

# **SEDRA/SMITH**

---

## **INSTRUCTOR'S SOLUTIONS MANUAL FOR Microelectronic Circuits**

**INTERNATIONAL SEVENTH EDITION**

Adel S. Sedra  
*University of Waterloo*

New York            Oxford  
OXFORD UNIVERSITY PRESS

Oxford University Press is a department of the University of Oxford.  
It furthers the University's objective of excellence in research,  
scholarship, and education by publishing worldwide.

Oxford New York  
Auckland Cape Town Dar es Salaam Hong Kong Karachi  
Kuala Lumpur Madrid Melbourne Mexico City Nairobi  
New Delhi Shanghai Taipei Toronto

With offices in  
Argentina Austria Brazil Chile Czech Republic France Greece  
Guatemala Hungary Italy Japan Poland Portugal Singapore  
South Korea Switzerland Thailand Turkey Ukraine Vietnam

Copyright © 2017 by Oxford University Press

For titles covered by Section 112 of the US Higher Education  
Opportunity Act, please visit [www.oup.com/us/he](http://www.oup.com/us/he) for the  
latest information about pricing and alternate formats.

Published by Oxford University Press  
198 Madison Avenue, New York, New York 10016  
<http://www.oup.com>

Oxford is a registered trademark of Oxford University Press

All rights reserved. No part of this publication may be reproduced,  
stored in a retrieval system, or transmitted, in any form or by any means,  
electronic, mechanical, photocopying, recording, or otherwise,  
without the prior permission of Oxford University Press.

**ISBN: 978-0-19-933916-7**

Printing number: 9 8 7 6 5 4 3 2 1

Printed in the United States of America  
on acid-free paper

## **Contents**

**Exercise Solutions** (Chapters 1-17)

**Problem Solutions** (Chapters 1-17)

## Preface

This Instructor's Solution Manual (ISM) contains complete solutions for all exercises and end-of-chapter problems included in the book *Microelectronic Circuits, International Seventh Edition* by Adel S. Sedra and Kenneth C. Smith.

Most of the solutions are new; however, I have used and/or adapted some of the solutions from the ISM of the International Sixth Edition. Credit for these goes to the problem solvers listed therein.

This manual has greatly benefited from the careful work of the accuracy checkers listed below. These colleagues and friends worked diligently to ensure that the 2,030 solutions are free of error. Despite all of our combined efforts, however, there is little doubt that some errors remain, and for these I take full responsibility. I will be most grateful to instructors who discover errors and point them out to me. Please send all corrections and comments by email to: [sedra@uwaterloo.ca](mailto:sedra@uwaterloo.ca).

**Adel Sedra**  
**Waterloo, Ontario, Canada**  
**October 2015**

## Accuracy Checkers

- Professor Tony Chan Carusone, University of Toronto - Assisted by graduate students Jeffrey Wang and Luke Wang
- Professor Vincent Gaudet, University of Waterloo
- Professors Shahriar Mirabbasi and Mandana Amiri, University of British Columbia
- Professor Wai Tung Ng, University of Toronto
- Professor Olivier Trescases, University of Toronto
- Professor Amir Yazdani, Ryerson University

### Exercise 1-1

Ex: 1.1 When output terminals are open-circuited, as in Fig. 1.1a:

For circuit a.  $v_{oc} = v_s(t)$

For circuit b.  $v_{oc} = i_s(t) \times R_s$

When output terminals are short-circuited, as in Fig. 1.1b:

For circuit a.  $i_{sc} = \frac{v_s(t)}{R_s}$

For circuit b.  $i_{sc} = i_s(t)$

For equivalency

$$R_s i_s(t) = v_s(t)$$

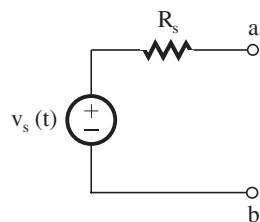


Figure 1.1a

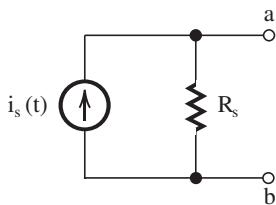
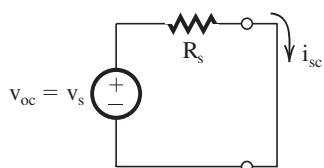


Figure 1.1b

Ex: 1.2



$$v_{oc} = 10 \text{ mV}$$

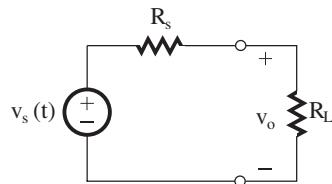
$$i_{sc} = 10 \mu\text{A}$$

$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{10 \text{ mV}}{10 \mu\text{A}} = 1 \text{ k}\Omega$$

Ex: 1.3 Using voltage divider:

$$v_o(t) = v_s(t) \times \frac{R_L}{R_s + R_L}$$

Given  $v_s(t) = 10 \text{ mV}$  and  $R_s = 1 \text{ k}\Omega$ .



If  $R_L = 100 \text{ k}\Omega$ :

$$v_o = 10 \text{ mV} \times \frac{100}{100 + 1} = 9.9 \text{ mV}$$

If  $R_L = 10 \text{ k}\Omega$ :

$$v_o = 10 \text{ mV} \times \frac{10}{10 + 1} \approx 9.1 \text{ mV}$$

If  $R_L = 1 \text{ k}\Omega$ :

$$v_o = 10 \text{ mV} \times \frac{1}{1 + 1} = 5 \text{ mV}$$

If  $R_L = 100 \Omega$ :

$$v_o = 10 \text{ mV} \times \frac{100}{100 + 1 \text{ K}} \approx 0.91 \text{ mV}$$

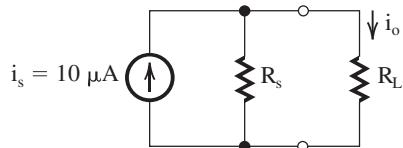
For  $v_o = 0.8v_s$ ,

$$\frac{R_L}{R_L + R_s} = 0.8$$

Since  $R_s = 1 \text{ k}\Omega$ ,

$$R_L = 4 \text{ k}\Omega$$

Ex: 1.4 Using current divider:



$$i_o = i_s \times \frac{R_s}{R_s + R_L}$$

Given  $i_s = 10 \mu\text{A}$ ,  $R_s = 100 \text{ k}\Omega$ .

For

$$R_L = 1 \text{ k}\Omega, i_o = 10 \mu\text{A} \times \frac{100}{100 + 1} = 9.9 \mu\text{A}$$

For

$$R_L = 10 \text{ k}\Omega, i_o = 10 \mu\text{A} \times \frac{100}{100 + 10} \approx 9.1 \mu\text{A}$$

For

$$R_L = 100 \text{ k}\Omega, i_o = 10 \mu\text{A} \times \frac{100}{100 + 100} = 5 \mu\text{A}$$

$$\text{For } R_L = 1 \text{ M}\Omega, i_o = 10 \mu\text{A} \times \frac{100 \text{ K}}{100 \text{ K} + 1 \text{ M}}$$

$$\approx 0.9 \mu\text{A}$$

$$\text{For } i_o = 0.8i_s, \frac{100}{100 + R_L} = 0.8$$

$$\Rightarrow R_L = 25 \text{ k}\Omega$$

## Exercise 1–2

Ex: 1.5  $f = \frac{1}{T} = \frac{1}{10^{-3}} = 1000 \text{ Hz}$

$$\omega = 2\pi f = 2\pi \times 10^3 \text{ rad/s}$$

Ex: 1.6 (a)  $T = \frac{1}{f} = \frac{1}{60} \text{ s} = 16.7 \text{ ms}$

(b)  $T = \frac{1}{f} = \frac{1}{10^{-3}} = 1000 \text{ s}$

(c)  $T = \frac{1}{f} = \frac{1}{10^6} \text{ s} = 1 \mu\text{s}$

Ex: 1.7 If 6 MHz is allocated for each channel, then 470 MHz to 806 MHz will accommodate

$$\frac{806 - 470}{6} = 56 \text{ channels}$$

Since the broadcast band starts with channel 14, it will go from channel 14 to channel 69.

Ex: 1.8  $P = \frac{1}{T} \int_0^T \frac{V^2}{R} dt$

$$= \frac{1}{T} \times \frac{V^2}{R} \times T = \frac{V^2}{R}$$

Alternatively,

$$P = P_1 + P_3 + P_5 + \dots$$

$$= \left( \frac{4V}{\sqrt{2}\pi} \right)^2 \frac{1}{R} + \left( \frac{4V}{3\sqrt{2}\pi} \right)^2 \frac{1}{R} + \left( \frac{4V}{5\sqrt{2}\pi} \right)^2 \frac{1}{R} + \dots$$

$$= \frac{V^2}{R} \times \frac{8}{\pi^2} \times \left( 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \dots \right)$$

It can be shown by direct calculation that the infinite series in the parentheses has a sum that approaches  $\pi^2/8$ ; thus  $P$  becomes  $V^2/R$  as found from direct calculation.

Fraction of energy in fundamental

$$= 8/\pi^2 = 0.81$$

Fraction of energy in first five harmonics

$$= \frac{8}{\pi^2} \left( 1 + \frac{1}{9} + \frac{1}{25} \right) = 0.93$$

Fraction of energy in first seven harmonics

$$= \frac{8}{\pi^2} \left( 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} \right) = 0.95$$

Fraction of energy in first nine harmonics

$$= \frac{8}{\pi^2} \left( 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81} \right) = 0.96$$

Note that 90% of the energy of the square wave is in the first three harmonics, that is, in the fundamental and the third harmonic.

Ex: 1.9 (a) D can represent 15 distinct values between 0 and +15 V. Thus,

$$v_A = 0 \text{ V} \Rightarrow D = 0000$$

$$v_A = 1 \text{ V} \Rightarrow D = 0001$$

$$v_A = 2 \text{ V} \Rightarrow D = 0010$$

$$v_A = 15 \text{ V} \Rightarrow D = 1111$$

$$(b) (i) +1 \text{ V} (ii) +2 \text{ V} (iii) +4 \text{ V} (iv) +8 \text{ V}$$

(c) The closest discrete value represented by

D is 5 V; thus D = 0101. The error is  $-0.2 \text{ V}$ , or  $-0.2/5.2 \times 100 = -4\%$ .

Ex: 1.10 Voltage gain =  $20 \log 100 = 40 \text{ dB}$

$$\text{Current gain} = 20 \log 1000 = 60 \text{ dB}$$

$$\begin{aligned} \text{Power gain} &= 10 \log A_p = 10 \log (A_v A_i) \\ &= 10 \log 10^5 = 50 \text{ dB} \end{aligned}$$

Ex: 1.11  $P_{dc} = 15 \times 8 = 120 \text{ mW}$

$$P_L = \frac{(6/\sqrt{2})^2}{1} = 18 \text{ mW}$$

$$P_{dissipated} = 120 - 18 = 102 \text{ mW}$$

$$\eta = \frac{P_L}{P_{dc}} \times 100 = \frac{18}{120} \times 100 = 15\%$$

Ex: 1.12  $v_o = 1 \times \frac{10}{10^6 + 10} \simeq 10^{-5} \text{ V} = 10 \mu\text{V}$

$$P_L = v_o^2 / R_L = \frac{(10 \times 10^{-6})^2}{10} = 10^{-11} \text{ W}$$

With the buffer amplifier:

$$\begin{aligned} v_o &= 1 \times \frac{R_i}{R_i + R_s} \times A_{v_o} \times \frac{R_L}{R_L + R_o} \\ &= 1 \times \frac{1}{1+1} \times 1 \times \frac{10}{10+10} = 0.25 \text{ V} \end{aligned}$$

$$P_L = \frac{v_o^2}{R_L} = \frac{0.25^2}{10} = 6.25 \text{ mW}$$

$$\text{Voltage gain} = \frac{v_o}{v_s} = \frac{0.25 \text{ V}}{1 \text{ V}} = 0.25 \text{ V/V}$$

$$= -12 \text{ dB}$$

$$\text{Power gain } (A_p) \equiv \frac{P_L}{P_i}$$

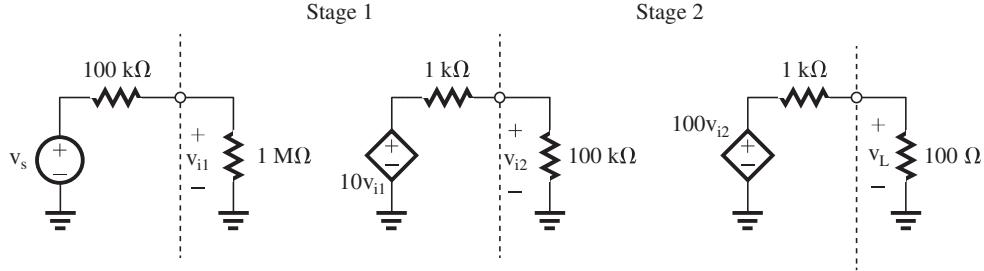
where  $P_L = 6.25 \text{ mW}$  and  $P_i = v_i i_i$ ,

$$v_i = 0.5 \text{ V} \text{ and}$$

$$i_i = \frac{1 \text{ V}}{1 \text{ M}\Omega + 1 \text{ M}\Omega} = 0.5 \mu\text{A}$$

### Exercise 1-3

This figure belongs to Exercise 1.15.



Thus,

$$P_i = 0.5 \times 0.5 = 0.25 \mu\text{W}$$

and

$$A_p = \frac{6.25 \times 10^{-3}}{0.25 \times 10^{-6}} = 25 \times 10^3$$

$$10 \log A_p = 44 \text{ dB}$$

Ex: 1.13 Open-circuit (no load) output voltage =  $A_{vo}v_i$

Output voltage with load connected

$$= A_{vo}v_i \frac{R_L}{R_L + R_o}$$

$$0.8 = \frac{1}{R_o + 1} \Rightarrow R_o = 0.25 \text{ k}\Omega = 250 \Omega$$

Ex: 1.14  $A_{vo} = 40 \text{ dB} = 100 \text{ V/V}$

$$P_L = \frac{v_o^2}{R_L} = \left( A_{vo}v_i \frac{R_L}{R_L + R_o} \right)^2 / R_L$$

$$= v_i^2 \times \left( 100 \times \frac{1}{1+1} \right)^2 / 1000 = 2.5 v_i^2$$

$$P_i = \frac{v_i^2}{R_i} = \frac{v_i^2}{10,000}$$

$$A_p \equiv \frac{P_L}{P_i} = \frac{2.5v_i^2}{10^{-4}v_i^2} = 2.5 \times 10^4 \text{ W/W}$$

$$10 \log A_p = 44 \text{ dB}$$

Ex: 1.15 Without stage 3 (see figure above)

$$\frac{v_L}{v_s} =$$

$$\left( \frac{1 \text{ M}\Omega}{100 \text{ k}\Omega + 1 \text{ M}\Omega} \right) (10) \left( \frac{100 \text{ k}\Omega}{100 \text{ k}\Omega + 1 \text{ k}\Omega} \right)$$

$$\times (100) \left( \frac{100}{100 + 1 \text{ k}\Omega} \right)$$

$$\frac{v_L}{v_s} = (0.909)(10)(0.9901)(100)(0.0909)$$

$$= 81.8 \text{ V/V}$$

Ex: 1.16 Refer the solution to Example 1.3 in the text.

$$\frac{v_{i1}}{v_s} = 0.909 \text{ V/V}$$

$$v_{i1} = 0.909 v_s = 0.909 \times 1 = 0.909 \text{ mV}$$

$$\frac{v_{i2}}{v_s} = \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s} = 9.9 \times 0.909 = 9 \text{ V/V}$$

$$v_{i2} = 9 \times v_s = 9 \times 1 = 9 \text{ mV}$$

$$\frac{v_{i3}}{v_s} = \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s} = 90.9 \times 9.9 \times 0.909$$

$$= 818 \text{ V/V}$$

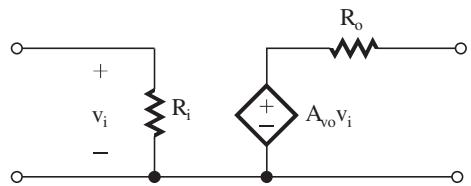
$$v_{i3} = 818 v_s = 818 \times 1 = 818 \text{ mV}$$

$$\frac{v_L}{v_s} = \frac{v_L}{v_{i3}} \times \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s}$$

$$= 0.909 \times 90.9 \times 9.9 \times 0.909 \simeq 744 \text{ V/V}$$

$$v_L = 744 \times 1 \text{ mV} = 744 \text{ mV}$$

Ex: 1.17 Using voltage amplifier model, the three-stage amplifier can be represented as



$$R_i = 1 \text{ M}\Omega$$

$$R_o = 10 \Omega$$

$$A_{vo} = A_{v1} \times A_{v2} \times A_{v3} = 9.9 \times 90.9 \times 1 = 900 \text{ V/V}$$

The overall voltage gain

$$\frac{v_o}{v_s} = \frac{R_o}{R_i + R_s} \times A_{vo} \times \frac{R_L}{R_L + R_o}$$

### Exercise 1-4

For  $R_L = 10 \Omega$ :

Overall voltage gain

$$= \frac{1 \text{ M}}{1 \text{ M} + 100 \text{ K}} \times 900 \times \frac{10}{10 + 10} = 409 \text{ V/V}$$

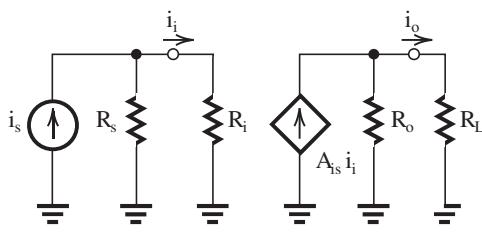
For  $R_L = 1000 \Omega$ :

Overall voltage gain

$$= \frac{1 \text{ M}}{1 \text{ M} + 100 \text{ K}} \times 900 \times \frac{1000}{1000 + 10} = 810 \text{ V/V}$$

$\therefore$  Range of voltage gain is from 409 V/V to 810 V/V.

Ex: 1.18



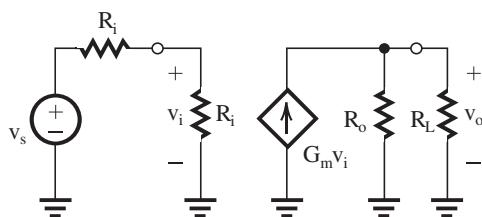
$$i_i = i_s \frac{R_s}{R_s + R_i}$$

$$i_o = A_{is} i_i \frac{R_o}{R_o + R_L} = A_{is} i_s \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

Thus,

$$\frac{i_o}{i_s} = A_{is} \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

Ex: 1.19



$$v_i = v_s \frac{R_i}{R_i + R_s}$$

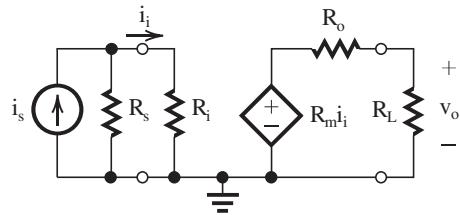
$$v_o = G_m v_i (R_o \parallel R_L)$$

$$= G_m v_s \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

Thus,

$$\frac{v_o}{v_s} = G_m \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

Ex: 1.20 Using the transresistance circuit model, the circuit will be



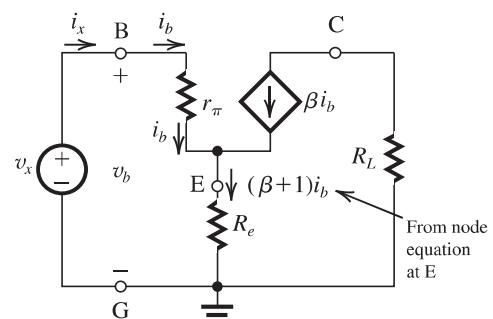
$$\frac{i_i}{i_s} = \frac{R_s}{R_i + R_s}$$

$$v_o = R_m i_i \times \frac{R_L}{R_L + R_o}$$

$$\frac{v_o}{i_i} = R_m \frac{R_L}{R_L + R_o}$$

$$\begin{aligned} \text{Now } \frac{v_o}{i_s} &= \frac{v_o}{i_i} \times \frac{i_i}{i_s} = R_m \frac{R_L}{R_L + R_o} \times \frac{R_s}{R_i + R_s} \\ &= R_m \frac{R_s}{R_s + R_i} \times \frac{R_L}{R_L + R_o} \end{aligned}$$

Ex: 1.21



$$v_b = i_b r_\pi + (\beta + 1) i_b R_e$$

$$= i_b [r_\pi + (\beta + 1) R_e]$$

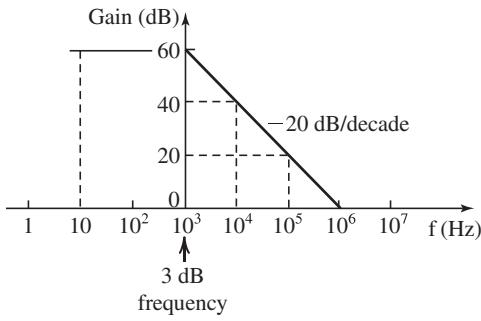
But  $v_b = v_x$  and  $i_b = i_x$ , thus

$$R_{in} \equiv \frac{v_x}{i_x} = \frac{v_b}{i_b} = r_\pi + (\beta + 1) R_e$$

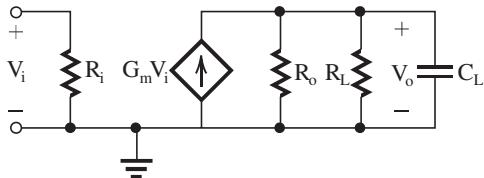
Ex: 1.22

	f	Gain
10 Hz		60 dB
10 kHz		40 dB
100 kHz		20 dB
1 MHz		0 dB

### Exercise 1-5



Ex: 1.23



$$V_o = G_m V_i [R_o \parallel R_L \parallel C_L]$$

$$= \frac{G_m V_i}{\frac{1}{R_o} + \frac{1}{R_L} + sC_L}$$

$$\text{Thus, } \frac{V_o}{V_i} = \frac{G_m}{\frac{1}{R_o} + \frac{1}{R_L}} \times \frac{1}{1 + \frac{sC_L}{\frac{1}{R_o} + \frac{1}{R_L}}}$$

$$\frac{V_o}{V_i} = \frac{G_m (R_L \parallel R_o)}{1 + sC_L (R_L \parallel R_o)}$$

which is of the STC LP type.

$$\text{DC gain} = G_m (R_L \parallel R_o) = 10 \times (R_L \parallel 50)$$

To obtain a dc gain of at least 40 dB (i.e., 100),  
 $10(R_L \parallel 50) \geq 100$

$$\Rightarrow R_L \geq 12.5 \text{ k}\Omega$$

$$\omega_0 = \frac{1}{C_L (R_L \parallel R_o)}$$

$$= \frac{1}{C_L (12.5 \parallel 50) \times 10^3}$$

For  $\omega_0$  to be at least  $2\pi \times 100 \times 10^3$ , the highest value allowed for  $C_L$  is

$$C_L = \frac{1}{2\pi \times 10^5 \times 10 \times 10^3} = 159.2 \text{ pF}$$

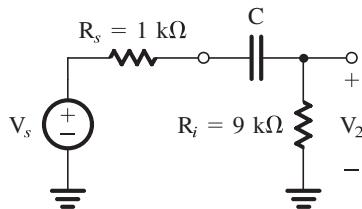
Ex: 1.24 Refer to Fig. E1.24

$$\frac{V_2}{V_s} = \frac{R_i}{R_s + \frac{1}{sC} + R_i} = \frac{R_i}{R_s + R_i} \frac{s}{s + \frac{1}{C(R_s + R_i)}}$$

which is an HP STC function.

$$f_{3\text{dB}} = \frac{1}{2\pi C(R_s + R_i)} \leq 100 \text{ Hz}$$

$$C \geq \frac{1}{2\pi (1+9)10^3 \times 100} = 0.16 \mu\text{F}$$



Ex: 1.25 T = 50 K

$$n_i = BT^{3/2} e^{-Eg/(2kT)}$$

$$= 7.3 \times 10^{15} (50)^{3/2} e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 50)}$$

$$\simeq 9.6 \times 10^{-39} / \text{cm}^3$$

T = 350 K

$$n_i = BT^{3/2} e^{-Eg/(2kT)}$$

$$= 7.3 \times 10^{15} (350)^{3/2} e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 350)}$$

$$= 4.15 \times 10^{11} / \text{cm}^3$$

Ex: 1.26 N\_D = 10^17 / cm^3

From Exercise 1.1, n\_i at

$$T = 350 \text{ K} = 4.15 \times 10^{11} / \text{cm}^3$$

$$n_n = N_D = 10^{17} / \text{cm}^3$$

$$p_n \cong \frac{n_i^2}{N_D}$$

$$= \frac{(4.15 \times 10^{11})^2}{10^{17}}$$

$$= 1.72 \times 10^6 / \text{cm}^3$$

Ex: 1.27 At 300 K, n\_i = 1.5 × 10<sup>10</sup> / cm<sup>3</sup>

$$p_p = N_A$$

Want electron concentration

$$= n_p = \frac{1.5 \times 10^{10}}{10^6} = 1.5 \times 10^4 / \text{cm}^3$$

$$\therefore N_A = p_p = \frac{n_i^2}{n_p}$$

### Exercise 1–6

$$= \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^4}$$

$$= 1.5 \times 10^{16} / \text{cm}^3$$

Ex: 1.28 (a)  $v_{n\text{-drift}} = -\mu_n E$

Here negative sign indicates that electrons move in a direction opposite to  $E$ .

We use

$$v_{n\text{-drift}} = 1350 \times \frac{1}{2 \times 10^{-4}} \therefore 1 \mu\text{m} = 10^{-4} \text{ cm}$$

$$= 6.75 \times 10^6 \text{ cm/s} = 6.75 \times 10^4 \text{ m/s}$$

(b) Time taken to cross  $2\mu\text{m}$

$$\text{length} = \frac{2 \times 10^{-6}}{6.75 \times 10^4} \simeq 30 \text{ ps}$$

(c) In n-type silicon, drift current density  $J_n$  is

$$J_n = qn\mu_n E$$

$$= 1.6 \times 10^{-19} \times 10^{16} \times 1350 \times \frac{1 \text{ V}}{2 \times 10^{-4}}$$

$$= 1.08 \times 10^4 \text{ A/cm}^2$$

(d) Drift current  $I_n = AJ_n$

$$= 0.25 \times 10^{-8} \times 1.08 \times 10^4$$

$$= 27 \mu\text{A}$$

Note that  $0.25 \mu\text{m}^2 = 0.25 \times 10^{-8} \text{ cm}^2$ .

$$\text{Ex: 1.29 } J_n = qD_n \frac{dn(x)}{dx}$$

From Fig. E1.5,

$$n_0 = 10^{17} / \text{cm}^3 = 10^5 / (\mu\text{m})^3$$

$$D_n = 35 \text{ cm}^2/\text{s} = 35 \times (10^4)^2 (\mu\text{m})^2/\text{s}$$

$$= 35 \times 10^8 (\mu\text{m})^2/\text{s}$$

$$\frac{dn}{dx} = \frac{10^5 - 0}{1} = 10^5 \mu\text{m}^{-4}$$

$$J_n = qD_n \frac{dn(x)}{dx}$$

$$= 1.6 \times 10^{-19} \times 35 \times 10^8 \times 10^5$$

$$= 56 \times 10^{-6} \text{ A}/\mu\text{m}^2$$

$$= 56 \mu\text{A}/\mu\text{m}^2$$

For  $I_n = 1 \text{ mA} = J_n \times A$

$$\Rightarrow A = \frac{1 \text{ mA}}{J_n} = \frac{10^3 \mu\text{A}}{56 \mu\text{A}/(\mu\text{m})^2} \simeq 18 \mu\text{m}^2$$

Ex: 1.30 Using Eq. (1.45),

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$$

$$D_n = \mu_n V_T = 1350 \times 25.9 \times 10^{-3}$$

$$\cong 35 \text{ cm}^2/\text{s}$$

$$D_p = \mu_p V_T = 480 \times 25.9 \times 10^{-3}$$

$$\cong 12.4 \text{ cm}^2/\text{s}$$

Ex: 1.31 Equation (1.50),

$$W = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0}$$

$$= \sqrt{\frac{2\epsilon_s}{q} \left( \frac{N_A + N_D}{N_A N_D} \right) V_0}$$

$$W^2 = \frac{2\epsilon_s}{q} \left( \frac{N_A + N_D}{N_A N_D} \right) V_0$$

$$V_0 = \frac{1}{2} \left( \frac{q}{\epsilon_s} \right) \left( \frac{N_A N_D}{N_A + N_D} \right) W^2$$

Ex: 1.32 In a p<sup>+</sup>n diode  $N_A \gg N_D$

$$\text{Equation (1.50), } W = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0}$$

We can neglect the term  $\frac{1}{N_A}$  as compared to  $\frac{1}{N_D}$ , thus

$$W \simeq \sqrt{\frac{2\epsilon_s}{qN_D} \cdot V_0}$$

$$\text{Equation (1.51), } x_n = W \frac{N_A}{N_A + N_D}$$

$$\simeq W \frac{N_A}{N_A}$$

$$= W$$

$$\text{Equation (1.52), } x_p = W \frac{N_D}{N_A + N_D}$$

since  $N_A \gg N_D$

$$\simeq W \frac{N_D}{N_A} = W \left( \frac{N_A}{N_D} \right)$$

$$\text{Equation (1.53), } Q_J = Aq \left( \frac{N_A N_D}{N_A + N_D} \right) W$$

$$\simeq Aq \frac{N_A N_D}{N_A} W$$

$$= Aq N_D W$$

$$\text{Equation (1.54), } Q_J = A \sqrt{2\epsilon_s q \left( \frac{N_A N_D}{N_A + N_D} \right) V_0}$$

$$\simeq A \sqrt{2\epsilon_s q \left( \frac{N_A N_D}{N_A} \right) V_0} \text{ since } N_A \gg N_D$$

$$= A \sqrt{2\epsilon_s q N_D V_0}$$

### Exercise 1-7

Ex: 1.33 In Example 1.10,  $N_A = 10^{18}/\text{cm}^3$  and  $N_D = 10^{16}/\text{cm}^3$

In the n-region of this pn junction

$$n_n = N_D = 10^{16}/\text{cm}^3$$

$$p_n = \frac{n_i^2}{n_n} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4/\text{cm}^3$$

As one can see from above equation, to increase minority-carrier concentration ( $p_n$ ) by a factor of 2, one must lower  $N_D (= n_n)$  by a factor of 2.

Ex: 1.34

$$\text{Equation (1.65)} I_S = Aq n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right).$$

Since  $\frac{D_p}{L_p}$  and  $\frac{D_n}{L_n}$  have approximately

similar values, if  $N_A \gg N_D$ , then the term  $\frac{D_n}{L_n N_A}$  can be neglected as compared to  $\frac{D_p}{L_p N_D}$ .

$$\therefore I_S \cong Aq n_i^2 \frac{D_p}{L_p N_D}$$

$$\begin{aligned} \text{Ex: 1.35 } I_S &= Aq n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) \\ &= 10^{-4} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2 \\ &\quad \times \left( \frac{10}{5 \times 10^{-4} \times \frac{10^{16}}{2}} + \frac{18}{10 \times 10^{-4} \times 10^{18}} \right) \\ &= 1.46 \times 10^{-14} \text{ A} \end{aligned}$$

$$\begin{aligned} I &= I_S (e^{V/V_T} - 1) \\ &\simeq I_S e^{V/V_T} = 1.45 \times 10^{-14} e^{0.605/(25.9 \times 10^{-3})} \\ &= 0.2 \text{ mA} \end{aligned}$$

$$\begin{aligned} \text{Ex: 1.36 } W &= \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 - V_F)} \\ &= \sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}} \left( \frac{1}{10^{18}} + \frac{1}{10^{16}} \right) (0.814 - 0.605)} \\ &= 1.66 \times 10^{-5} \text{ cm} = 0.166 \mu\text{m} \end{aligned}$$

$$\begin{aligned} \text{Ex: 1.37 } W &= \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)} \\ &= \sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}} \left( \frac{1}{10^{18}} + \frac{1}{10^{16}} \right) (0.814 + 2)} \end{aligned}$$

$$= 6.08 \times 10^{-5} \text{ cm} = 0.608 \mu\text{m}$$

Using Eq. (1.53),

$$\begin{aligned} Q_J &= Aq \left( \frac{N_A N_D}{N_A + N_D} \right) W \\ &= 10^{-4} \times 1.6 \times 10^{-19} \left( \frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}} \right) \times 6.08 \times 10^{-5} \text{ cm} \\ &= 9.63 \text{ pC} \end{aligned}$$

$$\begin{aligned} \text{Reverse current } I &= I_S = Aq n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) \\ &= 10^{-14} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2 \\ &\quad \times \left( \frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{18}} \right) \\ &= 7.3 \times 10^{-15} \text{ A} \end{aligned}$$

Ex: 1.38 Equation (1.72),

$$\begin{aligned} C_{j0} &= A \sqrt{\left( \frac{\epsilon_s q}{2} \right) \left( \frac{N_A N_D}{N_A + N_D} \right) \left( \frac{1}{V_0} \right)} \\ &= 10^{-4} \sqrt{\left( \frac{1.04 \times 10^{-12} \times 1.6 \times 10^{-19}}{2} \right)} \\ &\quad \sqrt{\left( \frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}} \right) \left( \frac{1}{0.814} \right)} \\ &= 3.2 \text{ pF} \end{aligned}$$

Equation (1.71),

$$\begin{aligned} C_j &= \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} \\ &= \frac{3.2 \times 10^{-12}}{\sqrt{1 + \frac{2}{0.814}}} \\ &= 1.72 \text{ pF} \end{aligned}$$

$$\text{Ex: 1.39 } C_d = \frac{dQ}{dV} = \frac{d}{dV} (\tau_T I)$$

$$\begin{aligned} &= \frac{d}{dV} [\tau_T \times I_S (e^{V/V_T} - 1)] \\ &= \tau_T I_S \frac{d}{dV} (e^{V/V_T} - 1) \\ &= \tau_T I_S \frac{1}{V_T} e^{V/V_T} \\ &= \frac{\tau_T}{V_T} \times I_S e^{V/V_T} \\ &\cong \left( \frac{\tau_T}{V_T} \right) I \end{aligned}$$

### Exercise 1-8

Ex: 1.40 Equation (1.75),

$$\begin{aligned}\tau_p &= \frac{L_p^2}{D_p} \\ &= \frac{(5 \times 10^{-4})^2}{10} \\ &= 25 \text{ ns} \\ \text{Equation (1.81), } C_d &= \left( \frac{\tau_T}{V_T} \right) I\end{aligned}$$

In Example 1.10,  $N_A = 10^{18}/\text{cm}^3$ ,

$$N_D = 10^{16}/\text{cm}^3$$

Assuming  $N_A \gg N_D$ ,

$$\tau_T \approx \tau_p = 25 \text{ ns}$$

$$\begin{aligned}\therefore C_d &= \left( \frac{25 \times 10^{-9}}{25.9 \times 10^{-3}} \right) 0.1 \times 10^{-3} \\ &= 96.5 \text{ pF}\end{aligned}$$

## Chapter 1–1

1.1 (a)  $I = \frac{V}{R} = \frac{5 \text{ V}}{1 \text{ k}\Omega} = 5 \text{ mA}$

(b)  $R = \frac{V}{I} = \frac{5 \text{ V}}{1 \text{ mA}} = 5 \text{ k}\Omega$

(c)  $V = IR = 0.1 \text{ mA} \times 10 \text{ k}\Omega = 1 \text{ V}$

(d)  $I = \frac{V}{R} = \frac{1 \text{ V}}{100 \text{ }\Omega} = 0.01 \text{ A} = 10 \text{ mA}$

Note: Volts, millamps, and kilohms constitute a consistent set of units.

1.2 (a)  $P = I^2R = (20 \times 10^{-3})^2 \times 1 \times 10^3$

$= 0.4 \text{ W}$

Thus, R should have a  $\frac{1}{2}$ -W rating.

(b)  $P = I^2R = (40 \times 10^{-3})^2 \times 1 \times 10^3$

$= 1.6 \text{ W}$

Thus, the resistor should have a 2-W rating.

(c)  $P = I^2R = (1 \times 10^{-3})^2 \times 100 \times 10^3$

$= 0.1 \text{ W}$

Thus, the resistor should have a  $\frac{1}{8}$ -W rating.

(d)  $P = I^2R = (4 \times 10^{-3})^2 \times 10 \times 10^3$

$= 0.16 \text{ W}$

Thus, the resistor should have a  $\frac{1}{4}$ -W rating.

(e)  $P = V^2/R = 20^2/(1 \times 10^3) = 0.4 \text{ W}$

Thus, the resistor should have a  $\frac{1}{2}$ -W rating.

(f)  $P = V^2/R = 11^2/(1 \times 10^3) = 0.121 \text{ W}$

Thus, a rating of  $\frac{1}{8}$  W should theoretically suffice,

though  $\frac{1}{4}$  W would be prudent to allow for

inevitable tolerances and measurement errors.

1.3 (a)  $V = IR = 5 \text{ mA} \times 1 \text{ k}\Omega = 5 \text{ V}$

$P = I^2R = (5 \text{ mA})^2 \times 1 \text{ k}\Omega = 25 \text{ mW}$

(b)  $R = V/I = 5 \text{ V}/1 \text{ mA} = 5 \text{ k}\Omega$

$P = VI = 5 \text{ V} \times 1 \text{ mA} = 5 \text{ mW}$

(c)  $I = P/V = 100 \text{ mW}/10 \text{ V} = 10 \text{ mA}$

$R = V/I = 10 \text{ V}/10 \text{ mA} = 1 \text{ k}\Omega$

(d)  $V = P/I = 1 \text{ mW}/0.1 \text{ mA}$

$= 10 \text{ V}$

$R = V/I = 10 \text{ V}/0.1 \text{ mA} = 100 \text{ k}\Omega$

(e)  $P = I^2R \Rightarrow I = \sqrt{P/R}$

$I = \sqrt{1000 \text{ mW}/1 \text{ k}\Omega} = 31.6 \text{ mA}$

$V = IR = 31.6 \text{ mA} \times 1 \text{ k}\Omega = 31.6 \text{ V}$

Note: V, mA, kΩ, and mW constitute a consistent set of units.

1.4 See figure on next page, which shows that there are 17 possible resistance values: 5.7, 6.7, 8, 8.6, 10, 13.3, 14.3, 17.1, 20, 23.3, 28, 30, 40, 46.7, 50, 60, and 70 kΩ.

1.5 Shunting the 10 kΩ by a resistor of value of R result in the combination having a resistance  $R_{eq}$ ,

$$R_{eq} = \frac{10R}{R + 10}$$

Thus, for a 1% reduction,

$$\frac{R}{R + 10} = 0.99 \Rightarrow R = 990 \text{ k}\Omega$$

For a 5% reduction,

$$\frac{R}{R + 10} = 0.95 \Rightarrow R = 190 \text{ k}\Omega$$

For a 10% reduction,

$$\frac{R}{R + 10} = 0.90 \Rightarrow R = 90 \text{ k}\Omega$$

For a 50% reduction,

$$\frac{R}{R + 10} = 0.50 \Rightarrow R = 10 \text{ k}\Omega$$

Shunting the 10 kΩ by

(a) 1 MΩ results in

$$R_{eq} = \frac{10 \times 1000}{1000 + 10} = \frac{10}{1.01} = 9.9 \text{ k}\Omega$$

a 1% reduction;

(b) 100 kΩ results in

$$R_{eq} = \frac{10 \times 100}{100 + 10} = \frac{10}{1.1} = 9.09 \text{ k}\Omega$$

a 9.1% reduction;

(c) 10 kΩ results in

$$R_{eq} = \frac{10}{10 + 10} = 5 \text{ k}\Omega$$

a 50% reduction.

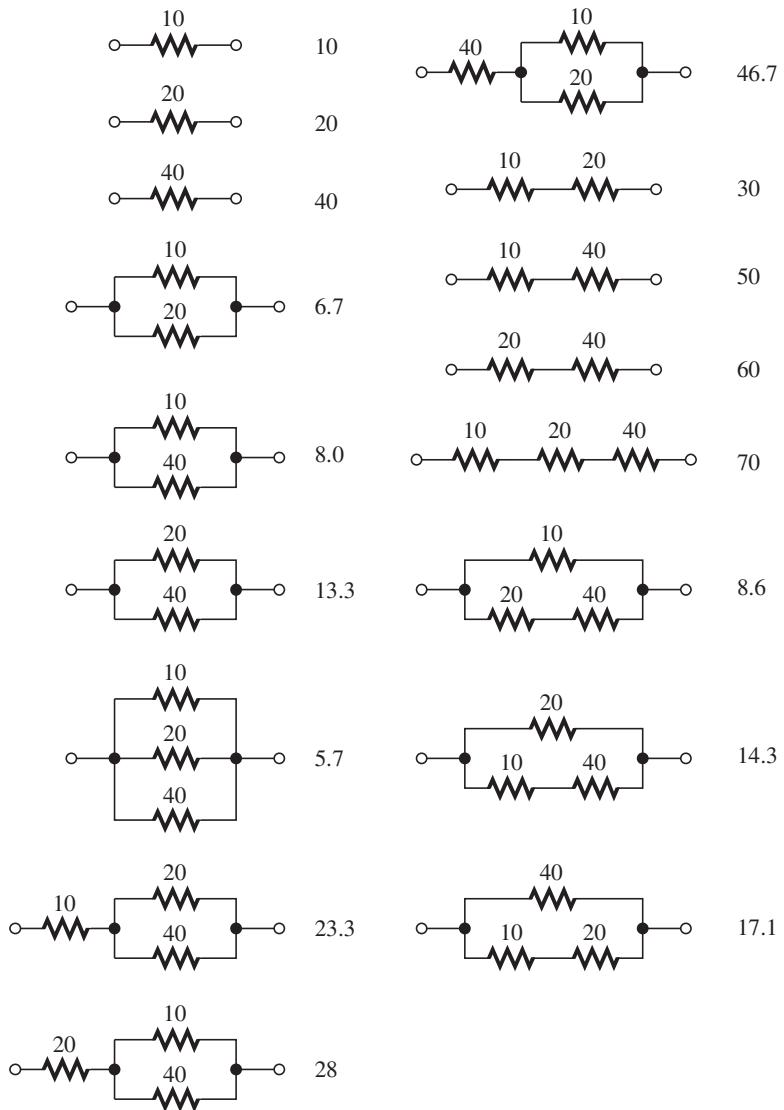
1.6  $V_o = V_{DD} \frac{R_2}{R_1 + R_2}$

To find  $R_o$ , we short-circuit  $V_{DD}$  and look back into node X,

$$R_o = R_2 \parallel R_1 = \frac{R_1 R_2}{R_1 + R_2}$$

## Chapter 1–2

This figure belongs to 1.4.



1.7 Use voltage divider to find  $V_O$

$$V_O = 5 \frac{2}{2+3} = 2 \text{ V}$$

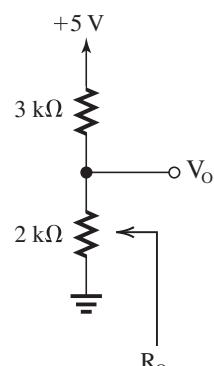
Equivalent output resistance  $R_O$  is

$$R_O = (2 \text{ k}\Omega \parallel 3 \text{ k}\Omega) = 1.2 \text{ k}\Omega$$

The extreme values of  $V_O$  for  $\pm 5\%$  tolerance resistor are

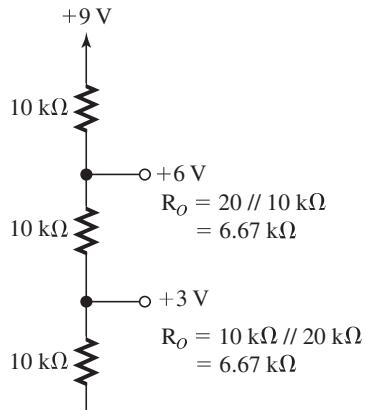
$$V_{O\min} = 5 \frac{2(1 - 0.05)}{2(1 - 0.05) + 3(1 + 0.05)} \\ = 1.88 \text{ V}$$

$$V_{O\max} = 5 \frac{2(1 + 0.05)}{2(1 + 0.05) + 3(1 - 0.05)} \\ = 2.12 \text{ V}$$

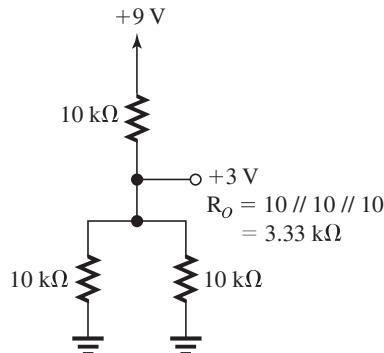


The extreme values of  $R_O$  for  $\pm 5\%$  tolerance resistors are  $1.2 \times 1.05 = 1.26 \text{ k}\Omega$  and  $1.2 \times 0.95 = 1.14 \text{ k}\Omega$ .

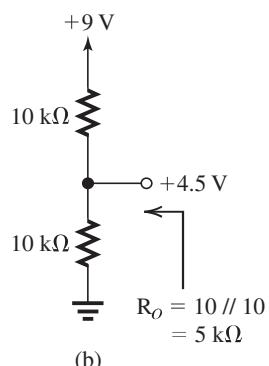
1.8



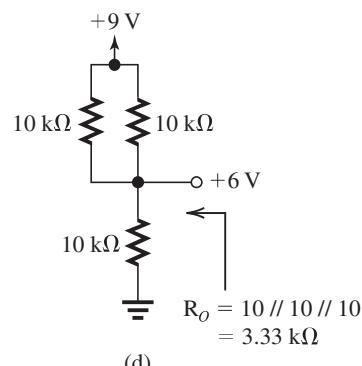
(a)



(c)



(b)



(d)

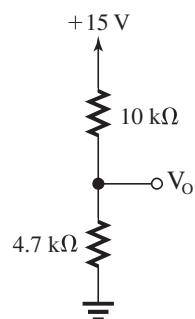
Voltage generated:

$+3\text{ V}$  [two ways: (a) and (c) with (c) having lower output resistance]

$+4.5\text{ V}$  (b)

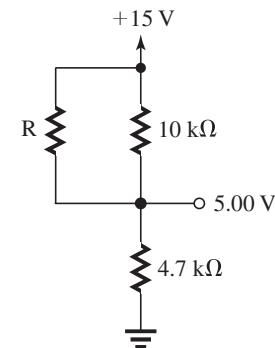
$+6\text{ V}$  [two ways: (a) and (d) with (d) having a lower output resistance]

1.9



$$V_O = 15 \frac{4.7}{10 + 4.7} = 4.80\text{ V}$$

To increase  $V_O$  to 10.00 V, we shunt the 10-kΩ resistor by a resistor  $R$  whose value is such that  $10 // R = 2 \times 4.7$ .



Thus

$$\frac{1}{10} + \frac{1}{R} = \frac{1}{9.4}$$

$$\Rightarrow R = 156.7 \approx 157\text{ k}\Omega$$

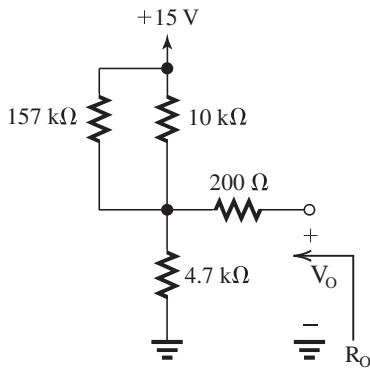
Now,

$$R_O = 10\text{ k}\Omega // R // 4.7\text{ k}\Omega$$

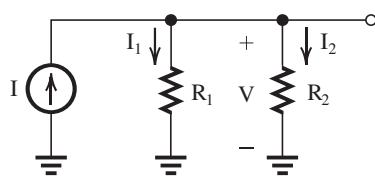
$$= 9.4 // 4.7 = \frac{9.4}{3} = 3.133\text{ k}\Omega$$

To make  $R_O = 3.33$ , we add a series resistance of approximately 200 Ω, as shown on the next page.

This figure belongs to 1.9.



1.10



$$V = I(R_1 \parallel R_2)$$

$$= I \frac{R_1 R_2}{R_1 + R_2}$$

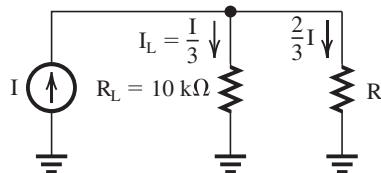
$$I_1 = \frac{V}{R_1} = I \frac{R_2}{R_1 + R_2}$$

$$I_2 = \frac{V}{R_2} = I \frac{R_1}{R_1 + R_2}$$

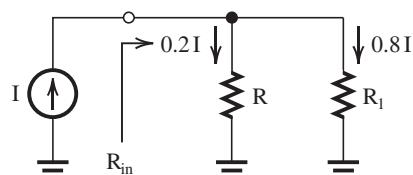
1.11 Connect a resistor  $R$  in parallel with  $R_L$ . To make  $I_L = I/3$  (and thus the current through  $R$ ,  $2I/3$ ),  $R$  should be such that

$$10I/3 = 2IR/3$$

$$\Rightarrow R = 5 \text{ k}\Omega$$



1.12



To make the current through  $R$  equal to  $0.2I$ , we shunt  $R$  by a resistance  $R_1$  having a value such

that the current through it will be  $0.8I$ ; thus

$$0.2IR = 0.8IR_1 \Rightarrow R_1 = \frac{R}{4}$$

The input resistance of the divider,  $R_{in}$ , is

$$R_{in} = R \parallel R_1 = R \parallel \frac{R}{4} = \frac{1}{5}R$$

Now if  $R_1$  is 10% too high, that is, if

$$R_1 = 1.1 \frac{R}{4}$$

the problem can be solved in two ways:

(a) Connect a resistor  $R_2$  across  $R_1$  of value such that  $R_2 \parallel R_1 = R/4$ , thus

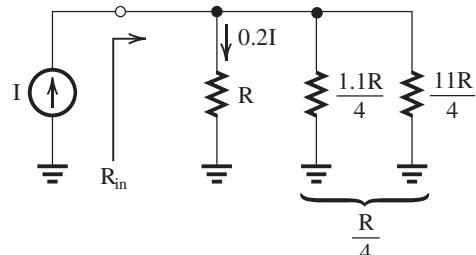
$$\frac{R_2(1.1R/4)}{R_2 + (1.1R/4)} = \frac{R}{4}$$

$$1.1R_2 = R_2 + \frac{1.1R}{4}$$

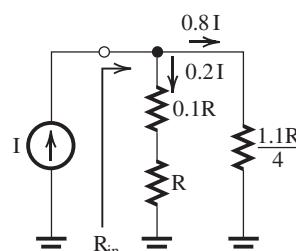
$$\Rightarrow R_2 = \frac{11R}{4} = 2.75R$$

$$R_{in} = R \parallel \frac{1.1R}{4} \parallel \frac{11R}{4}$$

$$= R \parallel \frac{R}{4} = \frac{R}{5}$$



(b) Connect a resistor in series with the load resistor  $R$  so as to raise the resistance of the load branch by 10%, thereby restoring the current division ratio to its desired value. The added series resistance must be 10% of  $R$  (i.e.,  $0.1R$ ).

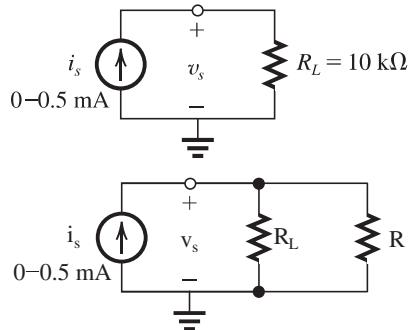


$$R_{in} = 1.1R \parallel \frac{1.1R}{4}$$

$$= \frac{1.1R}{5}$$

that is, 10% higher than in case (a).

1.13 For  $R_L = 10 \text{ k}\Omega$ , when signal source generates 0–0.5 mA, a voltage of 0–2 V may appear across the source



To limit  $v_s \leq 1 \text{ V}$ , the net resistance has to be  $\leq 2 \text{ k}\Omega$ . To achieve this we have to shunt  $R_L$  with a resistor  $R$  so that  $(R \parallel R_L) \leq 2 \text{ k}\Omega$ .

$$R \parallel R_L \leq 2 \text{ k}\Omega.$$

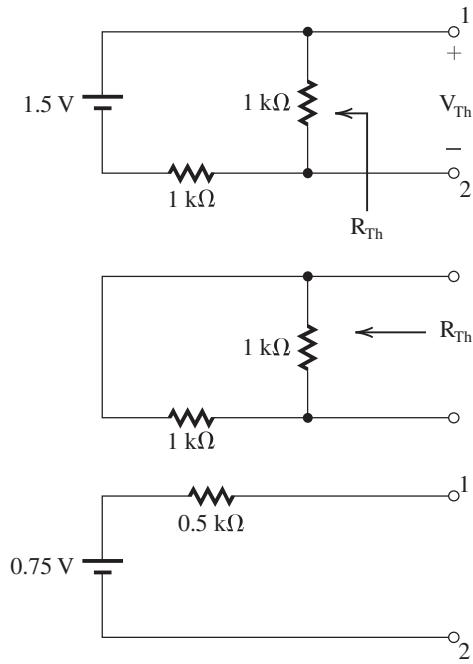
$$\frac{RR_L}{R+R_L} \leq 2 \text{ k}\Omega$$

For  $R_L = 10 \text{ k}\Omega$

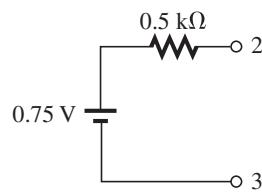
$$R \leq 2.5 \text{ k}\Omega$$

The resulting circuit needs only one additional resistance of  $2 \text{ k}\Omega$  in parallel with  $R_L$  so that  $v_s \leq 1 \text{ V}$ . The circuit is a current divider, and the current through  $R_L$  is now 0–0.1 mA.

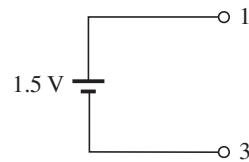
1.14 (a) Between terminals 1 and 2:



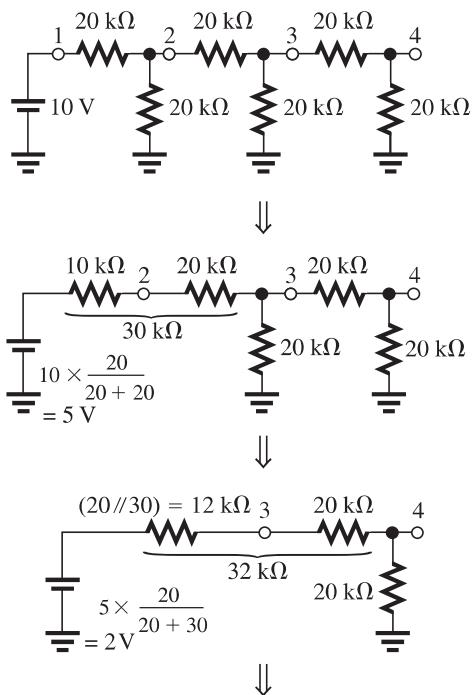
(b) Same procedure is used for (b) to obtain



(c) Between terminals 1 and 3, the open-circuit voltage is 1.5 V. When we short circuit the voltage source, we see that the Thévenin resistance will be zero. The equivalent circuit is then



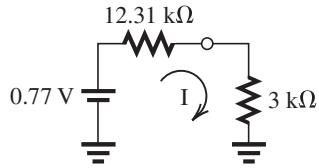
1.15



Thévenin equivalent:  $(20//32) = 12.31 \text{ k}\Omega$

$$2 \times \frac{20}{20+32} = 0.77 \text{ V}$$

(cont d on the next page)



Now, when a resistance of  $3\text{ k}\Omega$  is connected between node 4 and ground,

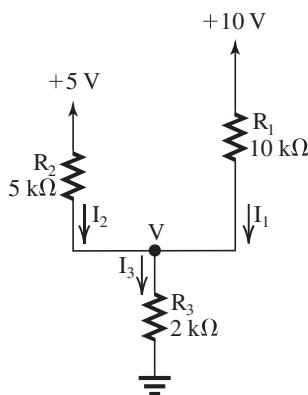
$$\begin{aligned} I &= \frac{0.77}{12.31 + 3} \\ &= 0.05 \text{ mA} \end{aligned}$$

1.16 (a) Node equation at the common mode yields

$$I_3 = I_1 + I_2$$

Using the fact that the sum of the voltage drops across  $R_1$  and  $R_3$  equals 10 V, we write

$$\begin{aligned} 10 &= I_1 R_1 + I_3 R_3 \\ &= 10I_1 + (I_1 + I_2) \times 2 \\ &= 12I_1 + 2I_2 \end{aligned}$$



That is,

$$12I_1 + 2I_2 = 10 \quad (1)$$

Similarly, the voltage drops across  $R_2$  and  $R_3$  add up to 5 V, thus

$$5 = I_2 R_2 + I_3 R_3$$

$$= 5I_2 + (I_1 + I_2) \times 2$$

which yields

$$2I_1 + 7I_2 = 5 \quad (2)$$

Equations (1) and (2) can be solved together by multiplying Eq. (2) by 6:

$$12I_1 + 42I_2 = 30 \quad (3)$$

Now, subtracting Eq. (1) from Eq. (3) yields

$$\begin{aligned} 40I_2 &= 20 \\ \Rightarrow I_2 &= 0.5 \text{ mA} \end{aligned}$$

Substituting in Eq. (2) gives

$$\begin{aligned} 2I_1 &= 5 - 7 \times 0.5 \text{ mA} \\ \Rightarrow I_1 &= 0.75 \text{ mA} \\ I_3 &= I_1 + I_2 \\ &= 0.75 + 0.5 \end{aligned}$$

$$\begin{aligned} &= 1.25 \text{ mA} \\ V &= I_3 R_3 \\ &= 1.25 \times 2 = 2.5 \text{ V} \end{aligned}$$

To summarize:

$$\begin{aligned} I_1 &= 0.75 \text{ mA} & I_2 &= 0.5 \text{ mA} \\ I_3 &= 1.25 \text{ mA} & V &= 2.5 \text{ V} \end{aligned}$$

(b) A node equation at the common node can be written in terms of  $V$  as

$$\frac{10 - V}{R_1} + \frac{5 - V}{R_2} = \frac{V}{R_3}$$

Thus,

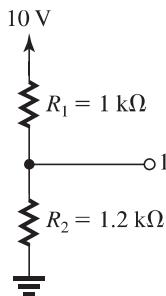
$$\begin{aligned} \frac{10 - V}{10} + \frac{5 - V}{5} &= \frac{V}{2} \\ \Rightarrow 0.8V &= 2 \\ \Rightarrow V &= 2.5 \text{ V} \end{aligned}$$

Now,  $I_1$ ,  $I_2$ , and  $I_3$  can be easily found as

$$\begin{aligned} I_1 &= \frac{10 - V}{10} = \frac{10 - 2.5}{10} \\ &= 0.75 \text{ mA} \\ I_2 &= \frac{5 - V}{5} = \frac{5 - 2.5}{5} \\ &= 0.5 \text{ mA} \\ I_3 &= \frac{V}{R_3} = \frac{2.5}{2} = 1.25 \text{ mA} \end{aligned}$$

Method (b) is much preferred, being faster, more insightful, and less prone to errors. In general, one attempts to identify the lowest possible number of variables and write the corresponding minimum number of equations.

1.17 Find the Thévenin equivalent of the circuit to the left of node 1.

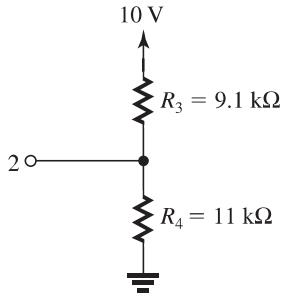


Between node 1 and ground,

$$R_{Th} = (1 \text{ k}\Omega \parallel 1.2 \text{ k}\Omega) = 0.545 \text{ k}\Omega$$

$$V_{Th} = 10 \times \frac{1.2}{1 + 1.2} = 5.45 \text{ V}$$

Find the Thévenin equivalent of the circuit to the right of node 2.

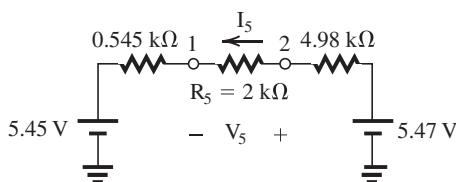


Between node 2 and ground,

$$R_{Th} = 9.1 \text{ k}\Omega \parallel 11 \text{ k}\Omega = 4.98 \text{ k}\Omega$$

$$V_{Th} = 10 \times \frac{11}{11 + 9.1} = 5.47 \text{ V}$$

The resulting simplified circuit is



$$I_5 = \frac{5.47 - 5.45}{4.98 + 2 + 0.545}$$

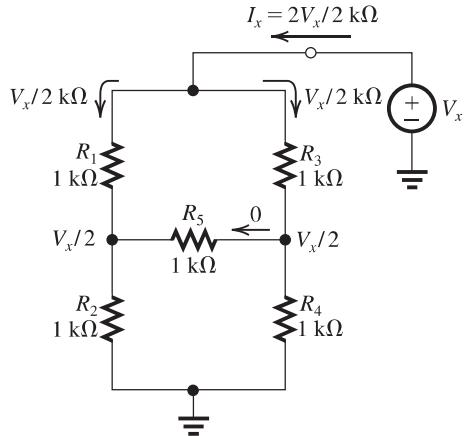
$$= 2.66 \mu\text{A}$$

$$V_5 = 2.66 \mu\text{A} \times 2 \text{ k}\Omega \\ = 5.32 \text{ mV}$$

1.18 From the symmetry of the circuit, there will be no current in  $R_5$ . (Otherwise the symmetry would be violated.) Thus each branch will carry a

current  $V_x/2 \text{ k}\Omega$  and  $I_x$  will be the sum of the two currents,

$$I_x = \frac{2V_x}{2 \text{ k}\Omega} = \frac{V_x}{1 \text{ k}\Omega}$$



Thus,

$$R_{eq} \equiv \frac{V_x}{I_x} = 1 \text{ k}\Omega$$

Now, if  $R_4$  is raised to  $1.2 \text{ k}\Omega$ , the symmetry will be broken. To find  $I_5$  we use Thévenin's theorem as shown in the figures on the next page. Thus,

$$I_5 = \frac{0.545V_x - 0.5V_x}{0.5 + 1 + 0.545} = 0.022V_x$$

$$V_1 = \frac{V_x}{2} + 0.022 V_x \times 0.5$$

$$= 0.5V_x \times 1.022 = 0.511V_x$$

$$V_2 = V_1 + I_5 R_5 = 0.533V_x$$

$$I_1 = \frac{V_x - V_1}{1 \text{ k}\Omega} = 0.489V_x$$

$$I_2 = \frac{V_x - V_2}{1 \text{ k}\Omega} = 0.467V_x$$

$$I_x = I_1 + I_2 = 0.956V_x$$

$$\Rightarrow R_{eq} \equiv \frac{V_x}{I_x} = 1.05 \text{ k}\Omega$$

1.19 Refer to Fig. P1.19. Using the voltage divider rule at the input side, we obtain

$$\frac{v_\pi}{v_s} = \frac{r_\pi}{r_\pi + R_s} \quad (1)$$

At the output side, we find  $v_o$  by multiplying the current  $g_m v_\pi$  by the parallel equivalent of  $r_o$  and  $R_L$ ,

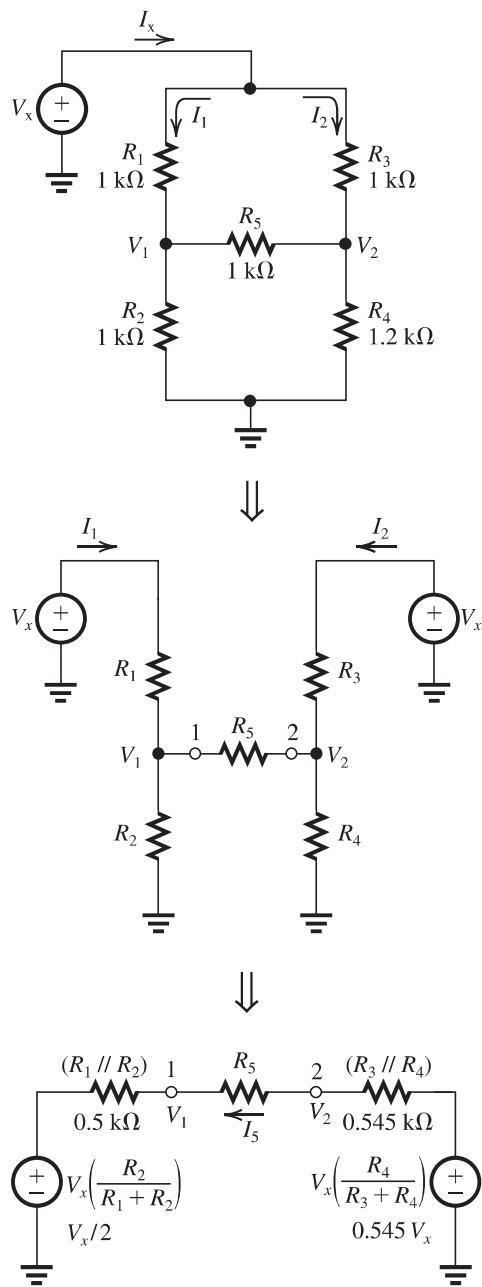
$$v_o = -g_m v_\pi (r_o \parallel R_L) \quad (2)$$

Finally,  $v_o/v_s$  can be obtained by combining Eqs. (1) and (2) as

$$\frac{v_o}{v_s} = -\frac{r_\pi}{r_\pi + R_s} g_m (r_o \parallel R_L)$$

Chapter 1–8

This figure belongs to Problem 1.18.



$$1.20 \text{ (a) } T = 10^{-4} \text{ ms} = 10^{-7} \text{ s}$$

$$f = \frac{1}{T} = 10^7 \text{ Hz}$$

$$\omega = 2\pi f = 6.28 \times 10^7 \text{ rad/s}$$

$$\text{(b) } f = 1 \text{ GHz} = 10^9 \text{ Hz}$$

$$T = \frac{1}{f} = 10^{-9} \text{ s}$$

$$\omega = 2\pi f = 6.28 \times 10^9 \text{ rad/s}$$

$$\text{(c) } \omega = 6.28 \times 10^2 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = 10^2 \text{ Hz}$$

$$T = \frac{1}{f} = 10^{-2} \text{ s}$$

$$\text{(d) } T = 10 \text{ s}$$

$$f = \frac{1}{T} = 10^{-1} \text{ Hz}$$

$$\omega = 2\pi f = 6.28 \times 10^{-1} \text{ rad/s}$$

$$\text{(e) } f = 60 \text{ Hz}$$

$$T = \frac{1}{f} = 1.67 \times 10^{-2} \text{ s}$$

$$\omega = 2\pi f = 3.77 \times 10^2 \text{ rad/s}$$

$$\text{(f) } \omega = 1 \text{ krad/s} = 10^3 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = 1.59 \times 10^2 \text{ Hz}$$

$$T = \frac{1}{f} = 6.28 \times 10^{-3} \text{ s}$$

$$\text{(g) } f = 1900 \text{ MHz} = 1.9 \times 10^9 \text{ Hz}$$

$$T = \frac{1}{f} = 5.26 \times 10^{-10} \text{ s}$$

$$\omega = 2\pi f = 1.194 \times 10^{10} \text{ rad/s}$$

1.21 (a)  $Z = 1 \text{ k}\Omega$  at all frequencies

$$\text{(b) } Z = 1/j\omega C = -j\frac{1}{2\pi f \times 10 \times 10^{-9}}$$

$$\text{At } f = 60 \text{ Hz, } Z = -j265 \text{ k}\Omega$$

$$\text{At } f = 100 \text{ kHz, } Z = -j159 \text{ }\Omega$$

$$\text{At } f = 1 \text{ GHz, } Z = -j0.016 \text{ }\Omega$$

$$\text{(c) } Z = 1/j\omega C = -j\frac{1}{2\pi f \times 10 \times 10^{-12}}$$

$$\text{At } f = 60 \text{ Hz, } Z = -j0.265 \text{ G}\Omega$$

$$\text{At } f = 100 \text{ kHz, } Z = -j0.16 \text{ M}\Omega$$

$$\text{At } f = 1 \text{ GHz, } Z = -j15.9 \text{ }\Omega$$

$$\text{(d) } Z = j\omega L = j2\pi f L = j2\pi f \times 10 \times 10^{-3}$$

$$\text{At } f = 60 \text{ Hz, } Z = j3.77 \text{ }\Omega$$

$$\text{At } f = 100 \text{ kHz, } Z = j6.28 \text{ k}\Omega$$

$$\text{At } f = 1 \text{ GHz, } Z = j62.8 \text{ M}\Omega$$

$$\text{(e) } Z = j\omega L = j2\pi f L = j2\pi f (1 \times 10^{-6})$$

$$\text{f} = 60 \text{ Hz, } Z = j0.377 \text{ m}\Omega$$

$$\text{f} = 100 \text{ kHz, } Z = j0.628 \text{ }\Omega$$

$$\text{f} = 1 \text{ GHz, } Z = j6.28 \text{ k}\Omega$$

## Chapter 1-9

$$1.22 \text{ (a)} Z = R + \frac{1}{j\omega C}$$

$$= 10^3 + \frac{1}{j2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}}$$

$$= (1 - j1.59) \text{ k}\Omega$$

$$\text{(b)} Y = \frac{1}{R} + j\omega C$$

$$= \frac{1}{10^4} + j2\pi \times 10 \times 10^3 \times 0.01 \times 10^{-6}$$

$$= 10^{-4}(1 + j6.28) \Omega$$

$$Z = \frac{1}{Y} = \frac{10^4}{1 + j6.28}$$

$$= \frac{10^4(1 - j6.28)}{1 + 6.28^2}$$

$$= (247.3 - j1553) \Omega$$

$$\text{(c)} Y = \frac{1}{R} + j\omega C$$

$$= \frac{1}{100 \times 10^3} + j2\pi \times 10 \times 10^3 \times 100 \times 10^{-12}$$

$$= 10^{-5}(1 + j0.628)$$

$$Z = \frac{10^5}{1 + j0.628}$$

$$= (71.72 - j45.04) \text{ k}\Omega$$

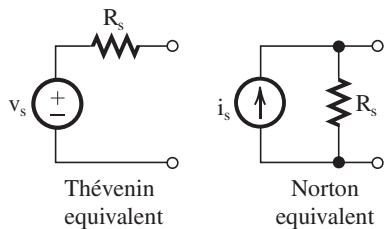
$$\text{(d)} Z = R + j\omega L$$

$$= 100 + j2\pi \times 10 \times 10^3 \times 10 \times 10^{-3}$$

$$= 100 + j6.28 \times 100$$

$$= (100 + j628), \Omega$$

1.23



$$v_{oc} = v_s$$

$$i_{sc} = i_s$$

$$v_s = i_s R_s$$

Thus,

$$R_s = \frac{v_{oc}}{i_{sc}}$$

$$\text{(a)} v_s = v_{oc} = 1 \text{ V}$$

$$i_s = i_{sc} = 0.1 \text{ mA}$$

$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{1 \text{ V}}{0.1 \text{ mA}} = 10 \text{ k}\Omega$$

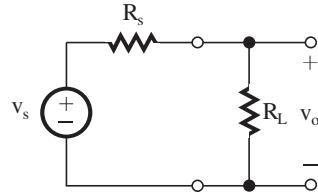
$$\text{(b)} v_s = v_{oc} = 0.1 \text{ V}$$

$$i_s = i_{sc} = 1 \mu\text{A}$$

$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{0.1 \text{ V}}{1 \mu\text{A}} = 0.1 \text{ M}\Omega = 100 \text{ k}\Omega$$

1.24 The observed output voltage is 1 mV/°C, which is one half the voltage specified by the sensor, presumably under open-circuit conditions: that is, without a load connected. It follows that that sensor internal resistance must be equal to  $R_L$ , that is, 5 kΩ.

1.25



$$\frac{v_o}{v_s} = \frac{R_L}{R_L + R_s}$$

$$v_o = v_s \left/ \left( 1 + \frac{R_s}{R_L} \right) \right.$$

Thus,

$$\frac{v_s}{1 + \frac{R_s}{100}} = 40 \quad (1)$$

and

$$\frac{v_s}{1 + \frac{R_s}{10}} = 10 \quad (2)$$

Dividing Eq. (1) by Eq. (2) gives

$$\frac{1 + (R_s / 10)}{1 + (R_s / 100)} = 4$$

$$\Rightarrow R_s = 50 \text{ k}\Omega$$

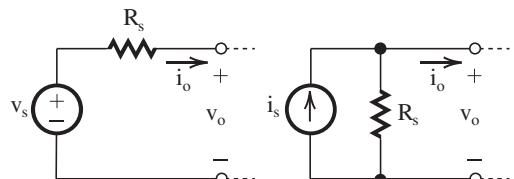
Substituting in Eq. (2) gives

$$v_s = 60 \text{ mV}$$

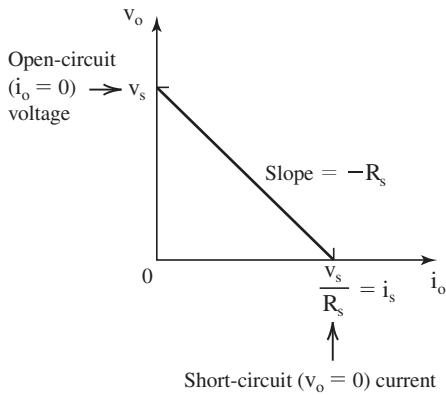
The Norton current  $i_s$  can be found as

$$i_s = \frac{v_s}{R_s} = \frac{60 \text{ mV}}{50 \text{ k}\Omega} = 1.2 \mu\text{A}$$

1.26



$$v_o = v_s - i_o R_s$$



1.27

Case	$\omega$ (rad/s)	f (Hz)	T(s)
a	$3.14 \times 10^{10}$	$5 \times 10^9$	$0.2 \times 10^{-9}$
b	$2 \times 10^9$	$3.18 \times 10^8$	$3.14 \times 10^{-9}$
c	$6.28 \times 10^{10}$	$1 \times 10^{10}$	$1 \times 10^{-10}$
d	$3.77 \times 10^2$	60	$1.67 \times 10^{-2}$
e	$6.28 \times 10^4$	$1 \times 10^4$	$1 \times 10^{-4}$
f	$6.28 \times 10^5$	$1 \times 10^5$	$1 \times 10^{-5}$

1.28 (a)  $v = 10 \sin(2\pi \times 10^3 t)$ , V

(b)  $v = 120\sqrt{2} \sin(2\pi \times 60)$ , V

(c)  $v = 0.1 \sin(2000t)$ , V

(d)  $v = 0.1 \sin(2\pi \times 10^3 t)$ , V

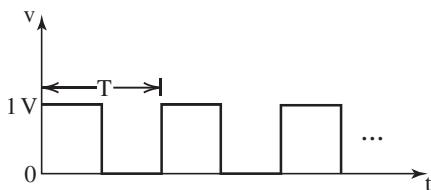
1.29 (a)  $V_{\text{peak}} = 117 \times \sqrt{2} = 165$  V

(b)  $V_{\text{rms}} = 33.9/\sqrt{2} = 24$  V

(c)  $V_{\text{peak}} = 220 \times \sqrt{2} = 311$  V

(d)  $V_{\text{peak}} = 220 \times \sqrt{2} = 311$  kV

1.30 Comparing the given waveform to that described by Eq. (1.2), we observe that the given waveform has an amplitude of 0.5 V (1 V peak-to-peak) and its level is shifted up by 0.5 V (the first term in the equation). Thus the waveform looks as follows:



Average value = 0.5 V

Peak-to-peak value = 1 V

Lowest value = 0 V

Highest value = 1 V

$$\text{Period } T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 10^{-3} \text{ s}$$

$$\text{Frequency } f = \frac{1}{T} = 1 \text{ kHz}$$

1.31 The two harmonics have the ratio  $126/98 = 9/7$ . Thus, these are the 7th and 9th harmonics. From Eq. (1.2), we note that the amplitudes of these two harmonics will have the ratio 7 to 9, which is confirmed by the measurement reported. Thus the fundamental will have a frequency of  $98/7$ , or 14 kHz, and peak amplitude of  $63 \times 7 = 441$  mV. The rms value of the fundamental will be  $441/\sqrt{2} = 312$  mV. To find the peak-to-peak amplitude of the square wave, we note that  $4V/\pi = 441$  mV. Thus,

Peak-to-peak amplitude

$$= 2V = 441 \times \frac{\pi}{2} = 693 \text{ mV}$$

$$\text{Period } T = \frac{1}{f} = \frac{1}{14 \times 10^3} = 71.4 \mu\text{s}$$

1.32

Decimal	Binary
0	0
6	110
11	1011
28	11100
59	111011

1.33

$b_3$	$b_2$	$b_1$	$b_0$	Value Represented
0	0	0	0	+0
0	0	0	1	+1
0	0	1	0	+2
0	0	1	1	+3
0	1	0	0	+4
0	1	0	1	+5
0	1	1	0	+6
0	1	1	1	+7
1	0	0	0	-0
1	0	0	1	-1
1	0	1	0	-2
1	0	1	1	-3
1	1	0	0	-4
1	1	0	1	-5
1	1	1	0	-6
1	1	1	1	-7

Note that there are two possible representations of zero: 0000 and 1000. For a 0.5-V step size, analog signals in the range  $\pm 3.5$  V can be represented.

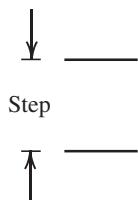
Input	Steps	Code
+2.5 V	+5	0101
-3.0 V	-6	1110
+2.7	+5	0101
-2.8	-6	1110

1.34 (a) For  $N$  bits there will be  $2^N$  possible levels, from 0 to  $V_{FS}$ . Thus there will be  $(2^N - 1)$  discrete steps from 0 to  $V_{FS}$  with the step size given by

$$\text{Step size} = \frac{V_{FS}}{2^N - 1}$$

This is the analog change corresponding to a change in the LSB. It is the value of the resolution of the ADC.

(b) The maximum error in conversion occurs when the analog signal value is at the middle of a step. Thus the maximum error is



$$\frac{1}{2} \times \text{step size} = \frac{1}{2} \frac{V_{FS}}{2^N - 1}$$

This is known as the quantization error.

$$(c) \frac{5 \text{ V}}{2^N - 1} \leq 2 \text{ mV}$$

$$2^N - 1 \geq 2500$$

$$2^N \geq 2501 \Rightarrow N = 12,$$

For  $N = 12$ ,

$$\text{Resolution} = \frac{5}{2^{12} - 1} = 1.2 \text{ mV}$$

$$\text{Quantization error} = \frac{1.2}{2} = 0.6 \text{ mV}$$

1.35 (a) When  $b_i = 1$ , the  $i$ th switch is in position 1 and a current  $(V_{ref}/2^i R)$  flows to the output. Thus  $i_O$  will be the sum of all the currents corresponding to 1 bits, that is,

$$i_O = \frac{V_{ref}}{R} \left( \frac{b_1}{2^1} + \frac{b_2}{2^2} + \cdots + \frac{b_N}{2^N} \right)$$

(b)  $b_N$  is the LSB

$b_1$  is the MSB

$$(c) i_{O\max} = \frac{10 \text{ V}}{10 \text{ k}\Omega} \left( \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} \right)$$

$$= 0.99609375 \text{ mA}$$

Corresponding to the LSB changing from 0 to 1 the output changes by  $(10/10) \times 1/2^8 = 3.91 \mu\text{A}$ .

1.36 There will be 44,100 samples per second with each sample represented by 16 bits. Thus the throughput or speed will be  $44,100 \times 16 = 7.056 \times 10^5$  bits per second.

$$1.37 (a) A_v = \frac{v_o}{v_i} = \frac{10 \text{ V}}{100 \text{ mV}} = 100 \text{ V/V}$$

$$\text{or } 20 \log 100 = 40 \text{ dB}$$

$$A_i = \frac{i_o}{i_l} = \frac{v_o/R_L}{i_l} = \frac{10 \text{ V}/100 \Omega}{100 \mu\text{A}} = \frac{0.1 \text{ A}}{100 \mu\text{A}}$$

$$= 1000 \text{ A/A}$$

$$\text{or } 20 \log 1000 = 60 \text{ dB}$$

$$A_p = \frac{v_o i_o}{v_i i_l} = \frac{v_o}{v_i} \times \frac{i_o}{i_l} = 100 \times 1000$$

$$= 10^5 \text{ W/W}$$

$$\text{or } 10 \log 10^5 = 50 \text{ dB}$$

$$(b) A_v = \frac{v_o}{v_i} = \frac{1 \text{ V}}{10 \mu\text{V}} = 1 \times 10^5 \text{ V/V}$$

$$\text{or } 20 \log 1 \times 10^5 = 100 \text{ dB}$$

$$A_i = \frac{i_o}{i_l} = \frac{v_o/R_L}{i_l} = \frac{1 \text{ V}/10 \text{ k}\Omega}{100 \text{ nA}}$$

$$= \frac{0.1 \text{ mA}}{100 \text{ nA}} = \frac{0.1 \times 10^{-3}}{100 \times 10^{-9}} = 1000 \text{ A/A}$$

$$\text{or } 20 \log A_i = 60 \text{ dB}$$

$$A_p = \frac{v_o i_o}{v_i i_l} = \frac{v_o}{v_i} \times \frac{i_o}{i_l}$$

$$= 1 \times 10^5 \times 1000$$

$$= 1 \times 10^8 \text{ W/W}$$

$$\text{or } 10 \log A_p = 80 \text{ dB}$$

$$(c) A_v = \frac{v_o}{v_i} = \frac{5 \text{ V}}{1 \text{ V}} = 5 \text{ V/V}$$

$$\text{or } 20 \log 5 = 14 \text{ dB}$$

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{i_i} = \frac{5 V / 10 \Omega}{1 \text{ mA}}$$

$$= \frac{0.5 \text{ A}}{1 \text{ mA}} = 500 \text{ A/A}$$

or  $20 \log 500 = 54 \text{ dB}$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i}$$

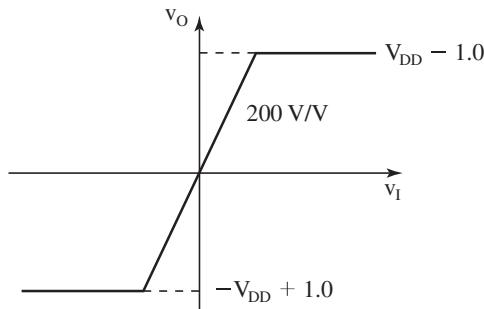
$$= 5 \times 500 = 2500 \text{ W/W}$$

or  $10 \log A_p = 34 \text{ dB}$

1.38 For  $\pm 5 \text{ V}$  supplies:

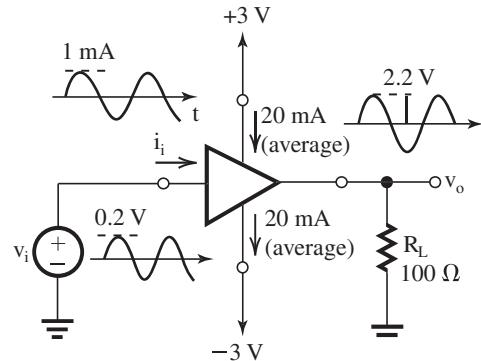
The largest undistorted sine-wave output is of 4-V peak amplitude or  $4/\sqrt{2} = 2.8 \text{ V}_{\text{rms}}$ . Input needed is  $14 \text{ mV}_{\text{rms}}$ .

For  $\pm 10\text{-V}$  supplies, the largest undistorted sine-wave output is of 9-V peak amplitude or  $6.4 \text{ V}_{\text{rms}}$ . Input needed is  $32 \text{ mV}_{\text{rms}}$ .



For  $\pm 15\text{-V}$  supplies, the largest undistorted sine-wave output is of 14-V peak amplitude or  $9.9 \text{ V}_{\text{rms}}$ . The input needed is  $9.9 \text{ V}/200 = 49.5 \text{ mV}_{\text{rms}}$ .

1.39



$$A_v = \frac{v_o}{v_i} = \frac{2.2}{0.2}$$

$$= 11 \text{ V/V}$$

or  $20 \log 11 = 20.8 \text{ dB}$

$$A_i = \frac{i_o}{i_i} = \frac{2.2 \text{ V} / 100 \Omega}{1 \text{ mA}}$$

$$= \frac{22 \text{ mA}}{1 \text{ mA}} = 22 \text{ A/A}$$

or  $20 \log A_i = 26.8 \text{ dB}$

$$A_p = \frac{p_o}{p_i} = \frac{(2.2/\sqrt{2})^2 / 100}{\frac{0.2}{\sqrt{2}} \times \frac{10^{-3}}{\sqrt{2}}}$$

$$= 242 \text{ W/W}$$

or  $10 \log A_p = 23.8 \text{ dB}$

Supply power =  $2 \times 3 \text{ V} \times 20 \text{ mA} = 120 \text{ mW}$

$$\text{Output power} = \frac{v_{\text{rms}}^2}{R_L} = \frac{(2.2/\sqrt{2})^2}{100 \Omega} = 24.2 \text{ mW}$$

$$\text{Input power} = \frac{24.2}{242} = 0.1 \text{ mW} \text{ (negligible)}$$

Amplifier dissipation  $\simeq$  Supply power - Output power

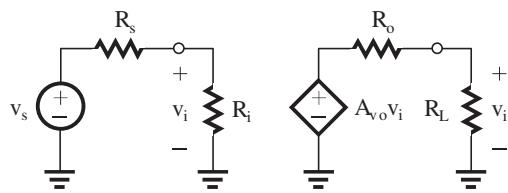
$$= 120 - 24.2 = 95.8 \text{ mW}$$

$$\text{Amplifier efficiency} = \frac{\text{Output power}}{\text{Supply power}} \times 100$$

$$= \frac{24.2}{120} \times 100 = 20.2\%$$

$$1.40 \quad v_o = A_{v_o} v_i \frac{R_L}{R_L + R_o}$$

$$= A_{v_o} \left( v_s \frac{R_i}{R_i + R_s} \right) \frac{R_L}{R_L + R_o}$$



Thus,

$$\frac{v_o}{v_s} = A_{v_o} \frac{R_i}{R_i + R_s} \frac{R_L}{R_L + R_o}$$

(a)  $A_{v_o} = 100, R_i = 10R_s, R_L = 10R_o$ :

$$\frac{v_o}{v_s} = 100 \times \frac{10R_s}{10R_s + R_s} \times \frac{10R_o}{10R_o + R_o}$$

$$= 82.6 \text{ V/V or } 20 \log 82.6 = 38.3 \text{ dB}$$

(b)  $A_{v_o} = 100, R_i = R_s, R_L = R_o$ :

$$\frac{v_o}{v_s} = 100 \times \frac{1}{2} \times \frac{1}{2} = 25 \text{ V/V or } 20 \log 25 = 28 \text{ dB}$$

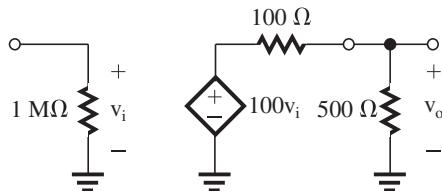
(c)  $A_{vo} = 100 \text{ V/V}$ ,  $R_i = R_s/10$ ,  $R_L = R_o/10$ :

$$\frac{v_o}{v_s} = 100 \frac{R_s/10}{(R_s/10) + R_s} \frac{R_o/10}{(R_o/10) + R_o}$$

$$= 0.826 \text{ V/V} \text{ or } 20 \log 0.826 = -1.7 \text{ dB}$$

$$\begin{aligned}\frac{v_o}{v_s} &= \frac{R_L}{R_L + R_s} \\ &= \frac{100 \Omega}{100 \Omega + 100 \text{ k}\Omega} \\ &\simeq 0.001 \text{ V/V}\end{aligned}$$

1.41

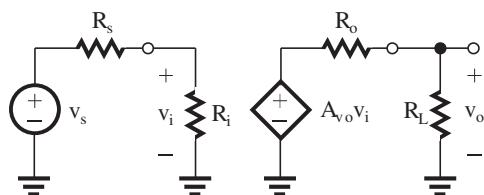


$$20 \log A_{vo} = 40 \text{ dB} \Rightarrow A_{vo} = 100 \text{ V/V}$$

$$\begin{aligned}A_v &= \frac{v_o}{v_i} \\ &= 100 \times \frac{500}{500 + 100} \\ &= 83.3 \text{ V/V} \\ \text{or } 20 \log 83.3 &= 38.4 \text{ dB} \\ A_p &= \frac{v_o^2/500 \Omega}{v_i^2/1 \text{ M}\Omega} = A_v^2 \times 10^4 = 1.39 \times 10^7 \text{ W/W} \\ \text{or } 10 \log (1.39 \times 10^7) &= 71.4 \text{ dB.}\end{aligned}$$

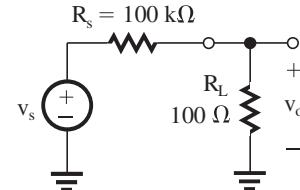
For a peak output sine-wave current of 20 mA, the peak output voltage will be  $20 \text{ mA} \times 500 \Omega = 10 \text{ V}$ . Correspondingly  $v_i$  will be a sine wave with a peak value of  $10 \text{ V}/A_v = 10/83.3$ , or an rms value of  $10/(83.3 \times \sqrt{2}) = 0.085 \text{ V}$ . Corresponding output power =  $(10/\sqrt{2})^2/500 \Omega = 0.1 \text{ W}$

1.42



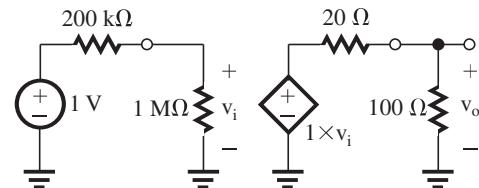
$$\begin{aligned}\frac{v_o}{v_s} &= \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 100 \text{ k}\Omega} \times 1000 \times \frac{100 \Omega}{100 \Omega + 1 \text{ k}\Omega} \\ &= \frac{10}{110} \times 1000 \times \frac{100}{1100} = 8.26 \text{ V/V}\end{aligned}$$

The signal loses about 90% of its strength when connected to the amplifier input (because  $R_i = R_s/10$ ). Also, the output signal of the amplifier loses approximately 90% of its strength when the load is connected (because  $R_L = R_o/10$ ). Not a good design! Nevertheless, if the source were connected directly to the load,



which is clearly a much worse situation. Indeed inserting the amplifier increases the gain by a factor  $8.3/0.001 = 8300$ .

1.43



$$\begin{aligned}v_o &= 1 \text{ V} \times \frac{1 \text{ M}\Omega}{1 \text{ M}\Omega + 200 \text{ k}\Omega} \\ &\times 1 \times \frac{100 \Omega}{100 \Omega + 20 \Omega} \\ &= \frac{1}{1.2} \times \frac{100}{120} = 0.69 \text{ V}\end{aligned}$$

$$\text{Voltage gain} = \frac{v_o}{v_s} = 0.69 \text{ V/V} \text{ or } -3.2 \text{ dB}$$

$$\begin{aligned}\text{Current gain} &= \frac{v_o/100 \Omega}{v_s/1.2 \text{ M}\Omega} = 0.69 \times 1.2 \times 10^4 \\ &= 8280 \text{ A/A} \quad \text{or } 78.4 \text{ dB}\end{aligned}$$

$$\text{Power gain} = \frac{v_o^2/100 \Omega}{v_s^2/1.2 \text{ M}\Omega} = 5713 \text{ W/W}$$

$$\text{or } 10 \log 5713 = 37.6 \text{ dB}$$

(This takes into account the power dissipated in the internal resistance of the source.)

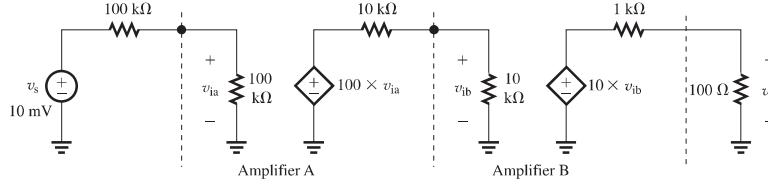
1.44 (a) Case S-A-B-L (see figure on next page):

$$\frac{v_o}{v_s} = \frac{v_o}{v_{ib}} \times \frac{v_{ib}}{v_{ia}} \times \frac{v_{ia}}{v_s} =$$

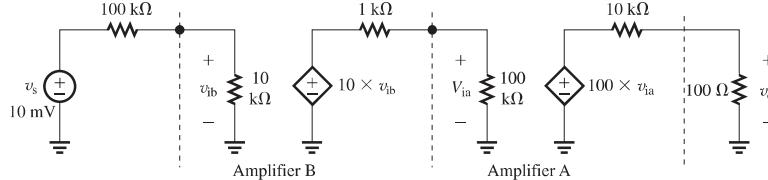
$$\left( 10 \times \frac{100}{100 + 1000} \right) \times \left( 100 \times \frac{10}{10 + 10} \right) \times \left( \frac{100}{100 + 100} \right)$$

### Chapter 1–14

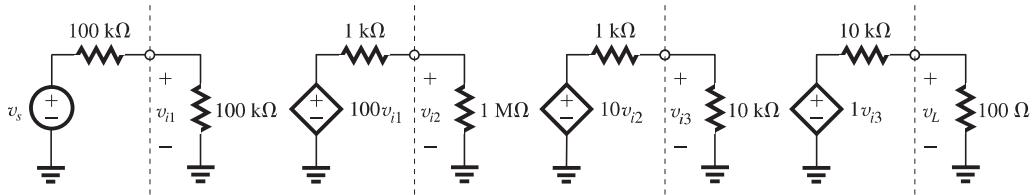
This figure belongs to 1.44, part (a).



This figure belongs to 1.44, part (b).



This figure belongs to Problem 1.45.



$$\frac{v_o}{v_s} = 22.7 \text{ V/V and gain in dB } 20 \log 22.7 =$$

27.1 dB

(b) Case S-B-A-L (see figure above):

$$\frac{v_o}{v_s} = \frac{v_o}{v_{ia}} \cdot \frac{v_{ia}}{v_{ib}} \cdot \frac{v_{ib}}{v_s}$$

$$= \left( 100 \times \frac{100}{100 + 10 \text{ K}} \right) \times \\ \left( 10 \times \frac{100 \text{ K}}{100 \text{ K} + 1 \text{ K}} \right) \times \\ \left( \frac{10 \text{ K}}{10 \text{ K} + 100 \text{ K}} \right)$$

$$\frac{v_o}{v_s} = 0.89 \text{ V/V and gain in dB is } 20 \log 0.89 = -1 \text{ dB. Obviously, case a is preferred because it provides higher voltage gain.}$$

1.45 In Example 1.3, when the first and the second stages are interchanged, the circuit looks like the figure above, and

$$\frac{v_{i1}}{v_s} = \frac{100 \text{ kΩ}}{100 \text{ kΩ} + 100 \text{ kΩ}} = 0.5 \text{ V/V}$$

$$A_{v1} = \frac{v_{i2}}{v_{i1}} = 100 \times \frac{1 \text{ MΩ}}{1 \text{ MΩ} + 1 \text{ kΩ}}$$

$$= 99.9 \text{ V/V}$$

$$A_{v2} = \frac{v_{i3}}{v_{i2}} = 10 \times \frac{10 \text{ kΩ}}{10 \text{ kΩ} + 1 \text{ kΩ}}$$

$$= 9.09 \text{ V/V}$$

$$A_{v3} = \frac{v_L}{v_{i3}} = 1 \times \frac{100 \text{ Ω}}{100 \text{ Ω} + 10 \text{ Ω}} = 0.909 \text{ V/V}$$

$$\text{Total gain } A_v = \frac{v_L}{v_s} = A_{v1} \times A_{v2} \times A_{v3}$$

$$= 99.9 \times 9.09 \times 0.909 = 825.5 \text{ V/V}$$

The voltage gain from source to load is

$$\frac{v_L}{v_s} = \frac{v_L}{v_{i1}} \times \frac{v_{i1}}{v_s} = A_v \cdot \frac{v_{i1}}{v_s}$$

$$= 825.5 \times 0.5$$

$$= 412.7 \text{ V/V}$$

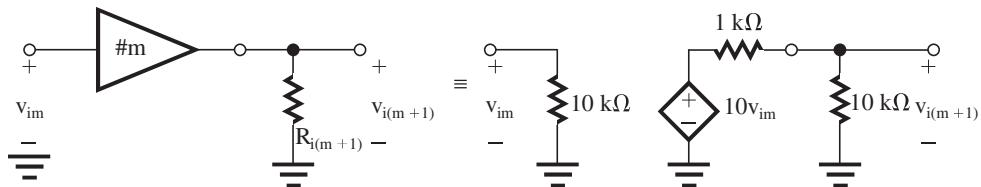
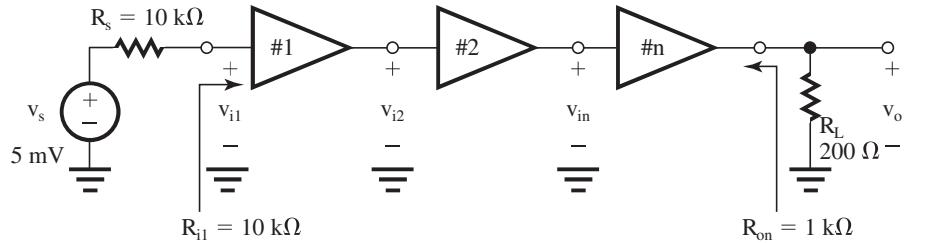
The overall voltage has reduced appreciably. This is because the input resistance of the first stage,  $R_{in}$ , is comparable to the source resistance  $R_s$ . In Example 1.3 the input resistance of the first stage is much larger than the source resistance.

1.46 Each of stages #1, 2, ..., (n – 1) can be represented by the equivalent circuit:

$$\frac{v_o}{v_s} = \frac{v_{i1}}{v_s} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i3}}{v_{i2}} \times \cdots \times \frac{v_{in}}{v_{in-1}} \times \frac{v_o}{v_{in}}$$

where

This figure belongs to 1.46.



$$\frac{v_{i1}}{v_s} = \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 10 \text{ k}\Omega} = 0.5 \text{ V/V}$$

$$\frac{v_o}{v_{in}} = 10 \times \frac{200 \Omega}{1 \text{ k}\Omega + 200 \Omega} = 1.67 \text{ V/V}$$

$$\begin{aligned} \frac{v_{i2}}{v_{i1}} &= \frac{v_{i3}}{v_{i2}} = \dots = \frac{v_{in}}{v_{i(n-1)}} = 10 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 10 \text{ k}\Omega} \\ &= 9.09 \text{ V/V} \end{aligned}$$

Thus,

$$\frac{v_o}{v_s} = 0.5 \times (9.09)^{n-1} \times 1.67 = 0.833 \times (9.09)^{n-1}$$

For  $v_s = 5 \text{ mV}$  and  $v_o = 3 \text{ V}$ , the gain  $\frac{v_o}{v_s}$  must be  $\geq 600$ , thus

$$0.833 \times (9.09)^{n-1} \geq 600$$

$$\Rightarrow n = 4$$

Thus four amplifier stages are needed, resulting in

$$\frac{v_o}{v_s} = 0.833 \times (9.09)^3 = 625.7 \text{ V/V}$$

and correspondingly

$$v_o = 625.7 \times 5 \text{ mV} = 3.13 \text{ V}$$

$$1.47 \text{ (a) Required voltage gain } \equiv \frac{v_o}{v_s}$$

$$= \frac{2 \text{ V}}{0.005 \text{ V}} = 400 \text{ V/V}$$

(b) The smallest  $R_i$  allowed is obtained from

$$0.1 \mu\text{A} = \frac{5 \text{ mV}}{R_s + R_i} \Rightarrow R_s + R_i = 50 \text{ k}\Omega$$

Thus  $R_i = 40 \text{ k}\Omega$ .

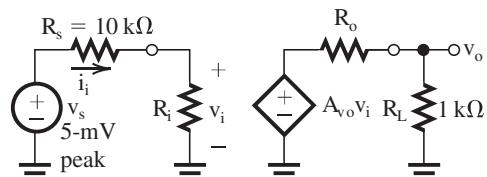
For  $R_i = 40 \text{ k}\Omega$ .  $i_i = 0.1 \mu\text{A}$  peak, and

$$\begin{aligned} \text{Overall current gain} &= \frac{v_o/R_L}{i_i} = \frac{2 \text{ V}/1 \text{ k}\Omega}{0.1 \mu\text{A}} \\ &= \frac{2 \text{ mA}}{0.1 \mu\text{A}} = 2 \times 10^4 \text{ A/A} \end{aligned}$$

$$\begin{aligned} \text{Overall power gain} &\equiv \frac{v_{o(\text{rms})}^2 / R_L}{v_{s(\text{rms})} \times i_{i(\text{rms})}} \\ &= \frac{\left(\frac{2}{\sqrt{2}}\right)^2 / 1000}{\left(\frac{5 \times 10^{-3}}{\sqrt{2}}\right) \times \left(\frac{0.1 \times 10^{-6}}{\sqrt{2}}\right)} \\ &= 8 \times 10^6 \text{ W/W} \end{aligned}$$

(This takes into account the power dissipated in the internal resistance of the source.)

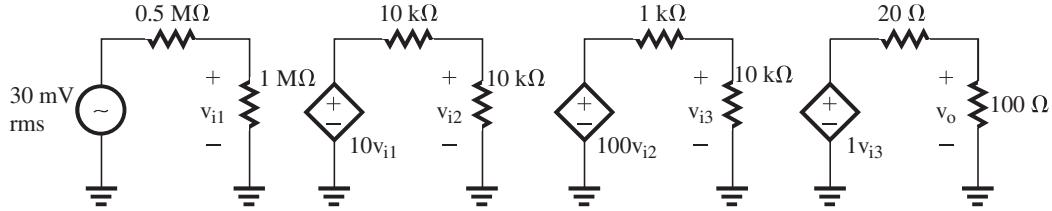
(c) If  $(A_{vo} v_i)$  has its peak value limited to 3 V, the largest value of  $R_o$  is found from



$$= 3 \times \frac{R_L}{R_L + R_o} = 2 \Rightarrow R_o = \frac{1}{2} R_L = 500 \Omega$$

(If  $R_o$  were greater than this value, the output voltage across  $R_L$  would be less than 2 V.)

This figure belongs to 1.48.



(d) For  $R_i = 40 \text{ k}\Omega$  and  $R_o = 500 \Omega$ , the required value  $A_{vo}$  can be found from

$$400 \text{ V/V} = \frac{40}{40 + 10} \times A_{vo} \times \frac{1}{1 + 0.5}$$

$$\Rightarrow A_{vo} = 750 \text{ V/V}$$

(e)  $R_i = 100 \text{ k}\Omega (1 \times 10^5 \Omega)$

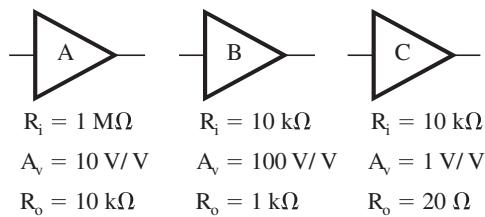
$$R_o = 100 \Omega (1 \times 10^2 \Omega)$$

$$400 = \frac{100}{100 + 10} \times A_{vo} \times \frac{1000}{1000 + 100}$$

$$\Rightarrow A_{vo} = 484 \text{ V/V}$$

#### 1.48 Deliver 0.5 W to a 100-Ω load.

Source is 30 mV rms with 0.5-MΩ source resistance. Choose from these three amplifier types:



Choose order to eliminate loading on input and output:

A, first, to minimize loading on 0.5-MΩ source

B, second, to boost gain

C, third, to minimize loading at 100-Ω output.

We first attempt a cascade of the three stages in the order A, B, C (see figure above), and obtain

$$\frac{v_{i1}}{v_s} = \frac{1 \text{ M}\Omega}{1 \text{ M}\Omega + 0.5 \text{ M}\Omega} = \frac{1}{1.5}$$

$$\Rightarrow v_{i1} = 30 \times \frac{1}{1.5} = 20 \text{ mV}$$

$$\frac{v_{i2}}{v_{i1}} = 10 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 10 \text{ k}\Omega} = 5$$

$$\Rightarrow v_{i2} = 20 \times 5 = 100 \text{ mV}$$

$$\frac{v_{i3}}{v_{i2}} = 100 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 1 \text{ k}\Omega} = 90.9$$

$$\Rightarrow v_{i3} = 100 \text{ mV} \times 90.9 = 9.09 \text{ V}$$

$$\frac{v_o}{v_{i3}} = 1 \times \frac{100 \Omega}{100 \Omega + 20 \Omega} = 0.833$$

$$\Rightarrow v_o = 9.09 \times 0.833 = 7.6 \text{ V}$$

$$P_o = \frac{v_{\text{rms}}^2}{R_L} = \frac{7.6^2}{100} = 0.57 \text{ W}$$

which exceeds the required 0.5 W. Also, the signal throughout the amplifier chain never drops below 20 mV (which is greater than the required minimum of 10 mV).

#### 1.49 From the equivalent circuit of the output side of a voltage amplifier [Fig. 1.16(b)]:

$$v_o = (A_{vo} v_i) \frac{R_L}{R_L + R_o}$$

$$200 = (A_{vo} v_i) \frac{1000}{1000 + R_o} \quad (1)$$

$$195 = (A_{vo} v_i) \frac{780}{780 + R_o} \quad (2)$$

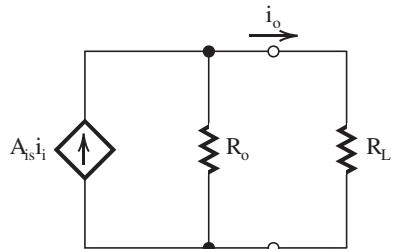
Dividing Eq. (2) by Eq. (1), we have

$$\frac{195}{200} = 0.78 \frac{1000 + R_o}{780 + R_o}$$

$$\Rightarrow R_o = 100 \Omega$$

$$(A_{vo} v_i) = 200[(1000 + 100)/1000] = 220 \text{ mV}$$

#### 1.50 The equivalent circuit at the output side of a current amplifier loaded with a resistance $R_L$ is shown. Since



$$i_o = (A_{is}i_i) \frac{R_o}{R_o + R_L}$$

we can write

$$1 = (A_{is}i_i) \frac{R_o}{R_o + 1} \quad (1)$$

and

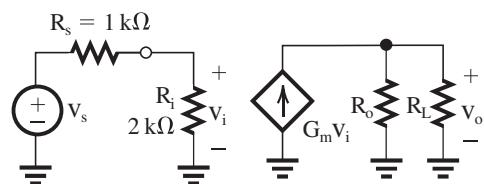
$$0.5 = (A_{is}i_i) \frac{R_o}{R_o + 12} \quad (2)$$

Dividing Eq. (1) by Eq. (2), we have

$$2 = \frac{R_o + 12}{R_o + 1} \Rightarrow R_o = 10 \text{ k}\Omega$$

$$A_{is}i_i = 1 \times \frac{10 + 1}{10} = 1.1 \text{ mA}$$

1.51



$$G_m = 60 \text{ mA/V}$$

$$R_o = 20 \text{ k}\Omega$$

$$R_L = 1 \text{ k}\Omega$$

$$v_i = v_s \frac{R_i}{R_s + R_i}$$

$$= v_s \frac{2}{1+2} = \frac{2}{3} v_s$$

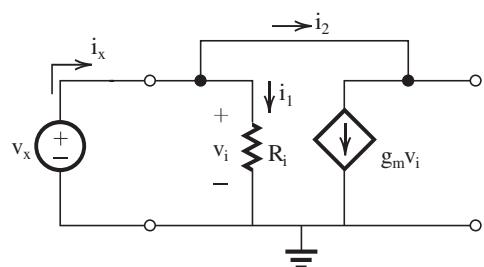
$$v_o = G_m v_i (R_L \parallel R_o)$$

$$= 60 \frac{20 \times 1}{20 + 1} v_i$$

$$= 60 \frac{20}{21} \times \frac{2}{3} v_s$$

$$\text{Overall voltage gain} \equiv \frac{v_o}{v_s} = 38.1 \text{ V/V}$$

1.52

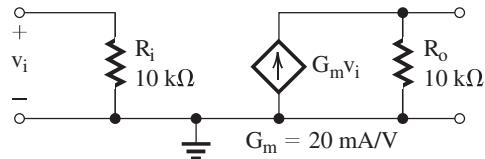


$$\left. \begin{array}{l} i_x = i_1 + i_2 \\ i_1 = v_i / R_i \\ i_2 = g_m v_i \\ v_i = v_x \end{array} \right\} \begin{array}{l} i_x = v_x / R_i + g_m v_x \\ i_x = v_x \left( \frac{1}{R_i} + g_m \right) \\ \frac{v_x}{i_x} = \frac{1}{1/R_i + g_m} \\ = \frac{R_i}{1 + g_m R_i} = R_{in} \end{array}$$

1.53 To obtain the weighted sum of  $v_1$  and  $v_2$

$$v_o = 10v_1 + 20v_2$$

we use two transconductance amplifiers and sum their output currents. Each transconductance amplifier has the following equivalent circuit:



Consider first the path for the signal requiring higher gain, namely  $v_2$ . See figure at top of next page.

The parallel connection of the two amplifiers at the output and the connection of  $R_L$  means that the total resistance at the output is

$$10 \text{ k}\Omega \parallel 10 \text{ k}\Omega \parallel 10 \text{ k}\Omega = \frac{10}{3} \text{ k}\Omega. \text{ Thus the component of } v_o \text{ due to } v_2 \text{ will be}$$

$$v_{o2} = v_2 \frac{10}{10 + 10} \times G_{m2} \times \frac{10}{3}$$

$$= v_2 \times 0.5 \times 20 \times \frac{10}{3} = 33.3v_2$$

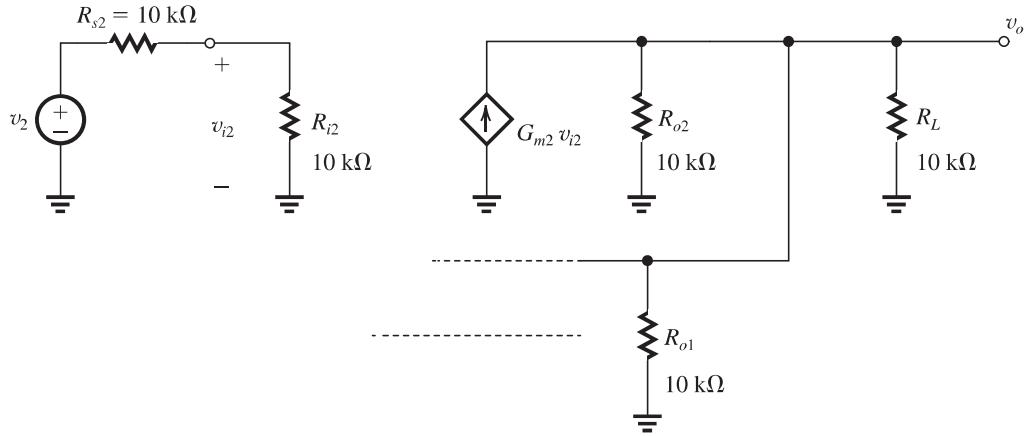
To reduce the gain seen by  $v_2$  from 33.3 to 20, we connect a resistance  $R_p$  in parallel with  $R_L$ ,

$$\left( \frac{10}{3} \parallel R_p \right) = 2 \text{ k}\Omega$$

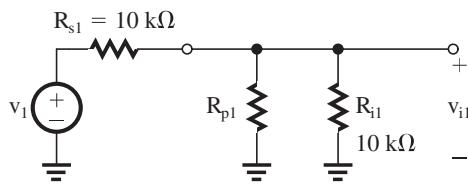
$$\Rightarrow R_p = 5 \text{ k}\Omega$$

We next consider the path for  $v_1$ . Since  $v_1$  must see a gain factor of only 10, which is half that seen by  $v_2$ , we have to reduce the fraction of  $v_1$  that appears at the input of its transconductance amplifier to half that that appears at the input of the  $v_2$  transconductance amplifier. We just saw that  $0.5 v_2$  appears at the input of the  $v_2$  transconductance amplifier. Thus, for the  $v_1$  transconductance amplifier, we want  $0.25v_1$  to appear at the input. This can be achieved by shunting the input of the  $v_1$  transconductance

This figure belongs to Problem 1.53.



amplifier by a resistance  $R_{p1}$  as in the following figure.



The value of  $R_{p1}$  can be found from

$$\frac{(R_{p1} \parallel R_{i1})}{(R_{p1} \parallel R_{i1}) + R_{s1}} = 0.25$$

Thus,

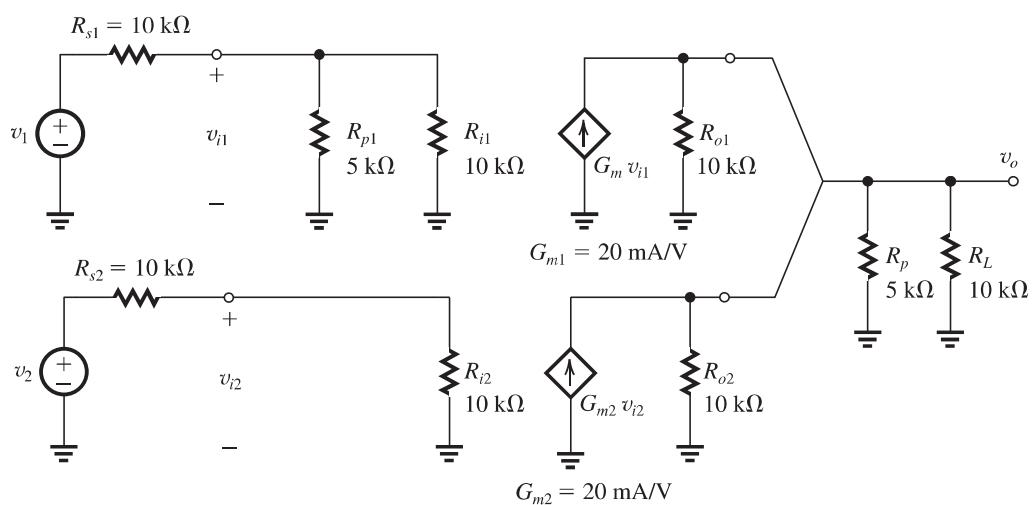
$$1 + \frac{R_{s1}}{(R_{p1} \parallel R_{i1})} = 4$$

$$\Rightarrow R_{p1} \parallel R_{i1} = \frac{R_{s1}}{3} = \frac{10}{3}$$

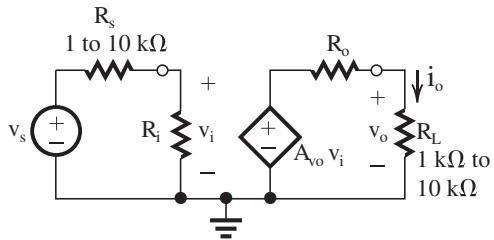
$$R_{p1} \parallel 10 = \frac{10}{3}$$

$$\Rightarrow R_{p1} = 5 \text{ k}\Omega$$

The final circuit will be as follows:



## 1.54 Voltage amplifier:



For  $R_s$  varying in the range 1 kΩ to 10 kΩ and  $\Delta v_o$  limited to 10%, select  $R_i$  to be sufficiently large:

$$R_i \geq 10 R_{s\max}$$

$$R_i = 10 \times 10 \text{ k}\Omega = 100 \text{ k}\Omega = 1 \times 10^5 \Omega$$

For  $R_L$  varying in the range 1 kΩ to 10 kΩ, the load voltage variation limited to 10%, select  $R_o$  sufficiently low:

$$R_o \leq \frac{R_{L\min}}{10}$$

$$R_o = \frac{1 \text{ k}\Omega}{10} = 100 \Omega = 1 \times 10^2 \Omega$$

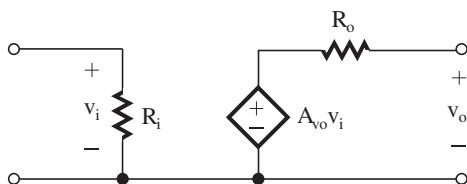
Now find  $A_{v_o}$ :

$$v_{o\min} = 10 \text{ mV} \times \frac{R_i}{R_i + R_{s\max}} \times A_{v_o} \frac{R_{L\min}}{R_o + R_{L\min}}$$

$$1 = 10 \times 10^{-3} \times \frac{100 \text{ k}\Omega}{100 \text{ k}\Omega + 10 \text{ k}\Omega}$$

$$\times A_{v_o} \times \frac{1 \text{ k}\Omega}{100 \Omega + 1 \text{ k}\Omega}$$

$$\Rightarrow A_{v_o} = 121 \text{ V/V}$$

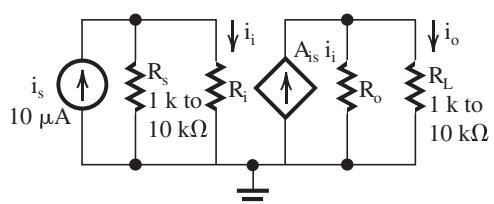


Values for the voltage amplifier equivalent circuit are

$$R_i = 1 \times 10^5 \Omega, A_{v_o} = 121 \text{ V/V}, \text{ and}$$

$$R_o = 1 \times 10^2 \Omega$$

## 1.55 Current amplifier:



For  $R_s$  varying in the range 1 kΩ to 10 kΩ and load current variation limited to 10%, select  $R_i$  to be sufficiently low:

$$R_i \leq \frac{R_{s\min}}{10}$$

$$R_i = \frac{1 \text{ k}\Omega}{10} = 100 \Omega = 1 \times 10^2 \Omega$$

For  $R_L$  varying in the range 1 kΩ to 10 kΩ and the load current variation limited to 10%,  $R_o$  is selected sufficiently large:

$$R_o \geq 10 R_{L\max}$$

$$R_o = 10 \times 10 \text{ k}\Omega$$

$$= 100 \text{ k}\Omega = 1 \times 10^5 \Omega$$

Now we find  $A_{is}$ :

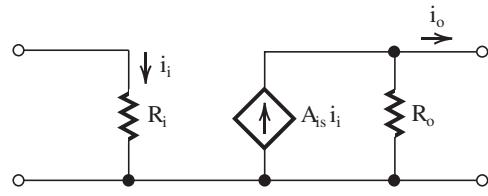
$$i_{o\min} = 10 \mu\text{A} \times \frac{R_{s\min}}{R_{s\min} + R_i} \times A_{is} \times \frac{R_o}{R_o + R_{L\max}}$$

$$1 \times 10^{-3} = 10 \times 10^{-6} \frac{1 \text{ k}\Omega}{1 \text{ k}\Omega + 100 \Omega}$$

$$\times A_{is} \frac{100 \text{ k}\Omega}{100 \text{ k}\Omega + 10 \text{ k}\Omega}$$

$$\Rightarrow A_{is} = 121 \text{ A/A}$$

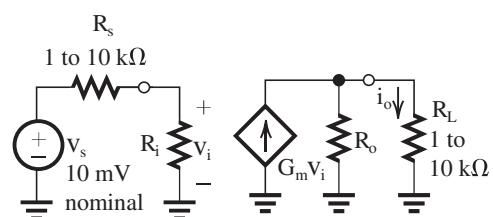
Current amplifier equivalent circuit:



$$R_i = 1 \times 10^2 \Omega, A_{is} = 121 \text{ A/A},$$

$$R_o = 1 \times 10^5 \Omega$$

## 1.56 Transconductance amplifier:



For  $R_s$  varying in the range 1 to 10 k $\Omega$ , and  $\Delta i_o$  limited to 10%, we have to select  $R_i$  sufficiently large;

$$R_i \geq 10R_{s\max}$$

$$R_i = 100 \text{ k}\Omega = 1 \times 10^5 \Omega$$

For  $R_L$  varying in the range 1 to 10 k $\Omega$ , the change in  $i_o$  can be kept to 10% if  $R_o$  is selected sufficiently large;

$$R_o \geq R_{L\max}$$

$$\text{Thus } R_o = 100 \text{ k}\Omega = 1 \times 10^5 \Omega$$

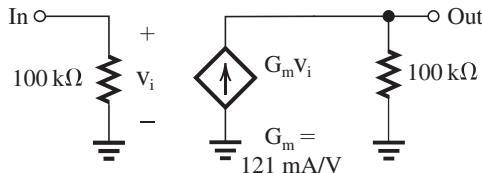
For  $v_s = 10 \text{ mV}$ ,

$$i_{o\min} = 10^{-2} \frac{R_i}{R_i + R_{s\max}} G_m \frac{R_o}{R_o + R_{L\max}}$$

$$10^{-3} = 10^{-2} \frac{100}{100 + 10} G_m \frac{100}{100 + 10}$$

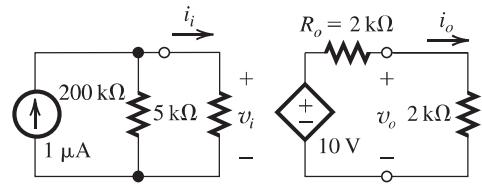
$$G_m = 1.21 \times 10^{-1} \text{ A/V}$$

$$= 121 \text{ mA/V}$$



$$1.57 \quad R_o = \frac{\text{Open-circuit output voltage}}{\text{Short-circuit output current}} = \frac{10 \text{ V}}{5 \text{ mA}} = 2 \text{ k}\Omega$$

$$v_o = 10 \times \frac{2}{2+2} = 5 \text{ V}$$



$$A_v = \frac{v_o}{v_i} = \frac{10(2/4)}{1 \times 10^{-6} \times (200 \parallel 5) \times 10^3}$$

$$1025 \text{ V/V or } 60.2 \text{ dB}$$

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{v_i/R_i}$$

$$= \frac{v_o}{v_i} \frac{R_i}{R_L} = 1025 \times \frac{5 \text{ k}\Omega}{2 \text{ k}\Omega}$$

$$= 2562.5 \text{ A/A or } 62.8 \text{ dB}$$

The overall current gain can be found as

$$\frac{i_o}{i_s} = \frac{v_o/R_L}{1 \mu\text{A}} = \frac{5 \text{ V}/2 \text{ k}\Omega}{1 \mu\text{A}}$$

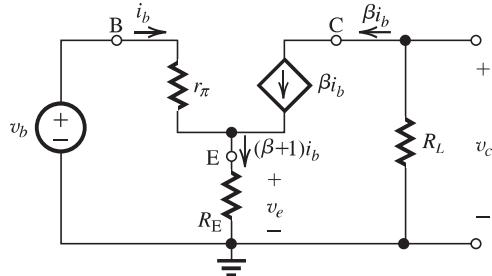
$$= \frac{2.5 \text{ mA}}{1 \mu\text{A}} = 2500 \text{ A/A}$$

or 68 dB.

$$A_p = \frac{v_o^2/R_L}{i_s^2 R_i} = \frac{5^2/(2 \times 10^3)}{\left(10^{-6} \times \frac{200}{200+5}\right)^2 5 \times 10^3}$$

$$= 2.63 \times 10^6 \text{ W/W or } 64.2 \text{ dB}$$

1.58



The node equation at E yields the current through  $R_E$  as  $(\beta i_b + i_b) = (\beta + 1)i_b$ . The voltage  $v_c$  can be found in terms of  $i_b$  as

$$v_c = -\beta i_b R_L \quad (1)$$

The voltage  $v_b$  can be related to  $i_b$  by writing for the input loop:

$$v_b = i_b r_\pi + (\beta + 1)i_b R_E$$

Thus,

$$v_b = [r_\pi + (\beta + 1)R_E]i_b \quad (2)$$

Dividing Eq. (1) by Eq. (2) yields

$$\frac{v_c}{v_b} = -\frac{\beta R_L}{r_\pi + (\beta + 1)R_E} \quad \text{Q.E.D}$$

The voltage  $v_e$  is related to  $i_b$  by

$$v_e = (\beta + 1)i_b R_E$$

That is,

$$v_e = [(\beta + 1)R_E]i_b \quad (3)$$

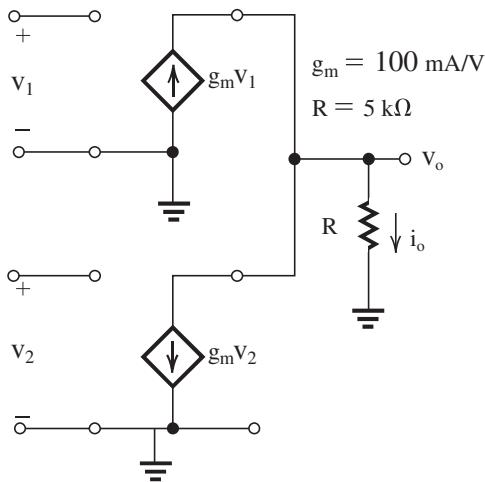
Dividing Eq. (3) by Eq. (2) yields

$$\frac{v_e}{v_b} = \frac{(\beta + 1)R_E}{(\beta + 1)R_E + r_\pi}$$

Dividing the numerator and denominator by  $(\beta + 1)$  gives

$$\frac{v_e}{v_b} = \frac{R_E}{R_E + [r_\pi/(\beta + 1)]} \quad \text{Q.E.D}$$

1.59



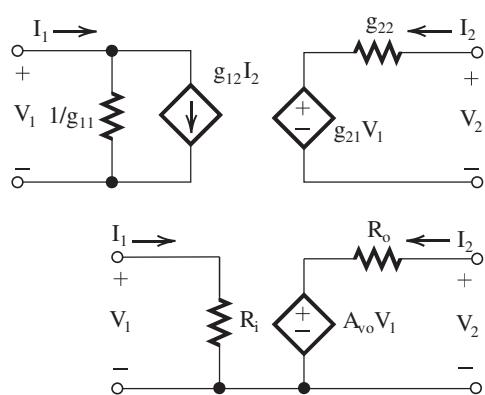
$$i_o = g_m v_1 - g_m v_2$$

$$v_o = i_o R_L = g_m R (v_1 - v_2)$$

$$v_1 = v_2 = 1 \text{ V} \quad \therefore v_o = 0 \text{ V}$$

$$\begin{aligned} v_1 &= 1.01 \text{ V} \\ v_2 &= 0.99 \text{ V} \end{aligned} \quad \therefore v_o = 100 \times 5 \times 0.02 = 10 \text{ V}$$

1.60



The correspondences between the current and voltage variables are indicated by comparing the two equivalent-circuit models above. At the outset we observe that at the input side of the  $g$ -parameter model, we have the controlled current source  $g_{12} I_2$ . This has no correspondence in the equivalent-circuit model of Fig. 1.16(a). It represents internal feedback, internal to the

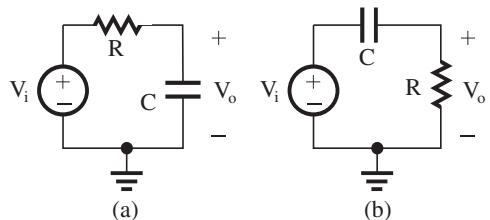
amplifier circuit. In developing the model of Fig. 1.16(a), we assumed that the amplifier is unilateral (i.e., has no internal feedback, or that the input side does not know what happens at the output side). If we neglect this internal feedback, that is, assume  $g_{12} = 0$ , we can compare the two models and thus obtain:

$$R_i = 1/g_{11}$$

$$A_{vo} = g_{21}$$

$$R_o = g_{22}$$

1.61 Circuits of Fig. 1.22:



$$\text{For (a)} \quad V_o = V_i \left( \frac{1/sC}{1/sC + R} \right)$$

$$\frac{V_o}{V_i} = \frac{1}{1 + sCR}$$

which is of the form shown for the low-pass function in Table 1.2 with  $K = 1$  and  $\omega_0 = 1/RC$ .

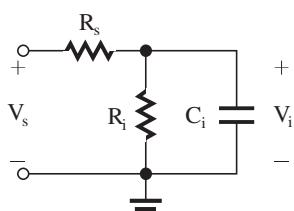
$$\text{For (b)} \quad V_o = V_i \left( \frac{R}{R + \frac{1}{sC}} \right)$$

$$\frac{V_o}{V_i} = \frac{sRC}{1 + sCR}$$

$$\frac{V_o}{V_i} = \frac{s}{s + \frac{1}{RC}}$$

which is of the form shown in Table 1.2 for the high-pass function, with  $K = 1$  and  $\omega_0 = 1/RC$ .

1.62



$$\begin{aligned} \frac{V_i}{V_s} &= \frac{\frac{R_i \frac{1}{sC_i}}{R_i + \frac{1}{sC_i}}}{R_s + \left( \frac{\frac{R_i \frac{1}{sC_i}}{R_i + \frac{1}{sC_i}}}{R_i + \frac{1}{sC_i}} \right)} = \frac{\frac{R_i}{1 + sC_i R_i}}{R_s + \left( \frac{R_i}{1 + sC_i R_i} \right)} \\ &= \frac{R_i}{R_s + sC_i R_i R_s + R_i} \\ \frac{V_i}{V_s} &= \frac{R_i}{(R_s + R_i) + sC_i R_i R_s} = \frac{\frac{R_i}{(R_s + R_i)}}{1 + s \left( \frac{C_i R_i R_s}{R_s + R_i} \right)} \end{aligned}$$

which is a low-pass STC function with

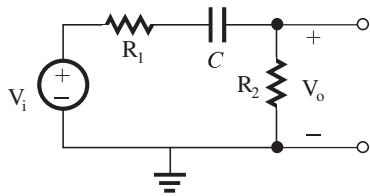
$$K = \frac{R_i}{R_s + R_i} \text{ and } \omega_0 = 1/[C_i(R_i \parallel R_s)].$$

For  $R_s = 10 \text{ k}\Omega$ ,  $R_i = 40 \text{ k}\Omega$ , and  $C_i = 5 \text{ pF}$ ,

$$\omega_0 = \frac{1}{5 \times 10^{-12} \times (40 \parallel 10) \times 10^3} = 25 \text{ Mrad/s}$$

$$f_0 = \frac{25}{2\pi} = 4 \text{ MHz}$$

1.63 Using the voltage-divider rule.



$$T(s) = \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1 + \frac{1}{sC}}$$

$$T(s) = \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{s}{s + \frac{1}{C(R_1 + R_2)}} \right)$$

which from Table 1.2 is of the high-pass type with

$$K = \frac{R_2}{R_1 + R_2} \quad \omega_0 = \frac{1}{C(R_1 + R_2)}$$

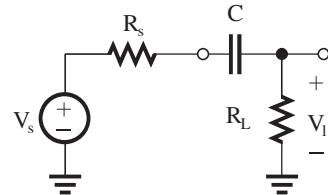
As a further verification that this is a high-pass network and  $T(s)$  is a high-pass transfer function, see that as  $s \Rightarrow 0$ ,  $T(s) \Rightarrow 0$ ; and as  $s \rightarrow \infty$ ,  $T(s) = R_2/(R_1 + R_2)$ . Also, from the circuit, observe as  $s \rightarrow \infty$ ,  $(1/sC) \rightarrow 0$  and  $V_o/V_i = R_2/(R_1 + R_2)$ . Now, for  $R_1 = 10 \text{ k}\Omega$ ,  $R_2 = 40 \text{ k}\Omega$  and  $C = 1 \mu\text{F}$ ,

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \times 1 \times 10^{-6} (10 + 40) \times 10^3}$$

$$= 3.18 \text{ Hz}$$

$$|T(j\omega_0)| = \frac{K}{\sqrt{2}} = \frac{40}{10 + 40} \frac{1}{\sqrt{2}} = 0.57 \text{ V/V}$$

1.64 Using the voltage-divider rule,



$$\frac{V_l}{V_s} = \frac{R_L}{R_L + R_s + \frac{1}{sC}}$$

$$= \frac{R_L}{R_L + R_s} \frac{s}{s + \frac{1}{C(R_L + R_s)}}$$

which is of the high-pass STC type (see Table 1.2) with

$$K = \frac{R_L}{R_L + R_s} \quad \omega_0 = \frac{1}{C(R_L + R_s)}$$

For  $f_0 \leq 100 \text{ Hz}$

$$\frac{1}{2\pi C(R_L + R_s)} \leq 100$$

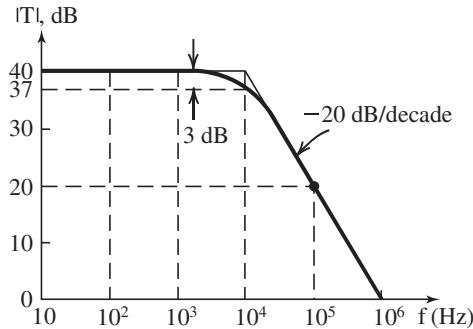
$$\Rightarrow C \geq \frac{1}{2\pi \times 100(20 + 5) \times 10^3}$$

Thus, the smallest value of  $C$  that will do the job is  $C = 0.064 \mu\text{F}$  or  $64 \text{ nF}$ .

1.65 The given measured data indicate that this amplifier has a low-pass STC frequency response with a low-frequency gain of 40 dB, and a 3-dB frequency of  $10^4 \text{ Hz}$ . From our knowledge of the Bode plots for low-pass STC networks [Fig. 1.23(a)], we can complete the table entries and sketch the amplifier frequency response.

$f$ (Hz)	$ T $ (dB)	$\angle T$ ( $^\circ$ )
0	40	0
100	40	0
1000	40	0
$10^4$	37	$-45^\circ$
$10^5$	20	$-90^\circ$
$10^6$	0	$-90^\circ$

This figure belongs to 1.65.



1.66 Since the overall transfer function is that of three identical STC LP circuits in cascade (but with no loading effects, since the buffer amplifiers have infinite input and zero output resistances) the overall gain will drop by 3 dB below the value at dc at the frequency for which the gain of each STC circuit is 1 dB down. This frequency is found as follows: The transfer function of each STC circuit is

$$T(s) = \frac{1}{1 + \frac{s}{\omega_0}}$$

where

$$\omega_0 = 1/CR$$

Thus,

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_{1\text{dB}}}{\omega_0}\right)^2}} = -1$$

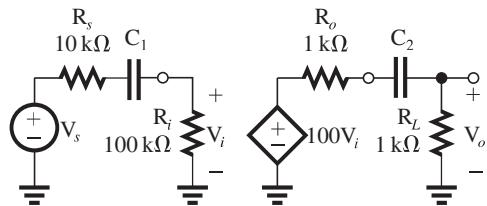
$$\Rightarrow 1 + \left(\frac{\omega_{1\text{dB}}}{\omega_0}\right)^2 = 10^{0.1}$$

$$\omega_{1\text{dB}} = 0.51\omega_0$$

$$\omega_{1\text{dB}} = 0.51/CR$$

1.67 For the input circuit, the corner frequency  $f_{01}$  is found from

$$f_{01} = \frac{1}{2\pi C_1(R_s + R_i)}$$



For  $f_{01} \leq 100$  Hz,

$$\frac{1}{2\pi C_1(10 + 100) \times 10^3} \leq 100$$

$$\Rightarrow C_1 \geq \frac{1}{2\pi \times 110 \times 10^3 \times 10^2} = 1.4 \times 10^{-8} \text{ F}$$

Thus we select  $C_1 = 1 \times 10^{-7} \text{ F} = 0.1 \mu\text{F}$ . The actual corner frequency resulting from  $C_1$  will be

$$f_{01} = \frac{1}{2\pi \times 10^{-7} \times 110 \times 10^3} = 14.5 \text{ Hz}$$

For the output circuit,

$$f_{02} = \frac{1}{2\pi C_2(R_o + R_L)}$$

For  $f_{02} \leq 100$  Hz,

$$\frac{1}{2\pi C_2(1 + 1) \times 10^3} \leq 100$$

$$\Rightarrow C_2 \geq \frac{1}{2\pi \times 2 \times 10^3 \times 10^2} = 0.8 \times 10^{-6}$$

Select  $C_2 = 1 \times 10^{-6} = 1 \mu\text{F}$ .

This will place the corner frequency at

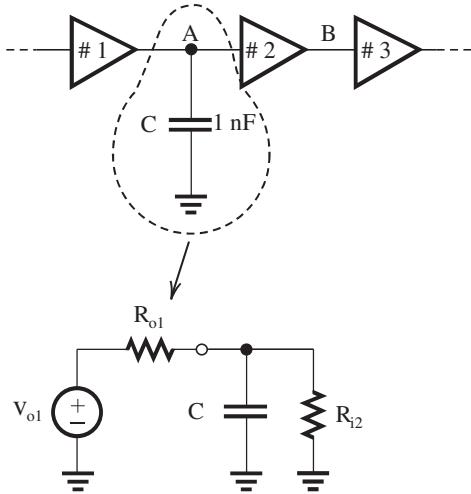
$$f_{02} = \frac{1}{2\pi \times 10^{-6} \times 2 \times 10^3} = 80 \text{ Hz}$$

$$T(s) = 100 \frac{s}{\left(1 + \frac{s}{2\pi f_{01}}\right)\left(1 + \frac{s}{2\pi f_{02}}\right)}$$

1.68 Since when C is connected to node A the 3-dB frequency is reduced by a large factor, the value of C must be much larger than whatever parasitic capacitance originally existed at node A (i.e., between A and ground). Furthermore, it must be that C is now the dominant determinant of the amplifier 3-dB frequency (i.e., it is dominating over whatever may be happening at node B or anywhere else in the amplifier). Thus, we can write

$$200 \text{ kHz} = \frac{1}{2\pi C(R_{o1} \parallel R_{i2})}$$

$$\Rightarrow (R_{o1} \parallel R_{i2}) = \frac{1}{2\pi \times 200 \times 10^3 \times 1 \times 10^{-9}} = 0.8 \text{ k}\Omega$$



Now  $R_{i2} = 100 \text{ k}\Omega$ .

Thus  $R_{o1} \approx 0.8 \text{ k}\Omega$

Similarly, for node B,

$$20 \text{ kHz} = \frac{1}{2\pi C(R_{o2} \parallel R_{i3})}$$

$$\Rightarrow R_{o2} \parallel R_{i3} = \frac{1}{2\pi \times 20 \times 10^3 \times 1 \times 10^{-9}} = 7.96 \text{ k}\Omega$$

$$R_{o2} = 8.65 \text{ k}\Omega$$

The designer should connect a capacitor of value  $C_p$  to node B where  $C_p$  can be found from

$$10 \text{ kHz} = \frac{1}{2\pi C_p(R_{o2} \parallel R_{i3})}$$

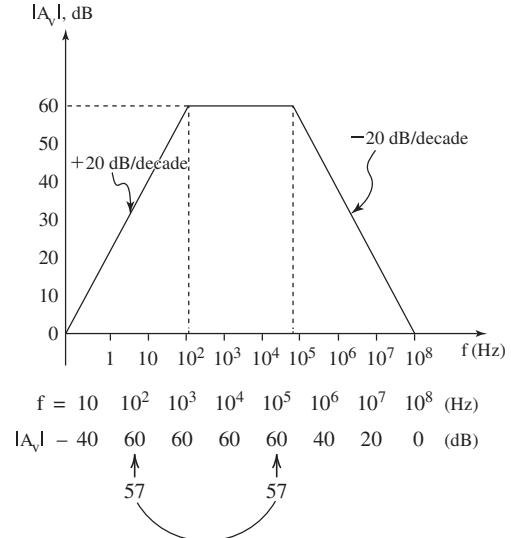
$$\Rightarrow C_p = \frac{1}{2\pi \times 10 \times 10^3 \times 7.96 \times 10^3} = 2 \text{ nF}$$

Note that if she chooses to use node A, she would need to connect a capacitor 10 times larger!

1.69 The LP factor  $1/(1+jf/10^5)$  results in a Bode plot like that in Fig. 1.23(a) with the 3-dB frequency  $f_0 = 10^3 \text{ Hz}$ . The high-pass factor  $1/(1+10^2/jf)$  results in a Bode plot like that in Fig. 1.24(a) with the 3-dB frequency

$$f_0 = 10^2 \text{ Hz.}$$

The Bode plot for the overall transfer function can be obtained by summing the dB values of the two individual plots and then shifting the resulting plot vertically by 60 dB (corresponding to the factor 1000 in the numerator). The result is as follows:



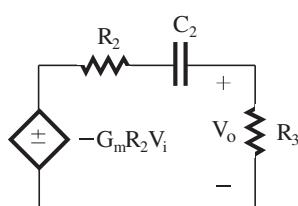
$$\text{Bandwidth} = 10^5 - 10^2 = 99,900 \text{ Hz}$$

$$1.70 T_i(s) = \frac{V_i(s)}{V_s(s)} = \frac{1/sC_1}{1/sC_1 + R_1} = \frac{1}{sC_1 R_1 + 1}$$

LP with a 3-dB frequency

$$f_{0i} = \frac{1}{2\pi C_1 R_1} = \frac{1}{2\pi 10^{-11} 10^5} = 159 \text{ kHz}$$

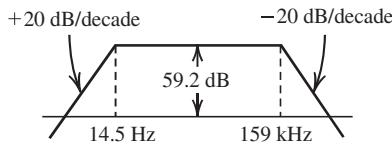
For  $T_o(s)$ , the following equivalent circuit can be used:



$$\begin{aligned} T_o(s) &= \frac{V_o}{V_i} = -G_m R_2 \frac{R_3}{R_2 + R_3 + 1/sC_2} \\ &= -G_m (R_2 \parallel R_3) \frac{s}{s + \frac{1}{C_2 (R_2 + R_3)}} \end{aligned}$$

which is an HP, with

$$\begin{aligned}
 \text{3-dB frequency} &= \frac{1}{2\pi C_2(R_2 + R_3)} \\
 &= \frac{1}{2\pi 100 \times 10^{-9} \times 110 \times 10^3} = 14.5 \text{ Hz} \\
 \therefore T(s) &= T_i(s)T_o(s) \\
 &= \frac{1}{1 + \frac{s}{2\pi \times 159 \times 10^3}} \times -909.1 \times \frac{s}{s + (2\pi \times 14.5)}
 \end{aligned}$$



$$\text{Bandwidth} = 159 \text{ kHz} - 14.5 \text{ Hz} \simeq 159 \text{ kHz}$$

$$1.71 V_i = V_s \frac{R_i}{R_s + R_i} \quad (1)$$

(a) To satisfy constraint (1), namely,

$$V_i \geq \left(1 - \frac{x}{100}\right) V_s$$

we substitute in Eq. (1) to obtain

$$\frac{R_i}{R_s + R_i} \geq 1 - \frac{x}{100} \quad (2)$$

Thus

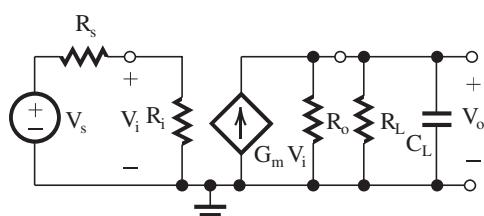
$$\frac{R_s + R_i}{R_i} \leq \frac{1}{1 - \frac{x}{100}}$$

$$\frac{R_s}{R_i} \leq \frac{1}{1 - \frac{x}{100}} - 1 = \frac{\frac{x}{100}}{1 - \frac{x}{100}}$$

which can be expressed as

$$\frac{R_i}{R_s} \geq \frac{1 - \frac{x}{100}}{\frac{x}{100}}$$

resulting in



$$R_i \geq R_s \left( \frac{100}{x} - 1 \right) \quad (3)$$

(b) The 3-dB frequency is determined by the parallel RC circuit at the output

$$f_0 = \frac{1}{2\pi} \omega_0 = \frac{1}{2\pi} \frac{1}{C_L (R_L \parallel R_o)}$$

Thus,

$$f_0 = \frac{1}{2\pi C_L} \left( \frac{1}{R_L} + \frac{1}{R_o} \right)$$

To obtain a value for  $f_0$  greater than a specified value  $f_{3dB}$  we select  $R_o$  so that

$$\frac{1}{2\pi C_L} \left( \frac{1}{R_L} + \frac{1}{R_o} \right) \geq f_{3dB}$$

$$\frac{1}{R_o} \geq 2\pi C_L f_{3dB} - \frac{1}{R_L}$$

$$R_o \leq \frac{1}{2\pi f_{3dB} C_L - \frac{1}{R_L}} \quad (4)$$

(c) To satisfy constraint (c), we first determine the dc gain as

$$\text{dc gain} = \frac{R_i}{R_s + R_i} G_m (R_o \parallel R_L)$$

For the dc gain to be greater than a specified value  $A_0$ ,

$$\frac{R_i}{R_s + R_i} G_m (R_o \parallel R_L) \geq A_0$$

The first factor on the left-hand side is (from constraint (2)) greater or equal to  $(1 - x/100)$ . Thus

$$G_m \geq \frac{A_0}{\left(1 - \frac{x}{100}\right) (R_o \parallel R_L)} \quad (5)$$

Substituting  $R_s = 10 \text{ k}\Omega$  and  $x = 10\%$  in (3) results in

$$R_i \geq 10 \left( \frac{100}{100} - 1 \right) = 90 \text{ k}\Omega$$

Substituting  $f_{3dB} = 2 \text{ MHz}$ ,  $C_L = 20 \text{ pF}$ , and

$R_L = 10 \text{ k}\Omega$  in Eq. (4) results in

$$\begin{aligned}
 R_o &\leq \frac{1}{2\pi \times 2 \times 10^6 \times 20 \times 10^{-12} - \frac{1}{10^4}} \\
 &= 6.61 \text{ k}\Omega
 \end{aligned}$$

Substituting  $A_0 = 100$ ,  $x = 10\%$ ,  $R_L = 10 \text{ k}\Omega$ , and  $R_o = 6.61 \text{ k}\Omega$ , Eq. (5) results in

$$G_m \geq \frac{100}{\left(1 - \frac{10}{100}\right)(10 \parallel 6.61) \times 10^3} = 27.9 \text{ mA/V}$$

1.72 Using the voltage divider rule, we obtain

$$\frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$$

where

$$Z_1 = R_1 \parallel \frac{1}{sC_1} \text{ and } Z_2 = R_2 \parallel \frac{1}{sC_2}$$

It is obviously more convenient to work in terms of admittances. Therefore we express  $V_o/V_i$  in the alternate form

$$\frac{V_o}{V_i} = \frac{Y_1}{Y_1 + Y_2}$$

and substitute  $Y_1 = (1/R_1) + sC_1$  and  $Y_2 = (1/R_2) + sC_2$  to obtain

$$\begin{aligned} \frac{V_o}{V_i} &= \frac{\frac{1}{R_1} + sC_1}{\frac{1}{R_1} + \frac{1}{R_2} + s(C_1 + C_2)} \\ &= \frac{C_1}{C_1 + C_2} \frac{s + \frac{1}{C_1 R_1}}{s + \frac{1}{(C_1 + C_2)} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)} \end{aligned}$$

This transfer function will be independent of frequency ( $s$ ) if the second factor reduces to unity.

This in turn will happen if

$$\frac{1}{C_1 R_1} = \frac{1}{C_1 + C_2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

which can be simplified as follows:

$$\frac{C_1 + C_2}{C_1} = R_1 \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \quad (1)$$

$$1 + \frac{C_2}{C_1} = 1 + \frac{R_1}{R_2}$$

or

$$C_1 R_1 = C_2 R_2$$

When this condition applies, the attenuator is said to be compensated, and its transfer function is given by

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

which, using Eq. (1), can be expressed in the alternate form

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{R_1}{R_2}} = \frac{R_2}{R_1 + R_2}$$

Thus when the attenuator is compensated ( $C_1 R_1 = C_2 R_2$ ), its transmission can be determined either by its two resistors  $R_1$ ,  $R_2$  or by its two capacitors  $C_1$ ,  $C_2$ , and the transmission is not a function of frequency.

1.73 The HP STC circuit whose response determines the frequency response of the amplifier in the low-frequency range has a phase angle of  $5.7^\circ$  at  $f = 100 \text{ Hz}$ . Using the equation for  $\angle T(j\omega)$  from Table 1.2, we obtain

$$\tan^{-1} \frac{f_0}{100} = 5.7^\circ \Rightarrow f_0 = 10 \text{ Hz}$$

The LP STC circuit whose response determines the amplifier response at the high-frequency end has a phase angle of  $-5.7^\circ$  at  $f = 1 \text{ kHz}$ . Using the relationship for  $\angle T(j\omega)$  given in Table 1.2, we obtain for the LP STC circuit,

$$-\tan^{-1} \frac{10^3}{f_0} = -5.7^\circ \Rightarrow f_0 \simeq 10 \text{ kHz}$$

At  $f = 100 \text{ Hz}$ , the drop in gain is due to the HP STC network, and thus its value is

$$20 \log \frac{1}{\sqrt{1 + \left( \frac{10}{100} \right)^2}} = -0.04 \text{ dB}$$

Similarly, at the drop in gain  $f = 1 \text{ kHz}$  is caused by the LP STC network. The drop in gain is

$$20 \log \frac{1}{\sqrt{1 + \left( \frac{1000}{10,000} \right)^2}} = -0.04 \text{ dB}$$

The gain drops by 3 dB at the corner frequencies of the two STC networks, that is, at  $f = 10 \text{ Hz}$  and  $f = 10 \text{ kHz}$ .

1.74 Use the expression in Eq. (1.2), with

$$B = 7.3 \times 10^{15} \text{ cm}^{-3} \text{ K}^{-3/2};$$

$$k = 8.62 \times 10^{-5} \text{ eV/K}; \text{ and } E_g = 1.12 \text{ V}$$

we have

$$T = -55^\circ \text{C} = 218 \text{ K};$$

$$n_i = 2.68 \times 10^6 \text{ cm}^{-3}; \frac{N}{n_i} = 1.9 \times 10^{16}$$

That is, one out of every  $1.9 \times 10^{16}$  silicon atoms is ionized at this temperature.

$T = 0^\circ\text{C} = 273 \text{ K}$ :

$$n_i = 1.52 \times 10^9 \text{ cm}^{-3}; \frac{N}{n_i} = 3.3 \times 10^{13}$$

$T = 20^\circ\text{C} = 293 \text{ K}$ :

$$n_i = 8.60 \times 10^9 \text{ cm}^{-3}; \frac{N}{n_i} = 5.8 \times 10^{12}$$

$T = 75^\circ\text{C} = 348 \text{ K}$ :

$$n_i = 3.70 \times 10^{11} \text{ cm}^{-3}; \frac{N}{n_i} = 1.4 \times 10^{11}$$

$T = 125^\circ\text{C} = 398 \text{ K}$ :

$$n_i = 4.72 \times 10^{12} \text{ cm}^{-3}; \frac{N}{n_i} = 1.1 \times 10^{10}$$

1.75 Use Eq. (1.2) to find  $n_i$ ,

$$n_i = BT^{3/2} e^{-E_g/2kT}$$

Substituting the values given in the problem,

$$\begin{aligned} n_i &= 3.56 \times 10^{14} (300)^{3/2} e^{-1.42/(2 \times 8.62 \times 10^{-5} \times 300)} \\ &= 2.2 \times 10^6 \text{ carriers/cm}^3 \end{aligned}$$

1.76 Since  $N_A \gg n_i$ , we can write

$$p_p \approx N_A = 5 \times 10^{18} \text{ cm}^{-3}$$

Using Eq. (1.3), we have

$$n_p = \frac{n_i^2}{p_p} = 45 \text{ cm}^{-3}$$

1.77  $T = 27^\circ\text{C} = 273 + 27 = 300 \text{ K}$

At 300 K,  $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$

Phosphorus-doped Si:

$$n_n \simeq N_D = 10^{17} \text{ cm}^{-3}$$

$$p_n = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

Hole concentration  $= p_n = 2.25 \times 10^3 \text{ cm}^{-3}$

$T = 125^\circ\text{C} = 273 + 125 = 398 \text{ K}$

$$\text{At } 398 \text{ K, } n_i = BT^{3/2} e^{-E_g/2kT}$$

$$\begin{aligned} &= 7.3 \times 10^{15} \times (398)^{3/2} e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 398)} \\ &= 4.72 \times 10^{12} \text{ cm}^{-3} \end{aligned}$$

$$p_n = \frac{n_i^2}{N_D} = 2.23 \times 10^8 \text{ cm}^{-3}$$

At 398 K, hole concentration is

$$p_n = 2.23 \times 10^8 \text{ cm}^{-3}$$

1.78 Hole concentration in intrinsic Si  $= n_i$

$$\begin{aligned} n_i &= BT^{3/2} e^{-E_g/2kT} \\ &= 7.3 \times 10^{15} (300)^{3/2} e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 300)} \\ &= 1.5 \times 10^{10} \text{ holes/cm}^3 \end{aligned}$$

In phosphorus-doped Si, hole concentration drops below the intrinsic level by a factor of  $10^8$ .

$\therefore$  Hole concentration in P-doped Si is

$$p_n = \frac{1.5 \times 10^{10}}{10^8} = 1.5 \times 10^2 \text{ cm}^{-3}$$

Now,  $n_n \simeq N_D$  and  $p_n n_n = n_i^2$

$$\begin{aligned} n_n &= n_i^2 / p_n = \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^2} \\ &= 1.5 \times 10^{18} \text{ cm}^{-3} \end{aligned}$$

$$N_D = n_n = 1.5 \times 10^{18} \text{ atoms/cm}^3$$

1.79 (a) The resistivity of silicon is given by Eq. (1.41):

For intrinsic silicon,

$$\rho = n = n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$

Using  $\mu_n = 1350 \text{ cm}^2/\text{V} \cdot \text{s}$  and

$\mu_p = 480 \text{ cm}^2/\text{V} \cdot \text{s}$ , and  $q = 1.6 \times 10^{-19} \text{ C}$  we have

$$\rho = 2.28 \times 10^5 \Omega\text{-cm.}$$

Using  $R = \rho \cdot \frac{L}{A}$  with  $L = 0.001 \text{ cm}$  and

$A = 3 \times 10^{-8} \text{ cm}^2$ , we have

$$R = 7.6 \times 10^9 \Omega.$$

(b)  $n_n \approx N_D = 5 \times 10^{16} \text{ cm}^{-3}$ ;

$$p_n = \frac{n_i^2}{n_n} = 4.5 \times 10^3 \text{ cm}^{-3}$$

Using  $\mu_n = 1200 \text{ cm}^2/\text{V} \cdot \text{s}$  and

$$\mu_p = 400 \text{ cm}^2/\text{V} \cdot \text{s}$$
, we have

$$\rho = 0.10 \Omega\text{-cm}; R = 3.33 \text{ k}\Omega.$$

(c)  $n_n \approx N_D = 5 \times 10^{18} \text{ cm}^{-3}$ ;

$$p_n = \frac{n_i^2}{n_n} = 45 \text{ cm}^{-3}$$

Using  $\mu_n = 1200 \text{ cm}^2/\text{V} \cdot \text{s}$  and

$$\mu_p = 400 \text{ cm}^2/\text{V} \cdot \text{s}$$
, we have

$$\rho = 1.0 \times 10^{-3} \Omega\text{-cm}; R = 33.3 \Omega.$$

As expected, since  $N_D$  is increased by 100, the resistivity decreases by the same factor.

(d)  $p_p \approx N_A = 5 \times 10^{16} \text{ cm}^{-3}$ ;  $n_p = \frac{n_i^2}{n_n}$

$$= 4.5 \times 10^3 \text{ cm}^{-3}$$

$$\rho = 0.31 \Omega\text{-cm}; R = 10.42 \text{ k}\Omega$$

(e) Since  $\rho$  is given to be  $2.8 \times 10^{-6} \Omega\text{-cm}$ , we directly calculate  $R = 9.33 \times 10^{-2} \Omega$ .

### 1.80 Cross-sectional area of Si bar

$$= 5 \times 4 = 20 \mu\text{m}^2$$

Since  $1 \mu\text{m} = 10^{-4} \text{ cm}$ , we get

$$= 20 \times 10^{-8} \text{ cm}^2$$

$$\text{Current } I = Aq(\mu_p + n\mu_n)E$$

$$= 20 \times 10^{-8} \times 1.6 \times 10^{-19}$$

$$(10^{16} \times 500 + 10^4 \times 1200) \times \frac{1 \text{ V}}{10 \times 10^{-4}}$$

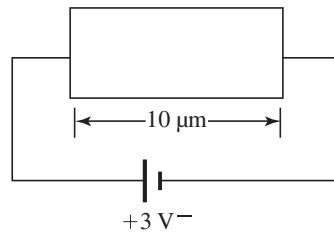
$$= 160 \mu\text{A}$$

### 1.81 Electric field:

$$E = \frac{3 \text{ V}}{10 \mu\text{m}} = \frac{3 \text{ V}}{10 \times 10^{-6} \text{ m}}$$

$$= \frac{3 \text{ V}}{10 \times 10^{-4} \text{ cm}}$$

$$= 3000 \text{ V/cm}$$



$$v_{p\text{-drift}} = \mu_p E = 480 \times 3000$$

$$= 1.44 \times 10^6 \text{ cm/s}$$

$$v_{n\text{-drift}} = \mu_n E = 1350 \times 3000$$

$$= 4.05 \times 10^6 \text{ cm/s}$$

$$\frac{v_n}{v_p} = \frac{4.05 \times 10^6}{1.44 \times 10^6} = 2.8125 \quad \text{or}$$

$$v_n = 2.8125 v_p$$

Or, alternatively, it can be shown as

$$\frac{v_n}{v_p} = \frac{\mu_n E}{\mu_p E} = \frac{\mu_n}{\mu_p} = \frac{1350}{480}$$

$$= 2.8125$$

1.82

$$p_{n0} = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^3$$

From Fig. P1.82,

$$\frac{dp}{dx} = -\frac{10^8 p_{n0} - p_{n0}}{W} \simeq -\frac{10^8 p_{n0}}{50 \times 10^{-7}}$$

since  $1 \text{ nm} = 10^{-7} \text{ cm}$

$$\frac{dp}{dx} = -\frac{10^8 \times 2.25 \times 10^4}{50 \times 10^{-7}}$$

$$= -4.5 \times 10^{17}$$

Hence

$$J_p = -qD_p \frac{dp}{dx}$$

$$= -1.6 \times 10^{-19} \times 12 \times (-4.5 \times 10^{17})$$

$$= 0.864 \text{ A/cm}^2$$

1.83 Use Eq. (1.45):  $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$

$D_n = \mu_n V_T$  and  $D_p = \mu_p V_T$  where  $V_T = 25.9 \text{ mV}$ .

Doping Concentration (carriers/cm <sup>3</sup> )	$\mu_n$ cm <sup>2</sup> /V · s	$\mu_p$ cm <sup>2</sup> /V · s	$D_n$ cm <sup>2</sup> /s	$D_p$ cm <sup>2</sup> /s
Intrinsic	1350	480	35	12.4
$10^{16}$	1200	400	31	10.4
$10^{17}$	750	260	19.4	6.7
$10^{18}$	380	160	9.8	4.1

Depletion width

$$W = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0} \leftarrow \text{Eq. (1.50)}$$

$$W =$$

$$\sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}} \left( \frac{1}{10^{17}} + \frac{1}{10^{16}} \right) \times 0.754}$$

$$= 0.328 \times 10^{-4} \text{ cm} = 0.328 \mu\text{m}$$

Use Eqs. (1.51) and (1.52) to find  $x_n$  and  $x_p$ :

$$x_n = W \frac{N_A}{N_A + N_D} = 0.328 \times \frac{10^{17}}{10^{17} + 10^{16}}$$

$$= 0.298 \mu\text{m}$$

$$x_p = W \frac{N_D}{N_A + N_D} = 0.328 \times \frac{10^{16}}{10^{17} + 10^{16}}$$

$$= 0.03 \mu\text{m}$$

Use Eq. (1.53) to calculate charge stored on either side:

$$Q_J = Aq \left( \frac{N_A N_D}{N_A + N_D} \right) W, \text{ where junction area}$$

$$= 100 \mu\text{m}^2 = 100 \times 10^{-8} \text{ cm}^2$$

$$Q_J = 100 \times 10^{-8} \times 1.6 \times 10^{-19} \left( \frac{10^{17} \cdot 10^{16}}{10^{17} + 10^{16}} \right)$$

$$\times 0.328 \times 10^{-4}$$

$$\text{Hence, } Q_J = 4.8 \times 10^{-14} \text{ C}$$

1.86 Equation (1.50):

$$W = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0},$$

Since  $N_A \gg N_D$ , we have

$$W \simeq \sqrt{\frac{2\epsilon_s}{q} \frac{1}{N_D} V_0}$$

$$V_0 = \frac{qN_D}{2\epsilon_s} \cdot W^2$$

1.85 From Table 1.3,

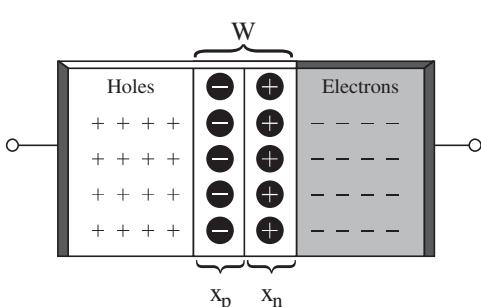
$V_T$  at 300 K = 25.9 mV

Using Eq. (1.46), built-in voltage  $V_0$  is obtained:

$$V_0 = V_T \ln \left( \frac{N_A N_D}{n_i^2} \right) = 25.9 \times 10^{-3} \times$$

$$\ln \left( \frac{10^{17} \times 10^{16}}{(1.5 \times 10^{10})^2} \right)$$

$$= 0.754 \text{ V}$$



Here  $W = 0.2 \mu m = 0.2 \times 10^{-4} \text{ cm}$

$$\text{So } V_0 = \frac{1.6 \times 10^{-19} \times 10^{16} \times (0.2 \times 10^{-4})^2}{2 \times 1.04 \times 10^{-12}}$$

$$= 0.31 \text{ V}$$

$$Q_J = Aq \left( \frac{N_A N_D}{N_A + N_D} \right) W \cong Aq N_D W$$

since  $N_A \gg N_D$ , we have  $Q_J = 3.2 \text{ fC}$ .

$$1.87 \quad V_0 = V_T \ln \left( \frac{N_A N_D}{n_i^2} \right)$$

If  $N_A$  or  $N_D$  is increased by a factor of 10, then new value of  $V_0$  will be

$$V_0 = V_T \ln \left( \frac{10 N_A N_D}{n_i^2} \right)$$

The change in the value of  $V_0$  is

$$V_T \ln 10 = 59.6 \text{ mV.}$$

1.88 Using Eq. (1.46) with  $N_A = 10^{17} \text{ cm}^{-3}$ ,  $N_D = 10^{16} \text{ cm}^{-3}$ , and  $n_i = 1.5 \times 10^{10}$ , we have  $V_0 = 754 \text{ mV}$

Using Eq. (1.55) with  $V_R = 5 \text{ V}$ , we have  $W = 0.907 \mu \text{m}$ .

Using Eq. (1.56) with  $A = 1 \times 10^{-6} \text{ cm}^2$ , we have  $Q_J = 13.2 \times 10^{-14} \text{ C}$ .

1.89 Equation (1.55):

$$\begin{aligned} W &= \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)} \\ &= \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0 \left( 1 + \frac{V_R}{V_0} \right)} \\ &= W_0 \sqrt{1 + \frac{V_R}{V_0}} \end{aligned}$$

Equation (1.56):

$$\begin{aligned} Q_J &= A \sqrt{2\epsilon_s q \left( \frac{N_A N_D}{N_A + N_D} \right) \cdot (V_0 + V_R)} \\ &= A \sqrt{2\epsilon_s q \left( \frac{N_A N_D}{N_A + N_D} \right) V_0 \cdot \left( 1 + \frac{V_R}{V_0} \right)} \\ &= Q_{J0} \sqrt{1 + \frac{V_R}{V_0}} \end{aligned}$$

1.90 Equation (1.63):

$$I = Aq n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) (e^{V/V_T} - 1)$$

$$\text{Here } I_p = Aq n_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

$$I_n = Aq n_i^2 \frac{D_n}{L_n N_A} (e^{V/V_T} - 1)$$

$$\frac{I_p}{I_n} = \frac{D_p}{D_n} \cdot \frac{L_n}{L_p} \cdot \frac{N_A}{N_D}$$

$$= \frac{10}{20} \times \frac{10}{5} \times \frac{10^{18}}{10^{16}}$$

$$\frac{I_p}{I_n} = 100$$

$$\text{Now } I = I_p + I_n = 100 I_n + I_n \equiv 1 \text{ mA}$$

$$I_n = \frac{1}{101} \text{ mA} = 0.0099 \text{ mA}$$

$$I_p = 1 - I_n = 0.9901 \text{ mA}$$

1.91 Equation (1.65):

$$I_S = Aq n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$$

$$A = 100 \mu \text{m}^2 = 100 \times 10^{-8} \text{ cm}^2$$

$$I_S = 100 \times 10^{-8} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2$$

$$\left( \frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{17}} \right)$$

$$= 7.85 \times 10^{-17} \text{ A}$$

$$I \cong I_S e^{V/V_T}$$

$$= 7.85 \times 10^{-17} \times e^{750/25.9}$$

$$\cong 0.3 \text{ mA}$$

$$1.92 \quad n_i = B T^{3/2} e^{-E_g/2kT}$$

At 300 K,

$$n_i = 7.3 \times 10^{15} \times (300)^{3/2} \times e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 300)}$$

$$= 1.4939 \times 10^{10} / \text{cm}^2$$

$$n_i^2 (\text{at 300 K}) = 2.232 \times 10^{20}$$

At 305 K,

$$n_i = 7.3 \times 10^{15} \times (305)^{3/2} \times e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 305)} \\ = 2.152 \times 10^{10}$$

$$n_i^2 (\text{at } 305 \text{ K}) = 4.631 \times 10^{20}$$

$$\text{so } \frac{n_i^2 (\text{at } 305 \text{ K})}{n_i^2 (\text{at } 300 \text{ K})} = 2.152$$

Thus  $I_S$  approximately doubles for every 5°C rise in temperature.

$$C_{j0} = 100 \times 10^{-8}$$

$$\sqrt{\left(\frac{1.04 \times 10^{-12} \times 1.6 \times 10^{-19}}{2}\right) \left(\frac{10^{17} \times 10^{16}}{10^{17} + 10^{16}}\right) \frac{1}{0.754}} \\ = 31.6 \text{ fF}$$

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} = \frac{31.6 \text{ fF}}{\sqrt{1 + \frac{3}{0.754}}} \\ = 14.16 \text{ fF}$$

1.93 Equation (1.63):

$$I = Aq n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) (e^{V/V_T} - 1)$$

$$\text{So } I_p = Aq n_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

$$I_n = Aq n_i^2 \frac{D_n}{L_n N_A} (e^{V/V_T} - 1)$$

For p+–n junction  $N_A \gg N_D$ , thus  $I_p \gg I_n$  and

$$I \approx I_p = Aq n_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

For this case using Eq. (1.65):

$$I_S \approx Aq n_i^2 \frac{D_p}{L_p N_D} = 10^4 \times 10^{-8} \times 1.6 \times 10^{-19} \\ \times (1.5 \times 10^{10})^2 \frac{10}{10 \times 10^{-4} \times 10^{17}} \\ = 3.6 \times 10^{-16} \text{ A}$$

$$I = I_S (e^{V/V_T} - 1) = 1.0 \times 10^{-3}$$

$$3.6 \times 10^{-16} (e^{V/(25.9 \times 10^{-3})} - 1) = 1.0 \times 10^{-3}$$

$$\Rightarrow V = 0.742 \text{ V}$$

1.94 Equation (1.72):

$$C_{j0} = A \sqrt{\left(\frac{\epsilon_s q}{2}\right) \left(\frac{N_A N_D}{N_A + N_D}\right) \left(\frac{1}{V_0}\right)}$$

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 25.9 \times 10^{-3} \times \ln\left(\frac{10^{17} \times 10^{16}}{(1.5 \times 10^{10})^2}\right)$$

$$= 0.754 \text{ V}$$

$$1.95 \text{ Equation (1.73), } C_j = \frac{C_{j0}}{\left(1 + \frac{V_R}{V_0}\right)^m}$$

$$\text{For } V_R = 1 \text{ V, } C_j = \frac{0.4 \text{ pF}}{\left(1 + \frac{1}{0.75}\right)^{1/3}} \\ = 0.3 \text{ pF}$$

$$\text{For } V_R = 10 \text{ V, } C_j = \frac{0.4 \text{ pF}}{\left(1 + \frac{10}{0.75}\right)^{1/3}}$$

$$= 0.16 \text{ pF}$$

1.96 Equation (1.67):

$$\alpha = A \sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D}}$$

Equation (1.69):

$$C_j = \frac{\alpha}{2\sqrt{V_0 + V_R}}$$

Substitute for  $\alpha$  from Eq. (1.67):

$$C_j = \frac{A \sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D}}}{2\sqrt{V_0 + V_R}} \times \frac{\sqrt{\epsilon_s}}{\sqrt{\epsilon_s}} \\ = A \epsilon_s \times \frac{1}{\sqrt{\frac{2\epsilon_s}{q} \left(\frac{N_A + N_D}{N_A N_D}\right) (V_0 + V_R)}} \\ = \epsilon_s A \frac{1}{\sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D}\right) (V_0 + V_R)}} \\ = \frac{\epsilon_s A}{W}$$

1.97 Equation (1.75):

$$\tau_p = \frac{L_p^2}{D_p} = \frac{(10 \times 10^{-4})^2}{10}$$

(Note:  $1 \mu m = 10^{-4} \text{ cm.}$ )

$$\tau_p = 100 \text{ ns}$$

$$Q_p = \tau_p I_p \text{ (Eq. 1.76)}$$

$$= 100 \times 10^{-9} \times 0.1 \times 10^{-3}$$

$$= 10 \times 10^{-12} \text{ C}$$

$$C_d = \left( \frac{\tau_p}{V_T} \right) I$$

$$= \left( \frac{100 \times 10^{-9}}{25.9 \times 10^{-3}} \right) \times 0.1 \times 10^{-3}$$

$$= 386 \text{ pF}$$

1.98 Equation (1.81):

$$C_d = \left( \frac{\tau_T}{V_T} \right) I$$

$$5 \text{ pF} = \left( \frac{\tau_T}{25.9 \times 10^{-3}} \right) \times 1 \times 10^{-3}$$

$$\tau_T = 5 \times 10^{-12} \times 25.9$$

$$= 129.5 \text{ ps}$$

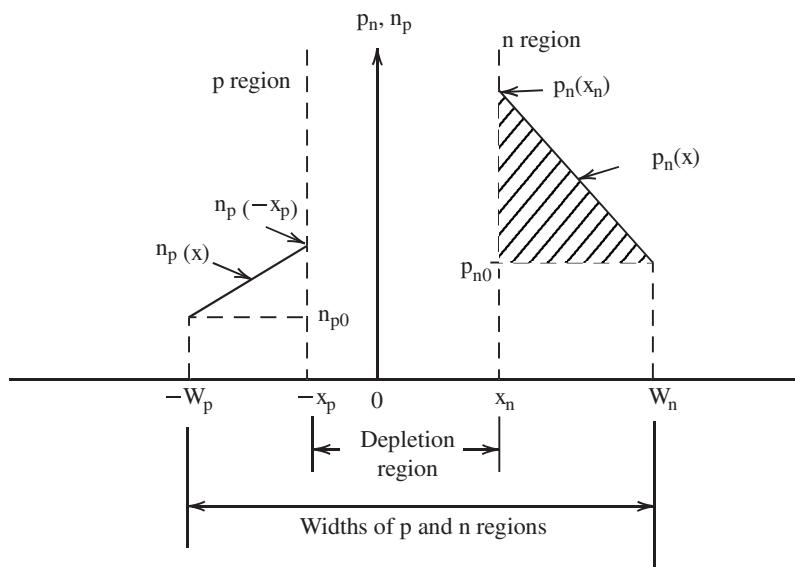
For  $I = 0.1 \text{ mA}$ :

$$C_d = \left( \frac{\tau_T}{V_T} \right) \times I$$

$$= \left( \frac{129.5 \times 10^{-12}}{25.9 \times 10^{-3}} \right) \times 0.1 \times 10^{-3} = 0.5 \text{ pF}$$

1.99

(a)



(b) The current  $I = I_p + I_n$ .

Find current component  $I_p$ :

$$p_n(x_n) = p_{n0} e^{V/V_T} \text{ and } p_{n0} = \frac{n_i^2}{N_D}$$

$$I_p = AJ_p = AqD_p \frac{dp}{dx}$$

$$\frac{dp}{dx} = \frac{p_n(x_n) - p_{n0}}{W_n - x_n} = \frac{p_{n0} e^{V/V_T} - p_{n0}}{W_n - x_n}$$

$$= p_{n0} \frac{(e^{V/V_T} - 1)}{W_n - x_n}$$

$$= \frac{n_i^2}{N_D} \frac{(e^{V/V_T} - 1)}{(W_n - x_n)}$$

$$\therefore I_p = AqD_p \frac{dp}{dx}$$

$$= Aq n_i^2 \frac{D_p}{(W_n - x_n) N_D} \times (e^{V/V_T} - 1)$$

Similarly,

$$I_n = Aq n_i^2 \frac{D_n}{(W_p - x_p) N_A} \times (e^{V/V_T} - 1)$$

$$I = I_p + I_n$$

$$= Aq n_i^2 \left[ \frac{D_p}{(W_n - x_n) N_D} + \frac{D_n}{(W_p - x_p) N_A} \right] \times (e^{V/V_T} - 1)$$

The excess charge,  $Q_p$ , can be obtained by multiplying the area of the shaded triangle of the  $p_n(x)$  distribution graph by  $Aq$ .

$$\begin{aligned}
 Q_p &= Aq \times \frac{1}{2} [p_n(x_n) - p_{n0}] (W_n - x_n) \\
 &= \frac{1}{2} Aq [p_{n0} e^{V/V_T} - p_{n0}] (W_n - x_n) \\
 &= \frac{1}{2} Aq p_{n0} (e^{V/V_T} - 1) (W_n - x_n) \\
 &= \frac{1}{2} Aq \frac{n_i^2}{N_D} (W_n - x_n) (e^{V/V_T} - 1) \\
 &= \frac{1}{2} \frac{(W_n - x_n)^2}{D_p} \cdot I_p \\
 &\simeq \frac{1}{2} \frac{W_n^2}{D_p} \cdot I_p \text{ for } W_n \gg x_n
 \end{aligned}$$

(c) For  $Q \simeq Q_p$ ,  $I \simeq I_p$ ,

$$Q \simeq \frac{1}{2} \frac{W_n^2}{D_p} I$$

Thus,  $\tau_T = \frac{1}{2} \frac{W_n^2}{D_p}$ , and

$$C_d = \frac{dQ}{dV} = \tau_T \frac{dI}{dV}$$

$$\text{But } I = I_S (e^{V/V_T} - 1)$$

$$\frac{dI}{dV} = \frac{I_S e^{V/V_T}}{V_T} \simeq \frac{I}{V_T}$$

$$\text{so } C_d \cong \tau_T \cdot \frac{I}{V_T}.$$

$$(d) C_d = \frac{1}{2} \frac{W_n^2}{10} \frac{1 \times 10^{-3}}{25.9 \times 10^{-3}} = 8 \times 10^{-12} F$$

Solve for  $W_n$ :

$$W_n = 0.64 \mu m$$

## Exercise 2-1

Ex: 2.1 The minimum number of terminals required by a single op amp is 5: two input terminals, one output terminal, one terminal for positive power supply, and one terminal for negative power supply.

The minimum number of terminals required by a quad op amp is 14: each op amp requires two input terminals and one output terminal (accounting for 12 terminals for the four op amps). In addition, the four op amps can all share one terminal for positive power supply and one terminal for negative power supply.

Ex: 2.2 Relevant equations are:

$$v_3 = A(v_2 - v_1); v_{Id} = v_2 - v_1,$$

$$v_{Icm} = \frac{1}{2}(v_1 + v_2)$$

(a)

$$v_1 = v_2 - \frac{v_3}{A} = 0 - \frac{2}{10^3} = -0.002 \text{ V} = -2 \text{ mV}$$

$$v_{Id} = v_2 - v_1 = 0 - (-0.002) = +0.002 \text{ V}$$

$$= 2 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(-2 \text{ mV} + 0) = -1 \text{ mV}$$

$$(b) -10 = 10^3(5 - v_1) \Rightarrow v_1 = 5.01 \text{ V}$$

$$v_{Id} = v_2 - v_1 = 5 - 5.01 = -0.01 \text{ V} = -10 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(5.01 + 5) = 5.005 \text{ V}$$

$$\simeq 5 \text{ V}$$

(c)

$$v_3 = A(v_2 - v_1) = 10^3(0.998 - 1.002) = -4 \text{ V}$$

$$v_{Id} = v_2 - v_1 = 0.998 - 1.002 = -4 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(1.002 + 0.998) = 1 \text{ V}$$

(d)

$$-3.6 = 10^3[v_2 - (-3.6)] = 10^3(v_2 + 3.6)$$

$$\Rightarrow v_2 = -3.6036 \text{ V}$$

$$v_{Id} = v_2 - v_1 = -3.6036 - (-3.6)$$

$$= -0.0036 \text{ V} = -3.6 \text{ mV}$$

$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}[-3.6 + (-3.6036)]$$

$$\simeq -3.6 \text{ V}$$

Ex: 2.3 From Fig. E2.3 we have:  $v_3 = \mu v_d$  and  $v_d = (G_m v_2 - G_m v_1)R = G_m R(v_2 - v_1)$

Therefore:

$$v_3 = \mu G_m R(v_2 - v_1)$$

That is, the open-loop gain of the op amp is  $A = \mu G_m R$ . For  $G_m = 10 \text{ mA/V}$  and

$\mu = 100$ , we have:

$$A = 100 \times 10 \times 10 = 10^4 \text{ V/V}, \text{ or equivalently, } 80 \text{ dB.}$$

Ex: 2.4 The gain and input resistance of the inverting amplifier circuit shown in Fig. 2.5 are  $\frac{R_2}{R_1}$  and  $R_1$ , respectively. Therefore, we have:

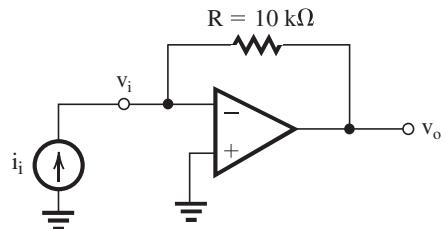
$R_1 = 100 \text{ k}\Omega$  and

$$-\frac{R_2}{R_1} = -10 \Rightarrow R_2 = 10 R_1$$

Thus:

$$R_2 = 10 \times 100 \text{ k}\Omega = 1 \text{ M}\Omega$$

Ex: 2.5



From Table 1.1 we have:

$$R_m = \left. \frac{v_o}{i_i} \right|_{i_o=0}; \text{ that is, output is open circuit}$$

The negative input terminal of the op amp (i.e.,  $v_i$ ) is a virtual ground, thus  $v_i = 0$ :

$$v_o = v_i - R_i = 0 - R_i = -R_i$$

$$R_m = \left. \frac{v_o}{i_i} \right|_{i_o=0} = -\frac{R_i}{i_i} = -R \Rightarrow R_m = -R$$

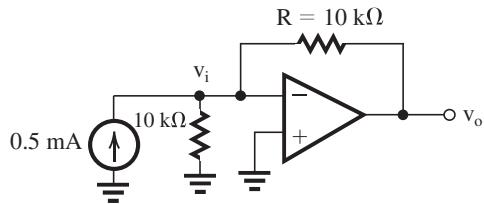
$$= -10 \text{ k}\Omega$$

$$R_i = \frac{v_i}{i_i} \text{ and } v_i \text{ is a virtual ground } (v_i = 0),$$

$$\text{thus } R_i = \frac{0}{i_i} = 0 \Rightarrow R_i = 0 \Omega$$

Since we are assuming that the op amp in this transresistance amplifier is ideal, the op amp has zero output resistance and therefore the output resistance of this transresistance amplifier is also zero. That is  $R_o = 0 \Omega$ .

### Exercise 2-2

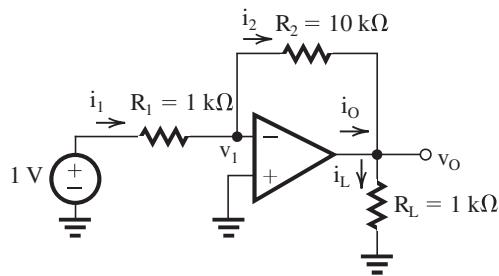


Connecting the signal source shown in Fig. E2.5 to the input of this amplifier, we have:

$v_i$  is a virtual ground that is  $v_i = 0$ , thus the current "owing through the  $10\text{-k}\Omega$  resistor connected between  $v_i$  and ground is zero. Therefore,

$$v_o = v_i - R \times 0.5 \text{ mA} = 0 - 10 \text{ k}\Omega \times 0.5 \text{ mA} = -5 \text{ V}$$

Ex: 2.6



$v_1$  is a virtual ground, thus  $v_1 = 0 \text{ V}$

$$i_1 = \frac{1 \text{ V} - v_1}{R_1} = \frac{1 - 0}{1 \text{ k}\Omega} = 1 \text{ mA}$$

Assuming an ideal op amp, the current "owing into the negative input terminal of the op amp is zero. Therefore,  $i_2 = i_1 \Rightarrow i_2 = 1 \text{ mA}$

$$v_o = v_1 - i_2 R_2 = 0 - 1 \text{ mA} \times 10 \text{ k}\Omega = -10 \text{ V}$$

$$i_L = \frac{v_o}{R_L} = \frac{-10 \text{ V}}{1 \text{ k}\Omega} = -10 \text{ mA}$$

$$i_O = i_L - i_2 = -10 \text{ mA} - 1 \text{ mA} = -11 \text{ mA}$$

$$\text{Voltage gain} = \frac{v_o}{1 \text{ V}} = \frac{-10 \text{ V}}{1 \text{ V}} = -10 \text{ V/V or } 20 \text{ dB}$$

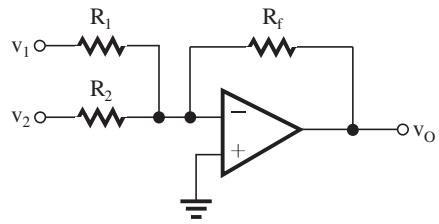
$$\text{Current gain} = \frac{i_L}{i_1} = \frac{-10 \text{ mA}}{1 \text{ mA}} = -10 \text{ A/A or } 20 \text{ dB}$$

Power gain

$$= \frac{P_L}{P_i} = \frac{-10(-10 \text{ mA})}{1 \text{ V} \times 1 \text{ mA}} = 100 \text{ W/W or } 20 \text{ dB}$$

Note that power gain in dB is  $10 \log_{10} \left| \frac{P_L}{P_i} \right|$ .

Ex: 2.7



For the circuit shown above we have:

$$v_o = - \left( \frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 \right)$$

Since it is required that  $v_o = -(v_1 + 5v_2)$ , we want to have:

$$\frac{R_f}{R_1} = 1 \quad \text{and} \quad \frac{R_f}{R_2} = 5$$

It is also desired that for a maximum output voltage of 10 V, the current in the feedback resistor not exceed 1 mA.

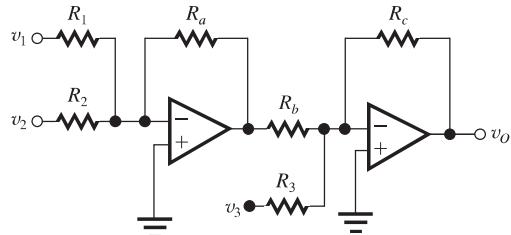
Therefore

$$\frac{10 \text{ V}}{R_f} \leq 1 \text{ mA} \Rightarrow R_f \geq \frac{10 \text{ V}}{1 \text{ mA}} \Rightarrow R_f \geq 10 \text{ k}\Omega$$

Let us choose  $R_f$  to be  $10 \text{ k}\Omega$ , then

$$R_1 = R_f = 10 \text{ k}\Omega \text{ and } R_2 = \frac{R_f}{5} = 2 \text{ k}\Omega$$

Ex: 2.8



$$v_o = \left( \frac{R_a}{R_1} \right) \left( \frac{R_c}{R_b} \right) v_1 + \left( \frac{R_a}{R_2} \right) \left( \frac{R_c}{R_b} \right) v_2 - \left( \frac{R_c}{R_3} \right) v_3$$

We want to design the circuit such that

$$v_o = 2v_1 + v_2 - 4v_3$$

Thus we need to have

$$\left( \frac{R_a}{R_1} \right) \left( \frac{R_c}{R_b} \right) = 2, \left( \frac{R_a}{R_2} \right) \left( \frac{R_c}{R_b} \right) = 1, \text{ and } \frac{R_c}{R_3} = 4$$

From the above three equations, we have to find six unknown resistors; therefore, we can arbitrarily choose three of these resistors. Let us choose  $R_a = R_b = R_c = 10 \text{ k}\Omega$ .

### Exercise 2-3

Then we have

$$R_3 = \frac{R_c}{4} = \frac{10}{4} = 2.5 \text{ k}\Omega$$

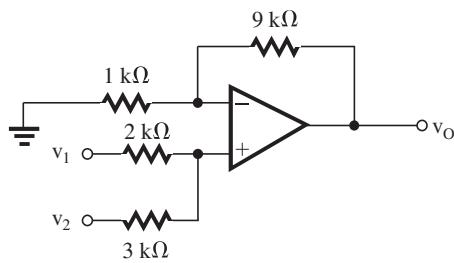
$$\left(\frac{R_a}{R_l}\right)\left(\frac{R_c}{R_b}\right) = 2, \Rightarrow \frac{10}{R_l} \times \frac{10}{10} = 2$$

$$\Rightarrow R_l = 5 \text{ k}\Omega$$

$$\left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) = 1 \Rightarrow \frac{10}{R_2} \times \frac{10}{10} = 1$$

$$\Rightarrow R_2 = 10 \text{ k}\Omega$$

Ex: 2.9 Using the superposition principle to find the contribution of  $v_1$  to the output voltage  $v_o$ , we set  $v_2 = 0$



$v_+$  (the voltage at the positive input of the op amp

$$\text{is: } v_+ = \frac{3}{2+3}v_1 = 0.6v_1$$

$$\text{Thus } v_o = \left(1 + \frac{9 \text{ k}\Omega}{1 \text{ k}\Omega}\right)v_+ = 10 \times 0.6v_1 = 6v_1$$

To find the contribution of  $v_2$  to the output voltage  $v_o$  we set  $v_1 = 0$ .

$$\text{Then } v_+ = \frac{2}{2+3}v_2 = 0.4v_2$$

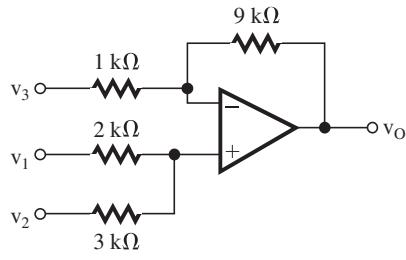
Hence

$$v_o = \left(1 + \frac{9 \text{ k}\Omega}{1 \text{ k}\Omega}\right)v_+ = 10 \times 0.4v_2 = 4v_2$$

Combining the contributions of  $v_1$  and  $v_2$

to  $v_o$ , we have  $v_o = 6v_1 + 4v_2$

Ex: 2.10



Using the superposition principle to find the contribution of  $v_1$  to  $v_o$ , we set  $v_2 = v_3 = 0$ . Then we have (refer to the solution of Exercise 2.9):  $v_o = 6v_1$

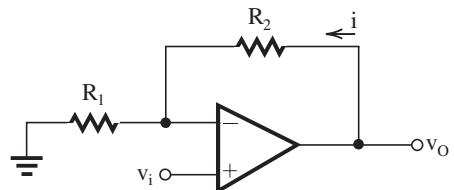
To find the contribution of  $v_2$  to  $v_o$ , we set  $v_1 = v_3 = 0$ , then:  $v_o = 4v_2$

To find the contribution of  $v_3$  to  $v_o$  we set  $v_1 = v_2 = 0$ , then

$$v_o = -\frac{9 \text{ k}\Omega}{1 \text{ k}\Omega} v_3 = -9v_3$$

Combining the contributions of  $v_1$ ,  $v_2$ , and  $v_3$  to  $v_o$  we have:  $v_o = 6v_1 + 4v_2 - 9v_3$ .

Ex: 2.11



$$\frac{v_o}{v_i} = 1 + \frac{R_2}{R_1} = 2 \Rightarrow \frac{R_2}{R_1} = 1 \Rightarrow R_1 = R_2$$

If  $v_o = 10 \text{ V}$ , then it is desired that  $i = 10 \mu\text{A}$ .

Thus,

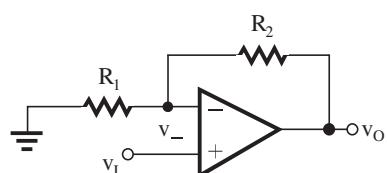
$$i = \frac{10 \text{ V}}{R_1 + R_2} = 10 \mu\text{A} \Rightarrow R_1 + R_2 = \frac{10 \text{ V}}{10 \mu\text{A}}$$

$R_1 + R_2 = 1 \text{ M}\Omega$  and

$$R_1 = R_2 \Rightarrow R_1 = R_2 = 0.5 \text{ M}\Omega$$

Ex: 2.12

(a)



$$v_i - v_- = v_o/A \Rightarrow v_- = v_i - v_o/A \quad (1)$$

But from the voltage divider across  $v_o$ ,

$$v_- = v_o \frac{R_1}{R_1 + R_2} \quad (2)$$

Equating Eq. (1) and Eq. (2) gives

$$v_o \frac{R_1}{R_1 + R_2} = v_i - \frac{v_o}{A}$$

### Exercise 2-4

which can be manipulated to the form

$$\frac{v_o}{v_i} = \frac{1 + (R_2/R_1)}{1 + (R_2/R_1)A}$$

(b) For  $R_1 = 1\text{ k}\Omega$  and  $R_2 = 9\text{ k}\Omega$  the ideal value for the closed-loop gain is  $1 + \frac{9}{1}$ , that is, 10. The actual closed-loop gain is  $G = \frac{10}{1 + 10/A}$ .

If  $A = 10^3$ , then  $G = 9.901$  and

$$\epsilon = \frac{G - 10}{10} \times 100 = -0.99\% \simeq -1\%$$

For  $v_i = 1\text{ V}$ ,  $v_o = G \times v_i = 9.901\text{ V}$  and

$$v_o = A(v_+ - v_-) \Rightarrow v_+ - v_- = \frac{v_o}{A} = \frac{9.901}{1000} \simeq 9.9\text{ mV}$$

If  $A = 10^4$ , then  $G = 9.99$  and  $\epsilon = -0.1\%$ .

For  $v_i = 1\text{ V}$ ,  $v_o = G \times v_i = 9.99\text{ V}$ ,

therefore,

$$v_+ - v_- = \frac{v_o}{A} = \frac{9.99}{10^4} = 0.999\text{ mV} \simeq 1\text{ mV}$$

If  $A = 10^5$ , then  $G = 9.999$  and  $\epsilon = -0.01\%$

For  $v_i = 1\text{ V}$ ,  $v_o = G \times v_i = 9.999$  thus,

$$v_+ - v_- = \frac{v_o}{A} = \frac{9.999}{10^5} = 0.09999\text{ mV}$$

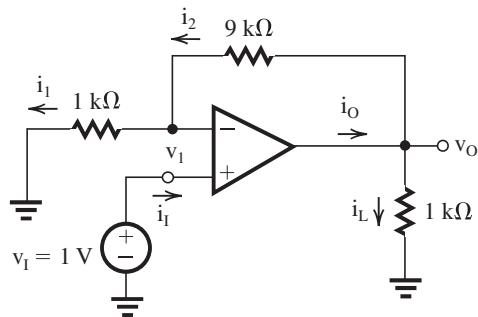
$\simeq 0.1\text{ mV}$

### Ex: 2.13

$$i_l = 0\text{ A}, v_i = v_1 = 1\text{ V}$$

$$i_1 = \frac{v_1}{1\text{ k}\Omega} = \frac{1\text{ V}}{1\text{ k}\Omega} = 1\text{ mA}$$

$$i_2 = i_1 = 1\text{ mA}$$



$$v_o = v_1 + i_2 \times 9\text{ k}\Omega = 1 + 1 \times 9 = 10\text{ V}$$

$$i_L = \frac{v_o}{1\text{ k}\Omega} = \frac{10\text{ V}}{1\text{ k}\Omega} = 10\text{ mA}$$

$$i_o = i_L + i_2 = 11\text{ mA}$$

$$\frac{v_o}{v_i} = \frac{10\text{ V}}{1\text{ V}} = 10\text{ V/V or } 20\text{ dB}$$

$$\frac{i_L}{i_1} = \frac{10\text{ mA}}{0} = \infty$$

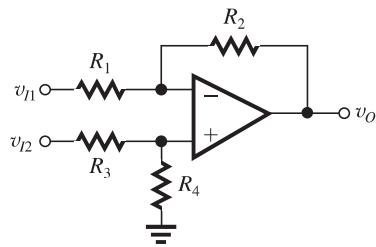
$$\frac{P_L}{P_i} = \frac{v_o \times i_L}{v_i \times i_1} = \frac{10 \times 10}{1 \times 0} = \infty$$

### Ex: 2.14

$$(a) \text{ Load voltage} = \frac{1\text{ k}\Omega}{1\text{ k}\Omega + 1\text{ M}\Omega} \times 1\text{ V} \simeq 1\text{ mV}$$

$$(b) \text{ Load voltage} = 1\text{ V}$$

### Ex: 2.15



$$(a) R_1 = R_3 = 2\text{ k}\Omega, R_2 = R_4 = 200\text{ k}\Omega$$

Since  $R_4/R_3 = R_2/R_1$  we have:

$$A_d = \frac{v_o}{v_{12} - v_{11}} = \frac{R_2}{R_1} = \frac{200}{2} = 100\text{ V/V}$$

$$(b) R_{id} = 2R_1 = 2 \times 2\text{ k}\Omega = 4\text{ k}\Omega$$

Since we are assuming the op amp is ideal,

$$R_o = 0\text{ }\Omega$$

### (c)

$$A_{cm} \equiv \frac{v_o}{v_{1cm}} = \frac{R_4}{R_3 + R_4} \left( 1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$

The worst-case common-mode gain (i.e., the largest  $A_{cm}$ ) occurs when the resistor tolerances are such that the quantity in parentheses is maximum. This in turn occurs when  $R_2$  and  $R_3$  are at their highest possible values (each one percent above nominal) and  $R_1$  and  $R_4$  are at their lowest possible values (each one percent below nominal), resulting in

$$A_{cm} = \frac{R_4}{R_3 + R_4} \left( 1 - \frac{1.01 \times 1.01}{0.99 \times 0.99} \right)$$

$$|A_{cm}| \simeq \frac{R_4}{R_3 + R_4} \times 0.04 \simeq \frac{200}{202} \times 0.04 \simeq 0.04\text{ V/V}$$

The corresponding CMRR is

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.04} = 2500$$

or 68 dB.

### Exercise 2-5

Ex: 2.16 We choose  $R_3 = R_1$  and  $R_4 = R_2$ . Then for the circuit to behave as a difference amplifier with a gain of 10 and an input resistance of  $20\text{ k}\Omega$ , we require

$$A_d = \frac{R_2}{R_1} = 10 \text{ and}$$

$$R_{ld} = 2R_1 = 20\text{ k}\Omega \Rightarrow R_1 = 10\text{ k}\Omega \text{ and}$$

$$R_2 = A_d R_1 = 10 \times 10\text{ k}\Omega = 100\text{ k}\Omega$$

Therefore,  $R_1 = R_3 = 10\text{ k}\Omega$  and

$$R_2 = R_4 = 100\text{ k}\Omega.$$

Ex: 2.17 Given  $v_{icm} = +5\text{ V}$

$$v_{Id} = 10 \sin \omega t \text{ mV}$$

$$2R_1 = 1\text{ k}\Omega, R_2 = 0.5\text{ M}\Omega$$

$$R_3 = R_4 = 10\text{ k}\Omega$$

$$v_{I1} = v_{icm} - \frac{1}{2}v_{Id} = 5 - \frac{1}{2} \times 0.01 \sin \omega t$$

$$= 5 - 0.005 \sin \omega t \text{ V}$$

$$v_{I2} = v_{icm} + \frac{1}{2}v_{Id}$$

$$= 5 + 0.005 \sin \omega t \text{ V}$$

$$v_-(\text{op amp } A_1) = v_{I1} = 5 - 0.005 \sin \omega t \text{ V}$$

$$v_-(\text{op amp } A_2) = v_{I2} = 5 + 0.005 \sin \omega t \text{ V}$$

$$v_{Id} = v_{I2} - v_{I1} = 0.01 \sin \omega t$$

$$v_{O1} = v_{I1} - R_2 \times \frac{v_{Id}}{2R_1}$$

$$= 5 - 0.005 \sin \omega t - 500\text{ k}\Omega \times \frac{0.01 \sin \omega t}{1\text{ k}\Omega}$$

$$= (5 - 5.005 \sin \omega t) \text{ V}$$

$$v_{O2} = v_{I2} + R_2 \times \frac{v_{Id}}{2R_1}$$

$$= (5 + 5.005 \sin \omega t) \text{ V}$$

$$v_+(\text{op amp } A_3) = v_{O2} \times \frac{R_4}{R_3 + R_4} = v_{O2} \frac{10}{10 + 10}$$

$$= \frac{1}{2}v_{O2} = \frac{1}{2}(5 + 5.005 \sin \omega t)$$

$$= (2.5 + 2.5025 \sin \omega t) \text{ V}$$

$$v_-(\text{op amp } A_3) = v_+(\text{op amp } A_3)$$

$$= (2.5 + 2.5025 \sin \omega t) \text{ V}$$

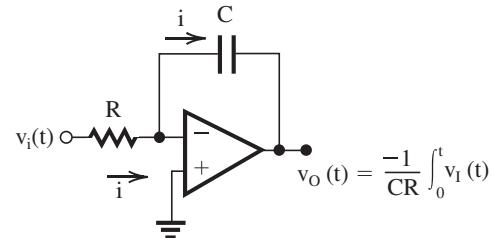
$$v_O = \frac{R_4}{R_3} \left( 1 + \frac{R_2}{R_1} \right) v_{Id}$$

$$\frac{10\text{ k}\Omega}{10\text{ k}\Omega} \left( 1 + \frac{0.5\text{ M}\Omega}{0.5\text{ k}\Omega} \right) \times 0.01 \sin \omega t$$

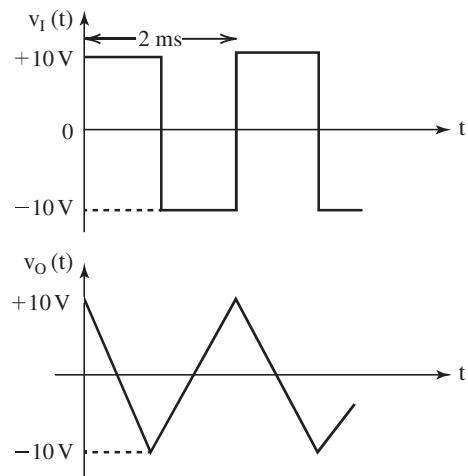
$$= 1(1 + 1000) \times 0.01 \sin \omega t$$

$$= 10.01 \sin \omega t \text{ V}$$

Ex: 2.18



The signal waveforms will be as shown.



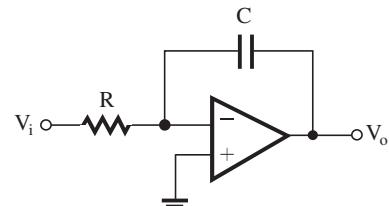
When  $v_i = +10\text{ V}$ , the current through the capacitor will be in the direction indicated,  $i = 10\text{ V}/R$ , and the output voltage will decrease linearly from  $+10\text{ V}$  to  $-10\text{ V}$ . Thus in  $(T/2)$  seconds, the capacitor voltage changes by  $20\text{ V}$ . The charge equilibrium equation can be expressed as

$$i(T/2) = C \times 20\text{ V}$$

$$\frac{10}{R} \frac{T}{2} = 20C \Rightarrow CR = \frac{10T}{40} = \frac{1}{4} \times 2 \times 10^{-3}$$

$$= 0.5\text{ ms}$$

Ex: 2.19



The input resistance of this inverting integrator is  $R$ ; therefore,  $R = 10\text{ k}\Omega$ .

## Exercise 2–6

Since the desired integration time constant is  $10^{-3}$  s, we have:  $CR = 10^{-3}$  s  $\Rightarrow$

$$C = \frac{10^{-3} \text{ s}}{10 \text{ k}\Omega} = 0.1 \mu\text{F}$$

From Eq. (2.27) the transfer function of this integrator is:

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR}$$

For  $\omega = 10$  rad/s, the integrator transfer function has magnitude

$$\left| \frac{V_o}{V_i} \right| = \frac{1}{1 \times 10^{-3}} = 100 \text{ V/V}$$

and phase  $\phi = 90^\circ$ .

For  $\omega = 1$  rad/s, the integrator transfer function has magnitude

$$\left| \frac{V_o}{V_i} \right| = \frac{1}{1 \times 10^{-3}} = 1000 \text{ V/V}$$

and phase  $\phi = 90^\circ$ .

The frequency at which the integrator gain magnitude is unity is

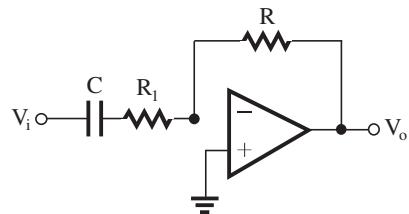
$$\omega_{int} = \frac{1}{CR} = \frac{1}{10^{-3}} = 1000 \text{ rad/s}$$

For  $\omega = 10^3$  rad/s, the differentiator transfer function has magnitude

$$\left| \frac{V_o}{V_i} \right| = 10^3 \times 10^{-2} = 10 \text{ V/V}$$

and phase  $\phi = -90^\circ$ .

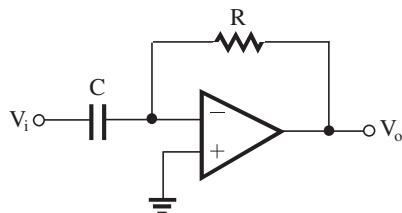
If we add a resistor in series with the capacitor to limit the high-frequency gain of the differentiator to 100, the circuit would be:



At high frequencies the capacitor C acts like a short circuit. Therefore, the high-frequency gain of this circuit is:  $\frac{R}{R_l}$ . To limit the magnitude of this high-frequency gain to 100, we should have:

$$\frac{R}{R_l} = 100 \Rightarrow R_l = \frac{R}{100} = \frac{1 \text{ M}\Omega}{100} = 10 \text{ k}\Omega$$

### Ex: 2.20



$C = 0.01 \mu\text{F}$  is the input capacitance of this differentiator. We want  $CR = 10^{-2}$  s (the time constant of the differentiator); thus,

$$R = \frac{10^{-2}}{0.01 \mu\text{F}} = 1 \text{ M}\Omega$$

From Eq. (2.33), the transfer function of the differentiator is

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -j\omega CR$$

Thus, for  $\omega = 10$  rad/s the differentiator transfer function has magnitude

$$\left| \frac{V_o}{V_i} \right| = 10 \times 10^{-2} = 0.1 \text{ V/V}$$

and phase  $\phi = -90^\circ$ .

### Ex: 2.21

Refer to the model in Fig. 2.28 and observe that

$$v_+ - v_- = V_{os} + v_2 - v_1 = V_{os} + v_{id}$$

and since  $v_o = v_3 = A(v_+ - v_-)$ , then

$$v_o = A(v_{id} + V_{os}) \quad (1)$$

where  $A = 10^4 \text{ V/V}$  and  $V_{os} = 5 \text{ mV}$ . From Eq. (1) we see that  $v_{id} = 0$  results in  $v_o = 50 \text{ V}$ , which is impossible; thus the op amp saturates and  $v_o = +10 \text{ V}$ . This situation pertains for  $v_{id} \geq -4 \text{ mV}$ . If  $v_{id}$  decreases below  $-4 \text{ mV}$ , the op-amp output decreases correspondingly. For instance,  $v_{id} = -4.5 \text{ mV}$  results in  $v_o = +5 \text{ V}$ ;  $v_{id} = -5 \text{ mV}$  results in  $v_o = 0 \text{ V}$ ;  $v_{id} = -5.5 \text{ mV}$  results in  $v_o = -5 \text{ V}$ ; and  $v_{id} = -6 \text{ mV}$  results in  $v_o = -10 \text{ V}$ , at which point the op amp saturates at the negative level of  $-10 \text{ V}$ . Further decreases in  $v_{id}$  have no effect on the output voltage. The result is the transfer characteristic sketched in Fig. E2.21. Observe that the linear range of the characteristic is now centered around  $v_{id} = -5 \text{ mV}$  rather than the ideal situation of  $v_{id} = 0$ ; this shift is obviously a result of the input offset voltage  $V_{os}$ .

### Exercise 2-7

Ex: 2.22 (a) The inverting amplifier of  $-1000$  V/V gain will exhibit an output dc offset voltage of  $\pm V_{OS}(1 + R_2/R_1) = \pm 3 \text{ mV} \times (1 + 1000) = \pm 3.03 \text{ V}$ . Now, since the op-amp saturation levels are  $\pm 10 \text{ V}$ , the room left for output signal swing is approximately  $\pm 7 \text{ V}$ . Thus to avoid op-amp saturation and the attendant clipping of the peak of the output sinusoid, we must limit the peak amplitude of the input sine wave to approximately  $7 \text{ V}/1000 = 7 \text{ mV}$ .

(b) If at room temperature ( $25^\circ\text{C}$ ),  $V_{OS}$  is trimmed to zero and (i) the circuit is operated at a constant temperature, the peak of the input sine wave can be increased to  $10 \text{ mV}$ . (ii) However, if the circuit is to operate in the temperature range of  $0^\circ\text{C}$  to  $75^\circ\text{C}$  (i.e., at a temperature that deviates from room temperature by a maximum of  $50^\circ\text{C}$ ), the input offset voltage will drift from a maximum of  $10 \mu\text{V}/^\circ\text{C} \times 50^\circ\text{C} = 500 \mu\text{V}$  or  $0.5 \text{ mV}$ . This will reduce the allowed peak amplitude of the input sinusoid to  $9.5 \text{ mV}$ .

### Ex: 2.23

(a) If the amplifier is capacitively coupled in the manner of Fig. 2.31(a), then the input offset voltage  $V_{OS}$  will see a unity-gain amplifier [Fig. 2.31(b)] and the cc offset voltage at the output will be equal to  $V_{OS}$ , that is,  $3 \text{ mV}$ . Thus, almost the entire output range of  $\pm 10 \text{ V}$  will be available for signal swing, allowing a sine-wave input of approximately  $10\text{-mV}$  peak without the risk of output clipping. Obviously, in this case there is no need for output trimming.

(b) We need to select a value of the coupling capacitor  $C$  that will place the 3-dB frequency of the resulting high-pass STC circuit at  $100 \text{ Hz}$ , thus

$$100 = \frac{1}{2\pi CR_1}$$

$$\Rightarrow C = \frac{1}{2\pi \times 100 \times 1 \times 10^3} = 1.6 \mu\text{F}$$

### Ex: 2.24 From Eq. (2.37) we have:

$$V_O = I_B R_2 \simeq I_B R_2$$

$$= 100 \text{ nA} \times 1 \text{ M}\Omega = 0.1 \text{ V}$$

From Eq. (2.39) the value of resistor  $R_3$  (placed in series with positive input to minimize the output offset voltage) is

$$R_3 = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \text{ k}\Omega \times 1 \text{ M}\Omega}{10 \text{ k}\Omega + 1 \text{ M}\Omega}$$

$$= 9.9 \text{ k}\Omega$$

$$R_3 = 9.9 \text{ k}\Omega \simeq 10 \text{ k}\Omega$$

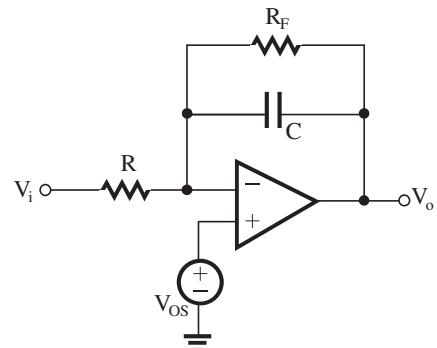
With this value of  $R_3$ , the new value of the output dc voltage [using Eq. (2.40)] is:

$$V_O = I_{OS} R_2 = 10 \text{ nA} \times 10 \text{ k}\Omega = 0.01 \text{ V}$$

### Ex: 2.25 Using Eq. (2.41) we have:

$$v_O = V_{OS} + \frac{V_{OS}}{CR} t \Rightarrow 12 = 2 \text{ mV} + \frac{2 \text{ mV}}{1 \text{ ms}} t$$

$$\Rightarrow t = \frac{12 \text{ V} - 2 \text{ mV}}{2 \text{ mV}} \times 1 \text{ ms} \simeq 6 \text{ s}$$



With the feedback resistor  $R_F$ , to have at least  $\pm 10 \text{ V}$  of output signal swing available, we have to make sure that the output voltage due to  $V_{OS}$  has a magnitude of at most  $2 \text{ V}$ . From Eq. (2.36), we know that the output dc voltage due to  $V_{OS}$  is

$$V_O = V_{OS} \left( 1 + \frac{R_F}{R} \right) \Rightarrow 2 \text{ V} = 2 \text{ mV} \left( 1 + \frac{R_F}{10 \text{ k}\Omega} \right)$$

$$1 + \frac{R_F}{10 \text{ k}\Omega} = 1000 \Rightarrow R_F \simeq 10 \text{ M}\Omega$$

The corner frequency of the resulting STC

$$\text{network is } \omega_0 = \frac{1}{CR_F}$$

We know  $RC = 1 \text{ ms}$  and

$$R = 10 \text{ k}\Omega \Rightarrow C = 0.1 \mu\text{F}$$

$$\text{Thus } \omega_0 = \frac{1}{0.1 \mu\text{F} \times 10 \text{ M}\Omega} = 1 \text{ rad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} = 0.16 \text{ Hz}$$

### Ex: 2.26

$$20 \log A_0 = 106 \text{ dB} \Rightarrow A_0 = 200,000 \text{ V/V}$$

$$f_t = 3 \text{ MHz}$$

$$f_b = f_t / A_0 = \frac{3 \text{ MHz}}{200,000} = 15 \text{ Hz}$$

### Exercise 2-8

At  $f_b$ , the open-loop gain drops by 3 dB below its value at dc; thus it becomes 103 dB.

For  $f \gg f_b$ ,  $|A| \approx f_t/f$ ; thus

$$\text{At } f = 300 \text{ Hz}, |A| = \frac{3 \text{ MHz}}{300 \text{ Hz}} = 10^4 \text{ V/V}$$

or 80 dB

$$\text{At } f = 3 \text{ kHz}, |A| = \frac{3 \text{ MHz}}{3 \text{ kHz}} = 10^3 \text{ V/V}$$

or 60 dB

At  $f = 12 \text{ kHz}$ , which is two octaves higher than 3 kHz, the gain will be  $2 \times 6 = 12 \text{ dB}$  below its value at 3 kHz; that is,  $60 - 12 = 48 \text{ dB}$ .

$$\text{At } f = 60 \text{ kHz}, |A| = \frac{3 \text{ MHz}}{60 \text{ kHz}} = 50 \text{ V/V}$$

or 34 dB

Ex: 2.27

$$A_0 = 10^6 \text{ V/V or } 120 \text{ dB}$$

The gain falls off at the rate of 20 dB/decade. Thus, it reaches 40 dB at a frequency four decades higher than  $f_b$ ,

$$10^4 f_b = 10 \text{ kHz} \Rightarrow f_b = 1 \text{ Hz}$$

The unity-gain frequency  $f_t$  will be two decades higher than 10 kHz, that is,

$$f_t = 100 \times 10 \text{ kHz} = 1 \text{ MHz}$$

Alternatively, we could have found  $f_t$  from

$$f_t = A_0 f_b = 10^6 \times 1 \text{ Hz} = 1 \text{ MHz}$$

At a frequency  $f \gg f_b$ ,

$$|A| = f_t/f$$

$$\text{For } f = 1 \text{ kHz}, |A| = \frac{1 \text{ MHz}}{1 \text{ kHz}} = 10^3 \text{ V/V or } 60 \text{ dB}$$

Ex: 2.28

$$20 \log A_0 = 106 \text{ dB} \Rightarrow A_0 = 200,000 \text{ V/V}$$

$$f_t = 2 \text{ MHz}$$

For a noninverting amplifier with a nominal dc gain of 100,

$$1 + \frac{R_2}{R_1} = 100$$

Since the nominal dc gain is much lower than  $A_0$ ,

$$f_{3\text{dB}} \approx f_t / \left( 1 + \frac{R_2}{R_1} \right)$$

$$= \frac{2 \text{ MHz}}{100} = 20 \text{ kHz}$$

Ex: 2.29 For the input voltage step of magnitude  $V$  the output waveform will still be given by the exponential waveform of Eq. (2.58) if

$$\omega_t V \leq SR$$

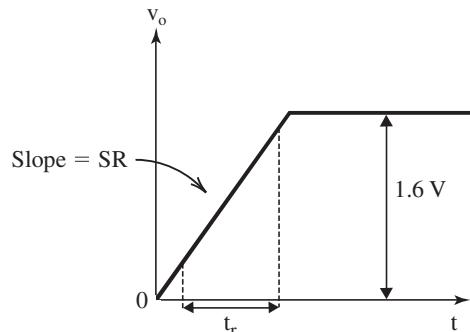
$$\text{that is, } V \leq \frac{SR}{\omega_t} \Rightarrow V \leq \frac{SR}{2\pi f_t} \text{ resulting in}$$

$$V \leq 0.16 \text{ V}$$

From Appendix F we know that the 10% to 90% rise time of the output waveform of the form of Eq. (2.58) is  $t_r \approx 2.2 \times \text{time constant} = \frac{2.2}{\omega_t}$ .

