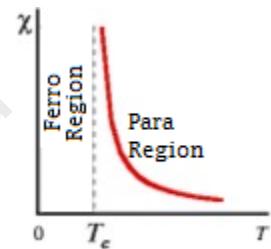
### Ferromagnetic materials:

The materials which when placed in a magnetic field become strongly magnetized in the direction of field are known as ferromagnetic materials.

Examples: Iron (Fe), Nickel (Ni), Cobalt (Co), Magnetite ( $\text{Fe}_3\text{O}_4$ ), etc

Properties:

1. When a ferromagnetic material is placed inside a magnetic field, it attracts the magnetic lines of forces very strongly as shown in fig.
2. They acquire relatively high magnetization in a weak field. Further even in the absence of applied field, the magnetic moments are enormous. This is called **spontaneous magnetization**.
3. The magnitude of susceptibility is very large and positive.
4. The susceptibility of a ferromagnetic substance decreases steadily with the rise of temperature in accordance with Curie-Weiss Law.  $\chi = \frac{C}{T - T_c}$ , where C is curie constant and  $T_c$  is curie's temperature.
5. Above certain temperature, called Curie temperature, ferromagnetic material changes to paramagnetic.



## 2. Weiss molecular field theory of ferromagnetism (5M)

In 1907, Weiss proposed the molecular field theory to explain qualitatively the salient features of ferromagnetism. He suggested that the field experienced by a given dipole in ferromagnetic material is not only due to the applied field ( $H$ ) but also due to a contribution from the neighbouring dipoles.

In ferromagnetic material, spontaneous magnetization is due to internal molecular field and these internal molecular fields tend to align the atomic dipoles in the same direction as its neighbours.

According to Weiss, internal molecular fields  $H_{int}$  are directly proportional to intensity of magnetizations.  $H_{int} \propto M$  or  $H_{int} = \gamma M$

Where ' $\gamma$ ' is molecular field constant known as Weiss constant.

$\therefore$  The effective magnetic field on the atomic dipole  $H_{eff} = H + H_{int}$

$$H_{eff} = H + \gamma M \quad \text{---- (1)}$$

The eq(1) explains the Curie-Weiss law by assuming that ferromagnetics are essentially paramagnetic materials having a very large molecular field.

### **Curie-Weiss law from molecular field theory:**

Let us consider a system with  $N$  spins per cubic meter, each giving rise to a magnetic moment of 1 Bohr magneton,  $\mu_m$  which may be either parallel or antiparallel to an external applied magnetic field. The magnetization of such a system is given by  $M = N\mu_m \tanh \left[ \left( \frac{\mu_0 \mu_m}{kT} \right) (H + \gamma M) \right]$  ---- (2)

Even in the case of ferromagnetic materials, at very high temperatures ( $T > T_c$ ), the thermal agitations are so great that internal field is not sufficient to maintain alignment of magnetic dipole moments. The behavior becomes analogous to paramagnetism, except that the field acting on the dipole is the internal field given by equation (1).

At sufficiently high temperatures, the term in square brackets in equation (2) will become small compared to unity. Then since  $\tanh \alpha \approx \alpha$  for  $\alpha \ll 1$ , we may approximate equation(2) by

$$\begin{aligned} M &= \frac{N\mu_0\mu_m^2}{kT} (H + \gamma M) \\ M &= \frac{N\mu_0\mu_m^2}{kT} H + \frac{N\mu_0\mu_m^2}{kT} \gamma M \\ M - \frac{N\mu_0\mu_m^2}{kT} \gamma M &= \frac{N\mu_0\mu_m^2}{kT} H \\ M \left( 1 - \frac{N\mu_0\mu_m^2}{kT} \gamma \right) &= \frac{N\mu_0\mu_m^2}{kT} H \\ M \left( T - \frac{N\mu_0\mu_m^2 \gamma}{k} \right) &= \frac{N\mu_0\mu_m^2}{k} H \end{aligned}$$

$$\text{Thus the susceptibility } \chi = \frac{M}{H} = \frac{N\mu_0\mu_m^2/k}{T - N\mu_0\mu_m^2\gamma/k}$$

This is in the form of Curie-Weiss law  $\chi = \frac{C}{T - T_c}$ . Where  $C = N\mu_0\mu_m^2/k$  and  $T_c = N\mu_0\mu_m^2\gamma/k = \gamma C$

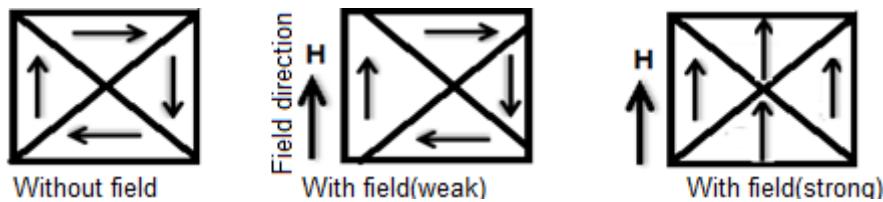
Thus ferromagnetic materials behave in the same fashion as paramagnetic material for  $T >> T_c$ .

### **3. Weiss domain theory of ferromagnetism**

According to Weiss, a virgin specimen of ferromagnetic material consists of a number of small regions called **domains**. These domains have spontaneous magnetization due to the parallel alignment of spin magnetic moments in each domain. But the direction of spontaneous magnetization varies from domain to domain and are oriented in such a way that the net magnetization of the specimen is zero as shown in figure(1).

When the magnetic field is applied, the magnetization occurs in the specimen by two ways.

- (i) By the motion of domain walls and (ii) By rotation of domain walls



**The motion of domain walls** takes place in weak magnetic field. The volume of domains oriented in the direction of magnetic field increases at the cost of those that are unfavorably oriented as shown in figure(2).

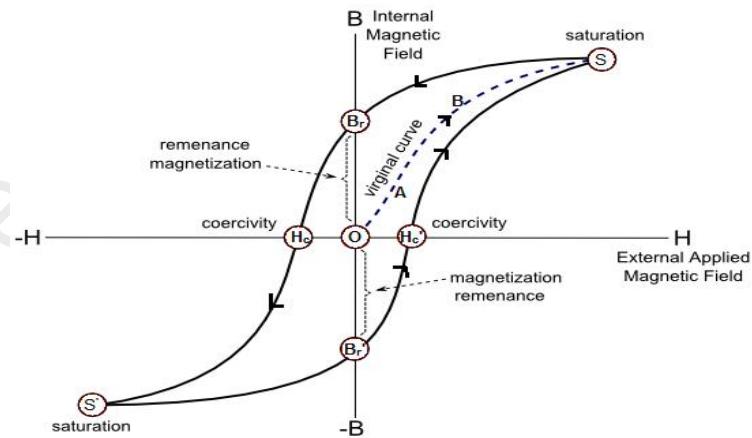
**The rotation of domain walls** takes place in strong magnetic fields. When the external field is high(strong) then the magnetization changes by means of rotation of the direction of magnetization towards the direction of applied magnetic field as shown in figure(3).

#### 4. Explain Hysteresis Curve on the basis of domain theory (5M)

When a ferromagnetic material is made to undergo through a cycle of magnetization, the variation of  $B$ (magnetic induction) with respect to  $H$ (applied field) can be represented by a closed *hysteresis loop* (or) *curve*. i.e., it refers to the lagging of magnetization behind the magnetic field.

##### Explanation on the basis of domain theory:

When a field is applied, domains where the magnetization is parallel or at a small angle with the field grow at the expense of those where the magnetization is antiparallel so that the boundary between domains is displaced.

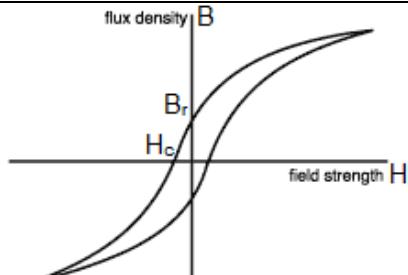
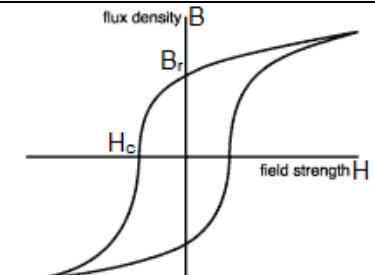


Initially OA the magnetization of the substance as whole proceeds by small reversible boundary displacement but the steeper part AB of the magnetization curve is due to larger, irreversible displacements. Above the knee of the curve BS magnetization proceeds by rotation of the direction of magnetization of whole domains and such a process is rather difficult and the increase in magnetization is relatively slow.

