National University of Computer and Emerging Sciences, Lahore Campus

Course:	Analog and Digital Communications	Course Code:	EE3003
Program:	BS (Electrical Engineering)	Semester:	Fall 2024
Exam:	Assignment 3	Page(s):	3
Chapter(s):	4 (Amplitude Modulation)	Section:	ALL
Submission Date:		CLO:	03
Instructor(s):	Dr. S. M. Sajid, Mohsin Yousuf	Marks:	100

Instructions: *The design parts should include the diagram along with mathematical equations.*

Question # 1 [CLO 03] **[5+5]**

DSB-SC Modulation and Demodulation

(a) An information signal is of the form

$$m(t) = \frac{\sin(2\pi t)}{t}$$

This signal amplitude modulates a carrier of frequency 10 Hz. **Sketch** the DSB-SC waveform and its spectra.

(b) **Analyze** the switching demodulator that uses the ring modulator as a switch with diagrams and mathematical equations.

Question # 2 [CLO 03] **[3+4+4]**

Dual-Tone Modulated AM Signal

Consider an AM signal with multi-tone modulation where the modulation signal $m(t)$ is given by

$$m(t) = \cos\omega_m t + 2\sin 3\omega_m t$$

and the carrier $c(t)$ is given by

$$c(t) = 3\cos\omega_c t\,; \quad where\ \omega_c \gg \omega_m$$

Find an expression for **(i)** $\varphi_{AM}(t)$, **(ii)** the spectrum of the AM modulated signal, $\Phi_{AM}(\omega)$.

In addition, **sketch** the spectrum of the dual tone modulated signal.

Question # 3 [CLO 03] **[3+2+5]**

Modulation with Multiplexing

Two signals $m_1(t)$ and $m_2(t)$, both band-limited to 5000 rad/s, are to be transmitted simultaneously over a channel by the multiplexing scheme shown in Figure 1. The signal at point 'b' is the multiplexed signal, which now modulates a carrier of frequency 30,000 rad/s. The modulated signal at point 'c' is transmitted over a channel.

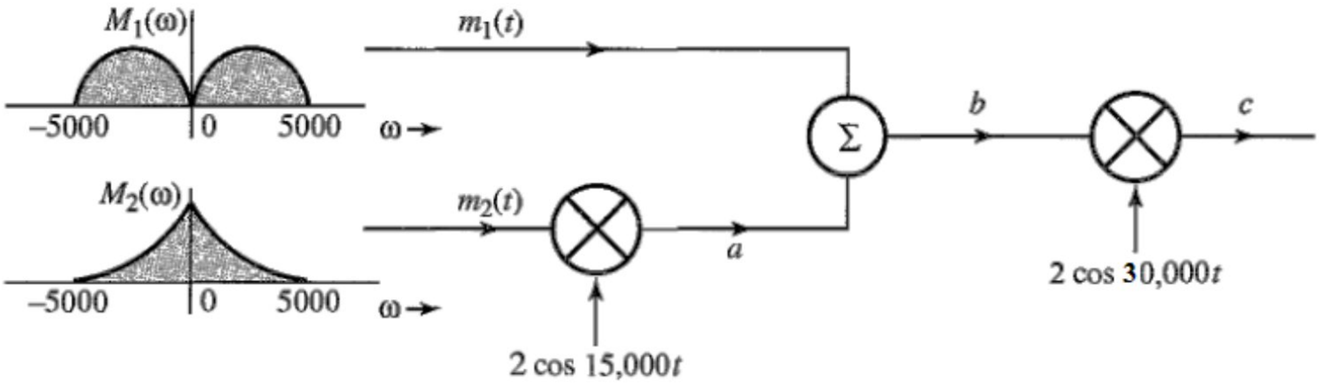


Figure 1. Multiplexing of two signals

(a) Sketch signal spectra at points a, b and c.

(b) Find the bandwidth of the channel for transmitting the multiplexed and modulated signal.

(c) Design a receiver to recover signals $m_1(t)$ and $m_2(t)$ from the modulated signal at point c.

Question # 4 [CLO 03] **[5+5]**

Design of DSB-SC modulator

Design a DSB-SC modulator to generate a modulated signal $k\, m(t) \cos \omega_c t$, where $m(t)$ is a signal band-limited to B Hz. Figure 2 shows a DSB-SC modulator available in the stock room. The carrier generator available generates $\cos^5 \omega_c t$ instead of $\cos \omega_c t$. Would it be possible to generate the desired signal using only this equipment. The filter can be utilized as deemed appropriate.

(a) Find an expression in time-domain at point b and the *filter* to obtain the required signal at point c.

(b) Determine the signal spectra at points b, and c indicting frequency bands occupied by these spectra.

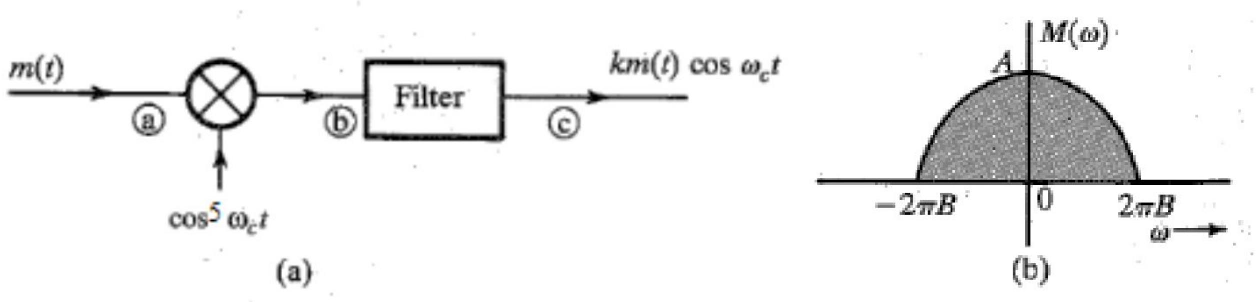
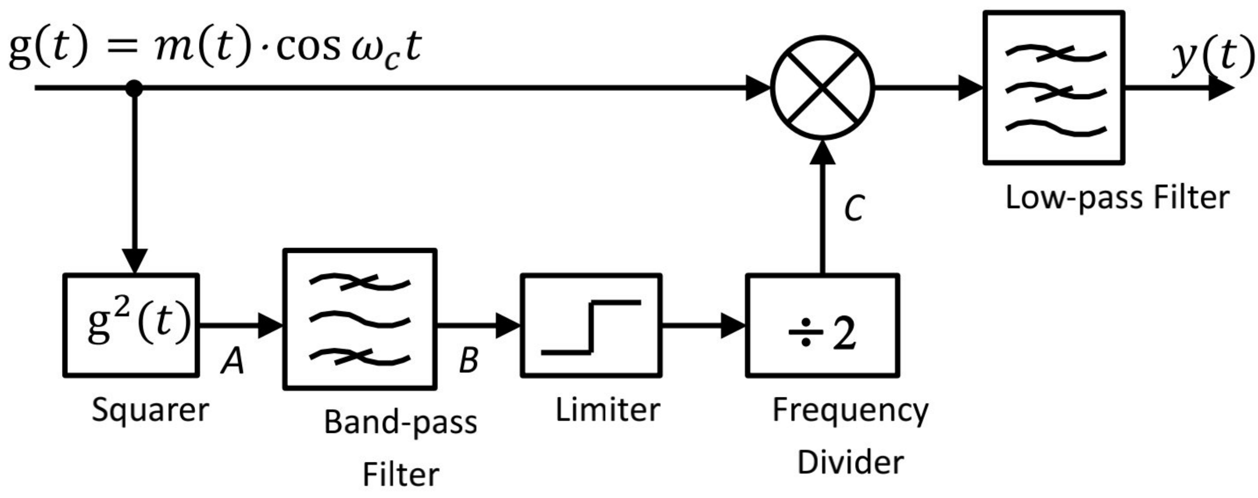


Figure 2.(a) DSB-SC modulator (b) Spectra of $m(t)$

Question # 5 [CLO 03] **[2+2+2+2+2]**

"Squaring Loop" Demodulation of DSB-SC

We have an incoming DSB-SC amplitude modulated signal of the form, $m(t) \cdot \cos \omega_c t$. A "squaring loop" is shown in the block diagram.