Thus,  $t_r \approx 0.35 \mu\text{s}$

If an input step of amplitude 1.6 V (10 times as large compared to the previous case) is applied, the output will be slew-rate limited and thus linearly rising with a slope equal to the slew rate, as shown in the following figure.



$$t_r = \frac{0.9 \times 1.6 - 0.1 \times 1.6}{1 \text{ V}/\mu\text{s}}$$

$$\Rightarrow t_r = 1.28 \mu\text{s}$$

Ex: 2.30 From Eq. (2.59) we have:

$$f_M = \frac{SR}{2\pi V_{O_{\max}}} = 15.915 \text{ kHz} \approx 15.9 \text{ kHz}$$

Using Eq. (2.60), for an input sinusoid with frequency  $f = 5 f_M$ , the maximum possible amplitude that can be accommodated at the output without incurring SR distortion is:

$$V_O = V_{O_{\max}} \left( \frac{f_M}{5f_M} \right) = 10 \times \frac{1}{5} = 2 \text{ V (peak)}$$

## Chapter 2–1

2.1 The minimum number of pins required by dual op amp is 8. Each op amp has 2 input terminals (4 pins) and one output terminal (2 pins). Another 2 pins are required for power supplies.

Similarly, the minimum number of pins required by quad op amp is 14:

$$4 \times 2 + 4 \times 1 + 2 = 14$$

2.2 Refer to Fig. P2.2.

$$v_+ = v_I \times \frac{1 \text{ k}\Omega}{1 \text{ k}\Omega + 1 \text{ M}\Omega} = v_I \frac{1}{1001}$$

$$v_O = A v_+ = A v_I \frac{1}{1001}$$

$$A = 1001 \frac{v_O}{v_I}$$

$$= 1001 \times \frac{4}{1}$$

$$A = 4004 \text{ V/V}$$

2.3

#	$v_1$	$v_2$	$v_d = v_2 - v_1$	$v_O$	$v_O/v_d$
1	0.00	0.00	0.00	0.00	—
2	1.00	1.00	0.00	0.00	—
3 (a)	1.00	(b)	1.00		
4	1.00	1.10	0.10	10.1	101
5	2.01	2.00	-0.01	-0.99	99
6	1.99	2.00	0.01	1.00	100
7	5.10	(c)	(d)	-5.10	

Experiments 4, 5, and 6 show that the gain is approximately 100 V/V.

The missing entry for experiment #3 can be predicted as follows:

$$(a) v_d = \frac{v_O}{A} = \frac{1.00}{100} = 0.01 \text{ V.}$$

$$(b) v_1 = v_2 - v_d = 1.00 - 0.01 = 0.99 \text{ V}$$

The missing entries for experiment #7:

$$(c) v_d = \frac{-5.10}{100} = -0.051 \text{ V}$$

$$(d) v_2 = v_1 + v_d = 5.10 - 0.051 = 5.049 \text{ V}$$

All the results seem to be reasonable.

$$2.4 i = G_m(v_2 - v_1)$$

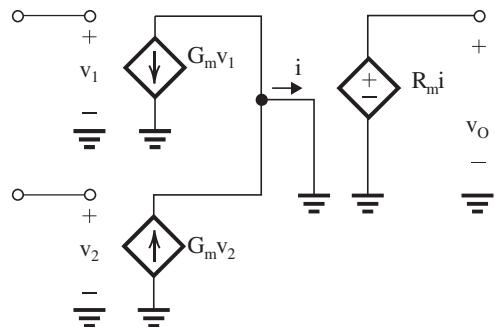
$$v_O = R_m i$$

$$= R_m G_m (v_2 - v_1)$$

$$\text{Gain } A = \frac{v_O}{v_2 - v_1} = R_m G_m$$

$$= 1 \times 10^6 \times 40 \times 10^{-3}$$

$$= 40,000 \text{ V/V}$$



2.5 Refer to Fig. E2.3.

$$v_d = R(G_{m2}v_2 - G_{m1}v_1)$$

$$v_O = v_3 = \mu v_d = \mu R(G_{m2}v_2 - G_{m1}v_1)$$

$$v_O =$$

$$\mu R(G_m v_2 + \frac{1}{2} \Delta G_m v_2 - G_m v_1 + \frac{1}{2} \Delta G_m v_1)$$

$$v_O = \mu R G_m \underbrace{(v_2 - v_1)}_{V_{Id}} + \frac{1}{2} \mu R \Delta G_m \underbrace{(v_1 + v_2)}_{2V_{Icm}}$$

$$\text{we have } v_O = A_d v_{Id} + A_{cm} v_{Icm}$$

$$\Rightarrow A_d = \mu R G_m, A_{cm} = \mu R \Delta G_m$$

$$\text{CMRR} = 20 \log \left| \frac{A_d}{A_{cm}} \right| = 20 \log \frac{G_m}{\Delta G_m}$$

For a CMRR  $\geq 60$  dB,

$$\frac{G_m}{\Delta G_m} \geq 1000 \Rightarrow \frac{\Delta G_m}{G_m} \leq 0.1\%$$

$$2.6 v_{cm} = 2 \sin(2\pi 60)t = \frac{1}{2}(v_1 + v_2)$$

$$v_d = 0.005 \sin(2\pi 1000)t = v_1 - v_2$$

$$v_1 = v_{cm} - v_d/2 \\ = \sin(120\pi)t - 0.0025 \sin(2000\pi t)$$

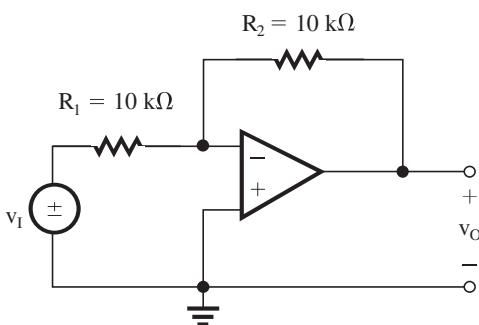
$$v_2 = v_{cm} + v_d/2 \\ = \sin(120\pi t) + 0.0025 \sin(2000\pi t)$$

2.7

Circuit	$v_o/v_i (\text{V/V})$	$R_{in} (\text{k}\Omega)$
a	$\frac{-100}{20} = -5$	20
b	-5	20
c	-5	20
d	-5	20

Note that in circuit (b) the  $20\text{-k}\Omega$  load resistance has no effect on the closed-loop gain because of the zero output resistance of the ideal op amp. In circuit (c), no current "ows" in the  $20\text{-k}\Omega$  resistor connected between the negative input terminal and ground (because of the virtual ground at the inverting input terminal). In circuit (d), zero current "ows" in the  $20\text{-k}\Omega$  resistor connected in series with the positive input terminal.

2.8



Closed-loop gain is

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} = -\frac{10 \text{ k}\Omega}{10 \text{ k}\Omega}$$

$$= -1 \text{ V/V}$$

For  $v_I = +1.00 \text{ V}$ ,

$$v_O = -1 \times 1.00$$

$$= -1.00 \text{ V}$$

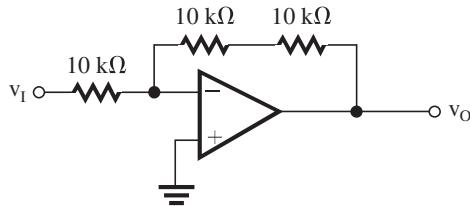
The two resistors are 1% resistors

$$\left| \frac{v_O}{v_I} \right|_{\min} = \frac{10(1 - 0.01)}{10(1 + 0.01)} = 0.98 \text{ V/V}$$

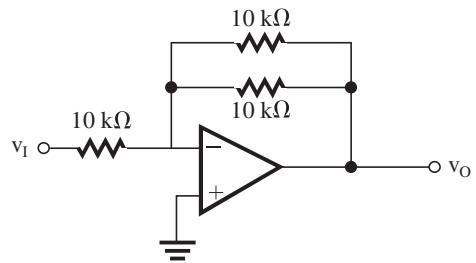
$$\left| \frac{v_O}{v_I} \right|_{\max} = \frac{10(1 + 0.01)}{10(1 - 0.01)} = 1.02 \text{ V/V}$$

Thus the measured output voltage will range from  $-0.98 \text{ V}$  to  $-1.02 \text{ V}$ .

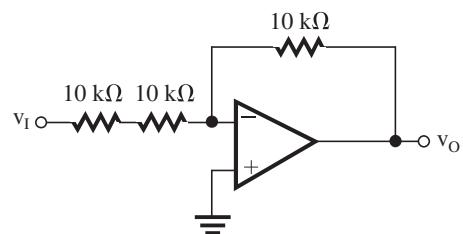
2.9 There are four possibilities:



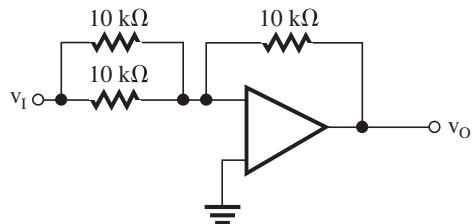
$$\frac{v_O}{v_I} = -2 \text{ V/V} \quad R_{in} = 10 \text{ k}\Omega \text{ (largest gain magnitude)}$$



$$\frac{v_O}{v_I} = -0.5 \text{ V/V} \quad R_{in} = 10 \text{ k}\Omega \text{ (smallest gain magnitude)}$$



$$\frac{v_O}{v_I} = -0.5 \text{ V/V} \quad R_{in} = 20 \text{ k}\Omega \text{ (smallest gain magnitude)}$$



$$\frac{v_O}{v_I} = -2 \text{ V/V} \quad R_{in} = 5 \text{ k}\Omega \text{ (largest gain magnitude)}$$

2.10

- (a)  $G = -1 \text{ V/V}$
- (b)  $G = -10 \text{ V/V}$
- (c)  $G = -0.1 \text{ V/V}$
- (d)  $G = -100 \text{ V/V}$
- (e)  $G = -10 \text{ V/V}$

 2.11 (a)  $G = -1 \text{ V/V}$ 

$$= \frac{-R_2}{R_1} \Rightarrow R_1 = R_2 = 10 \text{ k}\Omega$$

 (b)  $G = -2 \text{ V/V}$ 

$$= \frac{-R_2}{R_1} \Rightarrow R_1 = 10 \text{ k}\Omega, R_2 = 20 \text{ k}\Omega$$

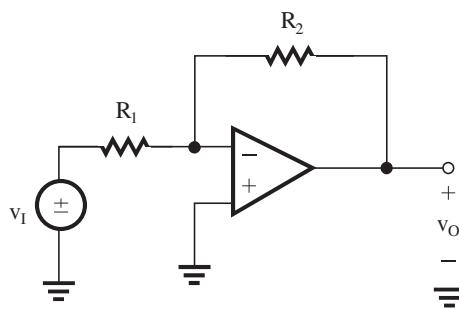
 (c)  $G = -5 \text{ V/V}$ 

$$= \frac{-R_2}{R_1} \Rightarrow R_1 = 10 \text{ k}\Omega, R_2 = 50 \text{ k}\Omega$$

(d)  $G = -100 \text{ V/V}$

$$= \frac{-R_2}{R_1} \Rightarrow R_1 = 10 \text{ k}\Omega, R_2 = 1 \text{ M}\Omega$$

2.12



$$\frac{v_O}{v_I} = -10 \text{ V/V} = -\frac{R_2}{R_1}$$

$$\Rightarrow R_2 = 10R_1$$

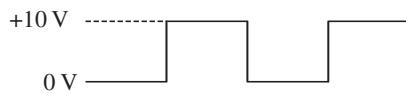
Total resistance used is  $110 \text{ k}\Omega$

$$\therefore R_1 + R_2 = 110 \text{ k}\Omega$$

$$R_1 + 10R_1 = 110 \text{ k}\Omega$$

$$R_1 = 10 \text{ k}\Omega \text{ and } R_2 = 10R_1 = 100 \text{ k}\Omega$$

2.13

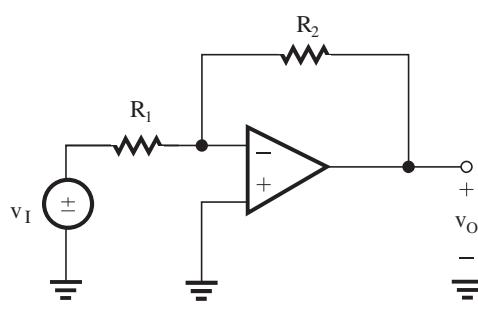


Average =  $+5 \text{ V}$

Highest =  $+10 \text{ V}$

Lowest =  $0 \text{ V}$

2.14 Gain is 46 dB



$$46 \text{ dB} = 20 \log |G|$$

$$|G| = 200$$

$$\therefore \frac{v_O}{v_I} = -200 \text{ V/V} = -\frac{R_2}{R_1}$$

$$\Rightarrow R_2 = 200R_1 \leq 1 \text{ M}\Omega$$

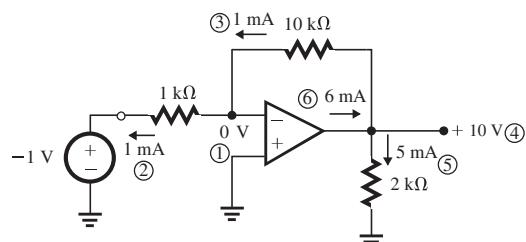
For the largest possible input resistance, choose

$$R_2 = 1 \text{ M}\Omega$$

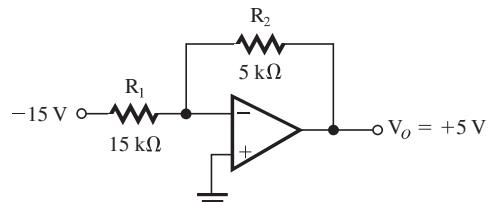
$$R_1 = \frac{1 \text{ M}\Omega}{200} = 5 \text{ k}\Omega$$

$$R_{in} = R_1 = 5 \text{ k}\Omega$$

2.15 The circled numbers indicate the order of the analysis steps. The additional current supplied by the op amp comes from the power supplies (not shown).



2.16 The circuit will be as follows:



If  $R_1$  and  $R_2$  have  $\pm 1\%$  tolerance, then  $V_O$  will exhibit  $\pm 2\%$  variability and thus will be in the range of  $5 \times 1.02 = 5.1 \text{ V}$  to  $5 \times 0.98 = 4.9 \text{ V}$ .

Variation of the  $-15\text{-V}$  supply by  $\pm 1\%$  results in a  $\pm 1\%$  variation in the output voltage. Thus the total variation in the output voltage can be  $\pm 3\%$ , resulting in  $V_O$  in the range  $4.85 \text{ V}$  to  $5.15 \text{ V}$ .

2.17  $R_1 = R_{1\text{nominal}}(1 \pm x\%)$

$R_2 = R_{2\text{nominal}}(1 \pm x\%)$

$$|G| = \frac{R_2}{R_1} = \frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} \frac{1 \pm x\%}{1 \pm x\%}$$

Thus,

$$|G|_{\max} \approx \frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} \times (1 + 2x\%)$$

$$|G|_{\min} \approx \frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} \times (1 - 2x\%)$$

Thus, the tolerance of the closed-loop gain is  $\pm 2x\%$ .

For  $G_{\text{nominal}} = -100$  and  $x = 1$ , the closed-loop gain will be in the range  $-98 \text{ V/V}$  to  $-102 \text{ V/V}$ .

2.18 (a) Choose  $R_1 = 1 \text{ k}\Omega$  and  $R_2 = 200 \text{ k}\Omega$ .

$$(b) G = -\frac{R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}}$$

$$= -\frac{200}{1 + \frac{1 + 200}{5000}} = -192.3 \text{ V/V}$$

To restore the gain to  $-200 \text{ V/V}$ , we need to change the effective value of  $R_1$  to  $R'_1$ ,

$$-200 = -\frac{200/R'_1}{1 + \frac{1 + 200/R'_1}{5000}}$$

Thus,

$$\frac{200}{R'_1} = 200 + \frac{200}{5000} \left(1 + \frac{200}{R'_1}\right)$$

$$0.96 \times \frac{200}{R'_1} = 200.04$$

$$\Rightarrow R'_1 = 0.960 \text{ k}\Omega$$

This effective value can be realized by shunting  $R_1$  ( $1 \text{ k}\Omega$ ) with  $R_a$ ,

$$\frac{1}{0.960} = \frac{1}{1} + \frac{1}{R_a} \Rightarrow R_a = 24 \text{ k}\Omega$$

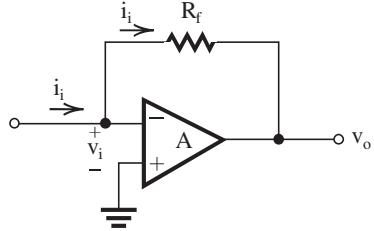
(c) From Appendix J, we find the closest available 1% resistor as either  $23.7 \text{ k}\Omega$  or  $24.3 \text{ k}\Omega$ .

2.19 Output voltage ranges from  $-10 \text{ V}$  to  $+10 \text{ V}$  and open-loop gain is  $5000 \text{ V/V}$

$\therefore$  Voltage at the inverting input terminal will vary from  $\frac{-10}{5000}$  to  $\frac{10}{5000}$  (i.e.,  $-2 \text{ mV}$  to  $+2 \text{ mV}$ ).

Thus the virtual ground will depart by a maximum of  $\pm 2 \text{ mV}$ .

2.20



(a) For  $A = \infty$ :  $v_i = 0$

$$v_o = -i_i R_f$$

$$R_m = \frac{v_o}{i_i} = -R_f$$

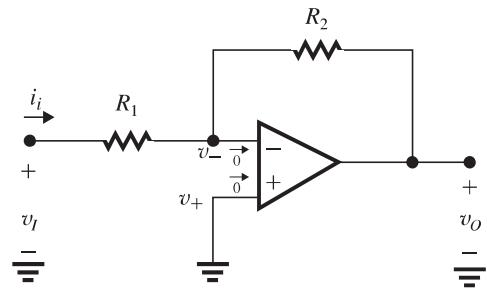
$$R_{in} = \frac{v_i}{i_i} = 0$$

(b) For  $A$  finite:  $v_i = -\frac{v_o}{A}$ ,  $v_o = v_i - i_i R_f$

$$\Rightarrow v_o = \frac{-v_o}{A} - i_i R_f \Rightarrow R_m = \frac{v_o}{i_i} = -\frac{R_f}{1 + \frac{1}{A}}$$

$$R_i = \frac{v_i}{i_i} = \frac{R_f}{1 + A}$$

2.21



$$v_o = -Av_- = v_- - i_i R_2$$

$$i_i R_2 = v_- (1 + A)$$

$$v_- = \frac{i_i R_2}{1 + A}$$

$$\text{Again } v_i = i_i R_1 + v_-$$

$$= i_i R_1 + i_i \frac{R_2}{1 + A}$$

$$\text{So } R_{in} = \frac{v_i}{i_i} = R_1 + \frac{R_2}{A + 1} \quad \text{Q.E.D.}$$

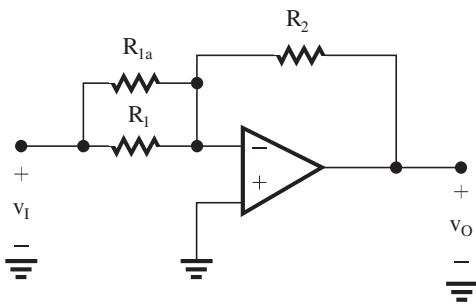
2.22 Closed-loop gain

$$G = \frac{v_o}{v_i} = \frac{-(R_2/R_1)}{1 + \frac{1 + R_2/R_1}{A}}$$

$$\text{Gain error } \epsilon = \frac{1 + (R_2/R_1)}{A} \times 100$$

$\epsilon$	0.1%	1%	10%
A	$1000\left(1 + \frac{R_2}{R_1}\right)$	$100\left(1 + \frac{R_2}{R_1}\right)$	$10\left(1 + \frac{R_2}{R_1}\right)$

Gain with resistance  $R_{1a}$



$$|G| \simeq \frac{\frac{R_2}{\left(\frac{R_1 R_{1a}}{R_1 + R_{1a}}\right)}}{1 + \frac{R_2}{R_1}}$$

$$= \frac{1 + \frac{R_1}{R_{1a}}}{1 + \frac{R_2}{R_1}}$$

where we have neglected the small effect of  $R_{1a}$  on the denominator.

$$|G| = \frac{\frac{R_2}{R_1} \left( \frac{R_1 + R_{1a}}{R_{1a}} \right)}{1 + \frac{R_2}{R_1}} = \frac{\frac{R_2}{R_1} \left( 1 + \frac{R_1}{R_{1a}} \right)}{1 + \frac{R_2}{R_1}}$$

To restore the gain to its nominal value of  $\frac{R_2}{R_1}$ , use

$$\frac{R_1}{R_{1a}} = \frac{1 + \frac{R_2}{R_1}}{A} = \frac{\epsilon}{100}$$

$$\text{So } R_{1a} = \frac{100R_1}{\epsilon}$$

$\epsilon$	0.1%	1%	10%
$R_{1a}$	$1000R_1$	$100R_1$	$10R_1$

2.23 Refer to Fig. P2.23.

$$\frac{V_o}{V_i} = \frac{-R_2/R'_1}{1 + \frac{1 + R_2/R'_1}{A}} \quad (1)$$

where

$$R'_1 = R_1 \parallel R_c$$

Thus,

$$\frac{1}{R'_1} = \frac{1}{R_1} + \frac{1}{R_c}$$

Substituting in Eq. (1),

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1} \frac{1 + \frac{R_1}{R_c}}{1 + \frac{R_2}{R_1} + \frac{R_2}{R_c}}$$

$$= \frac{1 + \frac{R_2}{R_1}}{1 + \frac{R_2}{R_1} + \frac{R_2}{R_c}}$$

To make  $\frac{V_o}{V_i} = -\frac{R_2}{R_1}$ , we have to make

$$\frac{R_1}{R_c} = \frac{1 + \frac{R_2}{R_1}}{A}$$

That is,

$$A \frac{R_1}{R_c} = 1 + G + G \frac{R_1}{R_c}$$

which yields

$$\frac{R_c}{R_1} = \frac{A - G}{1 + G} \quad \text{Q.E.D}$$

$$2.24 \text{ (a)} |G| = \frac{R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}} \quad (1)$$

If A is reduced by  $\Delta A$ , |G| will be correspondingly reduced by  $\Delta |G|$ ,

$$|G| - \Delta |G| = \frac{R_2/R_1}{1 + \frac{1 + R_2/R_1}{A - \Delta A}} \quad (2)$$

Dividing Eq. (2) by Eq. (1), we have

$$1 - \frac{\Delta |G|}{|G|} = \frac{1 + \frac{1 + R_2/R_1}{A}}{1 + \frac{1 + R_2/R_1}{A - \Delta A}}$$

Thus

$$\frac{\Delta |G|}{|G|} = \frac{\left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{A - \Delta A} - \frac{1}{A}\right)}{1 + \frac{1 + R_2/R_1}{A - \Delta A}}$$

$$= \frac{\left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{A}\right) \left(\frac{1}{1 - \frac{\Delta A}{A}} - 1\right)}{1 + \frac{R_2/R_1}{A - \Delta A}}$$

For  $\frac{\Delta A}{A} \ll 1$  and  $1 + \frac{R_2}{R_1} \ll A$ ,

$$\frac{\Delta |G|}{|G|} \simeq \frac{1 + \frac{R_2}{R_1}}{A} \frac{\Delta A}{A}$$

Thus,

$$\frac{\Delta |G| / |G|}{\Delta A / |A|} \simeq \frac{1 + R_2 / R_1}{A} \quad \text{Q.E.D}$$

$$(b) \frac{R_2}{R_1} = 100, \frac{\Delta A}{A} = 0.1, \frac{\Delta |G|}{|G|} \leq 0.001$$

$$A \geq (1 + 100) \times 0.1 / 0.001$$

That is,  $A_{\min} \simeq 10^4 \text{ V/V}$ .

2.25 (a) Equation (2.5):

$$G = \frac{v_o}{v_i} = \frac{-R_2 / R_1}{1 + \left(1 + \frac{R_2}{R_1}\right) / A} / A$$

$$\text{and } G_{\text{nominal}} = -\frac{R_2}{R_1}$$

Gain error

$$\epsilon = \left| \frac{G - G_{\text{nominal}}}{G_{\text{nominal}}} \right| = \left| \frac{G}{G_{\text{nominal}}} - 1 \right|$$

$$= \left| \frac{1}{1 + \left(1 + \frac{R_2 / R_1}{A}\right)} - 1 \right| = \left| \frac{-\frac{1 + R_2 / R_1}{A}}{1 + \frac{(1 + R_2 / R_1)}{A}} \right|$$

$$\epsilon = \frac{1}{1 + \frac{A}{1 + \frac{R_2}{R_1}}}$$

$$\frac{1}{\epsilon} = 1 + \frac{A}{1 + \frac{R_2}{R_1}}$$

Solving for A, we get

$$A = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{\epsilon} - 1\right)$$

$$= (1 - G_{\text{nominal}}) \left(\frac{1}{\epsilon} - 1\right)$$

$$(b) G_{\text{nominal}} = -100 = -\frac{R_2}{R_1}$$

$$R_{\text{in}} = 1 \text{ k}\Omega = R_1$$

$$R_1 = 1 \text{ k}\Omega$$

$$R_2 = 100R_1 = 100 \text{ k}\Omega$$

Again  $G_{\text{nominal}} = -100$  and  $\epsilon_{\max} = 10\%$

$$\therefore A = (1 - G_{\text{nominal}}) \left(\frac{1}{\epsilon} - 1\right)$$

$$= [1 - (-100)] \left(\frac{1}{0.1} - 1\right)$$

$$= 101 \times 9$$

$$A = 909 \text{ V/V}$$

2.26 From Example 2.2,

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right)$$

Here  $R_1 = R_2 = R_4 = 1 \text{ M}\Omega$

$$\therefore \frac{v_o}{v_i} = -\left(1 + 1 + \frac{1}{R_3}\right) = -\left(2 + \frac{1 \text{ M}}{R_3}\right)$$

$$\frac{v_o}{v_i} + 2 = -\frac{1 \text{ M}\Omega}{R_3}$$

$$R_3 = -\frac{1 \text{ M}\Omega}{\left(\frac{v_o}{v_i} + 2\right)}$$

$$(a) \frac{v_o}{v_i} = -100 \text{ V/V}$$

$$R_3 = -\frac{1 \text{ M}\Omega}{(-100 + 2)} = 10.2 \text{ k}\Omega$$

$$(b) \frac{v_o}{v_i} = -10 \text{ V/V}$$

$$R_3 = -\frac{1 \text{ M}\Omega}{(-10 + 2)} = 125 \text{ k}\Omega$$

$$(c) \frac{v_o}{v_i} = -2 \text{ V/V}, R_3 = -\frac{1 \text{ M}\Omega}{(-2 + 2)} = \infty$$

2.27

$$R_2 / R_1 = 500, R_2 = 100 \text{ k}\Omega \Rightarrow R_1 = 200 \Omega$$

$$(a) R_{\text{in}} = R_1 = 200 \Omega$$

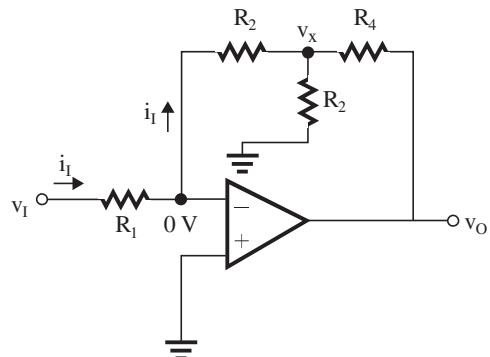
$$(b) \frac{v_o}{v_i} = \frac{-R_2}{R_1} \left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right) = -500$$

$$\text{If } R_2 = R_1 = R_4 = 100 \text{ k}\Omega \Rightarrow R_3 = \frac{100 \text{ k}\Omega}{500 - 2} \simeq 200 \Omega$$

$$R_{\text{in}} = R_1 = 100 \text{ k}\Omega$$

$$2.28 i_l = \frac{v_i}{R_1}, v_x = -i_l R_2 = -\frac{v_i}{R_1} R_2$$

$$\text{So } \frac{v_x}{v_i} = -\frac{R_2}{R_1}$$



Now, because of the virtual ground at the negative input terminal of the op amp,  $v_o$  appears across the series combination of  $R_4$  and  $(R_2 \parallel R_3)$ ; thus

$$\begin{aligned} v_x &= v_o \frac{(R_2 \parallel R_3)}{(R_2 \parallel R_3) + R_4} \\ &= v_o \frac{R_2 R_3}{R_2 R_3 + R_2 R_4 + R_3 R_4} \\ \frac{v_o}{v_x} &= \frac{R_2 R_3 + R_2 R_4 + R_3 R_4}{R_2 R_3} \\ &= 1 + \frac{R_4}{R_3} + \frac{R_4}{R_2} \\ \frac{v_o}{v_i} &= \frac{v_o}{v_x} \frac{v_x}{v_i} \\ &= -\frac{R_2}{R_1} \left( 1 + \frac{R_4}{R_3} + \frac{R_4}{R_2} \right) \quad \text{Q.E.D.} \end{aligned}$$

2.29 (a)  $R_l = R$

$$R_2 = (R \parallel R) + \frac{R}{2} = \frac{R}{2} + \frac{R}{2} = R$$

$$R_3 = (R_2 \parallel R) + \frac{R}{2} = (R \parallel R) + \frac{R}{2} = R$$

$$R_4 = (R_3 \parallel R) + \frac{R}{2} = (R \parallel R) + \frac{R}{2} = R$$

See Fig. 1.

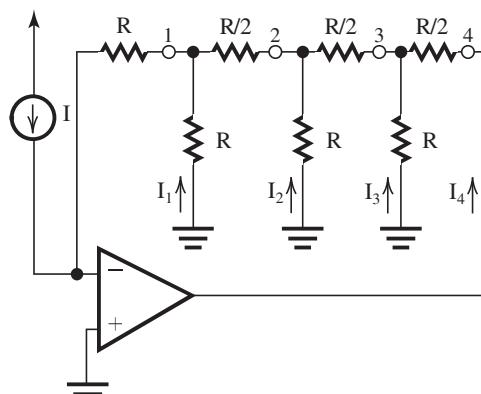


Figure 1

(b)

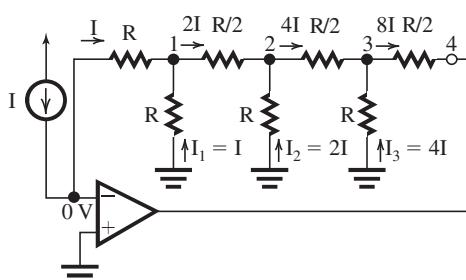


Figure 2

See Fig. 2. We utilize the results of (a) above as follows: At node 1 we have a resistance  $R$  to ground and, looking left, a resistance  $R$ . These two resistances must carry equal currents

$\Rightarrow I_1 = I$ . A node equation at node 1 results in the current through  $(R/2)$  as  $2I$ . At node 2, we have a resistance  $R$  to ground and an equal resistance looking to the left. These two resistances must carry equal currents  $\Rightarrow I_2 = 2I$ .

A node equation at 2 results in the current leaving node 2 as  $4I$ . We continue in the same fashion to find  $I_3 = 4I$  and the current from 3 to 4 as  $8I$ .

$$(c) V_1 = -I_1 R = -IR$$

$$V_2 = -I_2 R = -2IR$$

$$V_3 = -I_3 R = -4IR$$

$$V_4 = V_3 - \frac{R}{2} \times 8I = -8IR$$

$$2.30 (a) I_1 = \frac{1 \text{ V}}{10 \text{ k}\Omega} = 0.1 \text{ mA}$$

$$I_2 = I_1 = 0.1 \text{ mA}$$

$$-I_2 \times 10 \text{ k}\Omega = -I_3 \times 100 \Omega \equiv V_x$$

$$\therefore I_3 = I_2 \times \frac{10 \text{ k}\Omega}{100 \Omega}$$

$$= 0.1 \times 100$$

$$= 10 \text{ mA}$$

$$\text{Now } I_L = I_2 + I_3 = 10.1 \text{ mA}$$

$$\text{and } V_x = -I_2 \times 10 \text{ k}\Omega = -0.1 \times 10 \text{ k}\Omega = -1 \text{ V}$$

$$(b) V_x = R_L I_L + V_o = 10.1 R_L + V_o$$

$$R_L = \frac{V_x - V_o}{10.1 \text{ mA}} = \frac{-1 - (-13)}{10.1} = 1.19 \text{ k}\Omega$$

$$(c) 100 \Omega \leq R_L \leq 1 \text{ k}\Omega$$

$I_L$  stays fixed at 10.1 mA.

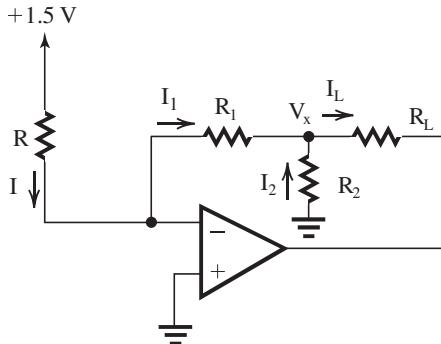
$$V_o = V_x - R_L I_L = -1 - R_L \times 10.1$$

$$R_L = 100 \Omega, V_o = -2.01 \text{ V}; R_L = 1 \text{ k}\Omega,$$

$$V_o = -11.1 \text{ V}$$

$$\therefore -11.1 \text{ V} \leq V_o \leq -2.01 \text{ V}$$

2.31



The circuit shown in Fig. P2.30 (redrawn above) provides a constant current through  $R_L$  that is independent of the value of  $R_L$ . The current through  $R_L$  is determined by the input current and the ratio of the two resistors  $R_1$  and  $R_2$ . Specifically,  $I_1 = I$ ,  $V_x = -I_1 R_1$ ,  $I_2 = -V_x / R_2 = -I_1 R_1 / R_2$ , thus  $I_1 (R_1 / R_2) = I (R_1 / R_2)$ , thus

$$I_L = I + I \frac{R_1}{R_2} = \left(1 + \frac{R_1}{R_2}\right) I$$

which is independent of the value of  $R_L$ . Now, for our specific design:

$$I_L = 3.1 \text{ mA}$$

$$I = 0.1 \text{ mA}$$

$$R = \frac{1.5\text{V}}{0.1 \text{ mA}} = 15 \text{ k}\Omega$$

$$3.1 = 0.1 \left(1 + \frac{R_1}{R_2}\right)$$

$$\Rightarrow \frac{R_1}{R_2} = 30$$

Choosing  $R_2 = 500 \Omega \Rightarrow R_1 = 15 \text{ k}\Omega$ . The circuit will work properly as long as the op amp does not saturate (i.e., as long as  $V_O \leq -10 \text{ V}$ ). But

$$V_O = V_x - I_L R_L$$

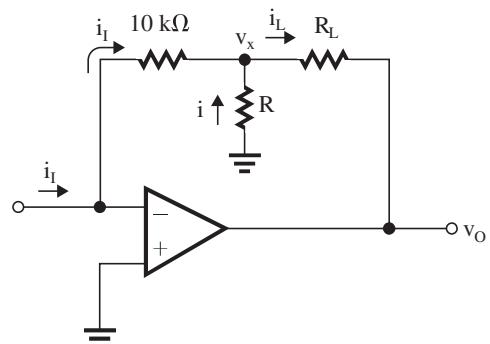
$$= -IR_1 - I_L R_L$$

$$= -0.1 \times 15 - 3.1 R_L$$

The maximum allowed value for  $R_L$  can now be found by substituting  $V_O = -10 \text{ V}$ , thus

$$R_L = 2.74 \text{ k}\Omega$$

2.32



$$(a) v_x = -i_L \times 10$$

$$i = -v_x / R = i_L (10 / R)$$

$$i_L = i_L + i = i_L \left(1 + \frac{10}{R}\right)$$

Thus,

$$\frac{i_L}{i_L} = 1 + \frac{10}{R}$$

$$\text{For } \frac{i_L}{i_L} = 11 \Rightarrow R = 1 \text{ k}\Omega.$$

(b)  $R_{in} = 0 \Omega$  (because of the virtual ground at the input).  $R_o = \infty \Omega$  (because  $i_L$  is independent of the value of  $R_L$ ).

(c) If  $i_L$  is in the direction shown in the figure above, the maximum allowable value of  $i_L$  will be determined by  $v_O$  reaching  $-12 \text{ V}$ , at which point

$$v_x = -i_{Lmax} \times 10$$

and

$$v_O = -12 = v_x - i_{Lmax} \times 1 = -10 i_{Lmax} - 11 i_{Lmax}$$

$$\Rightarrow i_{Lmax} = \frac{12}{21} = 0.57 \text{ mA}$$

If  $i_L$  is in a direction opposite to that shown in the figure, then

$$v_x = 10i_L$$

$$v_O = v_x + i_L R_L = 10i_L + 11i_L = 21i_L$$

The maximum value of  $i_L$  will result in  $v_O = +12 \text{ V}$ . Thus

$$12 = 21i_{Lmax} \Rightarrow i_{Lmax} = \frac{12}{21} = 0.57 \text{ mA}$$

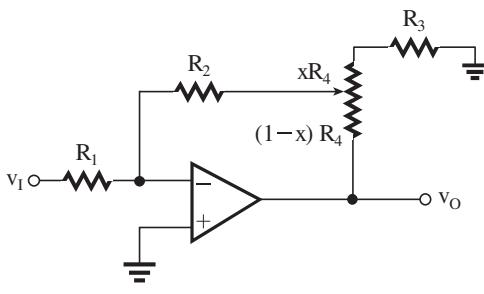
Thus, the allowable range of  $i_L$  is

$$-0.57 \text{ mA} \leq i_L \leq +0.57 \text{ mA}$$

(d) Since  $R_{in} = 0$ , the value of the source resistance will have no effect on the resulting  $i_L$ ,

$$i_L = 0.2 \times 11 = 2.2 \text{ mA}$$

2.33



To obtain an input resistance of  $100 \text{ k}\Omega$ , we select  $R_1 = 100 \text{ k}\Omega$ . From Example 2.2 we have

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} \left[ 1 + \frac{(1-x)R_4}{R_2} + \frac{(1-x)R_4}{R_3 + xR_4} \right]$$

The minimum gain magnitude is obtained when  $x = 1$ ,

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} = -1$$

Thus,  $R_2 = 100 \text{ k}\Omega$ .

The maximum gain magnitude is obtained when  $x = 0$ ,

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} \left[ 1 + \frac{R_4}{R_2} + \frac{R_4}{R_3} \right] = -100$$

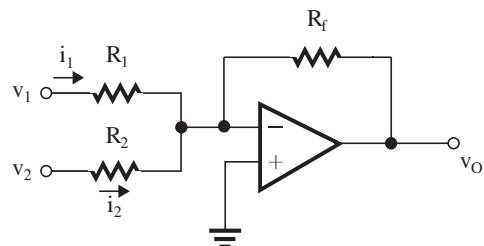
$$\Rightarrow 1 + \frac{100}{100} + \frac{100}{R_3} = 100$$

$$\Rightarrow R_3 = \frac{100}{98} = 1.02 \text{ k}\Omega$$

When the potentiometer is set exactly in the middle,  $x = 0.5$  and

$$\begin{aligned} \frac{v_O}{v_I} &= -\frac{R_2}{R_1} \left[ 1 + \frac{0.5R_4}{R_2} + \frac{0.5R_4}{R_3 + 0.5R_4} \right] \\ &= -\frac{100}{100} \left[ 1 + \frac{0.5 \times 100}{100} + \frac{0.5 \times 100}{1.02 + 0.5 \times 100} \right] \\ &= -2.48 \text{ V/V} \end{aligned}$$

2.34



The output of the weighted summer circuit is

$$v_O = -\left( \frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 \right)$$

$$v_O = -\left( 2v_1 + \frac{v_2}{2} \right)$$

$$i_1 = \frac{v_1}{R_1} \text{ and } i_2 = \frac{v_2}{R_2}$$

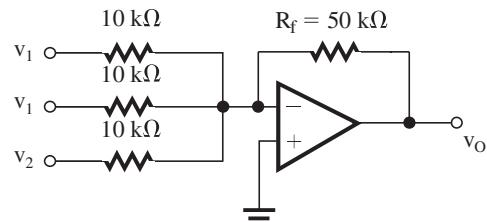
Since  $i_1, i_2 \leq 50 \mu\text{A}$  for 1-V input signals

$$\therefore R_1, R_2 \geq 20 \text{ k}\Omega$$

$$\text{Here } \frac{R_f}{R_1} = 2, \text{ if } R_1 = 20 \text{ k}\Omega, R_f = 40 \text{ k}\Omega$$

$$\frac{R_f}{R_2} = \frac{1}{2} \Rightarrow R_2 = 2R_f = 80 \text{ k}\Omega$$

2.35



$$v_O = -\left( \frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_1 + \frac{R_f}{R_3} v_2 \right)$$

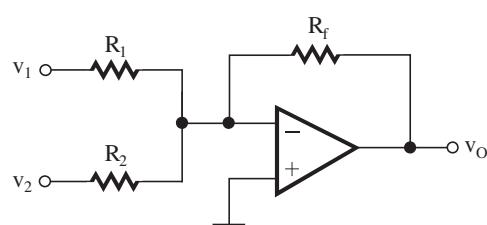
$$= -\left( \frac{50}{10} v_1 + \frac{50}{10} v_1 + \frac{50}{10} v_2 \right)$$

$$= -(10v_1 + 5v_2)$$

Now  $v_1 = 1 \text{ V}$  and  $v_2 = -1 \text{ V}$

$$v_O = -(10 \times 1 - 5) = -5 \text{ V}$$

2.36



For  $v_1$  and  $v_2$ , we assume

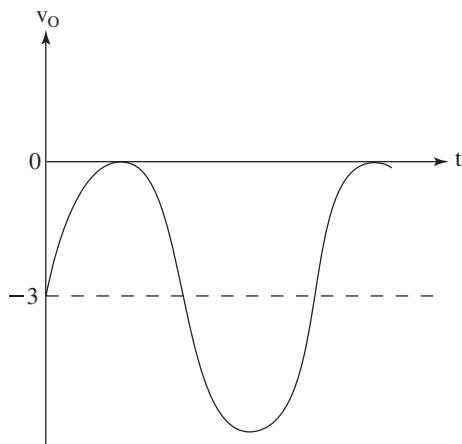
## Chapter 2-10

$$v_1 = 3 \sin \omega t$$

$$v_2 = 1.5 \text{ V}$$

The output signal required is

$$v_o = -3 \sin \omega t - 3$$



For the summer circuit, we should have

$$\frac{R_f}{R_1} = 1 \text{ and } \frac{R_f}{R_2} = 2$$

Select  $R_f = 2R_2 = 20 \text{ k}\Omega$ .

Thus  $R_2 = 10 \text{ k}\Omega$ ,

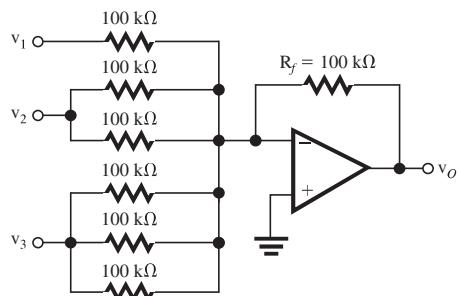
and  $R_1 = 20 \text{ k}\Omega$ .

$$2.37 \text{ (a) } v_o = -(v_1 + 2v_2 + 3v_3)$$

$$\frac{R_f}{R_1} = 1 \Rightarrow R_1 = 100 \text{ k}\Omega,$$

$$\frac{R_f}{R_2} = 2 \Rightarrow R_2 = 50 \text{ k}\Omega$$

$$\frac{R_f}{R_3} = 3 \Rightarrow R_3 = \frac{100}{3} \text{ k}\Omega$$

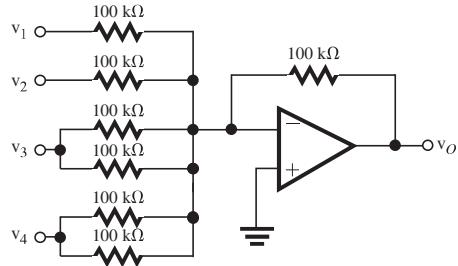


$$R_{i1} = 100 \text{ k}\Omega$$

$$R_{i2} = 50 \text{ k}\Omega$$

$$R_{i3} = 33.3 \text{ k}\Omega$$

$$(b) v_o = -(v_1 + v_2 + 2v_3 + 2v_4)$$



$$\frac{R_f}{R_1} = 1 \Rightarrow R_1 = 100 \text{ k}\Omega$$

$$\frac{R_f}{R_2} = 1 \Rightarrow R_2 = 100 \text{ k}\Omega$$

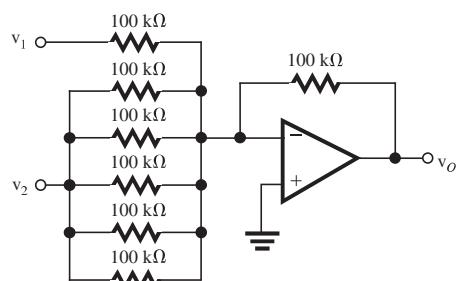
$$\frac{R_f}{R_3} = 2 \Rightarrow R_3 = \frac{100}{2} \text{ k}\Omega$$

$$\frac{R_f}{R_4} = 2 \Rightarrow R_4 = \frac{100}{2} \text{ k}\Omega$$

$$R_{i1} = 100 \text{ k}\Omega, R_{i2} = 100 \text{ k}\Omega,$$

$$R_{i3} = 50 \text{ k}\Omega, R_{i4} = 50 \text{ k}\Omega$$

$$(c) v_o = -(v_1 + 5v_2)$$



$$R_1 = 100 \text{ k}\Omega$$

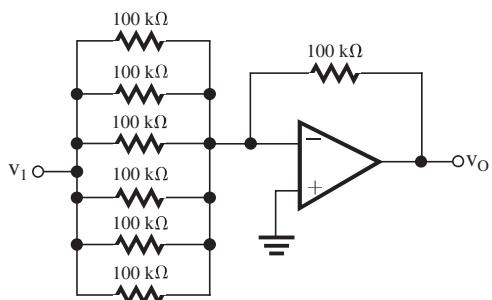
$$R_2 = \frac{100 \text{ k}\Omega}{5}$$

$$R_{i1} = 100 \text{ k}\Omega$$

$$R_{i2} = 20 \text{ k}\Omega$$

$$(d) v_o = -6v_1$$

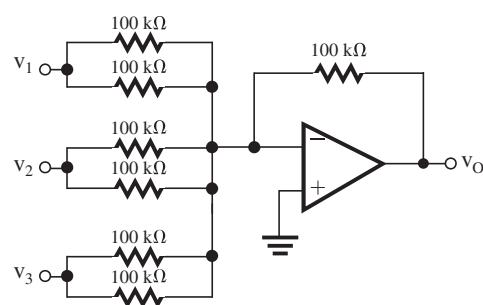
$$R_1 = \frac{100 \text{ k}\Omega}{6}$$



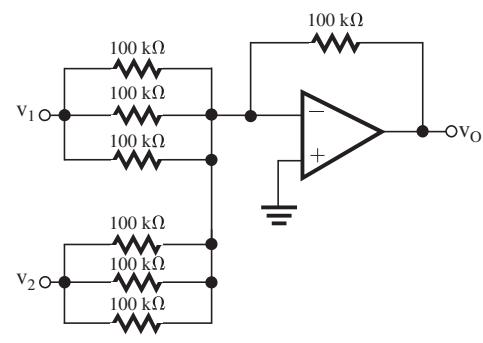
$$R_{i1} = \frac{100}{6} = 1.67 \text{ k}\Omega$$

Suggested configurations:

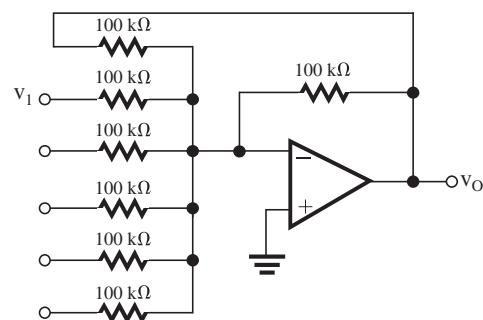
$$v_O = -(2v_1 + 2v_2 + 2v_3)$$



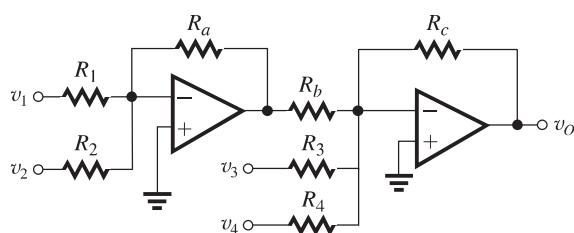
$$v_O = -(3v_1 + 3v_2)$$



In order to have coefficient = 0.5, connect one of the input resistors to  $v_O$ .  $\frac{v_O}{v_1} = 0.5$ .



2.38 Using the circuit of Fig. 2.11:



To obtain  $v_O = v_1 + 2v_2 - 3v_3 - 5v_4$ , we can arbitrarily select  $R_c = R_b$ , then

$$\frac{R_a}{R_1} = 1 \quad \text{and} \quad \frac{R_a}{R_2} = 2$$

If we select  $R_2 = 10 \text{ k}\Omega$ , then  $R_a = 20 \text{ k}\Omega$  and  $R_1 = 20 \text{ k}\Omega$ .

For the second summer, to obtain

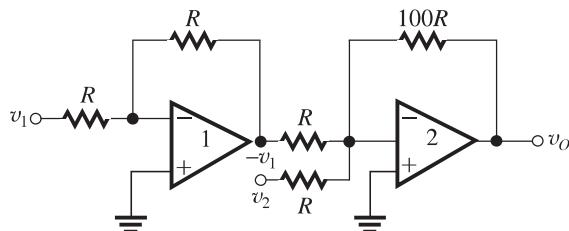
$$\frac{v_O}{v_3} = -3 \quad \text{then} \quad \frac{R_c}{R_3} = 3$$

and

$$\frac{v_O}{v_4} = -5 \quad \text{requires} \quad \frac{R_c}{R_4} = 5$$

We can select  $R_c = R_b = 60 \text{ k}\Omega$ , resulting in  $R_3 = 20 \text{ k}\Omega$  and  $R_4 = 12 \text{ k}\Omega$ .

2.39



Here the first inverting amplifier simply inverts  $v_1$ , resulting at its output in

$$-v_1 = -2 \sin(2\pi \times 60t) - 0.01 \sin(2\pi \times 1000t)$$

The second amplifier provides the sum of  $-v_1$  and  $v_2$ , that is,  $(v_2 - v_1)$  multiplied by a gain of 100. Thus

$$v_O = -100(v_2 - v_1)$$

$$= 1 \sin(2\pi \times 1000t), \text{ V}$$

The value of  $R$  can be selected (arbitrarily but conveniently) as  $R = 10 \text{ k}\Omega$ .

2.40 This is a weighted summer circuit:

$$v_O = -\left(\frac{R_f}{R_0}v_O + \frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

We may write:

$$v_0 = 5V \times a_0, \quad v_2 = 5V \times a_2,$$

$$v_1 = 5V \times a_1, \quad v_3 = 5V \times a_3,$$

$$v_o = -R_f \left( \frac{5}{80}a_0 + \frac{5}{40}a_1 + \frac{5}{20}a_2 + \frac{5}{10}a_3 \right)$$

$$v_o = -R_f \left( \frac{a_0}{16} + \frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{2} \right)$$

$$v_o = -\frac{R_f}{16} (2^0 a_0 + 2^1 a_1 + 2^2 a_2 + 2^3 a_3)$$

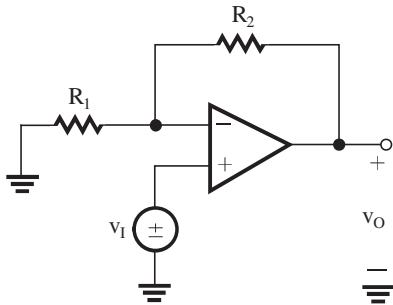
For  $-12 \text{ V} \leq v_o \leq 0$ ,

$$\frac{R_f}{16} (2^0 \times 1 + 2 \times 1 + 2^2 \times 1 + 2^3 \times 1) = 12$$

$$= \frac{15R_f}{16} = 12$$

$$\Rightarrow R_f = 12.8 \text{ k}\Omega$$

2.41



$$\frac{v_o}{v_i} = 1 + \frac{R_2}{R_l}$$

$$(a) \frac{v_o}{v_i} = 1 = 1 + \frac{R_2}{R_l}$$

Set  $R_2 = 0 \Omega$  and eliminate  $R_l$

$$(b) \frac{v_o}{v_i} = 2 = 1 + \frac{R_2}{R_l}$$

$$\frac{R_2}{R_l} = 1; \text{ set } R_l = R_2 = 10 \text{ k}\Omega$$

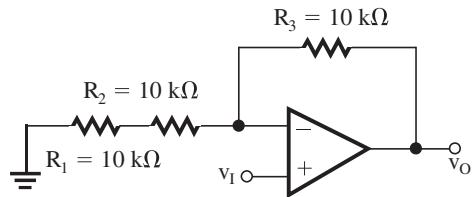
$$(c) \frac{v_o}{v_i} = 21 = 1 + \frac{R_2}{R_l}$$

$$\frac{R_2}{R_l} = 20; \text{ set } R_l = 10 \text{ k}\Omega, R_2 = 200 \text{ k}\Omega$$

$$(d) \frac{v_o}{v_i} = 100 = 1 + \frac{R_2}{R_l}$$

$$\frac{R_2}{R_l} = 99; \text{ set } R_l = 10 \text{ k}\Omega, R_2 = 990 \text{ k}\Omega$$

2.42

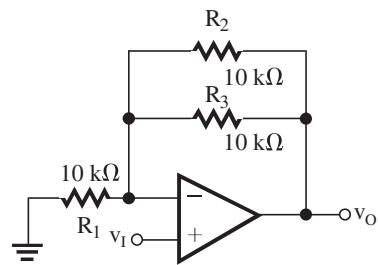


Short-circuit  $R_2$ :

$$\frac{v_o}{v_i} = 2$$

Short-circuit  $R_3$ :

$$\frac{v_o}{v_i} = 1$$



2.43  $V_- = V_+ = V$ ; thus the current in the moving-coil meter will be  $I = \frac{V}{R}$ , independent of the resistance of the meter. To obtain  $I = 100 \mu\text{A}$  when  $V = 10 \text{ V}$ , we select

$$R = \frac{10}{0.1 \text{ mA}} = 100 \text{ k}\Omega$$

The meter resistance does not affect the voltmeter calibration.

2.44 Refer to the circuit in Fig. P2.44:

(a) Using superposition, we first set  $v_{P1} = v_{P2}, \dots, = 0$ . The output voltage that results in response to  $v_{N1}, v_{N2}, \dots, v_{Nn}$  is

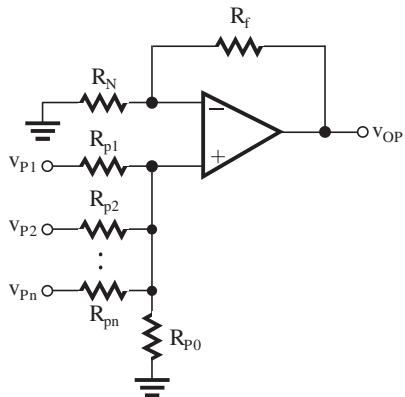
$$v_{ON} = - \left[ \frac{R_f}{R_{N1}} v_{N1} + \frac{R_f}{R_{N2}} v_{N2} + \dots + \frac{R_f}{R_{Nn}} v_{Nn} \right]$$

Then we set  $v_{N1} = v_{N2} = \dots = 0$ , then:

$$R_N = R_{N1} \parallel R_{N2} \parallel R_{N3} \parallel \dots \parallel R_{Nn}$$

The circuit simplifies to that shown below.

$$v_{OP} = \left( 1 + \frac{R_f}{R_N} \right) \times \left( v_{P1} \frac{1/R_{P1}}{\frac{1}{R_{P0}} + \frac{1}{R_{P1}} + \dots + \frac{1}{R_{Pn}}} + v_{P2} \frac{1/R_{P2}}{\frac{1}{R_{P0}} + \dots + \frac{1}{R_{Pn}}} + \dots + v_{Pn} \frac{1/R_{Pn}}{\frac{1}{R_{P0}} + \dots + \frac{1}{R_{Pn}}} \right)$$



$$v_{OP} = \left(1 + \frac{R_f}{R_N}\right) \left( v_{P1} \frac{R_p}{R_{P1}} + v_{P2} \frac{R_p}{R_{P2}} + \cdots + v_{Pn} \frac{R_p}{R_{Pn}} \right)$$

where

$$R_p = R_{P0} \parallel R_{P1} \parallel \cdots \parallel R_{Pn}$$

when all inputs are present:

$$v_O = v_{ON} + v_{OP}$$

$$= - \left( \frac{R_f}{R_{N1}} v_{N1} + \frac{R_f}{R_{N2}} v_{N2} + \cdots \right) + \left( 1 + \frac{R_f}{R_N} \right) \left( \frac{R_p}{R_{P1}} v_{P1} + \frac{R_p}{R_{P2}} v_{P2} + \cdots \right)$$

$$(b) v_O = -4v_{N1} + v_{P1} + 3v_{P2}$$

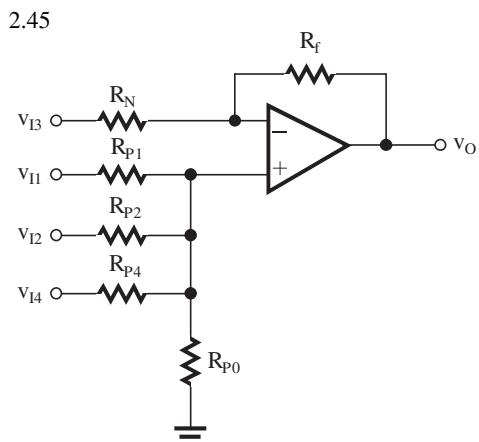
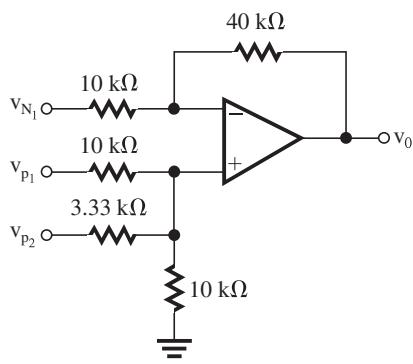
$$\frac{R_f}{R_{N1}} = 4, R_{N1} = 10 \text{ k}\Omega \Rightarrow R_f = 40 \text{ k}\Omega$$

$$\left( 1 + \frac{R_f}{R_N} \right) \left( \frac{R_p}{R_{P1}} \right) = 1 \Rightarrow 5 \frac{R_p}{R_{P1}} = 1 \quad (1)$$

$$\left( 1 + \frac{R_f}{R_N} \right) \left( \frac{R_p}{R_{P2}} \right) = 3 \Rightarrow 5 \frac{R_p}{R_{P2}} = 3 \quad (2)$$

Substituting for  $\frac{1}{R_p}$ ,  $\frac{1}{R_p} = \frac{1}{R_{P0}} + \frac{1}{R_{P1}} + \frac{1}{R_{P2}}$  in

Eqs. (1) and (2) and selecting (arbitrarily)  $R_{P0} = 10 \text{ k}\Omega$  results in  $R_{P1} = 10 \text{ k}\Omega$  and  $R_{P2} = 3.33 \text{ k}\Omega$ . The result is the following circuit:



Adapting the expression given in Problem 2.44 to the circuit above yields

$$v_O = -\frac{R_f}{R_N} v_{I3} + \left( 1 + \frac{R_f}{R_N} \right) \left( \frac{R_p}{R_{P1}} v_{I1} + \frac{R_p}{R_{P2}} v_{I2} + \frac{R_p}{R_{P4}} v_{I4} \right)$$

where  $R_p = R_{P0} \parallel R_{P1} \parallel R_{P2} \parallel R_{P3}$ .

We require

$$v_O = -9v_{I3} + v_{I1} + 2v_{I2} + 4v_{I4}$$

Equating the coefficients of  $v_{I3}$ , we have

$$\frac{R_f}{R_N} = 9$$

Selecting  $R_N = 10 \text{ k}\Omega \Rightarrow R_f = 90 \text{ k}\Omega$ .

Equating the coefficients of  $v_{I1}$  provides

$$\left( 1 + \frac{R_f}{R_N} \right) \frac{R_p}{R_{P1}} = 1$$

Thus,

$$10 \frac{R_p}{R_{P1}} = 1 \Rightarrow \frac{R_p}{R_{P1}} = 0.1 \quad (1)$$

Similarly, equating the coefficients of  $v_{I2}$  gives

$$10 \frac{R_p}{R_{P2}} = 2 \Rightarrow \frac{R_p}{R_{P2}} = 0.2 \quad (2)$$

and equating the coefficients of  $v_{I4}$  gives

$$10 \frac{R_p}{R_{P4}} = 4 \Rightarrow \frac{R_p}{R_{P4}} = 0.4 \quad (3)$$

Now, summing Eqs. (1), (2), and (3) provides

$$R_p \left( \frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P4}} \right) = 0.7 \quad (4)$$

But,

$$\frac{1}{R_p} = \frac{1}{R_{P0}} + \frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P4}}$$

Thus,

$$\frac{1}{R_p} - \frac{1}{R_{P0}} = \frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P4}} \quad (5)$$

Equations (4) and (5) can be combined to obtain

$$1 - \frac{R_p}{R_{P0}} = 0.7 \Rightarrow R_{P0} = \frac{R_p}{0.3} \quad (6)$$

Selecting

$$R_{P0} = 10 \text{ k}\Omega$$

$$\text{Equation (6)} \Rightarrow R_p = 3 \text{ k}\Omega$$

$$\text{Equation (1)} \Rightarrow R_{P1} = 30 \text{ k}\Omega$$

$$\text{Equation (2)} \Rightarrow R_{P2} = 15 \text{ k}\Omega$$

$$\text{Equation (3)} \Rightarrow R_{P4} = 7.5 \text{ k}\Omega$$

$$2.46 \quad v_+ = v_I \frac{R_4}{R_3 + R_4}$$

$$\frac{v_O}{v_I} = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4}{R_3 + R_4}\right) = \frac{1 + R_2/R_1}{1 + R_3/R_4}$$

2.47 Refer to Fig. P2.47. Setting  $v_2 = 0$ , we obtain the output component due to  $v_1$  as

$$v_{O1} = -10v_1$$

Setting  $v_1 = 0$ , we obtain the output component due to  $v_2$  as

$$v_{O2} = v_2 \left(1 + \frac{10R}{R}\right) \left(\frac{10R}{10R + R}\right) = 10v_2$$

The total output voltage is

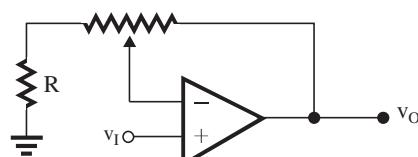
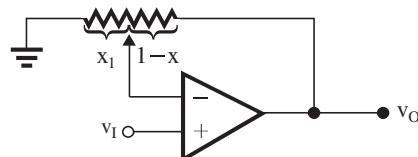
$$v_O = v_{O1} + v_{O2} = 10(v_2 - v_1)$$

$$\text{For } v_1 = 10 \sin(2\pi \times 60t) - 0.1 \sin(2\pi \times 1000t)$$

$$v_2 = 10 \sin(2\pi \times 60t) + 0.1 \sin(2\pi \times 1000t)$$

$$v_O = 2 \sin(2\pi \times 1000t)$$

2.48



$$\frac{v_O}{v_I} = 1 + \frac{R_2}{R_1} = 1 + \frac{1-x}{x} = 1 + \frac{1}{x} - 1$$

$$\therefore \frac{v_O}{v_I} = \frac{1}{x}$$

$$0 \leq x \leq 1 \Rightarrow \infty \geq \frac{v_O}{v_I} \geq 1$$

Add a resistor as shown:

$$\frac{v_O}{v_I} = 1 + \frac{(1-x) \times 10 \text{ k}\Omega}{x \times 10 \text{ k}\Omega + R}$$

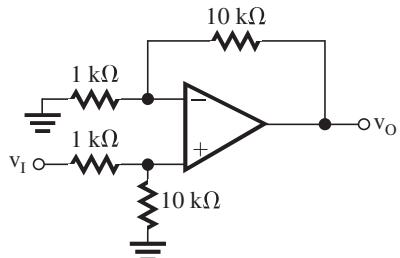
$$\text{For } \left(\frac{v_O}{v_I}\right)_{\max} = 11$$

$$x = 0, \frac{v_O}{v_I} = 11 = 1 + \frac{10 \text{ k}\Omega}{R}$$

$$10 = \frac{10 \text{ k}\Omega}{R}$$

$$R = 1 \text{ k}\Omega$$

2.49

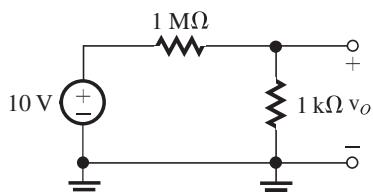


$$v_O = v_I \frac{10}{1+10} \left(1 + \frac{10}{1}\right)$$

$$v_O = 10v_I$$

$$R_{in} = 11 \text{ k}\Omega$$

2.50

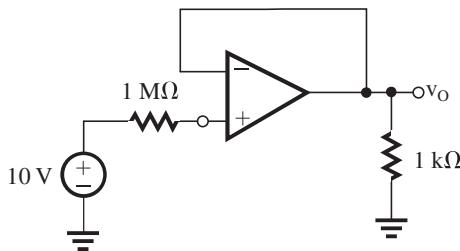


(a) Source is connected directly.

$$v_O = 10 \times \frac{1}{1001} \approx 10 \text{ mV}$$

$$i_L = \frac{v_O}{1 \text{ k}\Omega} = \frac{10 \text{ mV}}{1 \text{ k}\Omega} = 10 \mu\text{A}$$

Current supplied by the source is  $10 \mu\text{A}$ .



(b) Inserting a buffer.

$$v_O = 10 \text{ V}$$

$$i_L = \frac{10 \text{ V}}{1 \text{ k}\Omega} = 10 \text{ mA}$$

Current supplied by the source is 0.

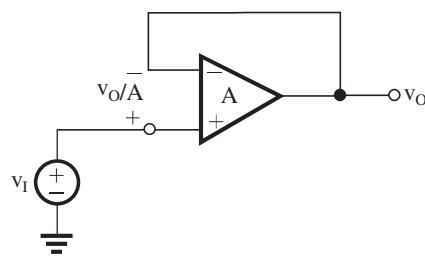
The load current  $i_L$  comes from the power supply of the op amp.

2.51

for a noninverting amplifier:

$$R_{in} = \infty \quad G = 1 + \frac{R_2}{R_1}$$

Case	Gain (V/V)	$R_{in}$	$R_1$	$R_2$
a	-10	10 kΩ	10 kΩ	100 kΩ
b	-1	100 kΩ	100 kΩ	100 kΩ
c	-2	100 kΩ	100 kΩ	200 kΩ
d	+1	∞	∞	0
e	+2	∞	100 kΩ	100 kΩ
f	+11	∞	10 kΩ	100 kΩ
g	-0.5	20 kΩ	20 kΩ	10 kΩ



$$v_O = v_I - \frac{v_O}{A}$$

$$\frac{v_O}{v_I} = \frac{1}{1 + \frac{1}{A}}$$

Error of gain magnitude

$$\left| \frac{\frac{v_O}{v_I} - 1}{1} \right| = -\frac{1}{A+1}$$

$A(\text{V/V})$	1000	100	10
$\frac{v_O}{v_I} (\text{V/V})$	0.999	0.990	0.909
Gain error	-0.1%	-1%	-9.1%

2.52 For an inverting amplifier

$$R_{in} = R_1, \quad G = -\frac{R_2}{R_1}$$

2.53 For a non inverting amplifier [Eq. (2.11)]:

$$G = \frac{G_0}{1 + \frac{G_0}{A}} \quad \epsilon = \frac{G_0 - G}{G_0} \times 100$$

for an inverting amplifier (Eq. 2.5):

$$G = \frac{G_0}{1 + \frac{1 - G_0}{A}} \quad \epsilon = \frac{|G_0| - |G|}{|G_0|} \times 100$$

Case	$G_0(\text{V/V})$	$A(\text{V/V})$	$G(\text{V/V})$	$\epsilon (\%)$
a	-1	10	-0.83	17
b	1	10	0.91	9
c	-1	100	-0.98	2
d	10	10	5	50
e	-10	100	-9	10
f	-10	1000	-9.89	1.1
g	+1	2	0.67	33

$$2.54 \quad G = \frac{G_0}{1 + \frac{G_0}{A}}, \quad \frac{G_0 - G}{G_0} \times 100 = \frac{G_0/A \times 100}{1 + \frac{G_0}{A}} \leq x$$

$$\text{or } \frac{1 + \frac{G_0}{A}}{\frac{G_0}{A}} \geq \frac{100}{x} \Rightarrow \frac{A}{G_0} \geq \underbrace{\left( \frac{100}{x} - 1 \right)}_F$$

$$\Rightarrow A \geq G_0 F, \text{ where } F = \frac{100}{x} - 1 \simeq \frac{100}{x}$$

x	0.01	0.1	1	10
F	$10^4$	$10^3$	$10^2$	10

Thus for:

$$x = 0.01: \begin{array}{c|ccccc} G_0 (\text{V/V}) & 1 & 10 & 10^2 & 10^3 & 10^4 \\ A(\text{V/V}) & 10^4 & 10^5 & 10^6 & \underbrace{10^7}_{\text{too high to}} & 10^8 \\ & & & & \text{be practical} & \end{array}$$

$$x = 0.1: \begin{array}{c|ccccc} G_0 (\text{V/V}) & 1 & 10 & 10^2 & 10^3 & 10^4 \\ A(\text{V/V}) & 10^3 & 10^4 & 10^5 & 10^6 & 10^7 \end{array}$$

$$x = 1: \begin{array}{c|ccccc} G_0 (\text{V/V}) & 1 & 10 & 10^2 & 10^3 & 10^4 \\ A(\text{V/V}) & 10^2 & 10^3 & 10^4 & 10^5 & 10^6 \end{array}$$

$$x = 10: \begin{array}{c|ccccc} G_0 (\text{V/V}) & 1 & 10 & 10^2 & 10^3 & 10^4 \\ A(\text{V/V}) & 10 & 10^2 & 10^3 & 10^4 & 10^5 \end{array}$$

2.55 Refer to Fig. P2.55. When potentiometer is set to the bottom:

$$v_o = v_+ = -15 + \frac{30 \times 25}{25 + 100 + 25} = -10 \text{ V}$$

and to the top:

$$v_o = -15 + \frac{30 \times 25}{25 + 100 + 25} = +10 \text{ V}$$

$$\Rightarrow -10 \text{ V} \leq v_o \leq +10 \text{ V}$$

Pot has 20 turns, and for each turn:

$$\Delta v_o = \frac{2 \times 10}{20} = 1 \text{ V}$$

2.56  $R_1 = R_3 = 5 \text{ k}\Omega$ ,  $R_2 = R_4 = 100 \text{ k}\Omega$

$$\text{Equation (2.15): } \frac{R_4}{R_3} = \frac{R_2}{R_1} = 20$$

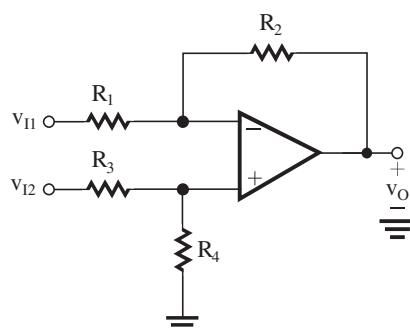
From Eq. (2.16),

$$v_o = \frac{R_2}{R_1} v_{id}$$

$$A_d = \frac{v_o}{v_{id}} = \frac{R_2}{R_1} = 20 \text{ V/V}$$

From Eq. (2.20)

$$R_{id} = 2R_1 = 2 \times 5 \text{ k}\Omega = 10 \text{ k}\Omega$$



The two resistance ratios  $\frac{R_2}{R_1}$  and  $\frac{R_4}{R_3}$  differ by 1%.

$$\therefore \frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3}$$

Now for this case,  $A_{cm}$  can be found from Eq. (2.19)

$$\begin{aligned} A_{cm} &= \left( \frac{R_4}{R_4 + R_3} \right) \left( 1 - \frac{R_2}{R_1} \times \frac{R_3}{R_4} \right) \\ &\simeq \frac{100}{100 + 5} \times \left( 1 - 0.99 \frac{R_4}{R_3} \times \frac{R_3}{R_4} \right) \end{aligned}$$

$$A_{cm} = 0.0095$$

Neglecting the effect of resistance variation on  $A_d$ ,

$$A_d = \frac{R_2}{R_1} = \frac{100}{5} = 20 \text{ V/V}$$

$$\text{CMRR} = 20 \log \left| \frac{A_d}{A_{cm}} \right|$$

$$= 20 \log \left| \frac{20}{0.0095} \right|$$

$$= 66.4 \text{ dB}$$

2.57 If we assume  $R_3 = R_1$ ,  $R_4 = R_2$ , then

$$\text{Eq. (2.20): } R_{id} = 2R_1 \Rightarrow R_1 = \frac{20}{2} = 10 \text{ k}\Omega$$

(Refer to Fig. 2.16.)

$$(a) A_d = \frac{R_2}{R_1} = 1 \text{ V/V} \Rightarrow R_2 = 10 \text{ k}\Omega$$

$$R_1 = R_2 = R_3 = R_4 = 10 \text{ k}\Omega$$

$$(b) A_d = \frac{R_2}{R_1} = 5 \text{ V/V} \Rightarrow R_2 = 50 \text{ k}\Omega = R_4$$

$$R_1 = R_3 = 10 \text{ k}\Omega$$

$$(c) A_d = \frac{R_2}{R_1} = 100 \text{ V/V} \Rightarrow R_2 = 1 \text{ M}\Omega = R_4$$

$$R_1 = R_3 = 10 \text{ k}\Omega$$

$$(d) A_d = \frac{R_2}{R_1} = 0.5 \text{ V/V} \Rightarrow R_2 = 5 \text{ k}\Omega = R_4$$

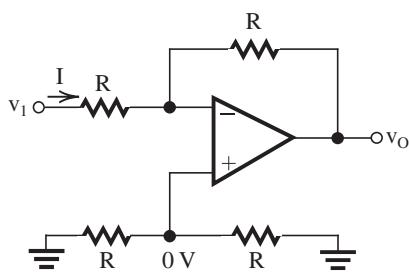
$$R_1 = R_3 = 10 \text{ k}\Omega$$

2.58 Refer to Fig. P2.58:

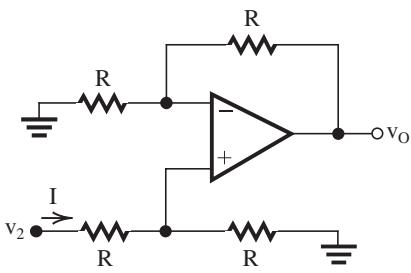
Considering that  $v_- = v_+$ :

$$v_1 + \frac{v_o - v_1}{2} = \frac{v_2}{2} \Rightarrow v_o = v_2 - v_1$$

$$v_1 \text{ only: } R_i = \frac{v_1}{I} = R$$

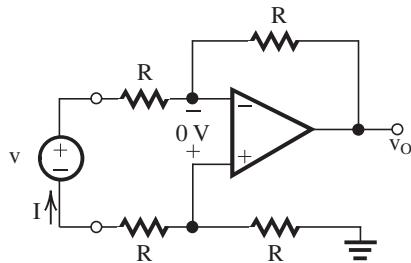


$$v_2 \text{ only: } R_i = \frac{v_2}{I} = 2R$$



v between 2 terminals:

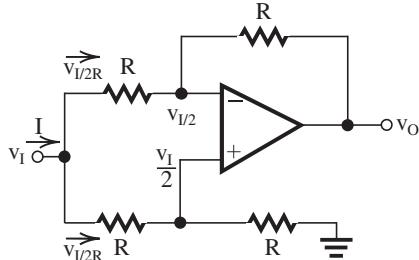
$$R_i = \frac{v}{I} = 2R$$



$v_I$  connected to both input terminals

$$I = \frac{v_I}{2R} + \frac{v_I}{2R} = \frac{v_I}{R}$$

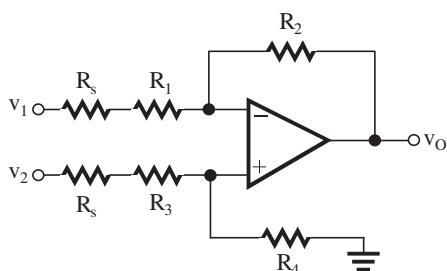
$$R_i = R$$



2.59 For an ideal difference amp, we need:

$$\frac{R_s + R_1}{R_2} = \frac{R_s + R_3}{R_4}$$

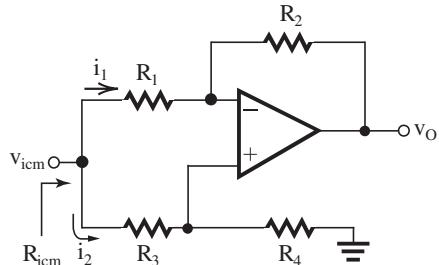
$$\frac{R_s/R_1 + 1}{R_2/R_1} = \frac{R_s/R_3 + 1}{R_4/R_3}$$



Since  $\frac{R_2}{R_1} = \frac{R_4}{R_3}$ :

$$\frac{R_s}{R_1} + 1 = \frac{R_s}{R_3} + 1 \Rightarrow R_1 = R_3 \Rightarrow R_2 = R_4$$

2.60



When  $R_2/R_1 = R_4/R_3$ , the output voltage  $v_O$  will be zero. Thus,

$$i_1 = \frac{v_{icm}}{R_1 + R_2}$$

and

$$i_2 = \frac{v_{icm}}{R_3 + R_4}$$

Thus,

$$i_1 = v_{icm} \left[ \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4} \right]$$

and

$$R_{icm} = (R_1 + R_2) \parallel (R_3 + R_4) \quad \text{Q.E.D.}$$

2.61 From Eq. (2.19),

$$A_{cm} \equiv \frac{v_O}{v_{icm}} = \left( \frac{R_4}{R_4 + R_3} \right) \left( 1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$

The second factor in this expression is the one that in effect determines  $A_{cm}$ . Now, for the circuit under consideration, all resistors are nominally equal to  $10\text{k}\Omega$ , but each has a tolerance of  $\pm x\%$ . Thus the second factor will have a maximum magnitude when  $R_2$  and  $R_3$  are at their lowest value and  $R_1$  and  $R_4$  are at their highest values, that is,

$$R_2 = R_3 = 10(1 - 0.01x)$$

$$R_1 = R_4 = 10(1 + 0.01x)$$

Substituting in the expression for  $A_{cm}$ , we have

$$A_{cm} = \frac{10(1 + 0.01x)}{20} \left[ 1 - \frac{(1 - 0.01x)^2}{(1 + 0.01x)^2} \right]$$

Now, for  $0.01x \ll 1$ ,

$$A_{cm} = 0.5(1 + 0.01x) \times 0.04x$$

$$A_{cm} \simeq 0.02x \text{ V/V}$$

$$A_d = 1$$

$$CMRR = \left| \frac{A_d}{A_{cm}} \right| = \frac{50}{x}$$

$$\text{or } 20 \log\left(\frac{50}{x}\right) \text{ dB}$$

x(%)	0.1	1	5
A <sub>cm</sub> (V/V)	0.002	0.02	0.1
CMRR (dB)	54	34	20

2.62 From Eq. (2.19),

$$A_{cm} = \left( \frac{R_4}{R_4 + R_3} \right) \left( 1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$

The second factor in this expression in effect determines A<sub>cm</sub>. The largest, |A<sub>cm</sub>|, will occur when R<sub>2</sub> and R<sub>3</sub> are at their lowest (or highest) values and R<sub>1</sub> and R<sub>4</sub> are at their highest (or lowest) values, as this will provide the maximum deviation of  $\left( \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$  from unity. Thus,

$$R_2 = R_{2\text{nominal}}(1 - \epsilon)$$

$$R_3 = R_{3\text{nominal}}(1 - \epsilon)$$

$$R_1 = R_{1\text{nominal}}(1 + \epsilon)$$

$$R_4 = R_{4\text{nominal}}(1 + \epsilon)$$

where

$$\frac{R_{2\text{nominal}}}{R_{1\text{nominal}}} = \frac{R_{4\text{nominal}}}{R_{3\text{nominal}}} = K$$

Substituting in the expression for A<sub>cm</sub>,

$$A_{cm} = \frac{R_{4\text{nominal}}(1 + \epsilon)}{R_{4\text{nominal}}(1 + \epsilon) + R_{3\text{nominal}}(1 - \epsilon)} \times \left[ 1 - \frac{(1 - \epsilon)^2}{(1 + \epsilon)^2} \right]$$

For  $\epsilon \ll 1$ ,

$$A_{cm} \simeq \frac{K}{K + 1} \times 4\epsilon$$

Since

$$A_d = K$$

$$CMRR = \left| \frac{A_d}{A_{cm}} \right| = \left( \frac{K + 1}{4\epsilon} \right)$$

which in dB becomes

$$CMRR = 20 \log \left[ \frac{K + 1}{4\epsilon} \right] \quad \text{Q.E.D.}$$

For A<sub>d</sub> = 100 V/V and  $\epsilon = 0.01$ ,

$$CMRR = 20 \log \left( \frac{101}{0.04} \right) = 68 \text{ dB}$$

To obtain CMRR = 80 dB,

$$\frac{101}{4\epsilon} = 10^4$$

$$\epsilon = \frac{101}{4 \times 10^4} \simeq 0.25 \times 10^{-2}$$

That is, the resistor tolerance should be a maximum of 0.25%.

2.63 See solution to Problem 2.62 above.

Specifically, if the resistors have a tolerance of x%, then

$$CMRR = 20 \log \left[ \frac{K + 1}{4(x/100)} \right]$$

where K is the nominal differential gain. Here we are required to obtain

$$K = 1000$$

$$R_{in} = 2 \text{ k}\Omega$$

$$\text{Thus, } R_1 = R_3 = 1 \text{ k}\Omega \text{ and } R_2 = R_4 = 1 \text{ M}\Omega.$$

To obtain a CMRR of 88 dB, we write

$$88 = 20 \log \left[ \frac{1001}{4(x/100)} \right]$$

$$\frac{1001 \times 100}{4x} = 25,118.86$$

$$\Rightarrow x \simeq 1\%$$

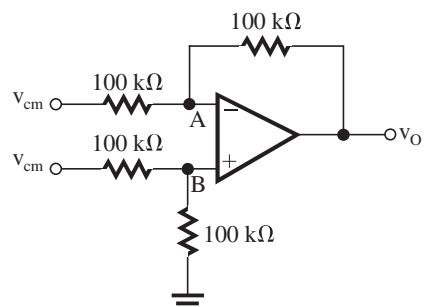
2.64 (a) Refer to Fig. P2.64 and Eq. (2.19):

$$A_{cm} = \frac{R_4}{R_3 + R_4} \left( 1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right) = \frac{100}{100 + 100} \left( 1 - \frac{100 \cdot 100}{100 \cdot 100} \right) = A_{cm} = 0$$

$$\text{Refer to 2.17: } \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

$$\Rightarrow A_d = \frac{R_2}{R_1} = 1$$

$$(b) v_A = v_B$$

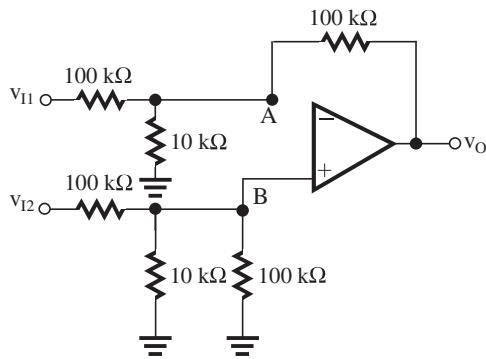


$$v_A = v_{cm} \frac{100}{100 + 100}$$

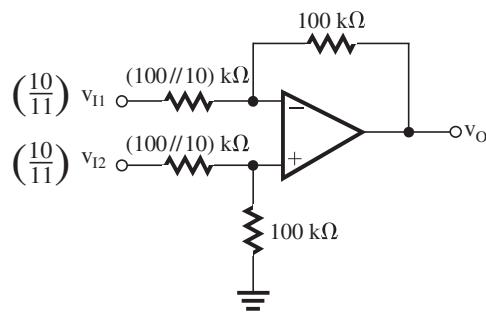
$$v_A = \frac{v_{cm}}{2} \text{ and } v_B = \frac{v_{cm}}{2}$$

$$\Rightarrow -5 \text{ V} \leq v_{cm} \leq 5 \text{ V}$$

(c) The circuit becomes as shown below:



Applying Thévenin's theorem to  $v_{I11}$  together with the associated (100-kΩ, 10-kΩ) voltage divider, and similarly to  $v_{I12}$  and the associated (100-kΩ, 10-kΩ) voltage divider, we obtain the following circuit:



$$v_O = \frac{100 \text{ k}\Omega}{(100 \parallel 10) \text{ k}\Omega} \left( \frac{10}{11} v_{I12} - \frac{10}{11} v_{I11} \right)$$

$$= 10(v_{I12} - v_{I11})$$

For  $v_{I11} = v_{I12} = v_{lcm}$ ,  $v_O = 0$ , thus

$$A_{cm} = 0$$

For  $v_{I12} - v_{I11} = v_{Id}$ ,

$$v_O = 10 v_{Id}$$

Thus,  $A_d = 10$ .

To obtain the input common-mode range, we note that for  $v_{I11} = v_{I12} = v_{lcm}$ ,

$$v_+ = v_- = \frac{10}{11} v_{lcm} \times \frac{100}{100 + (100 \parallel 10)} \\ = 0.833 v_{lcm}$$

For  $v_+$  and  $v_-$  in the range

$$-2.5 \text{ V} \leq v_+, v_- \leq +2.5 \text{ V}$$

the range of  $v_{lcm}$  will be

$$-3 \text{ V} \leq v_{lcm} \leq +3 \text{ V}$$

2.65 Refer to Fig. P2.65. Using superposition:

$$v_O = v_{O1} + v_{O2}$$

$$\text{Calculate } v_{O1}: v_+ = \frac{\beta v_{O1}}{2} = v_-$$

$$\frac{v_1 - \frac{\beta v_{O1}}{2}}{R} = \frac{\frac{\beta v_{O1}}{2} - v_O}{R} \Rightarrow v_{O1} = \frac{v_1}{\beta - 1}$$

Calculate  $v_{O2}$ :

$$v_- = \frac{v_{O2}}{2} = v_+ \Rightarrow v_2 - \frac{v_{O2}}{2} = \frac{v_{O2}}{2} - \beta v_{O2}$$

$$\Rightarrow v_{O2} = \frac{v_2}{1 - \beta}$$

$$v_O = v_{O1} + v_{O2} = \frac{v_1}{\beta - 1} + \frac{v_2}{1 - \beta}$$

$$= \frac{1}{1 - \beta} (v_2 - v_1)$$

$$A_d = \frac{v_O}{v_2 - v_1} = \frac{1}{1 - \beta} \quad \text{Q.E.D}$$

$$A_d = 10 \text{ V/V} \Rightarrow \beta = 0.9 = \frac{R_6}{R_5 + R_6}$$

$$R_{Id} = 2R = 2 \text{ M}\Omega \Rightarrow R = 1 \text{ M}\Omega$$

$$R_5 + R_6 \leq \frac{R}{100} \Rightarrow R_5 + R_6 \leq 10 \text{ k}\Omega$$

Selecting  $R_6 = 6.8 \text{ k}\Omega$

$$\frac{6.8}{6.8 + R_5} = 0.9 \Rightarrow R_5 = 756 \text{ }\Omega$$

2.66 See partial analysis on circuit diagram on next page.

From input loop:

$$v_{Id} = 2i_1 R_1 \quad (1)$$

From the loop containing  $R_2$ , +, -,  $R_2$ :

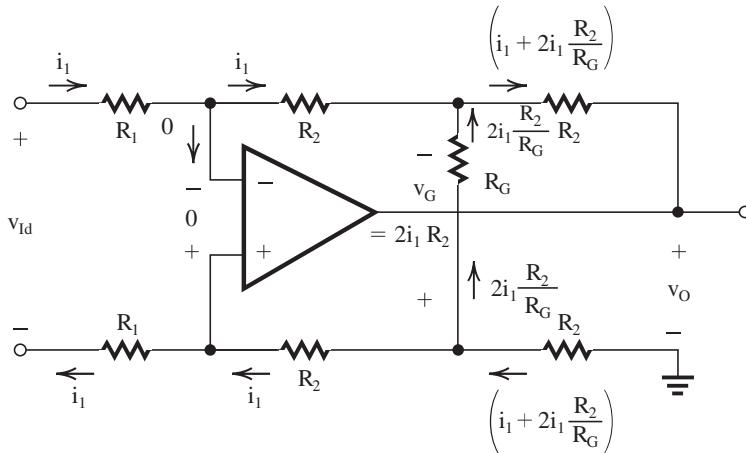
$$v_G = i_1 R_2 + 0 + i_1 R_2 = 2i_1 R_2$$

Thus, we can find the current through  $R_G$  as  $v_G/R_G = 2i_1(R_2/R_G)$ . Finally, from the loop containing  $v_O$ ,  $R_2$ ,  $R_G$ , and  $R_2$ :

$$v_O = -2i_1 \left( 1 + 2 \frac{R_2}{R_G} \right) R_2 - 2i_1 R_2$$

$$= -4i_1 R_2 \left( 1 + \frac{R_2}{R_G} \right)$$

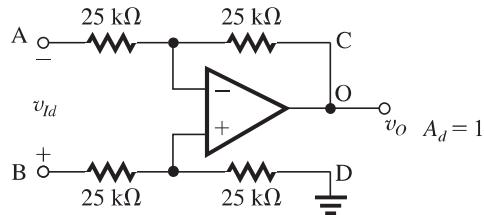
This figure belongs to Problem 2.66.



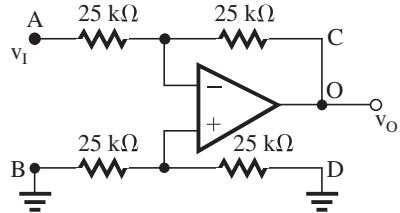
Substituting for \$2i\_1\$ from Eq. (1), we obtain the gain as

$$\frac{v_O}{v_{Id}} = -2 \left( \frac{R_2}{R_1} \right) \left( 1 + \frac{R_2}{R_G} \right) \quad \text{Q.E.D.}$$

2.67 (a) Refer to Eq. (2.17): \$A\_d = \frac{R\_2}{R\_1} = 1\$. Connect C and O together, and D to ground.

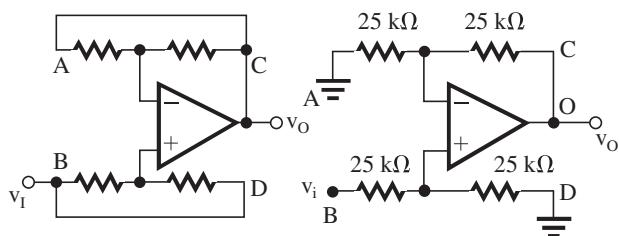


$$(b) (i) \frac{v_O}{v_i} = -1 \text{ V/V}$$



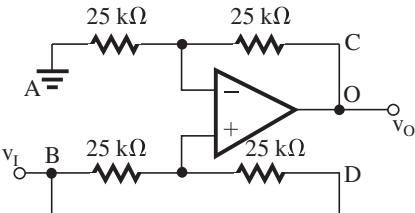
$$(ii) \frac{v_O}{v_i} = +1 \text{ V/V}$$

Two possibilities:

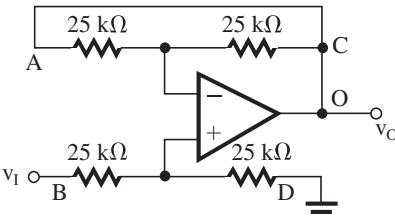


The circuit on the left ideally has infinite input resistance

$$(iii) \frac{v_O}{v_i} = +2 \text{ V/V}$$



$$(iv) \frac{v_O}{v_i} = +\frac{1}{2} \text{ V/V}$$



2.68 Refer to the figure on next page:

$$v_B = v_A = 3 + 0.05 \sin \omega t, \text{ V}$$

$$v_C = v_D = 3 - 0.05 \sin \omega t, \text{ V}$$

Current through \$R\_2\$, \$2R\_1\$, and \$R\_2\$ is

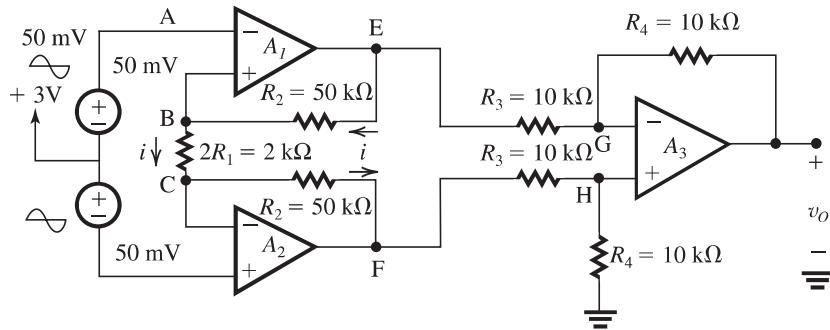
$$i = (v_B - v_C)/2R_1$$

$$= 0.05 \sin \omega t, \text{ mA}$$

$$v_E = v_B + iR_2 = 3 + 2.55 \sin \omega t, \text{ V}$$

$$v_F = v_C - iR_2 = 3 - 2.55 \sin \omega t, \text{ V}$$

This figure belongs to Problem 2.68.



$$v_G = v_H = \frac{1}{2}v_F = 1.5 - 1.275 \sin \omega t, \text{ V}$$

$$v_O = (v_F - v_E) \times 1 = -5.1 \sin \omega t, \text{ V}$$

2.69 (a) Refer to Fig. 2.20(a).

The gain of the first stage is  $\left(1 + \frac{R_2}{R_1}\right) = 101$ . If the op amps of the first stage saturate at  $\pm 12 \text{ V}$  then  $-12 \text{ V} \leq 101v_{\text{icm}} \leq +12 \text{ V}$

$$\Rightarrow -0.12 \text{ V} \leq v_{\text{icm}} \leq 0.12 \text{ V}$$

As explained in the text, the disadvantage of circuit in Fig. 2.20(a) is that  $v_{\text{icm}}$  is amplified by a gain equal to  $\left(1 + \frac{R_2}{R_1}\right)$  in the first stage and therefore a very small  $v_{\text{icm}}$  range is acceptable to avoid saturation.

(b) In Fig. 2.20(b), when  $v_{\text{icm}}$  is applied,  $v_-$  for both  $A_1$  and  $A_2$  is the same and therefore no current flows through  $2R_1$ . This means the voltage at the output of  $A_1$  and  $A_2$  is the same as  $v_{\text{icm}}$ .

$$\Rightarrow -12 \text{ V} \leq v_{\text{icm}} \leq 12 \text{ V}$$

This circuit allows for a much larger range of  $v_{\text{icm}}$ .

2.70 (a) Refer to the circuit in Fig. 2.20(a).

$$v_{I1} = v_{\text{icm}} - \frac{1}{2}v_{\text{id}}$$

$$v_{I2} = v_{\text{icm}} + \frac{1}{2}v_{\text{id}}$$

$$v_{O1} = \left(1 + \frac{R_2}{R_1}\right)v_{I1}$$

$$= \left(1 + \frac{R_2}{R_1}\right)v_{\text{icm}} - \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right)v_{\text{id}}$$

$$v_{O2} = \left(1 + \frac{R_2}{R_1}\right)v_{I2}$$

$$= \left(1 + \frac{R_2}{R_1}\right)v_{\text{icm}} + \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right)v_{\text{id}}$$

$$v_{Od} = v_{O2} - v_{O1} = \left(1 + \frac{R_2}{R_1}\right)v_{\text{id}}$$

$$v_{Ocm} = \frac{1}{2}(v_{O1} + v_{O2}) = \left(1 + \frac{R_2}{R_1}\right)v_{\text{icm}}$$

$$A_{d1} = 1 + \frac{R_2}{R_1}$$

$$A_{cm1} = 1 + \frac{R_2}{R_1}$$

$$\text{CMRR} = 20 \log \left| \frac{A_{d1}}{A_{cm1}} \right| = 0 \text{ dB}$$

(b) Refer to the circuit in Fig. 2.20(b):

$$v_{I1} = v_{\text{icm}} - \frac{1}{2}v_{\text{id}}$$

$$v_{I2} = v_{\text{icm}} + \frac{1}{2}v_{\text{id}}$$

$$v_-(A_1) = v_{I1} = v_{\text{icm}} - \frac{1}{2}v_{\text{id}}$$

$$v_-(A_2) = v_{I2} = v_{\text{icm}} + \frac{1}{2}v_{\text{id}}$$

Current through  $R_1$  in the upward direction is

$$i = \frac{v_-(A_2) - v_-(A_1)}{2R_1} = \frac{v_{\text{id}}}{2R_1}$$

$$v_{O1} = v_-(A_1) - iR_2 = v_{\text{icm}} - \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right)v_{\text{id}}$$

$$v_{O2} = v_-(A_2) + iR_2 = v_{\text{icm}} + \frac{1}{2}\left(1 + \frac{R_2}{R_1}\right)v_{\text{id}}$$

$$v_{Od} = v_{O2} - v_{O1} = \left(1 + \frac{R_2}{R_1}\right)v_{\text{id}}$$

$$v_{Ocm} = \frac{1}{2}(v_{O1} + v_{O2}) = v_{Icm}$$

$$A_{d1} = 1 + \frac{R_2}{R_l}$$

$$A_{cm1} = 1$$

$$CMRR = 20 \log \left| \frac{A_{d1}}{A_{cm1}} \right| = 20 \log \left( 1 + \frac{R_2}{R_l} \right)$$

Comment: In circuit (a), the first stage amplifies the differential signal and the common-mode signal equally. On the other hand, in circuit (b), the first stage amplifies the differential signal by  $\left( 1 + \frac{R_2}{R_l} \right)$  and the common-mode signal by unity, thus providing a substantial CMRR. Circuit (a) is useless as a differential amplifier!

2.71 Ideally,

$$A_d = \frac{R_4}{R_3} \left( 1 + \frac{R_2}{R_l} \right) \quad (1)$$

$$A_{cm} = 0$$

$$CMRR = \infty$$

For  $R_2 = R_3 = R_4 = 100 \text{ k}\Omega$ , and  $2R_l = 10 \text{ k}\Omega$ .

$$A_d = 1 \left( 1 + \frac{100}{5} \right) = 21 \text{ V/V}$$

$$A_{cm} = 0$$

$$CMRR = \infty$$

If all resistors have  $\pm 1\%$  tolerance, the differential gain will be slightly affected; Eq. (1) indicates that in the worst case  $A_d$  can deviate by approximately  $\pm 4\%$  of the nominal value. The common-mode gain, however, undergoes dramatic change because of the significant effect of resistor tolerances on the operation of the difference amplifier in the second stage. Equation (2.19) can be employed to evaluate the worst-case common-mode gain. For our case,

$$A_{cm2} = 0.5[1 - (1 \pm 0.04)] = \pm 0.02$$

The common-mode gain of the first stage will remain approximately unity. Thus the  $\pm 1\%$  resistor tolerances will mainly affect the common-mode gain of the instrumentation amplifier, increasing it in the worst case to

$$|A_{cm}| = 0.02$$

Correspondingly, the CMRR will be reduced to

$$CMRR = 20 \log \frac{|A_d|}{|A_{cm}|}$$

$$\simeq 20 \log \left( \frac{21}{0.02} \right) = 60.4 \text{ dB}$$

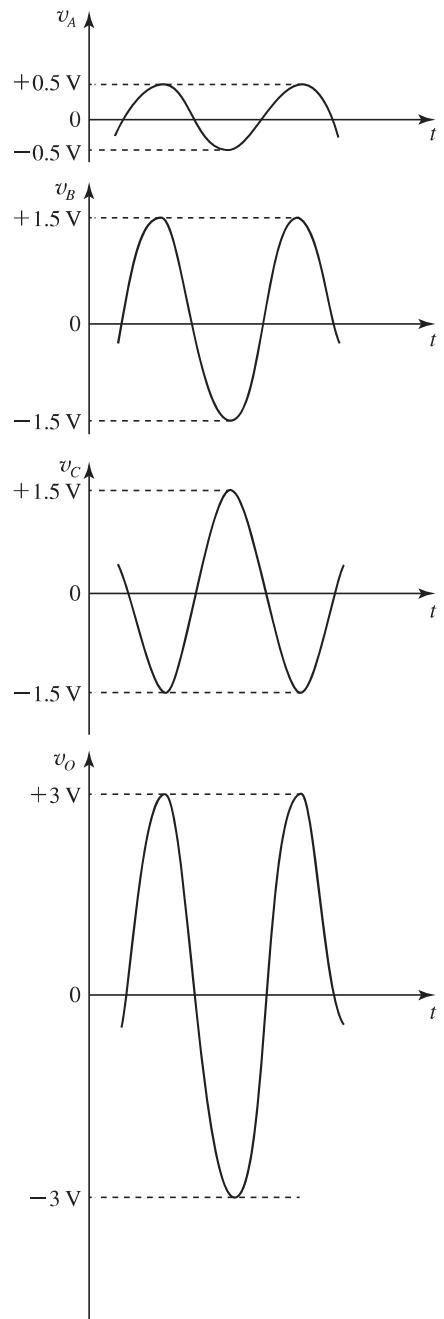
as opposed to the ideal infinite value!

If  $2R_l$  is reduced to  $1 \text{ k}\Omega$ ,  $A_d$  increases to  $201 \text{ V/V}$  while  $A_{cm}$  remains unchanged at  $0.02 \text{ V/V}$ . Thus, CMRR increases to about  $80 \text{ dB}$ . We conclude that increasing the gain of the first stage increases CMRR.

$$2.72 \text{ (a)} \frac{v_B}{v_A} = 1 + \frac{20}{10} = 3 \text{ V/V}$$

$$\frac{v_C}{v_A} = -\frac{30}{10} = -3 \text{ V/V}$$

$$\text{(b)} \quad v_O = v_B - v_C = 6v_A \Rightarrow \frac{v_O}{v_A} = 6 \text{ V/V}$$



(c)  $v_B$  and  $v_C$  can be  $\pm 14$  V, or 28 V P-P.

$-28 \text{ V} \leq v_O \leq 28 \text{ V}$ , or 56 V P-P

$$v_{O\text{rms}} = \frac{28}{\sqrt{2}} = 19.8 \text{ V}$$

2.73 See analysis on the circuit diagrams below:

Note that circuit (a) has the advantage of infinite input resistance. However, it has the limitation that the load impedance must be "floating". This constraint is removed in circuit (b), but the input resistance is finite ( $2R_1$ ).

$$2.74 \frac{V_o(s)}{V_i(s)} = -\frac{1}{sCR}$$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR}$$

$$\left| \frac{V_o}{V_i} \right| = \frac{1}{\omega CR} \quad \angle \phi = +90^\circ$$

For  $C = 1 \text{ nF}$  and  $R = 10 \text{ k}\Omega$ ,

$$CR = 1 \times 10^{-9} \times 10 \times 10^4 = 10^{-4} \text{ s}$$

$$(a) |V_o/V_i| = 1 \text{ at } \omega = \frac{1}{CR} = 10^4 \text{ rad/s.}$$

$$\text{Correspondingly, } f = \frac{10^4}{2\pi} = 1.59 \text{ kHz.}$$

(b) At  $f = 1.59 \text{ kHz}$ , the output sine wave leads the input sine wave by  $90^\circ$ .

(c) If the frequency is lowered by a factor of 10, the gain increases by a factor of 10 and, correspondingly, the output voltage increases by a factor of 10.

(d) The phase relation between input and output remains unchanged.

$$2.75 |T| = \frac{1}{\omega RC}. \text{ If } |T| = 100 \text{ V/V at}$$

$f = 10 \text{ kHz}$ , then for  $|T| = 1 \text{ V/V}$ ,  $f$  has to be  $10 \text{ kHz} \times 100 = 1 \text{ MHz}$ .

Also,

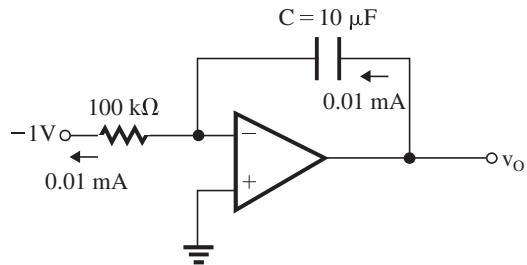
$$RC = \frac{1}{\omega_{\text{int}}} = \frac{1}{2\pi \times 1 \text{ MHz}} = 0.159 \mu\text{s}$$

2.76  $CR = 1 \text{ s}$  and  $R_{\text{in}} = 100 \text{ k}\Omega$ .

$$\Rightarrow R = 100 \text{ k}\Omega$$

$$C = \frac{1}{100 \times 10^3} = 10 \mu\text{F}$$

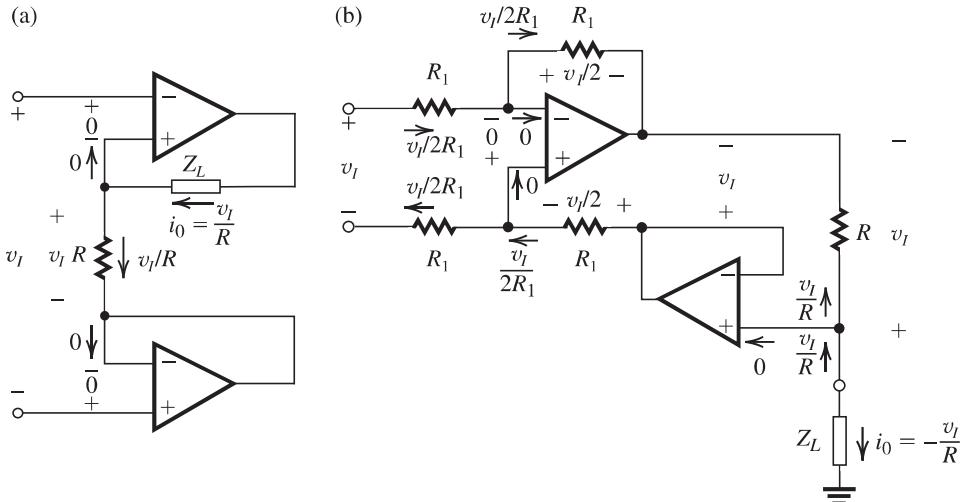
When a dc voltage of  $-1 \text{ V}$  is applied, a dc current of  $1 \text{ V}/100 \text{ k}\Omega = 0.01 \text{ mA}$  will flow as shown in the figure below.



The capacitor voltage  $v_O$  will rise linearly from its initial value of  $-10 \text{ V}$ . Thus,

$$\begin{aligned} v_O &= \frac{It}{C} - 10 \\ &= \frac{10^{-5}t}{10 \times 10^{-6}} - 10 = t - 10, \text{ V} \end{aligned}$$

This figure belongs to Problem 2.73.



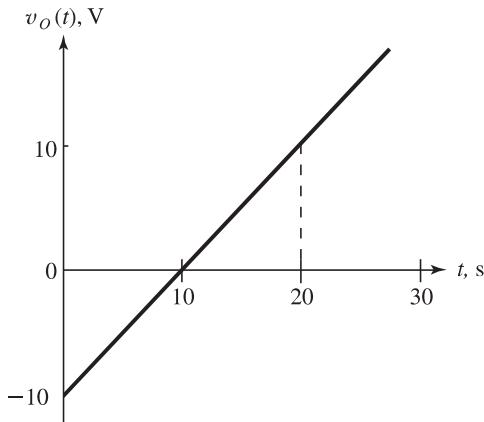
Thus  $v_O$  will reach 0 V at

$$t = 10 \text{ s}$$

and will reach 10 V at

$$t = 20 \text{ s}$$

See figure below.



$$2.77 |T| = \frac{1}{\omega RC} = 1 \text{ at } \omega_{int} = \frac{1}{RC}.$$

$$\text{For } \omega_{int} = 10 \text{ krad/s, } RC = \frac{1}{10^4} = 10^{-4} \text{ s. Now}$$

$R_{in} = 100 \text{ k}\Omega$ , thus  $R = 100 \text{ k}\Omega$  and

$$C = \frac{10^{-4} \text{ s}}{10^5 \Omega} = 10^{-9} \text{ F} = 1 \text{ nF. For a 2-V, } 100\text{-}\mu\text{s}$$

$$\text{input pulse, a current of } \frac{2 \text{ V}}{100 \text{ k}\Omega} = 0.02 \text{ mA}$$

"ows into R and C, causing  $v_O$  to decrease linearly from its initial value of 0 V according to

$$\begin{aligned} v_O &= -\frac{I}{C}t \\ &= \frac{-0.02 \times 10^{-3}}{1 \times 10^{-9}}t = -0.02 \times 10^6 t \end{aligned}$$

At  $t = 100 \mu\text{s}$ ,  $v_O$  becomes

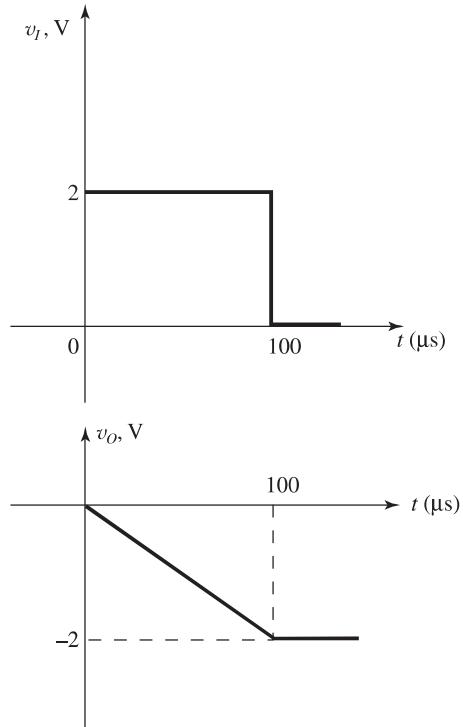
$$\begin{aligned} v_O(100 \mu\text{s}) &= -0.02 \times 10^6 \times 100 \times 10^{-6} \\ &= -2 \text{ V} \end{aligned}$$

and the output voltage then remains constant at this value. See figures on next column. When  $v_I = 2 \sin 10^4 t$ , the output will be a sine wave of the same frequency but phase-shifted by  $-270^\circ$  (or  $+90^\circ$ ), and its magnitude will be 2 V  $\times$  integrator gain at  $\omega = 10^4 \text{ rad/s}$ . The gain is given by

$$|T| = \frac{1}{\omega RC} = \frac{1}{10^4 \times 10^{-4}} = 1$$

Thus,

$$v_O = -2 \sin(10^4 t - 90^\circ)$$



$$2.78 2\pi \times 100 \times 10^3 = \frac{1}{CR} \Rightarrow CR = 1.59 \mu\text{s}$$

For  $R_{in} = 10 \text{ k}\Omega$ ,  $R = 10 \text{ k}\Omega$ , and

$$C = \frac{1.59 \times 10^{-6}}{10^4} = 159 \text{ pF}$$

To limit the dc gain to 40 dB (i.e., 100 V/V), we connect a resistance  $R_F$  across C (as in Fig. 2.25) with  $R_F = 100 R = 1 \text{ M}\Omega$ .

The resulting low-pass filter will have a 3-dB frequency of

$$\begin{aligned} f_{3\text{dB}} &= \frac{1}{2\pi CR_F} = \frac{1}{2\pi \times 159 \times 10^{-12} \times 10^6} \\ &= 1 \text{ kHz} \end{aligned}$$

When a 10-μs, 1-V pulse (see Fig. 1) is applied at the input, a current of  $1 \text{ V}/10 \text{ k}\Omega = 0.1 \text{ mA}$  "ows into the integrator. Now we consider two cases: with and without  $R_F$ .

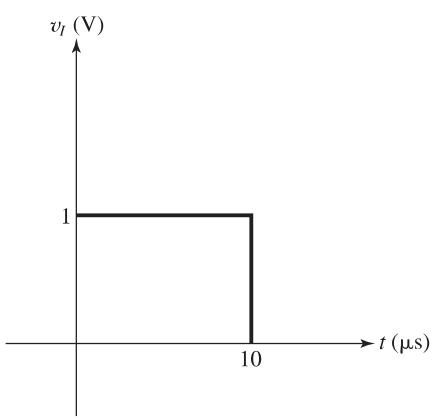


Figure 1

(a) For an integrator without  $R_F$ , the 0.1-mA current flows through  $C$  and the output voltage decreases linearly from 0 V as

$$\begin{aligned} v_O(t) &= -\frac{It}{C} \\ &= -\frac{0.1 \times 10^{-3}}{159 \times 10^{-12}} t \\ &= -0.63 \times 10^6 t, \text{ V} \end{aligned}$$

At the end of the pulse,  $t = 10 \mu s$ , resulting in

$$\begin{aligned} v_O(10 \mu s) &= -0.63 \times 10^6 \times 10 \times 10^{-6} \\ &= -6.3 \text{ V} \end{aligned}$$

See Fig. 2.

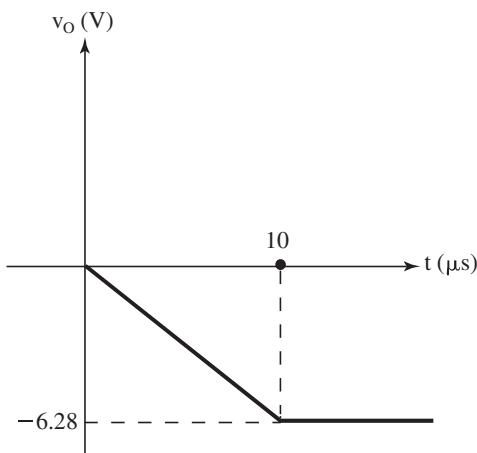


Figure 2

where  $t$  is in  $\mu s$ . At the end of the pulse,  $t = 10 \mu s$ ,

$$v_O(10 \mu s) = -100(1 - e^{-10/159}) = -6.1 \text{ V}$$

Beyond  $t = 10 \mu s$ , the capacitor discharges through  $R_F$ . Thus, including  $R_F$  results in the nonideal integrator response shown in Fig. 3.

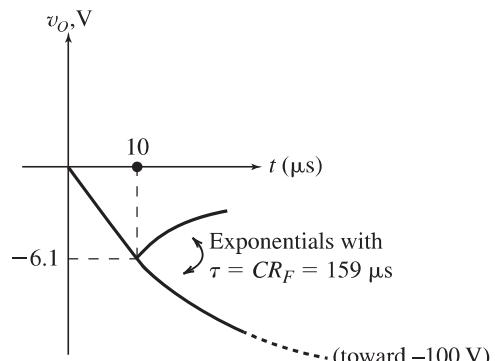


Figure 3

2.79 Each pulse provides a constant current of  $\frac{1 \text{ V}}{R}$  through the capacitor and thus deposits a charge of  $\frac{1 \text{ V}}{R} \times 10 \mu s$  on the capacitor, resulting in a change of the output voltage of

$$-\frac{1 \times 10 \times 10^{-6}}{RC} = -\frac{10^{-5}}{10^{-3}} = -0.01 \text{ V}$$

Therefore a total of 100 pulses are required to cause a change of  $-1 \text{ V}$  in  $v_O(t)$ .

(b) For an integrator with  $R_F$ , the 0.1-mA current flows through the parallel combination of  $C$  and  $R_F$ . The result is

$$v_O(t) = v_{O\text{final}} - (v_{O\text{final}} - v_{O\text{initial}})e^{-t/\tau}$$

where

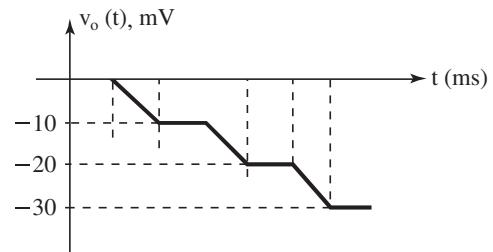
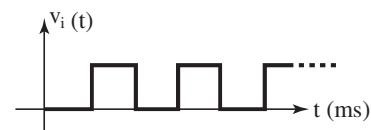
$$v_{O\text{final}} = -IR_F = -0.1 \times 10^{-3} \times 10^6 = -100 \text{ V}$$

$$v_{O\text{initial}} = 0$$

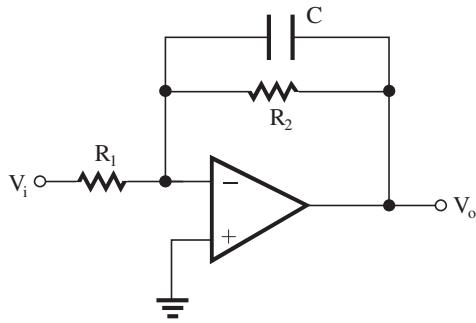
$$\tau = CR_F = 159 \times 10^{-12} \times 10^6 = 159 \mu s$$

Thus,

$$v_O(t) = -100(1 - e^{-t/159}), \text{ V}$$



2.80



Let  $Z_2 = R_2 \parallel \frac{1}{sC}$  and  $Z_1 = R_1$

$$\begin{aligned}\frac{V_o}{V_i} &= -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{1/R_1}{\frac{1}{R_2} + sC} \\ &= -\frac{(R_2/R_1)}{1+sCR_2}\end{aligned}$$

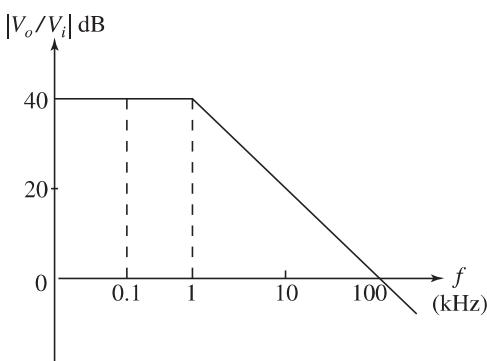
This function is of the STC low-pass type, having a dc gain of  $-\frac{R_2}{R_1}$  and a 3-dB frequency

$$\omega_0 = \frac{1}{CR_2}$$

$$R_{in} = R_1 = 10 \text{ k}\Omega$$

$$\text{dc gain} = 40 \text{ dB} = 100$$

$$\therefore 100 = \frac{R_2}{R_1} \Rightarrow R_2 = 100R_1 = 1 \text{ M}\Omega$$



3-dB frequency at 1 kHz

$$\therefore \omega_0 = 2\pi \times 1 \times 10^3 = \frac{1}{CR_2}$$

$$C = \frac{1}{2\pi \times 1 \times 10^3 \times 10^6} = 0.16 \text{ nF}$$

From the Bode plot shown in previous column, the unity-gain frequency is 100 kHz.

2.81 Equation (2.5) can be generalized as follows:

$$\frac{V_o}{V_i} = -\frac{Z_2/Z_1}{1 + \frac{1+Z_2/Z_1}{A}}$$

For  $Z_1 = R$ ,  $Z_2 = 1/sC$ , and  $A = A_0$ ,

$$\begin{aligned}\frac{V_o}{V_i} &= -\frac{1/sCR}{1 + \frac{1}{A_0} + \frac{1}{sA_0 CR}} \\ &= -\frac{1}{CR(1 + \frac{1}{A_0})} \frac{1}{s + \frac{1}{(A_0 + 1)CR}} \\ &= -\frac{A_0 / |(A_0 + 1)CR|}{s + \frac{1}{(A_0 + 1)CR}}\end{aligned}$$

which is low-pass STC function. The pole (or 3-dB) frequency is

$$\omega_p = \frac{1}{(A_0 + 1)CR}$$

The ideal integrator has  $\omega_p = 0$ . Observe that as  $A_0 \rightarrow \infty$ ,  $\omega_p \rightarrow 0$ . The dc gain is  $-A_0$ , which is the dc gain of the op amp.

If an ideal Miller integrator is fed with a  $-1\text{-V}$  pulse signal of width  $T = CR$ , the output voltage can be found as follows: The  $-1\text{-V}$  pulse will cause a current  $I = 1/V/R$  to be drawn through  $R$  and  $C$ . The capacitor voltage, which is  $v_o$ , will rise linearly according to

$$v_o = \frac{1}{C} It = \frac{1}{CR} t$$

Thus, at  $t = T$  (the end of the pulse) the output voltage reaches  $1 \text{ V}$  and then stays constant at this value.

If the integrator is made with an op amp having a finite  $A_0 = 1000$ , the response to the  $-1\text{-V}$  step will be that of an STC low-pass circuit. Thus,

$$v_O = v_{O\text{final}} - (v_{O\text{final}} - v_{O\text{initial}}) e^{-t/\tau}$$

where

$$v_{O\text{final}} = -1 \text{ V} \times \text{dc gain}$$

$$= -1 \text{ V} \times -1000$$

$$= 1000 \text{ V}$$

$$v_{O\text{initial}} = 0 \text{ V}$$

$$\tau = \frac{1}{\omega_p} = (A_0 + 1)CR = 1001CR$$

Thus,

$$v_O = 1000(1 - e^{-t/1000CR})$$

At  $t = T$ , which is equal to  $CR$ ,

$$v_O(T) = 1000(1 - e^{-0.001})$$

$$= -0.9995 \approx -1 \text{ V}$$

$$2.82 \frac{V_o}{V_i} = -sCR = -j\omega CR, \quad \left| \frac{V_o}{V_i} \right| = \omega CR$$

For  $R = 10 \text{ k}\Omega$  and  $C = 1 \text{ nF}$ ,

$$CR = 1 \times 10^{-9} \times 10 \times 10^3 = 10 \mu\text{s}$$

$$\frac{V_o}{V_i} = 1 \text{ at } \omega = \omega_0 = \frac{1}{CR} = \frac{1}{10 \times 10^{-6}}$$

$$= 100 \text{ krad/s}$$

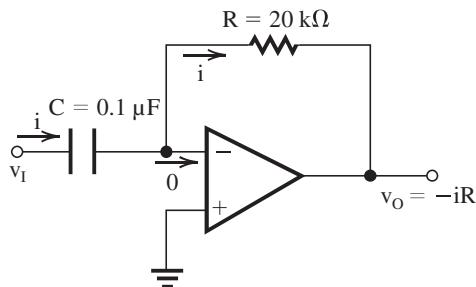
$$\text{or } f_0 = \frac{100}{2\tau} = 15.9 \text{ kHz}$$

$$\left| \frac{V_o}{V_i} \right| = \omega CR = \frac{\omega}{\omega_0} = \frac{f}{f_0}$$

For  $f = 10 f_0$ ;  $\left| \frac{V_o}{V_i} \right| = 10$ , and the output sine wave will have 10-V peak-to-peak amplitude. The  $(-j\omega)$  factor in the transfer function means inversion and  $+90^\circ$  phase shift, thus

$$v_O = -5 \sin(10^6 t + 90^\circ), \text{V}$$

2.83



$$i = C \frac{dv_I}{dt} = 0.1 \times 10^{-6} \times \text{slope}$$

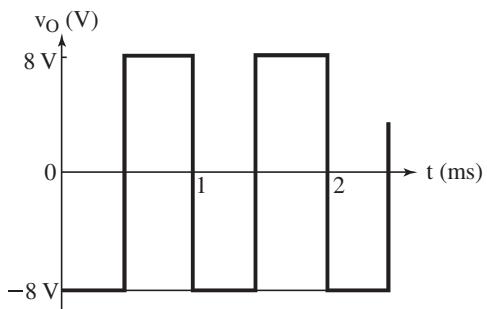
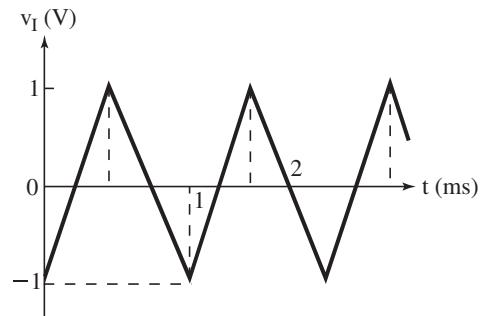
$$= 0.1 \times 10^{-6} \times \frac{2}{0.5 \times 10^{-3}}$$

$$= 0.4 \text{ mA}$$

The peak value of the output square wave is  $= iR$

$$= 0.4 \text{ mA} \times 20 \text{ k}\Omega$$

$$= 8 \text{ V}$$



The output wave has the same frequency as the input signal.

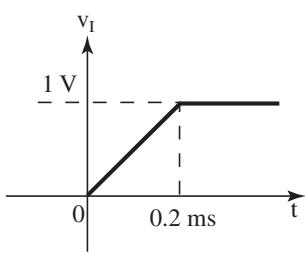
The average value of the output is zero.

To increase the value of the output to 12 V = 8 V × 1.5, the value of R has to be increased by 1.5 times:

$$20 \text{ k}\Omega \times 1.5 = 30 \text{ k}\Omega$$

2.84

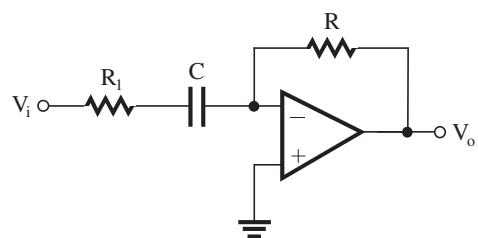
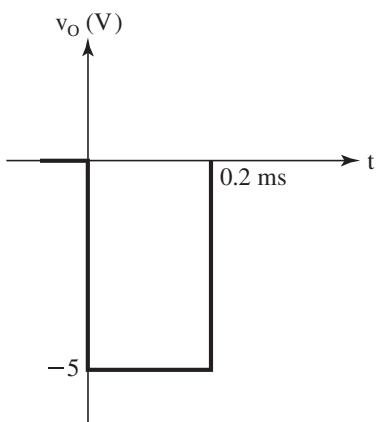
$$\omega_0 \text{ (unity-gain frequency)} = 10^3 \text{ rad/s}$$



$$\text{At } \omega = 0.1 \omega_0, \left| \frac{V_o}{V_i} \right| = 0.1 \text{ V/V}, \phi = 270^\circ$$

$$\text{At } \omega = 10 \omega_0, \left| \frac{V_o}{V_i} \right| = 10 \text{ V/V}, \phi = 270^\circ$$

When a series input resistor  $R_i$  is added as shown, then the high-frequency gain is limited to  $R_2/R_i$ . Thus,



$$R_i = \frac{R_2}{100} = \frac{100 \text{ k}\Omega}{100} = 1 \text{ k}\Omega$$

The circuit now has an STC high-pass response with a lower 3-dB frequency

$$\omega_{3\text{dB}} = \frac{1}{CR_i} = \frac{1}{10^{-8} \times 10^3} = 100 \text{ krad/s}$$

$$\frac{V_o}{V_i} = -\frac{100 \text{ s}}{s + \omega_{3\text{dB}}} = -\frac{100 \text{ s}}{s + 10^5}$$

For  $s = j\omega$ ,

$$\frac{V_o}{V_i} = \frac{-j\omega 100}{j\omega + 10^5} = -j \frac{100}{(10^5/\omega) + j}$$

At  $\omega = 10^4 \text{ rad/s}$ ,

$$\left| \frac{V_o}{V_i} \right| = \frac{100}{\sqrt{100+1}} = \frac{10}{\sqrt{1+0.01}} \approx 9.95 \text{ V/V}$$

$$\phi = 180^\circ + 90^\circ - \tan^{-1} 0.1 = 180^\circ + 84.3^\circ$$

Both these results differ slightly from the ideal values.

$$v_o = -CR \frac{dv_i}{dt}$$

Therefore:

For  $0 \leq t \leq 0.2 \text{ ms}$ :

$$v_o = -1 \text{ ms} \times \frac{1 \text{ V}}{0.2 \text{ ms}} = -5 \text{ V}$$

and  $v_o = 0$  otherwise.

$$2.85 \frac{V_o}{V_i} = -sCR = -10^{-3} \text{ s}$$

$$C = 10 \text{ nF} = 10^{-8} \text{ F}$$

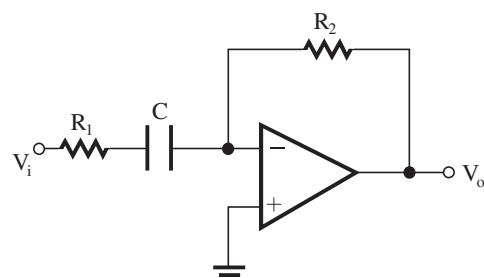
$$R = \frac{10^{-3} \text{ s}}{10^{-8} \text{ F}} = 100 \text{ k}\Omega$$

$$\frac{V_o}{V_i} = -j10^{-3}\omega = -j\frac{\omega}{10^3}$$

$$\left| \frac{V_o}{V_i} \right| = \frac{\omega}{10^3}$$

$$\phi = 180^\circ + 90^\circ \text{ (an inversion + a phase lead of } 90^\circ)$$

2.86



$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_i + \frac{1}{sC}}$$

Thus,

$$\frac{V_o}{V_i} = -\frac{(R_2/R_1)s}{s + \frac{1}{CR_1}}$$

which is that of an STC high-pass type.

$$\text{High-frequency gain } (s \rightarrow \infty) = -\frac{R_2}{R_1}$$

$$3\text{-dB frequency } (\omega_{3\text{dB}}) = \frac{1}{CR_1}$$

For a high-frequency input resistance of  $1\text{ k}\Omega$ , we select  $R_1 = 1\text{ k}\Omega$ . For a high-frequency gain of 40 dB,

$$\frac{R_2}{R_1} = 100 \Rightarrow R_2 = 100\text{ k}\Omega$$

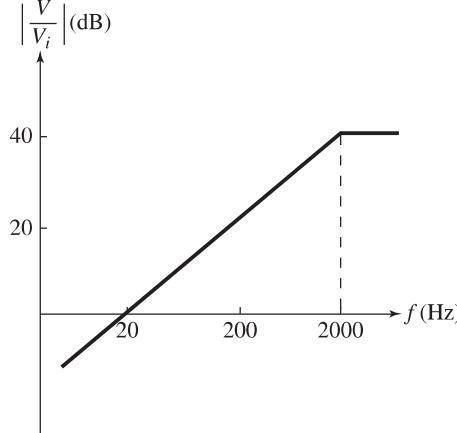
For  $f_{3\text{dB}} = 2\text{ kHz}$ ,

$$\frac{1}{2\pi CR_1} = 2 \times 10^3$$

$$\Rightarrow C = 79\text{ nF}$$

The magnitude of the transfer function reduces from 40 dB to unity (0 dB) in two decades. Thus

$$f \text{ (unity gain)} = \frac{f_{3\text{dB}}}{100} = \frac{2000}{100} = 20\text{ Hz}$$



2.87 Refer to the circuit in Fig. P2.87.

$$\begin{aligned} \frac{V_o}{V_i} &= -\frac{Z_2}{Z_1} = -\frac{1}{Z_1 Y_2} \\ &= -\frac{1}{\left(R_1 + \frac{1}{sC_1}\right)\left(\frac{1}{R_2} + sC_2\right)} \\ &= -\frac{R_2/R_1}{\left(1 + \frac{1}{sC_1 R_1}\right)(1 + sC_2 R_2)} \end{aligned}$$

$$\frac{V_o}{V_i}(j\omega) = -\frac{R_2/R_1}{[1 + (\omega_1/j\omega)][1 + j(\omega/\omega_2)]}$$

where

$$\omega_1 = \frac{1}{C_1 R_1} \quad \text{and} \quad \omega_2 = \frac{1}{C_2 R_2} \quad \text{Q.E.D.}$$

Assuming  $\omega_2 \gg \omega_1$ , then

(a) For  $\omega \ll \omega_1$ ,

$$\frac{V_o}{V_i} \simeq -\frac{R_2/R_1}{1 + (\omega_1/j\omega)}$$

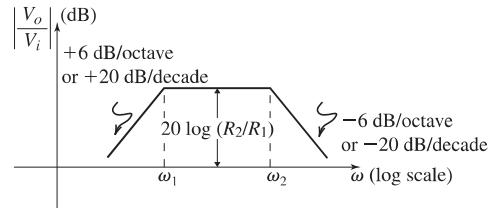
(b) For  $\omega_1 \ll \omega \ll \omega_2$

$$\frac{V_o}{V_i} \simeq -(R_2/R_1)$$

(c) For  $\omega \gg \omega_2$

$$\frac{V_o}{V_i} \simeq -\frac{R_2/R_1}{1 + j(\omega/\omega_2)}$$

The resulting Bode plot will be as shown:



$$\text{Design: Gain of 40 dB} \Rightarrow \frac{R_2}{R_1} = 100$$

$$f_1 = 200\text{ Hz} \Rightarrow \frac{1}{2\pi C_1 R_1} = 200$$

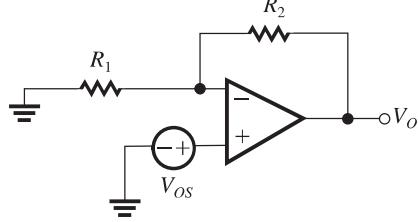
$$f_2 = 200\text{ kHz} \Rightarrow \frac{1}{2\pi C_2 R_2} = 200 \times 10^3$$

Input resistance (at  $\omega \gg \omega_1$ ) =  $2\text{ k}\Omega$

$$\Rightarrow R_1 = 2\text{ k}\Omega$$

Thus,  $R_1 = 2\text{ k}\Omega$ ,  $R_2 = 200\text{ k}\Omega$ ,  $C_1 \simeq 0.4\text{ }\mu\text{F}$ , and  $C_2 \simeq 4\text{ pF}$ .

2.88 Inverting configuration:



$$V_o = V_{OS} \left( 1 + \frac{R_2}{R_1} \right)$$

$$-0.2 = V_{OS} \left( 1 + \frac{R_2}{R_1} \right)$$

$$= V_{OS} \left( 1 + \frac{100}{2} \right)$$

$$\Rightarrow V_{OS} \simeq 4 \text{ mV}$$

 2.89  $V_{OS} = \pm 2 \text{ mV}$ 

$$\begin{aligned} V_o &= 0.01 \sin \omega t \times 100 + V_{OS} \times 100 \\ &= 1 \sin \omega t \pm 0.2 \text{ V} \end{aligned}$$

2.90 Input offset voltage = 3 mV

$$\begin{aligned} \text{Output dc offset voltage} \\ &= 3 \text{ mV} \times \text{closed loop gain} \end{aligned}$$

$$= 3 \text{ mV} \times 1000$$

$$= 3 \text{ V}$$

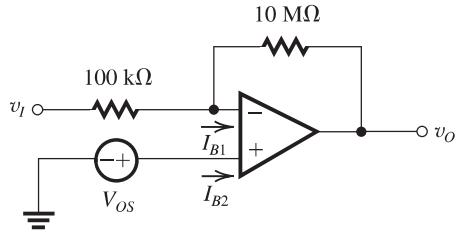
The maximum amplitude of an input sinusoid that results in an output peak amplitude of  $12 - 3 = 9 \text{ V}$  is given by

$$v_i = \frac{9}{1000} = 9 \text{ mV}$$

If amplifier is capacitively coupled then

$$v_{i \text{ max}} = \frac{12}{1000} = 12 \text{ mV}$$

2.91



$$(a) I_B = (I_{B1} + I_{B2})/2$$

Open input:

$$v_o = v_+ + R_2 I_{B1} = V_{OS} + R_2 I_{B1}$$

$$5.3 = V_{OS} + 10,000 I_{B1} \quad (1)$$

Input connected to ground:

$$v_o = v_+ + R_2 \left( I_{B1} + \frac{V_{OS}}{R_1} \right)$$

$$= V_{OS} \left( 1 + \frac{R_2}{R_1} \right) + R_2 I_{B1}$$

$$5 = V_{OS} \times 101 + 10,000 I_{B1} \quad (2)$$

Equations (1), (2)

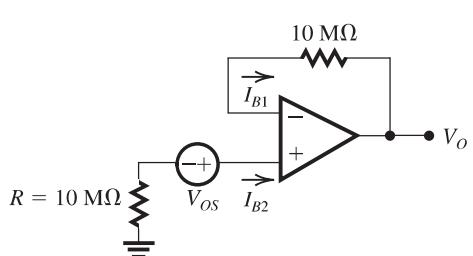
$$\Rightarrow 100V_{OS} = -0.3 \Rightarrow V_{OS} = -3 \text{ mV}$$

$$\Rightarrow I_{B1} = 530 \text{ nA}$$

$$I_B \simeq I_{B1} = 530 \text{ nA}$$

and both flow into the op-amp input terminals.

$$(b) V_{OS} = -3 \text{ mV}$$



(c) In this case, Since  $R$  is very large, we may ignore  $V_{OS}$  compared to the voltage drop across  $R$ .

$V_{OS} \ll RI_B$ , Also Eq. (2.46) holds:

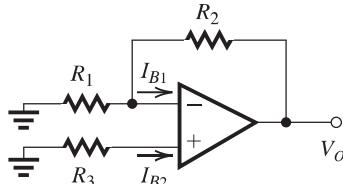
$$R_3 = R_1 \parallel R_2$$

therefore from Eq. (2.40):

$$V_O = I_{OS} \times R_2 \Rightarrow I_{OS} = \frac{-0.6}{10 \text{ M}\Omega}$$

$$I_{OS} = -60 \text{ nA}$$

2.92



$$R_2 = 100 \text{ k}\Omega$$

$$R_1 = \frac{100 \text{ k}\Omega}{9}$$

$$R_3 = 5 \text{ k}\Omega$$

$$I_{B1} = 2 \pm 0.1, \mu\text{A}, V_{OS} = 0$$

$$I_{B2} = 2 \pm 0.1, \mu\text{A}$$

From Eq. (2.38):

$$V_O = -I_{B2}R_3 + R_2 \left( I_{B1} - I_{B2} \frac{R_3}{R_1} \right)$$

Thus,

$$V_O = I_{B1}R_2 - I_{B2}R_3 \left( 1 + \frac{R_2}{R_1} \right) \quad (1)$$

The maximum value of  $V_O$  is obtained when  $I_{B1} = 2.1 \mu\text{A}$  and  $I_{B2} = 1.9 \mu\text{A}$ ,

$$V_{Omax} = 2.1 \times 100 - 1.9 \times 5 \left( 1 + \frac{100}{100/9} \right)$$

$$= 210 - 95 = 115 \text{ mV}$$

The minimum value of  $V_O$  is obtained when  $I_{B1} = 1.9 \mu\text{A}$  and  $I_{B2} = 2.1 \mu\text{A}$ ,

$$V_{Omin} = 1.9 \times 100 - 2.1 \times 5 \times 10$$

$$= 190 - 105 = 85 \text{ mV}$$

Thus the dc offset at the output will be in the range of 85 mV to 115 mV. The bulk of the dc offset at the output, that due to  $I_B$ , can be reduced to zero by making the dc resistances seen by the two input terminals equal. Currently, the positive input terminal sees a resistance  $R_3 = 5 \text{ k}\Omega$  and the negative input terminal sees a resistance equal to  $R_1 \parallel R_2 = \frac{100}{9} \parallel 100 = 10 \text{ k}\Omega$ . Thus the two resistances can be made equal by connecting a

5-k $\Omega$  resistance in series with  $R_3$ . The resulting dc offset voltage at the output will be

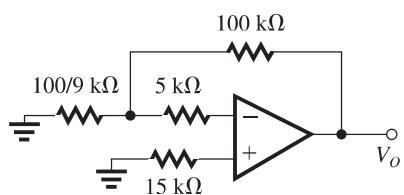
$$V_O = I_{OS}R_2 = 0.2 \times 100 = 20 \text{ mV}$$

Since  $I_{OS}$  can be of either polarity,

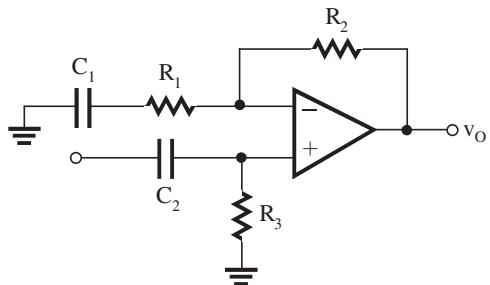
$$V_O = \pm 20 \text{ mV}$$

The same result could have been found by replacing  $R_3$  in Eq. (1) by  $(R_3 + R_4)$  where  $R_4 = 5 \text{ k}\Omega$ .

If the signal source resistance is 15 k $\Omega$ , then the resistances can be equalized by adding a 5-k $\Omega$  resistor in series with the negative input lead of the op amp.



2.93



$$R_2 = R_3 = 100 \text{ k}\Omega$$

$$1 + \frac{R_2}{R_1} = 100$$

$$R_1 = \frac{100 \text{ k}\Omega}{99} = 1.01 \text{ k}\Omega$$

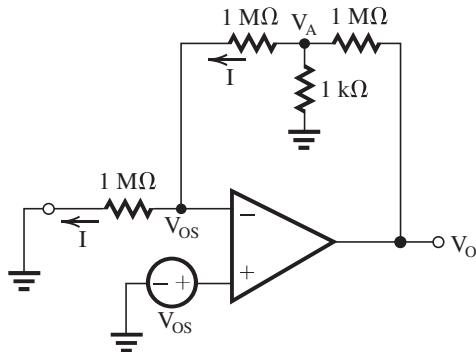
$$\frac{1}{R_1 C_1} = 2\pi \times 100 \Rightarrow C_1 = \frac{1}{1.01 \times 2\pi \times 10^5}$$

$$= 1.58 \mu\text{F}$$

$$\frac{1}{R_3 C_2} = 2\pi \times 10 \Rightarrow C_2 = \frac{1}{2\pi \times 10^6}$$

$$= 0.16 \mu\text{F}$$

2.94



$$v_- = v_+ = V_{OS}$$

$$V_A = 2V_{OS} = 6 \text{ mV}$$

$$I = \frac{V_{OS}}{1 \text{ M}\Omega} = \frac{6 \text{ mV}}{1 \text{ M}\Omega} = 6 \text{ nA}$$

$$V_O = V_A + 1 \text{ M}\Omega \times \left( I + \frac{V_A}{1 \text{ k}\Omega} \right)$$

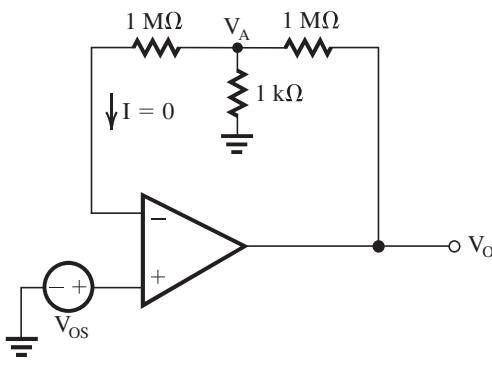
$$= 2V_{OS} + 1 \text{ M}\Omega \times \left( \frac{V_{OS}}{1 \text{ M}\Omega} + \frac{2V_{OS}}{1 \text{ k}\Omega} \right)$$

$$= 2003 V_{OS}$$

$$= 2003 \times 3 \text{ mV}$$

$$\approx 6 \text{ V}$$

For capacitively coupled input,



$$v_+ = v_- = V_{OS}$$

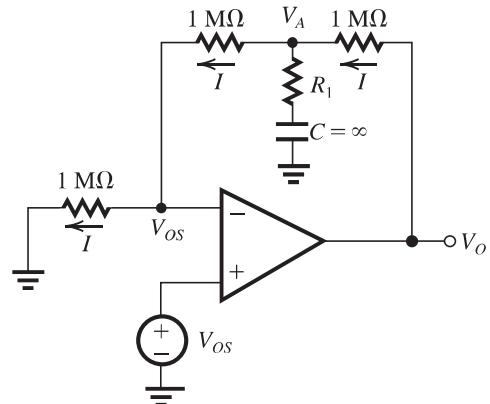
$$I = 0 \quad V_A = V_{OS}$$

$$V_O = V_A + 1 \text{ M}\Omega \times \frac{V_{OS}}{1 \text{ k}\Omega} = V_{OS} + 1000V_{OS}$$

$$= 1001V_{OS}$$

$$= 1001 \times 3 \text{ mV} \approx 3 \text{ V}$$

A large capacitor placed in series with the 1-kΩ resistor results in



$$v_+ = v_- = V_{OS}$$

No dc current flows through R<sub>1</sub>, C branch

$$\therefore V_O = V_A + V_{OS}$$

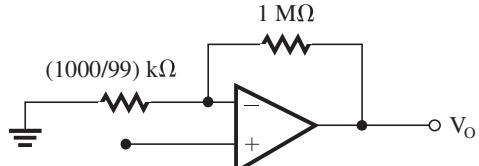
$$= 2V_{OS} + V_{OS}$$

$$= 3V_{OS}$$

$$= 3 \times 3 \text{ mV}$$

$$= 9 \text{ mV}$$

2.95



$$100 = 1 + \frac{R_2}{R_1} \Rightarrow R_1 = 10.1 \text{ k}\Omega$$

$$(a) V_O = 200 \times 10^{-9} \times 1 \times 10^6 = 0.2 \text{ V}$$

(b) Largest output offset is

$$V_O = 2 \text{ mV} \times 100 + 0.2 \text{ V} = 400 \text{ mV} = 0.4 \text{ V}$$

(c) For bias current compensation, we connect a resistor  $R_3$  in series with the positive input terminal of the op amp, with  $R_3 = R_1 \parallel R_2$ ,

$$R_3 = 10 \text{ k}\Omega \parallel 1 \text{ M}\Omega \simeq 10 \text{ k}\Omega$$

$$I_{OS} = \frac{200}{10} = 20 \text{ nA}$$

The offset current alone will result in an output offset voltage of

$$I_{OS} \times R_2 = 20 \times 10^{-9} \times 1 \times 10^6 = 20 \text{ mV}$$

$$(d) V_O = 200 \text{ mV} + 20 \text{ mV} = 220 \text{ mV} = 0.22 \text{ V}$$

2.96 At 0°C, we expect

$$\pm 20 \times 25 \times 1000 \mu\text{V} = \pm 500 \text{ mV} = \pm 0.5 \text{ V}$$

At 75°C, we expect

$$\pm 20 \times 50 \times 1000 \mu\text{V} = \pm 1 \text{ V}$$

We expect these quantities to have opposite polarities.

$$2.97 R_3 = R_1 \parallel R_2 = 10 \text{ k}\Omega \parallel 1 \text{ M}\Omega = 9.09 \text{ k}\Omega$$

Now, with the input grounded and assuming  $V_{OS} = 0$ , the measured +0.3-V at the output is entirely due to  $I_{OS}$ , that is,

$$0.3 = I_{OS} R_2 = I_{OS} \times 1 \text{ M}\Omega$$

Thus,

$$I_{OS} = 0.3 \mu\text{A}$$

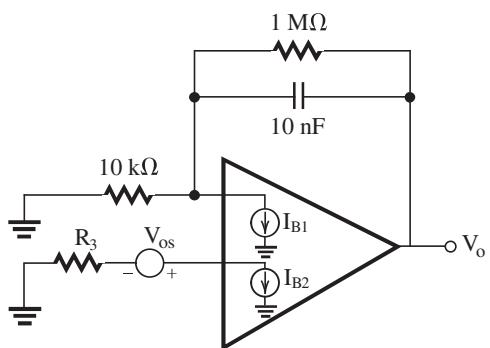
If  $V_{OS} = \pm 1 \text{ mV}$ , then it alone will result in an output voltage of

$$V_{OS} \left( 1 + \frac{R_2}{R_1} \right) = V_{OS} \times 101 = \pm 101 \text{ mV} \text{ or } \simeq \pm 0.1 \text{ V}$$

If  $V_{OS}$  is positive, 0.1 V of the output 0.3-V offset will be due to  $V_{OS}$ , leaving 0.2 V as the result of  $I_{OS}$ ; thus in this case,  $I_{OS} = \frac{0.2 \text{ V}}{1 \text{ M}\Omega} = 0.2 \mu\text{A}$ . On the other hand, if  $V_{OS}$  is negative, then -0.1 V of the output 0.3 V is due to  $V_{OS}$ , with the result that  $I_{OS}$  must be causing 0.4 V of output offset. In this

case,  $I_{OS} = \frac{0.4 \text{ V}}{1 \text{ M}\Omega} = 0.4 \mu\text{A}$ . Thus, the possible range of  $I_{OS}$  is 0.2 μA to 0.4 μA.

2.98



$$(a) R_3 = R \parallel R_F = 10 \text{ k}\Omega \parallel 1 \text{ M}\Omega$$

$$\Rightarrow R_3 = 9.9 \text{ k}\Omega$$

(b) As discussed in Section 2.8.2, the dc output voltage of the integrator when the input is

$$\text{grounded is } V_O = V_{OS} \left( 1 + \frac{R_F}{R} \right) + I_{OS} R_F$$

$$V_O = 2 \text{ mV} \left( 1 + \frac{1 \text{ M}\Omega}{10 \text{ k}\Omega} \right) + 20 \text{ nA} \times 1 \text{ M}\Omega$$

$$= 0.202 \text{ V} + 0.02 \text{ V}$$

$$V_O = 0.222 \text{ V}$$

$$2.99 f_t = A_0 f_b$$

$A_0$	$f_b$ (Hz)	$f_t$ (Hz)
$10^5$	$10^2$	$10^7$
$10^6$	1	$10^6$
$10^5$	$10^3$	$10^8$
$10^7$	$10^{-1}$	$10^6$
$2 \times 10^5$	10	$2 \times 10^6$

2.100 At very low frequencies, the gain is  $A_0$ , thus

$$20 \log A_0 = 98 \text{ dB} \Rightarrow A_0 \simeq 80,000 \text{ V/V}$$

At  $f = 100$  kHz, the gain is 40 dB or 100 V/V.  
Thus

$$f_t = 100 \text{ kHz} \times 100 = 10 \text{ MHz}$$

$$\text{Since } f_t = A_0 f_b \Rightarrow f_b = \frac{10 \text{ MHz}}{80,000} = 125 \text{ Hz.}$$

$$2.101 \quad f = 10 \text{ kHz} \quad |A| = 20 \times 10^3$$

$$f = 100 \text{ kHz} \quad |A| = 4 \times 10^3$$

Thus, a change of a decade in  $f$  does not result in a factor of 10 reduction in gain; in fact, the gain reduces by only a factor of 5. It follows that the first frequency (10 kHz) is less than  $f_b$ . Therefore, we must use the exact expression for  $|A|$ , that is,

$$|A| = \frac{A_0}{\sqrt{1 + (f/f_b)^2}}$$

Substituting the given data, we obtain

$$\frac{A_0}{\sqrt{1 + (10/f_b)^2}} = 20 \times 10^3 \quad (1)$$

$$\frac{A_0}{\sqrt{1 + (100/f_b)^2}} = 4 \times 10^3 \quad (2)$$

Dividing Eq. (1) by Eq. (2), we have

$$\sqrt{\frac{1 + (100/f_b)^2}{1 + (10/f_b)^2}} = 5$$

$$\Rightarrow 1 + \frac{100^2}{f_b^2} = 25 \left( 1 + \frac{10^2}{f_b^2} \right) = 25 + \frac{2500}{f_b^2}$$

$$\frac{10,000}{f_b^2} - \frac{2500}{f_b^2} = 24$$

$$f_b = \sqrt{\frac{7500}{24}} = 17.68 \text{ kHz}$$

Now, substituting in Eq. (1) yields

$$A_0 = 20 \times 10^3 \sqrt{1 + \left( \frac{10}{17.68} \right)^2} = 22,976 \text{ V/V}$$

and the unity-gain frequency is

$$\begin{aligned} f_t &= A_0 f_b = 22.976 \times 10^3 \times 17.68 \times 10^3 \\ &= 406.2 \text{ MHz} \end{aligned}$$

2.102 The gain drops by 20 dB at  $f \simeq 10f_b$ . Thus

$$(a) \quad A_0 = 2 \times 10^5 \text{ V/V}$$

$$f_b = \frac{5 \times 10^2}{10} = 50 \text{ Hz}$$

$$f_t = A_0 f_b = 2 \times 10^5 \times 50 = 10^7 \text{ Hz} = 10 \text{ MHz}$$

$$(b) \quad A_0 = 20 \times 10^5 \text{ V/V}$$

$$f_b = \frac{10}{10} = 1 \text{ Hz}$$

$$f_t = A_0 f_b = 20 \times 10^5 \times 1 = 2 \text{ MHz}$$

$$(c) \quad A_0 = 1800 \text{ V/V}$$

$$f_b = \frac{0.1 \text{ MHz}}{10} = 10 \text{ kHz}$$

$$f_t = A_0 f_b = 1800 \times 10 = 18 \text{ MHz}$$

$$(d) \quad A_0 = 100 \text{ V/V}$$

$$f_b = \frac{0.1 \text{ GHz}}{10} = 10 \text{ MHz}$$

$$f_t = A_0 f_b = 100 \times 10 = 1 \text{ GHz}$$

$$(e) \quad A_0 = 25 \text{ V/mV} = 25 \times 10^3 \text{ V/V}$$

$$f_b = \frac{250}{10} = 25 \text{ kHz}$$

$$f_t = A_0 f_b = 25 \times 10^3 \times 25 \times 10^3 = 625 \text{ MHz}$$

$$2.103 \quad G_{\text{nominal}} = -50 \Rightarrow \frac{R_2}{R_1} = 50$$

$$A_0 = 10^4$$

$$f_t = 10^6 \text{ Hz}$$

$$f_{3\text{dB}} \text{ of closed-loop amplifier} = \frac{f_t}{1 + \frac{R_2}{R_1}}$$

$$= \frac{10^6}{51} = 19.61 \text{ kHz}$$

$$G = -\frac{50}{1 + j \frac{f}{f_{3\text{dB}}}}$$

$$|G| = \frac{50}{\sqrt{1 + (f/f_{3dB})^2}}$$

$$\text{For } f = 0.1 f_{3dB}, |G| = \frac{50}{\sqrt{1.01}} = 49.75 \text{ V/V}$$

$$\text{For } f = 10 f_{3dB}, |G| = \frac{50}{\sqrt{1 + 100}} = 4.975 \text{ V/V}$$

which is a 20-dB reduction.

2.104  $f_t = 20 \text{ MHz}$  and closed-loop gain

$$1 + \frac{R_2}{R_1} = 100 \text{ V/V}$$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{20 \text{ MHz}}{100} = 200 \text{ kHz}$$

$$G(j\omega) = \frac{100}{1 + j \frac{f}{f_{3dB}}}$$

$$\Rightarrow \phi = -\tan^{-1} \frac{f}{f_{3dB}}$$

For  $\phi = -6^\circ$

$$f = f_{3dB} \times \tan 6^\circ = 21 \text{ kHz}$$

$$\phi = 84^\circ, f = f_{3dB} \times \tan 84^\circ = 1.9 \text{ MHz}$$

$$2.105 \text{ (a) } G = -50 \text{ V/V} \Rightarrow \frac{R_2}{R_1} = 50$$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{51}$$

$$\text{For } f_{3dB} = 100 \text{ kHz} \Rightarrow f_t = 100 \times 51 = 5.1 \text{ MHz}$$

$$\text{(b) } G = +50 \text{ V/V} \Rightarrow 1 + \frac{R_2}{R_1} = 50$$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{50}$$

$$\text{For } f_{3dB} = 100 \text{ kHz}, f_t = 5 \text{ MHz}$$

$$\text{(c) } G = +2 \text{ V/V} \Rightarrow 1 + \frac{R_2}{R_1} = 2$$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{2}$$

$$\text{For } f_{3dB} = 5 \text{ MHz}, f_t = 10 \text{ MHz}$$

$$\text{(d) } G = -2 \text{ V/V} \Rightarrow \frac{R_2}{R_1} = 2$$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{3}$$

$$\text{For } f_{3dB} = 5 \text{ MHz}, f_t = 15 \text{ MHz}$$

$$\text{(e) } G = -1000 \text{ V/V} \Rightarrow \frac{R_2}{R_1} = 1000$$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{1001}$$

$$\text{For } f_{3dB} = 10 \text{ kHz}, f_t = 10 \times 1001 = 10.1 \text{ MHz}$$

$$\text{(f) } G = +1 \text{ V/V} \Rightarrow \frac{R_2}{R_1} = 0$$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = f_t$$

$$\text{For } f_{3dB} = 1 \text{ MHz}, f_t = 1 \text{ MHz}$$

$$\text{(g) } G = -1 \text{ V/V} \Rightarrow \frac{R_2}{R_1} = 1$$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{2}$$

$$\text{For } f_{3dB} = 1 \text{ MHz}, f_t = 2 \text{ MHz}$$

$$2.106 \text{ } f_{3dB} = f_t = 1 \text{ MHz}$$

$$|G| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^2}} = \frac{1}{\sqrt{1 + f^2}} \text{ with}$$

$f$  in MHz

$$|G| = 0.99 \Rightarrow f = 0.142 \text{ MHz}$$

The follower behaves like a low-pass STC circuit  
with a time constant  $\tau = \frac{1}{2\pi \times 10^6} = \frac{1}{2\pi} \mu\text{s}$

$$t_r = 2.2\tau = 0.35 \mu\text{s.}$$

$$2.107 \cdot 1 + \frac{R_2}{R_1} = 10 \Rightarrow R_1 = 1 \text{ k}\Omega \text{ and } R_2 = 9 \text{ k}\Omega$$

$$1 + \left( \frac{f_{3dB}}{f_l} \right)^2 = \sqrt{2}$$

When a 100-mV (i.e., 0.1-V) step is applied at the input, the output will be

$$v_o = 0.1 \times 10(1 - e^{-t/\tau}), \text{ V}$$

where

$$\tau = \frac{1}{\omega_{3dB}}$$

$v_o$  reaches 1% of the 1-V final value at time  $t$ ,

$$1 - e^{-t/\tau} = 0.99$$

$$e^{-t/\tau} = 0.01$$

$$t = 4.6\tau$$

For  $t$  to be 200 ns,

$$\tau = \frac{200}{4.6} = 43.49 \text{ ns}$$

Thus we require a closed-loop 3-dB frequency

$$\omega_{3dB} = \frac{1}{\tau} \text{ or}$$

$$f_{3dB} = \frac{1}{2\pi\tau} = \frac{1}{2\pi \times 43.49 \times 10^{-9}} = 3.66 \text{ MHz}$$

Correspondingly, the op amp must have an  $f_l$  of

$$f_l = f_{3dB} \left( 1 + \frac{R_2}{R_1} \right) = 36.6 \text{ MHz}$$

2.108 (a) Assume two identical stages, each with a gain function:

$$G = \frac{G_0}{1 + j\frac{\omega}{\omega_1}} = \frac{G_0}{1 + j\frac{f}{f_l}}$$

$$G = \frac{G_0}{\sqrt{1 + \left(\frac{f}{f_l}\right)^2}}$$

$$\text{overall gain of the cascade is } \frac{G_0^2}{1 + \left(\frac{f}{f_l}\right)^2}$$

The gain will drop by 3 dB when

$$f_{3dB} = f_l \sqrt{\sqrt{2} - 1} \quad \text{Q.E.D}$$

$$(b) 40 \text{ dB} = 20 \log G_0 \Rightarrow G_0 = 100 = 1 + \frac{R_2}{R_1}$$

$$f_{3dB} = \frac{f_l}{1 + \frac{R_2}{R_1}} = \frac{2 \text{ MHz}}{100} = 20 \text{ kHz}$$

$$(c) \text{ Each stage should have 20-dB gain or } 1 + \frac{R_2}{R_1} = 10 \text{ and therefore a 3-dB frequency of}$$

$$f_l = \frac{2 \times 10^6}{10} = 2 \times 10^5 \text{ Hz}$$

$$\text{The overall } f_{3dB} = 2 \times 10^5 \sqrt{\sqrt{2} - 1}$$

$$= 128.7 \text{ kHz},$$

which is 6.4 times greater than the bandwidth achieved using a single op amp, as in case (b) above.

2.109  $f_l = 100 \times 5 = 500 \text{ MHz}$  if a single op amp is used.

With an op amp that has only  $f_l = 40 \text{ MHz}$ , the possible closed-loop gain at 5 MHz is

$$|A| = \frac{40}{5} = 8 \text{ V/V}$$

To obtain an overall gain of 100 would require three such amplifiers, cascaded. Now, if each of the stages has a low-frequency (dc) closed-loop gain  $K$ , then its 3-dB frequency will be  $\frac{40}{K} \text{ MHz}$ .

Thus for each stage the closed-loop gain is:

$$|G| = \frac{K}{\sqrt{1 + \left(\frac{f}{40}\right)^2}}$$

which at  $f = 5 \text{ MHz}$  becomes

$$|G_{5\text{MHz}}| = \frac{K}{\sqrt{1 + \left(\frac{K}{8}\right)^2}}$$

For overall gain of 100:  $100 = \left[ \frac{K}{\sqrt{1 + \left( \frac{K}{8} \right)^2}} \right]^3$

$$\Rightarrow K = 5.7$$

Thus for each cascade stage:  $f_{3dB} = \frac{40}{5.7}$

$$f_{3dB} = 7 \text{ MHz}$$

The 3-dB frequency of the overall amplifier,  $f_t$ , can be calculated as

$$\left[ \frac{5.7}{\sqrt{1 + \left( \frac{f_t}{7} \right)^2}} \right]^3 = \frac{(5.7)^3}{\sqrt{2}} \Rightarrow f_t = 3.6 \text{ MHz}$$

2.110 (a)  $\frac{R_2}{R_1} = K, \quad f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{1 + K}$

$$\text{GBP} = \text{Gain} \times f_{3dB}$$

$$\text{GBP} = K \frac{f_t}{1 + K} = \frac{K}{K + 1} f_t$$

(b)  $1 + \frac{R_2}{R_1} = K, \quad f_{3dB} = \frac{f_t}{K}$

$$\text{GBP} = K \frac{f_t}{K} = f_t$$

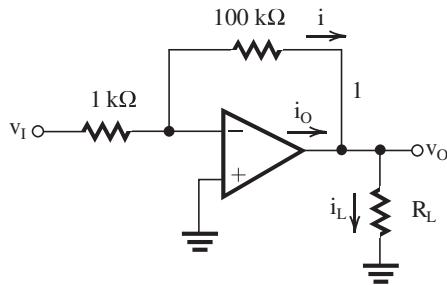
For the same closed-loop gain, the noninverting configuration realizes a higher GBP, and it is independent of the closed-loop gain and equal to  $f_t$  of the op amp.

2.111 The peak value of the largest possible sine wave that can be applied at the input without

output clipping is  $\frac{\pm 14 \text{ V}}{100} = 0.14 \text{ V} = 140 \text{ mV}$ .

Thus the rms value  $= \frac{140}{\sqrt{2}} \simeq 100 \text{ mV}$

2.112



(a)  $R_L = 1 \text{ k}\Omega$

for  $v_{O\max} = 10 \text{ V}$ :  $V_p = \frac{10}{100}$

$$V_p = 0.1 \text{ V}$$

When the output is at its peak,

$$i_L = \frac{10}{1 \text{ k}\Omega} = 10 \text{ mA}$$

$$i = \frac{-10}{100 \text{ k}\Omega} = -0.1 \text{ mA}; \text{ therefore}$$

$i_O = 10 + 0.1 = 10.1 \text{ mA}$  is well under  $i_{O\max} = 20 \text{ mA}$ .

At the negative peak of the output voltage,  $v_O = -10 \text{ V}$ ,  $i_L = -10 \text{ mA}$ ,  $i = 0.1 \text{ mA}$ , and  $i_O = -10.1 \text{ mA}$ , again well under the 20-mA maximum allowed.

(b)  $R_L = 200 \Omega$

If output is at its peak:  $i_L = \frac{10 \text{ V}}{0.2} = 50 \text{ mA}$

which exceeds  $i_{O\max} = 20 \text{ mA}$ . Therefore  $v_O$  cannot go as high as 10 V. Instead:

$$20 \text{ mA} = \frac{v_O}{200 \Omega} + \frac{v_O}{100 \text{ k}\Omega} \Rightarrow v_O = \frac{20}{5.01} \simeq 4 \text{ V}$$

$$v_p = \frac{4}{100} = 0.04 \text{ V} = 40 \text{ mV}$$

(c)  $R_L = ?, i_{O\max} = 20 \text{ mA} = \frac{10 \text{ V}}{R_{L\min}} + \frac{10 \text{ V}}{100 \text{ k}\Omega}$

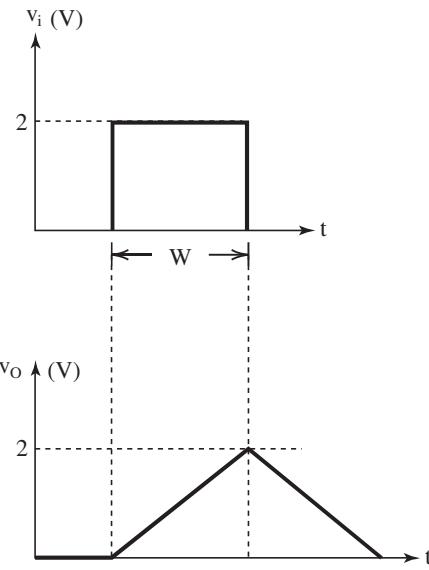
$$20 - 0.1 = \frac{10}{R_{L\min}} \Rightarrow R_{L\min} = 502 \Omega$$

2.113 Op-amp slew rate = 10 V/μs.

For the input pulse to rise 2 V, it will take  $\frac{2}{10} = 0.2 \mu\text{s}$ .

$\therefore$  The minimum pulse width =  $W = 0.2 \mu s$

The output will be a triangular with 2-V peak and  $10 V/\mu s$  slopes.



$$2.114 \text{ Slope of the triangle wave} = \frac{10 \text{ V}}{\text{T}/2} = \text{SR}$$

$$\text{Thus } \frac{10}{\text{T}} \times 2 = 20 \text{ V}/\mu s$$

$$\Rightarrow \text{T} = 1 \mu s \text{ or } f = \frac{1}{\text{T}} = 1 \text{ MHz}$$

For a sine wave  $v_o = \hat{V}_o \sin(2\pi \times 1 \times 10^6 t)$

$$\left. \frac{dv_o}{dt} \right|_{\max} = 2\pi \times 1 \times 10^6 \hat{V}_o = \text{SR}$$

$$\Rightarrow \hat{V}_o = \frac{20 \times 10^6}{2\pi \times 10^6 \times 1} = 3.18 \text{ V}$$

$$2.115 \ v_o = 10 \ \sin \omega t \Rightarrow \frac{dv_o}{dt}$$

$$= 10\omega \cos \omega t \Rightarrow \left. \frac{dv_o}{dt} \right|_{\max}$$

$$= 10\omega$$

The highest frequency at which this output is possible is that for which

$$\begin{aligned} \left. \frac{dv_o}{dt} \right|_{\max} &= \text{SR} \Rightarrow 10\omega_{\max} = 40 \times 10^6 \Rightarrow \omega_{\max} \\ &= 4 \times 10^6 \text{ rad/s} \\ \Rightarrow f_{\max} &= 637 \text{ kHz} \end{aligned}$$

$$2.116 \text{ (a)} \ V_i = 0.5, V_o = 10 \times 0.5 = 5 \text{ V}$$

Output distortion will be due to slew-rate limitation and will occur at the frequency for which  $\left. \frac{dv_o}{dt} \right|_{\max} = \text{SR}$

$$\omega_{\max} \times 5 = 10 \times 10^6$$

$$\omega_{\max} = 2 \times 10^6 \text{ rad/s and } f_{\max} = 318.3 \text{ kHz}$$

(b) The output will distort at the value of  $V_i$  that results in  $\left. \frac{dv_o}{dt} \right|_{\max} = \text{SR}$ .

$$\omega \times 10V_{i\max} = \text{SR}$$

$$V_{i\max} = \frac{10 \times 10^6}{2\pi \times 200 \times 10^3 \times 10} = 0.795 \text{ V}$$

$$(c) \ V_i = 50 \text{ mV} \quad V_o = 500 \text{ mV} = 0.5 \text{ V}$$

Slew rate begins at the frequency for which  $\omega \times 0.5 = \text{SR}$

$$\Rightarrow f = \frac{10 \times 10^6}{2\pi \times 0.5} = 3.18 \text{ MHz}$$

However, the small-signal 3-dB frequency is

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{20 \times 10^6}{10} = 2 \text{ MHz}$$

Thus the useful frequency range is limited to 2 MHz.

(d) For  $f = 50 \text{ kHz}$ , the slew-rate limitation occurs at the value of  $V_i$  given by

$$\begin{aligned} \omega_i \times 10V_i &= \text{SR} \Rightarrow V_i = \frac{10 \times 10^6}{2\pi \times 50 \times 10^3 \times 10} \\ &= 3.18 \text{ V} \end{aligned}$$

Such an input voltage, however, would ideally result in an output of 31.8 V, which exceeds  $V_{O\max}$ .

Thus  $V_{i\max} = \frac{V_{O\max}}{10} = 1 \text{ V peak.}$

### Exercise 3-1

Ex: 3.1 Refer to Fig. 3.3(a). For  $v_I \geq 0$ , the diode conducts and presents a zero voltage drop. Thus  $v_O = v_I$ . For  $v_I < 0$ , the diode is cut off, zero current flows through  $R$ , and  $v_O = 0$ . The result is the transfer characteristic in Fig. E3.1.

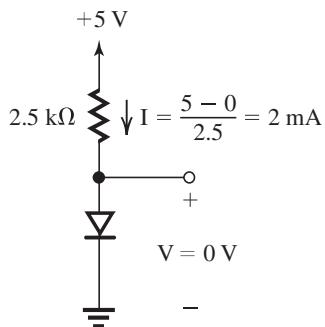
Ex: 3.2 See Fig. 3.3a and 3.3b. During the positive half of the sinusoid, the diode is forward biased, so it conducts resulting in  $v_D = 0$ . During the negative half cycle of the input signal  $v_I$ , the diode is reverse biased. The diode does not conduct, resulting in no current flowing in the circuit. So  $v_O = 0$  and  $v_D = v_I - v_O = v_I$ . This results in the waveform shown in Fig. E3.2.

$$\text{Ex: 3.3 } \hat{i}_D = \frac{\hat{v}_I}{R} = \frac{10 \text{ V}}{1 \text{ k}\Omega} = 10 \text{ mA}$$

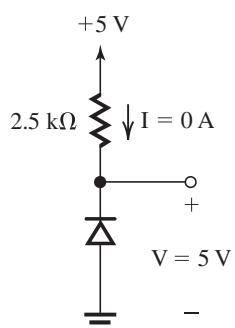
$$\begin{aligned} \text{dc component of } v_O &= \frac{1}{\pi} \hat{v}_O \\ &= \frac{1}{\pi} \hat{v}_I = \frac{10}{\pi} \\ &= 3.18 \text{ V} \end{aligned}$$

Ex: 3.4

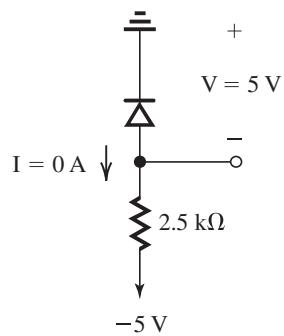
(a)



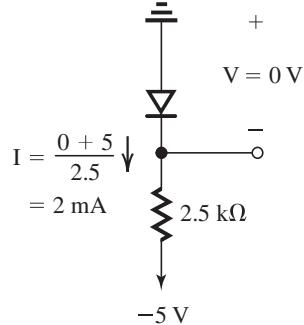
(b)



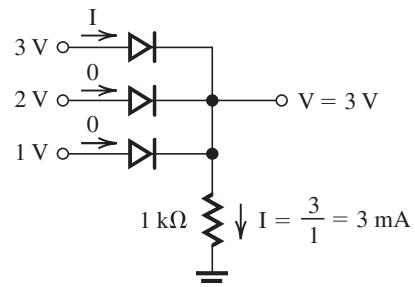
(c)



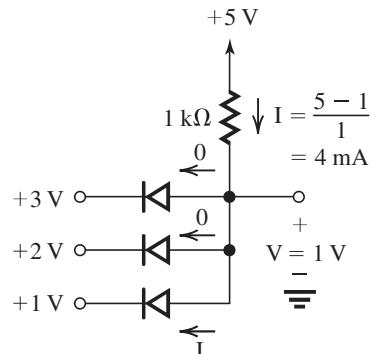
(d)



(e)



(f)



$$\text{Ex: 3.5 } V_{\text{avg}} = \frac{10}{\pi}$$

$$50 + R = \frac{10}{1 \text{ mA}} = \frac{10}{\pi} \text{ k}\Omega$$

$$\therefore R = 3.133 \text{ k}\Omega$$

### Exercise 3-2

Ex: 3.6

$$V_2 - V_1 = 2.3 V_T \log\left(\frac{I_2}{I_1}\right)$$

At room temperature  $V_T = 25 \text{ mV}$

$$\begin{aligned} V_2 - V_1 &= 2.3 \times 25 \times 10^{-3} \times \log\left(\frac{10}{0.1}\right) \\ &= 115 \text{ mV} \end{aligned}$$

$$\text{Ex: 3.7 } i = I_S e^{V/V_T}$$

$$1 \text{ (mA)} = I_S e^{0.7/V_T}$$

Dividing (1) by (2), we obtain

$$i \text{ (mA)} = e^{(v-0.7)/V_T}$$

$$\Rightarrow v = 0.7 + 0.025 \ln(i)$$

where  $i$  is in mA. Thus,

for  $i = 0.1 \text{ mA}$ ,

$$v = 0.7 + 0.025 \ln(0.1) = 0.64 \text{ V}$$

and for  $i = 10 \text{ mA}$ ,

$$v = 0.7 + 0.025 \ln(10) = 0.76 \text{ V}$$

Ex: 3.8  $\Delta T = 125 - 25 = 100^\circ\text{C}$

$$I_S = 10^{-14} \times 1.15^{\Delta T}$$

$$= 1.17 \times 10^{-8} \text{ A}$$

$$\text{Ex: 3.9 At } 20^\circ\text{C } I = \frac{1 \text{ V}}{1 \text{ M}\Omega} = 1 \mu\text{A}$$

Since the reverse leakage current doubles for every  $10^\circ\text{C}$  increase, at  $40^\circ\text{C}$

$$I = 4 \times 1 \mu\text{A} = 4 \mu\text{A}$$

$$\Rightarrow V = 4 \mu\text{A} \times 1 \text{ M}\Omega = 4.0 \text{ V}$$

$$@ 0^\circ\text{C } I = \frac{1}{4} \mu\text{A}$$

$$\Rightarrow V = \frac{1}{4} \times 1 = 0.25 \text{ V}$$

Ex: 3.10 a. Use iteration:

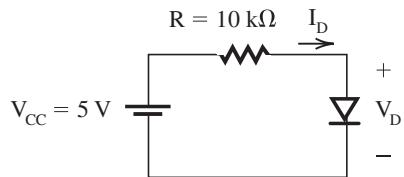
Diode has 0.7 V drop at 1 mA current.

Assume  $V_D = 0.7 \text{ V}$

$$I_D = \frac{5 - 0.7}{10 \text{ k}\Omega} = 0.43 \text{ mA}$$

Use Eq. (3.5) and note that

$$V_1 = 0.7 \text{ V}, \quad I_1 = 1 \text{ mA}$$



$$V_2 - V_1 = 2.3 \times V_T \log\left(\frac{I_2}{I_1}\right)$$

$$V_2 = V_1 + 2.3 \times V_T \log\left(\frac{I_2}{I_1}\right)$$

First iteration

$$\begin{aligned} V_2 &= 0.7 + 2.3 \times 25 \times 10^{-3} \log\left(\frac{0.43}{1}\right) \\ &= 0.679 \text{ V} \end{aligned}$$

Second iteration

$$I_2 = \frac{5 - 0.679}{10 \text{ k}\Omega} = 0.432 \text{ mA}$$

$$\begin{aligned} V_2 &= 0.7 + 2.3 \times 25.3 \times 10^{-3} \log\left(\frac{0.432}{1}\right) \\ &= 0.679 \text{ V} \simeq 0.68 \text{ V} \end{aligned}$$

we get almost the same voltage.

$\therefore$  The iteration yields

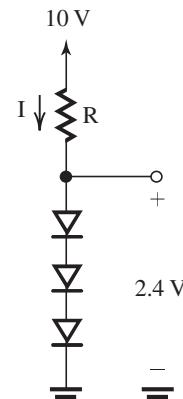
$$I_D = 0.43 \text{ mA}, \quad V_D = 0.68 \text{ V}$$

b. Use constant voltage drop model:

$$V_D = 0.7 \text{ V} \quad \text{constant voltage drop}$$

$$I_D = \frac{5 - 0.7}{10 \text{ k}\Omega} = 0.43 \text{ mA}$$

Ex: 3.11



Diodes have 0.7 V drop at 1 mA

$$\therefore 1 \text{ mA} = I_S e^{0.7/V_T} \quad (1)$$

At a current  $I$  (mA),

$$I = I_S e^{V_D/V_T} \quad (2)$$

Using (1) and (2), we obtain

$$I = e^{(V_D - 0.7)/V_T}$$

### Exercise 3-3

For an output voltage of 2.4 V, the voltage drop across each diode =  $\frac{2.4}{3} = 0.8 \text{ V}$

Now I, the current through each diode, is

$$I = e^{(0.8-0.7)/0.025}$$

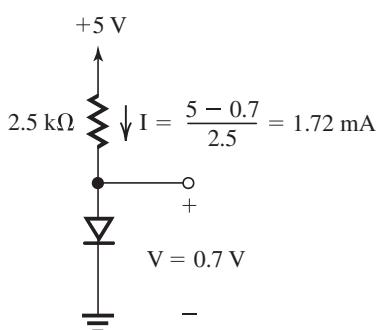
$$= 54.6 \text{ mA}$$

$$R = \frac{10 - 2.4}{54.6 \times 10^{-3}}$$

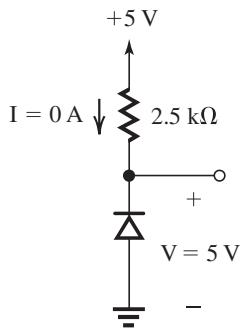
$$= 139 \Omega$$

Ex: 3.12

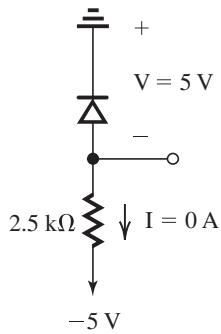
(a)



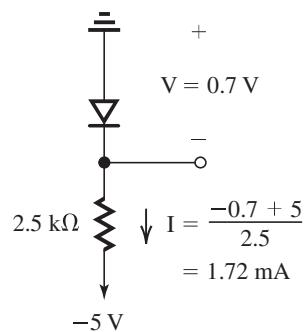
(b)



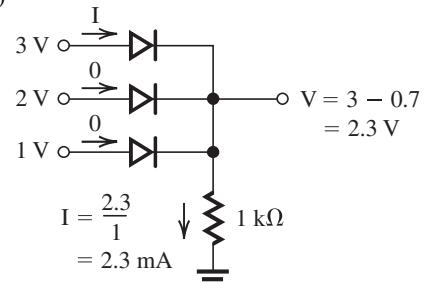
(c)



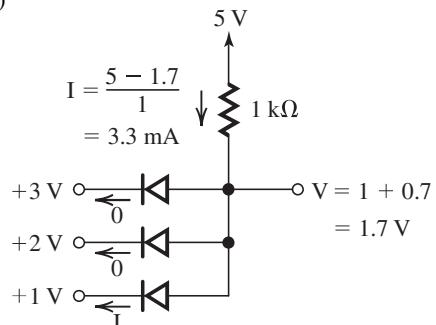
(d)



(e)



(f)



$$\text{Ex: 3.13 } r_d = \frac{V_T}{I_D}$$

$$I_D = 0.1 \text{ mA} \quad r_d = \frac{25 \times 10^{-3}}{0.1 \times 10^{-3}} = 250 \Omega$$

$$I_D = 1 \text{ mA} \quad r_d = \frac{25 \times 10^{-3}}{1 \times 10^{-3}} = 25 \Omega$$

$$I_D = 10 \text{ mA} \quad r_d = \frac{25 \times 10^{-3}}{10 \times 10^{-3}} = 2.5 \Omega$$

Ex: 3.14 For small signal model,

$$\Delta i_D = \Delta v_D / r_d \quad (1)$$

$$\text{where } r_d = \frac{V_T}{I_D}$$

For exponential model,

$$i_D = I_S e^{V/V_T}$$

### Exercise 3-4

$$\frac{i_{D2}}{i_{D1}} = e^{(V_2 - V_1)/V_T} = e^{\Delta V_D/V_T}$$

$$\Delta i_D = i_{D2} - i_{D1} = i_{D1}e^{\Delta V_D/V_T} - i_{D1}$$

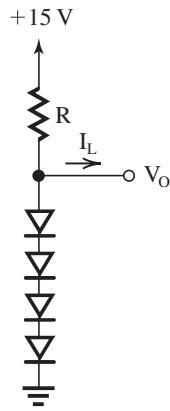
$$= i_{D1}(e^{\Delta V_D/V_T} - 1) \quad (2)$$

In this problem,  $i_{D1} = I_D = 1 \text{ mA}$ .

Using Eqs. (1) and (2) with  $V_T = 25 \text{ mV}$ , we obtain

	$\Delta V_D \text{ (mV)}$	$\Delta i_D \text{ (mA)}$ small signal	$\Delta i_D \text{ (mA)}$ exponential model
a	-10	-0.4	-0.33
b	-5	-0.2	-0.18
c	+5	+0.2	+0.22
d	+10	+0.4	+0.49

Ex: 3.15



a. In this problem,  $\frac{\Delta V_O}{\Delta i_L} = \frac{20 \text{ mV}}{1 \text{ mA}} = 20 \Omega$ .

$\therefore$  Total small-signal resistance of the four diodes  $= 20 \Omega$

$$\therefore \text{For each diode, } r_d = \frac{20}{4} = 5 \Omega.$$

$$\text{But } r_d = \frac{V_T}{I_D} \Rightarrow 5 = \frac{25 \text{ mV}}{I_D}.$$

$$\therefore I_D = 5 \text{ mA}$$

$$\text{and } R = \frac{15 - 3}{5 \text{ mA}} = 2.4 \text{ k}\Omega.$$

b. For  $V_O = 3 \text{ V}$ , voltage drop across each diode  $= \frac{3}{4} = 0.75 \text{ V}$

$$i_D = I_S e^{V/V_T}$$

$$I_S = \frac{i_D}{e^{V/V_T}} = \frac{5 \times 10^{-3}}{e^{0.75/0.025}} = 4.7 \times 10^{-16} \text{ A}$$

c. If  $i_D = 5 - i_L = 5 - 1 = 4 \text{ mA}$ .

Across each diode the voltage drop is

$$V_D = V_T \ln\left(\frac{i_D}{I_S}\right)$$

$$= 25 \times 10^{-3} \times \ln\left(\frac{4 \times 10^{-3}}{4.7 \times 10^{-16}}\right)$$

$$= 0.7443 \text{ V}$$

Voltage drop across 4 diodes

$$= 4 \times 0.7443 = 2.977 \text{ V}$$

so change in  $V_O = 2.977 - 3 = -23 \text{ mV}$ .

Ex: 3.16 For a zener diode

$$V_Z = V_{Z0} + I_Z r_Z$$

$$10 = V_{Z0} + 0.01 \times 50$$

$$V_{Z0} = 9.5 \text{ V}$$

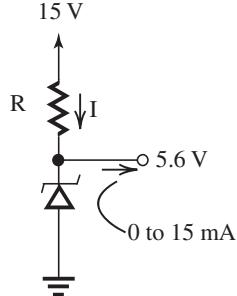
For  $I_Z = 5 \text{ mA}$ ,

$$V_Z = 9.5 + 0.005 \times 50 = 9.75 \text{ V}$$

For  $I_Z = 20 \text{ mA}$ ,

$$V_Z = 9.5 + 0.02 \times 50 = 10.5 \text{ V}$$

Ex: 3.17



The minimum zener current should be

$$5 \times I_{Zk} = 5 \times 1 = 5 \text{ mA}$$

Since the load current can be as large as 15 mA, we should select R so that with  $I_L = 15 \text{ mA}$ , a zener current of 5 mA is available. Thus the current should be 20 mA, leading to

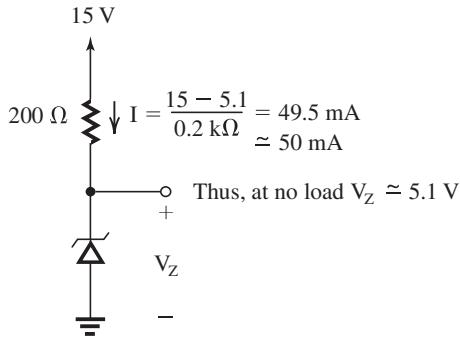
$$R = \frac{15 - 5.6}{20 \text{ mA}} = 470 \Omega$$

Maximum power dissipated in the diode occurs when  $I_L = 0$  is

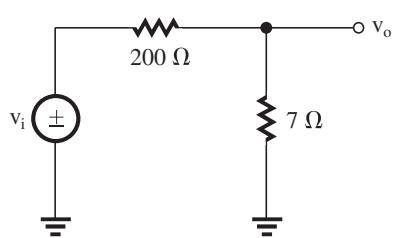
$$P_{\max} = 20 \times 10^{-3} \times 5.6 = 112 \text{ mW}$$

### Exercise 3-5

Ex: 3.18

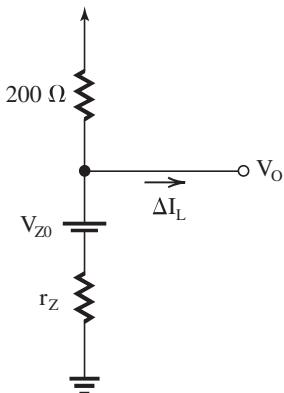


For line regulation:



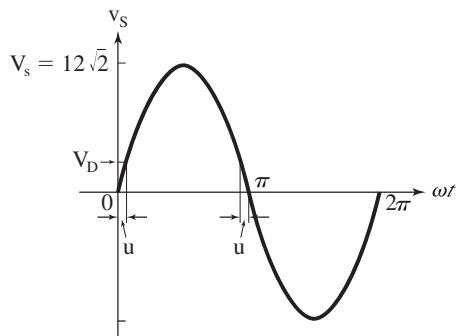
$$\text{Line regulation} = \frac{v_o}{v_i} = \frac{7}{200 + 7} = 33.8 \frac{\text{mV}}{\text{V}}$$

For load regulation:



$$\begin{aligned} \frac{\Delta v_o}{\Delta I_L} &= \frac{-\Delta I_L(r_Z \parallel 200 \Omega)}{\Delta I_L \text{ mA}} \\ &= -6.8 \frac{\text{mV}}{\text{mA}} \end{aligned}$$

Ex: 3.19



a. The diode starts conduction at

$$v_s = V_D = 0.7 \text{ V}$$

$$v_s = V_s \sin \omega t, \text{ here } V_s = 12\sqrt{2}$$

At  $\omega t = \theta$ ,

$$v_s = V_s \sin \theta = V_D = 0.7 \text{ V}$$

$$12\sqrt{2} \sin \theta = 0.7$$

$$\theta = \sin^{-1} \left( \frac{0.7}{12\sqrt{2}} \right) \approx 2.4^\circ$$

Conduction starts at  $\theta$  and stops at  $180 - \theta$ .

$$\therefore \text{Total conduction angle} = 180 - 2\theta = 175.2^\circ$$

$$\text{b. } v_{o,\text{avg}} = \frac{1}{2\pi} \int_{\theta}^{(\pi-\theta)} (V_s \sin \phi - V_D) d\phi$$

$$= \frac{1}{2\pi} [-V_s \cos \phi - V_D \phi]_{\theta}^{\phi=\pi-\theta}$$

$$= \frac{1}{2\pi} [V_s \cos \theta - V_s \cos (\pi - \theta) - V_D (\pi - 2\theta)]$$

But  $\cos \theta \approx 1$ ,  $\cos (\pi - \theta) \approx -1$ , and

$$\pi - 2\theta \approx \pi$$

$$v_{o,\text{avg}} = \frac{2V_s}{2\pi} - \frac{V_D}{2}$$

$$= \frac{V_s}{\pi} - \frac{V_D}{2}$$

For  $V_s = 12\sqrt{2}$  and  $V_D = 0.7 \text{ V}$

$$v_{o,\text{avg}} = \frac{12\sqrt{2}}{\pi} - \frac{0.7}{2} = 5.05 \text{ V}$$

c. The peak diode current occurs at the peak diode voltage.

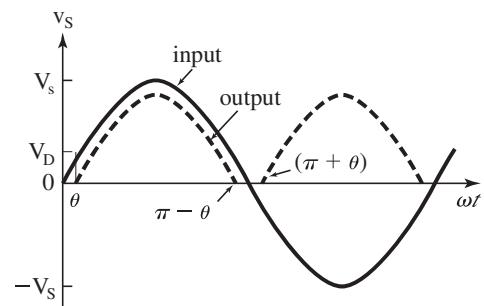
$$\therefore \hat{i}_D = \frac{V_s - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100}$$

$$= 163 \text{ mA}$$

$$\text{PIV} = +V_s = 12\sqrt{2}$$

$$\approx 17 \text{ V}$$

Ex: 3.20



a. As shown in the diagram, the output is zero between  $(\pi - \theta)$  to  $(\pi + \theta)$

$$= 2\theta$$

### Exercise 3–6

Here  $\theta$  is the angle at which the input signal reaches  $V_D$ .

$$\therefore V_s \sin \theta = V_D$$

$$\theta = \sin^{-1} \left( \frac{V_D}{V_s} \right)$$

$$2\theta = 2 \sin^{-1} \left( \frac{V_D}{V_s} \right)$$

b. Average value of the output signal is given by

$$\begin{aligned} V_O &= \frac{1}{2\pi} \left[ 2 \times \int_{\theta}^{(\pi-\theta)} (V_s \sin \phi - V_D) d\phi \right] \\ &= \frac{1}{\pi} [-V_s \cos \phi - V_D \phi]_{\phi=\theta}^{\pi-\theta} \\ &\simeq 2 \frac{V_s}{\pi} - V_D, \quad \text{for } \theta \text{ small.} \end{aligned}$$

c. Peak current occurs when  $\phi = \frac{\pi}{2}$ .

Peak current

$$= \frac{V_s \sin(\pi/2) - V_D}{R} = \frac{V_s - V_D}{R}$$

If  $v_s$  is 12 V(rms),

$$\text{then } V_s = \sqrt{2} \times 12 = 12\sqrt{2}$$

$$\text{Peak current} = \frac{12\sqrt{2} - 0.7}{100} \simeq 163 \text{ mA}$$

Nonzero output occurs for angle  $= 2(\pi - 2\theta)$

The fraction of the cycle for which  $v_O > 0$  is

$$\begin{aligned} &= \frac{2(\pi - 2\theta)}{2\pi} \times 100 \\ &= \frac{2 \left[ \pi - 2 \sin^{-1} \left( \frac{0.7}{12\sqrt{2}} \right) \right]}{2\pi} \times 100 \\ &\simeq 97.4 \% \end{aligned}$$

Average output voltage  $V_O$  is

$$V_O = 2 \frac{V_s}{\pi} - V_D = \frac{2 \times 12\sqrt{2}}{\pi} - 0.7 = 10.1 \text{ V}$$

Peak diode current  $\hat{i}_D$  is

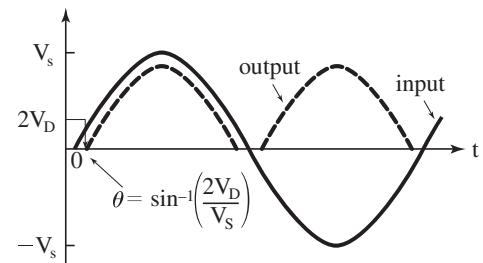
$$\begin{aligned} \hat{i}_D &= \frac{V_s - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100} \\ &= 163 \text{ mA} \end{aligned}$$

$$\text{PIV} = V_s - V_D + V_s$$

$$= 12\sqrt{2} - 0.7 + 12\sqrt{2}$$

$$= 33.2 \text{ V}$$

Ex: 3.21



$$(a) V_{O,\text{avg}} = \frac{1}{2\pi} \int (V_s \sin \phi - 2V_D) d\phi$$

$$= \frac{2}{2\pi} [-V_s \cos \phi - 2V_D \phi]_{\phi=\theta}^{\pi-\theta}$$

$$= \frac{1}{\pi} [V_s \cos \phi - V_s \cos(\pi - \theta) - 2V_D(\pi - 2\theta)]$$

But  $\cos \theta \approx 1$ ,

$$\cos(\pi - \theta) \approx -1$$

$\pi - 2\theta \approx \pi$ . Thus

$$\Rightarrow V_{O,\text{avg}} \simeq \frac{2V_s}{\pi} - 2V_D$$

$$= \frac{2 \times 12\sqrt{2}}{\pi} - 1.4 = 9.4 \text{ V}$$

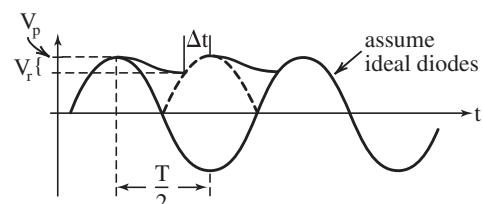
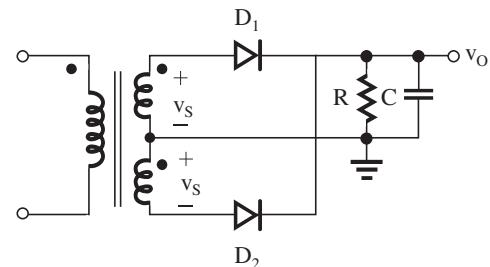
$$(b) \text{ Peak diode current} = \frac{\text{Peak voltage}}{R}$$

$$= \frac{V_s - 2V_D}{R} = \frac{12\sqrt{2} - 1.4}{100}$$

$$= 156 \text{ mA}$$

$$\text{PIV} = V_s - V_D = 12\sqrt{2} - 0.7 = 16.3 \text{ V}$$

Ex: 3.22 Full-wave peak rectifier:



The ripple voltage is the amount of voltage reduction during capacitor discharge that occurs

### Exercise 3-7

when the diodes are not conducting. The output voltage is given by

$$v_O = V_p e^{-t/RC}$$

$V_p - V_r = V_p e^{-\frac{T/2}{RC}}$  ← discharge is only half the period. We also assumed  $\Delta t \ll \frac{T}{2}$ .

$$V_r = V_p \left(1 - e^{-\frac{T/2}{RC}}\right)$$

$$e^{-\frac{T/2}{RC}} \approx 1 - \frac{T/2}{RC}, \quad \text{for } CR \gg T/2$$

$$\text{Thus } V_r \approx V_p \left(1 - 1 + \frac{T/2}{RC}\right)$$

$$V_r = \frac{V_p}{2fRC} \quad (\text{a}) \quad \text{Q.E.D.}$$

To find the average diode current, note that the charge supplied to C during conduction is equal to the charge lost during discharge.

$$Q_{\text{SUPPLIED}} = Q_{\text{LOST}}$$

$$i_{Cav} \Delta t = CV_r$$

$$(i_{D,av} - I_L) \Delta t = C \frac{V_p}{2fRC} = \frac{V_p}{2fR}$$

$$= \frac{V_p \pi}{\omega R}$$

$$i_{D,av} = \frac{V_p \pi}{\omega \Delta t R} + I_L$$

where  $\omega \Delta t$  is the conduction angle.

Note that the conduction angle has the same expression as for the half-wave rectifier and is given by Eq. (3.30),

$$\omega \Delta t \approx \sqrt{\frac{2V_r}{V_p}} \quad (\text{b})$$

Substituting for  $\omega \Delta t$ , we get

$$\Rightarrow i_{D,av} = \frac{\pi V_p}{\sqrt{\frac{2V_r}{V_p}} \cdot R} + I_L$$

Since the output is approximately held at  $V_p$ ,  $\frac{V_p}{R} \approx I_L$ . Thus

$$\Rightarrow i_{D,av} \approx \pi I_L \sqrt{\frac{V_p}{2V_r}} + I_L$$

$$= I_L \left[ 1 + \pi \sqrt{\frac{V_p}{2V_r}} \right] \quad \text{Q.E.D.}$$

If  $t = 0$  is at the peak, the maximum diode current occurs at the onset of conduction or at  $t = -\omega \Delta t$ .

During conduction, the diode current is given by

$$i_D = i_C + i_L$$

$$i_{D,max} = C \frac{dv_S}{dt} \Big|_{t=-\omega \Delta t} + i_L$$

$$\text{assuming } i_L \text{ is const. } i_L \approx \frac{V_p}{R} = I_L$$

$$= C \frac{d}{dt} (V_p \cos \omega t) + I_L$$

$$= -C \sin \omega t \times \omega V_p + I_L$$

$$= -C \sin(-\omega \Delta t) \times \omega V_p + I_L$$

For a small conduction angle

$$\sin(-\omega \Delta t) \approx -\omega \Delta t. \text{ Thus}$$

$$\Rightarrow i_{D,max} = C \omega \Delta t \times \omega V_p + I_L$$

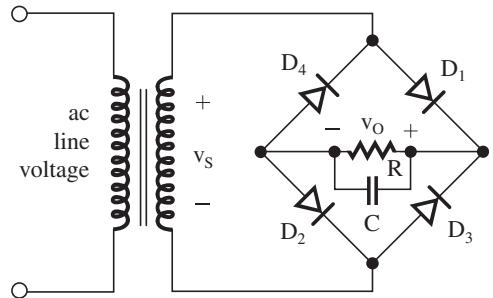
Sub (b) to get

$$i_{D,max} = C \sqrt{\frac{2V_r}{V_p}} \omega V_p + I_L$$

Substituting  $\omega = 2\pi f$  and using (a) together with  $V_p/R \approx I_L$  results in

$$i_{D,max} = I_L \left[ 1 + 2\pi \sqrt{\frac{V_p}{2V_r}} \right] \quad \text{Q.E.D.}$$

Ex: 3.23



The output voltage,  $v_O$ , can be expressed as

$$v_O = (V_p - 2V_D) e^{-t/RC}$$

At the end of the discharge interval

$$v_O = V_p - 2V_D - V_r$$

The discharge occurs almost over half of the time period  $\approx T/2$ .

$$\text{For time constant } RC \gg \frac{T}{2}$$

### Exercise 3-8

$$e^{-t/RC} \simeq 1 - \frac{T}{2} \times \frac{1}{RC}$$

$$\therefore V_p - 2V_D - V_r = (V_p - 2V_D) \left( 1 - \frac{T}{2} \times \frac{1}{RC} \right)$$

$$\Rightarrow V_r = (V_p - 2V_D) \times \frac{T}{2RC}$$

Here  $V_p = 12\sqrt{2}$  and  $V_r = 1$  V

$V_D = 0.8$  V

$$T = \frac{1}{f} = \frac{1}{60} \text{ s}$$

$$1 = (12\sqrt{2} - 2 \times 0.8) \times \frac{1}{2 \times 60 \times 100 \times C}$$

$$C = \frac{(12\sqrt{2} - 1.6)}{2 \times 60 \times 100} = 1281 \mu\text{F}$$

Without considering the ripple voltage, the dc output voltage

$$= 12\sqrt{2} - 2 \times 0.8 = 15.4 \text{ V}$$

If ripple voltage is included, the output voltage is

$$= 12\sqrt{2} - 2 \times 0.8 - \frac{V_r}{2} = 14.9 \text{ V}$$

$$I_L = \frac{14.9}{100 \Omega} \simeq 0.15 \text{ A}$$

The conduction angle  $\omega\Delta t$  can be obtained using Eq. (3.30) but substituting  $V_p = 12\sqrt{2} - 2 \times 0.8$ :

$$\omega\Delta t = \sqrt{\frac{2V_r}{V_p}} = \sqrt{\frac{2 \times 1}{12\sqrt{2} - 2 \times 0.8}}$$

$$= 0.36 \text{ rad} = 20.7^\circ$$

The average and peak diode currents can be calculated using Eqs. (3.34) and (3.35):

$$i_{Dav} = I_L \left( 1 + \pi \sqrt{\frac{V_p}{2V_r}} \right), \text{ where } I_L = \frac{14.9 \text{ V}}{100 \Omega},$$

$V_p = 12\sqrt{2} - 2 \times 0.8$ , and  $V_r = 1$  V; thus

$$i_{Dav} = 1.45 \text{ A}$$

$$i_{Dmax} = I \left( 1 + 2\pi \sqrt{\frac{V_p}{2V_r}} \right)$$

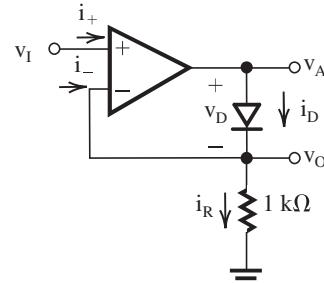
$$= 2.76 \text{ A}$$

PIV of the diodes

$$= V_S - V_{DO} = 12\sqrt{2} - 0.8 = 16.2 \text{ V}$$

To provide a safety margin, select a diode capable of a peak current of 3.5 to 4A and having a PIV rating of 20 V.

Ex: 3.24



The diode has 0.7 V drop at 1 mA current.

$$i_D = I_S e^{v_D/V_T}$$

$$\frac{i_D}{1 \text{ mA}} = e^{(v_D - 0.7)/V_T}$$

$$\Rightarrow v_D = V_T \ln\left(\frac{i_D}{1 \text{ mA}}\right) + 0.7 \text{ V}$$

For  $v_I = 10 \text{ mV}$ ,  $v_O = v_I = 10 \text{ mV}$

It is an ideal op amp, so  $i_+ = i_- = 0$ .

$$\therefore i_D = i_R = \frac{10 \text{ mV}}{1 \text{ k}\Omega} = 10 \mu\text{A}$$

$$v_D = 25 \times 10^{-3} \ln\left(\frac{10 \mu\text{A}}{1 \text{ mA}}\right) + 0.7 = 0.58 \text{ V}$$

$$v_A = v_D + 10 \text{ mV}$$

$$= 0.58 + 0.01$$

$$= 0.59 \text{ V}$$

For  $v_I = 1 \text{ V}$

$$v_O = v_I = 1 \text{ V}$$

$$i_D = \frac{v_O}{1 \text{ k}\Omega} = \frac{1}{1 \text{ k}\Omega} = 1 \text{ mA}$$

$$v_D = 0.7 \text{ V}$$

$$v_A = 0.7 \text{ V} + 1 \text{ k}\Omega \times 1 \text{ mA}$$

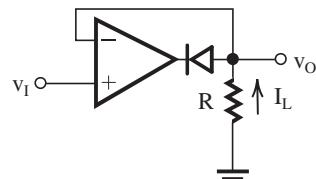
$$= 1.7 \text{ V}$$

For  $v_I = -1 \text{ V}$ , the diode is cut off.

$$\therefore v_O = 0 \text{ V}$$

$$v_A = -12 \text{ V}$$

Ex: 3.25



### Exercise 3-9

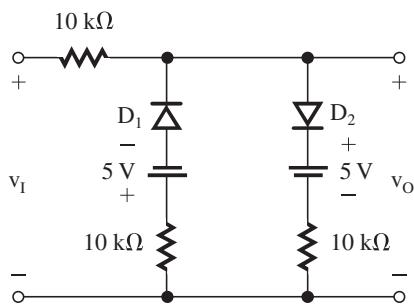
$v_I \geq 0 \sim$  diode is cut off, loop is open, and the opamp is saturated:

$$v_O = 0 \text{ V}$$

$v_I \leq 0 \sim$  diode conducts and closes the negative feedback loop:

$$v_O = v_I$$

Ex: 3.26



Both diodes are cut off

for  $-5V \leq v_I \leq +5V$

and  $v_O = v_I$

For  $v_I \leq -5 \text{ V}$ , diode D<sub>1</sub> conducts and

$$v_O = -5 + \frac{1}{2} (+v_I + 5)$$

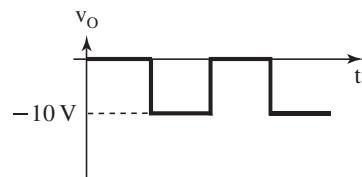
$$= \left( -2.5 + \frac{v_I}{2} \right) \text{ V}$$

For  $v_I \geq 5 \text{ V}$ , diode D<sub>2</sub> conducts and

$$v_O = +5 + \frac{1}{2} (v_I - 5)$$

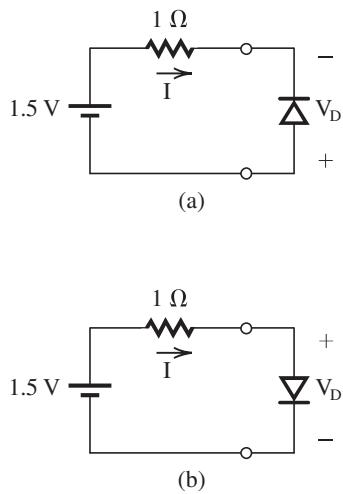
$$= \left( 2.5 + \frac{v_I}{2} \right) \text{ V}$$

Ex: 3.27 Reversing the diode results in the peak output voltage being clamped at 0 V:



Here the dc component of  $v_O = V_0 = -5 \text{ V}$

3.1



(a) The diode is reverse biased, thus

$$I = 0 \text{ A}$$

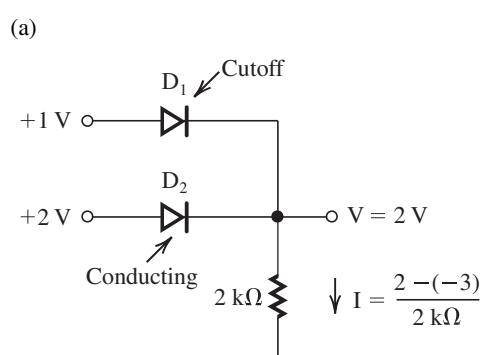
$$V_D = -1.5 \text{ V}$$

(b) The diode is forward biased, thus

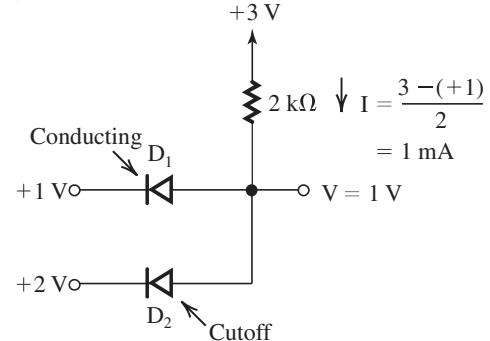
$$V_D = 0 \text{ V}$$

$$I = \frac{1.5 \text{ V}}{1 \Omega} = 1.5 \text{ A}$$

3.3



(b)



3.2 Refer to Fig. P3.2.

(a) Diode is conducting, thus

$$V = -3 \text{ V}$$

$$I = \frac{+3 - (-3)}{10 \text{ k}\Omega} = 0.6 \text{ mA}$$

(b) Diode is reverse biased, thus

$$I = 0$$

$$V = +3 \text{ V}$$

(c) Diode is conducting, thus

$$V = +3 \text{ V}$$

$$I = \frac{+3 - (-3)}{10 \text{ k}\Omega} = 0.6 \text{ mA}$$

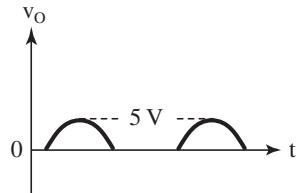
(d) Diode is reverse biased, thus

$$I = 0$$

$$V = -3 \text{ V}$$

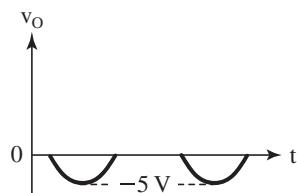
3.4

(a)



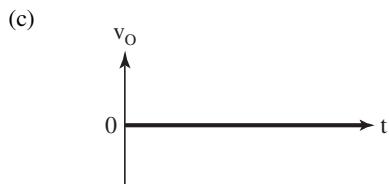
$$V_{p^+} = 5 \text{ V} \quad V_{p^-} = 0 \text{ V} \quad f = 1 \text{ kHz}$$

(b)

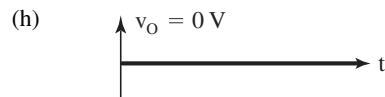


$$V_{p^+} = 0 \text{ V} \quad V_{p^-} = -5 \text{ V} \quad f = 1 \text{ kHz}$$

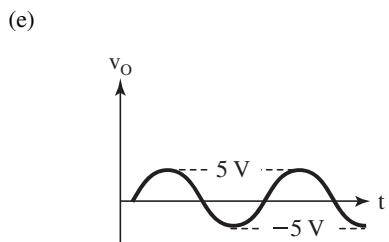
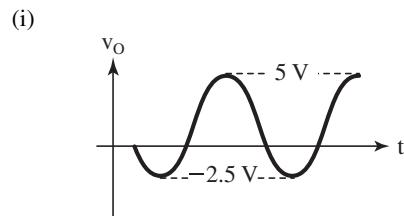
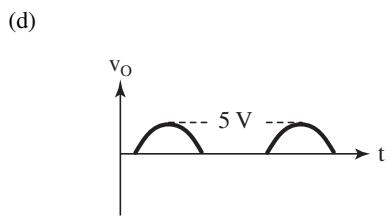
Chapter 3-2



$D_1$  shorts to ground when  $v_I > 0$  and is cut off when  $v_I < 0$  whereby the output follows  $v_I$ .

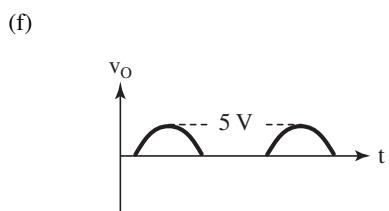
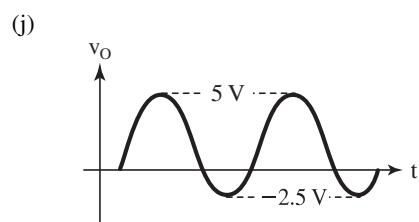


$v_O = 0 \text{ V} \sim$  The output is always shorted to ground as  $D_1$  conducts when  $v_I > 0$  and  $D_2$  conducts when  $v_I < 0$ .



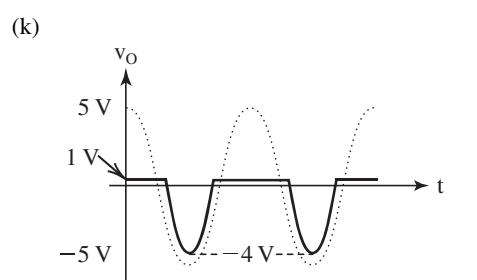
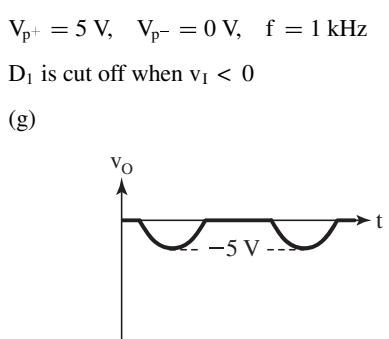
When  $v_I < 0$ ,  $D_1$  is conducting and the circuit becomes a voltage divider where the negative peak is

$$\frac{1 \text{ k}\Omega}{1 \text{ k}\Omega + 1 \text{ k}\Omega} \times -5 \text{ V} = -2.5 \text{ V}$$



When  $v_I > 0$ , the output follows the input as  $D_1$  is conducting.

When  $v_I < 0$ ,  $D_1$  is cut off and the circuit becomes a voltage divider.



When  $v_I > 0$ ,  $D_1$  is cut off and  $D_2$  is conducting. The output becomes 1 V.

