CS590 F23 Solution

a) $T(n) = 16\left(\frac{n}{4}\right) + \sqrt{n}$

$n^{\log_4 16}$ vs $n^{1/2}$ $\Rightarrow n^2$ vs $n^{1/2}$

$2 - \varepsilon \neq 1/2 \Rightarrow \varepsilon > 0$

MT case 1.

$T(n) = \Theta(n^2)$

b) $T(n) = 27T\left(\frac{n}{3}\right) + \Theta(n^4)$

$n^3$ vs $\Theta^4$ $\quad 3 + \varepsilon = 4 \Rightarrow \varepsilon > 0$

MT 3 $\quad 27\left(\frac{n}{3}\right)^4 = cn^4$

$c < 1$

$\therefore T(n) = \Theta(n^4)$

c) $T(n) = nT\left(\frac{n}{7}\right) + 4n^a$

a must be constant,

$f(n)$ must be finite.

$\therefore$ MT cannot be applied.

d) $T(n) = 8T\left(\frac{n}{2}\right) + \Theta(n^3)$

$n^3 = n^3 \quad k = 0$

$k \neq 2$,

$T(n) = \Theta(n^3 \lg n)$

#2

a) $T(n) = T(\frac{n}{3}) + T(\frac{n}{9}) + dn \qquad T(n) = \Omega(cn\lg n)$

$\geq c\frac{n}{3}\lg\frac{n}{3} + c\frac{n}{9}\lg\left(\frac{n}{9}\right) + dn$

$\geq c\frac{n}{3}\lg n - c\frac{n}{3}\lg 3 + c\frac{n}{9}\lg n - c\frac{n}{9}\lg 9 + dn$

$\geq c\left(\frac{n}{3}+\frac{n}{9}\right)\lg n - n\left(\frac{c}{3}\lg 3 + \frac{2c}{9}\lg 3 - d\right)$

$\geq \frac{4c}{9} n\lg n - n\left(\frac{5c}{9}\lg 3 - d\right)$

$T(n) = \Omega(n\lg n) \qquad c \geq \frac{9d}{5\lg 3}$

b) $T(n) = 4T(\frac{n}{5}) + \Theta(n^2) + \Theta(n\lg n)$, $T(n) = O(n^2)$

$= 4T(\frac{n}{3}) + \Theta(n^2)$

$\leq 4c\left(\frac{n}{3}\right)^2 + dn^2$

$\leq \frac{4}{9}cn^2 + dn^2 = \left(\frac{4}{9}c + d\right)n^2$

$c \leq \frac{4}{9}c + d$

$\frac{5}{9}c \leq d \qquad c \leq \frac{9}{5}d$

$T(n) = O(n^2)$

#2

c) $T(n) = 2T(n-3) + 3n \quad T(n) = O(n)$

$T(n) \le cn$

$cn \le 2c(n-3) + 3n$

$\le cn + (c+3)n - 6c$

cannot define $c$.

let $T(n) \le O(2^{n-2} - n)$

$T(n) = 2T(n-3) + 3n$

$\le 2c(2^{n-3}) - 2d(n-3) + 3n$

$\le c2^{n-2} - dn + \underbrace{(3-d)n}_{\le 0} + 6d$

$d \ge 3$.

The guess function is valid.

#3

Use counting sort.

running time will be always linear.

Merge sort will face a memory issue and will run in $\log -$.