When the applied field is reduced, there is a little change in the domain structure so that the magnetic induction or magnetization remains quite high, until high reverse fields are applied. Further even when the external field is zero, there is a residual magnetization in the specimen

and that can be destroyed by applying a high reverse field. Thus the reversible and irreversible domain wall movements give rise to hysteresis in the ferromagnetic materials.

## 5. Differentiate Hard and soft magnetic materials (5M)

Soft Magnetic Materials	Hard Magnetic Materials
 <p>flux density <math>B</math> field strength <math>H</math> <math>B_r</math> <math>H_c</math></p>	 <p>flux density <math>B</math> field strength <math>H</math> <math>B_r</math> <math>H_c</math></p>
<p>Soft magnetic materials are easy to magnetize and demagnetize. Hence these materials are used for making temporary magnets.</p>	<p>Hard magnetic materials are difficult to magnetize and demagnetize. Hence these materials are used for making permanent magnets.</p>
<p>They have low hysteresis loss due to small hysteresis loop area.</p>	<p>They have large hysteresis loss due to large hysteresis loop area.</p>
<p>Susceptibility and permeability are high.</p>	<p>Susceptibility and permeability are low.</p>
<p>Coercivity and retentivity values are less.</p>	<p>Coercivity and retentivity values are large.</p>
<p>Magnetic energy stored is less.</p>	<p>Magnetic energy stored is high.</p>
<p>The eddy current loss is less.</p>	<p>The eddy current loss is high.</p>
<p>Examples: Iron, silicon alloys, Ferrites, Garnets etc.</p>	<p>Examples: Carbon steel, Tungsten steel, Cu-Ni-Fe(Cunife), Cu-Ni-Co(Cunico), Al-Ni-Co(Alnico)</p>

## 6. Explain properties and application of Ferrites (5M)

Ferrimagnetic materials are also called as Ferrites. Ferrites are the modified structures of iron with no carbon and are composed of two or more sets of different transition metals. These materials have anti parallel magnetic moments of different magnitudes. In the absence of external magnetic field there is a net magnetization. In the presence of external magnetic field large magnetic moment arises. This property of the materials is called ferri magnetism and the materials which exhibit this property are called ferri magnetic materials.



Examples: Manganese-zinc ferrite, Nickel-zinc ferrite, Cobalt ferrite...

## Properties

The susceptibility ( $\chi$ ) is very large and positive. It is represented by,

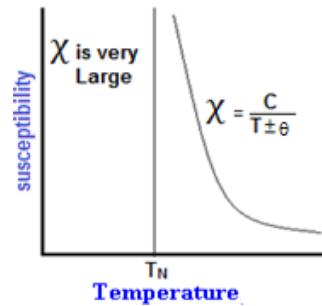
$$\chi = \frac{C}{T \pm \theta}, \text{ when } T > T_N$$

When  $T < T_N$ , they behave as ferrimagnetic materials

These materials have low eddy current losses and low hysteresis

## Applications of Ferrites

1. Ferrite is used in radio receivers to increase the sensitivity and selectivity of the receiver.
2. Ferrites are used as cores in audio and TV transformers.
3. Ferrites are used in digital computers and data processing circuits.
4. Ferrites are used to produce low frequency ultrasonic waves by magnetostriction principle.
5. Ferrites are widely used in non-reciprocal microwave devices. Examples for non-reciprocal microwave devices are Gyrator, Isolator and Circulator.
6. Ferrites are also used in power limiting and harmonic gyration devices.
7. Ferrites can also be used in the design of ferromagnetic amplifiers of microwave signals.
8. The rectangular shape ferrite cores can be used as a magnetic shift register.
9. Hard ferrites are used to make permanent magnets.
10. The permanent magnets (hard ferrites) are used in instruments like galvanometers, ammeter, voltmeter, flex meters, speedometers, wattmeter, compasses and recorders.



## NUMERICAL PROBLEMS

1. A paramagnetic material has a magnetic field intensity of  $10^4$  A/m. If the susceptibility at room temperature is  $3.7 \times 10^{-3}$ , calculate the magnetization and flux density in the material.

Solution: We know that  $\chi = \frac{M}{H} \Rightarrow M = \chi H = 3.7 \times 10^{-3} \times 10^4 = 37$  A/m

Flux density  $B = \mu_0 [H + M] = 4\pi \times 10^{-7} [10^4 + 37] = 0.012612$  Wb/m<sup>2</sup>

2. A magnetic field strength of  $2 \times 10^5$  ampere/m is applied to a paramagnetic material with relative permeability of 1.01. Calculate the values of B and M

Solution:

We know that  $\mu = \frac{B}{H}$

$$\mu_0 \mu_r = \frac{B}{H} \quad [\text{since } \mu = \mu_0 \mu_r]$$

Therefore magnetic flux density  $B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 1.01 \times 2 \times 10^5 = 0.2537 \text{ Wb/m}$

We know that  $\mu_r = 1 + \chi$  and  $\chi = \frac{M}{H}$

Therefore  $\mu_r = 1 + \frac{M}{H}$  or  $\mu_r - 1 = \frac{M}{H}$

Therefore Magnetization  $M = [\mu_r - 1]H = [1.01 - 1] 2 \times 10^5 = 2000 \text{ ampere/m}$

**Questions from previous papers**

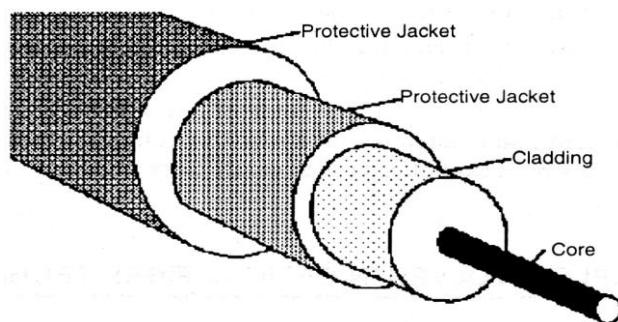
1. Draw the nature of magnetic dipole moments and variation of susceptibility with temperature graphs in Ferro, Ferri and anti-ferromagnetic materials. [June 2011, 3 marks]
2. Distinguish between soft and hard magnetic materials. [June-13, Jan 12, Dec-07&08, June-08, 4 marks]
3. Describe Weiss molecular field theory of Ferromagnetism. [June 2012, 5 marks]
4. Distinguish between Ferro, anti-Ferro and Ferri magnetic materials. [Jan 2013, 2 marks]
5. Explain Weiss molecular field theory of Ferro magnetism and obtain the Curie-Weiss law. [Jan 13, 5 m]
6. Explain Weiss molecular field theory and obtain the Curie-Weiss Law [April/May-2007, 3mark]
7. What are ferrites? Explain its spinal and inverse spinal structure. [June-2013, 5marks]

# Unit-V FIBER OPTICS

## 1. Explain different parts of optical Fiber

Physically, an optical fiber is a very thin and flexible medium having a cylindrical shape, consisting of three section core, cladding and protective jacket as follows

**(a) The Core.** The core is the innermost section having diameter approximately  $5 \mu\text{m}$ -  $100 \mu\text{m}$ . The fibers are generally silica ( $\text{SiO}_2$ ) based.