(a) If the input is $g(t) = m(t)\cdot\cos\omega_c t$, then **find** the output of the squarer block (that is, output at point "A" in the block diagram)?

(b) **Find** the output of the band-pass filter (that is, at point "B" in the block diagram)?

(c) **Demonstrate** the properties required of the band-pass filter.

(d) **Find** the output $y(t)$.

(e) **Demonstrate** the use of limiter in the block diagram.

Question # 6 [CLO 03] **[5+5]**

AM Power and Efficiency

Sketch the conventional AM signal $[A + m(t)]\cos\omega_c t$ for the periodic triangle signal $m(t)$ shown in Figure 3 corresponding to modulation index: $\mu = 0.8$, $\mu = 1$ and $\mu = 2$.

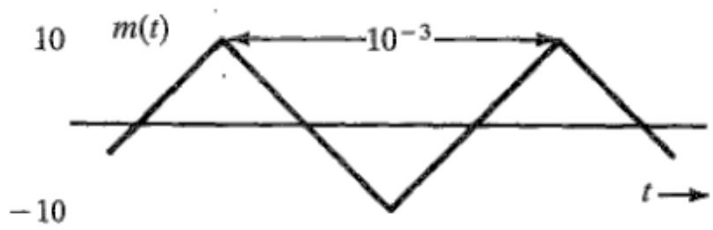


Figure 3. A periodic triangular message signal $m(t)$

For $\mu = 0.8$ case,

(a) **Find** the amplitude and power of the carrier.

(b) **Find** the sideband power and the power efficiency η.

Question # 7 [CLO 03] **[5+5]**

An amplitude modulated signal is given by

$$\varphi_{AM}(t) = 2.5\,[b + 3\cos\omega_m t]\cos\omega_c t\,, \quad for\ \omega_m \ll \omega_c$$

Find the power efficiency η of the AM signal for, **(i)** $b = 8$ and **(ii)** $b = 3$.

Question # 8 [CLO 03] **[2+3+3+2]**

Single Sideband (SSB)

Find $\varphi_{LSB}(t)$ and $\varphi_{USB}(t)$ for the modulating signal $m(t) = B\,\text{sinc}(2\pi Bt)$ with $B = 1000$ and carrier frequency $\omega_c = 10{,}000\pi$. Follow these steps.

(a) Sketch spectra of $m(t)$ and coressponding DSB-SC signal $2m(t)\cos\omega_c t$.

(b) Find and **sketch** the LSB spectrum.

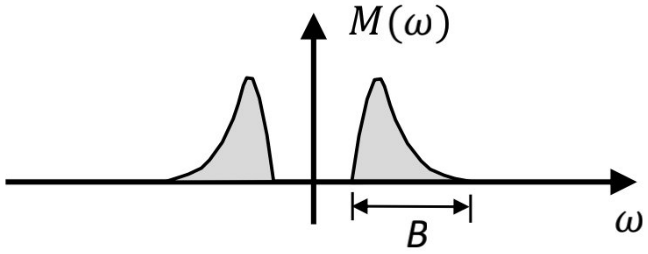
(c) Find and **sketch** the USB spectrum.

(d) Find the bandwidth of the SSB signal.

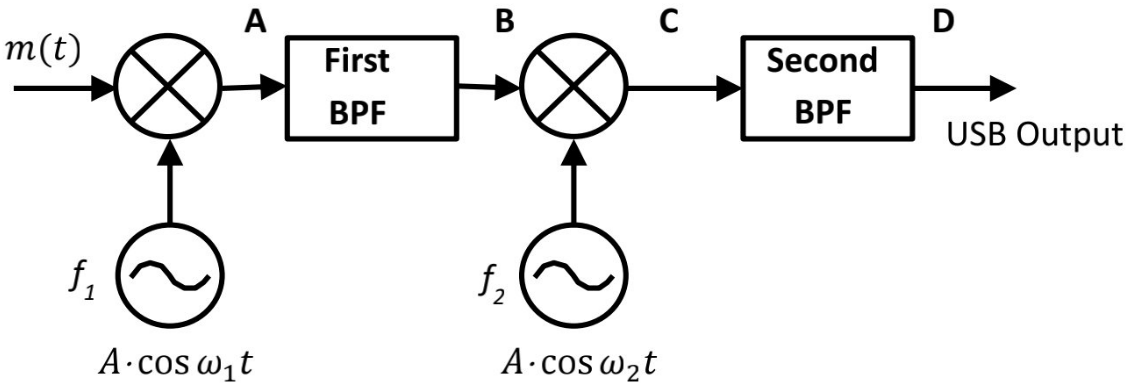
Question # 9 [CLO 03] **[2+3+3+2]**

Two-Step SSB Generation

Another method to generate SSB signals is the Two-Step SSB Generator. This approach can be used when the modulating function $m(t)$ has a two-sided frequency spectrum $M(\omega)$ with negligible energy around zero frequency (*i.e.*, around DC) as shown in the spectral plot $M(\omega)$ below. Note the spectrum $M(\omega)$ has a gap centered at $\omega = 0$ (meaning DC).



The two-step SSB modulator (or generator) produces an USB signal with carrier frequency $f_C = f_1 + f_2$, where the first oscillator is set at frequency f_1 and the second oscillator is at frequency f_2. It also requires the first band-pass filter (BPF) to have a lower cutoff frequency equal to f_1 and the lower cutoff frequency of the second BPF to be frequency $f_1 + f_2$.



Demonstrate the two-step SSB system's operation by showing the line spectra at points A, B, C and D along the component chain.

It is suggested to use tone modulation to show this; assume $m(t)$ *to be at frequency* ω_m.

Question # 10 [CLO 03] **[10]**

Vestigial Sideband (VSB)

A vestigial filter $H_i(\omega)$ has a transfer function as shown in Figure 4. The carrier frequency is $f_c = 11$ kHz and the baseband signal bandwidth is 5 kHz. **Find** the corresponding transfer function of the equalizer filter $H_o(\omega)$.