When  $v_I < 0$ ,  $D_1$  is conducting and  $D_2$  is cut off.  
The output becomes:

$$v_O = v_I + 1 \text{ V}$$

3.5

A	B	X	Y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

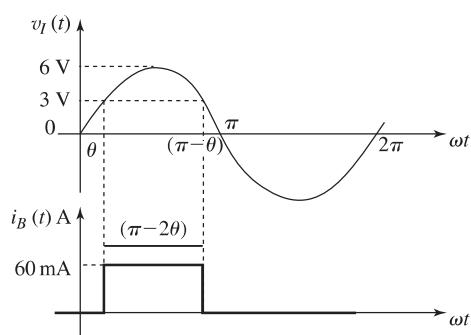
$$X = AB, \quad Y = A + B$$

X and Y are the same for

$$A = B$$

X and Y are opposite if  $A \neq B$

3.6



From Fig. P3.6 we see that when  $v_I < V_B$ ; that is,  $v_I < 3 \text{ V}$ ,  $D_1$  will be conducting the current I and  $i_B$  will be zero. When  $v_I$  exceeds the battery voltage ( $3 \text{ V}$ ),  $D_1$  cuts off and  $D_2$  conducts, thus steering I into the battery. Thus,  $i_B$  will have the waveform shown in the figure. Its peak value will be  $60 \text{ mA}$ . To obtain the average value, we first

These figures belong to Problem 3.9.

determine the conduction angle of  $D_2$ ,  $(\pi - 2\theta)$ , where

$$\theta = \sin^{-1}\left(\frac{3}{6}\right) = 30^\circ$$

Thus

$$\pi - 2\theta = 180^\circ - 60 = 120^\circ$$

The average value of  $i_B$  will be

$$|i_B|_{av} = \frac{60 \times 120^\circ}{360^\circ} = 20 \text{ mA}$$

If the peak value of  $v_I$  is reduced by 10%, i.e. from  $6 \text{ V}$  to  $5.4 \text{ V}$ , the peak value of  $i_B$  does not change. The conduction angle of  $D_2$ , however, changes since  $\theta$  now becomes

$$\theta = \sin^{-1}\left(\frac{3}{5.4}\right) = 33.75^\circ$$

and thus

$$\pi - 2\theta = 112.5^\circ$$

Thus the average value of  $i_B$  becomes

$$|i_B|_{av} = \frac{60 \times 112.5^\circ}{360^\circ} = 18.75 \text{ mA}$$

3.7 The case for the highest current in a single diode is when only one input is high:

$$V_Y = 5 \text{ V}$$

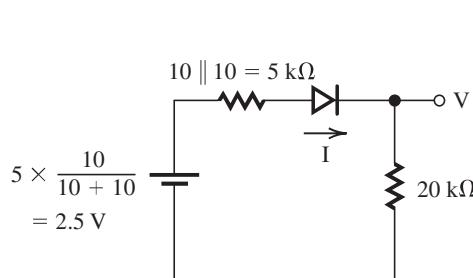
$$\frac{V_Y}{R} \leq 0.2 \text{ mA} \Rightarrow R \geq 25 \text{ k}\Omega$$

3.8 The maximum input current occurs when one input is low and the other two are high.

$$\frac{5 - 0}{R} \leq 0.2 \text{ mA}$$

$$R \geq 25 \text{ k}\Omega$$

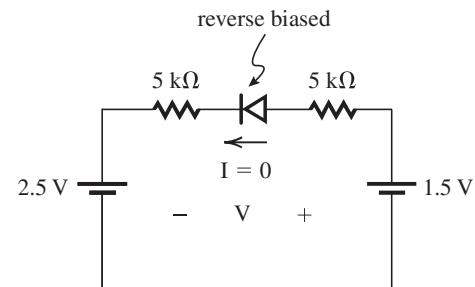
3.9 The analysis is shown on the circuit diagrams below.



$$I = \frac{2.5}{5 + 20} = 0.1 \text{ mA}$$

$$V = 0.1 \times 20 = 2 \text{ V}$$

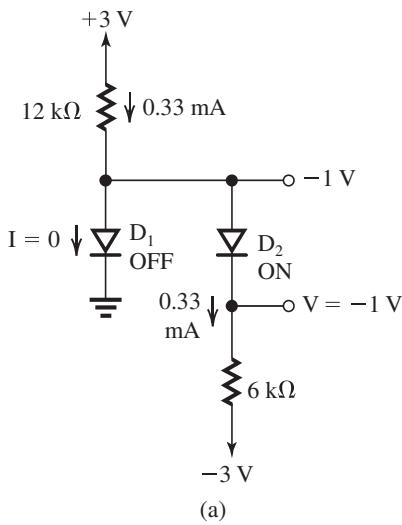
(a)



$$V = 1.5 - 2.5 = -1 \text{ V}$$

(b)

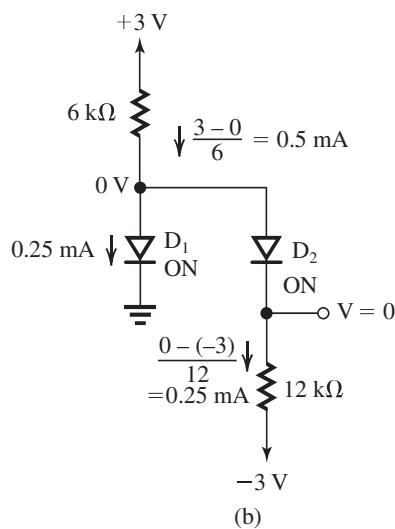
3.10



(a) If we assume that both \$D\_1\$ and \$D\_2\$ are conducting, then \$V = 0\text{ V}\$ and the current in \$D\_2\$ will be \$[0 - (-3)]/6 = 0.5\text{ mA}\$. The current in the \$12\text{ k}\Omega\$ resistor will be \$(3 - 0)/12 = 0.25\text{ mA}\$. A node equation at the common anodes node yields a negative current in \$D\_1\$. It follows that our assumption is wrong and \$D\_1\$ must be off. Now making the assumption that \$D\_1\$ is off and \$D\_2\$ is on, we obtain the results shown in Fig. (a):

$$I = 0$$

$$V = -1\text{ V}$$



(b) In (b), the two resistors are interchanged. With some reasoning, we can see that the current supplied through the \$6\text{-k}\Omega\$ resistor will exceed that drawn through the \$12\text{-k}\Omega\$ resistor, leaving sufficient current to keep \$D\_1\$ conducting. Assuming that \$D\_1\$ and \$D\_2\$ are both conducting gives the results shown in Fig. (b):

$$I = 0.25\text{ mA}$$

$$V = 0\text{ V}$$

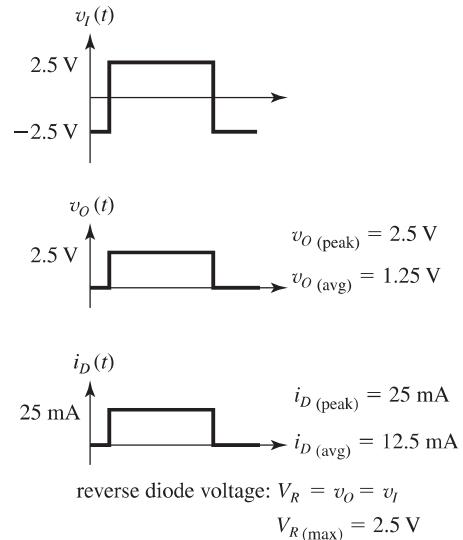
$$3.11 \quad R \geq \frac{120\sqrt{2}}{40} \geq 4.2\text{ k}\Omega$$

The largest reverse voltage appearing across the diode is equal to the peak input voltage:

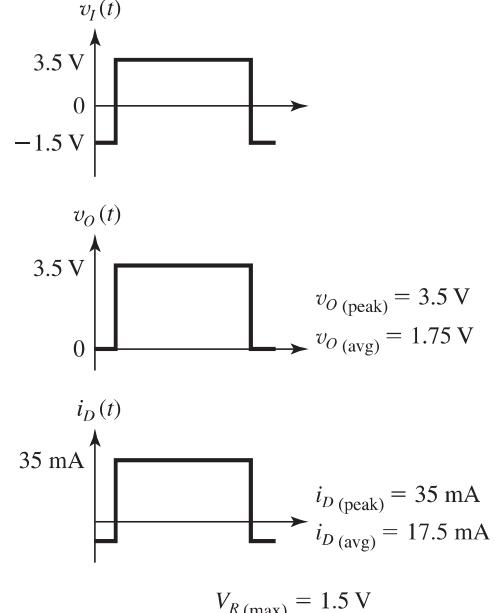
$$120\sqrt{2} = 169.7\text{ V}$$

3.12 For \$v\_I > 0\text{ V}\$: D is on, \$v\_O = v\_I\$, \$i\_D = v\_I/R\$

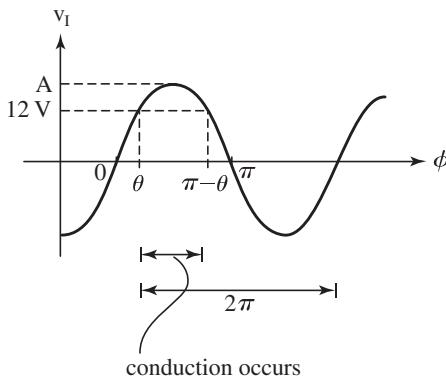
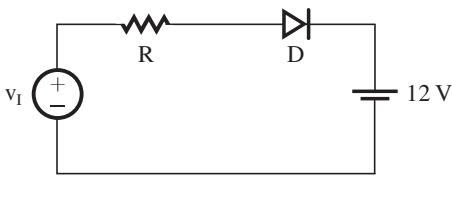
For \$v\_I < 0\text{ V}\$: D is off, \$v\_O = 0\$, \$i\_D = 0\$



3.13



3.14



$$v_I = A \sin \theta = 12 \text{ V} \quad \text{conduction through D occurs}$$

For a conduction angle  $(\pi - 2\theta)$  that is 25% of a cycle

$$\frac{\pi - 2\theta}{2\pi} = \frac{1}{4}$$

$$\theta = \frac{\pi}{4}$$

$$A = 12 / \sin \theta = 17 \text{ V}$$

$\therefore$  Peak-to-peak sine wave voltage

$$= 2A = 34 \text{ V}$$

Given the average diode current to be

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{A \sin \phi - 12}{R} d\phi = 100 \text{ mA}$$

$$\frac{1}{2\pi} \left( \frac{-17 \cos \phi - 12\phi}{R} \right)_{\phi=0.25\pi}^{\phi=0.75\pi} = 0.1$$

$$R = 8.3 \Omega$$

$$\text{Peak diode current} = \frac{A - 12}{R} = 0.6 \text{ A}$$

$$\text{Peak reverse voltage} = A + 12 = 29 \text{ V}$$

For resistors specified to only one significant digit and peak-to-peak voltage to the nearest volt, choose  $A = 17$  so the peak-to-peak sine wave voltage = 34 V and  $R = 8 \Omega$ .

Conduction starts at  $v_I = A \sin \theta = 12$

$$17 \sin \theta = 12$$

$$\theta = \left( \frac{\pi}{4} \right) \text{ rad}$$

Conduction stops at  $\pi - \theta$ .

$\therefore$  Fraction of cycle that current flows is

$$\frac{\pi - 2\theta}{2\pi} \times 100 = 25\%$$

Average diode current =

$$\frac{1}{2\pi} \left( \frac{-17 \cos \phi - 12\phi}{8} \right)_{\phi=\pi/4}^{\phi=3\pi/4} = 103 \text{ mA}$$

Peak diode current

$$= \frac{17 - 12}{8} = 0.625 \text{ A}$$

Peak reverse voltage =

$$A + 12 = 29 \text{ V}$$

3.15

V RED GREEN

3 V ON OFF - D<sub>1</sub> conducts

0 OFF OFF - No current flows

-3 V OFF ON - D<sub>2</sub> conducts

$$3.16 \quad V_T = \frac{kT}{q}$$

where

$$k = 1.38 \times 10^{-23} \text{ J/K} = 8.62 \times 10^{-5} \text{ eV/K}$$

$$T = 273 + x^\circ \text{C}$$

$$q = 1.60 \times 10^{-19} \text{ C}$$

Thus

$$V_T = 8.62 \times 10^{-5} + (273 \times x^\circ \text{C}), \text{ V}$$

x [°C]	V <sub>T</sub> [mV]
-55	18.8
0	23.5
+40	27.0
+125	34.3

for  $V_T = 25 \text{ mV}$  at  $17^\circ \text{C}$

$$3.17 \quad I_1 = I_S e^{0.7/V_T} = 10^{-3}$$

$$i_2 = I_S e^{0.5/V_T}$$

$$\frac{i_2}{i_1} = \frac{i_2}{10^{-3}} = e^{\frac{0.5 - 0.7}{0.025}}$$

$$i_2 = 0.335 \mu \text{A}$$

$$3.18 \quad i = I_S e^{v/0.025}$$

$$\therefore 10,000 I_S = I_S e^{v/0.025}$$

$$v = 0.230 \text{ V}$$

At  $v = 0.7 \text{ V}$ ,

$$i = I_S e^{0.7/0.025} = 1.45 \times 10^{12} I_S$$

## Chapter 3–6

$$3.19 \quad I = I_S e^{V_D/V_T}$$

$$10^{-3} = I_S e^{0.7/V_T} \quad (1)$$

For  $V_D = 0.71$  V,

$$I = I_S e^{0.71/V_T} \quad (2)$$

Combining (1) and (2) gives

$$I = 10^{-3} e^{(0.71 - 0.7)/0.025}$$

$$= 1.49 \text{ mA}$$

For  $V_D = 0.8$  V,

$$I = I_S e^{0.8/V_T} \quad (3)$$

Combining (1) and (3) gives

$$I = 10^{-3} \times e^{(0.8 - 0.7)/0.025}$$

$$= 54.6 \text{ mA}$$

Similarly, for  $V_D = 0.69$  V we obtain

$$I = 10^{-3} \times e^{(0.69 - 0.7)/0.025}$$

$$= 0.67 \text{ mA}$$

and for  $V_D = 0.6$  V we have

$$I = 10^{-3} e^{(0.6 - 0.7)/0.025}$$

$$= 18.3 \mu\text{A}$$

To increase the current by a factor of 10,  $V_D$  must be increased by  $\Delta V_D$ ,

$$10 = e^{\Delta V_D/0.025}$$

$$\Rightarrow \Delta V_D = 0.025 \ln 10 = 57.6 \text{ mV}$$

3.20  $I_S$  can be found by using  $I_S = I_D \cdot e^{-V_D/V_T}$ .

Let a decrease by a factor of 10 in  $I_D$  result in a decrease of  $V_D$  by  $\Delta V$ :

$$I_D = I_S e^{V_D/V_T}$$

$$\frac{I_D}{10} = I_S e^{(V_D - \Delta V)/V_T} = I_S e^{V_D/V_T} \cdot e^{-\Delta V/V_T}$$

Taking the ratio of the above two equations, we have

$$10 = e^{\Delta V/V_T} \Rightarrow \Delta V \simeq 60 \text{ mV}$$

Thus the result in each case is a decrease in the diode voltage by 60 mV.

$$(a) \quad V_D = 0.700 \text{ V}, \quad I_D = 1 \text{ A} \\ \Rightarrow I_S = 6.91 \times 10^{-13} \text{ A};$$

10% of  $I_D$  gives  $V_D = 0.64$  V

$$(b) \quad V_D = 0.650 \text{ V}, \quad I_D = 1 \text{ mA}$$

$$\Rightarrow I_S = 5.11 \times 10^{-15} \text{ A};$$

10% of  $I_D$  gives  $V_D = 0.59$  V

$$(c) \quad V_D = 0.650 \text{ V}, \quad I_D = 10 \mu\text{A}$$

$$\Rightarrow I_S = 5.11 \times 10^{-17} \text{ A};$$

10% of  $I_D$  gives  $V_D = 0.59$  V

$$(d) \quad V_D = 0.700 \text{ V}, \quad I_D = 100 \text{ mA}$$

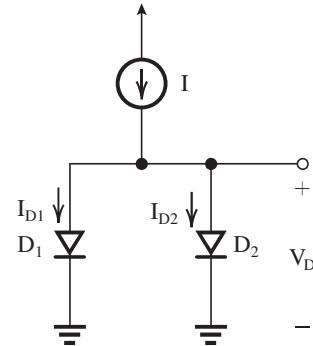
$$\Rightarrow I_S = 6.91 \times 10^{-14} \text{ A};$$

10% of  $I_D$  gives  $V_D = 0.64$  V

3.21 The voltage across three diodes in series is 2.0 V; thus the voltage across each diode must be 0.667 V. Using  $I_D = I_S e^{V_D/V_T}$ , the required current  $I$  is found to be 3.9 mA.

If 1 mA is drawn away from the circuit,  $I_D$  will be 2.9 mA, which would give  $V_D = 0.794$  V, giving an output voltage of 1.98 V. The change in output voltage is –22 mV.

3.22



$$I_{D1} = I_S e^{V_D/V_T} \quad (1)$$

$$I_{D2} = I_S e^{V_D/V_T} \quad (2)$$

Summing (1) and (2) gives

$$I_{D1} + I_{D2} = (I_{S1} + I_{S2}) e^{V_D/V_T}$$

But

$$I_{D1} + I_{D2} = I$$

Thus

$$I = (I_{S1} + I_{S2}) e^{V_D/V_T} \quad (3)$$

From Eq. (3) we obtain

$$V_D = V_T \ln \left( \frac{I}{I_{S1} + I_{S2}} \right)$$

Also, Eq. (3) can be written as

$$I = I_{S1} e^{V_D/V_T} \left( 1 + \frac{I_{S2}}{I_{S1}} \right) \quad (4)$$

Now using (1) and (4) gives

$$I_{D1} = \frac{I}{1 + (I_{S2}/I_{S1})} = I \frac{I_{S1}}{I_{S1} + I_{S2}}$$

We similarly obtain

$$I_{D2} = \frac{I}{1 + (I_{S1}/I_{S2})} = I \frac{I_{S2}}{I_{S1} + I_{S2}}$$

3.23 Connecting an identical diode in parallel would reduce the current in each diode by a factor of 2. Writing expressions for the currents, we have

$$I_D = I_S e^{V_D/V_T}$$

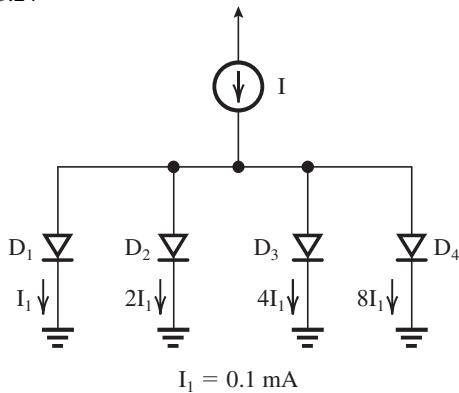
$$\frac{I_D}{2} = I_S e^{(V_D - \Delta V)/V_T} = I_S e^{V_D/V_T} \cdot e^{-\Delta V/V_T}$$

Taking the ratio of the above two equations, we have

$$2 = e^{\Delta V/V_T} \Rightarrow \Delta V = 17.3 \text{ mV}$$

Thus the result is a decrease in the diode voltage by 17.3 mV.

3.24



The junction areas of the four diodes must be related by the same ratios as their currents, thus

$$A_4 = 2A_3 = 4A_2 = 8A_1$$

With  $I_1 = 0.1 \text{ mA}$ ,

$$I = 0.1 + 0.2 + 0.4 + 0.8 = 1.5 \text{ mA}$$

3.25 We can write a node equation at the anodes:

$$I_{D2} = I_1 - I_2 = 7 \text{ mA}$$

$$I_{D1} = I_2 = 3 \text{ mA}$$

We can write the following equation for the diode voltages:

$$V = V_{D2} - V_{D1}$$

If  $D_2$  has saturation current  $I_S$ , then  $D_1$ , which is 10 times larger, has saturation current  $10I_S$ . Thus we can write

$$I_{D2} = I_S e^{V_{D2}/V_T}$$

$$I_{D1} = 10I_S e^{V_{D1}/V_T}$$

Taking the ratio of the two equations above, we have

$$\frac{I_{D2}}{I_{D1}} = \frac{7}{3} = \frac{1}{10} e^{(V_{D2} - V_{D1})/V_T} = \frac{1}{10} e^{V/V_T}$$

$$\Rightarrow V = 0.025 \ln\left(\frac{70}{3}\right) = 78.7 \text{ mV}$$

To instead achieve  $V = 60 \text{ mV}$ , we need

$$\frac{I_{D2}}{I_{D1}} = \frac{I_1 - I_2}{I_2} = \frac{1}{10} e^{0.06/0.025} = 1.1$$

Solving the above equation with  $I_1$  still at 10 mA, we find  $I_2 = 4.76 \text{ mA}$ .

3.26 We can write the following node equation at the diode anodes:

$$I_{D2} = 10 \text{ mA} - V/R$$

$$I_{D1} = V/R$$

We can write the following equation for the diode voltages:

$$V = V_{D2} - V_{D1}$$

We can write the following diode equations:

$$I_{D2} = I_S e^{V_{D2}/V_T}$$

$$I_{D1} = I_S e^{V_{D1}/V_T}$$

Taking the ratio of the two equations above, we have

$$\frac{I_{D2}}{I_{D1}} = \frac{10 \text{ mA} - V/R}{V/R} = e^{(V_{D2} - V_{D1})/V_T} = e^{V/V_T}$$

To achieve  $V = 50 \text{ mV}$ , we need

$$\frac{I_{D2}}{I_{D1}} = \frac{10 \text{ mA} - 0.05/R}{0.05/R} = e^{0.05/0.025} = 7.39$$

Solving the above equation, we have

$$R = 42 \Omega$$

3.27 For a diode conducting a constant current, the diode voltage decreases by approximately 2 mV per increase of 1°C.

$T = -20^\circ\text{C}$  corresponds to a temperature decrease of  $40^\circ\text{C}$ , which results in an increase of the diode voltage by 80 mV. Thus  $V_D = 770 \text{ mV}$ .

$T = +85^\circ\text{C}$  corresponds to a temperature increase of  $65^\circ\text{C}$ , which results in a decrease of the diode voltage by 130 mV. Thus  $V_D = 560 \text{ mV}$ .

3.28 For a diode conducting a constant current, the diode voltage decreases by approximately 2 mV per increase of 1°C.

A decrease in  $V_D$  by 100 mV corresponds to a junction temperature increase of  $50^\circ\text{C}$ .

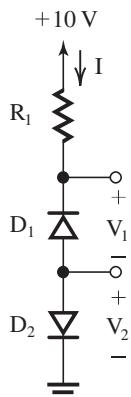
The power dissipation is given by

$$PD = (10 \text{ A})(0.6 \text{ V}) = 6 \text{ W}$$

The thermal resistance is given by

$$\frac{\Delta T}{P_D} = \frac{50^\circ\text{C}}{6 \text{ W}} = 8.33^\circ\text{C/W}$$

3.29



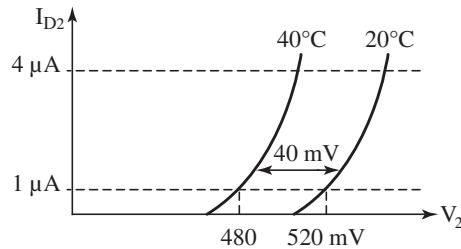
At  $20^\circ\text{C}$ :

$$V_{RI} = V_2 = 520 \text{ mV}$$

$$R_1 = 520 \text{ k}\Omega$$

$$I = \frac{520 \text{ mV}}{520 \text{ k}\Omega} = 1 \mu\text{A}$$

Since the reverse current doubles for every  $10^\circ\text{C}$  rise in temperature, at  $40^\circ\text{C}$ ,  $I = 4 \mu\text{A}$



$$V_2 = 480 + 2.3 \times 1 \times 25 \log 4$$

$$= 514.6 \text{ mV}$$

$$V_{RI} = 4 \mu\text{A} \times 520 \text{ k}\Omega = 2.08 \text{ V}$$

$$\text{At } 0^\circ\text{C}, I = \frac{1}{4} \mu\text{A}$$

$$V_2 = 560 - 2.3 \times 1 \times 25 \log 4$$

$$= 525.4 \text{ mV}$$

$$V_{RI} = \frac{1}{4} \times 520 = 0.13 \text{ V}$$

3.30 Given two different voltage/current measurements for a diode, we can write

$$I_{D1} = I_S e^{V_{D1}/V_T}$$

$$I_{D2} = I_S e^{V_{D2}/V_T}$$

Taking the ratio of the above two equations, we have

$$\begin{aligned} \frac{I_{D1}}{I_{D2}} &= I_S e^{(V_{D1}-V_{D2})/V_T} \Rightarrow V_{D1} - V_{D2} \\ &= V_T \ln\left(\frac{I_{D1}}{I_{D2}}\right) \end{aligned}$$

For  $I_D = 1 \text{ mA}$ , we have

$$\begin{aligned} \Delta V &= V_T \ln\left(\frac{1 \times 10^{-3} \text{ A}}{10 \text{ A}}\right) = -230 \text{ mV} \\ \Rightarrow V_D &= 570 \text{ mV} \end{aligned}$$

For  $I_D = 3 \text{ mA}$ , we have

$$\begin{aligned} \Delta V &= V_T \ln\left(\frac{3 \times 10^{-3} \text{ A}}{10 \text{ A}}\right) = -202 \text{ mV} \\ \Rightarrow V_D &= 598 \text{ mV} \end{aligned}$$

Assuming  $V_D$  changes by  $-2 \text{ mV}$  per  $1^\circ\text{C}$  increase in temperature, we have, for  $\pm 20^\circ\text{C}$  changes:

For  $I_D = 1 \text{ mA}$ ,  $530 \text{ mV} \leq V_D \leq 610 \text{ mV}$

For  $I_D = 3 \text{ mA}$ ,  $558 \text{ mV} \leq V_D \leq 638 \text{ mV}$

Thus the overall range of  $V_D$  is between  $530 \text{ mV}$  and  $638 \text{ mV}$ .

3.31 Given two different voltage/current measurements for a diode, we have

$$\begin{aligned} \frac{I_{D1}}{I_{D2}} &= I_S e^{(V_{D1}-V_{D2})/V_T} \Rightarrow V_{D1} - V_{D2} \\ &= V_T \ln\left(\frac{I_{D1}}{I_{D2}}\right) \end{aligned}$$

For the first diode, with  $I_D = 0.1 \text{ mA}$  and

$$V_D = 700 \text{ mV}, \text{ we have}$$

$I_D = 1 \text{ mA}$ :

$$\begin{aligned} \Delta V &= V_T \ln\left(\frac{1.0}{0.1}\right) = 57.6 \text{ mV} \\ \Rightarrow V_D &= 757.6 \text{ mV} \end{aligned}$$

$I_D = 3 \text{ mA}$ :

$$\Delta V = V_T \ln\left(\frac{3}{0.1}\right) = 85 \text{ mV} \Rightarrow V_D = 785 \text{ mV}$$

For the second diode, with

$I_D = 1 \text{ A}$  and  $V_D = 700 \text{ mV}$ , we have

$I_D = 1.0 \text{ mA}$ :

$$\begin{aligned} \Delta V &= V_T \ln\left(\frac{0.001}{1}\right) = -173 \text{ mV} \\ \Rightarrow V_D &= 527 \text{ mV} \end{aligned}$$

### Chapter 3–9

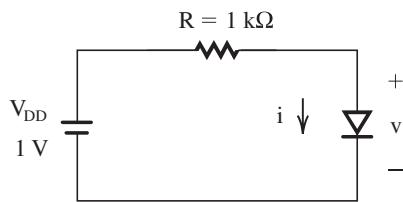
$$I_D = 3 \text{ mA}$$

$$\Delta V = V_T \ln\left(\frac{0.003}{1}\right) = -145 \text{ mV}$$

$$\Rightarrow V_D = 555 \text{ mV}$$

For both  $I_D = 1.0 \text{ mA}$  and  $I_D = 3 \text{ mA}$ , the difference between the two diode voltages is approximately 230 mV. Since, for a fixed diode current, the diode voltage changes with temperature at a constant rate (-2 mV per °C temp. increase), this voltage difference will be independent of temperature!

3.32



$$I_S = 10^{-15} \text{ A} = 10^{-12} \text{ mA}$$

Calculate some points

$$v = 0.6 \text{ V}, \quad i = I_S e^{v/V_T}$$

$$= 10^{-12} e^{0.6/0.025}$$

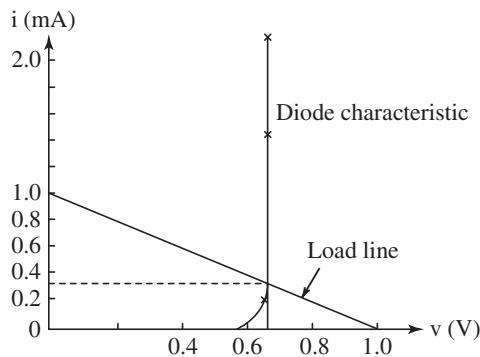
$$\approx 0.03 \text{ mA}$$

$$v = 0.65 \text{ V}, \quad i \approx 0.2 \text{ mA}$$

$$v = 0.7 \text{ V}, \quad i \approx 1.45 \text{ mA}$$

Make a sketch showing these points and load line and determine the operating point. The points for the load line are obtained using

$$I_D = \frac{V_{DD} - V_D}{R}$$



From this sketch one can see that the operating point must lie between  $v = 0.65 \text{ V}$  to  $v = 0.7 \text{ V}$

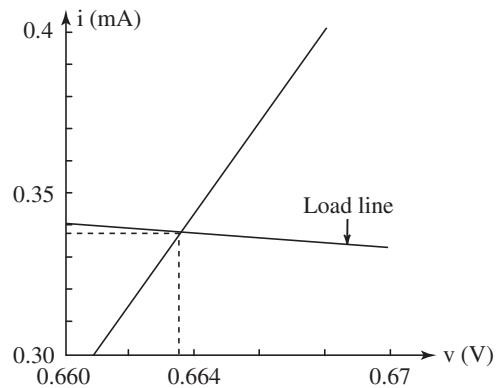
$$\text{For } i = 0.3 \text{ mA}, v = V_T \ln\left(\frac{i}{I_S}\right)$$

$$= 0.025 \times \ln\left(\frac{3}{10^{-12}}\right)$$

$$= 0.661 \text{ V}$$

$$\text{For } i = 0.4 \text{ mA}, v = 0.668 \text{ V}$$

Now we can refine the diagram to obtain a better estimate



From this graph we get the operating point

$$i = 0.338 \text{ mA}, v = 0.6635 \text{ V}$$

Now we compare graphical results with the exponential model.

At  $i = 0.338 \text{ mA}$

$$v = V_T \ln\left(\frac{i}{I_S}\right) = 0.025 \times \ln\left(\frac{0.338}{10^{-12}}\right)$$

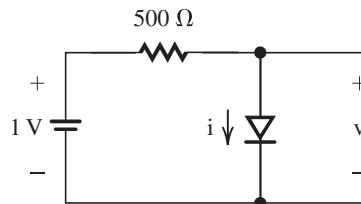
$$= 0.6637 \text{ V}$$

The difference between the exponential model and graphical results is  $= 0.6637 - 0.6635$

$$= 0.0002 \text{ V}$$

$$= 0.2 \text{ mV}$$

3.33



$$\text{a) } I_D = \frac{1 - 0.7}{0.5 \text{ k}\Omega} = 0.6 \text{ mA}$$

b) Diode has 0.7 V drop at 1 mA current. Use Eq. (3.5):

$$v_2 = v_1 + 2.3V_T \log\left(\frac{i_2}{i_1}\right)$$

$$\text{1. } v = 0.7 \text{ V}$$

$$i = \frac{1 - 0.7}{0.5 \text{ k}\Omega} = 0.6 \text{ mA}$$

$$2. v = 0.7 + 2.3 \times 0.025 \log\left(\frac{0.6}{1}\right)$$

$$= 0.6872 \text{ V}$$

$$i = \frac{1 - 0.6872}{0.5} = 0.6255 \text{ mA}$$

$$3. v = 0.7 + 2.3 \times 0.025 \log\left(\frac{0.6255}{1}\right)$$

$$= 0.6882 \text{ V}$$

$$i = \frac{1 - 0.6882}{0.5 \text{ k}\Omega} = 0.6235 \text{ mA}$$

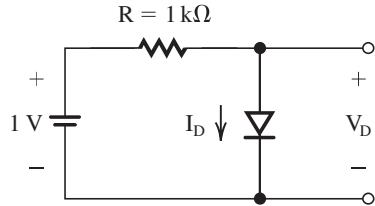
$$4. v = 0.7 + 2.3 \times 0.025 \log\left(\frac{0.6235}{1}\right)$$

$$= 0.6882 \text{ V}$$

$$i = \frac{1 - 0.6882}{0.5 \text{ k}\Omega} = 0.6235 \text{ mA}$$

Stop as we are getting the same result.

3.34



$$I_S = 10^{-15} \text{ A} = 10^{-12} \text{ mA}$$

Use the iterative analysis procedure:

$$1. V_D = 0.7 \text{ V}, I_D = \frac{1 - 0.7}{1 \text{ K}} = 0.3 \text{ mA}$$

$$2. V_D = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.025 \ln\left(\frac{0.3}{10^{-12}}\right) = 0.6607 \text{ V}$$

$$I_D = \frac{1 - 0.6607}{1 \text{ K}} = 0.3393 \text{ mA}$$

$$3. V_D = 0.025 \ln\left(\frac{0.3393}{10^{-12}}\right) = 0.6638 \text{ V}$$

$$I_D = \frac{1 - 0.6638}{1 \text{ K}} = 0.3362 \text{ mA}$$

$$4. V_D = 0.025 \ln\left(\frac{0.3362}{10^{-12}}\right) = 0.6635 \text{ V}$$

$$I_D = \frac{1 - 0.6635}{1 \text{ k}\Omega} = 0.3365 \text{ mA}$$

Stop here as we are getting almost same value of  $I_D$  and  $V_D$

3.35 We first find the value of  $I_S$  for the diode, given by  $I_S = I_{D0} e^{-V_D/V_T}$  with  $I_D = 1 \text{ mA}$  and  $V_D = 0.75 \text{ V}$ . This gives  $I_S = 9.36 \times 10^{-17} \text{ A}$ .

In order to have 3.3 V across the 4 series-connected diodes, each diode drop must be

0.825 V. Applying this voltage to the diode gives current  $I_D = 20.1 \text{ mA}$ . We can then find the resistor value using

$$R = \frac{15 \text{ V} - 3.3 \text{ V}}{20.1 \text{ mA}} = 582 \Omega$$

3.36 Constant voltage drop model:

$$\text{Using } V_D = 0.7 \text{ V}, \Rightarrow i_{D1} = \frac{V - 0.7}{R}$$

$$\text{Using } V_D = 0.6 \text{ V}, \Rightarrow i_{D2} = \frac{V - 0.6}{R}$$

For the difference in currents to be only 1%,

$$\Rightarrow i_{D2} = 1.01 i_{D1}$$

$$V - 0.6 = 1.01(V - 0.7)$$

$$V = 10.7 \text{ V}$$

For  $V = 3 \text{ V}$  and  $R = 1 \text{ k}\Omega$ :

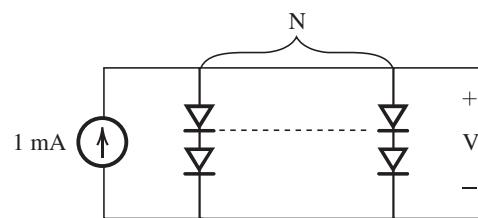
$$\text{At } V_D = 0.7 \text{ V}, i_{D1} = \frac{3 - 0.7}{1} = 2.3 \text{ mA}$$

$$\text{At } V_D = 0.6 \text{ V}, i_{D2} = \frac{3 - 0.6}{1} = 2.4 \text{ mA}$$

$$\frac{i_{D2}}{i_{D1}} = \frac{2.4}{2.3} = 1.04$$

Thus the percentage difference is 4%.

3.37 Available diodes have 0.7 V drop at 2 mA current since  $2V_D = 1.4 \text{ V}$  is close to 1.3 V, use N parallel pairs of diodes to split the 1 mA current evenly, as shown in the figure next.



The voltage drop across each pair of diodes is 1.3 V. ∴ Voltage drop across each diode

$$= \frac{1.3}{2} = 0.65 \text{ V. Using}$$

$$I_2 = I_1 e^{(V_2 - V_1)/V_T}$$

$$= 2e^{(0.65 - 0.7)/0.025}$$

$$= 0.2707 \text{ mA}$$

Thus current through each branch is 0.2707 mA.

The 1 mA will split in  $\frac{1}{0.2707} = 3.69$  branches.

Choose  $N = 4$ .

There are 4 pairs of diodes in parallel.

$\therefore$  We need 8 diodes.

$$\text{Current through each pair of diodes} = \frac{1 \text{ mA}}{4} = 0.25 \text{ mA}$$

$\therefore$  Voltage across each pair

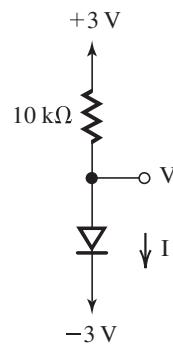
$$= 2 \left[ 0.7 + 0.025 \ln\left(\frac{0.25}{2}\right) \right]$$

$$= 1.296 \text{ V}$$

SPECIAL NOTE: There is another possible design utilizing only 6 diodes and realizing a voltage of 1.313 V. It consists of the series connection of 4 parallel diodes and 2 parallel diodes.

3.39

(a)

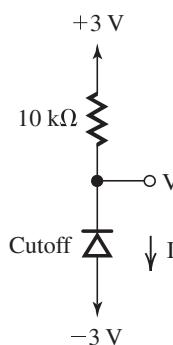


$$V = -3 + 0.7 = -2.3 \text{ V}$$

$$I = \frac{3 + 2.3}{10}$$

$$= 0.53 \text{ mA}$$

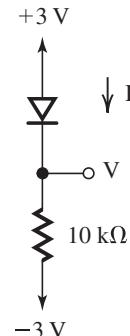
(b)



$$I = 0 \text{ A}$$

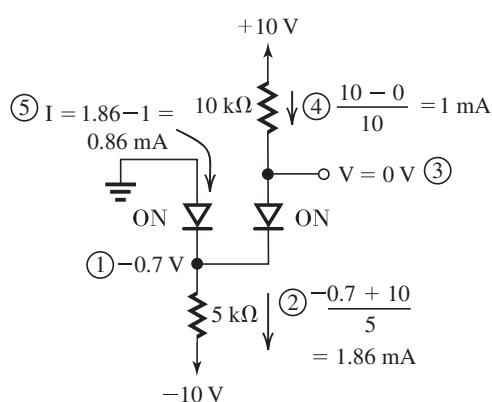
$$V = 3 - I(10) = 3 \text{ V}$$

(c)

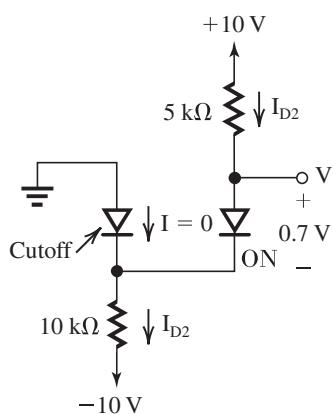


3.38 Refer to Example 3.2.

(a)



(b)



$$I_{D2} = \frac{10 - (-10) - 0.7}{15} = 1.29 \text{ mA}$$

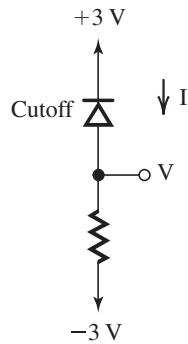
$$V_D = -10 + 1.29(10) + 0.7 = 3.6 \text{ V}$$

$$V = 3 - 0.7 = 2.3 \text{ V}$$

$$I = \frac{2.3 + 3}{10} = 0.53 \text{ mA}$$

(d)

3.41

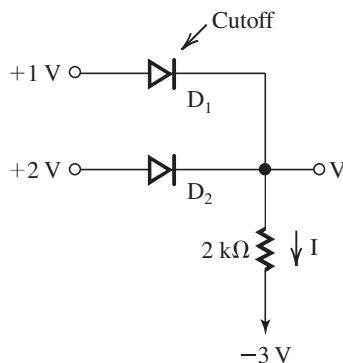


$$I = 0 \text{ A}$$

$$V = -3 \text{ V}$$

3.40

(a)



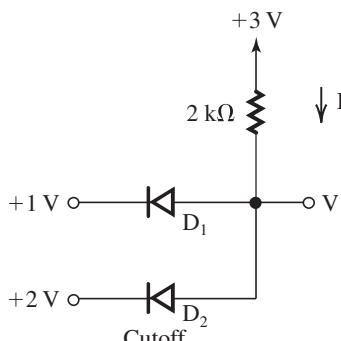
$$V = 2 - 0.7$$

$$= 1.3 \text{ V}$$

$$I = \frac{1.3 - (-3)}{2}$$

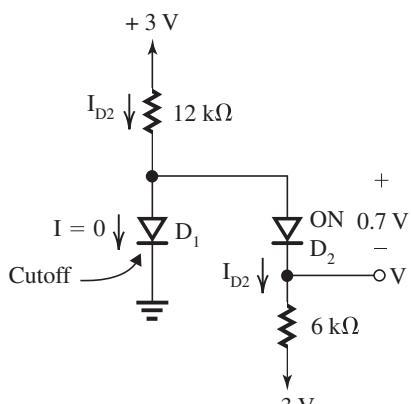
$$= 2.15 \text{ mA}$$

(b)

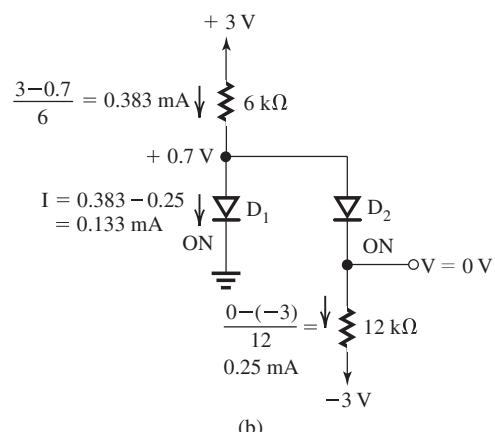


$$V = 1 + 0.7 = 1.7 \text{ V}$$

$$I = \frac{3 - 1.7}{2} = 0.65 \text{ mA}$$



(a)



(b)

$$(a) I_{D2} = \frac{3 - 0.7 - (-3)}{12 + 6} = 0.294 \text{ mA}$$

$$V = -3 + 0.294 \times 6 = -1.23 \text{ V}$$

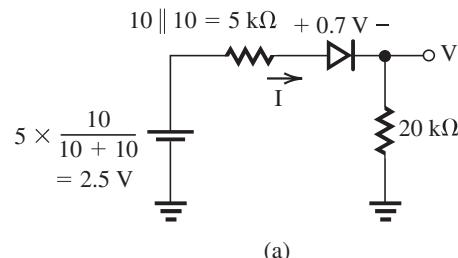
Check that  $D_1$  is off: Voltage at the anode of  $D_1 = V + V_{D2} = -1.23 + 0.7 = -0.53 \text{ V}$  which keeps  $D_1$  off.

(b) See analysis on Fig. (b).

$$I = 0.133 \text{ mA}$$

$$V = 0 \text{ V}$$

3.42



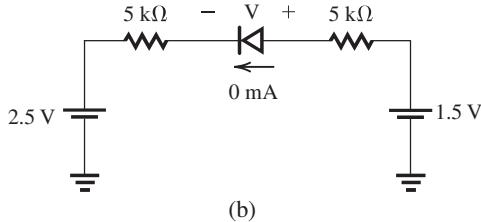
(a)

3.42

$$10 \parallel 10 = 5 \text{ k}\Omega$$

$$+ 0.7 \text{ V} -$$

$$5 \times \frac{10}{10 + 10} = 2.5 \text{ V}$$



$$(a) I = \frac{2.5 - 0.7}{5 + 20} = 0.072 \text{ mA}$$

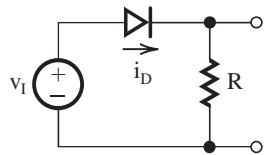
$$V = 0.072 \times 20 = 1.44 \text{ V}$$

(b) The diode will be cut off, thus

$$I = 0$$

$$V = 1.5 - 2.5 = -1 \text{ V}$$

3.43



$$i_{D,\text{peak}} = \frac{v_{I,\text{peak}} - 0.7}{R} \leq 40 \text{ mA}$$

$$R \geq \frac{120\sqrt{2} - 0.7}{40} = 4.23 \text{ k}\Omega$$

$$\text{Reverse voltage} = 120\sqrt{2} = 169.7 \text{ V.}$$

The design is essentially the same since the supply voltage  $\gg 0.7 \text{ V}$

3.44 Use the exponential diode model to find the percentage change in the current.

$$i_D = I_S e^{v/V_T}$$

$$\frac{i_{D2}}{i_{D1}} = e^{(V_2 - V_1)/V_T} = e^{\Delta v/V_T}$$

For  $+5 \text{ mV}$  change we obtain

$$\frac{i_{D2}}{i_{D1}} = e^{5/25} = 1.221$$

% change

$$\begin{aligned} &= \frac{i_{D2} - i_{D1}}{i_{D1}} \times 100 = \frac{1.221 - 1}{1} \times 100 \\ &= 22.1\% \end{aligned}$$

For  $-5 \text{ mV}$  change we obtain

$$\frac{i_{D2}}{i_{D1}} = e^{-5/25} = 0.818$$

$$\begin{aligned} \% \text{ change} &= \frac{i_{D2} - i_{D1}}{i_{D1}} \times 100 = \frac{0.818 - 1}{1} \times 100 \\ &= -18.1\% \end{aligned}$$

Maximum allowable voltage signal change when the current change is limited to  $\pm 10\%$  is found as follows:

The current varies from 0.9 to 1.1

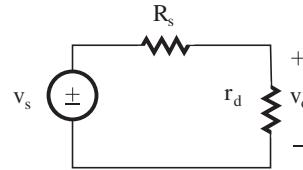
$$\frac{i_{D2}}{i_{D1}} = e^{\Delta v/V_T}$$

$$\text{For } 0.9, \Delta v = 25 \ln(0.9) = -2.63 \text{ mV}$$

$$\text{For } 1.1, \Delta v = 25 \ln(1.1) = +2.38 \text{ mV}$$

For  $\pm 10\%$  current change the voltage signal change is from  $-2.63 \text{ mV}$  to  $+2.38 \text{ mV}$

3.45 The dc current  $I$  flows through the diode giving rise to the diode resistance  $r_d = \frac{V_T}{I}$  and the small-signal equivalent circuit is represented by



$$v_o = v_s \times \frac{r_d}{r_d + R_s} = v_s \frac{V_T/I}{V_T/I + R_s} = v_s \frac{V_T}{V_T + IR_s}$$

$$\text{Now, } v_o = 10 \text{ mV} \times \frac{25 \text{ mV}}{25 \text{ mV} + 10^3 I}$$

$I$	$v_o$
1 mA	0.24 mV
0.1 mA	2.0 mV
1 $\mu$ A	9.6 mV

$$\begin{aligned} \text{For } v_o = \frac{1}{2} v_s = v_s \times \frac{0.025}{0.025 + 10^3 I} \\ \Rightarrow I = 25 \mu\text{A} \end{aligned}$$

3.46 As shown in Problem 3.45,

$$\frac{v_o}{v_i} = \frac{V_T}{V_T + R_s I} = \frac{0.025}{0.025 + 10^4 I} \quad (1)$$

Here  $R_s = 10 \text{ k}\Omega$

The current changes are limited  $\pm 10\%$ . Using exponential model, we get

$$\frac{i_{D2}}{i_{D1}} = e^{\Delta v/V_T} = 0.9 \text{ to } 1.1$$

$$\Delta v = 25 \times 10^{-3} \ln\left(\frac{i_{D2}}{i_{D1}}\right) \text{ and here}$$

$$\text{For } 0.9, \Delta v = -2.63 \text{ mV}$$

$$\text{For } 1.1, \Delta v = 2.38 \text{ mV}$$

The variation is  $-2.63 \text{ mV}$  to  $+2.38 \text{ mV}$  for  $\pm 10\%$  current variation. Thus the largest symmetrical

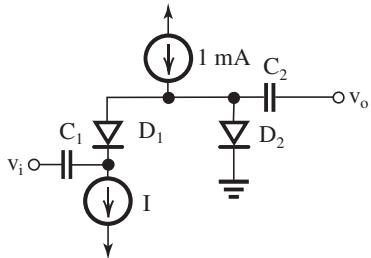
output signal allowed is 2.38 mV in amplitude. To obtain the corresponding input signal, we divide this by ( $v_o/v_i$ ):

$$\hat{v}_s = \frac{2.38 \text{ mV}}{v_o/v_i} \quad (2)$$

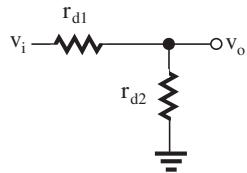
Now for the given values of ( $v_o/v_i$ ) calculate I and  $\hat{v}_s$  using Equations (1) and (2)

$\frac{v_o}{v_i}$	I in mA	$\hat{v}_s$ in mV
0.5	0.0025	4.76
0.1	0.0225	23.8
0.01	0.2475	238
0.001	2.4975	2380

3.47



When both  $D_1$  and  $D_2$  are conducting, the small-signal model is



where we have replaced the large capacitors  $C_1$  and  $C_2$  by short circuits:

$$\frac{v_o}{v_i} = \frac{r_{d2}}{r_{d1} + r_{d2}} = \frac{\frac{V_T}{1 \text{ m} - I}}{\frac{V_T}{I} + \frac{V_T}{1 \text{ m} - I}} = \frac{I}{1 \text{ m}}$$

Thus

$$\frac{v_o}{v_i} = I, \quad \text{where } I \text{ is in mA}$$

$$\text{Now } I = 0 \mu\text{A}, \frac{v_o}{v_i} = 0$$

$$I = 1 \mu\text{A}, \frac{v_o}{v_i} = 1 \times 10^{-3} = 0.001 \text{ V/V}$$

$$I = 10 \mu\text{A}, \frac{v_o}{v_i} = 10 \times 10^{-3} = 0.01 \text{ V/V}$$

$$I = 100 \mu\text{A}, \frac{v_o}{v_i} = 100 \times 10^{-3} = 0.1 \text{ V/V}$$

$$I = 500 \mu\text{A}, \frac{v_o}{v_i} = 500 \times 10^{-3} = 0.5 \text{ V/V}$$

$$I = 600 \mu\text{A}, \frac{v_o}{v_i} = 600 \times 10^{-3} = 0.6 \text{ V/V}$$

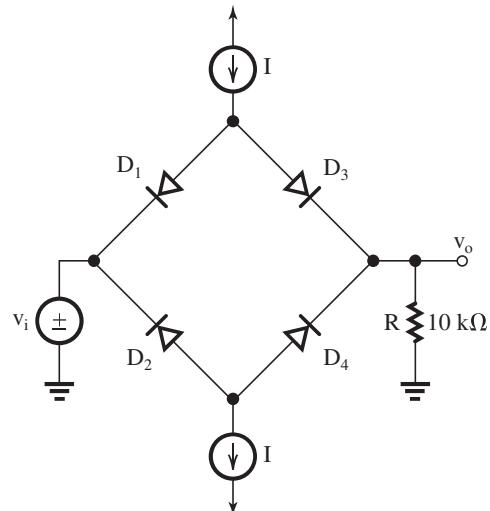
$$I = 900 \mu\text{A}, \frac{v_o}{v_i} = 900 \times 10^{-3} = 0.9 \text{ V/V}$$

$$I = 990 \mu\text{A}, \frac{v_o}{v_i} = 990 \times 10^{-3} = 0.99 \text{ V/V}$$

$$I = 1 \text{ mA}$$

$$= 1000 \mu\text{A}, \frac{v_o}{v_i} = 1000 \times 10^{-3} = 1 \text{ V/V}$$

3.48



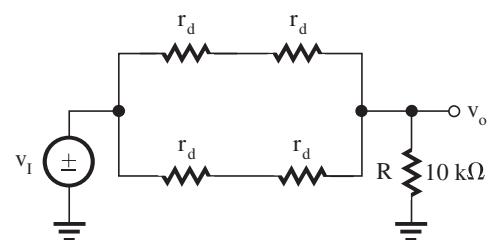
a. The current through each diode is  $\frac{I}{2}$ :

$$r_d = \frac{V_T}{\frac{I}{2}} = \frac{2V_T}{I} = \frac{0.05}{I}$$

From the equivalent circuit

$$\frac{v_o}{v_i} = \frac{R}{R + (2r_d \parallel 2r_d)} = \frac{R}{R + r_d}$$

I	$r_d$	$\frac{v_o}{v_i}$
0 μA	∞	0
1 μA	50 kΩ	0.167
10 μA	5 kΩ	0.667
100 μA	500 Ω	0.952
1 mA	50 Ω	0.995
10 mA	5 Ω	0.9995



Equivalent Circuit

b. For signal current to be limited to  $\pm 10\%$  of  $I$  ( $I$  is the biasing current), the change in diode voltage can be obtained from the equation

$$\frac{i_D}{I} = e^{\Delta v_D/v_T} = 0.9 \text{ to } 1.1$$

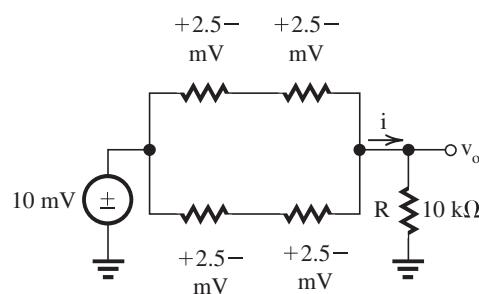
$$\Delta v_D = -2.63 \text{ mV to } +2.32 \text{ mV}$$

$$\simeq \pm 2.5 \text{ mV}$$

so the signal voltage across each diode is limited to 2.5 mV when the diode current remains within 10% of the dc bias current.

$$\therefore v_o = 10 - 2.5 - 2.5 = 5 \text{ mV}$$

$$\text{and } i = \frac{5 \text{ mV}}{10 \text{ k}\Omega} = 0.5 \mu\text{A}$$



The current through each diode

$$= \frac{0.5}{2} \mu\text{A} = 0.25 \mu\text{A}$$

The signal current  $i$  is  $0.5 \mu\text{A}$ , and this is 10% of the dc biasing current.

$$\therefore \text{DC biasing current } I = 0.5 \times 10 = 5 \mu\text{A}$$

c. Now  $I = 1 \text{ mA}$ .

$$\therefore I_D = 0.5 \text{ mA}$$

Maximum current derivation 10%.

$$\therefore i_d = \frac{0.5}{10} = 0.05 \text{ mA}$$

and  $i = 2i_d = 0.1 \text{ mA}$ .

$$\therefore \text{Maximum } v_o = i \times 10 \text{ k}\Omega$$

$$= 0.1 \times 10$$

$$= 1 \text{ V}$$

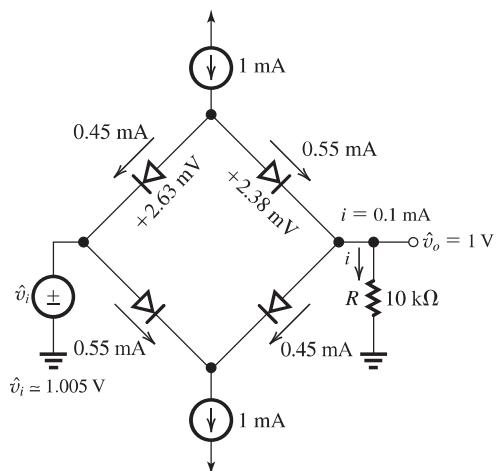
From the results of (a) above, for  $I = 1 \text{ mA}$ ,  $v_o/v_i = 0.995$ ; thus the maximum input signal will be

$$\hat{v}_i = \hat{v}_o / 0.995 = 1 / 0.995$$

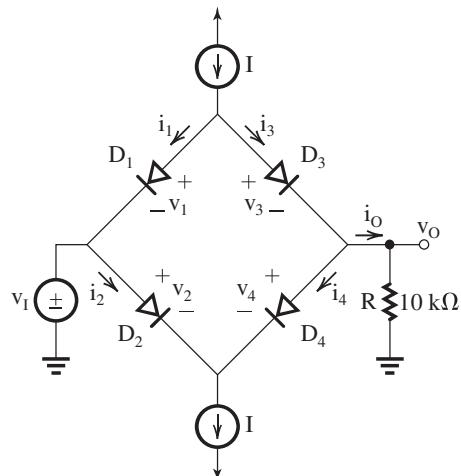
$$= 1.005 \text{ V}$$

The same result can be obtained from the figure above where the signal across the two series diodes is 5 mV, thus

$\hat{v}_i = \hat{v}_o + 5 \text{ mV} = 1 \text{ V} + 5 \text{ mV} = 1.005 \text{ V}$ . See also the figure below.



3.49



$$I = 1 \text{ mA}$$

Each diode exhibits 0.7 V drop at 1 mA current. Using diode exponential model we have

$$v_2 - v_1 = V_T \ln\left(\frac{i_2}{i_1}\right)$$

$$\text{and } v_1 = 0.7 \text{ V}, i_1 = 1 \text{ mA}$$

$$\Rightarrow v = 0.7 + V_T \ln\left(\frac{i}{1}\right)$$

$$= 700 + 25 \ln(i)$$

Calculation for different values of  $v_o$ :

$v_o = 0$ ,  $i_o = 0$ , and the current  $I = 1 \text{ mA}$  divide equally between the  $D_3$ ,  $D_4$  side and the  $D_1$ ,  $D_2$  side.

$$i_1 = i_2 = i_3 = i_4 = \frac{I}{2} = 0.5 \text{ mA}$$

$$v = 700 + 25 \ln(0.5) \simeq 683 \text{ mV}$$

$$v_1 = v_2 = v_3 = v_4 = 683 \text{ mV}$$

From the circuit, we have

$$v_I = -v_1 + v_3 + v_O = -683 + 683 + 0 = 0 \text{ V}$$

$$\text{For } v_O = 1 \text{ V}, i_O = \frac{1}{10 \text{ k}\Omega} = 0.1 \text{ mA}$$

Because of symmetry of the circuit, we obtain

$$i_3 = i_2 = \frac{I}{2} + \frac{i_O}{2} = 0.5 + 0.05 = 0.55 \text{ mA} \text{ and}$$

$$i_4 = i_1 = 0.45 \text{ mA}$$

$$v_3 = v_2 = 700 + 25 \ln\left(\frac{i_2}{1}\right) = 685 \text{ mV}$$

$$v_4 = v_1 = 700 + 25 \ln\left(\frac{i_4}{1}\right) = 680 \text{ mV}$$

$v_O$ (V)	$i_O$ (mA)	$i_3 = i_2$ (mA)	$i_4 = i_1$ (mA)	$v_3 = v_2$ (mV)	$v_4 = v_1$ (mV)	$v_I = -v_1 + v_3 + v_O$ (V)
0	0	0.5	0.5	683	683	0
+1	0.1	0.55	0.45	685	680	1.005
+2	0.2	0.6	0.4	~687	677	2.010
+5	0.5	0.75	0.25	~693	665	5.028
+9	0.9	0.95	0.05	~699	~625	9.074
+9.9	0.99	0.995	0.005	~700	568	10.09
9.99	0.999	0.9995	0.0005	~700	510	10.18
10	1	1	0	700	0	10.7

$$v_I = -v_1 + v_2 + v_O = -0.680$$

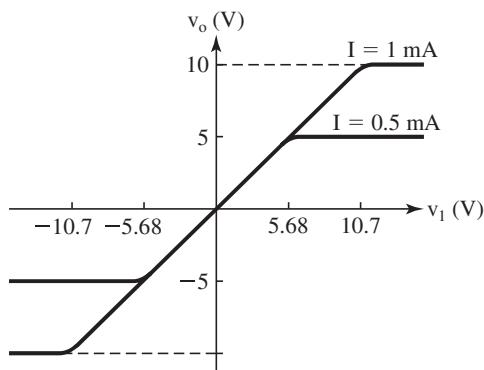
$$+0.685 + 1 = 1.005 \text{ V}$$

Similarly, other values are calculated as shown in the table. The largest values of  $v_O$  on positive and negative side are +10 V and -10 V, respectively. This restriction is imposed by the current  $I = 1 \text{ mA}$

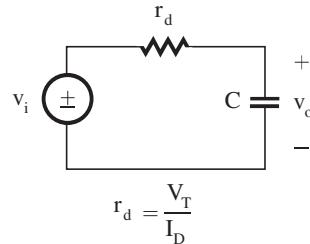
A similar table can be generated for the negative values. It is symmetrical.

For  $v_I > +10.7 \text{ V}$ ,  $v_O$  will be saturated at +10 V and it is because  $I = 1 \text{ mA}$ . Similarly, for  $v_I < -10.7 \text{ V}$ ,  $v_O$  will be saturated at -10 V.

For  $I = 0.5 \text{ mA}$ , the output will saturate at  $0.5 \text{ mA} \times 10 \text{ k}\Omega = 5 \text{ V}$ .



3.50 Representing the diode by the small-signal resistances, the circuit is



$$r_d = \frac{V_T}{I_D}$$

$$\frac{V_o}{V_i} = \frac{\frac{1}{sC}}{\frac{1}{r_d + \frac{1}{sC}}} = \frac{1}{1 + sCr_d}$$

$$\frac{V_o}{V_i} = \frac{1}{1 + j\omega Cr_d}$$

$$\text{Phase shift} = -\tan^{-1}\left(\frac{\omega Cr_d}{1}\right)$$

$$= -\tan^{-1}\left(\omega C \frac{V_T}{I}\right)$$

For phase shift of  $-45^\circ$ , we obtain

$$-45 = -\tan^{-1}\left(2\pi \times 100 \times 10^3 \times 10 \times 10^{-9} \times \frac{0.025}{I}\right)$$

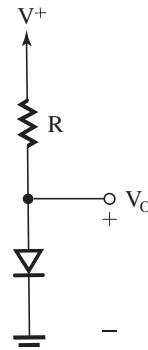
$$\Rightarrow I = 157 \mu\text{A}$$

Now  $I$  varies from  $\frac{157}{10} \mu\text{A}$  to  $157 \times 10 \mu\text{A}$

Range is  $15.7 \mu\text{A}$  to  $1570 \mu\text{A}$

Range of phase shift is  $-84.3^\circ$  to  $-5.71^\circ$

3.51

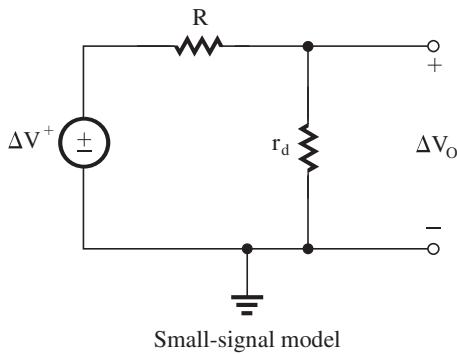


$$(a) \frac{\Delta V_o}{\Delta V^+} = \frac{r_d}{R + r_d} = \frac{V_T/I}{R + \frac{V_T}{I}}$$

$$= \frac{V_T}{IR + V_T}$$

$$\text{For no load, } I = \frac{V^+ - V_o}{R} = \frac{V^+ - 0.7}{R}$$

$$\therefore \frac{\Delta V_O}{\Delta V^+} = \frac{V_T}{V_T + (V^+ - 0.7)}$$



(b) If m diodes are in series, we obtain

$$\begin{aligned}\frac{\Delta V_O}{\Delta V^+} &= \frac{m r_d}{m r_d + R} = \frac{m V_T}{m V_T + I R} \\ &= \frac{m V_T}{m V_T + (V^+ - 0.7m)}\end{aligned}$$

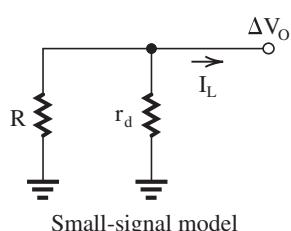
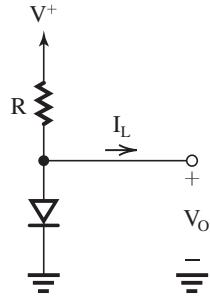
(c) For  $m = 1$

$$\frac{\Delta V_O}{\Delta V^+} = \frac{V_T}{V_T + V^+ - 0.7} = 1.75 \text{ mV/V}$$

For  $m = 4$

$$\frac{\Delta V_O}{\Delta V^+} = \frac{m V_T}{m V_T + 15 - m \times 0.7} = 8.13 \text{ mV/V}$$

3.52



(a) From the small-signal model

$$\Delta V_O = -I_L (r_d \parallel R)$$

$$\frac{\Delta V_O}{I_L} = -(r_d \parallel R)$$

$$(b) \text{ At no load, } I_D = \frac{V^+ - 0.7}{R}$$

$$r_d = \frac{V_T}{I_D}$$

$$\frac{\Delta V_O}{I_L} = -(r_d \parallel R) = -\frac{1}{\frac{1}{r_d} + \frac{1}{R}}$$

$$= -\frac{1}{\frac{I_D}{V^+ - 0.7} + \frac{I_D}{V_T}}$$

$$= -\frac{V_T}{I_D} \times \frac{1}{\frac{V_T}{V^+ - 0.7} + 1}$$

$$= -\frac{V_T}{I_D} \times \frac{V^+ - 0.7}{V_T + V^+ - 0.7}$$

$$\text{For } \frac{\Delta V_O}{I_L} \leq 5 \frac{\text{mV}}{\text{mA}}$$

$$\text{i.e., } \frac{V_T}{I_D} \times \frac{V^+ - 0.7}{V_T + V^+ - 0.7} \leq \frac{5 \text{ mV}}{\text{mA}}$$

$$\left[ \frac{25}{I_D} \times \frac{15 - 0.7}{0.025 + 15 - 0.7} \right] \leq 5 \frac{\text{mV}}{\text{mA}}$$

$$I_D \geq 4.99 \text{ mA}$$

$$I_D \simeq 5 \text{ mA}$$

$$R = \frac{V^+ - 0.7}{I_D} = \frac{15 - 0.7}{5 \text{ mA}}$$

$$R = 2.86 \text{ k}\Omega$$

Diode should be a 5-mA unit; that is, it conducts 5 mA at  $V_D = 0.7 \text{ V}$ , thus  $5 \times 10^{-3} = I_S e^{0.7/0.025}$ .

$$\Rightarrow I_S = 3.46 \times 10^{-15} \text{ A}$$

(c) For m diodes connected in series we have

$$I_D = \frac{V^+ - 0.7m}{R}$$

$$\text{and } r_d = \frac{V_T}{I_D}$$

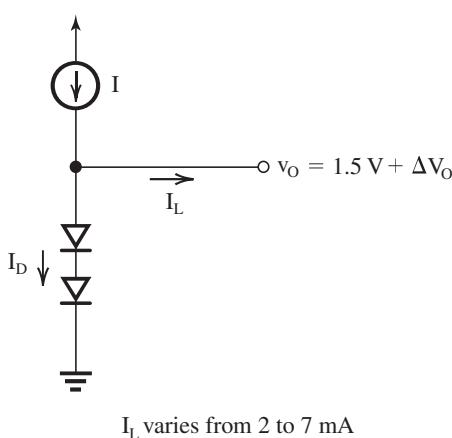
$$\text{So now } \frac{\Delta V_O}{I_L} = -(R \parallel m r_d) = -\frac{1}{\frac{1}{R} + \frac{1}{m r_d}}$$

$$= -\frac{1}{\frac{I_D}{V^+ - 0.7m} + \frac{I_D}{m V_T}}$$

$$= -\frac{m V_T}{I_D} \frac{m V_T}{V^+ - 0.7m + m V_T} + 1$$

$$= -\frac{m V_T}{I_D} \frac{V^+ - 0.7m}{V^+ - 0.7m + m V_T}$$

3.53



To supply a load current of 2 to 7 mA, the current  $I$  must be greater than 7 mA. So  $I$  can be only 10 mA or 15 mA.

Now let us evaluate  $\Delta V_o$  for both 10-mA and 15-mA sources. For the 10-mA source:

Since  $I_L$  varies from 2 to 7 mA, the current  $I_D$  will vary from 8 to 3 mA.

Correspondingly, the voltage across each diode changes by  $\Delta V_D$  where

$$\frac{3}{8} = e^{\Delta V_D / V_T}$$

$$\Rightarrow \Delta V_D = 25 \ln\left(\frac{3}{8}\right) = -24.5 \text{ mV}$$

and the output voltage changes by

$$\Delta V_o = 2 \times \Delta V_D = -49 \text{ mV}$$

With  $I = 15 \text{ mA}$ , the diodes current changes from 13 to 8 mA. Correspondingly, the voltage across each diode changes by  $\Delta V_D$  where

$$\frac{8}{13} = e^{\Delta V_D / V_T}$$

$$\Rightarrow \Delta V_D = 25 \ln\left(\frac{8}{13}\right) = -12.1 \text{ mV}$$

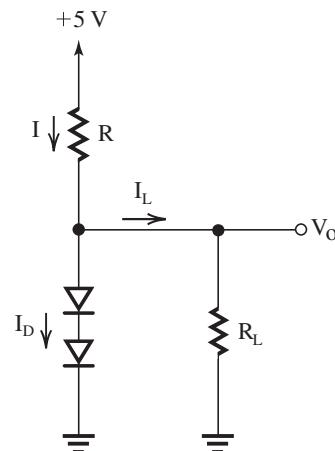
and the output voltage changes by

$$\Delta V_o = 2 \times \Delta V_D = -24.2 \text{ mV}$$

which is less than half that obtained with the 10-mA supply. Thus, from the point of view of reducing the change in  $V_o$  as  $I_L$  changes, we

choose the 15-mA supply. Note, however, that the price paid is increased power dissipation.

3.54



Diode has 0.7 V drop at 1 mA current

$$V_o = 1.5 \text{ V} \text{ when } R_L = 1.5 \text{ k}\Omega$$

$$I_D = I_S e^{V_o / V_T}$$

$$1 \times 10^{-3} = I_S e^{0.7 / 0.025}$$

$$\Rightarrow I_S = 6.91 \times 10^{-16} \text{ A}$$

$$\text{Voltage drop across each diode} = \frac{1.5}{2} = 0.75 \text{ V.}$$

$$\therefore I_D = I_S e^{V_o / V_T} = 6.91 \times 10^{-16} \times e^{0.75 / 0.025} \\ = 7.38 \text{ mA}$$

$$I_L = 1.5 / 1.5 = 1 \text{ mA}$$

$$I = I_D + I_L = 7.39 \text{ mA} + 1 \text{ mA}$$

$$= 8.39 \text{ mA}$$

$$\therefore R = \frac{5 - 1.5}{8.39 \text{ mA}} = 417 \Omega$$

Use a small-signal model to find voltage  $\Delta V_o$  when the value of the load resistor,  $R_L$ , changes:

$$r_d = \frac{V_T}{I_D} = \frac{0.025}{7.39} = 3.4 \Omega$$

When load is disconnected, all the current  $I$  flows through the diode. Thus

$$\Delta I_D = 1 \text{ mA}$$

$$\Delta V_o = \Delta I_D \times 2r_d$$

$$= 1 \times 2 \times 3.4$$

$$= 6.8 \text{ mV}$$

$$\text{With } R_L = 1 \text{ k}\Omega,$$

$$I_L \approx \frac{1.5 \text{ V}}{1} = 1.5 \text{ mA}$$

$$\Delta I_L = 0.5 \text{ mA}$$

$$\Delta I_D = -0.5 \text{ mA}$$

$$\Delta V_O = -0.5 \times 2 \times 3.4$$

$$= -3.4 \text{ mV}$$

With  $R_L = 750 \Omega$ ,

$$I_L \approx \frac{1.5}{0.75} = 2 \text{ mA}$$

$$\Delta I_L = 1 \text{ mA}$$

$$\Delta I_D = -1 \text{ mA}$$

$$\Delta V_O = -1 \times 2 \times 3.4$$

$$= -6.8 \text{ mV}$$

With  $R_L = 500 \Omega$ ,

$$I_L \approx \frac{1.5}{0.5} = 3 \text{ mA}$$

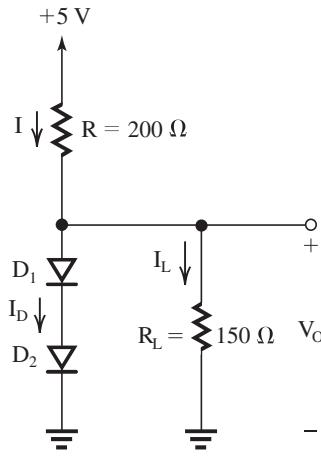
$$\Delta I_L = 2.0 \text{ mA}$$

$$\Delta I_D = -2.0 \text{ mA}$$

$$\Delta V_O = -2 \times 2 \times 3.4$$

$$= -13.6 \text{ mV}$$

3.55



(a) Iteration #1:

$$V_D = 0.7 \text{ V}$$

$$V_O = 2V_D = 1.4 \text{ V}$$

$$I_L = \frac{V_O}{R_L} = \frac{1.4}{0.15} = 9.33 \text{ mA}$$

$$I = \frac{5 - V_O}{R} = \frac{5 - 1.4}{0.2} = 18 \text{ mA}$$

$$I_D = I - I_L = 18 - 9.33 = 8.67 \text{ mA}$$

Iteration #2:

$$V_D = 0.7 + 0.025 \ln\left(\frac{8.67}{10}\right) = 0.696 \text{ V}$$

$$V_O = 1.393 \text{ V}$$

$$I_L = 9.287 \text{ mA}$$

$$I = \frac{5 - 1.393}{0.2} = 18.04 \text{ mA}$$

$$I_D = 18.04 - 9.287 = 8.753 \text{ mA}$$

Iteration #3:

$$V_D = 0.7 + 0.025 \ln\left(\frac{8.753}{10}\right) = 0.697$$

$$V_O = 1.393 \text{ V}$$

$$I_L = 9.287$$

$$I = 18.04 \text{ mA}$$

$$I_D = 8.753$$

No further iterations are necessary and

$$V_O = 1.39 \text{ V}$$

(b) With no load:

Iteration #1:

$$V_D = 0.7 \text{ V}$$

$$V_O = 1.4 \text{ V}$$

$$I = \frac{5 - 1.4}{0.2} = 18 \text{ mA}$$

$$I_D = I = 18 \text{ mA}$$

Iteration #2:

$$V_D = 0.7 + 0.025 \ln\left(\frac{18}{10}\right) = 0.715 \text{ V}$$

$$V_O = 1.429 \text{ V}$$

$$I = 17.85 \text{ mA}$$

$$I_D = 17.85 \text{ mA}$$

Iteration #3:

$$V_D = 0.7 + 0.025 \ln\left(\frac{17.85}{10}\right) = 0.714 \text{ V}$$

$$V_O = 1.43 \text{ V}$$

$$I = 17.86 \text{ mA}$$

$$I_D = 17.86 \text{ mA}$$

No further iterations are warranted and

$$V_O = 1.43 \text{ V}$$

$$(c) V_O = 1.39 - 0.1 = 1.29 \text{ V}$$

$$I_L = \frac{1.29}{0.15} = 8.6 \text{ mA}$$

$$V_D = \frac{1.29}{2} = 0.645 \text{ V}$$

$$I_D = 10 \times e^{(0.645-0.7)/0.025}$$

$$= 1.11 \text{ mA}$$

$$I = I_L + I_D = 8.6 + 1.11 = 9.71 \text{ mA}$$

$$V_{\text{Supply}} = V_O + IR = 1.29 + 9.71 \times 0.2$$

$$= 3.232 \text{ V}$$

which is a reduction of 1.768 V or  $-35.4\%$ .

$$(d) \text{ For } V_{\text{Supply}} = 5 + 1.786 = 6.786 \text{ V,}$$

Iteration #1:

$$V_D = 0.7 \text{ V}$$

$$V_O = 1.4 \text{ V}$$

$$I_L = 9.33 \text{ mA}$$

$$I = \frac{6.768 - 1.4}{0.2} = 26.84$$

$$I_D = I - I_L = 26.84 - 9.33 = 17.51 \text{ mA}$$

Iteration #2:

$$V_D = 0.7 + 0.025 \ln\left(\frac{17.51}{10}\right) = 0.714 \text{ V}$$

$$V_O = 1.428 \text{ V}$$

$$I_L = 9.52 \text{ mA}$$

$$I = 26.70 \text{ mA}$$

$$I_D = 17.18 \text{ mA}$$

Iteration #3:

$$V_D = 0.7 + 0.025 \ln\left(\frac{17.18}{10}\right) = 0.714 \text{ V}$$

$$V_O = 1.428 \text{ V}$$

No further iterations are needed and

$$V_O = 1.43 \text{ V}$$

(e) From the above we see that as  $V_{\text{Supply}}$  changes from 5 V to 3.232 V (a change of  $-35.4\%$ ) the output voltage changes from 1.39 V to 1.29 V (a change of  $-7.19\%$ ).

As  $V_{\text{Supply}}$  changes from 5 V to 6.786 V (a change of  $+35.4\%$ ) the output voltage changes from 1.39 V to 1.43 V (a change of  $+2.88\%$ ).

Thus the worst-case situation occurs when  $V_{\text{Supply}}$  is reduced, and

Percentage change in  $V_O$  per 1% change in

$$V_{\text{Supply}} = \frac{7.19}{35.4} = 0.2\%$$

$$3.56 \text{ V}_Z = V_{Z0} + I_{ZT}r_z$$

$$(a) 10 = 9.6 + 0.05 \times r_z$$

$$\Rightarrow r_z = 8 \Omega$$

$$\text{For } I_Z = 2I_{ZT} = 100 \text{ mA,}$$

$$V_Z = 9.6 + 0.1 \times 8 = 10.4 \text{ V}$$

$$P = 10.4 \times 0.1 = 1.04 \text{ W}$$

$$(b) 9.1 = V_{Z0} + 0.01 \times 30$$

$$\Rightarrow V_{Z0} = 8.8 \text{ V}$$

$$\text{At } I_Z = 2I_{ZT} = 20 \text{ mA,}$$

$$V_Z = 8.8 + 0.02 \times 30 = 9.4 \text{ V}$$

$$P = 9.4 \times 20 = 188 \text{ mW}$$

$$(c) 6.8 = 6.6 + I_{ZT} \times 2$$

$$\Rightarrow I_{ZT} = 0.1 \text{ A}$$

$$\text{At } I_Z = 2I_{ZT} = 0.2 \text{ A,}$$

$$V_Z = 6.6 + 0.2 \times 2 = 7 \text{ V}$$

$$P = 7 \times 0.2 = 1.4 \text{ W}$$

$$(d) 18 = 17.6 + 0.005 \times r_z$$

$$\Rightarrow r_z = 80 \Omega$$

At  $I_Z = 2I_{ZT} = 0.01 \text{ A}$ ,

$$V_Z = 17.6 + 0.01 \times 80 = 18.4 \text{ V}$$

$$P = 18.4 \times 0.01 = 0.184 \text{ W} = 184 \text{ mW}$$

$$(e) 7.5 = V_{Z0} + 0.2 \times 1.5$$

$$\Rightarrow V_{Z0} = 7.2 \text{ V}$$

At  $I_Z = 2I_{ZT} = 0.4 \text{ A}$ ,

$$V_Z = 7.2 + 0.4 \times 1.5 = 7.8 \text{ V}$$

$$P = 7.8 \times 0.4 = 3.12 \text{ W}$$

$$3.57 \quad V_Z = V_{Z0} + I_Z r_z$$

$$9.1 = V_{Z0} + 0.02 \times 10$$

$$\Rightarrow V_{Z0} = 8.9 \text{ V}$$

At  $I_Z = 10 \text{ mA}$ ,

$$V_Z = 8.9 + 0.01 \times 10 = 9.0 \text{ V}$$

At  $I_Z = 50 \text{ mA}$ ,

$$V_Z = 8.9 + 0.05 \times 10 = 9.4 \text{ V}$$

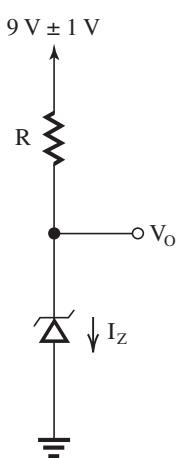
3.58 (a) Three 6.8-V zeners provide  $3 \times 6.8 = 20.4 \text{ V}$  with  $3 \times 10 = 30\Omega$  resistance. Neglecting  $R$ , we have

Load regulation =  $-30 \text{ mV/mA}$ .

(b) For 5.1-V zeners we use 4 diodes to provide 20.4 V with  $4 \times 30 = 120\Omega$  resistance.

Load regulation =  $-120 \text{ mV/mA}$

3.59



#### GIVEN PARAMETERS

$$V_Z = 6.8 \text{ V}, r_z = 5 \Omega$$

$$I_Z = 20 \text{ mA}$$

At knee,

$$I_{ZK} = 0.25 \text{ mA}$$

$$r_z = 750 \Omega$$

FIRST DESIGN: 9-V supply can easily supply current

Let  $I_Z = 20 \text{ mA}$ , well above knee.

$$\therefore R = \frac{9 - 6.8}{20} = 110 \Omega$$

$$\text{Line regulation} = \frac{\Delta V_o}{\Delta V_s} = \frac{r_z}{r_z + R}$$

$$= \frac{5}{5 + 110}$$

$$= 43.5 \frac{\text{mV}}{\text{V}}$$

SECOND DESIGN: limited current from 9-V supply

$$I_Z = 0.25 \text{ mA}$$

$$V_Z = V_{ZK} \simeq V_{Z0} - \text{calculate } V_{Z0} \text{ from}$$

$$V_Z = V_{Z0} + r_z I_{ZT}$$

$$6.8 = V_{Z0} + 5 \times 0.02$$

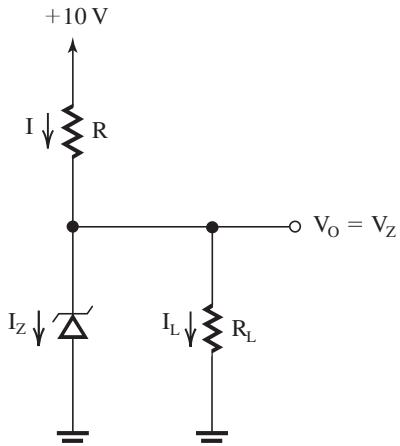
$$V_{Z0} = 6.7 \text{ V}$$

$$\therefore R = \frac{8 - 6.7}{0.25} = 5.2 \text{ k}\Omega$$

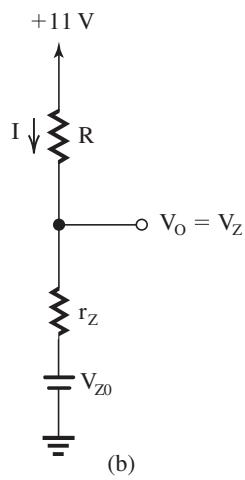
$$\text{LINE REGULATION} = \frac{\Delta V_o}{\Delta V_s} = \frac{750}{750 + 5200}$$

$$= 126 \frac{\text{mV}}{\text{V}}$$

3.60



(a)



$$= \frac{1.5 \parallel 0.03}{(1.5 \parallel 0.03) + 0.167} = 0.15$$

For  $\Delta V_S = +1\text{ V}$  (10% high),  $\Delta V_O = +0.15\text{ V}$  and  $V_O = 7.65\text{ V}$ .

For  $\Delta V_S = -1\text{ V}$  (10% low),  $\Delta V_O = -0.15\text{ V}$  and  $V_O = 7.35\text{ V}$ .

When the load is removed and  $V_S = 11\text{ V}$ , we can use the zener model to determine  $V_O$ . Refer to Fig. (b). To determine  $V_{Z0}$ , we use

$$V_Z = V_{Z0} + I_{ZT}r_z$$

$$7.5 = V_{Z0} + 0.01 \times 30$$

$$\Rightarrow V_{Z0} = 7.2\text{ V}$$

From Fig. (b) we have

$$I = \frac{11 - 7.2}{0.167 + 0.03} = 19.3\text{ mA}$$

Thus

$$V_O = V_{Z0} + Ir_z$$

$$= 7.2 + 0.0193 \times 30 = 7.78\text{ V}$$

To determine the smallest allowable value of  $R_L$  while  $V_S = 9\text{ V}$ , refer to Fig. (c). Note that  $I_Z = 0.5\text{ mA}$ , thus

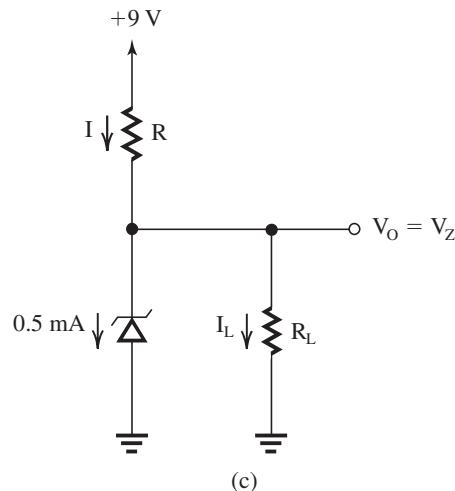
$$V_Z = V_{ZK} \approx V_{Z0} = 7.2\text{ V}$$

$$I = \frac{9 - 7.2}{0.167} = 10.69\text{ mA}$$

$$I_L = I - I_Z = 10.69 - 0.5 = 10.19\text{ mA}$$

$$R_L = \frac{V_O}{I_L} = \frac{7.2}{10.19} = 707\Omega$$

$$V_O = 7.2\text{ V}$$



To obtain  $V_O = 7.5\text{ V}$ , we must arrange for  $I_Z = 10\text{ mA}$  (the current at which the zener is specified).

Now,

$$I_L = \frac{V_O}{R_L} = \frac{7.5}{1.5} = 5\text{ mA}$$

Thus

$$I = I_Z + I_L = 10 + 5 = 15\text{ mA}$$

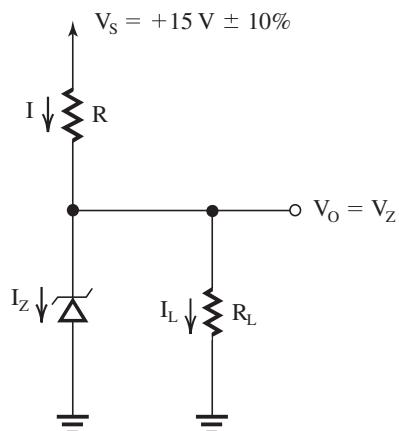
and

$$R = \frac{10 - V_O}{I} = \frac{10 - 7.5}{15} = 167\Omega$$

When the supply undergoes a change  $\Delta V_S$ , the change in the output voltage,  $\Delta V_O$ , can be determined from

$$\frac{\Delta V_O}{\Delta V_S} = \frac{(R_L \parallel r_z)}{(R_L \parallel r_z) + R}$$

3.61



$$V_Z = V_{Z0} + I_Z r_z$$

$$9.1 = V_{Z0} + 0.009 \times 40$$

$$\Rightarrow V_{Z0} = 8.74 \text{ V}$$

For  $I_Z = 10 \text{ mA}$ ,

$$V_Z = 8.74 + 0.01 \times 40 = 9.14 \text{ V}$$

$$I_L = \frac{9.14}{1 \text{ k}\Omega} = 9.14 \text{ mA}$$

$$I = I_Z + I_L = 10 + 9.14 = 19.14 \text{ mA}$$

$$R = \frac{15 - 9.14}{19.4} = 306 \Omega$$

Select  $R = 300 \Omega$

Denoting the resulting output voltage  $V_O$ , we obtain

$$I = \frac{15 - V_O}{0.3} \quad (1)$$

$$I_L = \frac{V_O}{1} \quad (2)$$

$$I_Z = \frac{V_O - V_{Z0}}{r_z} = \frac{V_O - 8.74}{0.04} \quad (3)$$

Since  $I = I_Z + I_L$ , we can use (1)–(3) to obtain  $V_O$ :

$$\frac{15 - V_O}{0.3} = \frac{V_O - 8.74}{0.04} + V_O$$

$$\Rightarrow V_O = 9.15 \text{ V}$$

$$\Delta V_O = \Delta V_S \frac{r_z \parallel R_L}{(r_z \parallel R_L) + R}$$

$$= \pm 1.5 \times \frac{(0.04 \parallel 1)}{(0.04 \parallel 1) + 0.3}$$

$$= \pm 0.17 \text{ V}$$

If  $I_L$  is reduced by 50%, then

$$I_L = \frac{1}{2} \times \frac{9.15}{1} = 4.6 \text{ mA}$$

$$I = \frac{15 - V_O}{0.3}$$

$$I_Z = \frac{V_O - 8.74}{0.04}$$

$$\frac{15 - V_O}{0.3} = \frac{V_O - 8.74}{0.04} + 4.6$$

$$\Rightarrow V_O = 9.31 \text{ V}$$

which is an increase of 0.16 V. When the supply voltage is low,

$$V_S = 13.5 \text{ V}$$

and  $R_L$  is at its lowest value, to maintain regulation, the zener current must be at least equal to  $I_{ZK}$ , thus

$$I_Z = 0.5 \text{ mA}$$

$$V_Z = V_{ZK} \simeq V_{Z0} \simeq 8.74$$

$$I = \frac{13.5 - 8.74}{0.3} = 15.87 \text{ mA}$$

$$I_L = I - I_Z = 15.87 - 0.5 = 15.37 \text{ mA}$$

$$R_L = \frac{V_Z}{I_L} = \frac{8.74}{15.37} = 589 \Omega$$

The lowest value of output voltage = 8.74 V

$$\text{Line regulation} = \frac{170 \text{ mV}}{1.5 \text{ V}}$$

$$= 113 \text{ mV/V}$$

$$\text{Load regulation} = -(r_z \parallel R)$$

$$= -(40 \parallel 300) = -35 \text{ mV/mA}$$

Or using the results obtained in this problem:

For a reduction in  $I_L$  of 4.6 mA,  $\Delta V_O = +0.16 \text{ V}$ , thus

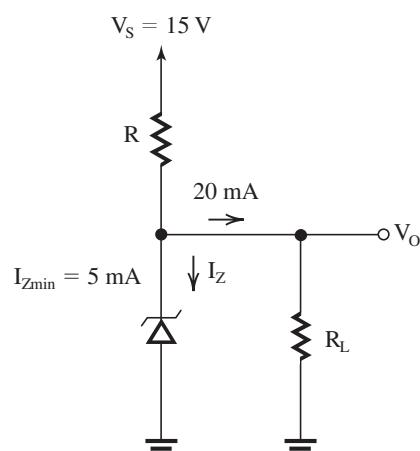
$$\text{Load regulation} = -\frac{160}{4.6} = -35 \text{ mV/mA}$$

$$3.62 \text{ (a)} \quad V_{ZT} = V_{Z0} + r_z I_{ZT}$$

$$10 = V_{Z0} + 7(0.025)$$

$$\Rightarrow V_{Z0} = 9.825 \text{ V}$$

(b) The minimum zener current of 5 mA occurs when  $I_L = 20 \text{ mA}$  and  $V_S$  is at its minimum of  $20(1 - 0.25) = 15 \text{ V}$ . See the circuit below:



$$R \leq \frac{15 - V_{Z0}}{20 + 5}$$

where we have used the minimum value of  $V_S$ , the maximum value of load current, and the minimum required value of zener diode current, and we assumed that at this current  $V_Z \approx V_{Z0}$ . Thus,

$$R \leq \frac{15 - 9.825 + 7}{25}$$

$$\leq 207 \Omega.$$

$$\therefore \text{use } R = 207 \Omega$$

$$(c) \text{ Line regulation} = \frac{7}{207 + 7} = 33 \frac{\text{mV}}{\text{V}}$$

$$\pm 25\% \text{ change in } v_S \equiv \pm 5 \text{ V}$$

$$V_O \text{ changes by } \pm 5 \times 33 = \pm 0.165 \text{ mV}$$

$$\text{corresponding to } \frac{\pm 0.165}{10} \times 100 = \pm 1.65\%$$

$$(d) \text{ Load regulation} = -(r_Z \parallel R)$$

$$= -(7 \parallel 207) = -6.77 \Omega$$

$$\text{or } -6.77 \text{ V/A}$$

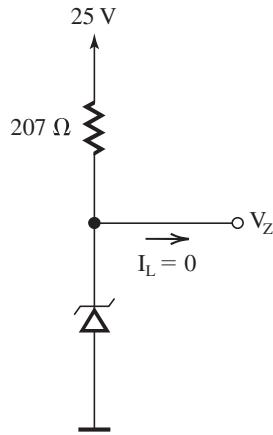
$$\Delta V_O = -6.77 \times 20 \text{ mA} = -135.4 \text{ mV}$$

$$\text{corresponding to } -\frac{0.1354}{10} \times 100 = -1.35\%$$

(e) The maximum zener current occurs at no load  $I_L = 0$  and the supply at its largest value of

$$20 + \frac{1}{4}(20) = 25 \text{ V.}$$

$$V_Z = V_{Z0} + r_Z I_Z$$



$$= 9.825 + 7 \times \frac{25 - V_Z}{207}$$

$$207V_Z = 207(9.825) + 7(25) - 7V_Z$$

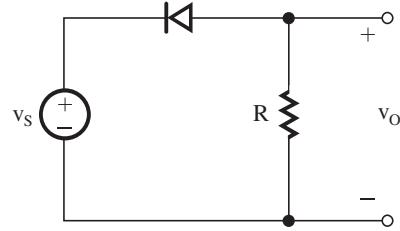
$$\Rightarrow V_Z = 10.32 \text{ V}$$

$$I_{Z\max} = \frac{25 - 10.32}{0.207} = 70.9 \text{ mA}$$

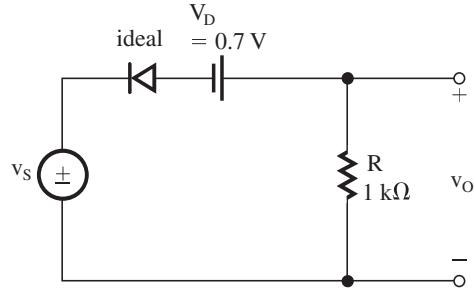
$$P_Z = 10.32 \times 70.9$$

$$= 732 \text{ mW}$$

3.63

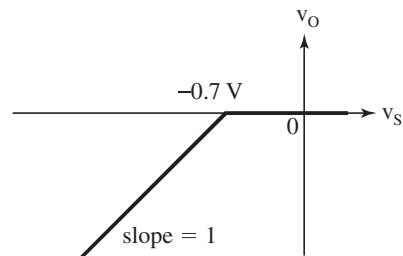


Using the constant voltage drop model:

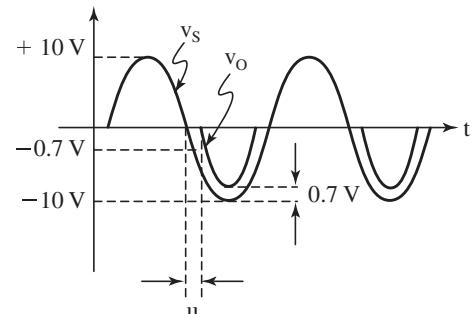


$$(a) V_O = V_S + 0.7 \text{ V, For } V_S \leq -0.7 \text{ V}$$

$$V_O = 0, \text{ for } V_S \geq -0.7 \text{ V}$$



(b)



(c) The diode conducts at an angle

$$\theta = \sin^{-1}\left(\frac{0.7}{10}\right) = 4^\circ \text{ and stops}$$

at  $\pi - \theta = 176^\circ$ Thus the conduction angle is  $\pi - 2\theta$  $= 172^\circ$  or 3 rad.

$$v_{O,\text{avg}} = \frac{-1}{2\pi} \int_{\theta}^{\pi-\theta} (10 \sin \phi - 0.7) d\phi$$

$$= \frac{-1}{2\pi} [-10 \cos \phi - 0.7\phi]_{\theta}^{\pi-\theta}$$

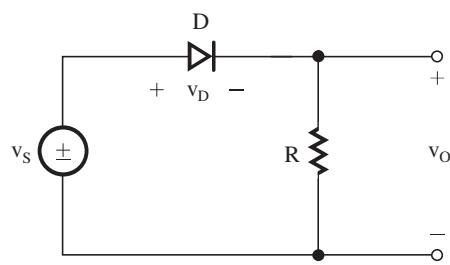
 $= -2.85 \text{ V}$ 

(d) Peak current in diode is

$$\frac{10 - 0.7}{1} = 9.3 \text{ mA}$$

(e) PIV occurs when  $v_s$  is at its peak and  $v_o = 0$ . $\text{PIV} = 10 \text{ V}$ 

3.64



$$i_D = I_S e^{v_D/V_T}$$

$$\frac{i_D}{i_D(1 \text{ mA})} = e^{[v_D - v_D(\text{at } 1 \text{ mA})]/V_T}$$

$$v_D - v_D(\text{at } 1 \text{ mA}) = V_T \ln \left[ \frac{i_D}{1 \text{ mA}} \right]$$

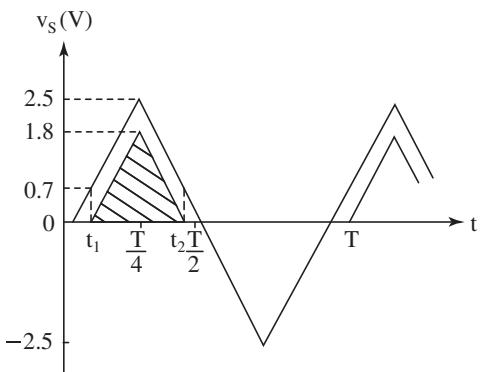
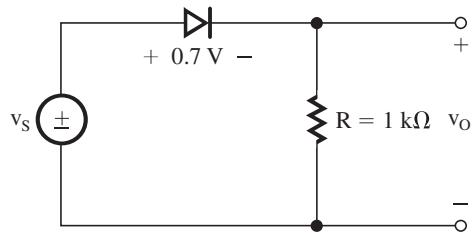
$$v_D = v_D(\text{at } 1 \text{ mA}) + V_T \ln \left[ \frac{v_o/R}{1} \right]$$

$$v_o = v_s - v_D$$

$$= v_s - v_D(\text{at } 1 \text{ mA}) - V_T \ln \left( \frac{v_o}{R} \right)$$

where  $R$  is in  $\text{k}\Omega$ .

3.65

First find  $t_1$  and  $t_2$ 

$$\frac{2.5}{T} = \frac{0.7}{t_1}$$

$$\Rightarrow t_1 = 0.07 T$$

$$t_2 = \frac{T}{2} - t_1$$

$$= \frac{T}{2} - 0.07 T$$

$$t_2 = 0.43 T$$

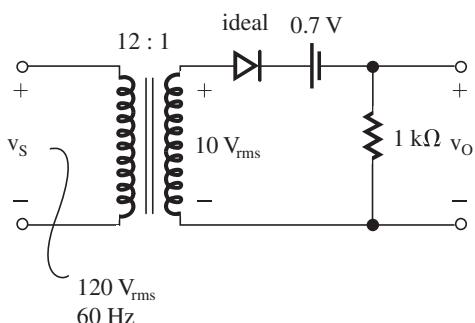
$$v_o(\text{ave.}) = \frac{1}{T} \times \text{area of shaded triangle}$$

$$= \frac{1}{T} \times (2.5 - 0.7) \times \left( \frac{T}{4} - t_1 \right)$$

$$= \frac{1}{T} \times 1.8 \times T \left( \frac{1}{4} - 0.07 \right)$$

$$= 0.324 \text{ V}$$

3.66



$$\hat{v}_o = 10\sqrt{2} - 0.7 = 13.44 \text{ V}$$

Conduction begins at

$$10\sqrt{2} \sin \theta = 0.7$$

$$\theta = \sin^{-1}\left(\frac{0.7}{10\sqrt{2}}\right) = 2.84^\circ$$

$$= 0.0495 \text{ rad}$$

Conduction ends at  $\pi - \theta$ .

$$\therefore \text{Conduction angle} = \pi - 2\theta = 3.04 \text{ rad}$$

The diode conducts for

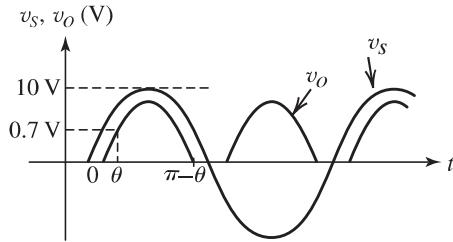
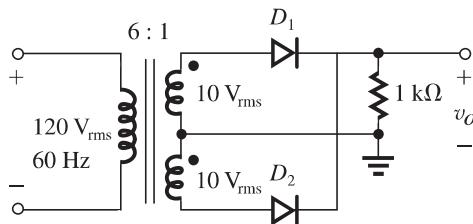
$$\frac{3.04}{2\pi} \times 100 = 48.4\% \text{ of the cycle}$$

$$v_{o,\text{avg}} = \frac{1}{2\pi} \int_{\theta}^{\pi-\theta} (10\sqrt{2}\sin\phi - 0.7) d\phi$$

$$= 4.15 \text{ V}$$

$$i_{D,\text{avg}} = \frac{v_{o,\text{avg}}}{R} = 4.15 \text{ mA}$$

3.67



$$\hat{v}_o = 10\sqrt{2} - V_D = 13.44 \text{ V}$$

$$\text{Conduction starts at } \theta = \sin^{-1} \frac{0.7}{10\sqrt{2}} = 2.84^\circ$$

$$2.84^\circ = 0.05 \text{ rad}$$

and ends at  $\pi - \theta$ . Conduction angle =  $\pi - 2\theta = 3.04$  rad in each half cycle. Thus the

$$\text{fraction of a cycle for which one of the two diodes conduct} = \frac{2(3.04)}{2\pi} \times 100 = 96.8\%$$

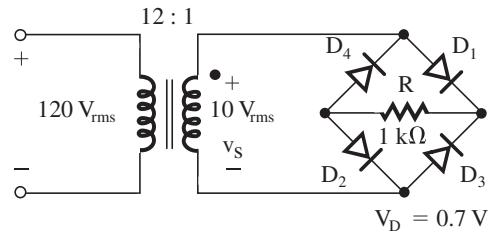
Note that during 96.8% of the cycle there will be conduction. However, each of the two diodes conducts for only half the time, i.e., for 48.4% of the cycle.

$$v_{o,\text{avg}} = \frac{1}{\pi} \int_{\theta}^{\pi-\theta} (10\sqrt{2}\sin\phi - 0.7) d\phi$$

$$= 8.3 \text{ V}$$

$$i_{L,\text{avg}} = \frac{8.3}{1 \text{ k}\Omega} = 8.3 \text{ mA}$$

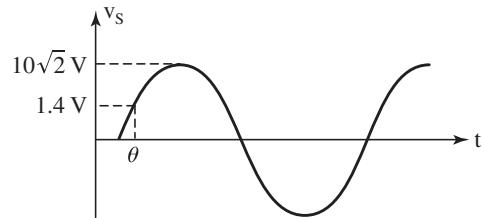
3.68



$$\text{Peak voltage across R} = 10\sqrt{2} - 2V_D$$

$$= 10\sqrt{2} - 1.4$$

$$= 12.74 \text{ V}$$



$$\theta = \sin^{-1} \frac{1.4}{10\sqrt{2}} = 5.68^\circ = 0.1 \text{ rad}$$

Fraction of cycle that D<sub>1</sub> & D<sub>2</sub> conduct is

$$\frac{\pi - 2\theta}{2\pi} \times 100 = 46.8\%$$

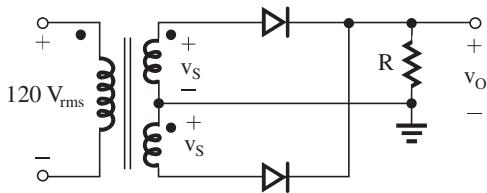
Note that D<sub>3</sub> & D<sub>4</sub> conduct in the other half cycle so that there is 2(46.8) = 93.6% conduction interval.

$$v_{o,\text{avg}} = \frac{2}{2\pi} \int_{\theta}^{\pi-\theta} (10\sqrt{2}\sin\phi - 2V_D) d\phi$$

$$\begin{aligned}
 &= \frac{1}{\pi} \left[ -12\sqrt{2} \cos \phi - 1.4\phi \right]_{\theta}^{\pi-\theta} \\
 &= \frac{2(12\sqrt{2} \cos \theta)}{\pi} - \frac{1.4(\pi - 2\theta)}{\pi} \\
 &= 7.65 \text{ V}
 \end{aligned}$$

$$i_{R,\text{avg}} = \frac{V_{O,\text{avg}}}{R} = \frac{7.65}{1} = 7.65 \text{ mA}$$

3.69



Refer to Fig. 3.24.

 For  $V_D \ll V_s$ , conduction angle  $\simeq \pi$ , and

$$V_{O,\text{avg}} = \frac{2}{\pi} V_s - V_D = \frac{2}{\pi} V_s - 0.7$$

 (a) For  $V_{O,\text{avg}} = 10 \text{ V}$ 

$$V_s = \frac{\pi}{2} \times 10.7 = 16.8 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{16.8} = 10.1 \text{ to } 1$$

 (b) For  $V_{O,\text{avg}} = 100 \text{ V}$ 

$$V_s = \frac{\pi}{2} \times 100.7 = 158.2 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{158.2} = 1.072 \text{ to } 1$$

3.70 Refer to Fig. 3.25

 For  $2V_D \ll V_s$ 

$$V_{O,\text{avg}} = \frac{2}{\pi} V_s - 2V_D = \frac{2}{\pi} V_s - 1.4$$

 (a) For  $V_{O,\text{avg}} = 10 \text{ V}$ 

$$10 \text{ V} = \frac{2}{\pi} \cdot V_s - 1.4$$

$$\therefore \hat{V}_s = 11.4 \left( \frac{\pi}{2} \right) = 17.9 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{17.9} = 9.5 \text{ to } 1$$

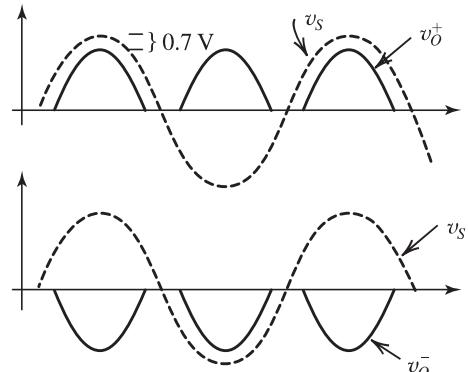
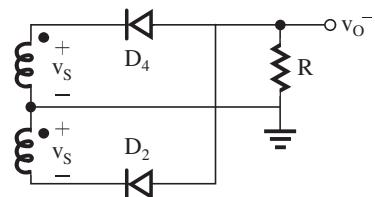
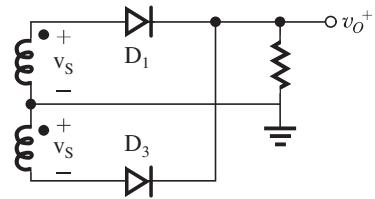
 (b) For  $V_{O,\text{avg}} = 100 \text{ V}$ 

$$100 \text{ V} = \frac{2}{\pi} \cdot V_s - 1.4$$

$$\Rightarrow V_s = 101.4 \left( \frac{\pi}{2} \right) = 159 \text{ V}$$

$$\text{Turns ratio} = \frac{120\sqrt{2}}{159} = 1.07 \text{ to } 1$$

3.71 The circuit is a full-wave rectifier with center tapped secondary winding. The circuit can be analyzed by looking at  $v_O^+$  and  $v_O^-$  separately.



$$V_{O,\text{avg}} = \frac{1}{2\pi} \int (V_s \sin \phi - 0.7) d\phi = 12$$

$$= \frac{2V_s}{\pi} - 0.7 = 12$$

where we have assumed  $V_s \gg 0.7 \text{ V}$  and thus the conduction angle (in each half cycle) is almost  $\pi$ .

$$V_s = \frac{12 + 0.7}{2} \pi = 19.95 \text{ V}$$

Thus voltage across secondary winding

$$= 2V_s \simeq 40 \text{ V}$$

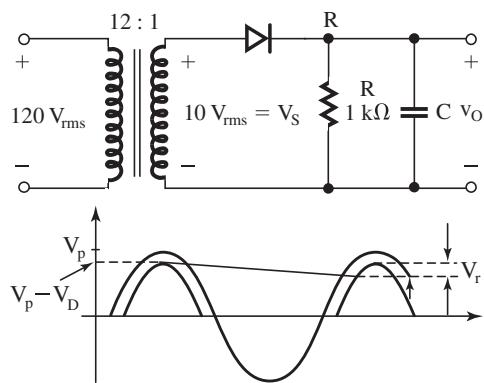
Looking at D<sub>4</sub>,

$$\begin{aligned} \text{PIV} &= V_S - V_O^- \\ &= V_S + (V_S - 0.7) \\ &= 2V_S - 0.7 \\ &= 39.2 \text{ V} \end{aligned}$$

If choosing a diode, allow a safety margin by moving a factor of 1.5, thus

$$\text{PIV} \simeq 60 \text{ V}$$

3.72



$$(i) V_r \cong (V_p - V_D) \frac{T}{CR} \quad [\text{Eq. (3.28)}]$$

$$0.1(V_p - V_D) = (V_p - V_D) \frac{T}{CR}$$

$$C = \frac{1}{0.1 \times 60 \times 10^3} = 166.7 \mu\text{F}$$

(ii) For

$$V_r = 0.01(V_p - V_D) = \frac{(V_p - V_D)T}{CR}$$

$$C = 1667 \mu\text{F}$$

$$(a) (i) v_{O, \text{avg}} = V_p - V_D - \frac{1}{2}V_T$$

$$= 10\sqrt{2} - 0.7 - \frac{1}{2}(10\sqrt{2} - 0.7)0.1$$

$$= (10\sqrt{2} - 0.7) \left(1 - \frac{0.1}{2}\right)$$

$$= 12.77 \text{ V}$$

$$(ii) v_{O, \text{avg}} = (10\sqrt{2} - 0.7) \left(1 - \frac{0.01}{2}\right)$$

$$= 13.37 \text{ V}$$

(b) (i) Using Eq (3.30), we have the conduction angle =

$$\omega\Delta t \cong \sqrt{2V_f / (V_p - V_D)}$$

$$= \sqrt{\frac{2 \times 0.1 (V_p - 0.7)}{(V_p - 0.7)}}$$

$$= \sqrt{0.2}$$

$$= 0.447 \text{ rad}$$

∴ Fraction of cycle for

$$\text{conduction} = \frac{0.447}{2\pi} \times 100$$

$$= 7.1\%$$

$$(ii) \omega\Delta t \cong \sqrt{2 \times 0.01 \frac{(V_p - 0.7)}{V_p - 0.7}} = 0.141 \text{ rad}$$

$$\text{Fraction of cycle} = \frac{0.141}{2\pi} \times 100 = 2.24\%$$

(c) (i) Use Eq (3.31):

$$i_{D, \text{avg}} = I_L \left( 1 + \pi \sqrt{\frac{2(V_p - V_D)}{V_r}} \right)$$

$$= \frac{v_{O, \text{avg}}}{R} \left( 1 + \pi \sqrt{\frac{2(V_p - V_D)}{0.1(V_p - V_D)}} \right)$$

$$= \frac{12.77}{10^3} \left( 1 + \pi \sqrt{\frac{2}{0.1}} \right)$$

$$= 192 \text{ mA}$$

$$(ii) i_{D, \text{avg}} = \frac{13.37}{10^3} \left( 1 + \pi \sqrt{200} \right)$$

$$= 607 \text{ mA}$$

(d) Adapting Eq. (3.32), we obtain

$$(i) i_{D, \text{peak}} = I_L \left( 1 + 2\pi \sqrt{\frac{2(V_p - V_D)}{V_r}} \right)$$

$$= \frac{12.77}{10^3} \left( 1 + 2\pi \sqrt{\frac{2}{0.1}} \right)$$

$$= 371 \text{ mA}$$

$$(ii) i_{D, \text{peak}} = \frac{13.37}{10^3} \left( 1 + 2\pi \sqrt{\frac{2}{0.01}} \right)$$

$$= 1201 \text{ mA} \simeq 1.2 \text{ A}$$

$$3.73 \text{ (i)} V_r = 0.1 (V_p - V_D) = \frac{(V_p - V_D)}{2fCR}$$

The factor of 2 accounts for discharge occurring only during half of the period,  $T/2 = \frac{1}{2f}$ .

$$C = \frac{1}{(2fR)0.1} = \frac{1}{2(60)10^3 \times 0.1} = 83.3 \mu F$$

$$\text{(ii)} C = \frac{1}{2(60) \times 10^3 \times 0.01} = 833 \mu F$$

$$\text{(a) (i)} V_O = V_p - V_D - \frac{1}{2}V_r$$

$$= (V_p - V_D) \left(1 - \frac{0.1}{2}\right)$$

$$= (13.44) \left(1 - \frac{0.1}{2}\right)$$

$$= 12.77 \text{ V}$$

$$\text{(ii)} V_O = (13.44) \left(1 - \frac{0.01}{2}\right) = 13.37 \text{ V}$$

$$\text{(b) (i)} \text{ Fraction of cycle} = \frac{2\omega\Delta t}{2\pi} \times 100$$

$$= \frac{\sqrt{2V_r/(V_p - V_D)}}{\pi} \times 100$$

$$= \frac{1}{\pi} \sqrt{2(0.1)} \times 100 = 14.2\%$$

$$\text{(ii)} \text{ Fraction of cycle} = \frac{2\sqrt{2}(0.01)}{2\pi} \times 100$$

$$= 4.5\%$$

(c) Use Eq. (3.34):

$$\text{(i)} i_{D, \text{avg}} = I_L \left(1 + \pi \sqrt{\frac{V_p - V_D}{2V_r}}\right)$$

$$= \frac{12.77}{1} \left(1 + \pi \sqrt{\frac{1}{2(0.1)}}\right) = 102.5 \text{ mA}$$

$$\text{(ii)} i_{D, \text{avg}} = \frac{13.37}{1} \left(1 + \pi \frac{1}{\sqrt{2(0.01)}}\right)$$

$$= 310 \text{ mA}$$

(d) Use Eq. (3.35):

$$\text{(i)} \hat{i}_D = I_L \left(1 + 2\pi \frac{1}{\sqrt{2(0.1)}}\right) = 192 \text{ mA}$$

$$\text{(ii)} \hat{i}_D = I_L \left(1 + 2\pi \frac{1}{\sqrt{0.02}}\right) = 607 \text{ mA}$$

$$3.74 \text{ (i)} V_r = 0.1 (V_p - V_D \times 2) = \frac{V_p - 2V_D}{2fCR}$$

$$C = \frac{(V_p - 2V_D)}{(V_p - 2V_D)} \frac{1}{2(0.1)fR} = 83.3 \mu F$$

$$\text{(ii)} C = \frac{1}{2(0.01)fR} = 833 \mu F$$

$$\text{(a)} V_O = V_p - 2V_D - \frac{1}{2}V_r$$

$$\text{(i)} V_O = V_p - 2V_D - \frac{1}{2} (V_p - 2V_D) \times 0.1$$

$$= (V_p - 2V_D) \times 0.95$$

$$= (10\sqrt{2} - 2 \times 0.7) \times 0.95 = 12.1 \text{ V}$$

$$\text{(ii)} V_O = (10\sqrt{2} - 2 \times 0.7) \times 0.995 = 12.68 \text{ V}$$

$$\text{(b) (i)} \text{ Fraction of cycle} = \frac{2\omega\Delta t}{2\pi} \times 100$$

$$= \frac{\sqrt{2(0.1)}}{\pi} \times 100 = 14.2\%$$

$$\text{(ii)} \text{ Fraction of cycle} = \frac{\sqrt{2(0.01)}}{\pi} \times 100 = 4.5\%$$

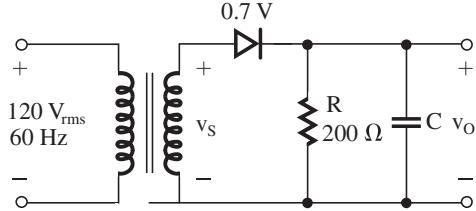
$$\text{(c) (i)} i_{D, \text{avg}} = \frac{12.1}{1} \left(1 + \pi \sqrt{\frac{1}{0.2}}\right) = 97 \text{ mA}$$

$$\text{(ii)} i_{D, \text{avg}} = \frac{12.68}{1} \left(1 + \pi / \sqrt{0.02}\right) = 249 \text{ mA}$$

$$\text{(d) (i)} \hat{i}_D = \frac{12.1}{1} \left(1 + 2\pi \sqrt{\frac{1}{0.2}}\right) = 182 \text{ mA}$$

$$\text{(ii)} \hat{i}_D = \frac{12.68}{1} \left(1 + 2\pi \sqrt{\frac{1}{0.02}}\right) = 576 \text{ mA}$$

3.75



$$V_O = 12 \text{ V} \pm 1 \text{ V} \text{ (ripple)}$$

$$R_L = 200 \Omega$$

$$\text{(a)} V_O = V_p - V_D - 1$$

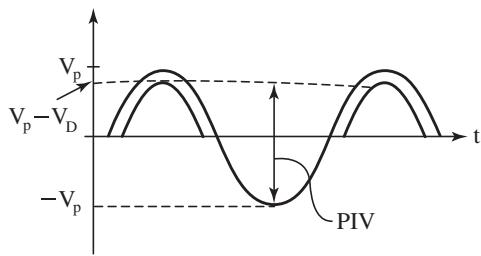
$$\Rightarrow V_p = 13 + 0.7 = 13.7 \text{ V}$$

$$V_{\text{rms}} = \frac{13.7}{\sqrt{2}} = 9.7 \text{ V}$$

$$\text{(b)} V_r = \frac{V_p - V_D}{fCR}$$

$$2 = \frac{13.7 - 0.7}{60 \times C \times 200}$$

$$\Rightarrow C = \frac{13}{2 \times 60 \times 200} = 542 \mu F$$



(c) When the diode is cut off, the maximum reverse voltage across it will occur when  $v_s = -V_p$ . At this time,  $v_o = V_o$  and the maximum reverse voltage will be

$$\text{Maximum reverse voltage} = V_o + V_p$$

$$= 12 + 13.7 = 25.7 \text{ V}$$

Using a factor of safety of 1.5 we obtain

$$\text{PIV} = 1.5 \times 25.7$$

$$= 38.5 \text{ V}$$

$$(d) i_{Dav} = I_L \left[ 1 + \pi \sqrt{\frac{2(V_p - V_D)}{V_r}} \right]$$

$$= \frac{V_o}{R_L} \left[ 1 + \pi \sqrt{\frac{2(V_p - V_D)}{V_r}} \right]$$

$$= \frac{12}{0.2} \left[ 1 + \pi \sqrt{\frac{2(13.7 - 0.7)}{2}} \right]$$

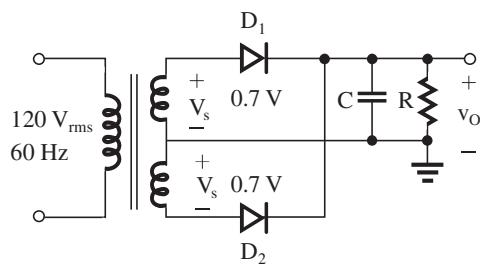
$$= 739 \text{ mA}$$

$$(e) i_{Dmax} = I_L \left[ 1 + 2\pi \sqrt{\frac{2(V_p - V_D)}{V_r}} \right]$$

$$= \frac{12}{0.2} \left[ 1 + 2\pi \sqrt{\frac{2(13.7 - 0.7)}{2}} \right]$$

$$= 1.42 \text{ A}$$

3.76

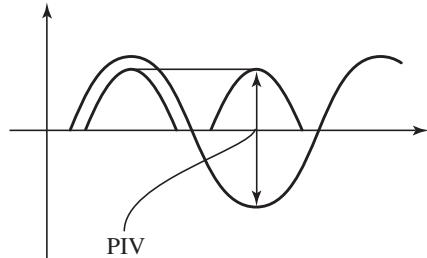


$$(a) V_o = V_p - V_D - 1$$

$$\Rightarrow V_p = V_o + V_D + 1 = 13 + 0.7 = 13.7 \text{ V}$$

$$V_{rms} = \frac{13.7}{\sqrt{2}} = 9.7 \text{ V}$$

This voltage appears across each half of the transformer secondary. Across the entire secondary we have  $2 \times 9.7 = 19.4 \text{ V}$  (rms).



$$(b) V_r = \frac{V_p - V_D}{2fCR}$$

$$2 = \frac{13.7 - 0.7}{2 \times 60 \times 200 \times C}$$

$$\Rightarrow C = \frac{12}{2 \times 2 \times 60 \times 200} = 271 \mu\text{F}$$

(c) Maximum reverse voltage across  $D_1$  occurs when  $v_s = -V_p$ . At this point  $v_o = V_o$ . Thus maximum reverse voltage =  $V_o + V_p = 12 + 13.7 = 25.7$ . The same applies to  $D_2$ .

In specifying the PIV for the diodes, one usually uses a factor of safety of about 1.5,

$$\text{PIV} = 1.5 \times 25.7 = 38.5 \text{ V}$$

$$(d) i_{Dav} = I_L \left[ 1 + \pi \sqrt{\frac{V_p - V_D}{2 V_r}} \right]$$

$$= \frac{12}{0.2} \left[ 1 + \pi \sqrt{\frac{13.7 - 0.7}{2 \times 2}} \right]$$

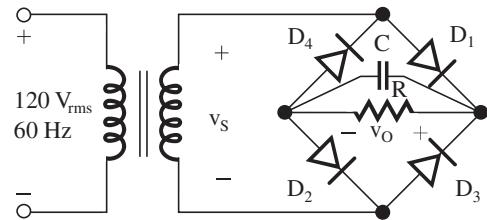
$$= 399 \text{ mA}$$

$$(e) i_{Dmax} = I_L \left[ 1 + 2\pi \sqrt{\frac{V_p - V_D}{2 V_r}} \right]$$

$$= \frac{12}{0.2} \left[ 1 + 2\pi \sqrt{\frac{13.7 - 0.7}{2 \times 2}} \right]$$

$$= 739 \text{ mA}$$

3.77



$$(a) V_O = V_p - 2V_D - 1$$

$$\Rightarrow V_p = V_O + 2V_D + 1 = 12 + 2 \times 0.7 + 1 = 14.4 \text{ V}$$

$$V_{rms} = \frac{14.4}{\sqrt{2}} = 10.2 \text{ V}$$

$$(b) V_r = \frac{V_p - 2 V_D}{2fCR}$$

$$\Rightarrow C = \frac{14.4 - 1.4}{2 \times 2 \times 60 \times 200} = 271 \mu\text{F}$$

(c) The maximum reverse voltage across  $D_1$  occurs when  $V_s = -V_p = -14.4 \text{ V}$ . At this time  $D_3$  is conducting, thus

$$\begin{aligned} \text{Maximum reverse voltage} &= -V_p + V_{D3} \\ &= -14.4 + 0.7 = -13.7 \text{ V} \end{aligned}$$

The same applies to the other three diodes. In specifying the PIV rating for the diode we use a factor of safety of 1.5 to obtain

$$\text{PIV} = 1.5 \times 13.7 = 20.5 \text{ V}$$

$$(d) i_{Dav} = I_L \left[ 1 + \pi \sqrt{\frac{V_p - 2 V_D}{2 V_r}} \right]$$

$$= \frac{12}{0.2} \left[ 1 + \pi \sqrt{\frac{14.4 - 1.4}{2 \times 2}} \right]$$

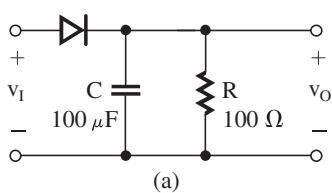
$$= 400 \text{ mA}$$

$$(e) i_{Dmax} = I_L \left[ 1 + 2\pi \sqrt{\frac{V_p - 2 V_D}{2 V_r}} \right]$$

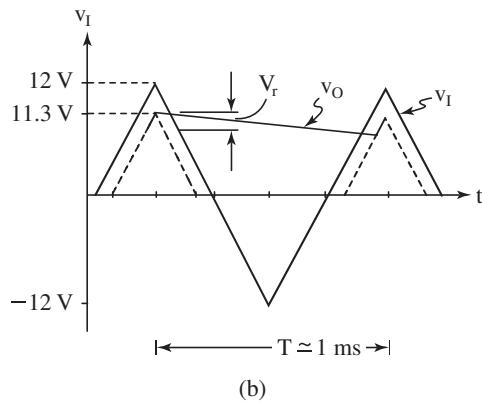
$$= \frac{12}{0.2} \left[ 1 + 2\pi \sqrt{\frac{14.4 - 0.7}{2 \times 2}} \right]$$

$$= 740 \text{ mA}$$

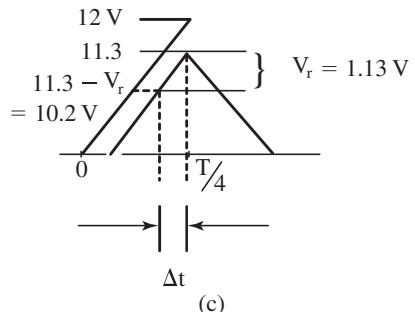
3.78



(a)



(b)



During the diode's off interval (which is almost equal to  $T$ ) the capacitor discharges and its voltage is given by

$$v_O(t) = 11.3 e^{-t/CR}$$

where  $C = 100 \mu\text{F}$  and  $R = 100 \Omega$ , thus

$$CR = 100 \times 10^{-6} \times 100 = 0.01 \text{ s}$$

At the end of the discharge interval,  $t \simeq T$  and

$$v_O = 11.3 e^{-T/CR}$$

Since  $T = 0.001 \text{ s}$  is much smaller than  $CR$ ,

$$v_O \simeq 11.3 \left( 1 - \frac{T}{CR} \right)$$

The ripple voltage  $V_r$  can be found as

$$\begin{aligned} V_r &= 11.3 - 11.3 \left( 1 - \frac{T}{CR} \right) \\ &= \frac{11.3T}{CR} = \frac{11.3 \times 0.001}{0.01} = 1.13 \text{ V} \end{aligned}$$

The average dc output voltage is

$$v_O = 11.3 - \frac{V_r}{2} = 11.3 - \frac{1.13}{2} = 10.74 \text{ V}$$

To obtain the interval during which the diode conducts,  $\Delta t$ , refer to Fig. (c).

$$\begin{aligned} \frac{12}{T/4} &= \frac{V_r}{\Delta t} \\ \Rightarrow \Delta t &= \frac{V_r \times (T/4)}{12} = \frac{1.13 \times 1}{12 \times 4} \\ &= 23.5 \mu\text{s} \end{aligned}$$

Now, using the fact that the charge gained by the capacitor when the diode is conducting is equal to the charge lost by the capacitor during its discharge interval, we can write

$$i_{Cav} \times \Delta t = C V_r$$

$$\Rightarrow i_{Cav} = \frac{C V_r}{\Delta t} = \frac{100 \times 10^{-6} \times 1.13}{23.5 \times 10^{-6}} = 4.8 \text{ A}$$

$$i_{Dav} = i_{Cav} + i_{Lav}$$

where  $i_{Lav}$  is the average current through  $R$  during the short interval  $\Delta t$ . This is approximately

$$\frac{11.3}{R} = \frac{11.3}{100} = 0.113 \text{ A}. \text{ Thus}$$

$$i_{Dav} = 4.8 + 0.113 = 4.913 \text{ A}$$

Finally, to obtain the peak diode current, we use

$$\begin{aligned} i_{Dmax} &= i_{Cmax} + i_{Lmax} \\ &= C \frac{dv_I}{dt} + \frac{11.3}{R} \\ &= C \times \frac{12}{T/4} + \frac{11.3}{R} \\ &= 100 \times 10^{-6} \times \frac{12 \times 4}{1 \times 10^{-3}} + \frac{11.3}{100} \\ &= 4.8 + 0.113 = 4.913 \text{ A} \end{aligned}$$

which is equal to the average value. This is a result of the linear  $v_I$  which gives rise to a constant capacitor current during the diode conduction interval. Thus  $i_{Cmax} = i_{Cav} = 4.8 \text{ A}$ . Also, the maximum value of  $i_L$  is approximately equal to its average value during the short interval  $\Delta t$ .

3.79 Refer to Fig. P3.71 and let a capacitor  $C$  be connected across each of the load resistors  $R$ . The two supplies  $v_O^+$  and  $v_O^-$  are identical. Each is a full-wave rectifier similar to that of the tapped-transformer circuit. For each supply,

$$V_O = 12 \text{ V}$$

$$V_r = 1 \text{ V (peak to peak)}$$

Thus

$$v_O = 12 \pm 0.5 \text{ V}$$

It follows that the peak value of  $v_S$  must be  $12.5 + 0.7 = 13.2 \text{ V}$  and the total rms voltage across the secondary will be

$$= \frac{2 \times 13.2}{\sqrt{2}} = 18.7 \text{ V (rms)}$$

$$\text{Transformer turns ratio} = \frac{120}{18.7} = 6.43:1$$

To deliver 100-mA dc current to each load,

$$R = \frac{12}{0.1} = 120 \Omega$$

Now, the value of  $C$  can be found from

$$V_r = \frac{V_p - 0.7}{2fCR}$$

$$1 = \frac{12.5}{2 \times 60 \times C \times 120}$$

$$\Rightarrow C = 868 \mu\text{F}$$

To specify the diodes, we determine  $i_{Dav}$  and  $i_{Dmax}$ ,

$$i_{Dav} = I_L(1 + \pi \sqrt{(V_p - 0.7)/2 V_r})$$

$$= 0.1(1 + \pi \sqrt{12.5/2})$$

$$= 785 \text{ mA}$$

$$i_{Dmax} = I_L(1 + 2\pi \sqrt{(V_p - 0.7)/2 V_r})$$

$$= 0.1(1 + 2\pi \sqrt{12.5/2})$$

$$= 1.671 \text{ A}$$

To determine the required PIV rating of each diode, we determine the maximum reverse voltage that appears across one of the diodes, say  $D_1$ . This occurs when  $v_S$  is at its maximum negative value  $-V_p$ . Since the cathode of  $D_1$  will be at  $+12.5 \text{ V}$ , the maximum reverse voltage across  $D_1$  will be  $12.5 + 13.2 = 25.7 \text{ V}$ . Using a factor of safety of 1.5, then each of the four diodes must have

$$\text{PIV} = 1.5 \times 25.7 = 38.6 \text{ V}$$

3.80 Refer to Fig. P3.80. When  $v_I$  is positive,  $v_A$  goes positive, turning on the diode and closing the negative feedback loop around the op amp. The result is that  $v_- = v_I$ ,  $v_O = 2v_- = 2v_I$ , and  $v_A = v_O + 0.7$ . Thus

$$(a) v_I = +1 \text{ V}, v_- = +1 \text{ V}, v_O = +2 \text{ V}, \text{ and } v_A = +2.7 \text{ V}$$

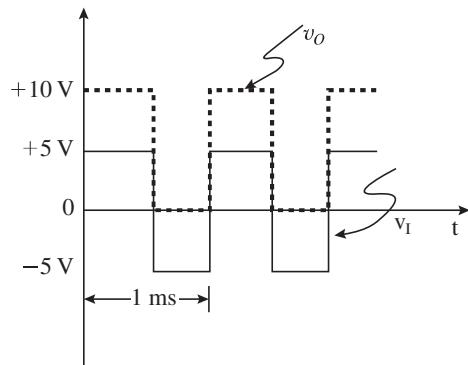
$$(b) v_I = +3 \text{ V}, v_- = +3 \text{ V}, v_O = +6 \text{ V}, \text{ and } v_A = +6.7 \text{ V}$$

When  $v_I$  goes negative,  $v_A$  follows, the diode turns off, and the feedback loop is opened. The op amp saturates with  $v_A = -13 \text{ V}$ ,  $v_- = 0 \text{ V}$  and  $v_O = 0 \text{ V}$ . Thus

$$(c) v_I = -1 \text{ V}, v_- = 0 \text{ V}, v_O = 0 \text{ V}, \text{ and } v_A = -13 \text{ V}$$

$$(d) v_I = -3 \text{ V}, v_- = 0 \text{ V}, v_O = 0 \text{ V}, \text{ and } v_A = -13 \text{ V}$$

Finally, if  $v_I$  is a symmetrical square wave of 1-kHz frequency, 5-V amplitude, and zero average, the output will be zero during the negative half cycles of the input and will equal twice the input during the positive half cycles. See figure.

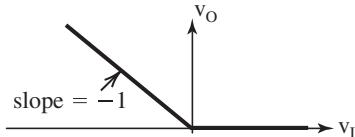


Thus,  $v_O$  is a square wave with 0-V and +10-V levels, i.e. 5-V average and, of course, the same frequency (1 kHz) as the input.

3.81  $v_I > 0$ :  $D_1$  conducts and  $D_2$  cutoff

$v_I < 0$ :  $D_1$  cutoff,

$$D_2 \text{ conducts } \sim \frac{v_O}{v_I} = -1$$



(a)  $v_I = +1 \text{ V}$

$$v_O = 0 \text{ V}$$

$$v_A = -0.7 \text{ V}$$

Keeps  $D_2$  off so no current "ows through R

$$\Rightarrow v_- = 0 \text{ V}$$

Virtual ground as feedback loop is closed through  $D_1$

(b)  $v_I = +3 \text{ V}$

$$v_O = 0 \text{ V}$$

$$v_A = -0.7 \text{ V}$$

$$v_- = 0 \text{ V}$$

(c)  $v_I = -1 \text{ V}$

$$v_O = +1 \text{ V}$$

$$v_A = 1.7 \text{ V}$$

$$v_- = 0 \text{ V}$$

~ Virtual ground as negative feedback loop is closed through  $D_2$  and R.

(d)  $v_I = -3 \text{ V} \Rightarrow v_O = +3 \text{ V}$

$$v_A = +3.7 \text{ V}$$

$$v_- = 0 \text{ V}$$

3.82 (a) See figure (a) on next page. For  $v_I \leq 3.5 \text{ V}$ ,  $i = 0$  and  $v_O = v_I$ . At  $v_I = 3.5 \text{ V}$ , the diode begins to conduct. At  $v_O = 3.7 \text{ V}$ , the diode is conducting  $i = 1 \text{ mA}$  and thus

$$v_I = v_O + i \times 1 \text{ k}\Omega = 4.7 \text{ V}$$

For  $v_I > 4.7 \text{ V}$  the diode current increases but the diode voltage remains constant at 0.7 V, thus  $v_O$  "attens and  $v_O$  vs.  $v_I$  becomes a horizontal line.

In practice, the diode voltage increases slowly and the line will have a small nonzero slope.

(b) See figure (b) on next page. Here  $v_O = v_I$  for  $v_I \geq 2.5 \text{ V}$ . At  $v_I = 2.5 \text{ V}$ ,  $v_O = 2.5 \text{ V}$  and the diode begins to conduct. The diode will be conducting 1 mA and exhibiting a drop of 0.7 at  $v_O = 2.3 \text{ V}$ . The corresponding value of  $v_I$

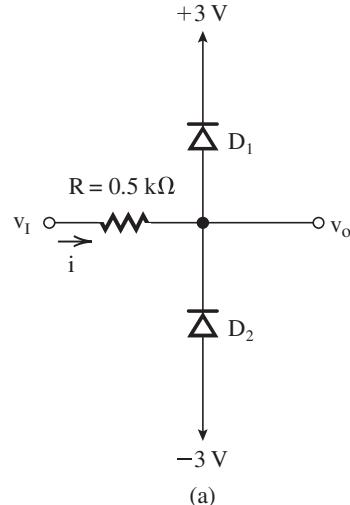
$$v_I = v_O - iR = 2.3 - 1 \times 1 = +1.3 \text{ V}$$

As  $v_I$  decreases below 1.3 V, the diode current increases, but the diode voltage remains constant at 0.7 V. Thus  $v_O$  "attens at about 2.3 V.

(c) See figure (c) on next page. For  $v_I \leq -2.5 \text{ V}$ , the diode is off, and  $v_O = v_I$ . At  $v_I = -2.5 \text{ V}$  the diode begins to conduct and its current reaches 1 mA at  $v_I = -1.3 \text{ V}$  (corresponding to  $v_O = -2.3 \text{ V}$ ). As  $v_I$  further increases, the diode current increases but its voltage remains constant at 0.7 V. Thus  $v_O$  "attens, as shown.

(d) See figure (d) on next page.

3.83



From Fig. (a) we see that for

$-3.5 \text{ V} \leq v_I \leq +3.5 \text{ V}$ , diodes  $D_1$  and  $D_2$  will be cut off and  $i = 0$ . Thus,  $v_O = v_I$ . For  $v_I \geq +3.5 \text{ V}$ , diode  $D_1$  begins to conduct and its voltage reaches 0.7 V (and thus  $v_O = +3.7 \text{ V}$ ) at  $i = 1 \text{ mA}$ . The corresponding value of  $v_I$  is

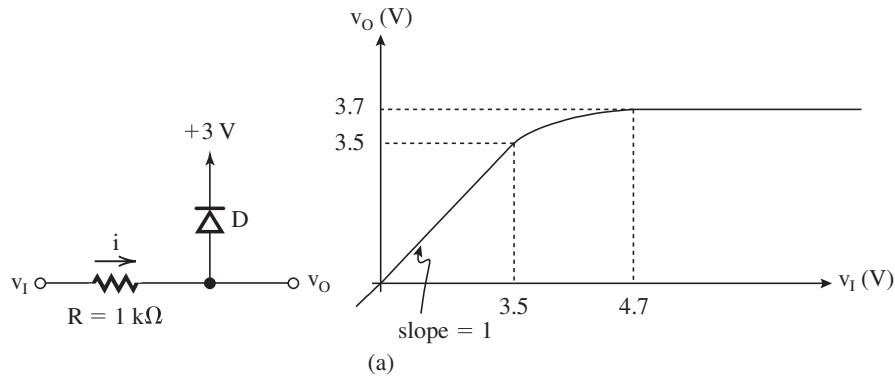
$$v_I = v_O - iR$$

$$v_I = 3.7 + 1 \times 0.5 = +4.2 \text{ V}$$

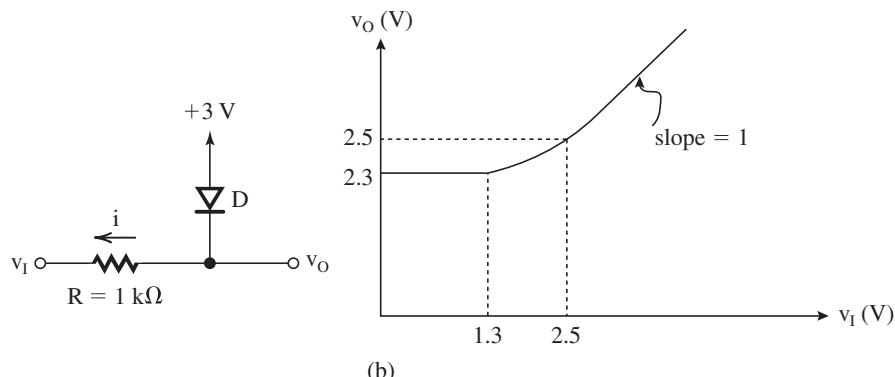
For  $v_I \geq 4.2 \text{ V}$ , the voltage of diode  $D_1$  remains 0.7 V and  $v_O$  saturates at +3.7 V.

A similar description applies for  $v_I \leq -3.5 \text{ V}$ . Here  $D_2$  conducts at  $v_I = -3.5 \text{ V}$  and its voltage becomes 0.7 V, and hence  $v_O = -3.7 \text{ V}$ , at  $i = 1 \text{ mA}$  (in the direction into  $v_I$ ) at  $v_I = -4.2 \text{ V}$ . For  $v_I \leq -4.2 \text{ V}$ ,  $v_O = -3.7 \text{ V}$ .

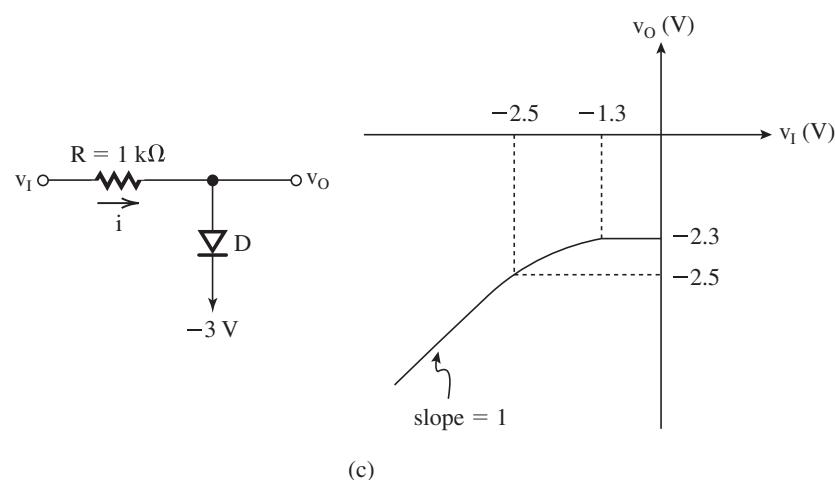
These figures belong to Problem 3.82.



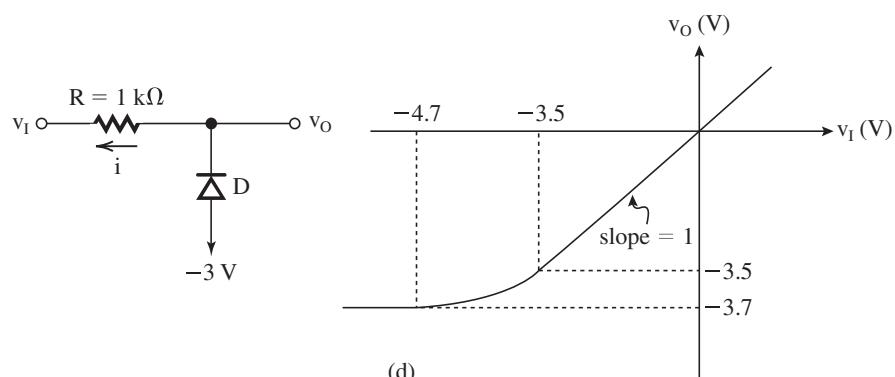
(a)



(b)



(c)



(d)

This figure belongs to Problem 3.83, part b.

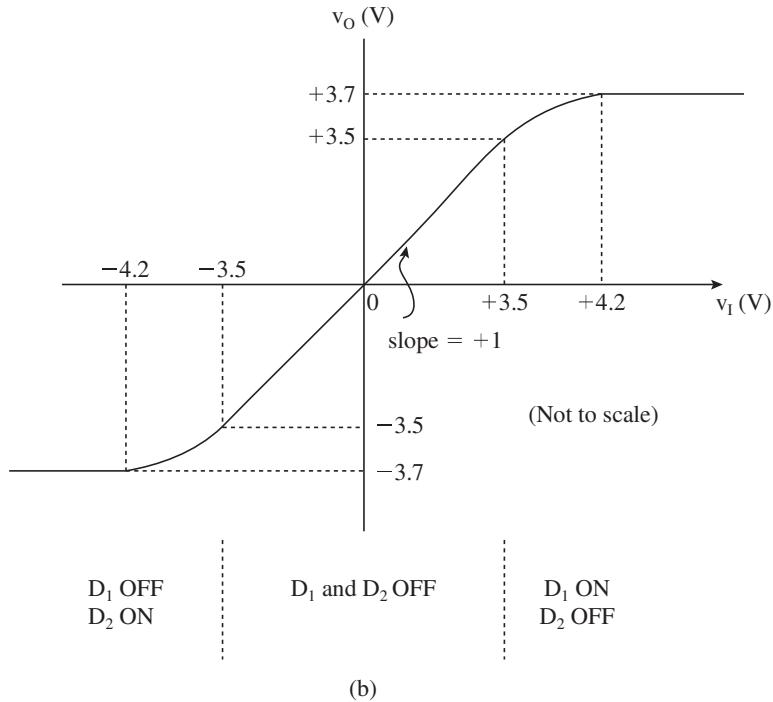
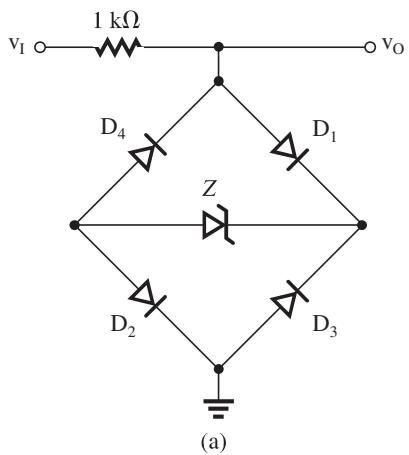
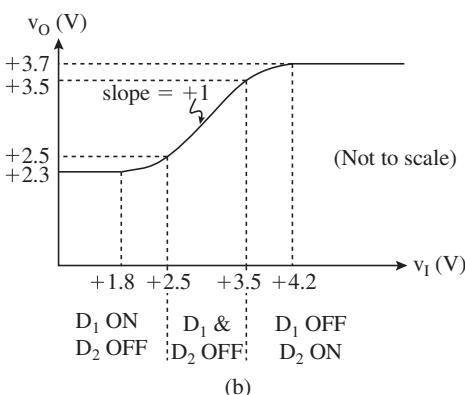
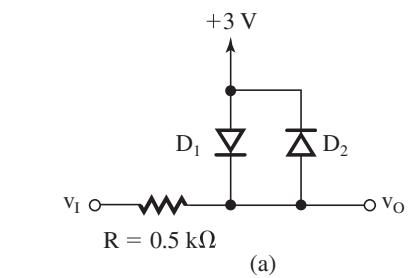


Figure (b) shows a sketch of the transfer characteristic of this double limiter.

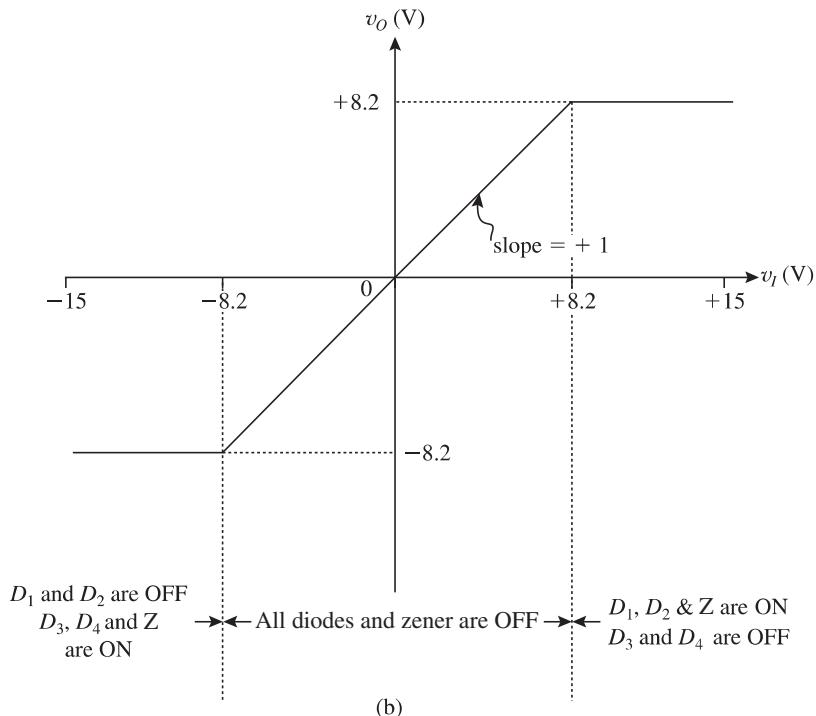
3.85

3.84 See figure.



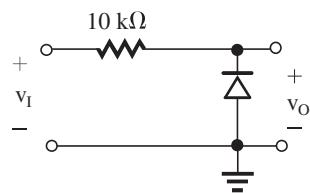
The limiter thresholds and the output saturation levels are found as  $2 \times 0.7 + 6.8 = 8.2$  V. The transfer characteristic is given in Fig. (b). See figure on next page.

This figure belongs to Problem 3.85, part b.

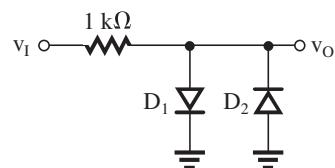


3.86

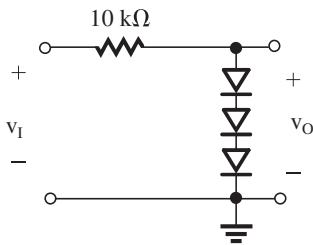
(a)



3.87



(b)



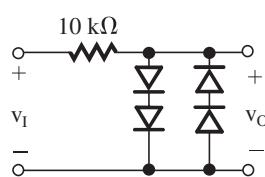
Diodes have 0.7 V drop at 1 mA current

$\therefore$  For diode  $D_1$

$$\frac{i_D}{1 \text{ mA}} = e^{(v_O - 0.7)/V_T}$$

$$i_D = 1 \times 10^{-3} e^{(v_O - 0.7)/V_T}$$

(c)



$$v_O = 0.7 + V_T \ln\left(\frac{i_D}{1 \text{ mA}}\right)$$

$$v_I = v_O + i_D \times 1 \text{ k}\Omega$$

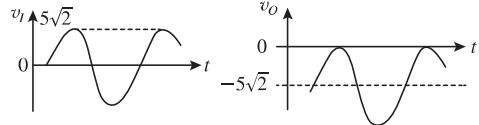
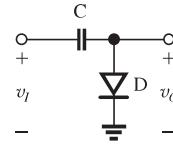
$v_I$  for the different values of  $v_O$ . For  $D_2$ ,  
 $v_I = v_O - i_D \times 1 \text{ k}\Omega$

	$v_o$ (V)	$v_I$ (V)
$D_1$ on	0.5 V	0.5003
	0.6	0.62
	0.7	1.7
	0.8	55.4
	0	0
	-0.5	-0.5003
	-0.6	-0.62
	-0.7	-1.7
	-0.8	-55.4

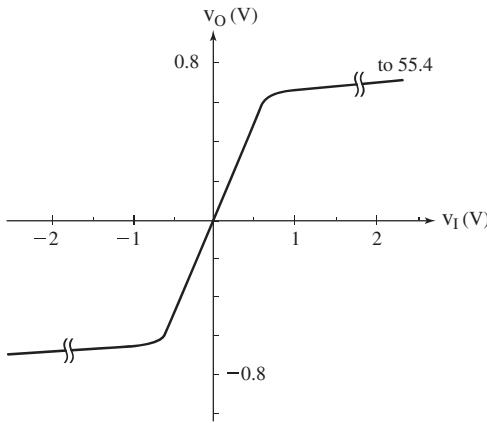
	$v_o$ (V)	$v_I$ (V)
$D_2$ on	0.5 V	0.5003
	0.6	0.62
	0.7	1.7
	0.8	55.4
	0	0
	-0.5	-0.5003
	-0.6	-0.62
	-0.7	-1.7
	-0.8	-55.4

3.89



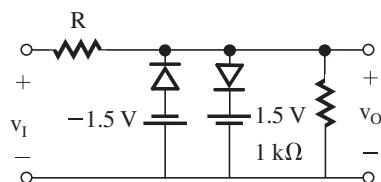
From the figure we see that

$$v_{Oav} = -5\sqrt{2} = -7.07 \text{ V}$$



It is a soft limiter with a gain  $K \approx 1$  and  
 $L_+ \approx 0.7 \text{ V}$ ,  $L_- \approx -0.7 \text{ V}$

3.88



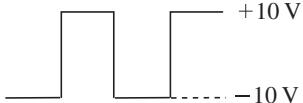
In the nonlimiting region

$$\frac{v_o}{v_i} = \frac{1000}{1000 + R} \geq 0.94$$

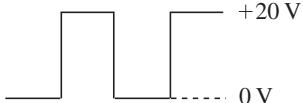
$$R \leq 63.8 \Omega$$

3.90

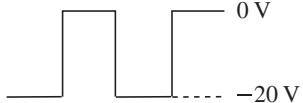
(a)



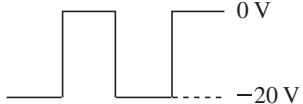
(b)



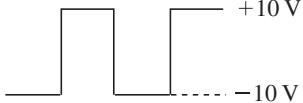
(c)



(d)



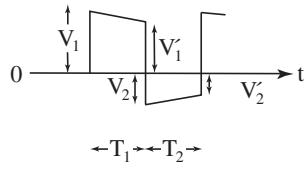
(e)



(f) Here there are two different time constants involved. To calculate the output levels, we shall consider the discharge and charge wave forms.

During  $T_1$ ,  $v_o = V_1 e^{-t/RC}$ 

$$\begin{aligned} \text{At } t = T_1 = T, v_o &= V'_1 \\ &= V_1 e^{-T/RC} \end{aligned}$$



where for  $T \ll CR$

$$V'_1 \simeq V_1(1 - T/CR) = V_1(1 - \alpha)$$

where  $\alpha \ll 1$

During the interval  $T_2$ , we have

$$|v_O| = |V_2| e^{-t/(CR/2)}$$

At the end of  $T_2$ ,  $t = T$ , and  $v_O = |V'_2|$

From (1) and (2) we find that

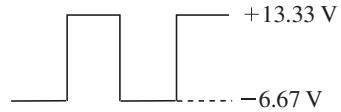
$$V_1 = 2|V_2|$$

Then using (1) and neglecting  $\alpha V_1$  yields

$$3|V_2| = 20 \Rightarrow |V_2| = 6.67 \text{ V}$$

$$V_1 = 13.33 \text{ V}$$

The result is



(g)

where

$$|V'_2| = |V_2| e^{-T/(CR/2)}$$

$$\simeq |V_2| \left(1 - \frac{T}{RC/2}\right) = |V_2|(1 - 2\alpha)$$

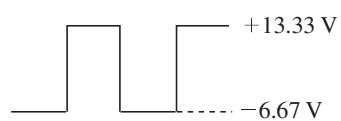
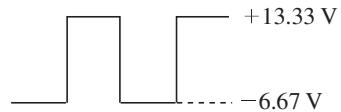
Now

$$V'_1 + |V'_2| = 20 \Rightarrow V_1 + |V_2| - \alpha V_1 = 20 \quad (1)$$

and

$$|V'_2| + V_1 = 20 \Rightarrow V_1 + |V_2| - 2\alpha |V_2| = 20 \quad (2)$$

(h) Using a method similar to that employed for case (f) above, we obtain



### Exercise 4-1

Ex: 4.1  $i_C = I_S e^{v_{BE}/V_T}$

$$v_{BE2} - v_{BE1} = V_T \ln \left[ \frac{i_{C2}}{i_{C1}} \right]$$

$$v_{BE2} = 700 + 25 \ln \left[ \frac{0.1}{1} \right]$$

$$= 642 \text{ mV}$$

$$v_{BE3} = 700 + 25 \ln \left[ \frac{10}{1} \right]$$

$$= 758 \text{ mV}$$

Ex: 4.2  $\therefore \alpha = \frac{\beta}{\beta + 1}$

$$\frac{50}{50+1} < \alpha < \frac{150}{150+1}$$

$$0.980 < \alpha < 0.993$$

Ex: 4.3  $I_C = I_E - I_B$

$$= 1.460 \text{ mA} - 0.01446 \text{ mA}$$

$$= 1.446 \text{ mA}$$

$$\alpha = \frac{I_C}{I_E} = \frac{1.446}{1.460} = 0.99$$

$$\beta = \frac{I_C}{I_B} = \frac{1.446}{0.01446} = 100$$

$$I_C = I_S e^{v_{BE}/V_T}$$

$$I_S = \frac{I_C}{e^{v_{BE}/V_T}} = \frac{1.446}{e^{700/25}}$$

$$= \frac{1.446}{e^{28}} \text{ mA} = 10^{-15} \text{ A}$$

Ex: 4.4  $\beta = \frac{\alpha}{1 - \alpha}$  and  $I_C = 10 \text{ mA}$

$$\text{For } \alpha = 0.99, \beta = \frac{0.99}{1 - 0.99} = 99$$

$$I_B = \frac{I_C}{\beta} = \frac{10}{99} = 0.1 \text{ mA}$$

$$\text{For } \alpha = 0.98, \beta = \frac{0.98}{1 - 0.98} = 49$$

$$I_B = \frac{I_C}{\beta} = \frac{10}{49} = 0.2 \text{ mA}$$

Ex: 4.5 Given:

$$I_S = 10^{-16} \text{ A}, \beta = 100, I_C = 1 \text{ mA}$$

We write

$$\begin{aligned} I_{SE} &= I_{SC}/\alpha = I_S = \left(1 + \frac{1}{\beta}\right) \\ &= 10^{-16} \times 1.01 = 1.01 \times 10^{-16} \text{ A} \end{aligned}$$

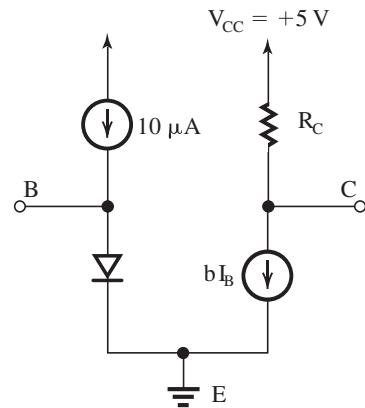
$$I_{SB} = \frac{I_S}{\beta} = \frac{10^{-16}}{100} = 10^{-18} \text{ A}$$

$$V_{BE} = V_T \ln \left[ \frac{I_C}{I_S} \right] = 25 \ln \left[ \frac{1 \text{ mA}}{10^{-16}} \right]$$

$$= 25 \times 29.9336$$

$$= 748 \text{ mV}$$

Ex: 4.6



$$v_{BE} = 690 \text{ mV}$$

$$I_C = 1 \text{ mA}$$

For active range  $V_C \geq V_B$ ,

$$\begin{aligned} R_{Cmax} &= \frac{V_{CC} - 0.690}{I_C} \\ &= \frac{5 - 0.69}{1} \\ &= 4.31 \text{ kΩ} \end{aligned}$$

Ex: 4.7  $I_S = 10^{-15} \text{ A}$

$$Area_C = 100 \times Area_E$$

$$I_{SC} = 100 \times I_S = 10^{-13} \text{ A}$$

Ex: 4.8  $i_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T}$

for  $i_C = 0$

$$I_S e^{v_{BE}/V_T} = I_{SC} e^{v_{BC}/V_T}$$

$$\frac{I_{SC}}{I_S} = \frac{e^{v_{BE}/V_T}}{e^{v_{BC}/V_T}}$$

$$= e^{(v_{BE} - v_{BC})/V_T}$$

$$\therefore V_{CE} = V_{BE} - V_{BC} = V_T \ln \left[ \frac{I_{SC}}{I_S} \right]$$

For collector Area = 100 × Emitter area

$$V_{CE} = 25 \ln \left[ \frac{100}{1} \right] = 115 \text{ mV}$$

### Exercise 4-2

$$\text{Ex: 4.9 } I_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T}$$

$$I_B = \frac{I_S}{\beta} e^{v_{BE}/V_T} + I_{SC} e^{v_{BC}/V_T}$$

$$\beta_{\text{forced}} = \left. \frac{I_C}{I_B} \right|_{\text{sat}} < \beta$$

$$= \beta \frac{I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T}}{I_S e^{v_{BE}/V_T} + \beta I_{SC} e^{v_{BC}/V_T}}$$

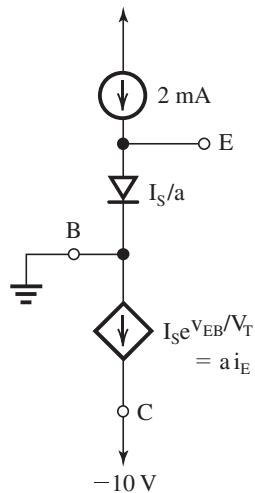
$$= \beta \frac{I_S e^{(v_{BE}-v_{BC})/V_T} - I_{SC}}{I_S e^{(v_{BE}-v_{BC})/V_T} + \beta I_{SC}}$$

$$= \beta \frac{e^{V_{CE\text{sat}}/V_T} - I_{SC}/I_S}{e^{V_{CE\text{sat}}/V_T} + \beta I_{SC}/I_S} \quad \text{Q.E.D.}$$

$$\beta_{\text{forced}} = 100 \frac{e^{200/25} - 100}{e^{200/25} + 100 \times 100}$$

$$= 100 \times 0.2219 \approx 22.2$$

Ex: 4.10



$$I_E = \frac{I_S}{\alpha} e^{v_{BE}/V_T}$$

$$2 \text{ mA} = \frac{51}{50} 10^{-14} e^{v_{BE}/V_T}$$

$$V_{BE} = 25 \ln \left[ \frac{2}{10^3} \times \frac{50}{51} \times 10^{14} \right]$$

$$= 650 \text{ mV}$$

$$I_C = \frac{\beta}{\beta + 1} I_E = \frac{50}{51} \times 2$$

$$= 1.96 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{1.96}{50} \Rightarrow 39.2 \mu\text{A}$$

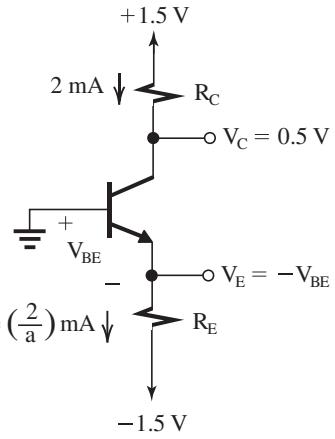
$$\text{Ex: 4.11 } I_C = I_S e^{v_{BE}/V_T} = 1.5 \text{ A}$$

$$\therefore V_{BE} = V_T \ln [1.5 / 10^{-11}]$$

$$= 25 \times 25.734$$

$$= 643 \text{ mV}$$

Ex: 4.12



$$R_C = \frac{1.5 - V_C}{I_C} = \frac{1.5 - 0.5}{2}$$

$$= 0.5 \text{ k}\Omega = 500 \Omega$$

Since at  $I_C = 1 \text{ mA}$ ,  $V_{BE} = 0.8 \text{ V}$ , then at  $I_C = 2 \text{ mA}$ ,

$$V_{BE} = 0.8 + 0.025 \ln \left( \frac{2}{1} \right)$$

$$= 0.8 + 0.017$$

$$= 0.817 \text{ V}$$

$$V_E = -V_{BE} = -0.817 \text{ V}$$

$$I_E = \frac{2 \text{ mA}}{\alpha} = \frac{2}{0.99} = 2.02 \text{ mA}$$

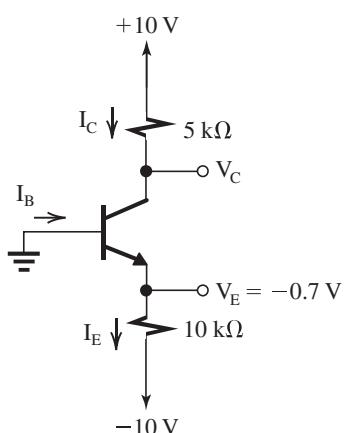
$$I_E = \frac{V_E - (-1.5)}{R_E}$$

Thus,

$$R_E = \frac{-0.817 + 1.5}{2.02} = 0.338 \text{ k}\Omega$$

$$= 338 \Omega$$

Ex: 4.13



### Exercise 4-3

$$I_E = \frac{V_E - (-10)}{10} = \frac{-0.7 + 10}{10}$$

$$= 0.93 \text{ mA}$$

Assuming active-mode operation,

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.93}{50 + 1} = 0.0182 \text{ mA}$$

$$= 18.2 \mu\text{A}$$

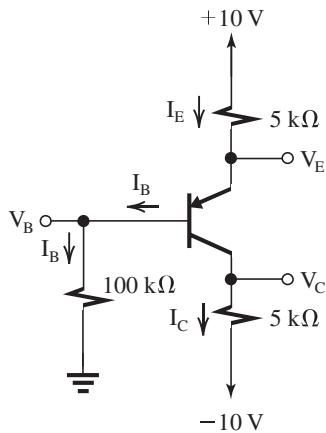
$$I_C = I_E - I_B = 0.93 - 0.0182 = 0.91 \text{ mA}$$

$$V_C = 10 - I_C \times 5$$

$$= 10 - 0.91 \times 5 = 5.45 \text{ V}$$

Since  $V_C > V_B$ , the transistor is operating in the active mode, as assumed.

Ex: 4.14



$$V_B = 1.0 \text{ V}$$

Thus,

$$I_B = \frac{V_B}{100 \text{ k}\Omega} = 0.01 \text{ mA}$$

$$V_E = +1.7 \text{ V}$$

Thus,

$$I_E = \frac{10 - V_E}{5 \text{ k}\Omega} = \frac{10 - 1.7}{5} = 1.66 \text{ mA}$$

and

$$\beta + 1 = \frac{I_E}{I_B} = \frac{1.66}{0.01} = 166$$

$$\Rightarrow \beta = 165$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{165}{165 + 1} = 0.994$$

Assuming active-mode operation,

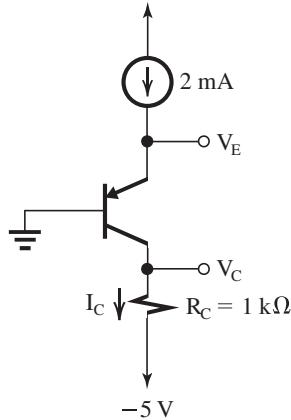
$$I_C = \alpha I_E = 0.994 \times 1.66 = 1.65 \text{ mA}$$

and

$$V_C = -10 + 1.65 \times 5 = -1.75 \text{ V}$$

Since  $V_C < V_B$ , the transistor is indeed operating in the active mode.

Ex: 4.15



The transistor is operating at a constant emitter current. Thus, a change in temperature of  $+30^\circ\text{C}$  results in a change in  $V_{EB}$  by

$$\Delta V_{EB} = -2 \text{ mV} \times 30 = -60 \text{ mV}$$

Thus,

$$\Delta V_E = -60 \text{ mV}$$

Since the collector current remains unchanged at  $\alpha I_E$ , the collector voltage does not change:

$$\Delta V_C = 0 \text{ V}$$

Ex: 4.16 Refer to Fig. 4.19(a):

$$i_C = I_S e^{v_{BE}/V_T} + \frac{V_{CE}}{r_o} \quad (1)$$

Now using Eqs. (4.21) and (4.22), we can express  $r_o$  as

$$r_o = \frac{V_A}{I_S e^{v_{BE}/V_T}}$$

Substituting in Eq. (1), we have

$$i_C = I_S e^{v_{BE}/V_T} \left( 1 + \frac{V_{CE}}{V_A} \right)$$

which is Eq. (4.18). Q.E.D.

$$\text{Ex: 4.17 } r_o = \frac{V_A}{I_C} = \frac{100}{I_C}$$

At  $I_C = 0.1 \text{ mA}$ ,  $r_o = 1 \text{ M}\Omega$

At  $I_C = 1 \text{ mA}$ ,  $r_o = 100 \text{ k}\Omega$

At  $I_C = 10 \text{ mA}$ ,  $r_o = 10 \text{ k}\Omega$

### Exercise 4-4

Ex: 4.18  $\Delta I_C = \frac{\Delta V_{CE}}{r_o}$

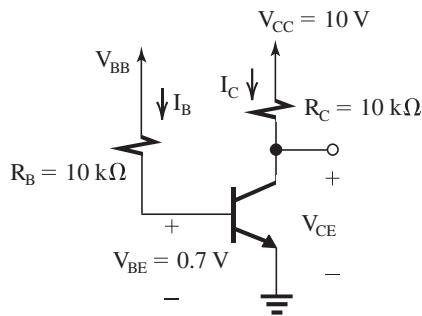
where

$$r_o = \frac{V_A}{I_C} = \frac{100}{1} = 100 \text{ k}\Omega$$

$$\Delta I_C = \frac{11 - 1}{100} = 0.1 \text{ mA}$$

Thus,  $I_C$  becomes 1.1 mA.

Ex: 4.19



(a) For operation in the active mode with  $V_{CE} = 5 \text{ V}$ ,

$$I_C = \frac{V_{CC} - V_C}{R_C} = \frac{10 - 5}{10} = 0.5 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5}{50} = 0.01 \text{ mA}$$

$$V_{BB} = V_{BE} + I_B R_B \\ = 0.7 + 0.01 \times 10 = 0.8 \text{ V}$$

(b) For operation at the edge of saturation,

$$V_{CE} = 0.3 \text{ V}$$

$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.3}{10} = 0.97 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.97}{50} = 0.0194 \text{ mA}$$

$$V_{BB} = V_B + I_B R_B$$

$$= 0.7 + 0.0194 \times 10 = 0.894 \text{ V}$$

(c) For operation deep in saturation with  $\beta_{\text{forced}} = 10$ , we have

$$V_{CE} \simeq 0.2 \text{ V}$$

$$I_C = \frac{10 - 0.2}{10} = 0.98 \text{ mA}$$

$$I_B = \frac{I_C}{\beta_{\text{forced}}} = \frac{0.98}{10} = 0.098 \text{ mA}$$

$$V_{BB} = V_B + I_B R_B$$

$$= 0.7 + 0.098 \times 10 = 1.68 \text{ V}$$

Ex: 4.20 For  $V_{BB} = 0 \text{ V}$ ,  $I_B = 0$  and the transistor is cut off. Thus,

$$I_C = 0$$

and

$$V_C = V_{CC} = +10 \text{ V}$$

Ex: 4.21 Refer to the circuit in Fig. 4.22 and let  $V_{BB} = 1.7 \text{ V}$ . The current  $I_B$  can be found from

$$I_B = \frac{V_{BB} - V_B}{R_B} = \frac{1.7 - 0.7}{10} = 0.1 \text{ mA}$$

Assuming operation in the active mode,

$$I_C = \beta I_B = 50 \times 0.1 = 5 \text{ mA}$$

Thus,

$$V_C = V_{CC} - R_C I_C$$

$$= 10 - 1 \times 5 = 5 \text{ V}$$

which is greater than  $V_B$ , verifying that the transistor is operating in the active mode, as assumed.

(a) To obtain operation at the edge of saturation,  $R_C$  must be increased to the value that results in  $V_{CE} = 0.3 \text{ V}$ :

$$R_C = \frac{V_{CC} - 0.3}{I_C} \\ = \frac{10 - 0.3}{5} = 1.94 \text{ k}\Omega$$

(b) Further increasing  $R_C$  results in the transistor operating in saturation. To obtain saturation-mode operation with  $V_{CE} = 0.2 \text{ V}$  and  $\beta_{\text{forced}} = 10$ , we use

$$I_C = \beta_{\text{forced}} \times I_B \\ = 10 \times 0.1 = 1 \text{ mA}$$

The value of  $R_C$  required can be found from

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} \\ = \frac{10 - 0.2}{1} = 9.8 \text{ k}\Omega$$

Ex: 4.22 Refer to the circuit in Fig. 4.23(a) with the base voltage raised from 4 V to  $V_B$ . If at this value of  $V_B$ , the transistor is at the edge of saturation then,

$$V_C = V_B - 0.4 \text{ V}$$

Since  $I_C \simeq I_E$ , we can write

$$\frac{10 - V_C}{R_C} = \frac{V_E}{R_E} = \frac{V_B - 0.7}{R_E}$$

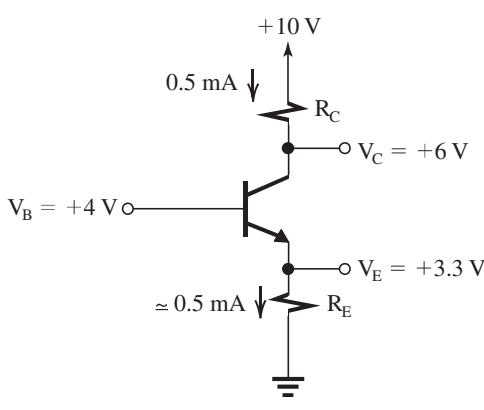
### Exercise 4-5

Thus,

$$\frac{10 - (V_B - 0.4)}{4.7} = \frac{V_B - 0.7}{3.3}$$

$$\Rightarrow V_B = +4.7 \text{ V}$$

Ex: 4.23



To establish a reverse-bias voltage of 2 V across the CBJ,

$$V_C = +6 \text{ V}$$

From the figure we see that

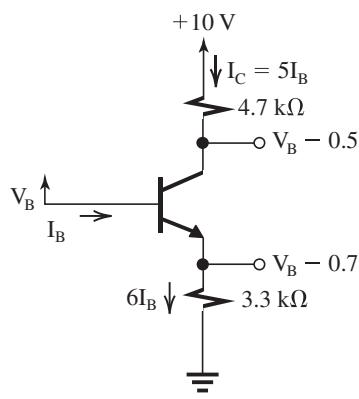
$$R_C = \frac{10 - 6}{0.5} = 8 \text{ k}\Omega$$

and

$$R_E = \frac{3.3}{0.5} = 6.6 \text{ k}\Omega$$

where we have assumed  $\alpha \approx 1$ .

Ex: 4.24



The figure shows the circuit with the base voltage at  $V_B$  and the BJT operating in saturation with  $V_{CE} = 0.2 \text{ V}$  and  $\beta_{\text{forced}} = 5$ .

$$I_C = 5I_B = \frac{10 - (V_B - 0.5)}{4.7} \quad (1)$$

$$I_E = 6I_B = \frac{V_B - 0.7}{3.3} \quad (2)$$

Dividing Eq. (1) by Eq. (2), we have

$$\frac{5}{6} = \frac{10.5 - V_B}{V_B - 0.7} \times \frac{3.3}{4.7}$$

$$\Rightarrow V_B = +5.18 \text{ V}$$

Ex: 4.25 Refer to the circuit in Fig. 4.26(a). The largest value for  $R_C$  while the BJT remains in the active mode corresponds to

$$V_C = +0.4 \text{ V}$$

Since the emitter and collector currents remain unchanged, then from Fig. 4.26(b) we obtain

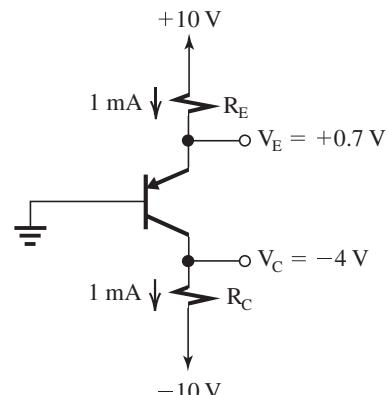
$$I_C = 4.6 \text{ mA}$$

Thus,

$$R_C = \frac{V_C - (-10)}{I_C}$$

$$= \frac{+0.4 + 10}{4.6} = 2.26 \text{ k}\Omega$$

Ex: 4.26



For a 4-V reverse-biased voltage across the CBJ,

$$V_C = -4 \text{ V}$$

Refer to the figure.

$$I_C = 1 \text{ mA} = \frac{V_C - (-10)}{R_C}$$

$$\Rightarrow R_C = \frac{-4 + 10}{1} = 6 \text{ k}\Omega$$

$$R_E = \frac{10 - V_E}{I_E}$$

Assuming  $\alpha = 1$ ,

$$R_E = \frac{10 - 0.7}{1} = 9.3 \text{ k}\Omega$$

### Exercise 4-6

Ex: 4.27 Refer to the circuit in Fig. 4.27:

$$I_B = \frac{5 - 0.7}{100} = 0.043 \text{ mA}$$

To ensure that the transistor remains in the active mode for  $\beta$  in the range 50 to 150, we need to select  $R_C$  so that for the highest collector current possible, the BJT reaches the edge of saturation, that is,  $V_{CE} = 0.3 \text{ V}$ . Thus,

$$V_{CE} = 0.3 = 10 - R_C I_{Cmax}$$

where

$$I_{Cmax} = \beta_{max} I_B$$

$$= 150 \times 0.043 = 6.45 \text{ mA}$$

Thus,

$$R_C = \frac{10 - 0.3}{6.45} = 1.5 \text{ k}\Omega$$

For the lowest  $\beta$ ,

$$I_C = \beta_{min} I_B$$

$$= 50 \times 0.043 = 2.15 \text{ mA}$$

and the corresponding  $V_{CE}$  is

$$V_{CE} = 10 - R_C I_C = 10 - 1.5 \times 2.15$$

$$= 6.775 \text{ V}$$

Thus,  $V_{CE}$  will range from 0.3 V to 6.8 V.

Ex: 4.28 Refer to the solution of Example 4.10.

$$\begin{aligned} I_E &= \frac{V_{BB} - V_{BE}}{R_E + [R_{BB}/(\beta + 1)]} \\ &= \frac{5 - 0.7}{3 + (33.3/51)} = 1.177 \text{ mA} \end{aligned}$$

$$I_C = \alpha I_E = 0.98 \times 1.177 = 1.15 \text{ mA}$$

Thus the current is reduced by

$$\Delta I_C = 1.28 - 1.15 = 0.13 \text{ mA}$$

which is a -10% change.

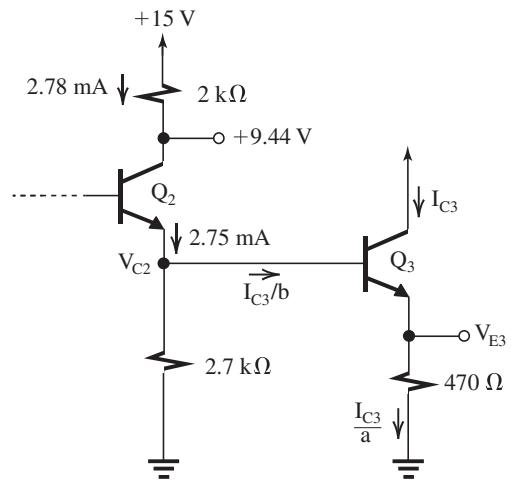
Ex: 4.29 Refer to the circuit in Fig. 4.30(b). The total current drawn from the power supply is

$$I = 0.103 + 1.252 + 2.78 = 4.135 \text{ mA}$$

Thus, the power dissipated in the circuit is

$$P = 15 \text{ V} \times 4.135 \text{ mA} = 62 \text{ mW}$$

Ex: 4.30



From the figure we see that

$$V_{E3} = \frac{I_{C3}}{\alpha} \times 0.47$$

$$V_{C2} = V_{E3} + 0.7 = \frac{I_{C3}}{\alpha} \times 0.47 + 0.7 \quad (1)$$

A node equation at the collector of Q2 yields

$$2.75 = \frac{V_{C2}}{2.7} + \frac{I_{C3}}{\beta}$$

Substituting for  $V_{C2}$  from Eq. (1), we obtain

$$2.75 = \frac{(0.47 I_{C3}/\alpha) + 0.7}{2.7} + \frac{I_{C3}}{\beta}$$

Substituting  $\alpha = 0.99$  and  $\beta = 100$  and solving for  $I_{C3}$  results in

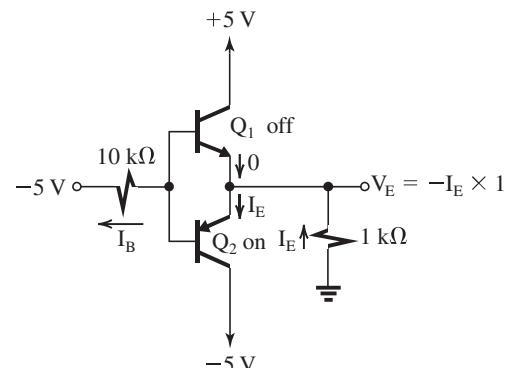
$$I_{C3} = 13.4 \text{ mA}$$

Now,  $V_{E3}$  and  $V_{C2}$  can be determined:

$$V_{E3} = \frac{I_{C3}}{\alpha} \times 0.47 = \frac{13.4}{0.99} \times 0.47 = +6.36 \text{ V}$$

$$V_{C2} = V_{E3} + 0.7 = +7.06 \text{ V}$$

Ex: 4.31



### Exercise 4-7

From the figure we see that  $Q_1$  will be off and  $Q_2$  will be on. Since the base of  $Q_2$  will be at a voltage higher than  $-5$  V, transistor  $Q_2$  will be operating in the active mode. We can write a loop equation for the loop containing the  $10\text{-k}\Omega$  resistor, the EBJ of  $Q_2$  and the  $1\text{-k}\Omega$  resistor:

$$-I_E \times 1 - 0.7 - I_B \times 10 = -5$$

Substituting  $I_B = I_E/(\beta + 1) = I_E/101$  and rearranging gives

$$I_E = \frac{5 - 0.7}{\frac{10}{101} + 1} = 3.9 \text{ mA}$$

Thus,

$$V_E = -3.9 \text{ V}$$

$$V_{B2} = -4.6 \text{ V}$$

$$I_B = 0.039 \text{ mA}$$

Ex: 4.32 With the input at  $+10$  V, there is a strong possibility that the conducting transistor

$Q_1$  will be saturated. Assuming this to be the case, the analysis steps will be as follows:

$$V_{CEsat}|_{Q_1} = 0.2 \text{ V}$$

$$V_E = 5 \text{ V} - V_{CEsat} = +4.8 \text{ V}$$

$$I_{E1} = \frac{4.8 \text{ V}}{1 \text{ k}\Omega} = 4.8 \text{ mA}$$

$$V_{BE1} = V_E + V_{BE1} = 4.8 + 0.7 = +5.5 \text{ V}$$

$$I_{B1} = \frac{10 - 5.5}{10} = 0.45 \text{ mA}$$

$$I_{C1} = I_{E1} - I_{B1} = 4.8 - 0.45 = 4.35 \text{ mA}$$

$$\beta_{forced} = \frac{I_C}{I_B} = \frac{4.35}{0.45} = 9.7$$

which is lower than  $\beta_{min}$ , verifying that  $Q_1$  is indeed saturated.

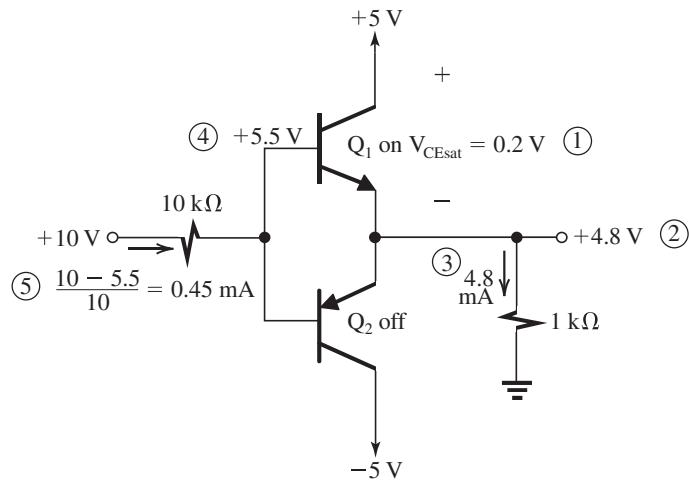
Finally, since  $Q_2$  is off,

$$I_{C2} = 0$$

$$\text{Ex: 4.33 } V_O = +10 - BV_{CEO} = 10 - 70$$

$$= -60 \text{ V}$$

This figure belongs to Exercise 4.32.



## Chapter 4–1

- 4.1  
 1. Active  
 2. Saturation  
 3. Active  
 4. Saturation  
 5. Active  
 6. Cutoff

4.2 The EB junctions have a 4:1 area ratio.

$$I_C = I_S e^{V_{BE}/V_T}$$

$$0.5 \times 10^{-3} = I_{S1} \times e^{0.75/0.025}$$

$$\Rightarrow I_{S1} = 4.7 \times 10^{-17} \text{ A}$$

$$I_{S2} = 4I_{S1} = 1.87 \times 10^{-16} \text{ A}$$

$$4.3 \quad I_{C1} = 10^{-13} e^{700/25} = 0.145 \text{ A} = 145 \text{ mA}$$

$$I_{C2} = 10^{-18} e^{700/25} = 1.45 \mu\text{A}$$

For the first transistor 1 to conduct a current of 1.45  $\mu\text{A}$ , its  $V_{BE}$  must be

$$V_{BE1} = 0.025 \ln\left(\frac{1.45 \times 10^{-6}}{10^{-13}}\right)$$

$$= 0.412 \text{ V}$$

$$4.4 \quad \frac{I_{S1}}{I_{S2}} = \frac{A_{E1}}{A_{E2}} = \frac{200 \times 200}{0.4 \times 0.4} = 250,000$$

$$I_{C1} = I_{S1} e^{V_{BE1}/V_T}$$

$$I_{C2} = I_{S2} e^{V_{BE2}/V_T}$$

For  $I_{C1} = I_{C2}$  we have

$$e^{(V_{BE2}-V_{BE1})/V_T} = \frac{I_{S1}}{I_{S2}} = 250,000$$

$$V_{BE2} - V_{BE1} = 0.025 \ln(250,000)$$

$$= 0.31 \text{ V}$$

4.5 Old technology:

$$10^{-3} = 2 \times 10^{-15} e^{V_{BE}/V_T}$$

$$V_{BE} = 0.025 \ln\left(\frac{10^{-3}}{2 \times 10^{-15}}\right) = 0.673 \text{ V}$$

New technology:

$$10^{-3} = 2 \times 10^{-18} e^{V_{BE}/V_T}$$

$$V_{BE} = 0.025 \ln\left(\frac{10^{-3}}{2 \times 10^{-18}}\right) = 0.846 \text{ V}$$

$$4.6 \quad I_B = 10 \mu\text{A}$$

$$I_C = 800 \mu\text{A}$$

$$\beta = \frac{I_C}{I_B} = 80$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{80}{81} = 0.988$$

4.7

$\alpha$	0.5	0.8	0.9	0.95	0.98	0.99	0.995	0.999
$\beta = \frac{\alpha}{1-\alpha}$	1	4	9	19	49	99	199	999

4.8

$\beta$	1	2	10	20	50	100	200	500	1000
$\alpha = \frac{\beta}{\beta+1}$	0.5	0.67	0.91	0.95	0.98	0.99	0.995	0.998	0.999

$$4.9 \quad \beta = \frac{\alpha}{1-\alpha} \quad (1)$$

$$\alpha \rightarrow \alpha + \Delta\alpha$$

$$\beta \rightarrow \beta + \Delta\beta$$

$$\beta + \Delta\beta = \frac{\alpha + \Delta\alpha}{1 - \alpha - \Delta\alpha} \quad (2)$$

Subtracting Eq. (1) from Eq. (2) gives

$$\begin{aligned} \Delta\beta &= \frac{\alpha + \Delta\alpha}{1 - \alpha - \Delta\alpha} - \frac{\alpha}{1 - \alpha} \\ \Delta\beta &= \frac{\Delta\alpha}{(1 - \alpha - \Delta\alpha)(1 - \alpha)} \end{aligned} \quad (3)$$

Dividing Eq. (3) by Eq. (1) gives

$$\frac{\Delta\beta}{\beta} = \left(\frac{\Delta\alpha}{\alpha}\right) \left(\frac{1}{1 - \alpha - \Delta\alpha}\right)$$

For  $\Delta\alpha \ll 1$ , the second factor on the right-hand side is approximately equal to  $\beta$ . Thus

$$\frac{\Delta\beta}{\beta} \simeq \beta \left(\frac{\Delta\alpha}{\alpha}\right) \quad \text{Q.E.D.}$$

For  $\frac{\Delta\beta}{\beta} = -10\%$  and  $\beta = 100$ ,

$$\frac{\Delta\alpha}{\alpha} \simeq \frac{-10\%}{100} = -0.1\%$$

4.10 Transistor is operating in active region:

$$\beta = 50 \rightarrow 300$$

$$I_B = 10 \mu\text{A}$$

$$I_C = \beta I_B = 0.5 \text{ mA} \rightarrow 3 \text{ mA}$$

$$I_E = (\beta + 1) I_B = 0.51 \text{ mA} \rightarrow 3.01 \text{ mA}$$

Maximum power dissipated in transistor is

$$\begin{aligned} I_B \times 0.7 \text{ V} + I_C \times V_C \\ = 0.01 \times 0.7 + 3 \times 10 \simeq 30 \text{ mW} \end{aligned}$$

## Chapter 4–2

4.11 For  $i_B = 10 \mu A$ ,

$$i_C = i_E - i_B = 1000 - 10 = 990 \mu A$$

$$\beta = \frac{i_C}{i_B} = \frac{990}{10} = 99$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{99}{100} = 0.99$$

For  $i_B = 20 \mu A$ ,

$$i_C = i_E - i_B = 1000 - 20 = 980 \mu A$$

$$\beta = \frac{i_C}{i_B} = \frac{980}{20} = 49$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{49}{50} = 0.98$$

For  $i_B = 50 \mu A$ ,

$$i_C = i_E - i_B = 1000 - 50 = 950 \mu A$$

$$\beta = \frac{i_C}{i_B} = \frac{950}{50} = 19$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{19}{20} = 0.95$$

$$4.12 \quad i_C = I_S e^{v_{BE}/V_T}$$

$$= 5 \times 10^{-15} e^{0.7/0.025} = 7.2 \text{ mA}$$

$i_B$  will be in the range  $\frac{7.2}{50}$  mA to  $\frac{7.2}{200}$  mA, that is,  $144 \mu A$  to  $36 \mu A$ .

$i_E$  will be in the range  $(7.2 + 0.144)$  mA to  $(7.2 + 0.036)$  mA, that is,  $7.344$  mA to  $7.236$  mA.

4.14 First we determine  $I_S$ ,  $\beta$ , and  $\alpha$ :

$$1 \times 10^{-3} = I_S e^{700/25}$$

$$\Rightarrow I_S = 6.91 \times 10^{-16} \text{ A}$$

$$\beta = \frac{I_C}{I_B} = \frac{1 \text{ mA}}{10 \mu A} = 100$$

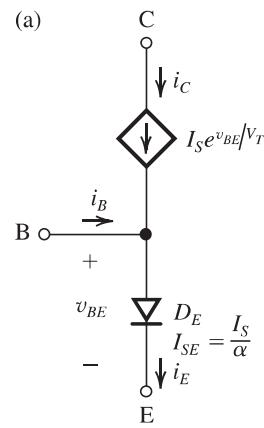
$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

Then we can determine  $I_{SE}$  and  $I_{SB}$ :

$$I_{SE} = \frac{I_S}{\alpha} = 6.98 \times 10^{-16} \text{ A}$$

$$I_{SB} = \frac{I_S}{\beta} = 6.91 \times 10^{-18} \text{ A}$$

The figure below and on next page shows the four large-signal models, corresponding to Fig. 4.5(a) to (d), together with their parameter values.



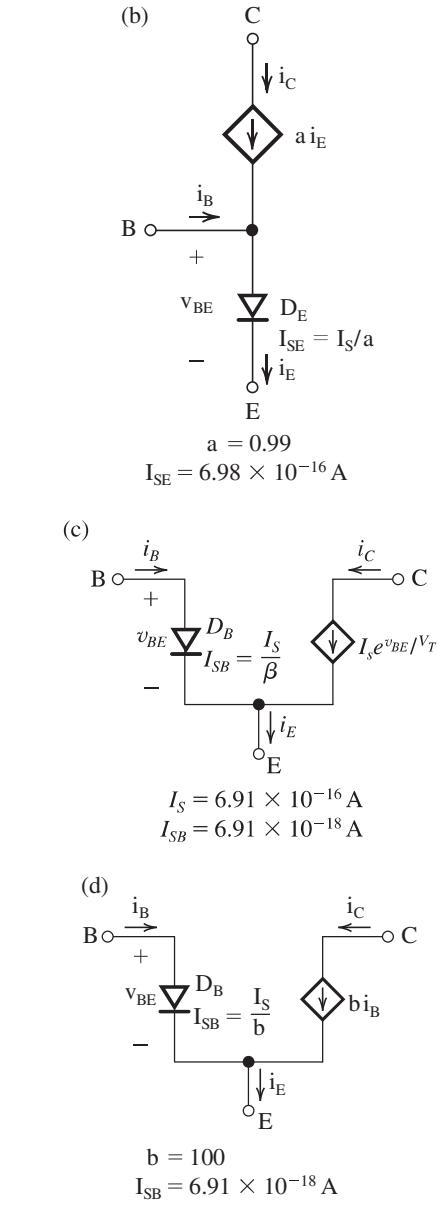
$$I_S = 6.91 \times 10^{-16} \text{ A}$$

$$I_{SE} = 6.98 \times 10^{-16} \text{ A}$$

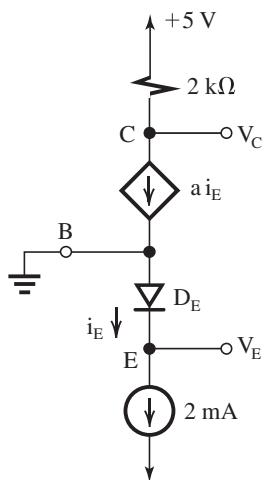
4.13 See table below.

This table belongs to Problem 4.13.

Transistor	a	b	c	d	e
$V_{BE}$ (mV)	700	690	580	780	820
$I_C$ (mA)	1.000	1.000	0.230	10.10	73.95
$I_B$ ( $\mu A$ )	10	20	5	120	1050
$I_E$ (mA)	1.010	1.020	0.235	10.22	75
$\alpha$	0.99	0.98	0.979	0.988	0.986
$\beta$	100	50	46	84	70
$I_S$ (A)	$6.9 \times 10^{-16}$	$1.0 \times 10^{-15}$	$1.9 \times 10^{-14}$	$2.8 \times 10^{-16}$	$4.2 \times 10^{-16}$



4.15



The figure shows the circuit, where

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

$$I_{SE} = \frac{I_s}{\alpha} = \frac{5 \times 10^{-15}}{0.99} = 5.05 \times 100^{-15} \text{ A}$$

The voltage at the emitter  $V_E$  is

$$V_E = -V_{DE}$$

$$= -V_T \ln(I_E/I_{SE})$$

$$= -0.025 \ln\left(\frac{2 \times 10^{-3}}{5.05 \times 10^{-15}}\right)$$

$$= -0.668 \text{ V}$$

The voltage at the collector  $V_C$  is found from

$$V_C = 5 - I_C \times 2$$

$$= 5 - 0.99 \times 2 \times 2 = 1.04 \text{ V}$$

4.16 Refer to the circuit in Fig. 4.6(b).

$$I_{SB} = \frac{I_s}{\beta} = \frac{5 \times 10^{-15}}{50} = 10^{-16} \text{ A}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5 \times 10^{-3}}{50} = 10^{-5} \text{ A}$$

$$V_B = V_{BE} = V_T \ln\left(\frac{I_B}{I_{SB}}\right)$$

$$= 0.025 \ln\left(\frac{10^{-5}}{10^{-16}}\right)$$

$$= 0.633 \text{ V}$$

We can determine  $R_B$  from

$$R_B = \frac{V_{CC} - V_B}{I_B}$$

$$= \frac{15 - 0.633}{10^{-5}} = 1.44 \text{ M}\Omega$$

To obtain  $V_{CE} = 1 \text{ V}$ , we select  $R_C$  according to

$$R_C = \frac{V_{CC} - V_{CE}}{I_C}$$

$$= \frac{15 - 1}{0.5} = 28 \text{ k}\Omega$$

4.17  $I_s = 10^{-15} \text{ A}$

Thus, a forward-biased EBJ conducting a current of 1 mA will have a forward voltage drop  $V_{BE}$ :

$$V_{BE} = V_T \ln\left(\frac{I}{I_s}\right)$$

$$= 0.025 \ln\left(\frac{10^{-3}}{10^{-15}}\right) = 0.691 \text{ V}$$

$$I_{SC} = 100I_s = 10^{-13} \text{ A}$$

## Chapter 4–4

Thus, a forward-biased CBJ conducting a 1-mA current will have a forward voltage drop  $V_{BC}$ :

$$V_{BC} = V_T \ln\left(\frac{1 \times 10^{-3}}{1 \times 10^{-13}}\right) = 0.576 \text{ V}$$

When forward-biased with 0.5 V, the emitter-base junction conducts

$$\begin{aligned} I &= I_S e^{0.5/0.025} \\ &= 10^{-15} e^{0.5/0.025} = 0.49 \mu\text{A} \end{aligned}$$

and the CBJ conducts

$$\begin{aligned} I &= I_{SC} e^{0.5/0.025} \\ &= 10^{-13} e^{0.5/0.025} = 48.5 \mu\text{A} \end{aligned}$$

4.18 Dividing Eq. (4.14) by Eq. (4.15) and substituting  $i_C/i_B = \beta_{\text{forced}}$  gives

$$\beta_{\text{forced}} = \frac{I_S e^{V_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T}}{(I_S/\beta) e^{V_{BE}/V_T} + I_{SC} e^{V_{BC}/V_T}}$$

Dividing the numerator and denominator of the right-hand side by  $I_{SC} e^{V_{BC}/V_T}$  and replacing  $V_{BE} - V_{BC}$  by  $V_{CE\text{sat}}$  gives

$$\beta_{\text{forced}} = \frac{\left(\frac{I_S}{I_{SC}}\right) e^{V_{CE\text{sat}}/V_T} - 1}{\frac{1}{\beta} \left(\frac{I_S}{I_{SC}}\right) e^{V_{CE\text{sat}}/V_T} + 1}$$

This equation can be used to obtain  $e^{V_{CE\text{sat}}/V_T}$  and hence  $V_{CE\text{sat}}$  as

$$\begin{aligned} \left(\frac{I_S}{I_{SC}}\right) e^{V_{CE\text{sat}}/V_T} &= \frac{1 + \beta_{\text{forced}}}{1 - \beta_{\text{forced}}/\beta} \\ \Rightarrow V_{CE\text{sat}} &= V_T \ln\left[\frac{I_{SC}}{I_S} \frac{1 + \beta_{\text{forced}}}{1 - \beta_{\text{forced}}/\beta}\right] \quad \text{Q.E.D.} \end{aligned}$$

For  $\beta = 100$  and  $I_{SC}/I_S = 100$ , we can use this equation to obtain  $V_{CE\text{sat}}$  corresponding to the given values of  $\beta_{\text{forced}}$ . The results are as follows:

$\beta_{\text{forced}}$	50	10	5	1
$V_{CE\text{sat}}$ (V)	0.231	0.178	0.161	0.133

This table belongs to Problem 4.19.

4.19 The equations utilized are

$$V_{BC} = V_{BE} - V_{CE} = 0.7 - V_{CE}$$

$$i_{BC} = I_{SC} e^{V_{BC}/V_T} = 10^{-13} e^{V_{BC}/0.025}$$

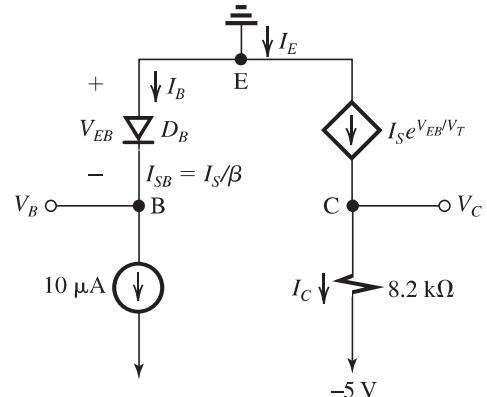
$$i_{BE} = I_{SB} e^{V_{BE}/V_T} = 10^{-17} e^{0.7/0.025}$$

$$i_B = i_{BC} + i_{BE}$$

$$i_C = I_S e^{V_{BE}/V_T} - i_{BC} = 10^{-15} e^{0.7/0.025} - i_{BC}$$

Performing these calculations for  $V_{CE} = 0.4 \text{ V}$ ,  $0.3 \text{ V}$ , and  $0.2 \text{ V}$ , we obtain the results shown in the table below.

4.20



The emitter-base voltage  $V_{EB}$  is found as the voltage drop across the diode  $D_B$ , whose scale current is  $I_{SB} = I_S/\beta$ , it is conducting a 10-μA current. Thus,

$$V_{EB} = V_T \ln\left(\frac{10 \mu\text{A}}{I_{SB}}\right)$$

where

$$I_{SB} = \frac{I_S}{\beta} = \frac{10^{-14}}{50} = 2 \times 10^{-16} \text{ A}$$

$$V_{EB} = 0.025 \ln\left(\frac{10 \times 10^{-6}}{2 \times 10^{-16}}\right)$$

$$= 0.616 \text{ V}$$

Thus,

$$V_B = -V_{EB} = -0.616 \text{ V}$$

The collector current can be found as

$$I_C = \beta I_B$$

$$= 50 \times 10 = 500 \mu\text{A} = 0.5 \text{ mA}$$

$v_{CE}$ (V)	$v_{BC}$ (V)	$i_{BC}$ ( $\mu\text{A}$ )	$i_{BE}$ ( $\mu\text{A}$ )	$i_B$ ( $\mu\text{A}$ )	$i_C$ (mA)	$i_C/i_B$
0.4	0.3	0.016	14.46	14.48	1.446	100
0.3	0.4	0.89	14.46	15.35	1.445	94
0.2	0.5	48.5	14.46	62.96	1.398	29

The collector voltage can now be obtained from

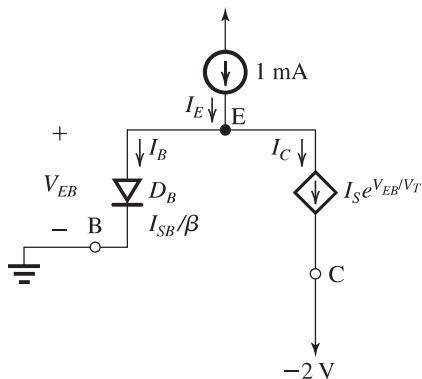
$$V_C = -5 + I_C \times 8.2 = -5 + 0.5 \times 8.2 = -0.9 \text{ V}$$

The emitter current can be found as

$$I_E = I_B + I_C = 10 + 500 = 510 \mu\text{A}$$

$$= 0.51 \text{ mA}$$

4.21



Referring to the figure, we see that

$$I_E = I_B + I_C = \frac{I_C}{\beta} + I_C$$

Thus,

$$I_C = \frac{I_E}{1 + \frac{1}{\beta}} = \frac{1}{1 + \frac{1}{10}} = 0.909 \text{ mA}$$

$$I_B = 0.091 \text{ mA}$$

For direction of flow, refer to the figure.

$$V_{EB} = V_T \ln\left(\frac{I_B}{I_{SB}}\right)$$

where

$$I_{SB} = \frac{I_S}{\beta} = \frac{10^{-15}}{10} = 10^{-16} \text{ A}$$

$$V_{EB} = 0.025 \ln\left(\frac{0.091 \times 10^{-3}}{10^{-16}}\right)$$

$$= 0.688 \text{ V}$$

Thus,

$$V_E = V_B + V_{EB} = 0 + 0.688 = 0.688 \text{ V}$$

If a transistor with  $\beta = 1000$  is substituted,

$$I_C = \frac{I_E}{1 + \frac{1}{\beta}} = \frac{1}{1 + \frac{1}{1000}} = 0.999 \text{ mA}$$

Thus,  $I_C$  changes by  $0.999 - 0.909 = 0.09 \text{ mA}$ , a 9.9% increase.

4.22

$$I_B = \frac{I_E}{\beta + 1} = \frac{5}{20 + 1} = 0.238 \text{ A} = 238 \text{ mA}$$

$$I_C = I_S e^{V_{EB}/V_T}$$

$$\alpha I_E = I_S e^{V_{EB}/V_T}$$

where

$$\alpha = \frac{20}{21} = 0.95$$

$$I_S = \alpha I_E e^{-V_{EB}/V_T}$$

$$= 0.95 \times 5 e^{-(0.8/0.025)}$$

$$= 6 \times 10^{-14} \text{ A}$$

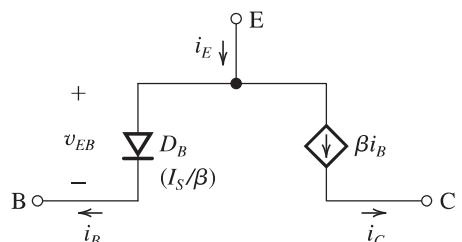
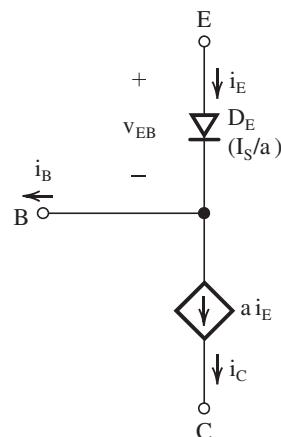
A transistor that conducts  $I_C = 1 \text{ mA}$  with  $V_{EB} = 0.70 \text{ V}$  has a scale current

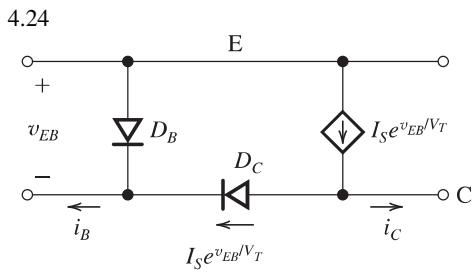
$$I_S = 1 \times 10^{-3} e^{-0.70/0.025} = 6.9 \times 10^{-16} \text{ A}$$

The emitter-base junction areas of these two transistors will have the same ratio as that of their scale currents, thus

$$\frac{\text{EBJ area of first transistor}}{\text{EBJ area of second transistor}} = \frac{6 \times 10^{-14}}{6.9 \times 10^{-16}} = 87$$

4.23 The two missing large-signal equivalent circuits for the pnp transistor are those corresponding to the npn equivalent circuits in Fig. 4.5(b) and 4.5(d). They are shown in the figure.





4.25 (a) Refer to Fig. P4.25(a).

$$I_1 = \frac{10.7 - 0.7}{5 \text{ k}\Omega} = 2 \text{ mA}$$

Assuming operation in the active mode,

$$I_C = \alpha I_1 \simeq I_1 = 2 \text{ mA}$$

$$V_2 = -10.7 + I_C \times 5$$

$$= -10.7 + 2 \times 5 = -0.7 \text{ V}$$

Since  $V_2$  is lower than  $V_B$ , which is 0 V, the transistor is operating in the active mode, as assumed.

(b) Refer to Fig. P4.25(b).

Since  $V_C = -4 \text{ V}$  is lower than  $V_B = -2.7 \text{ V}$ , the transistor is operating in the active mode.

$$I_C = \frac{-4 - (-10)}{2.4 \text{ k}\Omega} = 2.5 \text{ mA}$$

$$I_E = \frac{I_C}{\alpha} \simeq I_C = 2.5 \text{ mA}$$

$$V_3 = +12 - I_E \times 5.6 = 12 - 2.5 \times 5.6 = -2 \text{ V}$$

(c) Refer to Fig. P4.25(c) and use

$$I_C = \frac{0 - (-10)}{20} = 0.5 \text{ mA}$$

Assuming active-mode operation, and utilizing the fact that  $\beta$  is large,  $I_B \simeq 0$  and

$$V_4 \simeq 2 \text{ V}$$

Since  $V_C < V_B$ , the transistor is indeed operating in the active region.

$$I_5 = I_E = \frac{I_C}{\alpha} \simeq I_C = 0.5 \text{ mA}$$

(d) Refer to Fig. P4.25(d). Since the collector is connected to the base with a 10-k $\Omega$  resistor and  $\beta$  is assumed to be very high, the voltage drop across the 10-k $\Omega$  resistor will be close to zero and the base voltage will be equal to that of the collector:

$$V_B = V_7$$

This also implies active-mode operation. Now,

$$V_E = V_B - 0.7$$

Thus,

$$V_E = V_7 - 0.7$$

$$I_6 = \frac{V_E - (-10)}{3}$$

$$= \frac{V_7 - 0.7 + 10}{3} = \frac{V_7 + 9.3}{3} \quad (1)$$

Since  $I_B = 0$ , the collector current will be equal to the current through the 9.1-k $\Omega$  resistor,

$$I_C = \frac{+10 - V_7}{9.1} \quad (2)$$

Since  $\alpha \approx 1$ ,  $I_C = I_E = I_6$  resulting in

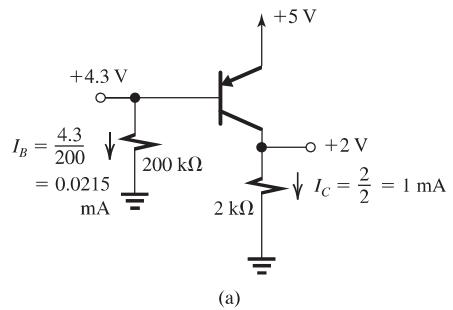
$$\frac{10 - V_7}{9.1} = \frac{V_7 + 9.3}{3}$$

$$\Rightarrow V_7 = -4.5 \text{ V}$$

and

$$I_6 = \frac{V_7 + 9.3}{3} = \frac{-4.5 + 9.3}{3} = 1.6 \text{ mA}$$

4.26 (a)



(a)

Since  $V_C$  is lower than  $V_B$ , the transistor is operating in the active region. From the figure corresponding to Fig. P4.26(a), we see that

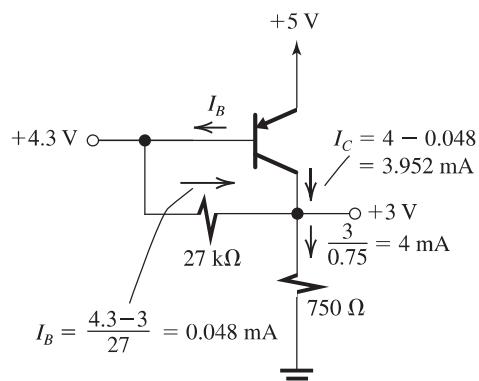
$$I_C = 1 \text{ mA}$$

$$I_B = 0.0215 \text{ mA}$$

Thus,

$$\beta \equiv \frac{I_C}{I_B} = \frac{1}{0.0215} = 46.5$$

(b)

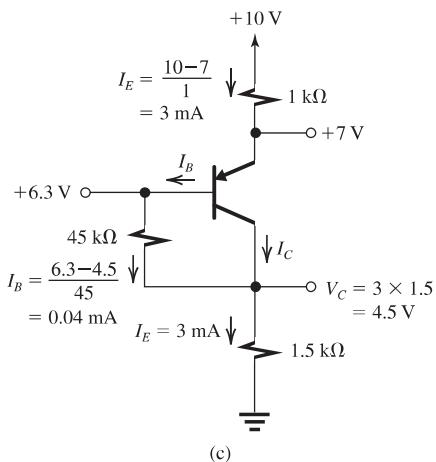


(b)

Observe that with  $V_C$  at 3 V and  $V_B$  at 4.3 V, the transistor is operating in the active region. Refer to the analysis shown in the figure, which leads to

$$\beta \equiv \frac{I_C}{I_B} = \frac{3.952}{0.048} = 82.3$$

(c)



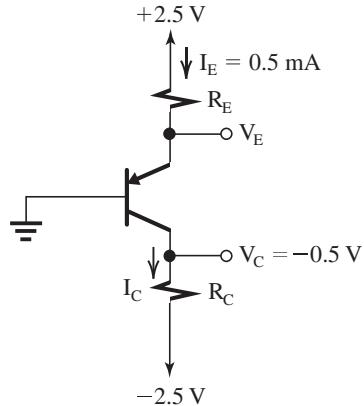
Observe that the transistor is operating in the active region and note the analysis performed on the circuit diagram. Thus,

$$I_C = I_E - I_B = 3 - 0.04 = 2.96 \text{ mA}$$

and

$$\beta \equiv \frac{I_C}{I_B} = \frac{2.96}{0.04} = 74$$

4.27



From the figure we see that  $V_C = -0.5 \text{ V}$  is lower than the base voltage ( $V_B = 0 \text{ V}$ ); thus the transistor will be operating in the active mode.

$$I_C = \alpha I_E = \left( \frac{\beta}{\beta + 1} \right) I_E = \frac{100}{100 + 1} \times 0.5 = 0.495 \text{ mA}$$

$$R_C = \frac{V_C - (-2.5)}{I_C} = \frac{-0.5 + 2.5}{0.495} = 4.04 \text{ k}\Omega \simeq 4 \text{ k}\Omega$$

The transistor  $V_{EB}$  can be found from

$$V_{EB} = 0.64 + V_T \ln \left( \frac{0.5 \text{ mA}}{0.1 \text{ mA}} \right)$$

$$= 0.68 \text{ V}$$

Thus,

$$V_E = +0.68 \text{ V}$$

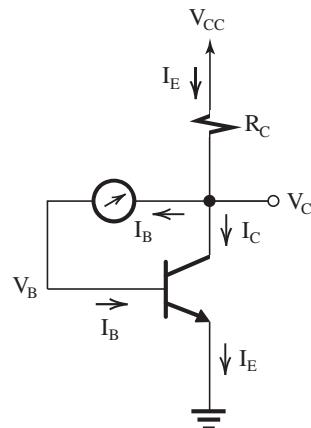
and

$$R_E = \frac{2.5 - 0.68}{0.5} = 3.64 \text{ k}\Omega$$

The maximum allowable value for  $R_C$  while the transistor remains in the active mode corresponds to  $V_C = +0.4 \text{ V}$ . Thus,

$$R_{C\max} = \frac{0.4 - (-2.5)}{0.495} = 5.86 \text{ k}\Omega$$

4.28



Since the meter resistance is small,  $V_C \simeq V_B$  and the transistor is operating in the active region. To obtain  $I_E = 1 \text{ mA}$ , we arrange that  $V_{BE} = 0.7 \text{ V}$ . Since  $V_C \simeq V_B$ ,  $V_C$  must be set to 0.7 by selecting  $R_C$  according to

$$V_C = 0.7 = V_{CC} - I_E R_C$$

Thus,

$$0.7 = 9 - 1 \times R_C$$

$$\Rightarrow R_C = 8.3 \text{ k}\Omega$$

Since the meter reads full scale when the current "owing through it (in this case,  $I_B$  is  $50 \mu\text{A}$ ), a full-scale reading corresponds to

$$\beta \equiv \frac{I_C}{I_B} \simeq \frac{1 \text{ mA}}{50 \mu\text{A}} = 20$$

If the meter reads 1/5 of full scale, then  $I_B = 10 \mu\text{A}$  and

$$\beta = \frac{1 \text{ mA}}{10 \mu\text{A}} = 100$$

A meter reading of 1/10 full scale indicates that

$$\beta = \frac{1 \text{ mA}}{5 \mu\text{A}} = 200$$

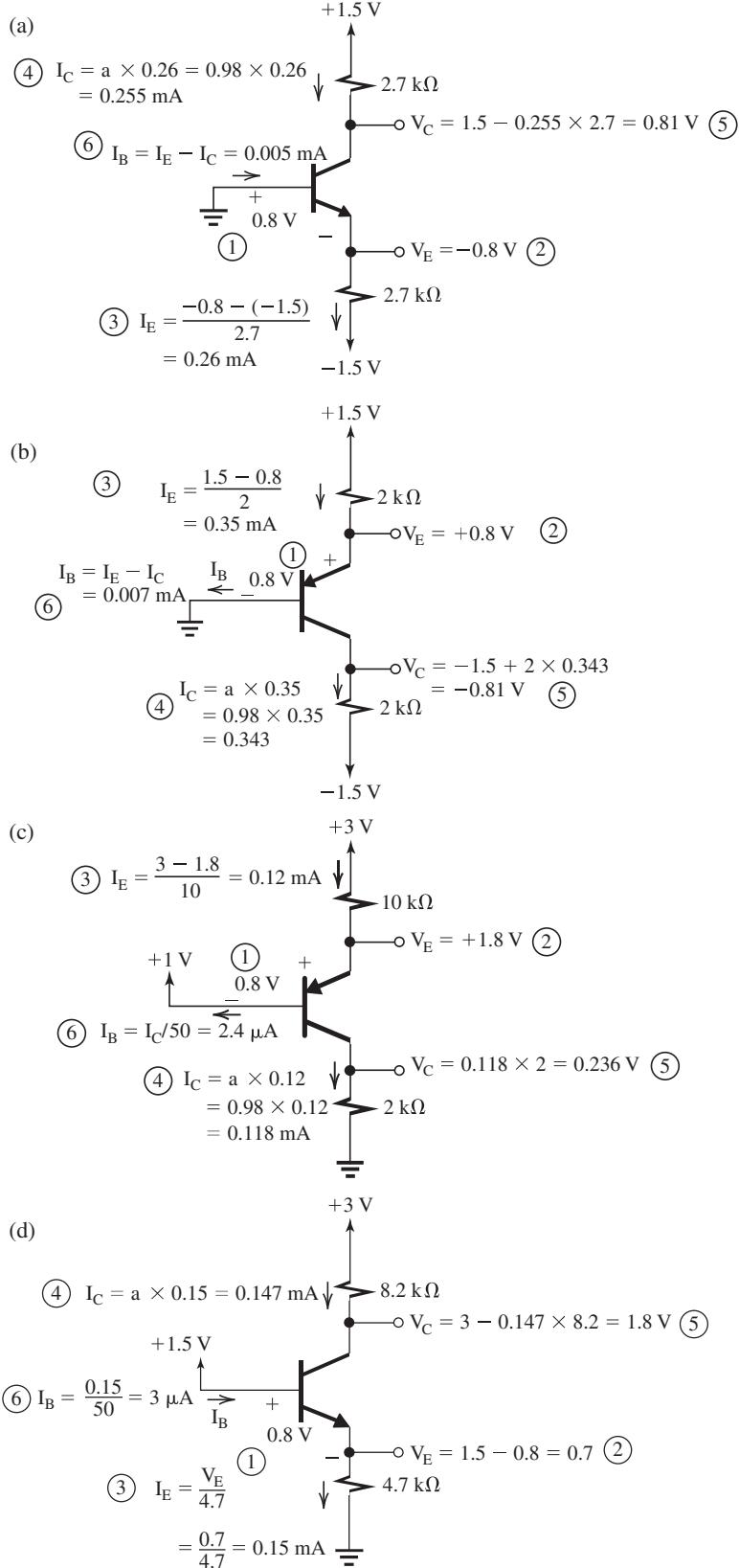
4.29 Refer to Fig. 4.15(a) with  $R_C = 5.1 \text{ k}\Omega$  and  $R_E = 6.8 \text{ k}\Omega$ . Assuming  $V_{BE} \simeq 0.7 \text{ V}$ , then  $V_E = -0.7 \text{ V}$ , and

$$I_E = \frac{-0.7 - (-15)}{6.8} = 2.1 \text{ mA}$$

$$I_C = \alpha I_E \simeq 2.1 \text{ mA}$$

$$V_C = 15 - 2.1 \times 5.1 \simeq 4.3 \text{ V}$$

4.30



In all circuits shown in Fig. P4.30, we assume active-mode operation and verify that this is the case at the end of the solution. The solutions are indicated on the corresponding circuit diagrams; the order of the steps is shown by the circled numbers.