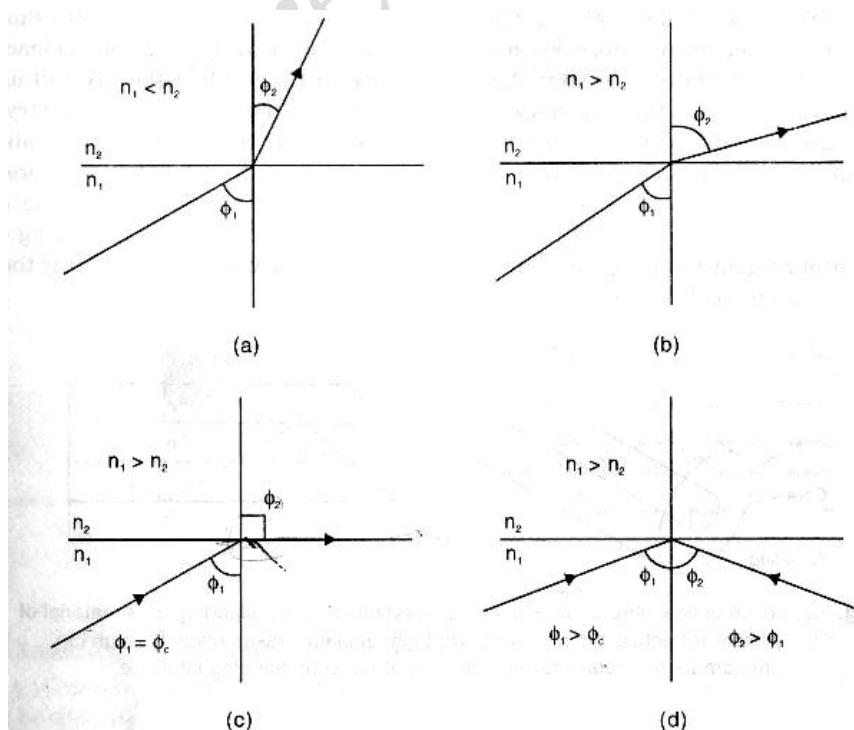


**(b)The Cladding.** The core is surrounded by cladding of slightly lower refractive index. Diameter usually  $125 \mu\text{m}$

**(c)The Protective Jacket.** The outermost section is called the jacket consisting of primary coating ( $250 \mu\text{m}$  in diameter) followed by another layer i.e., secondary coating. These are made of plastic or polymer and are provided for protection against moisture, abrasion, crushing and other environmental dangers etc.

## 2. Explain total Internal Reflection.

When light rays propagate across an interface between two optically different media, a portion of light is moving into the second medium with a changed direction of propagation. If  $n_1$  and  $n_2$  represent the refractive indices of first and second semi-infinite media forming an interface respectively, tow situations can occur.



1) If  $n_2 > n_1$  the second medium said to be optically denser than the first medium or one can say the first medium is optically lighter than the second medium. In this situation, the refracted ray is bent towards the normal to the interface as the ray of light goes from air into glass.

2) If  $n_2 < n_1$ , then the ray goes into an optically lighter medium then bends away from the normal. For example from glass into air.

Above figures shows:- (a) ray of light passes from optically rarer to denser medium (b) ray of light passes from optically denser to rarer medium (c) angle of incidence in the denser medium is equal to critical angle of low media concerned (d) angle of incidence in the denser medium is greater than critical angle. Where  $\Phi_1$  and  $\Phi_2$  represent the angles of incidence and refraction respectively. From Fig (b) we find that if the angle of incidence  $\Phi_1$  is increased, the refracted ray bends more and more away from the normal and at a particular angle of incidence, the refracted ray passes perpendicular to the normal i.e., grazing along the interface (Fig.c). This particular angle of incidence is known as the critical angle ( $\Phi_c$ ). When the angle of incidence is increased beyond this critical angle, no refracted ray exists and all the incident energy is reflected back into the optically denser medium as illustrated in Fig(d). This phenomenon where all the incident energy is reflected back to the first medium is called as **total internal reflection**. Phenomenon of total internal reflection occurs when angle of incidence in the optically denser medium is greater than critical angle ( $\Phi_c$ ) ....

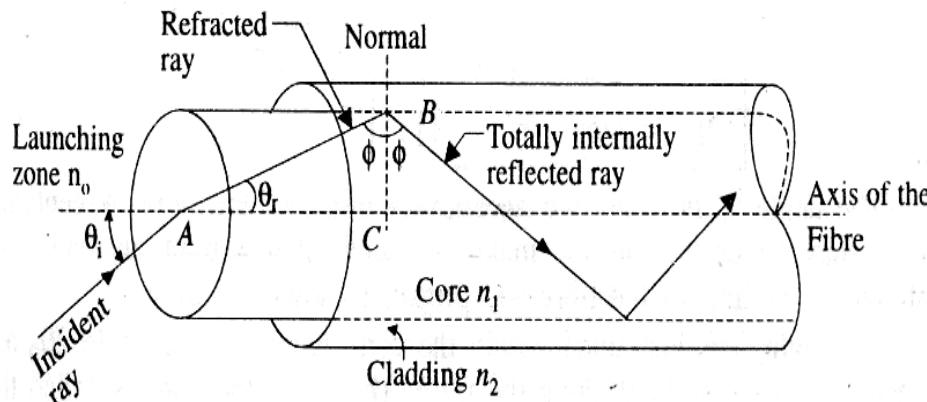
According to Snell's Law  $n_1 \sin \Phi_1 = n_2 \sin \Phi_2$  If  $\Phi_1 = \Phi_c$  and  $\Phi_2 = 90^\circ$  then

$$\Phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \quad \dots \dots \dots (1)$$

### 3. Derive Acceptance angle and Numerical aperture of optical fiber.

Acceptance angle may be defined as the maximum angle that a light ray can have relative to the axis of the fiber and propagate down the fiber.

Let us consider an optical fiber into which light is launched. The end at which light enters the fiber is called the launching end. The fig shows the conditions at the launching end. Let refractive index of core be ' $n_1$ ' and the refractive index of the cladding be ' $n_2$ ', let  $n_0$  be the refractive index of the medium from which light is launched into the fiber



Let a light ray enter the fiber at an angle  $\theta_i$  to the axis of fiber. The light refracts at an angle  $\theta_r$  and strikes core and cladding surface at angle  $\Phi$ . If  $\Phi$  is greater than critical angle  $\Phi_c$  the ray undergoes total internal reflection. As long as the angle  $\Phi$  is greater than  $\Phi_c$  the light will stay within the fiber.

Let us now compute the maximum incident angle ( $\theta_{\max}$ )

Apply Snell's law to the launching face of the fiber we get  $\frac{\sin \theta_i}{\sin \theta_r} = \frac{n_1}{n_0}$  ----- (1)

From  $\Delta$  le ABC it is seen that  $\theta_r + \Phi_c + 90^\circ = 180^\circ$  or  $\theta_r = 90^\circ - \Phi_c$  ----- (2)

Substitute Eq (2) in Eq (1) then  $\frac{\sin \theta_i}{\sin(90^\circ - \Phi_c)} = \frac{n_1}{n_0}$  or  $\frac{\sin \theta_i}{\cos \Phi_c} = \frac{n_1}{n_0}$

For  $\theta_{\max} \sin \theta_{\max} = \frac{n_1}{n_0} \times \cos \Phi_c$  ----- (3)

But from critical angle  $\sin \Phi_c = \frac{n_2}{n_1}$  therefore  $\cos \Phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$

Substitute  $\cos \Phi_c$  in equation (3) then  $\sin \theta_{\max} = \frac{n_1}{n_0} \times \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$

For air medium  $\sin \theta_{\max} = \sqrt{n_1^2 - n_2^2}$  or  $\theta_{\max} = \sin^{-1} \sqrt{n_1^2 - n_2^2}$

### Numerical Aperture

Numerical aperture determines the light gathering ability of the fiber. It is measure of the amount of light that can be accepted by a fiber. Numerical aperture depends only refractive index of the fiber. A larger NA implies that a fiber will accept large amount of light form the source.