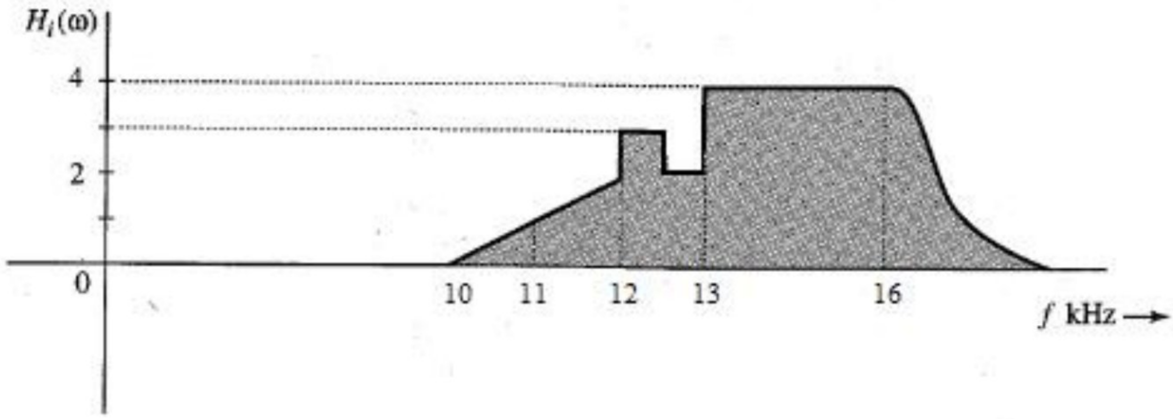


Figure 4. Transfer function of a Vestigial filter

~~ADC~~

ASSIGNMENT #03

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Q.No.1

(a)

$m(t) = \frac{\sin(2\pi t)}{t}$; $f_c = 10\,Hz$

As $\frac{\sin(t)}{t} = \text{sinc}(t)$

Hence, $m(t) = 2\pi \cdot \frac{\sin(2\pi t)}{2\pi t}$

$\Rightarrow m(t) = 2\pi \,\text{sinc}(2\pi t)$; modulating signal

As $\omega_c = 2\pi f_c$

$= 2\pi(10)$

$\omega_c = 20\pi$ rad/s

$\Rightarrow \cos 20\pi t$; carrier signal

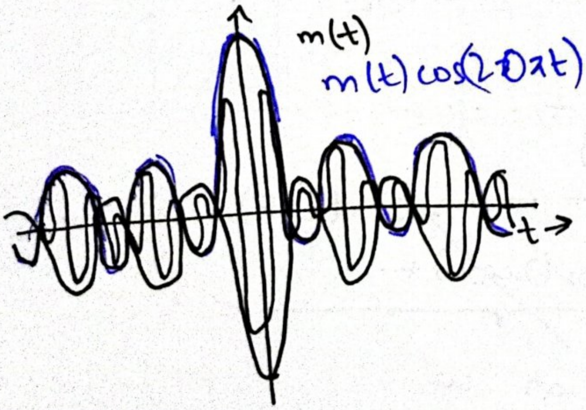
Modulated signal $\Rightarrow m(t)\cos\omega_c t$

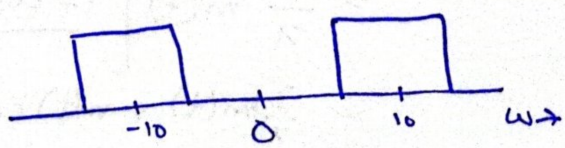
$\Rightarrow 2\pi\,\text{sinc}(2\pi t)\cos 20\pi t$

As $\text{sinc}(2\pi t) \Leftrightarrow \text{rect}\left(\frac{f}{2}\right)$

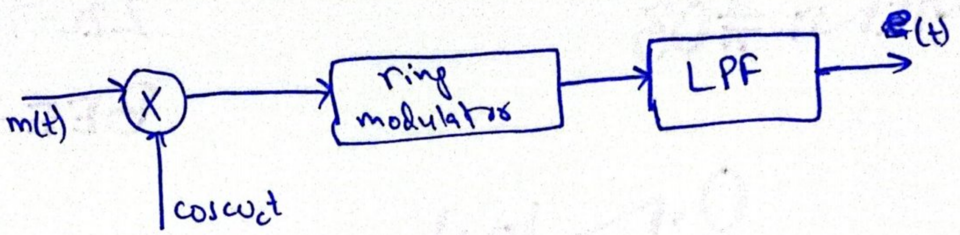
$2\pi\,\text{sinc}(2\pi t)\cos(2\pi t) \Leftrightarrow \frac{1}{2}\left[\text{rect}\left(\frac{f-f_c}{2}\right) + \text{rect}\left(\frac{f+f_c}{2}\right)\right]$

$2\pi\,\text{sinc}(2\pi t)\cos(2\pi t) \Leftrightarrow \frac{1}{2}\left[\text{rect}\left(\frac{f-10}{2}\right) + \text{rect}\left(\frac{f+10}{2}\right)\right]$

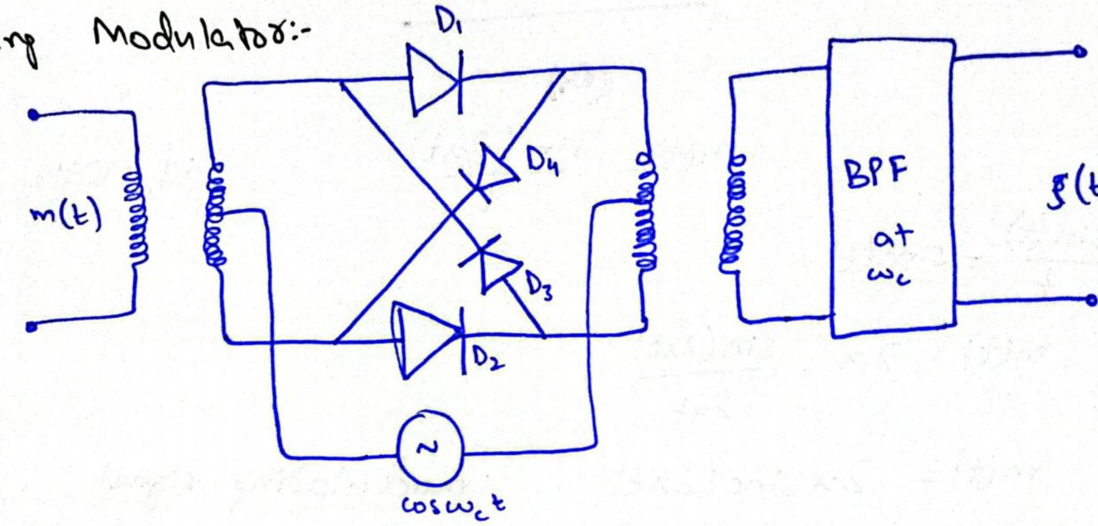




Switching Demodulator using Ring Modulator:-



Ring Modulator:-



⇒ During +ive half cycle of carrier, D_1 & D_3 conducts while D_2 & D_4 are open

⇒ During -ive half cycle of carrier, D_2 & D_4 conduct while D_1 & D_3 are open

⇒ The input signal is $m(t)\cos\omega_c t$.