4.31 Refer to the circuit in Fig. P4.31. Since  $V_C = 0.5$  V is greater than  $V_B$ , the transistor will be operating in the active mode. The transistor  $V_{BE}$  can be found from

$$V_{BE} = 0.8 + 0.025 \ln\left(\frac{0.2 \text{ mA}}{1 \text{ mA}}\right)$$

$$= 0.76 \text{ V}$$

Thus,

$$V_E = -0.76 \text{ V}$$

$$I_E = \frac{I_C}{\alpha} = I_C \left( \frac{\beta + 1}{\beta} \right) = 0.2 \times \frac{101}{100} \\ = 0.202 \text{ mA}$$

The required value of  $R_E$  can be found from

$$R_E = \frac{V_E - (-1.5)}{I_E}$$

$$R_E = \frac{-0.76 + 1.5}{0.202} = 3.66 \text{ k}\Omega$$

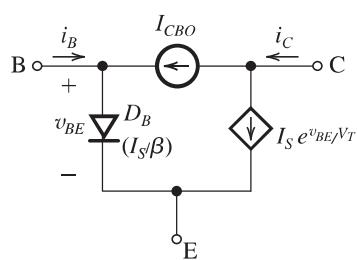
To establish  $V_C = 0.5$  V, we select  $R_C$  according to

$$R_C = \frac{1.5 - 0.5}{0.2} = 5 \text{ k}\Omega$$

4.32  $I_{CBO}$  approximately doubles for every  $10^\circ\text{C}$  rise in temperature. A change in temperature from  $25^\circ\text{C}$  to  $125^\circ\text{C}$  that is, an increase of  $100^\circ\text{C}$  results in 10 doublings or, equivalently, an increase by a factor of  $2^{10} = 1024$ . Thus  $I_{CBO}$  becomes

$$I_{CBO} = 10 \text{ nA} \times 1024 = 10.24 \mu\text{A}$$

4.33



From the figure we can write

$$I_B = \left( \frac{I_S}{\beta} \right) e^{V_{BE}/V_T} - I_{CBO} \quad (1)$$

$$I_C = I_S e^{V_{BE}/V_T} + I_{CBO} \quad (2)$$

$$I_E = I_S \left( 1 + \frac{1}{\beta} \right) e^{V_{BE}/V_T} \quad (3)$$

When the base is left open-circuited,  $i_B = 0$  and Eq. (1) yields

$$I_{CBO} = \left( \frac{I_S}{\beta} \right) e^{V_{BE}/V_T}$$

or equivalently,

$$I_S e^{V_{BE}/V_T} = \beta I_{CBO} \quad (4)$$

Substituting for  $I_S e^{V_{BE}/V_T}$  in Eqs. (2) and (3) gives

$$i_C = i_E = (\beta + 1) I_{CBO}$$

4.34 Since the BJT is operating at a constant emitter current, its  $|V_{BE}|$  decreases by 2 mV for every  $^\circ\text{C}$  rise in temperature. Thus,

$$|V_{BE}| \text{ at } 0^\circ\text{C} = 0.7 + 0.002 \times 25 = 0.75 \text{ V}$$

$$|V_{BE}| \text{ at } 100^\circ\text{C} = 0.7 - 0.002 \times 75 = 0.55 \text{ V}$$

4.35 (a) If the junction temperature rises to  $50^\circ\text{C}$ , which is an increase of  $30^\circ\text{C}$ , the EB voltage decreases to

$$v_{EB} = 692 - 2 \times 30 = 632 \text{ mV}$$

(b) First, we evaluate  $V_T$  at  $20^\circ\text{C}$  and at  $50^\circ\text{C}$ :

$$V_T = \frac{kT}{q}$$

where  $k = 8.62 \times 10^{-5} \text{ eV/K}$ .

Thus,

$$\text{At } 20^\circ\text{C}, T = 293 \text{ K and } V_T = 8.62 \times 10^{-5} \times 293 = 25.3 \text{ mV}$$

$$\text{At } 50^\circ\text{C}, T = 323 \text{ K and } V_T = 8.62 \times 10^{-5} \times 323 = 27.8 \text{ mV}$$

If the transistor is operated at  $v_{BE} = 700 \text{ mV}$ , then

(i) At  $20^\circ\text{C}$ ,  $i_E$  becomes

$$i_E = 0.5e^{(700-692)/25.3} = 0.69 \text{ mA}$$

(ii) At  $50^\circ\text{C}$ ,  $i_E$  becomes

$$i_E = 0.5e^{(700-632)/27.8} = 5.77 \text{ mA}$$

$$4.36 v_{BE} = 0.7 \text{ V at } i_C = 10 \text{ mA}$$

For  $v_{BE} = 0.5 \text{ V}$ ,

$$i_C = 10e^{(0.5-0.7)/0.025} = 3.35 \mu\text{A}$$

At a current  $I_C$  and a BE voltage  $V_{BE}$ , the slope of the  $i_C-v_{BE}$  curve is  $I_C/V_T$ . Thus,

$$\text{Slope at } V_{BE} \text{ of } 700 \text{ mV} = \frac{10 \text{ mA}}{25 \text{ mV}} = 400 \text{ mA/V}$$

$$\text{Slope at } V_{BE} \text{ of } 500 \text{ mV} = \frac{3.35 \mu\text{A}}{25 \text{ mV}} = 0.134 \text{ mA/V}$$

$$\text{Ratio of slopes} = \frac{400}{0.134} \simeq 3000$$

4.37 Use Eq. (4.18):

$$i_C = I_S e^{v_{BE}/V_T} \left( 1 + \frac{v_{CE}}{V_A} \right)$$

with  $I_S = 10^{-15} \text{ A}$  and  $V_A = 100 \text{ V}$ , to get

$$i_C = 10^{-15} e^{v_{BE}/0.025} \left( 1 + \frac{v_{CE}}{100} \right)$$

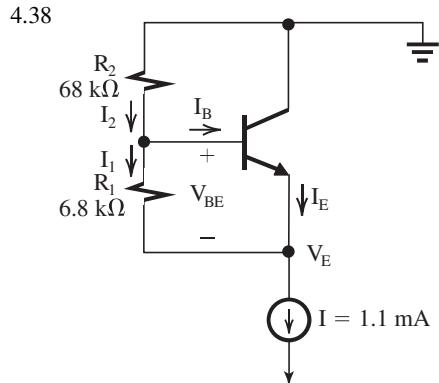
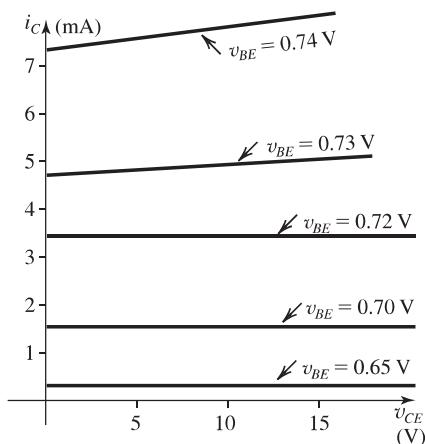
$v_{BE}$	0.65 V	0.70 V	0.72 V	0.73 V	0.74 V
$v_{CE}$ (V)	$i_C$ (mA)				
0	0.196	1.45	3.21	4.81	7.16
15	0.225	1.67	3.70	5.52	8.24

To find the intercept of the straight-line characteristics on the  $i_C$  axis, we substitute  $v_{CE} = 0$  and evaluate

$$i_C = 10^{-15} e^{v_{BE}/V_T} \text{ A}$$

for the given value of  $v_{BE}$ . The slope of each straight line is equal to this value divided by 100 V ( $V_A$ ). Thus we obtain

$v_{BE}$ (V)	0.65	0.70	0.72	0.73	0.74
Intercept (mA)	0.2	1.45	3.22	4.80	7.16
Slope (mA/V)	0.002	0.015	0.032	0.048	0.072



At  $25^\circ\text{C}$ , assume  $I_E = 1 \text{ mA}$ . Thus,

$$V_{BE} = 0.68 \text{ V}$$

$$I_1 = \frac{V_{BE}}{R_1} = \frac{0.68 \text{ V}}{6.8 \text{ k}\Omega} = 0.1 \text{ mA}$$

$$I_E = I - I_1 = 1.1 - 0.1 = 1 \text{ mA}$$

which is the value assumed.

$$I_2 = I_1 + I_B = I_1 + \frac{I_E}{\beta + 1}$$

$$= 0.1 + \frac{1}{101} = 0.11 \text{ mA}$$

Note that the currents in  $R_1$  and  $R_2$  differ only by the small base current, 0.01 mA. Had  $I_1$  and  $I_2$  been equal, then we would have had

$$I_1 R_1 = V_{BE}$$

$$I_2 R_2 \simeq I_1 R_2 = V_{BE} \frac{R_2}{R_1}$$

$$V_E = -(I_1 R_1 + I_2 R_2)$$

$$= -V_{BE} \left( 1 + \frac{R_2}{R_1} \right) \quad (1)$$

$$= -V_{BE} \left( 1 + \frac{6.8}{0.68} \right) = -11 \text{ V}_{BE} = -7.48 \text{ V}$$

which gives this circuit the name  $V_{BE}$  multiplier. A more accurate value of  $V_E$  can be obtained by taking  $I_B$  into account:

$$V_E = -(I_1 R_1 + I_2 R_2)$$

$$= - \left( V_{BE} + \frac{R_2}{R_1} V_{BE} + I_B R_2 \right)$$

$$= - \left( 1 + \frac{R_2}{R_1} \right) V_{BE} - I_B R_2 \quad (2)$$

$$= -7.48 - 0.01 \times 68 = -8.16 \text{ V}$$

As temperature increases, an approximate estimate for the temperature coefficient of  $V_E$  can be obtained by assuming that  $I_E$  remains constant and ignoring the temperature variation of  $\beta$ . Thus, we would be neglecting the temperature change of the  $(I_B R_2)$  terms in Eq. (2). From Eq. (2) we

can obtain the temperature coefficient of  $V_E$  by utilizing the fact the  $V_{BE}$  changes by  $-2.2 \text{ mV}/^\circ\text{C}$ . Thus,

Temperature coefficient of  $V_E$

$$= - \left( 1 + \frac{R_2}{R_1} \right) \times -2.2 \\ = -11 \times -2.2 = +24.2 \text{ mV}/^\circ\text{C}$$

At  $75^\circ\text{C}$ , which is a temperature increase of  $50^\circ\text{C}$ ,

$$V_E = -8.16 + 24.2 \times 50 = -6.95 \text{ V}$$

As a check on our assumption of constant  $I_E$ , let us find the value of  $I_E$  at  $75^\circ\text{C}$ :

$$I_l(75^\circ\text{C}) = \frac{V_{BE}(75^\circ\text{C})}{R_1} \\ = \frac{0.68 - 2.2 \times 10^{-3} \times 50}{6.8} \\ = 0.084 \text{ mA} \\ I_E(75^\circ\text{C}) = I - I_l(75^\circ\text{C}) \\ = 1.1 - 0.084 = 1.016 \text{ mA}$$

which is reasonably close to the assumed value of 1 mA.

$$4.39 \quad r_o = \frac{V_A}{I_C} = \frac{50 \text{ V}}{1 \text{ mA}}$$

Thus,

$$\text{At } I_C = 1 \text{ mA}, \quad r_o = \frac{50 \text{ V}}{1 \text{ mA}} = 50 \text{ k}\Omega$$

$$\text{At } I_C = 100 \mu\text{A}, \quad r_o = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k}\Omega$$

$$4.40 \quad r_o = 1/\text{slope}$$

$$= 1/(0.8 \times 10^{-5})$$

$$= 125 \text{ k}\Omega$$

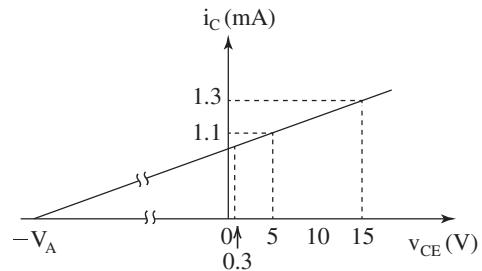
$$r_o = \frac{V_A}{I_C}$$

$$125 \text{ k}\Omega = \frac{V_A}{1 \text{ mA}} \Rightarrow V_A = 125 \text{ V}$$

At  $I_C = 10 \text{ mA}$ ,

$$r_o = \frac{V_A}{I_C} = \frac{125 \text{ V}}{10 \text{ mA}} = 12.5 \text{ k}\Omega$$

4.41



Slope of  $i_C-v_{CE}$  line corresponding to  $v_{BE} = 710 \text{ mV}$  is

$$\text{Slope} = \frac{1.3 - 1.1}{15 - 5} = \frac{0.2 \text{ mA}}{10 \text{ V}} = 0.02 \text{ mA/V}$$

Near saturation,  $V_{CE} = 0.3 \text{ V}$ , thus

$$i_C = 1.1 - 0.02 \times (5 - 0.3)$$

$$= 1.006 \simeq 1 \text{ mA}$$

$i_C$  will be 1.2 mA at,

$$v_{CE} = 5 + \frac{1.2 - 1.1}{0.02} = 10 \text{ V}$$

The intercept of the  $i_C-v_{CE}$  straight line on the  $i_C$  axis will be at

$$i_C = 1.1 - 5 \times 0.02 = 1 \text{ mA}$$

Thus, the Early voltage is obtained as

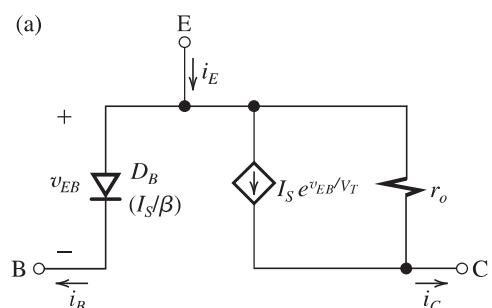
$$\text{Slope} = \frac{i_C(\text{at } v_{CE} = 0)}{V_A}$$

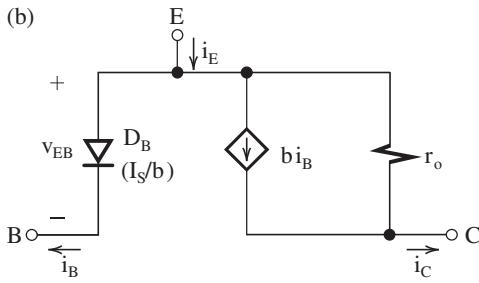
$$\Rightarrow V_A = \frac{1}{0.02} = 50 \text{ V}$$

$$r_o = \frac{V_A}{I_C} = \frac{50 \text{ V}}{1 \text{ mA}} = 50 \text{ k}\Omega$$

which is the inverse of the slope of the  $i_C-v_{CE}$  line.

4.42 The equivalent circuits shown in the figure correspond to the circuits in Fig. 4.19.





$$4.43 \quad \beta = \frac{i_C}{i_B} = \frac{1 \text{ mA}}{10 \mu\text{A}} = 100$$

$$\beta_{ac} = \left. \frac{\Delta i_C}{\Delta i_B} \right|_{V_{CE} \text{ constant}} = \frac{0.08 \text{ mA}}{1.0 \mu\text{A}} = 80$$

$$\Delta i_C = \Delta i_B \times \beta_{ac} + \frac{\Delta V_{CE}}{r_o}$$

where

$$r_o = \frac{V_A}{I_C} = \frac{100}{1} = 100 \text{ k}\Omega$$

Thus,

$$\begin{aligned} \Delta i_C &= 2 \times 80 + \frac{2}{100} \times 10^3 = 180 \mu\text{A} \\ &= 0.18 \text{ mA} \end{aligned}$$

4.44 Refer to the circuit in Fig. P4.44.

(a) For active-mode operation with  $V_C = 2 \text{ V}$ :

$$I_C = \frac{V_{CC} - V_C}{R_C} = \frac{10 - 2}{1} = 8 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{8}{50} = 0.16 \text{ mA}$$

$$V_{BB} = I_B R_B + V_{BE}$$

$$= 0.16 \times 10 + 0.7 = 2.3 \text{ V}$$

(b) For operation at the edge of saturation:

$$V_{CE} = 0.3 \text{ V}$$

$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.3}{1} = 9.7 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{9.7}{50} = 0.194 \text{ mA}$$

$$V_{BB} = I_B R_B + V_{BE}$$

$$= 0.194 \times 10 + 0.7 = 2.64 \text{ V}$$

(c) For operation deep in saturation with  $\beta_{forced} = 10$ :

$$V_{CE} = 0.2 \text{ V}$$

$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.2}{1} = 9.8 \text{ mA}$$

$$I_B = \frac{I_C}{\beta_{forced}} = \frac{9.8}{10} = 0.98 \text{ mA}$$

$$V_{BB} = I_B R_B + V_{BE}$$

$$= 0.98 \times 10 + 0.7 = 10.5 \text{ V}$$

4.45 Refer to the circuit in Fig. P4.44 (with  $V_{BB} = V_{CC}$ ) and to the BJT equivalent circuit of Fig. 4.21.

$$I_C = \frac{V_{CC} - 0.2}{R_C}$$

$$I_B = \frac{V_{CC} - 0.7}{R_B}$$

$$\beta_{forced} \equiv \frac{I_C}{I_B}$$

Thus,

$$\beta_{forced} = \left( \frac{V_{CC} - 0.2}{V_{CC} - 0.7} \right) \left( \frac{R_B}{R_C} \right) \quad (1)$$

$$P_{dissipated} = V_{CC}(I_C + I_B)$$

$$= V_{CC}(\beta_{forced} I_B + I_B)$$

$$= (\beta_{forced} + 1)V_{CC}I_B \quad (2)$$

For  $V_{CC} = 5 \text{ V}$  and  $\beta_{forced} = 10$  and  $P_{dissipated} \leq 20 \text{ mW}$ , we can proceed as follows.

Using Eq. (1) we can determine  $(R_B / R_C)$ :

$$\begin{aligned} 10 &= \left( \frac{5 - 0.2}{5 - 0.7} \right) \left( \frac{R_B}{R_C} \right) \\ &\Rightarrow \frac{R_B}{R_C} = 8.96 \end{aligned} \quad (3)$$

Using Eq. (2), we can find  $I_B$ :

$$(10 + 1) \times 5 \times I_B \leq 20 \text{ mW}$$

$$\Rightarrow I_B \leq 0.36 \text{ mA}$$

Thus,

$$\frac{V_{CC} - 0.7}{R_B} \leq 0.36 \text{ mA}$$

$$\Rightarrow R_B \geq 11.9 \text{ k}\Omega$$

From the table of 1% resistors in Appendix J we select

$$R_B = 12.1 \text{ k}\Omega$$

Substituting in Eq. (3), we have

$$R_C = 1.35 \text{ k}\Omega$$

From the table of 1% resistors in Appendix J we select

$$R_C = 1.37 \text{ k}\Omega$$

For these values:

$$I_C = \frac{5 - 0.2}{1.37} = 3.5 \text{ mA}$$

$$I_B = \frac{5 - 0.7}{12.1} = 0.36 \text{ mA}$$

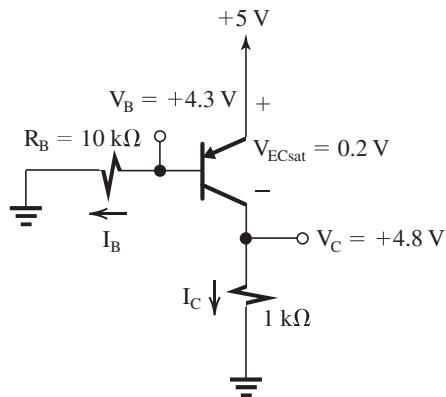
Thus,

$$\beta_{\text{forced}} = \frac{3.5}{0.36} = 9.7$$

$$P_{\text{dissipated}} = V_{CC}(I_C + I_B)$$

$$= 5 \times 3.86 = 19.2 \text{ mW}$$

4.46



Assume saturation-mode operation. From the figure we see that

$$I_C = \frac{V_C}{1 \text{ k}\Omega} = \frac{4.8}{1} = 4.8 \text{ mA}$$

$$I_B = \frac{V_B}{R_B} = \frac{4.3}{10} = 0.43 \text{ mA}$$

Thus,

$$\beta_{\text{forced}} \equiv \frac{I_C}{I_B} = \frac{4.8}{0.43} = 11.2$$

Since 11.2 is lower than the transistor  $\beta$  of 50, we have verified that the transistor is operating in saturation, as assumed.

$$V_C = V_{CC} - V_{ECsat} = 5 - 0.2 = 4.8 \text{ V}$$

To operate at the edge of saturation,

$$V_{EC} = 0.3 \text{ V} \quad \text{and} \quad I_C/I_B = \beta = 50$$

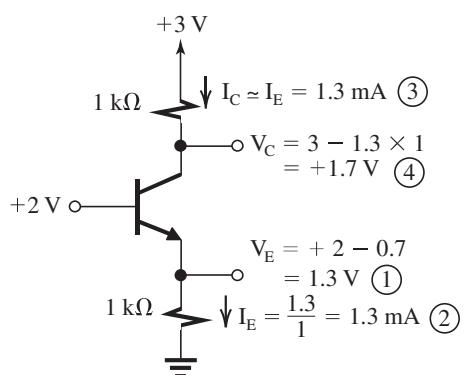
Thus,

$$I_C = \frac{5 - 0.3}{1} = 4.7 \text{ mA}$$

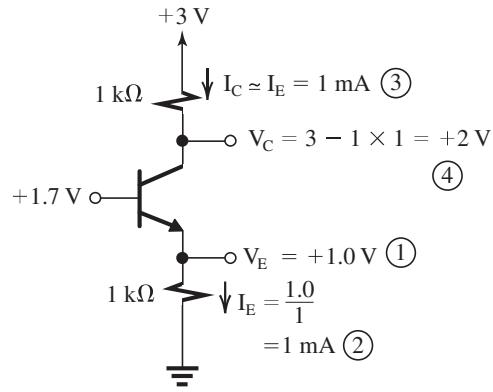
$$I_B = \frac{I_C}{\beta} = \frac{4.7}{50} = 0.094 \text{ mA}$$

$$R_B = \frac{4.3}{I_B} = \frac{4.3}{0.094} = 45.7 \text{ k}\Omega$$

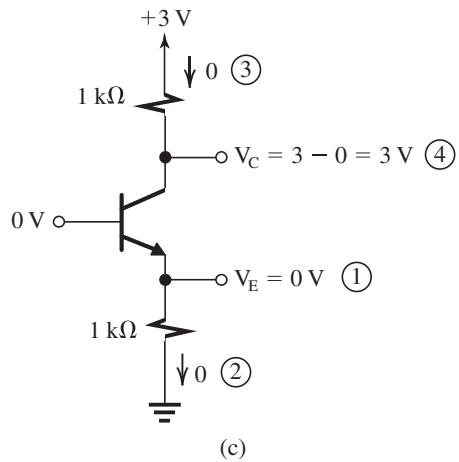
4.47



(a)



(b)



(c)

The analysis and the results are given on the circuit diagrams of Figs. 1 through 3 (see preceding page). The circled numbers indicate the order of the analysis steps.

4.48

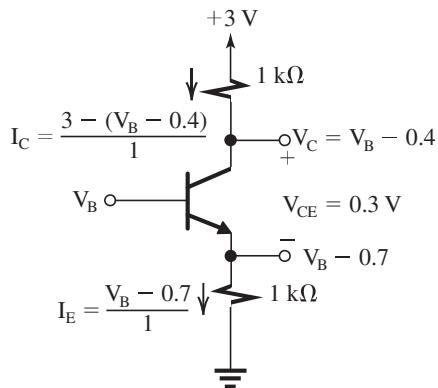


Figure 1

Figure 1 shows the circuit with the value of  $V_B$  that results in operation at the edge of saturation. Since  $\beta$  is very high,

$$I_C \approx I_E$$

$$\frac{3 - (V_B - 0.4)}{1} = \frac{V_B - 0.7}{1}$$

$$\Rightarrow V_B = 2.05 \text{ V}$$

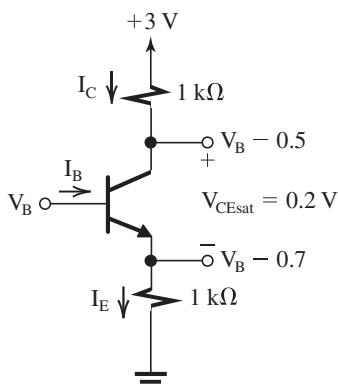


Figure 2

Figure 2 shows the circuit with the value of  $V_B$  that results in the transistor operating in saturation, with

$$I_E = \frac{V_B - 0.7}{1} = V_B - 0.7$$

$$I_C = \frac{3 - (V_B - 0.5)}{1} = 3.5 - V_B$$

$$I_B = I_E - I_C = 2V_B - 4.2$$

For  $\beta_{\text{forced}} = 2$ ,

$$\frac{I_C}{I_B} = 2$$

$$\frac{3.5 - V_B}{2V_B - 4.2} = 2$$

$$\Rightarrow V_B = 2.38 \text{ V}$$

4.49 Refer to the circuit in Fig. P4.49.

(a) For  $V_B = -1 \text{ V}$ ,

$$V_E = V_B - V_{BE} = -1 - 0.7 = -1.7 \text{ V}$$

$$I_E = \frac{V_E - (-3)}{1} = \frac{-1.7 + 3}{1} = 1.3 \text{ mA}$$

Assuming active-mode operation, we have

$$I_C = \alpha I_E \approx I_E = 1.3 \text{ mA}$$

$$V_C = +3 - I_C \times 1 = 3 - 1.3 = +1.7 \text{ V}$$

Since  $V_C > V_B - 0.4$ , the transistor is operating in the active mode as assumed.

(b) For  $V_B = 0 \text{ V}$ ,

$$V_E = 0 - V_{BE} = -0.7 \text{ V}$$

$$I_E = \frac{-0.7 - (-3)}{1} = 2.3 \text{ mA}$$

Assuming operation in the active mode, we have

$$I_C = \alpha I_E \approx I_E = 2.3 \text{ mA}$$

$$V_C = +3 - I_C \times 1 = 3 - 2.3 = +0.7 \text{ V}$$

Since  $V_C > V_B - 0.4$ , the BJT is operating in the active mode, as assumed.

(c) For  $V_B = +1 \text{ V}$ ,

$$V_E = 1 - 0.7 = +0.3 \text{ V}$$

$$I_E = \frac{0.3 - (-3)}{1} = 3.3 \text{ mA}$$

Assuming operation in the active mode, we have

$$I_C = \alpha I_E \approx I_E = 3.3 \text{ mA}$$

$$V_C = 3 - 3.3 \times 1 = -0.3 \text{ V}$$

Now  $V_C < V_B - 0.4$ , indicating that the transistor is operating in saturation, and our original assumption is incorrect. It follows that

## Chapter 4–15

$$V_C = V_E + V_{CEsat}$$

$$= 0.3 + 0.2 = 0.5 \text{ V}$$

$$I_C = \frac{3 - V_C}{1} = \frac{3 - 0.5}{1} = 2.5 \text{ mA}$$

$$I_B = I_E - I_C = 3.3 - 2.5 = 0.8 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.5}{0.8} = 3.1$$

(d) When  $V_B = 0 \text{ V}$ ,  $I_E = 2.3 \text{ mA}$ . The emitter current becomes 0.23 mA at

$$V_B = -3 + 0.23 \times 1 + 0.7 = -2.07 \text{ V}$$

(e) The transistor will be at the edge of conduction when  $I_E \approx 0$  and  $V_{BE} = 0.5 \text{ V}$ , that is,

$$V_B = -3 + 0.5 = -2.5 \text{ V}$$

In this case,

$$V_E = -3 \text{ V}$$

$$V_C = +3 \text{ V}$$

(f) The transistor reaches the edge of saturation when  $V_{CE} = 0.3 \text{ V}$  but  $I_C = \alpha I_E \approx I_E$ :

$$V_E = V_B - 0.7$$

$$I_E = \frac{V_B - 0.7 - (-3)}{1} = V_B + 2.3$$

$$V_C = V_E + 0.3 = V_B - 0.4$$

$$I_C = \frac{3 - V_C}{1} = \frac{3 - V_B + 0.4}{1} = 3.4 - V_B$$

Since

$$I_C \approx I_E$$

$$3.4 - V_B = V_B + 2.3$$

$$V_B = 0.55 \text{ V}$$

For this value,

$$V_E = 0.55 - 0.7 = -0.15 \text{ V}$$

$$V_C = -0.15 + 0.3 = +0.15 \text{ V}$$

(g) For the transistor to operate in saturation with  $\beta_{\text{forced}} = 2$ ,

$$V_E = V_B - 0.7$$

$$I_E = \frac{V_B - 0.7 - (-3)}{1} = V_B + 2.3$$

$$V_C = V_E + V_{CEsat} = V_B - 0.7 + 0.2 = V_B - 0.5$$

$$I_C = \frac{3 - (V_B - 0.5)}{1} = 3.5 - V_B$$

$$I_B = I_E - I_C = 2 V_B - 1.2$$

$$\frac{I_C}{I_B} = \frac{3.5 - V_B}{2 V_B - 1.2} = 2$$

$$\Rightarrow V_B = +1.18 \text{ V}$$

4.50 Refer to the circuit in Fig. P4.50.

$$V_E = 1 \text{ V}$$

$$I_E = \frac{3 - 1}{5} = 0.4 \text{ mA}$$

$$V_B = V_E - 0.7 = 0.3 \text{ V}$$

$$I_B = \frac{V_B}{50 \text{ k}\Omega} = \frac{0.3}{50} = 0.006 \text{ mA}$$

$$I_C = I_E - I_B = 0.4 - 0.006 = 0.394 \text{ mA}$$

$$V_C = -3 + 5 \times 0.394 = -1.03 \text{ V}$$

Observe that  $V_C < V_B$ , confirming our implicit assumption that the transistor is operating in the active region.

$$\beta = \frac{I_C}{I_B} = \frac{0.394}{0.006} = 66$$

$$\alpha = \frac{I_C}{I_E} = \frac{0.394}{0.4} = 0.985$$

4.51

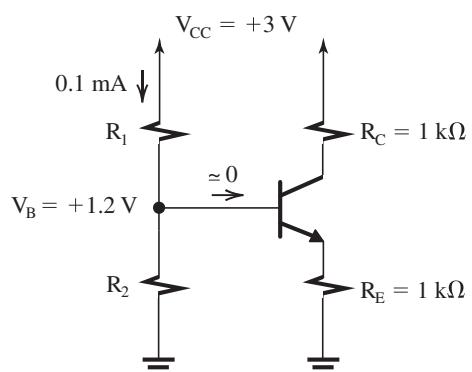


Figure 1

From Fig. 1 we see that

$$R_1 + R_2 = \frac{V_{CC}}{0.1 \text{ mA}} = \frac{3}{0.1} = 30 \text{ k}\Omega$$

$$V_{CC} \frac{R_2}{R_1 + R_2} = 1.2$$

$$3 \times \frac{R_2}{30} = 1.2$$

$$\Rightarrow R_2 = 12 \text{ k}\Omega$$

$$R_1 = 30 - 12 = 18 \text{ k}\Omega$$

For  $\beta = 100$ , to obtain the collector current, we replace the voltage divider with its Thévenin equivalent, consisting of

$$V_{BB} = 3 \times \frac{R_2}{R_1 + R_2} = 3 \times \frac{12}{18 + 12} = 1.2 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 12 \parallel 18 = 7.2 \text{ k}\Omega$$

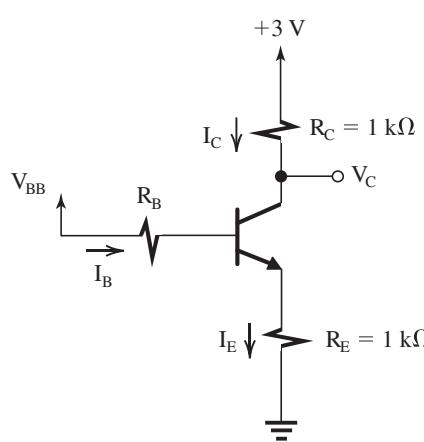


Figure 2

Refer to Fig. 2. Assuming active-mode operation, we can write a loop equation for the base–emitter loop:

$$V_{BB} = I_B R_B + V_{BE} + I_E R_E$$

$$1.2 = \frac{I_E}{\beta + 1} \times 7.2 + 0.7 + I_E \times 1$$

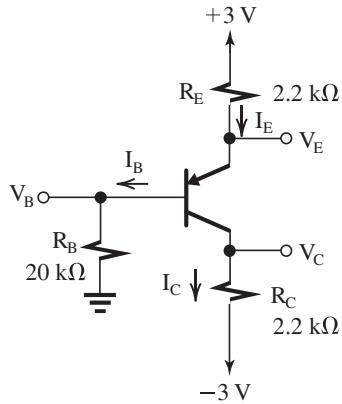
$$\Rightarrow I_E = \frac{1.2 - 0.7}{1 + \frac{7.2}{101}} = 0.47 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.47 = 0.46 \text{ mA}$$

$$V_C = +3 - 0.46 \times 1 = +2.54 \text{ V}$$

Since  $V_B = I_E R_E + V_{BE} = 0.47 + 0.7 = 1.17 \text{ V}$ , we see that  $V_C > V_B - 0.4$ , and thus the transistor is operating in the active region, as assumed.

4.52



Writing a loop equation for the EBJ loop, we have

$$3 = I_E R_E + V_{EB} + I_B R_B \quad (1)$$

$$= I_E \times 2.2 + 0.7 + \frac{I_E}{\beta + 1} \times 20$$

$$\Rightarrow I_E = \frac{3 - 0.7}{2.2 + \frac{41}{41}} = 0.86 \text{ mA}$$

$$V_E = 3 - 0.86 \times 2.2 = +1.11 \text{ V}$$

$$V_B = V_E - 0.7 = +0.41 \text{ V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = \frac{40}{41} \times 0.86 = 0.84 \text{ mA}$$

$$V_C = -3 + 0.84 \times 2.2 = -1.15 \text{ V}$$

Since  $V_C < V_B + 0.4$ , the transistor is operating in the active mode, as assumed. Now, if  $R_B$  is increased to  $100 \text{ k}\Omega$ , the loop equation [Eq. (1)] yields

$$I_E = \frac{3 - 0.7}{2.2 + \frac{100}{41}} = 0.5 \text{ mA}$$

$$V_E = 3 - 0.5 \times 2.2 = +1.9 \text{ V}$$

$$V_B = V_E - V_{EB} = 1.9 - 0.7 = +1.2 \text{ V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = \frac{40}{41} \times 0.5 = 0.48 \text{ mA}$$

$$V_C = -3 + 0.48 \times 2.2 = -1.9 \text{ V}$$

Since  $V_C < V_B + 0.4$ , the transistor is operating in the active mode, as assumed.

If with  $R_B = 100 \text{ k}\Omega$ , we need the voltages to remain at the values obtained with  $R_B = 20 \text{ k}\Omega$ , the transistor must have a  $\beta$  value determined as follows. For  $I_E$  to remain unchanged,

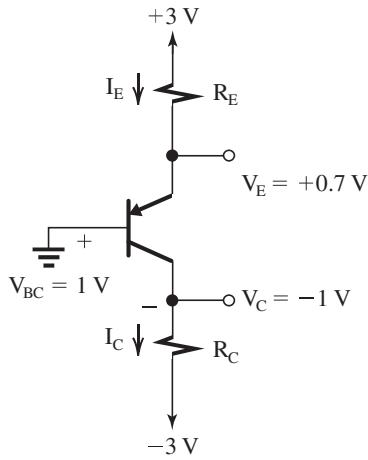
$$\frac{3 - 0.7}{2.2 + \frac{20}{41}} = \frac{3 - 0.7}{2.2 + \frac{100}{\beta + 1}}$$

$$\Rightarrow \frac{20}{41} = \frac{100}{\beta + 1}$$

$$\beta + 1 = \frac{410}{2} = 205$$

$$\beta = 204$$

4.53



Refer to the figure. To obtain  $I_E = 0.5 \text{ mA}$  we select  $R_E$  according to

$$R_E = \frac{3 - 0.7}{0.5} = 4.6 \text{ k}\Omega$$

To obtain  $V_C = -1 \text{ V}$ , we select  $R_C$  according to

$$R_C = \frac{-1 - (-3)}{0.5} = 4 \text{ k}\Omega$$

where we have utilized the fact that  $\alpha \simeq 1$  and thus  $I_C \simeq I_E = 0.5 \text{ mA}$ . From the table of 5% resistors in Appendix J we select

$$R_E = 4.7 \text{ k}\Omega \quad \text{and} \quad R_C = 3.9 \text{ k}\Omega$$

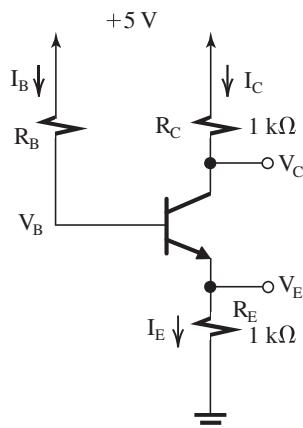
For these values,

$$I_E = \frac{3 - 0.7}{4.7} = 0.49 \text{ mA}$$

$$I_C \simeq I_E = 0.49 \text{ mA}$$

$$V_{BC} = 0 - V_C = -(-3 + 0.49 \times 3.9) = -1.1 \text{ V}$$

4.54



A loop equation for the EB loop yields

$$5 = I_B R_B + V_{BE} + I_E R_E$$

$$\Rightarrow I_E = \frac{5 - 0.7}{R_E + \frac{R_B}{\beta + 1}}$$

$$I_E = \frac{4.3}{1 + \frac{100}{101}}$$

(a) For  $R_B = 100 \text{ k}\Omega$ ,

$$I_E = \frac{4.3}{1 + \frac{100}{101}} = 2.16 \text{ mA}$$

$$V_E = I_E R_E = 2.16 \times 1 = 2.16 \text{ V}$$

$$V_B = V_E + 0.7 = 2.86 \text{ V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = 0.99 \times 2.16 = 2.14 \text{ mA}$$

$$V_C = 5 - 2.14 \times 1 = +2.86 \text{ V}$$

Since  $V_C > V_B - 0.4$ , the transistor is operating in the active region, as assumed.

(b) For  $R_B = 10 \text{ k}\Omega$ ,

$$I_E = \frac{4.3}{1 + \frac{10}{101}} = 3.91 \text{ mA}$$

$$V_E = 3.91 \times 1 = 3.91 \text{ V}$$

$$V_B = 3.91 + 0.7 = 4.61 \text{ V}$$

Assuming active-mode operation, we obtain

$$I_C = \alpha I_E = 0.99 \times 3.91 = 3.87 \text{ mA}$$

$$V_C = 5 - 3.87 = +1.13 \text{ V}$$

Since  $V_C < V_B - 0.4$ , the transistor is operating in saturation, contrary to our original assumption. Therefore, we need to redo the analysis assuming saturation-mode operation, as follows:

$$V_B = V_E + 0.7$$

$$V_C = V_E + V_{CEsat} = V_E + 0.2$$

$$\begin{aligned} I_B &= \frac{5 - V_B}{R_B} \\ &= \frac{5 - V_E - 0.7}{10} = \frac{4.3 - V_E}{10} \end{aligned} \quad (1)$$

$$\begin{aligned} I_C &= \frac{5 - V_C}{R_C} = \frac{5 - V_E - 0.2}{1} \\ &= 4.8 - V_E \end{aligned} \quad (2)$$

$$I_E = \frac{V_E}{R_E} = \frac{V_E}{1} = V_E \quad (3)$$

Substituting from Eqs. (1), (2), and (3) into

$$I_E = I_B + I_C$$

gives

$$V_E = 0.43 - 0.1 V_E + 4.8 - V_E$$

$$\Rightarrow V_E = 2.5 \text{ V}$$

$$V_C = 2.7 \text{ V}$$

$$V_B = 3.2 \text{ V}$$

$$I_B = \frac{5 - 3.2}{10} = 0.18 \text{ mA}$$

$$I_C = \frac{5 - 2.7}{1} = 2.3 \text{ mA}$$

Thus,

$$\frac{I_C}{I_B} = \frac{2.3}{0.18} = 12.8$$

which is lower than the value of  $\beta$ , verifying saturation-mode operation.

(c) For  $R_B = 1 \text{ k}\Omega$ , we assume saturation-mode operation:

$$V_B = V_E + 0.7$$

$$V_C = V_E + 0.2$$

$$I_B = \frac{5 - (V_E + 0.7)}{1} = 4.3 - V_E$$

$$I_C = \frac{5 - (V_E + 0.2)}{1} = 4.8 - V_E$$

$$I_E = \frac{V_E}{1} = V_E$$

These values can be substituted into

$$I_E = I_B + I_C$$

to obtain

$$V_E = 4.3 - V_E + 4.8 - V_E$$

$$\Rightarrow V_E = 3 \text{ V}$$

$$V_B = 3.7 \text{ V}$$

$$V_C = 3.2 \text{ V}$$

Now checking the currents,

$$I_B = \frac{5 - 3.7}{1} = 1.3 \text{ mA}$$

$$I_C = \frac{5 - 3.2}{1} = 1.8 \text{ mA}$$

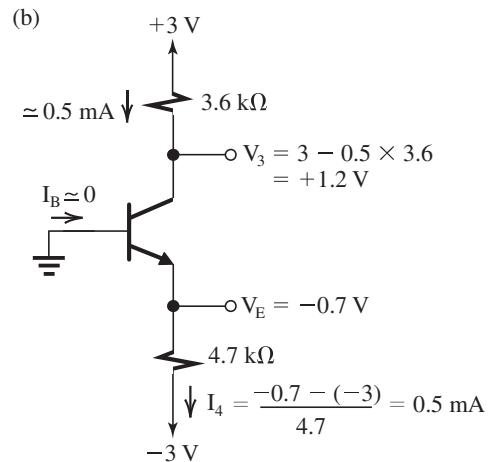
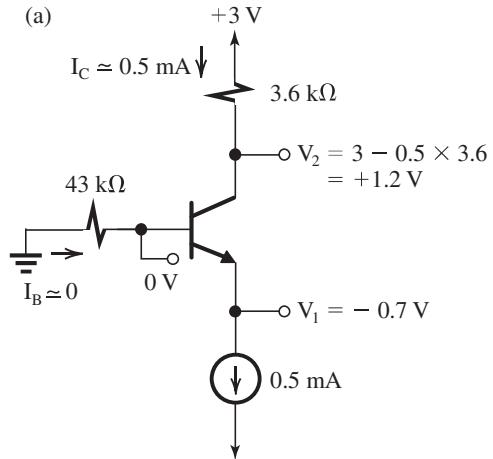
Thus, the transistor is operating at a forced  $\beta$  of

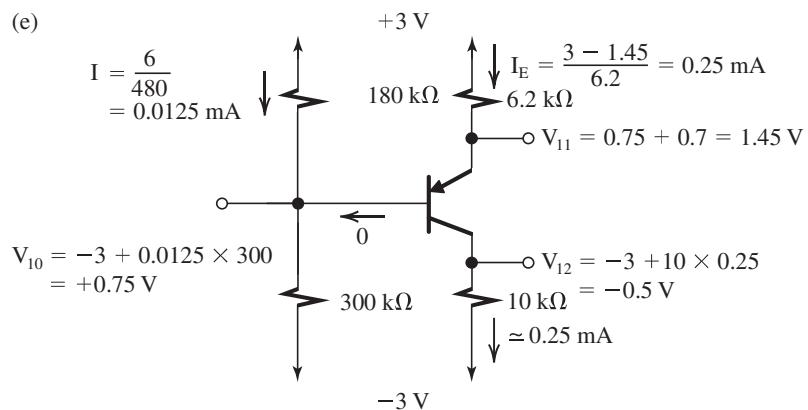
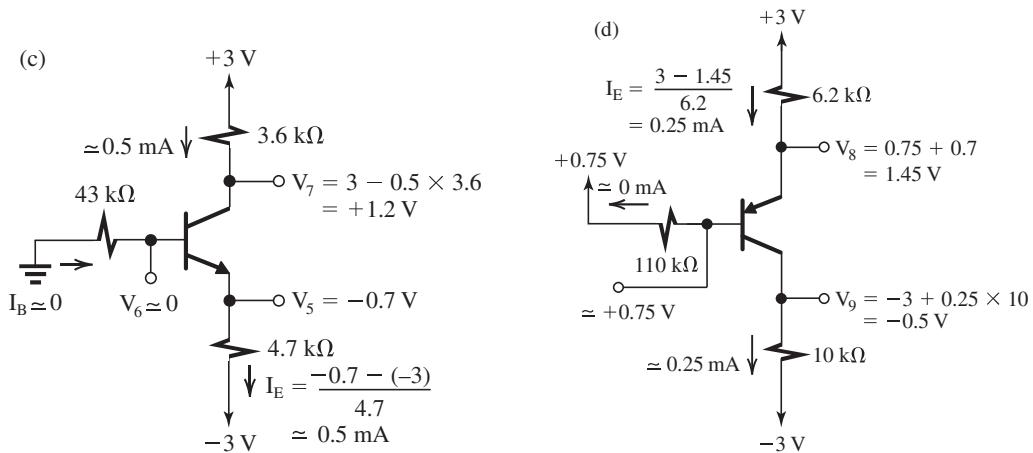
$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{1.8}{1.3} = 1.4$$

which is much lower than the value of  $\beta$ , confirming operation in saturation.

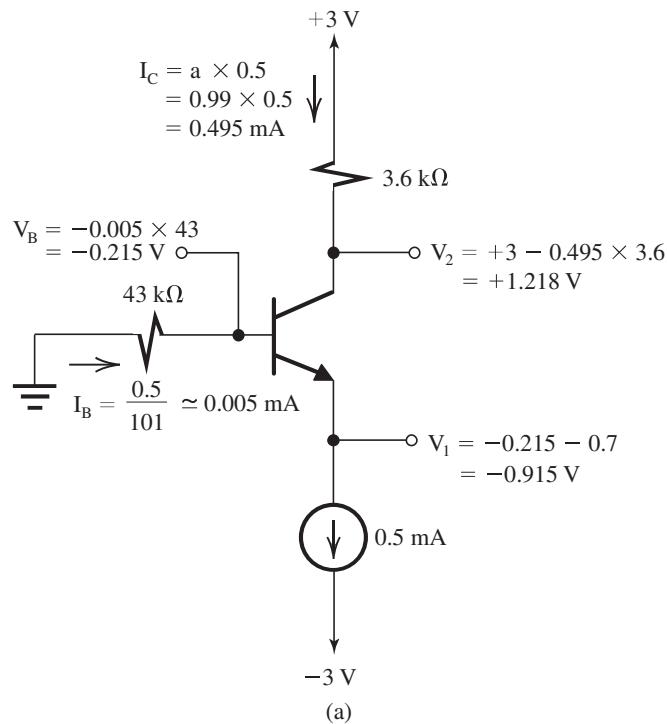
#### 4.55

For the solutions and answers to parts (a) through (e), see the corresponding circuit diagrams.



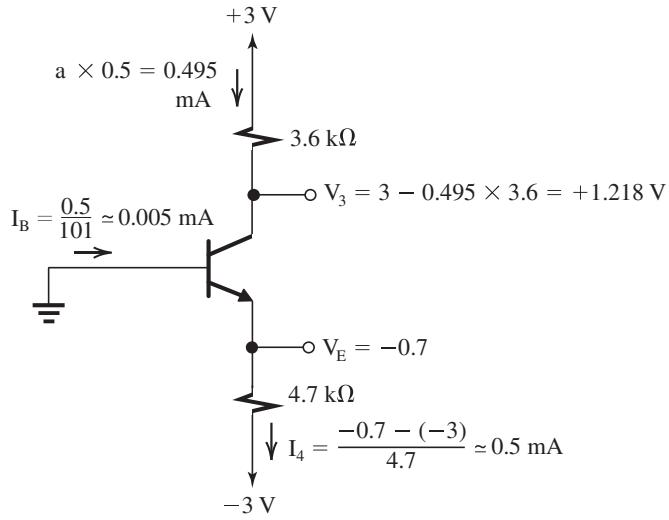


4.56 (a)



See solution and answer on the figure, which corresponds to Fig. P4.55(a).

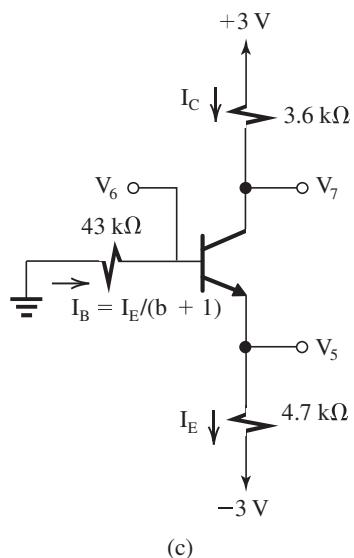
(b)



(b)

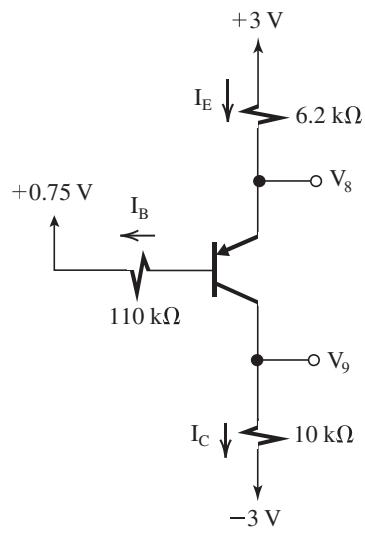
See solution and answer on the figure, which corresponds to Fig. P4.55(b).

(c)



(c)

(d)



(d)

Writing an equation for the loop containing the BEJ of the transistor leads to

$$I_E = \frac{3 - 0.7}{4.7 + \frac{43}{101}} = 0.449 \text{ mA}$$

$$V_5 = -3 + 0.449 \times 4.7 = -0.9 \text{ V}$$

$$V_6 = -0.9 + 0.7 = -0.2 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 0.449 = 0.444 \text{ mA}$$

$$V_7 = 3 - 0.444 \times 3.6 = +1.4 \text{ V}$$

An equation for the loop containing the EBJ of the transistor yields

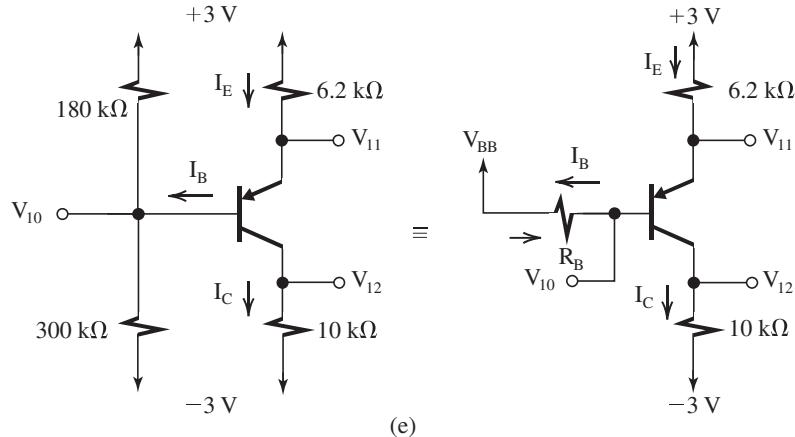
$$I_E = \frac{3 - 0.75 - 0.7}{6.2 + \frac{110}{101}} = 0.213 \text{ mA}$$

$$V_8 = +3 - 0.213 \times 6.2 = +1.7 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 0.213 = 0.21 \text{ mA}$$

$$V_9 = -3 + 0.21 \times 10 = -0.9 \text{ V}$$

This figure belongs to Problem 4.56, part (e).



(e) See figure above.

First, we use Thévenin's theorem to replace the voltage divider feeding the base with V<sub>BB</sub> and R<sub>B</sub>:

$$V_{BB} = -3 + \frac{6}{480} \times 300 = +0.75 \text{ V}$$

$$R_B = 180 \parallel 300 = 112.5 \text{ k}\Omega$$

Next we write an equation for the loop containing the EBJ to obtain

$$I_E = \frac{3 - 0.75 - 0.7}{6.2 + \frac{112.5}{101}} = 0.212 \text{ mA}$$

$$V_{11} = +3 - 0.212 \times 6.2 = +1.7 \text{ V}$$

$$V_{10} = 1.7 - 0.7 = +1 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 0.212 = 0.21 \text{ mA}$$

$$V_{12} = -3 + 0.21 \times 10 = -0.9 \text{ V}$$

Since V<sub>C</sub> = 2 V is lower than V<sub>B</sub>, which is +2.3 V, the transistor will be operating in the active mode. Thus,

$$I_C = \beta I_B = 50 \times 0.023 = 1.15 \text{ mA}$$

To obtain V<sub>C</sub> = 2 V, we select R<sub>C</sub> according to

$$R_C = \frac{V_C}{I_C} = \frac{2 \text{ V}}{1.15 \text{ mA}} = 1.74 \text{ k}\Omega$$

Now, if the transistor is replaced with another having  $\beta = 100$ , then

$$I_C = 100 \times 0.023 = 2.3 \text{ mA}$$

which would imply

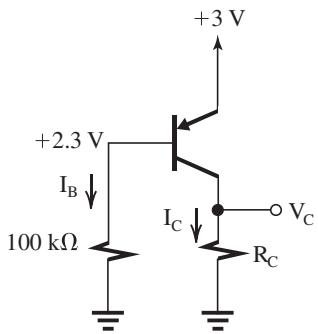
$$V_C = 2.3 \times 1.74 = 4 \text{ V}$$

which is impossible because the base is at 2.3 V. Thus the transistor must be in the saturation mode and

$$V_C = V_E - V_{ECsat}$$

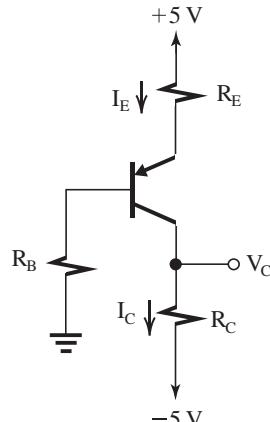
$$= 3 - 0.2 = 2.8 \text{ V}$$

4.57



$$I_B = \frac{2.3 \text{ V}}{100 \text{ k}\Omega} = 0.023 \text{ mA}$$

4.58



We required  $I_E$  to be nominally 1 mA (i.e., at  $\beta = 100$ ) and to remain within  $\pm 10\%$  as  $\beta$  varies from 50 to 150. Writing an equation for the loop containing the EBJ results in

$$I_E = \frac{5 - 0.7}{R_E + \frac{R_B}{\beta + 1}}$$

Thus,

$$\frac{4.3}{R_E + \frac{R_B}{101}} = 1 \quad (1)$$

$$\frac{4.3}{R_E + \frac{R_B}{51}} = I_{E\min} \quad (2)$$

$$\frac{4.3}{R_E + \frac{R_B}{151}} = I_{E\max} \quad (3)$$

If we set  $I_{E\min} = 0.9$  mA and solve Eqs. (1) and (2) simultaneously, we obtain

$$R_E = 3.81 \text{ k}\Omega$$

$$R_B = 49.2 \text{ k}\Omega$$

Substituting these values in Eqs. (2) and (3) gives

$$I_{E\min} = 0.9 \text{ mA}$$

$$I_{E\max} = 1.04 \text{ mA}$$

Obviously, this is an acceptable design.

Alternatively, if we set  $I_{E\max}$  in Eq. (3) to 1.1 mA and solve Eqs. (1) and (3) simultaneously, we obtain

$$R_E = 3.1 \text{ k}\Omega$$

$$R_B = 119.2 \text{ k}\Omega$$

Substituting these values in Eqs. (2) and (3) gives

$$I_{E\min} = 0.8 \text{ mA}$$

$$I_{E\max} = 1.1 \text{ mA}$$

Obviously this is not an acceptable design ( $I_{E\min}$  is 20% lower than nominal).

Therefore, we shall use the first design.

Specifying the resistor values to the nearest kilohm results in

$$R_E = 4 \text{ k}\Omega$$

$$R_B = 50 \text{ k}\Omega$$

To obtain the value of  $R_C$ , we note that at the nominal emitter current value of 1 mA,  $V_C = -1$  V,

$$I_C = \alpha I_E = 0.99 \text{ mA}$$

$$R_C = \frac{-1 - (-5)}{0.99} = 4.04 \text{ k}\Omega$$

Specified to the nearest kilohm,

$$R_C = 4 \text{ k}\Omega$$

Finally, for our design we need to determine the range obtained for collector current and collector voltage for  $\beta$  ranging from 50 to 150 with a nominal value of 100. We compute the nominal value of  $I_E$  from

$$I_{Enominal} = \frac{4.3}{4 + \frac{50}{101}} = 0.96 \text{ mA}$$

We utilize Eqs. (2) and (3) to compute  $I_{E\min}$  and  $I_{E\max}$ ,

$$I_{E\min} = \frac{4.3}{4 + \frac{50}{51}} = 0.86 \text{ mA}$$

$$I_{E\max} = \frac{4.3}{4 + \frac{50}{151}} = 0.99 \text{ mA}$$

Thus,

$$\frac{I_{E\max}}{I_{Enominal}} = \frac{0.99}{0.96} = 1.03$$

$$\frac{I_{E\min}}{I_{Enominal}} = \frac{0.86}{0.96} = 0.9$$

which meet our specifications. The collector currents are

$$I_{Cnominal} = 0.99 \times 0.96 = 0.95 \text{ mA}$$

$$I_{Cmin} = 0.99 \times 0.86 = 0.85 \text{ mA}$$

$$I_{Cmax} = 0.99 \times 0.99 = 0.98 \text{ mA}$$

and the collector voltages are

$$V_{Cnominal} = -5 + 0.95 \times 4 = -1.2 \text{ V}$$

$$V_{Cmin} = -5 + 0.85 \times 4 = -1.6 \text{ V}$$

$$V_{Cmax} = -5 + 0.98 \times 4 = -1.1 \text{ V}$$

4.59 Figure 1 on next page shows the circuit with  $\beta = \infty$ ; the required voltage values are indicated. The resistor values are obtained as follows:

$$V_2 = -0.7 \text{ V}$$

$$R_I = \frac{V_2 - (-5)}{0.5 \text{ mA}}$$

$$\Rightarrow R_I = 8.6 \text{ k}\Omega$$

This figure belongs to Problem 4.59.

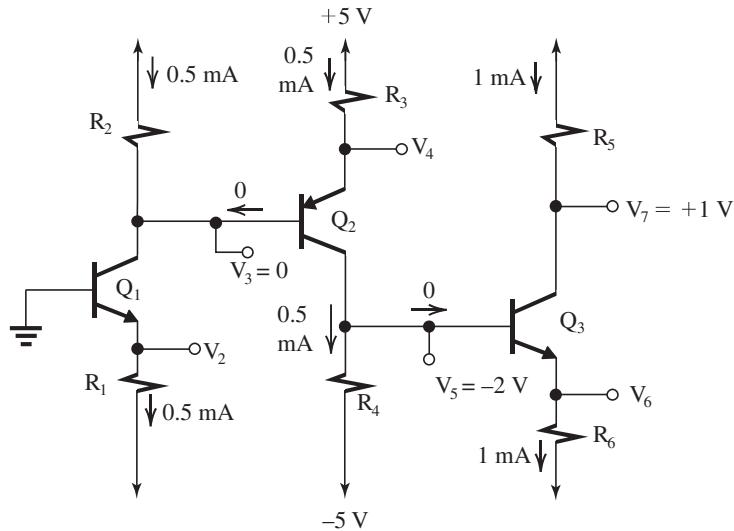


Figure 1

This figure belongs to Problem 4.59.

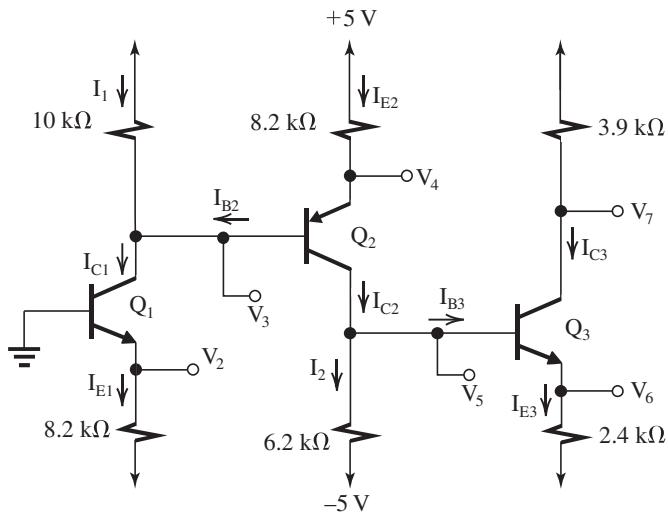


Figure 2

$$R_2 = \frac{5 - V_3}{0.5} = \frac{5 - 0}{0.5} = 10 \text{ k}\Omega$$

$$V_4 = 0 + 0.7 = 0.7 \text{ V}$$

$$R_3 = \frac{5 - V_4}{0.5} = \frac{5 - 0.7}{0.5} = 8.6 \text{ k}\Omega$$

$$R_4 = \frac{V_5 - (-5)}{0.5} = \frac{-2 + 5}{0.5} = 6 \text{ k}\Omega$$

$$V_6 = V_5 - 0.7 = -2 - 0.7 = -2.7 \text{ V}$$

$$R_6 = \frac{V_6 - (-5)}{1} = \frac{-2.7 + 5}{1} = 2.3 \text{ k}\Omega$$

$$R_5 = \frac{5 - V_7}{1} = \frac{5 - 1}{1} = 4 \text{ k}\Omega$$

Consulting the table of 5% resistors in Appendix J, we select the following resistor values:

$$R_1 = 8.2 \text{ k}\Omega \quad R_2 = 10 \text{ k}\Omega \quad R_3 = 10 \text{ k}\Omega$$

$$R_4 = 6.2 \text{ k}\Omega \quad R_5 = 3.9 \text{ k}\Omega \quad R_6 = 2.4 \text{ k}\Omega$$

The circuit with the selected resistor values is shown in Fig. 2. Analysis of the circuit proceeds as follows:

$$V_2 = -0.7 \text{ V}$$

$$I_{E1} = \frac{V_2 - (-5)}{8.2} = \frac{-0.7 + 5}{8.2} = 0.524 \text{ mA}$$

$$I_{C1} = \alpha I_{E1} = 0.99 \times 0.524 = 0.52 \text{ mA}$$

The current  $I_1$  through the  $10\text{-k}\Omega$  resistor is given by

$$I_1 = I_{C1} - I_{B2} = I_{C1} - \frac{I_{E2}}{101}$$

Noting that the voltage drop across the  $10\text{-k}\Omega$  resistor is equal to  $(I_{E2} \times 8.2 + 0.7)$ , we can write

$$I_1 \times 10 = 8.2I_{E2} + 0.7$$

Thus,

$$10 \left( 0.52 - \frac{I_{E2}}{101} \right) = 8.2I_{E2} + 0.7$$

$$\Rightarrow I_{E2} = 0.542 \text{ mA}$$

$$V_4 = 5 - 0.542 \times 8.2 = 0.56 \text{ V}$$

$$V_3 = 0.56 - 0.7 = -0.14 \text{ V}$$

$$I_{C2} = \alpha I_{E2} = 0.99 \times 0.542 = 0.537 \text{ mA}$$

$$I_2 = I_{C2} - I_{B3} = 0.537 - \frac{I_{E3}}{101}$$

Since the voltage drop across the  $6.2\text{-k}\Omega$  resistor is equal to  $(0.7 + I_{E3} \times 2.4)$ ,

$$I_2 \times 6.2 = 0.7 + 2.4I_{E3}$$

$$6.2 \left( 0.537 - \frac{I_{E3}}{101} \right) = 0.7 + 2.4I_{E3}$$

$$\Rightarrow I_{E3} = 1.07 \text{ mA}$$

$$V_6 = -5 + 1.07 \times 2.4 = -2.43 \text{ V}$$

$$V_5 = V_6 + 0.7 = -1.73$$

$$I_{C3} = \alpha \times I_{E3} = 0.99 \times 1.07 = 1.06 \text{ mA}$$

$$V_7 = -3.9 \times 1.06 = 0.87 \text{ V}$$

4.60 Refer to the circuit in Fig. P4.60.

(a) For  $v_I = 0$ , both transistors are cut off and all currents are zero. Thus

$$V_B = 0 \text{ V} \quad \text{and} \quad V_E = 0 \text{ V}$$

(b) For  $v_I = +2 \text{ V}$ ,  $Q_1$  will be conducting and  $Q_2$  will be cut off, and the circuit reduces to that in Fig. 1.

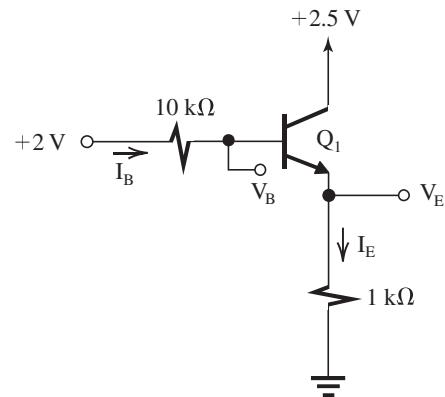


Figure 1

Since  $V_B$  will be lower than  $+2 \text{ V}$ ,  $V_C$  will be higher than  $V_B$  and the transistor will be operating in the active mode. Thus,

$$I_E = \frac{2 - 0.7}{10} = 1.1 \text{ mA}$$

$$V_E = +1.1 \text{ V}$$

$$V_B = 1.8 \text{ V}$$

(c) For  $v_I = -2.5 \text{ V}$ ,  $Q_1$  will be off and  $Q_2$  will be on, and the circuit reduces to that in Fig. 2.

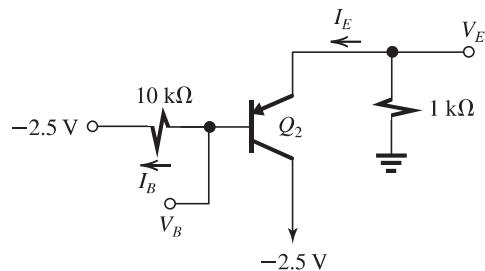


Figure 2

Since  $V_B > -2.5 \text{ V}$ ,  $V_C$  will be lower than  $V_B$  and  $Q_2$  will be operating in the active region. Thus

$$I_E = \frac{2.5 - 0.7}{10} = 1.5 \text{ mA}$$

$$V_E = -I_E \times 1 = -1.5 \text{ V}$$

$$V_B = -1.5 - 0.7 = -2.2 \text{ V}$$

(d) For  $v_I = -5$  V,  $Q_1$  will be off and  $Q_2$  will be on, and the circuit reduces to that in Fig. 3.

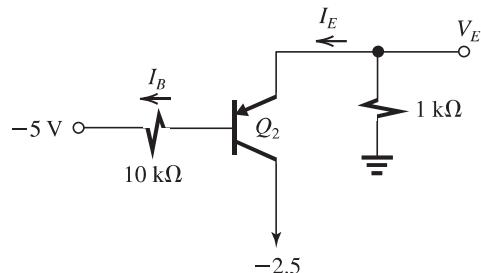


Figure 3

Here we do not know whether  $Q_2$  is operating in the active mode or in saturation. Assuming active-mode operation, we obtain

$$I_E = \frac{5 - 0.7}{1 + \frac{10}{51}} = 3.6 \text{ mA}$$

$$V_E = -3.6 \text{ V}$$

$$V_B = -4.3 \text{ V}$$

which is impossible, indicating that our original assumption is incorrect and that  $Q_2$  is saturated. Assuming saturation-mode operation, we obtain

$$V_E = V_C + V_{ECsat} = -2.5 + 0.2 = -2.3 \text{ V}$$

$$I_E = \frac{-V_E}{1 \text{ k}\Omega} = 2.3 \text{ mA}$$

$$V_B = V_E - 0.7 = -3 \text{ V}$$

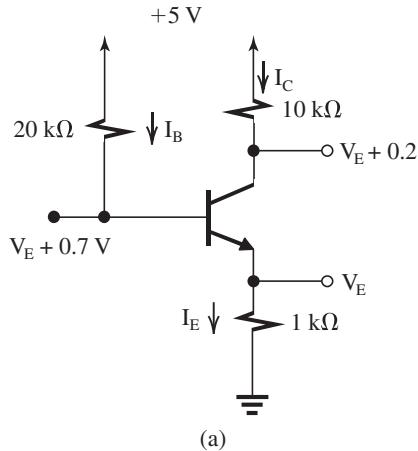
$$I_B = \frac{-3 - (-5)}{10} = 0.2 \text{ mA}$$

$$I_C = I_E - I_B = 2.3 - 0.2 = 2.1 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.1}{0.2} = 10.5$$

which is lower than  $\beta$ , verifying that  $Q_2$  is operating in saturation.

4.61 (a)



Assuming saturation-mode operation, the terminal voltages are interrelated as shown in the figure, which corresponds to Fig. P4.61(a). Thus we can write

$$I_E = \frac{V_E}{1} = V_E$$

$$I_C = \frac{5 - (V_E + 0.2)}{10} = 0.5 - 0.1(V_E + 0.2)$$

$$I_B = \frac{5 - (V_E + 0.7)}{20} = 0.25 - 0.05(V_E + 0.7)$$

Now, imposing the constraint

$$I_E = I_C + I_B$$

results in

$$V_E = 0.5 - 0.1(V_E + 0.2) + 0.25 - 0.05(V_E + 0.7)$$

$$\Rightarrow V_E = 0.6 \text{ V}$$

$$V_C = 0.8 \text{ V}$$

$$V_B = 1.3 \text{ V}$$

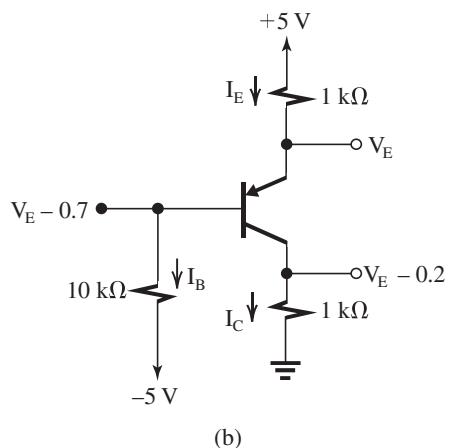
$$I_C = \frac{5 - 0.8}{10} = 0.42 \text{ mA}$$

$$I_B = \frac{5 - 1.3}{20} = 0.185 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{0.42}{0.185} = 2.3$$

which is less than the value of  $\beta_1$  verifying saturation-mode operation.

(b)



Assuming saturation-mode operation, the terminal voltages are interrelated as shown in the

figure, which corresponds to Fig. P4.61(b). We can obtain the currents as follows:

$$I_E = \frac{5 - V_E}{1} = 5 - V_E$$

$$I_C = \frac{V_E - 0.2}{1} = V_E - 0.2$$

$$I_B = \frac{V_E - 0.7 - (-5)}{10} = 0.1 V_E + 0.43$$

Imposing the constraint

$$I_E = I_B + I_C$$

results in

$$5 - V_E = V_E - 0.2 + 0.1 V_E + 0.43$$

$$\Rightarrow V_E = +2.27 \text{ V}$$

$$V_C = +2.07 \text{ V}$$

$$V_B = 1.57 \text{ V}$$

$$I_C = \frac{2.07}{1} = 2.07 \text{ mA}$$

$$I_B = \frac{1.57 - (-5)}{10} = 0.657 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.07}{0.657} = 3.2$$

which is lower than the value of  $\beta$ , verifying saturation-mode operation.

(c) We shall assume that both  $Q_3$  and  $Q_4$  are operating in saturation. To begin the analysis

shown in the figure, which corresponds to Fig. P4.61(c), we denote the voltage at the emitter of  $Q_3$  as  $V$  and then obtain the voltages at all other nodes in terms of  $V$ , utilizing the fact that a saturated transistor has  $|V_{CE}| = 0.2 \text{ V}$  and of course  $|V_{BE}| = 0.7 \text{ V}$ . Note that the choice of the collector node to begin the analysis is arbitrary; we could have selected any other node and denoted its voltage as  $V$ . We next draw a circle around the two transistors to define a supernode. A node equation for the supernode will be

$$I_{E3} + I_{C4} = I_{B3} + I + I_{E4} \quad (1)$$

where

$$I_{E3} = \frac{5 - (V + 0.2)}{10} = 0.48 - 0.1V \quad (2)$$

$$I_{C4} = \frac{5 - (V - 0.5)}{30} = 0.183 - 0.033V \quad (3)$$

$$I_{B3} = \frac{5 - (V - 0.5)}{10} = 0.1V - 0.05 \quad (4)$$

$$I = \frac{V}{20} = 0.05V \quad (5)$$

$$I_{E4} = \frac{V - 0.7}{10} = 0.1V - 0.07 \quad (6)$$

Substituting from Eqs. (2)–(6) into Eq. (1) gives

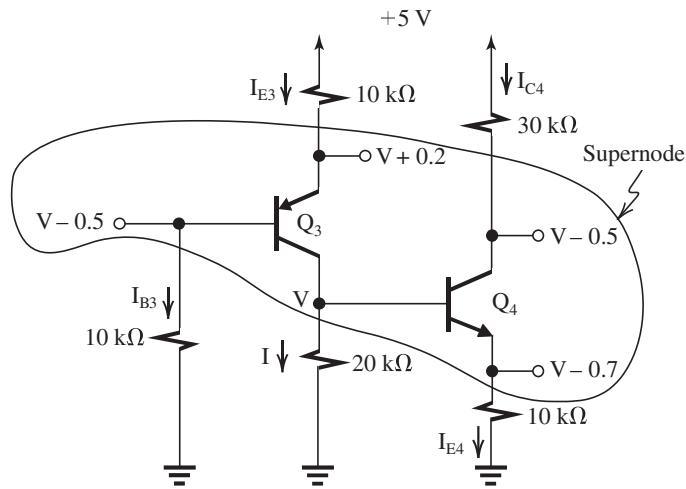
$$\begin{aligned} & 0.48 - 0.1V + 0.183 - 0.033V \\ & = 0.1V - 0.05 + 0.05V + 0.1V - 0.07 \end{aligned}$$

$$\Rightarrow V = 2.044 \text{ V}$$

Thus

$$V_{C3} = V = 2.044 \text{ V}$$

This figure belongs to Problem 4.61, part (c).



(c)

$$V_{C4} = V - 0.5 = 1.54 \text{ V}$$

Next we determine all currents utilizing Eqs. (2)–(6):

$$I_{E3} = 0.276 \text{ mA} \quad I_{C4} = 0.116 \text{ mA}$$

$$I_{B3} = 0.154 \text{ mA} \quad I = 0.102 \text{ mA}$$

$$I_{E4} = 0.134$$

The base current of  $Q_4$  can be obtained from

$$I_{B4} = I_{E4} - I_{C4} = 0.134 - 0.116 = 0.018 \text{ mA}$$

Finally, the collector current of  $Q_3$  can be found as

$$I_{C3} = I + I_{B4} = 0.102 + 0.018 = 0.120$$

The forced  $\beta$  values can now be found as

$$\beta_{\text{forced}3} = \frac{I_{C3}}{I_{B3}} = \frac{0.120}{0.154} = 0.8$$

$$\beta_{\text{forced}4} = \frac{I_{C4}}{I_{B4}} = \frac{0.116}{0.018} = 6.4$$

Both  $\beta_{\text{forced}}$  values are well below the  $\beta$  value of 50, verifying that  $Q_3$  and  $Q_4$  are in deep saturation.

4.62 (a) Consider first the case  $\beta = \infty$  and  $R$  open circuited. The circuit is shown in Fig. 1 below, where  $\beta = \infty$  and  $R$  is open circuited. Since  $V_{D1} = V_{BE1}$ , we have

$$V_1 = V_{E1}$$

This figure belongs to Problem 4.62, part (a).

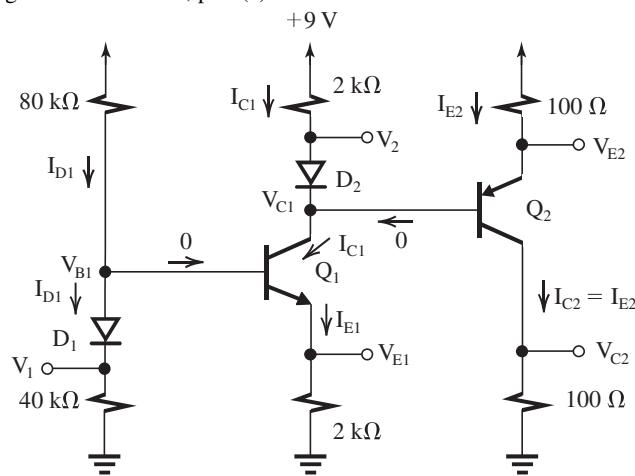


Figure 1  $\beta = \infty$ , and  $R$  is open circuited

Thus,

$$I_{D1} \times 40 = I_{E1} \times 2$$

$$\Rightarrow I_{D1} = 0.05I_{E1}$$

But

$$I_{D1} = \frac{9 - 0.7}{80 + 40} = 0.069 \text{ mA} \simeq 0.07 \text{ mA}$$

Thus,

$$I_{E1} = \frac{0.069}{0.05} = 1.38 \text{ mA} \simeq 1.4 \text{ mA}$$

$$V_{E1} = I_{E1} \times 2 = 2.77 \text{ V} \simeq 2.8 \text{ V}$$

$$V_{B1} = V_{E1} + 0.7 = 3.5 \text{ V}$$

$$I_{C1} = I_{E1} = 1.38 \text{ mA} \simeq 1.4 \text{ mA}$$

$$V_2 = 9 - I_{C1} \times 2 = 9 - 1.38 \times 2 \simeq 6.2 \text{ V}$$

$$V_{C1} = V_2 - V_{D2} = 6.2 - 0.7 = 5.5 \text{ V}$$

$$V_{E2} = V_2 = 6.2 \text{ V}$$

$$I_{E2} = \frac{9 - 6.2}{100 \Omega} = 28 \text{ mA}$$

$$I_{C2} = I_E = 28 \text{ mA}$$

$$V_{C2} = 28 \times 0.1 = 2.8 \text{ V}$$

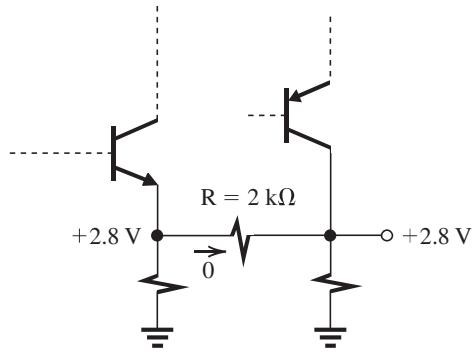


Figure 2

Now connecting the resistance  $R = 2 \text{ k}\Omega$  between  $C_1$  and  $E_2$  (see Fig. 2) both of which at 2.8 V, will result in zero current through  $R$ ; thus all voltages and currents remain unchanged.

(b) We next consider the situation with  $\beta = 100$ , first with R disconnected. The circuit is shown in Fig. 3 below.

Once again we observe that  $V_{E1} = V_1$ , thus

$$I_{E1} \times 2 = I_{D1} \times 40$$

$$\Rightarrow I_{D1} = 0.05I_{E1}$$

The base current of  $Q_1$  is  $I_{E1}/101 \simeq 0.01I_{E1}$ . Thus, the current through the 80-k $\Omega$  resistor is  $0.05I_{E1} + 0.01I_{E1} = 0.06I_{E1}$  and

$$V_{B1} = V_{E1} + 0.7 = 2I_{E1} + 0.7$$

$$0.06I_{E1} = \frac{9 - V_{B1}}{80} = \frac{9 - (2I_{E1} + 0.7)}{80}$$

$$\Rightarrow I_{E1} = 1.22 \text{ mA}$$

$$V_{E1} = 1.22 \times 2 = 2.44 \text{ V}$$

$$V_{B1} = 2.44 + 0.7 = 3.14 \text{ V}$$

$$I_{C1} = \alpha I_{E1} = 0.99 \times 1.22 = 1.21 \text{ mA}$$

Observing that  $V_{E2} = V_2$ , we see that the voltage drops across the  $2\text{-k}\Omega$  resistor and the  $100\text{-}\Omega$  resistor are equal, thus

$$I_{D2} \times 2 = I_{E2} \times 0.1$$

$$\Rightarrow I_{D2} = 0.05I_{E2}$$

As the base current of  $Q_2$  is approximately  $0.01I_{E2}$ , a node equation at  $C_1$  yields

$$I_{D2} = I_{C1} - 0.01I_{E2}$$

Thus,

$$0.05I_{E2} \equiv I_{C1} - 0.01I_{E2}$$

$$\Rightarrow 0.06J_{E2} = J_{C1}$$

$$I_{E2} = \frac{I_{C1}}{0.06} = \frac{1.21}{0.06} = 20.13 \text{ mA}$$

$$I_{D2} = 0.05 \times 20.13 = 1 \text{ mA}$$

$$V_{C1} = 9 - 1 \times 2 - 0.7 = 6.3 \text{ V}$$

$$V_{E2} = 6.3 + 0.7 = 7 \text{ V}$$

$$I_{C2} = \alpha I_{E2} = 0.99 \times 20.13 = 20 \text{ mA}$$

$$V_{C2} = 20 \times 0.1 = 2 \text{ V}$$

Finally, with the resistance  $R$  connected between  $E_1$  and  $C_2$ , it will conduct a current that we can initially estimate as

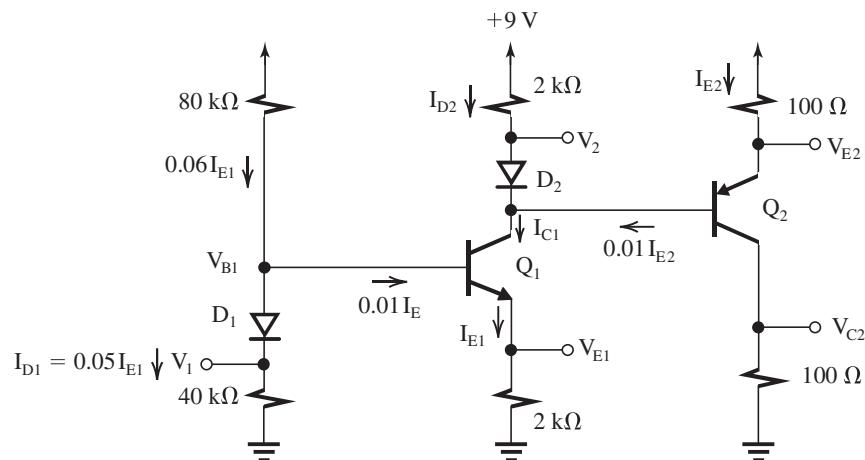


Figure 3

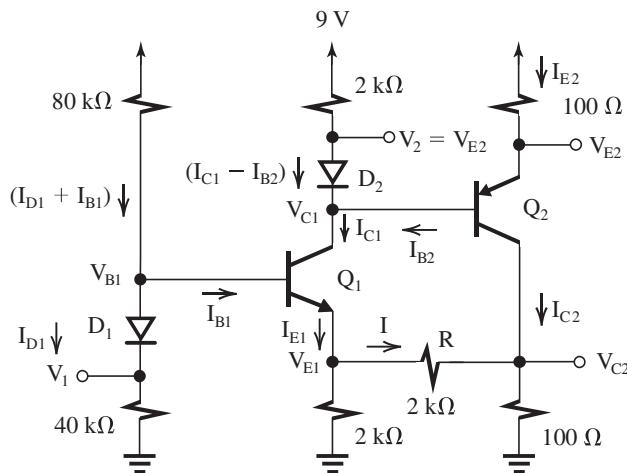


Figure 4

$$I = \frac{V_{E1} - V_{C2}}{R} = \frac{2.44 - 2}{2} = 0.22 \text{ mA}$$

This is a substantial amount compared to  $I_{E1} = 1.22 \text{ mA}$ , requiring that we redo the analysis with  $R$  in place. The resulting circuit is shown in Fig. 4 above.

Denoting the emitter voltage of  $Q_1$ ,  $V_{E1}$ , and the current through  $R$  as  $I$ , the analysis proceeds as follows:

$$V_1 = V_{E1}$$

$$I_{D1} = \frac{V_1}{40} = \frac{V_{E1}}{40} = 0.025V_{E1}$$

$$I_{E1} = \frac{V_{E1}}{2} + I = 0.5V_{E1} + I$$

$$I_{B1} = \frac{I_{E1}}{101} = 0.005V_{E1} + 0.01I$$

$$I_{80\text{k}\Omega} = I_{D1} + I_{B1} = 0.03V_{E1} + 0.01I$$

$$V_{C2} = V_{E1} - I \times 2 = V_{E1} - 2I$$

$$I_{C2} = -I + \frac{V_{C2}}{0.1} = 10V_{E1} - 21I$$

$$I_{B2} = \frac{I_{C2}}{101} = 0.1V_{E1} - 0.21I$$

$$I_{C1} = \alpha I_{E1} = 0.495V_{E1} + 0.99I$$

$$I_{D2} = I_{C1} - I_{B2} = 0.395V_{E1} + 1.2I$$

$$I_{E2} = \frac{I_{C2}}{\alpha} = 10.1V_{E1} - 21.2I$$

$$I_{D2} \times 2 = I_{E2} \times 0.1$$

$$2(0.395V_{E1} + 1.2I) = 0.1(10.1V_{E1} - 21.2I)$$

$$\Rightarrow I = 0.05V_{E2}$$

$$\text{Voltage drop across } 80\text{-k}\Omega \text{ resistor} = (0.03V_{E1} + 0.01I) \times 80 = 9 - V_{E1} - 0.7$$

$$\text{Substituting } I = 0.05V_{E2} \text{ gives}$$

$$V_{E1} = 2.41 \text{ V}$$

$$I = 0.12 \text{ mA}$$

$$\text{Substituting these quantities in the equations above gives}$$

$$V_{B1} = 2.41 + 0.7 = 3.11 \text{ V}$$

$$I_{E1} = 1.325 \text{ mA}$$

$$I_{C1} = 1.31 \text{ mA}$$

$$I_{D1} = 1.09 \text{ mA}$$

$$V_{C1} = 9 - 1.09 \times 2 - 0.7 = 6.12 \text{ V}$$

$$V_{E2} = 6.82 \text{ V}$$

$$I_{E2} = \frac{9 - 6.82}{0.1} = 21.8 \text{ mA}$$

$$I_{C2} = 0.99 \times 21.8 = 21.6 \text{ mA}$$

$$V_{C2} = 2.17 \text{ V}$$

### Exercise 5-1

Ex: 5.1

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5 \text{ pF/m}}{4 \text{ nm}} = 8.625 \text{ fF}/\mu\text{m}^2$$

$$\mu_n = 450 \text{ cm}^2/\text{V}\cdot\text{s}$$

$$k'_n = \mu_n C_{ox} = 388 \mu\text{A}/\text{V}^2$$

$$V_{OV} = (V_{GS} - V_t) = 0.5 \text{ V}$$

$$g_{DS} = \frac{1}{1 \text{ k}\Omega} = k'_n \frac{W}{L} V_{OV} \Rightarrow \frac{W}{L} = 5.15$$

$$L = 0.18 \mu\text{m}, \text{ so } W = 0.93 \mu\text{m}$$

$$\text{Ex: 5.2 } C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5 \text{ pF/m}}{4 \text{ nm}} = 8.6 \text{ fF}/\mu\text{m}^2$$

$$\mu_n = 450 \text{ cm}^2/\text{V}\cdot\text{s}$$

$$k'_n = \mu_n C_{ox} = 387 \mu\text{A}/\text{V}^2$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 = 0.3 \text{ mA}, \frac{W}{L} = 20$$

$$\therefore V_{OV} = 0.28 \text{ V}$$

$$V_{DS, \min} = V_{OV} = 0.28 \text{ V, for saturation}$$

$$\text{Ex: 5.3 } I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 \text{ in saturation}$$

Change in  $I_D$  is:

- (a) double  $L$ , 0.5
- (b) double  $W$ , 2
- (c) double  $V_{OV}$ ,  $2^2 = 4$
- (d) double  $V_{DS}$ , no change (ignoring length modulation)

Case (c) would cause leaving saturation if

$$V_{DS} < 2V_{OV}$$

Ex: 5.4 For saturation  $V_{DS} \geq V_{OV}$ , so  $V_{DS}$  must be changed to  $2V_{OV}$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2, \text{ so } I_D \text{ increases by a factor of 4.}$$

$$\text{Ex: 5.5 } V_{OV} = 0.5 \text{ V}$$

$$g_{DS} = k'_n \frac{W}{L} V_{OV} = \frac{1}{1 \text{ k}\Omega}$$

$$\therefore k_n = k'_n \frac{W}{L} = \frac{1}{1 \times 0.5} = 2 \text{ mA/V}^2$$

For  $V_{DS} = 0.5 \text{ V} = V_{OV}$ , the transistor operates in saturation, and

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 = 0.25 \text{ mA}$$

Similarly,  $V_{DS} = 1 \text{ V}$  results in saturation-mode operation and  $I_D = 0.25 \text{ mA}$ .

$$\text{Ex: 5.6 } V_A = V'_A L = 50 \times 0.8 = 40 \text{ V}$$

$$\lambda = \frac{1}{V_A} = 0.025 \text{ V}^{-1}$$

$$V_{DS} = 1 \text{ V} > V_{OV} = 0.5 \text{ V}$$

$$\Rightarrow \text{Saturation: } I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 (1 + \lambda V_{DS})$$

$$I_D = \frac{1}{2} \times 200 \times \frac{16}{0.8} \times 0.5^2 (1 + 0.025 \times 1)$$

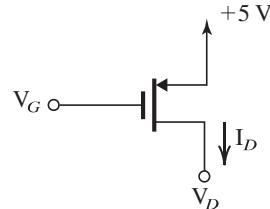
$$= 0.51 \text{ mA}$$

$$r_o = \frac{V_A}{I_D} = \frac{40}{0.5} = 80 \text{ k}\Omega$$

where  $I_D$  is the value of  $I_D$  without channel-length modulation taken into account.

$$r_o = \frac{\Delta V_{DS}}{\Delta I_O} \Rightarrow \Delta I_O = \frac{2 \text{ V}}{80 \text{ k}\Omega} = 0.025 \text{ mA}$$

Ex: 5.7



$$V_{tp} = -1 \text{ V}$$

$$k'_p = 60 \mu\text{A}/\text{V}^2$$

$$\frac{W}{L} = 10 \Rightarrow k_p = 600 \mu\text{A}/\text{V}^2$$

(a) Conduction occurs for  $V_{SG} \geq |V_{tp}| = 1 \text{ V}$

$$\Rightarrow V_G \leq 5 - 1 = 4 \text{ V}$$

(b) Triode region occurs for  $V_{DG} \geq |V_{tp}| = 1 \text{ V}$

$$\Rightarrow V_D \geq V_G + 1$$

(c) Conversely, for saturation

$$V_{DG} \leq |V_{tp}| = 1 \text{ V}$$

$$\Rightarrow V_D \leq V_G + 1$$

(d) Given  $\lambda \cong 0$

$$I_D = \frac{1}{2} k'_p \frac{W}{L} |V_{OV}|^2 = 75 \mu\text{A}$$

$$\therefore |V_{OV}| = 0.5 \text{ V} = V_{SG} - |V_{tp}|$$

$$\Rightarrow V_{SG} = |V_{OV}| + |V_{tp}| = 1.5 \text{ V}$$

$$V_G = 5 - |V_{SG}| = 3.5 \text{ V}$$

$$V_D \leq V_G + 1 = \leq 4.5 \text{ V}$$

### Exercise 5-2

(e) For  $\lambda = -0.02 \text{ V}^{-1}$  and  $|V_{ov}| = 0.5 \text{ V}$ ,

$$I_D = 75 \mu\text{A} \text{ and } r_o = \frac{1}{|\lambda| I_D} = 667 \text{ k}\Omega$$

(f) At  $V_D = 3 \text{ V}$ ,  $V_{SD} = 2 \text{ V}$

$$I_D = \frac{1}{2} k_n \frac{W}{L} |V_{ov}|^2 (1 + |\lambda| |V_{SD}|)$$

$$= 75 \mu\text{A} (1.04) = 78 \mu\text{A}$$

At  $V_D = 0 \text{ V}$ ,  $V_{SD} = 5 \text{ V}$

$$I_D = 75 \mu\text{A} (1.10) = 82.5 \mu\text{A}$$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{3 \text{ V}}{4.5 \mu\text{A}} = 667 \text{ k}\Omega$$

which is the same value found in (c).

Ex: 5.8

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{ov}^2 \Rightarrow 0.3 = \frac{1}{2} \times \frac{60}{1000}$$

$$\times \frac{120}{3} V_{ov}^2 \Rightarrow$$

$$V_{ov} = 0.5 \text{ V} \Rightarrow V_{GS} = V_{ov} + V_t = 0.5 + 1$$

$$= 1.5 \text{ V}$$

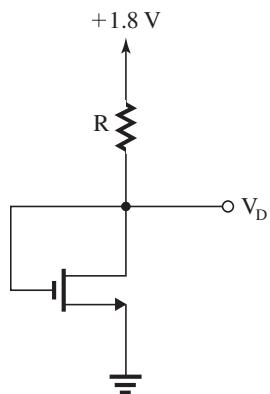
$$V_S = -1.5 \text{ V} \Rightarrow R_S = \frac{V_S - V_{SS}}{I_D}$$

$$= \frac{-1.5 - (-2.5)}{0.3}$$

$$R_S = 3.33 \text{ k}\Omega$$

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{2.5 - 0.4}{0.3} = 7 \text{ k}\Omega$$

Ex: 5.9



$$V_{tn} = 0.5 \text{ V}$$

$$\mu_n C_{ox} = 0.4 \text{ mA/V}^2$$

$$\frac{W}{L} = \frac{0.72 \mu\text{m}}{0.18 \mu\text{m}} = 4.0$$

$$\lambda = 0$$

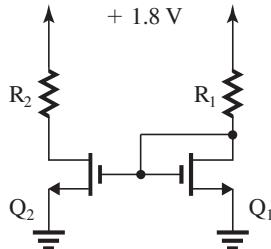
Saturation mode ( $V_{GD} = 0 < V_{tn}$ ):

$$V_D = 0.7 \text{ V} = 1.8 - I_D R_D$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_D - V_{tn})^2 = 0.032 \text{ mA}$$

$$\therefore R = \frac{1.8 - 0.7}{0.032 \text{ mA}} = 34.4 \text{ k}\Omega$$

Ex: 5.10



Since  $Q_2$  is identical to  $Q_1$  and their  $V_{GS}$  values are the same,

$$I_{D2} = I_{D1} = 0.032 \text{ mA}$$

For  $Q_2$  to operate at the triode-saturation boundary, we must have

$$V_{D2} = V_{ov} = 0.2 \text{ V}$$

$$\therefore R_2 = \frac{1.8 \text{ V} - 0.2 \text{ V}}{0.032 \text{ mA}} = 50 \text{ k}\Omega$$

Ex: 5.11  $R_D = 12.4 \times 2 = 24.8 \text{ k}\Omega$

$V_{GS} = 5 \text{ V}$ , assume triode region:

$$\left. \begin{aligned} I_D &= k_n \frac{W}{L} \left[ (V_{GS} - V_t)V_{DS} - \frac{1}{2} V_{DS}^2 \right] \\ I_D &= \frac{V_{DD} - V_{DS}}{R} \end{aligned} \right\} \Rightarrow$$

$$\frac{5 - V_{DS}}{24.8} = 1 \times \left[ (5 - 1)V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$\Rightarrow V_{DS}^2 - 8.08V_{DS} + 0.4 = 0$$

$$\Rightarrow V_{DS} = 0.05 \text{ V} < V_{ov} \Rightarrow \text{triode region}$$

$$I_D = \frac{5 - 0.05}{24.8} = 0.2 \text{ mA}$$

Ex: 5.12 As indicated in Example 5.6,

$V_D \geq V_G - V_t$  for the transistor to be in the saturation region.

$$V_{Dmin} = V_G - V_t = 5 - 1 = 4 \text{ V}$$

$$I_D = 0.5 \text{ mA} \Rightarrow R_{Dmax} = \frac{V_{DD} - V_{Dmin}}{I_D}$$

$$= \frac{10 - 4}{0.5} = 12 \text{ k}\Omega$$

### Exercise 5-3

Ex: 5.13

$$I_D = 0.32 \text{ mA} = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 = \frac{1}{2} \times 1 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.8 \text{ V}$$

$$V_{GS} = 0.8 + 1 = 1.8 \text{ V}$$

$$V_G = V_S + V_{GS} = 1.6 + 1.8 = 3.4 \text{ V}$$

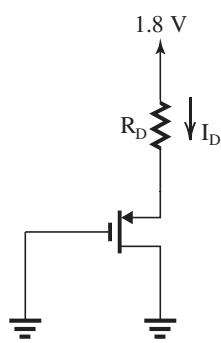
$$R_{G2} = \frac{V_G}{I} = \frac{3.4}{1 \mu\text{A}} = 3.4 \text{ M}\Omega$$

$$R_{G1} = \frac{5 - 3.4}{1 \mu\text{A}} = 1.6 \text{ M}\Omega$$

$$R_S = \frac{V_S}{0.32} = 5 \text{ k}\Omega$$

$$V_D = 3.4 \text{ V}, \text{ then } R_D = \frac{5 - 3.4}{0.32} = 5 \text{ k}\Omega$$

Ex: 5.14



$$V_{tp} = -0.4 \text{ V}$$

$$k'_p = 0.1 \text{ mA/V}^2$$

$$\frac{W}{L} = \frac{10 \mu\text{m}}{0.18 \mu\text{m}} \Rightarrow k_p = 5.56 \text{ mA/V}^2$$

$$V_{SG} = |V_{tp}| + |V_{ov}|$$

$$= 0.4 + 0.6 = 1 \text{ V}$$

$$V_S = +1 \text{ V}$$

Since  $V_{DG} = 0$ , the transistor is operating in saturation, and

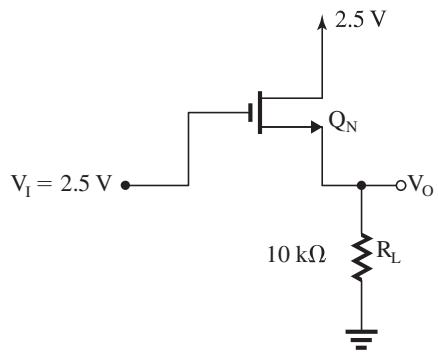
$$I_D = \frac{1}{2} k'_p V_{OV}^2 = 1 \text{ mA}$$

$$\therefore R = \frac{1.8 - 1}{1} = 0.8 \text{ k}\Omega = 800 \Omega$$

Ex: 5.15  $v_I = 0$ : since the circuit is perfectly symmetrical,  $v_O = 0$  and therefore  $V_{GS} = 0$ , which implies that the transistors are turned off and  $I_{DN} = I_{DP} = 0$ .

$v_I = 2.5 \text{ V}$ : if we assume that the NMOS is turned on, then  $v_O$  would be less than 2.5 V, and this implies that PMOS is off ( $V_{SGP} < 0$ ).

$$I_{DN} = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$$



$$I_{DN} = \frac{1}{2} \times 1(2.5 - V_O - 1)^2$$

$$I_{DN} = 0.5(1.5 - V_O)^2$$

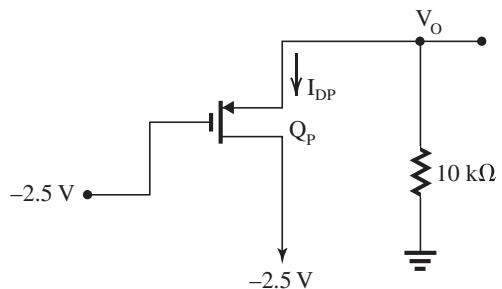
$$\text{Also: } V_O = R_L I_{DN} = 10 I_{DN}$$

$$I_{DN} = 0.5(1.5 - 10 I_{DN})^2$$

$$\Rightarrow 100 I_{DN}^2 - 32 I_{DN} + 2.25 = 0 \Rightarrow I_{DN}$$

$$= 0.104 \text{ mA}$$

$$I_{DP} = 0, V_O = 10 \times 0.104 = 1.04 \text{ V}$$



$V_I = -2.5 \text{ V}$ : Again if we assume that  $Q_P$  is turned on, then  $V_O > -2.5 \text{ V}$  and  $V_{GS1} < 0$ , which implies that the NMOS  $Q_N$  is turned off.

$$I_{DN} = 0$$

Because of the symmetry,

$$I_{DP} = 0.104,$$

$$V_O = -I_{DP} \times 10 \text{ k}\Omega$$

$$= -1.04 \text{ V}$$

$$\text{Ex: 5.16 } V_t = 0.8 + 0.4 \left[ \sqrt{0.7 + 3} - \sqrt{0.7} \right]$$

$$= 1.23 \text{ V}$$

$$\text{Ex: 5.17 } v_{DSmin} = v_{GS} + |V_t|$$

$$= 1 + 2 = 3 \text{ V}$$

$$I_D = \frac{1}{2} \times 2 [1 - (-2)]^2$$

$$= 9 \text{ mA}$$

## Chapter 5–1

$$5.1 \quad C_{ox} = 9 \text{ fF}/\mu\text{m}^2, \quad V_{OV} = 0.2 \text{ V}$$

$$L = 0.36 \mu\text{m}, \quad V_{DS} = 0 \text{ V}$$

$$W = 3.6 \mu\text{m}$$

$$Q = C_{ox} \cdot W \cdot L \cdot V_{OV} = 2.33 \text{ fC}$$

$$5.2 \quad k'_n = \mu_n C_{ox}$$

$$= \frac{m^2}{V \cdot s} \frac{F}{m^2} = \frac{F}{V \cdot s} = \frac{C/V}{V \cdot s} = \frac{C}{s} \frac{1}{V^2}$$

$$= \frac{A}{V^2}$$

Since  $k_n = k'_n W/L$  and  $W/L$  is dimensionless,  $k_n$  has the same dimensions as  $k'_n$ ; that is,  $A/V^2$ .

5.3 With  $V_{DS}$  small, compared to  $V_{OV}$ , Eq. (5.13a) applies:

$$r_{DS} = \frac{1}{(\mu_n C_{ox}) \left( \frac{W}{L} \right) (V_{OV})}$$

(a)  $V_{OV}$  is doubled  $\rightarrow r_{DS}$  is halved. factor = 0.5

(b)  $W$  is doubled  $\rightarrow r_{DS}$  is halved. factor = 0.5

(c)  $W$  and  $L$  are doubled  $\rightarrow r_{DS}$  is unchanged. factor = 1.0

(d) If oxide thickness  $t_{ox}$  is halved, and

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

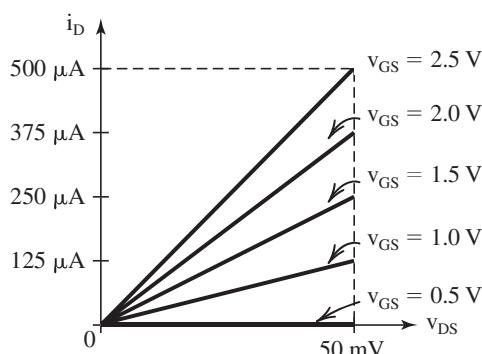
then  $C_{ox}$  is doubled. If  $W$  and  $L$  are also halved,  $r_{DS}$  is halved, factor = 0.5.

$$5.4 \quad k_n = 5 \text{ mA/V}^2, \quad V_{tn} = 0.5 \text{ V},$$

small  $V_{DS}$

$$i_D = k_n (V_{GS} - V_t) V_{DS} = k_n V_{OV} V_{DS}$$

$$g_{DS} = \frac{1}{r_{DS}} = k_n V_{OV}$$



$V_{GS}$ (V)	$V_{OV}$ (V)	$g_{DS}$ (mA/V)	$r_{DS}$ ( $\Omega$ )
0.5	0	0	$\infty$
1.0	0.5	2.5	400
1.5	1.0	5.0	200
2.0	1.5	7.5	133
2.5	2.0	10	100

5.5 The transistor size will be minimized if  $W/L$  is minimized. To start with, we minimize  $L$  by using the smallest feature size,

$$L = 0.18 \mu\text{m}$$

$$r_{DS} = \frac{1}{k'_n (W/L) (V_{GS} - V_t)}$$

$$r_{DS} = \frac{1}{k'_n (W/L) V_{OV}}$$

Two conditions need to be met for  $V_{OV}$  and  $r_{DS}$

Condition 1:

$$r_{DS,1} = \frac{1}{400 \times 10^{-6} (W/L) V_{OV,1}}$$

$$= 250 \Rightarrow (W/L) V_{OV,1} = 10$$

Condition 2:

$$r_{DS,2} = \frac{1}{400 \times 10^{-6} (W/L) V_{OV,2}}$$

$$= 1000 \Rightarrow (W/L) V_{OV,2} = 2.5$$

If condition 1 is met, condition 2 will be met since the over-drive voltage can always be reduced to satisfy this requirement. For condition 1, we want to decrease  $W/L$  as much as possible (so long as it is greater than or equal to 1), while still meeting all of the other constraints. This requires our using the largest possible  $V_{GS,1}$  voltage.

$V_{GS,1} = 1.8 \text{ V}$  so  $V_{OV,1} = 1.8 - 0.5 = 1.3 \text{ V}$ , and

$$W/L = \frac{10}{V_{OV,1}} = \frac{10}{1.3} = 7.69$$

Condition 2 now can be used to find  $V_{GS,2}$

$$V_{OV,2} = \frac{2.5}{W/L} = \frac{2.5}{7.69} = 0.325$$

$$\Rightarrow V_{GS,2} = 0.825 \text{ V} \Rightarrow 0.825 \text{ V} \leq V_{GS} \leq 1.8 \text{ V}$$

## Chapter 5–2

$$\begin{aligned}
 5.6 \quad r_{ds} &= 1/\left.\frac{\partial i_D}{\partial v_{DS}}\right|_{v_{DS}=v_{OV}} \\
 &= \left[ \frac{\partial}{\partial v_{DS}} \left( k_n \left( V_{OV} v_{DS} - \frac{1}{2} v_{DS}^2 \right) \right) \right]^{-1} \\
 &= \left[ k_n \left( \frac{\partial}{\partial v_{DS}} \right) (v_{OV} v_{DS}) - 1/2 \frac{\partial}{\partial v_{DS}} (v_{DS}^2) \right]^{-1} \\
 &= \left[ k_n \left( V_{OV} - \frac{1}{2} \cdot 2V_{DS} \right) \right]^{-1} \\
 &= \frac{1}{k_n (V_{OV} - V_{DS})}
 \end{aligned}$$

If  $V_{DS} = 0 \Rightarrow r_{ds} = \frac{1}{k_n V_{OV}}$

If  $V_{DS} = 0.2V_{OV} \Rightarrow r_{ds} = \frac{1.25}{V_{OV}}$

If  $V_{DS} = 0.5V_{OV} \Rightarrow r_{ds} = \frac{1}{k_n (V_{OV} - 0.5V_{OV})}$   
 $= 1/k_n (0.5V_{OV}) = \frac{2}{k_n V_{OV}}$

If  $V_{DS} = 0.8V_{OV} \Rightarrow r_{ds} = \frac{1}{k_n (V_{OV} - 0.8V_{OV})}$   
 $= 1/k_n (0.2V_{OV}) = \frac{5}{k_n V_{OV}}$

If  $V_{DS} = V_{OV}$ ,

$$r_{ds} = \frac{1}{0} \Rightarrow \infty$$

5.7  $V_{DS\ sat} = V_{OV}$

$$V_{OV} = V_{GS} - V_t = 1 - 0.5 = 0.5 \text{ V}$$

$$\Rightarrow V_{DS\ sat} = 0.5 \text{ V}$$

In saturation:

$$i_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2 = \frac{1}{2} k_n V_{OV}^2$$

$$i_D = \frac{1}{2} \times \frac{4 \text{ mA}}{\text{V}^2} \times (0.5 \text{ V})^2$$

$$i_D = 0.5 \text{ mA}$$

5.8  $L_{min} = 0.25 \mu \text{m}$

$$t_{ox} = 6 \text{ nm}$$

$$\mu_n = 460 \frac{\text{cm}^2}{\text{V} \cdot \text{s}} = 460 \times 10^{-4} \frac{\text{m}^2}{\text{V} \cdot \text{s}}$$

$$(a) C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5 \text{ pF/m}}{6 \text{ nm}}$$

$$= 5.75 \times 10^{-3} \frac{\text{F}}{\text{m}^2} \left( \frac{\text{pF}}{\mu \text{m}^2} \right)$$

$$k'_n = \mu_n C_{ox} = 265 \mu \text{A/V}^2$$

$$(b) \text{ For } \frac{W}{L} = \frac{20}{0.25}, k_n = 21.2 \text{ mA/V}^2$$

$$\therefore 0.5 \text{ mA} = I_D = \frac{1}{2} k_n V_{OV}^2$$

$$V_{OV} = 0.22 \text{ V}$$

$$V_{GS} = 0.72 \text{ V}$$

$$V_{DS} \geq 0.22 \text{ V}$$

$$(c) g_{DS} = \frac{1}{100 \Omega} = k_n V_{OV}$$

$$\therefore V_{OV} = 0.47 \text{ V.}$$

$$V_{GS} = 0.97 \text{ V.}$$

5.9  $V_{tp} = -0.7 \text{ V}$

(a)  $|V_{SG}| = |V_{tp}| + |V_{ov}|$

$$= 0.7 + 0.4 = 1.1 \text{ V}$$

$$\Rightarrow V_G = -1.1 \text{ V}$$

(b) For the p-channel transistor to operate in saturation, the drain voltage must not exceed the gate voltage by more than  $|V_{tp}|$ . Thus

$$v_{Dmax} = -1.1 + 0.7 = -0.4 \text{ V}$$

Put differently,  $V_{SD}$  must be at least equal to  $|V_{ov}|$ , which in this case is 0.4 V. Thus  $v_{Dmax} = -0.4 \text{ V}$ .

(c) In (b), the transistor is operating in saturation, thus

$$I_D = \frac{1}{2} k_p |V_{ov}|^2$$

$$0.5 = \frac{1}{2} \times k_p \times 0.4^2$$

$$\Rightarrow k_p = 6.25 \text{ mA/V}^2$$

For  $V_D = -20 \text{ mV}$ , the transistor will be operating in the triode region. Thus

$$I_D = k_p \left[ v_{SD} |V_{ov}| - \frac{1}{2} v_{SD}^2 \right]$$

$$= 6.25 \left[ 0.02 \times 0.4 - \frac{1}{2} (0.02)^2 \right]$$

$$= 0.05 \text{ mA}$$

For  $V_D = -2 \text{ V}$ , the transistor will be operating in saturation, thus

$$I_D = \frac{1}{2} k_p |V_{ov}|^2 = \frac{1}{2} \times 6.25 \times 0.4^2 = 0.5 \text{ mA}$$

5.10  $i_D = \frac{1}{2} k'_n \frac{W}{L} |V_{ov}|^2 \quad k'_n = \mu_n C_{ox}$

For equal drain currents:

$$\mu_n C_{ox} \frac{W_n}{L} = \mu_p C_{ox} \frac{W_p}{L}$$

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = \frac{1}{0.4} = 2.5$$

### Chapter 5–3

5.11 For small  $v_{DS}$ ,  $i_D \simeq k'_n \frac{W}{L_1} (V_{GS} - V_t) V_{DS}$ ,

$$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{k'_n \frac{W}{L} (V_{GS} - V_t)}$$

$$= \frac{1}{100 \times 10^{-6} \times 20 \times (5 - 0.7)}$$

$$r_{DS} = 116.3 \Omega \quad V_{DS} = r_{DS} \times i_D = 116.3 \text{ mV}$$

For the same performance of a p-channel device:

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 2.5 \Rightarrow \frac{W_p}{L} = \frac{W_n}{L} \times 2.5$$

$$= 20 \times 2.5 \Rightarrow \frac{W_p}{L} = 50$$

5.12  $t_{ox} = 6 \text{ nm}$ ,  $\mu_n = 460 \text{ cm}^2/\text{V}\cdot\text{s}$ ,  
 $V_t = 0.5 \text{ V}$ , and  $W/L = 10$ .

$$k_n = \mu_n C_{ox} \frac{W}{L} = 460 \times 10^{-4} \times \frac{3.45 \times 10^{-11}}{6 \times 10^{-9}} \times 10$$

$$= 2.645 \text{ mA/V}^2$$

$$(a) \quad V_{GS} = 2.5 \text{ V} \quad \text{and} \quad V_{DS} = 1 \text{ V}$$

$$v_{OV} = V_{GS} - V_t = 2 \text{ V}$$

Thus  $V_{DS} < v_{OV} \Rightarrow$  triode region,

$$I_D = k_n \left[ V_{DS} v_{OV} - \frac{1}{2} V_{DS}^2 \right]$$

$$= 2.645 \left[ 1 \times 2 - \frac{1}{2} \times 1 \right] = 4 \text{ mA}$$

$$(b) \quad V_{GS} = 2 \text{ V} \quad \text{and} \quad V_{DS} = 1.5 \text{ V}$$

$$v_{OV} = V_{GS} - V_t = 2 - 0.5 = 1.5 \text{ V}$$

Thus,  $V_{DS} = v_{OV} \Rightarrow$  saturation region,

$$i_D = \frac{1}{2} k_n v_{OV}^2 = \frac{1}{2} \times 2.645 \times 1.5^2$$

$$= 3 \text{ mA}$$

$$(c) \quad V_{GS} = 2.5 \text{ V} \quad \text{and} \quad V_{DS} = 0.2 \text{ V}$$

$$v_{OV} = 2.5 - 0.5 = 2 \text{ V}$$

Thus,  $V_{DS} < v_{OV} \Rightarrow$  triode region,

$$i_D = k_n \left[ V_{DS} v_{OV} - \frac{1}{2} V_{DS}^2 \right]$$

$$= 2.645 [0.2 \times 2 - \frac{1}{2} 0.2^2] = 1 \text{ mA}$$

$$(d) \quad V_{GS} = V_{DS} = 2.5 \text{ V}$$

$$v_{OV} = 2.5 - 0.5 = 2 \text{ V}$$

Thus,  $V_{DS} > v_{OV} \Rightarrow$  saturation region,

$$i_D = \frac{1}{2} k_n v_{OV}^2$$

$$= \frac{1}{2} \times 2.645 \times 2^2 = 5.3 \text{ mA}$$

5.13 See Table below.

This table belongs to 5.13.

$L (\mu\text{m})$	0.5	0.25	0.18	0.13
$t_{ox} (\text{nm})$	10	5	3.6	2.6
$C_{ox} \left( \frac{\text{fF}}{\mu\text{m}^2} \right)$ $\epsilon_{ox} = 34.5 \text{ pF/m}$	3.45	6.90	9.58	13.3
$k'_n \left( \frac{\mu\text{A}}{\text{V}^2} \right)$ $(\mu_n = 500 \text{ cm}^2/\text{V}\cdot\text{s})$	173	345	479	665
$k_n \left( \frac{\text{mA}}{\text{V}^2} \right)$ for $\frac{W}{L} = 10$	1.73	3.45	4.79	6.65
$A(\mu\text{m}^2)$ for $\frac{W}{L} = 10$	2.50	0.625	0.324	0.169
$V_{DD}(\text{V})$	5	2.5	1.8	1.3
$V_t(\text{V})$	0.7	0.5	0.4	0.4
$I_D(\text{mA})$ for $V_{GS} = V_{DS} = V_{DD}$ , $I_D = \frac{1}{2} k_n (V_{DD} - V_t)^2$	16	6.90	4.69	2.69
$P(\text{mW}) \quad P = V_{DD} I_D$	80	17.3	8.44	3.50
$\frac{P}{A} \left( \frac{\text{mW}}{\mu\text{m}^2} \right)$	32	27.7	26.1	20.7
Devices Chip	n	4n	7.72n	14.8n

This figure belongs to 5.14, part (a).

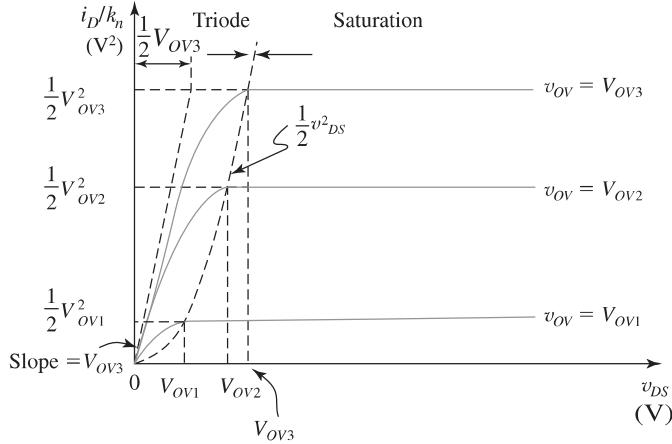


Fig. 1

$$5.14 \quad i_D = k_n \left[ v_{OV} v_{DS} - \frac{1}{2} v_{DS}^2 \right]$$

$$\frac{i_D}{k_n} = v_{OV} v_{DS} - \frac{1}{2} v_{DS}^2 \quad (1)$$

Figure 1 shows graphs for  $i_D/k_n$  versus  $v_{DS}$  for various values of  $v_{OV}$ . Since the right-hand side of Eq. (1) does not have any MOSFET parameters, these graphs apply for any n-channel MOSFET with the assumption that  $\lambda = 0$ . They also apply to p-channel devices with  $v_{DS}$  replaced by  $v_{SD}$ ,  $k_n$  by  $k_p$ , and  $v_{OV}$  with  $|v_{OV}|$ . The slope of each graph at  $v_{DS} = 0$  is found by differentiating Eq. (1) relative to  $v_{DS}$  with  $v_{OV} = V_{OV}$  and then substituting  $v_{DS} = 0$ . The result is

$$\frac{d(i_D/k_n)}{d v_{DS}} \Big|_{v_{DS}=0, v_{OV}=V_{OV}} = V_{OV}$$

Figure 1 shows the tangent at  $v_{DS} = 0$  for the graph corresponding to  $v_{OV} = V_{OV3}$ . Observe that it intersects the horizontal line  $i_D/k_n = \frac{1}{2} V_{OV3}^2$  at  $v_{DS} = \frac{1}{2} V_{OV3}$ . Finally, observe that the curve representing the boundary between the triode region and the saturation region has the equation

$$i_D/k_n = \frac{1}{2} v_{DS}^2$$

Figure 2 shows the graph for the relationship

$$i_D/k_n = \frac{1}{2} v_{OV}^2$$

which describes the MOSFETs operation in the saturation region, that is,

$$v_{DS} \geq v_{OV}$$

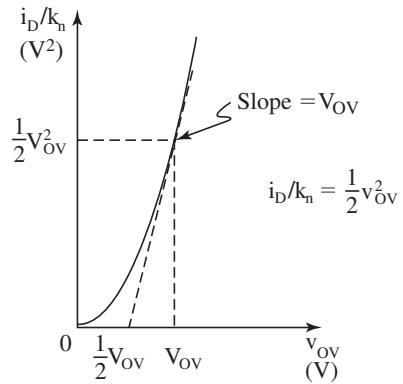


Fig. 2

Here also observe that this relationship (and graph) is universal and represents any MOSFET. The slope at  $v_{OV} = V_{OV}$  is

$$\frac{d(i_D/k_n)}{d v_{OV}} \Big|_{v_{OV}=V_{OV}} = V_{OV}$$

Replacing  $k_n$  by  $k_p$  and  $v_{OV}$  by  $|v_{OV}|$  adapts this graph to PMOS transistors.

### 5.15 For triode-region operation with $v_{DS}$ small,

$$i_D \simeq k_n(v_{GS} - V_t)v_{DS}$$

Thus

$$r_{DS} \equiv \frac{v_{DS}}{i_D} = \frac{1}{k_n(v_{GS} - V_t)}$$

$$1 = \frac{1}{k_n(1.2 - 0.8)} = \frac{1}{0.4 k_n}$$

$$\Rightarrow k_n = 2.5 \text{ mA/V}$$

## Chapter 5–5

$$r_{DS} = \frac{1}{2.5(V_{GS} - 0.8)} \quad (\text{k}\Omega)$$

$$0.2 = \frac{1}{2.5(V_{GS} - 0.8)}$$

$$\Rightarrow V_{GS} = 2.8 \text{ V}$$

For a device with twice the value of  $W$ ,  $k_n$  will be twice as large and the resistance values will be half as large:  $500 \Omega$  and  $100 \Omega$ , respectively.

$$5.16 \quad V_{in} = 0.5 \text{ V}, \quad k_n = 1.6 \text{ mA/V}^2$$

$$I_D = 0.05 = \frac{1}{2} \times 1.6 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.25 \text{ V and } V_{DS} \geq 0.25 \text{ V}$$

$$V_{GS} = 0.5 + 0.25 = 0.75 \text{ V}$$

$$I_D = 0.2 = \frac{1}{2} \times 1.6 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.5 \text{ V and } V_{DS} \geq 0.5 \text{ V}$$

$$V_{GS} = 0.5 + 0.5 = 1 \text{ V}$$

$$5.17 \quad k'_n = 0.4 \text{ mA/V}^2 \quad \text{and} \quad V_t = 0.5 \text{ V}$$

For  $V_{GS} = V_{DS} = 1.8 \text{ V}$ , the MOSFET is operating in saturation. Thus, to obtain

$I_D = 2 \text{ mA}$ , we write

$$2 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times (1.8 - 0.5)^2$$

$$\Rightarrow \frac{W}{L} = 5.92$$

For  $L = 0.18 \mu\text{m}$

$$W = 1.07 \mu\text{m}$$

5.18 For  $V_{GS} = V_{DS} = 1 \text{ V}$ , the MOSFET is operating in saturation,

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$0.4 = \frac{1}{2} k_n (1 - V_t)^2 \quad (1)$$

$$0.1 = \frac{1}{2} k_n (0.8 - V_t)^2 \quad (2)$$

Dividing Eq. (1) by Eq. (2) and taking square roots gives

$$2 = \frac{1 - V_t}{0.8 - V_t}$$

$$\Rightarrow V_t = 0.6 \text{ V}$$

Substituting in Eq. (1), we have

$$0.4 = \frac{1}{2} k_n \times 0.4^2$$

$$\Rightarrow k_n = 5 \text{ mA/V}^2$$

$$5.19 \quad i_D = k_n (V_{GS} - V_t) V_{DS}$$

$$25 = k_n (1 - V_t) \times 0.05 \quad (1)$$

$$50 = k_n (1.5 - V_t) \times 0.05 \quad (2)$$

Dividing Eq. (2) by Eq. (1), we have

$$2 = \frac{1.5 - V_t}{1 - V_t}$$

$$\Rightarrow V_t = 0.5 \text{ V}$$

Substituting in Eq. (1) yields

$$25 = k_n \times 0.5 \times 0.05$$

$$\Rightarrow k_n = 1000 \mu\text{A/V}^2$$

For  $k'_n = 50 \mu\text{A/V}^2$

$$\frac{W}{L} = 20$$

For  $V_{GS} = 2 \text{ V}$  and  $V_{DS} = 0.1 \text{ V}$ ,

$$i_D = k_n \left[ (V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

$$= 1 \left[ (2 - 0.5) \times 0.1 - \frac{1}{2} \times 0.1^2 \right]$$

$$= 0.145 \text{ mA} = 145 \mu\text{A}$$

For  $V_{GS} = 2 \text{ V}$ , pinch-off will occur for

$$V_{DS} = V_{GS} - V_t = 2 - 0.5 = 1.5 \text{ V}$$

and the resulting drain current will be

$$i_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 1 \times (2 - 0.5)^2$$

$$= 1.125 \text{ mA}$$

5.20 For the channel to remain continuous,

$$V_{DS} \leq V_{GS} - V_t$$

Thus for  $V_{GS} = 1.0 \text{ V}$  to  $1.8 \text{ V}$  and  $V_t = 0.4$ ,

$$V_{DS} \leq 1 - 0.4$$

That is,  $V_{DSmax} = 0.6 \text{ V}$ .

$$5.21 \quad \frac{W}{L} = \frac{20}{1} = 20 \quad k'_n = 100 \mu\text{A/V}^2$$

$$k_n = k'_n \left( \frac{W}{L} \right) = 100 \times 20 = 2000 \mu\text{A/V}^2$$

$$= 2 \text{ mA/V}^2$$

For operation as a linear resistance,

$$i_D = k_n(v_{GS} - V_t)v_{DS}$$

and

$$\begin{aligned} r_{DS} &\equiv \frac{v_{DS}}{i_D} = \frac{1}{k_n(v_{GS} - V_t)} \\ &= \frac{1}{2(v_{GS} - 0.8)} \end{aligned}$$

At  $v_{GS} = 1.0$  V,

$$r_{DS} = \frac{1}{2(1 - 0.8)} = 2.5 \text{ k}\Omega$$

At  $v_{GS} = 4.8$  V,

$$r_{DS} = \frac{1}{2(4.8 - 0.8)} = 0.125 \text{ k}\Omega$$

Thus,  $r_{DS}$  will vary in the range of 2.5 kΩ to 125 Ω.

(a) If W is halved,  $k_n$  will be halved and  $r_{DS}$  will vary in the range of 5 kΩ to 250 Ω.

(b) If L is halved,  $k_n$  will be doubled and  $r_{DS}$  will vary in the range of 1.25 kΩ to 62.5 Ω.

(c) If both W and L are halved,  $k_n$  will remain unchanged and  $r_{DS}$  will vary in the original range of 2.5 kΩ to 125 Ω.

**5.22 (a)** Refer to Fig. P5.22. For saturation-mode operation of an NMOS transistor,  $v_{DG} \geq -V_{tn}$ ; thus  $v_{DG} = 0$  results in saturation-mode operation. Similarly, for a p-channel MOSFET, saturation-mode operation is obtained for  $v_{GD} \geq -|V_{tp}|$ , which includes  $v_{GD} = 0$ . Thus, the diode-connected MOSFETs of Fig. P5.22 have the i-v relationship

$$i = \frac{1}{2}k' \left( \frac{W}{L} \right) (v - |V_t|)^2 \quad (1)$$

where  $k'$  represents  $k'_n$  in the NMOS case and  $k'_p$  in the PMOS case.

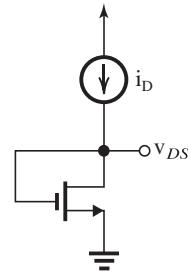
**(b)** If either of the MOSFETs in Fig. P5.22 is biased to operate at  $v = |V_t| + |V_{ov}|$ , then its incremental resistance  $r$  at the bias point can be obtained by differentiating Eq. (1) relative to v and then substituting  $v = |V_t| + |V_{ov}|$  as follows:

$$\frac{\partial i}{\partial v} = k' \left( \frac{W}{L} \right) (v - |V_t|)$$

$$\frac{\partial i}{\partial v} \Big|_{v=|V_t|+V_{ov}} = k' \left( \frac{W}{L} \right) V_{ov}$$

$$r = 1 \Big/ \left[ \frac{\partial i}{\partial v} \right] = 1 \Big/ \left( k' \frac{W}{L} V_{ov} \right) \quad \text{Q.E.D}$$

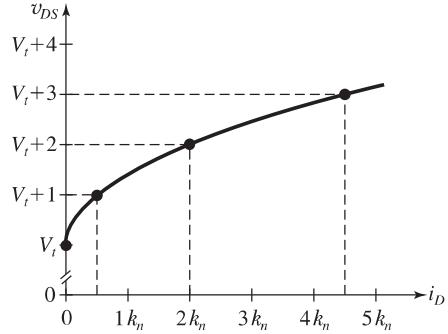
5.23



$$v_{DS} = v_{GS}$$

$$i_D = \frac{1}{2}k_n(v_{DS} - V_t)^2$$

$$\therefore v_{DS} = \sqrt{\frac{2i_D}{k_n}} + V_t$$



$$5.24 \quad V_{DS} = V_D - V_S \quad V_{GS} = V_G - V_S$$

$$V_{ov} = V_{GS} - V_t = V_{GS} - 1.0$$

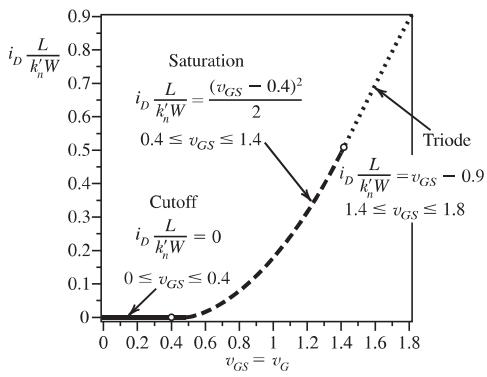
According to Table 5.1, three regions are possible.

Case	$V_S$	$V_G$	$V_D$	$V_{GS}$	$V_{ov}$	$V_{DS}$	Region of operation
a	+1.0	+1.0	+2.0	0	-1.0	+1.0	Cutoff
b	+1.0	+2.5	+2.0	+1.5	+0.5	+1.0	Sat.
c	+1.0	+2.5	+1.5	+1.5	+0.5	+0.5	Sat.
d	+1.0	+1.5	0	+0.5	-0.5	-1.0	Sat.*
e	0	+2.5	1.0	+2.5	+1.5	+1.0	Triode
f	+1.0	+1.0	+1.0	0	-1.0	0	Cutoff
g	-1.0	0	0	+1.0	0	+1.0	Sat.
h	-1.5	0	0	+1.5	+0.5	+1.5	Sat.
i	-1.0	0	+1.0	+1.0	0	+2.0	Sat.
j	+0.5	+2.0	+0.5	+1.5	+0.5	0	Triode

\* With the source and drain interchanged.

5.25 The cutoff–saturation boundary is determined by  $v_{GS} = V_t$ , thus  $v_{GS} = 0.4$  V at the boundary.

The saturation–triode boundary is determined by  $v_{GD} = V_t$ , and  $v_{DS} = V_{DD} = 1$  V, and since  $v_{GS} = v_{GD} + v_{DS}$ , one has  $v_{GS} = 0.4 + 1.0 = 1.4$  V at the boundary.



5.26 (a) Let  $Q_1$  have a ratio  $(W/L)$  and  $Q_2$  have a ratio  $1.03 (W/L)$ . Thus

$$I_{D1} = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (1 - V_t)^2$$

$$I_{D2} = \frac{1}{2} k'_n \left( \frac{W}{L} \right) \times 1.03 \times (1 - V_t)^2$$

Thus,

$$\frac{I_{D2}}{I_{D1}} = 1.03$$

That is, a 3% mismatch in the  $W/L$  ratios results in a 3% mismatch in the drain currents.

(b) Let  $Q_1$  have a threshold voltage  $V_t = 0.6$  V and  $Q_2$  have a threshold voltage  $V_t + \Delta V_t = 0.6 + 0.01 = 0.61$  V.

Thus

$$I_{D1} = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (1 - 0.6)^2$$

$$I_{D2} = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (1 - 0.61)^2$$

and

$$\frac{I_{D2}}{I_{D1}} = \frac{(1 - 0.61)^2}{(1 - 0.6)^2} = 0.95$$

That is, a 10-mV mismatch in the threshold voltage results in a 5% mismatch in drain currents.

$$5.27 r_o = \frac{V_A}{i_D} = \frac{20}{i_D}, 0.1 \text{ mA} \leq i_D \leq 1 \text{ mA}$$

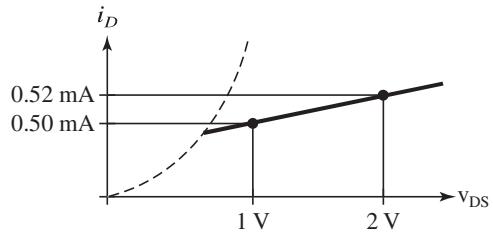
$$\Rightarrow 20 \text{ k}\Omega \leq r_o \leq 200 \text{ k}\Omega$$

$$r_o = \frac{\Delta v_{DS}}{\Delta i_D} \Rightarrow \Delta i_D = \frac{\Delta v_{DS}}{r_o} = \frac{1}{r_o}$$

$$\text{At } i_D = 0.1 \text{ mA}, \Delta i_D = 5 \mu\text{A}, \frac{\Delta i_D}{i_D} = 5\%$$

$$\text{At } i_D = 1 \text{ mA}, \Delta i_D = 50 \mu\text{A}, \frac{\Delta i_D}{i_D} = 5\%$$

5.28



$$r_o = \frac{\Delta v_{DS}}{\Delta i_D} \Big|_{v_{GS} \text{ const.}} = \frac{1}{0.02} = 50 \text{ k}\Omega$$

$$V_A \cong I_D r_o = 0.5 \times 50 = 25 \text{ V}$$

$$\lambda = \frac{1}{V_A} = 0.04 \text{ V}^{-1}$$

5.29  $V_A = V'_A L$ , where  $V'_A$  is completely process dependent. Also,  $r_o = \frac{V_A}{i_D}$ . Therefore, to achieve desired  $r_o$  (which is 5 times larger), we should increase  $L$  ( $L = 5 \times 1 = 5 \mu\text{m}$ ).

To keep  $I_D$  unchanged, the  $\frac{W}{L}$  ratio must stay unchanged. Therefore:

$$W = 5 \times 10 = 50 \mu\text{m} \text{ (so } \frac{W}{L} \text{ is kept at 10)} \\ V_A = r_o i_D = 100 \text{ k}\Omega \times 0.2 \text{ mA} = 20 \text{ V (for the standard device)}$$

$$V_A = 5 \times 20 = 100 \text{ V (for the new device)}$$

5.30  $L = 1.5 \mu\text{m} = 3 \times$  minimum. Thus

$$\lambda = \frac{0.03 \text{ V}^{-1}}{3} = 0.01 \text{ V}^{-1}$$

If  $v_{DS}$  is increased from 1 V to 5 V, the drain current will change from

$$I_D = 100 \mu\text{A} = I'_D (1 + \lambda \times 1) = 1.01 I'_D$$

to

$$I_D + \Delta I_D = I'_D (1 + \lambda \times 5) = 1.05 I'_D$$

## Chapter 5–8

where  $I'_D$  is the drain current without channel-length modulation taken into account. Thus

$$I'_D = \frac{100}{1.01}$$

and

$$100 + \Delta I_D = 1.05 I'_D = \frac{1.05 \times 100}{1.01} = 104 \mu A$$

$$\Rightarrow \Delta I_D = 4 \mu A \text{ or } 4\%$$

To reduce  $\Delta I_D$  by a factor of 2, we need to reduce  $\lambda$  by a factor of 2, which can be obtained by doubling the channel length to  $3 \mu m$ .

$$5.31 \quad V_A = V'_A L = 20 \times 1.5 = 30 V$$

$$\lambda = \frac{1}{V_A} = \frac{1}{30} = 0.033 V^{-1}$$

$$I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2 (1 + \lambda V_{DS})$$

$$= \frac{1}{2} \times 0.2 \times \left( \frac{15}{1.5} \right) \times 0.5^2 (1 + 0.033 \times 2)$$

$$= 0.267 mA$$

$$r_o = \frac{V_A}{\frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2} = \frac{30}{\frac{1}{2} \times 0.2 \times \left( \frac{15}{1.5} \right) \times 0.5^2}$$

$$= 120 k\Omega$$

$$\Delta I_D = \frac{\Delta V_{DS}}{r_o} = \frac{1 V}{120 k\Omega} = 0.008 mA$$

5.32 Quadrupling W and L keeps the current  $I_D$  unchanged. However, the quadrupling of L increases  $V_A$  by a factor of 4 and hence increases  $r_o$  by a factor of 4.

Halving  $V_{OV}$  results in decreasing  $I_D$  by a factor of 4. Thus, this alone increases  $r_o$  by a factor of 4. The overall increase in  $r_o$  is by a factor of  $4 \times 4 = 16$ .

5.33

NMOS	1	2	3	4
$\lambda$	$0.05 V^{-1}$	$0.02 V^{-1}$	$0.1 V^{-1}$	$0.01 V^{-1}$
$V_A$	20 V	50 V	10 V	100 V
$I_D$	0.5 mA	2 mA	0.1 mA	0.2 mA
$r_o$	40 k $\Omega$	25 k $\Omega$	100 k $\Omega$	500 k $\Omega$

5.34 Refer to the circuit in Fig. P5.26 and let  $V_{D1} = 2 V$  and  $V_{D2} = 2.5 V$ . If the two devices are matched,

$$I_{D1} = \frac{1}{2} k_n (1 - V_t)^2 \left( 1 + \frac{2}{V_A} \right)$$

$$I_{D2} = \frac{1}{2} k_n (1 - V_t)^2 \left( 1 + \frac{2.5}{V_A} \right)$$

$$\Delta I_D = I_{D2} - I_{D1} = \frac{1}{2} k_n (1 - V_t)^2 \left( \frac{0.5}{V_A} \right)$$

$$\frac{\Delta I_D}{\frac{1}{2} k_n (1 - V_t)^2} \simeq 0.01 = \frac{0.5}{V_A}$$

$\Rightarrow V_A = 50 V$  (or larger to limit the mismatch in  $I_D$  to 1%).

If  $V'_A = 100 V/\mu m$ , the minimum required channel length is  $0.5 \mu m$ .

$$5.35 \quad V_{tp} = 0.8 V, \quad |V_A| = 40 V$$

$$|v_{GS}| = 3 V, \quad |v_{DS}| = 4 V$$

$$i_D = 3 mA$$

$$|V_{ov}| = |v_{GS}| - |V_{tp}| = 2.2 V$$

$|v_{DS}| > |V_{ov}| \Rightarrow$  saturation mode

$$v_{GS} = -3 V$$

$$v_{SG} = +3 V$$

$$v_{DS} = -4 V$$

$$v_{SD} = 4 V$$

$$V_{tp} = -0.8 V$$

$$V_A = -40 V$$

$$\lambda = -0.025 V^{-1}$$

$$i_D = \frac{1}{2} k_p (v_{GS} - V_{tp})^2 (1 + \lambda v_{DS})$$

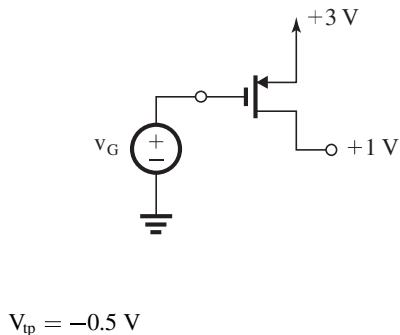
$$3 = \frac{1}{2} k_p [-3 - (-0.8)]^2 (1 - 0.025 \times -4)$$

$$\Rightarrow k_p = 1.137 mA/V^2$$

$$5.36 \quad PMOS \text{ with } V_{tp} = -1 V$$

Case	$V_S$	$V_G$	$V_D$	$V_{SG}$	$ V_{ov} $	$V_{SD}$	Region of operation
a	+2	+2	0	0	0	2	Cutoff
b	+2	+1	0	+1	0	2	Cutoff-Sat.
c	+2	0	0	+2	1	2	Sat.
d	+2	0	+1	+2	1	1	Sat-Triode
e	+2	0	+1.5	+2	1	0.5	Triode
f	+2	0	+2	+2	1	0	Triode

5.37



As  $v_G$  reaches  $+2.5 \text{ V}$ , the transistor begins to conduct and enters the saturation region, since  $v_{DG}$  will be negative. The transistor continues to operate in the saturation region until  $v_G$  reaches  $0.5 \text{ V}$ , at which point  $v_{DG}$  will be  $0.5 \text{ V}$ , which is equal to  $|V_{tp}|$ , and the transistor enters the triode region. As  $v_G$  goes below  $0.5 \text{ V}$ , the transistor continues to operate in the triode region.

5.38 Case a, assume, sat,

$$\frac{(1 - V_t)^2}{(1.5 - V_t)^2} = \frac{100}{400} \Rightarrow V_t = 0.5,$$

$$V_{GD} \leq V_t$$

 $\therefore$  sat;Case b same procedure, except use  $V_{SG}$  and  $V_{SD}$ .

$$\frac{(2 - 1V_t1)^2}{(3 - 1V_t1)^2} = \frac{50}{450} \Rightarrow |V_t| = 1.5,$$

$$V_{GD} \geq -1.5 \text{ V} \quad \therefore \text{sat}$$

$$\text{Case c} - \frac{(2 - |V_t|)^2}{(3 - |V_t|)^2} = \frac{200}{800} \Rightarrow |V_t| = 1.0,$$

$$V_{GD} \geq -1.0 \text{ V} \quad \therefore \text{sat}$$

Case d

$$\frac{\text{sat}}{\text{triode}} \frac{\frac{1}{2}k_n(2 - V_t)^2}{k_n \left[ (4 - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right]} = \frac{72}{270}$$

(after failing assumption that both cases are sat.)

5.39 Refer to the circuits in Fig. P5.39.

$$(a) V_1 = V_{DS} = V_{GS} = 1 \text{ V}$$

$$(b) V_2 = +1 - V_{DS} = 1 - 1 = 0 \text{ V}$$

$$(c) V_3 = V_{SD} = V_{SG} = 1 \text{ V}$$

$$(d) V_4 = +1.25 - V_{SG} = 1.25 - 1 = 0.25 \text{ V}$$

Now place a resistor  $R$  in series with the drain. For the circuits in (a) and (b) to remain in saturation,  $V_D$  must not fall below  $V_G$  by more than  $V_t$ . Thus,

$$IR \leq V_t$$

$$R_{\max} = \frac{V_t}{I} = \frac{0.5}{0.1} = 5 \text{ k}\Omega$$

For the circuits in (c) and (d) to remain in saturation,  $V_D$  must not exceed  $V_G$  by more than  $|V_t|$ . Thus

This table belongs to 5.38.

Case	Transistor	$V_S$ (V)	$V_G$ (V)	$V_D$ (V)	$I_D$ ( $\mu\text{A}$ )	Type	Mode	$\mu C_{ox} \frac{W}{L}$ ( $\mu \text{ A/V}^2$ )	$V_t$ (V)
a	1	0	1	2.5	100	NMOS	Sat.	800	0.5
		0	1.5	2.5	400		Sat.		
b	2	5	3	-4.5	50	PMOS	Sat.	400	-1.5
		5	2	-0.5	450		Sat.		
c	3	5	3	4	200	PMOS	Sat.	400	-1
		5	2	0	800		Sat.		
d	4	-2	0	0	72	NMOS	Sat.	100	+0.8
		-4	0	-3	270		Triode		

$$IR \leq |V_t|$$

which yields  $R_{\max} = 5 \text{ k}\Omega$ .

Now place a resistor  $R_S$  in series with the MOSFET source. The voltage across the current source becomes

$$(a) V_{CS} = 2.5 - V_{DS} - IR_S \quad (1)$$

To keep  $V_{CS}$  at least at 0.5 V, the maximum  $R_S$  can be found from

$$0.5 = 2.5 - 1 - 0.1 \times R_{S\max}$$

$$\Rightarrow R_{S\max} = 10 \text{ k}\Omega$$

$$V_I = 2.5 - 0.5 = 2 \text{ V}$$

$$(b) V_{CS} = 1 - V_{DS} - IR_S - (-1.5)$$

$$= 2.5 - V_{DS} - IR_S$$

which is identical to Eq. (1). Thus

$$R_{S\max} = 10 \text{ k}\Omega$$

$$V_2 = -1.5 + 0.5 = -1 \text{ V}$$

$$(c) V_{CS} = 2.5 - IR_S - V_{SD}$$

which yields

$$R_{S\max} = 10 \text{ k}\Omega$$

$$V_3 = 2.5 - 0.5 = 2 \text{ V}$$

$$(d) V_{CS} = 1.25 - IR_S - V_{SD} - (-1.25)$$

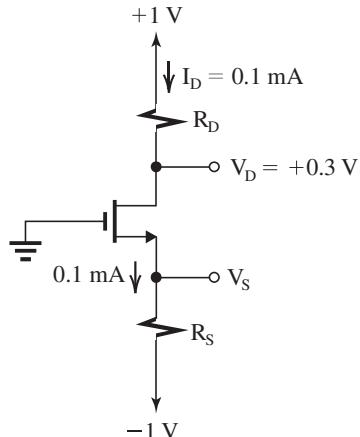
$$= 2.5 - V_{SD} - IR_S$$

which yields

$$R_{S\max} = 10 \text{ k}\Omega$$

$$V_4 = -1.25 + 0.5 = -0.75 \text{ V}$$

5.40



Since  $V_{DG} > 0$ , the MOSFET is in saturation.

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.4 \times \frac{5}{0.4} \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

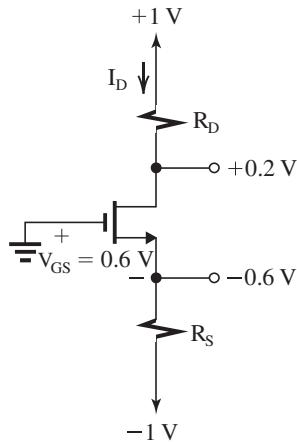
$$V_{GS} = V_t + V_{OV} = 0.5 + 0.2 = 0.7 \text{ V}$$

$$V_S = 0 - V_{GS} = -0.7 \text{ V}$$

$$R_S = \frac{V_S - (-1)}{I_D} = \frac{-0.7 + 1}{0.1} = 3 \text{ k}\Omega$$

$$R_D = \frac{1 - V_D}{I_D} = \frac{1 - 0.3}{0.1} = \frac{0.7}{0.1} = 7 \text{ k}\Omega$$

5.41



Since  $V_{DG} > 0$ , the MOSFET is operating in saturation. Thus

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 4 \times (0.6 - 0.4)^2$$

$$= 0.08 \text{ mA}$$

$$R_D = \frac{1 - V_D}{I_D} = \frac{1 - 0.2}{0.08} = \frac{0.8}{0.08} = 10 \text{ k}\Omega$$

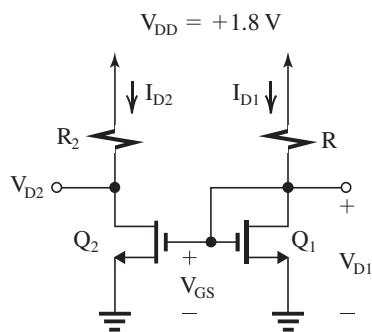
$$R_S = \frac{-0.6 - (-1)}{I_D} = \frac{-0.6 + 1}{0.08} = 5 \text{ k}\Omega$$

For  $I_D$  to remain unchanged from 0.08 mA, the MOSFET must remain in saturation. This in turn can be achieved by ensuring that  $V_D$  does not fall below  $V_G$  (which is zero) by more than  $V_t$  (0.4 V). Thus

$$1 - I_D R_{D\max} = -0.4$$

$$R_{D\max} = \frac{1.4}{0.08} = 17.5 \text{ k}\Omega$$

5.42



$$(a) I_{D1} = 50 \mu A$$

$$0.05 = \frac{1}{2} \times 0.4 \times \frac{1.44}{0.36} V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.25 V$$

$$V_{GS1} = V_t + V_{OV}$$

$$= 0.5 + 0.25 = 0.75 V$$

$$V_{DI} = V_{GS1} = 0.75 V$$

$$R = \frac{V_{DD} - V_{DI}}{I_{D1}} = \frac{1.8 - 0.75}{0.05} = 21 k\Omega$$

(b) Note that both transistors operate at the same  $V_{GS}$  and  $V_{OV}$ , and

$$I_{D2} = 0.5 mA$$

But

$$I_{D2} = \frac{1}{2} k_n \left( \frac{W_2}{L_2} \right) V_{OV}^2$$

$$0.5 = \frac{1}{2} \times 0.4 \times \frac{W_2}{0.36} \times 0.25^2$$

$$\Rightarrow W_2 = 14.4 \mu m$$

which is 10 times  $W_1$ , as needed to provide  $I_{D2} = 10I_{D1}$ . Since  $Q_2$  is to operate at the edge of saturation,

$$V_{DS2} = V_{OV}$$

Thus,

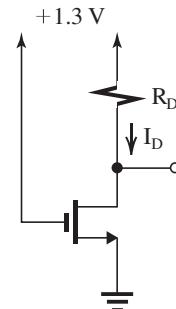
$$V_{D2} = 0.25 V$$

and

$$R_2 = \frac{V_{DD} - V_{D2}}{I_{D2}}$$

$$= \frac{1.8 - 0.25}{0.5} = 3.1 k\Omega$$

5.43



$$I_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 0.4 \times \frac{W}{L} (1.3 - 0.4)^2$$

$$= 0.162 \left( \frac{W}{L} \right)$$

$$V_D = 1.3 - I_D R_D = 1.3 - 0.162 \left( \frac{W}{L} \right) R_D$$

For the MOSFET to be at the edge of saturation, we must have

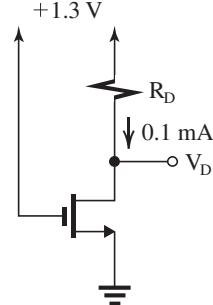
$$V_D = V_{OV} = 1.3 - 0.4 = 0.9$$

Thus

$$0.9 = 1.3 - 0.162 \left( \frac{W}{L} \right) R_D$$

$$\Rightarrow \left( \frac{W}{L} \right) R_D \simeq 2.5 k\Omega \quad Q.E.D$$

5.44



$$V_{OV} = V_{GS} - V_t$$

$$= 1.3 - 0.4 = 0.9$$

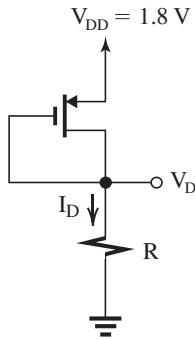
To operate at the edge of saturation, we must have

$$V_D = V_{OV} = 0.9 V$$

Thus,

$$R_D = \frac{1.3 - 0.9}{0.1} = 4 k\Omega$$

5.45



$$I_D = 180 \mu A \quad \text{and} \quad V_D = 1 V$$

$$R = \frac{V_D}{I_D} = \frac{1}{0.18} = 5.6 k\Omega$$

Transistor is operating in saturation with  $|V_{ov}| = 1.8 - V_D - |V_t| = 1.8 - 1 - 0.5 = 0.3 V$ :

$$I_D = \frac{1}{2} k'_p \frac{W}{L} |V_{ov}|^2$$

$$180 = \frac{1}{2} \times 100 \times \frac{W}{L} \times 0.3^2$$

$$\Rightarrow \frac{W}{L} = 40$$

$$W = 40 \times 0.18 = 7.2 \mu m$$

5.46 Refer to Fig. P5.46. Both  $Q_1$  and  $Q_2$  are operating in saturation at  $I_D = 0.5 mA$ . For  $Q_1$ ,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W_1}{L_1} V_{OV1}^2$$

$$0.5 = \frac{1}{2} \times 0.25 \times \frac{W_1}{L_1} (1 - 0.5)^2$$

$$\Rightarrow \frac{W_1}{L_1} = 16$$

$$W_1 = 16 \times 0.25 = 4 \mu m$$

For  $Q_2$ , we have

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W_2}{L_2} \right) V_{OV2}^2$$

$$0.5 = \frac{1}{2} \times 0.25 \times \frac{W_2}{L_2} (1.8 - 1 - 0.5)^2$$

$$\Rightarrow \frac{W_2}{L_2} = 44.4$$

$$W_2 = 44.4 \times 0.25 = 11.1$$

$$R = \frac{2.5 - 1.8}{0.5} = 1.4 k\Omega$$

5.47 Refer to the circuit in Fig. P5.47. All three transistors are operating in saturation with  $I_D = 90 \mu A$ . For  $Q_1$ ,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W_1}{L_1} (V_{GS1} - V_t)^2$$

$$90 = \frac{1}{2} \times 90 \times \frac{W_1}{L_1} (0.8 - 0.5)^2$$

$$\Rightarrow \frac{W_1}{L_1} = 22.2$$

$$W_1 = 22.2 \times 0.5 = 11.1 \mu m$$

For  $Q_2$ ,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W_2}{L_2} (V_{GS2} - V_t)^2$$

$$90 = \frac{1}{2} \times 90 \times \frac{W_2}{L_2} (1.5 - 0.8 - 0.5)^2$$

$$\Rightarrow \frac{W_2}{L_2} = 50$$

$$W_2 = 50 \times 0.5 = 25 \mu m$$

For  $Q_3$ ,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W_3}{L_3} (V_{GS3} - V_t)^2$$

$$90 = \frac{1}{2} \times 90 \times \frac{W_3}{L_3} (2.5 - 1.5 - 0.5)^2$$

$$\Rightarrow \frac{W_3}{L_3} = 8$$

$$W_3 = 8 \times 0.5 = 4 \mu m$$

5.48 Refer to the circuits in Fig. 5.24 (page 338):

$$V_{GS} = 5 - 6I_D$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 1.5 \times (5 - 6I_D - 1.5)^2$$

which results in the following quadratic equation in  $I_D$ :

$$36I_D^2 - 43.33I_D + 12.25 = 0$$

The physically meaningful root is

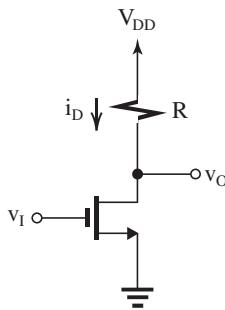
$$I_D = 0.45 mA$$

This should be compared to the value of 0.5 mA found in Example 5.6. The difference of about 10% is relatively small, given the large variations in  $k_n$  and  $V_t$  (50% increase in each). The new value of  $V_D$  is

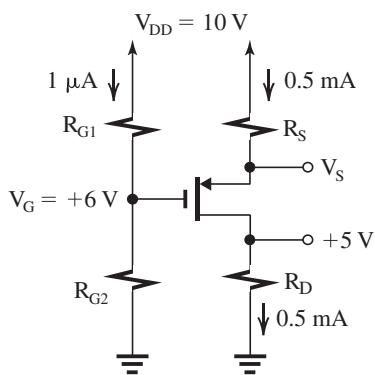
$$V_D = V_{DD} - R_D I_D = 10 - 6 \times 0.45 = +7.3 \text{ V}$$

as compared to +7 V found in Example 5.6. We conclude that this circuit is quite tolerant to variations in device parameters.

5.50



5.49



Refer to the circuit in the figure above,

$$R_{G1} = \frac{V_{DD} - V_G}{1 \mu\text{A}}$$

$$= \frac{10 - 6}{1} = 4 \text{ M}\Omega$$

$$R_{G2} = \frac{6}{1 \mu\text{A}} = 6 \text{ M}\Omega$$

$$R_D = \frac{5 \text{ V}}{0.5 \text{ mA}} = 10 \text{ k}\Omega$$

To determine  $V_S$ , we use

$$I_D = \frac{1}{2} k_p \left( \frac{W}{L} \right) (V_{SG} - |V_t|)^2$$

$$0.5 = \frac{1}{2} \times 4 \times (V_{SG} - 1.5)^2$$

$$\Rightarrow V_{SG} = 2 \text{ V}$$

Thus,

$$V_S = V_G + V_{SG} = 6 + 2 = 8 \text{ V}$$

$$R_S = \frac{10 - 8}{0.5} = 4 \text{ k}\Omega$$

Assuming linear operation in the triode region, we can write

$$i_D = \frac{v_O}{r_{DS}} = \frac{50 \text{ mV}}{50 \Omega} = 1 \text{ mA}$$

$$i_D = k'_n \left( \frac{W}{L} \right) (v_{GS} - V_t) v_{DS}$$

$$1 = 0.5 \times \frac{W}{L} \times (1.3 - 0.4) \times 0.05$$

$$\Rightarrow \frac{W}{L} = 44.4$$

$$R = \frac{V_{DD} - v_O}{i_D} = \frac{1.3 - 0.05}{1}$$

$$= 1.25 \text{ k}\Omega$$

5.51 (a) Refer to Fig. P5.51(a): Assuming saturation-mode operation, we have

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$2 = \frac{1}{2} \times 4 V_{OV}^2$$

$$\Rightarrow V_{OV} = 1 \text{ V}$$

$$V_{GS} = |V_t| + V_{OV} = 1 + 1 = 2 \text{ V}$$

$$V_1 = 0 - V_{GS} = -2 \text{ V}$$

$$V_2 = 5 - 2 \times 2 = +1 \text{ V}$$

Since  $V_{DG} = +1 \text{ V}$ , the MOSFET is indeed in saturation.

Refer to Fig. P5.51(b): The transistor is operating in saturation, thus

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$2 = \frac{1}{2} \times 4 \times V_{OV}^2 \Rightarrow V_{OV} = 1 \text{ V}$$

$$V_{GS} = 2 \text{ V}$$

$$\Rightarrow V_3 = 2 \text{ V}$$

Refer to Fig. P5.51(c): Assuming saturation-mode operation, we have

$$I_D = \frac{1}{2} k_p |V_{ov}|^2$$

$$2 = \frac{1}{2} \times 4 \times |V_{ov}|^2$$

$$\Rightarrow |V_{ov}| = 1 \text{ V}$$

$$V_{SG} = |V_t| + |V_{ov}| = 1 + 1 = 2 \text{ V}$$

$$V_4 = V_{SG} = 2 \text{ V}$$

$$V_5 = -5 + I_D \times 1.5$$

$$= -5 + 2 \times 1.5 = -2 \text{ V}$$

Since  $V_{DG} < 0$ , the MOSFET is indeed in saturation.

Refer to Fig. P5.51(d): Both transistors are operating in saturation at equal  $|V_{ov}|$ . Thus

$$2 = \frac{1}{2} \times 4 \times |V_{ov}|^2 \Rightarrow |V_{ov}| = 1 \text{ V}$$

$$V_{SG} = |V_t| + |V_{ov}| = 2 \text{ V}$$

$$V_6 = 5 - V_{SG} = 5 - 2 = 3 \text{ V}$$

$$V_7 = +5 - 2 \text{ V}_{SG} = 5 - 2 \times 2 = 1 \text{ V}$$

(b) Circuit (a): The 2-mA current source can be replaced with a resistance  $R$  connected between the MOSFET source and the  $-5\text{-V}$  supply with

$$R = \frac{V_1 - (-5)}{2 \text{ mA}} = \frac{-2 + 5}{2} = 1.5 \text{ k}\Omega$$

Circuit (b): The 2-mA current source can be replaced with a resistance  $R$ ,

$$R = \frac{5 - V_3}{2 \text{ mA}} = \frac{5 - 2}{2} = 1.5 \text{ k}\Omega$$

Circuit (c): The 2-mA current source can be replaced with a resistance  $R$ ,

$$R = \frac{5 - V_4}{2 \text{ mA}} = \frac{5 - 2}{2} = 1.5 \text{ k}\Omega$$

Circuit (d): The 2-mA current source can be replaced with a resistance  $R$ ,

$$R = \frac{V_7}{2 \text{ mA}} = \frac{1}{2} = 0.5 \text{ k}\Omega$$

We use the nearest 1% resistor, which is  $499 \Omega$ .

5.52 (a) Refer to Fig. P5.52(a): The MOSFET is operating in saturation. Thus

$$I_D = \frac{1}{2} k_n V_{ov}^2$$

$$10 = \frac{1}{2} \times 500 \times V_{ov}^2 \Rightarrow V_{ov} = 0.2 \text{ V}$$

$$V_{GS} = V_t + V_{ov} = 0.8 + 0.2 = 1 \text{ V}$$

$$V_1 = 0 - V_{GS} = -1 \text{ V}$$

(b) Refer to Fig. P5.52(b): The MOSFET is operating in saturation. Thus

$$100 = \frac{1}{2} \times 500 \times V_{ov}^2 \Rightarrow V_{ov} = 0.63 \text{ V}$$

$$V_{GS} = 0.8 + 0.63 = 1.43 \text{ V}$$

$$V_2 = -1.43 \text{ V}$$

(c) Refer to Fig. P5.52(c). The MOSFET is operating in saturation. Thus

$$1 = \frac{1}{2} \times 0.5 \times V_{ov}^2 \Rightarrow V_{ov} = 2 \text{ V}$$

$$V_{GS} = 0.8 + 2 = 2.8 \text{ V}$$

$$V_3 = -2.8 \text{ V}$$

(d) Refer to Fig. P5.52(d). The MOSFET is operating in saturation. Thus

$$10 = \frac{1}{2} \times 500 \times V_{ov}^2 \Rightarrow V_{ov} = 0.2 \text{ V}$$

$$V_{GS} = 0.8 + 0.2 = 1 \text{ V}$$

$$V_4 = 1 \text{ V}$$

(e) Refer to Fig. P5.52(e). The MOSFET is operating in saturation. Thus

$$1 = \frac{1}{2} \times 0.5 \times V_{ov}^2 \Rightarrow V_{ov} = 2 \text{ V}$$

$$V_{GS} = 0.8 + 2 = 2.8 \text{ V}$$

$$V_5 = V_{GS} = 2.8 \text{ V}$$

(f) Refer to Fig. P5.52(f). To simplify our solution, we observe that this circuit is that in Fig. P5.56(d) with the  $10\text{-}\mu\text{A}$  current source replaced with a  $400\text{-k}\Omega$  resistor. Thus  $V_G = V_4 = +1 \text{ V}$  and, as a check,  $I_D = \frac{5 - 1}{400} = 0.01 \text{ mA} = 10 \mu\text{A}$ .

(g) Refer to Fig. P5.52(g). Our work is considerably simplified by observing that this circuit is similar to that in Fig. P5.52(e) with the

1-mA current source replaced with a 2.2-k $\Omega$  resistor. Thus  $V_7 = V_5 = 2.8$  V and, as a check,  $I_D = \frac{5 - 2.8}{2.2} = 1$  mA.

(h) Refer to Fig. P5.52(h). Our work is considerably simplified by observing that this circuit is similar to that in Fig. P5.52(a) with the 10- $\mu$ A current source replaced with a 400-k $\Omega$  resistor. Thus  $V_8 = V_1 = -1$  V and, as a check,  $I_D = \frac{-1 + 5}{400} = 0.01$  mA = 10  $\mu$ A.

5.53 (a) Refer to the circuit in Fig. P5.53(a). Transistor  $Q_1$  is operating in saturation. Assume that  $Q_2$  also is operating in saturation,

$$V_{GS2} = 0 - V_2 = -V_2$$

and

$$V_2 = -2.5 + I_D \times 1$$

$$\Rightarrow I_D = V_2 + 2.5$$

Now,

$$I_D = \frac{1}{2}k_n(V_{GS2} - V_t)^2$$

Substituting  $I_D = V_2 + 2.5$  and  $V_{GS2} = -V_2$ ,

$$V_2 + 2.5 = \frac{1}{2} \times 1.5(-V_2 - 0.9)^2$$

$$\frac{2}{1.5}(V_2 + 2.5) = V_2^2 + 1.8V_2 + 0.81$$

$$V_2^2 + 0.467V_2 - 2.523 = 0$$

$$\Rightarrow V_2 = -1.84$$

Thus,

$$I_D = V_2 + 2.5 = -1.84 + 2.5 = 0.66$$

and

$$V_{GS2} = 1.84$$

Since  $Q_1$  is identical to  $Q_2$  and is conducting the same  $I_D$ , then

$$V_{GS1} = 1.84$$

$$\Rightarrow V_1 = 2.5 - 1.84 = 0.66$$

which confirms that  $Q_1$  is operating in saturation, as assumed.

(b) Refer to the circuit in Fig. P5.53(b). From symmetry, we see that

$$V_4 = 2.5$$

Now, compare the part of the circuit consisting of  $Q_2$  and the 1-k $\Omega$  resistor. We observe the similarity of this part with the circuit between the gate of  $Q_2$  and ground in Fig. P5.53(a). It follows that for the circuit in Fig. P5.53(b), we can use the solution of part (a) above to write

$$I_{D2} = 0.66 \text{ mA} \quad \text{and} \quad V_{GS2} = 1.84 \text{ V}$$

Thus,

$$V_5 = V_4 - V_{GS2} = 2.5 - 1.84 = 0.66$$

Since  $Q_1$  is conducting an equal  $I_D$  and has the same  $V_{GS}$ ,

$$I_{D1} = 0.66 \text{ mA} \quad \text{and} \quad V_{GS1} = 1.84 \text{ V}$$

$$\Rightarrow V_3 = V_4 + V_{GS1} = 2.5 + 1.84 = 3.34 \text{ V}$$

We could, of course, have used the circuit symmetry, observed earlier, to write this final result.

5.54

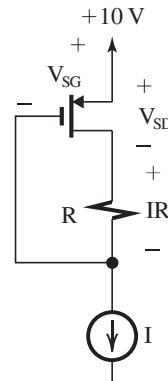


Fig. 1

(a) From Fig. 1 we see that

$$V_{DG} = IR$$

Since for the PMOS transistor to operate in saturation,

$$V_{DG} \leq |V_{tp}|$$

It follows the

$$IR \leq |V_{tp}| \quad \text{Q.E.D}$$

(b) (i)  $R = 0$ , the condition above is satisfied and

$$I_D = I = \frac{1}{2} k_p |V_{ov}|^2$$

$$0.1 = \frac{1}{2} \times 0.2 \times |V_{ov}|^2$$

$$\Rightarrow |V_{ov}| = 1 \text{ V}$$

$$V_{SG} = |V_{tp}| + |V_{ov}| = 1 + 1 = 2 \text{ V}$$

$$V_G = 10 - 2 = 8 \text{ V}$$

$$V_D = V_G = 8 \text{ V}$$

$$V_{SD} = 2 \text{ V}$$

(ii)  $R = 10 \text{ k}\Omega$

$$IR = 0.1 \times 10 = 1 \text{ V}$$

which just satisfies the condition for saturation-mode operation in (a) above.

Obviously  $I_D$  and  $|V_{ov}|$  will be the same as in (i) above.

$$V_{SG} = 2 \text{ V}$$

$$V_G = 8 \text{ V}$$

$$V_D = V_G + IR = 8 + 1 = 9 \text{ V}$$

$$V_{SD} = 1 \text{ V}$$

(iii)  $R = 30 \text{ k}\Omega$

$$IR = 0.1 \times 30 = 3 \text{ V}$$

which is greater than  $|V_{tp}|$ . Thus the condition in (a) above is not satisfied and the MOSFET is operating in the triode region. From Fig. 2,

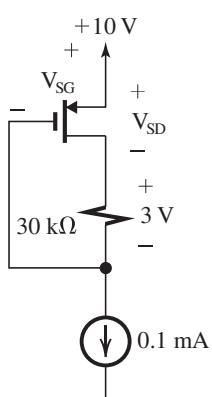


Fig. 2

From Fig. 2, we see that

$$V_{SD} = V_{SG} - 3$$

Now, for triode-mode operation,

$$I_D = k_p \left[ (V_{SG} - |V_{tp}|)V_{SD} - \frac{1}{2}V_{SD}^2 \right]$$

$$0.1 = 0.2 \left[ (V_{SG} - 1)(V_{SG} - 3) - \frac{1}{2}(V_{SG} - 3)^2 \right]$$

$$\Rightarrow V_{SG}^2 - 2V_{SG} - 4 = 0$$

$$\Rightarrow V_{SG} = 3.24 \text{ V}$$

$$V_{SD} = V_{SG} - 3 = 0.24 \text{ V}$$

(iv)  $R = 100 \text{ k}\Omega$

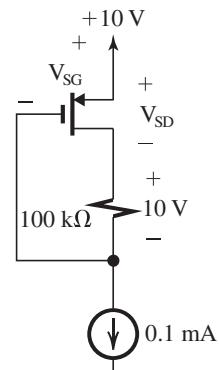


Fig. 3

Here also (see Fig. 3) the MOSFET will be operating in the triode region, and

$$V_{SD} = V_{SG} - 10 \text{ V}$$

Since we expect  $V_{SD}$  to be very small, we can neglect the  $V_{SD}^2$  term in the expression for  $I_D$  and write

$$I_D \simeq k_p (V_{SG} - |V_{tp}|)V_{SD}$$

$$0.1 = 0.2(V_{SG} - 1)(V_{SG} - 10)$$

$$\Rightarrow V_{SG}^2 - 11V_{SG} + 9.5 = 0$$

$$\Rightarrow V_{SG} = 10.055 \text{ V}$$

$$V_{SD} = V_{SG} - 10 = 0.055 \text{ V}$$

5.55 (a) Refer to the circuit in Fig. P5.55(a). Since the two NMOS transistors are identical and have the same  $I_D$ , their  $V_{GS}$  values will be equal. Thus

$$V_{GS} = \frac{3}{2} = 1.5 \text{ V}$$

$$V_2 = 1.5 \text{ V}$$

$$V_{OV} = V_{GS} - V_t = 1.0 \text{ V}$$

$$I_1 = I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$= \frac{1}{2} \times 270 \times \frac{3}{1} \times 1$$

$$= 405 \mu\text{A}$$

(b) Refer to the circuit in Fig. P5.55(b). Here  $Q_N$  and  $Q_P$  have the same  $I_D = I_3$ . Thus

$$I_3 = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OVN}^2 \quad (1)$$

$$I_3 = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right) V_{OVP}^2 \quad (2)$$

Equating Eqs. (1) and (2) and using  $\mu_n C_{ox} = 3\mu_p C_{ox}$  gives  $3V_{OVN}^2 = V_{OVP}^2$ :

$$|V_{OVP}| = \sqrt{3} V_{OVN}$$

Now,

$$V_{GSN} = V_{OVN} + V_t = V_{OVN} + 0.5$$

$$V_{SGP} = |V_{OVP}| + |V_t| = \sqrt{3} V_{OVN} + 0.5$$

But

$$V_{SGP} + V_{GSN} = 3$$

$$(\sqrt{3} + 1)V_{OVN} + 1 = 3$$

$$\Rightarrow V_{OVN} = 0.732 \text{ V}$$

$$V_{OVP} = 1.268 \text{ V}$$

$$V_{GSN} = 1.232 \text{ V}$$

$$V_{SGP} = 1.768 \text{ V}$$

$$V_4 = V_{GSN} = 1.232 \text{ V}$$

$$I_3 = \frac{1}{2} \times 270 \times \frac{3}{1} \times 0.732^2 = 217 \mu\text{A}$$

(c) Refer to Fig. P5.55(c). Here the width of the PMOS transistor is made 3 times larger than that of the NMOS transistor. This compensates for the factor 3 in the process transconductance parameter, resulting in  $k_p = k_n$ , and the two transistors are matched. The solution will be identical to that for (a) above with

$$V_5 = \frac{3}{2} = 1.5 \text{ V}$$

$$I_6 = 405 \mu\text{A}$$

5.56 Refer to the circuit in Fig. P5.56. First consider  $Q_1$  and  $Q_2$ . Both are operating in saturation and since they are identical, they have equal  $V_{GS}$ :

$$V_{GS1} = V_{GS2} = \frac{5}{2} = 2.5 \text{ V}$$

Thus,

$$\begin{aligned} I_{D2} &= I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS1} - V_t)^2 \\ &= \frac{1}{2} \times 50 \times \frac{10}{1} (2.5 - 1)^2 \\ &= 562.5 \mu\text{A} \end{aligned}$$

Now,  $Q_3$  has the same  $V_{GS}$  at  $Q_1$  and is matched to  $Q_1$ . Thus if we assume that  $Q_3$  is operating in saturation, we have

$$I_{D3} = I_{D1} = 562.5 \mu\text{A}$$

Thus,

$$I_2 = 562.5 \mu\text{A}$$

This is the same current that flows through  $Q_4$ , which is operating in saturation and is matched to  $Q_3$ . Thus

$$V_{GS4} = V_{GS3} = V_{GS1} = 2.5 \text{ V}$$

Thus,

$$V_2 = 5 - V_{GS4} = 2.5 \text{ V}$$

This is equal to the voltage at the gate of  $Q_3$ ; thus  $Q_3$  is indeed operating in saturation, as assumed.

If  $Q_3$  and  $Q_4$  have  $W = 100 \mu\text{m}$ , nothing changes for  $Q_1$  and  $Q_2$ . However,  $Q_3$ , which has the same  $V_{GS}$  as  $Q_1$  but has 10 times the width, will have a drain current 10 times larger than  $Q_1$ .

Thus

$$I_{D2} = I_{D3} = 10 I_{D1} = 10 \times 562.5 \mu\text{A}$$

$$= 5.625 \text{ mA}$$

Transistor  $Q_4$  will carry  $I_2$  but will retain the same  $V_{GS}$  as before, thus  $V_2$  remains unchanged at 2.5 V.

5.57 Refer to the circuit in Fig. P5.57.

(a)  $Q_1$  and  $Q_2$  are matched. Thus, from symmetry, we see that the 200- $\mu\text{A}$  current will split equally between  $Q_1$  and  $Q_2$ :

$$I_{D1} = I_{D2} = 100 \mu\text{A}$$

$$V_1 = V_2 = 2.5 - 0.1 \times 20 = 0.5 \text{ V}$$

To find  $V_3$ , we determine  $V_{GS}$  of either  $Q_1$  and  $Q_2$  (which, of course, are equal),

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{GS} - V_t)^2$$

$$100 = \frac{1}{2} \times 125 \times 20 \times (V_{GS} - 0.7)^2$$

$$\Rightarrow V_{GS} = 0.983 \text{ V}$$

Thus,

$$V_3 = -0.983 \text{ V}$$

(b) With  $V_{GS1} = V_{GS2}$ , but  $(W/L)_1 = 1.5(W/L)_2$ , transistor  $Q_1$  will carry a current 1.5 times that in  $Q_2$ , that is,

$$I_{D1} = 1.5 I_{D2}$$

But,

$$I_{D1} + I_{D2} = 200 \mu\text{A}$$

Thus

$$I_{D1} = 120 \mu\text{A}$$

$$I_{D2} = 80 \mu\text{A}$$

$$V_1 = 2.5 - 0.12 \times 20 = 0.1 \text{ V}$$

$$V_2 = 2.5 - 0.08 \times 20 = 0.9 \text{ V}$$

To find  $V_3$ , we find  $V_{GS}$  from the  $I_D$  equation for either  $Q_1$  or  $Q_2$ ,

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{GS} - V_t)^2$$

$$120 = \frac{1}{2} \times 125 \times 20 \times (V_{GS} - 0.7)^2$$

$$\Rightarrow V_{GS} = 1.01 \text{ V}$$

$$V_3 = -1.01 \text{ V}$$

5.58 Using Eq. (5.30), we can write

$$V_t = V_{t0} + \gamma [\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}]$$

where

$$V_{t0} = 1.0 \text{ V}$$

$$\gamma = 0.5 \text{ V}^{1/2}$$

$$2\phi_f = 0.6 \text{ V}$$

and

$$V_{SB} = 0 \text{ to } 4 \text{ V}$$

At

$$V_{SB} = 0, \quad V_t = V_{t0} = 1.0 \text{ V}$$

At

$$V_{SB} = 4 \text{ V},$$

$$V_t = 1 + 0.5[\sqrt{0.6 + 4} - \sqrt{0.6}]$$

$$= 1.69 \text{ V}$$

If the gate oxide thickness is increased by a factor of 4,  $C_{ox}$  will decrease by a factor of 4 and Eq. (5.31) indicates that  $\gamma$  will increase by a factor of 4, becoming 2. Thus at  $V_{SB} = 4 \text{ V}$ ,

$$V_t = 1 + 2[\sqrt{0.6 + 4} - \sqrt{0.6}]$$

$$= 3.74 \text{ V}$$

$$5.59 |V_t| = |V_{t0}| + \gamma [\sqrt{2\phi_f + |V_{SB}|} - \sqrt{2\phi_f}]$$

$$= 0.7 + 0.5[\sqrt{0.75 + 3} - \sqrt{0.75}]$$

$$= 1.24 \text{ V}$$

Thus,

$$V_t = -1.24 \text{ V}$$

$$5.60 \text{ (a)} i_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (V_{GS} - V_t)^2$$

$$\frac{\partial i_D}{\partial T} = \frac{1}{2} \frac{\partial k'_n}{\partial T} \left( \frac{W}{L} \right) (V_{GS} - V_t)^2$$

$$-k'_n \left( \frac{W}{L} \right) (V_{GS} - V_t) \frac{\partial V_t}{\partial T}$$

$$\frac{\partial i_D / i_D}{\partial T} = \frac{\partial k'_n / k'_n}{\partial T} - \frac{2}{V_{GS} - V_t} \frac{\partial V_t}{\partial T}$$

For

$$\frac{\partial V_t}{\partial T} = -2 \text{ mV/}^\circ\text{C} = -0.002 \text{ V/}^\circ\text{C}$$

and

$$\frac{\partial i_D / i_D}{\partial T} = -0.002/^\circ\text{C}, \quad V_{GS} = 5 \text{ V}$$

and

$$V_t = 1 \text{ V}$$

$$-0.002 = \frac{\partial k'_n / k'_n}{\partial T} - \frac{2 \times -0.002}{5 - 1}$$

$$\Rightarrow \frac{\partial k'_n/k'_n}{\partial T} = -0.003/\text{°C}$$

or  $-0.3\%/\text{°C}$

$$5.61 \quad i_D = k_n \left[ (v_{GS} - V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2 \right],$$

for  $v_{DS} \leq v_{GS} - V_{tn}$

$$i_D = \frac{1}{2}k_n(v_{GS} - V_{tn})^2(1 + \lambda v_{DS}),$$

for  $v_{DS} \geq v_{GS} - V_{tn}$

For our case,

$$V_{tn} = -2 \text{ V}, \quad k_n = 0.2 \text{ mA/V}^2, \quad \lambda = 0.02 \text{ V}^{-1}$$

and  $v_{GS} = 0$ . Thus

$$i_D = 0.2 \left( 2v_{DS} - \frac{1}{2}v_{DS}^2 \right), \quad \text{for } v_{DS} \leq 2 \text{ V}$$

$$i_D = 0.4(1 + 0.02 v_{DS}), \quad \text{for } v_{DS} \geq 2 \text{ V}$$

For  $v_{DS} = 1 \text{ V}$ ,

$$i_D = 0.2 \left( 2 - \frac{1}{2} \right) = 0.3 \text{ mA}$$

For  $v_{DS} = 2 \text{ V}$ ,

$$i_D = 0.4(1 + 0.02 \times 2) = 0.416 \text{ mA}$$

For  $v_{DS} = 3 \text{ V}$ ,

$$i_D = 0.4(1 + 0.02 \times 3) = 0.424 \text{ mA}$$

For  $v_{DS} = 10 \text{ V}$ ,

$$i_D = 0.4(1 + 0.02 \times 10) = 0.48 \text{ mA}$$

If the device width  $W$  is doubled,  $k_n$  is doubled, and each of the currents above will be doubled. If both  $W$  and  $L$  are doubled,  $k_n$  remains unchanged. However,  $\lambda$  is divided in half; thus for  $v_{DS} = 2 \text{ V}$ ,  $i_D$  becomes  $0.408 \text{ mA}$ ; for  $v_{DS} = 3 \text{ V}$ ,  $i_D$  becomes  $0.412 \text{ mA}$ ; and for  $v_{DS} = 10 \text{ V}$ ,  $i_D$  becomes  $0.44 \text{ mA}$ .

### Exercise 6-1

Ex: 6.1 Refer to Fig. 6.2(a) and 6.2(b).

Coordinates of point A:  $V_t$  and  $V_{DD}$ ; thus 0.4 V and 1.8 V. To determine the coordinates of point B, we use Eqs. (6.7) and (6.8) as follows:

$$\begin{aligned} V_{OV}|_B &= \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} \\ &= \frac{\sqrt{2 \times 4 \times 17.5 \times 1.8 + 1} - 1}{4 \times 17.5} \\ &= 0.213 \text{ V} \end{aligned}$$

Thus,

$$V_{GS}|_B = V_t + V_{OV}|_B = 0.4 + 0.213 = 0.613 \text{ V}$$

and

$$V_{DS}|_B = V_{OV}|_B = 0.213 \text{ V}$$

Thus, coordinates of B are 0.613 V and 0.213 V. At point C, the MOSFET is operating in the triode region, thus

$$i_D = k_n \left[ (V_{GS}|_C - V_t) v_{DS}|_C - \frac{1}{2} v_{DS}^2 |_C \right]$$

If  $v_{DS}|_C$  is very small,

$$\begin{aligned} i_D &\simeq k_n (V_{GS}|_C - V_t) v_{DS}|_C \\ &= 4(1.8 - 0.4) v_{DS}|_C \\ &= 5.6 v_{DS}|_C, \text{ mA} \end{aligned}$$

But

$$i_D = \frac{V_{DD} - v_{DS}|_C}{R_D} \simeq \frac{V_{DD}}{R_D} = \frac{1.8}{17.5} = 0.1 \text{ mA}$$

Thus,  $v_{DS}|_C = \frac{0.1}{5.6} = 0.018 \text{ V} = 18 \text{ mV}$ , which

is indeed very small, as assumed.

Ex: 6.2 Refer to Example 6.1 and Fig. 6.4(a).

Design 1:

$$V_{OV} = 0.2 \text{ V}, V_{GS} = 0.6 \text{ V}$$

$$I_D = 0.8 \text{ mA}$$

Now,

$$A_v = -k_n V_{OV} R_D$$

Thus,

$$-10 = -0.4 \times 10 \times 0.2 \times R_D$$

$$\Rightarrow R_D = 12.5 \text{ k}\Omega$$

$$V_{DS} = V_{DD} - R_D I_D$$

$$= 1.8 - 12.5 \times 0.08 = 0.8 \text{ V}$$

Design 2:

$$R_D = 17.5 \text{ k}\Omega$$

$$A_v = -k_n V_{OV} R_D$$

$$-10 = -0.4 \times 10 \times V_{OV} \times 17.5$$

Thus,

$$V_{OV} = 0.14 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.4 + 0.14 = 0.54 \text{ V}$$

$$I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2$$

$$= \frac{1}{2} \times 0.4 \times 10 \times 0.14^2 = 0.04 \text{ mA}$$

$$R_D = 17.5 \text{ k}\Omega$$

$$V_{DS} = V_{DD} - R_D I_D$$

$$= 1.8 - 17.5 \times 0.04 = 1.1 \text{ V}$$

Ex: 6.3

$$A_v = -\frac{I_C R_C}{V_T}$$

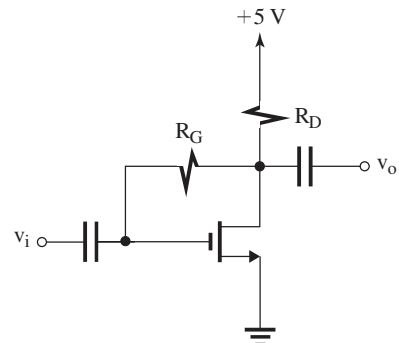
$$-320 = -\frac{1 \times R_C}{0.025} \Rightarrow R_C = 8 \text{ k}\Omega$$

$$V_C = V_{CC} - I_C R_C$$

$$= 10 - 1 \times 8 = 2 \text{ V}$$

Since the collector voltage is allowed to decrease to  $+0.3 \text{ V}$ , the largest negative swing allowed at the output is  $2 - 0.3 = 1.7 \text{ V}$ . The corresponding input signal amplitude can be found by dividing  $1.7 \text{ V}$  by the gain magnitude (320 V/V), resulting in  $5.3 \text{ mV}$ .

Ex: 6.4



Refer to the solution of Example 6.3. From Eq. (6.47),  $A_v \equiv \frac{v_o}{v_i} = -g_m R_D$  (note that  $R_L$  is absent).

Thus,

$$g_m R_D = 25$$

Substituting for  $g_m = k_n V_{OV}$ , we have

$$k_n V_{OV} R_D = 25$$

### Exercise 6-2

where  $k_n = 1 \text{ mA/V}^2$ , thus

$$V_{OV}R_D = 25 \quad (1)$$

Next, consider the bias equation

$$V_{GS} = V_{DS} = V_{DD} - R_D I_D$$

Thus,

$$V_t + V_{OV} = V_{DD} - R_D I_D$$

Substituting  $V_t = 0.7 \text{ V}$ ,  $V_{DD} = 5 \text{ V}$ , and

$$I_D = \frac{1}{2}k_n V_{OV}^2 = \frac{1}{2} \times 1 \times V_{OV}^2 = \frac{1}{2}V_{OV}^2$$

we obtain

$$0.7 + V_{OV} = 5 - \frac{1}{2}V_{OV}^2 R_D \quad (2)$$

Equations (1) and (2) can be solved to obtain

$$V_{OV} = 0.319 \text{ V}$$

and

$$R_D = 78.5 \text{ k}\Omega$$

The dc current  $I_D$  can be now found as

$$I_D = \frac{1}{2}k_n V_{OV}^2 = 50.9 \mu\text{A}$$

To determine the required value of  $R_G$  we use Eq. (6.48), again noting that  $R_L$  is absent:

$$R_{in} = \frac{R_G}{1 + g_m R_D}$$

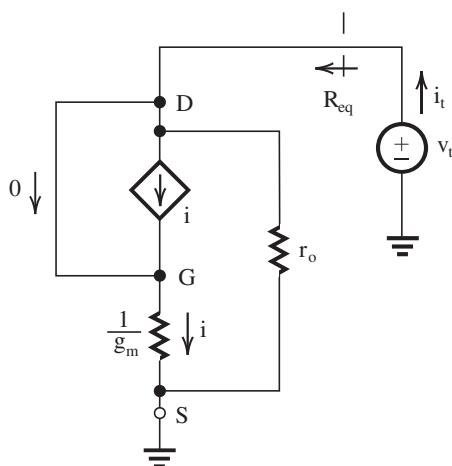
$$0.5 \text{ M}\Omega = \frac{R_G}{1 + 25}$$

$$\Rightarrow R_G = 13 \text{ M}\Omega$$

Finally, the maximum allowable input signal  $\hat{v}_i$  can be found as follows:

$$\hat{v}_i = \frac{V_t}{|A_v| + 1} = \frac{0.7 \text{ V}}{25 + 1} = 27 \text{ mV}$$

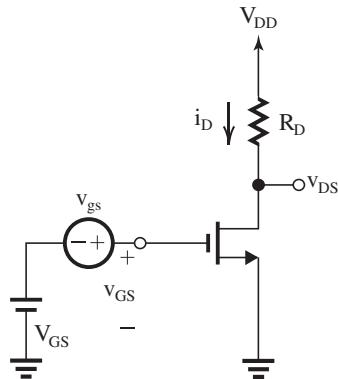
Ex: 6.5



$$i_t = \frac{v_t}{r_o} + i = \frac{v_t}{r_o} + g_m v_t$$

$$\therefore R_{eq} = \frac{v_t}{i_t} = r_o \parallel \frac{1}{g_m}$$

Ex: 6.6



$$V_{DD} = 5 \text{ V}$$

$$V_{GS} = 2 \text{ V}$$

$$V_t = 1 \text{ V}$$

$$\lambda = 0$$

$$k'_n = 20 \mu\text{A/V}^2$$

$$R_D = 10 \text{ k}\Omega$$

$$\frac{W}{L} = 20$$

$$(a) V_{GS} = 2 \text{ V} \Rightarrow V_{OV} = 1 \text{ V}$$

$$I_D = \frac{1}{2}k'_n \frac{W}{L} V_{OV}^2 = 200 \mu\text{A}$$

$$V_{DS} = V_{DD} - I_D R_D = +3 \text{ V}$$

$$(b) g_m = k'_n \frac{W}{L} V_{OV} = 400 \mu\text{A/V} = 0.4 \text{ mA/V}$$

$$(c) A_v = \frac{V_{ds}}{V_{gs}} = -g_m R_D = -4 \text{ V/V}$$

$$(d) v_{gs} = 0.2 \sin \omega t \text{ V}$$

$$v_{ds} = -0.8 \sin \omega t \text{ V}$$

$$v_{DS} = V_{DS} + v_{ds} \Rightarrow 2.2 \text{ V} \leq v_{DS} \leq 3.8 \text{ V}$$

(e) Using Eq. (6.28), we obtain

$$i_D = \frac{1}{2}k_n(V_{GS} - V_t)^2 + k_n(V_{GS} - V_t)v_{gs} + \frac{1}{2}k_n v_{gs}^2$$

$$i_D = 200 + 80 \sin \omega t$$

$$+ 8 \sin^2 \omega t, \mu\text{A}$$

### Exercise 6-3

$$= [200 + 80 \sin \omega t + (4 - 4 \cos 2\omega t)] \\ = 204 + 80 \sin \omega t - 4 \cos 2\omega t, \mu A$$

$I_D$  shifts by 4  $\mu A$ .

Thus,

$$2HD = \frac{\hat{i}_{2\omega}}{\hat{i}_\omega} = \frac{4 \mu A}{80 \mu A} = 0.05 (5\%)$$

Ex: 6.7

$$(a) g_m = \frac{2I_D}{V_{OV}}$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 = \frac{1}{2} \times 60 \times 40 \times (1.5 - 1)^2$$

$$I_D = 300 \mu A = 0.3 mA, V_{OV} = 0.5 V$$

$$g_m = \frac{2 \times 0.3}{0.5} = 1.2 mA/V$$

$$r_o = \frac{V_A}{I_D} = \frac{15}{0.3} = 50 k\Omega$$

$$(b) I_D = 0.5 mA \Rightarrow g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$= \sqrt{2 \times 60 \times 40 \times 0.5 \times 10^3}$$

$$g_m = 1.55 mA/V$$

$$r_o = \frac{V_A}{I_D} = \frac{15}{0.5} = 30 k\Omega$$

Ex: 6.8

$$I_D = 0.1 mA, g_m = 1 mA/V, k'_n = 50 \mu A/V^2$$

$$g_m = \frac{2I_D}{V_{OV}} \Rightarrow V_{OV} = \frac{2 \times 0.1}{1} = 0.2 V$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{k'_n V_{OV}^2}$$

$$= \frac{2 \times 0.1}{50} = 100 \times 0.2^2$$

Ex: 6.9

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV}$$

Same bias conditions, so same  $V_{OV}$  and also same  $L$  and  $g_m$  for both PMOS and NMOS.

$$\mu_n C_{ox} W_n = \mu_p C_{ox} W_p \Rightarrow \frac{\mu_p}{\mu_n} = 0.4 = \frac{W_n}{W_p}$$

$$\Rightarrow \frac{W_p}{W_n} = 2.5$$

Ex: 6.10

$$I_D = \frac{1}{2} k'_p \frac{W}{L} (V_{SG} - |V_t|)^2$$

$$= \frac{1}{2} \times 60 \times \frac{16}{0.8} \times (1.6 - 1)^2$$

$$I_D = 216 \mu A$$

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 216}{1.6 - 1} = 720 \mu A/V$$

$$= 0.72 mA/V$$

$$\lambda = 0.04 \Rightarrow V'_A = \frac{1}{\lambda} = \frac{1}{0.04} = 25 V/\mu m$$

$$r_o = \frac{V'_A \times L}{I_D} = \frac{25 \times 0.8}{0.216} = 92.6 k\Omega$$

Ex: 6.11

$$g_m r_o = \frac{2I_D}{V_{OV}} \times \frac{V_A}{I_D} = \frac{2V_A}{V_{OV}}$$

$$V'_A \times L = V_A$$

$$L = 0.8 \mu m \Rightarrow g_m r_o = \frac{2 \times 12.5 \times 0.8}{0.2}$$

$$= 100 V/V$$

Ex: 6.12

$$\text{Given: } g_m = \left. \frac{\partial i_C}{\partial V_{BE}} \right|_{i_C = I_C}$$

$$\text{where } I_C = I_S e^{V_{BE}/V_T}$$

$$\frac{\partial i_C}{\partial V_{BE}} = \frac{I_S e^{V_{BE}/V_T}}{V_T} = \frac{I_C}{V_T}$$

Thus,

$$g_m = \frac{I_C}{V_T}$$

Ex: 6.13

$$g_m = \frac{I_C}{V_T} = \frac{0.5 mA}{25 mV} = 20 mA/V$$

Ex: 6.14

$$I_C = 0.5 mA (\text{constant})$$

$$\beta = 50 \quad \beta = 200$$

$$g_m = \frac{I_C}{V_T} = \frac{0.5 mA}{25 mV}$$

$$= 20 mA/V = 20 mA/V$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5}{50} = \frac{0.5}{200}$$

$$= 10 \mu A = 2.5 \mu A$$

$$r_\pi = \frac{\beta}{g_m} = \frac{50}{20} = \frac{200}{20}$$

$$= 2.5 k\Omega = 10 k\Omega$$

### Exercise 6-4

Ex: 6.15

$$\beta = 100 \quad I_C = 1 \text{ mA}$$

$$g_m = \frac{I_C}{25 \text{ mV}} = 40 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} \approx \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$i_c = \beta i_b = \beta \frac{v_{be}}{r_\pi}$$

$$= \left( \frac{\beta}{r_\pi} \right) v_{be} = g_m v_{be}$$

$$i_e = i_b + \beta i_b = (\beta + 1) i_b = (\beta + 1) \frac{v_{be}}{r_\pi}$$

$$= \frac{v_{be}}{r_\pi(\beta + 1)} = \frac{v_{be}}{r_e}$$

Ex: 6.18

Ex: 6.16

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA/V}$$

$$A_v = \frac{V_{ce}}{V_{be}} = -g_m R_C$$

$$= -40 \times 10$$

$$= -400 \text{ V/V}$$

$$V_C = V_{CC} - I_C R_C$$

$$= 15 - 1 \times 10 = 5 \text{ V}$$

$$v_C(t) = V_C + v_c(t)$$

$$= (V_{CC} - I_C R_C) + A_v v_{be}(t)$$

$$= (15 - 10) - 400 \times 0.005 \sin \omega t$$

$$= 5 - 2 \sin \omega t$$

$$i_B(t) = I_B + i_b(t)$$

where

$$I_B = \frac{I_C}{\beta} = \frac{1 \text{ mA}}{100} = 10 \mu\text{A}$$

$$\text{and } i_b(t) = \frac{g_m v_{be}(t)}{\beta}$$

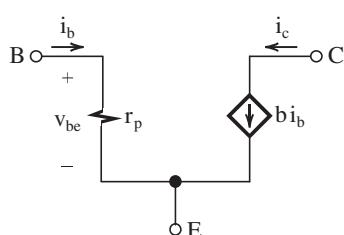
$$= \frac{40 \times 0.005 \sin \omega t}{100}$$

$$= 2 \sin \omega t, \mu\text{A}$$

Thus,

$$i_B(t) = 10 + 2 \sin \omega t, \mu\text{A}$$

Ex: 6.17



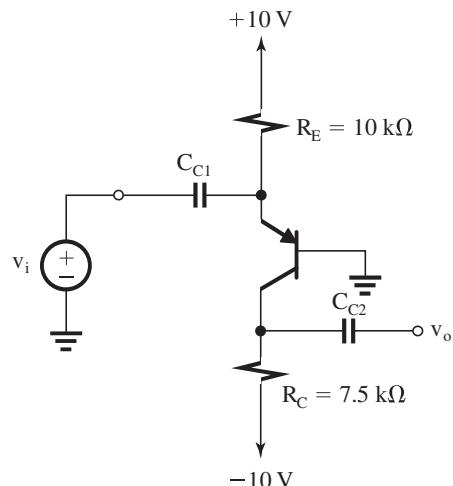
$$i_b = \frac{v_{be}}{r_e} - g_m v_{be}$$

$$= v_{be} \left( \frac{1}{r_e} - g_m \right)$$

$$= v_{be} \left( \frac{1}{r_{\pi/\beta+1}} - \frac{\beta}{r_\pi} \right)$$

$$= v_{be} \left( \frac{\beta + 1}{r_\pi} - \frac{\beta}{r_\pi} \right) = \frac{v_{be}}{r_\pi}$$

Ex: 6.19



### Exercise 6-5

$$I_E = \frac{10 - 0.7}{10} = 0.93 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.93 \\ = 0.92 \text{ mA}$$

$$V_C = -10 + I_C R_C \\ = -10 + 0.92 \times 7.5 = -3.1 \text{ V}$$

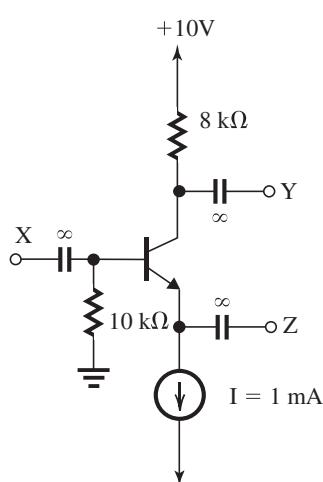
$$A_v = \frac{v_o}{v_i} = \frac{\alpha R_C}{r_e}$$

$$\text{where } r_e = \frac{25 \text{ mV}}{0.93 \text{ mA}} = 26.9 \Omega$$

$$A_v = \frac{0.99 \times 7.5 \times 10^3}{26.9} = 276.2 \text{ V/V}$$

$$\text{For } \hat{v}_i = 10 \text{ mV}, \hat{v}_o = 276.2 \times 10 = 2.76 \text{ V}$$

Ex: 6.20



$$I_E = 1 \text{ mA}$$

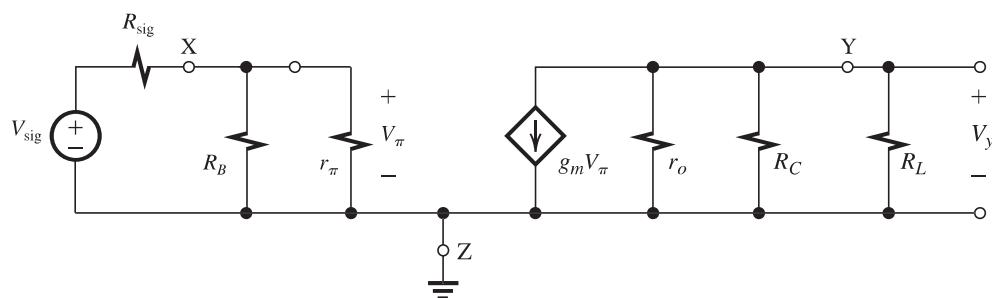
$$I_C = \frac{100}{101} \times 1 = 0.99 \text{ mA}$$

$$I_B = \frac{1}{101} \times 1 = 0.0099 \text{ mA}$$

$$(a) V_C = 10 - 8 \times 0.99 = 2.08 \simeq 2.1 \text{ V}$$

$$V_B = -10 \times 0.0099 = -0.099 \simeq -0.1 \text{ V}$$

This figure belongs to Exercise 6.20c.



$$V_E = -0.1 - 0.7 = -0.8 \text{ V}$$

$$(b) g_m = \frac{I_C}{V_T} = \frac{0.99}{0.025} \simeq 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} \simeq 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.99} = 101 \simeq 100 \text{ k}\Omega$$

$$(c) R_{\text{sig}} = 2 \text{ k}\Omega \quad R_B = 10 \text{ k}\Omega \quad r_\pi = 2.5 \text{ k}\Omega$$

$$g_m = 40 \text{ mA/V}$$

$$R_C = 8 \text{ k}\Omega \quad R_L = 8 \text{ k}\Omega \quad r_o = 100 \text{ k}\Omega$$

$$\frac{V_y}{V_{\text{sig}}} = \frac{V_\pi}{V_{\text{sig}}} \times \frac{V_y}{V_\pi}$$

$$= \frac{R_B \| r_\pi}{(R_B \| r_\pi) + R_{\text{sig}}} \times -g_m (R_C \| R_L \| r_o)$$

$$= \frac{10 \| 2.5}{(10 \| 2.5) + 2} \times -40(8 \| 8 \| 100)$$

$$-0.5 \times 40 \times 3.846 = -77 \text{ V/V}$$

If  $r_o$  is neglected,  $\frac{V_y}{V_{\text{sig}}} = -80$ , for an error of 3.9%.

Ex: 6.21

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.25}{0.25} = 2 \text{ mA/V}$$

$$R_{\text{in}} = \infty$$

$$A_{vo} = -g_m R_D = -2 \times 20 = -40 \text{ V/V}$$

$$R_o = R_D = 20 \text{ k}\Omega$$

$$A_v = A_{vo} \frac{R_L}{R_L + R_o} = -40 \times \frac{20}{20 + 20}$$

$$= -20 \text{ V/V}$$

$$G_v = A_v = -20 \text{ V/V}$$

$$\hat{v}_i = 0.1 \times 2V_{OV} = 0.2 \times 2 \times 0.25 = 0.05 \text{ V}$$

$$\hat{v}_o = 0.05 \times 20 = 1 \text{ V}$$

### Exercise 6-6

Ex: 6.22

$$I_C = 0.5 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$$R_{in} = r_\pi = 5 \text{ k}\Omega$$

$$A_v = -g_m R_C = -20 \times 10 = -200 \text{ V/V}$$

$$R_o = R_C = 10 \text{ k}\Omega$$

$$A_v = A_{v_o} \frac{R_L}{R_L + R_o} = -200 \times \frac{5}{5 + 10} \\ = -66.7 \text{ V/V}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_v = \frac{5}{5 + 5} \times -66.7$$

$$= -33.3 \text{ V/V}$$

$$\hat{v}_\pi = 5 \text{ mV} \Rightarrow \hat{v}_{sig} = 2 \times 5 = 10 \text{ mV}$$

$$\hat{v}_o = 10 \times 33.3 = 0.33 \text{ V}$$

Although a larger fraction of the input signal reaches the amplifier input, linearity considerations cause the output signal to be in fact smaller than in the original design!

Ex: 6.23 Refer to the solution to Exercise 6.21. If  $\hat{v}_{sig} = 0.2 \text{ V}$  and we wish to keep  $\hat{v}_{gs} = 50 \text{ mV}$ ,

then we need to connect a resistance  $R_s = \frac{3}{g_m}$  in the source lead. Thus,

$$R_s = \frac{3}{2 \text{ mA/V}} = 1.5 \text{ k}\Omega$$

$$G_v = A_v = -\frac{R_D \parallel R_L}{\frac{1}{g_m} + R_s}$$

$$= -\frac{20 \parallel 20}{0.5 + 1.5} = -5 \text{ V/V}$$

$$\hat{v}_o = G_v \hat{v}_{sig} = 5 \times 0.2 = 1 \text{ V (unchanged)}$$

Ex: 6.24

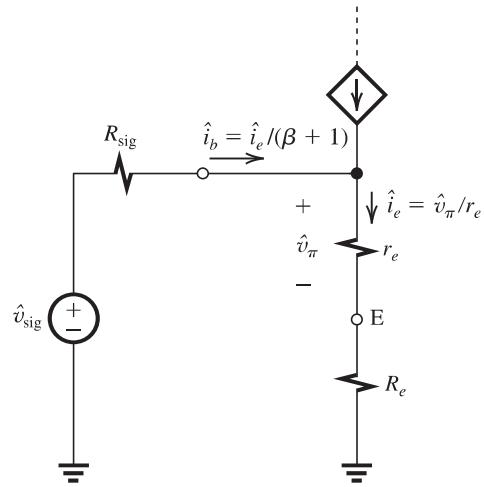
From the following figure we see that

$$\hat{v}_{sig} = \hat{i}_b R_{sig} + \hat{v}_\pi + \hat{i}_e R_e$$

$$= \frac{\hat{i}_e}{\beta + 1} R_{sig} + \hat{v}_\pi + \hat{i}_e R_e$$

$$= \frac{\hat{v}_\pi}{(\beta + 1)r_e} R_{sig} + \hat{v}_\pi + \frac{\hat{v}_\pi}{r_e} R_e$$

$$\hat{v}_{sig} = \hat{v}_\pi \left( 1 + \frac{R_e}{r_e} + \frac{R_{sig}}{r_\pi} \right) \quad \text{Q.E.D}$$



For  $I_C = 0.5 \text{ mA}$  and  $\beta = 100$ ,

$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{0.99 \times 25}{0.5} \simeq 50 \text{ }\Omega$$

$$r_\pi = (\beta + 1)r_e \simeq 5 \text{ k}\Omega$$

For  $\hat{v}_{sig} = 100 \text{ mV}$ ,  $R_{sig} = 10 \text{ k}\Omega$  and with  $\hat{v}_\pi$  limited to 10 mV, the value of  $R_e$  required can be found from

$$100 = 10 \left( 1 + \frac{R_e}{50} + \frac{10}{5} \right)$$

$$\Rightarrow R_e = 350 \text{ }\Omega$$

$$R_{in} = (\beta + 1)(r_e + R_e) = 101 \times (50 + 350)$$

$$= 40.4 \text{ k}\Omega$$

$$G_v = -\beta \frac{R_C \parallel R_L}{R_{sig} + (\beta + 1)(r_e + R_e)} \\ = -100 \frac{10}{10 + 101 \times 0.4} = -19.8 \text{ V/V}$$

Ex: 6.25

$$\frac{1}{g_m} = R_{sig} = 100 \text{ }\Omega$$

$$\Rightarrow g_m = \frac{1}{0.1 \text{ k}\Omega} = 10 \text{ mA/V}$$

But

$$g_m = \frac{2I_D}{V_{OV}}$$

Thus,

$$10 = \frac{2I_D}{0.2}$$

$$\Rightarrow I_D = 1 \text{ mA}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times g_m R_D$$

$$= 0.5 \times 10 \times 2 = 10 \text{ V/V}$$

### Exercise 6-7

Ex: 6.26

$$I_C = 1 \text{ mA}$$

$$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

$$R_{in} = r_e = 25 \Omega$$

$$A_vo = g_m R_C = 40 \times 5 = 200 \text{ V/V}$$

$$R_o = R_C = 5 \text{ k}\Omega$$

$$A_v = A_{vo} \frac{R_L}{R_L + R_o} = 200 \times \frac{5}{5+5} = 100 \text{ V/V}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times A_v$$

$$= \frac{25}{25+5000} \times 100 = 0.5 \text{ V/V}$$

Ex: 6.27

$$R_{in} = r_e = 50 \Omega$$

$$\Rightarrow I_E = \frac{V_T}{r_e} = \frac{25 \text{ mV}}{50 \Omega} = 0.5 \text{ mA}$$

$$I_C \simeq I_E = 0.5 \text{ mA}$$

$$G_v = \frac{R_C \| R_L}{r_e + R_{sig}}$$

$$40 = \frac{R_C \| R_L}{(50+50)\Omega}$$

$$R_C \| R_L = 4 \text{ k}\Omega$$

Ex: 6.28 Refer to Fig. 6.41(c).

$$R_o = 100 \Omega$$

Thus,

$$\frac{1}{g_m} = 100 \Omega \Rightarrow g_m = 10 \text{ mA/V}$$

But

$$g_m = \frac{2I_D}{V_{ov}}$$

Thus,

$$I_D = \frac{10 \times 0.25}{2} = 1.25 \text{ mA}$$

$$\hat{v}_o = \hat{v}_i \times \frac{R_L}{R_L + R_o} = 1 \times \frac{1}{1+0.1} = 0.91 \text{ V}$$

$$\hat{v}_{gs} = \hat{v}_i \frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_L} = 1 \times \frac{0.1}{0.1+1} = 91 \text{ mV}$$

Ex: 6.29

$$R_o = 200 \Omega$$

$$\frac{1}{g_m} = 200 \Omega$$

$$\Rightarrow g_m = 5 \text{ mA/V}$$

But

$$g_m = k'_n \left( \frac{W}{L} \right) V_{ov}$$

Thus,

$$5 = 0.4 \times \frac{W}{L} \times 0.25$$

$$\Rightarrow \frac{W}{L} = 50$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{ov}^2$$

$$= \frac{1}{2} \times 0.4 \times 50 \times 0.25^2$$

$$= 0.625 \text{ mA}$$

$$R_L = 1 \text{ k}\Omega \text{ to } 10 \text{ k}\Omega$$

Correspondingly,

$$G_v = \frac{R_L}{R_L + R_o} = \frac{R_L}{R_L + 0.2}$$

will range from

$$G_v = \frac{1}{1+0.2} = 0.83 \text{ V/V}$$

to

$$G_v = \frac{10}{10+0.2} = 0.98 \text{ V/V}$$

Ex: 6.30

$$I_C = 5 \text{ mA}$$

$$r_e = \frac{V_T}{I_E} \simeq \frac{V_T}{I_C} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5 \Omega$$

$$R_{sig} = 10 \text{ k}\Omega \quad R_L = 1 \text{ k}\Omega$$

$$R_{in} = (\beta + 1)(r_e + R_L)$$

$$= 101 \times (0.005 + 1)$$

$$= 101.5 \text{ k}\Omega$$

$$G_{vo} = 1 \text{ V/V}$$

$$R_{out} = r_e + \frac{R_{sig}}{\beta + 1}$$

$$= 5 + \frac{10,000}{101} = 104 \Omega$$

$$G_v = \frac{R_L}{R_L + r_e + \frac{R_{sig}}{\beta + 1}} = \frac{R_L}{R_L + R_{out}}$$

$$= \frac{1}{1+0.104} = 0.91 \text{ V/V}$$

### Exercise 6-8

$$v_\pi = v_{\text{sig}} \frac{r_e}{r_e + R_L + \frac{R_{\text{sig}}}{\beta + 1}}$$

$$\hat{v}_{\text{sig}} = \hat{v}_\pi \left[ 1 + \frac{R_L}{r_e} + \frac{R_{\text{sig}}}{(\beta + 1) r_e} \right]$$

$$\hat{v}_{\text{sig}} = 5 \left[ 1 + \frac{1000}{5} + \frac{10,000}{101 \times 5} \right] = 1.1 \text{ V/V}$$

Correspondingly,

$$\hat{v}_o = G_v \times 1.1 = 0.91 \times 1.1 = 1 \text{ V}$$

Ex: 6.31

$$I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$$

$$0.5 = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 2 \text{ V}$$

If  $V_t = 1.5 \text{ V}$ , then

$$I_D = \frac{1}{2} \times 1 \times (2 - 1.5)^2 = 0.125 \text{ mA}$$

$$\Rightarrow \frac{\Delta I_D}{I_D} = \frac{0.125 - 0.5}{0.5} = -0.75 = -75\%$$

Ex: 6.32

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 2}{0.5} = 6 \text{ k}\Omega$$

$$\rightarrow R_D = 6.2 \text{ k}\Omega$$

$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 \Rightarrow 0.5 = \frac{1}{2} \times 1 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 1 \text{ V}$$

$$\Rightarrow V_{GS} = V_{OV} + V_t = 1 + 1 = 2 \text{ V}$$

$$\Rightarrow V_S = -2 \text{ V}$$

$$R_S = \frac{V_S - V_{SS}}{I_D} = \frac{-2 - (-5)}{0.5} = 6 \text{ k}\Omega$$

$$\rightarrow R_S = 6.2 \text{ k}\Omega$$

If we choose  $R_D = R_S = 6.2 \text{ k}\Omega$ , then  $I_D$  will change slightly:

$$I_D = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2. \text{ Also}$$

$$V_{GS} = -V_S = 5 - R_S I_D$$

$$2 I_D = (4 - 6.2 I_D)^2$$

$$\Rightarrow 38.44 I_D^2 - 51.6 I_D^2 + 16 = 0$$

$$\Rightarrow I_D = 0.49 \text{ mA}, 0.86 \text{ mA}$$

$I_D = 0.86$  results in  $V_S > 0$  or  $V_S > V_G$ , which is not acceptable. Therefore  $I_D = 0.49 \text{ mA}$  and

$$V_S = -5 + 6.2 \times 0.49 = -1.96 \text{ V}$$

$$V_D = 5 - 6.2 \times 0.49 = +1.96 \text{ V}$$

$R_G$  should be selected in the range of  $1 \text{ M}\Omega$  to  $10 \text{ M}\Omega$  to have low current.