The Numerical Aperture (NA) is defined as the sin of the acceptance angle. Thus

$$\begin{aligned} NA &= \sin \theta_{\max} \\ NA &= \sin(\sin^{-1} \sqrt{n_1^2 - n_2^2}) = \sqrt{n_1^2 - n_2^2} \end{aligned}$$

Consider

$$n_1^2 - n_2^2 = (n_1 + n_2)(n_1 - n_2) = \left( \frac{n_1 + n_2}{2} \right) \left( \frac{n_1 - n_2}{n_1} \right) \times 2n_1$$

Approximating  $\left( \frac{n_1 + n_2}{2} \right) = n_1$  we can express the above relation as

$$n_1^2 - n_2^2 = 2n_1^2 \Delta \quad \text{Where } \Delta \text{ is called } \textit{fractional refractive index} \quad \Delta = \left( \frac{n_1 - n_2}{n_1} \right)$$

$$\text{Therefore NA} = n_1 \sqrt{2\Delta}$$

#### 4. Write short notes on classification on optical fibers based on refractive index Profiles.

##### Refractive index profile

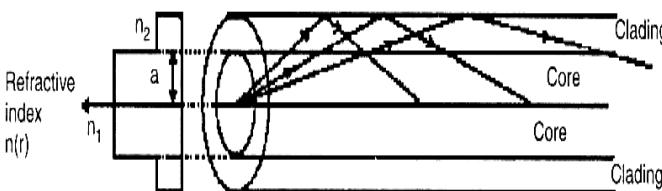
**Step-index fiber**- The optical fiber with a core of constant refractive index  $n_1$  and a cladding of slightly lower refractive index  $n_2$  is known as Step Index Fiber. This is because the refractive index profile for this type of fiber makes a step change at the core cladding interface.

**Graded Index fiber**:- Graded index fiber do not have a constant refractive index in the core but a decreasing core indeed with radial distance from a maximum value of  $n_1$  at the axis to a constant value  $n_2$  beyond the core radius.

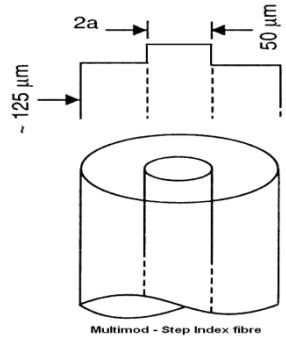
Above two fibers further classified into three parts

- (a) step-index multimode (b) Graded-index multimode (c) step-index single mode

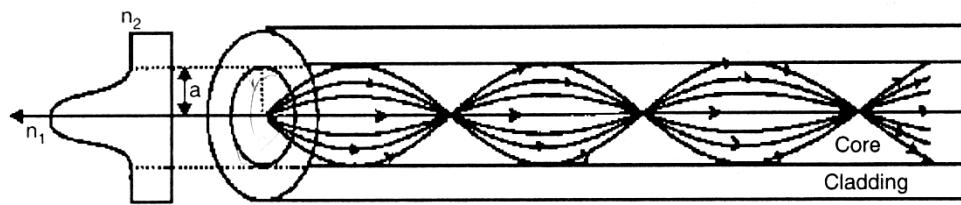
**Step-index multimode**:- The fiber has a uniformed refractive index in the core and has a step jump of refractive index at the core cladding interface. Light is traveling in the fiber in straight lines, reflecting off the core/cladding interface. Since each mode or angle of light travels a different path, the light is dispersed while traveling through the fiber and it limits the bandwidth the step index fiber.



Core diameter is  $50\mu\text{m}$  and cladding diameter is  $125\mu\text{m}$

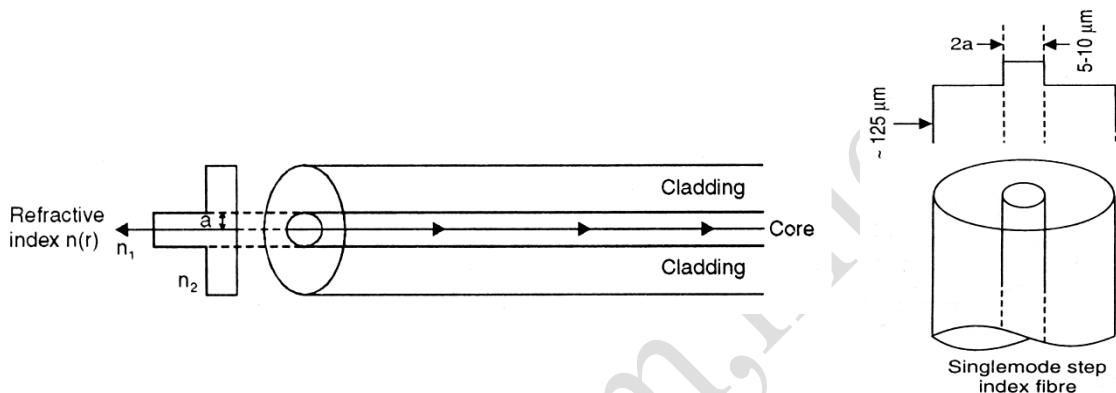


**Graded-index multimode fiber**:- In this fiber core refractive index varies continuously. Since the light travels faster in lower index of refraction glass, the light travel faster as it approaches the outside the core. Graded index fiber is a multimode fiber as it supports more than one mode and rays of light follow sinusoidal paths. This means that low order modes stay close to the centre of the fiber while high order modes spend more time near the edge of the core. Low order modes travel in the high index part of the core and so travel slowly, whereas high order modes spend predominantly more time in the low index part of the core and so travel faster. Thus, a properly constructed index profile will compensate for the different path lengths of each mode and so they all reach the end of the fiber almost at the same time and this will eliminates multimode dispersion and so increase the bandwidth capacity of the fiber by as much as 100 times that of step index multimode fiber. Schematic representation of this fiber with typical core and cladding diameters, refractive index profile and mode of operation is shown in below fig. The propagation characteristics of a given fiber depend on the various structural parameters of the fiber. These parameters can be combined into a normalized parameter (V-number).



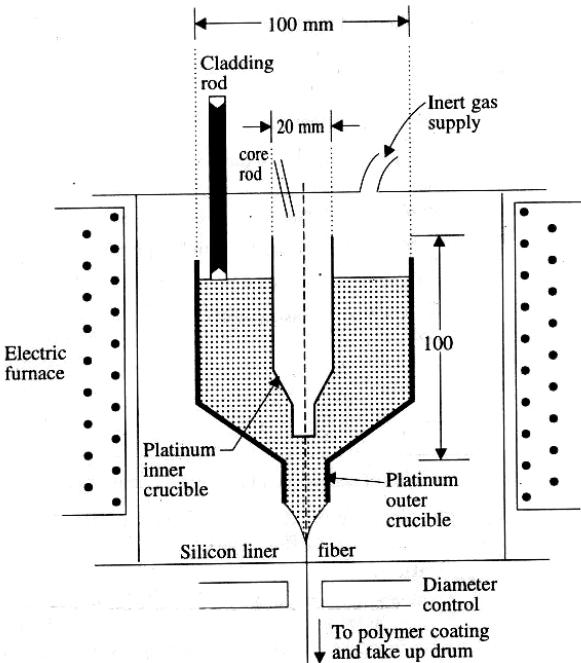
**Step-index single mode:** - In step index fiber core diameter of  $5\text{-}10 \mu\text{m}$ . Light travels in SMF along a single path that is along the axis. A single mode step index fiber is designed to have a V number between 0 and 2.4. The

NA and fractional refractive index very small for single mode step index fiber. The single mode fibre is having very small diameter, causing all light to travel in only one mode. In single mode modal dispersion disappears and bandwidth of the fiber increases. High speed networks are using it either to support gigabit data rates or long distance links.



## 5. Explain Double-Crucible method

The double crucible consists of two concentric platinum crucibles having thin orifices at the bottom. The raw material for core-glass is placed in the inner crucible and for cladding is fed to the outer crucible. The double crucible arrangement is mounted vertically (Fig.) in a furnace. The furnace is maintained at a suitable temperature to take the raw material into molten state. The fibers are drawn through the thin orifices at the bottom of the crucibles. As both the materials are drawn simultaneously, a filament of core glass surrounded by a tube of cladding glass is obtained in this process. The thickness of the fibre is monitored and the fibre is then coated with a polymer. Subsequently, it is passed through a plastic extrusion die to form a plastic sheath over the fiber.



## 5. Write a short note on losses in optical fiber

Optical losses Extrinsic Fiber Losses These losses are specific to geometry and handling of the fibers and are not functions of the fiber material itself.