⇒ The carrier "$\cos\omega_c t$" causes periodic switching of input signal. Therefore, input $m(t)\cos\omega_c t$ is multiplied with pulse train $w(t)$

$$w(t) = \frac{4}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \dots\right)$$

The output become

$$\Rightarrow m(t)\cos\omega_c t\, w(t)$$

$$= m(t)\cos\omega_c t\left[\frac{4}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \dots\right)\right]$$

$$= \frac{4}{\pi}m(t)\cos^2\omega_c t - \frac{4}{3\pi}m(t)\cos\omega_c t\cos 3\omega_c t + \dots$$

$$= \frac{4}{\pi}m(t)\left[\frac{1+\cos 2\omega_c t}{2}\right] + \dots$$

$$= \frac{2}{\pi}[m(t) + m(t)\cos 2\omega_c t] + \dots \quad \to 0 \text{ (LPF)}$$

$$= \frac{2}{\pi}m(t) + \underbrace{\cancel{\frac{2}{\pi}m(t)\cos(2\omega_c t) + \dots}}$$

$\boxed{\text{Output: } \frac{2}{\pi}m(t)}$ centered at ω_c

Q. No. 2

$m(t) = \cos\omega_m t + 2\sin 3\omega_m t$

$c(t) = 3\cos\omega_c t \quad ; \quad \omega_c >> \omega_m$

$\phi_{AM}(t) = [A + m(t)]\cos\omega_c t$

$= [3 + \cos\omega_m t + 2\sin 3\omega_m t]\cos\omega_c t$

$= 3[1 + \frac{1}{3}\cos\omega_m t + \frac{2}{3}\sin 3\omega_m t]\cos\omega_c t\}$

$m_1 = \frac{1}{3}$ and $m_2 = \frac{2}{3}$

Total modulation index:-

$m = \sqrt{m_1^2 + m_2^2}$

$= \sqrt{(\frac{1}{3})^2 + (\frac{2}{3})^2}$

$m = 0.74$

Hence, ϕ_{AM} satisfies the detection criteria. $(0 \le m \le 1)$

$\phi_{AM}(t) = 3(1 + \frac{1}{3}\cos\omega_m t + \frac{2}{3}\sin 3\omega_m t)\cos\omega_c t$

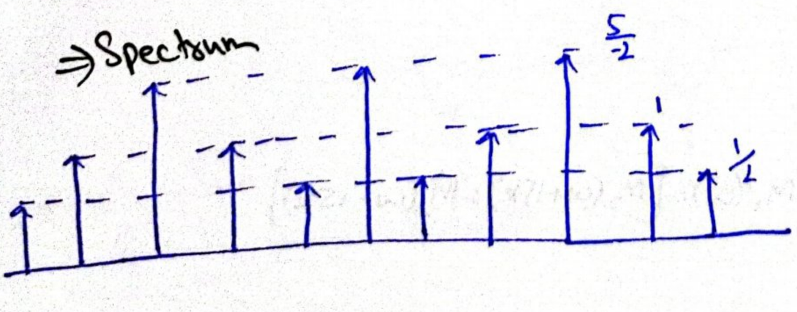
$= 3\cos\omega_c t + \cos(\omega_m t)\cos(\omega_c t) + 2\sin(3\omega_m t)\cos(\omega_c t)$

$\Rightarrow \phi_{AM}(\omega) = \frac{3}{2}\Big[\delta(\omega-\omega_c) + \delta(\omega+\omega_c) + 1(\delta(\omega-(\omega_c+\omega_m)) + \delta(\omega+(\omega_c+\omega_m))$

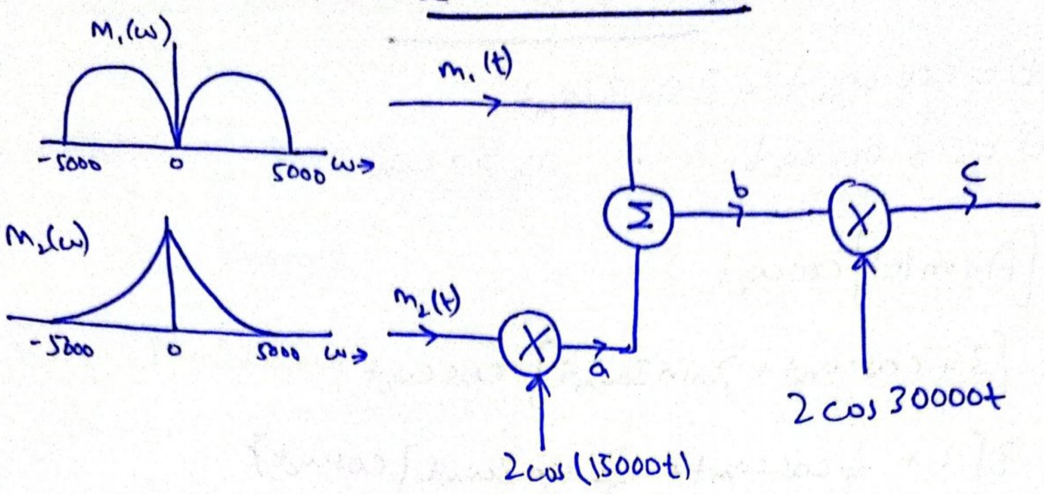
$+ 1(\delta(\omega-(\omega_c-\omega_m)) + \delta(\omega+(\omega_c-\omega_m)) + \frac{1}{2}(\delta(\omega-(\omega_c+2\omega_m))$

$+ \delta(\omega+(\omega_c+2\omega_m)) + \frac{1}{2}(\delta(\omega-(\omega_c-2\omega_m)) + \delta(\omega+(\omega_c-2\omega_m$

$\Rightarrow$ Spectrum

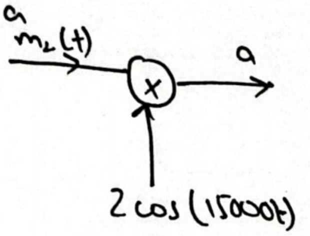


Q. No. 3



(a)

At point a



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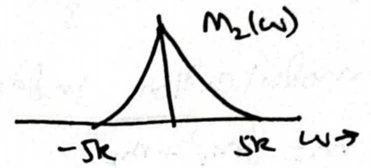
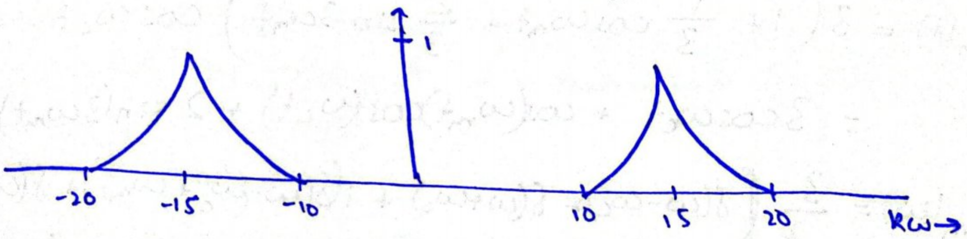


At $a \Rightarrow$ $m_2(t)\, 2\cos 15000t$

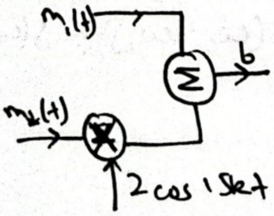
where

$$m_2(t)\, 2\cos 15000t \Leftrightarrow \frac{2}{2}\left[M_2(\omega+15k) + M_2(\omega-15k)\right]$$