Ex: 6.33

$$I_D = 0.5 \text{ mA} = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2$$

$$\Rightarrow V_{OV}^2 = \frac{0.5 \times 2}{1} = 1$$

$$\Rightarrow V_{OV} = 1 \text{ V} \Rightarrow V_{GS} = 1 + 1 = 2 \text{ V}$$

$$= V_D \Rightarrow R_D = \frac{5 - 2}{0.5} = 6 \text{ k}\Omega$$

$\Rightarrow R_D = 6.2 \text{ k}\Omega$  (standard value). For this  $R_D$  we have to recalculate  $I_D$ :

$$I_D = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2$$

$$= \frac{1}{2} (V_{DD} - R_D I_D - 1)^2$$

$$(V_{GS} = V_D = V_{DD} - R_D I_D)$$

$$I_D = \frac{1}{2} (4 - 6.2 I_D)^2 \Rightarrow I_D \cong 0.49 \text{ mA}$$

$$V_D = 5 - 6.2 \times 0.49 = 1.96 \text{ V}$$

Ex: 6.34 Refer to Example 6.12.

(a) For design 1,  $R_E = 3 \text{ k}\Omega$ ,  $R_I = 80 \text{ k}\Omega$ , and  $R_2 = 40 \text{ k}\Omega$ . Thus,  $V_{BB} = 4 \text{ V}$ .

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_1 \parallel R_2}{\beta + 1}}$$

For the nominal case,  $\beta = 100$  and

$$I_E = \frac{4 - 0.7}{3 + \frac{40 \parallel 80}{101}} = 1.01 \cong 1 \text{ mA}$$

For  $\beta = 50$ ,

$$I_E = \frac{4 - 0.7}{3 + \frac{40 \parallel 80}{51}} = 0.94 \text{ mA}$$

For  $\beta = 150$ ,

$$I_E = \frac{4 - 0.7}{3 + \frac{40 \parallel 80}{151}} = 1.04 \text{ mA}$$

Thus,  $I_E$  varies over a range approximately 10% of the nominal value of 1 mA.

(b) For design 2,  $R_E = 3.3 \text{ k}\Omega$ ,  $R_I = 8 \text{ k}\Omega$ , and  $R_2 = 4 \text{ k}\Omega$ . Thus,  $V_{BB} = 4 \text{ V}$ . For the nominal case,  $\beta = 100$  and

### Exercise 6-9

$$I_E = \frac{4 - 0.7}{3.3 + \frac{4\parallel 8}{101}} = 0.99 \simeq 1 \text{ mA}$$

For  $\beta = 50$ ,

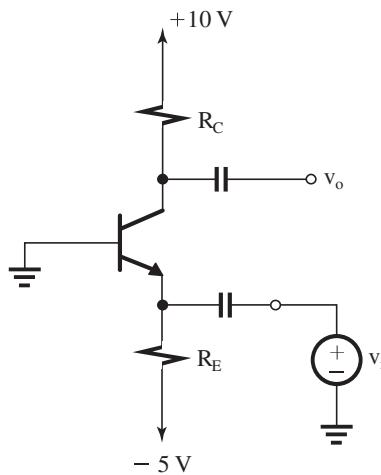
$$I_E = \frac{4 - 0.7}{3.3 + \frac{4\parallel 8}{51}} = 0.984 \text{ mA}$$

For  $\beta = 150$ ,

$$I_E = \frac{4 - 0.7}{3.3 + \frac{4\parallel 8}{151}} = 0.995 \text{ mA}$$

Thus,  $I_E$  varies over a range of 1.1% of the nominal value of 1 mA. Note that lowering the resistances of the voltage divider considerably decreases the dependence on the value of  $\beta$ , a highly desirable result obtained at the expense of increased current and hence power dissipation.

Ex: 6.35 Refer to Fig. 6.53. Since the circuit is to be used as a common-base amplifier, we can dispense with  $R_B$  altogether and ground the base; thus  $R_B = 0$ . The circuit takes the form shown in the figure below.



To establish  $I_E = 1 \text{ mA}$ ,

$$I_E = \frac{5 - V_{BE}}{R_E}$$

$$1 \text{ mA} = \frac{5 - 0.7}{R_E}$$

$$\Rightarrow R_E = 4.3 \text{ k}\Omega$$

The voltage gain  $\frac{v_o}{v_i} = g_m R_C$ , where  $g_m = \frac{I_C}{V_T} =$

$40 \text{ mA/V}$ . To maximize the voltage gain, we select  $R_C$  as large as possible, consistent with obtaining a  $\pm 2\text{-V}$  signal swing at the collector.

To maintain active-mode operation at all times, the collector voltage should not be allowed to fall below the value that causes the CBJ to become forward biased, namely,  $-0.4 \text{ V}$ . Thus, the lowest possible dc voltage at the collector is  $-0.4 \text{ V} + 2\text{V} = +1.6 \text{ V}$ . Correspondingly,

$$R_C = \frac{10 - 1.6}{I_C} \simeq \frac{10 - 1.6}{1 \text{ mA}} = 8.4 \text{ k}\Omega$$

Ex: 6.36 Refer to Fig. 6.54. For  $I_E = 1 \text{ mA}$  and  $V_C = 2.3 \text{ V}$ ,

$$I_E = \frac{V_{CC} - V_C}{R_C}$$

$$1 = \frac{10 - 2.3}{R_C}$$

$$\Rightarrow R_C = 7.7 \text{ k}\Omega$$

Now, using Eq. (6.147), we obtain

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}}$$

$$1 = \frac{10 - 0.7}{7.7 + \frac{R_B}{101}}$$

$$\Rightarrow R_B = 162 \text{ k}\Omega$$

Selecting standard 5% resistors (Appendix J), we use

$$R_B = 160 \text{ k}\Omega \quad \text{and} \quad R_C = 7.5 \text{ k}\Omega$$

The resulting value of  $I_E$  is found as

$$I_E = \frac{10 - 0.7}{7.5 + \frac{160}{101}} = 1.02 \text{ mA}$$

and the collector voltage will be

$$V_C = V_{CC} - I_E R_C = 2.3 \text{ V}$$

Ex: 6.37 Refer to Fig. 6.55(b).

$V_S = 3.5$  and  $I_D = 0.5 \text{ mA}$ ; thus

$$R_S = \frac{V_S}{I_D} = \frac{3.5}{0.5} = 7 \text{ k}\Omega$$

$V_{DD} = 15 \text{ V}$  and  $V_D = 6 \text{ V}$ ; thus

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{15 - 6}{0.5 \text{ mA}} = 18 \text{ k}\Omega$$

To obtain  $V_{OV}$ , we use

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.5 = \frac{1}{2} \times 4V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.5 \text{ V}$$

### Exercise 6–10

Thus,

$$V_{GS} = V_t + V_{OV} = 1 + 0.5 = 1.5 \text{ V}$$

We now can obtain the dc voltage required at the gate,

$$V_G = V_S + V_{GS} = 3.5 + 1.5 = 5 \text{ V}$$

Using a current of  $2 \mu\text{A}$  in the voltage divider, we have

$$R_{G2} = \frac{5 \text{ V}}{2 \mu\text{A}} = 2.5 \text{ M}\Omega$$

The voltage drop across  $R_{G1}$  is 10 V, thus

$$R_{G1} = \frac{10 \text{ V}}{2 \mu\text{A}} = 5 \text{ M}\Omega$$

This completes the bias design. To obtain  $g_m$  and  $r_o$ , we use

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.5} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

Ex: 6.38 Refer to Fig. 6.55(a) and (c) and to the values found in the solution to Exercise 6.37 above.

$$R_{in} = R_{G1} \parallel R_{G2} = 5 \parallel 2.5 = 1.67 \text{ M}\Omega$$

$$R_o = R_D \parallel r_o = 18 \parallel 200 = 16.5 \text{ k}\Omega$$

$$\begin{aligned} G_v &= -\frac{R_{in}}{R_{in} + R_{sig}} g_m (r_o \parallel R_D \parallel R_L) \\ &= -\frac{1.67}{1.67 + 0.1} \times 2 \times (200 \parallel 18 \parallel 20) \\ &= -17.1 \text{ V/V} \end{aligned}$$

Ex: 6.39 To reduce  $v_{gs}$  to half its value, the unbypassed  $R_s$  is given by

$$R_s = \frac{1}{g_m}$$

From the solution to Exercise 6.37 above,  $g_m = 2 \text{ mA/V}$ . Thus

$$R_s = \frac{1}{2} = 0.5 \text{ k}\Omega$$

Neglecting  $r_o$ ,  $G_v$  is given by

$$\begin{aligned} G_v &= -\frac{R_{in}}{R_{in} + R_{sig}} \times -\frac{R_D \parallel R_L}{\frac{1}{g_m} + R_s} \\ &= -\frac{1.67}{1.67 + 0.1} \times \frac{18 \parallel 20}{0.5 + 0.5} \\ &= -8.9 \text{ V/V} \end{aligned}$$

Ex: 6.40 Refer to Fig. 6.56(a). For  $V_B = 5 \text{ V}$  and  $50 \mu\text{A}$  current through  $R_{B2}$ , we have

$$R_{B2} = \frac{5 \text{ V}}{0.05 \text{ mA}} = 100 \text{ k}\Omega$$

The base current is

$$I_B = \frac{I_E}{\beta + 1} \approx \frac{0.5 \text{ mA}}{100} = 5 \mu\text{A}$$

The current through  $R_{B1}$  is

$$I_{R_{B1}} = I_B + I_{R_{B2}} = 5 + 50 = 55 \mu\text{A}$$

Since the voltage drop across  $R_{B1}$  is  $V_{CC} - V_B = 10 \text{ V}$ , the value of  $R_{B1}$  can be found from

$$R_{B1} = \frac{10 \text{ V}}{0.055 \mu\text{A}} = 182 \text{ k}\Omega$$

The value of  $R_E$  can be found from

$$I_E = \frac{V_B - V_{BE}}{R_E}$$

$$\Rightarrow R_E = \frac{5 - 0.7}{0.5} = 8.6 \text{ k}\Omega$$

The value of  $R_C$  can be found from

$$V_C = V_{CC} - I_C R_C$$

$$6 = 15 - 0.99 \times 0.5 \times R_C$$

$$R_C \approx 18 \text{ k}\Omega$$

This completes the bias design. The values of  $g_m$ ,  $r_\pi$ , and  $r_o$  can be found as follows:

$$g_m = \frac{I_C}{V_T} \approx \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} \approx \frac{100}{0.5} = 200 \text{ k}\Omega$$

Ex: 6.41 Refer to Fig. 6.56(b) and to the solution of Exercise 6.40 above.

$$R_{in} = R_{B1} \parallel R_{B2} \parallel r_\pi$$

$$= 182 \parallel 100 \parallel 5 = 4.64 \text{ k}\Omega$$

$$R_o = R_C \parallel r_o = 18 \parallel 200 = 16.51 \text{ k}\Omega$$

$$G_v = -\frac{R_{in}}{R_{in} + R_{sig}} g_m (R_C \parallel R_L \parallel r_o)$$

$$G_v = -\frac{4.64}{4.64 + 10} \times 20 \times (18 \parallel 20 \parallel 200)$$

$$= -57.3 \text{ V/V}$$

Ex: 6.42 Refer to the solutions of Exercises 6.40 and 6.41 above. With  $R_E$  included (i.e., left unbypassed), the input resistance becomes [refer to Fig. 6.57(b)]

### Exercise 6–11

$$R_{in} = R_{B1} \parallel R_{B2} \parallel [(\beta + 1)(r_e + R_e)]$$

Thus,

$$10 = 182 \parallel 100 \parallel [101(0.05 + R_e)]$$

$$\text{where we have substituted } r_e = \frac{V_T}{I_E} =$$

$\frac{25}{0.5} = 50 \Omega$ . The value of  $R_e$  is found from the equation above to be

$$R_e = 67.7 \Omega$$

The overall voltage gain can be found from

$$G_v = -\alpha \frac{R_{in}}{R_{in} + R_{sig}} \frac{R_C \parallel R_L}{r_e + R_e}$$

$$G_v = -0.99 \times \frac{10}{10 + 10} \frac{18 \parallel 20}{0.05 + 0.0677} \\ = -39.8 \text{ V/V}$$

Ex: 6.43 Refer to Fig. 6.58.

$$R_{in} = 50 \Omega = r_e \parallel R_E \simeq r_e$$

$$r_e = 50 \Omega = \frac{V_T}{I_E}$$

$$\Rightarrow I_E = 0.5 \text{ mA}$$

$$I_C = \alpha I_E \simeq I_E = 0.5 \text{ mA}$$

$$V_C = V_{CC} - R_C I_C$$

For  $V_C = 1 \text{ V}$  and  $V_{CC} = 5 \text{ V}$ , we have

$$1 = 5 - R_C \times 0.5$$

$$\Rightarrow R_C = 8 \text{ k}\Omega$$

To obtain the required value of  $R_E$ , we note that the voltage drop across it is  $(V_{EE} - V_{BE}) = 4.3 \text{ V}$ . Thus,

$$R_E = \frac{4.3}{0.5} = 8.6 \text{ k}\Omega$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} g_m (R_C \parallel R_L)$$

$$= \frac{50 \Omega}{50 \Omega + 50 \Omega} \times 20(8 \parallel 8) \\ = 40 \text{ V/V} \\ \hat{v}_o = 40 \hat{v}_{sig} = 40 \times 10 \text{ mV} = 0.4 \text{ V}$$

Ex: 6.44 Refer to Fig. 6.59. Consider first the bias design of the circuit in Fig. 6.59(a). Since the required  $I_E = 1 \text{ mA}$ , the base current

$$I_B = \frac{I_E}{\beta + 1} = \frac{1}{101} \simeq 0.01 \text{ mA. For a dc voltage drop across } R_B \text{ of } 1 \text{ V, we obtain}$$

$$R_B = \frac{1 \text{ V}}{0.01 \text{ mA}} = 100 \text{ k}\Omega$$

The result is a base voltage of  $-1 \text{ V}$  and an emitter voltage of  $-1.7 \text{ V}$ . The required value of  $R_E$  can now be determined as

$$R_E = \frac{-1.7 - (-5)}{I_E} = \frac{3.3}{1 \text{ mA}} = 3.3 \text{ k}\Omega$$

$$R_{in} = R_B \parallel [(\beta + 1)[r_e + (R_E \parallel r_o \parallel R_L)]]$$

$$\text{where } r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

$$R_{in} = 100 \parallel [(100 + 1)[0.025 + (3.3 \parallel 100 \parallel 1)]]$$

$$= 44.3 \text{ k}\Omega$$

$$\frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{44.3}{44.3 + 50} = 0.469 \text{ V/V}$$

$$\frac{v_o}{v_i} = \frac{R_E \parallel r_o \parallel R_L}{r_e + (R_E \parallel r_o \parallel R_L)} = 0.968 \text{ V/V}$$

$$G_v \equiv \frac{v_o}{v_{sig}} = 0.469 \times 0.968 = 0.454 \text{ V/V}$$

$$R_{out} = r_o \parallel R_E \parallel \left[ r_e + \frac{R_B \parallel R_{sig}}{\beta + 1} \right]$$

$$= 100 \parallel 3.3 \parallel \left[ 0.025 + \frac{100 \parallel 50}{101} \right]$$

$$= 320 \Omega$$

6.1 Coordinates of point A:  $v_{GS} = V_t = 0.5$  V and  $v_{DS} = V_{DD} = 5$  V.

To obtain the coordinates of point B, we first use Eq. (6.6) to determine  $V_{GS}|_B$  as

$$\begin{aligned} V_{GS}|_B &= V_t + \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} \\ &= 0.5 + \frac{\sqrt{2 \times 10 \times 20 \times 5 + 1} - 1}{10 \times 20} \\ &= 0.5 + 0.22 = 0.72 \text{ V} \end{aligned}$$

The vertical coordinate of point B is  $V_{DS}|_B$ ,

$$V_{DS}|_B = V_{GS}|_B - V_t = V_{OV}|_B = 0.22 \text{ V}$$

$$6.2 \quad V_{DS}|_B = V_{OV}|_B = 0.5 \text{ V}$$

Thus,

$$I_D|_B = \frac{1}{2} k_n V_{DS}|_B^2 = \frac{1}{2} \times 5 \times 0.5^2 = 0.625 \text{ mA}$$

The value of  $R_D$  required can now be found as

$$\begin{aligned} R_D &= \frac{V_{DD} - V_{DS}|_B}{I_D|_B} \\ &= \frac{5 - 0.5}{0.625} = 7.2 \text{ k}\Omega \end{aligned}$$

If the transistor is replaced with another having twice the value of  $k_n$ , then  $I_D|_B$  will be twice as large and the required value of  $R_D$  will be half that used before, that is, 3.6 kΩ.

6.3 Bias point Q:  $V_{OV} = 0.2$  V and  $V_{DS} = 1$  V.

$$\begin{aligned} I_{DQ} &= \frac{1}{2} k_n V_{OV}^2 \\ &= \frac{1}{2} \times 10 \times 0.04 = 0.2 \text{ mA} \\ R_D &= \frac{V_{DD} - V_{DS}}{I_{DQ}} = \frac{5 - 1}{0.2} = 20 \text{ k}\Omega \end{aligned}$$

Coordinates of point B:

Equation (6.6):

$$\begin{aligned} V_{GS}|_B &= V_t + \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} \\ &= 0.5 + \frac{\sqrt{2 \times 10 \times 20 \times 5 + 1} - 1}{10 \times 20} \\ &= 0.5 + 0.22 = 0.72 \text{ V} \end{aligned}$$

Equations (6.7) and (6.8):

$$V_{DS}|_B = \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} = 0.22 \text{ V}$$

$$A_v = -k_n R_D V_{OV}$$

$$= -10 \times 20 \times 0.2 = -40 \text{ V/V}$$

The lowest instantaneous voltage allowed at the output is  $V_{DS}|_B = 0.22$  V. Thus the maximum allowable negative signal swing at the output is  $V_{DSQ} - 0.22 = 1 - 0.22 = 0.78$  V. The

corresponding peak input signal is

$$\hat{v}_{gs} = \frac{0.78 \text{ V}}{|A_v|} = \frac{0.78}{40} = 19.5 \text{ mV}$$

6.4 From Eq. (6.18):

$$|A_{v\max}| = \frac{V_{DD} - V_{OV}|_B}{V_{OV}|_B/2}$$

$$14 = \frac{2 - V_{OV}|_B}{V_{OV}|_B/2}$$

$$\Rightarrow V_{OV}|_B = 0.25 \text{ V}$$

Now, using Eq. (6.15) at point B, we have

$$A_v|_B = -k_n V_{OV}|_B R_D$$

Thus,

$$-14 = -k_n R_D \times 0.25$$

$$\Rightarrow k_n R_D = 56$$

To obtain a gain of  $-12$  V/V at point Q:

$$-12 = -k_n R_D V_{OV}|_Q$$

$$= -56 V_{OV}|_Q$$

Thus,

$$V_{OV}|_Q = \frac{12}{56} = 0.214 \text{ V}$$

To obtain the required  $V_{DS}|_Q$ , we use Eq. (6.17),

$$A_v = -\frac{V_{DD} - V_{DS}|_Q}{V_{OV}|_Q/2}$$

$$-12 = -\frac{2 - V_{DS}|_Q}{0.214/2}$$

$$\Rightarrow V_{DS}|_Q = 0.714 \text{ V}$$

6.5  $R_D = 20 \text{ k}\Omega$

$$k'_n = 200 \mu \text{A/V}^2$$

$$V_{RD} = 1.5 \text{ V}$$

$$V_{GS} = 0.7 \text{ V}$$

$$A_v = -10 \text{ V/V}$$

$$A_v = -k_n V_{OV} R_D$$

$$V_{RD} = I_D R_D = \frac{1}{2} k_n V_{OV}^2 R_D$$

$$\frac{A_v}{V_{RD}} = \frac{-2}{V_{OV}} = \frac{-10}{1.5}$$

$$\therefore V_{OV} = 0.30 \text{ V}$$

$$V_t = V_{GS} - V_{OV} = 0.40 \text{ V}$$

$$k_n = \frac{A_v}{V_{OV} R_D} = \frac{-10}{-0.3 \times 20}$$

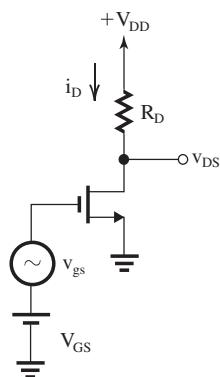
$$= 1.67 \text{ mA/V}^2$$

## Chapter 6–2

$$k_n = k'_n \frac{W}{L} = 1.67 \text{ mA/V}^2$$

$$\therefore \frac{W}{L} = 8.33$$

6.6



$$V_{DD} = 5 \text{ V}, \quad k'_n \frac{W}{L} = 1 \frac{\text{mA}}{\text{V}^2}$$

$$R_D = 24 \text{ k}\Omega, \quad V_t = 1 \text{ V}$$

(a) Endpoints of saturation transfer segment:

Point A occurs at  $V_{GS} = V_t = 1 \text{ V}$ ,  $i_D = 0$

Point A = (1 V, 5 V) ( $V_{GS}$ ,  $V_{DS}$ )

Point B occurs at sat/triode boundary ( $V_{GD} = V_t$ )

$$V_{GD} = 1 \text{ V} \Rightarrow V_{GS} - [5 - i_D R_D] = 1$$

$$V_{GS} - 5 + \left(\frac{1}{2}\right)(1)(24) [V_{GS} - 1]^2 - 1 = 0$$

$$12V_{GS}^2 - 23V_{GS} + 6 = 0$$

$$V_{GS} = 1.605 \text{ V}$$

$$i_D = 0.183 \text{ mA} \quad V_{DS} = 0.608 \text{ V}$$

Point B = (+1.61 V, 0.61 V)

(b) For  $V_{OV} = V_{GS} - V_t = 0.5 \text{ V}$ , we have

$$V_{GS} = 1.5 \text{ V}$$

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 1(1.5 - 1)^2$$

$$I_D = 0.125 \text{ mA} \quad V_{DS} = +2.00 \text{ V}$$

Point Q = (1.50 V, 2.00 V)

$$A_v = -k_n V_{OV} R_D = -12 \text{ V/V}$$

(c) From part (a) above, the maximum instantaneous input signal while the transistor

remains in saturation is 1.61 V and the corresponding output voltage is 0.61 V. Thus, the maximum amplitude of input sine wave is  $(1.61 - 1.5) = 0.11 \text{ V}$ . That is,  $v_{GS}$  ranges from  $1.5 - 0.11 = 1.39 \text{ V}$ , at which

$$i_D = \frac{1}{2} \times 1 \times (1.39 - 1)^2 = 0.076 \text{ mA}$$

and

$$v_{DS} = 5 - 0.076 \times 24 = 3.175 \text{ V}$$

and  $v_{GS} = 1.5 + 0.11 = 1.61 \text{ V}$  at which  $v_{DS} = 0.61 \text{ V}$ .

Thus, the large-signal gain is

$$\frac{0.61 - 3.175}{1.61 - 1.39} = -11.7 \text{ V/V}$$

whose magnitude is slightly less (-2.5%) than the incremental or small-signal gain (-12 V/V). This is an indication that the transfer characteristic is not a straight line.

## 6.7 At sat/triode boundary

$$v_{GS}|_B = V_{GS} + \hat{v}_{gs}$$

$$v_{DS}|_B = V_{DS} - \hat{v}_o$$

( $\hat{v}_o$  = max downward amplitude), we get

$$\begin{aligned} v_{DS}|_B &= v_{GS}|_B - V_t = V_{GS} + \frac{\hat{v}_o}{|A_v|} - V_t \\ &= V_{DS} - \hat{v}_o \\ V_{OV} + \frac{\hat{v}_o}{|A_v|} &= V_{DS} - \hat{v}_o \\ \hat{v}_o &= \frac{V_{DS} - V_{OV}}{1 + \frac{1}{|A_v|}} \end{aligned} \quad (1)$$

For  $V_{DD} = 5 \text{ V}$ ,  $V_{OV} = 0.5 \text{ V}$ , and

$$k'_n \frac{W}{L} = 1 \text{ mA/V}^2, \text{ we use}$$

$$A_v = \frac{-2(V_{DD} - V_{DS})}{V_{OV}}$$

and Eq. (1) to obtain

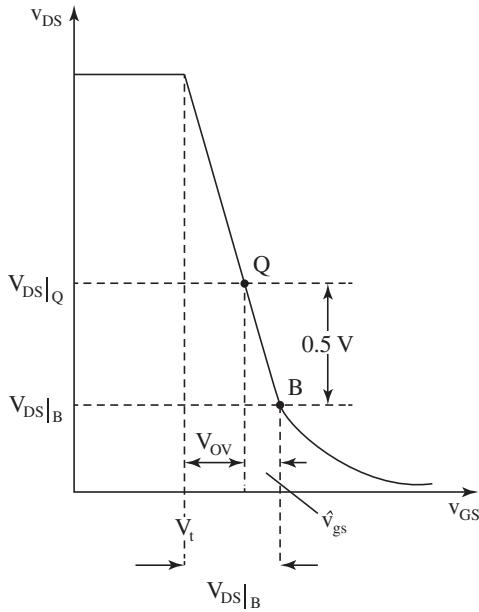
$V_{DS}$	$A_v$	$\hat{v}_o$	$\hat{v}_i$
1 V	-16	471 mV	29.4 mV
1.5 V	-14	933 mV	66.7 mV
2 V	-12	1385 mV	115 mV
2.5 V	-10	1818 mV	182 mV

For  $V_{DS} = 1 \text{ V}$ ,  $A_v = -16 = -k_n V_{OV} R_D$

$$\therefore R_D = 32 \text{ k}\Omega$$

$$I_D R_D = 4 \text{ V}, I_D = 0.125 \text{ mA}$$

## 6.8



To obtain maximum gain while allowing for a  $-0.5\text{-V}$  signal swing at the output, we bias the MOSFET at point Q where

$$V_{DS}|_Q = V_{DS}|_B + 0.5 \text{ V} \quad (1)$$

as indicated in the figure above. Now,  $V_{DS}|_B$  is given by Eq. (6.8) [together with Eq. (6.7)],

$$V_{DS}|_B = \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} \quad (2)$$

From the figure we see that

$$V_{DS}|_B = V_{OV} + \hat{v}_{gs}$$

where  $V_{OV} = 0.2 \text{ V}$  (given) and

$$\begin{aligned} \hat{v}_{gs} &= \frac{0.5 \text{ V}}{|A_v|} \\ &= \frac{0.5}{k_n R_D V_{OV}} = \frac{0.5}{k_n R_D \times 0.2} = \frac{2.5}{k_n R_D} \end{aligned} \quad (3)$$

Thus,

$$V_{DS}|_B = 0.2 + \frac{2.5}{k_n R_D}$$

Substituting for  $V_{DS}|_B$  from Eq. (2), we obtain

$$\frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} = 0.2 + \frac{2.5}{k_n R_D}$$

Substituting  $V_{DD} = 5 \text{ V}$ , rearranging the equation to obtain a quadratic equation in  $k_n R_D$ , and

solving the resulting quadratic equation results in

$$k_n R_D = 213.7$$

which can be substituted into Eq. (2) to obtain

$$V_{DS}|_B = 0.212 \text{ V}$$

The value of  $V_{DS}$  at the bias point can now be found from Eq. (1) as

$$V_{DS}|_Q = 0.212 + 0.5 = 0.712 \text{ V}$$

(b) The gain achieved can be found as

$$A_v = -k_n R_D V_{OV}$$

$$= -213.7 \times 0.2 = -42.7 \text{ V/V}$$

$$\hat{v}_{gs} = \frac{0.5}{|A_v|} = \frac{0.5}{42.7} = 11.7 \text{ mV}$$

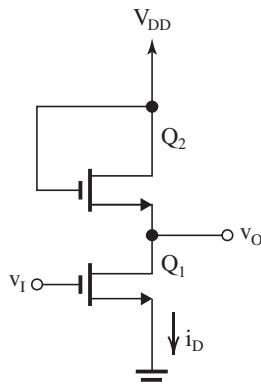
$$(c) I_D = 100 \mu\text{A}$$

$$\begin{aligned} R_D &= \frac{V_{DD} - V_{DS}|_Q}{I_D} \\ &= \frac{5 - 0.712}{0.1} = 42.88 \text{ k}\Omega \end{aligned}$$

$$(d) k_n = \frac{213.7}{42.88} = 4.98 \text{ mA/V}^2$$

$$\frac{W}{L} = \frac{4.98}{0.2} = 24.9$$

## 6.9



given  $V_{t1} = V_{t2} = V_t$

$$\text{For } Q_2, i_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_2 [V_{DD} - v_O - V_t]^2$$

$$\text{For } Q_1, i_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_1 [v_I - V_t]^2$$

For  $V_t \leq v_I \leq v_O + V_t$ ,

equate  $i_{D1}$  and  $i_{D2}$

$$\begin{aligned}
 & \left(\frac{W}{L}\right)_2 [V_{DD} - v_O + V_t]^2 \\
 &= \left(\frac{W}{L}\right)_1 [v_I - V_t]^2 \\
 [V_{DD} - v_O - V_t] &= \sqrt{\frac{(W/L)_1}{(W/L)_2}} \cdot [v_I - V_t] \\
 v_O &= V_{DD} - V_t + V_t \sqrt{\frac{(W/L)_1}{(W/L)_2}} \\
 &\quad - v_I \sqrt{\frac{(W/L)_1}{(W/L)_2}} \\
 \text{For } \sqrt{\frac{(W/L)_1}{(W/L)_2}} &= \sqrt{\frac{\left(\frac{50}{0.5}\right)}{\left(\frac{5}{0.5}\right)}} = \sqrt{10}, \\
 A_v &= -\sqrt{10} = -3.16 \text{ V/V}
 \end{aligned}$$

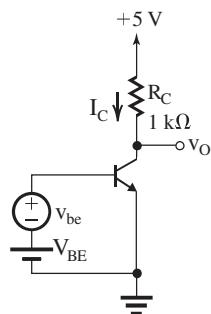
6.10 Refer to Fig. 6.6.

$$\begin{aligned}
 A_v &= -\frac{V_{CC} - V_{CE}}{V_T} \\
 &= -\frac{5 - 1}{0.025} = -160 \text{ V/V}
 \end{aligned}$$

The transistor enters saturation when  $v_{CE} \leq 0.3 \text{ V}$ , thus the maximum allowable output voltage swing is  $1 - 0.3 = 0.7 \text{ V}$ . The corresponding maximum input signal permitted  $\hat{v}_{be}$  is

$$\hat{v}_{be} = \frac{0.7 \text{ V}}{|A_v|} = \frac{0.7}{160} = 4.4 \text{ mV}$$

6.11



For  $I_C = 0.5 \text{ mA}$ , we have

$$A_v = -\frac{I_C R_C}{V_T} = -\frac{0.5}{0.025} = -20 \text{ V/V}$$

$$V_{CE} = V_{CC} - I_C R_C$$

$$= 5 - 0.5 = 4.5 \text{ V}$$

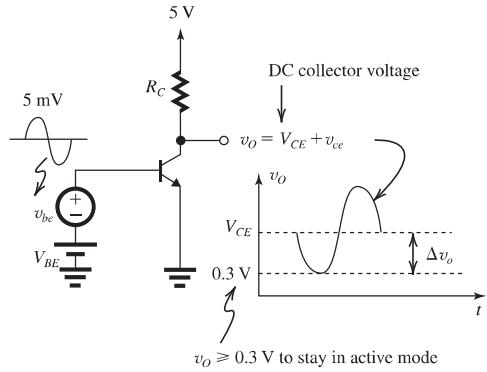
$$\max +\Delta v_O = 5 - 4.5 = 0.5 \text{ V}$$

$$\max -\Delta v_O = 4.5 - 0.3 = 4.2 \text{ V}$$

Similarly,

$I_C$ (mA)	$V_{CE}$ (V)	$A_v$ (V/V)	POS $\Delta v_O$ (V)	Neg $\Delta v_O$ (V)
0.5	4.5	-20	0.5	4.2
1.0	4.0	-40	1.0	3.7
2.5	2.5	-100	2.5	2.2
4.0	1.0	-160	4.0	0.7
4.5	0.5	-180	4.5	0.2

6.12



$$A_v = -\frac{I_C R_C}{V_T} = -\frac{V_{CC} - V_{CE}}{V_T}$$

On the verge of saturation

$$V_{CE} - \hat{v}_{ce} = 0.3 \text{ V}$$

For linear operation,  $v_{ce} = A_v v_{be}$

$$V_{CE} - |A_v| \hat{v}_{be} = 0.3$$

$$(5 - I_C R_C) - |A_v| \times 5 \times 10^{-3} = 0.3$$

But

$$|A_v| = \frac{I_C R_C}{V_T}$$

Thus,

$$I_C R_C = |A_v| V_T$$

and

$$5 - |A_v| V_T - |A_v| \times 5 \times 10^{-3} = 0.3$$

$$|A_v| (0.025 + 0.005) = 5 - 0.3$$

$$|A_v| = 156.67. \text{ Note } A_v \text{ is negative.}$$

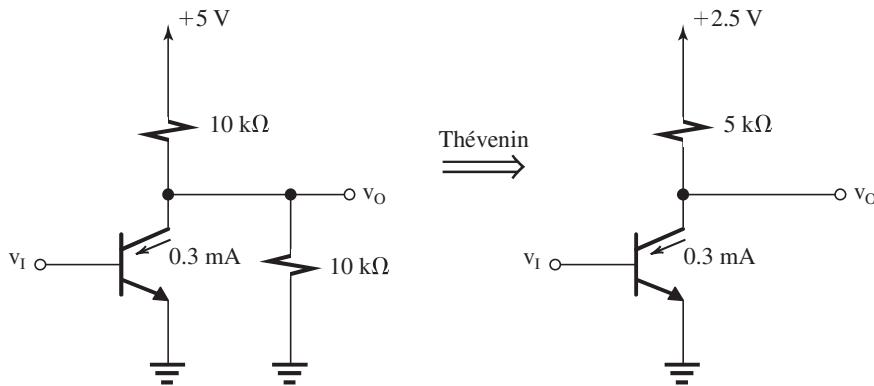
$$\therefore A_v = -156.67 \text{ V/V}$$

Now we can find the dc collector voltage.

Referring to the sketch of the output voltage, we see that

$$V_{CE} = 0.3 + |A_v| 0.005 = 1.08 \text{ V}$$

This figure belongs to Problem 6.13.



6.13 See figure above

$$A_v = -\frac{I_C R_C}{V_T} = -\frac{0.3 \times 5}{0.025} = -60 \text{ V/V}$$

6.14 To obtain an output signal of peak amplitude  $P$  volts and maximum gain, we bias the transistor at

$$V_{CE} = V_{CEsat} + P$$

The resulting gain will be

$$A_v = -\frac{V_{CC} - V_{CE}}{V_T}$$

which results in  $V_{CC}$  of

$$V_{CC} = V_{CE} + |A_v|V_T$$

Thus the minimum required  $V_{CC}$  will be

$$V_{CCmin} = V_{CEsat} + P + |A_v|V_T$$

but we have to make sure that the amplifier can support a positive peak amplitude of  $P$ , that is,

$$|A_v|V_T \geq P$$

In the results obtained, tabulated below,  $V_{CEsat} = 0.3 \text{ V}$  and  $V_{CC}$  is the nearest 0.5 V to  $V_{CCmin}$ .

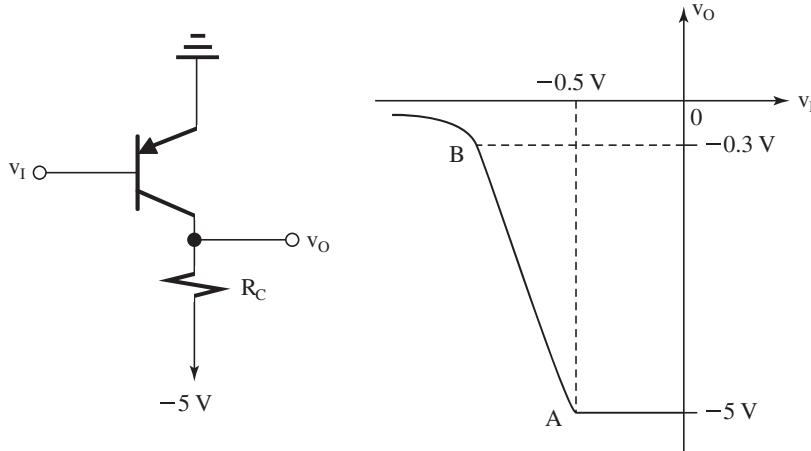
Case	$A_v (\text{V/V})$	$P (\text{V})$	$ A_v V_T$	$V_{CCmin}$	$V_{CC}$
a	-20	0.2	0.5	1.0	1.0
b	-50	0.5	1.25	2.05	2.5
c	-100	0.5	2.5	3.3	3.5
d	-100	1.0	2.5	3.8	4.0
e	-200	1.0	5.0	6.3	6.5
f	-500	1.0	12.5	13.8	14.0
g	-500	2.0	12.5	14.8	15.0

6.15 (a) See figure below

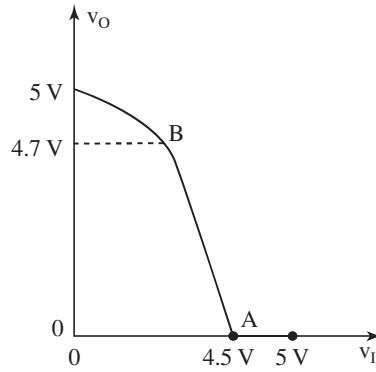
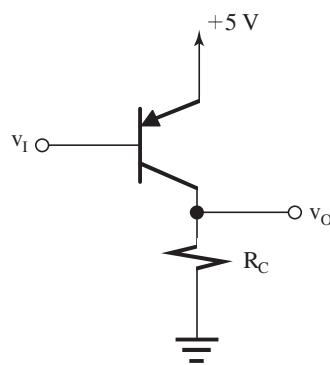
(b) See figure on next page

Note that in part (b) the graph is shifted right by +5 V and up by +5 V.

This figure belongs to Problem 6.15(a).



This figure belongs to Problem 6.15(b).



$$6.16 \quad i_C = I_S e^{v_{BE}/V_T} \left( 1 + \frac{V_{CE}}{V_A} \right)$$

$$I_C = I_S e^{v_{BE}/V_T} \left( 1 + \frac{V_{CE}}{V_A} \right)$$

$$v_{CE} = V_{CC} - R_C i_C$$

$$V_{CE} = V_{CC} - R_C I_C$$

$$A_v = \left. \frac{dv_{CE}}{dv_{BE}} \right|_{v_{BE}=V_{BE}, v_{CE}=V_{CE}}$$

$$= -R_C I_S \left( 1 + \frac{V_{CE}}{V_A} \right) e^{v_{BE}/V_T} \left( \frac{1}{V_T} \right)$$

$$= -R_C I_S e^{v_{BE}/V_T} \left( \frac{dv_{CE}}{dv_{BE}} \right) \left( \frac{1}{V_A} \right)$$

$$= -R_C I_C \frac{1}{V_T} - R_C \frac{I_C}{1 + \frac{V_{CE}}{V_A}} \left( \frac{1}{V_A} \right) A_v$$

Thus,

$$A_v = \frac{-I_C R_C / V_T}{1 + \frac{I_C R_C}{V_A + V_{CE}}} \quad \text{Q.E.D}$$

Substituting  $I_C R_C = V_{CC} - V_{CE}$ , we obtain

$$A_v = - \frac{(V_{CC} - V_{CE}) / V_T}{\left[ 1 + \frac{V_{CC} - V_{CE}}{V_A + V_{CE}} \right]} \quad \text{Q.E.D}$$

For  $V_{CC} = 5 \text{ V}$ ,  $V_{CE} = 3 \text{ V}$ , and  $V_A = 100 \text{ V}$ ,

$$A_v \text{ (without the Early effect)} = - \frac{5 - 3}{0.025} \\ = -80 \text{ V/V}$$

$$A_v \text{ (with the Early effect)} = \frac{-80}{1 + \frac{2}{100 + 3}} \\ = -78.5 \text{ V/V}$$

6.17 (a) Using Eq. (6.23) yields

$$|A_{v \max}| = \frac{V_{CC} - 0.3}{V_T} = \frac{3 - 0.3}{0.025} = 108 \text{ V/V}$$

(b) Using Eq. (6.22) with  $A_v = -60$  yields

$$-60 = - \frac{V_{CC} - V_{CE}}{V_T} = - \frac{3 - V_{CE}}{0.025}$$

$$\Rightarrow V_{CE} = 1.5 \text{ V}$$

(c)  $I_C = 0.5 \text{ mA}$

$$I_C R_C = V_{CC} - V_{CE} = 3 - 1.5 = 1.5 \text{ V}$$

$$R_C = \frac{1.5}{0.5} = 3 \text{ k}\Omega$$

(d)  $I_C = I_S e^{v_{BE}/V_T}$

$$0.5 \times 10^{-3} = 10^{-15} e^{v_{BE}/0.025}$$

$$\Rightarrow V_{BE} = 0.673 \text{ V}$$

(e) Assuming linear operation around the bias point, we obtain

$$v_{ce} = A_v \times v_{be}$$

$$= -60 \times 5 \sin \omega t = -300 \sin \omega t, \text{ mV}$$

$$= -0.3 \sin \omega t, \text{ V}$$

$$(f) i_c = \frac{-v_{ce}}{R_C} = 0.1 \sin \omega t, \text{ mA}$$

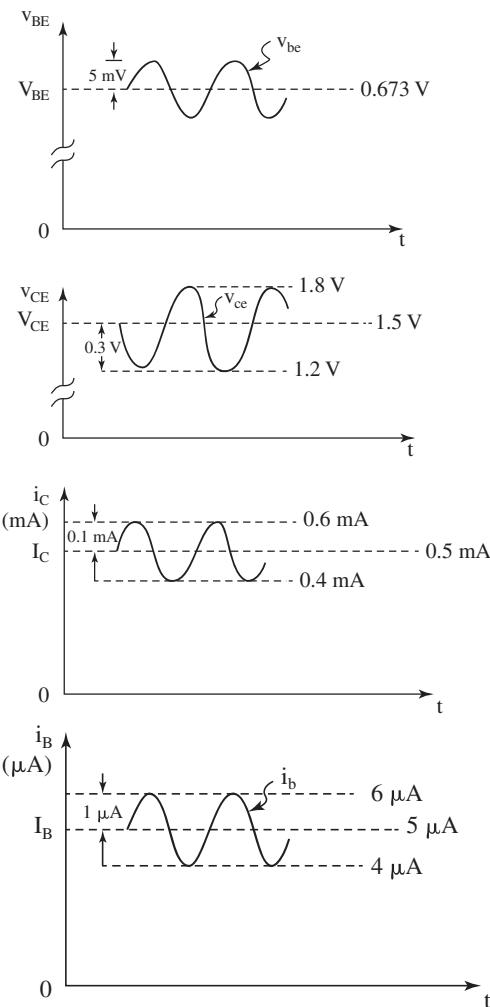
$$(g) I_B = \frac{I_C}{\beta} = \frac{0.5 \text{ mA}}{100} = 0.005 \text{ mA}$$

$$i_b = \frac{i_c}{\beta} = \frac{0.1}{100} \sin \omega t = 0.001 \sin \omega t, \text{ mA}$$

$$(h) \text{ Small-signal input resistance } \equiv \frac{\hat{v}_{be}}{\hat{v}_b}$$

$$= \frac{5 \text{ mV}}{0.001 \text{ mA}} = 5 \text{ k}\Omega$$

(i)



$$6.18 \quad A_v = -\left(\frac{I_C}{V_T}\right)R_C$$

But

$$A_v \equiv \frac{\Delta V_O}{\Delta V_{BE}} = \frac{-\Delta i_C R_C}{\Delta V_{BE}} = -g_m R_C$$

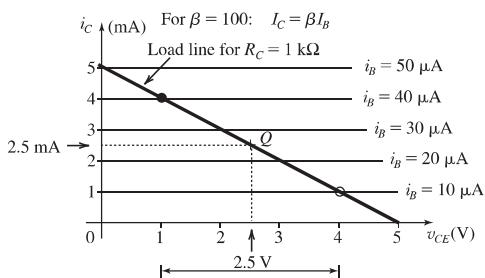
Thus,

$$g_m = I_C / V_T$$

 For a transistor biased at  $I_C = 0.5 \text{ mA}$ , we have

$$g_m = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

6.19



$$\text{Peak-to-peak } v_C \text{ swing} = 4 - 1 = 3 \text{ V}$$

 For point Q at  $V_{CC}/2 = 2.5 \text{ V}$ , we obtain

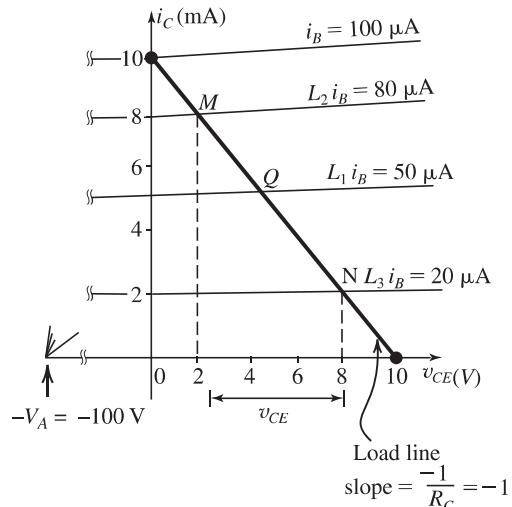
$$V_{CE} = 2.5 \text{ V}, \quad I_C = 2.5 \text{ mA}$$

$$I_B = 25 \mu\text{A}$$

$$I_B = \frac{V_{BB} - 0.7}{R_B} = 25 \mu\text{A}$$

$$\Rightarrow V_{BB} = I_B R_B + 0.7 = 2.5 + 0.7 = 3.2 \text{ V}$$

6.20 See the graphical construction that follows.



For this circuit:

$$V_{CC} = 10 \text{ V}, \quad \beta = 100,$$

$$R_C = 1 \text{ k}\Omega, \quad V_A = 100 \text{ V},$$

$$I_B = 50 \mu\text{A} \text{ (dc bias)},$$

$$\text{At } v_{CE} = 0, i_C = \beta i_B$$

$$\therefore I_C = 50 \times 100$$

$$= 5 \text{ mA (dc bias)}$$

 Given the base bias current of 50 mA, the dc or bias point of the collector current  $I_C$ , and voltage  $V_{CE}$  can be found from the intersection of the load

## Chapter 6–8

line and the transistor line  $L_1$  of  $i_B = 50 \mu A$ . Specifically:

$$\text{Eq. of } L_1 \Rightarrow i_C = I_C (1 + v_{CE}/V_A)$$

$$= 5 (1 + v_{CE}/100)$$

$$= 5 + 0.05v_{CE}$$

$$\text{Load line} \Rightarrow i_C = \frac{V_{CC} - v_{CE}}{R_C} = 10 - v_{CE}$$

$$\therefore 10 - v_{CE} = 5 + 0.05v_{CE}$$

$$v_{CE} = v_{CE} = 4.76 \text{ V}$$

$$I_C = i_C = 10 - v_{CE} = 5.24 \text{ mA}$$

Now for a signal of  $30\text{-}\mu\text{A}$  peak superimposed on  $I_B = 50 \mu\text{A}$ , the operating point moves along the load line between points N and M. To obtain the coordinates of point M, we solve the load line and line  $L_2$  to find the intersection M, and the load line and line  $L_3$  to find N:

For point M:

$$i_C = 8 + (8/100)v_{CE} \text{ and } i_C = 10 - v_{CE}$$

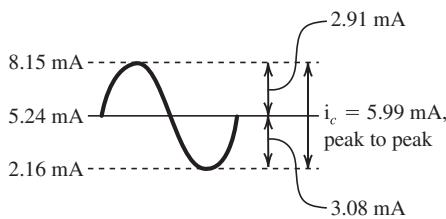
$$\therefore i_C|_M = 8.15 \text{ mA}, \quad v_{CE}|_M = 1.85 \text{ V}$$

For point N:

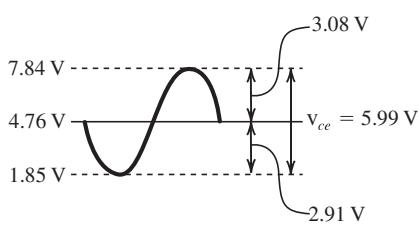
$$i_C = 2 + 0.02v_{CE} \text{ and } i_C = 10 - v_{CE}$$

$$v_{CE}|_N = 7.84 \text{ V}, \quad i_C|_N = 2.16 \text{ mA}$$

Thus the collector current varies as follows:



And the collector voltage varies as follows:



6.21 Substituting  $v_{gs} = V_{gs} \sin \omega t$  in Eq. (6.28),

$$i_D = \frac{1}{2}k_n(V_{GS} - V_t)^2 + k_n(V_{GS} - V_t)V_{gs} \sin \omega t \\ + \frac{1}{2}k_nV_{gs}^2 \sin^2 \omega t$$

$$= \frac{1}{2}k_n(V_{GS} - V_t)^2 + k_n(V_{GS} - V_t)V_{gs} \sin \omega t \\ + \frac{1}{2}k_nV_{gs}^2 \left(\frac{1}{2} - \frac{1}{2} \cos 2\omega t\right)$$

Second-harmonic distortion

$$= \frac{\frac{1}{4}k_nV_{gs}^2}{k_n(V_{GS} - V_t)V_{gs}} \times 100 \\ = \frac{1}{4} \frac{V_{gs}}{V_{OV}} \times 100 \quad \text{Q.E.D}$$

For  $V_{gs} = 10 \text{ mV}$ , to keep the second-harmonic distortion to less than 1%, the minimum overdrive voltage required is

$$V_{OV} = \frac{1}{4} \times \frac{0.01 \times 100}{1} = 0.25 \text{ V}$$

$$6.22 \quad I_D = \frac{1}{2}k_nV_{OV}^2 = \frac{1}{2} \times 10 \times 0.2^2 = 0.2 \text{ mA}$$

$$v_{GS} = V_{GS} + v_{gs}, \text{ where } v_{gs} = 0.02 \text{ V}$$

$$v_{OV} = 0.2 + 0.02 = 0.22 \text{ V}$$

$$i_D = \frac{1}{2}k_nV_{OV}^2 = \frac{1}{2} \times 10 \times 0.22^2 = 0.242 \text{ mA}$$

Thus,

$$i_d = 0.242 - 0.2 = 0.042 \text{ mA}$$

For

$$v_{gs} = -0.02 \text{ V}, \quad v_{OV} = 0.2 - 0.02 = 0.18 \text{ V}$$

$$i_D = \frac{1}{2}k_nV_{OV}^2 = \frac{1}{2} \times 10 \times 0.18^2 = 0.162 \text{ mA}$$

Thus,

$$i_d = 0.2 - 0.162 = 0.038 \text{ mA}$$

Thus, an estimate of  $g_m$  can be obtained as follows:

$$g_m = \frac{0.042 + 0.038}{0.04} = 2 \text{ mA/V}$$

Alternatively, using Eq. (6.33), we can write

$$g_m = k_nV_{OV} = 10 \times 0.2 = 2 \text{ mA/V}$$

which is an identical result.

$$6.23 \quad (a) \quad I_D = \frac{1}{2}k_n(V_{GS} - V_t^2)$$

$$= \frac{1}{2} \times 5(0.6 - 0.4)^2 = 0.1 \text{ mA}$$

$$V_{DS} = V_{DD} - I_D R_D = 1.8 - 0.1 \times 10 = 0.8 \text{ V}$$

$$(b) \quad g_m = k_nV_{OV} = 5 \times 0.2 = 1 \text{ mA/V}$$

$$(c) \quad A_v = -g_m R_D = -1 \times 10 = -10 \text{ V/V}$$

$$(d) \quad \lambda = 0.1 \text{ V}^{-1}, \quad V_A = \frac{1}{\lambda} = 10 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$A_v = -g_m(R_D \parallel r_o)$$

$$= -1(10 \parallel 100) = -9.1 \text{ V/V}$$

$$6.24 A_v = -10 = -g_m R_D = -g_m \times 20$$

$$g_m = 0.5 \text{ mA/V}$$

To allow for a  $-0.2\text{-V}$  signal swing at the drain while maintaining saturation-region operation, the minimum voltage at the drain must be at least equal to  $V_{OV}$ . Thus

$$V_{DS} = 0.2 + V_{OV}$$

Since

$$A_v = -\frac{\frac{V_{DD} - V_{DS}}{\frac{1}{2}V_{OV}}}{2}$$

$$-10 = -\frac{1.8 - 0.2 - V_{OV}}{0.5V_{OV}}$$

$$\Rightarrow V_{OV} = 0.27 \text{ V}$$

The value of  $I_D$  can be found from

$$g_m = \frac{2I_D}{V_{OV}}$$

$$0.5 = \frac{2 \times I_D}{0.27}$$

$$\Rightarrow I_D = 0.067 \text{ mA}$$

The required value of  $k_n$  can be found from

$$I_D = \frac{1}{2}k_n V_{OV}^2$$

$$0.067 = \frac{1}{2}k_n \times 0.27^2$$

$$\Rightarrow k_n = 1.83 \text{ mA/V}^2$$

Since  $k'_n = 0.2 \text{ mA/V}^2$ , the W/L ratio must be

$$\frac{W}{L} = \frac{k_n}{k'_n} = \frac{1.83}{0.2} = 9.14$$

Finally,

$$V_{GS} = V_t + V_{OV} = 0.4 + 0.27 = 0.67 \text{ V}$$

$$6.25 A_v = -g_m R_D$$

Upon substituting for  $g_m$  from Eq. (6.42), we can write

$$A_v = -\frac{2I_D R_D}{V_{OV}} = -\frac{2(V_{DD} - V_{DS})}{V_{OV}} \quad \text{Q.E.D} \quad (1)$$

$$v_{GS}|_{max} = V_{GS} + \hat{v}_i = V_t + V_{OV} + \hat{v}_i$$

$$v_{DS}|_{min} = V_{DS} - |A_v|\hat{v}_i$$

To just maintain saturation-mode operation,

$$v_{GS}|_{max} = v_{DS}|_{min} + V_t$$

which results in

$$V_{OV} + \hat{v}_i = V_{DS} - |A_v|\hat{v}_i$$

Substituting for  $|A_v|$  from Eq. (1) yields

$$V_{OV} + \hat{v}_i = V_{DS} - \frac{2(V_{DD} - V_{DS})}{V_{OV}} \hat{v}_i$$

$$V_{DS}[1 + 2(\hat{v}_i/V_{OV})]$$

$$= V_{OV} + \hat{v}_i + 2V_{DD}(\hat{v}_i/V_{OV})$$

$$\Rightarrow V_{DS} = \frac{V_{OV} + \hat{v}_i + 2V_{DD}(\hat{v}_i/V_{OV})}{1 + 2(\hat{v}_i/V_{OV})} \quad \text{Q.E.D}$$

For

$$V_{DD} = 2.5 \text{ V}, \hat{v}_i = 20 \text{ mV} \text{ and } m = 15$$

$$V_{OV} = m\hat{v}_i = 15 \times 20 = 0.3 \text{ V}$$

$$V_{DS} = \frac{0.3 + 0.02 + 2 \times 2.5 \times (0.02/0.3)}{1 + 2(0.02/0.3)}$$

$$= 0.576 \text{ V}$$

$$A_v = -\frac{2(V_{DD} - V_{DS})}{V_{OV}} = -\frac{2(2.5 - 0.576)}{0.3}$$

$$= -12.82 \text{ V/V}$$

$$\hat{v}_o = |A_v|\hat{v}_i = 12.82 \times 20 \text{ mV} = 0.256 \text{ V}$$

To operate at  $I_D = 200 \mu\text{A} = 0.2 \text{ mA}$ ,

$$R_D = \frac{2.5 - 0.576}{0.2} = 9.62 \text{ k}\Omega$$

$$I_D = \frac{1}{2}k_n V_{OV}^2$$

$$0.2 = \frac{1}{2}k_n \times 0.3^2$$

$$\Rightarrow k_n = 4.44 \text{ mA/V}^2$$

The required W/L ratio can now be found as

$$\frac{W}{L} = \frac{k_n}{k'_n} = \frac{4.44}{0.1} = 44.4$$

$$6.26 \text{ Given } \mu_n = 500 \text{ cm}^2/\text{V}\cdot\text{s},$$

$$\mu_p = 250 \text{ cm}^2/\text{V}\cdot\text{s}, \text{ and } C_{ox} = 0.4 \text{ fF}/\mu\text{m}^2,$$

$$k'_n = \mu_n C_{ox} = 20 \mu\text{A/V}^2$$

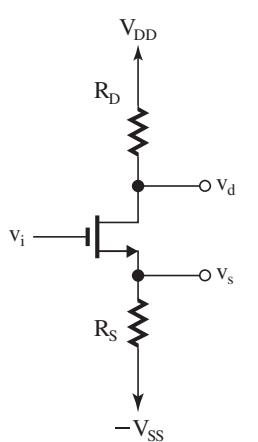
$$k'_p = 10 \mu\text{A/V}^2$$

See table on next page.

Case type	$I_D$ (mA)	$ V_{GS} $ (V)	$ V_t $ (V)	$ V_{OV} $ (V)	$W$ ( $\mu\text{m}$ )	$L$ ( $\mu\text{m}$ )	$\frac{W}{L}$	$k' \frac{W}{L}$ (mA/V <sup>2</sup> )	$g_m$ (mA/V)
a (N)	①	③	②	1	100	①	100	2	2
b (N)	①	1.2	⑥.7	⑤.5	⑤0	0.125	400	8	4
c (N)	⑩	—	—	②	250	①	250	5	10
d (N)	⑥.5	—	—	⑥.5	—	—	200	4	2
e (N)	⑥.1	—	—	1.41	⑩	②	5	0.1	0.141
f (N)	0.1	⑧.8	⑧.8	1	④0	④	10	0.2	0.2
g (P)	⑥.5	—	—	2	—	—	②5	0.25	0.5
h (P)	1	③	①	2	—	—	50	⑥.5	1
i (P)	⑩	—	—	1	④000	②	2000	20	20
j (P)	⑩	—	—	④	—	—	125	1.25	5
k (P)	0.05	—	—	①	③0	③	10	0.1	0.1
l (P)	1	—	—	⑤	—	—	8	⑥.08	0.4

Note: The circled entries are the givens.

6.27



$$v_i = (g_m v_{gs}) \left( \frac{1}{g_m} + R_S \right)$$

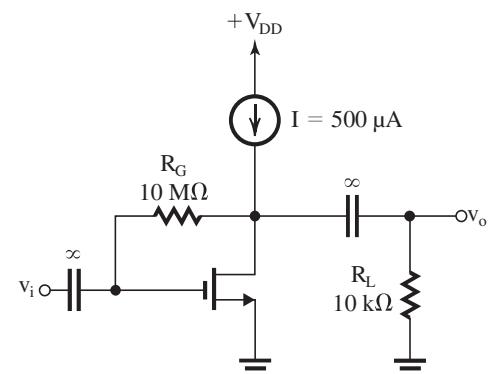
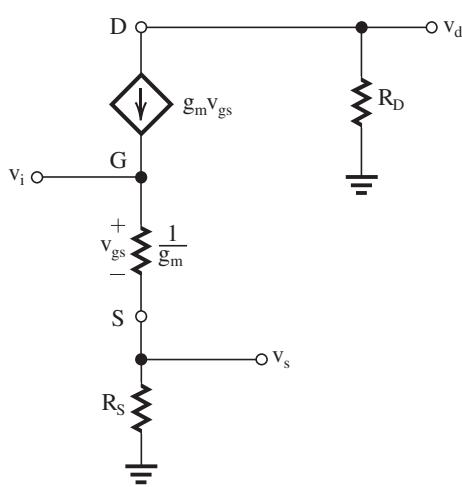
$$v_d = -g_m v_{gs} R_D$$

$$v_s = +g_m v_{gs} R_S$$

$$\therefore \frac{v_s}{v_i} = \frac{R_S}{\frac{1}{g_m} + R_S} = \frac{+g_m R_S}{1 + g_m R_S}$$

$$\frac{v_d}{v_i} = \frac{-R_D}{\frac{1}{g_m} + R_S} = \frac{-g_m R_D}{1 + g_m R_S}$$

6.28



$$V_t = 0.5 \text{ V}$$

$$V_A = 50 \text{ V}$$

Given  $V_{DS} = V_{GS} = 1 \text{ V}$ . Also,  $I_D = 0.5 \text{ mA}$ .

$$V_{OV} = 0.5 \text{ V}, g_m = \frac{2I_D}{V_{OV}} = 2 \text{ mA/V}$$



$$(d) R_{in} = 10 \text{ M}\Omega \parallel 5 \text{ M}\Omega = 3.33 \text{ M}\Omega$$

$$\frac{v_{gs}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{3.33}{3.33 + 0.2}$$

$$= 0.94 \text{ V/V}$$

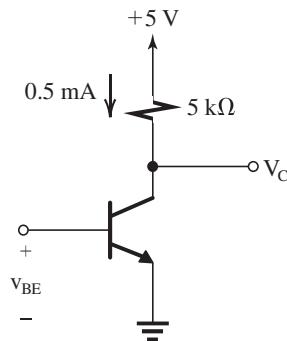
$$\frac{v_o}{v_{gs}} = -g_m(200 \parallel 16 \parallel 16)$$

$$= -2 \times 7.69 = -15.38 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = \frac{v_{gs}}{v_{sig}} \times \frac{v_o}{v_{gs}} = -0.94 \times 15.38$$

$$= -14.5 \text{ V/V}$$

6.32



6.31 (a) Using the exponential characteristic:

$$i_c = I_C e^{v_{be}/V_T} - I_C$$

$$\text{giving } \frac{i_c}{I_C} = e^{v_{be}/V_T} - 1$$

(b) Using small-signal approximation:

$$i_c = g_m v_{be} = \frac{I_C}{V_T} \cdot v_{be}$$

$$\text{Thus, } \frac{i_c}{I_C} = \frac{v_{be}}{V_T}$$

See table below.

For signals at  $\pm 5 \text{ mV}$ , the error introduced by the small-signal approximation is 10%.

The error increases to above 20% for signals at  $\pm 10 \text{ mV}$ .

With  $v_{BE} = 0.700 \text{ V}$

$$V_C = V_{CC} - R_C I_C$$

$$= 5 - 5 \times 0.5 = 2.5 \text{ V}$$

$$\text{For } v_{BE} = 705 \text{ mV} \Rightarrow v_{be} = 5 \text{ mV}$$

$$i_c = I_C e^{v_{be}/V_T}$$

$$= 0.5 \times e^{5/25} = 0.611 \text{ mA}$$

$$v_C = V_{CC} - R_C i_C = 5 - 5 \times 0.611 = 1.95 \text{ V}$$

$$v_{ce} = v_C - V_C = 1.95 - 2.5 = -0.55 \text{ V}$$

$$\text{Voltage gain, } A_v = \frac{v_{ce}}{v_{be}} = -\frac{0.55}{5 \text{ mV}}$$

$$= -110 \text{ V/V}$$

Using small-signal approximation, we write

$$A_v = -g_m R_C$$

where

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$A_v = -20 \times 5 = -100 \text{ V/V}$$

Thus, the small-signal approximation at this signal level ( $v_{be} = 5 \text{ mV}$ ) introduces an error of -9.1% in the gain magnitude.

6.33 At  $I_C = 0.5 \text{ mA}$ ,

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20 \text{ mA/V}} = 5 \text{ k}\Omega$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C}$$

where

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{100 + 1} = 0.99$$

$$r_e = \frac{0.99 \times 25 \text{ mV}}{0.5 \text{ mA}} \simeq 50 \text{ }\Omega$$

At  $I_C = 50 \mu\text{A} = 0.05 \text{ mA}$ ,

$$g_m = \frac{I_C}{V_T} = \frac{0.05}{0.025} = 2 \text{ mA/V}$$

$v_{be}$ (mV)	$i_c/I_C$ Exponential	$i_c/I_C$ Small signal	Error (%)
+1	+0.041	+0.040	-2.4
-1	-0.039	-0.040	+2.4
+2	+0.083	+0.080	-3.6
-2	-0.077	-0.080	+3.9
+5	+0.221	+0.200	-9.7
-5	-0.181	-0.200	+10.3
+8	+0.377	+0.320	-15.2
-8	-0.274	-0.320	+16.8
+10	+0.492	+0.400	-18.7
-10	-0.330	-0.400	+21.3
+12	+0.616	+0.480	-22.1
-12	-0.381	-0.480	+25.9

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{2 \text{ mA/V}} = 50 \text{ k}\Omega$$

$$r_e = \frac{\alpha V_T}{I_C} = \frac{0.99 \times 25 \text{ mV}}{0.5 \text{ mA}} \simeq 500 \text{ }\Omega$$

6.34 For  $g_m = 30 \text{ mA/V}$ ,

$$g_m = \frac{I_C}{V_T} \Rightarrow I_C = g_m V_T = 30 \times 0.025 = 0.75 \text{ mA}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{\beta}{30 \text{ mA/V}}$$

For  $r_\pi \geq 3 \text{ k}\Omega$ , we require

$$\beta \geq 90$$

That is,  $\beta_{\min} = 90$ .

$$6.35 \quad g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_e = \frac{\alpha}{g_m} = \frac{0.99}{40 \text{ mA/V}} \simeq 25 \text{ }\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40 \text{ mA/V}} = 2.5 \text{ k}\Omega$$

$$A_v = -g_m R_C = -40 \times 5 = -200 \text{ V/V}$$

$$\hat{v}_o = |A_v| \hat{V}_{be} = 200 \times 5 \text{ mV} = 1 \text{ V}$$

$$6.36 \quad V_{CC} = 3 \text{ V}, \quad V_C = 1 \text{ V}, \quad R_C = 2 \text{ k}\Omega$$

$$I_C = \frac{3 - 1}{2} = 1 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$v_{be} = 0.005 \sin \omega t$$

$$i_c = g_m v_{be} = 0.2 \sin \omega t, \text{ mA}$$

$$i_C(t) = I_C + i_c = 1 + 0.2 \sin \omega t, \text{ mA}$$

$$v_C(t) = V_{CC} - R_C i_C$$

$$= 3 - 2(1 + 0.2 \sin \omega t)$$

$$= 1 - 0.4 \sin \omega t, \text{ V}$$

$$i_B(t) = i_C(t)/\beta$$

$$= 0.01 + 0.002 \sin \omega t, \text{ mA}$$

$$A_v = \frac{v_C}{v_{be}} = -\frac{0.4}{0.005} = -80 \text{ V/V}$$

6.37 Since  $\hat{V}_{be}$  is the maximum value for acceptable linearity, the largest signal at the collector will be obtained by designing for maximum gain magnitude. This in turn is achieved by biasing the transistor at the lowest  $V_{CE}$  consistent with the transistor remaining in the active mode at the negative peak of  $v_o$ . Thus

$$V_{CE} - |A_v| \hat{V}_{be} = 0.3$$

where we have assumed  $V_{CEsat} = 0.3 \text{ V}$ . Since

$$V_{CE} = V_{CC} - I_C R_C$$

and

$$|A_v| = g_m R_C = \frac{I_C}{V_T} R_C$$

then

$$V_{CC} - I_C R_C - \frac{\hat{V}_{be}}{V_T} I_C R_C = 0.3$$

which can be manipulated to yield

$$I_C R_C = \frac{V_{CC} - 0.3}{1 + \frac{\hat{V}_{be}}{V_T}} \quad (1)$$

Since the voltage gain is given by

$$A_v = -\frac{I_C R_C}{V_T}$$

then

$$A_v = \frac{V_{CC} - 0.3}{V_T + \hat{V}_{be}}$$

For  $V_{CC} = 3 \text{ V}$  and  $\hat{V}_{be} = 5 \text{ mV}$ ,

$$I_C R_C = \frac{3 - 0.3}{1 + \frac{5}{25}} = 2.25 \text{ V}$$

Thus,

$$V_{CE} = V_{CC} - I_C R_C$$

$$= 3 - 2.25 = 0.75 \text{ V}$$

$$\hat{V}_o = V_{CE} - 0.3 = 0.75 - 0.3 = 0.45 \text{ V}$$

$$A_v = -\frac{3 - 0.3}{0.025 + 0.005} = -90 \text{ V/V}$$

Check:

$$A_v = -g_m R_C = -\frac{I_C R_C}{V_T} = -\frac{2.25}{0.025} = -90 \text{ V/V}$$

$$\hat{V}_o = |A_v| \times \hat{V}_{be} = 90 \times 5 = 450 \text{ mV} = 0.45 \text{ V}$$

6.38

Transistor	a	b	c	d	e	f	g
$\alpha$	1.000	0.990	0.980	1	0.990	0.900	0.940
$\beta$	$\infty$	100	50	$\infty$	100	9	15.9
$I_C (\text{mA})$	1.00	0.99	1.00	1.00	0.248	4.5	17.5
$I_E (\text{mA})$	1.00	1.00	1.02	1.00	0.25	5	18.6
$I_B (\text{mA})$	0	0.010	0.020	0	0.002	0.5	1.10
$g_m (\text{mA/V})$	40	39.6	40	40	9.92	180	700
$r_e (\Omega)$	25	25	24.5	25	100	5	1.34
$r_\pi (\Omega)$	$\infty$	2.525 k	1.25 k	$\infty$	10.1 k	50	22.7

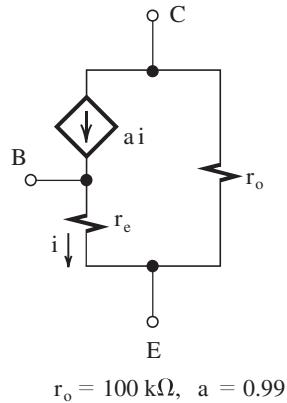
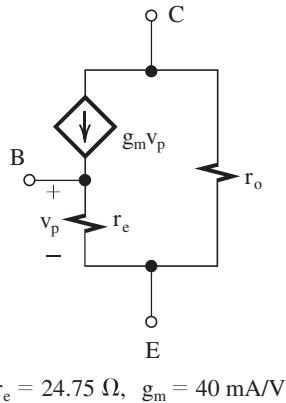
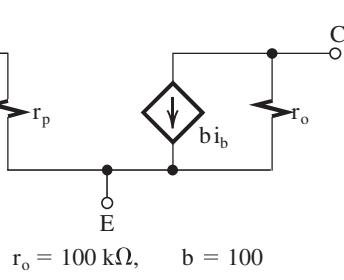
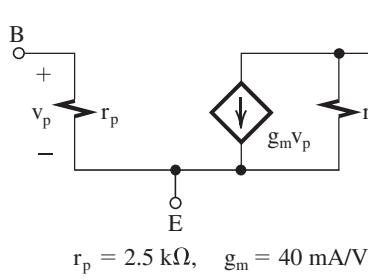
$$6.39 \quad I_C = 1 \text{ mA}, \quad \beta = 100, \quad V_A = 100 \text{ V}$$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40 \text{ mA/V}} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

These figures belong to Problem 6.39.

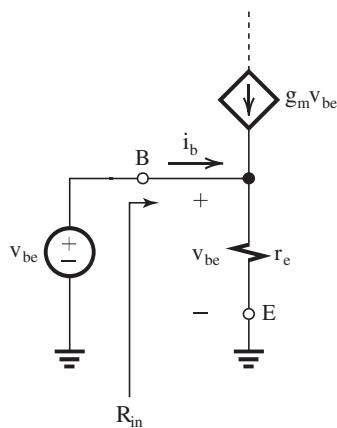


$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{100 + 1} = 0.99$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{0.99 \times 25 \text{ mV}}{1 \text{ mA}} = 24.75 \Omega$$

6.40

$$\begin{aligned} i_b &= v_{be} \left( \frac{g_m}{\alpha} - g_m \right) \\ &= g_m v_{be} \frac{1 - \alpha}{\alpha} \\ &= \frac{g_m v_{be}}{\beta} \\ R_{in} &\equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = r_\pi \quad \text{Q.E.D} \end{aligned}$$



$$i_b = \frac{v_{be}}{r_e} - g_m v_{be}$$

$$= v_{be} \left( \frac{1}{r_e} - g_m \right)$$

Since

$$r_e = \frac{\alpha}{g_m}$$

6.41 Refer to Fig. 6.26.

$$\begin{aligned} i_c &= \alpha i_e = \alpha \frac{v_{be}}{r_e} = \frac{\alpha}{r_e} v_{be} \\ &= g_m v_{be} \quad \text{Q.E.D} \end{aligned}$$

6.42 The large-signal model of Fig. 4.5(d) is shown in Fig. 1.

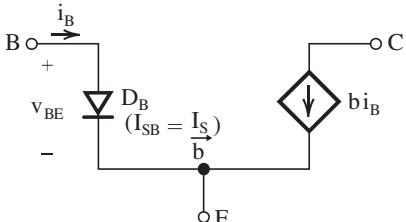


Figure 1

For  $v_{BE}$  undergoing an incremental change  $v_{be}$  from its equilibrium value of  $V_{BE}$ , the current  $i_B$

changes from  $I_B$  by an increment  $i_b$ , which is related to  $v_{be}$  by the incremental resistance of  $D_B$  at the bias current  $I_B$ . This resistance is given by  $V_T/I_B$ , which is  $r_\pi$ .

The collector current  $\beta i_B$  changes from  $\beta I_B$  to  $\beta(I_B + i_b)$ . The incremental changes around the equilibrium or bias point are related to each other by the circuit shown in Fig. 2,

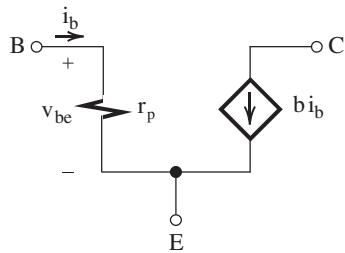


Figure 2

which is the hybrid- $\pi$  model of Fig. 6.24(b). Q.E.D.

6.43 The large-signal T model of Fig. 4.5(b) is shown below in Fig. 1.

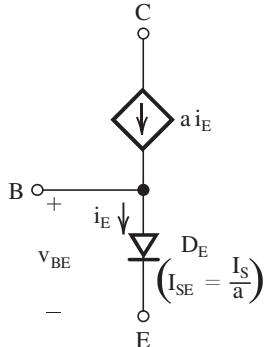


Figure 1

If  $i_E$  undergoes an incremental change  $i_e$  from its equilibrium or bias value  $I_E$ , the voltage  $v_{BE}$  will correspondingly change by an incremental amount  $v_{be}$  (from its equilibrium or bias value  $V_{BE}$ ), which is related to  $i_e$  by the incremental resistance of diode  $D_E$ . The latter is equal to  $V_T/I_E$ , which is  $r_e$ .

The incremental change  $i_e$  in  $i_E$  gives rise to an incremental change  $\alpha i_e$  in the current of the controlled source.

The incremental quantities can be related by the equivalent circuit model shown in Fig. 2,

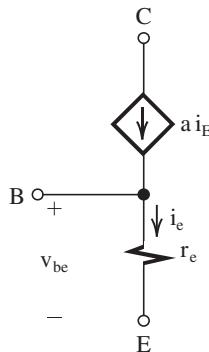


Figure 2

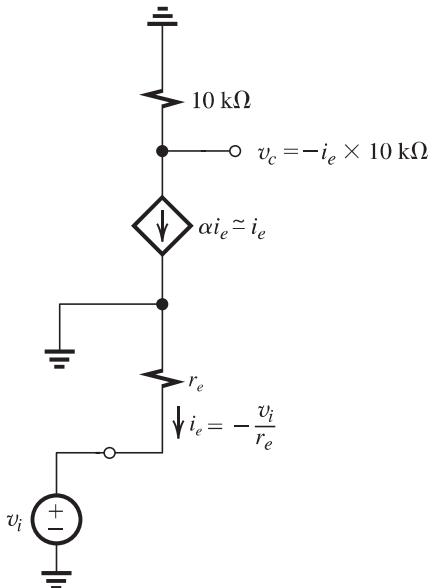
which is the small-signal T model of Fig. 6.26(b). Q.E.D.

6.44 Refer to Fig. P6.44:

$$V_C = 3 - 0.2 \times 10 = 1 \text{ V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.2 \text{ mA}} = 125 \Omega$$

Replacing the BJT with the T model of Fig. 6.26(b), we obtain the equivalent circuit shown below.



$$v_c = -i_e \times 10 \text{ k}\Omega$$

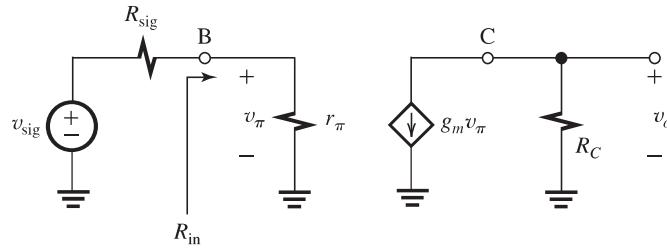
where

$$i_e = -\frac{v_i}{r_e} = -\frac{v_i}{0.125 \text{ k}\Omega}$$

Thus,

$$\begin{aligned} \frac{v_c}{v_i} &= \frac{10 \text{ k}\Omega}{0.125 \text{ k}\Omega} \\ &= 80 \text{ V/V} \end{aligned}$$

This figure belongs to Problem 6.45.



6.45

$$R_{in} \equiv \frac{v_\pi}{i_b} = r_\pi$$

$$\frac{v_\pi}{v_{sig}} = \frac{r_\pi}{r_\pi + R_{sig}}$$

$$v_o = -g_m v_\pi R_C$$

$$\frac{v_o}{v_\pi} = -g_m R_C$$

The overall voltage gain can be obtained as follows:

$$\frac{v_o}{v_{sig}} = \frac{v_o}{v_\pi} \frac{v_\pi}{v_{sig}}$$

$$= -g_m R_C \frac{r_\pi}{r_\pi + R_{sig}}$$

$$= -g_m r_\pi \frac{R_C}{r_\pi + R_{sig}}$$

$$= -\frac{\beta R_C}{r_\pi + R_{sig}} \quad Q.E.D.$$

6.46  $v_{ce} = |A_v| v_{be}$

$$|A_v| = g_m R_C = 50 \times 2 = 100 \text{ V/V}$$

For  $v_{ce}$  being 1 V peak to peak,

$$v_{be} = \frac{1 \text{ V}}{100} = 0.01 \text{ V peak to peak}$$

$$i_b = \frac{v_{be}}{r_\pi}$$

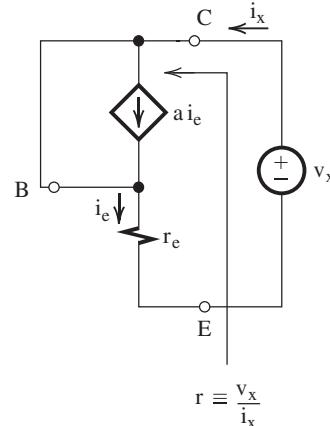
where

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{50} = 2 \text{ k}\Omega$$

Thus,

$$i_b = \frac{0.01 \text{ V}}{2 \text{ k}\Omega} = 0.005 \text{ mA peak to peak}$$

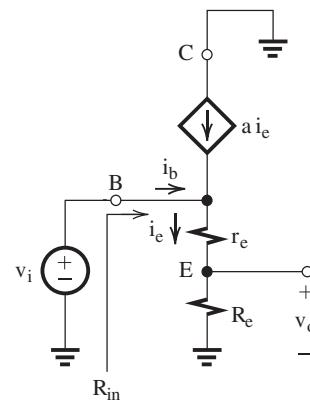
6.47 Replacing the BJT with the T model of Fig. 6.26(b), we obtain the circuit shown in next column above.



Since  $v_x$  appears across  $r_e$  and  $i_x = i_e = \frac{v_x}{r_e}$ , the small-signal resistance  $r$  is given by

$$r \equiv \frac{v_x}{i_x} = \frac{v_x}{i_e} = r_e$$

6.48 Refer to Fig. P6.48. Replacing the BJT with the T model of Fig. 6.26(b) results in the following amplifier equivalent circuit:



$$R_{in} \equiv \frac{v_i}{i_b} = \frac{v_i}{(1 - \alpha)i_e}$$

From the circuit we see that

$$i_e = \frac{v_i}{r_e + R_e}$$

Thus,

$$R_{in} = \frac{r_e + R_e}{1 - \alpha}$$

But

$$1 - \alpha = \frac{1}{\beta + 1}$$

Thus,

$$R_{in} = (\beta + 1)(r_e + R_e) \quad Q.E.D.$$

From the equivalent circuit, we see that  $v_o$  and  $v_i$  are related by the ratio of the voltage divider formed by  $r_e$  and  $R_e$ :

$$\frac{v_o}{v_i} = \frac{R_e}{R_e + r_e} \quad Q.E.D.$$

6.49 Refer to Fig. P6.49.

$$\alpha = \frac{\beta}{\beta + 1} = \frac{200}{201} = 0.995$$

$$I_C = \alpha \times I_E = 0.995 \times 10 = 9.95 \text{ mA}$$

$$V_C = I_C R_C = 9.95 \times 0.1 \text{ k}\Omega = 0.995 \text{ V} \simeq 1 \text{ V}$$

Replacing the BJT with its hybrid- $\pi$  model results in the circuit shown below.

$$g_m = \frac{I_C}{V_T} \simeq \frac{10 \text{ mA}}{0.025 \text{ V}} = 400 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{400} = 0.5 \text{ k}\Omega$$

$$R_{ib} = r_\pi = 0.5 \text{ k}\Omega$$

$$R_{in} = 10 \text{ k}\Omega \parallel 0.5 \text{ k}\Omega = 0.476 \text{ k}\Omega$$

$$\frac{v_\pi}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{0.476}{0.476 + 1} = 0.322 \text{ V/V}$$

$$\frac{v_o}{v_\pi} = -g_m R_C = -400 \times 0.1 = -40 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = -40 \times 0.322 = -12.9 \text{ V/V}$$

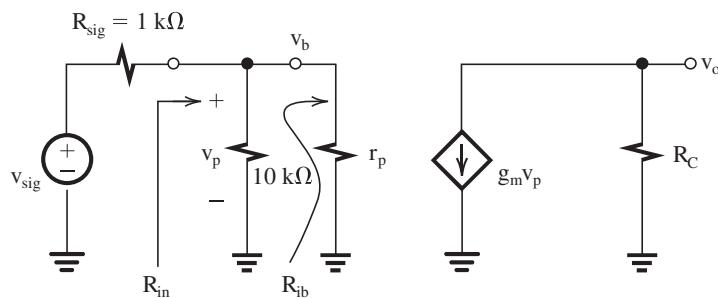
For

$$v_o = \pm 0.4 \text{ V/V}$$

$$v_b = v_\pi = \frac{\pm 0.4}{-40} = \mp 0.01 \text{ V} = \mp 10 \text{ mV}$$

$$v_{sig} = \frac{\pm 0.4}{-12.9} = \mp 31 \text{ mV}$$

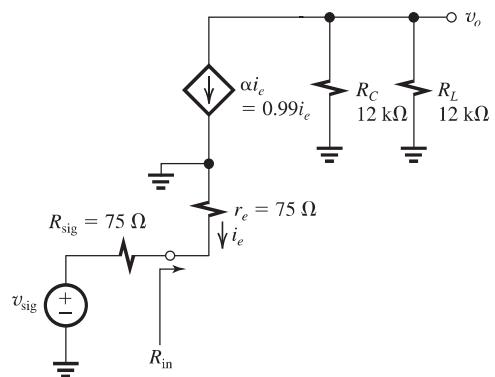
This figure belongs to Problem 6.49.



6.50 Refer to Fig. P6.50. The transistor is biased at  $I_E = 0.33 \text{ mA}$ . Thus

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.33 \text{ mA}} = 75 \Omega$$

Replacing the BJT with its T model results in the following amplifier equivalent circuit.



The input resistance  $R_{in}$  can be found by inspection to be

$$R_{in} = r_e = 75 \Omega$$

To determine the voltage gain ( $v_o/v_i$ ) we first find  $i_e$ :

$$i_e = -\frac{v_i}{R_{sig} + r_e} = -\frac{v_i}{150 \Omega} = -\frac{v_i}{0.15 \text{ k}\Omega}$$

The output voltage  $v_o$  is given by

$$v_o = -\alpha i_e (R_C \parallel R_L)$$

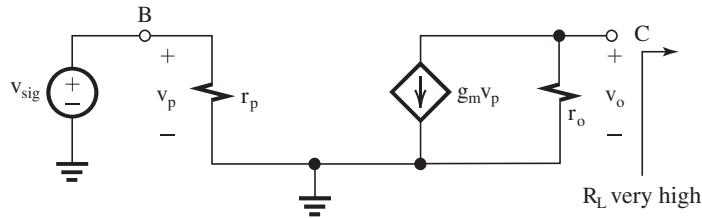
$$= -0.99 i_e \times (12 \parallel 12) = -0.99 \times 6 i_e$$

$$= -0.99 \times 6 \times \frac{-v_i}{0.15}$$

Thus,

$$\frac{v_o}{v_i} = 39.6 \text{ V/V}$$

This figure belongs to Problem 6.51.



6.51 The largest possible voltage gain is obtained when  $R_L \rightarrow \infty$ , in which case

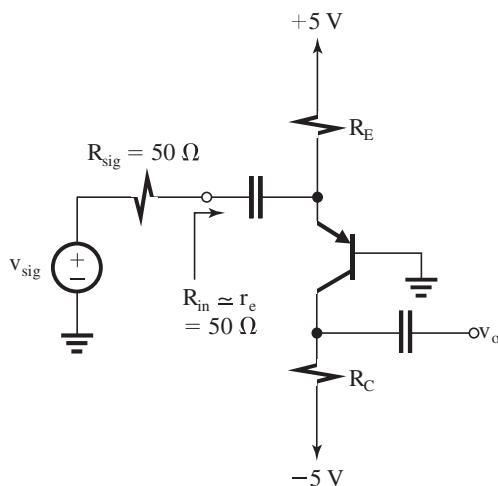
$$\frac{v_o}{v_{sig}} = -g_m r_o = -\frac{I_C}{V_T} \frac{V_A}{I_C}$$

$$= -\frac{V_A}{V_T}$$

$$\text{For } V_A = 25 \text{ V}, \frac{v_o}{v_{sig}} = -\frac{25}{0.025} = -1000 \text{ V/V}$$

$$\text{For } V_A = 125 \text{ V}, \frac{v_o}{v_{sig}} = -\frac{125}{0.025} = -5000 \text{ V/V}$$

6.52



$$r_e = 50 \Omega = \frac{V_T}{I_E}$$

$$\Rightarrow I_E = 0.5 \text{ mA}$$

Thus,

$$\frac{5 - V_E}{R_E} = 0.5 \text{ mA}$$

where

$$V_E \approx 0.7 \text{ V}$$

$$\Rightarrow R_E = 8.6 \text{ k}\Omega$$

To obtain maximum gain and the largest possible signal swing at the output for  $v_{eb}$  of 10 mV, we select a value for  $R_C$  that results in

$$V_C + |A_v| \times 0.01 \text{ V} = +0.4 \text{ V}$$

which is the highest allowable voltage at the collector while the transistor remains in the active region. Since

$$V_C = -5 + I_C R_C \approx -5 + 0.5 R_C$$

then

$$-5 + 0.5 R_C + g_m R_C \times 0.01 = 0.4$$

Substituting  $g_m = 20 \text{ mA/V}$  results in

$$R_C = 7.7 \text{ k}\Omega$$

The overall voltage gain achieved is

$$\begin{aligned} \frac{v_o}{v_{sig}} &= \frac{R_C}{R_{in} + R_{sig}} \times g_m R_C \\ &= \frac{50}{50 + 50} \times 20 \times 7.7 \\ &= 77 \text{ V/V} \end{aligned}$$

6.53 Refer to Fig. P6.53 on next page. Since  $\beta$  is very large, the dc base current can be neglected. Thus the dc voltage at the base is determined by the voltage divider,

$$V_B = 5 \frac{100}{100 + 100} = 2.5 \text{ V}$$

and the dc voltage at the emitter will be

$$V_E = V_B - 0.7 = 1.8 \text{ V}$$

The dc emitter current can now be found as

$$I_E = \frac{V_E}{R_E} = \frac{1.8}{3.6} = 0.5 \text{ mA}$$

and

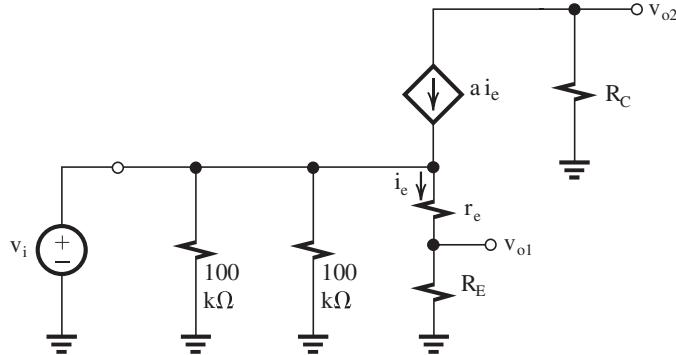
$$I_C \approx I_E = 0.5 \text{ mA}$$

Replacing the BJT with the T model of Fig. 6.26(b) results in the equivalent circuit model for the amplifier shown on next page.

$$i_e = \frac{v_i}{R_E + r_e}$$

$$v_{o1} = i_e R_E = v_i \frac{R_E}{R_E + r_e}$$

This figure belongs to Problem 6.53.



$$\frac{v_{o1}}{v_i} = \frac{R_E}{R_E + r_e} \quad \text{Q.E.D.}$$

$$v_{o2} = -\alpha i_e R_C = -\alpha \frac{v_i}{R_E + r_e} R_C$$

$$\frac{v_{o2}}{v_i} = -\frac{\alpha R_C}{R_E + r_e} \quad \text{Q.E.D.}$$

For  $\alpha \approx 1$ ,

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

$$\frac{v_{o1}}{v_i} = \frac{3.6}{3.6 + 0.05} = 0.986 \text{ V/V}$$

$$\frac{v_{o2}}{v_i} = -\frac{3.3}{3.6 + 0.05} = 0.904 \text{ V/V}$$

If  $v_{o1}$  is connected to ground,  $R_E$  will in effect be short-circuited at signal frequencies, and  $v_{o2}/v_i$  will become

$$\frac{v_{o2}}{v_i} = -\frac{\alpha R_C}{r_e} = -\frac{3.3}{0.05} = -66 \text{ V/V}$$

6.54 See figures below and on next page.

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_{vo} \frac{R_L}{R_L + R_o}$$

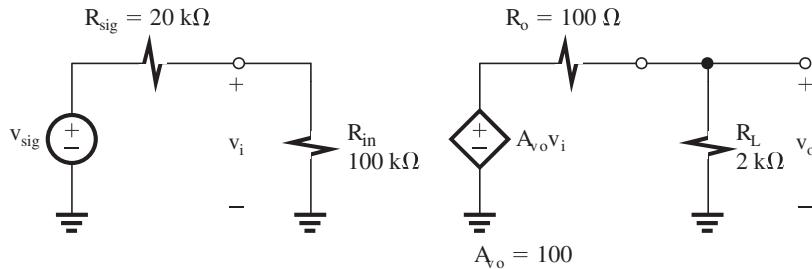
$$= \frac{100}{100 + 20} \times 100 \times \frac{2}{2 + 0.1}$$

$$= 79.4 \text{ V/V}$$

$$i_o = \frac{v_o}{R_L}$$

$$i_i = \frac{v_{sig}}{R_{sig} + R_{in}}$$

This figure belongs to Problem 6.54.



$$\begin{aligned} \frac{i_o}{i_i} &= \frac{v_o}{v_{sig}} \frac{R_{sig} + R_{in}}{R_L} \\ &= G_v \frac{R_{sig} + R_{in}}{R_L} \\ &= 79.4 \times \frac{20 + 100}{2} = 4762 \text{ A/A} \end{aligned}$$

$$6.55 \text{ (a)} \frac{R_{in}}{R_{in} + R_{sig}} = 0.95$$

$$\frac{R_{in}}{R_{in} + 100} = 0.95$$

$$\Rightarrow R_{in} = 1.9 \text{ M}\Omega$$

(b) With  $R_L = 2 \text{ k}\Omega$ ,

$$v_o = A_{vo} v_i \frac{2}{2 + R_o}$$

With  $R_L = 1 \text{ k}\Omega$ ,

$$v_o = A_{vo} v_i \frac{1}{1 + R_o}$$

Thus the change in  $v_o$  is

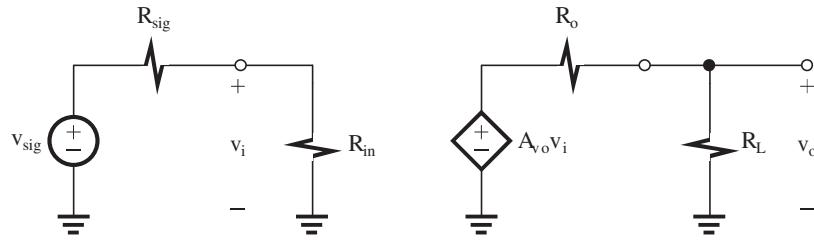
$$\Delta v_o = A_{vo} v_i \left[ \frac{2}{2 + R_o} - \frac{1}{1 + R_o} \right]$$

To limit this change to 5% of the value with  $R_L = 2 \text{ k}\Omega$ , we require

$$\left[ \frac{2}{2 + R_o} - \frac{1}{1 + R_o} \right] / \left( \frac{2}{2 + R_o} \right) = 0.05$$

$$\Rightarrow R_o = \frac{1}{9} \text{ k}\Omega = 111 \Omega$$

This figure belongs to Problem 6.55.



$$\begin{aligned}
 (c) \quad G_v &= 10 = \frac{R_{in}}{R_{in} + R_{sig}} A_{vo} \frac{R_L}{R_L + R_o} \\
 &= \frac{1.9}{1.9 + 0.1} \times A_{vo} \times \frac{2}{2 + 0.111} \\
 \Rightarrow A_{vo} &= 11.1 \text{ V/V}
 \end{aligned}$$

The values found above are limit values; that is, we require

$$R_{in} \geq 1.9 \text{ M}\Omega$$

$$R_o \leq 111 \Omega$$

$$A_{vo} \geq 11.1 \text{ V/V}$$

6.56 The circuit in Fig. 1(b) (see figure below) is that in Fig. P6.56, with the output current source expressed as  $G_m v_i$ . Thus, for equivalence, we write

$$G_m = \frac{A_{vo}}{R_o}$$

These figures belong to Problem 6.56.

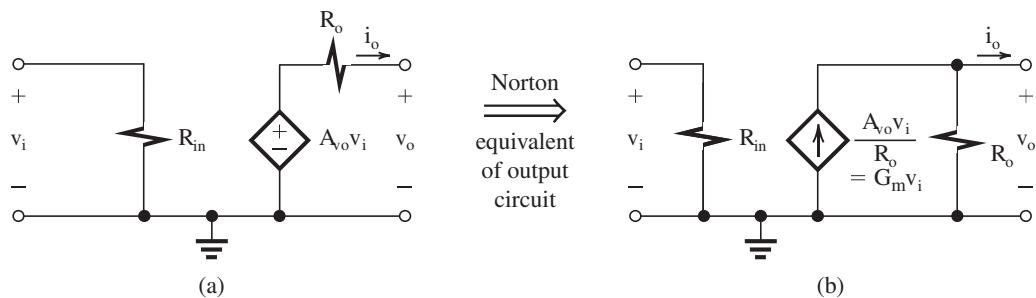


Figure 1

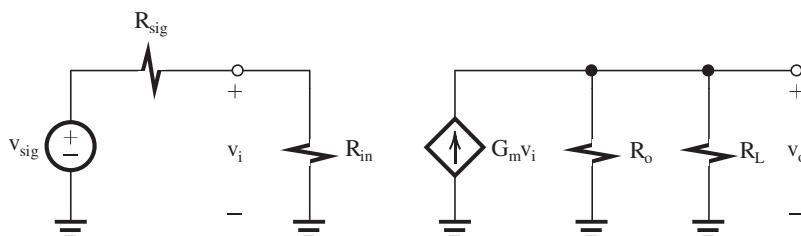


Figure 2

To determine  $G_m$  (at least conceptually), we short-circuit the output of the equivalent circuit in Fig. 1(b). The short-circuit current will be

$$i_o = G_m v_i$$

Thus  $G_m$  is defined as

$$G_m = \left. \frac{i_o}{v_i} \right|_{R_L=0}$$

and is known as the short-circuit transconductance. From Fig. 2 below,

$$\frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}}$$

$$v_o = G_m v_i (R_o \parallel R_L)$$

Thus,

$$\frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} G_m (R_o \parallel R_L)$$

6.57

$$G_{vo} = \left. \frac{v_o}{v_{sig}} \right|_{R_L=\infty}$$

Now, setting  $R_L = \infty$  in the equivalent circuit in Fig. 1(b), we can determine  $G_{vo}$  from

$$G_{vo} = \left. \frac{R_{in}}{R_{in} + R_{sig}} \right|_{R_L=\infty} A_{vo}$$

Denoting  $R_{in}$  with  $R_L = \infty$  as  $R_i$ , we can express  $G_{vo}$  as

$$G_{vo} = \frac{R_i}{R_i + R_{sig}} A_{vo} \quad \text{Q.E.D.}$$

From the equivalent circuit in Fig. 1(a), the overall voltage  $G_v$  can be obtained as

$$G_v = G_{vo} \frac{R_L}{R_L + R_{out}} \quad \text{Q.E.D.}$$

This figure belongs to Problem 6.57.

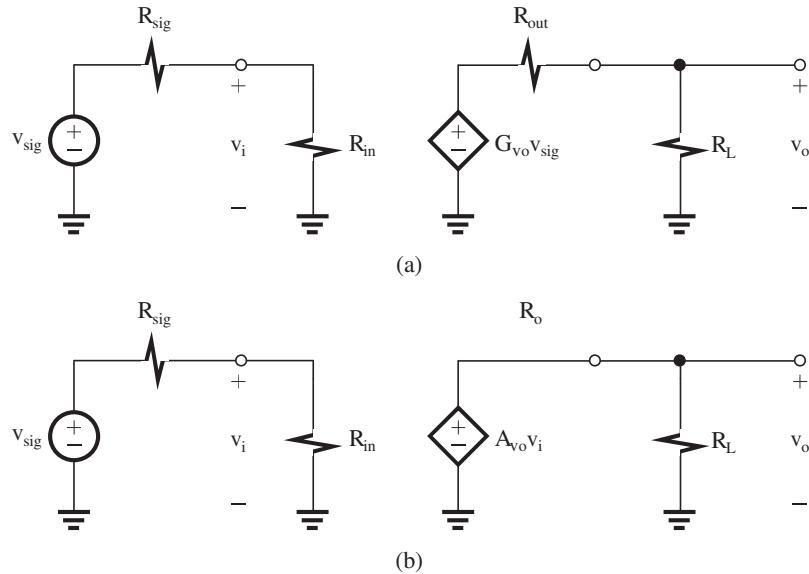


Figure 1

This figure belongs to Problem 6.58.

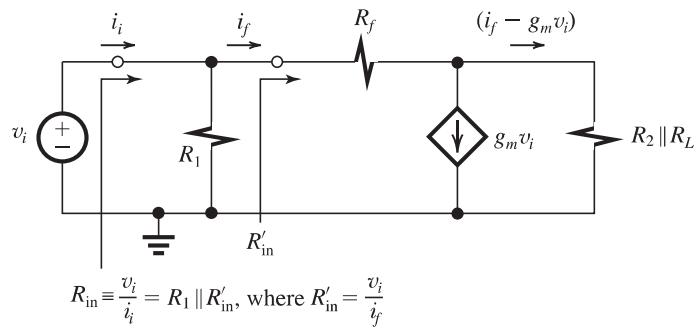


Figure 1

6.58 Refer to Fig. P6.58. To determine  $R_{in}$ , we simplify the circuit as shown in Fig. 1, where

$$R_{in} \equiv \frac{v_i}{i_i} = R_1 \parallel R'_{in}, \quad \text{where } R'_{in} \equiv \frac{v_i}{i_f}$$

$$v_i = i_f R_f + (i_f - g_m v_i)(R_2 \parallel R_L)$$

Thus,

$$v_i[1 + g_m(R_2 \parallel R_L)] = i_f [R_f + (R_2 \parallel R_L)]$$

$$R'_{in} \equiv \frac{v_i}{i_f} = \frac{R_f + (R_2 \parallel R_L)}{1 + g_m(R_2 \parallel R_L)}$$

and

$$\begin{aligned} R_{in} &= R_1 \parallel R'_{in} \\ &= R_1 \parallel \left[ \frac{R_f + (R_2 \parallel R_L)}{1 + g_m(R_2 \parallel R_L)} \right] \quad \text{Q.E.D.} \end{aligned}$$

To determine  $A_{vo}$ , we open-circuit  $R_L$  and use the circuit in Fig. 2, where

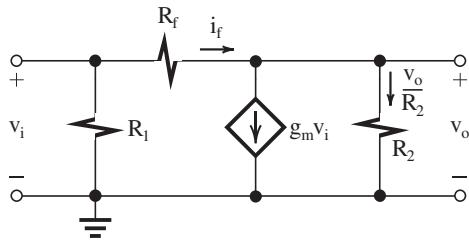


Figure 2

$$i_f = g_m v_i + \frac{v_o}{R_2}$$

$$v_i = i_f R_f + v_o = \left( g_m v_i + \frac{v_o}{R_2} \right) R_f + v_o$$

$$v_i (1 - g_m R_f) = v_o \left( 1 + \frac{R_f}{R_2} \right)$$

Thus,

$$A_{vo} \equiv \frac{v_o}{v_i} = \frac{1 - g_m R_f}{1 + \frac{R_f}{R_2}}$$

which can be manipulated to the form

$$A_{vo} = -g_m R_2 \frac{1 - 1/g_m R_f}{1 + (R_2/R_f)} \quad \text{Q.E.D.}$$

Finally, to obtain  $R_o$  we short-circuit  $v_i$  in the circuit of Fig. P6.58. This will disable the controlled source  $g_m v_i$ . Thus, looking between the output terminals (behind  $R_L$ ), we see  $R_2$  in parallel with  $R_f$ ,

$$R_o = R_2 \parallel R_f \quad \text{Q.E.D.}$$

For  $R_1 = 100 \text{ k}\Omega$ ,  $R_f = 1 \text{ M}\Omega$ ,  $g_m = 100 \text{ mA/V}$

$R_2 = 100 \Omega$  and  $R_L = 1 \text{ k}\Omega$

$$R_{in} = 100 \parallel \frac{1000 + (0.1 \parallel 1)}{1 + 100(0.1 \parallel 1)} = 100 \parallel 99.1 \\ = 49.8 \text{ k}\Omega$$

Without  $R_f$  present (i.e.,  $R_f = \infty$ ),  $R_{in} = 100 \text{ k}\Omega$  and

$$A_{vo} = -100 \times 0.1 \frac{1 - (1/100 \times 1000)}{1 + \frac{0.1}{1000}} \\ \simeq -10 \text{ V/V}$$

Without  $R_f$ ,  $-A_{vo} = 10 \text{ V/V}$  and

$$R_o = 0.1 \parallel 1000 \simeq 0.1 \text{ k}\Omega = 100 \Omega$$

Without  $R_f$ ,  $R_o = 100 \Omega$ .

Thus the only parameter that is significantly affected by the presence of  $R_f$  is  $R_{in}$ , which is reduced by a factor of 2!

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_{vo} \frac{R_L}{R_L + R_o}$$

With  $R_f$ ,

$$G_v = \frac{49.8}{49.8 + 100} \times -10 \times \frac{1}{1 + 0.1} \\ = -3 \text{ V/V}$$

Without  $R_f$ ,

$$G_v = \frac{100}{100 + 100} \times -10 \times \frac{1}{1 + 0.1} = -4.5 \text{ V/V}$$

$$6.59 \quad R_{sig} = 1 \text{ M}\Omega, R_L = 10 \text{ k}\Omega$$

$$g_m = 2 \text{ mA/V}, R_D = 10 \text{ k}\Omega$$

$$G_v = -g_m (R_D \parallel R_L)$$

$$= -2(10 \parallel 10) = -10 \text{ V/V}$$

$$6.60 \quad R_D = 2R_L = 30 \text{ k}\Omega$$

$$V_{OV} = 0.25 \text{ V}$$

$$G_v = -g_m (R_D \parallel R_L)$$

$$-10 = -g_m (30 \parallel 15)$$

$$\Rightarrow g_m = 1 \text{ mA/V}$$

$$g_m = \frac{2I_D}{V_{OV}}$$

$$1 = \frac{2 \times I_D}{0.25}$$

$$\Rightarrow I_D = 0.125 \text{ mA} = 125 \mu\text{A}$$

If  $R_D$  is reduced to  $15 \text{ k}\Omega$ ,

$$G_v = -g_m (R_D \parallel R_L)$$

$$= -1 \times (15 \parallel 15) = -7.5 \text{ V/V}$$

$$6.61 \quad R_{in} = \infty$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$$

$$320 = \frac{1}{2} \times 400 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.4 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.32}{0.4} = 1.6 \text{ mA/V}$$

$$A_{vo} = -g_m R_D = -1.6 \times 10 = -16 \text{ V/V}$$

$$R_o = R_D = 10 \text{ k}\Omega$$

$$G_v = A_{vo} \frac{R_L}{R_L + R_o}$$

$$= -16 \times \frac{10}{10 + 10} = -8 \text{ V/V}$$

$$\text{Peak value of } v_{sig} = \frac{0.2 \text{ V}}{8} = 25 \text{ mV.}$$

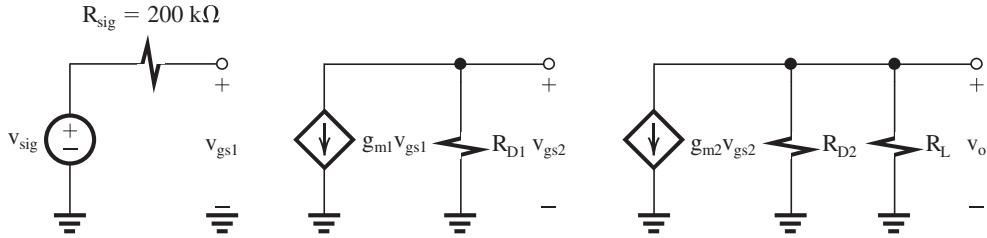
6.62 (a) See figure on next page.

$$(b) g_{m1} = g_{m2} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.3}{0.2} = 3 \text{ mA/V}$$

$$R_{D1} = R_{D2} = 10 \text{ k}\Omega$$

$$R_L = 10 \text{ k}\Omega$$

This figure belongs to Problem 6.62.



$$\begin{aligned}
 G_v &= \frac{v_{gs2}}{v_{gs1}} \times \frac{v_o}{v_{gs2}} \\
 &= -g_{m1}R_{D1} \times -g_{m2}(R_{D2} \parallel R_L) \\
 &= 3 \times 10 \times 3 \times (10 \parallel 10) \\
 &= 450 \text{ V/V}
 \end{aligned}$$

$$6.63 \quad g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20 \text{ mA/V}} = 5 \text{ k}\Omega$$

$$R_{in} = r_\pi = 5 \text{ k}\Omega$$

$$R_o = R_C = 10 \text{ k}\Omega$$

$$A_{vo} = -g_m R_C = -20 \times 10 = -200 \text{ V/V}$$

$$\begin{aligned}
 A_v &= A_{vo} \frac{R_L}{R_L + R_o} = -200 \times \frac{10}{10 + 10} \\
 &= -100 \text{ V/V}
 \end{aligned}$$

$$\begin{aligned}
 G_v &= \frac{R_{in}}{R_{in} + R_{sig}} A_v \\
 &= \frac{5}{5 + 10} \times -100 \\
 &= -33.3 \text{ V/V}
 \end{aligned}$$

For  $\hat{v}_\pi = 5 \text{ mV}$ ,  $\hat{v}_{sig}$  can be found from

$$\hat{v}_\pi = \hat{v}_{sig} \times \frac{R_{in}}{R_{in} + R_{sig}} = \hat{v}_{sig} \times \frac{5}{5 + 10}$$

$$\Rightarrow \hat{v}_{sig} = 15 \text{ mV}$$

Correspondingly,  $\hat{v}_o$  will be

$$\begin{aligned}
 \hat{v}_o &= G_v \hat{v}_{sig} \\
 &= 15 \times 33.3 = 500 \text{ mV} = 0.5 \text{ V}
 \end{aligned}$$

$$6.64 \quad |G_v| = \frac{R'_L}{(R_{sig}/\beta) + (1/g_m)}$$

$$\begin{aligned}
 R'_L &= 10 \text{ k}\Omega, R_{sig} = 10 \text{ k}\Omega, g_m = \frac{I_C}{V_T} \\
 &= \frac{1}{0.025} = 40 \text{ mA/V}
 \end{aligned}$$

Nominal  $\beta = 100$

$$\begin{aligned}
 \text{(a) Nominal } |G_v| &= \frac{10}{(10/100) + 0.025} \\
 &= 80 \text{ V/V}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } \beta = 50, |G_v| &= \frac{10}{(10/50) + 0.025} \\
 &= 44.4 \text{ V/V}
 \end{aligned}$$

$$\begin{aligned}
 \beta = 150, |G_v| &= \frac{10}{(10/150) + 0.025} \\
 &= 109.1 \text{ V/V}
 \end{aligned}$$

Thus,  $|G_v|$  ranges from 44.4 V/V to 109.1 V/V.

(c) For  $|G_v|$  to be within  $\pm 20\%$  of nominal (i.e., ranging between 64 V/V and 96 V/V), the corresponding allowable range of  $\beta$  can be found as follows:

$$64 = \frac{10}{(10/\beta_{min}) + 0.025}$$

$$\Rightarrow \beta_{min} = 76.2$$

$$96 = \frac{10}{(10/\beta_{max}) + 0.025}$$

$$\Rightarrow \beta_{max} = 126.3$$

(d) By varying  $I_C$ , we vary the term  $1/g_m$  in the denominator of the  $|G_v|$  expression. If  $\beta$  varies in the range 50 to 150 and we wish to keep  $|G_v|$  within  $\pm 20\%$  of a new nominal value of  $|G_v|$  given by

$$|G_v|_{nominal} = \frac{10}{(10/100) + (1/g_m)}$$

then

$$0.8 |G_v|_{nominal} = \frac{10}{(10/50) + (1/g_m)}$$

That is,

$$\frac{8}{0.1 + (1/g_m)} = \frac{10}{0.2 + (1/g_m)}$$

$$\Rightarrow \frac{1}{g_m} = 0.3 \text{ or } g_m = 3.33 \text{ mA/V}$$

$$|G_v|_{nominal} = \frac{10}{0.1 + 0.3} = 25 \text{ V/V}$$

$$|G_v|_{min} = \frac{10}{0.2 + 0.3}$$

$$= 20 \text{ V/V} (-20\% \text{ of nominal})$$

We need to check the value obtained for  $\beta = 150$ ,

$$|G_v|_{max} = \frac{10}{10/150 + 0.3} = 27.3 \text{ V/V}$$

which is less than the allowable value of  $1.2 |G_v|_{\text{nominal}} = 30 \text{ V/V}$ . Thus, the new bias current is

$$I_C = g_m \times V_T = 3.33 \times 0.025 = 0.083 \text{ mA}$$

$$|G_v|_{\text{nominal}} = 25 \text{ V/V}$$

6.65 (a) See figure below.

$$(b) R_{C1} = R_{C2} = 10 \text{ k}\Omega \quad R_{\text{sig}} = 10 \text{ k}\Omega$$

$$R_L = 10 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{I_C}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = \frac{100}{10} = 10 \text{ k}\Omega$$

$$\frac{v_{\pi 1}}{v_{\text{sig}}} = \frac{r_{\pi 1}}{r_{\pi 1} + R_{\text{sig}}} = \frac{10}{10 + 10} = 0.5 \text{ V/V}$$

$$\frac{v_{\pi 2}}{v_{\pi 1}} = -g_{m1}(R_{C1} \parallel r_{\pi 2}) = -10(10 \parallel 10)$$

$$= -50 \text{ V/V}$$

$$\frac{v_o}{v_{\pi 2}} = -g_{m2}(R_{C2} \parallel R_L)$$

$$= -10(10 \parallel 10) = -50 \text{ V/V}$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{v_o}{v_{\pi 2}} \times \frac{v_{\pi 2}}{v_{\pi 1}} \times \frac{v_{\pi 1}}{v_{\text{sig}}}$$

$$= -50 \times -50 \times 0.5$$

$$= 1250 \text{ V/V}$$

$$6.66 g_m|_{\text{effective}} = \frac{g_m}{1 + g_m R_s}$$

$$2 = \frac{5}{1 + 5R_s}$$

$$\Rightarrow R_s = 0.3 \text{ k}\Omega = 300 \Omega$$

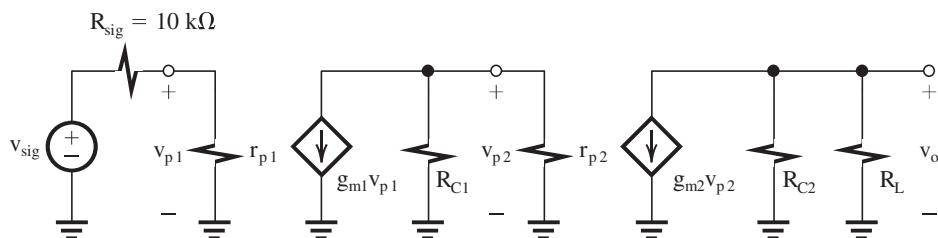
6.67 Including  $R_s$  reduced the gain by a factor of 2, thus

$$1 + g_m R_s = 2$$

$$\Rightarrow g_m = \frac{1}{R_s} = \frac{1}{0.5} = 2 \text{ mA/V}$$

The gain without  $R_s$  is  $-20 \text{ V/V}$ . To obtain a gain of  $-16 \text{ V/V}$ , we write

This figure belongs to Problem 6.65.



$$16 = \frac{20}{1 + g_m R_s} = \frac{20}{1 + 2R_s}$$

$$\Rightarrow R_s = 125 \Omega$$

$$6.68 g_m = \frac{I_C}{V_T} = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

$$r_e \simeq \frac{1}{g_m} = 50 \Omega$$

$$R_{\text{in}} = (\beta + 1)(r_e + R_e)$$

$$= 101(50 + 250) = 30.3 \text{ k}\Omega$$

$$A_{vo} = -\frac{\alpha R_C}{r_e + R_e} = -\frac{0.99 \times 12}{0.3} \simeq -40 \text{ V/V}$$

$$R_o = R_C = 12 \text{ k}\Omega$$

$$A_v = A_{vo} \frac{R_L}{R_L + R_o}$$

$$= -40 \times \frac{12}{12 + 12} = -20 \text{ V/V}$$

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \times A_v$$

$$= \frac{30.3}{30.3 + 10} \times -20 = -15 \text{ V/V}$$

$$\hat{v}_\pi = 5 \text{ mV} \Rightarrow \hat{v}_{\text{sig}} = \hat{v}_\pi \left( \frac{R_{\text{in}} + R_{\text{sig}}}{R_{\text{in}}} \right)$$

$$\hat{v}_{\text{sig}} = 5 \times \frac{30.3 + 10}{30.3} = 6.65 \text{ mV}$$

$$\hat{v}_o = \hat{v}_{\text{sig}} \times |G_v|$$

$$= 6.65 \times 15 \simeq 100 \text{ mV}$$

$$6.69 R_{\text{in}} = (\beta + 1)(r_e + R_e)$$

$$15 = 75(r_e + R_e)$$

$$r_e + R_e = \frac{15 \text{ k}\Omega}{75} = 200 \Omega$$

$$\hat{v}_\pi = \hat{v}_{\text{sig}} \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \frac{r_e}{r_e + R_e}$$

$$5 = 150 \times \frac{15}{15 + 30} \left( \frac{r_e}{r_e + R_e} \right)$$

$$\Rightarrow \frac{r_e}{r_e + R_e} = 0.1$$

But  $r_e + R_e = 200 \Omega$ , thus

$$r_e = 20 \Omega$$

which requires a bias current  $I_E$  of

$$I_E = \frac{V_T}{r_e} = \frac{25 \text{ mV}}{20 \Omega} = 1.25 \text{ mA}$$

$$I_C \simeq I_E = 1.25 \text{ mA}$$

$$R_e = 180 \Omega$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times \frac{-\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$= \frac{15}{15+30} \times \frac{-0.99 \times 6}{0.2}$$

$$\simeq -10 \text{ V/V}$$

$$\hat{v}_0 = 0.15 \times |G_v| = 1.5 \text{ V}$$

6.70 Using Eq. (6.113), we have

$$G_v = -\beta \frac{R_C \parallel R_L}{R_{sig} + (\beta + 1)(r_e + R_e)}$$

$$\simeq -\frac{R_C \parallel R_L}{(R_{sig}/\beta) + (r_e + R_e)}$$

$$|G_v| = \frac{10}{(10/\beta) + 0.025 + R_e}$$

Without  $R_e$ ,

$$|G_v| = \frac{10}{(10/\beta) + 0.025}$$

For the nominal case,  $\beta = 100$ ,

$$|G_v|_{nominal} = \frac{10}{0.1 + 0.025} = 80 \text{ V/V}$$

For  $\beta = 50$ ,

$$|G_v|_{low} = \frac{10}{0.2 + 0.025} = 44.4 \text{ V/V}$$

For  $\beta = 150$ ,

$$|G_v|_{high} = \frac{10}{(1/15) + 0.025} = 109.1 \text{ V/V}$$

Thus,  $|G_v|$  ranges from 44.4 V/V to 109.1 V/V with a nominal value of 80 V/V. This is a range of -44.5% to +36.4% of nominal.

To limit the range of  $|G_v|$  to  $\pm 20\%$  of a new nominal value, we connect a resistance  $R_e$  and find its value as follows. With  $R_e$ ,

$$|G_v|_{nominal} = \frac{10}{(10/100) + 0.025 + R_e}$$

$$= \frac{10}{0.125 + R_e}$$

Now,  $\beta = 50$ ,

$$|G_v|_{low} = \frac{10}{0.225 + R_e}$$

To limit this value to  $-20\%$  of  $|G_v|_{nominal}$ , we use

$$\frac{10}{0.225 + R_e} = 0.8 \times \frac{10}{0.125 + R_e}$$

$$\Rightarrow R_e = 0.275 \text{ k}\Omega = 275 \Omega$$

With this value of  $R_e$ ,

$$|G_v|_{nominal} = \frac{10}{0.125 + 0.275} = 25 \text{ V/V}$$

$$|G_v|_{low} = \frac{10}{0.225 + 0.275} = 20 \text{ V/V} (-20\% \text{ of nominal})$$

$$|G_v|_{high} = \frac{10}{(1/15) + 0.025 + 0.275} = 27.3 \text{ V/V} (+9.1\% \text{ of nominal})$$

$$6.71 R_{in} = \frac{1}{g_m} = \frac{1}{2 \text{ mA/V}} = 0.5 \text{ k}\Omega$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times g_m (R_D \parallel R_L)$$

$$= \frac{0.5}{0.5 + 0.75} \times 2(5 \parallel 5)$$

$$= 2 \text{ V/V}$$

For  $R_{in} = R_{sig} = 0.75 \text{ k}\Omega$

$$\frac{1}{g_m} = 0.75 \Rightarrow g_m = 1.33 \text{ mA/V}$$

Since  $g_m = \sqrt{2k_n I_D}$ , then to change  $g_m$  by a factor

$$\frac{1.33}{2} = 0.67, I_D \text{ must be changed by a factor of } (0.67)^2 = 0.45.$$

6.72 Refer to the circuit in Fig. P6.72. Since  $R_{sig} \gg r_e$ , most of  $i_{sig}$  flows into the emitter of the BJT. Thus

$$i_e \simeq i_{sig}$$

and

$$i_c = \alpha i_e \simeq i_{sig}$$

Thus,

$$v_o = i_c R_C = i_{sig} R_C$$

6.73 For  $R_{in} = R_{sig} = 50 \Omega$ ,

$$r_e = 50 \Omega$$

and, with  $\alpha \simeq 1$ ,

$$I_C \simeq \frac{V_T}{r_e} = \frac{25 \text{ mV}}{50 \Omega} = 0.5 \text{ mA}$$

$$g_m = I_C / V_T = 20 \text{ mA/V}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} g_m (R_C \parallel R_L)$$

$$G_v = \frac{50}{50 + 50} \times 20 \times (10 \parallel 10) = 50 \text{ V/V}$$

$$6.74 R_{in} = r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.2 \text{ mA}} = 125 \Omega$$

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} g_m (R_C \parallel R_L)$$

$$= \frac{0.125}{0.125 + 0.5} \times 8(10 \parallel 10) = 8 \text{ V/V}$$

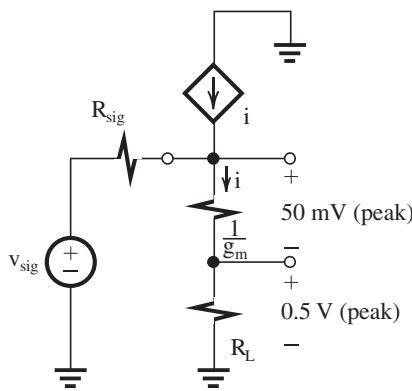
$$\hat{v}_\pi = \hat{v}_{\text{sig}} \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}$$

$$10 = \hat{v}_{\text{sig}} \frac{0.125}{0.125 + 0.5}$$

$$\Rightarrow \hat{v}_{\text{sig}} = 50 \text{ mV}$$

$$\hat{v}_o = G_v \hat{v}_{\text{sig}} = 8 \times 50 = 400 \text{ mV} = 0.4 \text{ V}$$

6.75



From the figure above, we have

$$\frac{1}{g_m} = 0.1 \times R_L$$

$$= 0.1 \times 2 = 0.2 \text{ k}\Omega$$

$$g_m = 5 \text{ mA/V}$$

$$g_m = \sqrt{2k_n I_D}$$

$$5 = \sqrt{2 \times 5 \times I_D}$$

$$I_D = 2.5 \text{ mA}$$

At the peak of the sine wave,

$$i_d = \frac{0.5 \text{ V}}{2 \text{ k}\Omega} = 0.25 \text{ mA}, \text{ thus}$$

$$i_{D\text{max}} = I_D + 0.25 = 2.75 \text{ mA}$$

$$i_{D\text{min}} = I_D - 0.25 = 2.25 \text{ mA}$$

$$\hat{v}_{\text{sig}} = \hat{v}_{\text{gs}} + \hat{v}_o = 0.05 + 0.5 = 0.55 \text{ V}$$

$$6.76 \quad A_v = \frac{R_L}{R_L + R_o}$$

$$A_v|_{\text{nominal}} = \frac{2}{2 + R_o}$$

$$A_v|_{\text{low}} = \frac{1.5}{1.5 + R_o}$$

$$A_v|_{\text{high}} = \frac{5}{5 + R_o}$$

For  $A_v|_{\text{high}} = 1.1 A_v|_{\text{nominal}}$

$$\frac{5}{5 + R_o} = \frac{1.1 \times 2}{2 + R_o}$$

$$\Rightarrow R_o = 0.357 \text{ k}\Omega$$

$$A_v|_{\text{nominal}} = \frac{2}{2.357} = 0.85 \text{ V/V}$$

$$A_v|_{\text{high}} = \frac{5}{5.357}$$

$$= 0.93$$

(+10% above nominal)

$$A_v|_{\text{low}} = \frac{1.5}{1.5 + 0.357}$$

$$= 0.81 (-5\% \text{ from nominal})$$

$$\frac{1}{g_m} = R_o = 0.357 \text{ k}\Omega$$

$$\Rightarrow g_m = 2.8 \text{ mA/V}$$

To find  $I_D$ , we use

$$g_m = \sqrt{2k_n I_D}$$

$$\Rightarrow I_D = g_m^2 / 2k_n$$

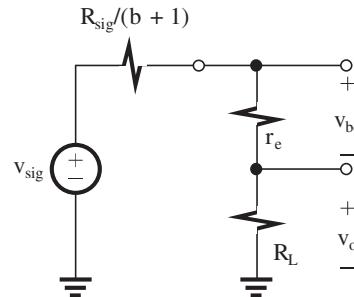
$$= \frac{2.8^2}{2 \times 2.5} = 1.6 \text{ mA}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$1.6 = \frac{1}{2} \times 2.5 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 1.13 \text{ V}$$

6.77



$$\hat{v}_o = 0.5 \text{ V}$$

$$R_L = 2 \text{ k}\Omega$$

$$\hat{v}_{be} = 5 \text{ mV}$$

From the figure above we see that

$$\frac{r_e}{R_L} = \frac{5 \text{ mV}}{500 \text{ mV}}$$

$$\Rightarrow r_e = \frac{R_L}{100} = 20 \Omega$$

$$I_E = \frac{V_T}{r_e} = \frac{25 \text{ mV}}{20 \Omega} = 1.25 \text{ mA}$$

At the peak of the output sine wave, we have

$$\hat{i}_e = \frac{\hat{v}_o}{R_L} = \frac{0.5}{2} = 0.25 \text{ mA}$$

Thus,

$$i_{E\max} = 1.25 + 0.25 = 1.5 \text{ mA}$$

and

$$i_{E\min} = 1.25 - 0.25 = 1.0 \text{ mA}$$

From the figure, we have

$$\begin{aligned} G_v &= \frac{v_o}{v_{sig}} = \frac{R_L}{R_L + r_e + \frac{R_{sig}}{\beta + 1}} \\ &= \frac{2}{2 + 0.02 + \frac{200}{101}} = 0.5 \text{ V/V} \end{aligned}$$

Thus,

$$\hat{v}_{sig} = \frac{\hat{v}_o}{G_v} = \frac{0.5 \text{ V}}{0.5 \text{ V/V}} = 1 \text{ V}$$

$$6.78 \quad I_C = 2 \text{ mA}$$

$$r_e = \frac{V_T}{I_E} \approx \frac{V_T}{I_C} = \frac{25}{2} = 12.5 \Omega$$

$$(a) R_{in} = (\beta + 1)(r_e + R_L)$$

$$= 101 \times (12.5 + 500) = 51.76 \text{ k}\Omega$$

$$\begin{aligned} \frac{v_b}{v_{sig}} &= \frac{R_{in}}{R_{in} + R_{sig}} = \frac{51.76}{51.76 + 10} \\ &= 0.84 \text{ V/V} \end{aligned}$$

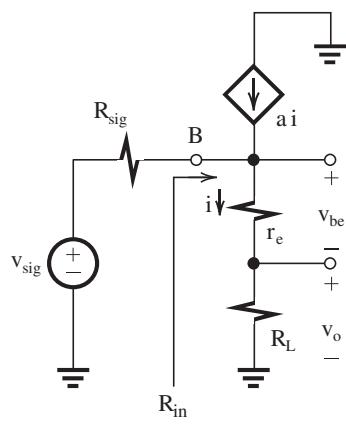
$$\frac{v_o}{v_{sig}} = \frac{v_b}{v_{sig}} \times \frac{v_o}{v_b}$$

$$= 0.84 \times \frac{R_L}{R_L + r_e}$$

$$= 0.84 \times \frac{0.5}{0.5 + 0.0125}$$

$$= 0.82 \text{ V/V}$$

(b)



$$\hat{v}_{be} = 10 \text{ mV}$$

$$\hat{v}_o = \frac{R_L}{r_e} \times \hat{v}_{be}$$

$$= \frac{500}{12.5} \times 10$$

$$= 400 \text{ mV} = 0.4 \text{ V}$$

$$\hat{v}_{sig} = \frac{\hat{v}_o}{G_v} = \frac{0.4}{0.82} = 0.488 \text{ V}$$

$$(c) G_{vo} = 1$$

$$\begin{aligned} R_{out} &= r_e + \frac{R_{sig}}{\beta + 1} = 12.5 + \frac{10,000}{101} \\ &= 111.5 \Omega \end{aligned}$$

Thus,

$$\begin{aligned} G_v &= G_{vo} \frac{R_L}{R_L + R_{out}} \\ &= 1 \times \frac{500}{500 + 111.5} = 0.82 \text{ V/V} \end{aligned}$$

which is the same value obtained in (a) above.

For  $R_L = 250 \Omega$ ,

$$\begin{aligned} G_v &= G_{vo} \frac{R_L}{R_L + R_{out}} \\ &= 1 \times \frac{250}{250 + 111.5} = 0.69 \text{ V/V} \end{aligned}$$

$$6.79 \quad R_{out} = r_e + \frac{R_{sig}}{\beta + 1}$$

$$150 = r_e + \frac{5000}{\beta + 1} \quad (1)$$

$$250 = r_e + \frac{10,000}{\beta + 1} \quad (2)$$

Subtracting Eq. (1) from Eq. (2), we have

$$100 = \frac{5000}{\beta + 1}$$

$$\beta + 1 = 50$$

Substituting in Eq. (1) yields

$$\begin{aligned} 150 &= r_e + \frac{5000}{50} \\ \Rightarrow r_e &= 50 \Omega \\ G_v &= \frac{R_L}{R_L + r_e + \frac{R_{sig}}{\beta + 1}} \\ &= \frac{1000}{1000 + 50 + \frac{10,000}{50}} = 0.8 \text{ V/V} \end{aligned}$$

6.80 (a) Refer to Fig. P6.80.

$$\frac{v_c}{v_{sig}} = \frac{-i_c R_C}{i_b R_B + i_c (r_e + R_E)}$$

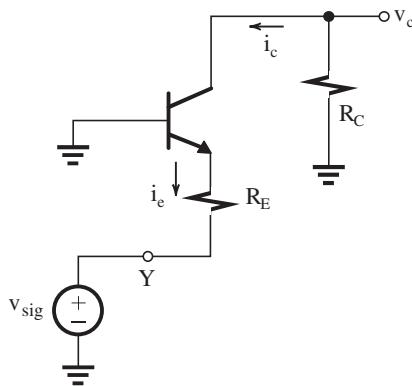
$$= -\frac{i_c}{i_b} \frac{R_C}{R_B + \left(\frac{i_c}{i_b}\right)(r_e + R_E)}$$

$$= -\beta \frac{R_C}{R_B + (\beta + 1)(r_e + R_E)}$$

$$\frac{v_e}{v_{sig}} = \frac{-i_e R_E}{i_b R_B + i_e (r_e + R_E)}$$

$$= \frac{R_E}{\frac{R_B}{\beta + 1} + r_e + R_E}$$

(b)



$$i_e = -\frac{v_{sig}}{r_e + R_E}$$

$$i_c = -i_e R_C = -\alpha i_e R_C$$

$$\frac{v_c}{v_{sig}} = \frac{-i_e R_C}{i_e (r_e + R_E)} = \alpha \frac{R_C}{r_e + R_E}$$

6.81 With the Early effect neglected, we can write

$$G_v = -100 \text{ V/V}$$

With the Early effect taken into account, the effective resistance in the collector is reduced from  $R_C = 10 \text{ k}\Omega$  to  $(R_C \parallel r_o)$ , where

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

$$(R_C \parallel r_o) = 10 \parallel 100 = 9.1 \text{ k}\Omega$$

Thus,  $G_v$  becomes

$$G_v = -100 \times \frac{9.1 \text{ k}\Omega}{10 \text{ k}\Omega}$$

$$= -91 \text{ V/V}$$

6.82 Adapting Eq. (6.114) gives

$$G_v = -\beta \frac{R_C \parallel R_L \parallel r_o}{R_{sig} + (\beta + 1)r_e}$$

$$= -\frac{R_C \parallel R_L \parallel r_o}{\frac{R_{sig}}{\beta + 1} + \frac{1}{\beta} r_e}$$

$$= -\frac{R_C \parallel R_L \parallel r_o}{\frac{R_{sig}}{\beta} + \frac{1}{g_m}}$$

Thus,

$$|G_v| = \frac{10 \parallel r_o}{0.1 + \frac{1}{g_m}} \quad (1)$$

where  $r_o$  and  $\frac{1}{g_m}$  are in kilohms and are given by

$$r_o = \frac{V_A}{I_C} = \frac{25 \text{ V}}{I_C \text{ mA}} \quad (2)$$

$$\frac{1}{g_m} = \frac{V_T}{I_C} = \frac{0.025 \text{ V}}{I_C \text{ mA}} \quad (3)$$

$I_C \text{ (mA)}$	$1/g_m \text{ (k}\Omega\text{)}$	$r_o \text{ (k}\Omega\text{)}$	$ G_v  \text{ (V/V)}$
0.1	0.250	250	27.5
0.2	0.125	125	41.2
0.5	0.050	50	55.6
1.0	0.025	25	57.1
1.25	0.020	20	55.6

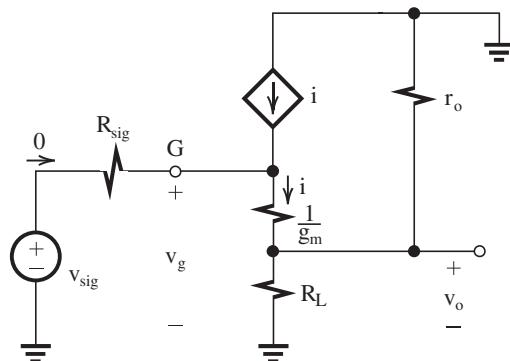
Observe that initially  $|G_v|$  increases as  $I_C$  is increased. However, above about 1 mA this trend reverses because of the effect of  $r_o$ . From the table we see that gain of 50 is obtained for  $I_C$  between 0.2 and 0.5 mA and also for  $I_C$  above 1.25 mA. Practically speaking, one normally uses the low value to minimize power dissipation. The required value of  $I_C$  is found by substituting for  $r_o$  and  $1/g_m$  from Eqs. (2) and (3), respectively, in Eq. (1) and equating  $G_v$  to 50. The result (after some manipulations) is the quadratic equation.

$$I_C^2 - 2.25I_C + 0.625 = 0$$

The two roots of this equation are  $I_C = 0.325 \text{ mA}$  and  $1.925 \text{ mA}$ ; our preferred choice is

$$I_C = 0.325 \text{ mA.}$$

6.83



$$v_g = v_{\text{sig}}$$

Noting that  $r_o$  appears in effect in parallel with  $R_L$ ,  $v_o$  is obtained as the ratio of the voltage divider formed by  $(1/g_m)$  and  $(R_L \parallel r_o)$ ,

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{v_o}{v_g} = \frac{(R_L \parallel r_o)}{(R_L \parallel r_o) + \frac{1}{g_m}} \quad \text{Q.E.D.}$$

With  $R_L$  removed,

$$G_v = \frac{r_o}{r_o + \frac{1}{g_m}} = 0.98 \quad (1)$$

With  $R_L = 500 \Omega$ ,

$$G_v = \frac{(500 \parallel r_o)}{(500 \parallel r_o) + \frac{1}{g_m}} = 0.49 \quad (2)$$

From Eq. (1), we have

$$\frac{1}{g_m} = \frac{r_o}{49}$$

Substituting in Eq. (2) and solving for  $r_o$  gives

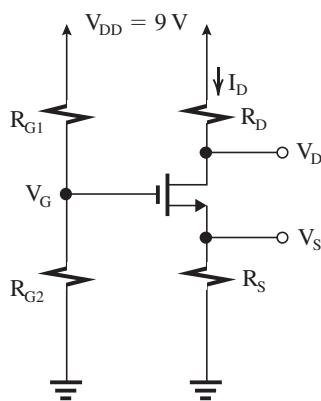
$$r_o = 25,000 \Omega = 25 \text{ k}\Omega$$

Thus

$$\frac{1}{g_m} = \frac{25,000}{49} \Omega$$

$$\Rightarrow g_m = 1.96 \text{ mA/V}$$

6.84



$$I_D = 1 \text{ mA}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$1 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 1 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 1 + 1 = 2 \text{ V}$$

$$\text{Now, selecting } V_S = \frac{V_{DD}}{3} = 3 \text{ V}$$

$$I_D R_S = 3$$

$$R_S = \frac{3}{1} = 3 \text{ k}\Omega$$

Also,

$$I_D R_D = \frac{V_{DD}}{3} = 3 \text{ V}$$

$$\Rightarrow R_D = \frac{3}{1} = 3 \text{ k}\Omega$$

$$V_G = V_S + V_{GS}$$

$$= 3 + 2 = 5 \text{ V}$$

Thus the voltage drop across  $R_{G2}(5 \text{ V})$  is larger than that across  $R_{G1}(4 \text{ V})$ . So we select

$$R_{G2} = 22 \text{ M}\Omega$$

and determine  $R_{G1}$  from

$$\frac{R_{G1}}{R_{G2}} = \frac{4 \text{ V}}{5 \text{ V}}$$

$$\Rightarrow R_{G1} = 0.8 R_{G2} = 0.8 \times 22$$

$$= 17.6 \text{ M}\Omega$$

Using only two significant figures, we have

$$R_{G1} = 18 \text{ M}\Omega$$

Note that this will cause  $V_G$  to deviate slightly from the required value of 5 V.

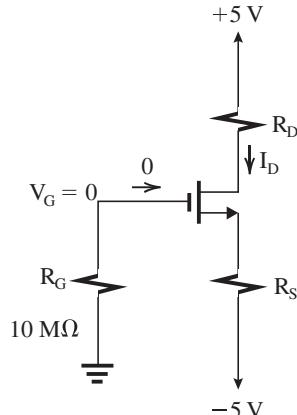
Specifically,

$$V_G = V_{DD} \frac{R_{G2}}{R_{G2} + R_{G1}}$$

$$= 9 \times \frac{22}{22 + 18} = 4.95 \text{ V}$$

It can be shown (after simple but somewhat tedious analysis) that the resulting  $I_D$  will be  $I_D = 0.986 \text{ mA}$ , which is sufficiently close to the desired 1 mA. Since  $V_D = V_{DD} - I_D R_D \simeq +6 \text{ V}$  and  $V_G \simeq 5 \text{ V}$ , and the drain voltage can go down to  $V_G - V_t = 4 \text{ V}$ , the drain voltage is 2 V above the value that causes the MOSFET to leave the saturation region.

6.85



For  $I_D = 0.5 \text{ mA}$

$$0.5 = \frac{1}{2} k_n V_{OV}^2$$

$$= \frac{1}{2} \times 1 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 1 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 1 + 1 = 2 \text{ V}$$

Since

$$V_G = 0 \text{ V}, \quad V_S = -V_{GS} = -2 \text{ V}$$

which leads to

$$R_S = \frac{V_S - (-5)}{I_C} = \frac{-2 + 5}{0.5} = 6 \text{ k}\Omega$$

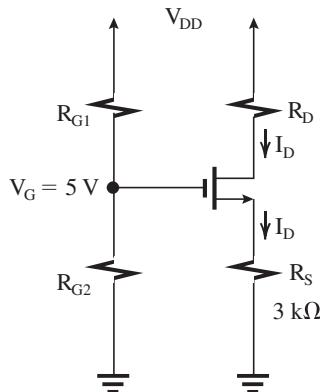
$V_D$  is required to be halfway between cutoff ( $+5 \text{ V}$ ) and saturation ( $0 - V_t = -1 \text{ V}$ ). Thus

$$V_D = +2 \text{ V}$$

and

$$R_D = \frac{5 - 2}{0.5} = 6 \text{ k}\Omega$$

6.86



$$V_S = I_D R_S = 3I_D$$

$$V_{GS} = 5 - V_S = 5 - 3I_D$$

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 2(5 - 3I_D - 1)^2$$

$$= 16 - 24I_D + 9I_D^2$$

$$9I_D^2 - 25I_D + 16 = 0$$

$$I_D = 1.78 \text{ mA or } 1 \text{ mA}$$

The first answer is physically meaningless, as it would result in  $V_S = 5.33 \text{ V}$ , which is greater than  $V_G$ , implying that the transistor is cut off. Thus,  $I_D = 1 \text{ mA}$ .

If a transistor for which  $k_n = 3 \text{ mA/V}^2$  is used, then

$$I_D = \frac{1}{2} \times 3(5 - 3I_D - 1)^2$$

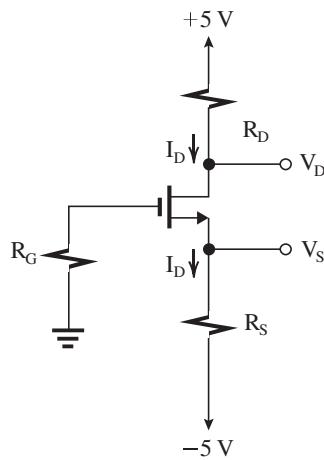
$$= 1.5(16 - 24I_D + 9I_D^2)$$

$$9I_D^2 - 24.67I_D + 16 = 0$$

whose physically meaningful solution is

$$I_D = 1.05 \text{ mA}$$

6.87



$$I_D = 0.5 \text{ mA} = \frac{1}{2} \times 4(V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 1.5 \text{ V}$$

Since  $V_G = 0 \text{ V}$ ,  $V_S = -1.5 \text{ V}$ , and

$$R_S = \frac{-1.5 - (-5)}{0.5} = 7 \text{ k}\Omega$$

Maximum gain is obtained by using the largest possible value of  $R_D$ , that is, the lowest possible value of  $V_D$  that is consistent with allowing negative voltage signal swing at the drain of 1 V. Thus

$$V_D - 1 = v_{Dmin} = V_G - V_t = 0 - 1$$

$$\Rightarrow V_D = 0 \text{ V}$$

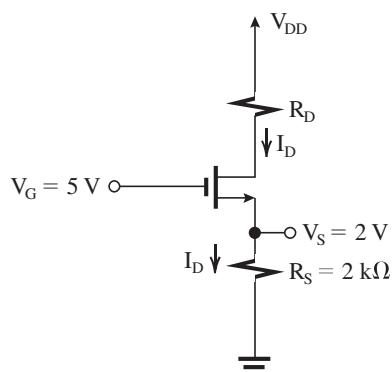
where we have assumed that the signal voltage at the gate is small. Now,

$$V_D = 0 = V_{DD} - I_D R_D$$

$$0 = 5 - 0.5 \times R_D$$

$$\Rightarrow R_D = 10 \text{ k}\Omega$$

6.88



$$I_D = \frac{2 \text{ V}}{2 \text{ k}\Omega} = 1 \text{ mA}$$

But

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$1 = \frac{1}{2} \times 2(V_G - V_S - V_t)^2$$

$$1 = (5 - 2 - V_t)^2$$

$$V_t = 2 \text{ V}$$

If  $V_t = 1.5 \text{ V}$ , then we have

$$V_S = I_D R_S = 2I_D$$

$$V_{GS} = V_G - V_S = 5 - 2I_D$$

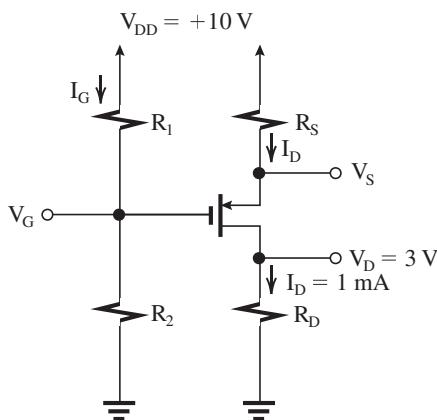
$$I_D = \frac{1}{2} \times 2(5 - 2I_D - 1.5)^2$$

$$4I_D^2 - 15I_D + 12.25 = 0$$

$$I_D = 1.2 \text{ mA}$$

$$V_S = 2.4 \text{ V}$$

6.89



$$I_D = 1 \text{ mA} \text{ and } V_D = 3 \text{ V}$$

Thus,

$$R_D = \frac{V_D}{I_D} = \frac{3 \text{ V}}{1 \text{ mA}} = 3 \text{ k}\Omega$$

For the transistor to operate 1 V from the edge of saturation

$$V_D = V_G + |V_t| - 1$$

Thus,

$$3 = V_G + |V_t| - 1$$

$$V_G + |V_t| = 4 \text{ V}$$

$$(a) |V_t| = 1 \text{ V} \text{ and } k_p = 0.5 \text{ mA/V}^2$$

$$V_G = 3 \text{ V}$$

$$R_2 = \frac{V_G}{I_G} = \frac{3 \text{ V}}{10 \mu\text{A}} = 0.3 \text{ M}\Omega$$

$$R_1 = \frac{V_{DD} - V_G}{I_G} = \frac{7 \text{ V}}{10 \mu\text{A}} = 0.7 \text{ M}\Omega$$

$$V_D = 3 \text{ V}$$

$$R_D = 3 \text{ k}\Omega$$

$$I_D = \frac{1}{2} k_p (V_{SG} - |V_t|)^2$$

$$1 = \frac{1}{2} \times 0.5(V_{SG} - 1)^2$$

$$\Rightarrow V_{SG} = 3 \text{ V}$$

$$V_S = V_G + 3 = 3 + 3 = 6 \text{ V}$$

$$R_S = \frac{V_{DD} - V_S}{I_D}$$

$$= \frac{10 - 6}{1} = 4 \text{ k}\Omega$$

$$(b) |V_t| = 2 \text{ V} \text{ and } k_p = 1.25 \text{ mA/V}^2$$

$$V_G = 4 - |V_t| = 2 \text{ V}$$

$$R_2 = \frac{V_G}{I_G} = \frac{2 \text{ V}}{10 \mu\text{A}} = 0.2 \text{ M}\Omega$$

$$R_1 = \frac{V_{DD} - V_G}{I_G} = \frac{8 \text{ V}}{10 \mu\text{A}} = 0.8 \text{ M}\Omega$$

$$V_D = 3 \text{ V}$$

$$R_D = 3 \text{ k}\Omega$$

$$I_D = \frac{1}{2} k_p (V_{SG} - |V_t|)^2$$

$$1 = \frac{1}{2} \times 1.25(V_{SG} - 2)^2$$

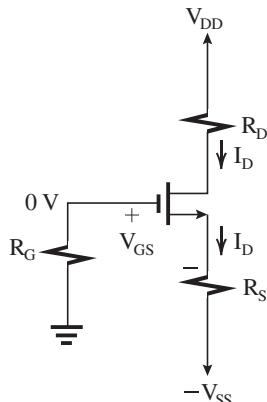
$$V_{SG} = 3.265 \text{ V}$$

$$V_S = V_G + 3.265 = 2 + 3.265$$

$$= 5.265 \text{ V}$$

$$R_S = \frac{10 - 5.265}{1} = 4.7 \text{ k}\Omega$$

6.90



$$(a) V_{GS} + I_D R_S = V_{SS}$$

But

$$I_D = \frac{1}{2} k_n \left( \frac{W}{L} \right) (V_{GS} - V_t)^2$$

$$= K(V_{GS} - V_t)^2$$

$$\Rightarrow V_{GS} = V_t + \sqrt{\frac{I_D}{K}}$$

Thus,

$$V_t + \sqrt{\frac{I_D}{K}} + I_D R_S = V_{SS}$$

Differentiating relative to K, we have

$$0 + \frac{1}{2\sqrt{I_D/K}} \left[ \frac{1}{K} \frac{\partial I_D}{\partial K} - \frac{I_D}{K^2} \right] + R_S \frac{\partial I_D}{\partial K} = 0$$

$$\frac{\partial I_D}{\partial K} \frac{K}{I_D} = \frac{1}{1 + 2\sqrt{K I_D} R_S}$$

$$S_K^{I_D} = 1/[1 + 2\sqrt{K I_D} R_S] \quad Q.E.D$$

$$(b) K = 100 \mu A/V^2, \frac{\Delta K}{K} = \pm 0.1, \text{ and}$$

$$V_t = 1 \text{ V. We require } I_D = 100 \mu \text{ A and}$$

$$\frac{\Delta I_D}{I_D} = \pm 0.01. \text{ Thus,}$$

$$S_K^{I_D} = \frac{\Delta I_D / I_D}{\Delta K / K} = \frac{0.01}{0.10} = 0.1$$

Substituting in the expression derived in (a),

$$0.1 = \frac{1}{1 + 2\sqrt{0.1 \times 0.1 R_S}}$$

$$\Rightarrow R_S = 45 \text{ k}\Omega$$

To find  $V_{GS}$ ,

$$I_D = K(V_{GS} - V_t)^2$$

$$100 = 100(V_{GS} - 1)^2$$

$$V_{GS} = 2 \text{ V}$$

$$V_{GS} + I_D R_S = V_{SS}$$

$$2 + 0.1 \times 45 = 6.5 \text{ V}$$

(c) For  $V_{SS} = 5 \text{ V}$  and  $V_{GS} = 2 \text{ V}$ ,

$$I_D R_S = 3 \text{ V}$$

$$R_S = \frac{3}{0.1} = 30 \text{ k}\Omega$$

$$S_K^{I_D} = \frac{1}{1 + 2\sqrt{0.1 \times 0.1 \times 30}} = \frac{1}{7}$$

$$\frac{\Delta I_D}{I_D} = \frac{1}{7} \times \frac{\Delta K}{K} = \frac{1}{7} \times \pm 10\% = \pm 1.4\%$$

6.91 (a) With a fixed  $V_{GS}$ ,

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$\frac{\partial I_D}{\partial V_t} = -k_n (V_{GS} - V_t)$$

$$S_{V_t}^{I_D} \equiv \frac{\partial I_D}{\partial V_t} \frac{V_t}{I_D} = -\frac{k_n (V_{GS} - V_t) V_t}{I_D}$$

$$= -\frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$= -\frac{2V_t}{V_{GS} - V_t} = -\frac{2V_t}{V_{OV}} \quad Q.E.D$$

For  $V_t = 0.5 \text{ V}$ ,  $\frac{\Delta V_t}{V_t} = \pm 5\%$ , and  $V_{OV} = 0.25 \text{ V}$ , we have

$$\frac{\Delta I_D}{I_D} = S_{V_t}^{I_D} \left( \frac{\Delta V_t}{V_t} \right)$$

$$= -\frac{2 \times 0.5}{0.25} \times \pm 5\%$$

$$= \mp 20\%$$

(b) For fixed bias at the gate  $V_G$  and a resistance  $R_S$  in the source lead, we have

$$V_G = V_{GS} + I_D R_S$$

where  $V_{GS}$  is obtained from

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$\Rightarrow V_{GS} = V_t + \sqrt{\frac{2I_D}{k_n}}$$

Thus

$$V_t + \sqrt{\frac{2I_D}{k_n}} + I_D R_S = V_G$$

Differentiating relative to  $V_t$ , we have

$$1 + \frac{1}{2\sqrt{2I_D/k_n}} \frac{2}{k_n} \frac{\partial I_D}{\partial V_t} + R_S \frac{\partial I_D}{\partial V_t} = 0$$

$$\frac{\partial I_D}{\partial V_t} \left[ \frac{1}{\sqrt{2k_n I_D}} + R_S \right] = -1$$

$$\frac{\partial I_D}{\partial V_t} = -\frac{1}{\frac{1}{\sqrt{2k_n I_D}} + R_S}$$

$$S_{V_t}^{I_D} = \frac{\partial I_D}{\partial V_t} \frac{V_t}{I_D} = -\frac{V_t}{\sqrt{\frac{I_D}{2k_n}} + I_D R_S}$$

But

$$I_D = \frac{1}{2} k_n V_{OV}^2 \Rightarrow V_{OV} = \sqrt{\frac{2I_D}{k_n}}$$

Thus

$$S_{V_t}^{I_D} = -\frac{2V_t}{V_{OV} + 2I_D R_S} \quad Q.E.D$$

For  $V_t = 0.5 \text{ V}$ ,  $\frac{\Delta V_t}{V_t} = \pm 5\%$ , and

$V_{OV} = 0.25 \text{ V}$ , to limit  $\frac{\Delta I_D}{I_D}$  to  $\pm 5\%$  we require

$$S_{V_t}^{I_D} = 1$$

Thus

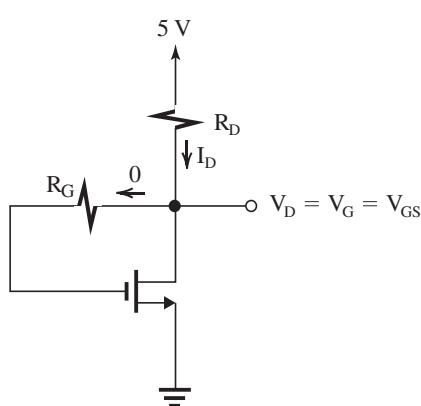
$$-1 = -\frac{2 \times 0.5}{0.25 + 2I_D R_S}$$

$$\Rightarrow I_D R_S = 0.375 \text{ V}$$

For  $I_D = 0.1 \text{ mA}$ ,

$$R_S = \frac{0.375}{0.1} = 3.75 \text{ k}\Omega$$

6.92

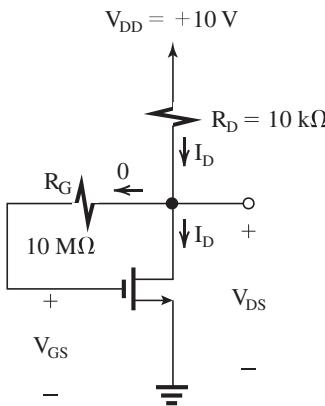


$$I_D = 0.2 = \frac{1}{2} \times 10(V_{GS} - V_t)^2$$

$$\Rightarrow V_{GS} = 1.2 \text{ V}$$

$$R_D = \frac{5 - 1.2}{0.2} = 19 \text{ k}\Omega$$

6.93



$$V_{GS} = V_{DD} - I_D R_D$$

$$= 10 - 10I_D$$

(a)  $V_t = 1 \text{ V}$  and  $k_n = 0.5 \text{ mA/V}^2$

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

$$I_D = \frac{1}{2} \times 0.5(10 - 10I_D - 1)^2$$

$$\Rightarrow I_D^2 - 1.84I_D + 0.81 = 0$$

$$I_D = 1.11 \text{ mA or } 0.73 \text{ mA}$$

The first root results in  $V_D = -0.11 \text{ V}$ , which is physically meaningless. Thus

$$I_D = 0.73 \text{ mA}$$

$$V_G = V_D = 10 - 10 \times 0.73 = 2.7 \text{ V}$$

(b)  $V_t = 2 \text{ V}$  and  $k_n = 1.25 \text{ mA/V}^2$

$$I_D = \frac{1}{2} \times 1.25(10 - 10I_D - 2)^2$$

$$\Rightarrow I_D^2 - 1.616I_D + 0.64 = 0$$

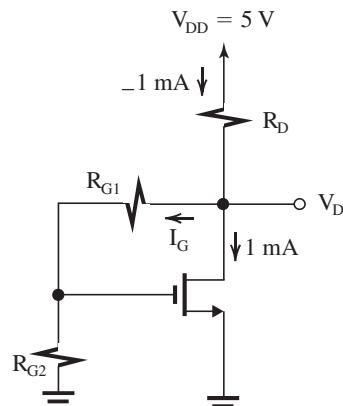
$$I_D = 0.92 \text{ mA or } 0.695 \text{ mA}$$

The first root can be shown to be physically meaningless, thus

$$I_D = 0.695 \text{ mA}$$

$$V_G = V_D = 10 - 10 \times 0.695 = 3.05 \text{ V}$$

6.94



$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$1 = \frac{1}{2} \times 8V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.5 \text{ V}$$

Since the transistor leaves the saturation region of operation when  $V_D < V_{OV}$ , we select

$$V_D = V_{OV} + 2$$

$$V_D = 2.5 \text{ V}$$

Since  $I_G \ll I_D$ , we can write

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 2.5}{1} = 2.5 \text{ k}\Omega$$

$$V_{GS} = V_t + V_{OV} = 0.8 + 0.5 = 1.3 \text{ V}$$

Thus the voltage drop across  $R_{G2}$  is 1.3 V and that across  $R_{G1}$  is  $(2.5 - 1.3) = 1.2$  V. Thus  $R_{G2}$  is the larger of the two resistances, and we select  $R_{G2} = 22 \text{ M}\Omega$  and find  $R_{G1}$  from

$$\frac{R_{G1}}{R_{G2}} = \frac{1.2}{1.3} \Rightarrow R_{G1} = 20.3 \text{ M}\Omega$$

Specifying all resistors to two significant digits, we have  $R_D = 2.5 \text{ k}\Omega$ ,  $R_{G1} = 22 \text{ M}\Omega$ , and  $R_{G2} = 20 \text{ M}\Omega$ .

$$6.95 \frac{R_{B1}}{R_{B1} + R_{B2}} \times 3 = 0.710$$

$$\Rightarrow \frac{R_{B2}}{R_{B1}} = 3.225$$

Given that  $R_{B1}$  and  $R_{B2}$  are 1% resistors, the maximum and minimum values of the ratio  $R_{B2}/R_{B1}$  will be  $3.225 \times 1.02 = 3.2895$  and  $3.225 \times 0.98 = 3.1605$ . The resulting  $V_{BE}$  will be 0.699 V and 0.721 V, respectively.

Correspondingly,  $I_C$  will be

$$I_{C\max} = 1 \times e^{(0.710-0.699)/0.025}$$

$$= 1.55 \text{ mA}$$

and

$$I_{C\min} = 1 \times e^{(0.710-0.721)/0.025}$$

$$I_{C\min} = 0.64 \text{ mA}$$

$V_{CE}$  will range from

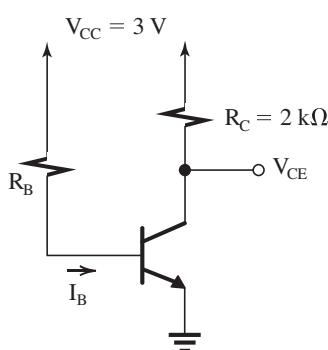
$$V_{CE\min} = 3 - 1.55 \times 2 = -0.1 \text{ V}$$

which is impossible, implying that the transistor will saturate at this value of dc bias!

$$V_{CE\max} = 3 - 0.64 \times 2 = 1.72 \text{ V}$$

It should be clear that this biasing arrangement is useless, since even the small and inevitable tolerances in  $R_{B1}$  and  $R_{B2}$  caused such huge variations in  $I_C$  that in one extreme the transistor left the active mode of operation altogether!

6.96



To obtain  $I_C = 1 \text{ mA}$ , we write

$$I_B = \frac{I_C}{\beta} = \frac{1 \text{ mA}}{100} = 0.01 \text{ mA}$$

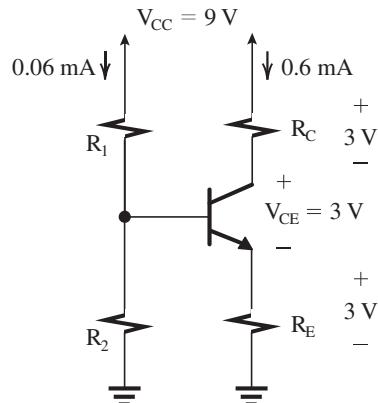
Thus,

$$R_B = \frac{V_{CC} - V_{BE}}{I_B} \simeq \frac{3 - 0.7}{0.01} = 230 \text{ k}\Omega$$

Since  $\beta$  ranges from 50 to 150 and  $I_B$  is fixed at 0.01 mA, the collector current  $I_C$  will range from  $0.01 \times 50 = 0.5 \text{ mA}$  to  $0.01 \times 150 = 1.5 \text{ mA}$ .

Correspondingly,  $V_{CE}$  will range from  $(3 - 0.5 \times 2) = 1 \text{ V}$  to  $(3 - 1.5 \times 2) = 0 \text{ V}$ . The latter value implies that the high- $\beta$  transistor will leave the active region of operation and saturate. Obviously, this bias method is very intolerant of the inevitable variations in  $\beta$ . Thus it is not a good method for biasing the BJT.

6.97



Initial design:  $\beta = \infty$

$$R_C = R_E = \frac{3 \text{ V}}{0.6} = 5 \text{ k}\Omega$$

$$R_1 + R_2 = \frac{9}{0.06} = 150 \text{ k}\Omega$$

$$V_B = V_E + V_{BE} = 3 + 0.7 = 3.7 \text{ V}$$

$$R_2 = \frac{3.7}{0.06} = 61.7 \text{ k}\Omega$$

$$R_1 = 150 - 61.7 = 88.3 \text{ k}\Omega$$

Using 5% resistors from Appendix J, and selecting  $R_1$  and  $R_2$  so as to obtain a  $V_{BB}$  that is slightly higher than 3.7 V, we write

$$R_1 = 82 \text{ k}\Omega \text{ and } R_2 = 62 \text{ k}\Omega$$

$$R_E = 5.1 \text{ k}\Omega \text{ and } R_C = 5.1 \text{ k}\Omega$$

$$V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2} = 9 \times \frac{62}{62 + 82} = 3.875$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

where

$$R_B = R_1 \parallel R_2 = 62 \parallel 82 = 35.3 \text{ k}\Omega$$

$$I_E = \frac{3.875 - 0.7}{5.1 + \frac{35.3}{91}} = 0.58 \text{ mA}$$

$$V_E = 0.58 \times 5.1 = 3.18$$

$$V_B = 3.88 \text{ V}$$

$$I_C = \alpha I_E = \frac{90}{91} \times 0.58 = 0.57 \text{ mA}$$

$$V_C = 6.1 \text{ V}$$

$$I_{R2} = \frac{V_B}{R_2} = \frac{3.88}{62} = 0.063 \text{ mA}$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.58}{91} = 0.006 \text{ mA and}$$

$$I_{R1} = 0.069 \text{ mA}$$

6.98 Refer to Fig. 6.52.

$$(a) I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

$$I_E|_{\text{nominal}} = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{101}}$$

$$I_E|_{\text{high}} = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{151}}$$

$$I_E|_{\text{low}} = \frac{V_{BE} - V_{BE}}{R_E + \frac{R_B}{51}}$$

Let s constrain  $I_E|_{\text{low}}$  to be equal to  $I_E|_{\text{nominal}} \times 0.95$  and then check  $I_E|_{\text{high}}$ :

$$0.95 \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{101}} = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{51}}$$

$$0.95 = \frac{1 + \frac{R_B/R_E}{101}}{1 + \frac{R_B/R_E}{51}}$$

$$\Rightarrow \frac{R_B}{R_E} = 5.73$$

For this value,

$$I_E|_{\text{nominal}} = 0.946 \left( \frac{V_{BB} - V_{BE}}{R_E} \right)$$

$$I_E|_{\text{low}} = 0.90 \left( \frac{V_{BB} - V_{BE}}{R_E} \right) = 0.95 I_E|_{\text{nominal}}$$

$$I_E|_{\text{high}} = 0.963 \left( \frac{V_{BE} - V_{BE}}{R_E} \right) = 1.02 I_E|_{\text{nominal}}$$

Thus, the maximum allowable ratio is

$$\frac{R_B}{R_E} = 5.73$$

$$(b) I_E = \frac{V_{BB} - V_{BE}}{R_E \left( 1 + \frac{R_B/R_E}{\beta + 1} \right)}$$

$$I_E R_E = \frac{V_{BB} - V_{BE}}{1 + \frac{5.73}{\beta + 1}}$$

$$\frac{V_{CC}}{3} = \frac{V_{BB} - V_{BE}}{1 + \frac{5.73}{101}}$$

$$\Rightarrow V_{BB} = V_{BE} + 0.352 V_{CC}$$

$$(c) V_{CC} = 5 \text{ V}$$

$$V_{BB} = 0.7 + 0.352 \times 5 = 2.46 \text{ V}$$

$$R_E = \frac{V_{CC}/3}{I_E} = \frac{5/3}{0.5} = 3.33 \text{ k}\Omega$$

$$R_B = 5.73 \times R_E = 19.08 \text{ k}\Omega$$

$$V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2}$$

$$2.46 = 5 \frac{R_2}{R_1 + R_2}$$

$$2.46 R_1 = 5 \frac{R_1 R_2}{R_1 + R_2} = 5 R_B$$

$$= 5 \times 19.08$$

$$\Rightarrow R_1 = 38.8 \text{ k}\Omega$$

$$R_2 = 1 / \left( \frac{1}{R_B} - \frac{1}{R_1} \right) = 37.5 \text{ k}\Omega$$

$$(d) V_{CE} = V_{CC} - R_C I_G$$

$$1 = 5 - R_C \times 0.99 \times 0.5$$

$$\Rightarrow R_C = 8.1 \text{ k}\Omega$$

Check design:

$$V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2} = 5 \times \frac{37.5}{37.5 + 38.8}$$

$$= 2.46 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 37.5 \parallel 38.8 = 19.07 \text{ k}\Omega$$

$$I_E|_{\text{nominal}} = \frac{2.46 - 0.7}{3.33 + \frac{19.07}{101}} = 0.5 \text{ mA}$$

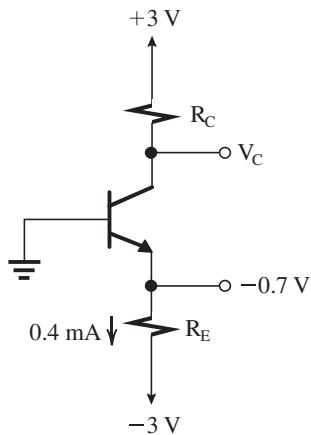
$$I_E|_{\text{low}} = \frac{2.46 - 0.7}{3.33 + \frac{19.07}{51}} = 0.475 \text{ mA}$$

which is 5% lower than  $I_E|_{\text{nominal}}$ , and

$$I_E|_{\text{high}} = \frac{2.46 - 0.7}{3.33 + \frac{19.07}{151}} = 0.509 \text{ mA}$$

which is 1.8% higher than  $I_E|_{\text{nominal}}$ .

6.99



$$R_E = \frac{-0.7 - (-3)}{0.4} = 5.75 \text{ k}\Omega$$

To maximize gain while allowing for  $\pm 1 \text{ V}$  signal swing at the collector, design for the lowest possible  $V_C$  consistent with

$$V_C - 1 = -0.7 + V_{CE\text{sat}}$$

$$= -0.7 + 0.3 = -0.4 \text{ V}$$

$$V_C = 0.6 \text{ V}$$

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{3 - 0.6}{0.39} = 6.2 \text{ k}\Omega$$

As temperature increases from  $25^\circ\text{C}$  to  $125^\circ\text{C}$ , (i.e., by  $100^\circ\text{C}$ ),  $V_{BE}$  decreases by  $2 \text{ mV} \times 100 =$

$$-200 \text{ mV}. \text{ Thus } I_E \text{ increases by } \frac{0.2 \text{ V}}{R_E} =$$

$$\frac{0.2 \text{ V}}{5.75 \text{ k}\Omega} = 0.035 \text{ mA} \text{ to become } 0.435 \text{ mA. The collector current becomes}$$

$$I_C = \frac{\beta}{\beta + 1} \times 0.435$$

where  $\beta$  is the increased value of 150,

$$I_C = \frac{150}{151} \times 0.435 \text{ mA} = 0.432 \text{ mA}$$

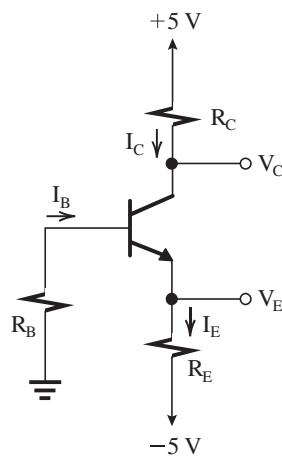
Thus,

$$\Delta I_C = 0.432 - 0.39 = 0.042 \text{ mA}$$

for a percentage increase of

$$\frac{\Delta I_C}{I_C} \times 100 = \frac{0.042}{0.39} \times 100 = 10.8\%$$

6.100



Required:  $I_C = 0.5 \text{ mA}$  and  $V_C = V_E + 2$ .

(a)  $\beta = \infty$

$$V_B = 0$$

$$V_E = -0.7 \text{ V}$$

$$I_E = 0.5 = \frac{V_E - (-5)}{R_E} = \frac{4.3}{R_E}$$

$$\Rightarrow R_E = 8.6 \text{ k}\Omega$$

$$V_C = V_E + 2 = -0.7 + 2 = +1.3 \text{ V}$$

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{5 - 1.3}{0.5} = 7.4 \text{ k}\Omega$$

(b)  $\beta_{\min} = 50$

$$I_{B\max} = \frac{I_E}{51} = \frac{0.5}{51} \simeq 0.01 \text{ mA}$$

$$I_E R_E = 0.5 \times 8.6 = 4.3 \text{ V}$$

$$I_{B\max} R_{B\max} = 0.1 I_E R_E = 0.43 \text{ V}$$

$$R_{B\max} = \frac{0.43}{0.01} = 43 \text{ k}\Omega$$

(c) Standard 5% resistors:

$$R_B = 43 \text{ k}\Omega$$

$$R_E = 8.2 \text{ k}\Omega$$

$$R_C = 7.5 \text{ k}\Omega$$

(d)  $\beta = \infty$ :

$$V_B = 0, \quad V_E = -0.7 \text{ V}$$

$$I_E = \frac{-0.7 - (-5)}{8.2} = 0.52 \text{ mA}$$

$$I_C = 0.52 \text{ mA}$$

$$V_C = 5 - 0.52 \times 7.5 = 1.1 \text{ V}$$

$\beta = 50$ :

$$I_E = \frac{5 - 0.7}{8.2 + \frac{43}{51}} = 0.48 \text{ mA}$$

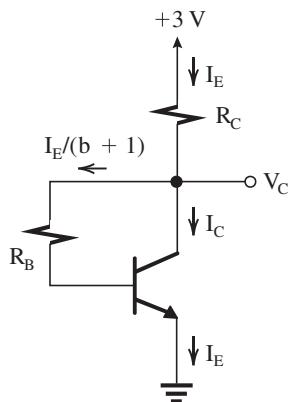
$$V_E = -5 + 0.48 \times 8.2 = -1.064 \text{ V}$$

$$V_B = -0.364 \text{ V}$$

$$I_C = \alpha I_E = \frac{50}{51} \times 0.48 = 0.47 \text{ mA}$$

$$V_C = 5 - 0.47 \times 7.5 = 1.475 \text{ V}$$

6.101



$$V_C = V_{CEsat} + 1 \text{ V}$$

$$= 1.3 \text{ V}$$

$$I_E = \frac{3 - 1.3}{R_C} = 0.5 \text{ mA}$$

$$\Rightarrow R_C = 3.4 \text{ k}\Omega$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.5}{101} \simeq 0.005 \text{ mA}$$

$$V_C = V_{BE} + I_B R_B$$

$$1.3 = 0.7 + 0.005 \times R_B$$

$$\Rightarrow R_B = 120 \text{ k}\Omega$$

Standard 5% resistors:

$$R_C = 3.3 \text{ k}\Omega$$

$$R_B = 120 \text{ k}\Omega$$

If the actual BJT has  $\beta = 50$ , then

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}} = \frac{3 - 0.7}{3.3 + \frac{120}{51}} = 0.41 \text{ mA}$$

$$V_C = 3 - I_E R_C = 3 - 0.41 \times 3.3 = 1.65 \text{ V}$$

Allowable negative signal swing at the collector is as follows:

$$V_C - V_{CEsat} = 1.65 - 0.3 = 1.35 \text{ V}$$

An equal positive swing is just possible. For  $\beta = 150$ :

$$I_E = \frac{3 - 0.7}{3.3 + \frac{120}{151}} = 0.56 \text{ mA}$$

$$V_C = 3 - I_E R_C = 3 - 0.56 \times 3.3 = 1.15 \text{ V}$$

Allowable negative signal swing at the collector =  $1.15 - 0.3 = 0.85 \text{ V}$ . An equal positive swing is possible.

6.102

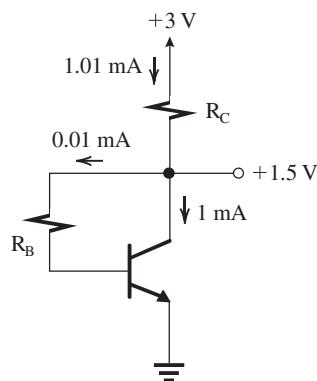


Figure 1

(a) From the circuit diagram of Fig. 1, we can write

$$R_C = \frac{3 - 1.5}{1.01 \text{ mA}} \simeq 1.5 \text{ k}\Omega$$

$$1.5 = 0.01 R_B + V_{BE}$$

$$= 0.01 R_B + 0.7$$

$$\Rightarrow R_B = 80 \text{ k}\Omega$$

(b) Selecting 5% resistors, we have

$$R_C = 1.5 \text{ k}\Omega$$

$$R_B = 82 \text{ k}\Omega$$

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta + 1}}$$

$$= \frac{3 - 0.7}{1.5 + \frac{82}{101}} = 0.99 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.99 = 0.98 \text{ mA}$$

$$V_C = 3 - 1.5 \times 0.99 = 1.52 \text{ V}$$

(c)  $\beta = \infty$ :

$$I_C = I_E = \frac{V_{CC} - V_{BE}}{R_C} = \frac{3 - 0.7}{1.5} = 1.53 \text{ mA}$$

$$V_C = 0.7 \text{ V}$$

(d) From the circuit diagram of Fig. 2, we can write

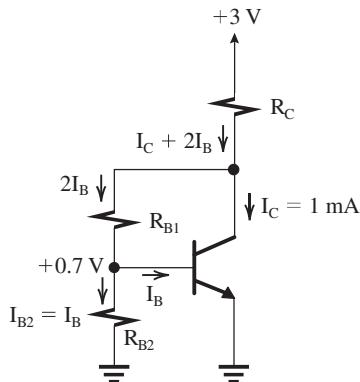


Figure 2

$$I_B = \frac{I_C}{\beta} = \frac{1}{100} = 0.01 \text{ mA}$$

$$R_{B2} = \frac{0.7}{I_B} = \frac{0.7}{0.01} = 70 \text{ k}\Omega$$

$$1.5 = 2I_B R_{B1} + 0.7$$

$$0.8 = 2 \times 0.01 \times R_{B1}$$

$$R_{B1} = 40 \text{ k}\Omega$$

$$R_C = \frac{3 - 1.5}{I_C + 2I_B} = \frac{1.5}{1.02} = 1.47 \text{ k}\Omega$$

For  $\beta = \infty$ :

$$I_B = 0, \quad I_{B2} = \frac{0.7}{R_{B2}} = \frac{0.7}{70} = 0.01 \text{ mA}$$

$$I_{B1} = I_{B2} = 0.01 \text{ mA}$$

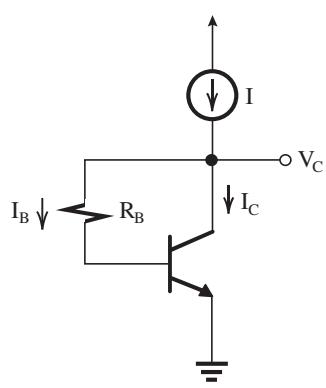
$$V_C = 0.01R_{B1} + 0.7 = 0.01 \times 40 + 0.7$$

$$= 1.1 \text{ V}$$

$$I_C + 0.01 = \frac{3 - 1.1}{R_C} = \frac{3 - 1.1}{1.47} = 1.29$$

$$I_C = 1.28 \text{ mA}$$

6.103



$$I_C = 1 \text{ mA}$$

$$I = I_C + I_B$$

$$= I_C + \frac{I_C}{\beta}$$

$$= 1 \left( 1 + \frac{1}{\beta} \right)$$

$$I = 1.01 \text{ mA}$$

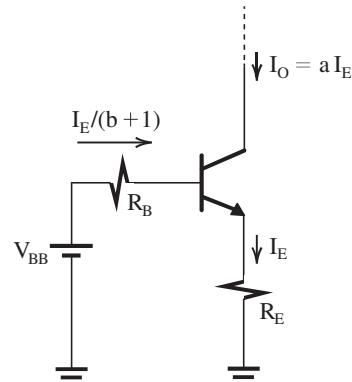
$$V_C = 1.5 \text{ V} = I_B R_B + V_{BE}$$

$$1.5 = 0.01 \times R_B + 0.7$$

$$R_B = 80 \text{ k}\Omega$$

6.104 Refer to the circuit in Fig. P6.104.

Replacing  $V_{CC}$  together with the voltage divider ( $R_1, R_2$ ) by its Thévenin equivalent results in the circuit shown below.



where

$$V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2}$$

and

$$R_B = (R_1 \parallel R_2)$$

Now,

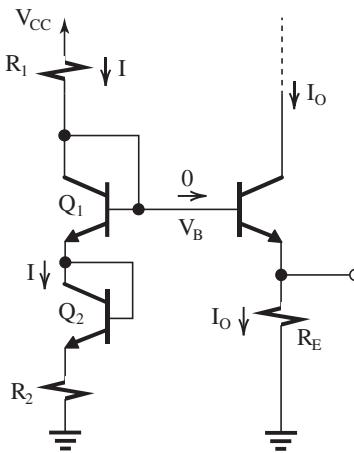
$$V_{BB} = \frac{I_E}{\beta + 1} R_B + V_{BE} + I_E R_E$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + (R_1 \parallel R_2)/(\beta + 1)}$$

$$I_C = \alpha I_E$$

$$= \alpha \frac{V_{CC} [R_2 / (R_1 + R_2)] - V_{BE}}{R_E + (R_1 \parallel R_2) / (\beta + 1)}$$

6.105



$$I = \frac{V_{CC} - V_{BE1} - V_{BE2}}{R_1 + R_2}$$

$$V_B = IR_2 + V_{BE2} + V_{BE1}$$

$$V_{E3} = V_B - V_{BE3}$$

$$V_{E3} = IR_2 + V_{BE2} + V_{BE1} - V_{BE3}$$

$$= (V_{CC} - V_{BE1} - V_{BE2}) \frac{R_2}{R_1 + R_2} + V_{BE1} \\ + V_{BE2} - V_{BE3}$$

$$I_O = \frac{V_E}{R_E} = \frac{\alpha}{R_E} \left[ (V_{CC} - V_{BE1} - V_{BE2}) \frac{R_2}{R_1 + R_2} + V_{BE1} + V_{BE2} - V_{BE3} \right]$$

Now, for  $R_1 = R_2$  and the currents in all junctions equal,

$$V_{BE1} = V_{BE2} = V_{BE3} = V_{BE}$$

$$I_O = \frac{1}{R_E} \left[ (V_{CC} - 2V_{BE}) \times \frac{1}{2} + V_{BE} \right]$$

$$I_O = \frac{V_{CC}}{2R_E} \quad \text{Q.E.D}$$

Thus,

$$I_O R_E = \frac{V_{CC}}{2}$$

$$V_B = \frac{V_{CC}}{2} + V_{BE}$$

$$I = (V_B - 2V_{BE})/R_2 = \left( \frac{V_{CC}}{2} - V_{BE} \right) / R_2$$

But since  $I$  must be equal to  $I_O$ , we have

$$\frac{V_{CC}}{2R_E} = \frac{V_{CC}/2 - V_{BE}}{R_2}$$

Thus,

$$R_1 = R_2 = R_E \left( \frac{V_{CC} - 2V_{BE}}{V_{CC}} \right)$$

For  $V_{CC} = 10$  V and  $V_{BE} = 0.7$  V,

$$R_1 = R_2 = R_E \left( \frac{10 - 1.4}{10} \right) = 0.86R_E$$

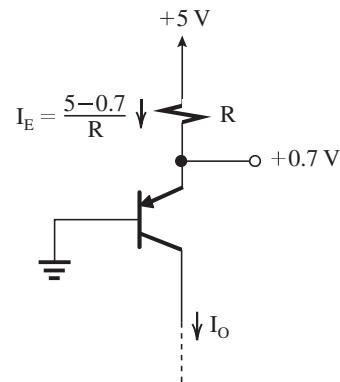
To obtain  $I_O = 0.5$  mA,

$$0.5 = \frac{V_{CC}}{2R_E} = \frac{10}{2R_E}$$

$$\Rightarrow R_E = 10 \text{ k}\Omega$$

$$R_1 = R_2 = 8.6 \text{ k}\Omega$$

6.106



$$I_O = \alpha I_E \simeq 0.5 \text{ mA}$$

$$I_E = 0.5 \text{ mA}$$

$$\Rightarrow R = \frac{5 - 0.7}{0.5} = 8.6 \text{ k}\Omega$$

$$v_{Cmax} = 0.7 - V_{ECsat} = 0.7 - 0.3$$

$$= +0.4 \text{ V}$$

6.107 Refer to the equivalent circuit in Fig. 6.55(b).

$$G_v = -\frac{R_{in}}{R_{in} + R_{sig}} g_m (R_D \parallel R_L \parallel r_o)$$

$$= -\frac{R_G}{R_G + R_{sig}} g_m (R_D \parallel R_L \parallel r_o)$$

$$= -\frac{10}{10+1} \times 3 \times (10 \parallel 20 \parallel 100)$$

$$= -17 \text{ V/V}$$

6.108 (a) Refer to Fig. P6.108. The dc circuit can be obtained by opening all coupling and bypass capacitors, resulting in the circuit shown in Fig. 1 on next page.

See analysis on figure.

$$V_{GS} = 2 - 1 = 1 \text{ V}$$

$$V_{OV} = V_{GS} - V_t = 1 - 0.7 = 0.3 \text{ V}$$

Since  $V_D$  at 2.5 V is 1.2 V higher than  $V_S + V_{OV} = 1 + 0.3 = 1.3$  V, the transistor is indeed operating in saturation. (Equivalent

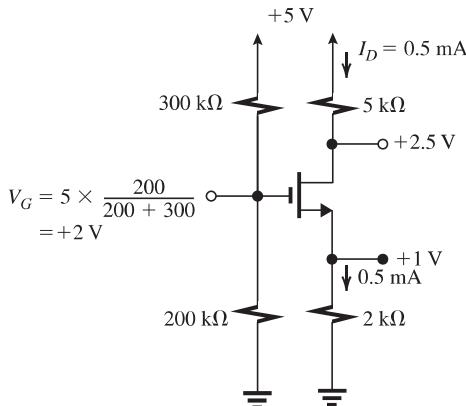


Figure 1

$V_D = 2.5 \text{ V}$  is higher than  $V_G - V_t = 1.3 \text{ V}$  by 1.2 V.)

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.5 = \frac{1}{2} k_n \times 0.3^3$$

$$\Rightarrow k_n = 11.1 \text{ mA/V}^2$$

(b) The amplifier small-signal equivalent-circuit model is shown in Fig. 2 below.

$$R_{in} = R_{G1} \parallel R_{G2} = 300 \text{ k}\Omega \parallel 200 \text{ k}\Omega = 120 \text{ k}\Omega$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.3} = 3.33 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{50}{0.5} = 100 \text{ k}\Omega$$

$$G_v = -\frac{R_{in}}{R_{in} + R_{sig}} g_m (r_o \parallel R_D \parallel R_L)$$

$$= -\frac{120}{120 + 120} \times 3.33 \times (100 \parallel 5 \parallel 5)$$

$$= -4.1 \text{ V/V}$$

$$(c) V_G = 2 \text{ V}, \quad V_D = 2.5 \text{ V}$$

$$\hat{v}_{GS} = 2 + \hat{v}_{gs}, \quad \hat{v}_{DS} = 2.5 - |A_v| \hat{v}_{gs}$$

where

$$|A_v| = g_m (r_o \parallel R_D \parallel R_L) = 8.1 \text{ V/V}$$

This figure belongs to Problem 6.108.

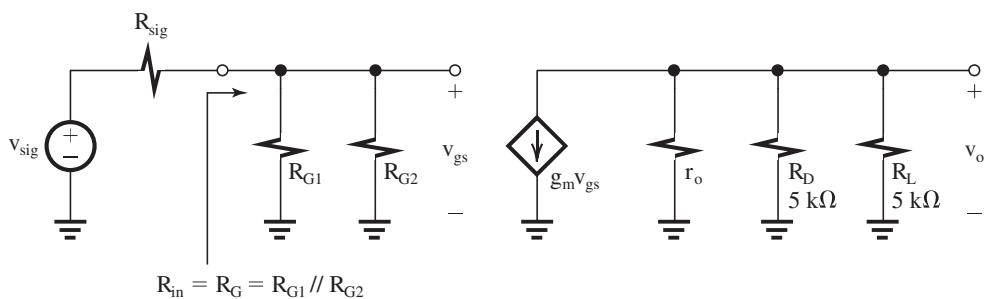


Figure 2

To remain in saturation,

$$\hat{v}_{DS} \geq \hat{v}_{GS} - V_t$$

$$2.5 - 8.1 \hat{v}_{gs} \geq 2 + \hat{v}_{gs} - 0.7$$

This is satisfied with equality at

$$\hat{v}_{gs} = \frac{2.5 - 1.3}{9.1} = 0.132 \text{ V}$$

The corresponding value of  $\hat{v}_{sig}$  is

$$\hat{v}_{sig} = \hat{v}_{gs} \left( \frac{120 + 120}{120} \right) = 2 \times 0.132 = 0.264 \text{ V}$$

The corresponding amplitude at the output will be

$$|G_v| \hat{v}_{sig} = 4.1 \times 0.264 = 1.08 \text{ V}$$

(d) To be able to double  $\hat{v}_{sig}$  without leaving saturation, we must reduce  $\hat{v}_{gs}$  to half of what would be its new value; that is, we must keep  $\hat{v}_{gs}$  unchanged. This in turn can be achieved by connecting an unbypassed  $R_s$  equal to  $1/g_m$ ,

$$R_s = \frac{1}{3.33 \text{ mA/V}} = 300 \text{ }\Omega$$

Since  $\hat{v}_{gs}$  does not change, the output voltage also will not change, thus  $\hat{v}_o = 1.08 \text{ V}$ .

6.109 Refer to Fig. P6.109.

(a) DC bias:

$$|V_{ov}| = 0.3 \text{ V} \Rightarrow V_{SG} = |V_{tp}| + |V_{ov}| = 1 \text{ V}$$

Since  $V_G = 0 \text{ V}$ ,  $V_S = V_{SG} = +1 \text{ V}$ , and

$$I_D = \frac{2.5 - 1}{R_s} = 0.3 \text{ mA}$$

$$\Rightarrow R_s = \frac{1.5}{0.3} = 5 \text{ k}\Omega$$

(b)  $G_v = -g_m R_D$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.3}{0.3} = 2 \text{ mA/V}$$

Thus,

$$-10 = -2R_D \Rightarrow R_D = 5 \text{ k}\Omega$$

(c)  $v_G = 0 \text{ V (dc)} + v_{\text{sig}}$

$$v_{G\min} = -\hat{v}_{\text{sig}}$$

$$\hat{v}_D = V_D + |G_v| \hat{v}_{\text{sig}}$$

where

$$V_D = -2.5 + I_D R_D = -2.5 + 0.3 \times 5 = -1 \text{ V}$$

To remain in saturation,

$$\hat{v}_D \leq \hat{v}_G + |V_{tp}|$$

$$-1 + 10 \hat{v}_{\text{sig}} \leq -\hat{v}_{\text{sig}} + 0.7$$

Satisfying this constraint with equality gives

$$\hat{v}_{\text{sig}} = 0.154 \text{ V}$$

and the corresponding output voltage

$$\hat{v}_d = |G_v| \hat{v}_{\text{sig}} = 1.54 \text{ V}$$

(d) If  $\hat{v}_{\text{sig}} = 50 \text{ mV}$ , then

$$V_D + |G_v| \hat{v}_{\text{sig}} = -\hat{v}_{\text{sig}} + |V_{tp}|$$

where

$$V_D = -2.5 + I_D R_D = -2.5 + 0.3 R_D$$

and

$$|G_v| = g_m R_D = 2 R_D$$

Thus

$$-2.5 + 0.3 R_D + 2 R_D \hat{v}_{\text{sig}} = -\hat{v}_{\text{sig}} + |V_{tp}|$$

$$-2.5 + 0.3 R_D + 2 R_D \times 0.05 = -0.05 + 0.7$$

$$0.4 R_D = 3.15$$

$$\Rightarrow R_D = 7.875 \text{ k}\Omega$$

$$G_v = -g_m R_D = -2 \times 7.875 = -15.75 \text{ V/V}$$

6.110 Refer to Fig. P6.110.

$$R_{i2} = \frac{1}{g_{m2}} = 50 \text{ }\Omega$$

$$\Rightarrow g_{m2} = \frac{1}{50} \text{ A/V} = 20 \text{ mA/V}$$

If  $Q_1$  is biased at the same point as  $Q_2$ , then

$$g_{m1} = g_{m2} = 20 \text{ mA/V}$$

$$i_{d1} = g_{m1} \times 5 \text{ (mV)}$$

$$= 20 \times 0.005 = 0.1 \text{ mA}$$

$$v_{d1} = i_{d1} \times 50 \text{ }\Omega$$

$$= 0.1 \times 50 = 5 \text{ mV}$$

$$v_o = i_{d1} R_D = 1 \text{ V}$$

$$R_D = \frac{1 \text{ V}}{0.1 \text{ mA}} = 10 \text{ k}\Omega$$

6.111 (a) Refer to the circuit of Fig. P6.111(a):

$$A_{vo} \equiv \frac{v_{o1}}{v_i} = \frac{10}{10 + \frac{1}{g_m}} = \frac{10}{10 + \frac{1}{10}} = 0.99 \text{ V/V}$$

$$R_o = \frac{1}{g_m} \parallel 10 \text{ k}\Omega = 0.1 \parallel 10 = 99 \text{ }\Omega$$

(b) Refer to Fig. P6.111(b):

$$R_{in} = 10 \text{ k}\Omega \parallel \frac{1}{g_m} = 10 \parallel 0.1 = 99 \text{ }\Omega$$

$$\frac{v_o}{v_{i2}} = \frac{5 \parallel 2}{1/g_m} = 10(5 \parallel 2) = 14.3 \text{ V/V}$$

$$(c) v_{i2} = (A_{vo} v_i) \frac{R_{in}}{R_{in} + R_o}$$

$$= 0.99 \times v_i \times \frac{99}{99 + 99}$$

$$\simeq 0.5 v_i$$

$$v_o = 14.3 \times v_{i2} = 14.3 \times 0.5 v_i$$

$$\frac{v_o}{v_i} = 7.15 \text{ V/V}$$

6.112 (a) DC bias: Refer to the circuit in Fig. P6.112 with all capacitors eliminated:

$$R_{in} \text{ at gate} = R_G = 10 \text{ M}\Omega$$

$V_G = 0$ , thus  $V_S = -V_{GS}$ , where  $V_{GS}$  can be obtained from

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$0.4 = \frac{1}{2} \times 5 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.4 \text{ V}$$

$$V_{GS} = V_t + 0.4 = 0.8 + 0.4 = 1.2 \text{ V}$$

$$V_S = -1.2 \text{ V}$$

$$R_S = \frac{-1.2 - (-5)}{0.4} = 9.5 \text{ k}\Omega$$

To remain in saturation, the minimum drain voltage must be limited to  $V_G - V_t = 0 - 0.8 = -0.8 \text{ V}$ . Now, to allow for 0.8-V negative signal swing, we must have

$$V_D = 0 \text{ V}$$

and

$$R_D = \frac{5 - 0}{0.4} = 12.5 \text{ k}\Omega$$

$$(b) g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.4} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{40}{0.4} = 100 \text{ k}\Omega$$

(c) If terminal Z is connected to ground, the circuit becomes a CS amplifier,

$$G_v = -\frac{v_y}{v_{sig}} = \frac{R_G}{R_G + R_{sig}} \times -g_m(r_o \parallel R_D \parallel R_L)$$

$$= -\frac{10}{10 + 1} \times 2 \times (100 \parallel 12.5 \parallel 10)$$

$$= -9.6 \text{ V/V}$$

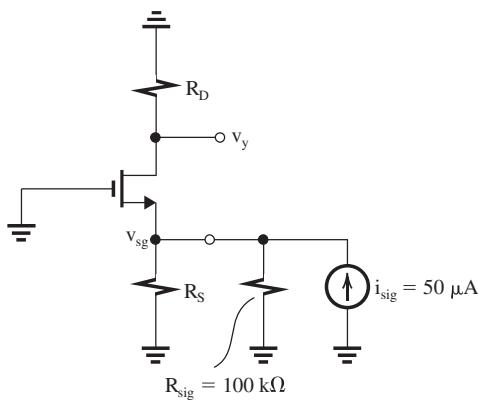
(d) If terminal Y is grounded, the circuit becomes a CD or source-follower amplifier:

$$\begin{aligned} \frac{v_z}{v_x} &= \frac{(R_S \parallel r_o)}{(R_S \parallel r_o) + \frac{1}{g_m}} \\ &= \frac{(9.5 \parallel 100)}{(9.5 \parallel 100) + \frac{1}{2}} = 0.946 \text{ V/V} \end{aligned}$$

Looking into terminal Z, we see  $R_o$ :

$$\begin{aligned} R_o &= R_S \parallel r_o \parallel \frac{1}{g_m} \\ &= 9.5 \parallel 100 \parallel \frac{1}{2} = 473 \Omega \end{aligned}$$

(e) If X is grounded, the circuit becomes a CG amplifier.



The figure shows the circuit prepared for signal calculations.

$$\begin{aligned} v_{sg} &= i_{sig} \times \left[ R_{sig} \parallel R_S \parallel \frac{1}{g_m} \right] \\ &= 50 \times 10^{-3} (\text{mA}) \left[ 100 \parallel 9.5 \parallel \frac{1}{2} \right] (\text{kΩ}) \\ &= 0.024 \text{ V} \end{aligned}$$

$$\begin{aligned} v_y &= (g_m R_D) v_{sg} \\ &= (2 \times 12.5) \times 0.024 = 0.6 \text{ V} \end{aligned}$$

6.113 (a) DC bias:

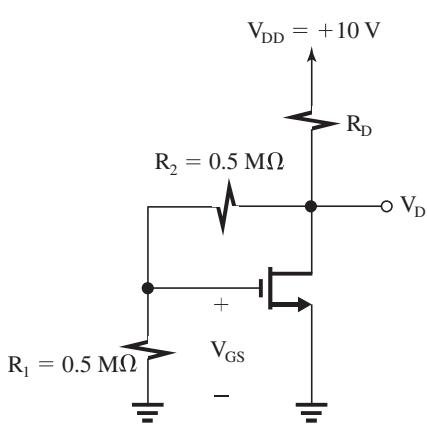


Figure 1

$$V_{OV} = 0.2 \text{ V}$$

$$V_{GS} = V_t + V_{OV}$$

$$= 0.6 + 0.2 = 0.8 \text{ V}$$

From the voltage divider ( $R_1, R_2$ : see Fig. 1), we can write

$$V_{GS} = \frac{R_1}{R_1 + R_2} V_D = \frac{0.5}{0.5 + 0.5} V_D = 0.5 V_D$$

Thus

$$V_D = 2V_{GS} = 1.6 \text{ V}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2$$

$$= \frac{1}{2} \times 5 \times 0.2^2 = 0.1 \text{ mA}$$

$$I_{\text{divider}} = \frac{V_D}{1 \text{ M}\Omega} = \frac{1.6 \text{ V}}{1 \text{ M}\Omega} = 1.6 \mu\text{A}$$

$$I_{R_D} = 0.1 + 0.0016 \simeq 0.102 \text{ mA}$$

$$R_D = \frac{V_{DD} - V_D}{I_{R_D}} = \frac{10 - 1.6}{0.102} = 82.4 \text{ k}\Omega$$

$$(b) g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

(c) Replacing the MOSFET with its T model results in the amplifier equivalent circuit shown in Fig. 2. At the output node,

$$v_o = i[R_D \parallel (R_1 + R_2)]$$

$$v_o = iR'_D \quad (1)$$

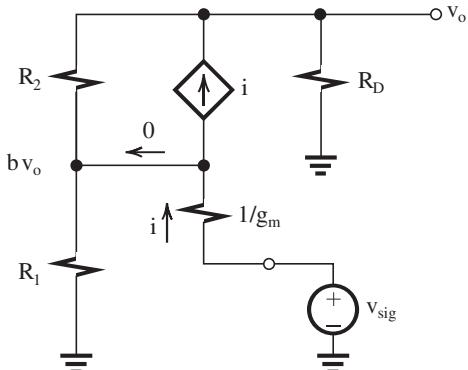


Figure 2

where  $R'_D = R_D \parallel (R_1 + R_2)$ . The voltage at the gate is a fraction  $\beta$  of  $v_o$  with

$$\beta = \frac{R_1}{R_1 + R_2}$$

Now, the current  $i$  can be found from

$$i = \frac{v_{sig} - \beta v_o}{1/g_m} = g_m v_{sig} - \beta g_m v_o \quad (2)$$

Substituting for  $i$  from Eq. (2) into Eq. (1) yields

$$v_o = (g_m v_{sig} - \beta g_m v_o) R'_D$$

Thus

$$\frac{v_o}{v_{sig}} = \frac{g_m R'_D}{1 + \beta g_m R'_D}$$

$$\begin{aligned}
&= \frac{1/\beta}{1 + \frac{1/\beta}{g_m R'_D}} \\
&= \frac{1 + (R_2/R_1)}{1 + \frac{1 + R_2/R_1}{g_m R'_D}} \quad \text{Q.E.D} \tag{3}
\end{aligned}$$

The input resistance  $R_{in}$  can be obtained as follows:

$$R_{in} = \frac{V_{sig}}{i}$$

Substituting for  $i$  from Eq. (1) yields

$$R_{in} = \frac{V_{sig}}{V_o} R'_D$$

and replacing  $\frac{V_{sig}}{V_o}$  by the inverse of the gain expression in Eq. (3) gives

$$\begin{aligned}
R_{in} &= R'_D \left[ \frac{1}{g_m R'_D} + \frac{1}{1 + (R_2/R_1)} \right] \\
R_{in} &= \frac{1}{g_m} \left[ 1 + g_m R'_D \frac{R_1}{R_1 + R_2} \right] \quad \text{Q.E.D}
\end{aligned}$$

(d) Substituting numerical values:

$$\begin{aligned}
\frac{V_o}{V_{sig}} &= \frac{1 + (0.5/0.5)}{1 + \frac{1 + (0.5/0.5)}{1 \times (82.4 \parallel 1000)}} \\
&= \frac{2}{1 + \frac{2}{76.13}} = 1.95 \text{ V/V}
\end{aligned}$$

Note that the gain  $\simeq 1 + \frac{R_2}{R_1} = 2$ , similar to that of an op amp connected in the noninverting configuration!

$$\begin{aligned}
R_{in} &= \frac{1}{1} \left[ 1 + 1 \times (82.4 \parallel 1000) \frac{0.5}{0.5 + 0.5} \right] \\
&= 39.1 \text{ k}\Omega
\end{aligned}$$

6.114 (a) DC bias:

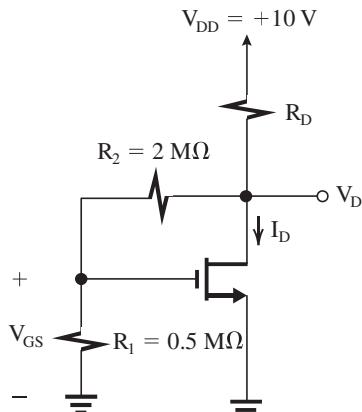


Figure 1

$$\begin{aligned}
V_{GS} &= V_t + V_{OV} \\
&= 0.6 + 0.2 = 0.8 \text{ V}
\end{aligned}$$

From the voltage divider ( $R_1, R_2$ : see Fig. 1), we can write

$$V_{GS} = V_D \frac{R_1}{R_1 + R_2} = V_D \frac{0.5}{0.5 + 2}$$

$$V_D = 5V_{GS} = 5 \times 0.8 = 4 \text{ V}$$

$$I_D = \frac{1}{2} k_n V_{OV}^2 \left( 1 + \frac{V_{DS}}{V_A} \right)$$

$$I_D = \frac{1}{2} \times 5 \times 0.2^2 \left( 1 + \frac{4}{60} \right)$$

$$= 0.107 \text{ mA}$$

The current in the voltage divider is

$$I = \frac{V_D}{R_1 + R_2} = \frac{4}{2.5} = 1.6 \mu\text{A} = 0.0016 \text{ mA}$$

Thus the current through  $R_D$  will be  $(0.107 + 0.0016) \simeq 0.109 \text{ mA}$  and

$$R_D = \frac{V_{DD} - V_D}{0.109} = \frac{10 - 4}{0.109} = 55 \text{ k}\Omega$$

$$(b) g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.107}{0.2} = 1.07 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{60}{0.107} = 561 \text{ k}\Omega$$

(c) Upon replacing the MOSFET with its hybrid- $\pi$  model, we obtain the small-signal equivalent circuit of the amplifier, shown in Fig. 2 on the next page.

Node equation at the output:

$$\frac{V_o}{R_D} + \frac{V_o}{r_o} + \frac{V_o - V_{gs}}{R_2} + g_m V_{gs} = 0$$

$$V_o \left( \frac{1}{R_D} + \frac{1}{r_o} + \frac{1}{R_2} \right) = -g_m \left( 1 - \frac{1}{g_m R_2} \right) V_{gs}$$

Thus,

$$V_o = -\overbrace{g_m (R_D \parallel r_o \parallel R_2)}^A \left( 1 - \frac{1}{g_m R_2} \right) V_{gs} \tag{1}$$

Next, we express  $V_{gs}$  in terms of  $V_{sig}$  and  $V_o$  using superposition:

$$V_{gs} = V_{sig} \frac{R_2}{R_1 + R_2} + V_o \frac{R_1}{R_1 + R_2} \tag{2}$$

Substituting for  $V_{gs}$  from Eq. (2) into Eq. (1) yields

$$V_o = -AV_{sig} \frac{R_2}{R_1 + R_2} - AV_o \frac{R_1}{R_1 + R_2}$$

where

$$A = g_m (R_D \parallel r_o \parallel R_2) \left( 1 - \frac{1}{g_m R_2} \right)$$

Thus,

$$V_o \left( 1 + A \frac{R_1}{R_1 + R_2} \right) = -A \frac{R_2}{R_1 + R_2} V_{sig}$$

$$\frac{V_o}{V_{sig}} = \frac{-A \frac{R_2}{R_1 + R_2}}{1 + A \frac{R_1}{R_1 + R_2}}$$

This figure belongs to Problem 6.114(c).

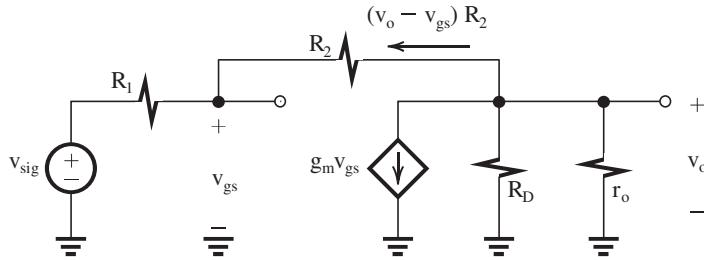


Figure 2

$$= \frac{-R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}}$$

Thus,

$$\frac{v_o}{v_{\text{sig}}} = -\frac{R_2/R_1}{1 + \frac{1 + R_2/R_1}{g_m(R_D \parallel r_o \parallel R_2)(1 - 1/g_m R_2)}}$$

Q.E.D

Substituting numerical values yields

$$\begin{aligned} \frac{v_o}{v_{\text{sig}}} &= \\ &= -\frac{2/0.5}{1 + \frac{1 + (2/0.5)}{1.07(55 \parallel 561 \parallel 2000)(1 - 1/1.07 \times 2000)}} \\ &= -\frac{4}{\left(1 + \frac{5}{52.6}\right)} \\ &= -3.65 \text{ V/V} \end{aligned}$$

Note that the gain is nearly equal to  $-R_2/R_1 = -4$ , which is the gain of an op amp connected in the inverting configuration.

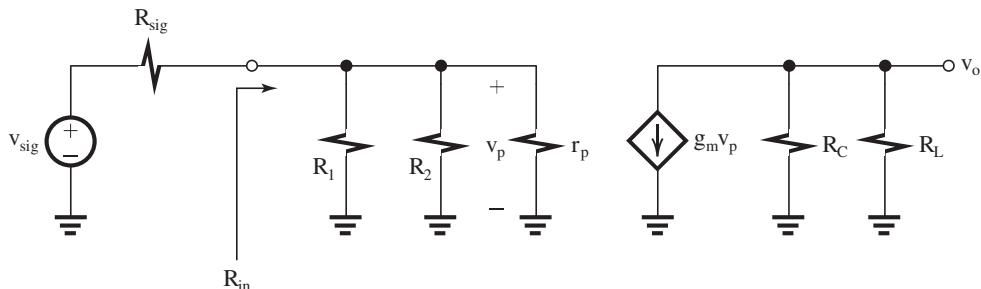
6.115 Refer to the circuit of Fig. P6.115.

$$I_C = \frac{\alpha(V_{BB} - V_{BE})}{R_E + \frac{R_B}{\beta + 1}}$$

where

$$V_{BB} = V_{CC} \frac{R_2}{R_2 + R_1} = 15 \times \frac{15}{15 + 27} = 5.357 \text{ V}$$

This figure belongs to Problem 6.115.



$$R_B = R_1 \parallel R_2 = 15 \parallel 27 = 9.643 \text{ k}\Omega$$

$$I_C = \frac{0.99(5.357 - 0.7)}{2.4 + \frac{9.643}{101}} = 1.85 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1.85 \text{ mA}}{0.025 \text{ V}} = 74 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{74} = 1.35 \text{ k}\Omega$$

Replacing the BJT with its hybrid- $\pi$  model results in the equivalent circuit shown at the bottom of the page:

$$R_{\text{in}} = R_1 \parallel R_2 \parallel r_\pi = R_B \parallel r_\pi = 9.643 \parallel 1.35$$

$$= 1.18 \text{ k}\Omega$$

$$\frac{v_\pi}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{1.18}{1.18 + 2} = 0.371 \text{ V/V}$$

$$\frac{v_o}{v_\pi} = -g_m(R_C \parallel R_L)$$

$$= -74(3.9 \parallel 2) = -97.83$$

$$\frac{v_o}{v_{\text{sig}}} = -0.371 \times 97.83 = -36.3 \text{ V/V}$$

6.116 Refer to the circuit of Fig. P6.116.

DC design:

$$V_B = 5 \text{ V}, \quad V_{BE} = 0.7 \text{ V}$$

$$V_E = 4.3 \text{ V}$$

For

$$I_E = 2 \text{ mA}, \quad R_E = \frac{V_E}{I_E} = \frac{4.3}{2} = 2.15 \text{ k}\Omega$$

$$I_{R_2} = 0.2 \text{ mA}, \quad R_2 = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{2}{101} \simeq 0.02 \text{ mA}$$

$$I_{R_1} = I_{R_2} + I_B = 0.2 + 0.02 = 0.22 \text{ mA}$$

$$R_1 = \frac{V_{CC} - V_B}{I_{R_1}} = \frac{15 - 5}{0.22} = 45.5 \text{ k}\Omega$$

Choosing 5% resistors:

$$R_E = 2.2 \text{ k}\Omega, \quad R_1 = 47 \text{ k}\Omega, \quad R_2 = 24 \text{ k}\Omega$$

For these values,

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

where

$$V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2} = 15 \times \frac{24}{24 + 47} = 5.07 \text{ V}$$

$$R_B = R_1 \parallel R_2 = 47 \parallel 24 = 15.89 \text{ k}\Omega$$

$$I_E = \frac{5.07 - 0.7}{2.2 + \frac{15.89}{101}} = 1.85 \text{ mA}$$

$$V_B = I_E R_E + V_{BE} = 1.85 \times 2.2 + 0.7 = 4.8 \text{ V}$$

$$I_C = \alpha I_E = 0.99 \times 1.85 = 1.84 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1.84}{0.025} = 73.4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{73.4} = 1.36 \text{ k}\Omega$$

$$R_{in} = R_1 \parallel R_2 \parallel r_\pi = 47 \parallel 24 \parallel 1.36 = 1.25 \text{ k}\Omega$$

$$\frac{v_\pi}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{1.25}{1.25 + 2} = 0.385 \text{ V/V}$$

For an overall gain of  $-40 \text{ V/V}$ ,

$$\frac{v_o}{v_\pi} = -\frac{40}{0.385} = -104 \text{ V/V}$$

But

$$\frac{v_o}{v_\pi} = -g_m(R_C \parallel R_L)$$

$$-104 = -73.4 (R_C \parallel 2)$$

$$(R_C \parallel 2) = 1.416$$

$$R_C = 4.86 \text{ k}\Omega$$

We can select either  $4.7 \text{ k}\Omega$  or  $5.1 \text{ k}\Omega$ . With  $4.7 \text{ k}\Omega$ , the gain will be

$$\frac{v_o}{v_{sig}} = -0.385 \times 73.4 \times (4.7 \parallel 2) = -39.6 \text{ V/V}$$

which is slightly lower than the required  $-40 \text{ V/V}$ , and we will obtain

$$V_C = 15 - 4.7 \times 1.84 = 6.4 \text{ V}$$

allowing for about 2 V of negative signal swing at the collector. If we choose  $5.1 \text{ k}\Omega$ , the gain will be

$$\frac{v_o}{v_{sig}} = -0.385 \times 73.4 \times (5.1 \parallel 2) = -40.6 \text{ V/V}$$

which is slightly higher than the required gain, and we will obtain

$$V_C = 15 - 5.1 \times 1.84 = 5.6 \text{ V}$$

which allows for only 1.2-V negative signal swing.