There are three basic types: • bending losses • launching losses • connector losses

### Bending Losses

Bending losses are the result of distortion of the fiber from the ideal straight-line configuration. While the light is traveling inside the fiber, part of the wave front on the outside of the bend must travel faster than the part of the smaller inner radius of the bend. Since this is not possible, a portion of the wave must be radiated away. Losses are greater for bends with smaller radii, particularly for kinks or micro-bends in a fiber. An important cause of attenuation is due to micro-bending of the fiber. Micro-bending is due to irregularly distributed undulations in the fiber with radii of curvature of a few millimeters and deviations from the mean line of a few micrometers

### Launching Losses

The term launching loss refers to an optical fiber not being able to propagate all the incoming light rays from an optical source. These occur during the process of coupling light into the fiber (e.g., losses at the interface stages). Rays launched outside the angle of acceptance excite only dissipative radiation modes in the fiber. In general, elaborate techniques have been developed to realize efficient coupling between the light source and the fiber, mainly achieved by means of condensers and focusing lenses. The focused input beam of light needs to be carefully matched by fiber parameters for good coupling

### connector losses

Connector losses are associated with the coupling of the output of one fiber with the input of another fiber, or couplings with detectors or other components. The significant losses may arise in fiber connectors and splices of the cores of the joined fibers having unequal diameters or misaligned centers, or if their axes are tilted. Mismatching of fiber diameters causes losses that can be approximated by  $-10 \log(d/D)$ . There are other connection losses such as offsets or tilts or air gaps between fibers, and poor surface finishes.

## Numerical Problems

- The numerical aperture of an optical fiber is 0.5 and refractive index of core is 1.54(a) find R.I. of cladding (b) calculate the change in core cladding refractive index per unit R.I. of the core. [Dec 2008, 3 marks]**

**Solution:**  $NA = \sqrt{n_1^2 - n_2^2}$

$$NA^2 = n_1^2 - n_2^2 \longrightarrow$$

$$2.1216$$

$$\therefore n_2 = 1.45$$

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.54 - 1.45}{1.54} = 0.0584$$

$$n_2^2 = (1.54)^2 - (0.5)^2 =$$

- Calculate the numerical aperture and acceptance angle of optical fibre of refractive indices for core and cladding as 1.62 and 1.52 respectively.**

**Solution:**  $NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.62^2 - 1.52^2} = 0.56$

$$\text{Acceptance Angle } \alpha_{i-\max} = \sin^{-1}(NA) = \sin^{-1}(0.56) = 34.06^\circ$$

- The refractive indices for core and cladding for a step index fibre are 1.52 and 1.41 respectively. Calculate (i)critical angle (ii)numerical aperture and (iii)the maximum incidence angle.**

**Solution:** critical angle  $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.41}{1.52}\right) = 68.06^\circ$

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.52^2 - 1.41^2} = 0.5677 = 0.568$$
$$\text{Acceptance Angle } \alpha_{i\text{-max}} = \sin^{-1}(NA) = \sin^{-1}(0.568) = 34.59^\circ$$

**IMPORTANT THEORY QUESTION**

1. Describe the classification of optical fibers. [Jan 2012, 4 marks]
2. Describe the fiber drawing process (double crucible method) [Dec-2007 & 2008, 3 marks]
3. What are the advantages of fiber communication? [May/June-2008, 2 marks]
4. Obtain the expression for numerical aperture in fibre optics [Apr/May-2007, 3 marks]

## Unit-IV LASERS

### 1. Write Characteristics of the laser

When compared with any conventional light (sun light or tube light), laser possesses few outstanding characteristics, they are

**1. Directionality:** During the propagation of a laser, its angular spreading will be less and occupies less area where it incidents. Hence, it possesses high degree of directionality.

**2. Monochromaticity:** The property of exhibiting a single wavelength by a light is monochromaticity. The light from a laser source is highly monochromatic. Even it is sent through a prism a single line will be appeared in the optical spectrum.

**3. Intensity:** Due to its high directionality many beams of light incident in a small area, therefore the intensity is high. Hence its brightness is more. So it is used for the welding.

**4. Coherence:** Laser beam possesses high degree of both spatial and temporal coherence.

The property of existing either zero or constant phase difference between two or more waves is known as coherence. Coherence is of two types 1. Spatial coherence 2. Temporal coherence.

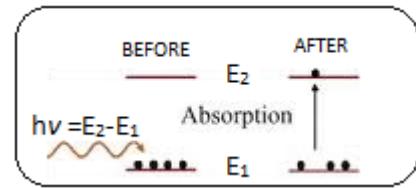
Spatial coherence: Consider the wavefront from a laser propagates along x-axis, the phase difference between two points  $x_1$  and  $x_2$  along the direction of propagation remains constant at all times. This is called spatial coherence.

Temporal coherence: If the phase difference between successive wavefronts as they pass through a given point at regular interval of time remains constant then the wave is said to have temporal coherence.

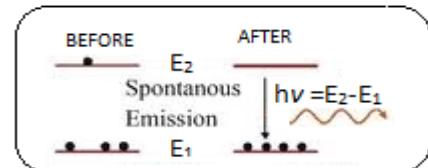
### 2. Write a short note on Basic definitions in Laser

To understand the working part of a laser system, one should have the knowledge of few fundamental concepts. They are

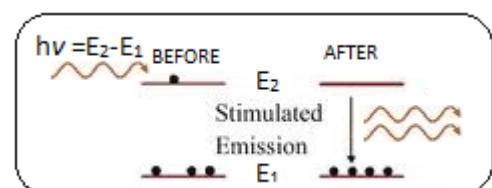
**1. Stimulated Absorption:** consider a system in which two active energy levels are present whose energies are  $E_1$  and  $E_2$  where  $E_1$  is ground state and  $E_2$  is excited state as shown in figure. Usually atoms are in the ground stage as long as external forces are not applied. When a photon of energy  $h\nu = E_2 - E_1$  is incident on the atom lying in ground stage then it excites to higher state  $E_2$ . This phenomenon is known as stimulated Absorption.



**2. Spontaneous Emission:** Let us assume that the system is in the excited state  $E_2$ . After the life time the atom de-excites to its ground stage spontaneously emitting a photon of energy  $h\nu = E_2 - E_1$  as shown in figure. This phenomenon is known as spontaneous emission. The photons emitted have various wavelength and they are out of phase. Thus the photons are incoherent.



**3. Stimulated Emission:** Let us assume that the atom is in the excited state  $E_2$ . If a photon of energy  $h\nu = E_2 - E_1$  is incident on it before the life time, it stimulates the atom from  $E_2$  to  $E_1$  then a photon of energy  $h\nu$  releases along with the incident stimulating photon as shown in figure. These two photons will have same energy and phase. This phenomenon is known as stimulated emission. The emitted photons have the single wavelength and they are in phase. Thus the photons are coherent.



**4. Life Time:** the duration of time spent by an atom in the excited state is known as life time of that energy level.

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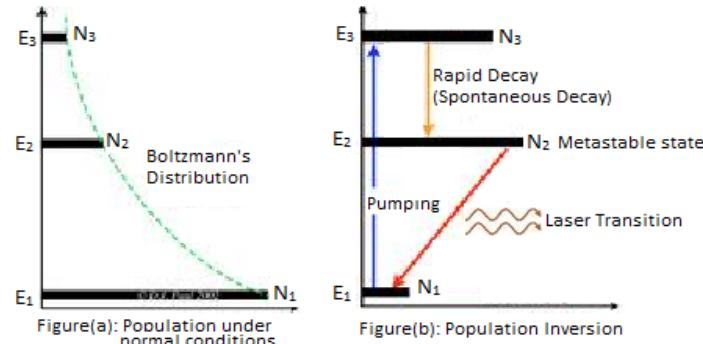
**5. Metastable state:** The excited state, which has long life time, is known as metastable state. According to Heisenberg's uncertainty principle metastable state is an excited state of an atomic system whose energy level width is very small so that the life time of the electrons is very large. Thus it is easy to achieve population inversion in metastable state to start lasing action.