$$m_2(t)\, 2\cos 15000t \Leftrightarrow M_2(\omega+15k) + M_2(\omega-15k)$$

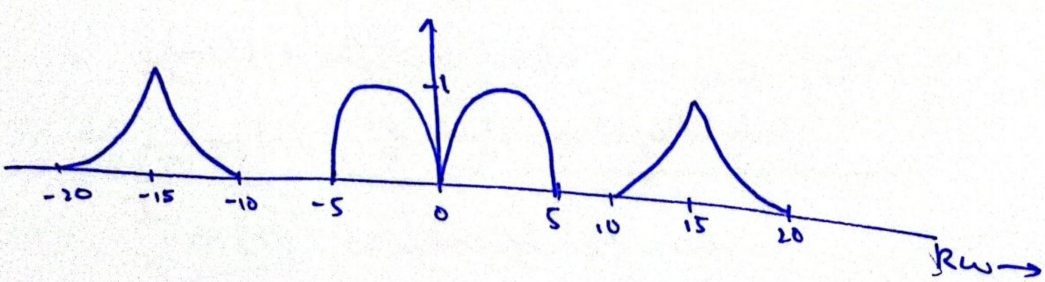


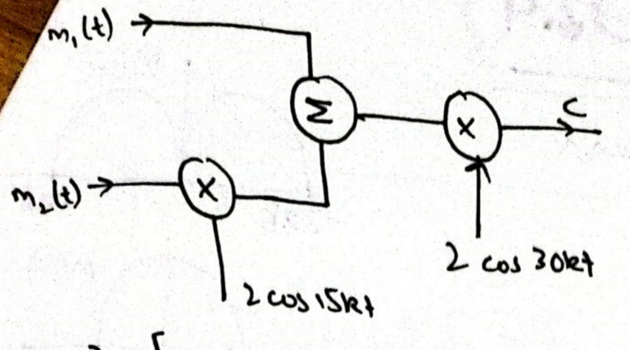
At point b



At $b \Rightarrow$ $m_1(t) + 2m_2(t)\cos 15kt$

where $m_1(t) + m_2(t)\, 2\cos 15kt \Leftrightarrow M_1(\omega) + \left[M_2(\omega+15k) + M_2(\omega-15k)\right]$



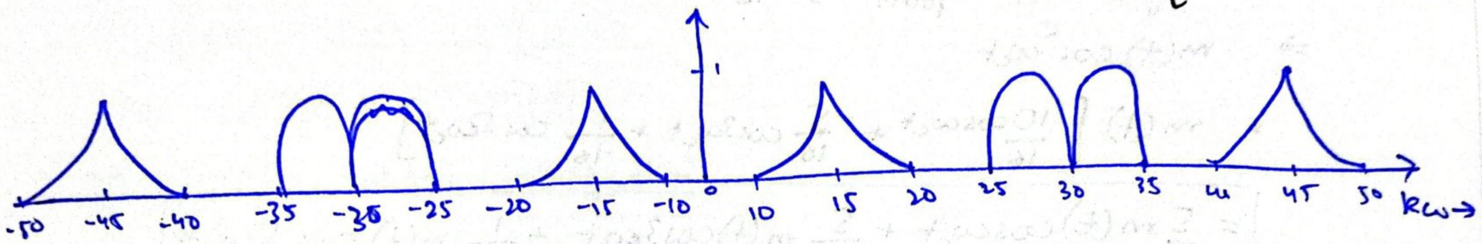


At $c \Rightarrow [m_1(t) + m_2(t)\, 2\cos 15kt]\, 2\cos 30kt$

$\Rightarrow 2m_1(t)\cos 30kt + 4\, m_2(t)\cos 15kt \cos 30kt$

where

$$2m_1(t)\cos 3kt + 4m_2(t)\cos 15kt \cos 30kt \Longleftrightarrow \frac{2}{2}[M_1(\omega+30k) + M_1(\omega-30k)] + \frac{4}{2}[M_2(\omega+15k)+M_2(\omega-15k)]$$
$$\left[\frac{1}{2}(M_2(\omega+30k)+M_2(\omega-30k))\right]$$



(b)

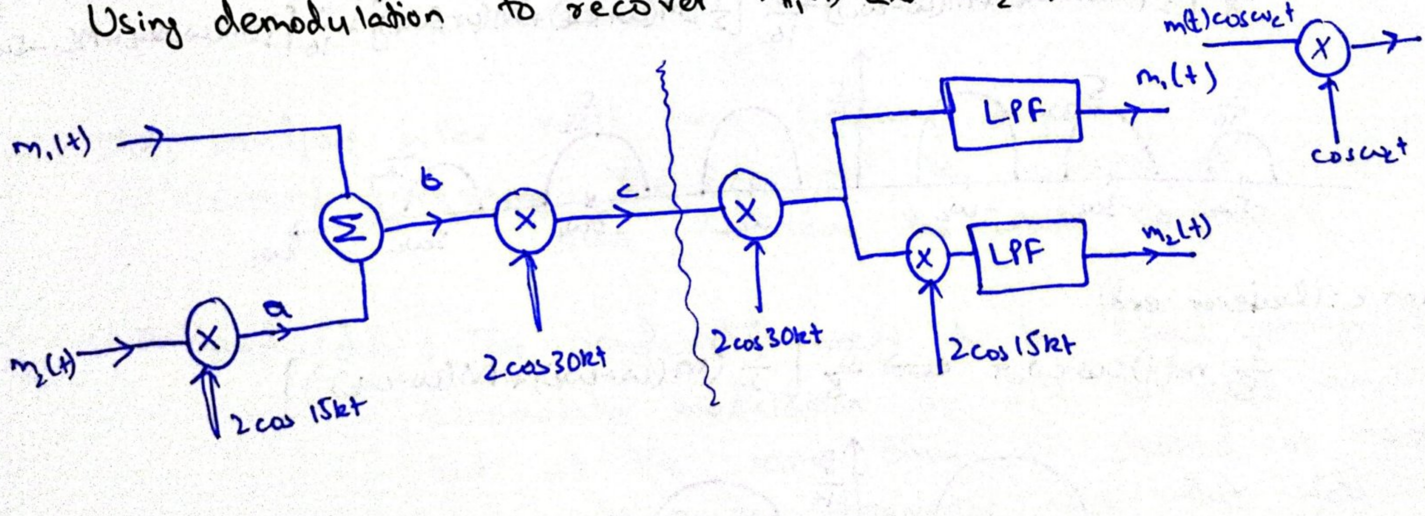
The total bandwidth required at c is dif. b/w highest and lowest frequency. i.e

$\Rightarrow 50000 - 10000$

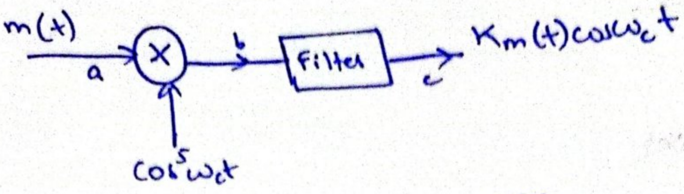
$= 40{,}000$ rad/s

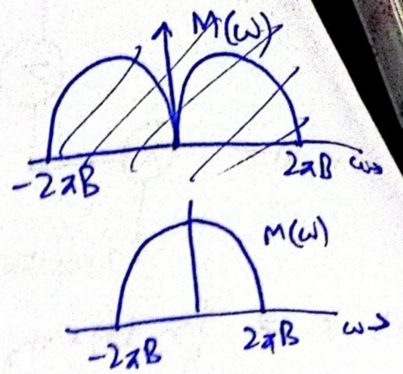
(C)

Using demodulation to recover $m_1(t)$ and $m_2(t)$ with a LPF



Q. No. 4





(a)

Desired modulated signal is $Km(t)\cos\omega_c t$

Carrier given $\cos^5\omega_c t$

As $\cos^5\omega_c t = \dfrac{10\cos\omega_c t + 5\cos 3\omega_c t + \cos 5\omega_c t}{16}$

The signal at point b is

$\Rightarrow m(t)\cos^5\omega_c t$

$= m(t)\left[\frac{10\cos\omega_c t}{16} + \frac{5}{16}\cos 3\omega_c t + \frac{1}{16}\cos 5\omega_c t\right]$

$$\boxed{= \frac{5}{8}m(t)\cos\omega_c t + \frac{5}{16}m(t)\cos 3\omega_c t + \frac{1}{16}m(t)\cos 5\omega_c t}$$

where only $\frac{5}{8}m(t)\cos\omega_c t$ is required at reciever end.