6.117 Refer to the circuit of Fig. P6.117.

DC voltage drop across  $R_B = 0.2 \text{ V}$ , and

$$I_B R_B = 0.2 \text{ V}$$

$$\frac{I}{\beta + 1} R_B = 0.2 \text{ V}$$

$$IR_B = 0.2 \times 101 \quad (1)$$

$$R_{in} = R_B \parallel r_\pi = 10 \text{ k}\Omega$$

$$R_B \parallel \frac{V_T}{I_B} = 10$$

$$R_B \parallel \frac{0.025}{I / (\beta + 1)} = 10$$

$$R_B \parallel \left( \frac{0.025 \times 101}{I} \right) = 10$$

$$\frac{R_B \times \frac{0.025 \times 101}{I}}{R_B + \frac{0.025 \times 101}{I}} = 10$$

$$\frac{0.025 \times 101 R_B}{IR_B + 0.025 \times 101} = 10 \quad (2)$$

Substituting for  $IR_B$  from Eq. (1) yields

$$\frac{0.025 \times 101 R_B}{0.2 \times 101 + 0.025 \times 101} = 10$$

$$\frac{0.025 R_B}{0.225} = 10$$

$$\Rightarrow R_B = 90 \text{ k}\Omega$$

$$I = \frac{0.2 \times 101}{90} = 0.22 \text{ mA}$$

To maximize the open-circuit voltage gain between base and collector while ensuring that the instantaneous collector voltage does not fall below  $(V_B - 0.4)$  when  $v_{be}$  is as high as 5 mV, we impose the constraint

$$V_C - |A_{vo}| \times 0.005 = V_B + 0.005 - 0.4$$

where

$$V_C = V_{CC} - I_C R_C$$

$$= 5 - 0.99 \times 0.22 R_C$$

$$= 5 - 0.22 R_C$$

$$|A_{vo}| = g_m R_C = \frac{0.99 \times 0.22}{0.025} R_C = 8.7 R_C$$

and

$$V_B = -\frac{0.22}{101} \times 90 = -0.2 \text{ V}$$

Thus,

$$5 - 0.22 R_C - 8.7 R_C \times 0.005 = -0.2 - 0.395$$

$$\Rightarrow R_C = 21.2 \text{ k}\Omega$$

Selecting 5% resistors, we find

$$R_B = 91 \text{ k}\Omega$$

$$R_C = 22 \text{ k}\Omega$$

and specifying I to one significant digit gives

$$I = 0.2 \text{ mA}$$

$$g_m = \frac{\alpha I_C}{V_T} \approx \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$A_{vo} = -g_m R_C = -8 \times 22 = -176 \text{ V/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{8} = 12.5 \text{ k}\Omega$$

$$R_{in} = R_B \parallel r_\pi = 91 \parallel 12.5 = 11 \text{ k}\Omega$$

$$G_v = -\frac{11}{20 + 11} \times 8(22 \parallel 20) \\ = -29.7 \text{ V/V}$$

6.118 Refer to the circuit of Fig. P6.118.

(a)  $I_E = 0.5 \text{ mA}$ . Writing a loop equation for the base-emitter circuit results in

$$I_B R_{sig} + V_{BE} + I_E R_E = 3$$

$$\frac{I_E}{\beta + 1} R_{sig} + V_{BE} + I_E R_E = 3$$

$$\frac{0.5}{101} \times 2.5 + 0.7 + 0.5 R_E = 3$$

$$\Rightarrow R_E = 4.6 \text{ k}\Omega$$

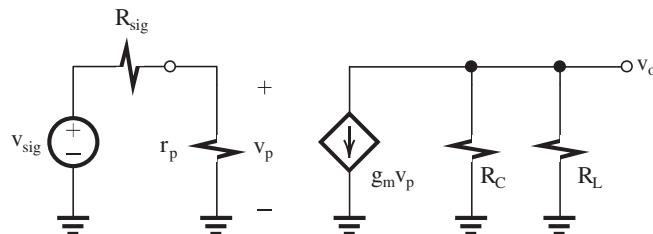
$$(b) I_C = \alpha I_E \approx 0.5 \text{ mA}$$

$$V_C = 0.5 = 3 - 0.5 R_C$$

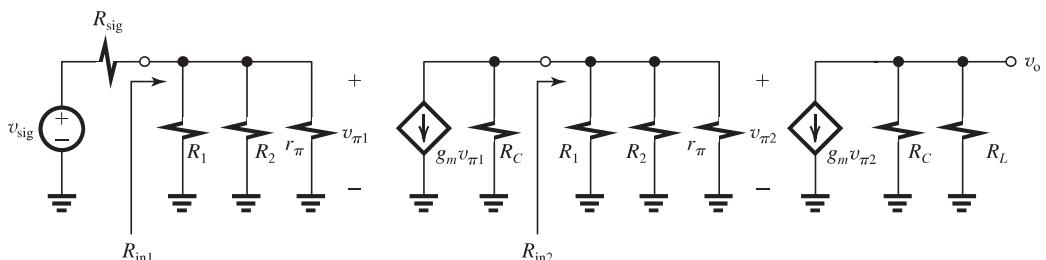
$$\Rightarrow R_C = 5 \text{ k}\Omega$$

$$(c) g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

This figure belongs to Problem 6.118.



This figure belongs to Problem 6.119.



$$(e) \frac{V_o}{V_{b2}} = \frac{V_o}{V_{\pi 2}} = -g_m(R_C \parallel R_L)$$

$$= -40(6.8 \parallel 2) = -61.8 \text{ V/V}$$

$$(f) \frac{V_o}{V_{\text{sig}}} = \frac{V_o}{V_{b2}} \times \frac{V_{b2}}{V_{b1}} \times \frac{V_{b1}}{V_{\text{sig}}} = -61.8$$

$$\times -69.2 \times 0.32 = 1368.5 \text{ V/V}$$

6.120 Refer to the circuit in Fig. P6.120:

$$I_E = 0.1 \text{ mA}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

Note that the emitter has a resistance  $R_e = 250 \Omega$ .

$$R_{in} = 200 \text{ k}\Omega \parallel (\beta + 1)(r_e + R_e)$$

$$= 200 \parallel [101 \times (0.25 + 0.25)]$$

$$= 200 \parallel 50.5 = 40.3 \text{ k}\Omega$$

$$\frac{V_b}{V_{\text{sig}}} = \frac{R_{in}}{R_{in} + R_{\text{sig}}} = \frac{40.3}{40.3 + 20} = 0.668 \text{ V/V}$$

$$\frac{V_o}{V_b} = -\alpha \frac{\text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$\simeq -\frac{20 \parallel 20}{0.25 + 0.25} = -20 \text{ V/V}$$

$$G_v = \frac{V_o}{V_{\text{sig}}} = -0.668 \times 20 = -13.4 \text{ V/V}$$

For  $V_{be}$  to be limited to 5 mV, the signal between base and ground will be 10 mV (because of the 5 mV across  $R_e$ ). The limit on  $V_{\text{sig}}$  can be obtained by dividing the 10 mV by  $V_b/V_{\text{sig}}$ ,

$$\hat{V}_{\text{sig}} = \frac{10 \text{ mV}}{0.668} = 15 \text{ mV}$$

Correspondingly, at the output we have

$$\hat{V}_o = |G_v| \hat{V}_{\text{sig}} = 13.4 \times 15 = 200 \text{ mV} = 0.2 \text{ V}$$

6.121 (a)

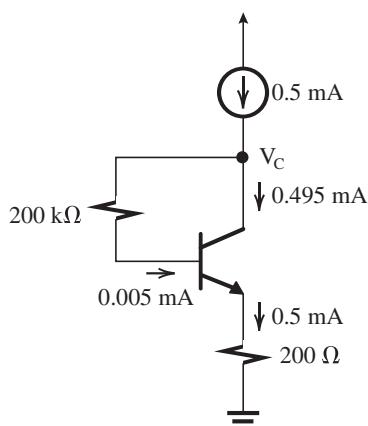


Figure 1

From Fig. 1 we see that

$$I_C = 0.495 \text{ mA}$$

$$V_C = I_B \times 200 \text{ k}\Omega + I_E \times 0.2 \text{ k}\Omega + V_{BE}$$

$$= 0.005 \times 200 + 0.5 \times 0.2 + 0.7$$

$$= 1.18 \text{ V}$$

(b)

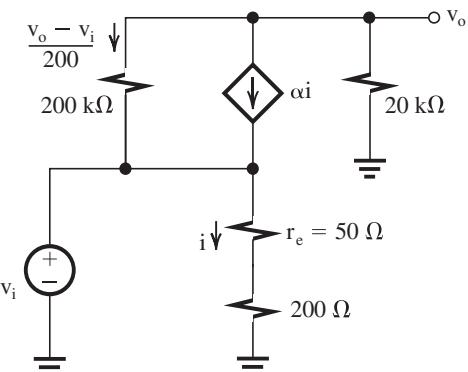


Figure 2

From Fig. 2, we have

$$g_m = \frac{I_C}{V_T} = \frac{0.495}{0.025} \simeq 20 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = 50 \Omega$$

$$i = \frac{V_i}{r_e + R_e} = \frac{V_i}{50 + 200}$$

$$= \frac{V_i}{250 \Omega} = \frac{V_i}{0.25 \text{ k}\Omega} = 4 V_i, \text{ mA}$$

Node equation at the output:

$$\frac{V_o}{20} + \alpha i + \frac{V_o - V_i}{200} = 0$$

$$\frac{V_o}{20} + 0.99 \times 4 V_i + \frac{V_o}{200} - \frac{V_i}{200} = 0$$

$$V_o \left( \frac{1}{20} + \frac{1}{200} \right) = -V_i \left( 4 \times 0.99 - \frac{1}{200} \right)$$

$$\frac{V_o}{V_i} = -71.9 \text{ V/V}$$

$$6.122 (a) I_E = \frac{3 - 0.7}{1 + \frac{100}{\beta + 1}}$$

$$\beta = 50:$$

$$I_E = \frac{2.3}{1 + \frac{100}{51}} = 0.78 \text{ mA}$$

$$V_E = I_E R_E = 0.78 \text{ V}$$

$$V_B = V_E + 0.7 = 1.48 \text{ V}$$

$$\beta = 200:$$

$$I_E = \frac{2.3}{1 + \frac{100}{201}} = 1.54 \text{ mA}$$

$$V_E = I_E R_E = 1.54 \text{ V}$$

$$V_B = V_E + 0.7 = 2.24 \text{ V}$$

$$(b) R_{in} = 100 \parallel (\beta + 1)[r_e + (1 \parallel 1)] \\ = 100 \parallel (\beta + 1)(r_e + 0.5)$$

$\beta = 50$ :

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.78 \text{ mA}} = 32.1 \Omega$$

$$R_{in} = 100 \parallel [51 \times (0.0321 + 0.5)] \\ = 21.3 \text{ k}\Omega$$

$\beta = 200$ :

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{1.54 \text{ mA}} = 16.2 \Omega$$

$$R_{in} = 100 \parallel [201 \times (0.0162 + 0.5)] \\ = 50.9 \text{ k}\Omega$$

$$(c) \frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}}$$

$$\frac{v_o}{v_b} = \frac{(1 \parallel 1)}{(1 \parallel 1) + r_e} = \frac{500}{500 + r_e} \quad (r_e \text{ in } \Omega)$$

$\beta = 50$ :

$$\frac{v_b}{v_{sig}} = \frac{21.3}{21.3 + 10} = 0.68 \text{ V/V}$$

$$\frac{v_o}{v_b} = \frac{500}{500 + 32.1} = 0.94 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = 0.68 \times 0.94 = 0.64 \text{ V/V}$$

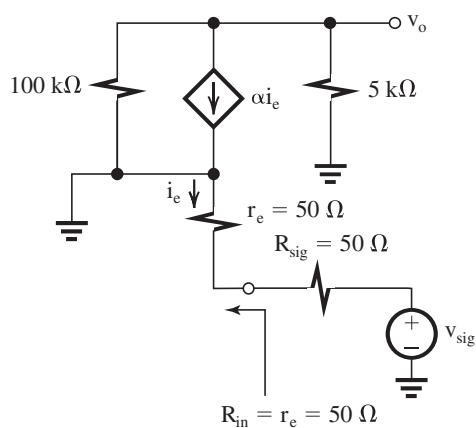
$\beta = 200$ :

$$\frac{v_b}{v_{sig}} = \frac{50.9}{50.9 + 10} = 0.836 \text{ V/V}$$

$$\frac{v_o}{v_b} = \frac{500}{500 + 16.2} = 0.969 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = 0.836 \times 0.969 = 0.81 \text{ V/V}$$

6.123 Refer to the circuit in Fig. P6.123.



The dc emitter current is equal to 0.5 mA, and  $I_C = \alpha I_E \simeq 0.5 \text{ mA}$ ; also,

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

$$R_{in} = r_e = 50 \Omega$$

$$i_e = \frac{-v_{sig}}{r_e + R_{sig}} = \frac{-v_{sig}}{50 + 50}$$

$$= \frac{-v_{sig}}{100 \Omega} = \frac{-v_{sig}}{0.1 \text{ k}\Omega}$$

At the output node,

$$v_o = -\alpha i_e (5 \parallel 100)$$

$$= \alpha \frac{v_{sig}}{0.1} (5 \parallel 100)$$

$$\frac{v_o}{v_{sig}} = \alpha \frac{5 \parallel 100}{0.1} \simeq 47.6 \text{ V/V}$$

6.124 Refer to the circuit in Fig. P6.124.

For dc analysis, open-circuit the two coupling capacitors. Then replace the 9-V source and the two 20-kΩ resistors by their Thévenin equivalent, namely, a 4.5-V source and a 10-kΩ series resistance. The latter can be added to the 10-kΩ resistor that is connected to the base. The result is the circuit shown in Fig. 1, which can be used to calculate  $I_E$ .

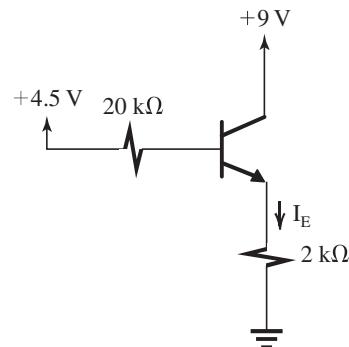


Figure 1

$$(a) I_E = \frac{4.5 - 0.7}{2 + \frac{20}{\beta + 1}}$$

$$= \frac{3.8}{2 + \frac{20}{101}} = 1.73 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 1.73 \text{ mA}$$

$$= 1.71 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = 68.4 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{1.73 \text{ mA}} = 14.5 \Omega$$

$$= 0.0145 \text{ k}\Omega$$

$$r_\pi = (\beta + 1)r_e = 101 \times 0.0145 = 1.4645 \text{ k}\Omega$$

(b) Replacing the BJT with its T model (without  $r_o$ ) and replacing the capacitors with short circuits results in the equivalent-circuit model shown in Fig. 2.

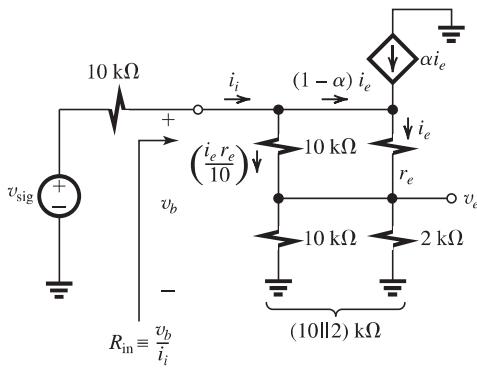


Figure 2

From Fig. 2 we see that

$$v_e = \left( i_e + i_e \frac{r_e}{10} \right) (10 \parallel 2)$$

$$v_b = v_e + i_e r_e = i_e (10 \parallel 2) \left( 1 + \frac{r_e}{10} \right) + i_e r_e$$

$$i_i = (1 - \alpha)i_e + i_e \frac{r_e}{10}$$

$$= \frac{i_e}{\beta + 1} + i_e \frac{r_e}{10}$$

We can now obtain  $R_{in}$  from

$$\begin{aligned} R_{in} &\equiv \frac{v_b}{i_i} = \frac{(10 \parallel 2) \left( 1 + \frac{r_e}{10} \right) + r_e}{\frac{1}{\beta + 1} + \frac{r_e}{10}} \\ &= \frac{(\beta + 1)(10 \parallel 2) \left( 1 + \frac{r_e}{10} \right) + (\beta + 1)r_e}{1 + (\beta + 1) \frac{r_e}{10}} \\ &= \frac{101 \times (10 \parallel 2) \times (1 + 0.00145) + 101 \times 0.0145}{1 + 101 \times 0.00145} \\ &= \frac{168.577 + 1.4645}{1 + 0.14645} = 148.3 \text{ k}\Omega \end{aligned}$$

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{148.3}{148.3 + 10} = 0.937$$

$$\begin{aligned} \frac{v_o}{v_b} &= \frac{v_e}{v_b} = \frac{i_e \left( 1 + \frac{r_e}{10} \right) (10 \parallel 2)}{i_e \left( 1 + \frac{r_e}{10} \right) (10 \parallel 2) + i_e r_e} \\ &= \frac{1.00145 \times (10 \parallel 2)}{1.00145 \times (10 \parallel 2) + 0.0145} \\ &= 0.991 \text{ V/V} \end{aligned}$$

$$G_v \equiv \frac{v_o}{v_{sig}} = 0.937 \times 0.991 = 0.93 \text{ V/V}$$

(c) When  $C_B$  is open-circuited, the equivalent circuit becomes that shown in Fig. 3.

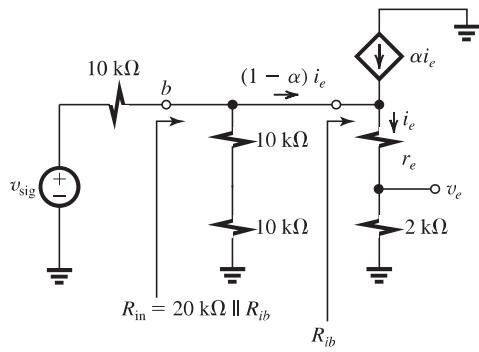


Figure 3

Thus,

$$\begin{aligned} R_{in} &= 20 \text{ k}\Omega \parallel R_{ib} \\ &= 20 \text{ k}\Omega \parallel (\beta + 1)(R_e + 2) \\ &= 20 \parallel 101 \times 2.0145 \\ &= 18.21 \text{ k}\Omega \end{aligned}$$

which is greatly reduced because of the absence of bootstrapping. The latter causes the lower node of the 10-kΩ base-biasing resistor to rise with the output voltage, thus causing a much reduced signal current in the 10-kΩ resistor and a correspondingly larger effective resistance across the amplifier input.

The reduced  $R_{in}$  will result in a reduction in  $v_b/v_{sig}$ ,

$$\frac{v_b}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{18.21}{28.21}$$

$$= 0.646 \text{ V/V}$$

$$\frac{v_o}{v_b} = \frac{2}{2 + 0.0145} = 0.993$$

$$G_v \equiv \frac{v_o}{v_{sig}} = 0.646 \times 0.993$$

$$= 0.64 \text{ V/V}$$

which is much reduced relative to the value obtained with bootstrapping.

### 6.125

(a) Applying Thévenin's theorem to the base-biasing circuit of  $Q_1$  results in the dc circuit shown below. From our partial analysis on the figure, we can write

$$I_{E1} = 0.1 \text{ mA}$$

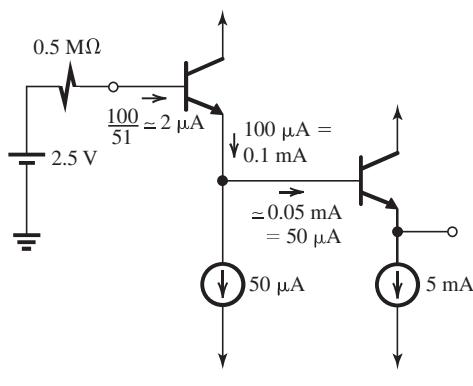
$$I_{E2} = 5 \text{ mA}$$

$V_{B1}$  can be obtained as

$$V_{B1} = 2.5 - 2 \mu A \times 0.5 M\Omega = 1.5 V$$

and  $V_{B2}$  can be found as

$$V_{B2} = V_{B1} - 0.7 = 0.8 V$$



(b) Refer to the circuit in Fig. P6.125. With a load resistance  $R_L = 1 k\Omega$  connected to the output terminal, the voltage gain  $v_o/v_{b2}$  can be found as

$$\frac{v_o}{v_{b2}} = \frac{R_L}{R_L + r_{e1}}$$

where

$$r_{e2} = \frac{25 mV}{5 mA} = 5 \Omega$$

$$\frac{v_o}{v_{b2}} = \frac{1000}{1000 + 5} = 0.995 V/V$$

$$R_{ib2} = (\beta_2 + 1)(r_{e2} + R_L)$$

$$= 101 \times 1.005 = 101.5 k\Omega$$

$$(c) R_{in} = 1 M\Omega \parallel 1 M\Omega \parallel (\beta + 1)(r_{e1} + R_{ib2})$$

where

$$r_e = \frac{V_T}{I_{E1}} = \frac{25 mV}{0.1 mA} = 250 \Omega = 0.25 k\Omega$$

$$R_{in} = 0.5 M\Omega \parallel [51 \times (0.25 + 101.5)] k\Omega$$

$$= 0.5 M\Omega \parallel 5.2 M\Omega$$

$$= 456 k\Omega$$

$$\frac{v_{e1}}{v_{b1}} = \frac{R_{ib}}{R_{ib} + r_{e1}} = \frac{101.5}{101.5 + 0.25}$$

$$= 0.9975 V/V$$

$$(d) \frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{456}{456 + 100} = 0.82 V/V$$

$$(e) \frac{v_o}{v_{sig}} = 0.82 \times 0.9975 \times 0.995 = 0.814 V/V$$

### Exercise 7-1

Ex: 7.1 In the current source of Example 7.1 (Fig. 7.1) we have  $I_O = 100 \mu A$  and we want to reduce the change in output current,  $\Delta I_O$ , corresponding to a 1-V change in output voltage,  $\Delta V_O$ , to 1% of  $I_O$ .

$$\text{That is, } \Delta I_O = \frac{\Delta V_O}{r_{o2}} = 0.01 I_O \Rightarrow \frac{1 \text{ V}}{r_{o2}}$$

$$= 0.01 \times 100 \mu A$$

$$r_{o2} = \frac{1 \text{ V}}{1 \mu A} = 1 \text{ M}\Omega$$

$$r_{o2} = \frac{V'_A \times L}{I_O} \Rightarrow 1 \text{ M}\Omega = \frac{20 \times L}{100 \mu A}$$

$$\Rightarrow L = \frac{100 \text{ V}}{20 \text{ V}/\mu \text{m}} = 5 \mu \text{m}$$

To keep  $V_{OV}$  of the matched transistors the same as that in Example 7.1,  $\frac{W}{L}$  of the transistor should remain the same. Therefore,

$$\frac{W}{5 \mu \text{m}} = \frac{10 \mu \text{m}}{1 \mu \text{m}} \Rightarrow W = 50 \mu \text{m}$$

So the dimensions of the matched transistors  $Q_1$  and  $Q_2$  should be changed to

$$W = 50 \mu \text{m} \text{ and } L = 5 \mu \text{m}$$

Ex: 7.2 For the circuit of Fig. 7.4 we have

$$I_2 = I_{REF} \frac{(W/L)_2}{(W/L)_1}, I_3 = I_{REF} \frac{(W/L)_3}{(W/L)_1}$$

$$\text{and } I_5 = I_4 \frac{(W/L)_5}{(W/L)_4}$$

Since all channel lengths are equal, that is,

$$L_1 = L_2 = \dots = L_5 = 1 \mu \text{m}$$

and

$$I_{REF} = 10 \mu A, I_2 = 60 \mu A, I_3 = 20 \mu A, I_4 = I_3 = 20 \mu A, \text{ and } I_5 = 80 \mu A,$$

we have

$$I_2 = I_{REF} \frac{W_2}{W_1} \Rightarrow \frac{W_2}{W_1} = \frac{I_2}{I_{REF}} = \frac{60}{10} = 6$$

$$I_3 = I_{REF} \frac{W_3}{W_1} \Rightarrow \frac{W_3}{W_1} = \frac{I_3}{I_{REF}} = \frac{20}{10} = 2$$

$$I_5 = I_4 \frac{W_5}{W_4} \Rightarrow \frac{W_5}{W_4} = \frac{I_5}{I_4} = \frac{80}{20} = 4$$

To allow the voltage at the drain of  $Q_2$  to go down to within 0.2 V of the negative supply voltage, we need  $V_{OV2} = 0.2 \text{ V}$ :

$$I_2 = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_2 V_{OV2}^2 = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_2 V_{OV2}^2$$

$$60 \mu A = \frac{1}{2} 200 \frac{\mu A}{V^2} \left( \frac{W}{L} \right)_2 (0.2)^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_2 = \frac{120}{200 \times (0.2)^2} = 15 \Rightarrow W_2 \\ = 15 \times L_2$$

$$W_2 = 15 \mu m, \frac{W_2}{W_1} = 6 \Rightarrow W_1 = \frac{W_2}{6} = 2.5 \mu m$$

$$\frac{W_3}{W_1} = 2 \Rightarrow W_3 = 2 \times W_1 = 5 \mu m$$

To allow the voltage at the drain of  $Q_5$  to go up to within 0.2 V of positive supply, we need

$$V_{OV5} = 0.2 \text{ V:}$$

$$I_5 = \frac{1}{2} k'_p \left( \frac{W}{L} \right)_5 V_{OV5}^2$$

$$80 \mu A = \frac{1}{2} 80 \frac{\mu A}{V^2} \left( \frac{W}{L} \right)_5 (0.2)^2 \Rightarrow$$

$$\left( \frac{W}{L} \right)_5 = \frac{2 \times 80}{80 \times (0.2)^2} = 50 \Rightarrow W_5 = 50 L_5$$

$$W_5 = 50 \mu m$$

$$\frac{W_5}{W_4} = 4 \Rightarrow W_4 = \frac{50 \mu m}{4} = 12.5 \mu m$$

Thus:

$$W_1 = 2.5 \mu m, W_2 = 15 \mu m, W_3 = 5 \mu m$$

$$W_4 = 12.5 \mu m, \text{ and } W_5 = 50 \mu m$$

Ex: 7.3 From Eq. (7.21) we have

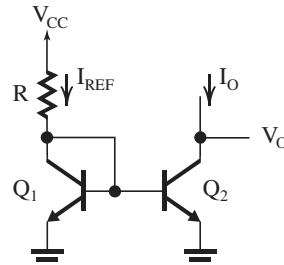
$$I_O = I_{REF} \left( \frac{m}{1 + \frac{m+1}{\beta}} \right) \left( 1 + \frac{V_O - V_{BE}}{V_A} \right)$$

$$I_O = 1 \text{ mA} \left( \frac{1}{1 + \frac{1+1}{100}} \right) \left( 1 + \frac{5 - 0.7}{100} \right) \\ = 1.02 \text{ mA}$$

$$I_O = 1.02 \text{ mA}$$

$$R_o = r_{o2} = \frac{V_A}{I_O} = \frac{100 \text{ V}}{1.02 \text{ mA}} = 98 \text{ k}\Omega \simeq 100 \text{ k}\Omega$$

Ex: 7.4



From Eq. (7.23), we have

$$I_O = \frac{I_{REF}}{1 + (2/\beta)} \left( 1 + \frac{V_O - V_{BE}}{V_A} \right)$$

## Exercise 7-2

where  $V_{BE} = V_T \ln\left(\frac{I_O}{I_S}\right)$   
 $= 0.025 \ln\left(\frac{0.5 \times 10^{-3}}{10^{-15}}\right) = 0.673 \text{ V}$

$$0.5 \text{ mA} = \frac{I_{REF}}{1 + (2/100)} \left(1 + \frac{2 - 0.673}{50}\right) \Rightarrow$$

$$I_{REF} = 0.5 \text{ mA} \frac{1.02}{1.026 \text{ mA}} = 0.497 \text{ mA}$$

$$I_{REF} = \frac{V_{CC} - V_{BE}}{R} \Rightarrow R = \frac{V_{CC} - V_{BE}}{I_{REF}}$$

$$R = \frac{5 - 0.673}{0.497 \text{ mA}} = 8.71 \text{ k}\Omega$$

$$V_{Omin} = V_{CEsat} = 0.3 \text{ V}$$

For  $V_O = 5 \text{ V}$ , From Eq. (7.23) we have

$$I_O = \frac{I_{REF}}{1 + (2/\beta)} \left(1 + \frac{V_O - V_{BE}}{V_A}\right)$$

$$I_O = \frac{0.497}{1 + (2/100)} \left(1 + \frac{5 - 0.673 \text{ V}}{50}\right) = 0.53 \text{ mA}$$

$$\text{Ex: } 7.5 \quad I_1 = I_2 = \dots = I_N = I_C|_{Q_{REF}}$$

At the input node,

$$I_{REF} = I_C|_{Q_{REF}} + I_B|_{Q_{REF}} + I_{B1} + \dots + I_{BN}$$

$$= I_C|_{Q_{REF}} + (N + 1) I_B|_{Q_{REF}}$$

$$= I_C|_{Q_{REF}} + \frac{(N + 1)}{\beta} I_C|_{Q_{REF}}$$

$$\Rightarrow I_C|_{Q_{REF}} = \frac{I_{REF}}{1 + \frac{N + 1}{\beta}}$$

Thus,

$$I_1 = I_2 = \dots = I_N = \frac{I_{REF}}{1 + \frac{N + 1}{\beta}} \quad \text{Q.E.D}$$

For  $\beta = 100$ , to limit the error to 10%,

$$0.1 = \frac{N + 1}{\beta} = \frac{N + 1}{100}$$

$$\Rightarrow N = 9$$

$$\text{Ex: } 7.6$$

$$R_{in} \simeq \frac{1}{g_{m1}}$$

Now,  $R_{in} = 1 \text{ k}\Omega$ , thus

$$g_{m1} = 1 \text{ mA/V}$$

But

$$g_{m1} = \sqrt{2(\mu_n C_{ox}) \left(\frac{W}{L}\right)_1 I_{D1}}$$

$$1 = \sqrt{2 \times 0.4 \times \left(\frac{W}{L}\right)_1 \times 0.1}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 12.5$$

To obtain

$$A_{is} = 5$$

$$5 = A_{is} = \frac{(W/L)_2}{(W/L)_1}$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 5 \times 12.5 = 62.5$$

$$R_o = r_{o2} = \frac{V_{A2}}{I_{D2}} = \frac{V_{A2}}{5I_{D1}}$$

Thus,

$$40 \text{ k}\Omega = \frac{V_{A2}}{5 \times 0.1}$$

$$\Rightarrow V_{A2} = 20 \text{ V}$$

But

$$V_{A2} = V'_{A2} L_2$$

$$20 = 20 \times L_2$$

$$\Rightarrow L_2 = 1 \mu\text{m}$$

Selecting  $L_1 = L_2$ , then

$$L_1 = L_2 = 1 \mu\text{m}$$

$$W_1 = 12.5 \mu\text{m}$$

$$W_2 = 62.5 \mu\text{m}$$

$$\text{Ex: } 7.7$$

Using Eq. (7.42):

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)} \cdot \sqrt{I_D}$$

For  $I_D = 10 \mu\text{A}$ , we have

$$g_m = \sqrt{2(387 \mu\text{A/V}^2)(10)(10 \mu\text{A})}$$

$$= 0.28 \text{ mA/V}$$

Using Eq. (7.46):

$$A_0 = V'_A \frac{\sqrt{2\mu_n C_{ox}(W/L)}}{\sqrt{I_D}}$$

$$= \frac{5 \text{ V}/\mu\text{m} \sqrt{2(387 \mu\text{A/V}^2)(10)(0.36)^2}}{\sqrt{10 \mu\text{A}}}$$

$$A_0 = 50 \text{ V/V}$$

Since  $g_m$  varies with  $\sqrt{I_D}$  and  $A_0$  with  $\frac{1}{\sqrt{I_D}}$ ,

for

$$I_D = 100 \mu\text{A} \Rightarrow g_m = 0.28 \text{ mA/V} \left(\frac{100}{10}\right)^{1/2}$$

$$= 0.88 \text{ mA/V}$$

### Exercise 7-3

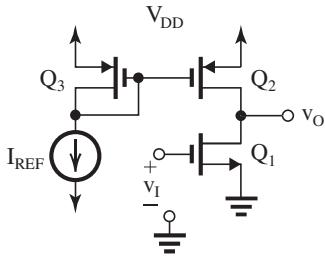
$$A_0 = 50 \left( \frac{10}{100} \right)^{1/2} = 15.8 \text{ V/V}$$

For  $I_D = 1 \text{ mA}$ , we have

$$g_m = 0.28 \text{ mA/V} \left( \frac{1}{0.010} \right)^{1/2} = 2.8 \text{ mA/V}$$

$$A_0 = 50 \left( \frac{0.010}{1} \right)^{1/2} = 5 \text{ V/V}$$

Ex: 7.8



Since all transistors have the same

$$\frac{W}{L} = \frac{7.2 \mu\text{m}}{0.36 \mu\text{m}},$$

we have

$$I_{\text{REF}} = I_{D3} = I_{D2} = I_{D1} = 100 \mu\text{A}$$

$$\begin{aligned} g_{m1} &= \sqrt{2\mu_n C_{\text{ox}} \left( \frac{W}{L} \right)_1} \sqrt{I_{D1}} \\ &= \sqrt{2(387 \mu\text{A/V}^2) \left( \frac{7.2}{0.36} \right)} (100 \mu\text{A}) \\ &= 1.24 \text{ mA/V} \end{aligned}$$

$$r_{o1} = \frac{V'_{A1} L_1}{I_{D1}} = \frac{5 \text{ V}/\mu\text{m} (0.36 \mu\text{m})}{0.1 \text{ mA}} = 18 \text{ k}\Omega$$

$$r_{o2} = \frac{|V'_{A2}| L_2}{I_{D2}} = \frac{6 \text{ V}/\mu\text{m} (0.36 \mu\text{m})}{0.1 \text{ mA}} = 21.6 \text{ k}\Omega$$

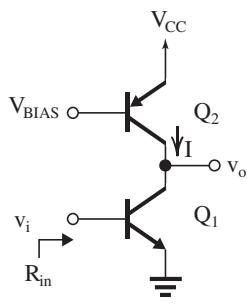
Voltage gain is

$$A_v = -g_{m1} (r_{o1} \parallel r_{o2})$$

$$A_v = -(1.24 \text{ mA/V}) (18 \text{ k}\Omega \parallel 21.6 \text{ k}\Omega)$$

$$= -12.2 \text{ V/V}$$

Ex: 7.9



$$I_{C1} = I = 100 \mu\text{A} = 0.1 \text{ mA}$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.1 \text{ mA}}{25 \text{ mV}} = 4 \text{ mA/V}$$

$$R_{\text{in}} = r_{\pi1} = \frac{\beta_1}{g_{m1}} = \frac{100}{4 \text{ mA/V}} = 25 \text{ k}\Omega$$

$$r_{o1} = \frac{V_A}{I} = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_A|}{I} = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k}\Omega$$

$$A_0 = g_{m1} r_{o1} = (4 \text{ mA/V}) (500 \text{ k}\Omega) = 2000 \text{ V/V}$$

$$A_v = -g_{m1} (r_{o1} \parallel r_{o2}) = -(4 \text{ mA/V}) \times (500 \text{ k}\Omega \parallel 500 \text{ k}\Omega) = -1000 \text{ V/V}$$

Ex: 7.10 Refer to Fig. 7.18(b),

$$v_o = i R_L$$

$$v_{\text{sig}} = i(R_s + R_{\text{in}})$$

Thus,

$$\frac{v_o}{v_{\text{sig}}} = \frac{R_L}{R_s + R_{\text{in}}} \quad \text{Q.E.D}$$

Ex: 7.11 Since  $g_m r_o \gg 1$ , we use Eq. (7.54),

$$R_{\text{in}} \simeq \frac{1}{g_m} + \frac{R_L}{g_m r_o}$$

$R_L$	0	$r_o$	$(g_m r_o) r_o$	$\infty$
$R_{\text{in}}$	$\frac{1}{g_m}$	$\frac{2}{g_m}$	$r_o$	$\infty$

Ex: 7.12 For  $g_m r_o \gg 1$ , we use Eq. (7.58),

$$R_{\text{out}} \simeq r_o + (g_m r_o) R_s$$

to obtain

$R_s$	0	$r_o$	$(g_m r_o) r_o$	$\infty$
$R_{\text{out}}$	$r_o$	$(g_m r_o) r_o$	$(g_m r_o)^2 r_o$	$\infty$

Ex: 7.13  $A_{v_o}$  remains unchanged at  $g_m r_o$ . With a load resistance  $R_L$  connected,

$$\begin{aligned} A_v &= A_{v_o} \frac{R_L}{R_L + R_o} \\ &= (g_m r_o) \frac{R_L}{R_L + (1 + g_m R_s) r_o} \end{aligned}$$

Ex: 7.14 Use Eq. (7.63)

$$R_{\text{in}} \simeq r_e \frac{r_o + R_L}{r_o + \frac{R_L}{\beta + 1}}$$

### Exercise 7-4

to obtain

$R_L$	0	$r_o$	$(\beta + 1)r_o$	$\infty$
$R_{in}$	$r_e$	$2 r_e$	$\frac{1}{2}r_\pi$	$r_\pi$

Ex: 7.15 Using Eq. (7.68),

$$R_{out} \simeq r_o + (g_m r_o) (R_e \parallel r_\pi)$$

we obtain

$R_e$	0	$r_e$	$r_\pi$	$r_o$	$\infty$
$R_{out}$	$r_o$	$2 r_o$	$\left(\frac{\beta}{2} + 1\right) r_o$	$(\beta + 1)r_o$	$(\beta + 1)r_o$

Ex: 7.16  $R_o = [1 + g_m(R_e \parallel r_\pi)]r_o$

where

$$g_m = 40 \text{ mA/V}, \quad r_\pi = \frac{\beta}{g_m} = 2.5 \text{ k}\Omega,$$

$$R_e = 0.5 \text{ k}\Omega, \text{ and } r_o = \frac{V_A}{I_C} = \frac{10}{1} = 10 \text{ k}\Omega$$

Thus,

$$R_o = [1 + 40(0.5 \parallel 2.5)] \times 10$$

$$= 177 \text{ k}\Omega$$

Without emitter degeneration,

$$R_o = r_o = 10 \text{ k}\Omega$$

Ex: 7.17 Since the CG transistor  $Q_2$  increases the output resistance by a factor approximately equal to  $g_{m2}r_{o2}$ ,

$$K \simeq g_{m2}r_{o2}$$

Ex: 7.18 If L is halved ( $L = \frac{0.55 \mu\text{m}}{2}$ ) and  $|V_A| = |V_{A1}| \cdot L$ , we obtain

$$|V_A| = 5 \text{ V}/\mu\text{m} \left(\frac{0.55 \mu\text{m}}{2}\right) = 1.375 \text{ V}$$

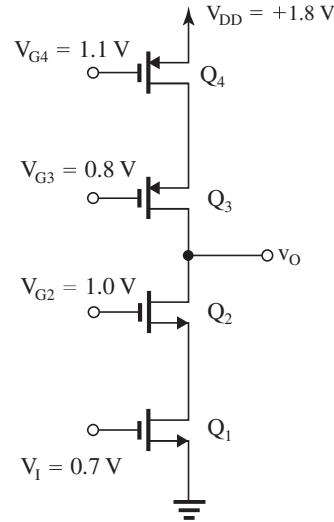
$$R_o = \frac{|V_A|}{|V_{ov}|/2} \cdot \frac{|V_A|}{I_D} = \frac{2(1.375 \text{ V})^2}{(0.3 \text{ V})(100 \mu\text{A})} = 126 \text{ k}\Omega$$

$$\text{Since } I_D = \frac{1}{2}(\mu_p C_{ox}) \left(\frac{W}{L}\right) |V_{ov}|^2 \left(1 + \frac{V_{SD}}{|V_A|}\right)$$

$$\frac{W}{L} = \frac{2(100 \mu\text{A})}{90 \mu\text{A}/\text{V}^2 (0.3 \text{ V})^2 \left(1 + \frac{0.3 \text{ V}}{1.375 \text{ V}}\right)}$$

$$\frac{W}{L} = 20.3$$

Ex: 7.19



If all transistors are matched and are obviously operating at the same  $I_D$ , then all  $|V_{ov}|$  will be equal and equal to that of  $Q_1$ , namely,  $|V_{ov}| = 0.7 - 0.5 = 0.2 \text{ V}$

$$V_{D1} = V_{S2} = V_{G2} - V_{tn} - V_{ov}$$

$$= 1.0 - 0.5 - 0.2 = 0.3 \text{ V}$$

The lowest  $v_{DS2}$  can go is  $|V_{ov}| = 0.2 \text{ V}$

$$\therefore v_{Omin} = V_{DS1} + V_{DS2} = 0.3 + 0.2 = 0.5 \text{ V}$$

Similarly,  $V_{SG4} = V_{SG3} = 0.7 \text{ V}$

$$V_{D4} = V_{S3} = V_{G3} + |V_t| + |V_{ov}|$$

$$= 0.8 + 0.5 + 0.2 = 1.5 \text{ V}$$

$v_{SD3}$  can go as low as  $|V_{ov}|$ , so

$$v_{Omax} = V_{D4} - v_{SD3min} = 1.5 - 0.2 = 1.3 \text{ V}$$

Ex: 7.20 Refer to Fig. 7.33.

$$g_{m1} = g_{m2} = g_{m3} = g_{m4} = \frac{2 I_D}{|V_{ov}|} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{|V_A|}{I_D} = \frac{2}{0.2} = 10 \text{ k}\Omega$$

$$R_{on} = (g_{m2}r_{o2})r_{o1} = (2 \times 10) \times 10 = 200 \text{ k}\Omega$$

$$R_{op} = (g_{m3}r_{o3})r_{o4} = (2 \times 10) \times 10 = 200 \text{ k}\Omega$$

$$R_o = R_{on} \parallel R_{op} = 200 \parallel 200 = 100 \text{ k}\Omega$$

$$A_v = -g_{m1}R_o = -2 \times 100 = -200 \text{ V/V}$$

Ex: 7.21  $g_{m1} = g_{m2} = g_m$

$$= \frac{I_D}{|V_{ov}|} = \frac{0.1 \text{ mA}}{(0.2/2) \text{ V}} = 1 \text{ mA/V}$$

### Exercise 7-5

$$\begin{aligned} r_{o1} &= r_{o2} = r_o \\ &= \frac{V_A}{I_D} = \frac{2 \text{ V}}{0.1 \text{ mA}} = 20 \text{ k}\Omega \\ \text{so, } g_m r_o &= 1 \text{ mA/V} (20 \text{ k}\Omega) = 20 \end{aligned}$$

(a) For  $R_L = 20 \text{ k}\Omega$ ,

$$R_{in2} = \frac{R_L + r_{o2}}{1 + g_{m2}r_{o2}} = \frac{20 \text{ k}\Omega + 20 \text{ k}\Omega}{1 + 20} = 1.9 \text{ k}\Omega$$

$$\begin{aligned} \therefore A_{v1} &= -g_{m1} (r_{o1} \parallel R_{in2}) \\ &= -1 \text{ mA/V} (20 \parallel 1.9) = -1.74 \text{ V/V} \\ \text{or} \end{aligned}$$

If we use the approximation of Eq. (7.83),

$$R_{in2} \approx \frac{R_L}{g_{m2}r_{o2}} + \frac{1}{g_{m2}} = \frac{20 \text{ k}\Omega}{20} + \frac{1}{1 \text{ mA/V}} = 2 \text{ k}\Omega$$

then

$$A_{v1} = -1 \text{ mA/V} (20 \text{ k}\Omega \parallel 2 \text{ k}\Omega) = -1.82 \text{ V/V}$$

Continuing, from Eq. (7.80),

$$\begin{aligned} A_v &= -g_{m1} [(g_{m2}r_{o2}r_{o1}) \parallel R_L] \\ A_v &= -1 \text{ mA/V} \{[(20)(20 \text{ k}\Omega)] \parallel 2 \text{ k}\Omega\} \end{aligned}$$

$$= -19.0 \text{ V/V}$$

$$A_{v2} = \frac{A_v}{A_{v1}} = \frac{-19.0}{-1.82} = 10.5 \text{ V/V}$$

(b) Now, for  $R_L = 400 \text{ k}\Omega$ ,

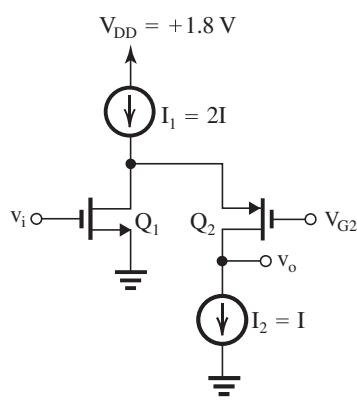
$$\begin{aligned} R_{in2} &\approx \frac{R_L}{g_{m2}r_{o2}} + \frac{1}{g_{m2}} = \frac{400 \text{ k}\Omega}{20} + \frac{1}{1 \text{ mA/V}} \\ &= 21 \text{ k}\Omega \end{aligned}$$

$$A_{v1} = -1 \text{ mA/V} (20 \text{ k}\Omega \parallel 21 \text{ k}\Omega) = -10.2 \text{ V/V}$$

$$\begin{aligned} A_v &= -1 \text{ mA/V} \{[(20)(20 \text{ k}\Omega)] \parallel 400 \text{ k}\Omega\} \\ &= -200 \text{ V/V} \end{aligned}$$

$$A_{v2} = \frac{A_v}{A_{v1}} = \frac{-200}{-10.2} = 19.6 \text{ V/V}$$

Ex: 7.22



(a)  $I_{D1} = I$  and  $I_{D2} = I$

Since  $V_{OV1} = V_{OV2} = 0.2 \text{ V}$ , we have

$$\frac{I_{D2}}{I_{D1}} = \frac{\frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_2 V_{OV2}^2}{\frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 V_{OV1}^2} = \frac{I}{I} = 1$$

Thus,

$$\begin{aligned} \frac{k'_p \left( \frac{W}{L} \right)_2}{k'_n \left( \frac{W}{L} \right)_1} &= 1 \Rightarrow \left( \frac{W}{L} \right)_2 = \frac{k'_n}{k'_p} \left( \frac{W}{L} \right)_1 \\ &= \frac{k'_n}{\frac{k'_n}{4}} \left( \frac{W}{L} \right)_1 \\ \text{or } \left( \frac{W}{L} \right)_2 &= 4 \left( \frac{W}{L} \right)_1 \end{aligned}$$

(b) The minimum voltage required across current source  $I_1$  would be  $|V_{OV}| = 0.2 \text{ V}$ , since it is made with a single transistor. If a 0.1-V<sub>pp</sub> signal swing is to be allowed at the drain of  $Q_1$ , the highest dc bias voltage would be

$$\begin{aligned} V_{DD} - |V_{OV}| - \frac{0.1 V_{pp}}{2} &= 1.8 - 0.2 - \frac{1}{2} (0.1) \\ &= 1.55 \text{ V} \end{aligned}$$

$$(c) V_{SG2} = |V_{OV}| + |V_{tp}| = 0.2 + 0.5 = 0.7 \text{ V}$$

$V_{G2}$  can be set at  $1.55 - 0.7 = 0.85 \text{ V}$ .

(d) Since current source  $I_2$  is implemented with a cascoded current source, the minimum voltage required across it for proper operation is  $2V_{OV} = 2(0.2 \text{ V}) = 0.4 \text{ V}$ .

(e) From parts (c) and (d), the allowable range of signal swing at the output is from  $0.4 \text{ V}$  to  $1.55 \text{ V} - V_{OV}$  or  $1.35 \text{ V}$ .

so,  $0.4 \text{ V} \leq v_o \leq 1.35 \text{ V}$ .

Ex: 7.23 Referring to Fig. 7.38,

$$R_{op} = (g_{m3}r_{o3})(r_{o4} \parallel r_{\pi3}) \text{ and}$$

$$R_{on} = (g_{m2}r_{o2})(r_{o1} \parallel r_{\pi2})$$

The maximum values of these resistances are obtained when  $r_o \gg r_{\pi}$  and are given by

$$R_{on} \Big|_{max} = (g_{m2}r_{o2})r_{\pi2}$$

$$R_{op} \Big|_{max} = (g_{m3}r_{o3})r_{\pi3}$$

Since  $g_m r_{\pi} = \beta$ ,

$$R_{on} \Big|_{max} = \beta_2 r_{o2}$$

$$R_{op} \Big|_{max} = \beta_3 r_{o3}$$

### Exercise 7-6

Since  $A_v = -g_{m1}(R_{on} \parallel R_{op})$ ,  
 $|A_{vmax}| = g_{m1}(\beta_2 r_{o2} \parallel \beta_3 r_{o3})$

Ex: 7.24 For the npn transistors,

$$g_{m1} = g_{m2} = \frac{|I_C|}{|V_T|} = \frac{0.2 \text{ mA}}{25 \text{ mV}} = 8 \text{ mA/V}$$

$$r_{\pi1} = r_{\pi2} = \frac{\beta}{g_m} = \frac{100}{8 \text{ mA/V}} = 12.5 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{|V_A|}{|I_C|} = \frac{5 \text{ V}}{0.2 \text{ mA}} = 25 \text{ k}\Omega$$

From Fig. 7.38,

$$\begin{aligned} R_{on} &= (g_{m2}r_{o2})(r_{o1} \parallel r_{\pi2}) \\ &= (8 \text{ mA/V})(25 \text{ k}\Omega)(25 \text{ k}\Omega \parallel 12.5 \text{ k}\Omega) \end{aligned}$$

$$R_{on} = 1.67 \text{ M}\Omega$$

For the pnp transistors,

$$g_{m3} = g_{m4} = \frac{|I_C|}{V_T} = \frac{0.2 \text{ mA}}{25 \text{ mV}} = 8 \text{ mA/V}$$

$$r_{\pi3} = r_{\pi4} = \frac{\beta}{g_m} = \frac{50}{8 \text{ mA/V}} = 6.25 \text{ k}\Omega$$

$$r_{o3} = r_{o4} = \frac{|V_A|}{|I_C|} = \frac{4 \text{ V}}{0.2 \text{ mA}} = 20 \text{ k}\Omega$$

$$\begin{aligned} R_{op} &= (g_{m3}r_{o3})(r_{o4} \parallel r_{\pi3}) \\ &= (8 \text{ mA/V})(20 \text{ k}\Omega)(20 \text{ k}\Omega \parallel 6.25 \text{ k}\Omega) \end{aligned}$$

$$R_{op} = 762 \text{ k}\Omega$$

$$A_v = -g_{m1}(R_{on} \parallel R_{op})$$

$$= -(8 \text{ mA/V})(1.67 \text{ M}\Omega \parallel 762 \text{ k}\Omega)$$

$$A_v = -4186 \text{ V/V}$$

$A_{vmax}$  occurs when  $r_{o1}$  and  $r_{o4}$  are  $\gg r_\pi$ .

Then

$$R_{on} = (g_{m2}r_{o2})r_{\pi2} = \beta_2 r_{o2}$$

$$R_{on} = 100(25 \text{ k}\Omega) = 2.5 \text{ M}\Omega$$

$$R_{op} = (g_{m3}r_{o3})r_{\pi3} = \beta_3 r_{o3}$$

$$R_{op} = 50(20 \text{ k}\Omega) = 1 \text{ M}\Omega$$

Finally,

$$A_{vmax} = -(8 \text{ mA/V})(2.5 \text{ M}\Omega \parallel 1.0 \text{ M}\Omega)$$

$$A_{vmax} = -5714 \text{ V/V}$$

Ex: 7.25 Refer to the circuit in Fig. 7.39. All transistors are operating at  $I_D = I_{REF} = 100 \mu\text{A}$  and equal  $V_{OV}$ , found from

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$100 = \frac{1}{2} \times 387 \times \frac{3.6}{0.36} \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.227 \text{ V}$$

$$V_{GS} = 0.227 + 0.5 = 0.727 \text{ V}$$

$$V_{Omin} = V_{G3} - V_{t3}$$

$$= V_{GS4} + V_{GS1} - V_{t3}$$

Thus,

$$V_{Omin} = 2V_{GS} - V_t$$

$$= V_t + 2 V_{OV}$$

$$= 0.5 + 2 \times 0.227 = 0.95 \text{ V}$$

$$g_m = \frac{2 I_D}{V_{OV}} = \frac{2 \times 0.1}{0.227} = 0.88 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A L}{I_D} = \frac{5 \times 0.36}{0.1} = 18 \text{ k}\Omega$$

$$\begin{aligned} R_o &= (g_{m3}r_{o3})r_{o2} = (0.88 \times 18) \times 18 \\ &= 285 \text{ k}\Omega \end{aligned}$$

Ex: 7.26 For the Wilson mirror from Eq. (7.94), we have

$$\frac{I_O}{I_{REF}} \approx \frac{1}{1 + \frac{2}{\beta^2}} = 0.9998$$

$$\text{Thus } \frac{|I_O - I_{REF}|}{I_{REF}} \times 100 = 0.02\%$$

whereas for the simple mirror from Eq. (7.18) we have

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + \frac{2}{\beta}} = 0.98$$

$$\text{Hence } \frac{|I_O - I_{REF}|}{I_{REF}} \times 100 = 2\%$$

For the Wilson current mirror, we have

$$R_o = \frac{\beta r_o}{2} = \frac{100 \times 100 \text{ k}\Omega}{2} = 5 \text{ M}\Omega$$

and for the simple mirror,  $R_o = r_o$

$$= 100 \text{ k}\Omega.$$

Ex: 7.27 For the two current sources designed in Example 7.6, we have

$$g_m = \frac{I_C}{V_T} = \frac{10 \mu\text{A}}{25 \text{ mV}} = 0.4 \frac{\text{mA}}{\text{V}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{10 \mu\text{A}} = 10 \text{ M}\Omega,$$

$$r_n = \frac{\beta}{g_m} = 250 \text{ k}\Omega$$

For the current source in Fig. 7.43(b), we have

$$R_o = r_{o2} = r_o = 10 \text{ M}\Omega$$

### Exercise 7-7

For the current source in Fig. 7.43, from Eq. (7.102), we have

$$R_{out} \simeq [1 + g_m (R_E \parallel r_n)] r_o$$

From Example 7.6,  $R_E = R_3 = 11.5 \text{ k}\Omega$ ;

therefore,

$$R_{out} \simeq \left[ 1 + 0.4 \frac{\text{mA}}{\text{V}} (11.5 \text{ k}\Omega \parallel 250 \text{ k}\Omega) \right] 10 \text{ M}\Omega$$

$$\therefore R_{out} = 54 \text{ M}\Omega$$

Ex: 7.28

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$g_{mb} = \chi g_m = 0.2 \times 2 = 0.4 \text{ mA/V}$$

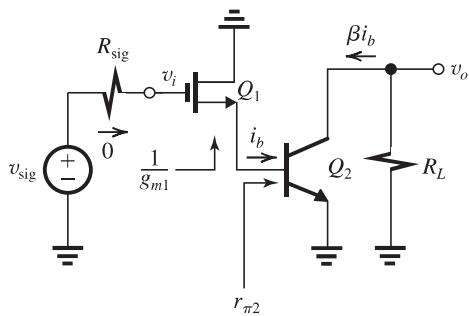
$$r_{o1} = r_{o3} = \frac{V_A}{I_D} = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$R_L = r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{mb}} = 25 \parallel 25 \parallel 2.5 \text{ k}\Omega$$

$$= 2.083 \text{ k}\Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + \frac{1}{g_m}} = \frac{2.083}{2.083 + \frac{1}{2}} = 0.81 \text{ V/V}$$

Ex: 7.29



$$g_{m1} = \sqrt{2k_n I_D}$$

$$= \sqrt{2 \times 8 \times 1}$$

$$= 4 \text{ mA/V}$$

$$\frac{1}{g_{m1}} = 0.25 \text{ k}\Omega$$

$$g_{m2} = 40 \text{ mA/V}$$

$$r_{\pi2} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$R_{in} = \infty$$

$$i_b = \frac{v_i}{\frac{1}{g_{m1}} + r_{\pi2}} = \frac{v_{sig}}{\frac{1}{g_{m1}} + r_{\pi2}} = \frac{v_{sig}}{0.25 + 2.5} \\ = \frac{v_{sig}}{2.75}$$

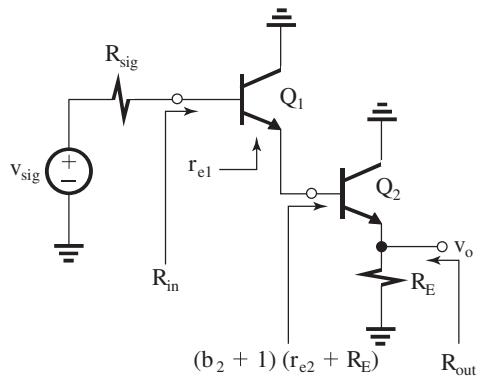
$$v_o = -\beta i_b R_L = -\frac{100 \times 4}{2.75} v_{sig}$$

$$G_v \equiv \frac{v_o}{v_{sig}} = -145.5 \text{ V/V}$$

These results apply for both  $R_{sig} = 4 \text{ k}\Omega$  and  $R_{sig} = 400 \text{ k}\Omega$ . If in the CC-CE amplifier of Example 7.7,  $R_{sig} = 400 \text{ k}\Omega$ ,  $G_v$  becomes

$$G_v = \frac{255}{255 + 400} \times 0.99 \times -160 \\ = -61.7 \text{ V/V}$$

Ex: 7.30



From the figure we can write

$$R_{in} = (\beta_1 + 1)[r_{e1} + (\beta_2 + 1)(r_{e2} + R_E)]$$

$$R_{out} = R_E \parallel \left[ r_{e2} + \frac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1} \right]$$

$$\frac{v_o}{v_{sig}} = \frac{R_E}{R_E + r_{e2} + \frac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1}}$$

For  $I_{E2} = 5 \text{ mA}$ ,  $\beta_1 = \beta_2 = 100$ ,  $R_E = 1 \text{ k}\Omega$ , and  $R_{sig} = 100 \text{ k}\Omega$ , we obtain

$$r_{e2} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5 \Omega$$

$$I_{E1} = \frac{5}{\beta_2 + 1} = \frac{5}{101} \simeq 0.05 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{0.05 \text{ mA}} = 500 \Omega$$

$$R_{in} = 101 \times (0.5 + 101 \times 1.005) = 10.3 \text{ M}\Omega$$

$$R_{out} = 1 \parallel \left[ 0.005 + \frac{0.5 + (100/101)}{101} \right] \simeq 20 \Omega$$

$$\frac{v_o}{v_{sig}} = \frac{1}{1 + 0.005 + \frac{0.5 + (100/101)}{101}}$$

$$= 0.98 \text{ V/V}$$

Ex: 7.31 Refer to Fig. 7.49.

$$r_e = 25 \Omega$$

$$R_{in} = (\beta_1 + 1)(2 r_e) = 101 \times 0.05 = 5.05 \text{ k}\Omega$$

Exercise 7-8

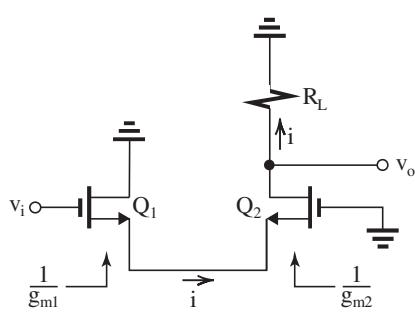
$$\frac{v_o}{v_i} = \frac{\alpha_2 R_L}{2 r_e} \approx \frac{5}{0.05} = 100 \text{ V/V}$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \times \frac{v_o}{v_i}$$

$$= \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \times \frac{v_o}{v_i}$$

$$= \frac{5.05}{5.05 + 5} \times 100 = 50 \text{ V/V}$$

Ex: 7.32



(a) From the figure we see that

$$i = \frac{v_i}{2/g_m} = \frac{1}{2} g_m v_i$$

and

$$v_o = i R_L$$

Thus,

$$\frac{v_o}{v_i} = \frac{1}{2} g_m R_L$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}}$$

Thus,

$$\frac{v_o}{v_i} = \frac{1}{2} \times \frac{2I}{V_{OV}} R_L = \frac{IR_L}{V_{OV}} \quad \text{Q.E.D}$$

(b) I = 0.1 mA and R<sub>L</sub> = 20 kΩ, to obtain a gain of 10 V/V,

$$10 = \frac{0.1 \times 20}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

The required W/L can be obtained from

$$I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.2 \times \left( \frac{W}{L} \right) \times 0.04$$

$$\Rightarrow \frac{W}{L} = 25$$

7.1 Referring to Fig. 7.1,  $V_{DD} = 1.3 \text{ V}$ ,

$$I_O = I_{REF} = 100 \mu\text{A}, L = 0.5 \mu\text{m}, W = 5 \mu\text{m}, V_A' = 5 \text{ V}/\mu\text{m}, V_t = 0.4 \text{ V}, k_n' = 500 \mu\text{A}/\text{V}^2$$

$$I_O = I_D = \frac{1}{2} k_n' \left( \frac{W}{L} \right) V_{OV}^2$$

$$V_{OV} = \sqrt{\frac{2I_D}{k_n' \left( \frac{W}{L} \right)}}$$

$$= \sqrt{\frac{2(100 \mu\text{A})}{(500 \mu\text{A}/\text{V}^2) \left( \frac{5}{0.5} \right)}} = 0.2 \text{ V}$$

$$V_{DS} = V_{GS} = V_t + V_{OV} = 0.4 + 0.2 = 0.6 \text{ V}$$

$$R = \frac{V_{DD} - V_{GS}}{I_{REF}} = \frac{1.8 - 0.6}{0.1 \text{ mA}} = 12 \text{ k}\Omega$$

The lowest  $V_O$  will be

$$V_{DS2} = V_{OV} = 0.2 \text{ V}$$

$$R_O = r_o = \frac{V_A' L}{I_D} = \frac{5 \text{ V}/\mu\text{m} \times 0.5 \mu\text{m}}{100 \mu\text{A}} = 25 \text{ k}\Omega$$

$$\Delta I_D \approx \frac{\Delta V_O}{r_o} = \frac{0.5 \text{ V}}{25 \text{ k}\Omega} = 20 \mu\text{A}$$

7.2 Refer to Fig. 7.1.

$$\frac{\Delta I_O}{I_O} = 10\%$$

$$\Delta I_O = 0.1 \times 150 = 15 \mu\text{A}$$

$$\Delta V_O = 1.8 - 0.3 = 1.5 \text{ V}$$

$$r_o = \frac{\Delta V_O}{\Delta I_O} = \frac{1.5 \text{ V}}{15 \mu\text{A}} = 100 \text{ k}\Omega$$

But

$$r_o = \frac{V_A}{I_O} = \frac{V_A' L}{I_O}$$

$$100 = \frac{10 \times L}{0.15} \Rightarrow L = 1.5 \mu\text{m}$$

$$\Rightarrow V_A = 15 \text{ V}$$

$$V_{OV} = V_{DS2\min} = 0.3 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.3 = 0.8 \text{ V}$$

$$I_D = \frac{1}{2} k_n' \left( \frac{W}{L} \right) V_{OV}^2 \left( 1 + \frac{V_{DS}}{V_A} \right)$$

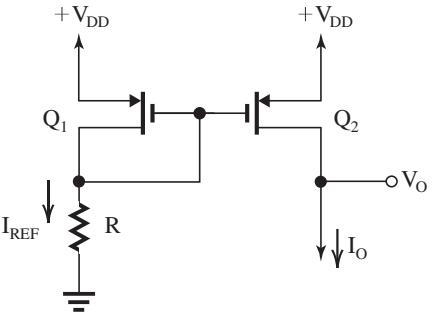
$$150 = \frac{1}{2} \times 400 \times \frac{W}{L} \times 0.09 \left( 1 + \frac{0.8}{15} \right)$$

$$\Rightarrow \frac{W}{L} = 7.91$$

$$W = 7.91 \times 1.5 = 11.9 \mu\text{m}$$

$$R = \frac{V_{DD} - V_{GS}}{I_{REF}} = \frac{1.8 - 0.8}{0.15} = 6.7 \text{ k}\Omega$$

7.3



$$\text{Set } |V_{OV}| = V_{DD} - V_{Omax}$$

$$= 1.3 - 1.1 = 0.2 \text{ V}$$

$$V_G = V_{DD} - |V_{tp}| - |V_{OV}|$$

$$= 1.3 - 0.4 - 0.2 = 0.7 \text{ V}$$

$$R = \frac{V_G}{I_{D1}} = \frac{0.7 \text{ V}}{80 \mu\text{A}} = 8.75 \text{ k}\Omega$$

$$I_D = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right) |V_{OV}|^2$$

thus

$$\frac{W}{L} = \frac{2I_D}{\mu_p C_{ox} |V_{OV}|^2} = \frac{2 \times 80 \mu\text{A}}{80 \mu\text{A}/\text{V}^2 \times 0.2^2} = 50$$

7.4 Referring to Fig. 7.2, if  $W_2 = 5 W_1$  and we let  $L_1 = L_2$ , then we obtain

$$I_O = I_{D2} = I_{REF} \frac{(W/L)_2}{(W/L)_1} = 20 \mu\text{A} \times 5 = 100 \mu\text{A}$$

$$V_{Omin} = V_{OV} = 0.2 \text{ V}$$

From Eq. (7.8):

$$I_O = \frac{(W/L)_2}{(W/L)_1} \cdot I_{REF} \left( 1 + \frac{V_O - V_{GS}}{V_{A2}} \right)$$

$$V_{GS} = V_t + V_{OV} = 0.5 \text{ V} + 0.2 \text{ V} = 0.7 \text{ V}$$

Thus,  $I_D$  equal  $5I_{REF}$  will be obtained at

$$V_O = V_{GS} = 0.7 \text{ V}$$

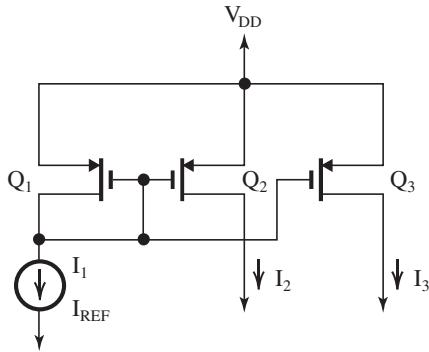
For  $V_O = V_{GS} + 1 = 1.7 \text{ V}$

$$I_O = 100 \left( 1 + \frac{1.7 - 0.7}{20} \right) = 105 \mu\text{A}$$

The corresponding increase in  $I_O$ ,  $\Delta I_O$  is, thus,  $5 \mu\text{A}$ .

7.5 Referring to the figure on the next page, suppose that  $Q_1$  has  $W = 10 \mu\text{m}$ ,  $Q_2$  has  $W = 20 \mu\text{m}$ , and  $Q_3$  has  $W = 40 \mu\text{m}$ .

This figure belongs to Problem 7.5.



(1) With  $Q_1$  diode connected,

$$I_2 = I_{\text{REF}} \frac{(W/L)_2}{(W/L)_1} = 100 \mu\text{A} \left( \frac{20}{10} \right) = 200 \mu\text{A}$$

$$I_3 = 100 \mu\text{A} \left( \frac{40}{10} \right) = 400 \mu\text{A}$$

(2) With  $Q_2$  diode connected, and  $W = 20 \mu\text{m}$ ,

$$I_1 = 100 \mu\text{A} \left( \frac{10}{20} \right) = 50 \mu\text{A}$$

$$I_3 = 100 \mu\text{A} \left( \frac{40}{20} \right) = 200 \mu\text{A}$$

(3) If  $Q_3$  with  $W = 40 \mu\text{m}$  is diode connected,

$$I_1 = 100 \mu\text{A} \left( \frac{10}{40} \right) = 25 \mu\text{A}$$

$$I_2 = 100 \mu\text{A} \left( \frac{20}{40} \right) = 50 \mu\text{A}$$

So, with only one transistor diode connected, we can get  $25 \mu\text{A}$ ,  $50 \mu\text{A}$ ,  $200 \mu\text{A}$ , and  $400 \mu\text{A}$ , or four different currents.

Now, if two transistors are diode connected, the effective width is the sum of the two widths.

(4) If  $Q_1$  and  $Q_2$  are diode connected, then

$W_{\text{eff}} = 20 + 10 = 30 \mu\text{m}$ , so that

$$I_3 = 100 \mu\text{A} \left( \frac{40}{30} \right) = 133 \mu\text{A}$$

(5) If  $Q_2$  and  $Q_3$  are diode connected, then

$W_{\text{eff}} = 20 + 40 = 60 \mu\text{m}$ , so that

$$I_1 = 100 \mu\text{A} \left( \frac{10}{60} \right) = 16.7 \mu\text{A}$$

(6) If  $Q_1$  and  $Q_3$  are diode connected,

$W_{\text{eff}} = 10 + 40 = 50 \mu\text{m}$ , so that

$$I_2 = 100 \mu\text{A} \left( \frac{20}{50} \right) = 40 \mu\text{A}$$

So three different currents are obtained with double-diode connects.

To find  $V_{SG}$ , we use the following for the diode-connected transistor(s):

$$I_D = \frac{1}{2} \mu_p C_{\text{ox}} \left( \frac{W}{L} \right) (V_{SG} - |V_{tp}|)^2$$

and substitute  $I_D = I_{\text{REF}} = 100 \mu\text{A}$ . Thus

$$100 = \frac{1}{2} \times 100 \times \left( \frac{W}{1 \mu\text{m}} \right) (V_{SG} - 0.6)^2$$

$$\Rightarrow V_{SG} = 0.6 + \sqrt{\frac{2}{W(\mu\text{m})}}$$

For the six cases above we obtain

$$(1) W = W_1 = 10 \mu\text{m} \Rightarrow V_{SG} = 1.05 \text{ V}$$

$$(2) W = W_2 = 20 \mu\text{m} \Rightarrow V_{SG} = 0.92 \text{ V}$$

$$(3) W = W_3 = 40 \mu\text{m} \Rightarrow V_{SG} = 0.82 \text{ V}$$

$$(4) W = W_1 + W_2 = 30 \mu\text{m} \Rightarrow V_{SG} = 0.86 \text{ V}$$

$$(5) W = W_2 + W_3 = 60 \mu\text{m} \Rightarrow V_{SG} = 0.78 \text{ V}$$

$$(6) W = W_1 + W_3 = 50 \mu\text{m} \Rightarrow V_{SG} = 0.80 \text{ V}$$

7.6 Refer to the circuit of Fig. P7.6. For  $Q_2$  to operate properly (i.e., in the saturation mode) for drain voltages as high as  $+0.8 \text{ V}$ , and provided its width is the minimum possible, we use

$$|V_{ov}| = 0.2 \text{ V}$$

Note that all three transistors  $Q_1$ ,  $Q_2$ , and  $Q_3$  will be operated at this value of overdrive voltage.

For  $Q_1$ ,

$$I_{D1} = I_{\text{REF}} = 20 \mu\text{A}$$

$$I_{D1} = \frac{1}{2} \mu_p C_{\text{ox}} \left( \frac{W}{L} \right)_1 |V_{ov}|^2$$

$$20 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_1 \times 0.04$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = 10$$

For  $L = 0.5 \mu\text{m}$ ,

$$W_1 = 5 \mu\text{m}$$

Now, for

$$I_2 = 100 \mu\text{A} = 5I_{\text{REF}}, \text{ we have}$$

$$\frac{(W/L)_2}{(W/L)_1} = 5$$

$$\Rightarrow \left( \frac{W}{L} \right)_2 = 5 \times 10 = 50$$

$$W_2 = 50 \times 0.5 = 25 \mu\text{m}$$

For

$I_3 = 40 \mu A = 2I_{REF}$ , we obtain

$$\frac{(W/L)_3}{(W/L)_1} = 2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = 20$$

$$W_3 = 10 \mu m$$

We next consider  $Q_4$  and  $Q_5$ . For  $Q_5$  to operate in saturation with the drain voltage as low as  $-0.8$  V, and for it to have the minimum possible  $W/L$ , we operate  $Q_5$  at

$$V_{OV} = 0.2 \text{ V}$$

This is the same overdrive voltage at which  $Q_4$  will be operating. Thus, we can write for  $Q_4$ ,

$$I_4 = I_3 = 40 \mu A$$

and using

$$I_{D4} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_4 V_{OV}^2$$

$$40 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_4 \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_4 = 5$$

$$W_4 = 2.5 \mu m$$

Finally, since

$$I_5 = 80 \mu A = 2 I_4,$$

$$\left(\frac{W}{L}\right)_5 = 2 \left(\frac{W}{L}\right)_4$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 10$$

$$W_5 = 5 \mu m$$

To find the value of  $R$ , we use

$$|V_{SG1}| = |V_{tp}| + |V_{OV1}|$$

$$= 0.5 + 0.2 = 0.7 \text{ V}$$

$$R = \frac{1 - |V_{SG1}|}{I_{REF}} = \frac{0.3 \text{ V}}{0.02 \text{ mA}}$$

$$= 15 \text{ k}\Omega$$

The output resistance of the current source  $Q_2$  is

$$r_{o2} = \frac{|V_{A2}|}{I_2} = \frac{|V'_{Ap}| \times L}{I_2}$$

$$= \frac{5 \times 0.5}{0.1 \text{ mA}} = 25 \text{ k}\Omega$$

The output resistance of the current sink  $Q_5$  is

$$r_{o5} = \frac{V_{A5}}{I_5} = \frac{V'_{An} \times L}{I_5}$$

$$= \frac{5 \times 0.5}{80} = 31.25 \text{ k}\Omega$$

7.7 Referring to Fig. P7.5, we obtain

$$V_{GS1} = V_{GS2} \text{ so that } \frac{I_{D2}}{I_{D1}} = \frac{(W/L)_2}{(W/L)_1} \text{ and}$$

$$I_{D2} = I_{REF} \frac{(W/L)_2}{(W/L)_1}$$

$$I_{D3} = I_{D2}$$

$$V_{GS3} = V_{GS4}, \text{ thus } \frac{I_{D4}}{I_{D3}} = \frac{(W/L)_4}{(W/L)_3}$$

$$I_O = I_{D4} = I_{REF} \frac{(W/L)_2}{(W/L)_1} \cdot \frac{(W/L)_4}{(W/L)_3}$$

7.8 (a) If  $I_S = 10^{-17}$  A and we ignore base currents, then

$$I_{REF} = I_S e^{V_{BE}/V_T} \text{ so that}$$

$$V_{BE} = V_T \ln\left(\frac{I_{REF}}{10^{-17}}\right)$$

For  $I_{REF} = 10 \mu A$ ,

$$V_{BE} = 0.025 \ln\left(\frac{10^{-5}}{10^{-17}}\right) = 0.691 \text{ V}$$

For  $I_{REF} = 10 \text{ mA}$ ,

$$V_{BE} = 0.025 \ln\left(\frac{10^{-2}}{10^{-17}}\right) = 0.863 \text{ V}$$

So for the range of

$$10 \mu A \leq I_{REF} \leq 10 \text{ mA},$$

$$0.691 \text{ V} \leq V_{BE} \leq 0.863 \text{ V}$$

(b) Accounting for finite  $\beta$ ,

$$I_O = I_{REF} \cdot \frac{1}{1 + 2/\beta}$$

For  $I_{REF} = 10 \mu A$ ,

$$I_O = \frac{10 \mu A}{1 + \frac{2}{50}} = 9.62 \mu A$$

For  $I_{REF} = 0.1 \text{ mA}$ ,

$$I_O = \frac{0.1 \text{ mA}}{1 + \frac{2}{100}} = 0.098 \text{ mA}$$

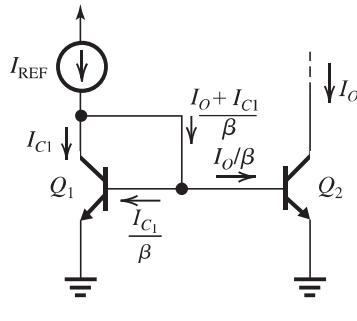
For  $I_{REF} = 1 \text{ mA}$ ,

$$I_O = \frac{1 \text{ mA}}{1 + \frac{2}{100}} = 0.98 \text{ mA}$$

For  $I_{REF} = 10 \text{ mA}$ ,

$$I_O = \frac{10 \text{ mA}}{1 + \frac{2}{50}} = 9.62 \text{ mA}$$

7.9



$$I_O = mI_{C1}$$

A node equation at the collector of  $Q_1$  yields

$$I_{\text{REF}} = I_{C1} + \frac{I_O + I_{C1}}{\beta}$$

Substituting  $I_{C1} = I_O/m$  results in

$$\frac{I_O}{I_{\text{REF}}} = \frac{m}{1 + \frac{m+1}{\beta}} \quad \text{Q.E.D.}$$

For  $\beta = 80$  and the error in the current transfer ratio to be limited to 10%, that is,

$$\frac{m}{1 + \frac{m+1}{\beta}} \geq 0.9m$$

$$\left(1 + \frac{m+1}{\beta}\right) \leq \frac{1}{0.9}$$

$$\frac{m+1}{\beta} \leq \frac{1}{0.9} - 1$$

$$m \leq \beta \left( \frac{1}{0.9} - 1 \right) - 1$$

$$m \leq 80 \left( \frac{1}{0.9} - 1 \right) - 1 = 7.88$$

Thus, the largest current transfer ratio possible is 7.88.

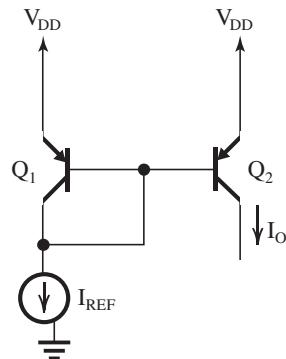
7.10 Nominally,  $I_O = I_{\text{REF}} = 1 \text{ mA}$

$$r_{o2} = \frac{V_{A2}}{I_O} = \frac{90}{1} = 90 \text{ k}\Omega$$

$$r_{o2} = \frac{\Delta V_O}{\Delta I_O} \Rightarrow \frac{10 - 1}{\Delta I_O} = 90 \Rightarrow \Delta I_O = 0.1 \text{ mA}$$

$$\frac{\Delta I_O}{I_O} = \frac{0.1}{1} = 10\% \text{ change}$$

7.11



For identical transistors, the transfer ratio is

$$\frac{I_O}{I_{\text{REF}}} = \frac{1}{1 + 2/\beta} = \frac{1}{1 + \frac{2}{50}} = 0.96$$

7.12 Equation (7.21) gives the current transfer ratio of an npn mirror with a nominal ratio of  $m$ :

$$I_O = I_{\text{REF}} \frac{m}{1 + \frac{m+1}{\beta}} \left( 1 + \frac{V_O - V_{BE}}{V_{A2}} \right)$$

This equation can be adapted for the pnp mirror of Fig. P7.12 by substituting  $m = 1$ , replacing  $V_O$  with the voltage across  $Q_3$ , namely  $(3 - V_O)$ , replacing  $V_{BE}$  with  $V_{EB}$ , and  $V_{A2}$  with  $|V_A|$ :

$$I_O = I_{\text{REF}} \frac{1 + [(3 - V_O - V_{EB})/|V_A|]}{1 + (2/\beta)} \quad (1)$$

Now, substituting  $I_O = 1 \text{ mA}$ ,  $V_O = 1 \text{ V}$ ,  $\beta = 50$ ,  $|V_A| = 50 \text{ V}$ , and

$$V_{EB} = V_T \ln \frac{I_O}{I_S} = 0.025 \ln \left( \frac{10^{-3}}{10^{-15}} \right) = 0.691 \text{ V}$$

results in

$$I_{\text{REF}} = \frac{1 \times (1 + 0.04)}{1 + \frac{3 - 1 - 0.691}{50}} = 1.013 \text{ mA}$$

$$R = \frac{V_{CC} - V_{EB}}{I_{\text{REF}}} = \frac{3 - 0.691}{1.013} = 2.28 \text{ k}\Omega$$

Maximum allowed voltage  $V_O = 3 - 0.3 = 2.7 \text{ V}$ . For  $V_O = 2.7 \text{ V}$ , Eq. (1) yields

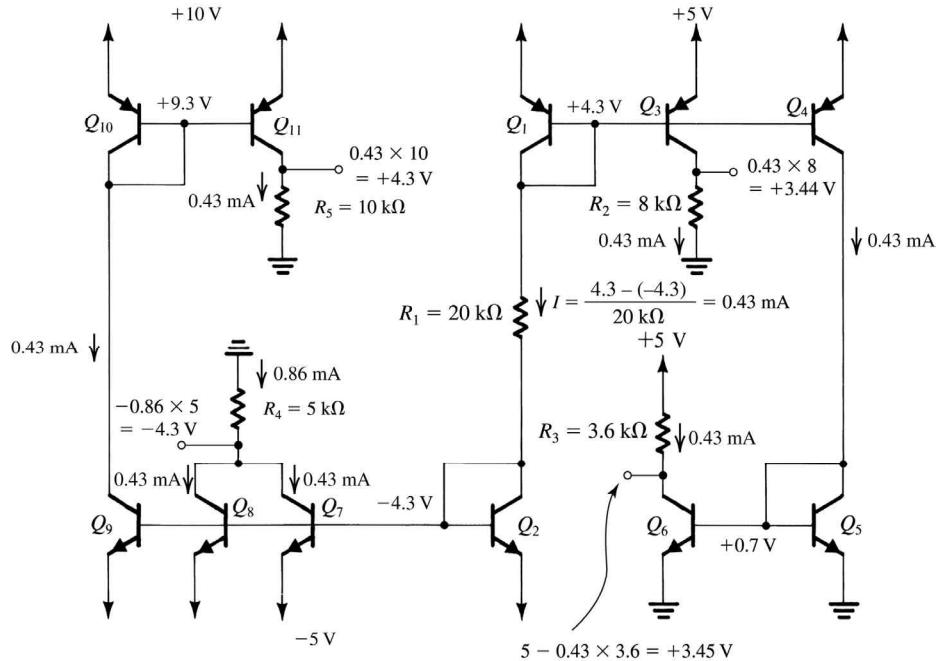
$$I_O = 1.013 \frac{1 + \frac{3 - 2.7 - 0.691}{50}}{1.04} = 0.966 \text{ mA}$$

For  $V_O = -5 \text{ V}$ , Eq. (1) yields

$$I_O = 1.013 \frac{1 + \frac{3 - (-5) - 0.691}{50}}{1.04} = 1.116 \text{ mA}$$

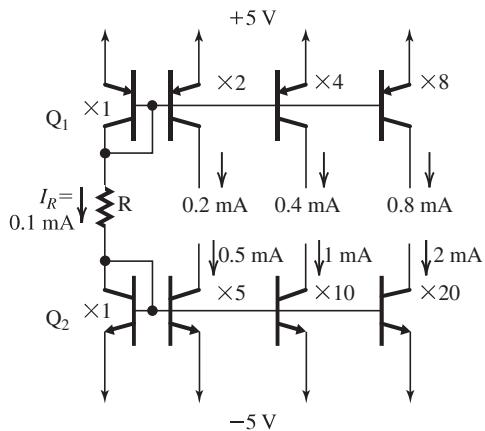
Thus, the change in  $I_O$  is 0.15 mA.

This figure belongs to Problem 7.13.



7.13 The solution is given in the circuit diagram. Note that the starting point is calculating the current I in the  $Q_1-R_1-Q_2$  branch. See figure above.

7.14 There are various ways this design could be achieved, but the most straightforward is the one shown:



Even without knowing exact circuitry, we can find the total power dissipation as approximately

$$P_T = P_{CC} + P_{EE}$$

$$P_T = 5 \text{ V} (0.1 + 0.2 + 0.4 + 0.8) \text{ mA}$$

$$+5 \text{ V} (0.1 + 0.5 + 1 + 2) \text{ mA}$$

$$P_T = 7.5 \text{ mW} + 18 \text{ mW} = 25.5 \text{ mW}$$

7.15 Refer to the circuit in Fig. P7.15.

$$V_2 = 2.7 - V_{EB} = 2.7 - 0.7 = +2 \text{ V}$$

$$V_3 = 0 + V_{EB} = +0.7 \text{ V}$$

Thus,  $Q_3$  and  $Q_4$  are operating in the active mode, and each is carrying a collector current of  $I/2$ . The same current is "owing" in  $Q_2$  and  $Q_1$ ; thus

$$V_1 = -2.7 + \frac{1}{2}IR$$

But

$$V_1 = -V_{BE1} = -0.7$$

Thus,

$$-0.7 = -2.7 + \frac{1}{2}IR$$

$$\Rightarrow IR = 4 \text{ V}$$

The current I splits equally between  $Q_5$  and  $Q_6$ ; thus

$$V_4 = -2.7 + \left(\frac{I}{2}\right) R = -2.7 + 2 = -0.7 \text{ V}$$

$$V_5 = -2.7 + \left(\frac{I}{2}\right) \left(\frac{R}{2}\right) = -2.7 + 1 = -1.7 \text{ V}$$

Thus,  $Q_5$  and  $Q_6$  are operating in the active mode as we have implicitly assumed.

Note that the values of  $V_1$ ,  $V_2$ ,  $V_3$ ,  $V_4$ , and  $V_5$  do not depend on the value of  $R$ . Only  $I$  depends on the value of  $R$ :

$$(a) R = 10 \text{ k}\Omega \Rightarrow I = \frac{4}{10} = 0.4 \text{ mA}$$

$$(b) R = 100 \text{ k}\Omega \Rightarrow I = \frac{4}{100} = 0.04 \text{ mA}$$

7.16 (a)

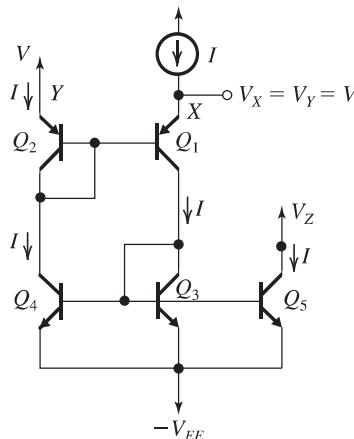


Figure 1

Figure 1 shows the current conveyor circuit with  $Y$  connected to a voltage  $V$ ,  $X$  fed with a current source  $I$ , and  $Z$  connected to a voltage  $V_Z$  that keeps  $Q_5$  operating in the active mode. Assuming that all transistors are operating in the active mode and that  $\beta \gg 1$ , so that we can neglect all base currents, we see that the current  $I$  through  $Q_1$  will "ow through the two-output mirror  $Q_3$ ,  $Q_4$ , and  $Q_5$ . The current  $I$  in  $Q_5$  will be drawn from  $Q_2$ , which forms a mirror with  $Q_1$ . Thus  $V_{EB2} = V_{EB1}$  and the voltage that appears at  $X$  will be equal to  $V$ . The current in  $Q_5$  will be equal to  $I$ , thus terminal  $Z$  sinks a constant current  $I$ .

(b)

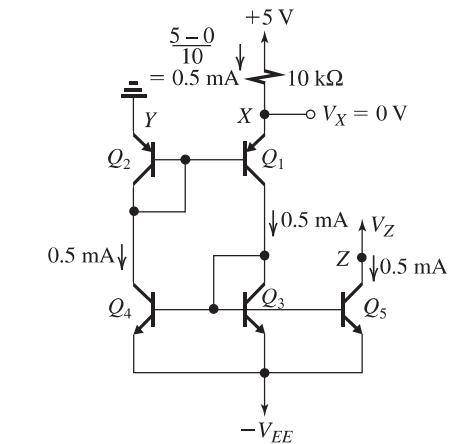


Figure 2

Figure 2 shows the special case of  $V = 0 \text{ V}$ . As before, the voltage at  $X$ ,  $V_X$ , will be equal to  $V$ . Thus

$$V_X = 0$$

That is, a virtual ground appears at  $X$ , and thus the current  $I$  that "ows into  $X$  can be found from

$$I = \frac{5 - V_X}{10 \text{ k}\Omega} = \frac{5 - 0}{10} = 0.5 \text{ mA}$$

This is the current that will be mirrored to the output, resulting in  $I_Z = 0.5 \text{ mA}$ .

$$7.17 A_{is} = 4 = \frac{(W/L)_2}{(W/L)_1}$$

Since  $L_1 = L_2$ , then

$$\frac{W_2}{W_1} = 4$$

$$R_{in} = r_{o1} \parallel \frac{1}{g_{m1}} \simeq \frac{1}{g_{m1}}$$

For

$$R_{in} = 500 \Omega \Rightarrow g_{m1} = 2 \text{ mA/V}$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}$$

Thus,

$$2 = \sqrt{2 \times 0.4 \times \left(\frac{W}{L}\right)_1 \times 0.2}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 25$$

$$R_o = r_{o2} = \frac{V_A}{I_{D2}} = \frac{V'_A L}{I_{D2}}$$

Thus,

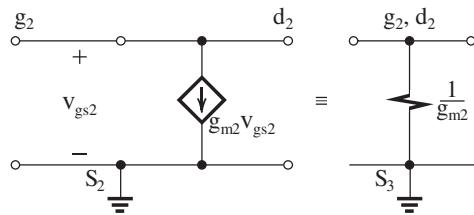
$$20 = \frac{20 L}{4 \times 0.2}$$

$$\Rightarrow L = 0.8 \mu m$$

$$W_1 = 25 \times 0.8 = 20 \mu m$$

$$W_2 = 4W_1 = 80 \mu m$$

7.18 Refer to Fig. P7.18. Consider first the diode-connected transistor  $Q_2$ . From the figure we



see that from a small-signal point of view it is equivalent to a resistance  $1/g_{m2}$ . Thus the voltage gain of  $Q_1$  will be

$$\frac{v_{d1}}{v_i} = -g_{m1} \times \frac{1}{g_{m2}} = -\frac{g_{m1}}{g_{m2}}$$

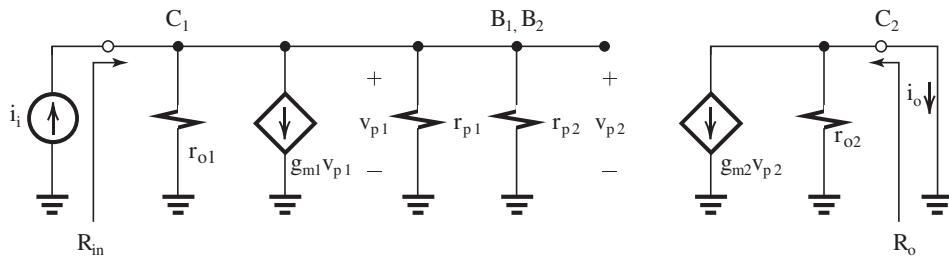
The signal current in the drain of  $Q_1$ ,  $g_{m1} v_i$ , will be mirror in the drain of  $Q_3$ ;

$$i_{d3} = g_{m1} v_i \frac{(W/L)_3}{(W/L)_2} = g_{m1} v_i \frac{W_3}{W_2}$$

which "ows through  $R_L$  and produces the output voltage  $v_o$ ,

$$v_o = i_{d3} R_L = g_{m1} v_i \frac{W_3}{W_2} R_L$$

This figure belongs to Problem 7.19.



Thus, the small-signal voltage gain will be

$$\frac{v_o}{v_i} = g_{m1} R_L (W_3/W_2)$$

7.19 Replacing  $Q_1$  and  $Q_2$  with their small-signal hybrid- $\pi$  models results in the equivalent circuit shown in the figure below. Observe that the controlled source  $g_{m1} v_{\pi 1}$  appears across its controlling voltage  $v_{\pi 1}$ ; thus the controlled source can be replaced with a resistance  $(1/g_{m1})$ . The input resistance  $R_{in}$  can now be obtained by inspection as

$$R_{in} = r_{o1} \parallel \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{\pi 2}$$

Since  $r_{o1} \gg r_{\pi 1}$ ,

$$R_{in} \simeq \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{\pi 2} \quad (1)$$

The short-circuit output current  $i_o$  is given by

$$i_o = g_{m2} v_{\pi 2}$$

Since  $v_{\pi 2} = v_{\pi 1} = i_i R_{in}$ , then the short-circuit current gain  $A_{is}$  is given by

$$A_{is} = \frac{i_o}{i_i} = g_{m2} R_{in}$$

$$= g_{m2} \left( \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{\pi 2} \right) \quad (2)$$

For situations where  $\beta_1$  and  $\beta_2$  are large, we can neglect  $r_{\pi 1}$  and  $r_{\pi 2}$  in Eqs. (1) and (2) to obtain

$$R_{in} \simeq 1/g_{m1}$$

$$A_{is} \simeq g_{m1}/g_{m2}$$

7.20 (a)

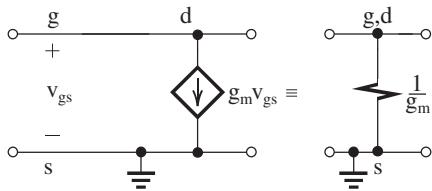


Figure 1

Replacing the MOSFET with its hybrid- $\pi$  model but neglecting  $r_o$  results in the equivalent circuit in Fig. 1. Observing that the controlled-source  $g_m v_{gs}$  appears across its control voltage  $v_{gs}$ , we can replace it by a resistance  $1/g_m$ , as indicated. Thus the small-signal resistance of the diode-connected MOS transistor is  $1/g_m$ . For the given values,

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D}$$

$$= \sqrt{2 \times 0.2 \times 10 \times 0.1} = 0.632 \text{ mA/V}$$

$$\frac{1}{g_m} = 1.6 \text{ k}\Omega$$

(b) Replacing the BJT with its hybrid- $\pi$  model results in the equivalent circuit in Fig. 2. Observing that the controlled-source  $g_m v_\pi$  appears across its control voltage  $v_\pi$ , we can replace it by a resistance  $1/g_m$ , as indicated. Next the two parallel resistances  $1/g_m$  and  $r_\pi$  can be combined as

$$\frac{\frac{1}{g_m} \times r_\pi}{\frac{1}{g_m} + r_\pi} = \frac{r_\pi}{1 + g_m r_\pi} = \frac{r_\pi}{\beta + 1} = r_e$$

Thus, the diode-connected BJT has a small-signal resistance  $r_e$ . For the given data,

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

This figure belongs to Problem 7.20, part (b).

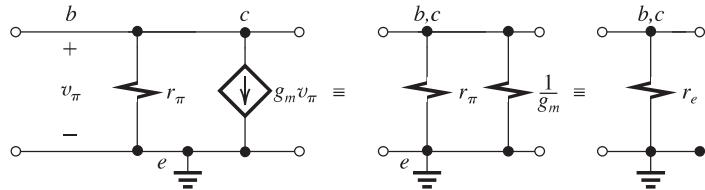


Figure 2

7.21 Refer to Fig. 7.11.

$$I_{C1} \approx I_{REF} = 0.1 \text{ mA}$$

$$V_{BE1} = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{0.1 \text{ mA}}\right)$$

$$= 0.642 \text{ V}$$

$$I_{C3} \approx I_{B1} + I_{B2} = 2 I_{B1} = 2 \times \frac{I_{C1}}{\beta}$$

$$= 2 \times \frac{0.1}{100} = 0.002 \text{ mA}$$

$$V_{BE3} = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{0.002 \text{ mA}}\right)$$

$$= 0.545 \text{ V}$$

$$V_x = V_{BE3} + V_{BE1} = 1.187 \text{ V}$$

If  $I_{REF}$  is increased to 1 mA,

$$V_{BE1} = 0.7$$

$$I_{C3} \approx 0.02 \text{ mA}$$

$$V_{BE3} = 0.7 - 0.025 \ln\left(\frac{1}{0.02}\right) = 0.6 \text{ V}$$

$$V_x = 1.3 \text{ V}$$

Thus,

$$\Delta V_x = 1.3 - 1.187 = 0.113 \text{ V}$$

When  $V_O = V_x$ , the Early effect on  $Q_1$  and  $Q_2$  will be the same, and

$$I_O = I_{REF}/(1 + 2/\beta^2)$$

Thus,  $I_O$  will be

$$I_{REF} = 100 \mu\text{A} \Rightarrow I_O = \frac{100}{1 + (2/100^2)} = 99.98 \mu\text{A}, \text{ for an error of } -0.02 \mu\text{A or } -0.02\%.$$

$$I_{REF} = 1 \text{ mA} \Rightarrow I_O = \frac{1}{1 + (2/100^2)} = 0.9998 \mu\text{A}, \text{ for an error of } -0.0002 \text{ mA or } -0.02\%. \text{ For proper current-source operation, the minimum required voltage at the output is the value needed to keep } Q_3 \text{ in the active region, which is approximately } 0.3 \text{ V.}$$



For  $I = 100 \mu A$ :

$$g_m = \frac{100 \mu A}{25 mV} = 4 mA/V$$

$$r_\pi = \frac{100}{4 mA/V} = 25 k\Omega$$

$$r_o = \frac{10 V}{100 \mu A} = 100 k\Omega$$

$$A_0 = 4 mA/V \times 100 k\Omega = 400 V/V$$

$$r_o = \frac{V_A}{I_C} = \frac{100 V}{0.5 mA} = 200 k\Omega$$

$$R_{in} = r_\pi = \frac{\beta}{g_m} = \frac{100}{20 mA/V} = 5 k\Omega$$

$$A_{vo} = -A_0 = -g_m r_o = -20 \times 200 = -4000 V/V$$

$$R_o = r_o = 200 k\Omega$$

To raise  $R_{in}$  by a factor of 5 by changing  $I$ , the value of  $I$  must be lowered by the same factor to  $I = 0.1 mA$ .

Now,  $g_m$  is reduced by a factor of 5 and  $r_o$  is increased by a factor of 5, keeping  $A_{vo}$  unchanged at  $-4000 V/V$ . However,  $R_o$  will be increased to

$$R_o = 5 \times 200 k\Omega = 1 M\Omega$$

If the amplifier is fed with a signal source having  $R_{sig} = 5 k\Omega$  and a  $100-k\Omega$  load resistance is connected to the output, the equivalent circuit shown below results.

$$\frac{V_o}{V_{sig}} = \frac{r_\pi}{r_\pi + R_{sig}} \times -g_m(r_o \parallel R_L)$$

$$= -\frac{25}{25+5} \times 4 (1000 k\Omega \parallel 100 k\Omega)$$

$$= -303 V/V$$

7.26

$$A_0 = \frac{2V_A}{V_{ov}} = \frac{2V_A L}{V_{ov}} = \frac{2 \times 10 \times 0.5}{0.2} = 50 V/V$$

$$g_m = \frac{2I_D}{V_{ov}}$$

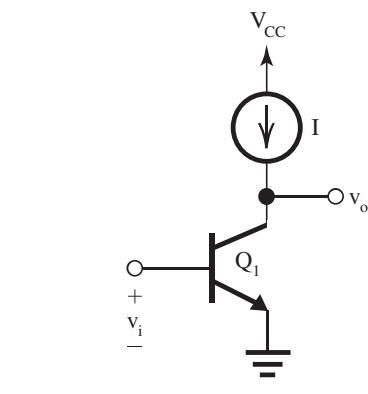
$$2 = \frac{2I_D}{0.2} \Rightarrow I_D = 0.2 mA$$

$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2$$

$$0.2 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 25$$

$$W = 12.5 \mu m$$



For  $I = 1 mA$ :

$$g_m = \frac{1 mA}{25 mV} = 40 mA/V$$

$$r_\pi = \frac{100}{40 mA/V} = 2.5 k\Omega$$

$$r_o = \frac{10 V}{1 mA} = 10 k\Omega$$

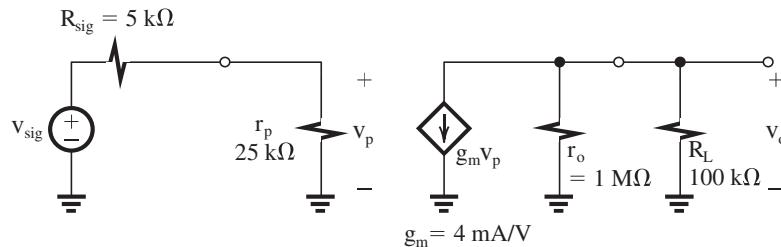
$$A_0 = 40 mA/V \times 10 k\Omega = 400 V/V$$

$I$	$g_m$	$r_\pi$	$r_o$	$A_0$
$10 \mu A$	$0.4 mA/V$	$250 k\Omega$	$1 M\Omega$	$400 V/V$
$100 \mu A$	$4.0 mA/V$	$25 k\Omega$	$100 k\Omega$	$400 V/V$
$1 mA$	$40 mA/V$	$2.5 k\Omega$	$10 k\Omega$	$400 V/V$

7.25 Refer to Fig. 7.13(b).

$$g_m = \frac{I_C}{V_T} = \frac{I}{V_T} = \frac{0.5 mA}{0.025 V} = 20 mA/V$$

This figure belongs to Problem 7.25.



$$7.27 \quad A_0 = \frac{2V_A}{V_{OV}} = \frac{2V'_A L}{V_{OV}}$$

$$20 = \frac{2 \times 5 \times L}{0.2}$$

$$\Rightarrow L = 0.4 \mu m$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}}$$

$$2 = \frac{2I}{0.2}$$

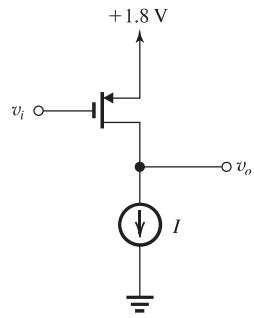
$$\Rightarrow I = 0.2 \text{ mA}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.04$$

$$\Rightarrow \frac{W}{L} = 25$$

7.28



The highest instantaneous voltage allowed at the drain is that which results in a voltage equal to ( $V_{OV}$ ) across the transistor. Thus

$$v_{Omax} = 1.8 - 0.2 = +1.6 \text{ V}$$

7.29 For the npn transistor,

$$g_m = \frac{I_C}{V_T} = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

For the NMOS transistor,

$$g_m = \frac{2I_D}{V_{OV}}$$

$$4 = \frac{2I_D}{0.25}$$

$$\Rightarrow I_D = 0.5 \text{ mA}$$

$$7.30 \quad g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

From Table K.1 (Appendix K), for the  $0.18\text{-}\mu\text{m}$  process we have

$$|V'_A| = 5 \text{ V}/\mu\text{m}, \mu_n C_{ox} = 387 \mu\text{A/V}^2$$

Thus, for our NMOS transistor whose  $L = 0.3 \mu\text{m}$ ,

$$V_A = 5 \times 0.3 = 1.5 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{1.5 \text{ V}}{0.1 \text{ mA}} = 15 \text{ k}\Omega$$

$$A_0 = g_m r_o = 1 \times 15 = 15 \text{ V/V}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$$

$$100 = \frac{1}{2} \times 387 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 13 \Rightarrow W = 3.9 \mu\text{m}$$

$$7.31 \quad g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.5} = 0.4 \text{ mA/V}$$

From Table J.1 (Appendix J), we find that for the  $0.5\text{-}\mu\text{m}$  process  $|V'_A| = 20 \text{ V}/\mu\text{m}$ . Thus for our  $1\text{-}\mu\text{-m}$  long transistor,  $V_A = 20 \text{ V}$ .

$$r_o = \frac{V_A}{I_D} = \frac{20 \text{ V}}{0.1 \text{ mA}} = 200 \text{ k}\Omega$$

$$A_0 = g_m r_o = 0.4 \times 200 = 80 \text{ V/V}$$

From Table J.1:

$$\mu_n C_{ox} = 190 \mu\text{A/V}^2$$

Now,

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$100 = \frac{1}{2} \times 190 \times \frac{W}{L} \times 0.25$$

$$\Rightarrow \frac{W}{L} = 4.21$$

$$\Rightarrow W = 4.21 \mu\text{m}$$

7.32 For the BJT cell:

$$g_m = \frac{I_C}{V_T} = \frac{I_C}{0.025 \text{ V}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{I_C}$$

$$A_0 = g_m r_o = \frac{V_A}{V_T} = \frac{100 \text{ V}}{0.025 \text{ V}} = 4000 \text{ V/V}$$

$$R_{in} = r_\pi = \frac{\beta}{g_m} = \frac{100}{g_m}$$

For the MOSFET cell:

$$g_m = \sqrt{2 \mu_n C_{ox} \left( \frac{W}{L} \right) I_D} = \sqrt{2 \times 0.2 \times 40 \times I_D}$$

$$= \sqrt{16I_D} = 4\sqrt{I_D} \text{ mA/V} \quad (I_D \text{ in mA})$$

$$r_o = \frac{V_A}{I_D} = \frac{10 \text{ V}}{I_D}$$

$$A_0 = g_m r_o = \frac{40}{\sqrt{I_D}} \text{ V/V} \quad (I_D \text{ in mA})$$

$$R_{in} = \infty$$

	BJT Cell		MOSFET Cell	
Bias current	$I_C = 0.1 \text{ mA}$	$I_C = 1 \text{ mA}$	$I_D = 0.1 \text{ mA}$	$I_D = 1 \text{ mA}$
$g_m (\text{mA/V})$	4	40	1.26	4
$r_o (\text{k}\Omega)$	1000	100	100	10
$A_0 (\text{V/V})$	4000	4000	126	40
$R_{in} (\text{k}\Omega)$	25	2.5	$\infty$	$\infty$

7.33 Using Eq. (7.46),

$$A_0 = \frac{V'_A \sqrt{2(\mu_n C_{ox})(WL)}}{\sqrt{I_D}}$$

$$18 = \frac{5\sqrt{2} \times 0.4 \times 8 \times 0.54 \times 0.54}{\sqrt{I_D}}$$

$$\Rightarrow I_D = 0.144 \text{ mA}$$

$$7.34 A_0 = \frac{2V_A}{V_{ov}} = \frac{2V'_A L}{V_{ov}} = \frac{2 \times 6 \times 0.5}{0.15} = 40 \text{ V/V}$$

$$I_D = \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{ov}^2$$

$$100 = \frac{1}{2} \times 400 \times \frac{W}{L} \times 0.15^2$$

$$\Rightarrow \frac{W}{L} = 22.2$$

Thus,

$$W = 22.2 \times 0.5 = 11.1 \mu\text{m}$$

$$g_m = \frac{2I_D}{V_{ov}} = \frac{2 \times 0.1}{0.15} = 1.33 \text{ mA/V}$$

$$r_o = \frac{V'_A L}{I_D} = \frac{6 \times 0.5}{0.1} = 30 \text{ k}\Omega$$

7.35

$$L = 0.36 \mu\text{m}, \quad V_{ov} = 0.25 \text{ V}, \quad I_D = 10 \mu\text{A}$$

$$(a) g_m = \frac{2I_D}{V_{ov}} = \frac{2 \times 10}{0.25} = 80 \mu\text{A/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V'_A L}{I_D}$$

From Appendix J, Table J.1,  $V'_A = 5 \text{ V}/\mu\text{m}$ ,

$$r_o = \frac{5 \times 0.36}{10} = 0.18 \text{ M}\Omega$$

$$A_0 = g_m r_o = 80 \times 0.18 = 14.4 \text{ V/V}$$

(b) If  $I_D$  is increased to  $100 \mu\text{A}$  (i.e., by a factor of 10),  $V_{ov}$  increases by a factor of  $\sqrt{10} = 3.16$  to

$$V_{ov} = 0.25 \times 3.16 = 0.79 \text{ V}$$

and  $g_m$  increases by a factor of  $\sqrt{10} = 3.16$  to

$$g_m = 80 \times 3.16 = 253 \mu\text{A/V} = 0.253 \text{ mA/V}$$

and  $r_o$  decreases by a factor of 10 to

$$r_o = \frac{0.18 \text{ M}\Omega}{10} = 18 \text{ k}\Omega$$

Thus,  $A_0$  becomes

$$A_0 = 0.253 \times 18 = 4.55 \text{ V/V}$$

(c) If the device is redesigned with a new value of  $W$  so that it operates at

$$V_{ov} = 0.25 \text{ V} \text{ for } I_D = 100 \mu\text{A},$$

$$g_m = \frac{2I_D}{V_{ov}} = \frac{0.2 \text{ mA}}{0.25 \text{ V}} = 0.8 \text{ mA/V}$$

$$r_o = \frac{V'_A L}{I_D} = \frac{5 \times 0.36}{0.1} = 18 \text{ k}\Omega$$

$$A_0 = g_m r_o = 0.8 \times 18 = 14.4 \text{ V/V}$$

(d) If the redesigned device in (c) is operated at  $10 \mu\text{A}$ ,  $V_{ov}$  decreases by a factor equal to  $\sqrt{10}$  to 0.08 V,  $g_m$  decreases by a factor of  $\sqrt{10}$  to 0.253 mA/V,  $r_o$  increases by a factor of 10 to 180 kΩ, and  $A_0$  becomes

$$0.253 \times 180 = 45.5 \text{ V/V}$$

which is an increase by a factor of  $\sqrt{10}$ .

(e) The lowest value of  $A_0$  is obtained with the first design when operated at  $I_D = 100 \mu\text{A}$ . The resulting  $A_0 = 4.55 \text{ V/V}$ . The highest value of  $A_0$  is obtained with the second design when operated at  $I_D = 10 \mu\text{A}$ . The resulting  $A_0 = 45.5 \text{ V/V}$ . If in any design  $W/L$  is held constant while  $L$  is increased by a factor of 10,  $g_m$  remains unchanged but  $r_o$  increases by a factor of 10, resulting in  $A_0$  increasing by a factor of 10.

$$7.36 A_0 = |A_{vo}| = 100$$

$$100 = \frac{2V_A}{V_{ov}} = \frac{2V_A}{0.2}$$

$$\Rightarrow V_A = 10 \text{ V}$$

Since  $V'_A = 20 \text{ V}/\mu\text{m}$ , we have

$$L = \frac{V_A}{V'_A} = \frac{10}{20} = 0.5 \mu\text{m}$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{ov}^2$$

$$50 = \frac{1}{2} \times 200 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 12.5$$

7.37 Refer to Fig. 7.15(a).

$$V_{SG2} = |V_{tp}| + |V_{ov}| = 0.5 + 0.3 = 0.8 \text{ V}$$

$$V_G = 2.5 - V_{SG2} = 2.5 - 0.8 = 1.7 \text{ V}$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 V_{OV1}^2$$

$$100 = \frac{1}{2} \times 200 \times \left( \frac{W}{L} \right)_1 \times 0.3^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = 11.1$$

$$I_{D2} = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_2 |V_{OV2}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_2 \times 0.3^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_2 = 22.2$$

$$A_v = -g_{m1} (r_{o1} \parallel r_{o2})$$

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.3} = 0.67 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{|V_A'|L}{I_D} = \frac{20 \times 0.5}{0.1} = 100 \text{ k}\Omega$$

$$A_v = -0.67 \times (100 \parallel 100) = -33.5 \text{ V/V}$$

7.38 Refer to Fig. 7.15. Since  $V'_{An} = |V'_{Ap}|$  and the channel lengths are equal,  $V_{An} = |V_{Ap}|$  and  $r_{o1} = r_{o2} = r_o$ . Thus

$$A_v = -g_{m1}(r_{o1} \parallel r_{o2}) = -g_{m1}(r_o/2)$$

$$-40 = -\frac{1}{2} g_{m1} r_o$$

$$\Rightarrow g_{m1} r_o = 80$$

$$A_0 = \frac{2V_{An}}{V_{OV}} = \frac{2V'_{An}L}{V_{OV}}$$

$$80 = \frac{2 \times 5 \times L}{0.25}$$

$$\Rightarrow L = 2 \mu\text{m}$$

$$V_{SG2} = |V_{tp}| + |V_{OV}| = 0.5 + 0.25 = 0.75 \text{ V}$$

$$V_G = V_{DD} - V_{SG2} = 1.8 - 0.75 = 1.05 \text{ V}$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 V_{OV1}^2$$

$$100 = \frac{1}{2} \times 400 \times \left( \frac{W}{L} \right)_1 \times 0.25^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = 8$$

$$I_{D2} = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_2 |V_{OV2}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_2 \times 0.25^2$$

$$\left( \frac{W}{L} \right)_2 = 32$$

7.39 Refer to Fig. P7.39. The gain of the first stage is

$$A_{v1} = -g_{m1}(r_{o1}/2)$$

where  $(r_{o1}/2)$  is the equivalent resistance at the output of  $Q_1$  and includes  $r_{o1}$  in parallel with the output resistance of the current-source load, which is equal to  $r_o$ . Similarly, the gain of the second stage is

$$A_{v2} = -g_{m2}(r_{o2}/2)$$

Now because  $V_{An} = |V_{Ap}| = |V_A|$  and both  $Q_1$  and  $Q_2$  are operating at equal currents  $I$ , we have

$$r_{o1} = r_{o2} = r_o$$

The overall voltage gain  $A_v$  will be

$$A_v = A_{v1} A_{v2}$$

$$A_v = \frac{1}{4} g_{m1} g_{m2} r_o^2$$

If the two transistors are operated at equal overdrive voltages,  $|V_{OV}|$ , both will have equal  $g_m$ ,

$$A_v = \frac{1}{4} (g_m r_o)^2$$

and

$$g_m r_o = \frac{2|V_A|}{|V_{OV}|} = \frac{2 \times 5}{|V_{OV}|} = \frac{10}{|V_{OV}|}$$

$$A_v = 400 = \frac{1}{4} \times \left[ \frac{10}{|V_{OV}|} \right]^2$$

$$\Rightarrow |V_{OV}| = 0.25 \text{ V}$$

7.40

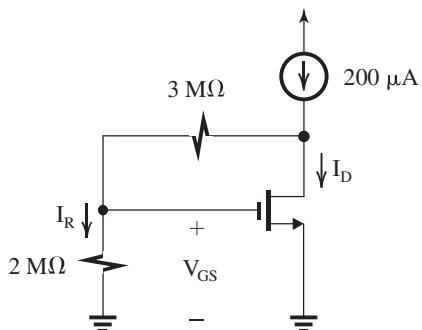


Figure 1

(a) Neglecting the dc current in the feedback network and the Early effect, we see from Fig. 1 that  $I_D = 200 \mu\text{A}$ . Now, using

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

we can determine  $V_{OV}$ :

$$0.2 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.45 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.45 = 0.95 \text{ V}$$

The current in the feedback network can now be found as

$$I_R = \frac{V_{GS}}{2 \text{ M}\Omega} = \frac{0.95}{2} = 0.475 \mu\text{A}$$

which indeed is much smaller than the  $200 \mu\text{A}$  delivered by the current source. Thus, we were justified in neglecting  $I_R$  above.

(b) Replacing the MOSFET with its hybrid- $\pi$  model, we obtain the equivalent circuit shown in Fig. 2.

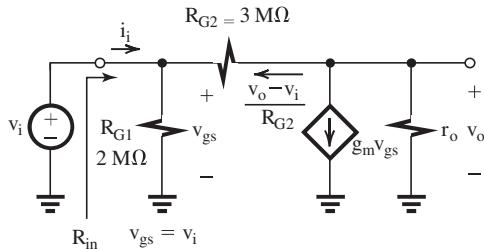


Figure 2

A node equation at the output node yields

$$\frac{v_o}{r_o} + g_m v_{gs} + \frac{v_o - v_i}{R_{G2}} = 0$$

where  $v_{gs} = v_i$ . Thus,

$$v_o \left( \frac{1}{r_o} + \frac{1}{R_{G2}} \right) = -v_i \left( g_m - \frac{1}{R_{G2}} \right)$$

$$\frac{v_o}{v_i} = - \left( g_m - \frac{1}{R_{G2}} \right) (r_o \parallel R_{G2})$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.45} = 0.894 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

$$\frac{v_o}{v_i} = - \left( 0.89 - \frac{1}{3000} \right) \times (100 \parallel 3000)$$

$$= -86.5 \text{ V/V}$$

To obtain the maximum allowable negative signal swing at the output, we first determine the dc voltage at the output by referring to Fig. 1,

$$\begin{aligned} V_{DS} &= V_{GS} \left( 1 + \frac{R_{G2}}{R_{G1}} \right) \\ &= 0.95 \times \left( 1 + \frac{3}{2} \right) = 2.375 \text{ V} \end{aligned}$$

The MOSFET will remain in saturation as long as  $V_{DG} \geq -V_t$ . Thus at the limit  $V_{DG} = -0.5 \text{ V}$ ,

$$V_{Gmax} = 0.5 + v_{Dmin}$$

$$V_{GS} + |\hat{v}_i| = 0.5 + V_{DS} - |\hat{v}_o|$$

$$0.95 + \frac{|\hat{v}_o|}{|A_v|} = 0.5 + 2.375 - |\hat{v}_o|$$

$$|\hat{v}_o| = \frac{0.5 + 2.375 - 0.95}{1 + |A_v|}$$

Substituting  $|A_v| = 86.5$ , we obtain

$$|\hat{v}_o| = 1.9 \text{ V}$$

An approximate value of  $|\hat{v}_o|$  could have been obtained from

$$v_{Omin} = V_{OV} = 0.45 \text{ V}$$

Thus,

$$V_{DS} - |\hat{v}_o| = V_{OV}$$

$$\Rightarrow |\hat{v}_o| = V_{DS} - V_{OV} = 2.375 - 0.45$$

$$= 1.925 \text{ V}$$

$$|\hat{v}_i| = \frac{|\hat{v}_o|}{86.5} = 22 \text{ mV}$$

(c) To determine  $R_{in}$ , refer to Fig. 2,

$$i_i = \frac{v_i}{R_{G1}} - \frac{v_o - v_i}{R_{G2}}$$

$$= \frac{v_i}{R_{G1}} - \frac{A_v v_i - v_i}{R_{G2}}$$

$$= v_i \left[ \frac{1}{R_{G1}} + \frac{(1 - A_v)}{R_{G2}} \right]$$

$$R_{in} = \frac{v_i}{i_i} = 1 \left/ \left[ \frac{1}{R_{G1}} + \frac{(1 - A_v)}{R_{G2}} \right] \right.$$

$$= 1 \left/ \left( \frac{1}{2} + \frac{(1 + 86.5)}{3} \right) \right. = 33.7 \text{ k}\Omega$$

7.41 From the results of Example 7.4, we see that the almost linear region of the transfer characteristic (i.e., region 3) is defined by  $V_{IA} = 0.89 \text{ V}$  and  $V_{IB} = 0.935 \text{ V}$ . Maximum output signal swing is achieved by biasing  $Q_1$  at the middle of this range; thus

$$V_I = 0.913 \text{ V}$$

The peak-to-peak amplitude at the output will be  $(V_{OA} - V_{OB}) = 2.47 - 0.335 = 2.135 \text{ V}$ . Thus the peak amplitude will be  $\frac{1}{2}(2.135) = 1.07 \text{ V}$ .

7.42 Refer to Fig. 7.16(a).

$$R_o = 100 \text{ k}\Omega = r_{o1} \parallel r_{o2}$$

But

$$r_{o1} = r_{o2} = \frac{|V_A|}{I_{REF}} = \frac{5}{I_{REF}}$$

Thus,

$$100 = \frac{1}{2} \times \frac{5}{I_{REF}}$$

$$\Rightarrow I_{REF} = 25 \mu A$$

$$A_v = -g_m R_o$$

$$-40 = -g_m \times 100$$

$$\Rightarrow g_m = 0.4 \text{ mA/V}$$

But

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}$$

$$0.4 = \sqrt{2 \times 0.4 \left(\frac{W}{L}\right)_1 \times 0.025}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 8$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$25 = \frac{1}{2} \times 400 \times 8 \times V_{OV1}^2$$

$$\Rightarrow V_{OV1} = 0.125 \text{ V}$$

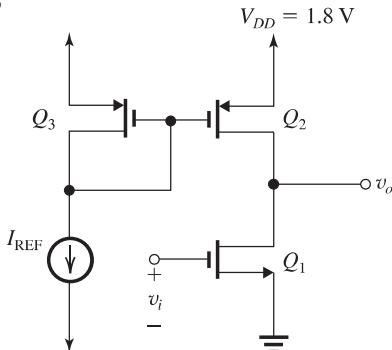
If  $Q_2$  and  $Q_3$  are operated at  $|V_{OV}| = 0.125 \text{ V}$ ,

$$I_{D2} = I_{D3} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_{2,3} |V_{OV}|^2$$

$$25 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{2,3} \times 0.125^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{2,3} = 32$$

7.43



For an output of 1.6 V,

$$V_{SD2min} = |V_{OV2}| = 1.8 - 1.6 = 0.2 \text{ V},$$

For an output of 0.2 V,

$$V_{DS1min} = 0.2 \text{ V},$$

thus

$$V_{OV1} = 0.2 \text{ V}$$

Since  $I_{D2} = I_{D3} = I_{D1} = 50 \mu A$

and  $I_D = \frac{1}{2} (\mu_p C_{ox}) (W/L) V_{OV}^2$ , we have

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \frac{2I_{D2}}{(\mu_p C_{ox})(V_{ov})^2}$$

$$= \frac{2(50 \mu A)}{(86 \mu A/V^2)(0.2 \text{ V})^2} = 29.1$$

For  $Q_1$ ,

$$\left(\frac{W}{L}\right)_1 = \frac{2(50 \mu A)}{(387 \mu A/V^2)(0.2 \text{ V})^2} = 6.46$$

$A_v$  must be at least  $-10 \text{ V/V}$

and  $A_v = -g_m (r_{o1} \parallel r_{o2})$

$$g_m = \frac{2I_D}{V_{OV1}} = \frac{2 \times 0.05}{0.2} = 0.5 \text{ mA/V}$$

$$r_{o1} \parallel r_{o2} = \frac{10}{0.5} = 20 \text{ k}\Omega$$

But

$$r_{o1} = \frac{V_{AI}}{I_{D1}} = \frac{V'_{An} L}{I_{D1}} = \frac{5L}{0.05} = 100L$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{|V'_{Ap}| L}{I_{D2}} = \frac{6L}{0.05} = 120L$$

Thus,

$$100L \parallel 120L = 20 \text{ k}\Omega$$

$$\Rightarrow L = 0.367 \mu \text{m}$$

If  $L$  is to be an integer multiple of  $0.18 \mu \text{m}$ , then

$$L = 0.54 \mu \text{m}$$

To raise the gain to  $20 \text{ V/V}$ ,  $r_{o1} \parallel r_{o2}$  has to be raised to  $40 \text{ k}\Omega$ , which requires

$$L = 2 \times 0.367 = 0.734$$

Again, to use a multiple of  $0.18 \mu \text{m}$  we select  $L = 0.9 \mu \text{m}$ . This represents an increase in  $L$  by a factor of  $\frac{0.90}{0.54} = \frac{5}{3}$ .  $W$ s will have to increase by the same factor. Thus, the area of each transistor will increase by a factor of  $\left(\frac{5}{3}\right)^2$  and the total area will increase as follows:

Initial total area =

$$\text{Area of } Q_1 + \text{Area of } Q_2 + \text{Area of } Q_3$$

$$= 6.46 \times 0.54^2 + 29.1 \times 0.54^2 + 29.1 \times 0.54^2$$

$$= 18.85 \mu \text{m}^2$$

$$\begin{aligned}\text{New total area} &= \\ &= 6.46 \times 0.9^2 + 29.1 \times 0.9^2 + 29.1 \times 0.9^2 \\ &= 52.37 \mu\text{m}^2\end{aligned}$$

Thus, the increase is by a factor of 2.78.

7.44 Refer to Fig. 7.16(a).

Note that  $Q_2, Q_3$  are not matched:

$$I_{D1} = 100 \mu\text{A}$$

$$(a) I_{D2} = I_{D1} = 100 \mu\text{A}$$

$$\frac{I_{D3}}{I_{D2}} = \frac{(W/L)_3}{(W/L)_2} = \frac{W_3}{W_2}$$

(Note that  $V_{SG2} = V_{SG3}$ )

$$\Rightarrow I_{D3} = 100 \mu\text{A} \frac{10}{40} = 25 \mu\text{A} \Rightarrow I_{REF} = 25 \mu\text{A}$$

(b) By referring to Fig. 7.16(d), you notice that in Segment III, both  $Q_1$  and  $Q_2$  are in saturation and the transfer characteristic is quite linear. The output voltage in this segment is limited between  $V_{OA}$  and  $V_{OB}$ : coordinates of point A:

$$V_{OA} = V_{DD} - |V_{OV3}|$$

$$|V_{OV3}|^2 = \frac{I_{D3}}{\frac{1}{2}k'_p \left(\frac{W}{L}\right)_3} = \frac{25}{\frac{1}{2} \times 50 \times \frac{10}{1}}$$

$$\Rightarrow |V_{OV3}| = 0.32 \text{ V}$$

$$V_{OA} = 3.3 - 0.32 = 2.98 \text{ V}$$

$$\text{At point B: } V_{OB} = V_{IB} - V_{tn}$$

Now we find the transfer equation for the linear section: (Refer to Example 7.4)

$$\begin{aligned}i_{D1} = i_{D2} &\Rightarrow (\text{Note that } |V_{OV2}| = |V_{OV3}|) \\ \frac{1}{2}k'_n \left(\frac{W}{L}\right)_1 (v_I - V_{tn})^2 \left(1 + \frac{v_O}{V_{An}}\right) \\ &= \frac{1}{2}k'_p \left(\frac{W}{L}\right)_2 V_{OV3}^2 \left(1 + \frac{V_{DD} - v_O}{|V_{AP}|}\right) \\ \frac{1}{2} \times 100 \times \frac{20}{1} (v_I - 0.8)^2 \left(1 + \frac{v_O}{100}\right) \\ &= \frac{1}{2} \times 50 \times \frac{40}{1} \times 0.32^2 \left(1 + \frac{3.3 - v_O}{50}\right) \\ (v_I - 0.8)^2 &= 0.32^2 \left(1.066 - \frac{v_O}{50}\right) / \left(1 + \frac{v_O}{100}\right) \\ (v_I - 0.8)^2 &= 0.11 \left(\frac{1 - 0.019v_O}{1 + 0.01v_O}\right) \\ &\simeq 0.11(1 - 0.03v_O)\end{aligned}$$

$$(v_I - 0.8)^2 = 0.11(1 - 0.03v_O) \quad (1)$$

Now if we solve for  $V_{OB} = V_{IB} - 0.8$

$$V_{OB}^2 + 0.0033V_{OB} - 0.11 = 0 \Rightarrow V_{OB} = 0.33 \text{ V}$$

Therefore the extreme values of  $v_O$  for which  $Q_1$  and  $Q_2$  are in saturation  $0.33 \text{ V} \leq v_O \leq 2.98 \text{ V}$

(c) From (b) we can find  $V_{IA}$  and  $V_{IB}$ :

$$V_{IB} = V_{OB} + V_t = 0.33 + 0.8 = 1.13 \text{ V}$$

If we solve (1) for  $V_{OA} = 2.98 \text{ V}$ , then

$$(V_{IA} - 0.8)^2 = 0.11(1 - 0.03 \times 2.98) \Rightarrow V_{IA}$$

$$= 1.116 \text{ V}$$

Large-signal voltage gain

$$= \frac{\Delta v_O}{\Delta v_I} = \frac{2.98 - 0.33}{1.13 - 1.116}$$

$$\frac{\Delta v_O}{\Delta v_I} = -189.3 \text{ V/V}$$

$$(d) v_O = \frac{V_{DD}}{2} = \frac{3.3}{2} = 1.65 \text{ V}$$

Differentiating both sides of (1) relative to  $v_I$ :

$$2(v_I - 0.8) = 0.11 \times (-0.03) \frac{\partial v_O}{\partial v_I}$$

$$\Rightarrow \frac{\partial v_O}{\partial v_I} = -606.1(v_I - 0.8)$$

For  $v_O = 1.65 \text{ V}$ , from ① we have

$$(v_I - 0.8)^2 = 0.11(1 - 0.03 \times 1.65) \Rightarrow v_I$$

$$= 1.123 \text{ V}$$

$$\left. \frac{\partial v_O}{\partial v_I} \right|_{v_I = 1.123} = -195.8 \text{ V/V}$$

$$(e) R_{out} = r_{o1} \parallel r_{o2}$$

$$r_{o1} = \frac{V_{An}}{I_{D1}} = \frac{100 \text{ V}}{0.1 \text{ mA}} = 1 \text{ M}\Omega$$

$$r_{o2} = \frac{V_{Ap}}{I_{D2}} = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k}\Omega$$

$$\Rightarrow R_{out} = 500 \text{ k}\Omega \parallel 1 \text{ M}\Omega$$

$$R_{out} = 333 \text{ k}\Omega$$

$$g_{m1} = \sqrt{2k'_n \left(\frac{W}{L}\right)_1 I_{D1}}$$

$$= \sqrt{2 \times 100 \times 10^{-6} \times \frac{20}{1} \times 100 \times 10^{-6}}$$

$$= 0.632 \text{ mA/V}$$

$$A_v = -g_{m1} (r_{o1} \parallel r_{o2}) = -210.6 \text{ V/V}$$

Comment: The three estimates of voltage gain obtained in (c), (d) and (e) are all reasonably close; about  $-200 \text{ V/V}$ .

7.45 (a)

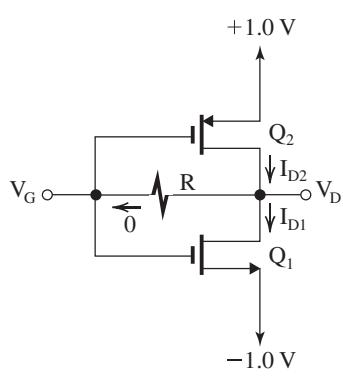


Figure 1

From Fig. 1 we see that since the dc currents into the gates are zero,

$$V_D = V_G$$

Also, since  $Q_1$  and  $Q_2$  are matched and carry equal drain currents,

$$I_{D1} = I_{D2} = I_D$$

$$V_{SG2} = V_{GS1} = 1 \text{ V}$$

and thus,

$$V_G = 0$$

Thus,

$$I_D = \frac{1}{2} \times 1 \times (1 - 0.5)^2 = 0.125 \text{ mA}$$

(b)

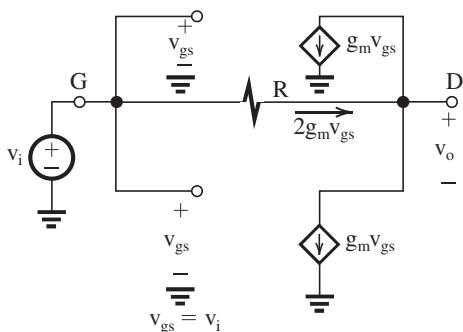


Figure 2

From Fig. 2 we see that

$$v_o = v_i - 2g_m v_{gs} R$$

But

$$v_{gs} = v_i$$

Thus,

$$A_v = \frac{v_o}{v_i} = 1 - 2g_m R$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.125}{1 - 0.5} = 0.5 \text{ mA/V}$$

$$A_v = 1 - 2 \times 0.5 \times 1000 = -999 \text{ V/V}$$

(c)

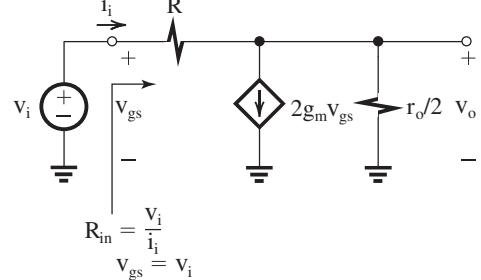


Figure 3

For the circuit in Fig. 3 we can write at the output

$$\frac{v_o}{r_o/2} + 2g_m v_{gs} + \frac{v_o - v_i}{R} = 0$$

Substituting  $v_{gs} = v_i$  and rearranging, we obtain

$$\frac{v_o}{v_i} = -2g_m \frac{\frac{1}{R} + \frac{2}{r_o}}{\frac{1}{r_o} + \frac{2g_m R}{2}}$$

But  $2g_m R \gg 1$ ; thus

$$A_v = \frac{v_o}{v_i} \simeq -2g_m \left( R \parallel \frac{r_o}{2} \right)$$

where

$$r_o = \frac{|V_A|}{I_D} = \frac{20}{0.125} = 160 \text{ k}\Omega$$

$$A_v = -2 \times 0.5 (1000 \parallel 80) = -74.1 \text{ V/V}$$

$$R_{in} = \frac{v_i}{i_i} = \frac{v_i}{(v_i - v_o)/R} = R \frac{1}{1 - \frac{v_o}{v_i}}$$

$$= \frac{R}{1 - A_v} = \frac{1000}{1 + 74.1} = 13.3 \text{ k}\Omega$$

$$(d) \frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{13.3}{20 + 13.3} = 0.4 \text{ V/V}$$

$$G_v = \frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} \times \frac{v_o}{v_i}$$

$$= 0.4 \times -74.1 = -29.6 \text{ V/V}$$

(e) Both  $Q_1$  and  $Q_2$  remain in saturation for output voltages that ensure that the minimum voltage across each transistor is equal to  $|V_{ov}| = 0.5 \text{ V}$ . Thus, the output voltage can range from  $-0.5 \text{ V}$  to  $+0.5 \text{ V}$ .

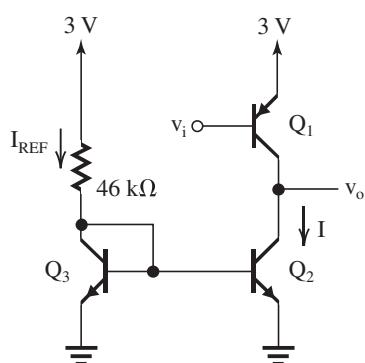
$$7.46 \text{ (a)} I_{\text{REF}} = I_{C3} = \frac{3 - V_{\text{BE}3}}{46 \text{ k}\Omega}$$

$$I_{\text{REF}} = \frac{3 - 0.7}{46}$$

$$= 0.05 \text{ mA}$$

$$\Rightarrow I_{C2} = 5I_{C3}$$

$$I_{C2} = I = 0.25 \text{ mA} \Rightarrow I = 0.25 \text{ mA}$$



$$(b) |V_A| = 50 \text{ V} \Rightarrow r_{o1} = \frac{|V_A|}{I} = \frac{30}{0.25} = 120 \text{ k}\Omega$$

$$r_{o2} = \frac{30}{0.25} = 120 \text{ k}\Omega$$

Total resistance at the collector of Q<sub>1</sub> is equal to  $r_{o1} \parallel r_{o2}$ , thus

$$r_{\text{tot}} = 120 \text{ k}\Omega \parallel 120 \text{ k}\Omega = 60 \text{ k}\Omega$$

$$(c) g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.25}{0.025} = 10 \text{ mA/V}$$

$$r_{\pi1} = \frac{\beta}{g_m} = \frac{50}{10} = 5 \text{ k}\Omega$$

$$(d) R_{\text{in}} = r_{\pi1} = 5 \text{ k}\Omega$$

$$R_o = r_{o1} \parallel r_{o2} = 120 \text{ k}\Omega \parallel 120 \text{ k}\Omega = 60 \text{ k}\Omega$$

$$A_v = -g_{m1}R_o = -10 \times 60 = -600 \text{ V/V}$$

7.47 Refer to Fig. 7.18.

$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o}$$

$$= \frac{20 + 20}{1 + 2 \times 20} = 980 \text{ }\Omega$$

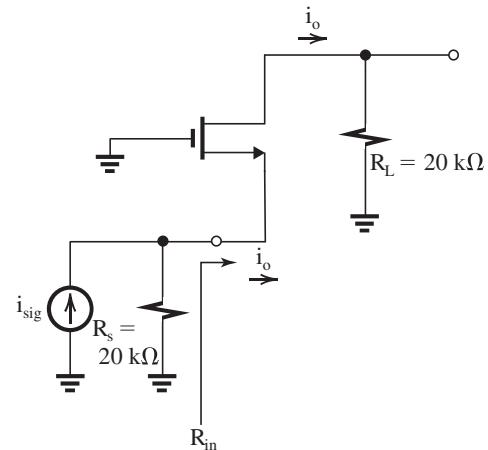
$$R_{\text{out}} = r_o + R_s + g_m r_o R_s$$

$$= 20 + 1 + 2 \times 20 \times 1 = 61 \text{ k}\Omega$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{R_L}{R_s + R_{\text{in}}}$$

$$= \frac{20}{1 + 0.98} = 10.1 \text{ V/V}$$

7.48



$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{20 + 20}{1 + 2 \times 20} = 980 \text{ }\Omega$$

Since  $i_s = i_o$ ,

$$\frac{i_o}{i_{\text{sig}}} = \frac{R_s}{R_s + R_{\text{in}}} = \frac{20}{20 + 0.98} = 0.95 \text{ A/A}$$

If  $R_L$  increases by a factor of 10,  $R_{\text{in}}$  becomes

$$R_{\text{in}} = \frac{20 + 200}{1 + 2 \times 20} = 5.37 \text{ k}\Omega$$

and the current gain becomes

$$\frac{i_o}{i_{\text{sig}}} = \frac{20}{20 + 5.37} = 0.79 \text{ A/A}$$

Thus an increase in  $R_L$  by a factor of 10 resulted in a decrease in the current gain from 0.95 A/A to 0.79 A/A, a change of only -17%. This indicates that the CG amplifier functions as an effective current buffer.

7.49 Refer to Fig. P7.49.

$$I_D = 0.2 \text{ mA} \quad V_{OV} = 0.2 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

$$R_{\text{out}} = r_o + R_s + g_m r_o R_s$$

$$500 = 100 + R_s(1 + 2 \times 100)$$

$$\Rightarrow R_s = \frac{400}{201} \simeq 2 \text{ k}\Omega$$

$$V_{BIAS} = I_D R_S + V_{GS}$$

$$= I_D R_S + V_t + V_{ov}$$

$$= 0.2 \times 2 + 0.5 + 0.2$$

$$= 1.1 \text{ V}$$

7.50 Refer to Fig. P7.50. To obtain maximum output resistance, we use the largest possible  $R_s$  consistent with  $I_D R_s \leq 0.3$  V. Thus

$$R_s = \frac{0.3 \text{ V}}{0.1 \text{ mA}} = 3 \text{ k}\Omega$$

Now, for  $Q_2$  we have

$$g_m = 1 \text{ mA/V} \quad \text{and} \quad V_A = 10 \text{ V}$$

Thus,

$$r_o = \frac{V_A}{I_D} = \frac{10 \text{ V}}{0.1 \text{ mA}} = 100 \text{ k}\Omega$$

$$\begin{aligned} R_{\text{out}} &= r_o + R_s + g_m r_o R_s \\ &= 100 + 3 + 1 \times 100 \times 3 \\ &= 403 \text{ k}\Omega \end{aligned}$$

7.51 Refer to Fig. P7.51.

$$(a) I_{D1} = I_{D2} = I_{D3} = 100 \mu\text{A}$$

Using  $I_{D1} = \frac{1}{2} k'_n (W/L)_1 V_{OV1}^2$ , we obtain

$$0.1 = \frac{1}{2} \times 4 \times V_{OV1}^2$$

$$\Rightarrow V_{OV1} = 0.224 \text{ V}$$

$$V_{GS1} = V_t + V_{OV1} = 0.8 + 0.224 = 1.024 \text{ V}$$

$$V_{BIAS} = V_{GS} + I_{D1} R_s$$

$$= 1.024 + 0.1 \times 0.05 = 1.03 \text{ V}$$

$$(b) g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.224} = 0.9 \text{ mA/V}$$

All transistors are operating at  $I_D = 0.1$  mA and have  $|V_A| = 20$  V. Thus all have equal values for  $r_o$ :

$$r_o = \frac{|V_A|}{I_D} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

(c) For  $Q_2$ ,  $R_L = r_{o2} = 200 \text{ k}\Omega$ ,

$$\begin{aligned} R_{\text{in}} &= \frac{r_o + R_L}{1 + g_m r_o} \\ &= \frac{200 + 200}{1 + 0.9 \times 200} = 2.2 \text{ k}\Omega \end{aligned}$$

$$(d) R_{\text{out}} = r_o + R_s + g_m r_o R_s$$

$$= 200 + 0.05 + 0.9 \times 200 \times 0.05$$

$$= 209 \text{ k}\Omega$$

$$(e) \frac{v_i}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_s} = \frac{2.2}{2.2 + 0.05} = 0.98 \text{ V/V}$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_{\text{in}}} = \frac{200}{2.2} = 90.9 \text{ V}$$

$$\frac{v_o}{v_{\text{sig}}} = 90.9 \times 0.98 = 89 \text{ V/V}$$

(f) The value of  $v_o$  can range from  $V_{BIAS} - V_t = 1.03 - 0.8 = 0.23$  V to  $(V_{DD} - V_{OV2})$ . Since  $I_{D2} = I_{D1}$  and  $k_n = k_p$ , then  $V_{OV2} = V_{OV1}$ . Thus the maximum value of  $v_o$  is  $3.3 - 0.224 = 3.076$  V. Thus the peak-to-peak value of  $v_o$  is  $3.076 - 0.23 = 2.85$  V.

Correspondingly, the peak-to-peak value of  $v_{\text{sig}}$  will be

$$v_{\text{sig}} (\text{peak to peak}) = \frac{2.85}{89} = 32 \text{ mV}$$

7.52 Given Eq. (7.63):

$$R_{\text{in}} \simeq r_e \frac{r_o + R_L}{r_o + \frac{R_L}{\beta + 1}}$$

We can write

$$\frac{R_{\text{in}}}{r_e} = \frac{1 + (R_L/r_o)}{1 + |R_L/(\beta + 1)r_o|} = \frac{1 + (R_L/r_o)}{1 + (R_L/101r_o)}$$

$R_L/r_o$	0	1	10	100	1000	$\infty$
$R_{\text{in}}/r_e$	1	2	10	50.8	91.8	101

Observe that the range of  $R_{\text{in}}$  is  $r_e$  to  $(\beta + 1)r_e$ .

7.53 Equation (7.66):

$$\begin{aligned} R_{\text{out}} &= r_o + (R_e \parallel r_\pi) + (R_e \parallel r_\pi) g_m r_o \\ &= r_o + (r_e \parallel r_\pi)(1 + g_m r_o) \end{aligned}$$

For  $g_m r_o \gg 1$ ,

$$\begin{aligned} R_{\text{out}} &\simeq r_o + g_m r_o (R_e \parallel r_\pi) \\ \frac{R_{\text{out}}}{r_o} &= 1 + \frac{g_m r_\pi R_e}{r_\pi + R_e} \\ &= 1 + \frac{\beta R_e}{(\beta + 1)r_e + R_e} \end{aligned}$$

Thus,

$$\frac{R_{\text{out}}}{r_o} = 1 + \frac{\beta (R_e/r_e)}{\beta + 1 + (R_e/r_e)}$$

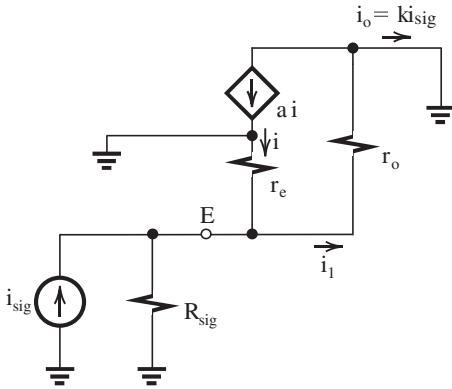
For  $\beta = 100$ ,

$$\frac{R_{\text{out}}}{r_o} = 1 + \frac{100(R_e/r_e)}{101 + (R_e/r_e)}$$

$R_e/r_e$	0	1	2	10	$\beta/2$	$\beta$	1000
$R_{\text{out}}/r_o$	1	2	2.9	10	34	51	92

Observe that  $R_{\text{out}}$  ranges from  $r_o$  to  $(\beta + 1)r_o$ , with the maximum value obtained for  $R_e = \infty$ .

7.54 Refer to Fig. P7.54. To obtain the short-circuit current gain  $k$ , we replace the BJT with its T model and short circuit the collector to



ground, resulting in the circuit shown in the figure.

At the emitter node we see that there are three parallel resistances to ground:  $r_e$ ,  $r_o$ , and  $R_{sig}$ . Thus,

$$i = -i_{sig} \frac{1/r_e}{\frac{1}{r_e} + \frac{1}{r_o} + \frac{1}{R_{sig}}}$$

and

$$i_l = i_{sig} \frac{1/r_o}{\frac{1}{r_e} + \frac{1}{r_o} + \frac{1}{R_{sig}}}$$

At the collector node, we can write

$$i_o \equiv ki_{sig} = i_l - \alpha i$$

Thus,

$$ki_{sig} = i_{sig} \frac{1/r_o + (\alpha/r_e)}{\frac{1}{r_e} + \frac{1}{r_o} + \frac{1}{R_{sig}}} \quad (1)$$

Now  $r_o \gg r_e$  and for the case  $R_{sig} \gg r_e$ , we obtain

$$k \approx \frac{\alpha/r_e}{1/r_e} = \alpha$$

For our case,

$$\alpha = \frac{\beta}{\beta + 1}$$

$$k = \alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

The output resistance  $R_{out}$  is given by

$$R_{out} = r_o + (R_{sig} \parallel r_\pi)(1 + g_m r_o)$$

where

$$r_o = \frac{V_A}{I_C} = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$g_m r_o = 4 \times 500 = 2000$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

Thus,

$$R_{out} = 500 + (10 \parallel 25) \times 2001 \\ = 14.8 \text{ M}\Omega$$

Thus the CB amplifier has a current gain of nearly unity and a very high output resistance: a near-ideal current buffer!

A more exact value of  $k$  can be obtained using Eq. (1);  $k = 0.975$ .

7.55 Refer to Fig. P7.55.

$$I = I_C = \alpha I_E = 0.99 \times \frac{5 - 0.7}{4.3} \approx 1 \text{ mA}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

$$R_{out} = r_o + (R_E \parallel r_\pi)(1 + g_m r_o)$$

where

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$g_m r_o = 40 \times 100 = 4000$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$R_E = 4.3 \text{ k}\Omega$$

Thus,

$$R_{out} = 100 + (4.3 \parallel 2.5) \times 4001 = 6.4 \text{ M}\Omega$$

For

$$\Delta V_C = 10 \text{ V}$$

$$\Delta I = \frac{10 \text{ V}}{6.4 \text{ M}\Omega} = 1.6 \mu\text{A}$$

A very small change indeed!

7.56 Refer to Fig. 7.27.

$$R_{out} = r_o + (R_E \parallel r_\pi)(1 + g_m r_o)$$

$$\simeq r_o + (R_E \parallel r_\pi)(g_m r_o)$$

$$\frac{R_{out}}{r_o} = 1 + g_m (R_E \parallel r_\pi)$$

$$= 1 + \frac{g_m r_\pi R_E}{r_\pi + R_E}$$

$$= 1 + \frac{\beta R_E}{(\beta/g_m) + R_E}$$

For our case  $\beta = 100$ ,

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}, \text{ thus}$$

$$\frac{R_{out}}{r_o} = 1 + \frac{100 R_E}{5 + R_E} \quad (1)$$

where  $R_E$  is in kilohms.

(a) For  $R_{out} = 5 r_o$ , Eq. (1) gives  
 $R_e = 0.208 \text{ k}\Omega = 208 \Omega$

(b) For  $R_{out} = 10 r_o$ , Eq. (1) gives  
 $R_e = 0.495 \text{ k}\Omega \simeq 500 \Omega$ .

(c) For  $R_{out} = 50 r_o$ , Eq. (1) gives  $R_e = 4.8 \text{ k}\Omega$ .  
From Eq. (1) we see that the maximum value of  $R_{out}/r_o$  is obtained with  $R_e = \infty$  and its value is 101, which is  $(\beta + 1)$ .

$$7.57 \quad 50 = g_m r_{o2}$$

$$= A_{o2} = \frac{2V_A}{V_{ov}}$$

$$V_A = 50 \times V_{ov}/2$$

$$= 25 \times 0.2 = 5 \text{ V}$$

$$V_A = V'_A L$$

$$5 = 5 \times L \Rightarrow L = 1 \mu\text{m}$$

7.58 Refer to Fig. 7.33(a).

$$g_{m1} = \frac{2I_D}{V_{ov}} = \frac{2I}{V_{ov}}$$

$$2 = \frac{2I}{0.25}$$

$$\Rightarrow I = 0.25 \text{ mA}$$

For identical transistors,

$$R_o = (g_m r_o) r_o = \frac{2V_A}{V_{ov}} \frac{V_A}{I} = \frac{2V_A^2}{V_{ov} I}$$

$$200 = \frac{2V_A^2}{0.25 \times 0.25}$$

$$\Rightarrow V_A = 2.5 \text{ V}$$

$$V_A = V'_A L$$

$$L = \frac{V_A}{V'_A} = \frac{2.5}{5} = 0.5 \mu\text{m}$$

To obtain W/L, we use

$$I_D = I = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{ov}^2$$

$$250 = \frac{1}{2} \times 400 \times \left( \frac{W}{L} \right) \times 0.25^2$$

$$\Rightarrow \frac{W}{L} = 20$$

To obtain maximum negative signal swing at the output, we select  $V_G$  so that the voltage at the drain of  $Q_1$  is the minimum permitted, which is equal to  $V_{ov}$  (i.e., 0.25 V). Thus

$$V_G = 0.25 + V_{GS2}$$

$$= 0.25 + V_{ov2} + V_t$$

$$= 0.25 + 0.25 + 0.5 = 1.0 \text{ V}$$

The minimum permitted output voltage is  
 $V_G - V_t = 1 - 0.5 = 0.5 \text{ V}$  or  $2V_{ov}$ .

7.59 Refer to Fig. 7.32

$$R_o = g_m r_{o3} r_{o4}$$

For identical transistors,

$$R_o = (g_m r_o) r_o = \frac{2|V_A|}{|V_{ov}|} \times \frac{|V_A|}{I}$$

Thus,

$$IR_o = \frac{2|V_A|^2}{V_{ov}} \quad \text{Q.E.D.}$$

$$(a) I = 0.1 \text{ mA}$$

$$0.1 \times R_o = \frac{2 \times 4^2}{0.2} = 160$$

$$R_o = 1.6 \text{ M}\Omega$$

To obtain the W/L values,

$$I = I_D = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_{3,4} |V_{ov}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_{3,4} \times 0.2^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_{3,4} = 50$$

$$(b) I = 0.5 \text{ mA}$$

$$0.5R_o = \frac{2 \times 4^2}{0.2} = 160$$

$$R_o = 320 \text{ k}\Omega$$

$$I = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_{3,4} |V_{ov}|^2$$

$$500 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_{3,4} |V_{ov}|^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_{3,4} = 250$$

7.60 Refer to Fig. 7.32.

$$R_o = (g_m r_{o3}) r_{o4}$$

For identical transistors,

$$R_o = (g_m r_o) r_o = \frac{2|V_A|}{|V_{ov}|} \times \frac{|V_A|}{I}$$

Thus,

$$IR_o = \frac{2|V_A|^2}{|V_{ov}|}$$

This table belongs to Problem 7.60.

	L = L <sub>min</sub> = 0.18 μm IR <sub>o</sub> = 8.1 V				L = 2L <sub>min</sub> = 0.36 μm IR <sub>o</sub> = 32.4 V				L = 3L <sub>min</sub> = 0.54 μm IR <sub>o</sub> = 72.9 V			
	g <sub>m</sub> (mA/V)	R <sub>o</sub> (kΩ)	A <sub>v</sub> (V/V)	2WL (μm <sup>2</sup> )	g <sub>m</sub> (mA/V)	R <sub>o</sub> (kΩ)	A <sub>v</sub> (V/V)	2WL (μm <sup>2</sup> )	g <sub>m</sub> (mA/V)	R <sub>o</sub> (kΩ)	A <sub>v</sub> (V/V)	2WL (μm <sup>2</sup> )
I = 0.01 mA W/L = n	0.1	810	-40.5	0.065 n	0.1	3,240	-162	0.26n	0.1	7,290	-364.5	0.58n
I = 0.1 mA W/L = 10 n	1.0	81	-40.5	0.65n	1.0	324	-162	2.6n	1.0	729	-364.5	5.8n
I = 1.0 mA W/L = 100 n	10.0	8.1	-40.5	6.5n	10.0	32.4	-162	26n	10.0	72.9	-364.5	58n

Substituting

$$|V_A| = |V'_A| L$$

$$IR_o = \frac{2|V'_A|^2}{|V_{ov}|} L^2 \quad \text{Q.E.D.}$$

Now, for

$$L = 0.18 \mu\text{m}, \quad IR_o = \frac{2 \times 5^2}{0.2} \times 0.18^2 = 8.1 \text{ V} \quad 7.62$$

$$L = 0.36 \mu\text{m}, \quad IR_o = \frac{2 \times 5^2}{0.2} \times 0.36^2 = 32.4 \text{ V}$$

$$L = 0.54 \mu\text{m}, \quad IR_o = \frac{2 \times 5^2}{0.2} \times 0.54^2 = 72.9 \text{ V}$$

To fill out the table above, we use

$$g_m = \frac{2I_D}{|V_{ov}|} = \frac{2I}{|V_{ov}|} = \frac{2I}{0.2} = 10I$$

$$A_v = g_m(R_o/2)$$

(a) The price paid is the increase in circuit area.

(b) As I is increased, g<sub>m</sub> increases and hence the current-driving capability of the amplifier, and as we will see later, its bandwidth.

(c) The circuit with the largest area (58n) as compared to the circuit with the smallest area (0.065n): A<sub>v</sub> is 364.5/40.5 = 9 times larger; g<sub>m</sub> is 100 times larger, but R<sub>o</sub> is 11.1 times lower.

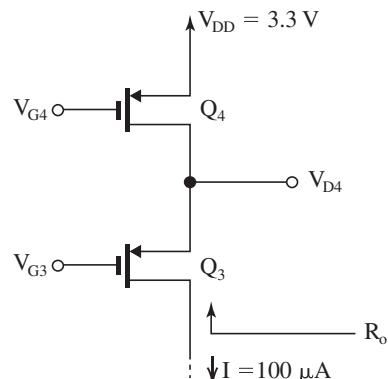
7.61 Refer to Fig. 7.33.

$$g_{ml} = \frac{2I_{D1}}{V_{OV1}} = \frac{2I}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

Since all transistors are operating at the same I<sub>D</sub> and |V<sub>ov</sub>|, all have equal values of g<sub>m</sub>. Also because all have equal |V<sub>A</sub>| = 4 V, all r<sub>o</sub>'s will be equal:

$$r_o = \frac{|V_A|}{I_D} = \frac{|V_A|}{I} = \frac{4}{0.2} = 20 \text{ k}\Omega$$

$$R_{on} = (g_m r_o) r_o = (2 \times 20) \times 20 = 800 \text{ k}\Omega$$



$$V_{SG4} = |V_{tp}| + |V_{ov}|$$

$$= 0.8 + 0.2 = 1 \text{ V}$$

Thus,

$$V_{G4} = V_{DD} - V_{SG4} = 3.3 - 1 = 2.3 \text{ V}$$

To obtain the largest possible signal swing at the output, we maximize the allowable positive signal swing by setting V<sub>D4</sub> at its highest possible value of V<sub>DD</sub> - |V<sub>ov</sub>| = 3.3 - 0.2 = 3.1 V. This will be obtained by selecting V<sub>GS</sub> as follows:

$$V_{G3} = V_{D4} - V_{SG3}$$

Since

$$V_{SG3} = V_{SG4} = 1 \text{ V}$$

$$V_{G3} = 3.1 - 1 = 2.1 \text{ V}$$

the highest allowable voltage at the output will be

$$V_{D3\max} = V_{G3} + |V_{tp}|$$

$$= 2.1 + 0.8 = 2.9 \text{ V}$$

Since both  $Q_3$  and  $Q_4$  carry the same current  $I = 100 \mu\text{A}$  and are operated at the same overdrive voltage,  $|V_{ov}| = 0.2 \text{ V}$ , their W/L ratios will be the same and can be found from

$$I_D = \frac{1}{2}\mu_p C_{ox} \left(\frac{W}{L}\right)_{3,4} |V_{ov}|^2$$

$$100 = \frac{1}{2} \times 60 \times \left(\frac{W}{L}\right)_{3,4} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 83.3$$

To obtain  $R_o$ , we first find  $g_m$  and  $r_o$  of both devices,

$$g_{m3,4} = \frac{2I_D}{|V_{ov}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o3,4} = \frac{|V_A|}{I_D} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$R_o = (g_{m3} r_{o3}) r_{o4} \\ = 1 \times 50 \times 50 = 2.5 \text{ M}\Omega$$

7.63 Refer to Fig. 7.33.

$$A_v = -g_m R_o$$

$$-280 = -1 \times R_o \Rightarrow R_o = 280 \text{ k}\Omega$$

$$g_{m1} = \frac{2I_D}{V_{ov}} = \frac{2I}{V_{ov}} \Rightarrow I = \frac{1}{2} g_{m1} V_{ov} \\ = \frac{1}{2} \times 1 \times 0.25 = 0.125 \text{ mA}$$

All four transistors are operated at the same value of  $I_D$  and the same value of  $|V_{ov}|$ . Also all have the same channel length and  $|V'_A|$ ; thus all  $r_o$  values are equal. Thus

$$R_{on} = R_{op} = 2R_o = 2 \times 280 = 560 \text{ k}\Omega$$

$$560 = (g_m r_o) r_o$$

$$= \frac{2|V_A|}{|V_{ov}|} \frac{|V_A|}{I} \\ = \frac{2|V_A|^2}{0.25 \times 0.125} \\ \Rightarrow V_A = 2.96 \text{ V}$$

$$L = \frac{V_A}{V'_A} = \frac{2.96}{5} = 0.6 \mu\text{m}$$

For each of the NMOS devices,

$$I_D = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2} V_{ov}^2$$

$$125 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{1,2} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{1,2} = 10$$

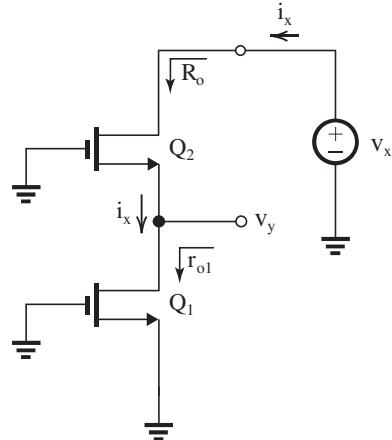
For each of the PMOS transistors,

$$I_D = \frac{1}{2}\mu_p C_{ox} \left(\frac{W}{L}\right)_{3,4} |V_{ov}|^2$$

$$125 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 40$$

7.64



While  $v_x$  appears across  $R_o$ ,  $v_y$  appears across  $r_{o1}$ , Thus,

$$\frac{v_y}{v_x} = \frac{r_{o1}}{R_o} \\ = \frac{r_{o1}}{r_{o1} + r_{o2} + g_{m2}r_{o2}r_{o1}}$$

For  $g_{m2}r_{o2} \gg 1$  and  $g_{m2}r_{o1} \gg 1$ ,

$$\frac{v_y}{v_x} \approx \frac{1}{g_{m2}r_{o2}}$$

7.65 Refer to Fig. P7.65.

(a) For the circuit in (a),

$$I = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right) V_{ova}^2 \quad (1)$$

For the circuit in (b),

$$I = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{4L}\right) V_{ovb}^2 \quad (2)$$

Comparing Eqs. (1) and (2) we see that

$$V_{ovb} = 2V_{ova} \quad \text{Q.E.D.}$$

Now,

$$g_m = \frac{2I_D}{V_{ov}}$$

Thus for the circuit in (a),

$$g_{ma} = \frac{2I}{V_{OVA}}$$

and for the circuit in (b),

$$g_{mb} = \frac{2I}{V_{OVB}} = \frac{2I}{2V_{OVA}} = \frac{I}{V_{OVA}}$$

Thus,

$$g_{mb} = \frac{1}{2} g_{ma} \quad \text{Q.E.D.}$$

Since the channel length in (b) is four times that in (a),

$$V_{Ab} = 4V_{Aa}$$

and

$$r_{ob} = 4r_{oa}$$

Thus

$$A_{va} = -g_{ma} r_{oa}$$

and

$$A_{vb} = -g_{mb} r_{ob}$$

$$= -\frac{1}{2} g_{ma} \times 4 r_{oa}$$

$$= 2A_{va} \quad \text{Q.E.D.}$$

(b) For the cascode circuit in (c) to have the same minimum voltage requirement at the drain as that for circuit (b), which is equal to  $V_{OVB} = 2V_{OVA}$ , we must operate each of the two transistors in the cascode amplifier at  $V_{OV} = V_{OVA}$ . Thus each of the two transistors in the cascode circuit will have  $g_m = g_{ma}$ . Also, each will have  $r_o = r_{oa}$ . Thus

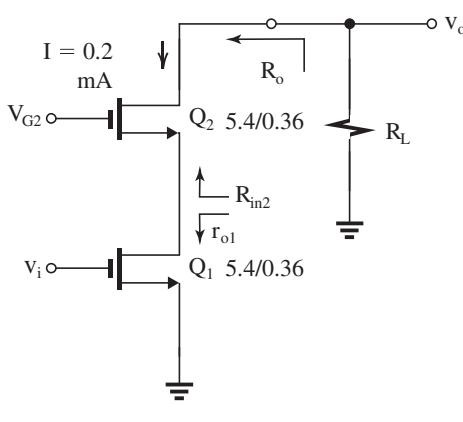
$$A_{vc} = -g_m R_o$$

$$\simeq -g_{ml}(g_m r_o) r_o$$

$$= -A_{va}^2$$

Obviously, the cascode delivers a much greater gain than that achieved by quadrupling the channel length of the CS amplifier.

7.66



$$g_{ml} = g_{m2} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D}$$

$$= \sqrt{2 \times 0.4 \times \frac{5.4}{0.36} \times 0.2} = 1.55 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I_D} = \frac{V'_A L}{I_D} = \frac{5 \times 0.36}{0.2} = 9 \text{ k}\Omega$$

$$R_o = r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1} = 9 + 9 + 1.55 \times 9 \times 9$$

$$= 143.6 \text{ k}\Omega$$

$$A_v = -g_{ml}(R_o \parallel R_L)$$

$$-100 = -1.55(R_o \parallel R_L)$$

$$\Rightarrow R_o \parallel R_L = 64.5 \text{ k}\Omega$$

$$\frac{1}{R_o} + \frac{1}{R_L} = \frac{1}{64.5}$$

$$\frac{1}{R_L} = \frac{1}{64.5} - \frac{1}{143.6} = \frac{1}{117}$$

$$\Rightarrow R_L = 117 \text{ k}\Omega$$

$$R_{in2} = \frac{r_{o2} + R_L}{1 + g_{m2} r_{o2}}$$

$$= \frac{9 + 143.6}{1 + 1.55 \times 9} = 10.2 \text{ k}\Omega$$

$$R_{d1} = r_{o1} \parallel R_{in2} = 9 \parallel 10.2 = 4.8 \text{ k}\Omega$$

$$A_I = -g_{ml} R_{d1} = -1.55 \times 4.8 = -7.41 \text{ V/V}$$

7.67 Refer to Fig. P7.67.

$$(a) R_1 = r_{o1} = r_o$$

$$R_2 \simeq (g_m r_o) r_o$$

$$R_3 = \frac{R_2 + r_o}{g_m r_o} = \frac{g_m r_o^2 + r_o}{g_m r_o} \simeq r_o$$

$$(b) i_1 = g_m v_i$$

$$i_2 = i_1 \frac{R_3}{R_3 + r_o} = g_m v_i \frac{r_o}{r_o + r_o} = \frac{1}{2} g_m v_i$$

$$i_3 = i_1 - i_2 = \frac{1}{2} g_m v_i$$

$$i_4 = i_3 = \frac{1}{2} g_m v_i$$

$$i_5 = i_4 = \frac{1}{2} g_m v_i$$

$$i_6 = 0 \text{ (because } v_{sg4} = 0)$$

$$i_7 = i_5 = \frac{1}{2} g_m v_i$$

$$(c) v_1 = -i_2 r_o = -\frac{1}{2} (g_m r_o) v_i$$

$$v_2 = -i_4 R_2 = -\frac{1}{2} g_m (g_m r_o) r_o v_i$$

$$= -\frac{1}{2} (g_m r_o)^2 v_i$$

$$v_3 = -i_5 R_1 = -\frac{1}{2} g_m v_i r_o = -\frac{1}{2} (g_m r_o) v_i$$

(d)  $v_i$  is a 5-mV peak sine wave (see Fig. 1).

$$\hat{v}_1 = -\frac{1}{2} \times 20 \times v_i = -10 \times 5 = -50 \text{ mV}$$

Thus,  $v_1$  is a 50-mV peak sine wave that is  $180^\circ$  out of phase with  $v_i$ .

$$\hat{v}_2 = -\frac{1}{2} \times 20^2 \times 5 = -1 \text{ V}$$

Thus,  $v_2$  is a 1-V peak sine wave,  $180^\circ$  out of phase relative to  $v_i$ .

$$\hat{v}_3 = -\frac{1}{2} \times 20 \times 5 = -50 \text{ mV}$$

Thus,  $v_3$  is a 50-mV peak sine wave,  $180^\circ$  out of phase relative to  $v_i$ .

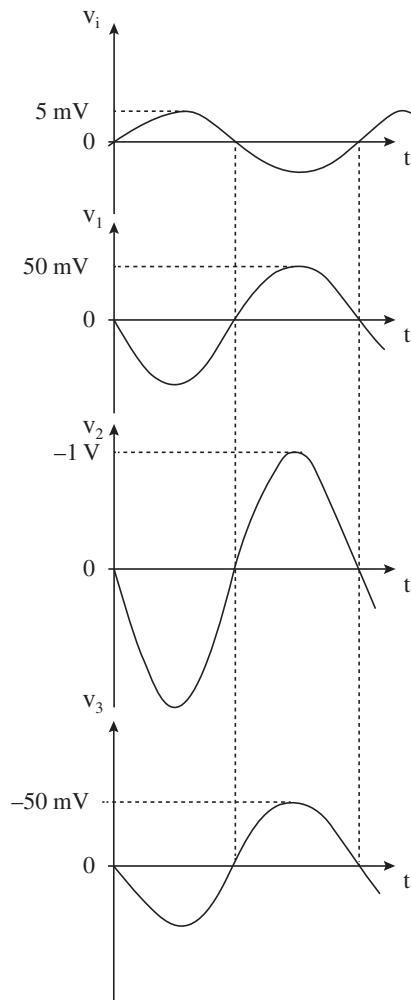
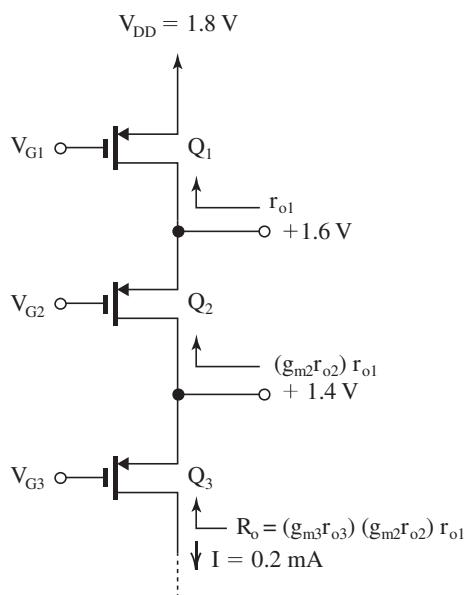


Figure 1

7.68



We design for a minimum voltage of  $|V_{ov}|$  across each of  $Q_1$  and  $Q_2$ .

$$V_{G1} = V_{DD} - V_{SG1} = V_{DD} - |V_{tp}| - |V_{ov}| \\ = 1.8 - 0.4 - 0.2 = 1.2 \text{ V}$$

$$V_{G2} = V_{S2} - V_{SG2}$$

$$= 1.6 - 0.4 - 0.2 = 1.0 \text{ V}$$

$$V_{G3} = V_{S3} - V_{SG3}$$

$$= 1.4 - 0.4 - 0.2 = 0.8 \text{ V}$$

All transistors carry the same  $I_D = 0.2 \text{ mA}$  and operate at the same value of  $|V_{ov}| = 0.2 \text{ V}$ . Thus, their W/L ratios will be equal,

$$0.2 = \frac{1}{2} \times 0.1 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 100$$

$$R_o = (g_m r_o)^2 r_o$$

where

$$g_m = \frac{2I_D}{|V_{ov}|} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_o = \frac{|V_A'| L}{I_D} = \frac{6 \times 0.4}{0.2} = 12 \text{ k}\Omega$$

$$R_o = (2 \times 12)^2 \times 12 = 6.91 \text{ M}\Omega$$

7.69 Refer to Fig. P7.69.

(a)  $R_{o1} = r_o$

$R_{o2} = r_o$

$$\begin{aligned}
 R_{o5} &= r_o \\
 R_{o4} &= (g_m r_o) r_o \\
 R_{o3} &= r_{o3} + (g_{m3} r_{o3})(R_{o1} \parallel R_{o2}) \\
 &= r_o + g_m r_o \times \frac{1}{2} r_o \\
 &\simeq r_o \left(1 + \frac{1}{2} g_m r_o\right) \simeq \frac{1}{2} (g_m r_o) r_o \\
 R_{in3} &= \frac{r_{o3} + R_{o4}}{1 + g_{m3} r_{o3}} \simeq \frac{r_o + g_m r_o r_o}{g_m r_o} \\
 &= \frac{1}{g_m} + r_o \simeq r_o
 \end{aligned}$$

(b)  $R_o = R_{o3} \parallel R_{o4}$

$$\begin{aligned}
 &= \frac{1}{2} (g_m r_o) r_o \parallel (g_m r_o) r_o \\
 &= \frac{1}{3} (g_m r_o) r_o
 \end{aligned}$$

(c) When  $v_o$  is short-circuited to ground,  $R_{in2}$  becomes equal to  $1/g_{m3}$ . This resistance will be much smaller than the two other resistances between the drain of  $Q_1$  and ground, namely,  $R_{o1} = r_o$  and  $R_{o2} = r_o$ . Thus the signal current in the drain of  $Q_1$ ,  $g_{m1} v_i$  will mostly flow into  $1/g_{m3}$ , that is, into the source of  $Q_3$  and out of the drain of  $Q_3$  to ground. Thus, the output short-circuit current will be equal to  $g_{m1} v_i$ ; thus the short-circuit transconductance  $G_m$  will be

$$G_m = g_{m1} \quad \text{Q.E.D.}$$

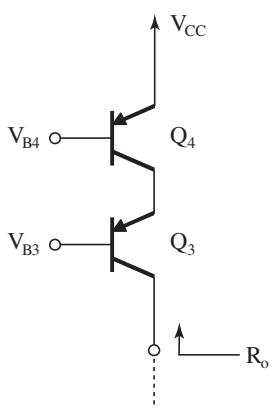
$$\begin{aligned}
 (\text{d}) \quad \frac{v_o}{v_i} &= -g_{m1} R_o \\
 &= -g_m \times \frac{1}{3} (g_m r_o) r_o \\
 &= -\frac{1}{3} (g_m r_o)^2
 \end{aligned}$$

For

$$g_m = 2 \text{ mA/V} \quad \text{and} \quad A_0 = 30$$

$$\frac{v_o}{v_i} = -\frac{1}{3} (30)^2 = -300 \text{ V/V}$$

7.70



$$\begin{aligned}
 R_o &= (g_{m3} r_{o3})(r_{o4} \parallel r_{\pi3}) \\
 I &= 0.2 \text{ mA} \\
 g_{m3} &= \frac{I}{V_T} = \frac{0.2}{0.025} = 8 \text{ mA/V} \\
 r_{o3} &= r_{o4} = \frac{V_A}{I} = \frac{5}{0.2} = 25 \text{ k}\Omega \\
 r_{\pi3} &= \frac{\beta}{g_{m3}} = \frac{50}{8} = 6.25 \text{ k}\Omega \\
 R_o &= (8 \times 25)(25 \parallel 6.25) \\
 &= 1 \text{ M}\Omega
 \end{aligned}$$

7.71 When Eq. (7.88) is applied to the case of identical pnp transistors, it becomes

$$R_o = (g_m r_o)(r_o \parallel r_{\pi})$$

Now,

$$g_m = \frac{I}{V_T} \quad r_o = \frac{|V_A|}{I}$$

$$g_m r_o = |V_A| / V_T$$

$$r_{\pi} = \frac{\beta}{g_m}$$

Thus,

$$\begin{aligned}
 IR_o &= \frac{|V_A|}{V_T} \frac{I r_o r_{\pi}}{r_o + r_{\pi}} \\
 &= \frac{|V_A|}{V_T} \frac{|V_A| r_{\pi}}{r_o + r_{\pi}} \\
 &= \frac{|V_A|}{V_T} \frac{|V_A|}{1 + \frac{r_o}{r_{\pi}}} \\
 &= \frac{|V_A|}{V_T} \frac{|V_A|}{1 + \frac{1}{\beta} g_m r_o} \\
 &= \frac{|V_A|}{V_T} \frac{1}{\frac{1}{|V_A|} + \frac{1}{\beta} \frac{1}{V_T}} \\
 &= \frac{|V_A|}{(V_T / |V_A|) + (1/\beta)} \quad \text{Q.E.D.}
 \end{aligned}$$

For  $|V_A| = 5 \text{ V}$  and  $\beta = 50$  we obtain

$$IR_o = \frac{5}{(0.025/5) + (1/50)} = 200 \text{ V}$$

I (mA)	0.1	0.5	1
R <sub>o</sub> (kΩ)	2000	400	200

7.72 The output resistance of the cascode amplifier (excluding the load) is

$$R_o = g_m r_o (r_o \parallel r_{\pi})$$

Thus,

$$\begin{aligned}
 A_v &= -g_m (R_o \parallel R_L) \\
 &= -g_m (R_o \parallel \beta r_o)
 \end{aligned}$$

For  $|V_A| = 100$  V,  $\beta = 50$ , and  $I = 0.2$  mA we obtain

$$g_m = \frac{I}{V_T} = \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{50}{8} = 6.25 \text{ k}\Omega$$

$$r_o = \frac{|V_A|}{I} = \frac{100}{0.2} = 500 \text{ k}\Omega$$

$$R_o = 8 \times 500 \times (500 \parallel 6.25)$$

$$= 24,691 \text{ k}\Omega$$

$$A_v = -8(24.7 \parallel 25) \times 10^3$$

$$= -99.4 \times 10^3 \simeq -10^5 \text{ V/V}$$

7.73 Refer to Fig. 7.38. When all transistors have equal  $\beta$  and  $r_o$ , and, since they conduct equal currents, they have equal  $g_m$ , then

$$R_{on} = R_{op} = g_m r_o (r_o \parallel r_\pi)$$

$$R_o = R_{on} \parallel R_{op} = \frac{1}{2} (g_m r_o) (r_o \parallel r_\pi)$$

$$A_v = -g_m R_o$$

$$= -\frac{1}{2} (g_m r_o) g_m (r_o \parallel r_\pi)$$

$$= -\frac{1}{2} (g_m r_o) \frac{g_m r_o r_\pi}{r_\pi + r_o}$$

$$= -\frac{1}{2} (g_m r_o) \frac{1}{\frac{1}{g_m r_o} + \frac{1}{g_m r_\pi}}$$

$$\text{Substituting } g_m r_o = \frac{|V_A|}{V_T} \text{ and } g_m r_\pi = \beta,$$

$$A_v = -\frac{1}{2} \frac{|V_A|/V_T}{(V_T/|V_A|) + (1/\beta)}$$

For  $|V_A| = 5$  V and  $\beta = 50$  we obtain

$$A_v = -\frac{1}{2} \frac{5/0.025}{(0.025/5) + (1/50)} = -4000 \text{ V/V}$$

7.74 (a) Refer to circuit in Fig. P7.74(a).

$$g_{m1} = \frac{I}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$$g_{m2} = g_{m1} = 4 \text{ mA/V}$$

$$r_{\pi1} = r_{\pi2} = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$R_{in} = r_{\pi1} = 25 \text{ k}\Omega$$

$$R_o = g_{m2} r_{o2} (r_{o1} \parallel r_{\pi2})$$

$$= (4 \times 50)(50 \parallel 25) = 3.33 \text{ M}\Omega$$

$$A_{vo} = -g_m R_o$$

$$= -4 \times 3.33 \times 10^3 = -13,320 \text{ V/V}$$

(b) Refer to the circuit in Fig. P7.74(b).

$$g_{m1} = \frac{I}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$$g_{m2} = \frac{2I_{D2}}{V_{OV}} = \frac{2I}{V_{OV}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{\pi1} = \frac{\beta}{g_{m1}} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_{o1} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$R_{in} = r_{\pi1} = 25 \text{ k}\Omega$$

$$R_o = g_{m2} r_{o2} r_{o1}$$

$$= 1 \times 50 \times 50 = 2.5 \text{ M}\Omega$$

$$A_{vo} = -g_{m1} R_o$$

$$= -4 \times 2.5 \times 10^3 = -10,000 \text{ V/V}$$

(c) Refer to the circuit in Fig. P7.74(c).

$$g_{m1} = g_{m2} = \frac{2I_D}{|V_{ov}|} = \frac{2I}{|V_{ov}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{|V_A|}{I_D} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$R_{in} = \infty$$

$$R_o = g_{m2} r_{o2} r_{o1}$$

$$= 1 \times 50 \times 50 = 2.5 \text{ M}\Omega$$

$$A_{vo} = -g_{m1} R_o = -1 \times 2.5 \times 10^3 = -2500 \text{ V/V}$$

(d) Refer to the circuit in Fig. P7.74(d).

$$g_{m1} = \frac{2I_D}{|V_{ov}|} = \frac{2I}{|V_{ov}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$g_{m2} = \frac{I}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$$r_{o1} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_A|}{I} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$r_{\pi2} = \frac{\beta}{g_{m2}} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$R_{in} = \infty$$

$$R_o = (g_{m2} r_{o2})(r_{o1} \parallel r_{\pi2})$$

$$= 4 \times 50(50 \parallel 25)$$

$$= 3.33 \text{ M}\Omega$$

$$A_{vo} = -g_{m1} R_o$$

$$= -1 \times 3.33 \times 10^6 = -3330 \text{ V/V}$$

Comment: The highest voltage gain (13,320 V/V) is obtained in circuit (a). However, the input resistance is only 25 kΩ. Of the two circuits with infinite input resistance (c and d), the circuit in (d) has the higher voltage gain. Observe that combining MOSFETs with BJTs results in

circuits superior to those with exclusively MOSFETs or BJTs.

7.75 (a) Refer to the circuit in Fig. P7.75(a).

$$\begin{aligned} g_{m1} &= \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D} \\ &= \sqrt{2 \times 0.4 \times 25 \times 0.1} \\ &= 1.41 \text{ mA/V} \\ r_{o1} &= \frac{V_A}{I_D} = \frac{1.8}{0.1} = 18 \text{ k}\Omega \\ g_{m2} &= \frac{I}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V} \\ r_{o2} &= \frac{V_A}{I} = \frac{1.8}{0.1} = 18 \text{ k}\Omega \\ r_{\pi2} &= \frac{\beta}{g_{m2}} = \frac{125}{4} = 31.25 \text{ k}\Omega \end{aligned}$$

$$G_m = g_{m1} = 1.41 \text{ mA/V}$$

$$R_o = g_{m2} r_{o2} (r_{o1} \parallel r_{\pi2})$$

$$= 4 \times 18 \times (18 \parallel 31.25) = 822.3 \text{ k}\Omega$$

$$A_{vo} = -G_m R_o = -1.41 \times 822.3 = -1159 \text{ V/V}$$

(b) Refer to circuit in Fig. P7.75(b).

$$g_{m1} = g_{m2} = \sqrt{2 \times 0.4 \times 25 \times 0.1}$$

$$= 1.41 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I} = \frac{1.8}{0.1} = 18 \text{ k}\Omega$$

$$G_m = g_{m1} = 1.41 \text{ mA/V}$$

$$R_o = g_{m2} r_{o2} r_{o1}$$

$$= 1.41 \times 18 \times 18 = 457 \text{ k}\Omega$$

$$A_{vo} = -G_m R_o = -1.41 \times 457 = -644 \text{ V/V}$$

We observe that the circuit with a cascode transistor provides higher gain.

7.76 Refer to Fig. 7.39.

$$\begin{aligned} I_O &= I_{REF} \frac{(W/L)_2}{(W/L)_1} \\ &= 20 \frac{40/1}{4/1} = 200 \mu\text{A} \end{aligned}$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$20 = \frac{1}{2} \times 160 \times \frac{4}{1} \times V_{OV1}^2$$

$$\Rightarrow V_{OV1} = 0.25 \text{ V}$$

$$V_{G2} = V_{GS1} = V_t + V_{OV1} = 0.6 + 0.25 = 0.85 \text{ V}$$

$$V_{OV4} = V_{OV1}$$

Thus,

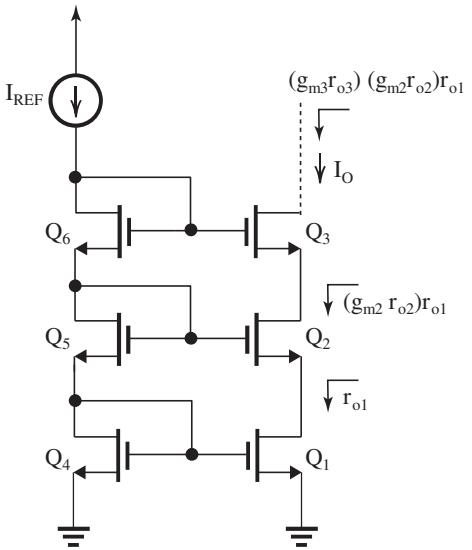
$$V_{GS4} = V_{GS1} = 0.85 \text{ V}$$

$$V_{G3} = 0.85 + 0.85 = 1.7 \text{ V}$$

The lowest voltage at the output while  $Q_3$  remains in saturation is

$$\begin{aligned} V_{Omin} &= V_{G3} - V_{t3} \\ &= 1.7 - 0.6 = 1.1 \text{ V} \\ g_{m2} &= g_{m3} = \frac{2I_{D2,3}}{V_{OV2,3}} = \frac{2 \times 0.2}{0.25} = 1.6 \text{ mA/V} \\ r_{o2} &= r_{o3} = \frac{V_A}{I_D} = \frac{10}{0.2} = 50 \text{ k}\Omega \\ R_o &= g_{m3} r_{o3} r_{o2} \\ &= 1.6 \times 50 \times 50 = 4 \text{ M}\Omega \end{aligned}$$

7.77



From the figure we see that

$$R_o = (g_{m3} r_{o3})(g_{m2} r_{o2}) r_{o1}$$

7.78 Refer to Eq. (7.95),

$$R_o = \beta_3 r_{o3} / 2$$

where

$$r_{o3} = \frac{V_A}{I} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

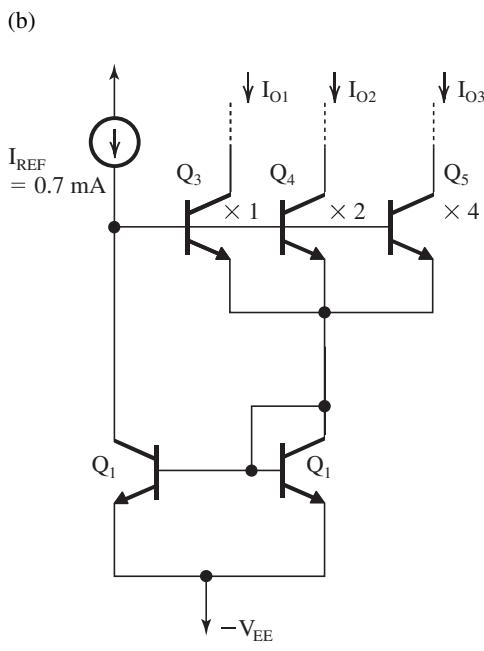
Thus,

$$R_o = \frac{100 \times 100}{2} = 5 \text{ M}\Omega$$

$$\Delta I_O = \frac{\Delta V_O}{R_o} = \frac{10 \text{ V}}{5 \text{ M}\Omega} = 2 \mu\text{A}$$

$$\frac{\Delta I_O}{I_O} = \frac{2 \mu\text{A}}{1 \text{ mA}} = 0.002 \quad \text{or } 0.2\%$$

$$7.79 \text{ (a)} \quad I_{O1} = I_{O2} = \frac{1}{2} \frac{I_{REF}}{1 + \frac{2}{\beta^2}}$$



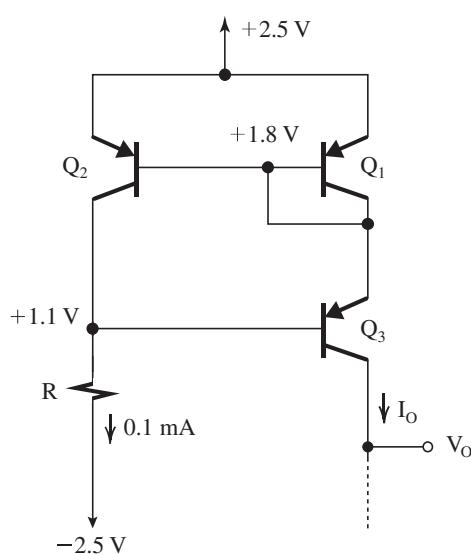
The figure shows the required circuit. Observe that the output transistor is split into three transistors having base-emitter junctions with area ratio 1:2:4. Thus

$$I_{O1} = \frac{0.1}{1 + \frac{2}{\beta^2}} = \frac{0.1}{1 + \frac{2}{50^2}} = 0.0999 \text{ mA}$$

$$I_{O2} = \frac{0.2}{1 + \frac{2}{50^2}} = 0.1998 \text{ mA}$$

$$I_{O4} = \frac{0.4}{1 + \frac{2}{50^2}} = 0.3997 \text{ mA}$$

7.80



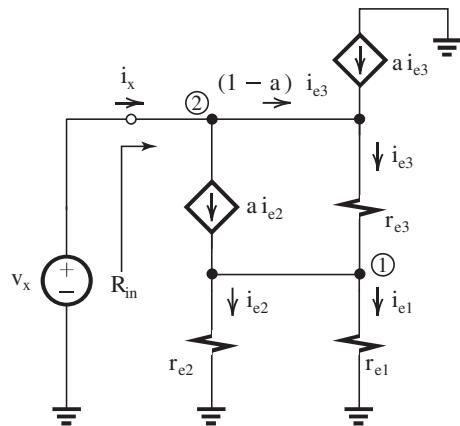
$$R = \frac{1.1 + 2.5}{0.1} = 36 \text{ k}\Omega$$

$V_{Omax}$  is limited by  $Q_3$  saturating. Thus

$$V_{Omax} = V_{E3} - V_{ECsat}$$

$$= 1.8 - 0.3 = 1.5 \text{ V}$$

7.81 Replacing each of the transistors in the Wilson mirror of Fig. 7.40 with its T model while neglecting  $r_o$  results in the circuit shown below.



Note that the diode-connected transistor  $Q_1$  reduces to a resistance  $r_{e1}$ . To determine  $R_{in}$ , we have applied a test voltage  $v_x$ . In the following we analyze the circuit to find  $i_x$  and hence  $R_{in}$ , as

$$R_{in} \equiv \frac{v_x}{i_x}$$

Note that all three transistors are operating at equal emitter currents, approximately equal to  $I_{REF}$ . Thus

$$r_{e1} = r_{e2} = r_{e3} = \frac{V_T}{I_{REF}}$$

Analysis of the circuit proceeds as follows. Since  $r_{e1} = r_{e2}$ , we obtain

$$i_{e2} = i_{e1} \quad (1)$$

Node equation at node 1:

$$i_{e3} + \alpha i_{e2} = i_{e1} + i_{e2}$$

Using Eq. (1) yields

$$i_{e3} = (2 - \alpha)i_{e1} \quad (2)$$

Node equation at node 2:

$$i_x = \alpha i_{e2} + (1 - \alpha)i_{e3}$$

Using Eqs. (1) and (2) yields

$$i_x = i_{e1}[\alpha + (1 - \alpha)(2 - \alpha)]$$

$$i_x = i_{e1}[2 - 2\alpha + \alpha^2] \quad (3)$$

Finally,  $v_x$  can be expressed as the sum of the voltages across  $r_{e3}$  and  $r_{e1}$ ,

$$v_x = i_{e3}r_e + i_{e1}r_e$$

Using Eq. (2) yields

$$v_x = i_{e1}r_e(3 - \alpha) \quad (4)$$

Dividing Eq. (4) by Eq. (3) yields

$$R_{in} = \frac{v_x}{i_x} = r_e \frac{3 - \alpha}{2 - 2\alpha + \alpha^2}$$

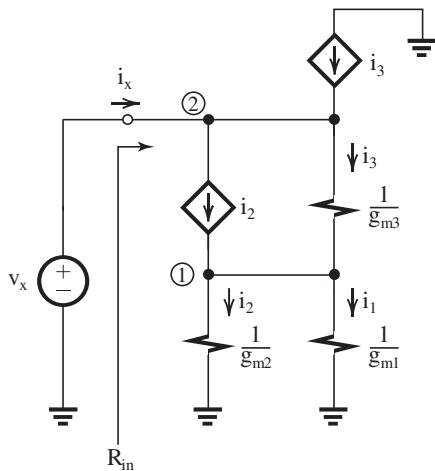
For  $\alpha \approx 1$ ,

$$R_{in} = 2r_e = 2 \frac{V_T}{I_{REF}} \quad \text{Q.E.D.}$$

Thus, for  $I_{REF} = 0.2$  mA,

$$R_{in} = 250 \Omega$$

7.82 Replacing each of the three transistors in the Wilson current mirror in Fig. 7.41(a) with its T model results in the circuit in the figure.



Here, we have applied a test voltage  $v_x$  to determine  $R_{in}$ ,

$$R_{in} \equiv \frac{v_x}{i_x}$$

Since all three transistors are identical and are operating at the same  $I_D$ ,

$$g_{m1} = g_{m2} = g_{m3}$$

Now from the figure we see that

$$i_1 = i_2$$

and

$$i_2 + i_3 = i_2 + i_1$$

Thus

$$i_3 = i_1 = i_2$$

A node equation at node 2 gives

$$i_x + i_3 = i_2 + i_3$$

Thus

$$i_x = i_2$$

The voltage  $v_x$  can be expressed as the sum of the voltages across  $1/g_{m3}$  and  $1/g_{m1}$ :

$$v_x = (i_3/g_{m3}) + (i_1/g_{m1})$$

Substituting  $i_3 = i_2$  and  $i_1 = i_2$ ,  $g_{m1} = g_{m3} = g_m$ , and

$$v_x = 2 i_2/g_m$$

But  $i_2 = i_x$ ; thus

$$v_x = 2 i_x/g_m$$

and thus

$$R_{in} = \frac{2}{g_m} \quad \text{Q.E.D.}$$

7.83 Refer to circuit in Fig. 7.41(a).

(a) Each of the three transistors is operating at  $I_D = I_{REF}$ . Thus

$$I_{REF} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$180 = \frac{1}{2} \times 400 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.3 \text{ V}$$

$$V_{G3} = V_{in} + V_{OV} = 0.5 + 0.3 = 0.8 \text{ V}$$

$$(b) Q_1 \text{ is operating at } V_{DS} = V_{GS} = 0.8 \text{ V}$$

$$Q_2 \text{ is operating at } V_{DS} = 2V_{GS} = 1.6 \text{ V}$$

Thus,

$$I_{REF} - I_O = \frac{\Delta V_{DS}}{r_o}$$

where

$$r_o = \frac{V_A}{I_{REF}} = \frac{18}{0.18} = 100 \text{ k}\Omega$$

$$I_{REF} - I_O = \frac{0.8}{100} = 0.008 \text{ mA} = 8 \mu\text{A}$$

$$I_O = 180 - 8 = 172 \mu\text{A}$$

(c) Refer to Fig. 7.41(c). Since  $Q_1$  and  $Q_2$  are now operating at equal  $V_{DS}$ , we estimate  $I_O = I_{REF} = 180 \mu\text{A}$ .

(d) The minimum allowable  $V_O$  is the value at which  $Q_3$  leaves the saturation region:

$$V_{Omin} = V_{G3} - V_t$$

$$= V_{GS3} + V_{GS1} - V_t$$

$$= 0.8 + 0.8 - 0.5 = 1.1 \text{ V}$$

(e) Diode-connected transistor  $Q_4$  has an incremental resistance  $1/g_{m4}$ . Reference to Fig. 7.41(b) indicates that the incremental resistance of  $Q_4$  would appear in series with the gate of  $Q_3$  and thus carries zero current. Thus including  $Q_4$

has no effect on the value of  $R_o$ , which can be found from Eq. (7.96):

$$R_o = g_m r_{o3} r_{o2}$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.18}{0.3} = 1.2 \text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{V_A}{I_{REF}} = \frac{18}{0.18} = 100 \text{ k}\Omega$$

Thus,

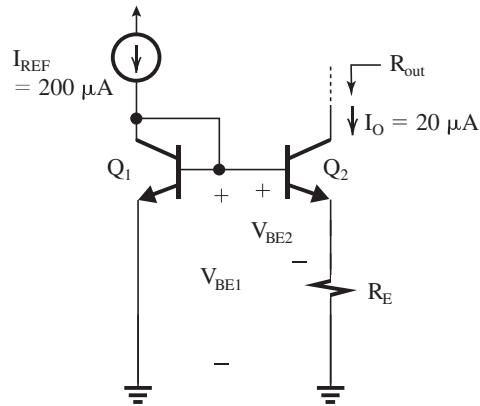
$$R_o = 1.2 \times 100 \times 100 = 12 \text{ M}\Omega$$

(f) For  $\Delta V_O = 1 \text{ V}$ , we obtain

$$\Delta I_O = \frac{\Delta V_O}{R_o} = \frac{1 \text{ V}}{12 \text{ M}\Omega} = 0.08 \mu\text{A}$$

$$\frac{\Delta I_O}{I_O} = \frac{0.04\%}{100\%} = 0.04\%$$

7.84



(a) Assuming  $\beta$  is high so that we can neglect base currents,

$$I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right)$$

Substituting  $I_O = 20 \mu\text{A}$  and  $I_{REF} = 200 \mu\text{A}$  results in

$$0.02 R_E = 0.025 \ln\left(\frac{200}{20}\right)$$

$$\Rightarrow R_E = 2.88 \text{ k}\Omega$$

$$(b) R_{out} = (R_E \parallel r_{\pi2}) + r_{o2} + g_m r_{o2} (R_E \parallel r_{\pi2})$$

where

$$g_m = \frac{0.02}{0.025} = 0.8 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I_O} = \frac{50}{0.02} = 2500 \text{ k}\Omega$$

$$r_{\pi2} = \frac{\beta}{g_m} = \frac{200}{0.8} = 250 \text{ k}\Omega$$

$$R_{out} = (2.9 \parallel 250) + 2500 + 0.8 \times 2500 \times (2.9 \parallel 250) \\ = 8.2 \text{ M}\Omega$$

A 5-V change in  $V_O$  gives rise to

$$\Delta I_O = \frac{5}{7.1} = 0.7 \mu\text{A}$$

7.85 Refer to Fig. 7.42.

(a) To obtain a current transfer ratio of 0.8 (i.e.,  $I_O/I_{REF} = 0.8$  and  $I_O = 80 \mu\text{A}$ ), we write

$$I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right)$$

$$0.08 R_E = 0.025 \ln\left(\frac{100}{80}\right)$$

$$\Rightarrow R_E = 69.7 \text{ }\Omega$$

$$g_m = \frac{0.08}{0.025} = 3.2 \text{ mA/V}$$

$$r_{o2} = \frac{50}{0.08} = 625 \text{ k}\Omega$$

$$r_{\pi2} = \infty \text{ (because } \beta = \infty)$$

$$R_{out} = R_E + r_{o2} + g_m r_{o2} R_E$$

$$= 0.069 + 625 + 3.2 \times 625 \times 0.0697$$

$$= 764.5 \text{ k}\Omega$$

Relative to the value of  $r_{o2}$ ,

$$\frac{R_{out}}{r_{o2}} = 1.22$$

(b) To obtain  $I_O/I_{REF} = 0.1$ , that is,  $I_O = 10 \mu\text{A}$ , we write

$$0.01 R_E = V_T \ln\left(\frac{100}{10}\right)$$

$$\Rightarrow R_E = 5.76 \text{ k}\Omega$$

$$g_m = \frac{0.01}{0.025} = 0.4 \text{ mA/V}$$

$$r_{o2} = \frac{50}{0.01} = 5000 \text{ k}\Omega$$

$$r_{\pi2} = \infty$$

$$R_{out} = R_E + r_{o2} + g_m r_{o2} R_E$$

$$= 5.76 + 5000 + 0.4 \times 5000 \times 5.76$$

$$= 16.5 \text{ M}\Omega$$

Compared to  $r_{o2}$ ,

$$\frac{R_{out}}{r_{o2}} = \frac{16.5}{5} = 3.3$$

(c) To obtain  $I_O/I_{REF} = 0.01$ , that is,  $I_O = 1 \mu\text{A}$ , we write

$$0.001 R_E = 0.025 \ln\left(\frac{100}{1}\right)$$

$$\Rightarrow R_E = 115 \text{ k}\Omega$$

$$g_m = \frac{0.001}{0.025} = 0.04 \text{ mA/V}$$

$$r_{o2} = \frac{50}{0.001} = 50 \times 10^3 \text{ k}\Omega$$

$$R_{out} = 115 + 50 \times 10^3 + 0.04 \times 50 \times 10^3 \times 115 \\ = 280 \text{ M}\Omega$$

Relative to the value of  $r_{o2}$ ,

$$\frac{R_{out}}{r_{o2}} = \frac{280}{50} = 5.6$$

7.86 (a) Refer to the circuit in Fig. P7.86. Neglecting the base currents, we see that all three transistors are operating at  $I_C = 10 \mu A$ , and thus

$$V_{BE1} = V_{BE2} = V_{BE3} = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{10 \mu \text{A}}\right) \\ = 0.585 \text{ V}$$

From the circuit we see that the voltage across R is  $V_{BE} = 0.585 \text{ V}$ , thus

$$I_O R = V_{BE}$$

$$R = \frac{0.585}{0.01} = 58.5 \text{ k}\Omega$$

$$(b) g_{m3} = \frac{0.01}{0.025} = 0.4 \text{ mA/V}$$

$$r_{o3} = \frac{40}{0.01} = 4000 \text{ k}\Omega$$

$$r_{\pi3} = \frac{\beta}{g_{m3}} = \frac{100}{0.4} = 250 \text{ k}\Omega$$

$$R_{out} = (R \parallel r_{\pi3}) + r_{o3} + g_{m3}r_{o3}(R \parallel r_{\pi3}) \\ = (58.5 \parallel 250) + 4000 + 0.4 \times 4000 \times (58.5 \parallel 250) \\ = 79.9 \text{ M}\Omega$$

7.87 Refer to the circuit in Fig. P7.87. Since  $Q_1$  and  $Q_2$  are matched and conducting equal currents I, their  $V_{GS}$  values will be equal. Thus from the loop  $Q_1$ ,  $Q_6$ , R, and  $Q_2$ , we see that

$$IR = V_{EB6}$$

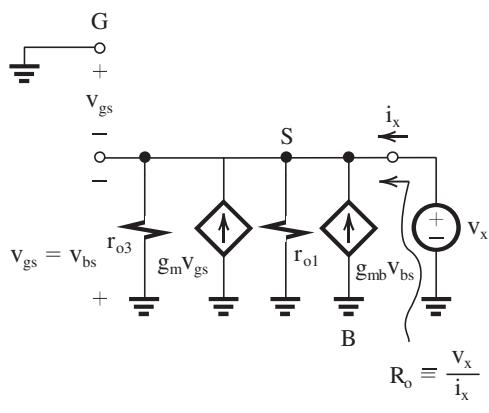
$$= V_T \ln\left(\frac{I}{I_S}\right) \quad \text{Q.E.D.}$$

Now to obtain  $I = 0.2 \text{ mA}$ , we write

$$0.2R = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{0.2 \text{ mA}}\right)$$

$$\Rightarrow R = 3.3 \text{ k}\Omega$$

7.88



The figure shows the equivalent circuit of the source follower prepared for finding  $R_o$ . Observe

that we have set  $v_i = 0$  and applied a test voltage  $v_x$ . We note that

$$v_{gs} = v_{bs} = -v_x \quad (1)$$

and

$$i_x = -g_{mb}v_{bs} + \frac{v_x}{r_{o1}} - g_m v_{gs} + \frac{v_x}{r_{o3}}$$

Thus,

$$i_x = g_{mb}v_x + \frac{v_x}{r_{o1}} + g_m v_x + \frac{v_x}{r_{o3}}$$

from which we obtain

$$R_o \equiv \frac{v_x}{i_x} = r_{o1} \parallel r_{o3} \parallel \frac{1}{g_m + g_{mb}} \quad \text{Q.E.D.}$$

7.89 The dc level shift provided by a source follower is equal to its  $V_{GS}$ . Thus, to obtain a dc level shift of 0.9 V, we write

$$V_{GS} = 0.9 \text{ V} = V_t + V_{OV}$$

$$\Rightarrow V_{OV} = 0.9 - 0.6 = 0.3 \text{ V}$$

To obtain the required bias current, we use

$$I = I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$= \frac{1}{2} \times 0.2 \times \frac{20}{0.5} \times 0.3^2$$

$$I = 0.36 \text{ mA} = 360 \mu \text{A}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.36}{0.3} = 2.4 \text{ mA/V}$$

$$g_{mb} = \chi g_m = 0.2 \times 2.4 = 0.48 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A L}{I_D} = \frac{20 \times 0.5}{0.36} = 27.8 \text{ k}\Omega$$

To determine  $A_{vo}$ , we note [refer to Fig. 7.45(b)] that the total effective resistance between the MOSFET source terminal and ground is

$$r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{mb}}. \text{ Denoting this resistance } R, \text{ we have}$$

$$R = r_o \parallel r_o \parallel \frac{1}{g_{mb}}$$

$$= 27.8 \parallel 27.8 \parallel \frac{1}{0.48}$$

$$= 1.81 \text{ k}\Omega$$

Thus, the open-circuit voltage gain is

$$A_{vo} = \frac{R}{R + \frac{1}{g_m}}$$

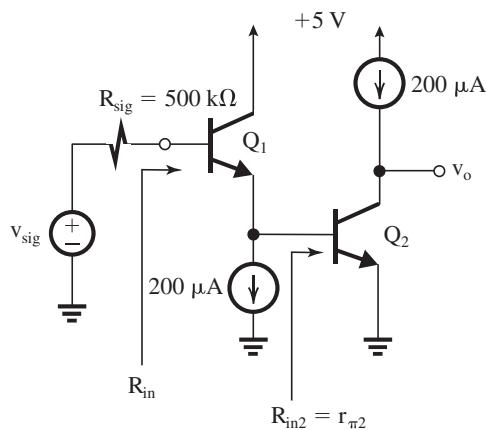
$$= \frac{1.81}{1.81 + \frac{1}{2.4}} = 0.81 \text{ V/V}$$

$$\begin{aligned} R_o &= R \parallel \frac{1}{g_m} \\ &= 1.81 \text{ k}\Omega \parallel \frac{1}{2.4 \text{ mA/V}} \\ &= 0.339 \text{ k}\Omega \end{aligned}$$

When a load resistance of  $2 \text{ k}\Omega$  is connected to the output, the total resistance between the output node and ground become  $R \parallel R_L = 1.81 \parallel 2 = 0.95 \text{ k}\Omega$ . Thus, the voltage gain becomes

$$A_v = \frac{0.95}{0.95 + \frac{1}{2.4}} = 0.7 \text{ V/V}$$

7.90



Each of  $Q_1$  and  $Q_2$  is operating at an  $I_C$  approximately equal to  $200 \mu\text{A}$ . Thus for both devices,

$$g_m = \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$r_e \approx \frac{1}{g_m} = 0.125 \text{ k}\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{8} = 12.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{0.2} = 250 \text{ k}\Omega$$

(a)  $R_{in2} = r_{\pi 2} = 12.5 \text{ k}\Omega$ 

$$R_{in} = (\beta_1 + 1)[r_{e1} + (r_{\pi 2} \parallel r_{o1})]$$

$$= 101[0.125 + (12.5 \parallel 250)]$$

$$= 1.215 \text{ M}\Omega$$

$$\frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{1.215}{1.215 + 0.5} = 0.71 \text{ V/V}$$

$$\frac{v_{e1}}{v_{b1}} = \frac{r_{\pi 2} \parallel r_{o1}}{(r_{\pi 2} \parallel r_{o1}) + r_{e1}} = 0.99 \text{ V/V}$$

$$\frac{v_o}{v_{b1}} = -g_{m2}r_{o2} = -8 \times 250 = -2000 \text{ V/V}$$

$$G_v = \frac{v_o}{v_{sig}} = 0.71 \times 0.99 \times -2000 = -1405 \text{ V/V}$$

(b) Increasing the bias current by a factor of 10 (i.e., to  $2 \text{ mA}$ ) results in

$$g_m = 80 \text{ mA/V}$$

$$r_e = 0.0125 \text{ k}\Omega$$

$$r_\pi = 1.25 \text{ k}\Omega$$

$$r_o = 25 \text{ k}\Omega$$

$$R_{in2} = r_{\pi 2} = 1.25 \text{ k}\Omega$$

$$R_{in} = 101[0.0125 + (1.25 \parallel 25)] = 121.5 \text{ k}\Omega$$

Thus,  $R_{in}$  has been reduced by a factor of 10.

$$\frac{v_{b1}}{v_{sig}} = \frac{121.5}{121.5 + 500}$$

$$= 0.2 \text{ V/V (considerably reduced)}$$

$$\frac{v_{e1}}{v_{b1}} = \frac{(1.25 \parallel 25)}{(1.25 \parallel 25) + 0.0125}$$

$$= 0.99 \text{ V/V (unchanged)}$$

$$\frac{v_o}{v_{b1}} = -g_{m2}r_o = -80 \times 25$$

$$= -2000 \text{ V/V (unchanged)}$$

$$G_v = \frac{v_o}{v_{sig}} = 0.2 \times 0.99 \times -2000 = -396 \text{ V/V}$$

which has been reduced by a factor of 3.5! All this reduction in gain is a result of the reduction in  $R_{in}$ .

7.91 From Fig. P7.91 we see that

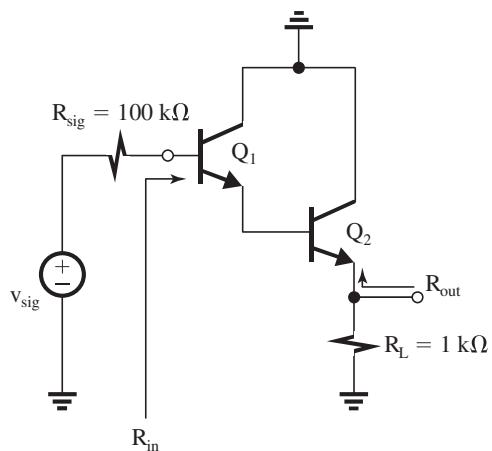
$$I_{E2} = 10 \text{ mA}$$

$$I_{E1} = \frac{I_{E2}}{\beta_2 + 1} \approx \frac{10}{100} = 0.1 \text{ mA}$$

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25 \text{ mV}}{10 \text{ mA}} = 2.5 \text{ }\Omega$$

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \text{ }\Omega$$

The Darlington follower circuit prepared for small-signal analysis is shown in the figure.



$$R_{in} = (\beta + 1)[r_{e1} + (\beta_2 + 1)(r_{e2} + R_L)] \\ = 101[0.25 + (101)(0.0025 + 1)] \\ = 10.25 \text{ M}\Omega$$

$$R_{out} = r_{e2} + \frac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1} \\ = 2.5 + \frac{250 + \frac{100 \times 10^3}{101}}{101} = 14.8 \text{ }\Omega$$

With  $R_L$  removed,

$$G_{vo} = \frac{v_o}{v_{sig}} = 1$$

With  $R_L$  connected,

$$G_v = \frac{v_o}{v_{sig}} = G_{vo} \frac{R_L}{R_L + R_{out}} \\ = 1 \times \frac{1}{1 + 0.0148} = 0.985$$

7.92

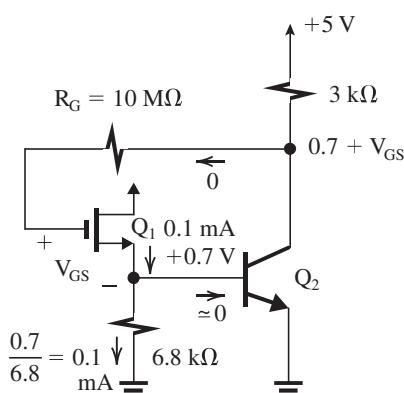


Figure 1

(a) From Fig. 1 we see that

$$I_{D1} \approx 0.1 \text{ mA/V}$$

But

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{ov}^2$$

$$0.1 = \frac{1}{2} \times 2 \times V_{ov}^2$$

$$\Rightarrow V_{ov} = 0.316 \text{ V}$$

$$V_{GS} = V_t + V_{ov} = 1.316 \text{ V}$$

Thus,

$$V_{C2} = V_{G2} = 0.7 + V_{GS} = 2.016 \text{ V}$$

$$I_{C2} = \frac{V_{CC} - V_{C2}}{3 \text{ k}\Omega} = \frac{5 - 2.016}{3} \approx 1 \text{ mA}$$

$$(b) g_{ml} = \frac{2I_{D1}}{V_{ov}} = \frac{2 \times 0.1}{0.316} = 0.632 \text{ mA/V}$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{1 \text{ mA}}{0.025} = 40 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{200}{40} = 5 \text{ k}\Omega$$

(c) Neglecting  $R_G$ , we can write

$$\frac{v_{b2}}{v_i} = \frac{r_{\pi 2} \parallel 6.8 \text{ k}\Omega}{(r_{\pi 2} \parallel 6.8 \text{ k}\Omega) + \frac{1}{g_{ml}}} \\ = 0.65 \text{ V/V}$$

$$\frac{v_o}{v_{b2}} = -g_{m2}(3 \parallel 1)$$

$$= -40 \times \frac{3}{4} = -30 \text{ V/V}$$

$$\frac{v_o}{v_i} = 0.65 \times -30 = -19.5 \text{ V/V}$$

(d)

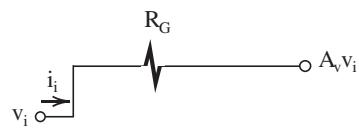


Figure 2

From Fig. 2 we can find  $i_d$  as

$$i_d = \frac{v_i - A_v v_i}{R_G} \\ = \frac{v_i + 19.4 v_i}{R_G}$$

Thus,

$$R_{in} \equiv \frac{v_i}{i_d} = \frac{R_G}{20.5} = \frac{10 \text{ M}\Omega}{20.5} = 487 \text{ k}\Omega$$

Thus the overall voltage gain becomes

$$\frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} \times A_v$$

$$= \frac{487}{487 + 500} \times -19.5$$

$$= -9.6 \text{ V/V}$$

(e) The suggested configuration, shown partially in Fig. 3, will have

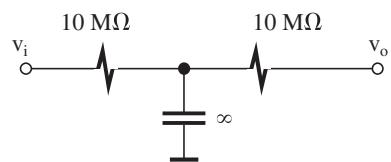


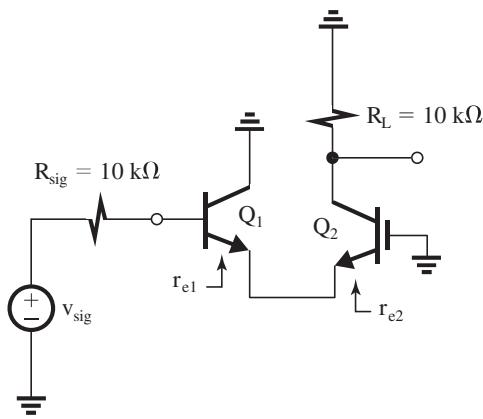
Figure 3

no effect on the dc bias of each transistor. However, it will have a profound effect on  $R_{in}$ , as  $R_{in}$  now is  $10 \text{ M}\Omega$ , and

$$\frac{v_o}{v_{sig}} = \frac{10}{10 + 0.5} \times -19.5 = -18.6 \text{ V/V}$$

This is nearly double the value we had before!