**6. Population:** The number of atoms per unit volume in an energy level is known as population of that energy level and is given by  $N = N_0 \exp \left[ \frac{-E}{K_B T} \right]$

Where  $N_0$  is the population in the ground energy level;  $K_B$  is Boltzmann constant;  $T$  is temperature.

**7. Population Inversion:** Consider a three level system in which three active energy levels  $E_1$ ,  $E_2$  &  $E_3$  are present and populations in those energy levels are  $N_1$ ,  $N_2$  &  $N_3$  respectively. In normal conditions  $E_1 < E_2 < E_3$  and  $N_1 > N_2 > N_3$  as shown in figure (a).

$E_1$  is the ground state; its lifetime is unlimited and it is most stable state.  $E_3$  is highest energy state; its lifetime is very less. Whereas  $E_2$  is an excited state and has more lifetime. Hence  $E_2$  is metastable state. When suitable form of energy is supplied to the system in a suitable way, then the atoms excite from ground state  $E_1$  to excited states ( $E_2$  &  $E_3$ ). Due to instability, excited atoms will come back to ground state after the lifetime of the respective energy states  $E_2$  and  $E_3$ . Because  $E_3$  is the most unstable state, atoms will fall into  $E_2$  immediately. At a stage the population in  $E_2$  will become more than the population in ground state. This situation is called population inversion and is shown in fig (b).



Figure(a): Population under normal conditions

Figure(b): Population Inversion

**8. Pumping:** The population inversion cannot be achieved thermally. To achieve population inversion suitable form of energy must be supplied. The process of supplying suitable form of energy to a system to achieve population inversion is called pumping.

In several ways pumping can be done. Most commonly used pumping methods are

1. Optical Pumping (Used in Ruby laser)
2. Electric discharge method (Used in He-Ne laser)
3. Direct conversion (used in semiconductor laser)
4. Chemical reaction (used in CO<sub>2</sub> laser)

**9. Active Medium:** A medium in which the population inversion is to be achieved is called as active medium or the gain medium for a laser. Laser systems are named based on the makeup of the gain medium, which may be a gas, liquid or solid.

**10. Resonant Cavity:** In a laser, the active system or the gain medium is enclosed in an optical cavity (or Resonant Cavity) usually made up of two parallel surfaces, one of which is perfectly reflecting and the other surface is partially reflecting. In this resonant cavity, the intensity of photons is raised tremendously through stimulated emission process.

### 3. Derive Einstein's coefficients

In order to understand Einstein's coefficients, the processes of absorption, spontaneous emission and stimulated emission must be discussed in terms of probability in detailed version.

**Absorption:** The rate of absorption is proportional to energy density  $\rho(v)$  of incident radiation and number of atoms in ground state( $N_1$ ).

Thus  $R_{12} \propto \rho(v)N_1$

$$R_{12} = B_{12} \rho(v)N_1$$

Where  $B_{12}$  is proportionality constant and is known as Einstein's coefficient of Absorption.

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**Spontaneous Emission:** The rate of spontaneous emission is proportional to number of atoms in energy level 2. This process is independent of energy density.

Thus  $(R_{21})_{spontaneous} \propto N_2$

$$(R_{21})_{spontaneous} = A_{21} N_2$$

Where  $A_{21}$  is proportionality constant and is known as Einstein's coefficient of spontaneous emission.

**Stimulated Emission:** The rate of stimulated emission is proportional to stimulated energy density  $U(v)$  and number of atoms in energy level 2.

Thus  $(R_{21})_{stimulated} \propto \rho(v)N_2$

$$(R_{21})_{stimulated} = B_{21} \rho(v)N_2$$

Where  $B_{21}$  is proportionality constant and is known as Einstein's coefficient of stimulated emission.

Under equilibrium condition, The rate of Absorption = The rate of emission (spon. + stim.)

$$B_{12}\rho(v)N_1 = A_{21}N_2 + B_{21}\rho(v)N_2$$

$$B_{12}\rho(v)N_1 - B_{21}\rho(v)N_2 = A_{21}N_2$$

$$\rho(v)[B_{12}N_1 - B_{21}N_2] = A_{21}N_2$$

$$\rho(v) = \frac{A_{21}N_2}{[B_{12}N_1 - B_{21}N_2]} = \frac{A_{21}N_2}{N_2 \left[ B_{12} \frac{N_1}{N_2} - B_{21} \right]} = \frac{A_{21}}{\left[ B_{12} \frac{N_1}{N_2} - B_{21} \right]}$$

According to Boltzmann's distribution law,  $\frac{N_1}{N_2} = e^{\frac{hv}{k_B T}}$

$$\text{Therefore } \rho(v) = \frac{A_{21}}{\left[ B_{12}e^{\frac{hv}{k_B T}} - B_{21} \right]} = \frac{A_{21}}{B_{21} \left[ \frac{B_{12}}{B_{21}} e^{\frac{hv}{k_B T}} - 1 \right]} = \frac{\frac{A_{21}}{B_{21}}}{\left[ \frac{B_{12}}{B_{21}} e^{\frac{hv}{k_B T}} - 1 \right]}$$

$$\text{But according to Planck's radiation law, } \rho(v) = \frac{\frac{8\pi h\nu^3}{c^3}}{\left[ \frac{hv}{e^{\frac{hv}{k_B T}} - 1} \right]}$$

$$\text{Comparing equation (1) and (2), we get } \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \text{ and } \frac{B_{12}}{B_{21}} = 1$$

$$\text{Therefore } \frac{A_{21}}{B_{21}} \propto \nu^3 \quad \text{and} \quad B_{12} = B_{21}$$

Above two equations are referred to as the **Einstein's relations**.

The ratio of spontaneous emission rate to stimulated emission rate is given by 'R'

$$R = \frac{\text{spontaneous emission rate}}{\text{stimulated emission rate}} = \frac{N_2 A_{21}}{N_2 \rho(v) B_{21}} = \frac{h\nu}{e^{\frac{hv}{k_B T}} - 1}$$

From above equation, we understand that to make R smaller, the energy density  $\rho(v)$  has to be made larger.

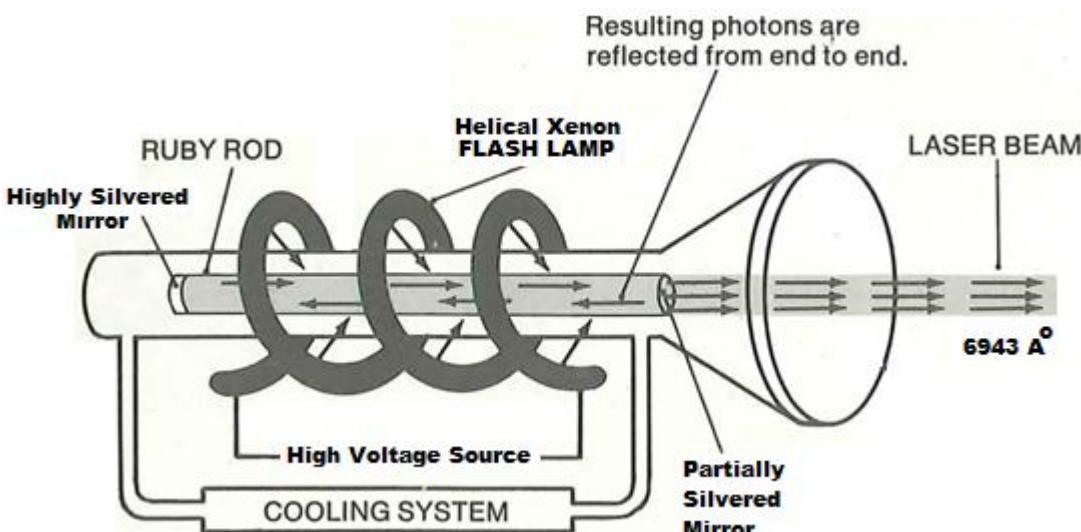
Now, let us consider the ratio of stimulated emission rate to stimulated absorption rate.

$$R' = \frac{\text{stimulated emission rate}}{\text{stimulated absorption rate}} = \frac{N_2 \rho(v) B_{21}}{N_1 \rho(v) B_{12}} = \frac{N_2}{N_1}$$

At thermal equilibrium,  $\frac{N_2}{N_1} \ll 1$

Thus at thermal equilibrium stimulated absorption predominates over stimulated emission. Instead if we create a situation that  $N_2 > N_1$ , stimulated emission will predominate over stimulated absorption. If stimulated emission predominates the photon density increases and LASER occurs. Therefore in order to get LASER action,  $N_2 > N_1$  and this condition is called **population inversion**.