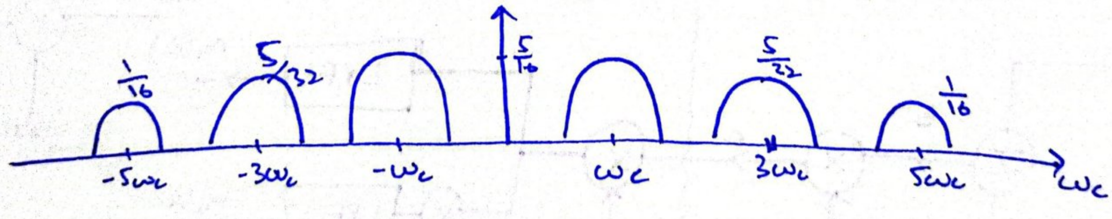
Hence, we use $\boxed{\text{Band pass filter centered at } \omega_c}$ to get $\frac{5}{8}m(t)\cos\omega_c t$.

(b)

At point b: $\frac{5}{8}m(t)\cos\omega_c t + \frac{5}{16}m(t)\cos 3\omega_c t + \frac{1}{16}m(t)\cos 5\omega_c t$

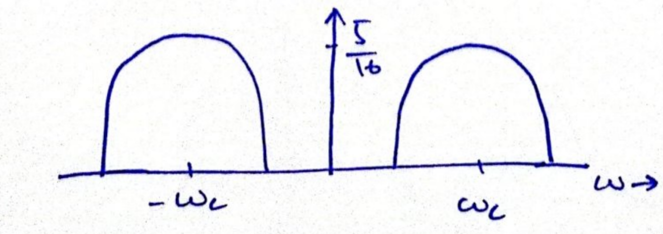
$\Longleftrightarrow$

$\frac{5}{8}\left[\frac{1}{2}(M(\omega+\omega_c) + M(\omega-\omega_c))\right] + \frac{5}{16}\left[\frac{1}{2}[M(\omega+3\omega_c) + M(\omega-3\omega_c)]\right] + \frac{1}{16}\left[\frac{1}{2}(M(\omega+5\omega_c) + M(\omega-5\omega_c\right.$



At point c: (Reciever end)

$\frac{5}{8}m(t)\cos\omega_c t \Longleftrightarrow \frac{5}{8}\left[\frac{1}{2}(M(\omega+\omega_c) + M(\omega-\omega_c))\right]$



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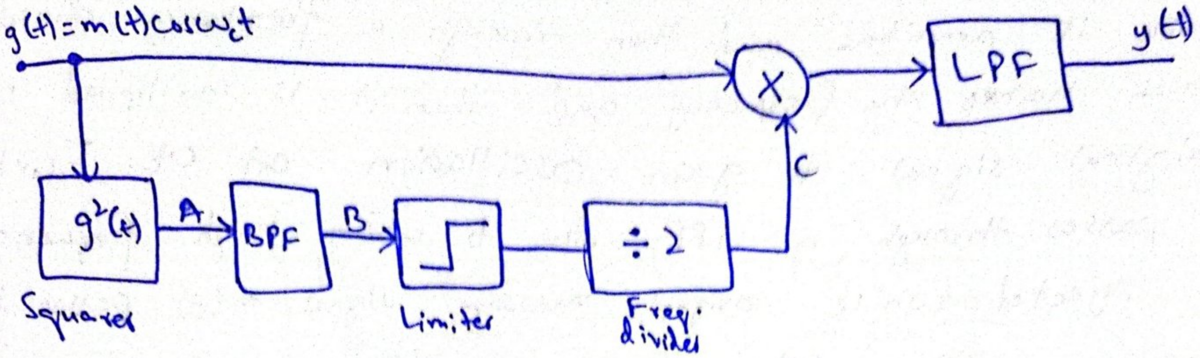


(a)

$g(t) = m(t) \cos \omega_c t$

Squaring $g^2(t) = [m(t) \cdot \cos \omega_c t]^2$

$g^2(t) = m^2(t) \cdot \cos^2(\omega_c t)$

As $\cos^2(\theta) = \dfrac{1+\cos(2\theta)}{2}$

$\Rightarrow g^2(t) = m^2(t) \dfrac{1+\cos(2\omega_c t)}{2}$

$$\boxed{g^2(t) = \frac{m^2(t)\,(1+\cos 2\omega_c t)}{2}}$$

(b)

The bandpass filter is tuned for only high frequency component ($2\omega_c$). Hence, it will not allow DC component to pass. i.e.

$\Rightarrow \cancel{\dfrac{m^2(t)}{2}}^{\to 0} + \dfrac{m^2(t)}{2} \cdot \cos(2\omega_c t)$

$$\Rightarrow \boxed{\frac{m^2(t)}{2} \cdot \cos(2\omega_c t)}$$

(c)

The bandpass filter must have following properties:-

→ Centered frequency: The BPF must be centered at $2\omega_c$ in order to pass the whole $\cos(2\omega_c t)$.

→ Bandwidth: The bandwidth needs to be broad enough to accomodate modulation in $m(t)$ to vary amplitude but at the same time, it should be narrow enough to reject frequency noise that falls outside of the desired range.

(d)

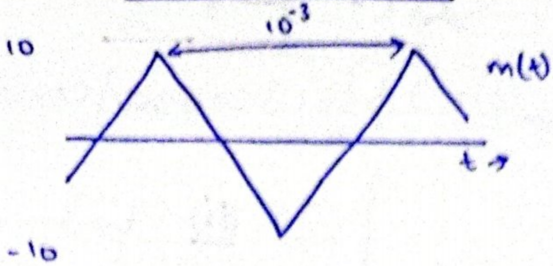
The signal at point B is passed through a limiter wh limits its amplitude and then through a frequency divider which halves the frequency and then it is multiplied with original signal to gain oscillation at ω_c. Further, it passes through a LPF due to which high frequencies are rejected while original message signal $m(t)$ passes through.

Hence, $y(t) = m(t)$

(e)

The limiter ensures that there is a constant amplitude before the signal passes through frequency divider. This is necessary because variations in amplitude could distort the signal, hence, the limiter preserves only the signal concerning the frequency for proper processing.

Q. 6

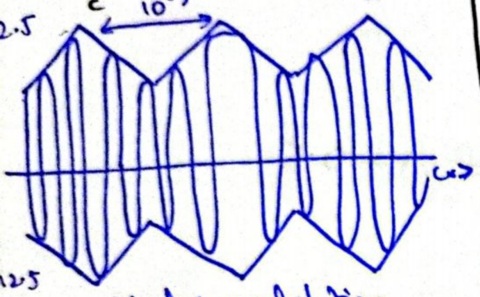


For $\mu = 0.8$

$\Rightarrow \mu = \frac{m_p}{A_c}$

$0.8 = \frac{10}{A_c}$

$\Rightarrow A_c = 12.5$



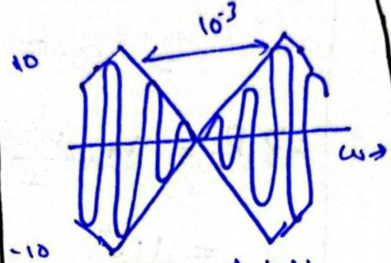
Under modulation

For $\mu = 1$

$\Rightarrow \mu = \frac{m_p}{A_c}$

$1 = \frac{10}{A_c}$

$\Rightarrow A_c = 10$



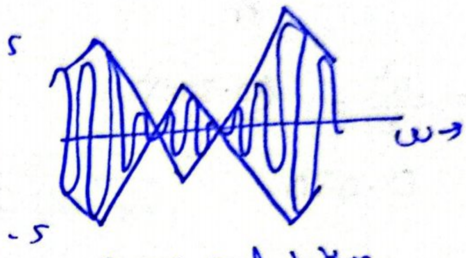
Ideal modulation.