7.93



The figure shows the circuit prepared for signal analysis.

$$G_v = \frac{v_o}{v_{sig}} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{\alpha R_L}{\frac{R_{sig}}{\beta + 1} + r_{e1} + r_{e2}}$$

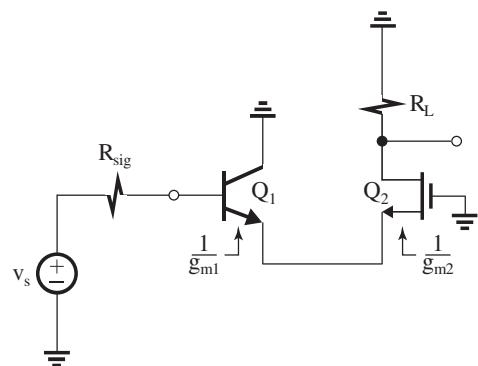
where

$$\alpha \approx 1$$

$$r_{e1} = r_{e2} = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

$$G_v = \frac{10}{\frac{10}{101} + 0.05 + 0.05} = 50.2 \text{ V/V}$$

7.94



From the figure we can determine the overall voltage gain as

$$G_v = \frac{v_o}{v_{sig}} = \frac{\text{Total resistance in the drain}}{\text{Total resistance in the sources}}$$

$$= \frac{R_L}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} = \frac{1}{2} g_m R_L$$

where

$$g_m = g_{m1} = g_{m2} = 5 \text{ mA/V}$$

$$G_v = \frac{1}{2} \times 5 \times 10 = 25 \text{ V/V}$$

7.95 Refer to Fig. P7.95. All transistors are operating at  $I_E = 0.5 \text{ mA}$ . Thus,

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

(a) Refer to Fig. P7.95(a).

$$\frac{v_o}{v_{sig}} = -\frac{\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$= \frac{-\alpha \times 10 \text{ k}\Omega}{10 \text{ k}\Omega + r_e}$$

For

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

$$G_v = \frac{-0.99 \times 10}{\frac{10}{101} + 0.05} = -66.4 \text{ V/V}$$

(b) Refer to Fig. P7.95(b).

$$i_{b1} = \frac{v_{sig}}{10 + (\beta + 1)r_e} = \frac{v_{sig}}{10 + 101 \times 0.05}$$

$$i_{c1} = \beta i_{b1} = \frac{100 v_{sig}}{10 + 101 \times 0.05}$$

$$i_{c2} = \alpha i_{c1} = \frac{0.99 \times 100 v_{sig}}{10 + 101 \times 0.05}$$

$$v_o = -i_{c2} \times 10$$

$$G_v \equiv \frac{v_o}{v_{sig}} = -\frac{10 \times 0.99 \times 100}{10 + 101 \times 0.05} = -65.8 \text{ V/V}$$

(c) Refer to Fig. P7.95(c).

$$G_v = \frac{v_o}{v_{sig}} = \frac{\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitters}}$$

$$= \frac{0.99 \times 10}{\frac{10}{\beta + 1} + 2 r_e} = \frac{0.99 \times 10}{\frac{10}{101} + 2 \times 0.05}$$

$$= 49.7 \text{ V/V}$$

(d) Refer to Fig. P7.95(d).

$$R_{in} (\text{at the base of } Q_1) = (\beta_1 + 1)[r_{e1} + r_{\pi 2}]$$

where

$$r_{e1} = 50 \Omega$$

$$r_{\pi 2} = (\beta + 1)r_{e2} = 101 \times 50 = 5.05 \text{ k}\Omega$$

Thus,

$$R_{in} = 101(0.05 + 5.05) = 515 \text{ k}\Omega$$

$$\frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{515}{515 + 10} = 0.98 \text{ V/V}$$

$$\frac{v_{b2}}{v_{b1}} = \frac{r_{\pi 2}}{r_{\pi 2} + r_{e1}} = \frac{5.05}{5.05 + 0.05} = 0.98 \text{ V/V}$$

$$\frac{v_o}{v_{b2}} = -g_m \times 10 \text{ k}\Omega$$

$$= -20 \times 10 = -200 \text{ V/V}$$

$$G_v = \frac{v_o}{v_{sig}} = 0.98 \times 0.98 \times -200 = -194 \text{ V/V}$$

(e) Refer to Fig. P7.95(e).

$$i_{b1} = \frac{v_{sig}}{10 + (\beta + 1)r_{e1}} = \frac{v_{sig}}{10 + 101 \times 0.05}$$

$$i_{c1} = \beta_1 i_{b1} = \frac{100 v_{sig}}{10 + 101 \times 0.05}$$

$$i_{e1} = i_{c1}$$

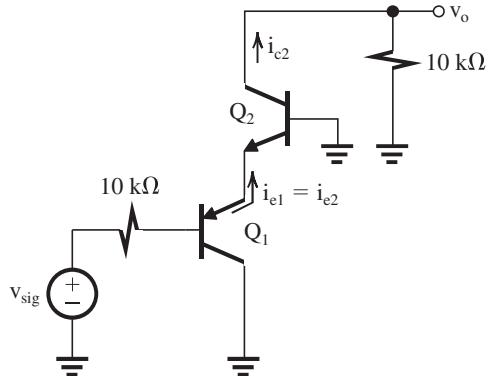
$$i_{c2} = \alpha i_{e2} = \alpha i_{c1} = \frac{0.99 \times 100 v_{sig}}{10 + 101 \times 0.05}$$

$$v_o = i_{c2} \times 10 = \frac{0.99 \times 100 \times 10 v_{sig}}{10 + 101 \times 0.05}$$

Thus,

$$G_v = \frac{v_o}{v_{sig}} = \frac{0.99 \times 100 \times 10}{10 + 101 \times 0.05} = 65.8 \text{ V/V}$$

(f)



$$i_{e1} = i_{e2} = \frac{v_{sig}}{\frac{10}{\beta_1 + 1} + r_{e1} + r_{e2}}$$

$$= \frac{v_{sig}}{\frac{10}{101} + 0.05 + 0.05}$$

$$i_{c2} = \alpha i_{e2} = \frac{0.99 v_{sig}}{\frac{10}{101} + 0.05 + 0.05}$$

$$v_o = i_{c2} \times 10 \text{ k}\Omega = \frac{0.99 \times 10 v_{sig}}{\frac{10}{101} + 0.05 + 0.05}$$

Thus,

$$G_v = \frac{v_o}{v_{sig}} = 49.7 \text{ V/V}$$

### Exercise 8-1

Ex: 8.1 Referring to Fig. 8.3,

If  $R_D$  is doubled to  $5\text{ k}\Omega$ ,

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} R_D$$

$$= 1.5 - \frac{0.4 \text{ mA}}{2} (5 \text{ k}\Omega) = 0.5 \text{ V}$$

$$V_{CM_{max}} = V_t + V_D = 0.5 + 0.5 = +1.0 \text{ V}$$

Since the currents  $I_{D1}$ , and  $I_{D2}$  are still 0.2 mA each,

$$V_{GS} = 0.82 \text{ V}$$

$$\text{So, } V_{CM_{min}} = V_{SS} + V_{CS} + V_{GS}$$

$$= -1.5 \text{ V} + 0.4 \text{ V} + 0.82 \text{ V} = -0.28 \text{ V}$$

So, the common-mode range is

$$-0.28 \text{ V to } +1.0 \text{ V}$$

Ex: 8.2 (a) The value of  $v_{id}$  that causes  $Q_1$  to conduct the entire current is  $\sqrt{2} V_{OV}$

$$\rightarrow \sqrt{2} \times 0.316 = 0.45 \text{ V}$$

$$\text{then, } V_{D1} = V_{DD} - I \times R_D$$

$$= 1.5 - 0.4 \times 2.5 = 0.5 \text{ V}$$

$$V_{D2} = V_{DD} = +1.5 \text{ V}$$

(b) For  $Q_2$  to conduct the entire current:

$$v_{id} = -\sqrt{2} V_{OV} = -0.45 \text{ V}$$

then,

$$V_{D1} = V_{DD} = +1.5 \text{ V}$$

$$V_{D2} = 1.5 - 0.4 \times 2.5 = 0.5 \text{ V}$$

(c) Thus the differential output range is

$$V_{D2} - V_{D1} : \text{from } 1.5 - 0.5 = +1 \text{ V}$$

$$\text{to } 0.5 - 1.5 = -1 \text{ V}$$

Ex: 8.3 Refer to answer table for Exercise 8.3 where values were obtained in the following way:

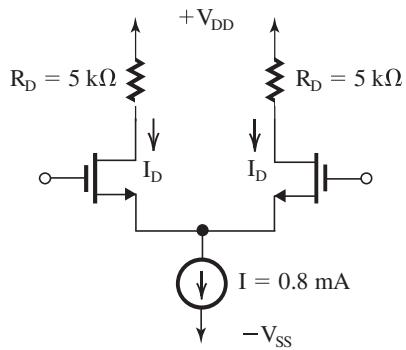
$$V_{OV} = \sqrt{I/k_n W/L} \Rightarrow \frac{W}{L} = \frac{I}{k_n V_{OV}^2}$$

$$g_m = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$\left(\frac{v_{id}/2}{V_{OV}}\right)^2 = 0.1 \rightarrow v_{id} = 2 V_{OV} \sqrt{0.1}$$

$$\text{Ex: 8.4 } I_D = \frac{I}{2} = \frac{0.8 \text{ mA}}{2} = 0.4 \text{ mA}$$

$$I_D = \frac{1}{2} k'_n \left(\frac{W}{L}\right) (V_{OV})^2$$



Thus,

$$V_{OV} = \sqrt{\frac{2I_D}{k'_n \left(\frac{W}{L}\right)}} = \sqrt{\frac{2(0.4 \text{ mA})}{0.2 (\text{mA/V}^2)(100)}} = 0.2 \text{ V}$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{0.4 \text{ mA} \times 2}{0.2 \text{ V}} = 4 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20 \text{ V}}{0.4 \text{ mA}} = 50 \text{ k}\Omega$$

$$A_d = g_m (R_D \parallel r_o)$$

$$A_d = (4 \text{ mA/V}) (5 \text{ k}\Omega \parallel 50 \text{ k}\Omega) = 18.2 \text{ V/V}$$

Ex: 8.5 With  $I = 200 \mu\text{A}$ , for all transistors,

$$I_D = \frac{I}{2} = \frac{200 \mu\text{A}}{2} = 100 \mu\text{A}$$

$$L = 2(0.18 \mu\text{m}) = 0.36 \mu\text{m}$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{|V_A| L}{I_D} = \frac{(10 \text{ V}/\mu\text{m})(0.36 \mu\text{m})}{0.1 \text{ mA}} = 36 \text{ k}\Omega$$

Since  $I_{D1} = I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) V_{OV}^2$ ,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2I_D}{\mu_n C_{ox} V_{OV}^2} = \frac{2(100 \mu\text{A})}{(400 \mu\text{A/V}^2)(0.2 \text{ V})^2} = 12.5$$

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{2I_D}{\mu_p C_{ox} |V_{OV}|^2} = \frac{2(100 \mu\text{A})}{(100 \mu\text{A/V}^2)(0.2)^2} = 50$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{(100 \mu\text{A})(2)}{0.2 \text{ V}} = 1 \text{ mA/V},$$

so

$$A_d = g_m (r_{o1} \parallel r_{o3}) = 1(\text{mA/V}) (36 \text{ k}\Omega \parallel 36 \text{ k}\Omega) = 18 \text{ V/V}$$

### Exercise 8-2

Ex: 8.6  $L = 2(0.18 \mu m) = 0.36 \mu m$

$$\text{All } r_o = \frac{|V_A| \cdot L}{I_D}$$

The drain current for all transistors is

$$I_D = \frac{I}{2} = \frac{200 \mu A}{2} = 100 \mu A$$

$$r_o = \frac{(10 V/\mu m)(0.36 \mu m)}{0.1 mA} = 36 k\Omega$$

Referring to Fig. 8.13(a),

Since  $I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) V_{OV}^2$  for all NMOS transistors,

$$\begin{aligned} \left(\frac{W}{L}\right)_1 &= \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 \\ &= \frac{2I_D}{\mu_n C_{ox} V_{OV}^2} = \frac{2(100 \mu A)}{400 \mu A/V^2(0.2 V)^2} = 12.5 \end{aligned}$$

$$\begin{aligned} \left(\frac{W}{L}\right)_5 &= \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 \\ &= \frac{2I_D}{\mu_p C_{ox} V_{OV}^2} = \frac{2(100 \mu A)}{100 \mu A/V^2(0.2 V)^2} = 50 \end{aligned}$$

For all transistors,

$$g_m = \frac{|I_D|}{|V_{OV}|/2} = \frac{(0.1 mA)(2)}{(0.2 V)} = 1 mA/V$$

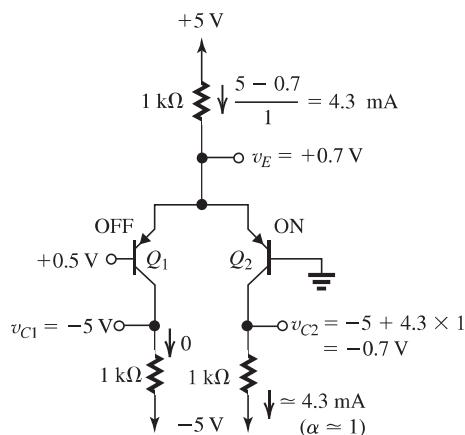
From Fig. 8.13(b),

$$\begin{aligned} R_{on} &= (g_m r_{o3}) r_{o1} = (1 \times 36) \times 36 \\ &= 1.296 M\Omega \end{aligned}$$

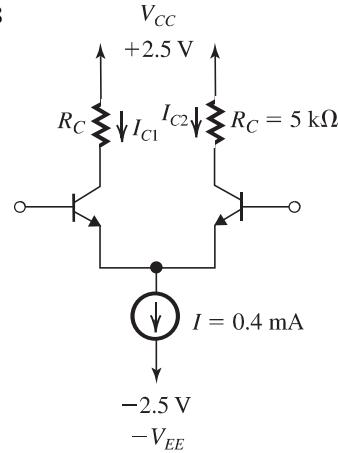
$$\begin{aligned} R_{op} &= (g_m r_{o5}) r_{o7} = (1 \times 36) \times 36 \\ &= 1.296 M\Omega \end{aligned}$$

$$\begin{aligned} A_d &= g_m (R_{on} \parallel R_{op}) \\ &= (1 mA/V) (1.296 M\Omega \parallel 1.296 M\Omega) \\ &= 648 V/V \end{aligned}$$

Ex: 8.7



Ex: 8.8



$$\begin{aligned} I_{C1} &= I_{C2} \simeq I_{E1} = I_{E2} = \frac{I}{2} = \frac{0.4 mA}{2} \\ &= 0.2 mA \end{aligned}$$

$$\begin{aligned} V_{CMmax} &\simeq V_C + 0.4 V \\ &= V_{CC} - I_C R_C + 0.4 V \\ &= 2.5 - 0.2 mA (5 k\Omega) + 0.4 V = +1.9 V \end{aligned}$$

$$V_{CMmin} = -V_{EE} + V_{CS} + V_{BE}$$

$$V_{CMmin} = -2.5 V + 0.3 V + 0.7 V = -1.5 V$$

Input common-mode range is  $-1.5 V$  to  $+1.9 V$

Ex: 8.9 Substituting  $i_{E1} + i_{E2} = I$  in Eqn. (8.45) yields

$$\begin{aligned} i_{E1} &= \frac{I}{1 + e^{(V_{B2} - V_{B1})/V_T}} \\ 0.99 I &= \frac{I}{1 + e^{(V_{B2} - V_{B1})/V_T}} \\ v_{B1} - v_{B2} &= -V_T \ln \left( \frac{1}{0.99} - 1 \right) \\ &= -25 \ln(1/99) \\ &= 25 \ln(99) = 115 mV \end{aligned}$$

Ex: 8.10 (a) The DC current in each transistor is  $0.5 mA$ . Thus  $V_{BE}$  for each will be

$$\begin{aligned} V_{BE} &= 0.7 + 0.025 \ln \left( \frac{0.5}{1} \right) \\ &= 0.683 V \end{aligned}$$

$$\Rightarrow v_E = 5 - 0.683 = +4.317 V$$

$$(b) g_m = \frac{I_C}{V_T} = \frac{0.5}{0.025} = 20 \frac{mA}{V}$$

$$\begin{aligned} (c) i_{C1} &= 0.5 + g_m \Delta v_{BE1} \\ &= 0.5 + 20 \times 0.005 \sin(2\pi \times 1000t) \\ &= 0.5 + 0.1 \sin(2\pi \times 1000t), mA \end{aligned}$$

### Exercise 8-3

$$i_{C2} = 0.5 - 0.1 \sin(2\pi \times 1000t), \text{ mA}$$

$$\begin{aligned} (d) v_{C1} &= (V_{CC} - I_C R_C) - 0.1 \\ &\times R_C \sin(2\pi \times 1000t) \\ &= (15 - 0.5 \times 10) - 0.1 \times 10 \sin(2\pi \times 1000t) \\ &= 10 - 1 \sin(2\pi \times 1000t), \text{ V} \\ v_{C2} &= 10 + 1 \sin(2\pi \times 1000t), \text{ V} \\ (e) v_{C2} - v_{C1} &= 2 \cdot \sin(2\pi \times 1000t), \text{ V} \\ (f) \text{ Voltage gain} &\equiv \frac{v_{C2} - v_{C1}}{v_{B1} - v_{B2}} \\ &= \frac{2 \text{ V peak}}{0.01 \text{ V peak}} = 200 \text{ V/V} \end{aligned}$$

Ex: 8.11 The transconductance for each transistor is

$$g_m = \sqrt{2\mu_n C_{ox} (W/L) I_D}$$

$$I_D = \frac{I}{2} = \frac{0.8 \text{ mA}}{2} = 0.4 \text{ mA}$$

Thus,

$$g_m = \sqrt{2 \times 0.2 \times 100 \times 0.4} = 4 \text{ mA/V}$$

The differential gain for matched

$$R_D \text{ values is } A_d = \frac{v_{O2} - v_{O1}}{v_{id}} = g_m R_D$$

If we ignore the 1% here, then we obtain

$$A_d = g_m R_D = (4 \text{ mA/V}) (5 \text{ k}\Omega) = 20 \text{ V/V}$$

$$\begin{aligned} |A_{cm}| &= \left( \frac{R_D}{2R_{SS}} \right) \left( \frac{\Delta R_D}{R_D} \right) \\ &= \left( \frac{5}{2 \times 25} \right) (0.01) = 0.001 \text{ V/V} \end{aligned}$$

$$\text{CMRR (dB)} = 20 \log \frac{|A_d|}{|A_{cm}|} = 20 \log \left( \frac{20}{0.001} \right)$$

$$= 86 \text{ dB}$$

Ex: 8.12 From Exercise 8.11,

$$W/L = 100, \mu_n C_{ox} = 0.2 \text{ mA/V}^2,$$

$$I_D = \frac{I}{2} = \frac{0.8 \text{ mA}}{2} = 0.4 \text{ mA}$$

$$\begin{aligned} g_m &= \sqrt{2\mu_n C_{ox} \left( \frac{W}{L} \right) I_D} \\ &= \sqrt{2 (0.2 \text{ mA/V}^2) (100) (0.4 \text{ mA})} \end{aligned}$$

$$g_m = 4 \text{ mA/V}$$

Using Eq. (8.88) and the fact that  $R_{SS} = 25 \text{ k}\Omega$ , we obtain

$$\begin{aligned} \text{CMRR} &= \frac{(2 g_m R_{SS})}{\left( \frac{\Delta g_m}{g_m} \right)} = \frac{2(4 \text{ mA/V})(25 \text{ k}\Omega)}{0.01} \\ &= 20,000 \end{aligned}$$

$$\text{CMRR (dB)} = 20 \log_{10} (20,000) = 86 \text{ dB}$$

Ex: 8.13 If the output of a MOS differential amplifier is taken single-endedly, then

$$|A_d| = \frac{1}{2} g_m R_D$$

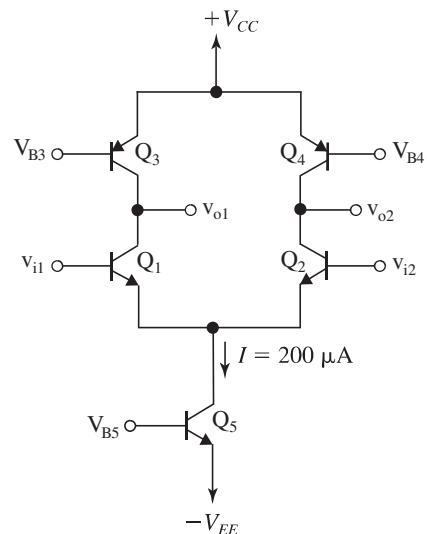
(that is, half the gain obtained with the output taken differentially), and from Fig. 8.25(d) we have

$$|A_{cm}| \approx \frac{R_D}{2R_{SS}}$$

Thus,

$$\text{CMRR} \equiv \frac{|A_d|}{|A_{cm}|} = g_m R_{SS} \quad \text{Q.E.D.}$$

### Ex: 8.14



$$I = 200 \mu A$$

Since  $\beta \gg 1$ ,

$$I_{C1} \approx I_{C2} \approx \frac{I}{2} = \frac{200 \mu A}{2} = 100 \mu A$$

$$g_{m1} = g_{m2} = g_m = \frac{I_C}{V_T} = \frac{100 \mu A}{25 mV} = 4 \text{ mA/V}$$

$$R_{C1} = R_{C2} = R_C = r_o = \frac{|V_A|}{I_C}$$

$$= \frac{10 \text{ V}}{100 \mu A} = 100 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{V_A}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{e1} = r_{e2} = r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 0.25 \text{ k}\Omega$$

$$\begin{aligned} |A_d| &= \frac{R_C \parallel r_o}{r_e} = \frac{100 \text{ k}\Omega \parallel 100 \text{ k}\Omega}{0.25 \text{ k}\Omega} \\ &= 200 \text{ V/V} \end{aligned}$$

$$R_{id} = 2r_\pi, \quad r_\pi = \frac{\beta}{g_m} = \frac{100}{4 \text{ mA/V}} = 25 \text{ k}\Omega$$

$$R_{id} = 2(25 \text{ k}\Omega) = 50 \text{ k}\Omega$$

### Exercise 8-4

$$R_{EE} = \frac{V_A}{I} = \frac{10 \text{ V}}{200 \mu\text{A}} = 50 \text{ k}\Omega$$

If the total load resistance is assumed to be mismatched by 1%, then we have

$$\begin{aligned} |A_{cm}| &= \frac{R_C}{2R_{EE}} \frac{\Delta R_C}{R_C} \\ &= \frac{100}{2 \times 50} \times 0.01 = 0.01 \text{ V/V} \end{aligned}$$

$$\begin{aligned} \text{CMRR (dB)} &= 20 \log_{10} \left| \frac{A_d}{A_{cm}} \right| = 20 \log_{10} \left| \frac{200}{0.01} \right| \\ &= 86 \text{ dB} \end{aligned}$$

Using Eq. (8.96), we obtain

$$\begin{aligned} R_{icm} &= \beta R_{EE} \cdot \frac{1 + \frac{R_C}{\beta r_o}}{1 + \frac{R_C + 2R_{EE}}{r_o}} \\ &= 100 \times 50 \times \frac{1 + \frac{100}{100 + 2 \times 50}}{1 + \frac{100}{100}} \\ R_{icm} &\simeq 1.68 \text{ M}\Omega \end{aligned}$$

Ex: 8.15 From Exercise 8.4:

$$V_{OV} = 0.2 \text{ V}$$

Using Eq. (8.101) we obtain  $V_{OS}$  due to  $\Delta R_D / R_D$  as:

$$\begin{aligned} V_{OS} &= \left( \frac{V_{OV}}{2} \right) \cdot \left( \frac{\Delta R_D}{R_D} \right) \\ &= \frac{0.2}{2} \times 0.02 = 0.002 \text{ V} \quad \text{i.e } 2 \text{ mV} \end{aligned}$$

To obtain  $V_{OS}$  due to  $\frac{\Delta(W/L)}{W/L}$ ,

use Eq. (8.106),

$$\begin{aligned} V_{OS} &= \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta(W/L)}{W/L} \right) \\ \Rightarrow V_{OS} &= \left( \frac{0.2}{2} \right) \times 0.02 = 0.002 \\ \Rightarrow 2 \text{ mV} \end{aligned}$$

The offset voltage arising from  $\Delta V_t$  is obtained from Eq. (8.109):

$$V_{OS} = \Delta V_t = 2 \text{ mV}$$

Finally, from Eq. (8.110) the total input offset is

$$V_{OS} =$$

$$\begin{aligned} &\left[ \left( \frac{V_{OV}}{2} \frac{\Delta R_D}{R_D} \right)^2 + \left( \frac{V_{OV}}{2} \frac{\Delta(W/L)}{W/L} \right)^2 + (\Delta V_t)^2 \right]^{1/2} \\ &= \sqrt{(2 \times 10^{-3})^2 + (2 \times 10^{-3})^2 + (2 \times 10^{-3})^2} \end{aligned}$$

$$= \sqrt{3 \times (2 \times 10^{-3})^2}$$

$$= 3.46 \text{ mV}$$

Ex: 8.16 From Eq. (8.120), we get

$$\begin{aligned} V_{OS} &= V_T \sqrt{\left( \frac{\Delta R_C}{R_C} \right)^2 + \left( \frac{\Delta I_S}{I_S} \right)^2} \\ &= 25 \sqrt{(0.02)^2 + (0.1)^2} \\ &= 2.55 \text{ mV} \\ I_B &= \frac{100}{2(\beta + 1)} = \frac{100}{2 \times 101} \cong 0.5 \mu\text{A} \\ I_{OS} &= I_B \left( \frac{\Delta \beta}{\beta} \right) \\ &= 0.5 \times 0.1 \mu\text{A} = 50 \text{ nA} \end{aligned}$$

Ex: 8.17  $I_D = \frac{1}{2} I = 0.4 \text{ mA}$

$$\begin{aligned} I_D &= \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_n V_{OV}^2 \\ 0.4 &= \frac{1}{2} \times 0.2 \times 100 \times V_{OV}^2 \\ \Rightarrow V_{OV} &= 0.2 \text{ V} \end{aligned}$$

$$g_{m1,2} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4 \text{ mA/V}$$

$$G_m = g_{m1,2} = 4 \text{ mA/V}$$

$$r_{o2} = \frac{V_{An}}{I_D} = \frac{20}{0.4} = 50 \text{ k}\Omega$$

$$r_{o4} = \frac{|V_{Ap}|}{I_D} = \frac{20}{0.4} = 50 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 50 \parallel 50 = 25 \text{ k}\Omega$$

$$A_d = G_m R_o = 4 \times 25 = 100 \text{ V/V}$$

$$\text{Ex: 8.18 } G_m = g_{m1,2} \cong \frac{I/2}{V_T} = \frac{0.4 \text{ mA}}{0.025 \text{ V}}$$

$$= 16 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{V_A}{I_C} = \frac{V_A}{I/2} = \frac{100}{0.4} = 250 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 250 \parallel 250 = 125 \text{ k}\Omega$$

$$A_d = G_m R_o = 16 \times 125 = 2000 \text{ V/V}$$

$$R_{id} = 2r_\pi = 2 \times \frac{\beta}{g_{m1,2}} = 2 \times \frac{160}{16} = 20 \text{ k}\Omega$$

Ex: 8.19

$$G_m = g_{m1} = g_{m2} \cong \frac{I/2}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_{o4} = \frac{V_A}{I/2} = \frac{100 \text{ V}}{0.5 \text{ mA}} = 200 \text{ k}\Omega$$

### Exercise 8–5

$$R_{o4} \simeq \beta_4 r_{o4} = 50 \times 200 = 10,000 \text{ k}\Omega = 10 \text{ M}\Omega$$

$$R_{o5} = \frac{1}{2} \beta_5 r_{o5}$$

where

$$r_{o5} = \frac{V_A}{I/2} = 200 \text{ k}\Omega$$

Thus,

$$R_{o5} = \frac{1}{2} \times 100 \times 200 = 10 \text{ M}\Omega$$

$$R_o = R_{o4} \parallel R_{o5} = 10 \parallel 10 = 5 \text{ M}\Omega$$

$$A_d = G_m R_o$$

$$= 20 \text{ mA/V} \times 5000 \text{ k}\Omega = 10^5 \text{ V/V or } 100 \text{ dB}$$

Ex: 8.20 From Exercise 8.17, we get

$$I_D = 0.4 \text{ mA}$$

$$V_{OV} = 0.2 \text{ V } g_{m1,2} = 4 \text{ mA/V}$$

$$G_m = 4 \text{ mA/V } A_d = 100 \text{ V/V}$$

Now,

$$R_{SS} = 25 \text{ k}\Omega$$

$$g_{m3} = \sqrt{2\mu_p C_{ox} \left(\frac{W}{L}\right)_p I_D}$$

$$= \sqrt{2 \times 0.1 \times 200 \times 0.4} = 4 \text{ mA/V}$$

$$|A_{em}| = \frac{1}{2g_{m3}R_{SS}} = \frac{1}{2 \times 4 \times 25} = 0.005 \text{ V/V}$$

$$CMRR = \frac{|A_d|}{|A_{em}|} = \frac{100}{0.005}$$

$$= 20,000 \text{ or } 20 \log 20,000 = 86 \text{ dB}$$

Ex: 8.21 From Exercise 8.18, we get

$$I = 0.8 \text{ mA, } I_C \simeq 0.4 \text{ mA, } V_A = 100 \text{ V}$$

$$g_{m1,2} = 16 \text{ mA/V, } G_m = 16 \text{ mA/V}$$

$$r_{o2} = r_{o4} = 250 \text{ k}\Omega, A_d = 2000 \text{ V/V}$$

Now,

$$R_{EE} = \frac{100 \text{ V}}{0.8 \text{ mA}} = 125 \text{ k}\Omega$$

Using Eq. (8.165),

$$|A_{em}| = \frac{r_{o4}}{\beta_3 R_{EE}} = \frac{250}{160 \times 125} = 0.0125 \text{ V/V}$$

$$CMRR = \frac{|A_d|}{|A_{em}|} = \frac{2000}{0.0125} = 160,000 \text{ V/V}$$

$$20 \log CMRR = 104 \text{ dB}$$

Ex: 8.22 Refer to Fig. (8.40).

(a) Using Eq. (8.170), we obtain

$$I_6 = \frac{(W/L)_6}{(W/L)_4} (I/2)$$

$$\Rightarrow 100 = \frac{(W/L)_6}{100} \times 50$$

$$\text{thus, } (W/L)_6 = 200$$

Using Eq. (8.171), we get

$$I_7 = \frac{(W/L)_7}{(W/L)_5} I$$

$$\Rightarrow 100 = \frac{(W/L)_7}{200} \times 100$$

$$\text{thus, } (W/L)_7 = 200$$

(b) For Q<sub>1</sub>,

$$\frac{I}{2} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$\Rightarrow V_{OV1} = \sqrt{\frac{50}{\frac{1}{2} \times 30 \times 200}} = 0.129 \text{ V}$$

Similarly for Q<sub>2</sub>, V<sub>OV2</sub> = 0.129 V

For Q<sub>6</sub>,

$$100 = \frac{1}{2} \times 90 \times 200 V_{OV6}^2$$

$$\Rightarrow V_{OV6} = 0.105 \text{ V}$$

$$(c) g_m = \frac{2I_D}{V_{OV}}$$

	I <sub>D</sub>	V <sub>OV</sub>	g <sub>m</sub>
Q <sub>1</sub>	50 μA	0.129 V	0.775 mA/V
Q <sub>2</sub>	50 μA	0.129 V	0.775 mA/V
Q <sub>6</sub>	100 μA	0.105 V	1.90 mA/V

$$(d) r_{o2} = 10/0.05 = 200 \text{ k}\Omega$$

$$r_{o4} = 10/0.05 = 200 \text{ k}\Omega$$

$$r_{o6} = 10/0.1 = 100 \text{ k}\Omega$$

$$r_{o7} = 10/0.1 = 100 \text{ k}\Omega$$

(e) Eq. (8.168):

$$A_l = -g_{m1} (r_{o2} \parallel r_{o4})$$

$$= -0.775 (200 \parallel 200) = -77.5 \frac{\text{V}}{\text{V}}$$

Eq. (8.169):

$$A_2 = -g_{m6} (r_{o6} \parallel r_{o7})$$

$$= -95 \text{ V/V}$$

### Exercise 8-6

Overall voltage gain is

$$A_I \times A_2 = -77.5 \times -95 = 7363 \text{ V/V}$$

Ex: 8.23  $R_{id} = 20.2 \text{ k}\Omega$

$$A_{vo} = 8513 \text{ V/V}$$

$$R_o = 152 \Omega$$

With  $R_S = 10 \text{ k}\Omega$  and  $R_L = 1 \text{ k}\Omega$ ,

$$G_v = \frac{20.2}{20.2 + 10} \times 8513 \times \frac{1}{(1 + 0.152)} \\ = 4943 \text{ V/V}$$

Ex: 8.24  $\frac{i_{e8}}{i_{b8}} = \beta_8 + 1 = 101$

$$\frac{i_{b8}}{i_{c7}} = \frac{R_5}{R_5 + R_{i4}} = \frac{15.7}{15.7 + 303.5} = 0.0492$$

$$\frac{i_{c7}}{i_{b7}} = \beta_7 = 100$$

$$\frac{i_{b7}}{i_{c5}} = \frac{R_3}{R_3 + R_{i3}} = \frac{3}{3 + 234.8} = 0.0126$$

$$\frac{i_{c5}}{i_{b5}} = \beta_5 = 100$$

$$\frac{i_{c2}}{i_1} = \frac{R_1 + R_2}{R_1 + R_2 + R_{i2}} = \frac{40}{40 + 5.05} = 0.888$$

$$\frac{i_{c2}}{i_1} = \beta_2 = 100$$

Thus the overall current gain is

$$\frac{i_{e8}}{i_1} = 101 \times 0.0492 \times 100 \times 0.0126 \times 100$$

$$\times 0.888 \times 100$$

$$= 55,599 \text{ A/A}$$

and the overall voltage gain is

$$\frac{V_o}{V_{id}} = \frac{R_6}{R_{i1}} \cdot \frac{i_{e8}}{i_1}$$

$$= \frac{3}{20.2} \times 55599 = 8257 \text{ V/V}$$

## Chapter 8–1

8.1 Refer to Fig. 8.2.

$$(a) \frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$0.08 = \frac{1}{2} \times 0.4 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$V_{GS} = V_{tn} + V_{OV} = 0.4 + 0.2 = 0.6 \text{ V}$$

$$(b) V_{CM} = 0$$

$$V_S = 0 - V_{GS} = -0.6 \text{ V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} = 0.08 \text{ mA}$$

$$V_{D1} = V_{D2} = V_{DD} - I_{D1,2} R_D$$

$$= 1 - 0.08 \times 5 = +0.6 \text{ V}$$

$$(c) V_{CM} = +0.4 \text{ V}$$

$$V_S = 0.4 - V_{GS} = 0.4 - 0.6 = -0.2 \text{ V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} = 0.08 \text{ mA}$$

$$V_{D1} = V_{D2} = V_{DD} - I_{D1,2} R_D$$

$$= 1 - 0.08 \times 5 = +0.6 \text{ V}$$

Since  $V_{CM} = 0.4 \text{ V}$  and  $V_D = 0.6 \text{ V}$ ,  $V_{GD} = -0.2 \text{ V}$ , which is less than  $V_{tn}$  (0.4 V), indicating that our implicit assumption of saturation-mode operation is justified.

$$(d) V_{CM} = -0.1 \text{ V}$$

$$V_S = -0.1 - V_{GS} = -0.1 - 0.6 = -0.7 \text{ V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} = 0.08 \text{ mA}$$

$$V_{D1} = V_{D2} = V_{DD} - I_{D1,2} R_D$$

$$= 1 - 0.08 \times 5 = +0.6 \text{ V}$$

(e) The highest value of  $V_{CM}$  for which  $Q_1$  and  $Q_2$  remain in saturation is

$$V_{CM\max} = V_{D1,2} + V_{tn}$$

$$= 0.6 + 0.4 = 1.0 \text{ V}$$

(f) To maintain the current-source operating properly, we need to keep a minimum voltage of 0.2 V across it, thus

$$V_{Smin} = -V_{SS} + V_{CS} = -1 + 0.2 = -0.8 \text{ V}$$

$$V_{CM\min} = V_{Smin} + V_{GS}$$

$$= -0.8 + 0.6$$

$$= -0.2 \text{ V}$$

8.2 Refer to Fig. P8.2.

(a) For  $v_{G1} = v_{G2} = 0 \text{ V}$ ,

$$I_{D1} = I_{D2} = \frac{1}{2} \times 0.5 = 0.25 \text{ mA}$$

$$I_{D1,2} = \frac{1}{2} k'_p \left( \frac{W}{L} \right) |V_{ov}|^2$$

$$0.25 = \frac{1}{2} \times 4 \times |V_{ov}|^2$$

$$\Rightarrow |V_{ov}| = 0.35 \text{ V}$$

$$V_{SG} = |V_{tp}| + |V_{ov}|$$

$$= 0.8 + 0.35 = 1.15 \text{ V}$$

$$V_S = 0 + V_{SG} = +1.15 \text{ V}$$

$$V_{D1} = V_{D2} = -V_{SS} + I_D R_D$$

$$= -2.5 + 0.25 \times 4$$

$$= -1.5 \text{ V}$$

Since for each of  $Q_1$  and  $Q_2$ ,

$$V_{SD} = 1.15 - (-1.5)$$

$$= 2.65 \text{ V}$$

which is greater than  $|V_{ov}|$ ,  $Q_1$  and  $Q_2$  are operating in saturation as implicitly assumed.

(b) The highest value of  $V_{CM}$  is limited by the need to keep a minimum of 0.4 V across the current source, thus

$$V_{CM\max} = +2.5 - 0.4 - V_{SG}$$

$$= +2.5 - 0.4 - 1.15 = +0.95 \text{ V}$$

The lowest value of  $V_{CM}$  is limited by the need to keep  $Q_1$  and  $Q_2$  in saturation, thus

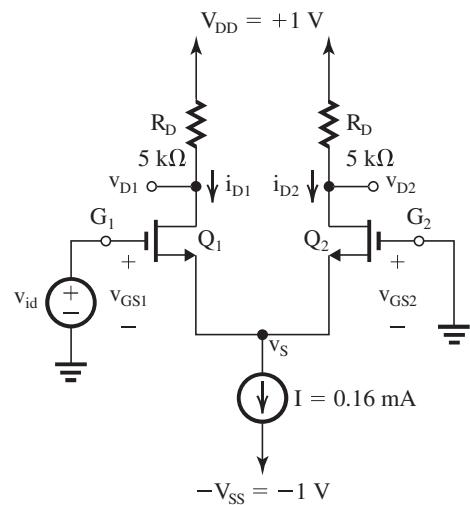
$$V_{CM\min} = V_{D1,2} - |V_{tp}|$$

$$= -1.5 - 0.8 = -2.3 \text{ V}$$

Thus,

$$-2.3 \text{ V} \leq V_{ICM} \leq +0.95 \text{ V}$$

### 8.3



(a) For  $i_{D1} = i_{D2} = 0.08 \text{ mA}$ ,

$$v_{G1} = v_{G2}$$

## Chapter 8–2

Thus,

$$v_{id} = 0 \text{ V}$$

$$i_{D1} = i_{D2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.08 = \frac{1}{2} \times 0.4 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$v_{GS1} = v_{GS2} = 0.2 + 0.4 = 0.6 \text{ V}$$

$$v_S = -0.6 \text{ V}$$

$$v_{D1} = v_{D2} = V_{DD} - i_{D1,2} R_D$$

$$= 1 - 0.08 \times 5 = 0.6 \text{ V}$$

$$v_{D2} - v_{D1} = 0 \text{ V}$$

(b) For  $i_{D1} = 0.12 \text{ mA}$  and  $i_{D2} = 0.04 \text{ mA}$ ,

$$i_{D2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (v_{GS2} - V_{tn})^2$$

$$0.04 = \frac{1}{2} \times 0.4 \times 10 \times (v_{GS2} - 0.4)^2$$

$$\Rightarrow v_{GS2} = 0.541 \text{ V}$$

Thus,

$$v_S = -0.541 \text{ V}$$

$$i_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (v_{GS1} - V_{tn})^2$$

$$0.12 = \frac{1}{2} \times 0.4 \times 10 (v_{id} - v_S - V_{tn})^2$$

$$= \frac{1}{2} \times 0.4 \times 10 (v_{id} + 0.541 - 0.4)^2$$

$$\Rightarrow v_{id} = 0.104 \text{ V}$$

$$v_{GS1} = 0.104 - (-0.541) = 0.645 \text{ V}$$

$$v_{D1} = V_{DD} - i_{D1} R_D$$

$$= 1 - 0.12 \times 5 = 0.4 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2} R_D$$

$$= 1 - 0.04 \times 5 = 0.8 \text{ V}$$

$$v_{D2} - v_{D1} = 0.8 - 0.4 = 0.4 \text{ V}$$

(c)  $i_{D1} = 0.16 \text{ mA}$  and  $i_{D2} = 0$  with  $Q_2$  just cutting off, thus

$$v_{GS2} = V_{tn} = 0.4 \text{ V}$$

$$\Rightarrow v_{S2} = -0.4 \text{ V}$$

$$i_{D1} = \frac{1}{2} \times 0.4 \times 10 (v_{GS1} - V_{tn})^2$$

$$0.16 = \frac{1}{2} \times 0.4 \times 10 (v_{id} + 0.4 - 0.4)^2$$

$$\Rightarrow v_{id} = 0.283 \text{ V}$$

which is  $\sqrt{2} V_{OV}$ , as derived in the text.

$$v_{GS1} = 0.283 - (-0.4) = 0.683 \text{ V}$$

$$v_{D1} = V_{DD} - i_{D1} R_D$$

$$= 1 - 0.16 \times 5 = +0.2 \text{ V}$$

Note that since  $v_{G1} = v_{id} = 0.283 \text{ V}$ ,  $Q_1$  is still operating in saturation, as implicitly assumed.

$$v_{D2} = V_{DD} - i_{D2} R_D$$

$$= 1 - 0 \times 5 = 1 \text{ V}$$

$$v_{D2} - v_{D1} = 1 - 0.2 = 0.8 \text{ V}$$

(d)  $i_{D1} = 0.04 \text{ mA}$  and  $i_{D2} = 0.12 \text{ mA}$ . Since this split of the current I is the complement of that in case (b) above, the value of  $v_{id}$  must be the negative of that found in (b). Thus,

$$v_{id} = -0.104 \text{ V}$$

$$v_{GS1} = 0.541 \text{ V}$$

$$v_S = -0.645 \text{ V}$$

$$v_{GS2} = 0.645 \text{ V}$$

$$v_{D1} = V_{DD} - i_{D1} R_D$$

$$= 1 - 0.04 \times 5 = 0.8 \text{ V}$$

$$v_{D2} = 1 - 0.12 \times 5 = 0.4 \text{ V}$$

$$v_{D2} - v_{D1} = -0.4 \text{ V}$$

(e)  $i_{D1} = 0$  ( $Q_1$  just cuts off) and  $i_{D2} = 0.16 \text{ mA}$ . This case is the complement of that in (c) above, thus

$$v_{GS1} = V_{tn} = 0.4 \text{ V}$$

$$v_{GS2} = 0.683 \text{ V}$$

$$v_S = -0.683 \text{ V}$$

$$v_{id} = -0.683 + 0.4 = -0.283 \text{ V}$$

which is  $-\sqrt{2} V_{OV}$ , as derived in the text.

$$v_{D1} = V_{DD} - i_{D1} R_D = 1 - 0 \times 5 = 1 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2} R_D = 1 - 0.16 \times 5 = 0.2 \text{ V}$$

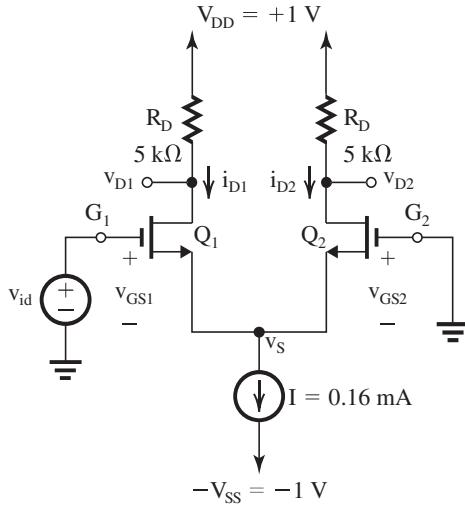
$$v_{D2} - v_{D1} = -0.8 \text{ V}$$

### Summary

A summary of the results is shown in the following table on the next page.

Case	$i_{D1}$ (mA)	$i_{D2}$ (mA)	$v_{id}$ (V)	$v_s$ (V)	$v_{D1}$ (V)	$v_{D2}$ (V)	$v_{D2} - v_{D1}$ (V)
a	0.08	0.08	0	-0.6	+0.6	+0.6	0
b	0.12	0.04	+0.104	-0.541	+0.4	+0.8	+0.4
c	0.16	0	+0.283	-0.4	+0.2	+1.0	+0.8
d	0.04	0.12	-0.104	-0.645	+0.8	+0.4	-0.4
e	0	0.16	-0.283	-0.683	+1.0	+0.2	-0.8

8.4



For  $i_{D1} = 0.09$  mA and  $i_{D2} = 0.07$  mA,

$$i_{D2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (v_{GS2} - V_{tn})^2$$

$$0.07 = \frac{1}{2} \times 0.4 \times 10 (v_{GS2} - 0.4)^2$$

$$\Rightarrow v_{GS2} = 0.587 \text{ V}$$

and

$$v_s = -0.587 \text{ V}$$

$$i_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (v_{GS1} - V_{tn})^2$$

$$0.09 = \frac{1}{2} \times 0.4 \times 10 (v_{GS1} - 0.4)^2$$

$$\Rightarrow v_{GS1} = 0.612 \text{ V}$$

$$v_{id} = v_s + v_{GS1} = -0.587 + 0.612$$

$$= 0.025 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2} R_D$$

$$= 1 - 0.07 \times 5 = 0.65 \text{ V}$$

$$v_{D1} = 1 - 0.09 \times 5 = 0.55 \text{ V}$$

$$v_{D2} - v_{D1} = 0.65 - 0.55 = 0.10 \text{ V}$$

$$\text{Voltage gain} = \frac{v_{D2} - v_{D1}}{v_{id}} = \frac{0.10}{0.025} = 4 \text{ V/V}$$

To obtain the complementary split in current, that is,  $i_{D1} = 0.07$  mA and  $i_{D2} = 0.09$  mA,

$$v_{id} = -0.025 \text{ V}$$

8.5 Refer to Fig. P8.2.

To determine  $V_{OV}$ ,

$$0.25 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.354 \text{ V}$$

With  $v_{G2} = 0$  and  $v_{G1} = v_{id}$ , to steer the current from one side of the differential pair to the other,  $v_{id}$  must be the ends of the range

$$-\sqrt{2} |V_{OV}| \leq v_{id} \leq \sqrt{2} |V_{OV}|$$

that is,

$$-0.5 \text{ V} \leq v_{id} \leq +0.5 \text{ V}$$

At  $v_{id} = -0.5$  V,  $Q_2$  just cuts off, thus

$$v_s = |V_{tp}| = 0.8 \text{ V}$$

and

$$v_{SG1} = 0.8 - (-0.5) = 1.3 \text{ V}$$

At this value of  $v_{SG1}$ ,

$$i_{D1} = \frac{1}{2} \times 4 \times (1.3 - 0.8)^2$$

$$= 0.5 \text{ mA}$$

which is the entire bias current.

$$v_{D1} = -2.5 + 0.5 \times 4 = -0.5 \text{ V}$$

Observe that since  $v_{G1} = v_{D1}$ ,  $Q_1$  is still operating in saturation, as implicitly assumed.

$$v_{D2} = -2.5 \text{ V}$$

At  $v_{id} = +0.5$  V,  $Q_1$  just cuts off, thus

$$v_{SG1} = |V_{tp}| = 0.8 \text{ V}$$

$$v_s = +0.5 + 0.8 = +1.3 \text{ V}$$

and thus

$$v_{SG2} = 1.3 \text{ V}$$

which results in

$$i_{D1} = \frac{1}{2} \times 4 (1.3 - 0.8)^2$$

$$= 0.5 \text{ mA}$$

which is the entire bias current. Here,

$$v_{D2} = -2.5 + 0.5 \times 4 = -0.5 \text{ V}$$

which verifies that  $Q_2$  is operating in saturation, as implicitly assumed.

8.6 Refer to the circuit in Fig. P8.6.

For  $v_{G1} = v_{G2} = 0 \text{ V}$ ,

$$I_{D1} = I_{D2} = \frac{0.4}{2} = 0.2 \text{ mA}$$

To obtain

$$V_{D1} = V_{D2} = +0.1 \text{ V}$$

$$V_{DD} - I_{D1,2} R_D = 0.1$$

$$0.9 - 0.2 R_D = 0.1$$

$$\Rightarrow R_D = 4 \text{ k}\Omega$$

For  $Q_1$  and  $Q_2$ ,

$$I_{D1,2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4 \left( \frac{W}{L} \right)_{1,2} \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_{1,2} = 44.4$$

For  $Q_3$ ,

$$0.4 = \frac{1}{2} \times 0.4 \times \left( \frac{W}{L} \right)_3 \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_3 = 88.8$$

Since  $Q_3$  and  $Q_4$  form a current mirror with  $I_{D3} = 4I_{D4}$ ,

$$\left( \frac{W}{L} \right)_4 = \frac{1}{4} \left( \frac{W}{L} \right)_3 = 22.2$$

$$V_{GS4} = V_{GS3} = V_{tn} + V_{OV} = 0.4 + 0.15$$

$$= 0.55 \text{ V}$$

$$R = \frac{0.9 - (-0.9) - 0.55}{0.1}$$

$$= 12.5 \text{ k}\Omega$$

The lower limit on  $V_{CM}$  is determined by the need to keep  $Q_3$  operating in saturation. For this to happen, the minimum value of  $V_{DS3}$  is

$$V_{OV} = 0.15 \text{ V. Thus,}$$

$$V_{ICMmin} = -V_{SS} + V_{OV3} + V_{GS1,2}$$

$$= -0.9 + 0.15 + 0.4 + 0.15$$

$$= -0.2 \text{ V}$$

The upper limit on  $V_{CM}$  is determined by the need to keep  $Q_1$  and  $Q_2$  in saturation, thus

$$V_{ICMmax} = V_{D1,2} + V_{tn}$$

$$= 0.1 + 0.4 = 0.5 \text{ V}$$

Thus,

$$-0.2 \text{ V} \leq V_{ICM} \leq +0.5 \text{ V}$$

8.7 Refer to Eq. (8.23). For

$$\begin{aligned} \left( \frac{v_{id}/2}{V_{OV}} \right)^2 &\leq k \\ \Rightarrow \left( \frac{v_{id}/2}{V_{OV}} \right) &\leq \sqrt{k} \end{aligned} \quad (1)$$

$$\Delta I = I \left( \frac{v_{id}/2}{V_{OV}} \right) \sqrt{1 - \left( \frac{v_{id}/2}{V_{OV}} \right)^2}$$

$$\Delta I_{max} = I \sqrt{k} \sqrt{1 - k}$$

Thus,

$$\frac{\Delta I_{max}}{I/2} = 2\sqrt{k(1-k)} \quad \text{Q.E.D.} \quad (2)$$

and the corresponding value of  $v_{id}$  is found from Eq. (2) as

$$v_{idmax} = 2\sqrt{k} V_{OV} \quad \text{Q.E.D.} \quad (3)$$

Equations (2) and (3) can be used to evaluate

$$\frac{\Delta I_{max}}{I/2} \text{ and } \frac{v_{idmax}}{V_{OV}}$$
 for various values of  $k$ :

$k$	0.01	0.1	0.2
$\frac{v_{idmax}}{V_{OV}}$	0.2	0.632	0.894
$\frac{\Delta I_{max}}{I/2}$	0.2	0.6	0.8

8.8 Switching occurs at

$$v_{id} = \sqrt{2} V_{OV}$$

Thus,

$$0.3 = \sqrt{2} V_{OV}$$

$$\Rightarrow V_{OV} = 0.212 \text{ V}$$

Now, to obtain full current switching at  $v_{id} = 0.5 \text{ V}$ ,  $V_{OV}$  must be increased to

$$V_{OV} = 0.212 \times \frac{0.5}{0.3} = 0.353 \text{ V}$$

Since  $I_D$  is proportional to  $V_{OV}^2$ , the current  $I_D$  and hence the bias current  $I$  must be increased by the ratio  $(0.353/0.212)^2$ , then  $I$  must be

$$I = 200 \times \left( \frac{0.353}{0.212} \right)^2 = 554.5 \mu\text{A}$$

## Chapter 8–5

8.9 Refer to Fig. 8.5.

$$g_m = \frac{2(I/2)}{V_{ov}} = \frac{I}{V_{ov}}$$

$$1 = \frac{I}{0.25}$$

$$\Rightarrow I = 0.25 \text{ mA}$$

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{ov}^2$$

$$\frac{1}{2} \times 0.25 = \frac{1}{2} \times 0.4 \times \left( \frac{W}{L} \right) 0.25^2$$

$$\Rightarrow \frac{W}{L} = 10$$

$$8.10 \quad 0.1 = \frac{1}{2} \times 0.2 \times 32 V_{ov}^2$$

$$\Rightarrow V_{ov} = 0.18 \text{ V}$$

$$g_m = \frac{2 \times (0.2/2)}{0.18} = 1.11 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$A_d = g_m (R_D \parallel r_o)$$

$$= 1.11 \times (10 \parallel 100) = 10.1 \text{ V/V}$$

8.11 For  $v_{id} = 0.1 \text{ V}$

$$\left( \frac{v_{id}/2}{V_{ov}} \right)^2 = 0.04$$

$$\frac{v_{id}/2}{V_{ov}} = 0.2$$

$$\frac{0.1/2}{V_{ov}} = 0.2$$

$$\Rightarrow V_{ov} = 0.25 \text{ V}$$

$$g_m = \frac{2 \times (I/2)}{V_{ov}}$$

$$2 = \frac{I}{0.25}$$

$$\Rightarrow I = 0.5 \text{ mA}$$

$$A_d = \frac{1 \text{ V}}{0.1 \text{ V}} = 10$$

$$g_m R_D = 10$$

$$\Rightarrow R_D = \frac{10}{2} = 5 \text{ k}\Omega$$

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{ov}^2$$

$$0.25 = \frac{1}{2} \times 0.2 \times \frac{W}{L} \times 0.25^2$$

$$\Rightarrow \frac{W}{L} = 40$$

8.12 Since the quiescent power dissipation is

$$P = (V_{DD} + V_{SS}) \times I$$

then the maximum allowable  $I$  is

$$I = \frac{1 \text{ mW}}{2 \text{ V}} = 0.5 \text{ mA}$$

We shall utilize this value. The value of  $V_{ov}$  can be found from

$$\sqrt{2} V_{ov} = 0.25 \text{ V}$$

$$\Rightarrow V_{ov} = \frac{0.25}{\sqrt{2}} = 0.18 \text{ V}$$

The realized value of  $g_m$  will be

$$g_m = \frac{2 \times (I/2)}{V_{ov}}$$

$$= \frac{0.5}{0.18} = 2.8 \text{ mA/V}$$

To obtain a differential gain  $A_d$  of 10 V/V,

$$A_d = g_m R_D$$

$$10 = 2.8 \times R_D$$

$$\Rightarrow R_D = 3.6 \text{ k}\Omega$$

Finally, the required value of  $W/L$  can be determined from

$$I_D = \frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{ov}^2$$

$$0.25 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.18^2$$

$$\Rightarrow \frac{W}{L} = 38.6$$

8.13 To limit the power dissipation to 1 mW,

$$P = (V_{DD} + V_{SS}) I$$

Thus, the maximum value we can use for  $I$  is

$$I = \frac{1 \text{ mW}}{2 \text{ V}} = 0.5 \text{ mA}$$

Using this value, we obtain

$$V_D = V_{DD} - \frac{I}{2} R_D$$

$$0.2 = 1 - 0.25 \times R_D$$

$$\Rightarrow R_D = 3.2 \text{ k}\Omega$$

$$A_d = g_m R_D$$

$$10 = g_m \times 3.2$$

$$g_m = \frac{10}{3.2} = 3.125 \text{ mA/V}$$

But

$$g_m = \frac{2 \times (I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$3.125 = \frac{0.5}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.16 \text{ V}$$

To obtain W/L, we use

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.25 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.16^2$$

$$\Rightarrow \frac{W}{L} = 48.8 \simeq 50$$

$$8.14 \text{ (a)} \quad A_d = g_m R_D$$

$$20 = g_m \times 47$$

$$\Rightarrow g_m = \frac{20}{47} = 0.426 \text{ mA/V}$$

$$(b) \quad g_m = \frac{2I_D}{V_{OV}} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$0.426 = \frac{I}{0.2}$$

$$\Rightarrow I = 0.085 \text{ mA} = 85 \mu\text{A}$$

(c) Across each  $R_D$  the dc voltage is

$$\frac{I}{2} R_D = \frac{0.085}{2} \times 47 = 2 \text{ V}$$

(d) The peak sine-wave signal across each gate source is 5 mV, thus at each drain the peak sine wave is

$$A_d \times 5 = 20 \times 5 = 100 \text{ mV} = 0.1 \text{ V}$$

(e) The minimum voltage at each drain will be

$$V_{Dmin} = V_{DD} - R_D I_D - V_{peak}$$

$$= V_{DD} - 2 - 0.1$$

For the transistor to remain in saturation

$$V_{Dmin} \geq V_{Gmax} - V_{tn}$$

where

$$V_{Gmax} = V_{CM} + V_{peak}(\text{input})$$

$$= 0.5 + 0.005 = 0.505 \text{ V}$$

Thus,

$$V_{DD} - 2.1 \geq 0.505 - 0.5$$

$$V_{DD} \geq 2.105 \text{ V}$$

Thus, the lowest value of  $V_{DD}$  is 2.21 V.

8.15 Since both circuits use the same supply voltages and dissipate equal powers, then their currents must be equal, that is,

$$I_D = I$$

where  $I_D$  is the bias current of the CS amplifier and  $I$  is the bias current of the differential pair. The gain of the CS amplifier is

$$|A| = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \left( \frac{W}{L} \right)_{CS} I_D}$$

Thus,

$$|A| = \sqrt{2\mu_n C_{ox} \left( \frac{W}{L} \right)_{CS} I_D R_D} \quad (1)$$

The gain of the differential amplifier is

$$A_d = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \left( \frac{W}{L} \right)_{diff} \left( \frac{I}{2} \right)}$$

Thus,

$$A_d = \sqrt{2\mu_n C_{ox} \left( \frac{W}{L} \right)_{diff} \left( \frac{I}{2} \right) R_D} \quad (2)$$

Equating the gains in Eqs. (1) and (2) and substituting  $I_D = I$  gives

$$\sqrt{\left( \frac{W}{L} \right)_{CS}} = \sqrt{\left( \frac{W}{L} \right)_{diff} \times \frac{1}{2}}$$

$$\Rightarrow \left( \frac{W}{L} \right)_{diff} = 2 \left( \frac{W}{L} \right)_{CS}$$

If all transistors have the same channel length, each of the differential pair transistors must be twice as wide as the transistor in the CS amplifier.

8.16 For a CS amplifier biased at a current  $I_D$  and utilizing a drain resistance  $R_D$ , the voltage gain is

$$|A| = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

Thus,

$$|A| = \sqrt{2\mu_n C_{ox} \frac{W}{L} \sqrt{I_D} R_D} \quad (1)$$

For a differential pair biased with a current  $I$  and utilizing drain resistances  $R_D$ , the differential gain is

$$A_d = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} \left(\frac{I}{2}\right)}$$

Thus

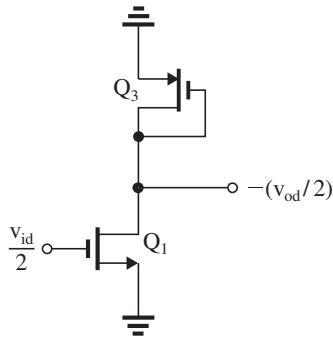
$$A_d = \sqrt{2\mu_n C_{ox} \frac{W}{L}} \sqrt{I/2} R_D \quad (2)$$

Equating the gains from Eqs. (1) and (2), we get

$$I = 2I_D$$

That is, the differential pair must be biased at a current twice that of the CS amplifier. Since both circuits use equal power supplies, the power dissipation of the differential pair will be twice that of the CS amplifier.

8.17



(a) The figure shows the differential half-circuit. Recalling that the incremental (small-signal) resistance of a diode-connected transistor is given by  $\left(\frac{1}{g_m} \parallel r_o\right)$ , the equivalent load resistance of  $Q_1$  will be

$$R_D = \frac{1}{g_{m3}} \parallel r_{o3}$$

and the differential gain of the amplifier in Fig. P8.17 will be

$$A_d \equiv \frac{v_{od}}{v_{id}} = g_{ml} \left[ \frac{1}{g_{m3}} \parallel r_{o3} \parallel r_{o1} \right]$$

Since both sides of the amplifier are matched, this expression can be written in a more general way as

$$A_d = g_{ml,2} \left[ \frac{1}{g_{m3,4}} \parallel r_{o3,4} \parallel r_{o1,2} \right]$$

(b) Neglecting  $r_{o1,2}$  and  $r_{o3,4}$  (much larger than  $1/g_{m3,4}$ ),

$$A_d \simeq \frac{g_{ml,2}}{g_{m3,4}}$$

$$= \frac{\sqrt{2\mu_n C_{ox} (W/L)_{1,2} (I/2)}}{\sqrt{2\mu_p C_{ox} (W/L)_{3,4} (I/2)}}$$

$$= \sqrt{\frac{\mu_n (W/L)_{1,2}}{\mu_p (W/L)_{3,4}}}$$

(c)  $\mu_n = 4\mu_p$  and all channel lengths are equal,

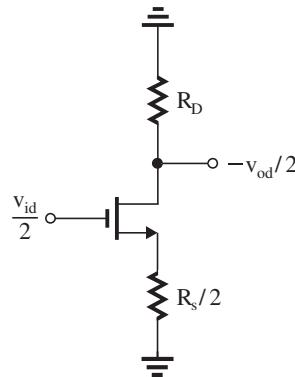
$$A_d = 2 \sqrt{\frac{W_{1,2}}{W_{3,4}}}$$

For  $A_d = 10$ ,

$$10 = 2 \sqrt{\frac{W_{1,2}}{W_{3,4}}}$$

$$\Rightarrow \frac{W_{1,2}}{W_{3,4}} = 25$$

8.18



From symmetry, a virtual ground appears at the mid point of  $R_s$ . Thus, the differential half circuit will be as shown in the figure, and

$$A_d \equiv \frac{v_{od}}{v_{id}} = \frac{R_D}{\frac{1}{g_m} + \frac{R_s}{2}}$$

For  $R_s = 0$ ,

$$A_d = \frac{R_D}{1/g_m} = g_m R_D,$$

as expected.

To reduce the gain to half this value, we use

$$\frac{R_s}{2} = \frac{1}{g_m}$$

$$\Rightarrow R_s = \frac{2}{g_m}$$

8.19 Refer to Fig. P8.19.

(a) With  $v_{G1} = v_{G2} = 0$ ,

$$v_{GS1} = v_{GS2} = V_{OV1,2} + V_{tn}$$

Thus

$$V_{S1} = V_{S2} = -(V_{OV1,2} + V_{tn})$$

(b) For the situation in (a),  $V_{DS}$  of  $Q_3$  is zero, thus zero current "ows in  $Q_3$ . Transistor  $Q_3$  will have an overdrive voltage of

$$\begin{aligned} V_{OV3} &= V_C - V_{S1,2} - V_{tn} \\ &= V_C + (V_{OV1,2} + V_{tn}) - V_{tn} \\ &= V_C + V_{OV1,2} \end{aligned}$$

(c) With  $v_{G1} = v_{id}/2$  and  $v_{G2} = -v_{id}/2$  where  $v_{id}$  is a small signal, a small signal will appear between drain and source of  $Q_3$ . Transistor  $Q_3$  will be operating in the triode region and its drain-source resistance  $r_{DS}$  will be given by

$$r_{DS} = \frac{1}{\mu_n C_{ox} \left( \frac{W}{L} \right)_3 V_{OV3}}$$

Thus,

$$R_s = \frac{1}{\mu_n C_{ox} \left( \frac{W}{L} \right)_3 V_{OV3}}$$

Now,

$$g_{m1,2} = (\mu_n C_{ox}) \left( \frac{W}{L} \right)_{1,2} V_{OV1,2}$$

$$g_{m3} = (\mu_n C_{ox}) \left( \frac{W}{L} \right)_3 V_{OV3}$$

$$\text{For } \left( \frac{W}{L} \right)_3 = \left( \frac{W}{L} \right)_{1,2},$$

$$\mu_n C_{ox} \left( \frac{W}{L} \right) = \frac{g_{m1,2}}{V_{OV1,2}}$$

Thus,

$$R_s = \frac{1}{\frac{g_{m1,2}}{V_{OV1,2}} \times V_{OV3}} = \frac{1}{g_{m1,2}} \frac{V_{OV1,2}}{V_{OV3}}$$

$$(d) (i) R_s = \frac{1}{g_{m1,2}}$$

$$V_{OV3} = V_{OV1,2}$$

But

$$V_{OV3} = V_C + V_{OV1,2}$$

$$\Rightarrow V_C = 0$$

$$(ii) R_s = \frac{0.5}{g_{m1,2}}$$

$$\Rightarrow V_{OV3} = 2 V_{OV1,2}$$

But

$$V_{OV3} = V_C + V_{OV1,2}$$

$$\Rightarrow V_C = V_{OV1,2}$$

8.20 Refer to Fig. P8.20.

(a) With  $v_{G1} = v_{G2} = 0$  V,

$$V_{S1} = V_{S2} = -V_{GS1,2} = -(V_t + V_{OV})$$

The current through  $Q_3$  and  $Q_4$  will be zero because the voltage across them ( $v_{DS3} + v_{DS4}$ ) is zero.

Because the voltages at their gates are zero and at their sources are  $-(V_t + V_{OV})$ , each of  $Q_3$  and  $Q_4$  will be operating at an overdrive voltage equal to  $V_{OV}$ . Thus each of  $Q_3$  and  $Q_4$  will have an  $r_{DS}$  given by

$$r_{DS3,4} = \frac{1}{\mu_n C_{ox} \left( \frac{W}{L} \right)_{3,4} V_{OV}} \quad (1)$$

Since

$$g_{m1,2} = \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{OV} \quad (2)$$

substituting from (2) into (1) gives

$$r_{DS3,4} = \frac{1}{g_{m1,2}} \frac{(W/L)_{1,2}}{(W/L)_{3,4}}$$

and since

$$R_s = r_{DS3} + r_{DS4}$$

then

$$R_s = \frac{2}{g_{m1,2}} \frac{(W/L)_{1,2}}{(W/L)_{3,4}} \quad (3)$$

(b) With  $v_{G1} = v_{id}/2$  and  $v_{G2} = -v_{id}/2$  where  $v_{id}$  is a small signal,

$$A_d \equiv \frac{V_{od}}{V_{id}}$$

$$= \frac{2 R_D}{\frac{1}{g_{m1}} + R_s + \frac{1}{g_{m2}}}$$

Using (3), we obtain

$$\begin{aligned} A_d &= \frac{R_D}{\frac{1}{g_{m1,2}} + \frac{1}{g_{m1,2}} \frac{(W/L)_{1,2}}{(W/L)_{3,4}}} \\ &= \frac{g_{m1,2} R_D}{1 + \frac{(W/L)_{1,2}}{(W/L)_{3,4}}} \end{aligned}$$

8.21 Refer to Fig. P8.21.

The value of R is found as follows:

$$\begin{aligned} R &= \frac{V_{G6} - V_{G7}}{I_{REF}} \\ &= \frac{0.8 - (-0.8)}{0.2} = 8 \text{ k}\Omega \end{aligned}$$

Since  $I = I_{REF}$ ,  $Q_3$  and  $Q_6$  are matched and are operating at

$$|V_{ov}| = 1.5 - 0.8 - 0.5 = 0.2 \text{ V}$$

Thus,

$$0.2 = \frac{1}{2} \times 0.1 \times \left(\frac{W}{L}\right)_{6,3} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_6 = 100$$

Each of  $Q_4$  and  $Q_5$  is conducting a dc current of  $(I/2)$  while  $Q_7$  is conducting a dc current  $I_{REF} = I$ . Thus  $Q_4$  and  $Q_5$  are matched and their W/L ratios are equal while  $Q_7$  has twice the (W/L) ratio of  $Q_4$  and  $Q_5$ . Thus,

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_{4,5} V_{ov4,5}^2$$

where

$$V_{ov4,5} = -0.8 - (-1.5) - 0.5 = 0.2 \text{ V}$$

thus,

$$0.1 = \frac{1}{2} \times 0.25 \times \left(\frac{W}{L}\right)_{4,5} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = 20$$

and

$$\left(\frac{W}{L}\right)_7 = 40$$

$$r_{o4} = r_{o5} = \frac{|V_{Ap}|}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{V_{An}}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$A_d = g_{m1,2}(r_{o1,2} \parallel r_{o4,5})$$

$$50 = g_{m1,2}(100 \parallel 100)$$

$$\Rightarrow g_{m1,2} = 1 \text{ mA/V}$$

But

$$g_{m1,2} = \frac{2(I/2)}{|V_{ov1,2}|}$$

$$1 = \frac{0.2}{|V_{ov1,2}|}$$

$$\Rightarrow |V_{ov1,2}| = 0.2 \text{ V}$$

The (W/L) ratio for  $Q_1$  and  $Q_2$  can now be determined from

$$0.1 = \frac{1}{2} \times 0.1 \times \left(\frac{W}{L}\right)_{1,2} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 50$$

A summary of the results is provided in the table below.

Transistor	W/L	$I_D(\text{mA})$	$ V_{GS} (\text{V})$
$Q_1$	50	0.1	0.7
$Q_2$	50	0.1	0.7
$Q_3$	100	0.2	0.7
$Q_4$	20	0.1	0.7
$Q_5$	20	0.1	0.7
$Q_6$	100	0.2	0.7
$Q_7$	40	0.2	0.7

## 8.22 Refer to Fig. 8.13.

All transistors have the same channel length and are carrying a dc current  $I/2$ . Thus all transistors have the same  $r_o = \frac{|V_A|}{I/2}$ . Also, all transistors are operating at the same  $|V_{ov}|$  and have equal dc currents, thus all have the same

$g_m = \frac{2(I/2)}{|V_{ov}|} = I/|V_{ov}|$ . Thus all transistors have equal intrinsic gain  $g_m r_o = 2|V_A|/|V_{ov}|$ . Now, the gain  $A_d$  is given by

$$\begin{aligned} A_d &= g_m (R_{on} \parallel R_{op}) \\ &= \frac{1}{2} g_m R_{on} \\ &= \frac{1}{2} g_m (g_m r_o) r_o = \frac{1}{2} (g_m r_o)^2 \end{aligned}$$

Thus,

$$\begin{aligned} A_d &= \frac{1}{2} \left[ \frac{2|V_A|}{|V_{ov}|} \right]^2 \\ &= 2(|V_A|/|V_{ov}|)^2 \quad \text{Q.E.D.} \end{aligned}$$

To obtain  $A_d = 500 \text{ V/V}$  while operating all transistors at  $|V_{ov}| = 0.2 \text{ V}$ , we use

$$500 = 2 \frac{|V_A|^2}{0.04}$$

$$\Rightarrow |V_A| = 3.16 \text{ V}$$

Since  $|V_A'| = 5 \text{ V}/\mu \text{m}$ , the channel length L (for all transistors) must be

$$3.16 = 5 \times L$$

$$L = 0.632 \mu \text{m}$$

To obtain the highest possible  $g_m$ , we operate at the highest possible I consistent with limiting the

power dissipation (in equilibrium) to 0.5 mW.  
Thus,

$$I = \frac{0.5 \text{ mW}}{(0.9 + 0.9)\text{V}} = 0.28 \text{ mA}$$

8.23 Refer to Fig. 8.15(a).

The current I will split equally between  $Q_1$  and  $Q_2$ . Thus,

$$I_{E1} = I_{E2} = 0.2 \text{ mA}$$

$$I_{C1} = I_{C2} = \alpha \times 0.2 = 0.99 \times 0.2 = 0.198 \text{ mA}$$

$$V_{BE1} = V_{BE2} = 0.7 + 0.025 \ln\left(\frac{0.198}{1}\right)$$

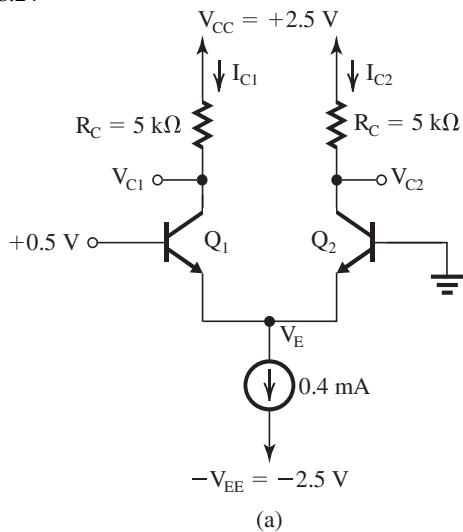
$$= 0.660 \text{ V}$$

$$V_{E1} = V_{E2} = -1 - 0.66 = -1.66 \text{ V}$$

$$V_{C1} = V_{C2} = V_{CC} - I_{C1,2} R_C$$

$$= 2.5 - 0.198 \times 5 = +1.51 \text{ V}$$

8.24



(a) For  $v_{BI} = +0.5 \text{ V}$ ,  $Q_1$  conducts all the current I (0.4 mA) while  $Q_2$  cuts off. Thus  $Q_1$  will have a  $V_{BE}$  obtained as follows:

$$V_{BE1} = 0.7 + 0.025 \ln\left(\frac{0.99 \times 0.4}{1}\right)$$

$$= 0.677 \text{ V}$$

Thus,

$$V_E = +0.5 - 0.677 = -0.177 \text{ V}$$

which indicates that  $V_{BE2} = +0.177 \text{ V}$ , too small to turn  $Q_2$  on.

$$V_{C1} = V_{CC} - I_{C1} R_C$$

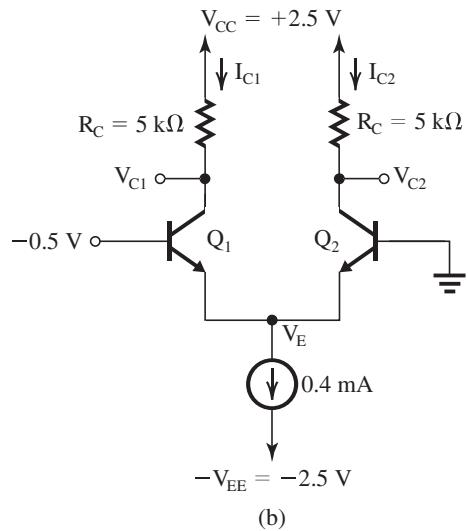
$$= 2.5 - 0.99 \times 0.4 \times 5$$

$$= +0.52 \text{ V}$$

$$V_{C2} = V_{CC} - I_{C2} \times R_C$$

$$= 2.5 - 0 \times 5 = 2.5 \text{ V}$$

Observe that  $Q_1$  is operating in the active mode, as implicitly assumed, and the current source has a voltage of 2.323 V across it, more than sufficient for its proper operation.



(b) With  $v_{BI} = -0.5 \text{ V}$ ,  $Q_1$  turns off and  $Q_2$  conducts all the bias current (0.4 mA) and thus exhibits a  $V_{BE}$  of 0.677 V, thus

$$V_E = -0.677 \text{ V}$$

which indicated that  $V_{BE1} = +0.177 \text{ V}$ , which is too small to turn  $Q_1$  on. Also, note that the current source has a voltage of  $-0.677 + 2.5 = 1.823 \text{ V}$  across it, more than sufficient for its proper operation.

$$V_{C1} = V_{CC} - I_{C1} R_C$$

$$= 2.5 - 0 \times 5 = 2.5 \text{ V}$$

$$V_{C2} = 2.5 - 0.99 \times 0.4 \times 5 = +0.52 \text{ V}$$

8.25 Refer to Fig. 8.15(a) and assume the current source I is implemented with a single BJT that requires a minimum of 0.3 V for proper operation. Thus, the minimum voltage allowed at the emitters of  $Q_1$  and  $Q_2$  is  $-2.5 \text{ V} + 0.3 \text{ V} = -2.2 \text{ V}$ . Now, since each of  $Q_1$  and  $Q_2$  is conducting a current of 0.2 mA, their  $V_{BE}$  voltages will be equal:

$$V_{BE1,2} = 0.7 + 0.025 \ln\left(\frac{0.99 \times 0.2}{1}\right)$$

$$= 0.660 \text{ V}$$

Thus, the minimum allowable  $V_{CM}$  is

$$V_{CM\min} = -2.2 + 0.660 = -1.54 \text{ V}$$

The upper limit on  $V_{CM}$  is dictated by the need to keep  $Q_1$  and  $Q_2$  operating in the active mode, thus

$$\begin{aligned} V_{CM\max} &= 0.4 + V_{C1,2} \\ &= 0.4 + (2.5 - 0.99 \times 0.2 \times 5) \\ &= +1.91 \text{ V} \end{aligned}$$

Thus, the input common-mode range is

$$-1.54 \text{ V} \leq V_{ICM} \leq 1.91 \text{ V}$$

8.26 (a) Refer to Fig. 8.15(a).

$$I_{E1} = I_{E2} = \frac{I}{2} = 10 \mu\text{A}$$

$$I_{C1} = I_{C2} = \alpha \times 10 = 0.98 \times 10 = 9.8 \mu\text{A}$$

$$\begin{aligned} V_{BE1} = V_{BE2} &= 0.690 + 0.025 \ln\left(\frac{9.8 \times 10^{-3}}{1}\right) \\ &= 0.574 \text{ V} \end{aligned}$$

Thus,

$$V_E = -0.574 \text{ V}$$

$$V_{C1} = V_{C2} = V_{CC} - I_C R_C$$

$$= 1.2 - 9.8 \times 10^{-3} \times 82$$

$$\simeq 0.4 \text{ V}$$

(b) Refer to Fig. 8.15(a).

The maximum value of  $V_{CM}$  is limited by the need to keep  $Q_1$  and  $Q_2$  in the active mode. This is achieved by keeping  $V_{CE1,2} \geq 0.3 \text{ V}$ .

Since  $V_{C1,2} = 0.4 \text{ V}$ ,

$$V_{E\max} = 0.4 - 0.3 = 0.1 \text{ V}$$

and

$$V_{CM\max} = V_{BE1,2} + V_{E\max}$$

$$V_{CM\max} = 0.574 + 0.1 = 0.674 \text{ V}$$

The minimum value of  $V_{CM}$  is dictated by the need to keep the current source operating properly, i.e. to keep 0.3 V across it, thus

$$V_{E\min} = -1.2 + 0.3 = -0.9 \text{ V}$$

and

$$V_{CM\min} = V_{E\min} + V_{BE1,2}$$

$$= -0.9 + 0.574 = -0.326 \text{ V}$$

Thus, the input common-mode range is

$$-0.326 \text{ V} \leq V_{ICM} \leq +0.674 \text{ V}$$

(c) Refer to Fig. 8.15(d).

$$i_{E1} = 11 \mu\text{A}, \quad i_{E2} = 9 \mu\text{A}$$

$$i_{C1} = 10.78 \mu\text{A}, \quad i_{C2} = 8.82 \mu\text{A}$$

$$\begin{aligned} v_{BE1} &= 0.69 + 0.025 \ln\left(\frac{10.78 \times 10^{-3}}{1}\right) \\ &= 0.5767 \text{ V} \end{aligned}$$

$$\begin{aligned} v_{BE2} &= 0.69 + 0.025 \ln\left(\frac{8.82 \times 10^{-3}}{1}\right) \\ &= 0.5717 \text{ V} \end{aligned}$$

Thus,

$$v_{BI} = v_{BE1} - v_{BE2}$$

$$= 0.5767 - 0.5717 = 0.005 \text{ V}$$

$$= 5 \text{ mV}$$

8.27 Refer to Fig. 8.15(a) with  $V_{CC}$  replaced by  $(V_{CC} + v_r)$ .

$$v_{C1} = (V_{CC} + v_r) - \alpha \frac{I}{2} R_C$$

$$= (V_{CC} - \alpha \frac{I}{2} R_C) + v_r$$

$$\begin{aligned} v_{C2} &= (V_{CC} + v_r) - \alpha \frac{I}{2} R_C \\ &= (V_{CC} - \alpha \frac{I}{2} R_C) + v_r \end{aligned}$$

$$v_{od} \equiv v_{C2} - v_{C1} = 0$$

Thus, while  $v_{C1}$  and  $v_{C2}$  will include a ripple component  $v_r$ , the difference output voltage  $v_{od}$  will be ripple free. Thus, the differential amplifier rejects the undesirable supply ripple.

8.28 Refer to Fig. 8.14.

$$(a) V_{CM\max} = V_{CC} - \frac{I}{2} R_C$$

$$(b) \text{ For } V_{CC} = 2 \text{ V} \text{ and } V_{CM\max} = 1 \text{ V},$$

$$1 = 2 - \frac{1}{2}(IR_C)$$

$$\Rightarrow IR_C = 2 \text{ V}$$

$$(c) I_B = \frac{I/2}{\beta + 1} \leq 2 \mu\text{A}$$

$$I \leq 2 \times 101 \times 2 = 404 \mu\text{A}$$

Select

$$I = 0.4 \text{ mA}$$

then

$$R_C = \frac{2}{0.4} = 5 \text{ k}\Omega$$

$$8.29 \quad \frac{\Delta i_{E1}}{I} = \frac{i_{E1} - (I/2)}{I}$$

$$= \frac{i_{E1}}{I} - 0.5$$

This table belongs to Problem 8.29.

$v_{id}$ (mV)	2	5	8	10	20	30	40
$\left[ \frac{\Delta i_{E1}}{I} / v_{id} \right] (V^{-1})$	9.99	9.97	9.92	9.87	9.50	8.95	8.30

This table belongs to Problem 8.30.

$v_{id}$ (mV)	2	5	10	15	20	25	30	35	40
$v_{od}$ (V)	0.2	0.498	0.987	1.457	1.90	2.311	2.685	3.022	3.320
$\text{Gain} = \frac{v_{od}}{v_{id}}$	100	99.7	98.7	97.1	95.0	92.4	89.5	86.3	83.0

Using Eq. (8.48), we obtain

$$\frac{\Delta i_{E1}}{I} = \frac{1}{1 + e^{-v_{id}/V_T}} - 0.5$$

Observe that for  $v_{id} < 10$  mV the proportional transconductance gain is nearly constant at about 10. The gain decreases as  $v_{id}$  further increases, indicating nonlinear operation. This is especially pronounced for  $v_{id} > 20$  mV.

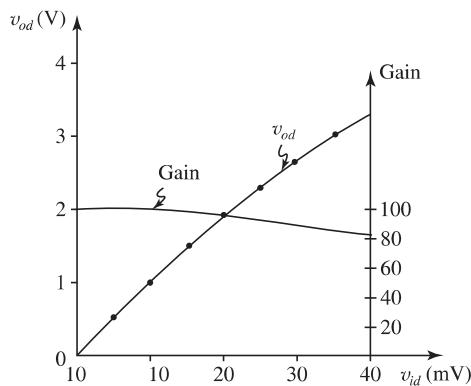
8.30 Refer to Fig. 8.14.

$$\begin{aligned} v_{od} &= v_{C2} - v_{C1} \\ &= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C) \\ &= R_C(i_{C1} - i_{C2}) \end{aligned}$$

Using Eqs. (8.48) and (8.49) and assuming  $\alpha \simeq 1$ , so that  $i_{C1} \simeq i_{E1}$  and  $i_{C2} \simeq i_{E2}$ , we get

$$\begin{aligned} v_{od} &= IR_C \left[ \frac{1}{1 + e^{-v_{id}/V_T}} - \frac{1}{1 + e^{v_{id}/V_T}} \right] \\ &= 5 \left[ \frac{1}{1 + e^{-v_{id}/V_T}} - \frac{1}{1 + e^{v_{id}/V_T}} \right] \end{aligned}$$

This relationship can be used to obtain the data in the table above.



The figure shows  $v_{od}$  versus  $v_{id}$  and the gain versus  $v_{id}$ . Observe that the transfer characteristic is nearly linear and the gain is nearly constant for  $v_{id} \leq 10$  mV. As  $v_{id}$  increases, the transfer characteristic bends and the gain is reduced. However, for  $v_{id}$  even as large as 20 mV, the gain is only 5% below its ideal value of 100.

8.31 Require  $v_{od} = 1$  V when  $v_{id} = 10$  mV and  $I = 1$  mA.

Using Eq. (8.48), we obtain

$$i_{E1} = \frac{1 \text{ (mA)}}{1 + e^{-10/V_T}} = 0.599 \text{ mA}$$

$$i_{E2} = I - i_{E1} = 1 - 0.599 = 0.401 \text{ mA}$$

$$v_{od} = v_{C2} - v_{C1}$$

$$= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C)$$

$$= (i_{C1} - i_{C2})R_C$$

$$\simeq (i_{E1} - i_{E2})R_C$$

$$= 0.198R_C$$

For  $v_{od} = 1$  V, we have

$$R_C = \frac{1}{0.198} = 5.05 \text{ k}\Omega$$

$$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2}R_C$$

$$= 5 - 0.5 \times 5.05 \simeq 2.5 \text{ V}$$

With a signal of 10 mV applied, the voltage at one collector rises to 3 V and at the other falls to 2 V. To ensure that the transistors remain in the active region, the maximum common-mode input voltage must be limited to  $(2 - 0.4) = +1.6$  V.

$$\begin{aligned} 8.32 \text{ (a)} \quad V_{BE} &= 0.69 + 0.025 \ln\left(\frac{0.1}{1}\right) \\ &= 0.632 \text{ V} \end{aligned}$$

(b) Using Eq. (8.48), we obtain

$$i_{C1} = \alpha i_{E1} \simeq \frac{I}{1 + e^{-V_{id}/V_T}}$$

For  $V_{id} = 20$  mV,

$$i_{C1} = \frac{200 \mu A}{1 + e^{-20/25}} = 138 \mu A$$

$$i_{C2} = 200 - 138 = 62 \mu A$$

(c) For  $V_{id} = 200$  mV while  $i_{C1} = 138 \mu A$  and  $i_{C2} = 62 \mu A$ : Since  $i_{C1}$  and  $i_{C2}$  have not changed,  $V_{BE1}$  and  $V_{BE2}$  also would not change. Thus,

$$V_{B1} - V_{B2} = V_{BE1} + i_{E1}R_e - i_{E2}R_e - V_{BE2}$$

$$= (V_{BE1} - V_{BE2}) + R_e (i_{E1} - i_{E2})$$

$$200 = 20 + R_e (i_{C1} - i_{C2})$$

$$= 20 + R_e (138 - 62)$$

$$\Rightarrow R_e = \frac{180 \text{ mV}}{76 \mu A} = 2.37 \text{ k}\Omega$$

(d) Without  $R_e$ ,

$$V_{id} = 20 \text{ mV} \rightarrow i_{C1} - i_{C2} = 76 \mu A$$

$$G_m = \frac{76 \mu A}{20 \text{ mV}} = 3.8 \text{ mA/V}$$

With  $R_e$ ,

$$V_{id} = 200 \text{ mV} \rightarrow i_{C1} - i_{C2} = 76 \mu A$$

$$G_m = \frac{76 \mu A}{200 \text{ mV}} = 0.38 \text{ mA/V}$$

Thus, the effective  $G_m$  has been reduced by a factor of 10, which is the same factor by which the allowable input signal has been increased while maintaining the same linearity.

$$8.33 \quad g_m = \frac{I_C}{V_T} = \frac{\alpha \times 0.2}{0.025} \simeq 8 \text{ mA/V}$$

$$R_{id} = 2r_\pi = 2 \frac{\beta}{g_m} = 2 \times \frac{160}{8} = 40 \text{ k}\Omega$$

$$8.34 \quad R_{id} = 2r_\pi = 20 \text{ k}\Omega$$

$$r_\pi = 10 \text{ k}\Omega$$

$$\frac{\beta}{g_m} = 10 \text{ k}\Omega$$

$$\frac{100}{g_m} = 10$$

$$\Rightarrow g_m = 10 \text{ mA/V}$$

$$A_d = 100 = g_m R_C$$

$$R_C = \frac{100}{g_m} = \frac{100}{10} = 10 \text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} \simeq \frac{I/2}{V_T}$$

$$\Rightarrow I = 2V_T g_m$$

$$= 2 \times 0.025 \times 10 = 0.5 \text{ mA}$$

$$8.35 \quad v_{id} = 10 \text{ mA/V}$$

Input signal to half-circuit = 5 mV. For  $I = 200 \mu A$ , the bias current of the half-circuit is 100  $\mu A$  and,

$$r_e = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

$$\text{Gain of half-circuit} = -\frac{R_C}{r_e} = -\frac{10}{0.25} = -40 \text{ V/V}$$

At each collector we expect a signal of  $40 \times 5 \text{ mV} = 200 \text{ mV}$ . Between the two collectors, the signal will be 400 mV.

$$8.36 \quad (a) \quad r_e = \frac{25 \text{ mV}}{0.25 \text{ mA}} = 100 \Omega$$

The 0.1-V differential input signal appears across  $(2r_e + 2R_e)$ , thus

$$i_e = \frac{100 \text{ mV}}{200 + 2 \times 400} = 0.1 \text{ mA}$$

$$v_{be} = 0.1 \times 100 = 10 \text{ mV}$$

(b) The total emitter current in one transistor is

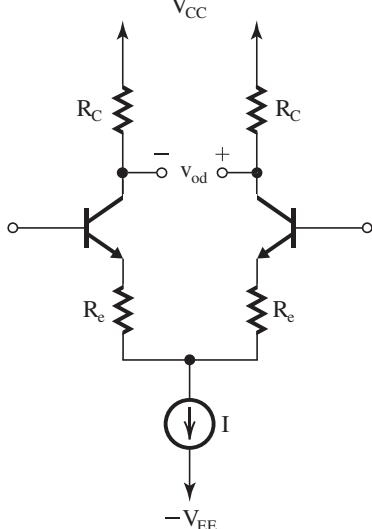
$$\frac{I}{2} + i_e = 0.35 \text{ mA} \text{ and in the other transistor}$$

$$\frac{I}{2} - i_e = 0.15 \text{ mA.}$$

(c) At one collector the signal voltage is  $-\alpha i_e R_C \simeq -i_e R_C = -0.1 \times 10 = -1 \text{ V}$  and at the other collector the signal voltage is +1 V.

$$(d) \quad \text{Voltage gain} = \frac{2 \text{ V}}{0.1 \text{ V}} = 20 \text{ V/V}$$

8.37



$v_{id} = 100 \text{ mV}$  appears across  $(2r_e + 2R_e)$ . Thus the signal across  $(r_e + R_e)$  is 50 mV. Since the signal across  $r_e$  is 5 mV, it follows that the signal across  $R_e$  must be  $50 - 5 = 45 \text{ mV}$  and thus

$$R_e = 9r_e$$

The input resistance  $R_{id}$  is

$$\begin{aligned} R_{id} &= (\beta + 1) (2r_e + 2R_e) \\ &= 2(100 + 1) (r_e + R_e) \\ &= 2 \times 101 \times (r_e + 9r_e) \\ &= 2 \times 101 \times 10r_e \end{aligned}$$

To obtain  $R_{id} = 100 \text{ k}\Omega$ ,

$$\begin{aligned} 100 &= 2 \times 101 \times 10 \times r_e \\ \Rightarrow r_e &\simeq 50 \Omega \end{aligned}$$

Since

$$r_e = \frac{V_T}{I_E},$$

$$50 = \frac{25 \text{ mV}}{I_E}$$

$$\Rightarrow I_E = 0.5 \text{ mA}$$

$$I = 1 \text{ mA}$$

$$R_e = 9r_e = 9 \times 50 = 450 \Omega$$

$$\text{Gain} = \frac{\alpha \times 2R_C}{2r_e + 2R_e}$$

$$\simeq \frac{R_C}{r_e + R_e}$$

But the gain required is

$$\text{Gain} = \frac{v_{od}}{v_{id}} = \frac{2 \text{ V}}{0.1 \text{ V}} = 20 \text{ V/V}$$

Thus,

$$20 = \frac{R_C}{0.05 + 0.45}$$

$$\Rightarrow R_C = 10 \text{ k}\Omega$$

The determination of a suitable value of  $V_{CC}$  requires information on the required input common-mode range (which is not specified). Suffice it to say that the dc voltage drop across  $R_C$  is 5 V and that each collector swings  $\pm 1 \text{ V}$ . A supply voltage  $V_{CC} = 10 \text{ V}$  will certainly be sufficient.

8.38 (a) The maximum allowable value of the bias current  $I$  is found as

$$I = \frac{P}{(V_{CC} + V_{EE})} = \frac{1 \text{ mW}}{5 \text{ V}} = 0.2 \text{ mA}$$

We choose to operate at this value of  $I$ . Thus

$$g_m = \frac{I_C}{V_T} = \frac{\alpha(0.2/2)}{0.025} \simeq 4 \text{ mA/V}$$

$$A_d = g_m R_C$$

$$60 = 4 \times R_C$$

$$\Rightarrow R_C = 15 \text{ k}\Omega$$

$$\begin{aligned} V_{C1} &= V_{C2} = V_{CC} - \frac{I}{2} R_C \\ &= 2.5 - \frac{0.2}{2} \times 15 \\ &= +1 \text{ V} \end{aligned}$$

$$(b) R_{id} = 2r_\pi = 2 \frac{\beta}{g_m}$$

$$= 2 \times \frac{100}{4} = 50 \text{ k}\Omega$$

$$(c) v_{od} = A_d \times v_{id}$$

$$= 60 \times 10 = 600 \text{ mV} = 0.6 \text{ V}$$

Thus, there will be  $\pm 0.3 \text{ V}$  signal swing at each collector. That is, the voltage at each collector will range between 0.7 V and +1.3 V.

(d) To maintain the BJT in the active mode at all times, the maximum allowable  $V_{CM}$  is limited to

$$V_{CM\max} = 0.4 + V_{Cmin}$$

$$= 0.4 + 0.7 = 1.1 \text{ V}$$

$$8.39 A_d = g_m R_C$$

$$= \frac{I_C}{V_T} R_C$$

$$\simeq \frac{(I/2)}{V_T} R_C$$

$$= \frac{IR_C}{2V_T}$$

$$= \frac{4}{2 \times 0.025} = 80 \text{ V/V}$$

$$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2} R_C$$

$$= 5 - 2 = 3 \text{ V}$$

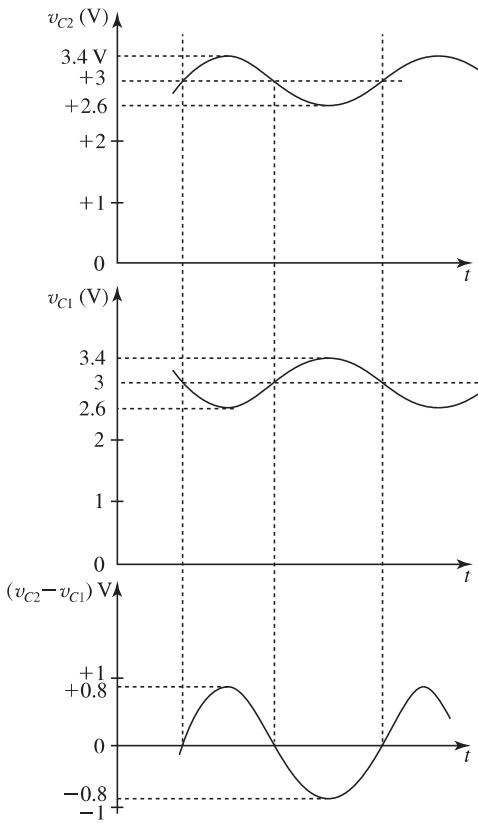
$$v_{C1} = 3 - 80 \times 0.005 \sin(\omega t)$$

$$= 3 - 0.4 \sin(\omega t)$$

$$v_{C2} = 3 + 0.4 \sin(\omega t)$$

$$v_{C2} - v_{C1} = 0.8 \sin(\omega t)$$

The waveforms are sketched in the figure on next page.



8.40 (a) Consider transistor Q<sub>1</sub>,

$$v_{C1\min} = (V_{CC} - \frac{I}{2}R_C) - A_d \left( \frac{\hat{v}_{id}}{2} \right) \quad (1)$$

where

$$A_d = g_m R_C \simeq \frac{I/2}{V_T} R_C$$

$$= \frac{IR_C}{2V_T}$$

Thus,

$$\frac{IR_C}{2} = A_d V_T \quad (2)$$

Substituting from (2) into (1), we obtain

$$v_{C1\min} = V_{CC} - A_d \left( V_T + \frac{\hat{v}_{id}}{2} \right) \quad (3)$$

Since

$$v_{BI} = V_{CM\max} + \frac{\hat{v}_{id}}{2}$$

to keep Q<sub>1</sub> in the active mode,

$$v_{BI} \leq 0.4 + v_{C1\min}$$

Thus,

$$V_{CM\max} + \frac{\hat{v}_{id}}{2} = 0.4 + V_{CC} - A_d \left( V_T + \frac{\hat{v}_{id}}{2} \right)$$

$$\Rightarrow V_{CM\max} = V_{CC} + 0.4 - \frac{\hat{v}_{id}}{2} - A_d \left( V_T + \frac{\hat{v}_{id}}{2} \right) \quad \text{Q.E.D.} \quad (4)$$

$$(b) V_{CC} = 2.5 \text{ V}, \quad \hat{v}_{id} = 10 \text{ mV},$$

$$A_d = 50 \text{ V/V},$$

$$V_{CM\max} = 2.5 + 0.4 - 0.005 - 50(25 + 5) \times 10^{-3}$$

$$\simeq 1.4 \text{ V}$$

$$\hat{v}_{od} = A_d \times \hat{v}_{id} = 50 \times 10 = 500 \text{ mV}$$

$$= 0.5 \text{ V}$$

Using Eq. (2), we obtain

$$IR_C = 2A_d V_T = 2 \times 50 \times 0.025$$

$$= 2.5 \text{ V}$$

To limit the power dissipation in the quiescent state to 1 mV, the bias current must be limited to

$$I = \frac{P_{\max}}{V_{CC} + V_{EE}} = \frac{1}{5} = 0.2 \text{ mA}$$

Using this value for I, we get

$$R_C = \frac{2.5}{0.2} = 12.5 \text{ k}\Omega$$

(c) To obtain V<sub>CMmax</sub> = 1 V, we use Eq. (4) to determine the allowable value of A<sub>d</sub>,

$$1 = 2.5 + 0.4 - 0.005 - A_d(25 + 5) \times 10^{-3}$$

$$\Rightarrow A_d = 63.2 \text{ V/V}$$

Thus, by reducing V<sub>CMmax</sub> from 1.4 V to 1 V, we are able to increase the differential gain from 50 V/V to 63.2 V/V.

8.41 See figure on next page. The circuit together with its equivalent half-circuit are shown in the figure.

$$A_d = g_{m1,2}(r_{o1,2} \parallel r_{o3,4})$$

For

$$r_{o1,2} = r_{o3,4} = \frac{V_A}{\alpha(I/2)} \simeq \frac{2V_A}{I}$$

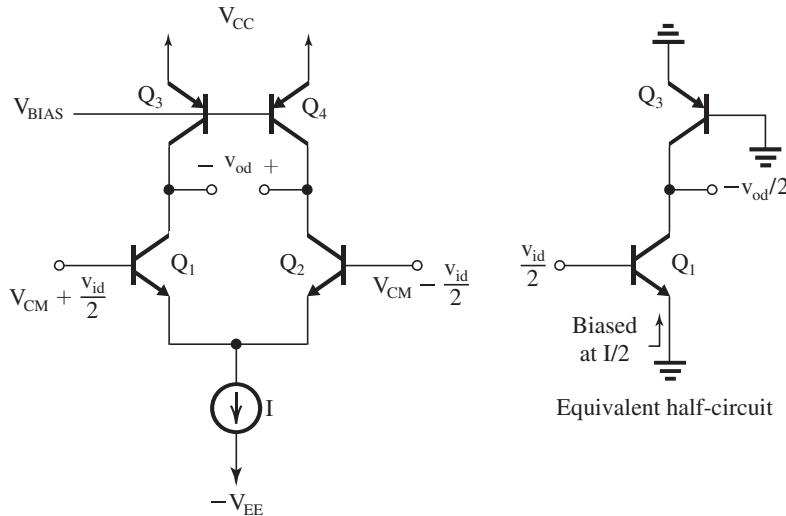
$$g_{m1,2} = \frac{I_{C1,2}}{V_T} \simeq \frac{I}{2V_T}$$

$$A_d = \frac{I}{2V_T} \left( \frac{2V_A}{I} \parallel \frac{2V_A}{I} \right)$$

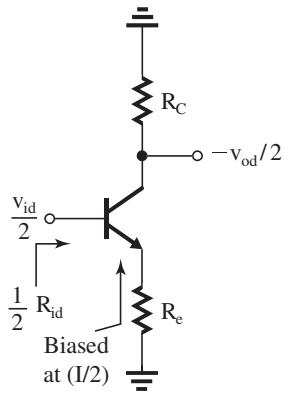
$$= \frac{I}{2V_T} \times \frac{V_A}{I} = \frac{V_A}{2V_T}$$

$$= \frac{20}{2 \times 0.025} = 400 \text{ V/V}$$

This figure belongs to Problem 8.41.



8.42



Both circuits have the same differential half-circuit shown in the figure. Thus, for both

$$A_d = \frac{\alpha R_C}{r_e + R_e}$$

$$\begin{aligned} R_{id} &= (\beta + 1)(2r_e + 2R_e) \\ &= 2(\beta + 1)(r_e + R_e) \end{aligned}$$

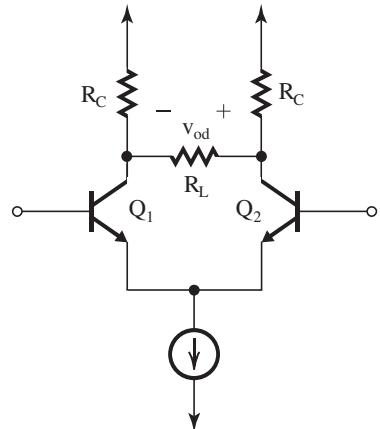
With  $V_{id} = 0$ , the dc voltage appearing at the top end of the bias current source will be

$$(a) V_{CM} - V_{BE} - \left(\frac{I}{2}\right)R_C$$

$$(b) V_{CM} - V_{BE}$$

Since circuit (b) results in a larger voltage across the current source and given that the minimum value of  $V_{CM}$  is limited by the need to keep a certain specified minimum voltage across the current source, we see that circuit (b) will allow a larger negative  $V_{CM}$ .

8.43



$$\begin{aligned} A_d &= \alpha \frac{\text{Total resistance between collectors}}{\text{Total resistance in the emitter circuit}} \\ &= \alpha \frac{(2R_C \parallel R_L)}{2r_e} \end{aligned}$$

8.44 Refer to Fig. P8.42(a).

$$\begin{aligned} \frac{I}{2}R_e &= 4V_T \\ \Rightarrow R_e &= \frac{8V_T}{I} \end{aligned} \quad (1)$$

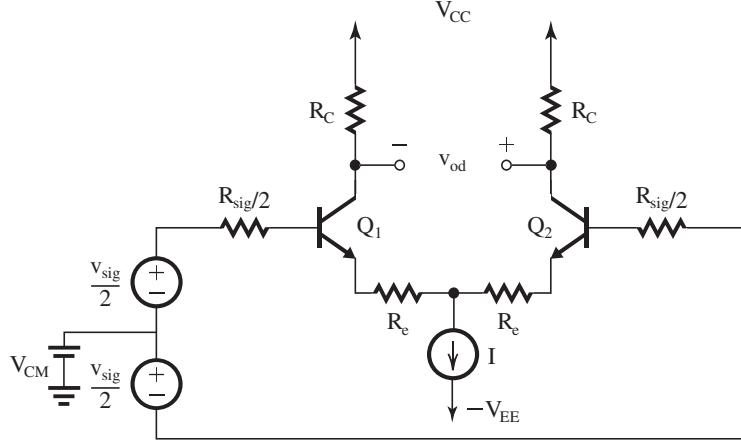
$$\alpha \left(\frac{I}{2}\right)R_C = 60V_T$$

$$R_C = \frac{120V_T}{\alpha I} \quad (2)$$

$$A_d = \alpha \frac{\text{Total resistance in collector circuit}}{\text{Total resistance in emitter circuit}}$$

$$A_d = \alpha \frac{2R_C}{2r_e + 2R_e} = \alpha \frac{R_C}{r_e + R_e}$$

This figure belongs to Problem 8.45.



Substituting for  $R_C$  from (2), for  $R_e$  from (1), and for  $r_e = V_T/(I/2)$ , we obtain

$$A_d = \frac{\alpha(120V_T/\alpha I)}{(2V_T/I) + (8V_T/I)}$$

$$= \frac{120}{2+8} = 12 \text{ V/V}$$

$$8.45 \frac{v_{id}}{v_{sig}} = \frac{R_{id}}{R_{id} + R_{sig}} \quad (1)$$

where

$$R_{id} = (\beta + 1)(2r_e + 2R_e)$$

thus,

$$\frac{v_{id}}{v_{sig}} = \frac{2(\beta + 1)(r_e + R_e)}{2(\beta + 1)(r_e + R_e) + R_{sig}} \quad (2)$$

$$\frac{v_{od}}{v_{id}} = \frac{\alpha \times \text{Total resistance between collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{2\alpha R_C}{2r_e + 2R_e}$$

$$\frac{v_{od}}{v_{id}} = \frac{\alpha R_C}{r_e + R_e} \quad (3)$$

Using (2) and (3), we get

$$G_v \equiv \frac{v_{od}}{v_{sig}} = \frac{2\alpha(\beta + 1)R_C}{2(\beta + 1)(r_e + R_e) + R_{sig}}$$

Since  $\alpha = \frac{\beta}{\beta + 1}$ ,  $\alpha(\beta + 1) = \beta$ , we have

$$G_v = \frac{2\beta R_C}{2(\beta + 1)(r_e + R_e) + R_{sig}} \quad (4)$$

If  $v_{id} = 0.5 v_{sig}$ , then from (1) we obtain

$$R_{id} = R_{sig}$$

Substituting for  $R_{sig} = R_{id} = 2(\beta + 1)(r_e + R_e)$  into Eq. (4) gives

$$G_v = \frac{2\beta R_C}{4(\beta + 1)(r_e + R_e)} = \frac{1}{2} \frac{\alpha R_C}{r_e + R_e} \quad (5)$$

If  $\beta$  is doubled to  $2\beta$  while  $R_{sig}$  remains at its old value, we get

$$R_{sig} = 2(\beta + 1)(r_e + R_e) \quad (6)$$

then the new value of  $G_v$  is obtained by replacing  $\beta$  by  $2\beta$  in Eq. (4) and substituting for  $R_{sig}$  from (5):

$$G_v = \frac{4\beta R_C}{2(2\beta + 1)(r_e + R_e) + 2(\beta + 1)(r_e + R_e)}$$

$$\simeq \frac{4R_C}{6(r_e + R_e)} = \frac{2}{3} \frac{R_C}{r_e + R_e}$$

Thus the gain increases from approximately  $\frac{1}{2} R_C / (r_e + R_e)$  to  $\frac{2}{3} R_C / (r_e + R_e)$ .

#### 8.46 Refer to Fig. P8.46.

$$\frac{v_o}{v_i} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{0.99 \times 25}{2r_e + 2 \times 0.25}$$

where

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

Thus,

$$\frac{v_o}{v_i} = \frac{0.99 \times 25}{2 \times 0.25 + 2 \times 0.25} \simeq 25 \text{ V/V}$$

$$R_{in} = (\beta + 1)(2r_e + 2R_e)$$

$$= 2 \times 101 \times (0.25 + 0.25)$$

$$= 101 \text{ k}\Omega$$

8.47 Refer to Fig. P8.47.

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

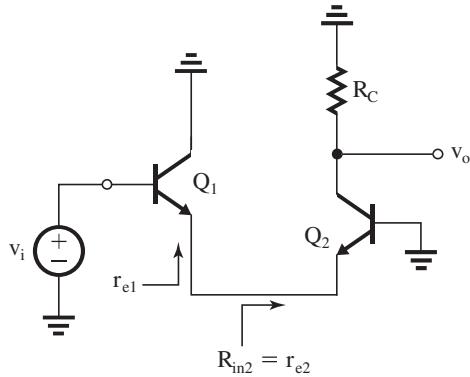
$$\begin{aligned} \frac{v_o}{v_i} &= \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}} \\ &= \frac{0.99 \times 25 \text{ k}\Omega}{2r_e + 500 \Omega} \\ &= \frac{0.99 \times 25 \text{ k}\Omega}{500 \Omega + 500 \Omega} \simeq 25 \text{ V/V} \end{aligned}$$

$$\begin{aligned} R_{in} &= (\beta + 1)(2r_e + 500 \Omega) \\ &= 101 \times (2 \times 250 \Omega + 500 \Omega) \\ &= 101 \text{ k}\Omega \end{aligned}$$

8.48 (a) Refer to the circuit in Fig. P8.48. As a differential amplifier, the voltage gain is found from

$$\begin{aligned} \frac{v_o}{v_i} &= \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}} \\ &= \frac{\alpha \times R_C}{2r_e} \\ &= \frac{\alpha R_C}{2r_e} \end{aligned}$$

(b) The circuit in Fig. P8.48 can be considered as the cascade connection of an emitter follower  $Q_1$  (biased at an emitter current  $I/2$ ) and a common-gate amplifier  $Q_2$  (also biased at an emitter current of  $I/2$ ). Referring to the figure below:



$$\frac{v_{e1,2}}{v_i} = \frac{r_{e2}}{r_{e1} + r_{e2}} = \frac{1}{2}$$

$$\frac{v_o}{v_{e1,2}} = \frac{\alpha R_C}{r_{e2}}$$

Thus,

$$\frac{v_o}{v_i} = \frac{1}{2} \times \frac{\alpha R_C}{r_{e2}} = \frac{\alpha R_C}{2r_e}$$

which is identical to the expression found in (a) above.

$$8.49 g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

$$= \sqrt{2 \times 3 \times 0.1} = 0.77 \text{ mA/V}$$

$$|A_d| = g_m R_D = 0.77 \times 10 = 7.7 \text{ V/V}$$

$$|A_{cm}| = \left( \frac{R_D}{2R_{SS}} \right) \left( \frac{\Delta R_D}{R_D} \right)$$

$$= \frac{10}{2 \times 100} \times 0.01 = 5 \times 10^{-4} \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = 1.54 \times 10^4 \text{ or } 83.8 \text{ dB}$$

8.50 Refer to Fig. P8.2.

$$I_D = 0.25 \text{ mA} = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right) |V_{OV}|^2$$

$$0.25 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.353 \text{ V}$$

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.25}{0.353} = 1.416 \text{ mA/V}$$

$$|A_d| = g_m R_D = 1.416 \times 4 = 5.67 \text{ V/V}$$

$$|A_{cm}| = \left( \frac{R_D}{2R_{SS}} \right) \left( \frac{\Delta R_D}{R_D} \right)$$

$$= \frac{4}{2 \times 30} \times 0.02$$

$$= 1.33 \times 10^{-3} \text{ V/V}$$

$$\text{CMRR} = 4252.5 \text{ or } 72.6 \text{ dB}$$

8.51 Refer to Fig. P8.51.

(a) Assume  $v_{id} = 0$  and the two sides of the differential amplifier are matched. Thus,

$$I_{D1} = I_{D2} = 0.5 \text{ mA}$$

$$I_{D1,2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.5 = \frac{1}{2} \times 2.5 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.632 \text{ V}$$

$$V_{CM} = V_{GS} + 1 \text{ mA} \times R_{SS}$$

$$= V_t + V_{OV} + 1 \times R_{SS}$$

$$= 0.7 + 0.632 + 1$$

$$= 2.332 \text{ V}$$

$$(b) g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5}{0.632} = 1.58 \text{ mA/V}$$

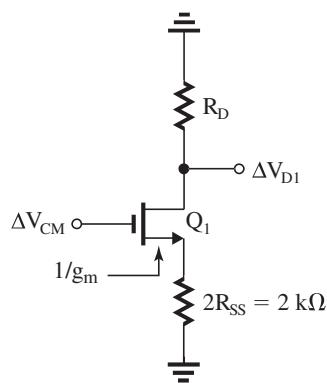
$$A_d = g_m R_D$$

$$8 = 1.38 \times R_D$$

$$\Rightarrow R_D = 5.06 \text{ k}\Omega$$

$$(c) V_{D1} = V_{D2} = V_{DD} - I_D R_D \\ = 5 - 0.5 \times 5.06 = 2.47 \text{ V}$$

(d)



The figure shows the common-mode half-circuit,

$$\frac{\Delta V_{D1}}{\Delta V_{CM}} = -\frac{R_D}{\frac{1}{g_m} + 2 R_{SS}}$$

$$\frac{\Delta V_{D1}}{\Delta V_{CM}} = -\frac{5.06}{\frac{1}{1.58} + 2} = -1.92 \text{ V/V}$$

(e) For  $Q_1$  and  $Q_2$  to enter the triode region

$$V_{CM} + \Delta V_{CM} = V_t + V_{D1} + \Delta V_{D1}$$

Substituting  $V_{CM} = 2.332$ ,  $V_t = 0.7 \text{ V}$ ,  $V_{D1} = 2.47 \text{ V}$ , and  $\Delta V_{D1} = -1.92 \Delta V_{CM}$  results in

$$2.332 + \Delta V_{CM} = 0.7 + 2.47 - 1.92 \Delta V_{CM}$$

$$\Rightarrow \Delta V_{CM} = 0.287 \text{ V}$$

With this change,  $V_{CM} = 2.619 \text{ V}$  and  $V_{D1,2} = 1.919 \text{ V}$ ; thus  $V_{CM} = V_t + V_{D1,2}$ .

8.52 The new deliberate mismatch  $\Delta R_D / R_D$  cancels the two existing mismatch terms in the expression for  $A_{cm}$  given in the problem statement so as to reduce  $A_{cm}$  to zero. Thus,

$$\frac{R_D}{2R_{SS}} \times \frac{\Delta R_D}{R_D} = -0.002$$

$$\frac{5}{2 \times 25} \times \frac{\Delta R_D}{R_D} = -0.002$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = -0.02 \text{ or } -2\%$$

(Note the sign of the change is usually determined experimentally.)

$$8.53 |A_{cm}| = \left( \frac{R_D}{2R_{SS}} \right) \frac{\Delta(W/L)}{W/L}$$

$$|A_d| = g_m R_D$$

$$CMRR = \frac{|A_d|}{|A_{cm}|} = 2g_m R_{SS} \sqrt{\frac{\Delta(W/L)}{W/L}}$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(0.1/2)}{0.2} = 0.5 \text{ mA/V}$$

For CMRR of 80 dB, the CMRR is  $10^4$ ; thus

$$10^4 = 2 \times 0.5 \times R_{SS} / 0.02$$

$$R_{SS} = 200 \text{ k}\Omega$$

For the current source transistor to have

$$r_o = 200 \text{ k}\Omega$$

$$200 = \frac{V_A' \times L}{0.1 \text{ mA}}$$

$$L = \frac{200 \times 0.1}{5} = 4 \mu\text{m}$$

8.54 It is required to raise the CMRR by 40 dB, that is, by a factor of 100. Thus, the cascoding of the bias current source must raise its output resistance  $R_{SS}$  by a factor of 100. Thus the cascode transistor must have  $A_0 = 100$ . Since

$$A_0 = g_m r_o = \frac{2I}{V_{OV}} \frac{V_A}{I} = \frac{2V_A}{V_{OV}}$$

$$100 = \frac{2V_A}{0.2}$$

$$\Rightarrow V_A = 10 \text{ V}$$

$$V_A = V_A' \times L$$

$$10 = 5 \times L$$

$$\Rightarrow L = 2 \mu\text{m}$$

8.55 Refer to Fig. P8.55,

$$(a) \frac{v_o}{v_{id}} = \alpha \frac{\text{Total resistance across which } v_o \text{ appears}}{\text{Total resistance in the emitter}} \\ = \alpha \times \frac{2 \text{ k}\Omega}{r_{e1} + r_{e2}}$$

To determine  $r_{e1} = r_{e2} = r_e = \frac{V_T}{I_E}$ , where  $I_E$  is the dc emitter current of each of  $Q_1$  and  $Q_2$ , we use

$$V_E = V_B - V_{BE} = 0 - 0.7$$

$$= -0.7 \text{ V}$$

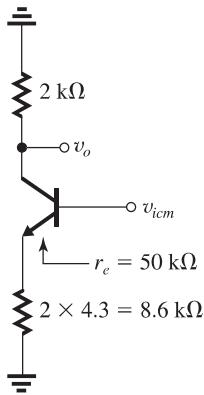
$$2I_E = \frac{-0.7 - (-5)}{4.3} = 1 \text{ mA}$$

$$I_E = 0.5 \text{ mA}$$

$$r_{e1} = r_{e2} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

$$\frac{v_o}{v_{id}} = \alpha \times \frac{2 \text{ k}\Omega}{0.1 \text{ k}\Omega} \simeq 20 \text{ V/V}$$

(b)



The common-mode half-circuit is shown in the figure,

$$\frac{v_o}{v_{icm}} = -\frac{\alpha \times 2 \text{ k}\Omega}{(0.05 + 8.6) \text{ k}\Omega}$$

$$\simeq -0.23 \text{ V/V}$$

$$\left| \frac{v_o}{v_{icm}} \right| = 0.23 \text{ V/V}$$

$$(c) \text{ CMRR} = \frac{|v_o/v_{id}|}{|v_o/v_{icm}|} = \frac{20}{0.23} = 86.5$$

or 38.7 dB

$$(d) v_o = -0.023 \sin 2\pi \times 60t + 0.2 \sin 2\pi \times 1000 t \text{ volts}$$

8.56

Figure (a) shows the differential half-circuit.

$$I_E = 0.5 \text{ mA}, \quad I_C = \alpha I_E \simeq 0.5 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$r_e = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

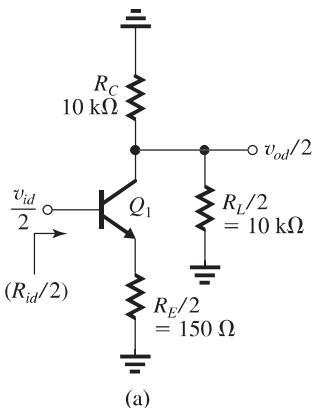
$$A_d = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$\simeq \frac{10 \text{ k}\Omega \parallel 10 \text{ k}\Omega}{(50 + 150) \Omega}$$

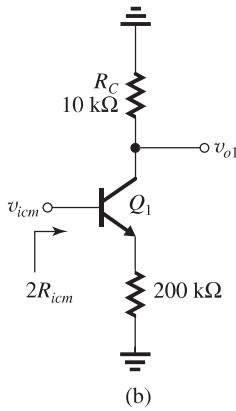
$$= \frac{5}{0.2} = 25 \text{ V/V}$$

We have neglected  $r_o$  because its equivalent value at the output will be  $r_o[1 + (R_e/r_e)] = 200[1 + (150/50)] = 800 \text{ k}\Omega$  which is much greater than the effective load resistance of  $5 \text{ k}\Omega$ .

$$R_{id} = 2 \times (\beta + 1)(50 \Omega + 150 \Omega) \\ = 2 \times 101 \times 0.2 (\text{k}\Omega) = 40.4 \text{ k}\Omega$$



(a)



(b)

$$|A_{cm}| \simeq \left( \frac{R_C}{2R_{ss}} \right) \left( \frac{\Delta R_C}{R_C} \right)$$

$$|A_{cm}| = \frac{10}{200} \times 0.02 = 0.001 \text{ V/V}$$

To obtain  $R_{icm}$ , we use Eq. (8.96):

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \frac{R_C + 2R_{EE}}{r_o}}$$

where  $2R_{EE} = 200 \text{ k}\Omega$ , thus  $R_{EE} = 100 \text{ k}\Omega$  and

$$R_{icm} = 100 \times 100 \frac{1 + (10/(100 \times 200))}{1 + \frac{10 + 200}{200}}$$

$$= 4.88 \text{ M}\Omega$$

$$8.57 \text{ (a)} \quad g_m = \frac{I_C}{V_T} \simeq \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$A_d = g_m R_C = 4 \times 25 = 100 \text{ V/V}$$

$$\text{(b)} \quad R_{id} = 2r_\pi = 2 \frac{\beta}{g_m} = 2 \times \frac{100}{4} = 50 \text{ k}\Omega$$

$$\text{(c)} \quad |A_{cm}| = \left( \frac{R_C}{2R_{EE}} \right) \left( \frac{\Delta R_C}{R_C} \right)$$

$$= \frac{25}{2 \times 500} \times 0.01$$

$$= 2.5 \times 10^{-4} \text{ V/V}$$

$$(d) \text{ CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{2.5 \times 10^{-4}} = 4 \times 10^5$$

or 112 dB

$$(e) r_o = \frac{V_A}{I_C} \simeq \frac{100}{0.1} = 1000 \text{ k}\Omega$$

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \frac{R_C + 2R_{EE}}{r_o}}$$

$$= 100 \times 500 \frac{1 + (25/(100 \times 1000))}{1 + \frac{25 + 1000}{1000}}$$

$$\simeq 25 \text{ M}\Omega$$

$$8.58 \quad R_{EE} = \frac{V_A}{I} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

For the transistors in the differential pair, we have

$$r_o = \frac{V_A}{I/2} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \frac{R_C + 2R_{EE}}{r_o}}$$

For  $R_C \ll r_o$ ,

$$R_{icm} \simeq \beta R_{EE} \left/ \left( 1 + \frac{2R_{EE}}{r_o} \right) \right.$$

$$= \frac{50 \times 100}{1 + \frac{2 \times 100}{200}} = 2.5 \text{ M}\Omega$$

8.59 For the differential-pair transistors, we have

$$I_C \simeq 0.25 \text{ mA}$$

$$g_m = \frac{0.25}{0.025} = 10 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{0.25} = 200 \text{ k}\Omega$$

$$(a) A_d = g_m R_C = 10 \times 5 = 50 \text{ V/V}$$

where we have neglected the effect of  $r_o$  since  $r_o \gg R_C$ .

(b) If the bias current is realized using a simple current source,

$$R_{EE} = r_o|_{\text{current source}} = \frac{V_A}{I} = \frac{50}{0.5} = 100 \text{ k}\Omega$$

$$|A_{cm}| = \left( \frac{R_C}{2R_{EE}} \right) \left( \frac{\Delta R_C}{R_C} \right)$$

$$= \left( \frac{5}{2 \times 100} \right) \times 0.1$$

$$= 2.5 \times 10^{-3} \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{50}{2.5 \times 10^{-3}} = 2 \times 10^4$$

or 86 dB

(c) If the bias current I is generated using a Wilson mirror,

$$R_{EE} = R_o|_{\text{Wilson mirror}}$$

$$= \frac{1}{2} \beta r_o$$

where  $r_o$  is that of the transistors in the Wilson mirror, then

$$r_o = \frac{50}{0.5} = 100 \text{ k}\Omega$$

$$R_{EE} = \frac{1}{2} \times 100 \times 100 = 5 \text{ M}\Omega$$

$$|A_{cm}| = \left( \frac{5}{2 \times 5,000} \right) \times 0.1$$

$$= 5 \times 10^{-5} \text{ V/V}$$

$$\text{CMRR} = \frac{50}{5 \times 10^{-5}} = 10^6$$

or 120 dB

8.60 See figure on next page.

$$v_{be1} = 2.5 \sin(\omega t), \text{ mV and}$$

$$v_{be2} = -2.5 \sin(\omega t), \text{ mV}$$

$$v_{C1} \simeq V_{CC} - \left( \frac{I}{2} \right) R_C - g_m R_C \times 2.5 \times 10^{-3} \sin(\omega t)$$

where

$$g_m = \frac{I/2}{V_T} = \frac{I \text{ mA}}{0.05 \text{ V}}$$

Thus,

$$v_{C1} = 5 - \frac{I}{2} \times 10 - \frac{I}{0.05} \times 10 \times 2.5 \times 10^{-3} \sin(\omega t)$$

$$= 5 - 5I - 0.5I \sin(\omega t)$$

Similarly,

$$v_{C2} = 5 - 5I + 0.5I \sin(\omega t)$$

To ensure operation in the active mode at all times with  $v_{CB} = 0 \text{ V}$ , we use

$$v_{C1\min} = 0.005$$

$$5 - 5.5I = 0.005$$

$$\Rightarrow I \simeq 0.9 \text{ mA}$$

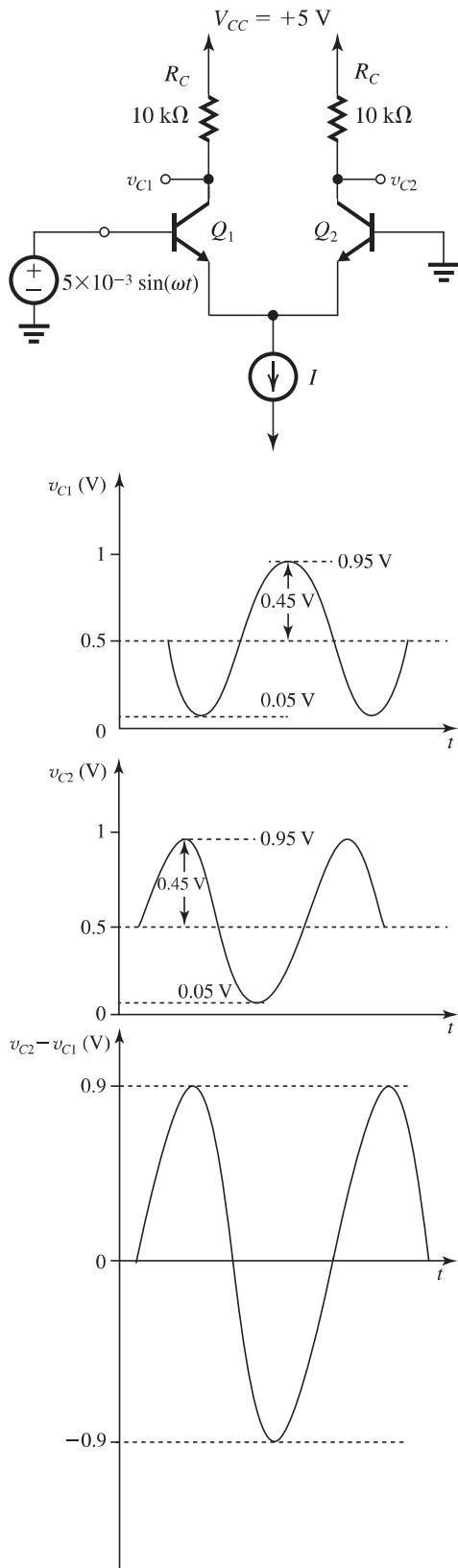
With this value of bias current, we obtain

$$g_m = \frac{0.9}{0.05} = 18 \text{ mA/V}$$

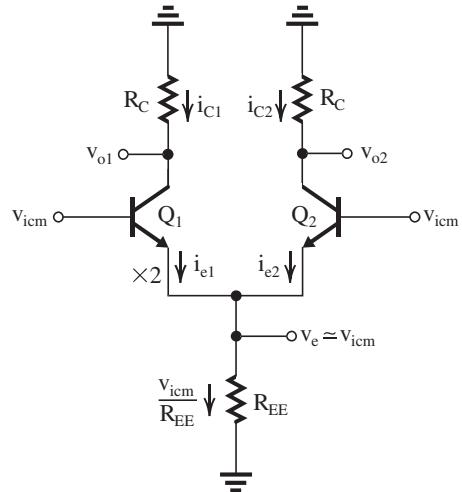
$$A_d = g_m R_C = 18 \times 10 = 180 \text{ V/V}$$

At each collector there will be a sine wave of  $180 \times 2.5 = 450 \text{ mV} = 0.45 \text{ V}$  amplitude superimposed on the dc bias voltage of  $5 - 0.45 \times 10 = 0.5 \text{ V}$ . Between the two collectors there will be a sine wave with 0.9 V peak amplitude. The second figure illustrates the waveforms obtained.

These figures belong to Problem 8.60.



8.61 If  $Q_1$  has twice the base-emitter junction area of  $Q_2$ , the bias current  $I$  will split  $\frac{2}{3} I$  in  $Q_1$  and  $\frac{1}{3} I$  in  $Q_2$ . This is because with  $B_1$  and  $B_2$  grounded the two transistors will have equal  $V_{BE}$ s. Thus their currents must be related by the ratio of their scale currents  $I_S$ , which are proportional to the junction areas.



With a common-mode input signal  $v_{icm}$  applied, as shown in the figure, the current ( $v_{icm}/R_{EE}$ ) will split between  $Q_1$  and  $Q_2$  in the same ratio as that of their base-emitter junction areas, thus

$$i_{e1} = \frac{2}{3} \frac{v_{icm}}{R_{EE}}$$

and

$$i_{e2} = \frac{1}{3} \frac{v_{icm}}{R_{EE}}$$

Thus,

$$v_{o1} = -i_{e1}R_C \approx -i_{e1}R_C = -\frac{2}{3} \frac{R_C}{R_{EE}} v_{icm}$$

and

$$v_{o2} = -\frac{1}{3} \frac{R_C}{R_{EE}} v_{icm}$$

With the output taken differentially, we have

$$v_{o2} - v_{o1} = \frac{1}{3} \frac{R_C}{R_{EE}} v_{icm}$$

$$A_{cm} = \frac{1}{3} \frac{R_C}{R_{EE}} = \frac{1}{3} \times \frac{12}{500} = 0.008 \text{ V/V}$$

8.62 If the output is taken single-endedly, then

$$|A_{cm}| = \frac{R_C}{2R_{EE}}$$

$$|A_d| = \frac{1}{2} g_m R_C$$

$$\text{CMRR}_s = \frac{|A_{cm}|}{|A_d|} = g_m R_{EE}$$

If the output is taken differentially, then

$$|A_{cm}| = \left( \frac{R_C}{2R_{EE}} \right) \left( \frac{\Delta R_C}{R_C} \right)$$

$$|A_d| = g_m R_C$$

$$\text{CMRR}_d = 2g_m R_{EE} / \left( \frac{\Delta R_C}{R_C} \right)$$

Thus,

$$\frac{\text{CMRR}_d}{\text{CMRR}_s} = \frac{2}{\Delta R_C / R_C}$$

$$20 \log \frac{2}{\Delta R_C / R_C} = 34 \text{ dB}$$

$$\Rightarrow \frac{\Delta R_C}{R_C} = 0.04 = 4\%$$

$$8.63 \quad g_m = \sqrt{2 k'_n (W/L) I_D}$$

$$= \sqrt{k'_n (W/L) I}$$

$$A_d = g_m R_D$$

$$V_{OV} = \frac{2 I_D}{g_m} = \frac{I}{g_m}$$

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right)$$

For  $I = 160 \mu\text{A}$ , we have

$$g_m = \sqrt{4 \times 0.16} = 0.8 \text{ mA/V}$$

$$A_d = 0.8 \times 10 = 8 \text{ V/V}$$

$$V_{OV} = \frac{0.16}{0.8} = 0.2 \text{ V}$$

$$V_{OS} = \frac{0.2}{2} \times 0.02 = 2 \text{ mV}$$

For  $I = 360 \mu\text{A}$ , we have

$$g_m = \sqrt{4 \times 0.36} = 1.2 \text{ mA/V}$$

$$A_d = 1.2 \times 10 = 12 \text{ V/V}$$

$$V_{OV} = \frac{0.36}{1.2} = 0.3 \text{ V}$$

$$V_{OS} = \frac{0.3}{2} \times 0.02 = 3 \text{ mV}$$

Thus by increasing the bias current, both the gain and the offset voltage increase, and by the same factor (1.5).

$$8.64 \quad (a) \quad g_m = \sqrt{2 k_n I_D} = \sqrt{k_n I}$$

$$A_d = g_m R_D = \sqrt{k_n I} R_D \quad (1)$$

$$V_{OV} = \sqrt{\frac{I/2}{\frac{1}{2} k_n}} = \sqrt{\frac{I}{k_n}}$$

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right)$$

Thus,

$$V_{OS} = \frac{1}{2} \sqrt{I/k_n} \left( \frac{\Delta R_D}{R_D} \right) \quad (2)$$

(b) For each value of  $V_{OS}$  we use Eq. (2) to determine  $I$  and then Eq. (1) to determine  $A_d$ . The results are as follows:

$V_{OS}$ (mV)	1	2	3	4	5
$I$ (mA)	0.04	0.16	0.36	0.64	1.00
$A_d$ (V/V)	4	8	12	16	20

We observe that by accepting a larger offset we are able to obtain a higher gain. Observe that the gain realized is proportional to the offset voltage one is willing to accept.

$$8.65 \quad V_{OV} = \sqrt{\frac{I/2}{\frac{1}{2} k'_n (W/L)}} = \sqrt{\frac{I}{k'_n (W/L)}}$$

$$= \sqrt{\frac{0.1}{0.2 \times 10}} = 0.224 \text{ V}$$

$$\frac{\Delta R_D}{R_D} = 0.04 \Rightarrow V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right)$$

$$= \frac{0.224}{2} \times 0.04 = 4.5 \text{ mV}$$

$$\frac{\Delta (W/L)}{(W/L)} = 0.04 \Rightarrow V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta (W/L)}{(W/L)} \right)$$

$$= \frac{0.224}{2} \times 0.04 = 4.5 \text{ mV}$$

$$\Delta V_t = 5 \text{ mV} \Rightarrow V_{OS} = \Delta V_t = 5 \text{ mV}$$

$$\text{Worst-case } V_{OS} = 4.5 + 4.5 + 5 = 14 \text{ mV}$$

If the three components are independent,

$$V_{OS} = \sqrt{4.5^2 + 4.5^2 + 5^2} = 8.1 \text{ mV}$$

8.66 The offset voltage due to  $\Delta V_t$  is

$$V_{OS} = \pm 5 \text{ mV}$$

The offset voltage due to  $\Delta R_D$  is

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right) = \frac{0.3}{2} \times 0.02 = 3 \text{ mV}$$

The offset voltage due to  $\Delta (W/L)$  is

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \frac{\Delta (W/L)}{(W/L)} = \frac{0.3}{2} \times 0.02 = 3 \text{ mV}$$

The worst-case offset voltage will be when all three components add up,

$$V_{OS} = 5 + 3 + 3 = 11 \text{ mV}$$

The major contribution to the total is the variability of  $V_t$ .

To compensate for a total offset of 11 mV by appropriately varying  $R_D$ , we need to change  $R_D$  by  $\Delta R_D$  obtained from

$$11 \text{ mV} = \left( \frac{V_{OV}}{2} \right) \times \frac{\Delta R_D}{R_D}$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = \frac{11 \times 2}{300} = 0.0733$$

or 7.33%

$$8.67 \quad V_{OS} = V_T \left( \frac{\Delta R_C}{R_C} \right)$$

$$= 25 \times 0.1 = 2.5 \text{ mV}$$

$$8.68 \quad V_{OS} = V_T \left( \frac{\Delta I_S}{I_S} \right)$$

$$= 25 \times 0.1 = 2.5 \text{ mV}$$

8.69 With both input terminals grounded, a mismatch  $\Delta R_C$  between the two collector resistors gives rise to an output voltage

$$V_O = \alpha \left( \frac{I}{2} \right) \Delta R_C \quad (1)$$

With a resistance  $R_E$  connected in the emitter of each transistor, the differential gain becomes

$$|A_d| = \frac{\alpha \times 2R_C}{2(r_e + R_E)} = \frac{\alpha R_C}{R_E + r_e} \quad (2)$$

The input offset voltage  $V_{OS}$  is obtained by dividing  $V_O$  in (1) by  $|A_d|$  in (2),

$$V_{OS} = \frac{I}{2}(r_e + R_E) \left( \frac{\Delta R_C}{R_C} \right)$$

$$\text{Since } r_e = \frac{V_T}{I/2},$$

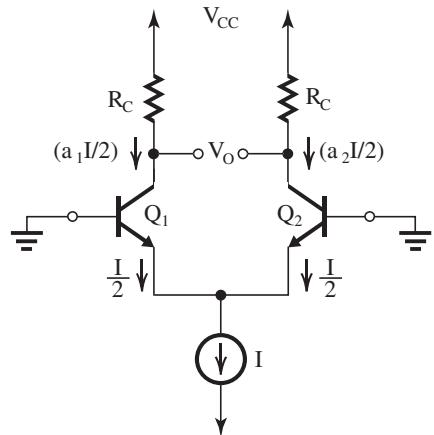
$$V_{OS} = (V_T + \frac{1}{2}IR_E) \left( \frac{\Delta R_C}{R_C} \right)$$

8.70 See figure.

The current  $I$  splits equally between the two emitters. However, the unequal  $\beta$ 's will mean unequal  $\alpha$ 's. Thus, the two collector currents will be unequal,

$$I_{C1} = \alpha_1 I/2$$

$$I_{C2} = \alpha_2 I/2$$



and the collector voltages will be unequal,

$$V_{C1} = V_{CC} - \alpha_1(I/2)R_C$$

$$V_{C2} = V_{CC} - \alpha_2(I/2)R_C$$

Thus a differential output voltage  $V_O$  develops:

$$V_O = V_{C2} - V_{C1}$$

$$= \frac{1}{2}IR_C(\alpha_1 - \alpha_2)$$

The input offset voltage  $V_{OS}$  can be obtained by dividing  $V_O$  by the differential gain  $A_d$ :

$$A_d = g_m R_C \simeq \frac{I/2}{V_T} R_C = \frac{IR_C}{2V_T}$$

Thus,

$$V_{OS} = V_T(\alpha_1 - \alpha_2)$$

Substituting, we obtain

$$\alpha_1 = \frac{\beta_1}{\beta_1 + 1}$$

and

$$\alpha_2 = \frac{\beta_2}{\beta_2 + 1}$$

$$V_{OS} = V_T \left( \frac{\beta_1}{\beta_1 + 1} - \frac{\beta_2}{\beta_2 + 1} \right)$$

$$= V_T \frac{\beta_1 \beta_2 + \beta_1 - \beta_1 \beta_2 - \beta_2}{(\beta_1 + 1)(\beta_2 + 1)}$$

$$= V_T \frac{\beta_1 - \beta_2}{(\beta_1 + 1)(\beta_2 + 1)}$$

$$\simeq V_T \frac{\beta_1 - \beta_2}{\beta_1 \beta_2}$$

$$= V_T \left( \frac{1}{\beta_2} - \frac{1}{\beta_1} \right) \quad \text{Q.E.D.}$$

For  $\beta_1 = 50$  and  $\beta_2 = 100$ , we have

$$V_{OS} = 25 \left( \frac{1}{100} - \frac{1}{50} \right) = -0.25 \text{ mV}$$

8.71 For the MOS amplifier:

$$\begin{aligned} V_{OS} &= \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right) \\ &= \frac{200}{2} \times 0.04 \\ &= 4 \text{ mV} \end{aligned}$$

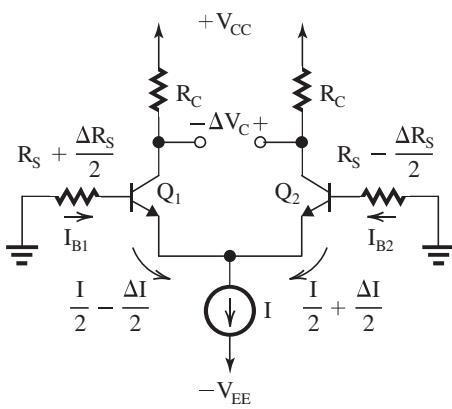
For the BJT amplifier:

$$\begin{aligned} V_{OS} &= V_T \left( \frac{\Delta R_C}{R_C} \right) \\ &= 25 \times 0.04 = 1 \text{ mV} \end{aligned}$$

If in the MOS amplifier the width of each device is increased by a factor of 4 while the bias current is kept constant,  $V_{OV}$  will be reduced by a factor of 2. Thus  $V_{OS}$  becomes

$$V_{OS} = 2 \text{ mV}$$

8.72



Consider only the incremental currents involved.

Assume the mismatch  $\Delta R_S$  is split between the two base (source) resistances. The emitter currents will be different, as shown.

Equating the voltage drop from each grounded input to the common emitters, we have

$$\begin{aligned} I_{B1} \left( R_S + \frac{\Delta R_S}{2} \right) + \left( \frac{I}{2} - \frac{\Delta I}{2} \right) r_e \\ = I_{B2} \left( R_S - \frac{\Delta R_S}{2} \right) + \left( \frac{I}{2} + \frac{\Delta I}{2} \right) r_e \end{aligned}$$

Subtracting out the  $\frac{I}{2} r_e$  terms, we have

$$\begin{aligned} I_{B1} \left( R_S + \frac{\Delta R_S}{2} \right) - \frac{\Delta I}{2} r_e \\ = I_{B2} \left( R_S - \frac{\Delta R_S}{2} \right) + \frac{\Delta I}{2} r_e \end{aligned}$$

In terms of the emitter currents, this becomes

$$\begin{aligned} \frac{\left( \frac{I}{2} - \frac{\Delta I}{2} \right)}{(\beta + 1)} \left( R_S + \frac{\Delta R_S}{2} \right) - \frac{\Delta I}{2} r_e \\ = \frac{\left( \frac{I}{2} + \frac{\Delta I}{2} \right)}{(\beta + 1)} \left( R_S - \frac{\Delta R_S}{2} \right) + \frac{\Delta I}{2} r_e \end{aligned}$$

Subtracting  $\frac{IR_S}{2(\beta + 1)}$  and  $-\frac{\Delta I \Delta R_S}{4(\beta + 1)}$  from each side, we obtain

$$\begin{aligned} \frac{I \Delta R_S}{4(\beta + 1)} - \frac{\Delta I R_S}{2(\beta + 1)} - \frac{I \Delta r_e}{2} \\ = -\frac{I \Delta R_S}{4(\beta + 1)} + \frac{\Delta I R_S}{2(\beta + 1)} + \frac{\Delta I r_e}{2} \end{aligned}$$

Combining terms, we have

$$\begin{aligned} \frac{I \Delta R_S}{2(\beta + 1)} &= \frac{\Delta I R_S}{(\beta + 1)} + \Delta I r_e \\ \Delta I \left( \frac{R_S}{(\beta + 1)} + r_e \right) &= \frac{I \Delta R_S}{2(\beta + 1)} \text{ so that} \end{aligned}$$

$$\Delta I = \frac{I \Delta R_S}{2(\beta + 1)} \cdot \frac{1}{\frac{R_S}{(\beta + 1)} + r_e}$$

$$\Delta V_C = \Delta I C R_C. \text{ If } \frac{\beta}{\beta + 1} \approx 1, \text{ we have}$$

$$\Delta V_C = \frac{I \Delta R_S R_C}{2(\beta + 1)} \cdot \frac{1}{\frac{R_S}{(\beta + 1)} + r_e}$$

Now  $V_{OS}$  can be obtained by dividing  $\Delta V_C$  by  $A_d = g_m R_C$ ,

$$\begin{aligned} V_{OS} &= \frac{\Delta V_C}{A_d} = \frac{\frac{I \Delta R_S R_C}{2(\beta + 1)} \cdot \frac{1}{\frac{R_S}{(\beta + 1)} + r_e}}{g_m R_C} \\ &= \frac{I \Delta R_S}{2(\beta + 1)} \cdot \frac{1}{g_m \left[ \frac{R_S}{(\beta + 1)} + r_e \right]} \end{aligned}$$

$$V_{OS} = \frac{I \Delta R_S}{2} \cdot \frac{1}{g_m R_S + (\beta + 1) r_e g_m}$$

Since  $(\beta + 1) r_e = r_\pi$  and  $r_\pi g_m = \beta$ , we have

$$V_{OS} = \frac{\left( \frac{I}{2\beta} \right) \cdot \Delta R_S}{1 + \frac{g_m R_S}{\beta}} \quad \text{Q.E.D.}$$

8.73 Since the only difference between the two sides of the differential pair is the mismatch in  $V_A$ , we can write

$$I_{C1} = I_C \left( 1 + \frac{V_{CE1}}{V_{AI}} \right)$$

$$\begin{aligned}
 I_{C2} &= I_C \left( 1 + \frac{V_{CE2}}{V_{A2}} \right) \\
 I_{C1} + I_{C2} &= \alpha I \\
 I_C \left( 2 + \frac{V_{CE1}}{V_{A1}} + \frac{V_{CE2}}{V_{A2}} \right) &= \alpha I \\
 \Rightarrow I_C &= \alpha I / \left( 2 + \frac{V_{CE1}}{V_{A1}} + \frac{V_{CE2}}{V_{A2}} \right) \\
 I_{C1} &= \frac{\alpha I}{2} \frac{1 + \frac{V_{CE1}}{V_{A1}}}{1 + \frac{V_{CE1}}{2V_{A1}} + \frac{V_{CE2}}{2V_{A2}}} \\
 \text{For } \frac{V_{CE1}}{V_{A1}} \ll 1 \text{ and } \frac{V_{CE2}}{V_{A2}} \ll 1 \text{ we have} \\
 I_{C1} &\simeq \frac{\alpha I}{2} \left( 1 + \frac{1}{2} \frac{V_{CE1}}{V_{A1}} - \frac{1}{2} \frac{V_{CE2}}{V_{A2}} \right) \\
 I_{C2} &\simeq \frac{\alpha I}{2} \left( 1 + \frac{1}{2} \frac{V_{CE2}}{V_{A2}} - \frac{1}{2} \frac{V_{CE1}}{V_{A1}} \right)
 \end{aligned}$$

The voltage  $V_O$  between the two collectors will be

$$\begin{aligned}
 V_O &= V_{C2} - V_{C1} \\
 &= I_{C1}R_C - I_{C2}R_C \\
 &= \frac{\alpha I}{2}R_C \times \left( \frac{V_{CE1}}{V_{A1}} - \frac{V_{CE2}}{V_{A2}} \right) \quad (1)
 \end{aligned}$$

Since we still have  $I_{C1} \simeq I_{C2} = \alpha \frac{I}{2}$ , the differential gain is still given by

$$A_d = g_m R_C = \frac{I_C R_C}{V_T} = \frac{\alpha I R_C}{2V_T} \quad (2)$$

Dividing (1) by (2) gives

$$V_{OS} = V_T \left( \frac{V_{CE1}}{V_{A1}} - \frac{V_{CE2}}{V_{A2}} \right)$$

As a first-order approximation, we can assume

$$V_{CE1} \simeq V_{CE2} = 10 \text{ V}$$

and substitute  $V_{A1} = 100 \text{ V}$  and  $V_{A2} = 200 \text{ V}$  to determine  $V_{OS}$  as

$$\begin{aligned}
 V_{OS} &= 25 \left( \frac{10}{100} - \frac{10}{200} \right) \\
 &= 25 \times 0.05 = 1.25 \text{ mV}
 \end{aligned}$$

8.74 Refer to Fig. P8.74.

$$(a) R_{C1} = 1.04 \times 5 = 5.20 \text{ k}\Omega$$

$$R_{C2} = 0.96 \times 5 = 4.80 \text{ k}\Omega$$

To equalize the total resistance in each collector, we adjust the potentiometer so that

$$R_{C1} + x \times 1 \text{ k}\Omega = R_{C2} + (1 - x) \times 1 \text{ k}\Omega$$

$$5.2 + x = 4.8 + 1 - x$$

$$\Rightarrow x = 0.3 \text{ k}\Omega$$

(b) If the area of  $Q_1$  and hence  $I_{S1}$  is 5% larger than nominal, then we have

$$I_{S1} = 1.05I_S$$

and the area of  $Q_2$  and hence  $I_{S2}$  is 5% smaller than nominal,

$$I_{S2} = 0.95I_S$$

Thus,

$$I_{E1} = 0.5 \times 1.05 = 0.525 \text{ mA}$$

$$I_{E2} = 0.5 \times 0.95 = 0.475 \text{ mA}$$

Assuming  $\alpha \simeq 1$ , we obtain

$$I_{C1} = 0.525 \text{ mA} \quad I_{C2} = 0.475 \text{ mA}$$

To reduce the resulting offset to zero, we adjust the potentiometer so that

$$V_{C1} = V_{C2}$$

$$\Rightarrow V_{CC} - (R_{C1} + x)I_{C1} = V_{CC} - (R_{C2} + 1 - x)I_{C2}$$

$$I_{C1}(R_{C1} + x) = I_{C2}(R_{C2} + 1 - x)$$

$$0.525(5 + x) = 0.475(5 + 1 - x)$$

$$\Rightarrow x = 0.225$$

$$8.75 \quad I_{B\max} = \frac{400}{2 \times 81} \simeq 2.5 \mu\text{A}$$

$$I_{B\min} = \frac{400}{2 \times 201} = 1 \mu\text{A}$$

$$I_{OS\max} = \frac{200}{81} - \frac{200}{201} \simeq 1.5 \mu\text{A}$$

8.76 A 2-mV input offset voltage corresponds to a difference  $\Delta R_C$  between the two collector resistances,

$$2 = V_T \frac{\Delta R_C}{R_C}$$

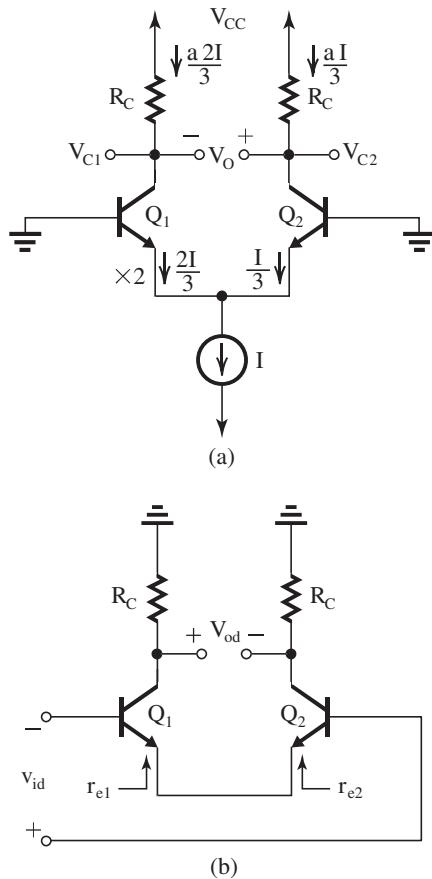
$$= 25 \times \frac{\Delta R_C}{20}$$

$$\Rightarrow \Delta R_C = 1.6 \text{ k}\Omega$$

Thus a 2-mV offset can be nulled out by adjusting one of the collector resistances by  $1.6 \text{ k}\Omega$ . If the adjustment mechanism raises one  $R_C$  and lowers the other, then each need to be adjusted by only  $(1.6 \text{ k}\Omega/2) = 0.8 \text{ k}\Omega$ .

If a potentiometer is used, the total resistance of the potentiometer must be at least  $1.6 \text{ k}\Omega$ . If specified to a single digit, we use  $2 \text{ k}\Omega$ .

8.77



From Fig. (a) we see that the transistor with twice the area ( $Q_1$ ) will carry twice the current in the other transistor ( $Q_2$ ). Thus

$$I_{E1} = \frac{2I}{3}, \quad I_{E2} = \frac{I}{3}$$

$$I_{C1} = \frac{\alpha 2I}{3}, \quad I_{C2} = \frac{\alpha I}{2}$$

Thus,

$$V_{C1} = V_{CC} - \frac{\alpha 2I}{3} R_C$$

$$V_{C2} = V_{CC} - \frac{\alpha I}{3} R_C$$

and the dc offset voltage at the output will be

$$V_O = V_{C2} - V_{C1}$$

$$V_O = \frac{1}{3} \alpha I R_C$$

To reduce this output voltage to zero, we apply a dc input voltage  $v_{id}$  in the direction shown in Fig. (b). The voltage  $v_{id}$  is required to produce  $v_{od}$  in the direction shown which is opposite in direction to  $V_O$  and of course  $|v_{od}| = |V_O|$ , thus

$$A_d v_{id} = \frac{1}{3} \alpha I R_C \quad (1)$$

The gain  $A_d$  is found as follows:

$$A_d = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{\alpha \times 2R_C}{r_{e1} + r_{e2}}$$

where

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{V_T}{2I/3} = \frac{3V_T}{2I} = \frac{1.5V_T}{I}$$

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{V_T}{I/3} = \frac{3V_T}{I}$$

thus,

$$A_d = \frac{2\alpha R_C}{4.5 V_T / I} = \frac{2\alpha I R_C}{4.5 V_T} \quad (2)$$

Substituting in Eq. (1) gives

$$v_{id} = 0.75 V_T = 18.75 \text{ mV}$$

Now, using large signal analysis:

$$v_{id} = V_{B2} - V_{B1} = (V_{B2} - V_E) - (V_{B1} - V_E)$$

$$I_{C1} = I_{S1} e^{(V_{B1} - V_E)/V_T} \quad (3)$$

$$I_{C2} = I_{S2} e^{(V_{B2} - V_E)/V_T} \quad (4)$$

where  $I_{S1} = 2 I_{S2}$ .

To make  $I_{C1} = I_{C2}$ ,

$$I_{S1} e^{(V_{B1} - V_E)/V_T} = I_{S2} e^{(V_{B2} - V_E)/V_T}$$

$$e^{(V_{B2} - V_{B1})/V_T} = 2$$

$$V_{B2} - V_{B1} = V_T \ln 2$$

Thus,

$$v_{id} = 17.3 \text{ mV}$$

which is reasonably close to the approximate value obtained using small-signal analysis.

 8.78  $G_m = 2 \text{ mA/V}$ 

With  $R_L = \infty$ ,

$$A_d = G_m R_o$$

and

$$v_o = G_m R_o v_{id}$$

With  $R_L = 20 \text{ k}\Omega$ ,

$$v_o = G_m R_o v_{id} \frac{R_L}{R_L + R_o}$$

$$= G_m R_o \frac{20}{20 + R_o} v_{id} = \frac{1}{2} G_m R_o v_{id}$$

Thus,

$$R_o = 20 \text{ k}\Omega$$

$$A_d (\text{with } R_L = \infty) = G_m R_o = 2 \times 20 = 40 \text{ V/V}$$

$$8.79 \quad G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}} = \frac{I}{0.25}$$

$$R_o = r_{o2} \parallel r_{o4}$$

For

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{|V'_A|L}{I/2}$$

$$= \frac{2 \times 5 \times 0.5}{I} = \frac{5}{I}$$

$$R_o = \frac{1}{2} \times \frac{5}{I} = \frac{2.5}{I}$$

Thus,

$$A_d = G_m R_o = \frac{I}{0.25} \times \frac{2.5}{I} = 10 \text{ V/V}$$

$$8.80 \quad \frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.2 \times 50 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.14 \text{ V}$$

$$g_{m1,2} = \frac{2 \times (I/2)}{V_{OV}} = \frac{2 \times 0.1}{0.14} = 1.4 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{|V'_A| \times L}{I/2} = \frac{5 \times 0.5}{0.1} = 25 \text{ k}\Omega$$

$$A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$$

$$= 1.4 \times (25 \parallel 25)$$

$$= 17.5 \text{ V/V}$$

$$8.81 \quad A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$$

$$g_{m1,2} = \sqrt{2k'_n \left( \frac{W}{L} \right) I_D}$$

$$= \sqrt{4I} = 2\sqrt{I}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I} = \frac{2 \times 5}{I} = \frac{10}{I}$$

$$A_d = 2\sqrt{I} \times \frac{1}{2} \times \frac{10}{I} = \frac{10}{\sqrt{I}}$$

$$20 = \frac{10}{\sqrt{I}}$$

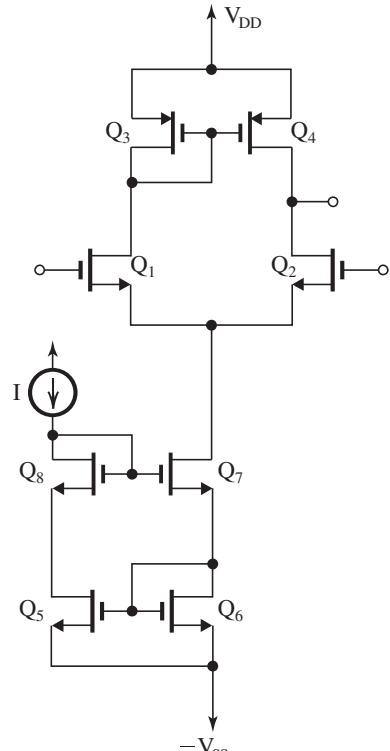
$$\Rightarrow I = 0.25 \text{ mA}$$

8.82 See figure.

For Q<sub>1</sub>, Q<sub>2</sub>, Q<sub>3</sub> and Q<sub>4</sub>:

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 5 \times V_{OV}^2$$



$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$V_{GS} = V_t + |V_{OV}|$$

$$= 0.5 + 0.2 = 0.7 \text{ V}$$

For Q<sub>5</sub>, Q<sub>6</sub>, Q<sub>7</sub>, and Q<sub>8</sub>:

$$I_D = 0.2 \text{ mA}$$

$$0.2 = \frac{1}{2} \times 5 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.28 \text{ V}$$

$$V_{GS} = 0.5 + 0.28 = 0.78 \text{ V}$$

From the figure we see that for each transistor to operate at V<sub>DS</sub> at least equal to V<sub>GS</sub>, the total power supply is given by

$$V_{DD} + V_{SS} = V_{DS4} + V_{DS2} + V_{DS7} + V_{DS6}$$

$$= V_{GS4} + V_{GS2} + V_{GS7} + V_{GS6}$$

$$= 0.7 + 0.7 + 0.78 + 0.78$$

$$= 2.96 \simeq 3.0 \text{ V}$$

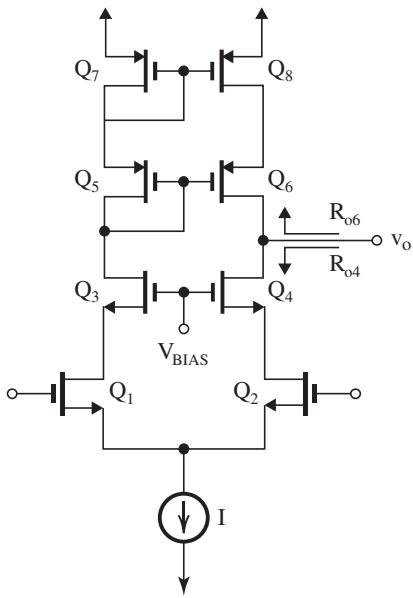
8.83

(a) See figure on next page.

(b)  $A_d = g_{m1,2}(R_{o4} \parallel R_{o6})$

$$g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$R_{o6} = g_{m6} r_{o6} r_{o8}$$



Since all transistors are operated at a bias current ( $I/2$ ) and have the same overdrive voltage  $|V_{ov}|$  and the same Early voltage,  $|V_A|$ , all have the same  $g_m = I/|V_{ov}|$  and the same

$$r_o = \frac{|V_A|}{I/2} = 2|V_A|/I. \text{ Thus,}$$

$$R_{o6} = g_m r_o^2$$

$$R_{o4} = g_m r_{o4} r_{o2} = g_m r_o^2$$

$$A_d = g_m (g_m r_o^2 \parallel g_m r_o^2)$$

$$= \frac{1}{2} (g_m r_o)^2$$

$$g_m r_o = \frac{I}{|V_{ov}|} \times \frac{2|V_A|}{I} = \frac{2|V_A|}{|V_{ov}|}$$

$$A_d = 2(|V_A|/|V_{ov}|)^2 \quad \text{Q.E.D.}$$

This figure belongs to Problem 8.84.

For  $|V_{ov}| = 0.2 \text{ V}$  and  $|V_A| = 10 \text{ V}$ , we have

$$A_d = 2 \left( \frac{10}{0.2} \right)^2 = 5000 \text{ V/V}$$

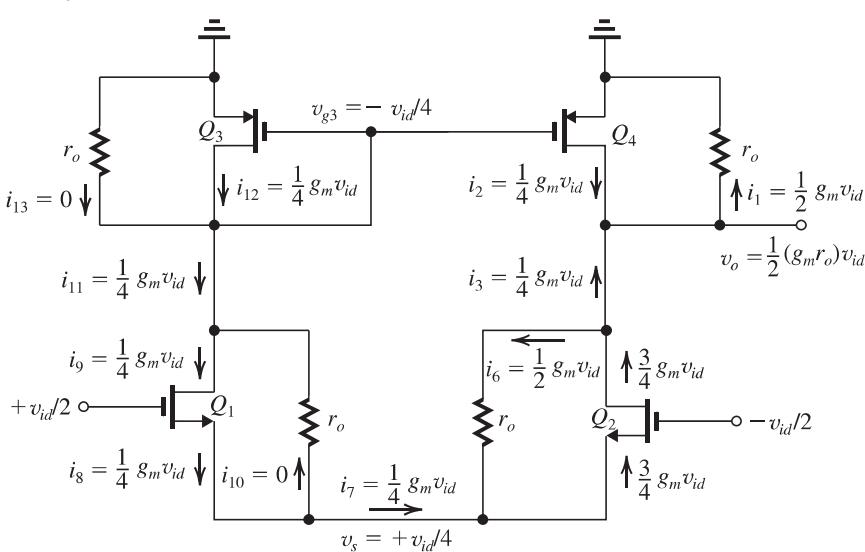
8.84 The currents  $i_1$  to  $i_{13}$  are shown on the circuit diagram. Observe that  $i_{11} = i_7 = i_3$  (the current that enters a transistor exits at the other end!). Also observe that the mirror  $Q_3$  and  $Q_4$  is indeed functioning properly as the drain currents of  $Q_3$  and  $Q_4$  are equal ( $i_{12} = i_2 = \frac{1}{4} g_m v_{id}$ ). However, the currents in their  $r_o$ s are far from being equal!

There are some inconsistencies that result from the approximations made to obtain the results shown in Fig. P8.84, namely,  $g_m r_o \gg 1$ . Note for instance that although we find the current in  $r_o$  of  $Q_2$  to be  $\frac{1}{2} g_m v_{id}$ , the voltages at the two ends of  $r_o$  are  $\frac{1}{2} (g_m r_o) v_{id}$  and  $v_{id}/4$ ; thus the current must be  $v_{id} \left( \frac{1}{2} g_m r_o - \frac{1}{4} \right) / r_o$ , which is approximately  $\frac{1}{2} g_m v_{id}$ .

The purpose of this problem is to show the huge imbalance that exists in this circuit. In fact,  $Q_1$  has  $|v_{gs}| = \frac{1}{4} v_{id}$  while  $Q_2$  has  $|v_{gs}| = \frac{3}{4} v_{id}$ . This imbalance results from the fact that the current mirror is not a balanced load. Nevertheless, we know that this circuit provides a reasonably high common-mode rejection.

$$8.85 G_m = g_{m1,2} = \frac{2(I/2)}{V_{ov1,2}} = \frac{0.2}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = \frac{V_{An}}{I/2} = \frac{20}{0.1} = 200 \text{ k}\Omega$$



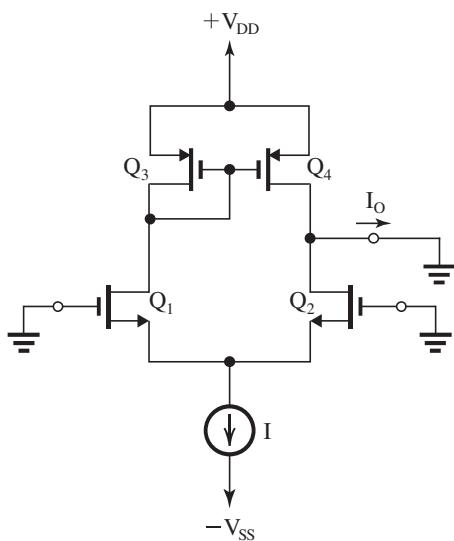
$$r_{o4} = \frac{|V_{Ap}|}{I/2} = \frac{12}{0.1} = 120 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 200 \parallel 120 = 75 \text{ k}\Omega$$

$$A_d = G_m R_o = 1 \times 75 = 75 \text{ V/V}$$

The gain is reduced by a factor of 2 with  $R_L = R_o = 75 \text{ k}\Omega$ .

8.86



(a) Let

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_A + \frac{1}{2} \Delta \left(\frac{W}{L}\right)_A$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_A - \frac{1}{2} \Delta \left(\frac{W}{L}\right)_A$$

$Q_1$  and  $Q_2$  have equal values of  $V_{GS}$  and thus of  $V_{OV}$ , thus

$$I_{D1} = \frac{1}{2} k'_n \left[ \left(\frac{W}{L}\right)_A + \frac{1}{2} \Delta \left(\frac{W}{L}\right)_A \right] V_{OV}^2$$

$$= \frac{1}{2} k'_n \left(\frac{W}{L}\right)_A \left[ 1 + \frac{1}{2} \frac{\Delta(W/L)_A}{(W/L)_A} \right] V_{OV}^2$$

Since, in the ideal case

$$I_{D1} = \frac{I}{2} = \frac{1}{2} k'_n \left(\frac{W}{L}\right)_A V_{OV}^2$$

$$I_{D1} = \frac{I}{2} \left[ 1 + \frac{1}{2} \frac{\Delta(W/L)_A}{(W/L)_A} \right]$$

Similarly, we can show that

$$I_{D2} = \frac{I}{2} \left[ 1 - \frac{1}{2} \frac{\Delta(W/L)_A}{(W/L)_A} \right]$$

The current mirror causes

$$I_{D4} = I_{D3} = I_{D1}$$

Thus,

$$I_O = I_{D4} - I_{D2}$$

$$= I_{D1} - I_{D2}$$

$$= \frac{I}{2} \frac{\Delta(W/L)_A}{(W/L)_A}$$

The input offset voltage is

$$V_{OS} = \frac{I_O}{G_m}$$

where

$$G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

Thus,

$$V_{OS} = (V_{OV}/2) \frac{\Delta(W/L)_A}{(W/L)_A} \quad \text{Q.E.D.}$$

$$(b) I_{D1} = I_{D2} = \frac{I}{2}$$

$$I_{D3} = I_{D1}$$

If the  $(W/L)$  ratios of the mirror transistors have a mismatch  $\Delta(W/L)_M$ , the current transfer ratio of the mirror will have an error of  $[\Delta(W/L)_M / (W/L)_M]$ . Thus

$$I_{D4} = I_{D3} \left[ 1 + \frac{\Delta(W/L)_M}{(W/L)_M} \right]$$

At the output node, we have

$$\begin{aligned} I_O &= I_{D4} - I_{D2} \\ &= I_{D3} \left[ 1 + \frac{\Delta(W/L)_M}{(W/L)_M} \right] - I_{D2} \\ &= I_{D1} \left[ 1 + \frac{\Delta(W/L)_M}{(W/L)_M} \right] - I_{D2} \\ &= \frac{I}{2} \frac{\Delta(W/L)_M}{(W/L)_M} \end{aligned}$$

and the corresponding  $V_{OS}$  will be

$$\begin{aligned} V_{OS} &= \frac{I_O}{G_m} = \frac{I_O}{I/V_{OV}} \\ &= \left(\frac{V_{OV}}{2}\right) \frac{\Delta(W/L)_M}{(W/L)_M} \quad \text{Q.E.D.} \end{aligned}$$

$$(c) V_{OS}|_{Q_1, Q_2 \text{ mismatch}} = \left(\frac{0.2}{2}\right) \times 0.02 = 2 \text{ mV}$$

$$V_{OS}|_{Q_3, Q_4 \text{ mismatch}} = \left(\frac{0.2}{2}\right) \times 0.02 = 2 \text{ mV}$$

Worst-case  $V_{OS} = 2 + 2 = 4 \text{ mV}$

8.87  $I_{E1} = I_{E2} = 0.25 \text{ mA}$

$$I_{C1} = I_{C2} \simeq 0.25 \text{ mA}$$

$$g_{m1,2} = \frac{I_{C1,2}}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$r_o = \frac{|V_A|}{I_C} = \frac{10 \text{ V}}{0.25 \text{ mA}} = 40 \text{ k}\Omega$$

$$R_{id} = 2 r_\pi = 2 \frac{\beta}{g_m} = 2 \times \frac{100}{10} = 20 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 40 \parallel 40 = 20 \text{ k}\Omega$$

$$G_m = g_{m1,2} = 10 \text{ mA/V}$$

$$A_d = G_m R_o = 10 \times 20 = 200 \text{ V/V}$$

If  $R_L = R_{id} = 20 \text{ k}\Omega$ , then

$$\begin{aligned} G_v &= 200 \times \frac{R_L}{R_L + R_o} \\ &= 200 \times \frac{20}{20 + 20} = 100 \text{ V/V} \end{aligned}$$

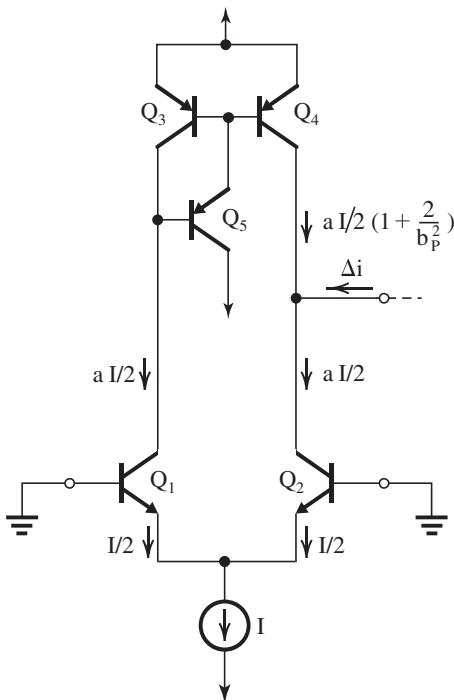
8.88 Using Eq. (8.145), we obtain

$$V_{OS} = -\frac{2V_T}{\beta_p}$$

$$-2 = -\frac{2 \times 25}{\beta_p}$$

$$\Rightarrow \beta_p = 25$$

8.89



The figure shows a BJT differential amplifier loaded in a base-current-compensated current mirror. To determine the systematic input offset voltage resulting from the error in the current-transfer ratio of the mirror, we ground the

two input terminals and determine the output current  $\Delta i$  as follows:

$$\begin{aligned} \Delta i &= I_{C2} - I_{C4} \\ &= \alpha \frac{I}{2} - \alpha \frac{I}{2} \frac{1}{1 + (2/\beta_p^2)} \\ &= \alpha \frac{I}{2} \left[ 1 - \frac{1}{1 + (2/\beta_p^2)} \right] \\ &\simeq -\alpha \frac{I}{2} \frac{2}{\beta_p^2} = -\alpha \frac{I}{\beta_p^2} \end{aligned}$$

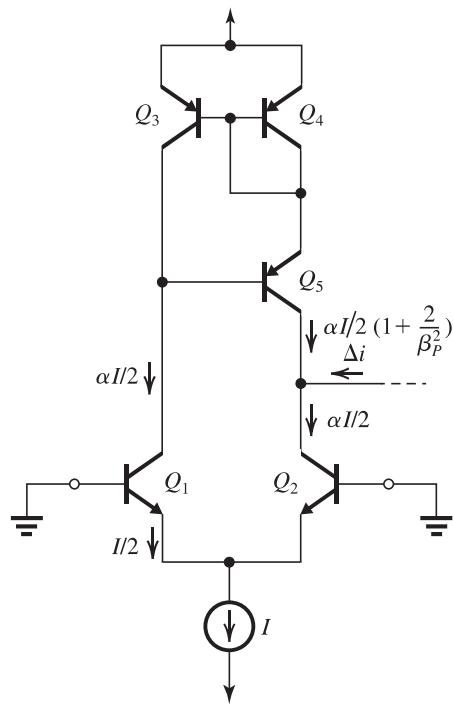
Dividing  $\Delta i$  by  $G_m = g_{m1,2} = \frac{\alpha I}{2V_T}$  gives

$$V_{OS} = -\frac{2V_T}{\beta_p^2}$$

For  $\beta_p = 50$ ,

$$V_{OS} = -\frac{2 \times 25}{50^2} = -20 \mu\text{V}$$

8.90



The figure shows a BJT differential amplifier loaded with a Wilson current mirror. To determine the systematic input offset voltage resulting from the error in the current-transfer ratio of the mirror,

we ground the two input terminals and determine the output current  $\Delta i$  as follows:

$$\Delta i = \alpha \frac{I}{2} - \alpha \frac{I}{2} \frac{1}{1 + (2/\beta_p^2)}$$

$$\simeq \alpha \frac{I}{2} \frac{2}{\beta_p^2} = \frac{\alpha I}{\beta_p^2}$$

Dividing  $\Delta i$  by  $G_m = g_{m1,2} = \frac{\alpha I/2}{V_T}$  provides the input offset voltage  $V_{OS}$ :

$$V_{OS} = -\frac{2V_T}{\beta_p^2}$$

For  $\beta_p = 50$ ,

$$V_{OS} = -\frac{2 \times 25}{50^2} = -20 \mu V$$

8.91 Refer to Fig. P8.91.

$$A_d = G_m R_o$$

where

$$G_m = g_{m1,2} \simeq \frac{I/2}{V_T}$$

and

$$R_o = R_{o4} \parallel R_{o7}$$

Here  $R_{o4}$  is the output resistance of the cascode amplifier (looking into the collector of  $Q_4$ ), thus

$$R_{o4} = g_{m4} r_{o4} (r_{o2} \parallel r_{\pi4})$$

Usually  $r_{\pi4} \ll r_{o2}$ ,

$$R_{o4} \simeq g_{m4} r_{\pi4} r_{o4} = \beta_4 r_{o4}$$

The resistance  $R_{o7}$  is the output resistance of the Wilson mirror and is given by

$$R_{o7} = \frac{1}{2} \beta_7 r_{o7}$$

Thus

$$R_o = (\beta_4 r_{o4}) \parallel \left( \frac{1}{2} \beta_7 r_{o7} \right)$$

Since all  $\beta$  and  $r_o$  are equal, we obtain

$$R_o = (\beta r_o) \parallel \left( \frac{1}{2} \beta r_o \right)$$

$$= \frac{1}{3} \beta r_o$$

and

$$A_d = \frac{1}{3} \beta g_m r_o \quad Q.E.D.$$

For  $\beta = 100$  and  $V_A = 20$  V, we have

$$g_m r_o = \frac{I_C}{V_T} \frac{V_A}{I_C} = \frac{V_A}{V_T} = \frac{20}{0.025} = 800$$

$$A_d = \frac{1}{3} \times 100 \times 800 = 2.67 \times 10^4 \text{ V/V}$$

8.92 Refer to Fig. P8.91.

$$\begin{aligned} V_{B7} &= +5 - V_{EB6} - V_{EB7} = 5 - 0.7 - 0.7 \\ &= +3.6 \text{ V} \end{aligned}$$

$$v_{Omax} = V_{B7} + 0.4 = +4 \text{ V}$$

(b) The dc bias voltage should be

$$V_O = v_{Omax} - 1.5$$

$$= 4 - 1.5 = +2.5 \text{ V}$$

(c) For  $v_O$  to swing negatively (i.e., below the dc bias value of 2.5 V) by 1.5 V, that is, to +1 V with  $Q_4$  remaining in saturation,  $V_{BIAS}$  should be

$$V_{BIAS} = v_{Omin} + 0.4$$

$$= 1.4 \text{ V}$$

(d) With  $V_{BIAS} = 1.4$  V, the bias voltage at the collectors of  $Q_1$  and  $Q_2$  is

$$V_{C1,2} = V_{BIAS} - V_{BE3,4}$$

$$= 1.4 - 0.7 = +0.7 \text{ V}$$

The upper limit on  $V_{CM}$  is 0.4 V above  $V_{C1,2}$ :

$$V_{CMmax} = 0.7 + 0.4 = +1.1 \text{ V}$$

8.93 To maximize the positive output voltage swing, we select  $V_{BIAS}$  as large as possible while maintaining the pnp current sources in saturation. For the latter to happen, we need a minimum of 0.3 V across each current source. Thus the maximum allowable voltage at the emitters of  $Q_3$  and  $Q_4$  is  $V_{CC} - 0.3 = 5 - 0.3 = +4.7$  V. Then, the maximum allowable value of  $V_{BIAS} = 4.7 - 0.7 = +4$  V. To keep  $Q_4$  in saturation,

$$v_{Omax} = V_{BIAS} + 0.4 = 4.4 \text{ V}$$

If the dc voltage at the output is 0 V, then the maximum positive voltage swing is 4.4 V. In the negative direction,

$$v_{Omin} = -V_{EE} + V_{BE7} + V_{BE5} - 0.4$$

$$= -5 + 0.7 + 0.7 - 0.4$$

$$= -4 \text{ V}$$

Thus,

$$-4 \text{ V} \leq v_O \leq +4.4 \text{ V}$$

$$G_m = g_{m1,2} \simeq \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$R_{o4} = \beta_4 r_{o4} = 50 \times \frac{|V_A|}{I/2}$$

$$= 50 \times \frac{100 \text{ V}}{0.25 \text{ mA}} = 20 \text{ M}\Omega$$

$$\begin{aligned} R_{o5} &= \frac{1}{2} \beta_5 r_{o5} = \frac{1}{2} \times 100 \times \frac{100}{0.25} \\ &= 20 \text{ M}\Omega \end{aligned}$$

$$R_o = R_{o4} \parallel R_{o5} = 20 \text{ M}\Omega \parallel 20 \text{ M}\Omega = 10 \text{ M}\Omega$$

$$A_d = G_m R_o = 10 \times 10,000 = 10^5 \text{ V/V}$$

8.94 The overdrive voltage,  $|V_{OV}|$ , at which  $Q_1$  and  $Q_2$  are operating is found from

$$\frac{I}{2} = \frac{1}{2} k'_p (W/L) |V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 6.4 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.18 \text{ V}$$

$$G_m = g_{m1,2} = \frac{2(I/2)}{|V_{OV}|}$$

$$= \frac{0.2}{0.18} = 1.13 \text{ mA/V}$$

$$r_{o2} = \frac{|V_{Ap}|}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{o4} = \frac{|V_{A_{npn}}|}{I/2} = \frac{30}{0.1} = 300 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 100 \text{ k}\Omega \parallel 300 \text{ k}\Omega = 75 \text{ k}\Omega$$

$$A_d = G_m R_o = 1.13 \times 75 = 85 \text{ V/V}$$

8.95 (a) For  $Q_1$  and  $Q_2$ ,

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.4 \times \left( \frac{W}{L} \right)_{1,2} \times 0.04$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = \left( \frac{W}{L} \right)_2 = 12.5$$

For  $Q_3$  and  $Q_4$ ,

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{3,4} |V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 0.1 \times \left( \frac{W}{L} \right)_{3,4} \times 0.04$$

$$\Rightarrow \left( \frac{W}{L} \right)_3 = \left( \frac{W}{L} \right)_4 = 50$$

$$(b) G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}} = \frac{0.2}{0.2} = 1 \text{ mA/V}$$

$$A_d = G_m R_o$$

$$50 = 1 \times R_o$$

$$\Rightarrow R_o = 50 \text{ k}\Omega$$

But

$$R_o = r_{o2} \parallel r_{o4}$$

and  $r_{o2} = r_{o4}$  ( $Q_2$  and  $Q_4$  have the same  $I_D = \frac{I}{2}$  and the same  $V_A$ ). Thus

$$r_{o2} = r_{o4} = 100 \text{ k}\Omega = \frac{|V_A|}{I/2}$$

$$|V_A| = \frac{I}{2} \times 100 \text{ k}\Omega = 10 \text{ V}$$

$$10 = |V_A| L = 20 \text{ L}$$

$$\Rightarrow L = 0.5 \text{ }\mu\text{m}$$

$$(c) v_{Omin} = V_{CM} - V_{tn}$$

$$= 0 - 0.5 = -0.5 \text{ V}$$

$$v_{Omax} = V_{DD} - |V_{OV}| = 1 - 0.2 = 0.8 \text{ V}$$

Thus,

$$-0.5 \text{ V} \leq v_O \leq 0.8 \text{ V}$$

$$(d) R_{SS} = \frac{|V_A|}{I} = \frac{10}{0.2} = 50 \text{ k}\Omega$$

The CMRR can be obtained using Eq. (8.159):

$$CMRR = (g_m r_o)(g_m R_{SS})$$

$$= (1 \times 100)(1 \times 50)$$

$$= 5000 \text{ or } 74 \text{ dB}$$

8.96 The CMRR is given by Eq. (8.158):

$$CMRR = [g_{m1,2}(r_{o2} \parallel r_{o4})] [2 g_{m3} R_{SS}]$$

(a) Current source is implemented with a simple current mirror:

$$R_{SS} = r_{o1} |_{QS} = \frac{|V_A|}{I}$$

$$g_{m1,2} = g_{m3} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I}$$

Thus,

$$CMRR = \frac{I}{V_{OV}} \times \frac{1}{2} \times \frac{2|V_A|}{I} \times 2 \times \frac{I}{V_{OV}} \times \frac{|V_A|}{I}$$

$$= 2 \left( \frac{V_A}{V_{OV}} \right)^2 \quad Q.E.D.$$

(b) Current source is implemented with the modified Wilson mirror in Fig. P8.82:

$$R_{SS} = g_{m7} r_{o7} r_{o9}$$

Transistor  $Q_7$  has the same  $k'(W/L)$  as  $Q_1$  and  $Q_2$ , but  $Q_7$  carries a current  $I$  twice that of  $Q_1$  and  $Q_2$ . Thus

$$V_{OV7} = \sqrt{2} V_{OV1,2} = \sqrt{2} V_{OV}$$

and

$$g_{m7} = \frac{2I}{V_{OV7}} = \frac{2I}{\sqrt{2}V_{OV}} = \frac{\sqrt{2}I}{V_{OV}}$$

$$r_{o7} = r_{o9} = \frac{V_A}{I}$$

Thus,

$$R_{SS} = \frac{\sqrt{2}I}{V_{OV}} \left( \frac{V_A}{I} \right)^2 = \frac{\sqrt{2}V_A^2}{V_{OV}I}$$

and

$$\begin{aligned} CMRR &= \frac{I}{V_{OV}} \times \frac{1}{2} \times \frac{2|V_A|}{I} \times 2 \times \frac{I}{V_{OV}} \times \frac{\sqrt{2}V_A^2}{V_{OV}I} \\ &= 2\sqrt{2} \left( \left| \frac{V_A}{V_{OV}} \right| \right)^3 \quad Q.E.D. \end{aligned}$$

For  $k'(W/L) = 4 \text{ mA/V}^2$  and  $I = 160 \mu\text{A}$ ,

$$0.080 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.2 \text{ V}$$

For  $|V_A| = 5 \text{ V}$ :

For case (a),

$$CMRR = 2 \times \left( \frac{5}{0.2} \right)^2 = 1250 \text{ or } 62 \text{ dB}$$

For case (b),

$$CMRR = 2\sqrt{2} \left( \frac{5}{0.2} \right)^3 = 4.42 \times 10^4$$

or 93 dB

$$8.97 G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{0.2}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 50 \text{ k}\Omega \parallel 50 \text{ k}\Omega$$

$$= 25 \text{ k}\Omega$$

$$A_d = G_m R_o = 1 \times 25 = 25 \text{ V/V}$$

$$R_{SS} = \frac{|V_A|}{I} = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$G_{mcm} = \frac{1}{2R_{SS}} = \frac{1}{2 \times 25} = 0.02 \text{ mA/V}$$

$$R_{im} = \frac{1}{g_{m3}} \parallel r_{o3}$$

where

$$g_{m3} = g_{m1} = g_{m2} = 1 \text{ mA/V}$$

$$r_{o3} = r_{o2} = r_{o4} = 50 \text{ k}\Omega$$

$$R_{im} = 1 \text{ k}\Omega \parallel 50 \text{ k}\Omega = 0.98 \text{ k}\Omega$$

$$\begin{aligned} A_m &= 1 \left/ \left( 1 + \frac{1}{g_{m3}r_{o3}} \right) \right. \\ &= 1 \left/ \left( 1 + \frac{1}{1 \times 50} \right) \right. = 0.98 \text{ A/A} \end{aligned}$$

$$R_{om} = r_{o4} = 50 \text{ k}\Omega$$

$$R_{o2} = r_{o2} + 2R_{SS} + 2g_{m2}r_{o2}R_{SS}$$

$$= 50 + 50 + 2 \times 1 \times 50 \times 25$$

$$= 2600 \text{ k}\Omega$$

$$A_{cm} = -(1 - A_m)G_{mcm}(R_{om} \parallel R_{o2})$$

$$A_{cm} = -(1 - 0.98) \times 0.02 \times (50 \parallel 2600)$$

$$= -0.0196 \text{ V/V}$$

$$CMRR = \left| \frac{A_d}{A_{cm}} \right| = \frac{25}{0.0196} = 1274$$

or 62.1 dB

Alternatively, using the approximate expression in Eq. (8.157), we obtain

$$A_{cm} \approx -\frac{1}{2g_{m3}R_{SS}} = -\frac{1}{2 \times 1 \times 25} = -0.02 \text{ V/V}$$

and

$$CMRR = \frac{25}{0.02} = 1250$$

or 61.9 dB

$$8.98 \text{ CMRR} = \left| \frac{A_d}{A_{cm}} \right|$$

CMRR = 60 dB or equivalently 1000. Thus,

$$1000 = \frac{50}{|A_{cm}|}$$

$$\Rightarrow |A_{cm}| = 0.05 \text{ V/V}$$

But from Eq. (8.153), we obtain

$$|A_{cm}| = (1 - A_m)G_{mcm}(R_{om} \parallel R_{o2})$$

Since  $R_{om} \ll R_{o2}$  and  $G_{mcm} = 1/2R_{SS}$ , we have

$$|A_{cm}| = (1 - A_m) \frac{R_{om}}{2R_{SS}}$$

$$0.05 = (1 - A_m) \times \frac{20}{2 \times 20}$$

$$\Rightarrow (1 - A_m) = 0.1$$

8.99 From Eq. (8.153), we have

$$A_{cm} = -(1 - A_m)G_{mcm}(R_{om} \parallel R_{o2})$$

where

$$G_{mcm} = \frac{1}{2R_{SS}} = \frac{1}{2 \times 45} = 0.011 \text{ mA/V}$$

Using the fact that  $R_{o2} \gg R_{om}$ , we obtain

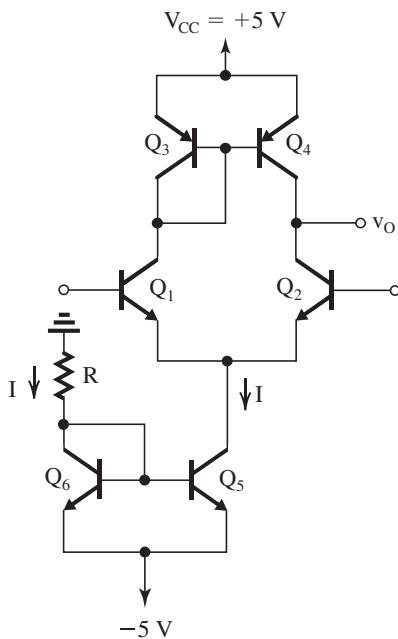
$$A_{cm} \simeq -(1 - 0.98) \times 0.011 \times 45$$

$$= -0.01 \text{ V/V}$$

$$\text{CMRR} = \left| \frac{A_d}{A_{cm}} \right| = \frac{30}{0.01} = 3000$$

or 69.5 dB

8.100



$$G_m = g_{m1,2} \simeq \frac{I/2}{V_T}$$

$$5 = \frac{I/2}{V_T}$$

$$\Rightarrow I = 0.25 \text{ mA}$$

Utilizing two matched transistors,  $Q_5$  and  $Q_6$ , the value of  $R$  can be found from

$$I = \frac{0 - (-5) - 0.7}{R} = 0.25 \text{ mA}$$

$$\Rightarrow R = 17.2 \text{ k}\Omega$$

$$R_{id} = 2r_\pi = 2 \frac{\beta}{g_m} = 2 \times \frac{100}{5} = 40 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4}$$

where

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{100}{0.125} = 800 \text{ k}\Omega$$

thus

$$R_o = 800 \text{ k}\Omega \parallel 800 \text{ k}\Omega = 400 \text{ k}\Omega$$

$$A_d = G_m R_o = 5 \times 400 = 2000 \text{ V/V}$$

$$I_B = \frac{I/2}{\beta + 1} \simeq \frac{0.125 \text{ mA}}{100} = 1.25 \mu\text{A}$$

The lower limit on  $V_{ICM}$  is determined by the lowest voltage allowed at the collector of  $Q_5$  while  $Q_5$  is in the active mode. This voltage is  $-5 + 0.3 = -4.7 \text{ V}$ . Thus

$$V_{ICM\min} = -4.7 + V_{BE1,2} = -4.7 + 0.7$$

$$= -4 \text{ V}$$

The upper limit on  $V_{ICM}$  is determined by the need to keep  $Q_1$  in the active mode. Thus

$$V_{ICM\max} = V_{C1} + 0.4$$

$$= 4.3 + 0.4 = 4.7 \text{ V}$$

Thus the input common-mode range is

$$-4 \text{ V} \leq V_{ICM} \leq +4.7 \text{ V}$$

The common-mode gain can be found using Eq. (8.165):

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

Here,

$$r_{o4} = \frac{|V_A|}{I/2} = \frac{100}{0.125} = 800 \text{ k}\Omega$$

$$\beta_3 = 100$$

$$R_{EE} = r_{o5} = \frac{|V_A|}{I} = \frac{100}{0.25} = 400 \text{ k}\Omega$$

Thus

$$A_{cm} = -\frac{800}{100 \times 400} = -0.02 \text{ V/V}$$

The CMRR can be found as

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{2000}{0.02} = 100,000$$

or 100 dB

8.101 See figure on next page.

From the solution to Problem 8.100, we know that  $I = 0.25 \text{ mA}$ . For the Widlar current source, use  $R = 2 \text{ k}\Omega$ . Thus

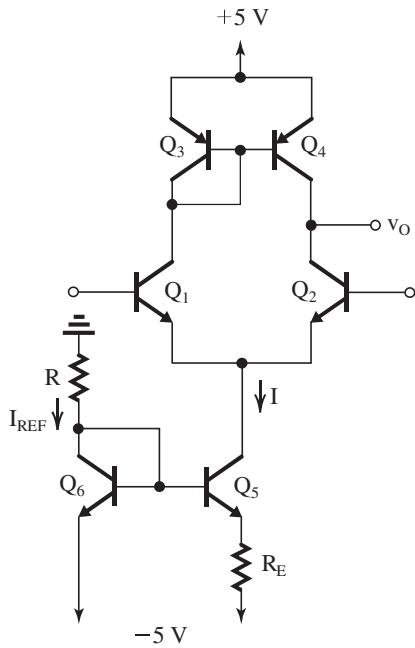
$$I_{REF} = \frac{5 - 0.7}{2} = 2.15 \text{ mA}$$

The value of  $R_E$  can be found from

$$IR_E = V_{BE6} - V_{BE5} = V_T \ln \left( \frac{I_{REF}}{I} \right)$$

$$0.25 \times R_E = 0.025 \ln \left( \frac{2.15}{0.25} \right)$$

$$R_E = 215 \text{ }\Omega$$



The output resistance of the Widlar current source is given by Eq. (7.102). Thus

$$R_{EE} = [1 + g_{m5}(R_E \parallel r_{\pi5})]r_{o5}$$

where

$$g_{m5} = \frac{I}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$r_{\pi5} = \frac{\beta}{g_{m5}} = \frac{100}{10} = 10 \text{ k}\Omega$$

$$r_{o5} = \frac{V_A}{I} = \frac{100}{0.25} = 400 \text{ k}\Omega$$

$$\begin{aligned} R_{EE} &= [1 + 10(0.215 \parallel 10)] \times 400 \\ &= 1.24 \text{ M}\Omega \end{aligned}$$

$R_{id}$ ,  $R_o$ ,  $A_d$ ,  $I_B$ , and the range of  $V_{ICM}$  will be the same as in Problem 8.100. The common-mode gain, however, will be lower:

$$\begin{aligned} A_{cm} &= -\frac{r_{o4}}{\beta_3 R_{EE}} \\ &= -\frac{800}{100 \times 1240} = 6.45 \times 10^{-3} \text{ V/V} \end{aligned}$$

and the CMRR will be

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{2000}{6.45 \times 10^{-3}} = 3.1 \times 10^5$$

or 110 dB

$$8.102 \quad G_m = g_{m1,2} \simeq \frac{I/2}{V_T} = \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$R_o = r_{o2} \parallel r_{o4}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{40}{0.2} = 200 \text{ k}\Omega$$

$$R_o = 200 \text{ k}\Omega \parallel 200 \text{ k}\Omega = 100 \text{ k}\Omega$$

$$A_d = G_m R_o = 8 \times 100 = 800 \text{ V/V}$$

$$R_{id} = 2r_\pi = 2\beta/g_m$$

$$= \frac{300}{8} = 37.5 \text{ k}\Omega$$

$$R_{EE} = \frac{|V_A|}{I} = \frac{40}{0.4} = 100 \text{ k}\Omega$$

The common-mode gain can be found using Eq. (8.165):

$$\begin{aligned} A_{cm} &= -\frac{r_{o4}}{\beta_3 R_{EE}} \\ &= -\frac{200}{150 \times 100} = -0.013 \text{ V/V} \end{aligned}$$

The CMRR can be obtained from

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{800}{0.013} = 60,000$$

or 96 dB

$$\begin{aligned} G_v &= \frac{R_{id}}{R_{id} + R_{sig}} \times A_d \\ &= \frac{37.5}{37.5 + 30} \times 800 = 444.4 \text{ V/V} \end{aligned}$$

8.103 Refer to Fig. P8.103. To determine the bias current  $I$ , which is the drain current of  $Q_7$ , we analyze the Wilson mirror circuit as follows: All four transistors,  $Q_5$ – $Q_8$ , are conducting equal currents ( $I$ ) and have the same  $V_{GS}$ ,

$$V_{GS} = V_t + V_{OV}$$

Thus

$$IR = 15 - (-5) - 2 V_{GS}$$

$$144I = 20 - 2 V_t - 2 V_{OV}$$

But

$$\begin{aligned} I &= \frac{1}{2} k'_n (W/L) V_{OV}^2 \\ &= \frac{1}{2} \times 2 \times V_{OV}^2 = V_{OV}^2 \end{aligned}$$

Thus

$$144 V_{OV}^2 = 20 - 2 \times 0.7 - 2 V_{OV}$$

$$144 V_{OV}^2 + 2 V_{OV} - 18.6 = 0$$

$$\Rightarrow V_{OV} = 0.35 \text{ V}$$

and

$$I = 0.35^2 = 0.12 \text{ mA}$$

$$(a) \quad R_{id} = 2r_\pi = 2\beta/g_m$$

where

$$g_m = g_{m1,2} \simeq \frac{I/2}{V_T} = \frac{0.06}{0.025} = 2.4 \text{ mA}$$

$$R_{id} = \frac{2 \times 100}{2.4} = 83.3 \text{ k}\Omega$$

$$(b) A_d = g_{m1,2} R_o$$

where

$$R_o = r_{o2} \parallel r_{o4}$$

But

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{60}{0.06} = 1 \text{ M}\Omega$$

$$R_o = 500 \text{ k}\Omega$$

$$A_d = 2.4 \times 500 = 1200 \text{ V/V}$$

(c)  $A_{cm}$  can be found from Eq. (8.165):

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

where  $R_{EE}$  is the output resistance of the Wilson mirror,

$$R_{EE} = g_{m7} r_{o7} r_{o5}$$

where

$$g_{m7} = \frac{2I}{V_{ov}} = \frac{2 \times 0.12}{0.35}$$

$$= 0.7 \text{ mA/V}$$

$$r_{o7} = r_{o5} = \frac{|V_A|}{I} = \frac{60}{0.12} = 500 \text{ k}\Omega$$

$$R_{EE} = 0.7 \times 500^2 = 175 \text{ M}\Omega$$

$$A_{cm} = -\frac{1}{100 \times 175} = 5.7 \times 10^{-5} \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{1200}{5.7 \times 10^{-5}} = 21 \times 10^6$$

or 146 dB

8.104 Refer to Fig. P8.104. To determine the bias current  $I$ , which is the current in the collector of  $Q_5$ , we first find the reference current through the 6.65-k $\Omega$  resistor:

$$I_{REF} = \frac{9 - (-5) - 0.7}{6.65} = 2 \text{ mA}$$

Assuming  $Q_5$  and  $Q_6$  are matched, we have

$$I = 2 \text{ mA}$$

$$(a) g_{m1,2} \simeq \frac{I/2}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$R_{id} = 2r_\pi = 2\beta/g_{m1,2}$$

$$= \frac{2 \times 100}{40} = 5 \text{ k}\Omega$$

$$(b) A_d = G_m R_o$$

where

$$G_m = g_{m1,2} = 40 \text{ mA/V}$$

$$R_o = r_{o2} \parallel r_{o4}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{60}{1} = 60 \text{ k}\Omega$$

$$R_o = 60 \text{ k}\Omega \parallel 60 \text{ k}\Omega = 30 \text{ k}\Omega$$

$$A_d = 40 \times 30 = 1200 \text{ V/V}$$

(c)  $A_{cm}$  can be found using Eq. (8.165),

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

where

$$R_{EE} = r_{o5} = \frac{|V_A|}{I} = \frac{60}{2} = 30 \text{ k}\Omega$$

$$A_{cm} = -\frac{60}{100 \times 30} = -0.02 \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{1200}{0.02} = 60,000$$

or 95.6 dB

8.105 Refer to Fig. 8.40.

$W_6$  can be determined using Eq. (8.172):

$$\frac{(W/L)_6}{(W/L)_4} = 2 \frac{(W/L)_7}{(W/L)_5}$$

$$\frac{(W/0.5)_6}{(10/0.5)} = 2 \frac{(60/0.5)}{(60/0.5)}$$

$$\Rightarrow W_6 = 20 \mu\text{m}$$

For all devices we can evaluate  $I_D$  as follows:

$$I_{D8} = I_{REF} = 225 \mu\text{A}$$

$$I_{D5} = I_{REF} \frac{(W/L)_5}{(W/L)_8} = I_{REF} = 225 \mu\text{A}$$

$$I = I_{D5} = 225 \mu\text{A}$$

$$I_{D1} = I_{D2} = \frac{1}{2} I_{D5} = 112.5 \mu\text{A}$$

$$I_{D3} = I_{D4} = I_{D1} = 112.5 \mu\text{A}$$

$$I_{D6} = I_{D7} = I_{REF} = 225 \mu\text{A}$$

With  $I_D$  in each device known, we can use

$$I_{Di} = \frac{1}{2} \mu C_{ox} \left( \frac{W}{L} \right)_i |V_{ovi}|^2$$

to determine  $|V_{ovi}|$  and then

$$|V_{GSi}| = |V_{ovi}| + |V_t|$$

The values of  $g_{mi}$  and  $r_{oi}$  can then be determined from

$$g_{mi} = \frac{2I_{Di}}{|V_{ovi}|}$$

$$r_{oi} = \frac{|V_A|}{I_{Di}}$$

$$A_l = -g_{mi}(r_{o2} \parallel r_{o4})$$

$$= -0.9 \times (80 \parallel 80) = -36 \text{ V/V}$$

The results for Problem 8.105 are summarized in the following table.

	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>	Q <sub>5</sub>	Q <sub>6</sub>	Q <sub>7</sub>	Q <sub>8</sub>
I <sub>D</sub> ( $\mu$ A)	112.5	112.5	112.5	112.5	225	225	225	225
V <sub>ov</sub>   (V)	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
V <sub>GS</sub>   (V)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
g <sub>m</sub> (mA/V)	0.9	0.9	0.9	0.9	1.8	1.8	1.8	1.8
r <sub>o</sub> (k $\Omega$ )	80	80	80	80	40	40	40	40

$$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$$

$$= -1.8 \times (40 \parallel 40) = -36 \text{ V/V}$$

$$A_0 = A_1 A_2 = -36 \times -36 = 1296 \text{ V/V}$$

The upper limit of V<sub>ICM</sub> is determined by the need to keep Q<sub>5</sub> in saturation, thus

$$V_{ICM\max} = V_{DD} - |V_{ov5}| - |V_{GS1}|$$

$$= 1.5 - 0.25 - 1 = +0.25 \text{ V}$$

The lower limit of V<sub>ICM</sub> is determined by the need to keep Q<sub>1</sub> and Q<sub>2</sub> in saturation, thus

$$V_{ICM\min} = V_{G3} - |V_t|$$

$$= -V_{SS} + |V_{GS3}| - |V_t|$$

$$= -1.5 + 1 - 0.75 = -1.25 \text{ V}$$

Thus

$$-1.25 \text{ V} \leq V_{ICM} \leq +0.25 \text{ V}$$

The output voltage range is

$$-V_{SS} + V_{ov6} \leq v_o \leq V_{DD} - |V_{ov7}|$$

that is,

$$-1.25 \text{ V} \leq v_o \leq +1.25 \text{ V}$$

### 8.106

$$(a) I_{D1} = I_{D2} = 100 = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{ov}^2$$

$$100 = \frac{1}{2} \times 400 \times \left( \frac{W}{L} \right)_{1,2} \times 0.04$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 = \left( \frac{W}{L} \right)_2 = 12.5$$

$$I_{D3} = I_{D4} = 100 = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_{3,4} |V_{ov}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_{3,4} \times 0.04$$

$$\Rightarrow \left( \frac{W}{L} \right)_3 = \left( \frac{W}{L} \right)_4 = 50$$

$$I_{D5} = I_{D7} = I_{D8} = 200 \mu\text{A}$$

Thus

$$200 = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{5,7,8} V_{ov}^2$$

$$= \frac{1}{2} \times 400 \times \left( \frac{W}{L} \right)_{5,7,8} \times 0.04$$

$$\left( \frac{W}{L} \right)_5 = \left( \frac{W}{L} \right)_7 = \left( \frac{W}{L} \right)_8 = 25$$

$$I_{D6} = 200 = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_6 |V_{ov}|^2$$

$$200 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_6 \times 0.04$$

$$\left( \frac{W}{L} \right)_6 = 100$$

The results are summarized in the following table:

Transistor	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>	Q <sub>5</sub>	Q <sub>6</sub>	Q <sub>7</sub>	Q <sub>8</sub>
W/L	12.5	12.5	50	50	25	100	25	25

Ideally, the dc voltage at the output is zero.

(b) The upper limit of V<sub>ICM</sub> is determined by the need to keep Q<sub>1</sub> and Q<sub>2</sub> in saturation, thus

$$V_{ICM\max} = V_{D1} + V_t$$

$$= V_{DD} - |V_{GS4}| + V_t$$

$$= 0.9 - |V_t| - |V_{ov4}| + V_t$$

$$= 0.9 - 0.2 = +0.7 \text{ V}$$

The lower limit of V<sub>ICM</sub> is determined by the need to keep Q<sub>5</sub> in saturation,

$$V_{ICM\min} = -0.9 + |V_{ov5}| + |V_{GS1}|$$

$$= -0.9 + 0.2 + 0.2 + 0.4 = -0.1 \text{ V}$$

Thus

$$-0.1 \text{ V} \leq V_{ICM} \leq +0.7 \text{ V}$$

$$(c) v_{omax} = V_{DD} - |V_{ov6}|$$

$$= 0.9 - 0.2 = +0.7 \text{ V}$$

$$\begin{aligned} v_{O\min} &= -V_{SS} + |V_{OV7}| \\ &= -0.9 + 0.2 = -0.7 \text{ V} \end{aligned}$$

Thus

$$-0.7 \text{ V} \leq v_O \leq +0.7 \text{ V}$$

$$(d) A_I = -g_{m1,2}(r_{o2} \parallel r_{o4})$$

where

$$g_{m1,2} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{0.1 \text{ mA}} = \frac{6}{0.1} = 60 \text{ k}\Omega$$

$$A_I = -1 \times (60 \parallel 60) = -30 \text{ V/V}$$

$$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$$

where

$$g_{m6} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_{o6} = r_{o7} = \frac{|V_A|}{0.2} = \frac{6}{0.2} = 30 \text{ k}\Omega$$

$$A_2 = -2 \times (30 \parallel 30) = -30 \text{ V/V}$$

$$A_0 = A_I A_2 = 30 \times 30 = 900 \text{ V/V}$$

8.107 (a) Increasing  $(W/L)_1$  and  $(W/L)_2$  by a factor of 4 reduces  $|V_{OV1,2}|$  by a factor of 2. Thus  $g_{m1,2} = 2I_D/|V_{OV1,2}|$  increase by a factor of 2.

(b)  $A_I$  is proportional to  $g_{m1,2}$ , thus  $A_I$  increases by a factor of 2 and the overall voltage gain increases by a factor of 2.

(c) Since the input offset voltage is proportional to  $|V_{OV1,2}|$ , it will decrease by a factor of 2. This, however, does not apply to  $V_{OS}$  due to  $\Delta V_t$ .

8.108 If  $(W/L)_7$  becomes 48/0.8,  $I_{D7}$  will become

$$I_{D7} = I_{D8} \frac{(W/L)_7}{(W/L)_8}$$

This figure belongs to Problem 8.109.

$$\begin{aligned} &= I_{REF} \frac{(48/0.8)}{(40/0.8)} \\ &= 90 \times 1.2 = 108 \mu\text{A} \end{aligned}$$

Thus  $I_{D7}$  will exceed  $I_{D6}$  by 18  $\mu\text{A}$ , which will result in a systematic offset voltage,

$$V_O = 18 \mu\text{A} (r_{o6} \parallel r_{o7})$$

where

$$r_{o6} = 111 \text{ k}\Omega$$

and  $r_{o7}$  now becomes

$$r_{o7} = \frac{10}{0.108} = 92.6 \text{ k}\Omega$$

Thus

$$V_O = 18 \times 10^{-3} \times (111 \parallel 92.6)$$

$$= 909 \text{ mV}$$

The corresponding input offset voltage will be

$$\begin{aligned} V_{OS} &= \frac{V_O}{A_0} \\ &= \frac{909}{1109} = 0.82 \text{ mV} \end{aligned}$$

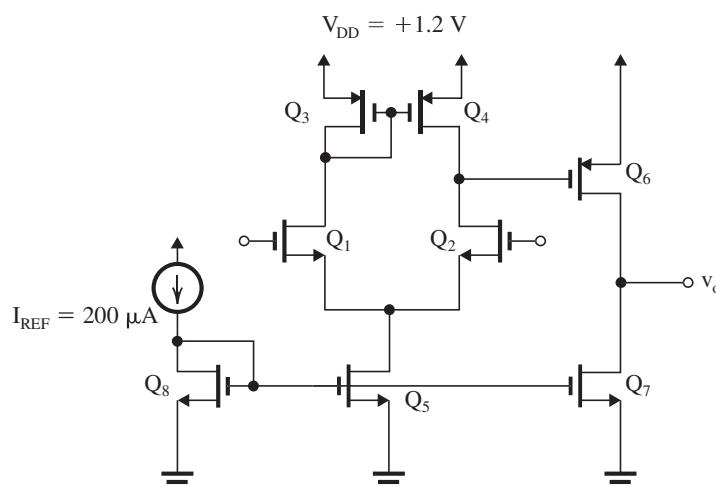
8.109

(a) With the two input terminals connected to a dc voltage of  $V_{DD}/2 = +0.6 \text{ V}$  and for  $Q_1 - Q_4$  to conduct a current of 200  $\mu\text{A}$ , we have

$$I_{D1,2} = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$200 = \frac{1}{2} \times 540 \times \left( \frac{W}{L} \right)_{1,2} \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_{1,2} = 32.9$$



$$I_{D3,4} = \frac{1}{2} k'_p \left( \frac{W}{L} \right)_{3,4} |V_{ov}|^2$$

$$200 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_{3,4} \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_{3,4} = 178$$

Transistor Q<sub>5</sub> must carry a current of 400 μA, thus

$$400 = \frac{1}{2} k'_n \left( \frac{W}{L} \right)_5 V_{ov}^2$$

$$= \frac{1}{2} \times 540 \times \left( \frac{W}{L} \right)_5 \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_5 = 65.8$$

Similarly, Q<sub>7</sub> is required to conduct a current of 400 μA, thus

$$\left( \frac{W}{L} \right)_7 = \left( \frac{W}{L} \right)_5 = 65.8$$

Transistor Q<sub>8</sub> conducts a current of 200 μA, thus

$$\left( \frac{W}{L} \right)_8 = \frac{1}{2} \left( \frac{W}{L} \right)_5 = 32.9$$

Finally, Q<sub>6</sub> must conduct a current equal to that of Q<sub>7</sub>, that is, 400 μA, thus

$$400 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_6 \times 0.15^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_6 = 356$$

The results are summarized in the following table:

Transistor	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>	Q <sub>5</sub>	Q <sub>6</sub>	Q <sub>7</sub>	Q <sub>8</sub>
I <sub>D</sub> (μA)	200	200	200	200	400	400	400	200
W/L	32.9	32.9	178	178	65.8	356	65.8	32.9

(b) The upper limit on V<sub>ICM</sub> is determined by the need to keep Q<sub>1</sub> and Q<sub>2</sub> in saturation, thus

$$\begin{aligned} V_{ICM\max} &= V_{D1,2} + |V_t| \\ &= V_{DD} - |V_t| - |V_{ov}| + |V_t| \\ &= 1.2 - 0.15 = 1.05 \text{ V} \end{aligned}$$

The lower limit on V<sub>ICM</sub> is determined by the need to keep Q<sub>5</sub> in saturation, thus

$$\begin{aligned} V_{ICM} &= |V_{ov5}| + V_{GS1,2} \\ &= 0.15 + 0.15 + 0.35 = 0.65 \text{ V} \end{aligned}$$

Thus

$$0.65 \text{ V} \leq V_{ICM} \leq 1.05 \text{ V}$$

Note that the input dc voltage in part (a) falls outside the allowable range of V<sub>ICM</sub>! Thus, part (a) should have specified a V<sub>ICM</sub> greater than 0.65 V. The results of part (a), however, will not change.

$$(c) 0.15 \text{ V} \leq v_o \leq (1.2 - 0.15)$$

that is,

$$0.15 \text{ V} \leq v_o \leq 1.05 \text{ V}$$

$$(d) g_{m1,2} = \frac{2 \times 0.2}{0.15} = 2.67 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{0.2 \text{ mA}} = \frac{1.8}{0.2} = 9 \text{ k}\Omega$$

$$A_1 = -g_{m1,2}(r_{o2} \parallel r_{o4}) = 2.67(9 \parallel 9)$$

$$= -12 \text{ V/V}$$

$$g_{m6} = \frac{2 \times 0.4}{0.15} = 5.33 \text{ mA/V}$$

$$r_{o6} = r_{o7} = \frac{|V_A|}{0.4 \text{ mA}} = \frac{1.8}{0.4} = 4.5 \text{ k}\Omega$$

$$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$$

$$= -5.33(4.5 \parallel 4.5) = 12 \text{ V/V}$$

$$A_0 = A_1 A_2 = -12 \times -12 = 144 \text{ V/V}$$

8.110 Refer to Fig. P8.110.

(a) With the inputs grounded and the output at 0 V dc, we have

$$I_{E1} = I_{E2} = \frac{1}{2} \times 0.4 = 0.2 \text{ mA}$$

$$I_{E3} = I_{E4} \simeq 0.2 \text{ mA}$$

$$I_{E5} \simeq 0.5 \text{ mA}$$

$$I_{E6} = 1 \text{ mA}$$

(b) The short-circuit transconductance of the first stage is

$$G_m = g_{m1,2} = \frac{I_{C1,2}}{V_T} \simeq \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

The voltage gain of the first stage can be obtained by multiplying G<sub>m</sub> by the total resistance at the output node of the stage, i.e., the common collectors of Q<sub>2</sub> and Q<sub>4</sub> and the base of Q<sub>5</sub>. Since r<sub>o2</sub> = r<sub>o4</sub> = ∞, the resistance at this node is equal to the input resistance of Q<sub>5</sub> which is R<sub>π5</sub>,

$$r_{\pi5} = \frac{\beta}{g_{m5}}$$

where

$$g_{m5} = \frac{I_{C5}}{V_T} = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

thus

$$r_{\pi5} = \frac{100}{20} = 5 \text{ k}\Omega$$

Thus the voltage gain of the first stage is given by

$$A_1 \equiv \frac{v_{b5}}{v_{id}} = -G_m r_{\pi5}$$

$$= -8 \times 5 = -40 \text{ V/V}$$

The voltage gain of the second stage is

$$A_2 \equiv \frac{v_{c5}}{v_{b5}} = -g_{m5} R_C$$

where  $R_{C5}$  is the total resistance in the collector of  $Q_5$ . Since  $r_{o5} = \infty$ ,  $R_{C5}$  is simply the input resistance of the emitter follower  $Q_6$ , we have

$$R_{C5} = R_{i6} = (\beta + 1)(r_{e6} + R_L)$$

where

$$r_{e6} = \frac{V_T}{I_{E6}} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

$$R_{i6} = (100 + 1)(0.025 + 1)$$

$$= 103.5 \text{ k}\Omega$$

Thus

$$A_2 = -20 \times 103.5 = -2070 \text{ V/V}$$

The gain of the third stage is given by

$$A_3 = \frac{v_o}{v_5} = \frac{R_L}{R_L + r_{e6}} = \frac{1}{1 + 0.025} = 0.976 \text{ V/V}$$

The overall voltage gain can now be obtained as

$$A_0 \equiv \frac{v_o}{v_{id}} = A_1 A_2 A_3$$

$$= -40 \times -2070 \times 0.976 = 8.07 \times 10^4 \text{ V/V}$$

8.111 See figure.

$$R_{in2} = 2(\beta + 1)(25 + 25)$$

$$= 2 \times 101 \times 50 \simeq 10 \text{ k}\Omega$$

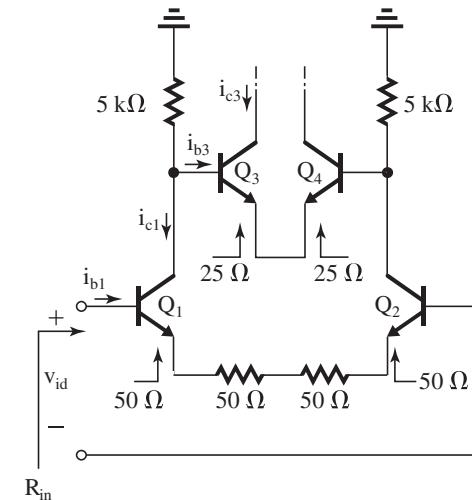
Effective load of first stage =  $R_{in2} \parallel (5 + 5)$

$$= 10 \parallel 10 = 5 \text{ k}\Omega$$

$$A_1 =$$

$$\alpha \frac{\text{Total resistance between collectors of } Q_1 \text{ and } Q_2}{\text{Total resistance in emitters of } Q_1 \text{ and } Q_2}$$

$$\simeq \frac{5 \text{ k}\Omega}{4 \times 50 \Omega} = 25 \text{ V/V}$$



$$R_{in} = (\beta + 1)(4 \times 50 \Omega)$$

$$= 101 \times 200 \simeq 20 \text{ k}\Omega$$

$$\frac{i_{c1}}{i_{b1}} = \beta_1 = 100$$

$$\frac{i_{b3}}{i_{c1}} = \frac{(5 + 5)}{(5 + 5) + R_{in2}} = \frac{10}{10 + 10} = 0.5$$

$$\frac{i_{c3}}{i_{b3}} = \beta_3 = 100$$

Thus

$$\frac{i_{c3}}{i_{b1}} = \frac{i_{c3}}{i_{b3}} \times \frac{i_{b3}}{i_{c1}} \times \frac{i_{c1}}{i_{b1}} = 100 \times 0.5 \times 100$$

$$= 5000 \text{ A/A}$$

8.112 Refer to Fig. 8.41. From Example 8.7, we obtain

$$I_{C1} = I_{C2} = 0.25 \text{ mA}$$

$$I_{C4} = I_{C5} = 1 \text{ mA}$$

$$I_{C7} = 1 \text{ mA}$$

$$I_{C8} = 5 \text{ mA}$$

Thus

$$r_{e1} = r_{e2} \simeq \frac{25 \text{ mV}}{0.25 \text{ mA}} = 100 \Omega$$

$$r_{e4} = r_{e5} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

With 100-Ω resistance in the emitter of each of  $Q_1$  and  $Q_2$ , we have

$$R_{id} = (\beta + 1)(2r_{e1,2} + 2R_{e1,2})$$

$$= 101 \times (2 \times 0.1 + 2 \times 0.1)$$

$$= 40.4 \text{ k}\Omega$$

Thus,  $R_{id}$  increases by a factor of 2. With  $25\text{-}\Omega$  resistance in the emitter of each of  $Q_4$  and  $Q_5$ , the input resistance of the second stage becomes

$$\begin{aligned} R_{i2} &= (\beta + 1)(2r_{e4,5} + 2R_{e4,5}) \\ &= 101(2 \times 0.025 + 2 \times 0.025) \\ &= 10.1 \text{ k}\Omega \end{aligned}$$

Thus,  $R_{i2}$  is increased by a factor of 2. The gain of the first stage will be

$$\frac{V_{o1}}{V_{id}} =$$

$$\frac{\alpha \times \text{Total resistance between the collectors of } Q_1 \text{ and } Q_2}{\text{Total resistance in emitters of } Q_1 \text{ and } Q_2}$$

$$\simeq \frac{40 \text{ k}\Omega \parallel 10 \text{ k}\Omega}{2 \times 0.1 + 2 \times 0.1} = 20 \text{ V/V}$$

Thus the gain of the first stage decreases but only slightly. Of course, the two  $100\text{-}\Omega$  resistances in the emitters reduce the gain but some of the reduction is mitigated by the increase in  $R_{i2}$ , which increases the effective load resistance of the first stage.