#### 4. Explain construction and working of Ruby Laser



The Ruby laser was the very first laser, developed by Maiman in 1960, which emitted deep red light at wavelength of  $6943 \text{ \AA}$  in the form of pulses.

Source of energy	:	Helical Xenon flash lamp
Active Medium	:	Ruby rod
Optical Cavity	:	Arrangement of silver polished surfaces on either side of the Ruby rod.

**Construction:** The Ruby laser consists of a single crystal of ruby in the form of a cylindrical rod of length between 5 to 20 cm and diameter 0.5 to 2 cm. Ruby rod is prepared from  $\text{Al}_2\text{O}_3$  doped with 0.05% of  $\text{Cr}_2\text{O}_3$ . Its two end faces are ground and made absolutely parallel to each other. One of the faces is highly silvered and the other end is partially silvered.

The ruby rod is placed along the axis of a high intensity xenon flash lamp of helical shape which itself surrounded by a reflector to maximize the intensity of the light incident on the ruby rod. The lamp gives out flashes of brilliant light when its two ends are connected to a pulsed high voltage source. During each pulse, the white light flash lasts for several milliseconds. From the white light, energy in the form of radiation is absorbed by the ruby for exciting the Chromium ions to higher energy levels. During the course of flash production enormous amount of heat is generated. The ruby rod is protected from heat by enclosing it in a hallow tube through which cold water circulation is maintained.

**Working:** When the light from the flash lamp is incident on the ruby rod, the Chromium ions absorb the radiation and get excited to the (higher energy level) H and these ions return back to their ground state G in two steps. First step is from higher state H to metastable state M, which is a shorter jump and energy emitted in this transition process to the crystal lattice as heat. This transition is called *radiationless transition*.

The chromium atoms returned to M level can remain in this state for several milliseconds. The accumulation of excited atoms at M level increases the population at M level and then transitions occur from M to G level emitting out the photons randomly. Due to continuous working of flash lamp, the chromium atoms are raised continuously to higher energy level H and then to M level.

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At a particular stage population of excited chromium atoms will be reversed and more chromium atoms at M than at G. At this position photon begin to interact with chromium atoms at level M to a significant extent. This results in stimulated emission of other identical photons and a cascade begins.

The photons travelling parallel to the axis of the ruby rod are used for stimulation while the photons travelling in any direction other than this will pass out from the ruby rod. In the meantime, photons moving back and forth inside the ruby and continue to build up until the intensity of radiation is great enough to come out in bursts through the partial silver polished surface and it serves as an output laser.

The output beam of wavelength is  $6943\text{ A}^\circ$  in the red region of visible spectrum.

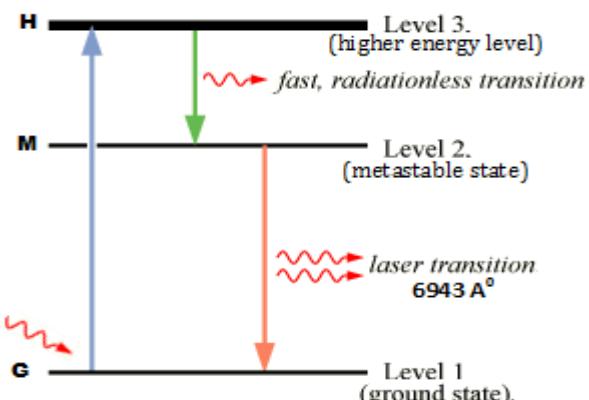


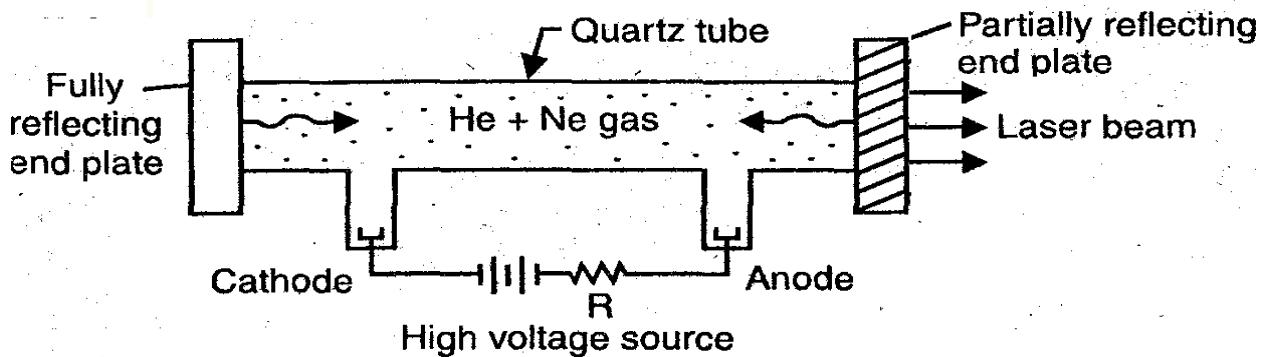
Fig: The energy level diagram of  $\text{Cr}^{3+}$  in the ruby crystal

## 5. Explain working of Helium-Neon (He-Ne) Laser

The Helium-Neon laser was first built in 1961 by Ali Javan. It is a gaseous laser system and is used to produce a continuous laser. This laser is highly directional, monochromatic, coherent and stable. But the output power is moderate when it is compared with the solid state laser. It is very useful in making holograms and interferometric experiments.

Source of energy	:	R.F. Oscillator
Active Medium	:	Helium and Neon gas mixture
Optical Cavity	:	Arrangement of Reflectors.

**Construction:** The Helium-Neon laser system consists of a gas discharge tube which is the active medium. The tube is made up of quartz and is filled with the mixture of Helium under a pressure of 1mm of mercury and Neon under a pressure of 0.1mm of mercury. The ratio of Helium-Neon mixture is 10:1. The power output from these lasers depends upon the length of the discharge tube and pressure of gas mixture. Further electrodes at the ends of the discharge tube are connected to a r.f. oscillator to produce electrical discharge in the He-Ne mixture so as to pump the He atoms to higher energy states



**Working:** By the electrical discharge in the gas tube, the ground level Helium atoms are excited to higher levels  $\text{He}_1$  and  $\text{He}_2$ . This process of excitation is called electron excitation. By resonance collision transfer method, the Helium atoms at  $\text{He}_2$  give up their excitation energy to the ground

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state Neon atoms. Thus the neon atoms are excited to their higher energy level Ne<sub>5</sub>. Meanwhile these Helium atoms are de-excited and returned to their ground state.