For $\mu = 2$

$\Rightarrow \mu = \frac{m_p}{A_c}$

$2 = \frac{10}{A_c}$

$\Rightarrow A_c = 5$



Over-modulation

(a) As $\mu = \frac{m_p}{A_c}$

$0.8 = \frac{10}{A_c}$

$\Rightarrow A_c = \frac{10}{0.8}$

Amplitude: $\boxed{A_c = 12.5\ V}$

As $P_c = \frac{A_c^2}{2}$

$= \frac{(12.5)^2}{2}$

Carrier Power: $\boxed{P_c = 78.125\,W}$

(b) As Sideband power $= \frac{P_c \mu^2}{3}$

$= \frac{(78.125)(0.8)^2}{3}$

$\boxed{P_{SB} = 16.66\,W}$

As Efficiency $= \frac{P_{SB}}{P_t} \times 100\%$

and $P_t = P_c + P_{SB}$

Hence

$\eta = \frac{P_{SB}}{P_c + P_{SB}} \times 100\%$

$= \frac{16.66}{78.125 + 16.66} \times 100\%$

$\boxed{\eta = 17.57\%}$

Q. No. 7

$\phi_{AM}(t) = 2.5[b + 3\cos\omega_m t]\cos\omega_c t$; $\omega_m << \omega_c$

(i) b = 8

$\phi_{AM}(t) = 2.5[8 + 3\cos\omega_m t]\cos\omega_c t$

$= (2.5)(8)[1 + \frac{3}{8}\cos\omega_m t]\cos\omega_c t$

$\Rightarrow$ Modulation index $\mu = \frac{3}{8} = 0.375$

Power efficiency $\eta = \frac{P_{SB}}{P_T} \times 100\%$

$\Rightarrow P_{SB} = \frac{\mu^2}{2} = \frac{(0.375)^2}{2}$

$= 0.070$

$\Rightarrow P_T = 1 + \frac{\mu^2}{2} = 1 + \frac{(0.375)^2}{2}$

$= 1.070$

$\Rightarrow \eta = \frac{0.070}{1.070} \times 100\%$

$\boxed{\eta = 6.569\%}$

(ii) b = 3

$\phi_{AM}(t) = 2.5[3 + 3\cos\omega_m t]\cos\omega_c t$

$= (2.5)(3)[1 + \cos\omega_m t]\cos\omega_c t$

$\Rightarrow \mu = 1$

$\Rightarrow P_{SB} = \frac{\mu^2}{2} = \frac{(1)^2}{2}$

$= 0.5$

$\Rightarrow P_T = 1 + \frac{\mu^2}{2} = 1 + 0.5$

$= 1.5$

$\Rightarrow \eta = \frac{0.5}{1.5} \times 100\%$

$\boxed{\eta = 33.3\%}$

Q. No. 8

$m(t) = B\,\text{sinc}(2\pi Bt)$; $B = 1000$; $\omega_c = 10000\pi$

(a)

$$m(t) = 1000\,\text{sinc}(2\pi(1000)t)$$
$$= 1000\,\text{sinc}(2000\pi t)$$
$$= 1000\,\frac{\sin(2000\pi t)}{2000\pi t} \qquad \because \text{sinc}\,\theta = \frac{\sin\theta}{\theta}$$
$$= \frac{1}{2}\,\frac{\sin(2000\pi t)}{\pi t}$$

As $\frac{\sin at}{\pi t} \Leftrightarrow \text{rect}\left(\frac{\omega}{2a}\right)$

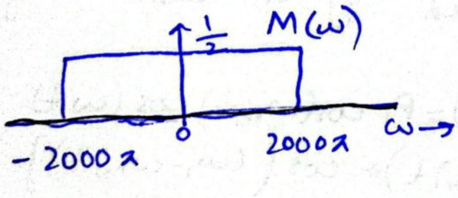
$$\frac{1}{2}\,\frac{\sin(2000\pi t)}{\pi t} \Leftrightarrow \frac{1}{2}\,\text{rect}\left(\frac{\omega}{2(2000\pi)}\right)$$

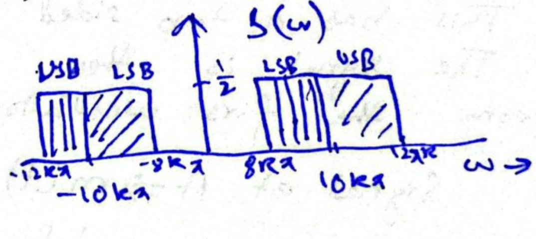
$$\Rightarrow M(\omega) = \frac{1}{2}\,\text{rect}\left(\frac{\omega}{4000\pi}\right)$$

$$S(t) = 2m(t)\cos\omega_c t$$

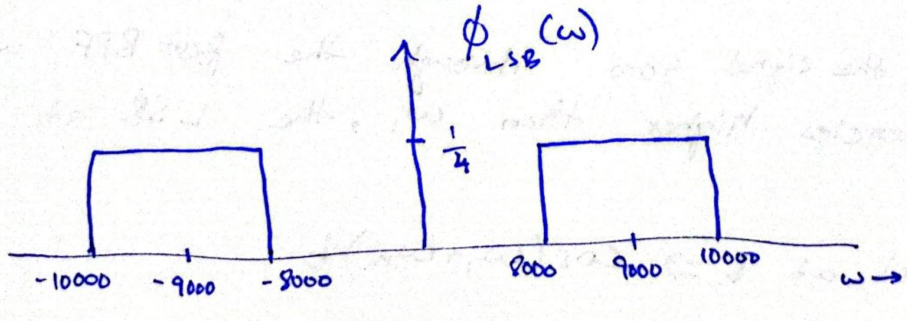
As $x(t)\cos\omega_0 t \Longleftrightarrow \frac{1}{2}[x(\omega-\omega_0) + x(\omega+\omega_0)]$

$$S(\omega) = M(\omega-\omega_c) + M(\omega+\omega_c)$$





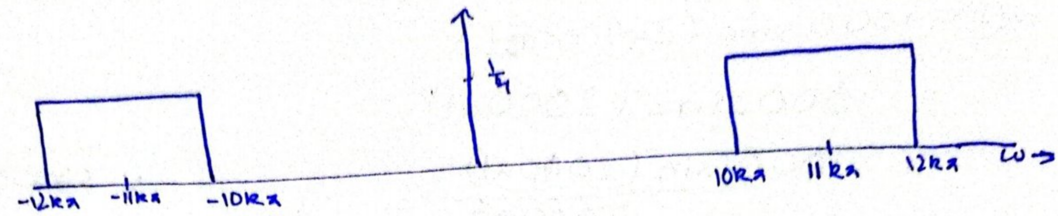
(b)



$$\phi_{LSB} = 1000\,\text{sinc}(2000\pi t)\cos 9000\pi t$$

(c)

$$\phi_{USB} = 1000 \text{ sinc}(2000\pi t)\cos 9000\pi t$$



(d)

Bandwidth of SSB signal would be B Hz which is 2000 rad/s.

Q. No 9

At point A:-

The signal $m(t)$ is modulated with first oscillator at a frequency f_1.

Assuming tone modulation, the modulating signal can be written as

$$m(t) = A\cos(\omega_m t)$$

This has a two sided spectrum centered about $\pm\omega_m$.

The signal is then multiplied by a carrier signal from the first oscillator.

Signal at $A \Rightarrow m(t)\cdot\cos(\omega_1 t) = A\cos(\omega_m t)\cos(\omega_1 t)$

$$\Rightarrow \frac{1}{2}\left[\cos((\omega_1+\omega_m)t) + \cos((\omega_1-\omega_m)t)\right]$$

At point B:-

When the signal goes through the first BPF which passes only frequencies higher than ω_1, the LSB at $(\omega_1-\omega_m)$ is eliminated.

So signal at $B \Rightarrow \cos(\omega_1+\omega_m)t$

Point C:-

The signal is added with the second oscillator at frequency f_2. The signal at Point B is multiplied by $\cos(\omega_2 t)$, so signal at point C becomes

$$[\cos((\omega_1+\omega_m)t)\cos(\omega_2 t)]$$

Using trignometric identity.

$$\cos((\omega_1+\omega m)t)\cos(\omega_2 t) = 2\left[\cos(\omega_1+\omega_m+\omega_2)t\right] + \cos\left(\cos[(\omega_1+\omega m-\omega_2)t]\right)$$

At Point D:-

The second BPF filters the output of the mixture and allows only the higher frequency component which represent the upper side band.

So,

$$\cos(\omega_1 t+\omega_m t+\omega_2 t)$$

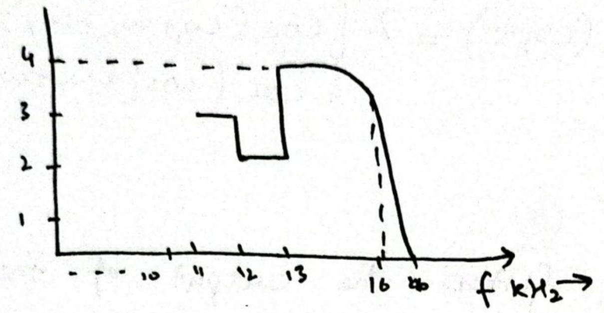
This is the end of last USB signal which has the carrier frequency ($f_c = f_1+f_2$).

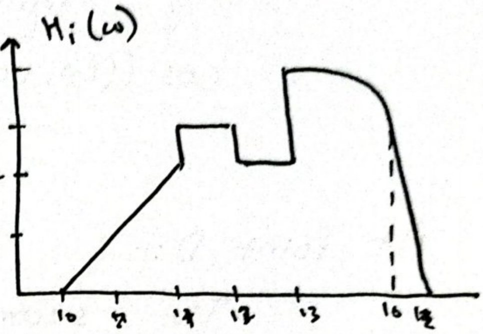
Q. No. 10

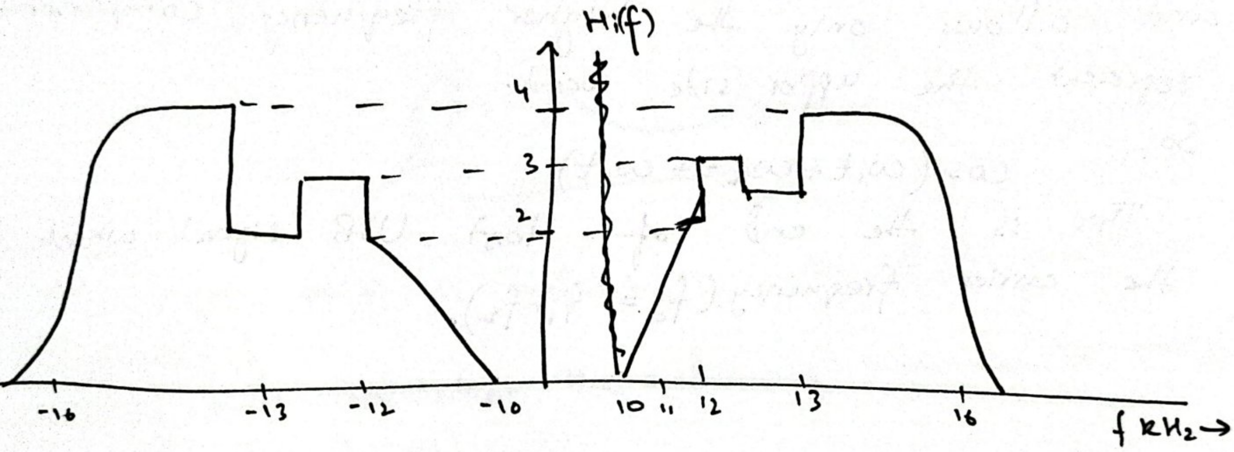
$f_c = 11\,kHz$; $B = 5\,kHz$

As $$H_o(\omega) = \frac{1}{H_i(\omega+\omega_c) + H_i(\omega-\omega_c)}$$

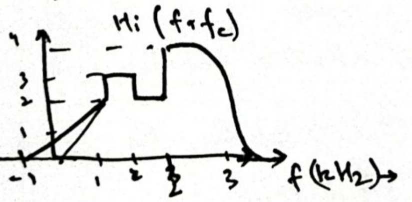
$|\omega| \le 2\pi B$

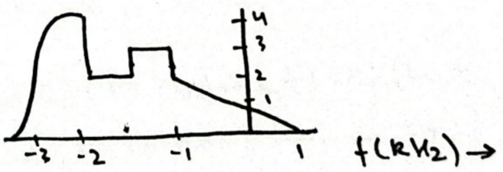


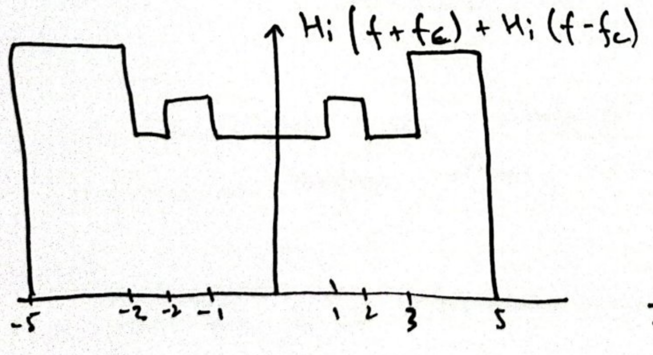


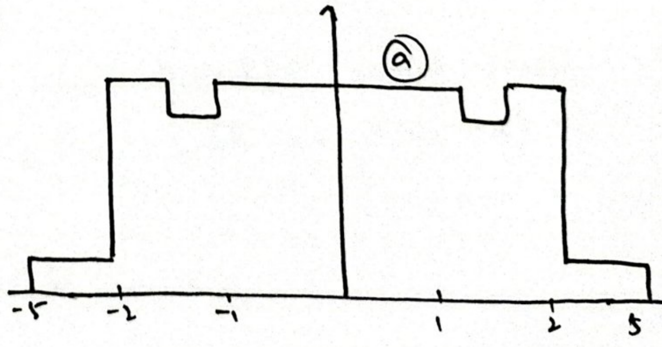


$H_i(f+f_c)$ is $(-f_c)$ centered version of $H_i(f)$









The figure (a) is $H_o(f) = \dfrac{1}{H_i(f+f_c) + H_i(f-f_c)}$; $|f| \le B$