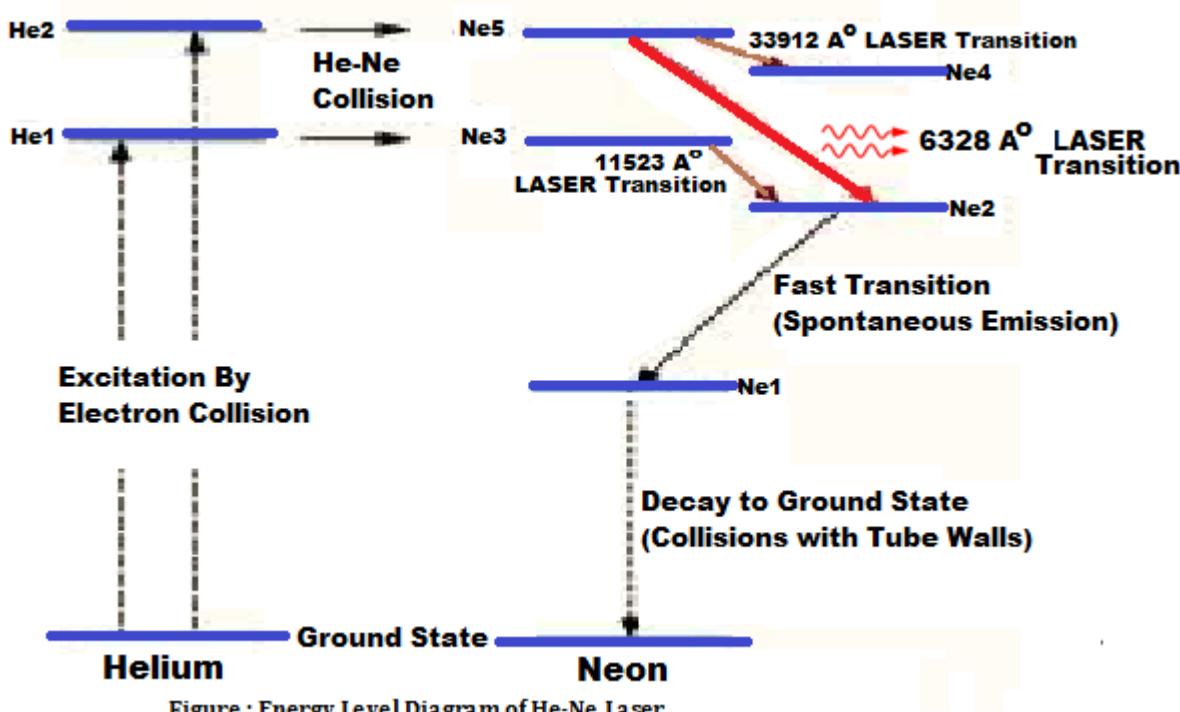
Similarly the He atoms at He<sub>1</sub> give up their excitation energy to the ground state neon atoms and the neon atoms are excited to another higher energy level Ne<sub>3</sub> as shown in figure. The He atoms are de-excited and returned to their ground state.

Since Ne<sub>3</sub> and Ne<sub>5</sub> are two meta-stable energy levels of Neon atoms, population inversion takes place at these levels. Any one of the spontaneously emitted photons will trigger the laser action. Thus three main types of laser transitions take place. They are

1. Transition from the Ne<sub>5</sub> level to the Ne<sub>4</sub> level which gives raise to radiation of wavelength 33912 Å<sup>0</sup> which is in infra-red region.
2. Transition from the Ne<sub>5</sub> level to the Ne<sub>2</sub> level which gives raise to radiation of wavelength 6328 Å<sup>0</sup> which is in visible region (Red light).
3. Transition from the Ne<sub>3</sub> level to the Ne<sub>2</sub> level which gives raise to radiation of wavelength 11523 Å<sup>0</sup> which is in infra-red region.

Some optical elements placed inside the laser system to absorb the infrared laser wavelengths 33912 Å<sup>0</sup> and 11523 Å<sup>0</sup>.

Hence the output of He-Ne laser contains only a single wavelength of 6328 Å<sup>0</sup>.



## 6. Explain construction of working of Semiconductor Laser

**Construction:** One of the examples of semiconductor lasers is gallium arsenide (GaAs). It is heavily doped semiconductor. Its n-region is formed by heavily doping with tellurium in a concentration of  $3 \times 10^{18}$  to  $5 \times 10^{18}$  atoms/cm<sup>3</sup> while its p-region is formed by doping with zinc in concentration around  $10^{19}$  atoms/cm<sup>3</sup>.

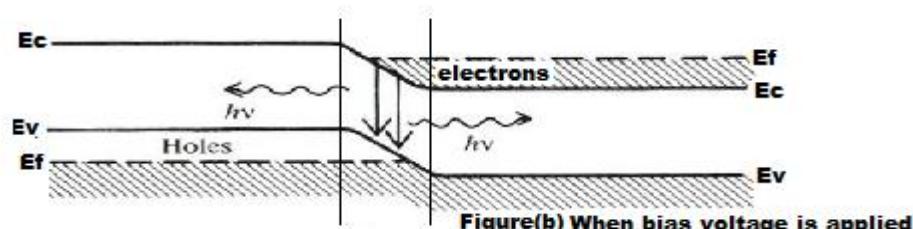
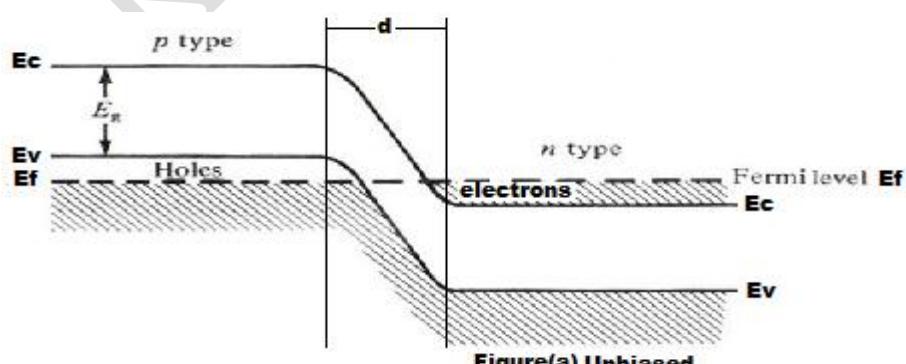
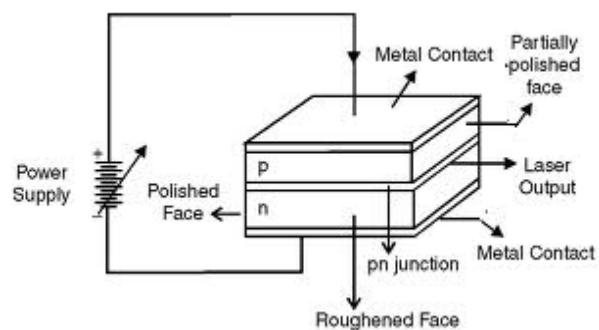
**Active medium:** The active medium in GaAs semiconductor laser is GaAS. But it is also commonly said that depletion region is the active medium in semiconductor laser. The thickness of the depletion layer is usually very small ( $0.1 \mu\text{m}$ ).

**Pumping Source:** Forward biasing is used as pumping source. The p-n junction is made forward biased that is p side is connected to positive terminal of the battery and n side to negative. Under the influence of forward biased electric field, conduction electrons will be injected from n side into junction area, while holes will enter the junction from the p side. Thus, there will be recombination of holes and electrons in depletion region and thus depletion region becomes thinner.

**Optical resonator system:** The two faces of semiconductor which are perpendicular to junction plane make a resonant cavity. The top and bottom faces of diode, which are parallel to junction plane are metallised so as to make external connections. The front and back faces are roughned to suppress the oscillations in unwanted direction.

### Working

When the junction is forward biased, at low voltage the electron and hole recombine and cause spontaneous emission. But when the forward voltage reaches a threshold value the carrier concentration rises to very high value. As a result the region "d" contains large number of electrons in the conduction band and at the same time large number of holes in the valence band. Thus the upper energy level has large number of electrons and the lower energy level has large number of vacancy, thus population inversion is achieved. The recombination of electron and hole leads to spontaneous emission and it stimulate the others to emit radiation. GaAs produces laser light of wavelength ranging from 7000 to 30,000 Å.



**Applications of Lasers**

<b>Medical applications</b>	<b>Welding and Cutting</b>	<b>Surveying</b>
<b>Garment industry</b>	<b>Laser nuclear fusion</b>	<b>Communication</b>
<b>Laser printing</b>	<b>CDs and optical discs</b>	<b>Spectroscopy</b>
<b>Heat treatment</b>	<b>Barcode scanners</b>	<b>Laser cooling</b>

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**Questions from previous examinations**

1. Distinguish between spontaneous and stimulated emission
2. What are Einstein coefficients and explain the concept of absorption, spontaneous emission and stimulated emission
3. By considering Einstein coefficients, show that population inversion is required ( $N_2 > N_1$ ) for laser action.
4. Draw a neat diagram to represent the components of a Ruby laser and explain its construction and working.
5. Explain the construction and working of He-Ne laser
6. Explain the construction and working of semiconductor laser
7. Mention few applications of lasers.