The gain of the second stage will now be

$$A_2 = \frac{V_{o2}}{V_{o1}} = -\alpha \frac{3 \text{ k}\Omega \parallel R_{i3}}{2 \times 0.025 + 2 \times 0.025}$$

From Example 8.8,  $R_{i3} = 234.8 \text{ k}\Omega$ , thus

$$A_2 \simeq -\frac{3 \parallel 234.8}{0.1} = -29.6 \text{ V/V}$$

which is about half the value without the two  $25\text{-}\Omega$  emitter resistances. The gain of the third stage remains unchanged at  $-6.42 \text{ V/V}$ , and the gain of the fourth stage remains unchanged at  $1 \text{ V/V}$ . Thus the overall voltage gain becomes

$$\begin{aligned} \frac{V_o}{V_{id}} &= A_1 A_2 A_3 A_4 \\ &= 20 \times -29.6 \times -6.42 \times 1 \\ &= 3800.6 \text{ V/V} \end{aligned}$$

which is less than half the gain obtained without the emitter resistances. This is the price paid for doubling  $R_{id}$ .

8.113 Refer to Fig. 8.41. With  $R_5$  replaced with a 1-mA constant-current source with a high output resistance, the total resistance in the collector of  $Q_7$  now becomes the input resistance of  $Q_8$ , which is

$$\begin{aligned} R_{i4} &= (\beta + 1)(r_{e8} + R_6) \\ &= 101 \times (0.005 + 3) = 303.5 \text{ k}\Omega \end{aligned}$$

Thus the gain of the third stage now becomes

$$\begin{aligned} A_3 &= -\alpha \frac{303.5}{2.3 + 0.025} \\ &\simeq -130.5 \text{ V/V} \end{aligned}$$

and the overall voltage gain increases to

$$\frac{V_o}{V_{id}} = 8513 \times \frac{130.5}{6.42} = 1.73 \times 10^5 \text{ V/V}$$

(b) The output resistance now becomes

$$\begin{aligned} R_o &= 3 \text{ k}\Omega \parallel \left( r_{e8} + \frac{\text{very large resistance}}{\beta + 1} \right) \\ &\simeq 3 \text{ k}\Omega \end{aligned}$$

When the amplifier is loaded with  $R_L = 100 \text{ }\Omega$ ,

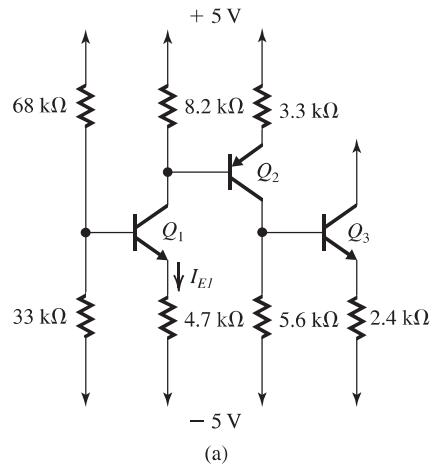
$$\begin{aligned} G_v &= 1.73 \times 10^5 \frac{R_L}{R_L + R_o} = \\ &1.73 \times 10^5 \times \frac{100}{3000 + 100} \\ G_v &= 5581 \text{ V/V} \end{aligned}$$

If the original amplifier is loaded in  $R_L = 100 \text{ }\Omega$ ,

$$G_v = 8513 \times \frac{100}{152 + 100} = 3378 \text{ V/V}$$

Thus, although the output resistance of the original amplifier is much lower than that of the modified one, the overall voltage gain realized when the original amplifier is loaded in  $100\text{-}\Omega$  resistance is much lower than that obtained with the modified design. Thus, replacing the  $15.7\text{-k}\Omega$  resistance with a constant-current source is an excellent modification to make!

### 8.114 (a)



Refer to Fig. (a) for the dc analysis. Replacing the  $68 \text{ k}\Omega$ - $33 \text{ k}\Omega$  divider network by its Thévenin

equivalent, we obtain

$$V_{BB} = -5 \text{ V} + \frac{33}{33+68} \times 10 \text{ V} \\ = -1.73 \text{ V}$$

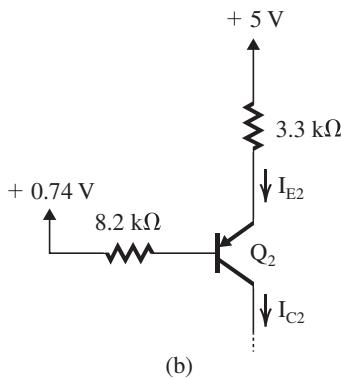
$$R_{BB} = 68 \text{ k}\Omega \parallel 33 \text{ k}\Omega = 22.2 \text{ k}\Omega$$

Now, we can determine  $I_{E1}$  from

$$I_{E1} = \frac{V_{BB} - (-5) - 0.7}{4.7 + \frac{R_{BB}}{\beta + 1}} \\ = \frac{-1.73 + 5 - 0.7}{4.7 + \frac{22.2}{101}} = 0.52 \text{ mA}$$

$$I_{C1} = \alpha_1 \times 0.52 = 0.99 \times 0.52 \simeq 0.52 \text{ mA}$$

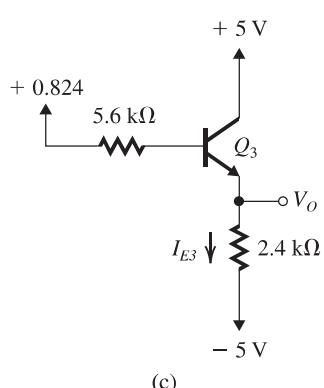
The collector current  $I_{C1}$  and the 8.2-k $\Omega$  resistor it feeds can be replaced by a Thévenin equivalent as shown in Fig. (b). Thus



$$I_{E2} = \frac{5 - 0.74 - 0.7}{3.3 + \frac{8.2}{101}} \\ = 1.05 \text{ mA}$$

$$I_{C2} \simeq 1.04 \text{ mA}$$

The collector current  $I_{C2}$  and the 5.6-k $\Omega$  resistance it feeds can be replaced by a Thévenin equivalent as shown in Fig. (c). Thus



$$I_{E3} = \frac{0.824 - 0.7 - (-5)}{2.4 + \frac{5.6}{101}} \\ = 2.1 \text{ mA}$$

$$V_O = -5 + 2.1 \times 2.4 = 0 \text{ V}$$

$$(b) R_{in} = 68 \text{ k}\Omega \parallel 33 \text{ k}\Omega \parallel r_{\pi 1}$$

where

$$r_{\pi 1} = \frac{\beta}{g_{m1}}$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.52}{0.025} = 20.8 \text{ mA/V}$$

$$r_{\pi 1} = \frac{100}{20.8} = 4.81 \text{ k}\Omega$$

$$R_{in} = 68 \text{ k}\Omega \parallel 33 \parallel 4.81 \simeq 4 \text{ k}\Omega$$

$$R_{out} = 2.4 \text{ k}\Omega \parallel \left( r_{e3} + \frac{5.6 \text{ k}\Omega}{\beta + 1} \right)$$

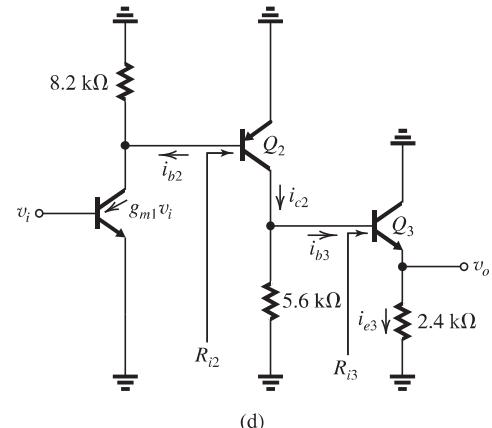
where

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{2.1 \text{ mA}} = 11.9 \text{ }\Omega$$

$$R_{out} = 2.4 \parallel \left( 0.0119 + \frac{5.6}{101} \right)$$

$$= 65.5 \text{ }\Omega$$

(c) Refer to Fig. (d)



$$i_{c1} = g_{m1}v_i = 20.8v_i$$

$$R_{i2} = r_{\pi 2} = \frac{\beta}{g_{m2}}$$

where

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{1.04 \text{ mA}}{0.025 \text{ V}} = 41.6 \text{ mA/V}$$

$$r_{\pi 2} = \frac{100}{41.6} = 2.4 \text{ k}\Omega$$

$$i_{b2} = g_{m1}v_i \frac{8.2}{8.2 + 2.4} = 16.1v_i$$

$$i_{c2} = \beta_2 i_{b2} = 100 \times 16.1v_i = 1610v_i$$

$$R_{i3} = (\beta + 1)(r_{e3} + 2.4 \text{ k}\Omega)$$

$$= 101(0.0119 + 2.4) = 243.6 \text{ k}\Omega$$

$$i_{b3} = i_{c2} \times \frac{5.6}{5.6 + 243.6} = 0.0225i_{c2}$$

$$= 0.0225 \times 1610v_i = 36.18v_i$$

$$i_{e3} = (\beta + 1)i_{b3}$$

$$= 101 \times 36.18 = 3654v_i$$

$$v_o = i_{e3} \times 2.4 \text{ k}\Omega$$

$$= 3654 \times 2.4v_i = 8770v_i$$

Thus

$$\frac{v_o}{v_i} = 8770 \text{ V/V}$$

8.115 From the figure we observe that the controlled source  $g_{m1} v_{gs1}$  can be replaced by a resistance  $1/g_{m1}$ , thus

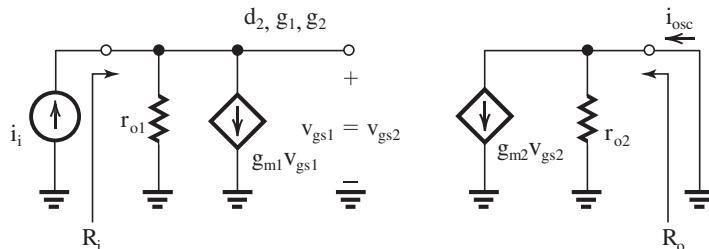
$$v_{gs1} = i_i \left( r_{o1} \parallel \frac{1}{g_{m1}} \right)$$

$$R_i \equiv \frac{v_{gs1}}{i_i} = \frac{1}{g_{m1}} \parallel r_{o1}$$

$$i_{osc} = g_{m2}v_{gs2} = g_{m2}v_{gs1} = g_{m2} \left( \frac{1}{g_{m1}} \parallel r_{o1} \right) i_i$$

$$A_{is} \equiv \frac{i_{osc}}{i_i} = g_{m2} - \frac{\frac{1}{g_{m1}}r_{o1}}{\frac{1}{g_{m1}} + r_{o1}}$$

This figure belongs to Problem 8.115.



This table belongs to Problem 8.116.

Transistor	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>	Q <sub>5</sub>	Q <sub>6</sub>	Q <sub>7</sub>	Q <sub>8</sub>
I <sub>D</sub>	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	I	$\frac{I}{2}$	I
g <sub>m</sub>	$\frac{I}{ V_{ov} }$	$\frac{2I}{ V_{ov} }$	$\frac{I}{ V_{ov} }$	$\frac{2I}{ V_{ov} }$				
r <sub>o</sub>	$\frac{2 V_A }{I}$	$\frac{ V_A }{I}$	$\frac{2 V_A }{I}$	$\frac{ V_A }{I}$				

$$= \frac{g_{m2}}{g_{m1}} \frac{1}{1 + \frac{1}{g_{m1}r_{o1}}}$$

Since  $g_{m1}r_o \gg 1$ ,

$$A_{is} \simeq \frac{g_{m2}}{g_{m1}} \left( 1 - \frac{1}{g_{m1}r_{o1}} \right)$$

$$= A_{is}|_{\text{ideal}} \left( 1 - \frac{1}{g_{m1}r_{o1}} \right)$$

where

$$A_{is}|_{\text{ideal}} = \frac{g_{m2}}{g_{m1}}$$

Finally, from inspection,

$$R_o = r_{o2}$$

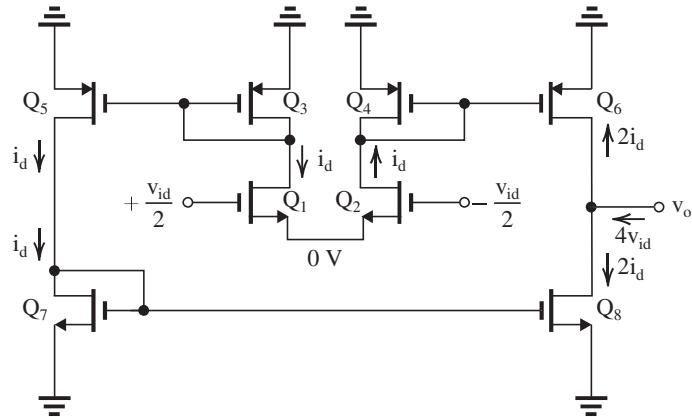
8.116 (a) Refer to Fig. P8.116. The current I<sub>D</sub> in each of the eight transistors can be found by inspection. Then, g<sub>m</sub> of each transistor can be determined as  $2I_D / |V_{ov}|$  and r<sub>o</sub> as  $|V_A| / I_D$ . The results are given in the table below:

(b) See figure on next page. Observe that at the output node the total signal current is  $4i_d$  where

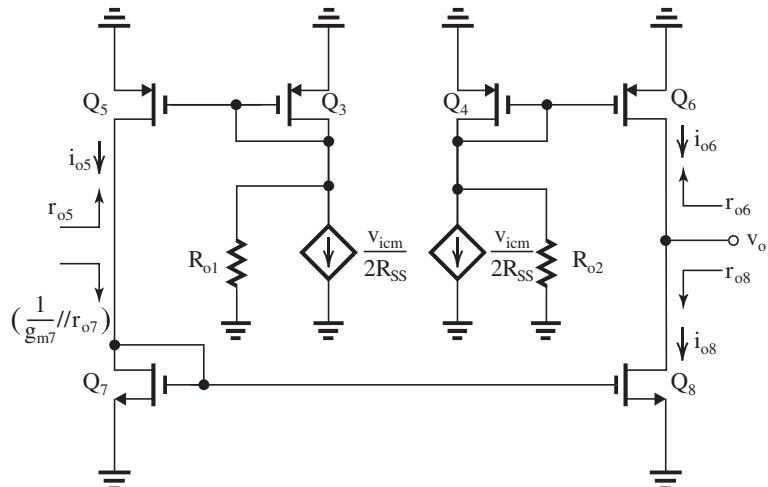
$$i_d = g_{m1,2} \frac{v_{id}}{2}$$

$$= \frac{I}{2|V_{ov}|} v_{id}$$

This figure belongs to Problem 8.116, part (b).



This figure belongs to Problem 8.116, part (c).



and since the output resistance is

$$R_o = r_{o6} \parallel r_{o8} = \frac{1}{2} \frac{|V_A|}{I}$$

then

$$v_o = 4 i_d R_o = 4 \times \frac{I}{2|V_{ov}|} \times \frac{1}{2} \frac{|V_A|}{I} \times v_{id}$$

Thus

$$A_d \equiv \frac{v_o}{v_{id}} = \frac{|V_A|}{|V_{ov}|} \quad \text{Q.E.D.}$$

(c) See figure. With  $v_{icm}$  applied to both input terminals, we can replace each of Q<sub>1</sub> and Q<sub>2</sub> with

an equivalent circuit composed of a controlled current,  $v_{icm} / 2R_{SS}$  in parallel with a very large output resistance ( $R_{o1}$  and  $R_{o2}$  which are equal). The resistances  $R_{o1}$  and  $R_{o2}$  will be much larger than the input resistance of each of the mirrors Q<sub>3</sub> – Q<sub>5</sub> and Q<sub>4</sub> – Q<sub>6</sub> and thus we can neglect  $R_{o1}$  and  $R_{o2}$  altogether. The short-circuit output current of the Q<sub>4</sub> – Q<sub>6</sub> mirror will be

$$\begin{aligned} i_{o6} &= \frac{g_{m6}}{g_{m4}} \left( 1 - \frac{1}{g_{m4} r_{o4}} \right) \frac{v_{icm}}{2R_{SS}} \\ &= \left( 1 - \frac{|V_{ov}|}{2|V_A|} \right) \left( \frac{v_{icm}}{R_{SS}} \right) \end{aligned}$$

and the output resistance will be  $r_{o6}$ . The short-circuit output current of the  $Q_3 - Q_5$  mirror will be

$$i_{o5} = \frac{g_{m5}}{g_{m3}} \left(1 - \frac{1}{g_{m3}r_{o3}}\right) \frac{v_{icm}}{2R_{SS}}$$

$$= \left(1 - \frac{|V_{ov}|}{2|V_A|}\right) \left(\frac{v_{icm}}{2R_{SS}}\right)$$

and the output resistance will be  $r_{o5}$ . Since  $r_{o5}$  is much larger than the input resistance of the  $Q_7 - Q_8$  mirror ( $\simeq 1/g_{m7}$ ), most of  $i_{o5}$  will "ow into  $Q_7$ , resulting in an output short-circuit current  $i_{o8}$ :

$$i_{o8} = \frac{g_{m8}}{g_{m7}} \left(1 - \frac{1}{g_{m7}r_{o7}}\right) i_{o5}$$

$$= 2 \left(1 - \frac{1}{g_{m7}r_{o7}}\right) i_{o5}$$

$$= \left(1 - \frac{|V_{ov}|}{2|V_A|}\right) \left(1 - \frac{1}{g_{m7}r_{o7}}\right) \frac{v_{icm}}{R_{SS}}$$

and the output resistance is  $r_{o8}$ . Thus, at the output node we have a net current

$$i_{o6} - i_{o8} = \left(1 - \frac{|V_{ov}|}{2|V_A|}\right) \left(\frac{1}{g_{m7}r_{o7}}\right) \left(\frac{v_{icm}}{R_{SS}}\right)$$

$$\simeq \left(\frac{1}{g_{m7}r_{o7}}\right) \left(\frac{v_{icm}}{R_{SS}}\right)$$

This current "ows into the output resistance ( $r_{o6} \parallel r_{o8}$ ) and thus produces an output voltage

$$v_o = \frac{r_{o6} \parallel r_{o8}}{R_{SS}} \frac{1}{g_{m7}r_{o7}} v_{icm}$$

and the common-mode gain becomes

$$|A_{cm}| = \frac{r_{o6} \parallel r_{o8}}{R_{SS}} \frac{1}{g_{m7}r_{o7}} \quad \text{Q.E.D.}$$

$$(d) R_{SS} = \frac{|V_A|}{I}$$

$$A_d = \left|\frac{V_A}{V_{ov}}\right|$$

$$|A_{cm}| = \frac{\frac{1}{2}|V_A|/I}{|V_A|/I} \frac{1}{|I/|V_{ov}| | 2|V_A|/I|}$$

$$|A_{cm}| = \frac{1}{2} \times \frac{1}{2} \frac{|V_{ov}|}{|V_A|} = \frac{1}{4} \left|\frac{V_{ov}}{V_A}\right|$$

$$\text{CMRR} = 4 \left|\frac{V_A}{V_{ov}}\right|^2 \quad \text{Q.E.D.}$$

(e) The upper limit on  $V_{ICM}$  is determined by  $Q_1$  and  $Q_2$  remaining in saturation, thus

$$V_{ICM_{max}} = V_{DD} - |V_{SG}| + |V_t|$$

$$= V_{DD} - |V_{ov}|$$

The lower limit on  $V_{ICM}$  is determined by the need to keep the bias current source in saturation, i.e. maintaining a minimum voltage across it of  $|V_{ov}|$ , thus

$$V_{ICM_{min}} = -V_{SS} + |V_{ov}| + |V_{GS}|$$

$$= -V_{SS} + 2|V_{ov}| + |V_t|$$

Thus

$$-V_{SS} + |V_t| + 2|V_{ov}| \leq V_{ICM} \leq V_{DD} - |V_{ov}|$$

The output linear range is

$$V_{DD} - |V_{ov}| \leq v_o \leq -V_{SS} + |V_{ov}|$$

### Exercise 9-1

$$\text{Ex: 9.1 } A_M = -\frac{R_G}{R_G + R_{sig}} g_m (R_D \parallel R_L)$$

$$= -\frac{10}{10 + 0.1} \times 2(10 \parallel 10)$$

$$= -9.9 \text{ V/V}$$

$$f_{p1} = \frac{1}{2\pi C_{C1}(R_{sig} + R_G)}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6} (0.1 + 10) \times 10^6}$$

$$= 0.016 \text{ Hz}$$

$$f_{p2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

$$= \frac{(2 + 0.1) \times 10^{-3}}{2\pi \times 1 \times 10^{-6}} = 334.2 \text{ Hz}$$

$$f_{p3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6} (10 + 10) \times 10^3}$$

$$= 8 \text{ Hz}$$

$$f_Z = \frac{1}{2\pi C_S R_S}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6} \times 10 \times 10^3}$$

$$= 15.9 \text{ Hz}$$

Since the highest-frequency pole is  $f_{p2} = 334.2$  and the next highest-frequency singularity is  $f_Z$  at 15.9 Hz, the lower 3-dB frequency  $f_L$  will be

$$f_L \simeq f_{p2} = 334.2 \text{ Hz}$$

Ex: 9.2 Refer to Fig. 9.10.

$$\tau_{C1} = C_{C1}[R_{sig} + (R_B \parallel r_\pi)]$$

$$= 1 \times 10^{-6} [5 + (100 \parallel 2.5)] \times 10^3$$

$$= 7.44 \text{ ms}$$

$$\tau_{CE} = C_E \left[ R_E \parallel \left( r_e + \frac{R_B \parallel R_{sig}}{\beta + 1} \right) \right]$$

$$\beta = g_m r_\pi = 40 \times 2.5 = 100$$

$$r_e \simeq 1/g_m = 25 \Omega$$

$$\tau_{CE} = 1 \times 10^{-6} \left[ 5 \parallel \left( 0.025 + \frac{100 \parallel 5}{101} \right) \right] \times 10^3$$

$$\tau_{CE} = 0.071 \text{ ms}$$

$$\tau_{C2} = C_{C2}(R_C + R_L)$$

$$= 1 \times 10^{-6} (8 + 5) \times 10^3$$

$$= 13 \text{ ms}$$

$$f_L = \frac{1}{2\pi} \left[ \frac{1}{\tau_{C1}} + \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C2}} \right]$$

$$= \frac{1}{2\pi} \left[ \frac{1}{7.44} + \frac{1}{0.071} + \frac{1}{13} \right] \times 10^3$$

$$= 2.28 \text{ kHz}$$

$$f_Z = \frac{1}{2\pi C_E R_E}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6} \times 5 \times 10^3} = 31.8 \text{ Hz}$$

Since  $f_Z$  is much lower than  $f_L$  it will have a negligible effect on  $f_L$ .

$$\text{Ex: 9.3 } C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11} \text{ F/m}}{10 \times 10^{-9} \text{ m}}$$

$$= 3.45 \times 10^{-3} \text{ F/m}^2$$

$$= 3.45 \text{ fF}/\mu\text{m}^2$$

$$C_{ov} = WL_{ov}C_{ox}$$

$$= 10 \times 0.05 \times 3.45 = 1.72 \text{ fF}$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + C_{ov}$$

$$= \frac{2}{3} \times 10 \times 1 \times 3.45 + 1.72$$

$$= 24.72 \text{ fF}$$

$$C_{gd} = C_{ov} = 1.72 \text{ fF}$$

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{V_{SB}}{V_0}}} = \frac{10}{\sqrt{1 + \frac{1}{0.6}}} = 6.1 \text{ fF}$$

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{V_{DB}}{V_0}}} = \frac{10}{\sqrt{1 + \frac{2+1}{0.6}}} = 4.1 \text{ fF}$$

$$\text{Ex: 9.4 } g_m = \sqrt{2k'_n(W/L)I_D}$$

$$= \sqrt{2 \times 0.16 \times (10/1) \times 0.1} = 0.566 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$= \frac{0.566 \times 10^{-3}}{2\pi(24.72 + 1.72) \times 10^{-15}}$$

$$f_T = 3.4 \text{ GHz}$$

$$\text{Ex: 9.5 } C_{de} = \tau_F g_m$$

where

$$\tau_F = 20 \text{ ps}$$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

### Exercise 9–2

Thus,

$$C_{de} = 20 \times 10^{-12} \times 40 \times 10^{-3} = 0.8 \text{ pF}$$

$$C_{je} \simeq 2C_{je0}$$

$$= 2 \times 20 = 40 \text{ fF}$$

$$C_\pi = C_{de} + C_{je}$$

$$= 0.8 + 0.04 = 0.84 \text{ pF}$$

$$C_\mu = \frac{C_{\mu 0}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^m}$$

$$= \frac{20}{\left(1 + \frac{2}{0.5}\right)^{0.33}} = 12 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(0.84 + 0.012) \times 10^{-12}} = 7.47 \text{ GHz}$$

$$\text{Ex: 9.6 } |h_{fe}| = 10 \text{ at } f = 50 \text{ MHz}$$

Thus,

$$f_T = 10 \times 50 = 500 \text{ MHz}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6}$$

$$= 12.7 \text{ pF}$$

$$C_\pi = 12.7 - 2 = 10.7 \text{ pF}$$

$$\text{Ex: 9.7 } C_\pi = C_{de} + C_{je}$$

$$10.7 = C_{de} + 2$$

$$\Rightarrow C_{de} = 8.7 \text{ pF}$$

Since  $C_{de}$  is proportional to  $g_m$  and hence  $I_C$ , at  $I_C = 0.1 \text{ mA}$ ,

$$C_{de} = 0.87 \text{ pF}$$

and

$$C_\pi = 0.87 + 2 = 2.87 \text{ pF}$$

$$f_T = \frac{4 \times 10^{-3}}{2\pi(2.87 + 2) \times 10^{-12}} = 130.7 \text{ MHz}$$

$$\text{Ex: 9.8 } A_M = -\frac{R_G}{R_G + R_{sig}} g_m R'_L$$

$$= -\frac{4.7}{4.7 + 0.01} \times 7.14 = -7.12 \text{ V/V}$$

$$\begin{aligned} f_H &= \frac{1}{2\pi C_{in}(R_{sig} \parallel R_G)} \\ &= \frac{1}{2\pi \times 4.26 \times 10^{-12} (0.01 \parallel 4.7) \times 10^6} \\ &= 3.7 \text{ MHz} \end{aligned}$$

$$\text{Ex: 9.9 } f_H = \frac{1}{2\pi C_{in}(R_{sig} \parallel R_G)}$$

$$1 \times 10^6 = \frac{1}{2\pi C_{in}(0.1 \parallel 4.7) \times 10^6}$$

$$\Rightarrow C_{in} = 1.625 \text{ pF}$$

But,

$$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$$

$$1.625 = 1 + C_{gd}(1 + 7.14)$$

$$\Rightarrow C_{gd} = 0.08 \text{ pF}$$

**Ex: 9.10** To reduce the midband gain to half the value found, we reduce  $R'_L$  by the same factor, thus

$$R'_L = 1.5 \text{ k}\Omega$$

But,

$$R'_L = r_o \parallel R_C \parallel R_L$$

$$1.5 = 100 \parallel 8 \parallel R_L$$

$$\Rightarrow R_L = 1.9 \text{ k}\Omega$$

$$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$$

$$= 7 + 1(1 + 40 \times 1.5)$$

$$= 68 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 68 \times 10^{-12} \times 1.65 \times 10^3}$$

$$= 1.42 \text{ MHz}$$

Thus, by accepting a reduction in gain by a factor of 2, the bandwidth is increased by a factor of  $1.42/0.754 = 1.9$ , approximately the same factor as the reduction in gain.

$$\text{Ex: 9.11 } f_t = |A_M| f_H$$

$$2 \times 10^9 = \frac{12.5}{2\pi(C_L + C_{gd}) \times 10 \times 10^3}$$

$$\Rightarrow C_L + C_{gd} = 99.5 \text{ fF}$$

$$C_L = 99.5 - 5 = 94.5 \text{ fF}$$

### Exercise 9-3

$$\text{Ex: 9.12 } T(s) = \frac{1000}{1 + \frac{s}{2\pi \times 10^5}}$$

$$GB = 1000 \times 100 \times 10^3 = 10^8 \text{ Hz}$$

$$\text{Ex: 9.13 } T(j\omega) = \frac{A_M}{\left(1 + j\frac{\omega}{\omega_{p1}}\right)\left(1 + j\frac{\omega}{\omega_{p2}}\right)}$$

$$|T| = \frac{|A_M|}{\sqrt{\left[1 + \left(\frac{\omega}{\omega_{p1}}\right)^2\right]\left[1 + \left(\frac{\omega}{\omega_{p2}}\right)^2\right]}}$$

$$|T| = \frac{|A_M|}{\sqrt{\left[1 + \left(\frac{\omega}{\omega_{p1}}\right)^2\right]\left[1 + \left(\frac{\omega}{k\omega_{p1}}\right)^2\right]}}$$

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{p1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{k\omega_{p1}}\right)^2\right]$$

For  $\omega_H = 0.9\omega_{p1}$ ,

$$2 = (1 + 0.81)\left(1 + \frac{0.81}{k^2}\right)$$

$$\Rightarrow k = 2.78$$

For  $\omega_H = 0.99\omega_{p1}$ ,

$$2 = (1 + 0.99^2)\left(1 + \frac{0.99^2}{k^2}\right)$$

$$\Rightarrow k = 9.88$$

$$\text{Ex: 9.14 } 2 = \left[1 + \left(\frac{\omega_H}{\omega_{p1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{k\omega_{p1}}\right)^2\right]$$

For  $k = 1$ ,

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{p1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{\omega_{p1}}\right)^2\right]$$

$$= \left[1 + \left(\frac{\omega_H}{\omega_{p1}}\right)^2\right]^2$$

$$\Rightarrow \omega_H = 0.64\omega_{p1}$$

The approximate value using Eq. (9.77) is

$$\omega_H \simeq 1 / \sqrt{\frac{2}{\omega_{p1}^2}} = \frac{\omega_{p1}}{\sqrt{2}}$$

$$= 0.71\omega_{p1}$$

For  $k = 2$ , the exact value of  $\omega_H$  is obtained from

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{p1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{2\omega_{p1}}\right)^2\right]$$

$$= 1 + \frac{5}{4}\left(\frac{\omega_H}{\omega_{p1}}\right)^2 + \frac{1}{4}\left(\frac{\omega_H}{\omega_{p1}}\right)^4$$

$$\left(\frac{\omega_H}{\omega_{p1}}\right)^4 + 5\left(\frac{\omega_H}{\omega_{p1}}\right)^2 - 4 = 0$$

$$\Rightarrow \omega_H = 0.84\omega_{p1}$$

The approximate value of  $\omega_H$  is obtained from Eq. (9.77),

$$\omega_H \simeq 1 / \sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{4\omega_{p1}^2}}$$

$$\Rightarrow \omega_H = 0.89\omega_{p1}$$

For  $k = 4$ , the exact value of  $\omega_H$  is obtained from

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{p1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{4\omega_{p1}}\right)^2\right]$$

$$= 1 + \frac{17}{16}\left(\frac{\omega_H}{\omega_{p1}}\right)^2 + \frac{1}{16}\left(\frac{\omega_H}{\omega_{p1}}\right)^4$$

$$\left(\frac{\omega_H}{\omega_{p1}}\right)^4 + 17\left(\frac{\omega_H}{\omega_{p1}}\right)^2 - 16 = 0$$

$$\Rightarrow \omega_H = 0.95\omega_{p1}$$

The approximate value of  $\omega_H$  is found from Eq. (9.77):

$$\omega_H \simeq 1 / \sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{16\omega_{p1}^2}}$$

$$= 0.97\omega_{p1}$$

$$\text{Ex: 9.15 } C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$$

$$= 20 + 5(1 + 1.25 \times 10)$$

$$= 87.5 \text{ fF}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 87.5 \times 10^{-15} \times 10 \times 10^3}$$

$$= 181.9 \text{ MHz}$$

This is greater than the value obtained in Example 9.8,  $f_H = 135.5 \text{ MHz}$ , by 34%. The value obtained in Example 9.8 is a better estimate of  $f_H$  as it takes into account the effect of  $C_L$ .

$$\text{Ex: 9.16 } |A_M| = g_m R'_L = 1.25 \times 10 = 12.5 \text{ V/V}$$

$$GB = |A_M| f_H$$

$$= 12.5 \times 135.5 = 1.69 \text{ GHz}$$

$$\text{Ex: 9.17 } |A_M| = \frac{1}{2} \times 12.5 = 6.25 \text{ V/V}$$

$$R_{gs} = 10 \text{ k}\Omega$$

### Exercise 9-4

$$\begin{aligned}
 R_{gd} &= R'_{sig}(1 + g_m R'_L) + R'_L \\
 &= 10(1 + 6.25) + 5 = 77.5 \text{ k}\Omega \\
 R_{C_L} &= R'_L = 5 \text{ k}\Omega \\
 \tau_{gs} &= C_{gs} R_{gs} = 20 \times 10^{-15} \times 10 \times 10^3 = 200 \text{ ps} \\
 \tau_{gd} &= C_{gd} R_{gd} = 5 \times 10^{-15} \times 77.5 \times 10^3 = 387.5 \text{ ps} \\
 \tau_{CL} &= C_L R_{CL} = 25 \times 10^{-15} \times 5 \times 10^3 = 125 \text{ ps} \\
 \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{CL} \\
 &= 200 + 387.5 + 125 \\
 &= 712.5 \text{ ps} \\
 f_H &= \frac{1}{2\pi \tau_H} \\
 &= \frac{1}{2\pi \times 712.5 \times 10^{-12}} = 223.4 \text{ MHz} \\
 GB &= 6.25 \times 223.4 = 1.4 \text{ GHz}
 \end{aligned}$$

$$\text{Ex: 9.18 } g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

Since  $I_D$  is increased by a factor of 4,  $g_m$  doubles:

$$g_m = 2 \times 1.25 = 2.5 \text{ mA/V}$$

Since  $R'_L$  is  $r_o/2$ , increasing  $I_D$  by a factor of four results in  $r_o$  and hence  $R'_L$  decreasing by a factor of 4, thus

$$\begin{aligned}
 R'_L &= \frac{1}{4} \times 10 = 2.5 \text{ k}\Omega \\
 |A_M| &= g_m R'_L = 2.5 \times 2.5 = 6.25 \text{ V/V} \\
 R_{gs} &= R'_{sig} = 10 \text{ k}\Omega \\
 R_{gd} &= R'_{sig}(1 + g_m R'_L) + R'_L \\
 &= 10(1 + 6.25) + 2.5 \\
 &= 75 \text{ k}\Omega \\
 R_{CL} &= R'_L = 2.5 \text{ k}\Omega \\
 \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{CL} \\
 &= C_{gs} R_{sig} + C_{gd} R_{gd} + C_L R_{CL} \\
 &= 20 \times 10^{-15} \times 10 \times 10^3 + 5 \times 10^{-15} \times 75 \\
 &\quad \times 10^3 + 25 \times 10^{-15} \times 2.5 \times 10^3 \\
 &= 200 + 375 + 62.5 \\
 &= 637.5 \text{ ps} \\
 f_H &= \frac{1}{2\pi \times 637.5 \times 10^{-12}} = 250 \text{ MHz} \\
 GB &= |A_M| f_H \\
 &= 6.25 \times 250 = 1.56 \text{ GHz}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex: 9.19 (a) } g_m &= 40 \text{ mA/V} \\
 r_\pi &= \frac{200}{40} = 5 \text{ k}\Omega \\
 r_{on} &= \frac{V_{An}}{I} = \frac{130}{1} = 130 \text{ k}\Omega \\
 r_{op} &= \frac{|V_{Ap}|}{I} = \frac{50}{1} = 50 \text{ k}\Omega \\
 R'_L &= r_{on} \parallel r_{op} = 130 \parallel 50 = 36.1 \text{ k}\Omega \\
 A_M &= -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m R'_L \\
 &= -\frac{5}{5 + 0.2 + 36} \times 40 \times 36.1 \\
 &= -175 \text{ V/V} \\
 \text{(b) } C_{in} &= C_\pi + C_\mu(1 + g_m R'_L) \\
 &= 16 + 0.3(1 + 40 \times 36.1) \\
 &= 450 \text{ pF} \\
 R'_{sig} &= r_\pi \parallel (r_x + R_{sig}) \\
 &= 5 \parallel (0.2 + 36) = 4.39 \text{ k}\Omega \\
 f_H &= \frac{1}{2\pi C_{in} R'_{sig}} \\
 &= \frac{1}{2\pi \times 450 \times 10^{-12} \times 4.39 \times 10^3} \\
 &= 80.6 \text{ kHz} \\
 \text{(c) } R_\pi &= R'_{sig} = 4.39 \text{ k}\Omega \\
 R_\mu &= R'_{sig}(1 + g_m R'_L) + R'_L \\
 &= 4.39(1 + 40 \times 36.1) + 36.1 \\
 &= 6.38 \text{ M}\Omega \\
 R_{CL} &= R'_L = 36.1 \text{ k}\Omega \\
 \tau_H &= C_\pi + C_\mu R_\mu + C_L R_{CL} \\
 &= 16 \times 4.39 + 0.3 \times 6.38 \times 10^3 + 5 \times 36.1 \\
 &= 70.2 + 1914 + 180.5 \\
 &= 2164.7 \text{ ns} \\
 f_H &= \frac{1}{2\pi \times 2164.7 \times 10^{-9}} \\
 &= 73.5 \text{ kHz} \\
 \text{(d) } f_Z &= \frac{g_m}{2\pi C_\mu} \\
 &= \frac{40 \times 10^{-3}}{2\pi \times 0.3 \times 10^{-12}} = 21.2 \text{ GHz} \\
 \text{(e) } GB &= 175 \times 73.5 = 12.9 \text{ MHz}
 \end{aligned}$$

### Exercise 9–5

$$\text{Ex: 9.20 } R_{in} = \frac{R_L + r_o}{1 + g_m r_o}$$

$$= \frac{500 + 20}{1 + 25} = 20 \text{ k}\Omega$$

$$G_v = \frac{R_L}{R_{sig} + R_{in}} = \frac{500}{10 + 20} = 16.7 \text{ V/V}$$

$$R_{gs} = R_{sig} \parallel R_{in} = 10 \parallel 20 = 6.7 \text{ k}\Omega$$

$$R_{gd} = R_L \parallel R_o$$

$$= 500 \parallel 280 = 179.5 \text{ k}\Omega$$

$$\tau_H = C_{gs} R_{gs} + (C_{gd} + C_L) R_{gd}$$

$$= 20 \times 10^{-15} \times 6.7 \times 10^3 + (5 + 25) \times 10^{-15}$$

$$\times 179.5 \times 10^3$$

$$= 134 + 5385$$

$$= 5519 \text{ ps}$$

$$f_H = \frac{1}{2\pi \times 5519 \times 10^{-12}} = 28.8 \text{ MHz}$$

Thus, while the midband gain has been increased substantially (by a factor of 9.7), the bandwidth has been substantially lowered (by a factor of 9.4). Thus, the high-frequency advantage of the CG amplifier is completely lost!

$$\text{Ex: 9.21 (a) } A_{CS} = -g_m (R_L \parallel r_o)$$

$$= -g_m (r_o \parallel r_o) = -\frac{1}{2} g_m r_o$$

$$= -\frac{1}{2} \times 40 = -20 \text{ V/V}$$

$$A_{cascode} = -g_m (R_L \parallel R_o)$$

$$= -g_m (r_o \parallel g_m r_o r_o)$$

$$\simeq -g_m r_o = -40 \text{ V/V}$$

Thus,

$$\frac{A_{cascode}}{A_{CS}} = 2$$

(b) For the CS amplifier,

$$\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd}$$

where

$$R_{gs} = R_{sig}$$

$$R_{gd} = R_{sig}(1 + g_m R'_L) + R'_L$$

$$\simeq R_{sig}(1 + g_m R'_L)$$

$$= R_{sig} \left( 1 + \frac{1}{2} g_m r_o \right)$$

$$= R_{sig} \left( 1 + \frac{1}{2} \times 40 \right) = 21 R_{sig}$$

$$\tau_H = C_{gs} R_{sig} + C_{gd} \times 21 R_{sig}$$

$$= C_{gs} R_{sig} + 0.25 C_{gs} \times 21 R_{sig}$$

$$= 6.25 C_{gs} R_{sig}$$

$$f_H = \frac{1}{2\pi \times 6.25 C_{gs} R_{sig}}$$

For the cascode amplifier,

$$\tau_H \simeq R_{sig} [C_{gs1} + C_{gd1}(1 + g_{m1} R_{d1})]$$

where

$$R_{d1} = r_{o1} \parallel R_{in2} = r_o \parallel \frac{r_o + r_o}{g_m r_o}$$

$$= r_o \parallel \frac{2}{g_m} = \frac{\frac{2}{g_m} r_o}{\frac{2}{g_m} + r_o}$$

$$= \frac{2 r_o}{2 + g_m r_o} = \frac{2 r_o}{2 + 40} = \frac{r_o}{21}$$

$$\tau_H = C_{gs} R_{sig} \left[ 1 + 0.25 \left( 1 + \frac{g_m r_o}{21} \right) \right]$$

$$= C_{gs} R_{sig} \left[ 1 + 0.25 \left( 1 + \frac{40}{21} \right) \right]$$

$$= 1.73 C_{gs} R_{sig}$$

$$f_H = \frac{1}{2\pi \times 1.73 C_{gs} R_{sig}}$$

Thus,

$$\frac{f_H(\text{cascode})}{f_H(\text{CS})} = \frac{6.25}{1.73} = 3.6$$

$$(c) \frac{f_t(\text{cascode})}{f_t(\text{CS})} = 2 \times 3.6 = 7.2$$

$$\text{Ex: 9.22 } g_m = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{40} = 5 \text{ k}\Omega$$

$$R_{in} = r_\pi + r_x = 5 + 0.2 = 5.2 \text{ k}\Omega$$

$$A_0 = g_m r_o$$

$$= 40 \times 130 = 5200 \text{ V/V}$$

$$R_{o1} = r_{o1} = 130 \text{ k}\Omega$$

$$R_{in2} = r_{e2} \frac{r_{o2} + R_L}{r_{o2} + R_L / (\beta_2 + 1)}$$

$$= 25 \frac{130 + 50}{130 + \frac{50}{201}}$$

$$= 35 \text{ }\Omega$$

$$R_o \simeq \beta_2 r_{o2} = 200 \times 130 = 26 \text{ M}\Omega$$

### Exercise 9–6

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m (R_o \parallel R_L)$$

$$= -\frac{5}{5 + 0.2 + 36} 40(26,000 \parallel 50)$$

$$A_M = -242 \text{ V/V}$$

$$R'_{sig} = r_{\pi 1} \parallel (r_{x1} + R_{sig})$$

$$= 5 \parallel (0.2 + 36) = 4.39 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{sig} = 4.39 \text{ k}\Omega$$

$$R_{c1} = r_{o1} \parallel R_{in2}$$

$$= 130 \text{ k}\Omega \parallel 35 \Omega \simeq 35 \Omega$$

$$R_{\mu 1} = R'_{sig}(1 + g_{m1} R_{c1}) + R_{c1}$$

$$= 4.39(1 + 40 \times 0.035) + 0.035$$

$$= 10.6 \text{ k}\Omega$$

$$r_H = C_{\pi 1} R_{\pi 1} + C_{\mu 1} R_{\mu 1} + C_{\pi 2} R_{c1}$$

$$+ (C_L + C_{\mu 2})(R_L \parallel R_o)$$

$$= 16 \times 4.39 + 0.3 \times 10.6 + 16 \times 0.035$$

$$+ (5 + 0.3)(50 \parallel 26,000)$$

$$= 70.24 + 3.18 + 0.56 + 264.5$$

$$= 338.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 338.5 \times 10^{-9}} = 470 \text{ kHz}$$

$$f_t = |A_M| f_H = 242 \times 470 = 113.8 \text{ MHz}$$

Thus, in comparison to the CE amplifier of Exercise 9.19, we see that  $|A_M|$  has increased from 175 V/V to 242 V/V,  $f_H$  has increased from 73.5 kHz to 470 kHz, and  $f_t$  has increased from 12.9 MHz to 113.8 MHz.

To have  $f_H$  equal to 1 MHz,

$$f_H = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 1 \times 10^6} = 159.2 \text{ ns}$$

Thus,

$$159.2 = 70.24 + 3.18 + 0.56$$

$$+ (C_L + C_{\mu})(50 \parallel 26000)$$

$$\Rightarrow C_L + C_{\mu} = 1.71 \text{ pF}$$

Thus,  $C_L$  must be reduced to 1.41 pF.

Ex: 9.23 From Eq. (9.120), we obtain

$$R_{gs} = \frac{R_{sig}}{g_m R'_L + 1} + \frac{R'_L}{g_m R'_L + 1} = \frac{R_{sig} + R'_L}{g_m R'_L + 1}$$

$$R_{gd} = R_{sig}$$

$$R_{CL} = \frac{R'_L}{g_m R'_L + 1}$$

Ex: 9.24 From Example 9.11, we get

$$r_H = b_1 = 104 \text{ ps}$$

$$f_H = \frac{1}{2\pi r_H}$$

$$= \frac{1}{2\pi \times 104 \times 10^{-12}} = 1.53 \text{ GHz}$$

This is lower than the exact value found in Example 9.11 (i.e., 1.86 GHz) by about 18%, still not a bad estimate!

Ex: 9.25  $g_m = 40 \text{ mA/V}$

$$r_e = 25 \text{ }\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$R'_{sig} = R_{sig} + r_x = R_{sig} = 1 \text{ k}\Omega$$

$$R'_L = R_L \parallel r_o = 1 \parallel 100 = 0.99 \text{ k}\Omega$$

$$A_M = \frac{R'_L}{R'_L + r_e + \frac{R'_{sig}}{\beta + 1}}$$

$$= \frac{0.99}{0.99 + 0.025 + (1/101)} = 0.97 \text{ V/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 400 \times 10^6}$$

$$= 15.9 \text{ pF}$$

$$C_\mu = 2 \text{ pF}$$

$$C_\pi = 13.9 \text{ pF}$$

$$f_Z = \frac{1}{2\pi C_\pi r_e} = \frac{1}{2\pi \times 13.9 \times 10^{-12} \times 25}$$

$$= 458 \text{ MHz}$$

$$b_1 = \frac{\left[ C_\pi + C_\mu \left( 1 + \frac{R'_L}{r_e} \right) \right] R'_{sig} + \left[ C_\pi + C_L \left( 1 + \frac{R'_{sig}}{r_\pi} \right) \right] R'_L}{1 + \frac{R'_L}{r_e} + \frac{R'_{sig}}{r_\pi}}$$

$$= \frac{\left[ 13.9 + 2 \left( 1 + \frac{0.99}{0.025} \right) \right] \times 1 + (13.9 + 0)0.99}{1 + \frac{0.99}{0.025} + \frac{1}{2.5}}$$

$$= 2.66 \times 10^{-9} \text{ s}$$

$$b_2 = \frac{C_\pi C_\mu R'_L R'_{sig}}{1 + \frac{R'_L}{r_e} + \frac{R'_{sig}}{r_\pi}} = \frac{13.9 \times 2 \times 0.99 \times 1}{1 + \frac{0.99}{0.025} + \frac{1}{2.5}}$$

$$= 0.671 \times 10^{-18}$$

### Exercise 9-7

$\omega_{P1}$  and  $\omega_{P2}$  are the roots of the equation

$$1 + b_1 s + b_2 s^2 = 0$$

Solving we obtain,

$$f_{P1} = 67 \text{ MHz}$$

$$f_{P2} = 563 \text{ MHz}$$

Since  $f_{P1} \ll f_{P2}$ ,

$$f_H \simeq f_{P1} = 67 \text{ MHz}$$

$$\text{Ex: 9.26 (a)} \quad I_{D1,2} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_{1,2} V_{OV}^2$$

$$0.4 = \frac{1}{2} \times 0.2 \times 100 V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4 \text{ mA/V}$$

$$(b) \quad A_d = g_m (R_D \parallel r_o)$$

where

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.4} = 50 \text{ k}\Omega$$

$$A_d = 4(5 \parallel 50) = 4 \times 4.545$$

$$= 18.2 \text{ V/V}$$

$$(c) \quad f_H = \frac{1}{2\pi(C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$$

$$= \frac{1}{2\pi(100 + 10 + 10) \times 10^{-15} \times 4.545 \times 10^3}$$

$$= 292 \text{ MHz}$$

$$(d) \quad \tau_{gs} = C_{gs} R_{sig} = 50 \times 10 = 500 \text{ ps}$$

$$\tau_{gd} = C_{gd} R_{gd} = C_{gd} [R_{sig}(1 + g_m R'_L) + R'_L]$$

$$= 10[10(1 + 18.2) + 4.545]$$

$$= 1965.5 \text{ ps}$$

$$\tau_{CL} = (C_L + C_{db}) R'_L = 110 \times 4.545 = 500 \text{ ps}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 500 + 1965.5 + 500 = 2965.5 \text{ ps}$$

$$f_H = \frac{1}{2\pi \times 2965.5 \times 10^{-12}}$$

$$= 53.7 \text{ MHz}$$

$$\text{Ex: 9.27} \quad f_Z = \frac{1}{2\pi R_{ss} C_{ss}}$$

$$= \frac{1}{2\pi \times 75 \times 10^3 \times 0.4 \times 10^{-12}}$$

$$= 5.3 \text{ MHz}$$

Thus, the 3-dB frequency of the CMRR is 5.3 MHz.

$$\text{Ex: 9.28} \quad A_d = g_{m1,2} (r_{o2} \parallel r_{o4})$$

where

$$g_{m1,2} = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$A_d = 20(200 \parallel 200) = 2000 \text{ V/V}$$

The dominant high-frequency pole is that introduced at the output node,

$$f_H = \frac{1}{2\pi C_L (r_{o2} \parallel r_{o4})}$$

$$= \frac{1}{2\pi \times 2 \times 10^{-12} \times 100 \times 10^3}$$

$$= 0.8 \text{ MHz}$$

$$\text{Ex: 9.29 (a)} \quad A_M = -g_m R'_L$$

where

$$R'_L = R_L \parallel r_o = 20 \parallel 20 = 10 \text{ k}\Omega$$

$$A_M = -2 \times 10 = -20 \text{ V/V}$$

$$\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R'_L$$

$$= C_{gs} R_{sig} + C_{gd} [R_{sig}(1 + g_m R'_L) + R'_L] + C_L R'_L$$

$$= 20 \times 20 + 5[20(1 + 20) + 10] + 5 \times 10$$

$$= 400 + 2150 + 50$$

$$= 2600 \text{ ps}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 2600 \times 10^{-12}}$$

$$= 61.2 \text{ MHz}$$

$$GB = |A_M| f_H$$

$$= 20 \times 61.2$$

$$= 1.22 \text{ GHz}$$

$$(b) \quad G_m = \frac{g_m}{1 + g_m R_s} = \frac{2}{1 + 2} = 0.67 \text{ mA/V}$$

$$R_o \simeq r_o (1 + g_m R_s)$$

$$= 20 \times 3 = 60 \text{ k}\Omega$$

$$R'_L = R_L \parallel R_o = 20 \parallel 60 = 15 \text{ k}\Omega$$

$$A_M = -G_m R'_L$$

$$= -0.67 \times 15 = -10 \text{ V/V}$$

$$R_{gd} = R_{sig}(1 + G_m R'_L) + R'_L$$

$$= 20(1 + 10) + 15$$

$$= 235 \text{ k}\Omega$$

### Exercise 9-8

$$R_{C_L} = R'_L = 15 \text{ k}\Omega$$

$$R_{gs} = \frac{R_{sig} + R_s + R_{sig}R_s/(r_o + R_L)}{1 + g_m R_s \left( \frac{r_o}{r_o + R_L} \right)}$$

where

$$R_s = \frac{2}{g_m} = 1 \text{ k}\Omega$$

$$R_{gs} = \frac{20 + 1 + \frac{20 \times 1}{20 + 20}}{1 + 2 \times \frac{20}{20 + 20}}$$

$$= 10.75 \text{ k}\Omega$$

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$$

$$= 20 \times 10.75 + 5 \times 235 + 5 \times 15$$

$$= 215 + 1175 + 75 = 1465 \text{ ps}$$

$$f_H = \frac{1}{2\pi \times 1465 \times 10^{-12}} = 109 \text{ MHz}$$

$$GB = 10 \times 109 = 1.1 \text{ GHz}$$

Ex: 9.30 Refer to Fig. 9.42(b).

$$A_M = \frac{2r_\pi}{2r_\pi + R_{sig}} \times \frac{1}{2} \times g_m R_L$$

where

$$g_m = 20 \text{ mA/V}$$

$$r_\pi = \frac{100}{20} = 5 \text{ k}\Omega$$

$$A_M = \frac{10}{10 + 10} \times \frac{1}{2} \times 20 \times 10 = 50 \text{ V/V}$$

$$f_{p1} = \frac{1}{2\pi \left( \frac{C_\pi}{2} + C_\mu \right) (2r_\pi \parallel R_{sig})}$$

$$= \frac{1}{2\pi \left( \frac{6}{2} + 2 \right) \times 10^{-12} (10 \parallel 10) \times 10^3}$$

$$= 6.4 \text{ MHz}$$

$$f_{p2} = \frac{1}{2\pi C_\mu R_L}$$

$$= \frac{1}{2\pi \times 2 \times 10^{-12} \times 10 \times 10^3}$$

$$= 8 \text{ MHz}$$

$$T(s) = \frac{50}{\left( 1 + \frac{s}{\omega_{p1}} \right) \left( 1 + \frac{s}{\omega_{p2}} \right)}$$

$$|T(j\omega)| = \frac{50}{\sqrt{\left[ 1 + \left( \frac{\omega}{\omega_{p1}} \right)^2 \right] \left[ 1 + \left( \frac{\omega}{\omega_{p2}} \right)^2 \right]}}$$

At  $\omega = \omega_H$ ,  $|T| = 50/\sqrt{2}$ , thus

$$2 = \left[ 1 + \left( \frac{\omega_H}{\omega_{p1}} \right)^2 \right] \left[ 1 + \left( \frac{\omega_H}{\omega_{p2}} \right)^2 \right]$$

$$= 1 + \left( \frac{\omega_H}{\omega_{p1}} \right)^2 + \left( \frac{\omega_H}{\omega_{p2}} \right)^2 + \left( \frac{\omega_H}{\omega_{p1}} \right)^2 \left( \frac{\omega_H}{\omega_{p2}} \right)^2$$

$$\frac{\omega_H^4}{\omega_{p1}^2 \omega_{p2}^2} + \omega_H^2 \left( \frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} \right) - 1 = 0$$

$$\frac{f_H^4}{f_{p1}^2 f_{p2}^2} + f_H^2 \left( \frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2} \right) - 1 = 0$$

$$\frac{f_H^4}{6.4^2 \times 8^2} + f_H^2 \left( \frac{1}{6.4^2} + \frac{1}{8^2} \right) - 1 = 0$$

$\Rightarrow f_H = 4.6 \text{ MHz}$  (Exact value)

Using Eq. (9.164), an approximate value for  $f_H$  can be obtained:

$$f_H \simeq 1 / \sqrt{\frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2}}$$

$$= 1 / \sqrt{\frac{1}{6.4^2} + \frac{1}{8^2}} = 5 \text{ MHz}$$

9.1 Refer to Fig. 9.3(b).

$$\frac{V_g}{V_{\text{sig}}} = \frac{R_G}{R_G + R_{\text{sig}} + \frac{1}{sC_{C1}}}$$

where

$$R_G = R_{G1} \parallel R_{G2} = 2 \text{ M}\Omega \parallel 1 \text{ M}\Omega = 667 \text{ k}\Omega$$

and

$$R_{\text{sig}} = 200 \text{ k}\Omega$$

$$\frac{V_g}{V_{\text{sig}}} = \frac{R_G}{R_G + R_{\text{sig}}} \frac{s}{s + \frac{1}{C_{C1}(R_G + R_{\text{sig}})}}$$

Thus,

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})}$$

We required

$$f_{P1} \leq 10 \text{ Hz}$$

thus we select  $C_{C1}$  so that

$$\frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})} \leq 10$$

$$C_{C1} \geq \frac{1}{2\pi \times 10 \times (667 + 200) \times 10^3} = 18.4 \text{ nF}$$

$$\Rightarrow C_{C1} = 20 \text{ nF}$$

9.2 Refer to Fig. 9.3(b).

$$V_o = -I_d \frac{\frac{R_D}{1+sC_{C2}} \times R_L}{R_D + R_L}$$

$$\frac{V_o}{I_d} = -\frac{R_D R_L}{R_D + R_L} \frac{s}{s + \frac{1}{C_{C2}(R_D + R_L)}}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

where

$$R_D = 10 \text{ k}\Omega \text{ and } R_L = 10 \text{ k}\Omega$$

To make  $f_{P3} \leq 10 \text{ Hz}$ ,

$$\frac{1}{2\pi C_{C2}(R_D + R_L)} \leq 10$$

$$\Rightarrow C_{C2} \geq \frac{1}{2\pi \times 10 \times (10 + 10) \times 10^3} = 0.8 \mu\text{F}$$

$$\text{Select, } C_{C2} = 0.8 \mu\text{F.}$$

9.3 Refer to Fig. 9.3(b).

$$I_s = \frac{V_g}{\frac{1}{g_m} + Z_s}$$

$$\begin{aligned} I_s &= \frac{g_m V_g Y_S}{Y_S + g_m} \\ \frac{I_s}{V_g} &= \frac{g_m \left( \frac{1}{R_S} + sC_S \right)}{g_m + \frac{1}{R_S} + sC_S} \\ &= g_m \frac{s + 1/C_S R_S}{s + \frac{g_m + 1/R_S}{C_S}} \end{aligned}$$

Thus,

$$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

$$f_Z = \frac{1}{2\pi C_S R_S}$$

where

$$g_m = 5 \text{ mA/V and } R_S = 1.8 \text{ k}\Omega$$

To make  $f_{P2} \leq 100 \text{ Hz}$ ,

$$\frac{g_m + 1/R_S}{2\pi C_S} \leq 100$$

$$\Rightarrow C_S \geq \frac{5 \times 10^{-3} + (1/1.8 \times 10^3)}{2\pi \times 100} = 8.8 \mu\text{F}$$

$$\text{Select } C_S = 10 \mu\text{F.}$$

Thus,

$$f_{P2} = \frac{5 \times 10^{-3} + (1/1.8 \times 10^3)}{2\pi \times 10 \times 10^{-6}} = 88.4 \text{ Hz}$$

and

$$f_Z = \frac{1}{2\pi \times 10 \times 10^{-6} \times 1.8 \times 10^3} = 8.84 \text{ Hz}$$

9.4 Refer to Fig. 9.3.

$$A_M = -\frac{R_G}{R_G + R_{\text{sig}}} \times g_m (R_D \parallel R_L)$$

where

$$R_G = R_{G1} \parallel R_{G2} = 47 \text{ M}\Omega \parallel 10 \text{ M}\Omega$$

$$= 8.246 \text{ M}\Omega$$

$$R_{\text{sig}} = 100 \text{ k}\Omega, g_m = 5 \text{ mA/V}, R_D = 4.7 \text{ k}\Omega \text{ and } R_L = 10 \text{ k}\Omega.$$

Thus,

$$A_M = -\frac{8.426}{8.426 + 0.1} \times 5(4.7 \parallel 10)$$

$$= -15.8 \text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})}$$

$$= \frac{1}{2\pi \times 0.01 \times 10^{-6} (8.426 + 0.1) \times 10^6}$$

$$= 1.9 \text{ Hz}$$

$$f_{p2} = \frac{g_m + 1/R_s}{2\pi C_s}$$

$$= \frac{5 \times 10^{-3} + 0.5 \times 10^{-3}}{2\pi \times 10 \times 10^{-6}} = 87.5 \text{ Hz}$$

$$f_z = \frac{1}{2\pi C_s R_s}$$

$$= \frac{1}{2\pi \times 10 \times 10^{-6} \times 2 \times 10^3} = 8 \text{ Hz}$$

$$f_{p3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6} (4.7 + 10) \times 10^3} = 10.8 \text{ Hz}$$

Since

$$f_{p2} \gg f_{p1}, f_{p3}, f_z,$$

$$f_L \approx f_{p2} = 87.5 \text{ Hz}$$

9.5

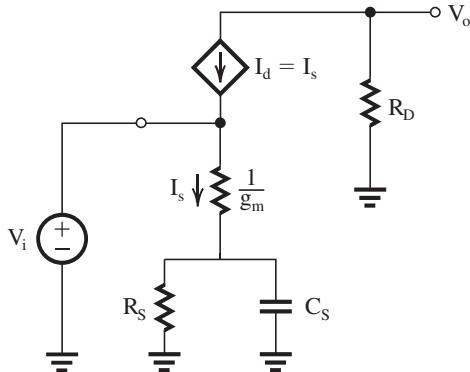


Figure 1

Replacing the MOSFET with its T model results in the circuit shown in Fig. 1.

$$(a) A_M \equiv \frac{V_o}{V_i} = -g_m R_D$$

$$-20 = -2 \times R_D$$

$$\Rightarrow R_D = 10 \text{ k}\Omega$$

$$(b) f_p = \frac{g_m + 1/R_s}{2\pi C_s}$$

$$100 = \frac{2 \times 10^{-3} + (1/4.5 \times 10^3)}{2\pi C_s}$$

$$\Rightarrow C_s = 3.53 \mu\text{F}$$

$$(c) f_z = \frac{1}{2\pi C_s R_s} =$$

$$\frac{1}{2\pi \times 3.53 \times 10^{-6} \times 4.5 \times 10^3} = 10 \text{ Hz}$$

(d) Since  $f_p \gg f_z$ ,

$$f_L \approx f_p = 100 \text{ Hz}$$

(e) The Bode plot for the gain is shown in Fig. 2.

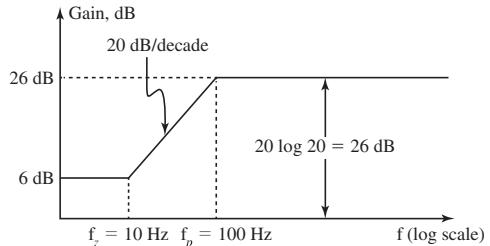


Figure 2

Observe that the dc gain is 6 dB, i.e. 2 V/V. This makes perfect sense since from Fig. 1 we see that at dc, capacitor  $C_s$  behaves as open circuit and the gain becomes

$$\begin{aligned} \text{DC gain} &= -\frac{R_D}{\frac{1}{g_m} + R_s} = -\frac{10 \text{ k}\Omega}{\left(\frac{1}{2} + 4.5\right)} \\ &= -2 \text{ V/V} \end{aligned}$$

9.6 See figure on next page. Replacing the MOSFET with its T model results in the circuit shown in the figure.

$$\begin{aligned} A_M &= -\frac{R_G}{R_G + R_{sig}} \times g_m (R_D \parallel R_L) \\ &= -\frac{2}{2 + 0.5} \times 3(20 \parallel 10) \\ &= -16 \text{ V/V} \end{aligned}$$

To minimize the total capacitance we select  $C_s$  so as to place  $f_{p2}$  (usually the highest-frequency low-frequency pole) at 100 Hz. Thus,

$$100 = \frac{g_m}{2\pi C_s}$$

$$= \frac{3 \times 10^{-3}}{2\pi C_s}$$

$$\Rightarrow C_s = 4.8 \mu\text{F}$$

Select  $C_s = 5 \mu\text{F}$ .

Of the two remaining poles, the one caused by  $C_{C2}$  has associated relatively low-valued resistances ( $R_D$  and  $R_L$  are much lower than  $R_G$ ), thus to minimize the total capacitance we place  $f_{p3}$  at 10 Hz and  $f_{p1}$  at 1 Hz. Thus,

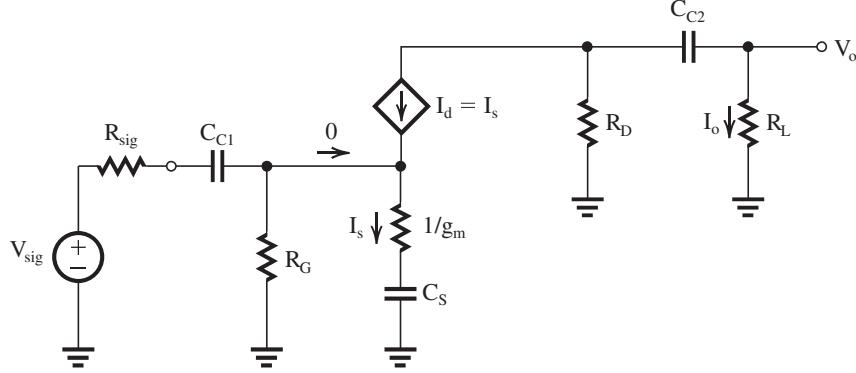
$$10 = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

$$\equiv \frac{1}{2\pi C_{C2}(20 + 10) \times 10^3}$$

$$\Rightarrow C_{C2} = 0.53 \mu\text{F}$$

Select  $C_{C2} = 1 \mu\text{F}$ .

This figure belongs to Problem 9.6.



$$1 = \frac{1}{2\pi C_{C1}(R_G + R_{sig})}$$

$$1 = \frac{1}{2\pi C_{C1}(2 + 0.5) \times 10^6}$$

$$\Rightarrow C_{C1} = 63.7 \text{ nF}$$

Select  $C_{C1} = 100 \text{ nF} = 0.1 \mu\text{F}$ .

With the selected capacitor values, we obtain

$$f_{p1} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 2.5 \times 10^6} = 0.64 \text{ Hz}$$

$$f_{p2} = \frac{3 \times 10^{-3}}{2\pi \times 5 \times 10^{-6}} = 95.5 \text{ Hz}$$

$f_Z = 0$  (dc)

$$f_{p3} = \frac{1}{2\pi \times 1 \times 10^{-6} (20 + 10) \times 10^3} = 5.3 \text{ Hz}$$

Since  $f_{p2} \gg f_{p1}$  and  $f_{p3}$ , we have

$$f_L \simeq f_{p2} = 95.5 \text{ Hz}$$

9.7 The amplifier in Fig. P9.7 will have the equivalent circuit in Fig. 9.9 except with  $R_E = \infty$  (i.e. omitted). Also, the equivalent circuits in Fig. 9.10 can be used to determine the three short-circuit time constants, again with  $R_E = \infty$ . Since the amplifier is operating at  $I_C \simeq I_E = 100 \mu\text{A} = 0.1 \text{ mA}$  and  $\beta = 100$ ,

$$r_e = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

$$g_m = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

Using the equivalent circuit in Fig. 9.10(b), we get

$$\tau_{CE} = C_E \left( r_e + \frac{R_B \parallel R_{sig}}{\beta + 1} \right)$$

To make  $C_E$  responsible for 80% of  $f_L$ , we use

$$\frac{1}{\tau_{CE}} = 0.8 \omega_L = 0.8 \times 2\pi f_L$$

$$\Rightarrow \tau_{CE} = \frac{1}{0.8 \times 2\pi \times 100} \simeq 2 \text{ ms}$$

Thus,

$$C_E \left[ 250 + \frac{(200 \parallel 20) \times 10^3}{101} \right] = 2 \times 10^{-3}$$

$$\Rightarrow C_E = 4.65 \mu\text{F}$$

Select,  $C_E = 5 \mu\text{F}$ .

Using the information in Fig. 9.10(a), we determine  $\tau_{C1}$  as

$$\tau_{C1} = C_{C1} [(R_B \parallel r_\pi) + R_{sig}]$$

To make the contribution of  $C_{C1}$  to the determination of  $f_L$  equal to 10%, we use

$$\frac{1}{\tau_{C1}} = 0.1 \omega_L = 0.1 \times 2\pi f_L$$

$$\Rightarrow \tau_{C1} = \frac{1}{0.1 \times 2\pi \times 100} = 15.92 \text{ ms}$$

Thus,

$$C_{C1} [(200 \parallel 25) \times 10^3 + 20 \times 10^3] = 15.92 \times 10^{-3}$$

$$\Rightarrow C_{C1} = 0.38 \mu\text{F}$$

Select  $C_{C1} = 0.5 \mu\text{F}$ .

For  $C_{C2}$  we use the information in Fig. 9.10(c) to determine  $\tau_{C2}$ :

$$\tau_{C2} = C_{C2} (R_C + R_L)$$

To make the contribution of  $C_{C2}$  to the determination of  $f_L$  equal to 10%, we use

$$\frac{1}{\tau_{C2}} = 0.1 \omega_L = 0.1 \times 2\pi f_L$$

$$\Rightarrow \tau_{C2} = \frac{1}{0.1 \times 2\pi \times 100} = 15.92 \text{ ms}$$

Thus,

$$C_{C2}(20 + 10) \times 10^3 = 15.92 \times 10^{-3}$$

$$\Rightarrow C_{C2} = 0.53 \mu F$$

Although, to be conservative we should select  $C_{C2} = 1 \mu F$ ; in this case we can select

$$C_{C2} = 0.5 \mu F$$

because the required value is very close to  $0.5 \mu F$  and because we have selected  $C_{C1}$  and  $C_E$  larger than the required values. The resulting  $f_L$  will be

$$f_L = \frac{1}{2\pi} \left[ \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C1}} + \frac{1}{\tau_{C2}} \right]$$

$$\tau_{CE} = 5 \times 10^{-6} \times \left[ 250 + \frac{(200 \parallel 20) \times 10^3}{101} \right]$$

$$= 2.15 \text{ ms}$$

$$\tau_{C1} = 0.5 \times 10^{-6} [(200 \parallel 25) \times 10^3 + 20 \times 10^3]$$

$$= 21.1 \text{ ms}$$

$$\tau_{C2} = 0.5 \times 10^{-6} (20 + 10) \times 10^3 = 15 \text{ ms}$$

$$f_L = \frac{1}{2\pi} \left[ \frac{10^3}{2.15} + \frac{10^3}{21.1} + \frac{10^3}{15} \right]$$

$$= 92.2 \text{ Hz}$$

which is lower (hence more conservative) than the required value of 100 Hz.

$$C_{\text{total}} = 5 + 0.5 + 0.5 = 6.0 \mu F$$

### 9.8 Refer to Fig. 9.9.

In the midband,

$$R_{in} = R_{B1} \parallel R_{B2} \parallel r_\pi$$

where

$$R_{B1} = 33 \text{ k}\Omega, R_{B2} = 22 \text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{0.3 \text{ mA}}{0.025 \text{ V}} = 12 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.3 \text{ mA}} = 83.3 \text{ }\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{12} = 10 \text{ k}\Omega$$

Thus,

$$R_{in} = 33 \parallel 22 \parallel 10 = 5.7 \text{ k}\Omega$$

$$A_M = -\frac{R_{in}}{R_{in} + R_{sig}} g_m (R_C \parallel R_L)$$

where

$$R_{sig} = 5 \text{ k}\Omega, R_C = 4.7 \text{ k}\Omega, \text{ and } R_L = 5.6 \text{ k}\Omega$$

Thus,

$$A_M = -\frac{5.7}{5.7 + 5} \times 12(4.7 \parallel 5.6)$$

$$= -16.3 \text{ V/V}$$

Using the method of short-circuit time constants and the information in Fig. 9.10, we obtain

$$\tau_{C1} = C_{C1} [(R_B \parallel r_\pi) + R_{sig}]$$

$$= C_{C1} (R_{in} + R_{sig})$$

$$= 1 \times 10^{-6} (5.7 + 5) \times 10^3 = 10.7 \text{ ms}$$

$$\tau_{CE} = C_E \left[ R_E \parallel \left( r_e + \frac{R_B \parallel R_{sig}}{\beta + 1} \right) \right]$$

$$= 20 \times 10^{-6} \left[ 3.9 \times 10^3 \parallel \left( 83.3 + \frac{(33 \parallel 22 \parallel 5) \times 10^3}{121} \right) \right]$$

$$= 2.2 \text{ ms}$$

$$\tau_{C2} = C_{C2} (R_C + R_L)$$

$$= 1 \times 10^{-6} (4.7 + 5.6) \times 10^3 = 10 \text{ ms}$$

$$f_L \approx \frac{1}{2\pi} \left( \frac{1}{\tau_{C1}} + \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C2}} \right)$$

$$= \frac{1}{2\pi} \left( \frac{1}{10.7 \times 10^{-3}} + \frac{1}{2.2 \times 10^{-3}} + \frac{1}{10.3 \times 10^{-3}} \right)$$

$$= 102.7 \text{ Hz}$$

9.9 Refer to the data given in the statement for Problem 9.8.

$$R_B = R_{B1} \parallel R_{B2} = 33 \text{ k}\Omega \parallel 22 \text{ k}\Omega = 13.2 \text{ k}\Omega$$

$$I_C \simeq I_E \simeq 0.3 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.3 \text{ mA}}{0.025 \text{ V}} = 12 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.3 \text{ mA}} = 83.3 \text{ }\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{12} = 10 \text{ k}\Omega$$

From Fig. 9.10, we have

$$\tau_{C1} = C_{C1} [(R_B \parallel r_\pi) + R_{sig}]$$

For  $C_{C1}$  to contribute 10% of  $f_L$ , we use

$$\frac{1}{\tau_{C1}} = 0.1 \omega_L = 0.1 \times 2\pi f_L$$

$$= 0.1 \times 2\pi \times 50$$

$$\Rightarrow \tau_{C1} = 31.8 \text{ ms}$$

Thus,

$$C_{C1} [(13.2 \parallel 10) + 5] \times 10^3 = 31.8 \times 10^{-3}$$

$$\Rightarrow C_{C1} = 3 \mu F$$

$$\tau_{C2} = C_{C2} (R_C + R_L)$$

For  $C_{C2}$  to contribute 10% of  $f_L$ , we use

$$\frac{1}{\tau_{C2}} = 0.1 \omega_L = 0.1 \times 2\pi f_L$$

$$= 0.1 \times 2\pi \times 50$$

$$\Rightarrow \tau_{C2} = 31.8 \text{ ms}$$

Thus,

$$C_{C2}(4.7 + 5.6) \times 10^3 = 31.8 \times 10^{-3}$$

$$\Rightarrow C_{C2} = 3.09 \mu F \simeq 3 \mu F$$

Finally,

$$\tau_{CE} = C_E \left[ R_E \parallel \left( r_e + \frac{R_B \parallel R_{sig}}{\beta + 1} \right) \right]$$

For  $C_E$  to contribute 80% of  $f_L$ , we use

$$\frac{1}{\tau_{CE}} = 0.8\omega_L = 0.8 \times 2\pi f_L$$

$$= 0.8 \times 2\pi \times 50$$

$$\Rightarrow \tau_{CE} = 3.98 \text{ ms}$$

Thus,

$$C_E \left[ 3900 \parallel \left( 83.3 + \frac{(13.2 \parallel 5) \times 1000}{121} \right) \right]$$

$$= 3.98 \times 10^{-3}$$

$$\Rightarrow C_E = 36.2 \mu F$$

9.10

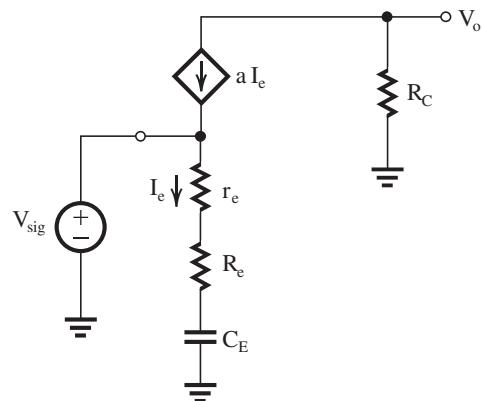


Figure 1

Replacing the BJT with its T model results in the circuit shown in Fig. 1.

$$(a) I_e = \frac{V_{sig}}{r_e + R_e + \frac{1}{sC_E}}$$

$$V_o = -\alpha I_e R_C$$

Thus,

$$\frac{V_o}{V_{sig}} = -\frac{\alpha R_C}{r_e + R_e} \frac{s}{s + \frac{1}{C_E(r_e + R_e)}} \quad (1)$$

From this expression we obtain

$$A_M = -\frac{\alpha R_C}{r_e + R_e} \quad (2)$$

and

$$f_L = f_P = \frac{1}{2\pi C_E(r_e + R_e)} \quad (3)$$

(b) From Eq. (2) we see that

$$|A_M| = \frac{\alpha R_C}{r_e} \frac{1}{1 + \frac{R_e}{r_e}}$$

Thus, including  $R_e$  reduces the gain magnitude by the factor  $\left(1 + \frac{R_e}{r_e}\right)$ .

(c) From Eq. (3), we obtain

$$f_L = \frac{1}{2\pi C_E r_e} \frac{1}{1 + \frac{R_e}{r_e}}$$

Thus, including  $R_e$  reduces  $f_L$  by the factor  $\left(1 + \frac{R_e}{r_e}\right)$ . This is the same factor by which the magnitude of the gain is reduced. Thus,  $R_e$  can be used to tradeoff gain for decreasing  $f_L$  (that is, increasing the amplifier bandwidth).

(d)  $I = 0.25 \text{ mA}$ ,  $R_C = 10 \text{ k}\Omega$ ,  $C_E = 10 \mu F$

$$r_e = \frac{V_T}{I} = \frac{25 \text{ mV}}{0.25 \text{ mA}} = 100 \Omega$$

For  $R_e = 0$ :

$$|A_M| = \frac{\alpha R_C}{r_e} \simeq \frac{10 \text{ k}\Omega}{100 \Omega} = 100 \text{ V/V}$$

$$f_L = \frac{1}{2\pi \times 10 \times 10^{-6} \times 100} = 159.2 \text{ Hz}$$

To lower  $f_L$  by a factor of 10, we use

$$1 + \frac{R_e}{r_e} = 10$$

$$\Rightarrow R_e = 900 \Omega$$

The gain now becomes

$$|A_M| = \frac{100}{1 + \frac{R_e}{r_e}} = \frac{100}{10} = 10 \text{ V/V}$$

See Fig. 2 for the Bode plot.

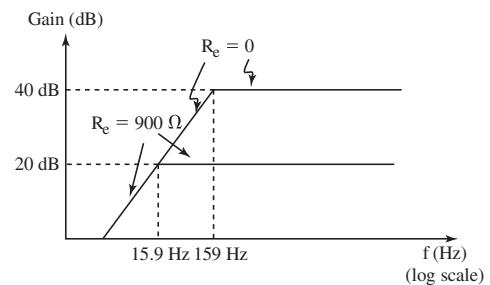


Figure 2

This figure belongs to Problem 9.11, part (a).

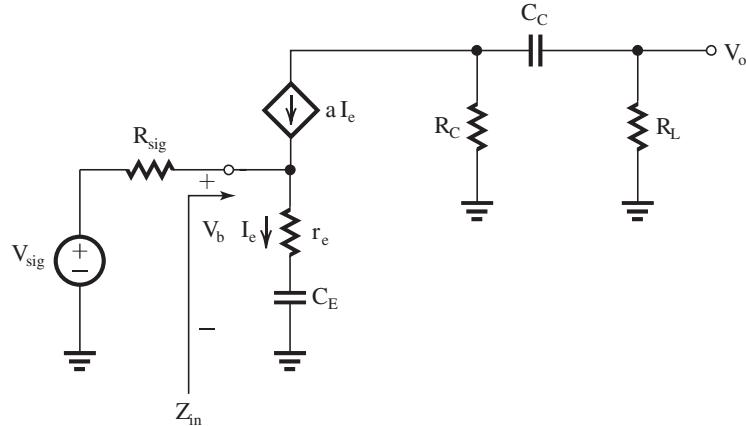


Figure 1

9.11 Replacing the BJT with its T model results in the equivalent circuit shown in Fig. 1 above.

(a) At midband,  $C_E$  and  $C_C$  act as short circuits. Thus

$$R_{\text{in}} = (\beta + 1)r_e$$

$$\begin{aligned} \frac{V_o}{V_{\text{sig}}} &= -\frac{(\beta + 1)r_e}{(\beta + 1)r_e + R_{\text{sig}}} g_m (R_C \parallel R_L) \\ &= -\frac{\beta(R_C \parallel R_L)}{(\beta + 1)r_e + R_{\text{sig}}} \end{aligned}$$

(b) Because the controlled current source  $\alpha I_e$  is ideal, it effectively separates the input circuit from the output circuit. The result is that the poles caused by  $C_E$  and  $C_C$  do not interact. The pole due to  $C_E$  will have frequency  $\omega_{PE}$ :

$$\omega_{PE} = \frac{1}{C_E \left[ r_e + \frac{R_{\text{sig}}}{\beta + 1} \right]}$$

and the pole due to  $C_C$  will have a frequency  $\omega_{PC}$

$$\omega_{PC} = \frac{1}{C_C (R_C + R_L)}$$

(c) The overall voltage transfer function can be expressed as

$$\frac{V_o}{V_{\text{sig}}} = A_M \frac{s}{s + \omega_{PE}} \frac{s}{s + \omega_{PC}}$$

$$(d) r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

$$\begin{aligned} A_M &= -\frac{100(10 \text{ k}\Omega \parallel 10 \text{ k}\Omega)}{101 \times 25 \times 10^{-3} \text{ k}\Omega + 10 \text{ k}\Omega} \\ &= -40 \text{ V/V} \end{aligned}$$

(e) To minimize the total capacitance we choose to make the pole caused by  $C_E$  the dominant one and make its frequency equal to  $f_L = 100 \text{ Hz}$ ,

$$2\pi \times 100 = \frac{1}{C_E \left[ 25 + \frac{10,000}{101} \right]}$$

$$\Rightarrow C_E = 12.83 \mu\text{F}$$

Placing the pole due to  $C_C$  at 10 Hz, we obtain

$$2\pi \times 10 = \frac{1}{C_C (10 + 10) \times 10^3}$$

$$\Rightarrow C_C = 0.8 \mu\text{F}$$

(f) A Bode plot for the gain magnitude is shown in Fig. 2.

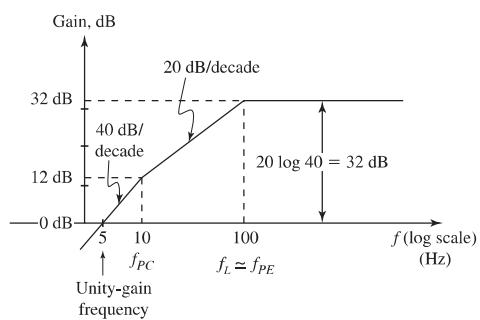


Figure 2

The gain at  $f_{P2}$  (10 Hz) is 12 dB. Since the gain decreases by 40 dB/decade or equivalently 12 dB/octave, it reaches 0 dB (unity magnitude) at  $f = f_{PC}/2 = 5 \text{ Hz}$ .

$$\begin{aligned}
 9.12 \quad C_{ox} &= \frac{\epsilon_{ox}}{t_{ox}} \\
 &= \frac{3.45 \times 10^{-11} \text{ F/m}}{8 \times 10^{-9} \text{ m}} = 0.43 \times 10^{-2} \text{ F/m}^2 \\
 &= 0.43 \times 10^{-2} \times 10^{-12} \text{ F}/\mu \text{m}^2 \\
 &= 4.3 \text{ fF}/\mu \text{m}^2
 \end{aligned}$$

$$\begin{aligned}
 k'_n &= \mu_n C_{ox} \\
 &= 450 \times 10^8 (\mu \text{m}^2/\text{V}\cdot\text{s}) \\
 &\quad \times 4.3 \times 10^{-15} \text{ F}/\mu \text{m}^2 \\
 &= 193.5 \mu \text{A/V}^2 \\
 I_D &= \frac{1}{2} k'_n \left( \frac{W}{L} \right) V_{OV}^2 (1 + \lambda V_{DS}) \\
 200 &= \frac{1}{2} \times 193.5 \times 20 \times V_{OV}^2 (1 + 0.05 \times 1.5) \\
 \Rightarrow V_{OV} &= 0.31 \text{ V} \\
 g_m &= \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.31} = 1.3 \text{ mA/V} \\
 \chi &= \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} \\
 &= \frac{0.5}{2\sqrt{0.65 + 1}} = 0.19 \\
 g_{mb} &= \chi g_m = 0.25 \text{ mA/V} \\
 r_o &= \frac{|V_A|}{I_D} = \frac{1}{|\lambda| I_D} = \frac{1}{0.05 \times 0.2} = 100 \text{ k}\Omega \\
 C_{gs} &= \frac{2}{3} WLC_{ox} + WL_{ov} C_{ox} \\
 &= \frac{2}{3} \times 20 \times 1 \times 4.3 + 20 \times 0.05 \times 4.3 \\
 &= 57.3 + 4.3 = 61.6 \text{ fF}
 \end{aligned}$$

$$\begin{aligned}
 C_{gd} &= WL_{ov} C_{ox} = 20 \times 0.05 \times 4.3 \\
 &= 4.3 \text{ fF} \\
 C_{sb} &= \frac{C_{sb0}}{\sqrt{1 + \frac{|V_{SB}|}{V_0}}} \\
 &= \frac{20}{\sqrt{1 + \frac{1}{0.7}}} = 12.8 \text{ fF} \\
 C_{db} &= \frac{C_{db0}}{\sqrt{1 + \frac{|V_{DB}|}{V_0}}} \\
 &= \frac{20}{\sqrt{1 + \frac{2.5}{0.7}}} = 9.4 \text{ fF} \\
 f_T &= \frac{g_m}{2\pi(C_{gs} + C_{gd})} \\
 &= \frac{1.3 \times 10^{-3}}{2\pi(61.6 + 4.3) \times 10^{-15}} = 3.1 \text{ GHz}
 \end{aligned}$$

$$9.13 \quad g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.3} = 1.33 \text{ mA/V}$$

$$\begin{aligned}
 f_T &= \frac{g_m}{2\pi(C_{gs} + C_{gd})} \\
 &= \frac{1.33 \times 10^{-3}}{2\pi \times (25 + 5) \times 10^{-15}} = 7.1 \text{ GHz}
 \end{aligned}$$

$$9.14 \quad f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

For  $C_{gs} \gg C_{gd}$

$$f_T \simeq \frac{g_m}{2\pi C_{gs}} \quad (1)$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov} C_{ox}$$

If the overlap component is small, we get

$$C_{gs} \simeq \frac{2}{3} WLC_{ox} \quad (2)$$

The transconductance  $g_m$  can be expressed as

$$g_m = \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV} \quad (3)$$

Substituting from (2) and (3) into (1), we obtain

$$\begin{aligned}
 f_T &= \frac{\mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}}{2\pi \times \frac{2}{3} WLC_{ox}} \\
 &= \frac{3\mu_n V_{OV}}{4\pi L^2}
 \end{aligned}$$

We note that for a given channel length,  $f_T$  can be increased by operating the MOSFET at a higher  $V_{OV}$ .

For  $L = 0.5 \mu \text{m}$  and  $\mu_n = 450 \text{ cm}^2/\text{V}\cdot\text{s}$ ,

we have

$$\begin{aligned}
 V_{OV} = 0.2 \text{ V} \Rightarrow f_T &= \frac{3 \times 450 \times 10^8 \times 0.2}{4\pi \times 0.5^2} \\
 &= 5.73 \text{ GHz}
 \end{aligned}$$

$$\begin{aligned}
 V_{OV} = 0.4 \text{ V} \Rightarrow f_T &= \frac{3 \times 450 \times 10^8 \times 0.4}{4\pi \times 0.5^2} \\
 &= 11.46 \text{ GHz}
 \end{aligned}$$

$$9.15 \quad f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

For  $C_{gs} \gg C_{gd}$

$$f_T \simeq \frac{g_m}{2\pi C_{gs}} \quad (1)$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov} C_{ox}$$

If the overlap component ( $WL_{ov} C_{ox}$ ) is small, we get

$$C_{gs} \simeq \frac{2}{3} WLC_{ox} \quad (2)$$

## Chapter 9–8

The transconductance  $g_m$  is given by

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D} \quad (3)$$

Substituting from (2) and (3) into (1), we get

$$\begin{aligned} f_T &= \frac{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D}}{2\pi \times \frac{2}{3} W L C_{ox}} \\ &= \frac{1.5}{\pi L} \sqrt{\frac{\mu_n I_D}{2C_{ox} WL}} \quad \text{Q.E.D.} \end{aligned}$$

We observe that for a given device,  $f_T$  is proportional to  $\sqrt{I_D}$ ; thus to obtain faster operation the MOSFET is operated at a higher  $I_D$ .

Also, we observe that  $f_T$  is inversely proportional to  $L\sqrt{WL}$ ; thus faster operation is obtained from smaller devices.

$$9.16 \quad A_0 = \frac{2V_A}{V_{OV}} = \frac{2V_A L}{V_{OV}}$$

$$A_0 = \frac{2 \times 5 \times L}{0.2} = 50L, \text{ V/V (L in } \mu\text{m)}$$

$$f_T \approx \frac{3\mu_n V_{OV}}{4\pi L^2} = \frac{3 \times 400 \times 10^8 \times 0.2}{4\pi L^2}$$

$$f_T = \frac{1.91}{L^2}, \text{ GHz (L in } \mu\text{m)}$$

The expressions for  $A_0$  and  $f_T$  can be used to obtain their values for different values of  $L$ . The results are given in the following table.

$L$	$L_{min}$ $0.13 \mu\text{m}$	$2L_{min}$ $0.26 \mu\text{m}$	$3L_{min}$ $0.39 \mu\text{m}$	$4L_{min}$ $0.52 \mu\text{m}$	$5L_{min}$ $0.65 \mu\text{m}$
$A_0$ (V/V)	6.5	13	19.5	26	32.5
$f_T$ (GHz)	113	28.3	12.6	7.1	4.5

$$9.17 \quad f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

where

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$C_\pi = 8 \text{ pF}$$

$$C_\mu = 1 \text{ pF}$$

Thus,

$$f_T = \frac{20 \times 10^{-3}}{2\pi \times (8 + 1) \times 10^{-12}} = 353.7 \text{ MHz}$$

$$f_\beta = \frac{f_T}{\beta} = \frac{353.7}{100} = 3.54 \text{ MHz}$$

9.18 See figure below.  $C_\pi = C_{de} + C_{je}$

where  $C_{de}$  is proportional to  $I_C$ .

At  $I_C = 0.5 \text{ mA}$ ,

$$8 = C_{de} + 2 \Rightarrow C_{de} = 6 \text{ pF}$$

$$\text{At } I_C = 0.25 \text{ mA, } C_{de} = \frac{1}{2} \times 6 = 3 \text{ pF, and } C_\pi = 3 + 2 = 5 \text{ pF.}$$

Also, at  $I_C = 0.25 \text{ mA}$ ,  $g_m = 10 \text{ mA/V}$ . Thus  $f_T$  at  $I_C = 0.25 \text{ mA}$  is

$$f_T = \frac{10 \times 10^{-3}}{2\pi(5 + 1) \times 10^{-12}} = 265.3 \text{ MHz}$$

$$9.19 \quad r_x = 100 \Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

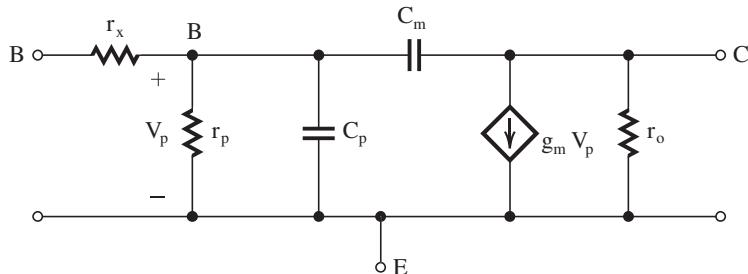
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{1} = 50 \text{ k}\Omega$$

$$C_{de} = \tau_F g_m = 30 \times 10^{-12} \times 40 \times 10^{-3} = 1.2 \text{ pF}$$

$$C_{je0} = 20 \text{ pF}$$

This figure belongs to Problem 9.18.



$$C_\pi = C_{de} + 2C_{je0} = 1.2 + 2 \times 0.02 = 1.24 \text{ pF}$$

$$C_\mu = \frac{C_{je0}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^m}$$

$$C_\mu = \frac{20}{\left(1 + \frac{2}{0.75}\right)^{0.5}} = 10.4 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(1.24 + 0.01) \times 10^{-12}}$$

$$= 5.1 \text{ GHz}$$

9.20 For  $f \gg f_\beta$ ,

$$|h_{fe}| = \frac{f_T}{f}$$

At  $f = 50 \text{ MHz}$  and  $I_C = 0.2 \text{ mA}$ ,

$$|h_{fe}| = 10 = \frac{f_T}{50}$$

$$\Rightarrow f_T = 500 \text{ MHz}$$

At  $f = 50 \text{ MHz}$  and  $I_C = 1.0 \text{ mA}$ ,

$$|h_{fe}| = 12 = \frac{f_T}{50}$$

$$\Rightarrow f_T = 600 \text{ MHz}$$

Now,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

where

$$C_\pi = C_{de} + C_{je}$$

$$= \tau_F g_m + C_{je}$$

$$C_\mu = 0.1 \text{ pF}$$

$$\text{At } I_C = 0.2 \text{ mA}, g_m = \frac{0.2}{0.025} = 8 \text{ mA/V, thus}$$

$$500 \times 10^6 = \frac{8 \times 10^{-3}}{2\pi(C_\pi + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_\pi = 2.45 \text{ pF}$$

$$\tau_F \times 8 \times 10^{-3} + C_{je} = 2.45 \times 10^{-12} \quad (1)$$

$$\text{At } I_C = 1 \text{ mA}, g_m = \frac{1}{0.025} = 40 \text{ mA/V, thus}$$

$$600 \times 10^6 = \frac{40 \times 10^{-3}}{2\pi(C_\pi + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_\pi = 10.51 \text{ pF}$$

$$\tau_F \times 40 \times 10^{-3} + C_{je} = 10.51 \times 10^{-12} \quad (2)$$

Solving (1) together with (2) yields

$$\tau_F = 252 \text{ ps}$$

$$C_{je} = 0.43 \text{ pF}$$

9.21 For  $f \gg f_\beta$ ,

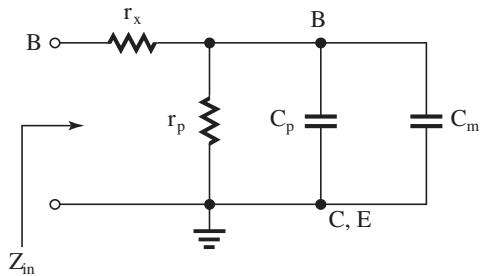
$$|h_{fe}| \simeq \frac{f_T}{f}$$

$$40 = \frac{2000 \text{ MHz}}{f}$$

$$\Rightarrow f = 50 \text{ MHz}$$

$$f_\beta = \frac{f_T}{\beta_0} = \frac{2000 \text{ MHz}}{200} = 10 \text{ MHz}$$

9.22



With the emitter and the collector grounded, the equivalent circuit takes the form shown in the figure, and the input impedance becomes

$$Z_{in} = r_x + \frac{1}{\frac{1}{r_\pi} + j\omega(C_\pi + C_\mu)} = r_x + \frac{r_\pi}{1 + j\omega(C_\pi + C_\mu)r_\pi}$$

Since  $\omega_\beta = \frac{1}{(C_\pi + C_\mu)r_\pi}$ , then

$$Z_{in} = r_x + \frac{r_\pi}{1 + j\left(\frac{\omega}{\omega_\beta}\right)} = r_x + r_\pi \frac{1 - j\left(\frac{\omega}{\omega_\beta}\right)}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

$$R_e(Z_{in}) = r_x + \frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

For the real part to be an estimate of  $r_x$  accurate to within 10%, we require

$$\frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} \leq 0.1r_x$$

$$\frac{1}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} \leq 0.1 \left(\frac{r_x}{r_\pi}\right)$$

But  $r_x \leq \frac{r_\pi}{10}$ , thus  $\frac{r_x}{r_\pi} \leq 0.1$ ,

## Chapter 9–10

$$\frac{1}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} \leq 0.1 \times 0.1$$

or, equivalently,

$$1 + \left(\frac{\omega}{\omega_\beta}\right)^2 \geq 100$$

$$\Rightarrow \omega \geq 10\omega_\beta$$

9.23 To complete the table below we use the following relationships:

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{I_E (\text{mA})}$$

$$g_m = \frac{I_C}{V_T} = \frac{\alpha I_E}{V_T} \approx \frac{I_E}{V_T} = \frac{I_E (\text{mA})}{0.025 \text{ V}}$$

$$r_\pi = \frac{\beta_0}{g_m (\text{mA/V})}, \text{ k}\Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$f_\beta = \frac{1}{2\pi(C_\pi + C_\mu)r_\pi}$$

$$f_\beta = \frac{f_T}{\beta_0}$$

$$9.24 C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$$

$$= 1 + 0.1(1 + 39)$$

$$= 5 \text{ pF}$$

$$f_{sdB} = \frac{1}{2\pi C_{in} R_{sig}}$$

$$= \frac{1}{2\pi \times 5 \times 10^{-12} R_{sig}}$$

For  $f_{sdB} > 1 \text{ MHz}$ ,

$$R_{sig} < \frac{1}{2\pi \times 5 \times 10^{-12} \times 1 \times 10^6} = 31.8 \text{ k}\Omega$$

$$9.25 \text{ (a)} V_o = -AV_i$$

If the current "owing through  $R_{sig}$  is denoted  $I_i$ , we obtain

$$Y_{in} = \frac{I_i}{V_i} = \frac{sC(V_i - V_o)}{V_i}$$

$$= sC\left(1 - \frac{V_o}{V_i}\right)$$

$$= sC(1 + A)$$

Thus,

$$C_{in} = C(1 + A)$$

$$(b) \frac{V_i(s)}{V_{sig}(s)} = \frac{1/sC_{in}}{R_{sig} + 1/sC_{in}}$$

$$= \frac{1}{1 + sC_{in}R_{sig}}$$

$$\frac{V_o(s)}{V_{sig}(s)} = -\frac{A}{1 + sC_{in}R_{sig}}$$

$$(c) \text{ DC gain} = 40 \text{ dB} = 100 \text{ V/V},$$

$$\Rightarrow A = 100 \text{ V/V}$$

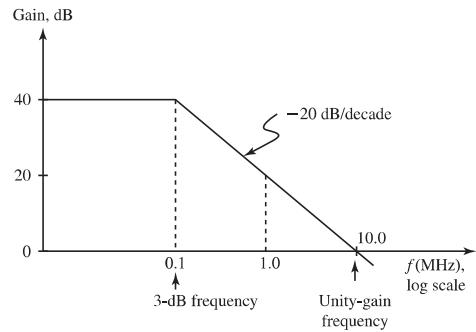
$$f_{3dB} = \frac{1}{2\pi C_{in} R_{sig}}$$

$$100 \times 10^3 = \frac{1}{2\pi C_{in} \times 1 \times 10^3}$$

$$\Rightarrow C_{in} = 1591.5 \text{ pF}$$

$$C = \frac{C_{in}}{A+1} = \frac{1591.5}{101} = 15.8 \text{ pF}$$

(d) The Bode plot is shown in the figure below.



This table belongs to Problem 9.23.

Transistor	$I_E$ (mA)	$r_e$ ( $\Omega$ )	$g_m$ (mA/V)	$r_\pi$ (k $\Omega$ )	$\beta_0$	$f_T$ (MHz)	$C_\mu$ (pF)	$C_\pi$ (pF)	$f_\beta$ (MHz)
(a)	2	12.5	80	12.5	100	500	2	23.5	5
(b)	1	25	40	3.13	125	500	2	10.7	4
(c)	1	25	40	2.5	100	500	2	10.7	5
(d)	10	2.5	400	0.25	100	500	2	125.3	5
(e)	0.1	250	4	25	100	150	2	2.2	1.5
(f)	1	25	40	0.25	10	500	2	10.7	50
(g)	1.25	20	50	0.2	10	800	1	9	80

From the figure we see that the gain reduces to unity two decades higher than  $f_{3dB}$ , that is at 10 MHz.

9.26 Refer to Example 9.3. If the transistor is replaced with another whose W is half that of the original transistor, we obtain

$$W_2 = \frac{1}{2}W_1$$

Since

$$C_{gs} = \frac{2}{3}WL_{ox} + WL_{ov}C_{ox}$$

then

$$C_{gs2} = \frac{1}{2}C_{gs1} = 0.5 \text{ pF}$$

Also,

$$C_{gd} = WL_{ov}C_{ox}$$

thus,

$$C_{gd2} = \frac{1}{2}C_{gd1} = 0.2 \text{ pF}$$

Since

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

then

$$g_{m2} = \frac{1}{\sqrt{2}} g_{m1} = 0.71 \text{ mA/V}$$

Since

$$I_D = \frac{1}{2}\mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

then

$$V_{OV2} = \sqrt{2}V_{OV1}$$

Finally,

$$r_{o2} = r_{o1} = 150 \text{ k}\Omega$$

Thus,

$$R'_{L2} = R'_{L1} = 7.14 \text{ k}\Omega$$

Thus,

$$C_{in2} = C_{gs2} + (g_{m2}R'_{L2} + 1)C_{gd2}$$

$$= 0.5 + (0.71 \times 7.14 + 1) \times 0.2$$

$$= 1.71 \text{ pF}$$

This should be compared to  $C_{in1} = 4.26 \text{ pF}$ . Thus,

$$f_{H2} = \frac{1}{2\pi C_{in2}(R_{sig} \parallel R_G)}$$

$$= \frac{1}{2\pi \times 1.71 \times 10^{-12} (0.1 \parallel 4.7) \times 10^6}$$

$$= 952 \text{ MHz}$$

in comparison to  $f_{H1} = 398 \text{ MHz}$

$$\begin{aligned} |A_{M2}| &= \frac{4.7}{4.7 + 0.1} \times g_{m2}R'_{L2} \\ &= \frac{4.7}{4.8} \times 0.71 \times 7.14 \\ &= 5 \text{ V/V} \end{aligned}$$

in comparison to  $|A_{M1}| = 7 \text{ V/V}$ .

$$GB_2 = 5 \times 952 = 4.73 \text{ GHz}$$

in comparison to  $GB_1 = 7 \times 398 = 2.79 \text{ GHz}$ .

$$9.27 f_H = \frac{1}{2\pi C_{in} R_{sig}}$$

For  $f_H \geq 6 \text{ MHz}$

$$C_{in} \leq \frac{1}{2\pi f_H R_{sig}} = \frac{1}{2\pi \times 6 \times 10^6 \times 1 \times 10^3}$$

$$C_{in} \leq 26.5 \text{ pF}$$

But,

$$\begin{aligned} C_{in} &= C_{gs} + (1 + g_m R'_L) C_{gd} \\ &= 5 + (1 + g_m R'_L) \times 1, \text{ pF} \\ &= 6 + g_m R'_L, \text{ pF} \end{aligned}$$

For  $C_{in} \leq 26.5 \text{ pF}$  we have

$$g_m R'_L \leq 20.5$$

$$R'_L \leq \frac{20.5}{5} = 4.1 \text{ k}\Omega$$

Corresponding to  $R'_L = 4.1 \text{ k}\Omega$ , we have

$$|A_M| = g_m R'_L = 20.5 \text{ V/V}$$

$$GB = |A_M| f_H$$

$$= 20.5 \times 6 = 123 \text{ MHz}$$

If  $f_H = 2 \text{ MHz}$ , we obtain

$$C_{in} = 26.5 \times 3 = 79.5 \text{ pF}$$

$$g_m R'_L = 79.5 - 6 = 73.5$$

Thus,

$$|A_M| = 73.5 \text{ V/V}$$

$$GB = 73.5 \times 2 = 147 \text{ MHz}$$

$$9.28 A_M = -\frac{R_{in}}{R_{in} + R_{sig}} g_m R'_L$$

where

$$R'_L = R_D \parallel R_L \parallel r_o$$

$$= 8 \text{ k}\Omega \parallel 10 \text{ k}\Omega \parallel 50 \text{ k}\Omega$$

$$= 4.1 \text{ k}\Omega$$

$$A_M = -\frac{100}{100 + 100} \times 3 \times 4.1$$

$$= -6.1 \text{ V/V}$$

$$\begin{aligned} C_{in} &= C_{gs} + C_{gd}(1 + g_m R'_L) \\ &= 1 + 0.2(1 + 3 \times 4.1) \end{aligned} \quad (1)$$

$$= 3.66 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

where

$$\begin{aligned} R'_{sig} &= R_{sig} \parallel R_{in} \\ &= 100 \text{ k}\Omega \parallel 100 \text{ k}\Omega = 50 \text{ k}\Omega \end{aligned}$$

$$\begin{aligned} f_H &= \frac{1}{2\pi \times 3.66 \times 10^{-12} \times 50 \times 10^3} \\ &= 870 \text{ kHz} \end{aligned} \quad (2)$$

To double  $f_H$  by changing  $R_{in}$ , Eq. (2) indicates that  $R'_{sig}$  must be halved:

$$R'_{sig} = 25 \text{ k}\Omega$$

which requires  $R_{in}$  to be changed to  $R_{in2}$ ,

$$25 \text{ k}\Omega = 100 \parallel R_{in2}$$

$$\Rightarrow R_{in2} = 33.3 \text{ k}\Omega$$

This change will cause  $|A_M|$  to become

$$\begin{aligned} |A_{M2}| &= \frac{33.3}{33.3 + 100} \times 3 \times 4.1 \\ &= 3.1 \text{ V/V} \end{aligned}$$

which is about half the original value.

To double  $f_H$  by changing  $R_L$ , Eq. (2) indicates that  $C_{in}$  must be halved:

$$C_{in2} = \frac{1}{2} \times 3.66 = 1.83 \text{ pF}$$

Using Eq. (1), we obtain

$$\begin{aligned} 1.83 &= 1 + 0.2(1 + g_m R'_{L2}) \\ \Rightarrow g_m R'_{L2} &= 3.15 \end{aligned}$$

Thus,

$$R'_{L2} = 1.05 \text{ k}\Omega$$

and  $R_{L2}$  can be found from

$$1.05 = R_L \parallel 8 \text{ k}\Omega \parallel 50 \text{ k}\Omega$$

$$\Rightarrow R_L = 1.24 \text{ k}\Omega$$

and the midband gain becomes

$$|A_{M2}| = \frac{100}{100 + 100} \times 3.15 = 1.6 \text{ V/V}$$

which is about a quarter of the original gain.

Clearly, changing  $R_{in}$  is the preferred course of action!

(1)

$$9.29 \text{ (a)} \quad A_M = -\frac{R_G}{R_G + R_{sig}} g_m R'_L$$

where

$$R'_L = R_D \parallel R_L \parallel r_o$$

$$= 20 \text{ k}\Omega \parallel 20 \text{ k}\Omega \parallel 100 \text{ k}\Omega$$

$$= 9.1 \text{ k}\Omega$$

$$\begin{aligned} A_M &= -\frac{2 \text{ M}\Omega}{2 \text{ M}\Omega + 0.5 \text{ M}\Omega} \times 5 \times 9.1 \\ &= -36.4 \text{ V/V} \end{aligned}$$

$$(b) \quad f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

where

$$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$$

$$= 3 + 0.5(1 + 5 \times 9.1)$$

$$= 26.25 \text{ pF}$$

and

$$R'_{sig} = R_{sig} \parallel R_G$$

$$= 500 \text{ k}\Omega \parallel 2000 \text{ k}\Omega$$

$$= 400 \text{ k}\Omega$$

Thus,

$$\begin{aligned} f_H &= \frac{1}{2\pi \times 26.25 \times 10^{-12} \times 400 \times 10^3} \\ &= 15.2 \text{ kHz} \end{aligned}$$

$$(c) \quad f_Z = \frac{g_m}{2\pi C_{gd}}$$

$$= \frac{5 \times 10^{-3}}{2\pi \times 0.5 \times 10^{-12}}$$

$$= 1.6 \text{ GHz}$$

$$9.30 \quad g_m = \sqrt{2\mu_n C_{ox} (W/L) I_{D1}}$$

$$= \sqrt{2 \times 0.09 \times 100 \times 0.1}$$

$$= 1.34 \text{ mA/V}$$

$$r_{o1} = \frac{|V_{A1}|}{I_{D1}} = \frac{12.8}{0.1} = 128 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{19.2}{0.1} = 192 \text{ k}\Omega$$

The total resistance at the output node,  $R'_L$ , is given by

$$R'_L = r_{o1} \parallel r_{o2} = 128 \text{ k}\Omega \parallel 192 \text{ k}\Omega$$

$$= 76.8 \text{ k}\Omega$$

$$A_M = -g_m R'_L$$

$$= -1.34 \times 76.8 = -103 \text{ V/V}$$

$$f_H = \frac{1}{2\pi C_{in} R_{sig}}$$

where

$$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$$

$$= 0.2 + 0.015(1 + 103)$$

$$= 1.76 \text{ pF}$$

Thus,

$$f_H = \frac{1}{2\pi \times 1.76 \times 10^{-12} \times 200 \times 10^3}$$

$$= 452 \text{ kHz}$$

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{1.34 \times 10^{-3}}{2\pi \times 0.015 \times 10^{-12}}$$

$$= 14.2 \text{ GHz}$$

$$9.31 \quad g_m R'_L = 50$$

$$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$$

$$= 10 + 1(1 + 50)$$

$$= 61 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 61 \times 10^{-12} \times 5 \times 10^3}$$

$$= 522 \text{ kHz}$$

$$9.32$$

$$A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_{sig} \parallel R_B)} g_m R'_L$$

where

$$R'_L = r_o \parallel R_C \parallel R_L$$

$$= 100 \text{ k}\Omega \parallel 10 \text{ k}\Omega \parallel 10 \text{ k}\Omega$$

$$= 4.76 \text{ k}\Omega$$

and

$$r_\pi = \beta/g_m = 100/40 = 2.5 \text{ k}\Omega$$

$$A_M = -\frac{100}{100 + 10} \times \frac{2.5}{2.5 + 0.1 + (10 \parallel 100)} \\ \times 40 \times 4.76$$

$$= -37 \text{ V/V}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

where

$$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$$

$$= 10 + 1 \times (1 + 40 \times 4.76)$$

$$= 201.4 \text{ pF}$$

and

$$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})]$$

$$= 2.5 \parallel [0.1 + (100 \parallel 10)]$$

$$= 2 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi \times 201.4 \times 10^{-12} \times 2 \times 10^3} \\ = 395 \text{ kHz}$$

$$9.33 \quad (a) \quad A_M =$$

$$-\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_{sig} \parallel R_B)} g_m R'_L$$

For  $R_B \gg R_{sig}$ ,  $r_x \ll R_{sig}$ ,  $R_{sig} \gg r_\pi$ ,

$$A_M \simeq -\frac{r_\pi}{R_{sig}} g_m R'_L = -\beta R'_L / R_{sig} \quad \text{Q.E.D.}$$

$$(b) \quad C_{in} = C_\pi + (g_m R'_L + 1) C_\mu$$

For  $g_m R'_L \gg 1$  and  $g_m R'_L C_\mu \gg C_\pi$ ,

$$C_{in} \simeq g_m R'_L C_\mu$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

where

$$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})]$$

$$\simeq r_\pi \parallel R_{sig} \simeq r_\pi$$

Thus,

$$f_H \simeq \frac{1}{2\pi g_m R'_L C_\mu r_\pi}$$

$$f_H = \frac{1}{2\pi C_\mu \beta R'_L} \quad \text{Q.E.D.}$$

$$(c) \quad GB = |A_M| f_H$$

$$= \beta \frac{R'_L}{R_{sig}} \frac{1}{2\pi C_\mu \beta R'_L} = \frac{1}{2\pi C_\mu R_{sig}} \quad \text{Q.E.D.}$$

For  $R_{sig} = 25 \text{ k}\Omega$  and  $C_\mu = 1 \text{ pF}$ ,

$$GB = \frac{1}{2\pi \times 1 \times 10^{-12} \times 25 \times 10^3} = 6.37 \text{ MHz}$$

For  $I_C = 1 \text{ mA}$  and  $\beta = 100$ ,

$$(i) \quad R'_L = 25 \text{ k}\Omega:$$

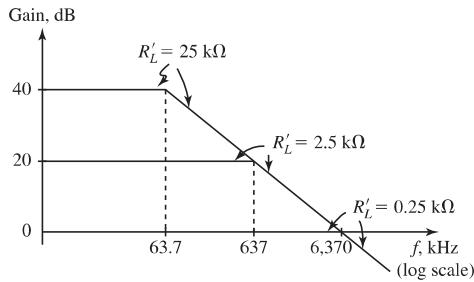
$$A_M = -100 \times \frac{25}{25} = -100 \text{ V/V}$$

$$f_H = \frac{GB}{|A_M|} = \frac{6.37 \text{ MHz}}{100 \text{ V/V}} = 63.7 \text{ kHz}$$

$$(ii) \quad R'_L = 2.5 \text{ k}\Omega:$$

$$A_M = -100 \times \frac{2.5}{25} = -10 \text{ V/V}$$

$$f_H = \frac{GB}{|A_M|} = \frac{6.37 \text{ MHz}}{10 \text{ V/V}} = 637 \text{ kHz}$$



The Bode plots are shown in the figure.

If the midband gain is unity,

$$f_H = GB = 6.37 \text{ MHz}$$

This is obtained when  $R'_L$  is

$$1 = 100 \times \frac{R'_L}{25}$$

$$\Rightarrow R'_L = 0.25 \text{ k}\Omega = 250 \text{ }\Omega$$

9.34 Refer to Example 9.4. Since  $I_E$  is doubled to 2 mA, we have

$$g_m = \frac{I_C}{V_T} = \frac{2 \text{ mA}}{25 \text{ mV}} = 80 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{80 \text{ mA/V}} = 1.25 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{2 \text{ mA}} = 50 \text{ k}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{80 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 16 \text{ pF}$$

$$C_\mu = 1 \text{ pF}$$

$$C_\pi = 15 \text{ pF}$$

$$r_x = 50 \text{ }\Omega$$

Also, now

$$R_B = 50 \text{ k}\Omega$$

$$R_C = 4 \text{ k}\Omega$$

The new value of  $A_M$  is

$$A_M = -\frac{R_B}{R_B + R_{\text{sig}}} \frac{r_\pi}{r_\pi + r_x + (R_B \parallel R_{\text{sig}})} (g_m R'_L)$$

where

$$R'_L = r_o \parallel R_C \parallel R_L$$

$$= 50 \parallel 4 \parallel 5 = 2.13 \text{ k}\Omega$$

Thus,

$$g_m R'_L = 80 \times 2.13 = 170 \text{ V/V}$$

and

$$A_M = -\frac{50}{50+5} \times \frac{1.25}{1.25 + 0.05 + (50 \parallel 5)} \times 170 \\ = -33 \text{ V/V}$$

and

$$20 \log |A_M| = 30.4 \text{ dB}$$

This should be compared to the previous value of 39 V/V (32 dB). To determine  $f_H$ , we first find  $C_{\text{in}}$ ,

$$C_{\text{in}} = C_\pi + C_\mu (1 + g_m R'_L)$$

$$= 15 + 1(1 + 170)$$

$$= 186 \text{ pF}$$

and the effective source resistance  $R'_{\text{sig}}$ ,

$$R'_{\text{sig}} = r_\pi \parallel [r_x + (R_B \parallel R_{\text{sig}})]$$

$$= 1.25 \parallel [0.05 + (50 \parallel 5)]$$

$$= 0.98 \text{ k}\Omega$$

Thus

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 186 \times 10^{-12} \times 0.98 \times 10^3} \\ = 873 \text{ kHz}$$

This should be compared to the previous value of 754 kHz. The gain-bandwidth product becomes

$$GB = |A_M| f_H = 33 \times 873 = 28.8 \text{ MHz}$$

This should be compared to the previous value of  $39 \times 754 = 29.4 \text{ MHz}$ . Thus, increasing the bias current by a factor of 2 results in an increase in  $f_H$  by a factor of 1.16—that is, by about 16%.

However, because of the attendant reduction in input resistance, the overall gain decreased by about the same factor and  $GB$  remained nearly constant. The price paid for the slight increase in  $f_H$  is an increase in power dissipation by a factor of about two.

$$9.35 R_{\text{in}} = \frac{R}{1 - K}$$

$$= \frac{100 \text{ k}\Omega}{1 - 0.9} = 1000 \text{ k}\Omega = 1 \text{ M}\Omega$$

9.36 Using Miller's theorem, we obtain

$$Z_{\text{in}} = \frac{Z}{1 - A}, \quad Z_{\text{out}} = \frac{Z}{1 - \frac{1}{A}}$$

For

$$Z = \frac{1}{j\omega C}$$

$$Z_{\text{in}} = \frac{1}{j\omega C(1 - A)} \Rightarrow C_{\text{in}} = C(1 - A)$$

$$Z_{\text{out}} = \frac{1}{j\omega C \left(1 - \frac{1}{A}\right)} \Rightarrow C_{\text{out}} = C \left(1 - \frac{1}{A}\right)$$

(a)  $A = -1000 \text{ V/V}$ ,  $C = 1 \text{ pF}$

$$C_{\text{in}} = 1(1 + 1000) = 1001 \text{ pF}$$

$$C_{\text{out}} = 1 \left( 1 + \frac{1}{1000} \right) = 1.001 \text{ pF}$$

(b)  $A = -10 \text{ V/V}$ ,  $C = 10 \text{ pF}$

$$C_{\text{in}} = 10(1 + 10) = 110 \text{ pF}$$

$$C_{\text{out}} = 10 \left( 1 + \frac{1}{10} \right) = 11 \text{ pF}$$

(c)  $A = -1 \text{ V/V}$ ,  $C = 10 \text{ pF}$

$$C_{\text{in}} = 10(1 + 1) = 20 \text{ pF}$$

$$C_{\text{out}} = 10(1 + 1) = 20 \text{ pF}$$

(d)  $A = +1 \text{ V/V}$ ,  $C = 10 \text{ pF}$

$$C_{\text{in}} = C(1 - 1) = 0$$

$$C_{\text{out}} = C(1 - 1) = 0$$

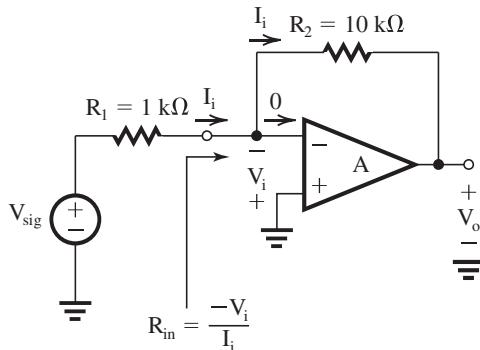
(e)  $A = +10 \text{ V/V}$ ,  $C = 10 \text{ pF}$

$$C_{\text{in}} = 10(1 - 10) = -90 \text{ pF}$$

$$C_{\text{out}} = 10 \left( 1 - \frac{1}{10} \right) = 9 \text{ pF}$$

The  $-90 \text{ pF}$  input capacitance can be used to cancel an equal ( $+90 \text{ pF}$ ) capacitance between the input node and ground.

9.37



This figure belongs to Problem 9.38.

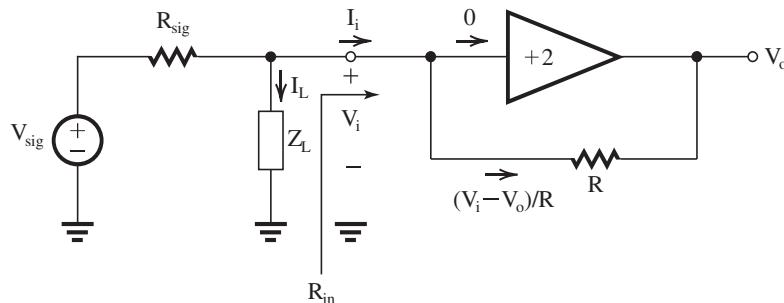


Figure 1

The first figure below (left) shows that

$$V_o = AV_i \quad (1)$$

From Miller's theorem, we have

$$R_{\text{in}} = \frac{R_2}{1 - \left( \frac{V_o}{-V_i} \right)} = \frac{R_2}{1 + \frac{V_o}{V_i}} = \frac{R_2}{1 + A} \quad (2)$$

Using the voltage divider rule at the input, we get

$$\begin{aligned} -V_i &= V_{\text{sig}} \frac{R_{\text{in}}}{R_{\text{in}} + R_l} \\ \Rightarrow V_i &= -V_{\text{sig}} \frac{R_{\text{in}}}{R_{\text{in}} + R_l} \end{aligned} \quad (3)$$

For each value of  $A$  we use Eq. (2) to determine  $R_{\text{in}}$ , Eq. (3) to determine  $V_i$  (for  $V_{\text{sig}} = 1 \text{ V}$ ), Eq. (1) to determine  $V_o$ , and finally we calculate the value of  $V_o/V_{\text{sig}}$ . The results are given in the table below.

$A$ (V/V)	$R_{\text{in}}$ (kΩ)	$V_i$ (V)	$V_o$ (V)	$V_o/V_{\text{sig}}$ (V/V)
10	$9.091 \times 10^{-1}$	-0.476	-4.76	-4.76
100	$9.901 \times 10^{-2}$	-0.0901	-9.01	-9.01
1000	$9.990 \times 10^{-3}$	$-9.89 \times 10^{-3}$	-9.89	-9.89
10,000	$9.999 \times 10^{-4}$	$-9.989 \times 10^{-4}$	-9.99	-9.99

9.38

(a) Refer to Fig. 1.

$$I_i = \frac{V_i - V_o}{R} = \frac{V_i - 2V_i}{R} = -\frac{V_i}{R}$$

Thus,

$$R_{\text{in}} \equiv \frac{V_i}{I_i} = -R$$

(b) Replacing the signal source with its equivalent Norton s form results in the circuit in Fig. 2. Observe that  $R_{eq} = \infty$  when  $R_{sig} = R$ . In this case,

$$I_L = \frac{V_{sig}}{R_{sig}} = \frac{V_{sig}}{R}$$

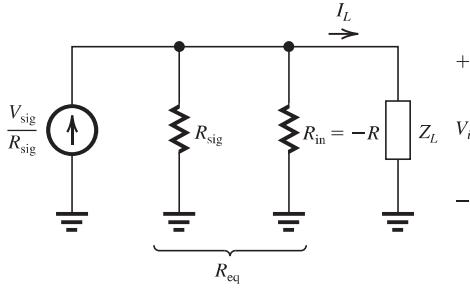


Figure 2

$$(c) \text{ If } Z_L = \frac{1}{sC},$$

$$V_i = I_L Z_L$$

$$= \frac{V_{sig}}{R} \times \frac{1}{sC}$$

$$= \frac{1}{sCR} V_{sig}$$

and

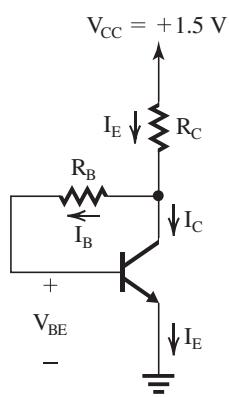
$$V_o = 2V_i = \frac{2}{sCR} V_{sig}$$

Thus,

$$\frac{V_o}{V_{sig}} = \frac{2}{sCR}$$

which is the transfer function of an ideal noninverting integrator.

9.39



(a) For the dc analysis, refer to the figure.

$$V_{CC} = I_E R_C + I_B R_B + V_{BE}$$

$$1.5 = I_E \times 1 + \frac{I_E}{\beta + 1} \times 47 + 0.7$$

$$\Rightarrow I_E = \frac{1.5 - 0.7}{1 + \frac{47}{101}} = 0.546 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.546 = 0.54 \text{ mA}$$

$$(b) g_m = \frac{I_C}{V_T} = \frac{0.54 \text{ mA}}{0.025 \text{ V}} = 21.6 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{21.6} = 4.63 \text{ k}\Omega$$

$$(c) \frac{V_o}{V_b} = -g_m (R_C \parallel R_L)$$

$$= -21.6(1 \parallel 1) = -10.8 \text{ V/V}$$

(d) Using Miller's theorem, the component of  $R_{in}$  due to  $R_B$  can be found as

$$R_{in1} = \frac{R_B}{1 - (V_o/V_b)}$$

$$= \frac{47 \text{ k}\Omega}{1 - (-10.8)} = 4 \text{ k}\Omega$$

$$R_{in} = R_{in1} \parallel r_\pi$$

$$= 4 \parallel 4.63 = 2.14 \text{ k}\Omega$$

$$(e) G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times \frac{V_o}{V_b}$$

$$= \frac{2.14}{2.14 + 1} \times -10.8 = -7.4 \text{ V/V}$$

$$(f) C_{in} = C_\pi + \left(1 + \left|\frac{V_o}{V_b}\right|\right) C_\mu$$

where

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{21.6 \times 10^{-3}}{2\pi \times 600 \times 10^6}$$

$$C_\pi + C_\mu = 5.73 \text{ pF}$$

$$C_\pi = 5.73 - 0.8 = 4.93 \text{ pF}$$

$$C_{in} = 4.93 + (1 + 10.8) \times 0.8$$

$$= 14.37 \text{ pF}$$

$$(g) R'_{sig} = R_{in} \parallel R_{sig}$$

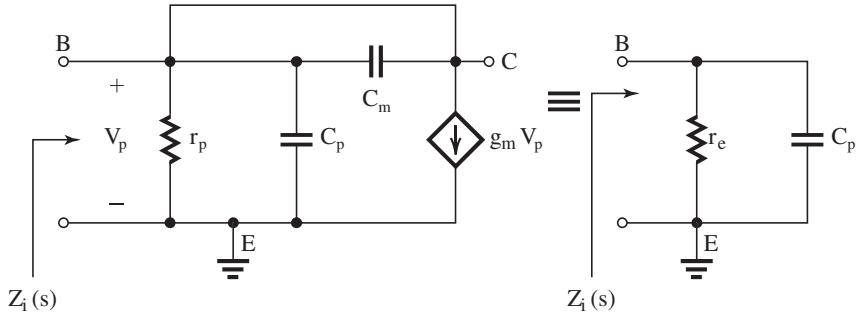
$$= 2.14 \text{ k}\Omega \parallel 1 \text{ k}\Omega = 0.68 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 14.37 \times 10^{-12} \times 0.68 \times 10^3}$$

$$= 16.3 \text{ MHz}$$

9.40



From the figure we see that the controlled current-source  $g_m V_\pi$  appears across its control voltage  $V_\pi$ , thus we can replace the current source with a resistance  $1/g_m$ . Now, the parallel equivalent of  $r_\pi$  and  $1/g_m$  is

$$\frac{r_\pi(1/g_m)}{r_\pi + \frac{1}{g_m}} = \frac{r_\pi}{g_m r_\pi + 1} = \frac{r_\pi}{\beta + 1} = r_e$$

Thus, the equivalent circuit simplifies to that of  $r_e$  in parallel with  $C_\pi$ ,

$$Z_i(s) = \frac{1}{\frac{1}{r_e} + sC_\pi} = \frac{r_e}{1 + sC_\pi r_e}$$

$$Z_i(j\omega) = \frac{r_e}{1 + j\omega C_\pi r_e}$$

$Z_i(j\omega)$  will have a  $45^\circ$  phase at

$$\omega_{45} C_\pi r_e = 1$$

$$\Rightarrow \omega_{45} = \frac{1}{C_\pi r_e}$$

Now,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

At high bias currents,

$$C_\pi \gg C_\mu$$

and

$$f_T \approx \frac{g_m}{2\pi C_\pi}$$

Since  $g_m \approx 1/r_e$ , we have

$$f_T \approx \frac{1}{2\pi C_\pi r_e}$$

Thus,

$$f_{45^\circ} \approx f_T = 400 \text{ MHz}$$

If the bias current is reduced to the value that results in  $C_\pi \approx C_\mu$ ,

$$f_T = \frac{g_m}{2\pi \times 2C_\pi} = \frac{g_m}{4\pi C_\pi}$$

Again,  $g_m \approx \frac{1}{r_e}$ , thus

$$f_T \approx \frac{1}{4\pi C_\pi r_e}$$

It follows that in this case,

$$f_{45^\circ} = \frac{1}{2} f_T = 200 \text{ MHz}$$

$$9.41 \quad A_M = -g_m R'_L$$

$$= -4 \times 20 = -80 \text{ V/V}$$

$$f_{sdB} = f_H = \frac{1}{2\pi(C_L + C_{gd})R'_L}$$

$$= \frac{1}{2\pi(2 + 0.1) \times 10^{-12} \times 20 \times 10^3}$$

$$= 3.8 \text{ MHz}$$

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{4 \times 10^{-3}}{2\pi \times 0.1 \times 10^{-12}} = 6.4 \text{ GHz}$$

$$f_t = |A_M| f_H$$

$$= 80 \times 3.8 = 304 \text{ MHz}$$

$$9.42 \quad f_t = \frac{g_m}{2\pi(C_L + C_{gd})}$$

$$C_L + C_{gd} = \frac{g_m}{2\pi f_t}$$

$$= \frac{2 \times 10^{-3}}{2\pi \times 2 \times 10^9} = 0.159 \text{ pF}$$

To reduce  $f_t$  to 1 GHz, an additional capacitance of 0.159 pF must be connected to the output node. (Doubling the effective capacitance at the output node reduces  $f_t$  by a factor of 2.)

9.43 Refer to Fig. P9.43. To determine  $g_{m1}$  we use

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}$$

$$= \sqrt{2 \times 0.090 \times \frac{100}{1.6} \times 0.1}$$

$$= 1.06 \text{ mA/V}$$

$$\begin{aligned}
 r_{o1} &= \frac{V_{AI}}{I_{D1}} = \frac{12.8}{0.1} = 128 \text{ k}\Omega \\
 r_{o2} &= \frac{|V_{A2}|}{I_{D2}} = \frac{19.2}{0.1} = 192 \text{ k}\Omega \\
 R'_L &= r_{o1} \parallel r_{o2} = 128 \parallel 192 = 76.8 \text{ k}\Omega \\
 A_M &= -g_m R'_L \\
 &= -1.06 \times 76.8 = -81.4 \text{ V/V} \\
 C_L &= C_{db1} + C_{db2} + C_{gd2} \\
 &= 20 + 36 + 15 = 71 \text{ fF} \\
 f_H &= \frac{1}{2\pi(C_L + C_{gd1})R'_L} \\
 f_H &= \frac{1}{2\pi(71 + 15) \times 10^{-15} \times 76.8 \times 10^3} \\
 &= 24.1 \text{ MHz} \\
 f_Z &= \frac{g_{m1}}{2\pi C_{gd1}} = \frac{1.06 \times 10^{-3}}{2\pi \times 0.015 \times 10^{-12}} \\
 &= 11.2 \text{ GHz}
 \end{aligned}$$

9.44 The equivalent circuit is shown in the figure.

$$g_m = \frac{I_C}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ V}} = 80 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{80} = 1.5 \text{ k}\Omega$$

$$(a) A_M = -\frac{r_\pi}{r_\pi + r_x} g_m R'_L$$

$$-10 = -\frac{1.5}{1.5 + 0.1} \times g_m R'_L$$

$$\Rightarrow g_m R'_L = 10.7 \text{ V/V}$$

$$R'_L = 0.133 \text{ k}\Omega$$

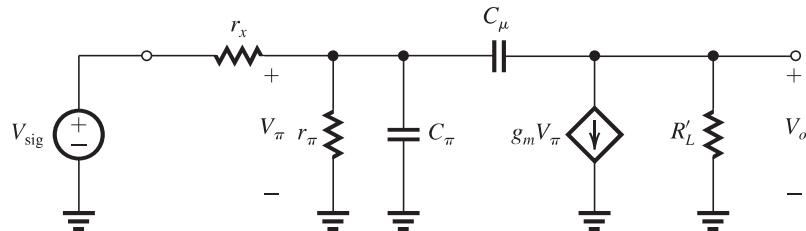
$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{80 \times 10^{-3}}{2\pi \times 2 \times 10^9} = 6.37 \text{ pF}$$

$$C_\pi = 6.37 - 1 = 5.37 \text{ pF}$$

$$C_{in} = C_\pi + (g_m R'_L + 1) C_\mu$$

This figure belongs to Problem 9.44.



$$\begin{aligned}
 C_{in} &= 5.37 + (10.7 + 1) \times 1 \\
 &= 17.07 \text{ pF} \\
 R'_{sig} &= r_\pi \parallel r_x = 1.5 \text{ k}\Omega \parallel 0.1 \text{ k}\Omega \\
 &= 0.094 \text{ k}\Omega \\
 f_H &= \frac{1}{2\pi C_{in} R'_{sig}} \\
 &= \frac{1}{2\pi \times 17.07 \times 10^{-12} \times 0.094 \times 10^3} \\
 &= 99.2 \text{ MHz} \\
 (b) \text{ If } |A_M| &\text{ is reduced to 1, we obtain} \\
 1 &= \frac{1.5}{1.6} \times g_m R'_L \\
 \Rightarrow g_m R'_L &= 1.07 \\
 C_{in} &= C_\pi + (g_m R'_L + 1) C_\mu \\
 &= 5.37 + (1.07 + 1) \times 1 \\
 &= 7.44 \text{ pF} \\
 f_H &= \frac{1}{2\pi \times 7.44 \times 10^{-12} \times 0.094 \times 10^3} \\
 &= 227.6 \text{ MHz}
 \end{aligned}$$

9.45 Figure 1 (next page) shows the amplifier high-frequency equivalent circuit. A node equation at the output provides

$$\left( \frac{1}{r_o} + sC_L \right) V_o + g_m V_\pi + sC_\mu (V_o - V_\pi) = 0$$

Replacing  $V_\pi$  by  $V_i$  and collecting terms results in

$$V_o \left[ \frac{1}{r_o} + s(C_L + C_\mu) \right] = -V_i (g_m - sC_\mu)$$

$$\Rightarrow \frac{V_o}{V_i} = -g_m r_o \frac{1 - s(C_\mu / g_m)}{1 + s(C_L + C_\mu) r_o} \quad \text{Q.E.D.}$$

For  $I_C = 200 \mu\text{A} = 0.2 \text{ mA}$  and  $V_A = 100 \text{ V}$ ,

$$g_m = \frac{I_C}{V_T} = \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.2} = 500 \text{ k}\Omega$$

Chapter 9–19

This figure belongs to Problem 9.45.

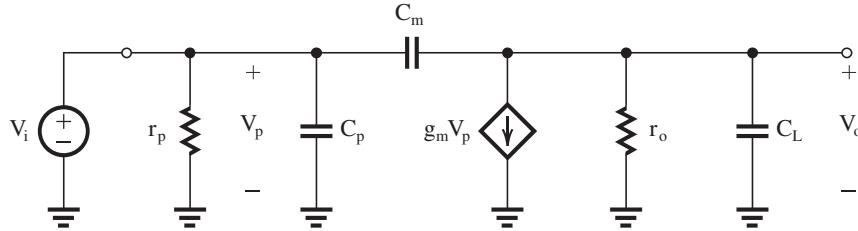


Figure 1

$$\text{DC gain} = -g_m r_o = 8 \times 500 = -4000 \text{ V/V}$$

$$\begin{aligned} f_{3\text{dB}} &= \frac{1}{2\pi(C_L + C_\mu)r_o} \\ &= \frac{1}{2\pi(1 + 0.2) \times 10^{-12} \times 500 \times 10^3} \\ &= 265.3 \text{ kHz} \\ f_z &= \frac{g_m}{2\pi C_\mu} \\ &= \frac{8 \times 10^{-3}}{2\pi \times 0.2 \times 10^{-12}} = 6.37 \text{ GHz} \\ f_t &= |A_{dc}| f_{3\text{dB}} \\ &= 4000 \times 265.3 = 1.06 \text{ GHz} \end{aligned}$$

The Bode plot is shown in Figure 2.

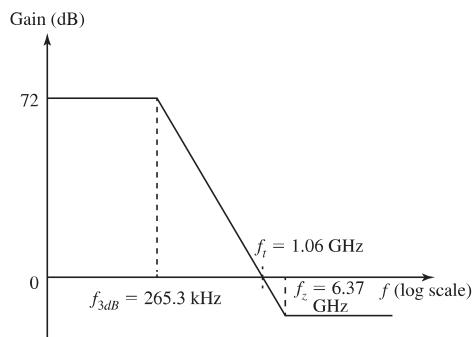


Figure 2

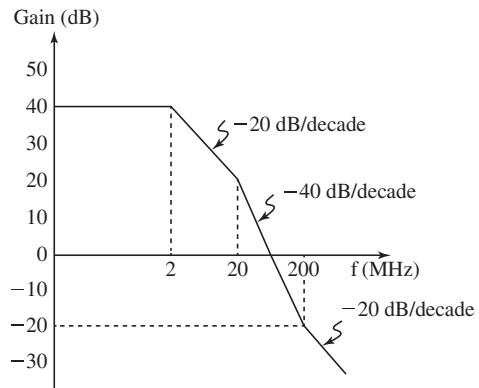
$$9.46 A_M = 40 \text{ dB} \Rightarrow 100 \text{ V/V}$$

$$A(s) = \frac{1 + s/(2\pi \times 200 \times 10^6)}{100 \left(1 + \frac{s}{2\pi \times 2 \times 10^6}\right) \left(1 + \frac{s}{2\pi \times 20 \times 10^6}\right)}$$

Since  $f_{p1} \ll f_{p2} \ll f_z$ , we have

$$f_{3\text{dB}} \simeq f_{p1} = 2 \text{ MHz}$$

The Bode plot is shown in the figure.



$$9.47 \text{ (a)} A(s) = 1000 \frac{1}{1 + s/(2\pi \times 10^5)}$$

(b)

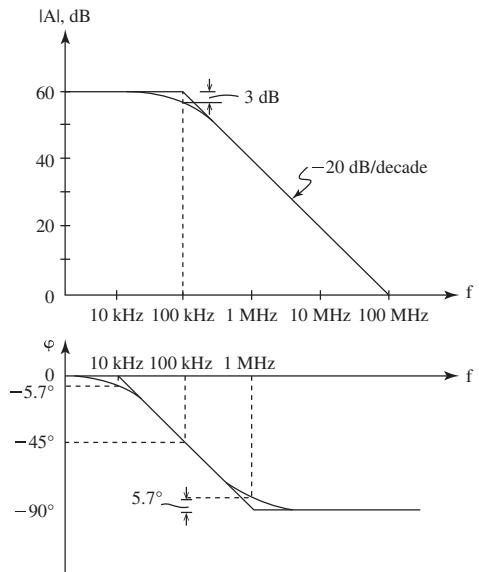


Figure 1

Figure 1 shows the Bode plot for the gain magnitude and phase.

(c)  $\text{GB} = 1000 \times 100 \text{ kHz} = 100 \text{ MHz}$

(d) The unity-gain frequency  $f_t$  is

$$f_t = 100 \text{ MHz}$$

(e)

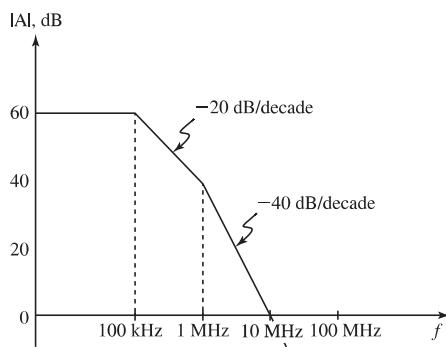


Figure 2

Figure 2 shows the magnitude response when a second pole at 1 MHz appears in the transfer function. The unity-gain frequency  $f_t$  now is

$$f_t = 10 \text{ MHz}$$

which is different from the gain-bandwidth product,

$$\text{GB} = 100 \text{ MHz}$$

9.48 Using the dominant-pole approximation,

$$\omega_H \simeq \omega_{P1}$$

Using the root-sum-of-squares formula, we get

$$\omega_H \simeq \frac{1}{\sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2}}} = \frac{\omega_{P1}}{\sqrt{1 + \left(\frac{\omega_{P1}}{\omega_{P2}}\right)^2}}$$

(a) For a difference of 10%,

$$\frac{\omega_{P1}}{\sqrt{1 + \left(\frac{\omega_{P1}}{\omega_{P2}}\right)^2}} = 0.9\omega_{P1}$$

$$\Rightarrow \frac{\omega_{P2}}{\omega_{P1}} = 4.26$$

(b) For a difference of 1%,

$$\frac{\omega_{P1}}{\sqrt{1 + \left(\frac{\omega_{P1}}{\omega_{P2}}\right)^2}} = 0.99\omega_{P1}$$

$$\Rightarrow \frac{\omega_{P2}}{\omega_{P1}} = 49.3$$

$$9.49 A(s) = -10^3 \frac{1 + \frac{s}{10^4}}{\left(1 + \frac{s}{10^3}\right)\left(1 + \frac{s}{10^5}\right)}$$

$$(a) \omega_H \simeq 10^3 \text{ rad/s}$$

$$(b) \omega_H = 1 \sqrt{\left(\frac{1}{10^6} + \frac{1}{10^{10}}\right)} - \frac{2}{10^8}$$

$$= 1010 \text{ Hz}$$

If the frequency of the finite zero is lowered to  $10^3$  rad/s the zero will cancel the pole at  $10^3$  rad/s and the transfer function becomes

$$A(s) = -10^3 \frac{1}{1 + \frac{s}{10^5}}$$

The 3-dB frequency now becomes

$$\omega_{3\text{dB}} = 10^5 \text{ rad/s}$$

9.50 If at  $\omega = 10^7$  rad/s the excess phase due to the 3 coincident poles (at frequency  $\omega_P$ ) is  $30^\circ$ , then each pole is contributing  $10^\circ$ . Thus,

$$\tan^{-1} \frac{10^7}{\omega_P} = 10^\circ$$

$$\omega_P = \frac{10^7}{\tan 10^\circ} = 5.67 \times 10^7 \text{ rad/s}$$

$$9.51 \tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_LR_{CL}$$

where

$$C_{gs} = 30 \text{ fF}$$

$$R_{gs} = R'_{sig} = 10 \text{ k}\Omega$$

$$C_{gd} = 5 \text{ fF}$$

$$R_{gd} = R'_{sig}(1 + g_mR'_L) + R'_L$$

$$= 10(1 + 2 \times 20) + 20$$

$$= 430 \text{ k}\Omega$$

$$C_L = 30 \text{ fF}$$

$$R_{CL} = R'_L = 20 \text{ k}\Omega$$

Thus,

$$\tau_H = 30 \times 10 + 5 \times 430 + 30 \times 20$$

$$= 3050 \text{ ps}$$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 3050 \times 10^{-12}}$$

$$= 52.2 \text{ MHz}$$

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{2 \times 10^{-3}}{2\pi \times 5 \times 10^{-15}} = 63.7 \text{ GHz}$$

$$\begin{aligned}
9.52 \quad A_M &= -g_m R'_L \\
&= -4 \times 20 = -80 \text{ V/V} \\
C_{in} &= C_{gs} + C_{gd}(g_m R'_L + 1) \\
&= 2 + 0.1(80 + 1) \\
&= 10.1 \text{ pF}
\end{aligned}$$

Using the Miller approximation, we obtain

$$\begin{aligned}
f_H &\simeq \frac{1}{2\pi C_{in} R'_{sig}} \\
&= \frac{1}{2\pi \times 10.1 \times 10^{-12} \times 20 \times 10^3} \\
&= 788 \text{ kHz}
\end{aligned}$$

Using the open-circuit time constants, we get

$$\begin{aligned}
\tau_{gs} &= C_{gs} R_{gs} = C_{gs} R'_{sig} \\
&= 2 \times 20 = 40 \text{ ns} \\
R_{gd} &= R'_{sig}(1 + g_m R'_L) + R'_L \\
&= 20(1 + 80) + 20 = 1640 \text{ k}\Omega \\
\tau_{gd} &= C_{gd} R_{gd} = 0.1 \times 1640 = 164 \text{ ns} \\
\tau_{CL} &= C_L R'_L \\
&= 2 \times 20 = 40 \text{ ns}
\end{aligned}$$

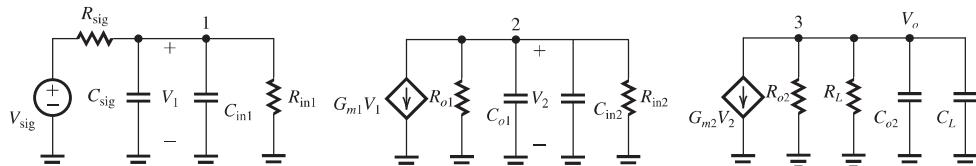
$$\begin{aligned}
\tau_H &= \tau_{gs} + \tau_{gd} + \tau_{CL} \quad (1) \\
&= 40 + 164 + 40 = 244 \text{ ns} \\
f_H &= \frac{1}{2\pi \tau_H} \\
&= \frac{1}{2\pi \times 244 \times 10^{-9}} = 652 \text{ kHz}
\end{aligned}$$

The estimate obtained using the open-circuit time constants is more appropriate as it takes into account the effect of  $C_L$ . We note from Eq. (1) that  $\tau_{CL}$  is 16.4% of  $\tau_H$ , thus  $C_L$  has a significant effect on the determination of  $f_H$ .

9.53 The figure shows the equivalent circuit of the two-stage amplifier where we have modeled each stage as a transconductance amplifier. At node 1:

$$\begin{aligned}
R_{eq1} &= R_{sig} \parallel R_{in1} \\
&= 10 \text{ k}\Omega \parallel 10 \text{ k}\Omega = 5 \text{ k}\Omega \\
C_{eq1} &= C_{in1} + C_{sig} = 10 + 10 = 20 \text{ pF}
\end{aligned}$$

This figure belongs to Problem 9.53.



Thus,

$$\begin{aligned}
f_{p1} &= \frac{1}{2\pi C_{eq1} R_{eq1}} \\
f_{p1} &= \frac{1}{2\pi \times 20 \times 10^{-12} \times 5 \times 10^3} \\
&= 1.59 \text{ MHz}
\end{aligned}$$

At node 2:

$$\begin{aligned}
R_{eq2} &= R_{o1} \parallel R_{in2} = 2 \text{ k}\Omega \parallel 10 \text{ k}\Omega \\
&= 1.67 \text{ k}\Omega \\
C_{eq2} &= C_{o1} + C_{in2} \\
&= 2 + 10 = 12 \text{ pF} \\
f_{p2} &= \frac{1}{2\pi C_{eq2} R_{eq2}} \\
&= \frac{1}{2\pi \times 12 \times 10^{-12} \times 1.67 \times 10^3} \\
&= 7.94 \text{ MHz}
\end{aligned}$$

At node 3:

$$\begin{aligned}
R_{eq3} &= R_{o2} \parallel R_L = 2 \text{ k}\Omega \parallel 1 \text{ k}\Omega = 0.67 \text{ k}\Omega \\
C_{eq3} &= C_{o2} + C_L = 2 + 7 = 9 \text{ pF} \\
f_{p3} &= \frac{1}{2\pi C_{eq3} R_{eq3}} \\
f_{p3} &= \frac{1}{2\pi \times 9 \times 10^{-12} \times 0.67 \times 10^3} \\
&= 26.4 \text{ MHz}
\end{aligned}$$

Thus, the three poles have frequencies 1.59 MHz, 7.94 MHz, and 26.4 MHz. Since the frequency of the second pole is more than two octaves higher than that of the first pole, the 3-dB frequency will be mostly determined by  $f_{p1}$ ,

$$f_{3dB} \simeq f_{p1} = 1.59 \text{ MHz}$$

A slightly better estimate of  $f_{3dB}$  can be determined using the root-sum-of-squares formula,

$$\begin{aligned}
f_{3dB} &= 1/\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2 + \left(\frac{1}{f_{p3}}\right)^2} \\
&= 1/\sqrt{\frac{1}{1.59^2} + \frac{1}{7.94^2} + \frac{1}{26.4^2}} \\
&= 1.56 \text{ MHz}
\end{aligned}$$

$$\begin{aligned}
 9.54 \quad \tau_{gs} &= C_{gs}R_{gs} = C_{gs}R'_{sig} \\
 &= 5 \times 10 = 50 \text{ ns} \\
 R_{gd} &= R'_{sig}(1 + g_m R'_L) + R'_L \\
 &= 10(1 + 5 \times 10) + 10 \\
 &= 520 \text{ k}\Omega \\
 \tau_{gd} &= C_{gd}R_{gd} = 1 \times 520 = 520 \text{ ns} \\
 R_{CL} &= C_L R_L \\
 &= 5 \times 10 = 50 \text{ ns} \\
 \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{CL} \\
 &= 50 + 520 + 50 = 620 \text{ ns} \\
 f_H &= \frac{1}{2\pi \tau_H} \quad (1) \\
 &= \frac{1}{2\pi \times 620 \times 10^{-9}} = 257 \text{ kHz}
 \end{aligned}$$

The interaction of  $R'_{sig}$  with the input capacitance contributes all of  $\tau_{gs}$  (50 ns) and a significant part of  $\tau_{gd}$ , namely

$$C_{gd}[R'_{sig}(1 + g_m R'_L)] = 1 \times 10(1 + 50) = 510 \text{ ns}$$

Thus, the total contribution of  $R'_{sig}$  is

$$50 + 510 = 560 \text{ ns}$$

which is  $\frac{560}{620} = 90.3\%$  of  $\tau_H$ . To double  $f_H$ , we must reduce  $\tau_H$  to half of its value:

$$\tau_H = \frac{1}{2} \times 620 = 310 \text{ ns}$$

Now,

$$\begin{aligned}
 \tau_H &= R'_{sig}[C_{gs} + C_{gd}(1 + g_m R'_L)] \\
 &\quad + C_{gd}R'_L + C_L R'_L \\
 310 &= R'_{sig}[5 + 1(1 + 50)] + 1 \times 10 + 5 \times 10 \\
 \Rightarrow R'_{sig} &= 4.46 \text{ k}\Omega
 \end{aligned}$$

9.55 To lower  $f_H$  from 135.5 MHz (see Example 9.8) to 100 MHz,  $\tau_H$  must be increased to

$$\begin{aligned}
 \tau_H &= \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 100 \times 10^6} \\
 &= 1591.5 \text{ ps}
 \end{aligned}$$

Now,

$$\begin{aligned}
 \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{CL} \\
 1591.5 &= 200 + 725 + \tau_{CL}
 \end{aligned}$$

$$\Rightarrow \tau_{CL} = 666.5$$

But,

$$\tau_{CL} = C'_L R'_L$$

$$665.5 = C'_L \times 10$$

$$\Rightarrow C'_L = 66.6 \text{ fF}$$

Thus, the original  $C_L$  of 25 fF must be increased by

$$66.6 - 25 = 41.6 \text{ fF}$$

9.56 We will assume that the value given in the problem statement is for  $R_{sig}$  (not  $R'_{sig}$ ):

$$\begin{aligned}
 R_{sig} &= 5 \text{ k}\Omega \\
 r_\pi &= \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega \\
 R'_{sig} &= r_\pi \parallel R_{sig} = 5 \text{ k}\Omega \parallel 5 \text{ k}\Omega \\
 &= 2.5 \text{ k}\Omega \\
 \tau_H &= C_\pi R_\pi = C_\pi R'_{sig} \\
 &= 10 \times 2.5 = 25 \text{ ns} \\
 \tau_\mu &= C_\mu R_\mu \\
 &= C_\mu [R'_{sig}(1 + g_m R'_L) + R'_L] \\
 &= 1 \times [2.5(1 + 20 \times 5) + 5] \\
 &= 257.5 \text{ ns} \\
 \tau_{CL} &= C_L R'_L \\
 &= 10 \times 5 = 50 \text{ ns} \\
 \tau_H &= \tau_\pi + \tau_\mu + \tau_{CL} \\
 &= 25 + 257.5 + 50 = 332.5 \text{ ns} \\
 f_H &= \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 332.5 \times 10^{-9}} = 479 \text{ kHz} \\
 A_M &= -\frac{r_\pi}{r_\pi + R_{sig}} g_m R'_L \\
 &= -\frac{5}{5+5} \times 20 \times 5 \\
 &= -50 \text{ V/V}
 \end{aligned}$$

9.57 Refer to Fig. 9.19(a). Since  $R_B$  is not specified, we assume that its value is very large.

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m R'_L$$

where

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

Thus,

$$\begin{aligned}
 A_M &= -\frac{2.5}{2.5 + 0.1 + 1} \times 40 \times 5 \\
 &= -138.9 \text{ V/V}
 \end{aligned}$$

Using the Miller approximation, we obtain

$$\begin{aligned}
 C_{in} &= C_\pi + C_\mu(1 + g_m R'_L) \\
 &= 10 + 0.3(1 + 40 \times 5) \\
 &= 70.3 \text{ pF} \\
 R'_{sig} &= r_\pi \parallel (r_x + R_{sig}) \\
 &= 2.5 \parallel (0.1 + 1) = 0.76 \text{ k}\Omega
 \end{aligned}$$

$$\begin{aligned} f_H &= \frac{1}{2\pi C_{in} R'_{sig}} \\ &= \frac{1}{2\pi \times 70.3 \times 10^{-12} \times 0.76 \times 10^3} \\ &= 2.98 \text{ MHz} \end{aligned}$$

Using the open-circuit time constants, we get

$$\begin{aligned} \tau_\pi &= C_\pi R_\pi = C_\pi R'_{sig} \\ &= 10 \times 0.76 = 7.6 \text{ ns} \\ \tau_\mu &= C_\mu [R'_{sig}(g_m R'_L + 1) + R'_L] \\ &= 0.3[0.76(40 \times 5 + 1) + 5] \\ &= 47.3 \text{ ns} \\ \tau_{C_L} &= C_L R'_L = 3 \times 5 = 15 \text{ ns} \\ \tau_H &= \tau_\pi + \tau_\mu + \tau_{C_L} \\ &= 7.6 + 47.3 + 15 = 69.9 \text{ ns} \\ f_H &= \frac{1}{2\pi \tau_H} \\ &= \frac{1}{2\pi \times 69.9 \times 10^{-9}} = 2.28 \text{ MHz} \end{aligned}$$

This is a more realistic estimate of  $f_H$  as it takes into account the effect of  $C_L$ .

$$\begin{aligned} 9.58 R'_L &= r_o \parallel R_L = 20 \text{ k}\Omega \parallel 12 \text{ k}\Omega \\ &= 7.5 \text{ k}\Omega \\ \tau_{gs} &= C_{gs} R_{gs} = C_{gs} R'_{sig} \\ &= 0.2 \times 100 = 20 \text{ ns} \\ \tau_{gd} &= C_{gd} [R'_{sig}(g_m R'_L + 1) + R'_L] \\ &= 0.2[100(1.5 \times 7.5 + 1) + 7.5] \\ &= 246.5 \text{ ns} \\ (\text{a}) \quad C_L &= 0 \\ \tau_{C_L} &= 0 \\ \tau_H &= \tau_{gs} + \tau_{gd} = 20 + 246.5 = 266.5 \text{ ns} \\ f_H &= \frac{1}{2\pi \times 266.5 \times 10^{-9}} = 597 \text{ kHz} \\ (\text{b}) \quad C_L &= 10 \text{ pF} \\ \tau_{C_L} &= C_L R'_L = 10 \times 7.5 = 75 \text{ ns} \\ \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{C_L} \\ &= 20 + 246.5 + 75 = 341.5 \text{ ns} \\ f_H &= \frac{1}{2\pi \tau_H} \\ &= \frac{1}{2\pi \times 341.5 \times 10^{-9}} = 466 \text{ kHz} \end{aligned}$$

$$\begin{aligned} (\text{c}) \quad C_L &= 50 \text{ pF} \\ \tau_{C_L} &= C_L R'_L = 50 \times 7.5 = 375 \text{ ns} \\ \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{C_L} \\ &= 20 + 246.5 + 375 \\ &= 641.5 \text{ ns} \\ f_H &= \frac{1}{2\pi \tau_H} \\ &= \frac{1}{2\pi \times 641.5 \times 10^{-9}} = 248 \text{ kHz} \end{aligned}$$

Using the Miller approximation, since  $C_L$  is not taken into account, then for all three cases we obtain

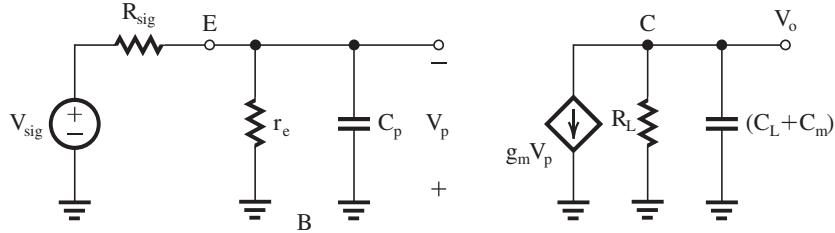
$$\begin{aligned} C_{in} &= C_{gs} + C_{gd}(g_m R'_L + 1) \\ &= 0.2 + 0.2(1.5 \times 7.5 + 1) \\ &= 2.65 \text{ pF} \\ f_H &= \frac{1}{2\pi C_{in} R'_{sig}} \\ &= \frac{1}{2\pi \times 2.65 \times 10^{-12} \times 100 \times 10^3} \\ &= 600 \text{ kHz} \end{aligned}$$

which is very close to the estimate obtained using the method of open-circuit time constants for the case  $C_L = 0$ . However, as  $C_L$  is increased, the estimate obtained using the Miller approximation becomes less and less realistic, which is due to the fact that it does not take  $C_L$  into account.

9.59 Refer to Fig. 9.26(c).

$$\begin{aligned} \frac{V_o}{V_{sig}} &= \frac{\frac{1}{g_m} g_m R_L}{\frac{1}{g_m} + R_{sig}} \\ &= \frac{\frac{1}{5} \times 5 \times 10}{\frac{1}{5} + 1} \\ &= 8.3 \text{ V/V} \\ f_{p1} &= \frac{1}{2\pi C_{gs} \left( R_{sig} \parallel \frac{1}{g_m} \right)} \\ f_{p1} &= \frac{1}{2\pi \times 4 \times 10^{-12} \left( 1 \parallel \frac{1}{5} \right) \times 10^3} \\ &= 239 \text{ MHz} \\ f_{p2} &= \frac{1}{2\pi (C_L + C_{gd}) R_L} \\ &= \frac{1}{2\pi (2 + 0.2) \times 10^{-12} \times 10 \times 10^3} \\ &= 7.23 \text{ MHz} \end{aligned}$$

This figure belongs to Problem 9.60.



Since  $f_{p1} \gg f_{p2}$ ,  $f_{p2}$  will be dominant and

$$f_H \simeq f_{p2} = 7.23 \text{ MHz}$$

9.60 See figure above. Replacing the BJT with its high-frequency T model while neglecting  $r_o$  and  $r_x$  results in the equivalent circuit shown in the figure.

(a) There are two separate poles, one at the input given by

$$f_{p1} = \frac{1}{2\pi C_\pi (R_{sig} \parallel r_e)}$$

and the other at the output, given by

$$f_{p2} = \frac{1}{2\pi(C_L + C_m)R_L} \quad \text{Q.E.D.}$$

(b)  $I_C = 1 \text{ mA}$ ,

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_e \simeq \frac{1}{g_m} = 25 \Omega$$

$$f_{p1} = \frac{1}{2\pi \times 10 \times 10^{-12} (1 \parallel 0.025) \times 10^3} \\ = 652.5 \text{ MHz}$$

$$f_{p2} = \frac{1}{2\pi(1+1) \times 10^{-12} \times 10 \times 10^3} \\ = 7.96 \text{ MHz}$$

Since  $f_{p2} \ll f_{p1}$ ,  $f_{p2}$  will be dominant and

$$f_H \simeq f_{p2} = 7.96 \text{ MHz}$$

$$9.61 \quad r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$R_{sig} = r_o/2 = 50 \text{ k}\Omega$$

$$R_L = r_o = 100 \text{ k}\Omega$$

$$g_m r_o = 1.5 \times 100 = 150$$

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{100 + 100}{1 + 150} = 1.32 \text{ k}\Omega$$

$$R_o = R_{sig} + r_o + g_m r_o R_{sig}$$

$$= 50 + 100 + 150 \times 50 = 7650 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs}(R_{in} \parallel R_{sig})$$

$$\tau_{gs} = 0.2(1.32 \parallel 50)$$

$$= 0.26 \text{ ns}$$

$$R_{gd} = R_L \parallel R_o$$

$$= 100 \parallel 7650 = 98.7 \text{ k}\Omega$$

$$\tau_{gd} = (C_{gd} + C_L + C_{db})R_{gd}$$

$$= (0.015 + 0.03 + 0.02) \times 98.7$$

$$= 6.42 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} = 0.26 + 6.42$$

$$= 6.68 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 6.68 \times 10^{-9}} = 23.8 \text{ MHz}$$

9.62 Refer to Example 9.9. To reduce  $f_H$  to 200 MHz,  $\tau_H$  must become

$$\tau_H = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 200 \times 10^6}$$

$$= 795.8 \text{ ps}$$

Since  $\tau_{gs}$  remains constant at 26.6 ps,  $\tau_{CL}$  must be increased to

$$\tau_{CL} = 795.8 - 26.6 = 769.2 \text{ ps}$$

But,

$$\tau_{CL} = (C_{gd} + C_L)R_{gd}$$

thus,

$$769.2 = (5 + C_L) \times 18.7$$

$$\Rightarrow C_L + 5 = 41.1 \text{ fF}$$

$$C_L = 36.1 \text{ fF}$$

Thus, the amount of additional capacitance to be connected at the output is

$$36.1 - 25 = 11.1 \text{ fF}$$

$$\begin{aligned}
 9.63 \quad R_o &= r_{o2} + r_{o1} + (g_{m2}r_{o2})r_{o1} \\
 &= 2r_o + g_m r_o^2 \\
 &= 2 \times 20 + 2 \times 20 \times 20 = 840 \text{ k}\Omega \\
 A_v &= -g_{m1}(R_o \parallel R_L) \\
 &= -2(840 \parallel 1000) \\
 &= -913 \text{ V/V}
 \end{aligned}$$

Using Eq. (9.109), we obtain

$$\begin{aligned}
 \tau_H &= R_{sig}[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})] \\
 &\quad + R_{d1}(C_{gd1} + C_{db1} + C_{gs2}) \\
 &\quad + (R_L \parallel R_o)(C_L + C_{gd2})
 \end{aligned}$$

where

$$\begin{aligned}
 R_{d1} &= r_{o1} \parallel R_{in2} \\
 R_{in2} &= \frac{r_{o2} + R_L}{1 + g_{m2}r_{o2}} \\
 &= \frac{20 + 1000}{1 + 2 \times 20} = 24.9 \text{ k}\Omega
 \end{aligned}$$

$$R_{d1} = 20 \parallel 24.9 = 11.1 \text{ k}\Omega$$

Thus,

$$\begin{aligned}
 \tau_H &= 100[20 + 5(1 + 2 \times 11.1)] \\
 &\quad + 11.1(5 + 5 + 20) \\
 &\quad + (1000 \parallel 840)(20 + 5) \\
 &= 13587 + 333 + 11413 \\
 &= 25,333 \text{ ps} = 25.33 \text{ ns} \\
 f_H &= \frac{1}{2\pi \times 25.33 \times 10^{-9}} = 6.28 \text{ MHz}
 \end{aligned}$$

$$9.64 \quad (a) \quad A_M = -g_m R'_L$$

where

$$R'_L = R_L \parallel r_o = 20 \parallel 20 = 10 \text{ k}\Omega$$

Thus,

$$A_M = -4 \times 10 = -40 \text{ V/V}$$

$$\tau_{gs} = C_{gs} R_{gs}$$

$$= C_{gs} R_{sig} = 2 \times 20 = 40 \text{ ns}$$

$$R_{gd} = R_{sig}(1 + g_m R'_L) + R'_L$$

$$= 20(1 + 4 \times 10) + 10$$

$$= 830 \text{ k}\Omega$$

$$\tau_{gd} = C_{gd} R_{gd} = 0.2 \times 830 = 166 \text{ ns}$$

$$\tau_{CL} = C_L R'_L = 1 \times 10 = 10 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 40 + 166 + 10 = 216 \text{ ns}$$

$$\begin{aligned}
 f_H &= \frac{1}{2\pi \tau_H} \\
 &= \frac{1}{2\pi \times 216 \times 10^{-9}} = 737 \text{ kHz} \\
 GB &\equiv |A_M| f_H = 40 \times 737 = 29.5 \text{ MHz} \\
 (b) \quad A_M &= -g_{m1}(R_o \parallel R_L) \\
 \text{where}
 \end{aligned}$$

$$\begin{aligned}
 R_o &= r_{o1} + r_{o2} + g_{m2}r_{o2}r_{o1} \\
 &= 2r_o + g_m r_o^2 \\
 &= 2 \times 20 + 4 \times 20 \times 20 = 1640 \text{ k}\Omega \\
 A_M &= -4(1640 \parallel 20) = -79 \text{ V/V}
 \end{aligned}$$

$$\begin{aligned}
 R_{in2} &= \frac{r_{o2} + R_L}{1 + g_{m2}r_{o2}} \\
 &= \frac{20 + 20}{1 + 4 \times 20} \simeq 0.49 \text{ k}\Omega \\
 R_{d1} &= r_{o1} \parallel R_{in2} = 20 \parallel 0.49 = 0.48 \text{ k}\Omega
 \end{aligned}$$

Using Eq. (9.109), we obtain

$$\begin{aligned}
 \tau_H &= R_{sig}[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})] \\
 &\quad + R_{d1}(C_{gd1} + C_{db1} + C_{gs2}) \\
 &\quad + (R_L \parallel R_o)(C_L + C_{gd2}) \\
 &= 20[2 + 0.2(1 + 4 \times 0.48)] \\
 &\quad + 0.48(0.2 + 0.2 + 2) \\
 &\quad + (20 \parallel 1640)(1 + 0.2)
 \end{aligned}$$

$$\tau_H = 51.7 + 1.15 + 23.7 = 76.6 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 76.6 \times 10^{-9}} = 2.08 \text{ MHz}$$

$$GB \equiv |A_M| f_H = 79 \times 2.08 = 164 \text{ MHz}$$

Note the increase in bandwidth and in GB.

$$9.65 \quad 20 \log |A_M| = 74 \text{ dB}$$

$$\Rightarrow |A_M| = 5000$$

$$R_o \simeq (g_m r_o) r_o$$

$$R_L = R_o$$

$$|A_M| = g_m (R_L \parallel R_o)$$

$$5000 = \frac{1}{2} g_m R_o$$

$$= \frac{1}{2} (g_m r_o)^2$$

$$\Rightarrow g_m r_o = 100$$

$$\frac{2V_A}{V_{OV}} = 100$$

$$\Rightarrow V_{OV} = \frac{2 \times 10}{100} = 0.2 \text{ V}$$

$$\begin{aligned} I_D &= \frac{1}{2}\mu_n C_{ox} \left( \frac{W}{L} \right) V_{ov}^2 \\ &= \frac{1}{2} \times 0.2 \times 50 \times 0.2^2 \\ &= 0.2 \text{ mA} \\ f_t &= \frac{g_m}{2\pi(C_L + C_{gd})} \end{aligned}$$

where

$$\begin{aligned} g_m &= \frac{2I_D}{V_{ov}} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V} \\ f_t &= \frac{2 \times 10^{-3}}{2\pi(1+0.1) \times 10^{-12}} \\ &= 289.4 \text{ MHz} \end{aligned}$$

If the cascode transistor is removed,

$$\begin{aligned} A_M &= -g_m(r_o \parallel R_L) \\ &\simeq -g_m r_o = -100 \text{ V/V} \end{aligned}$$

9.66 (a) From Fig. 9.29, we have

$$\begin{aligned} f_t &= \frac{g_m}{2\pi(C_L + C_{gd})} \\ &= \frac{\sqrt{2\mu_n C_{ox}(W/L)}}{2\pi(C_L + C_{gd})} \sqrt{I_D} \quad \text{Q.E.D.} \end{aligned} \quad (1)$$

$$(b) g_m = \sqrt{2\mu_n C_{ox}(W/L)} \sqrt{I_D} \quad (2)$$

$$V_{ov} = \sqrt{\frac{I_D}{\frac{1}{2}\mu_n C_{ox}(W/L)}} \quad (3)$$

$$r_o = \frac{V_A}{I_D} \quad (4)$$

$$R_o = (g_m r_o) r_o \quad (5)$$

$$\begin{aligned} A_M &= -g_m(R_o \parallel R_L) \\ &= -g_m(R_o \parallel R_o) \\ &= -\frac{1}{2} g_m R_o \end{aligned} \quad (6)$$

Substituting

$$\mu_n C_{ox} = 0.4 \text{ mA/V}^2, W/L = 20, C_L = 20 \text{ fF},$$

$C_{gd} = 5 \text{ fF}$  and  $V_A = 10 \text{ V}$  in Eqs. (1)–(6), we obtain the following results in the table below.

9.67 (a) For the CS amplifier,

$$\begin{aligned} |A_M| &= g_m(r_o \parallel r_o) = \frac{1}{2} g_m r_o \\ f_H &= \frac{1}{2\pi C_{in} R_{sig}} \\ &= \frac{1}{2\pi \left[ C_{gs} + C_{gd} \left( \frac{1}{2} g_m r_o + 1 \right) \right] R_{sig}} \end{aligned} \quad (1)$$

For the cascode amplifier,

$$\begin{aligned} |A_M| &= g_m(R_o \parallel r_o) \\ &= g_m[(g_m r_o) r_o \parallel r_o] \\ &\simeq g_m r_o \end{aligned}$$

Thus, the gain increases by a factor of 2.

$$f_H = \frac{1}{2\pi C_{in} R_{sig}}$$

where

$$\begin{aligned} C_{in} &= C_{gs} + C_{gd}(1 + g_m R_{d1}) \\ R_{d1} &= r_o \parallel R_{in2} \\ &= r_o \parallel \frac{R_L + r_o}{g_m r_o} \\ R_{d1} &= r_o \parallel \frac{r_o + r_o}{g_m r_o} \\ &= r_o \parallel \frac{2}{g_m} \simeq \frac{2}{g_m} \\ C_{in} &= C_{gs} + C_{gd} \left( 1 + g_m \times \frac{2}{g_m} \right) \\ &= C_{gs} + 3C_{gd} \\ f_H &= \frac{1}{2\pi(C_{gs} + 3C_{gd})R_{sig}} \end{aligned} \quad (2)$$

From (1) and (2), the ratio  $N$  of  $f_H$  of the cascode amplifier to  $f_H$  of the CS amplifier is

$$N = \frac{C_{gs} + C_{gd} \left( \frac{1}{2} g_m r_o + 1 \right)}{C_{gs} + 3C_{gd}}$$

This table belongs to Problem 9.66, part (b).

$I_D$ (mA)	$f_t$ (GHz)	$V_{ov}$ (V)	$g_m$ (mA/V)	$r_o$ ( $k\Omega$ )	$R_o$ ( $M\Omega$ )	$A_M$ (V/V)	$f_H$ (MHz)
0.1	8	0.16	1.26	100	12.6	-7938	1
0.2	11.5	0.22	1.80	50	4.5	-4050	2.8
0.5	18	0.35	2.83	20	1.13	-1600	11.3

Thus,

$$N \simeq \frac{C_{gs} + \frac{1}{2}(g_m r_o) C_{gd}}{C_{gs} + 3C_{gd}} \quad \text{Q.E.D.}$$

$$(b) 50 = \frac{1}{2} g_m r_o$$

$$\Rightarrow g_m r_o = 100$$

$$N = \frac{C_{gs} + \frac{1}{2} \times 100 \times 0.1 C_{gs}}{C_{gs} + 3 \times 0.1 C_{gs}}$$

$$= \frac{1+5}{1+0.3} = 4.6$$

$$(c) g_m r_o = \frac{2V_A}{V_{OV}}$$

$$100 = \frac{2 \times 10}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$= \frac{1}{2} \times 0.4 \times 10 \times 0.2^2$$

$$= 0.08 \text{ mA} = 80 \mu\text{A}$$

9.68 Refer to Fig. 9.30.

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m (\beta r_o \parallel R_L)$$

$$= -\frac{2.5}{2.5 + 0.05 + 5} 40 (100 \times 100 \parallel 2)$$

$$\simeq -26.5 \text{ (V/V)}$$

$$R'_{sig} = r_\pi \parallel (r_x + R_{sig})$$

$$= 2.5 \parallel (0.05 + 5) = 1.67 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{sig} = 1.67 \text{ k}\Omega$$

$$R_{c1} = r_{o1} \parallel \left[ r_{e2} \frac{r_{o2} + R_L}{r_{o2} + R_L / (\beta_2 + 1)} \right]$$

$$= 100 \parallel \left[ 0.025 \frac{100 + 2}{100 + \frac{2}{101}} \right]$$

$$= 25.5 \text{ }\Omega$$

$$R_{\mu 1} = R'_{sig} (1 + g_m R_{c1}) + R_{c1}$$

$$= 1.67(1 + 40 \times 0.0255) + 0.0255$$

$$= 3.4 \text{ k}\Omega$$

$$R_o = \beta_2 r_{o2} = 100 \times 100 = 10,000 \text{ k}\Omega$$

$$\tau_H = C_{\pi 1} R_{\pi 1} + C_{\mu 1} R_{\mu 1} + (C_{cs1} + C_{\pi 2}) R_{c1}$$

$$+ (C_L + C_{cs2} + C_{\mu 2})(R_L \parallel R_o)$$

$$= 10 \times 1.67 + 2 \times 3.4 + (0 + 10) \times 0.0255$$

$$+ (0 + 0 + 2)(2 \parallel 10,000)$$

$$= 16.7 + 6.8 + 0.255 + 4 = 27.8 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 27.8 \times 10^{-9}}$$

$$= 5.7 \text{ MHz}$$

9.69 (a) Gain from base to collector of  $Q_1 = -1$ .  
Thus,

$$C_{in} = C_{\pi 1} + C_{\mu 1}(1 + 1)$$

$$= C_{\pi 1} + 2C_{\mu 1}$$

$$f_{p1} = \frac{1}{2\pi R'_{sig} C_{in}}$$

$$= \frac{1}{2\pi R'_{sig} (C_{\pi 1} + 2C_{\mu 1})} \quad \text{Q.E.D.}$$

At the output node, the total capacitance is  $(C_L + C_{c2} + C_{\mu 2})$  and since  $r_o$  is large,  $R_o$  will be very large, thus the total resistance will be  $R_L$ . Thus the pole introduced at the output node will have a frequency  $f_{p2}$ ,

$$f_{p2} = \frac{1}{2\pi (C_L + C_{c2} + C_{\mu 2}) R_L} \quad \text{Q.E.D.}$$

$$(b) I = 1 \text{ mA}$$

$$g_m = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$(i) R_{sig} = 1 \text{ k}\Omega$$

$$R'_{sig} = r_\pi \parallel R_{sig} = 2.5 \parallel 1 = 0.71 \text{ k}\Omega$$

$$f_{p1} = \frac{1}{2\pi \times 0.71 \times 10^3 (10 + 2 \times 2) \times 10^{-12}}$$

$$= 16 \text{ MHz}$$

$$f_{p2} = \frac{1}{2\pi (0 + 0 + 2) \times 10^{-12} \times 2 \times 10^3}$$

$$\simeq 40 \text{ MHz}$$

$$f_H = 1 \sqrt{\frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2}}$$

$$= 1 \sqrt{\frac{1}{16^2} + \frac{1}{40^2}} = 14.9 \text{ MHz}$$

$$(ii) R_{sig} = 10 \text{ k}\Omega$$

$$R'_{sig} = r_\pi \parallel R_{sig} = 2.5 \parallel 10 = 2 \text{ k}\Omega$$

$$f_{p1} = \frac{1}{2\pi \times 2 \times 10^3 (10 + 4) \times 10^{-12}}$$

= 5.7 MHz

$f_{p2} = 40$  MHz

$$f_H = 1 / \sqrt{\frac{1}{5.7^2} + \frac{1}{40^2}} = 5.6 \text{ MHz}$$

9.70 Refer to Fig. 9.30.

$$I_C = 0.1 \text{ mA}$$

$$g_m = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_D} = \frac{100 \text{ V}}{0.1 \text{ mA}} = 1000 \text{ k}\Omega$$

$$r_e \simeq \frac{1}{g_m} = 0.25 \text{ k}\Omega$$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m (\beta r_o \parallel R_L)$$

$$= -\frac{r_\pi}{r_\pi + r_\pi} g_m (\beta r_o \parallel \beta r_o)$$

$$= -\frac{1}{4} \beta g_m r_o$$

$$= -\frac{1}{4} \times 100 \times 4 \times 1000$$

$$= -100,000 \text{ V/V}$$

$$R'_{sig} = r_\pi \parallel R_{sig} = r_\pi \parallel r_\pi = \frac{1}{2} r_\pi = 12.5 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{sig} = 12.5 \text{ k}\Omega$$

$$R_{c1} = r_o \parallel r_e \left( \frac{r_o + R_L}{r_o + \frac{R_L}{\beta + 1}} \right)$$

$$= 1000 \parallel 0.25 \left( \frac{r_o + \beta r_o}{r_o + \frac{\beta}{\beta + 1} r_o} \right)$$

$$\simeq 1000 \parallel 0.25 \times \frac{\beta + 1}{2}$$

$$= 1000 \parallel 12.5 = 12.3 \text{ k}\Omega$$

$$R_{\mu 1} = R'_{sig} (1 + g_{m1} R_{c1}) + R_{c1}$$

$$= 12.5 (1 + 4 \times 12.3) + 12.3 = 639.8 \text{ k}\Omega$$

$$R_o = \beta r_o = 100 \times 1000 = 100 \text{ M}\Omega$$

$$\begin{aligned} \tau_H &= C_{\pi 1} R_{\pi 1} + C_{\mu 1} R_{\mu 1} + (C_{cs1} + C_{\pi 2}) R_{c1} \\ &\quad + (C_L + C_{c2} + C_{\mu 2})(R_L \parallel R_o) \end{aligned}$$

To determine  $C_\pi$ , we use

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$1 \times 10^9 = \frac{4 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$\Rightarrow C_\pi + C_\mu = 0.64 \text{ pF}$$

$$C_\pi = 0.64 - 0.1 = 0.54 \text{ pF}$$

$$\tau_H = 0.54 \times 12.5 + 0.1 \times 639.8 + 0.54 \times 12.3$$

$$+ 0.1 \times \frac{1}{2} \times 100 \times 1000$$

$$= 6.8 + 64 + 6.6 + 5000 \text{ ns}$$

Obviously the last term, which is due to the pole at the output node, is dominant. The frequency of the output pole is

$$f_p = \frac{1}{2\pi \times 5000 \times 10^{-9}} = 31.8 \text{ kHz}$$

$$f_H \simeq f_p = 31.8 \text{ MHz}$$

Because the other poles are at much higher frequencies, an estimate of the unity-gain frequency can be found as

$$f_t = |A_M| f_p = 10^5 \times 31.8 \times 10^3 = 3.18 \text{ GHz}$$

This estimate of  $f_t$  is not very good (too high!). The other three poles have frequencies much lower than 3.18 GHz and will cause the gain to decrease faster, reaching the 0 dB value at a frequency lower than 3.18 GHz. Also note that  $f_T$  of the BJTs is 1 GHz and the models we use for the BJT do not hold at frequencies approaching  $f_T$ .

$$9.71 \quad A_M = \frac{R'_L}{R'_L + \frac{1}{g_m}}$$

where

$$R'_L = R_L \parallel r_o \parallel \frac{1}{g_{mb}}$$

$$= 2 \parallel 20 = 1.82 \text{ k}\Omega$$

$$A_M = \frac{1.82}{1.82 + \frac{1}{5}} = 0.91 \text{ V/V}$$

$$R_o = r_o \parallel \frac{1}{g_m} = 20 \parallel \frac{1}{5} \simeq 0.2 \text{ k}\Omega = 200 \text{ }\Omega$$

$$f_Z = \frac{g_m}{2\pi C_{gs}}$$

$$= \frac{5 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 398 \text{ MHz}$$

Next, we evaluate  $b_1$  and  $b_2$ :

$$\begin{aligned} b_1 &= \left( C_{gd} + \frac{C_{gs}}{g_m R'_L + 1} \right) R_{sig} + \left( \frac{C_{gs} + C_L}{g_m R'_L + 1} \right) R'_L \\ &= \left( 0.1 + \frac{2}{5 \times 1.82 + 1} \right) 20 + \left( \frac{2 + 1}{5 \times 1.82 + 1} \right) \times 1.82 \\ &= 5.96 + 0.54 = 6.50 \times 10^{-9} \text{ s} \\ b_2 &= \frac{(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}}{g_m R'_L + 1} R_{sig} R'_L \\ &= \frac{(2 + 0.1) \times 1 + 2 \times 0.1}{5 \times 1.82 + 1} \times 20 \times 1.82 \\ &= 8.3 \times 10^{-18} \\ Q &= \frac{\sqrt{b_2}}{b_1} = \frac{\sqrt{8.3}}{6.5} = 0.44 \end{aligned}$$

Thus, the poles are real and their frequencies can be obtained by finding the roots of the polynomial  $(1 + b_1 s + b_2 s^2)$

$$= 1 + 6.5 \times 10^{-9} s + 8.3 \times 10^{-18} s^2$$

which are

$$\omega_{p1} = 0.21 \times 10^9 \text{ rad/s}$$

and

$$\omega_{p2} = 0.57 \times 10^9 \text{ rad/s}$$

Thus,

$$f_{p1} = \frac{\omega_{p1}}{2\pi} = 33.4 \text{ MHz}$$

$$f_{p2} = \frac{\omega_{p2}}{2\pi} = 90.7 \text{ MHz}$$

Since the two poles are relatively close to each other, an estimate of  $f_H$  can be obtained using

$$\begin{aligned} f_H &= 1 / \sqrt{\frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2}} \\ &= 31.6 \text{ MHz} \end{aligned}$$

$$9.72 f_H \simeq f_{p1} \simeq \frac{1}{2\pi b_1}$$

where

$$b_1 = \left( C_{gd} + \frac{C_{gs}}{g_m R'_L + 1} \right) R_{sig} + \left( \frac{C_{gs} + C_L}{g_m R'_L + 1} \right) R'_L$$

For  $C_L = 0$ ,

$$b_1 = C_{gd} R_{sig} + \frac{C_{gs}}{g_m R'_L + 1} (R_{sig} + R'_L)$$

For  $R_{sig} \gg R'_L$ ,

$$\begin{aligned} b_1 &\simeq C_{gd} R_{sig} + \frac{C_{gs}}{g_m R'_L + 1} R_{sig} \\ &= \left( C_{gd} + \frac{C_{gs}}{g_m R'_L + 1} \right) R_{sig} \end{aligned}$$

and

$$f_H = \frac{1}{2\pi R_{sig} \left( C_{gd} + \frac{C_{gs}}{g_m R'_L + 1} \right)} \quad \text{Q.E.D.}$$

For the given numerical values,

$$\begin{aligned} f_H &= \frac{1}{2\pi \times 100 \times 10^3 \left[ 10 + \frac{2}{5 \times (2 \parallel 20) + 1} \right] \times 10^{-12}} \\ &= 156 \text{ kHz} \end{aligned}$$

(Note: An error was made in the first printing of the book and the values of  $C_{gs}$  and  $C_{gd}$  were exchanged. The above value of  $f_H$  corresponds to the numbers in the first printing.)

For  $C_{gs} = 10 \text{ pF}$  and  $C_{gd} = 2 \text{ pF}$ ,

$$\begin{aligned} f_H &= \frac{1}{2\pi \times 100 \times 10^3 \left[ 2 + \frac{10}{5 \times (2 \parallel 20) + 1} \right] \times 10^{-12}} \\ &= 532 \text{ kHz} \end{aligned}$$

9.73 Refer to Fig. 9.31(c). Replacing  $C_{gs}$  with an input capacitance between G and ground, we get

$$C_{eq} = C_{gs}(1 - K)$$

where

$$K = \frac{g_m R'_L}{1 + g_m R'_L}$$

then

$$C_{eq} = C_{gs} / (1 + g_m R'_L)$$

and the total input capacitance becomes

$$C_{in} = C_{gd} + C_{eq}$$

$$= C_{gd} + \frac{C_{gs}}{1 + g_m R'_L}$$

The frequency of the input pole is

$$f_{p1} = \frac{1}{2\pi R_{sig} \left( C_{gd} + \frac{C_{gs}}{1 + g_m R'_L} \right)}$$

$$f_H \simeq f_{p1}$$

9.74 With  $g_{mb} = 0$  and  $r_o$  large, we obtain

$$R'_L \simeq R_L$$

and

$$A_M = \frac{g_m R_L}{g_m R_L + 1}$$

For  $A_M = 0.9$ ,

$$0.9 = \frac{g_m R_L}{g_m R_L + 1}$$

$$\Rightarrow g_m R_L = 9$$

Now, for a maximally "at response,  $Q = 1/\sqrt{2}$ . Using the expression for  $Q$  in Eq. (9.129), we get

$$Q = \frac{\sqrt{g_m R_L + 1} \sqrt{[(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}]R_{sig}R_L}}{[C_{gs} + C_{gd}(g_m R_L + 1)]R_{sig} + (C_{gs} + C_L)R_L}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{9 + 1} \sqrt{[(10 + 1)10 + 10 \times 1] \times 100 \times R_L}}{[10 + 1(9 + 1)] \times 100 + (10 + 10)R_L}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{3}\sqrt{R_L}}{10\left(1 + \frac{R_L}{100}\right)}$$

$$\Rightarrow \left(\frac{R_L}{100}\right)^2 - 4\left(\frac{R_L}{100}\right) + 1 = 0$$

This equation results in two solutions,

$$R_L = 27 \text{ k}\Omega \text{ and } R_L = 373 \text{ k}\Omega$$

The second answer is not very practical as it implies the transistor is operating at  $g_m = 9/373 = 0.024 \text{ mA/V}$ , a very small transconductance!. We will pursue only the first answer. Thus,

$$R_L = 27 \text{ k}\Omega$$

$$g_m = 0.33 \text{ mA/V}$$

and the 3-dB frequency is found using Eq. (9.127):

$$f_{3dB} = f_0 = \frac{1}{2\pi\sqrt{b_2}}$$

$$\begin{aligned} \omega_{3dB} &= \sqrt{\frac{g_m R_L + 1}{R_{sig} R_L [(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}]}} \\ &= \sqrt{\frac{9 + 1}{100 \times 27 [(10 + 1) \times 10 + 10 \times 1] \times 10^6 \times 10^{-24}}} \end{aligned}$$

$$= 5.55 \text{ Mrad/s}$$

$$f_{3dB} = 884 \text{ kHz}$$

9.75 For a maximally "at response we have

$$Q = \frac{1}{\sqrt{2}}$$

$$\omega_0 = \omega_{3dB} = 2\pi \times 10^6 \text{ rad/s}$$

Thus, the transfer function will be

$$\begin{aligned} \frac{V_o(s)}{V_i(s)} &= \frac{\text{dc gain}}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \\ &= \frac{0.8}{s^2 + s \sqrt{2} \times 2\pi \times 10^6 + (2\pi \times 10^6)^2} \\ &= \frac{0.8}{s^2 + s 8.886 \times 10^6 s + 39.48 \times 10^{12}} \end{aligned}$$

9.76 Refer to Fig. 9.33.

$$I_C = 1 \text{ mA}$$

$$g_m = 40 \text{ mA/V}, \quad r_e = 25 \text{ }\Omega$$

$$r_\pi = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{20}{1} = 20 \text{ k}\Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$2 \times 10^9 = \frac{40 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$\Rightarrow C_\pi + C_\mu = 3.2 \text{ pF}$$

$$C_\pi = 3.2 - 0.1 = 3.1 \text{ pF}$$

$$R'_L = R_L \parallel r_o = 1 \parallel 20 = 0.95 \text{ k}\Omega$$

$$R'_{sig} = R_{sig} + r_x = 1 + 0.1 = 1.1 \text{ k}\Omega$$

$$A_M = \frac{R'_L}{R'_L + r_e + \frac{R'_{sig}}{\beta + 1}}$$

$$A_M = \frac{0.95}{0.95 + 0.025 + \frac{1.1}{101}}$$

$$= 0.96 \text{ V/V}$$

$$\begin{aligned} f_Z &= \frac{1}{2\pi C_\pi r_e} \\ &= \frac{1}{2\pi \times 3.1 \times 10^{-12} \times 25} \\ &\simeq 2 \text{ GHz} \end{aligned}$$

$$\begin{aligned} b_1 &= \frac{\left[C_\pi + C_\mu \left(1 + \frac{R'_L}{r_e}\right)\right] R'_{sig} + \left[C_\mu + C_L \left(1 + \frac{R'_{sig}}{r_\pi}\right)\right] R'_L}{1 + \frac{R'_L}{r_e} + \frac{R'_{sig}}{r_\pi}} \\ &= \frac{\left[3.1 + 0.1 \left(1 + \frac{0.95}{0.025}\right)\right] \times 1.1 + (3.1 + 0) \times 0.95}{1 + \frac{0.95}{0.025} + \frac{1.1}{2.5}} \end{aligned}$$

$$= 0.27 \times 10^{-9}$$

$$b_2 = \frac{[(C_\pi + C_\mu)C_L + C_\pi C_\mu]R'_L R'_\text{sig}}{1 + \frac{R'_L}{r_e} + \frac{R'_\text{sig}}{r_\pi}}$$

$$= \frac{(0 + 3.1 \times 0.1) \times 0.95 \times 1.1}{1 + \frac{0.95}{0.025} + \frac{1.1}{2.5}}$$

$$= 8.2 \times 10^{-21}$$

$$Q = \frac{\sqrt{b_2}}{b_1} = \frac{\sqrt{8.2 \times 10^{-21}}}{0.27 \times 10^{-9}} = 0.335$$

Thus, the poles are real and their frequencies can be found as the roots of the polynomial  $(1 + b_1 s + b_2 s^2)$

$$= 1 + 0.27 \times 10^{-9} s + 8.2 \times 10^{-21} s^2$$

$$= \left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right)$$

$$\Rightarrow \omega_{p1} = 4.25 \times 10^9 \text{ rad/s}$$

$$\omega_{p2} = 28.6 \times 10^9 \text{ rad/s}$$

Thus,

$$f_{p1} = 676 \text{ MHz}$$

$$f_{p2} = 4.6 \text{ GHz}$$

Thus,

$$f_{3dB} \simeq f_{p1} = 676 \text{ MHz}$$

$$9.77 \text{ I} = 0.4 \text{ mA}$$

$$(a) I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4 \times 16 V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.25 \text{ V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.25}$$

$$= 1.6 \text{ mA/V}$$

$$(b) r_o = \frac{V_A}{I_D} = \frac{20}{0.2}$$

$$= 100 \text{ k}\Omega$$

$$R_D \parallel r_o = 10 \parallel 100$$

$$= 9.1 \text{ k}\Omega$$

$$A_d = g_m (R_D \parallel r_o)$$

$$= 1.6 \times 9.1 = 14.5 \text{ V/V}$$

(c)  $R_{sig}$  small and the frequency response is determined by the output pole:

$$f_{p2} = \frac{1}{2\pi(C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$$

$$= \frac{1}{2\pi(100 + 5 + 5) \times 10^{-15} \times 9.1 \times 10^3}$$

$$= 159 \text{ MHz}$$

$$f_H \simeq 159 \text{ MHz}$$

$$(d) R_{sig} = 40 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs} R_{gs}$$

$$= C_{gs} R_{sig}$$

$$= 40 \times 10^{-15} \times 40 \times 10^3$$

$$= 1.6 \text{ ns}$$

$$R_{gd} = R_{sig}(g_m R'_L + 1) + R'_L$$

$$= 40(1.6 \times 9.1 + 1) + 9.1$$

$$= 631.5 \text{ k}\Omega$$

$$\tau_{gd} = C_{gd} R_{gd} = 5 \times 631.5 = 3.16 \text{ ns}$$

$$\tau_{CL} = (C_L + C_{db}) R'_L$$

$$= (100 + 5) \times 9.1$$

$$= 955.5 \text{ ps} = 0.96 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 1.6 + 3.16 + 0.96 = 5.72 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 5.72 \times 10^{-9}}$$

$$= 27.8 \text{ MHz}$$

9.78 The common-mode gain will have a zero at

$$f_Z = \frac{1}{2\pi R_{SS} C_{SS}}$$

$$= \frac{1}{2\pi \times 100 \times 10^3 \times 1 \times 10^{-12}}$$

$$= 1.59 \text{ MHz}$$

Thus, the CMRR will have two poles, one at  $f_Z$ , i.e. at 1.59 MHz, and the other at the dominant

pole of  $A_d$ , 20 MHz. Thus, the 3-dB frequency of CMRR will be approximately equal to  $f_z$ ,

$$f_{3dB} = 1.59 \text{ MHz}$$

9.79 At low frequencies,

$$A_d = 100 \text{ V/V}$$

$$A_{cm} = 0.1 \text{ V/V}$$

$$\text{CMRR} = \frac{A_d}{A_{cm}} = 1000 \text{ or } 60 \text{ dB}$$

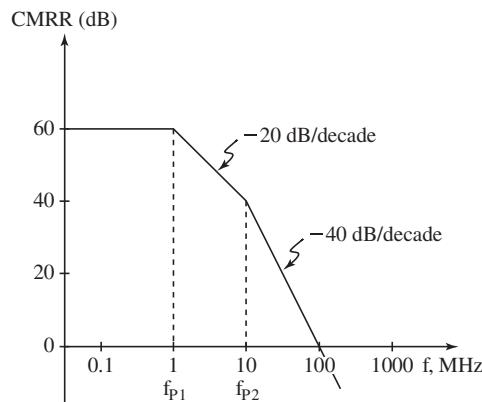
The first pole of CMRR is coincident with the zero of the common-mode gain,

$$f_{p1} = 1 \text{ MHz}$$

The second pole is coincident with the dominant pole of the differential gain,

$$f_{p2} = 10 \text{ MHz}$$

A sketch for the Bode plot for the gain magnitude is shown in the figure.



$$9.80 R_{ss} = \frac{V_A}{I} = \frac{40}{0.1} = 400 \text{ k}\Omega$$

$$C_{ss} = 100 \text{ fF}$$

$$f_z = \frac{1}{2\pi C_{ss} R_{ss}}$$

$$= \frac{1}{2\pi \times 100 \times 10^{-15} \times 400 \times 10^3} = 4 \text{ MHz}$$

If  $V_{ov}$  of the current source is reduced by a factor of 2 while  $I$  remains unchanged, ( $W/L$ ) must be increased by a factor of 4. Assume  $L$  remains unchanged,  $W$  must be increased by a factor of 4.

Since  $C_{ss}$  is proportional to  $W$ , its value will be quadrupled:

$$C_{ss} = 400 \text{ fF}$$

The output resistance  $R_{ss}$  will remain unchanged. Thus,  $f_z$  will decrease by a factor of 4 to become

$$f_z = 1 \text{ MHz}$$

$$9.81 g_m = \frac{I_C}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{10} = 10 \text{ k}\Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

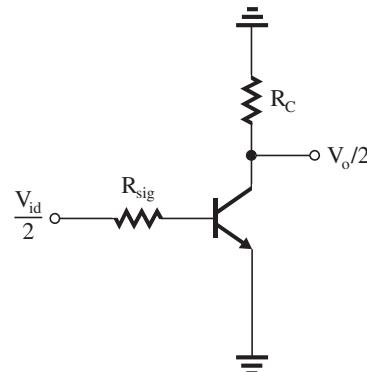
$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{10 \times 10^{-3}}{2\pi \times 500 \times 10^6}$$

$$= 3.2 \text{ pF}$$

$$C_\pi = 3.2 - 0.5 = 2.7 \text{ pF}$$

(a)



The figure on the next page shows the differential half-circuit and its high-frequency equivalent circuit.

$$(b) A_d \equiv \frac{V_o}{V_{id}} = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m R_C$$

$$= -\frac{10}{10 + 0.1 + 10} \times 10 \times 10$$

$$= -49.8 \text{ V/V}$$

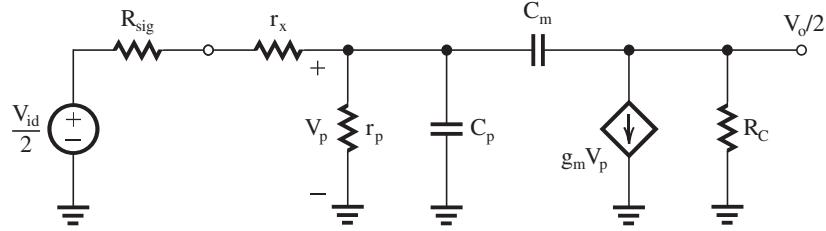
$$(c) C_{in} = C_\pi + C_\mu (1 + g_m R_C)$$

$$= 2.7 + 0.5(1 + 10 \times 10)$$

$$= 53.2 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

This figure belongs to Problem 9.81.



where

$$R'_\text{sig} = r_\pi \parallel (R_\text{sig} + r_x)$$

$$= 10 \parallel 10.1 \simeq 5 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi \times 53.2 \times 10^{-12} \times 5 \times 10^3} = 598 \text{ kHz}$$

$$\text{GB} = |A_d| f_H = 49.8 \times 598 = 29.8 \text{ MHz}$$

$$9.82 \quad g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}} = \frac{I}{V_{OV1,2}}$$

$$= \frac{0.2 \text{ mA}}{0.2 \text{ V}} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I_D} = \frac{10 \text{ V}}{0.1 \text{ mA}} = 100 \text{ k}\Omega$$

$$A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$$

$$= 1(100 \parallel 100) = 50 \text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi C_L R_o}$$

$$= \frac{1}{2\pi C_L (r_{o2} \parallel r_{o4})}$$

$$= \frac{1}{2\pi \times 0.2 \times 10^{-12} (100 \parallel 100) \times 10^3}$$

$$= \frac{1}{2\pi \times 0.2 \times 50 \times 10^{-9}} = 15.9 \text{ MHz}$$

$$f_{P2} = \frac{g_{m3}}{2\pi C_m}$$

where

$$g_{m3} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|} = 1 \text{ mA/V}$$

$$f_{P2} = \frac{1 \times 10^{-3}}{2\pi \times 0.1 \times 10^{-12}} = 1.59 \text{ GHz}$$

$$f_Z = \frac{2g_{m3}}{2\pi C_m} = 2f_{P2} = 2 \times 1.59 = 3.18 \text{ GHz}$$

$$9.83 \quad A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$$

$$g_{m1,2} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I}$$

$$A_d = \frac{I}{|V_{OV}|} \left( \frac{2|V_A|}{I} \parallel \frac{2|V_A|}{I} \right)$$

$$A_d = \frac{|V_A|}{|V_{OV}|}$$

$$f_{P1} = \frac{1}{2\pi C_L R_o}$$

where

$$R_o = r_{o2} \parallel r_{o4} = \frac{|V_A|}{I}$$

$$f_{P1} = \frac{I}{2\pi C_L |V_A|} \quad (1)$$

$$f_{P2} = \frac{g_{m3}}{2\pi C_m}$$

where

$$g_{m3} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|}$$

$$C_m = \frac{C_L}{4}$$

$$f_{P2} = \frac{4I}{2\pi C_L |V_{OV}|} \quad (2)$$

$$f_Z = \frac{2g_{m3}}{2\pi C_m} = 2f_{P2} = \frac{8I}{2\pi C_L |V_{OV}|} \quad (3)$$

Dividing (2) by (1), we obtain

$$\frac{f_{P2}}{f_{P1}} = 4 \frac{|V_A|}{|V_{OV}|} = 4 A_d \quad \text{Q.E.D.}$$

Since  $f_{P2} = 4 A_d f_{P1}$ , the unity-gain frequency  $f_t$  is equal to GB, thus

$$f_t = A_d f_{P1}$$

$$= \frac{|V_A|}{|V_{OV}|} \frac{I}{2\pi C_L |V_A|}$$

$$f_t = \frac{I/|V_{OV}|}{2\pi C_L}$$

$$= \frac{g_m}{2\pi C_L} \quad \text{Q.E.D.}$$

For the numerical values given, we have

$$A_d = \frac{20}{0.2} = 100 \text{ V/V}$$

$$g_m = \frac{I}{V_{ov}} = \frac{0.2}{0.2} = 1 \text{ mA/V}$$

$$f_{p1} = \frac{I}{2\pi C_L |V_A|}$$

$$= \frac{0.2 \times 10^{-3}}{2\pi \times 100 \times 10^{-15} \times 20} \\ = 15.9 \text{ MHz}$$

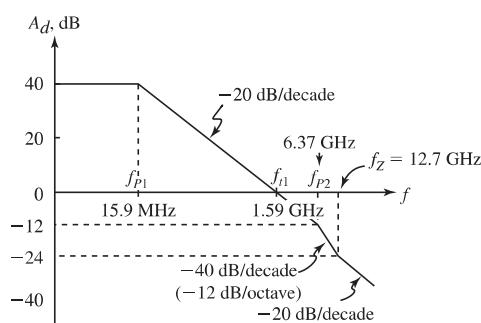
$$f_t = 15.9 \times 100 = 1.59 \text{ GHz}$$

$$f_{p2} = 4A_d f_{p1} = 4 \times 100 \times 15.9$$

$$= 6.37 \text{ GHz}$$

$$f_z = 2f_{p2} = 12.7 \text{ GHz}$$

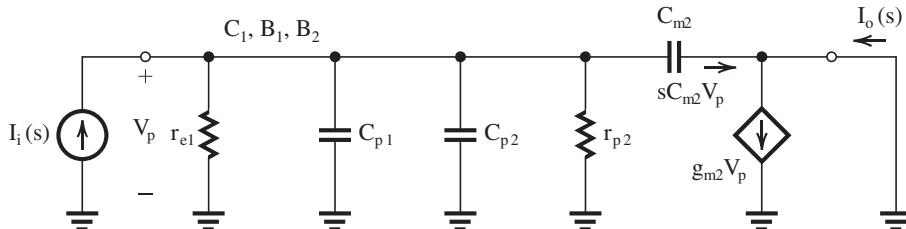
A sketch of the Bode plot for  $|A_d|$  is shown in the figure.



9.84 See figure below. The mirror high-frequency equivalent circuit is shown in the figure. Note that we have neglected  $r_x$  and  $r_o$ . The model of the diode-connected transistor  $Q_1$  reduces to  $r_{e1}$  in parallel with  $C_{\pi 1}$ .

To obtain the current-transfer function  $I_o(s)/I_i(s)$ , we first determine  $V_\pi$  in terms of  $I_i$ . Observe that

This figure belongs to Problem 9.84.



the short-circuit at the output causes  $C_{\mu 2}$  to appear in parallel with  $C_{\pi 1}$  and  $C_{\pi 2}$ . Thus,

$$V_\pi = \frac{1}{I_i(s) \left( \frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}} \right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})} \quad (1)$$

At the output node we have

$$I_o(s) = g_{m2}V_\pi - sC_{\mu 2}V_\pi \quad (2)$$

Combining Eqs. (1) and (2) gives

$$\frac{I_o(s)}{I_i(s)} = \frac{g_{m2} - sC_{\mu 2}}{\left( \frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}} \right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})}$$

Since the two transistors are operating at approximately equal dc bias currents, their small-signal parameters will be equal, thus

$$\begin{aligned} \frac{I_o(s)}{I_i(s)} &= \frac{g_m - sC_\mu}{\frac{1}{r_e} \left( 1 + \frac{1}{\beta + 1} \right) + s(2C_\pi + C_\mu)} \\ &= \frac{\frac{g_m r_e}{1 + \frac{1}{\beta + 1}}}{1 + s \left[ (2C_\pi + C_\mu) r_e / \left( 1 + \frac{1}{\beta + 1} \right) \right]} \\ &= \frac{\alpha}{1 + \frac{1}{\beta + 1}} \frac{1 - s(C_\mu / g_m)}{1 + s \left[ (2C_\pi + C_\mu) r_e / \left( 1 + \frac{1}{\beta + 1} \right) \right]} \\ &= \frac{1}{1 + 2/\beta} \frac{1 - s(C_\mu / g_m)}{1 + s \left[ (2C_\pi + C_\mu) r_e / \left( 1 + \frac{1}{\beta + 1} \right) \right]} \end{aligned}$$

Thus we see that the low-frequency transmission is

$$\frac{I_o}{I_i}(0) = \frac{1}{1 + \frac{2}{\beta}}$$

as expected. The pole is at  $f_p$ ,

$$f_p \approx \frac{1}{2\pi \left[ (2C_\pi + C_\mu) r_e / \left( 1 + \frac{1}{\beta} \right) \right]}$$

and the zero is at

$$f_Z = \frac{g_m}{2\pi C_\mu}$$

For the numerical values given,

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_e \simeq 25 \Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 100^6}$$

$$= 12.7 \text{ pF}$$

$$C_\pi = 12.7 - 2 = 10.7 \text{ pF}$$

$$f_P = \frac{1}{2\pi[(2 \times 10.7 + 2) \times 10^{-12} \times 25/1.01]}$$

$$= \frac{1.01 \times 10^{12}}{2\pi \times 23.4 \times 25}$$

$$= 274.8 \text{ MHz}$$

$$f_Z = \frac{g_m}{2\pi C_\mu}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 3.18 \text{ GHz}$$

9.85 Refer to Eqs. (9.146)–(9.153). For our case,

$$G_m = \frac{g_m}{1 + g_m R_s} \quad (1)$$

$R_o$  = very large

$$R'_L = R_L \parallel R_o = R_L$$

$$A_m = -G_m R_L = \frac{-g_m R_L}{1 + g_m R_s} \quad (2)$$

$$R_{gd} = R_{sig}(1 + G_m R_L) + R_L \quad (3)$$

$$R_{gs} = \frac{R_{sig} + R_s}{1 + g_m R_s} \quad (4)$$

$$\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

For the numerical values given:

$$(a) R_s = 0$$

$$G_m = g_m = 5 \text{ mA/V}$$

$$A_M = -g_m R_L = -5 \times 5 = -25 \text{ V/V}$$

$$R_{gd} = 100(1 + 5 \times 5) + 5 = 2605 \text{ k}\Omega$$

$$\tau_H = 10 \times 100 + 2 \times 2605 = 6.21 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 6.21 \times 10^{-9}} = 26.6 \text{ MHz}$$

$$GB = 26.6 \times 25 = 641 \text{ MHz}$$

$$(b) R_s = 100 \Omega$$

$$G_m = \frac{5}{1 + 5 \times 0.1} = 3.33 \text{ mA/V}$$

$$A_M = -3.33 \times 5 = -16.7 \text{ V/V}$$

$$R_{gd} = 100(1 + 3.33 \times 5) + 5$$

$$= 1771.7 \text{ k}\Omega$$

$$R_{gs} = \frac{100 + 0.1}{1 + 5 \times 0.1} = 66.7 \text{ k}\Omega$$

$$\tau_H = 10 \times 66.7 + 2 \times 1771.7 = 4.21 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 4.21 \times 10^{-9}} = 37.8 \text{ MHz}$$

$$GB = 631 \text{ MHz}$$

$$(c) R_s = 200 \Omega$$

$$G_m = \frac{5}{1 + 5 \times 0.2} = 2.5 \text{ mA/V}$$

$$A_M = -2.5 \times 5 = -12.5 \text{ V/V}$$

$$R_{gd} = 100(1 + 2.5 \times 5) + 5 = 1355 \text{ k}\Omega$$

$$R_{gs} = \frac{100 + 0.2}{1 + 5 \times 0.2} = 50.1 \text{ k}\Omega$$

$$\tau_H = 10 \times 50.1 + 2 \times 1355 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 3.21 \times 10^{-9}} = 49.6 \text{ MHz}$$

$$GB = 49.6 \times 12.5 = 620 \text{ MHz}$$

A summary of the results is provided in the following table:

	$R_s = 0$	$R_s = 100 \Omega$	$R_s = 200 \Omega$
$ A_M  (\text{V/V})$	25	16.7	12.5
$f_H$ (MHz)	26.6	37.8	49.6
GB (MHz)	641	631	620

Observe that increasing  $R_s$  trades off gain for bandwidth while GB remains approximately constant.

$$9.86 (a) GB = |A_M| f_H$$

$$= \frac{1}{2\pi C_{gd} R_{sig}}$$

$$= \frac{1}{2\pi \times 0.2 \times 10^{-12} \times 100 \times 10^3}$$

$$= 7.96 \text{ MHz}$$

$$(b) |A_M| = 20 \text{ V/V}$$

$$f_H = \frac{7.96}{20} = 398 \text{ kHz}$$

$$(c) A_0 = g_m r_o$$

$$100 = 5 \times r_o$$

$$\Rightarrow r_o = 20 \text{ k}\Omega$$

$$G_m = \frac{g_m}{1 + g_m R_s} = \frac{5}{1 + g_m R_s}$$

$$R_o = r_o(1 + g_m R_s) = 20(1 + g_m R_s)$$

$$R'_L = R_L \parallel R_o = 20 \parallel 20(1 + g_m R_s)$$

$$A_M = -G_m R'_L$$

$$20 = \frac{5}{1 + g_m R_s} [20 \parallel 20(1 + g_m R_s)]$$

$$4(1 + g_m R_s) = \frac{20 \times 20(1 + g_m R_s)}{20 + 20(1 + g_m R_s)}$$

$$\Rightarrow 1 + g_m R_s = 4$$

$$\Rightarrow R_s = \frac{3}{g_m} = 0.6 \text{ k}\Omega = 600 \text{ }\Omega$$

$$9.87 \text{ (a)} A_M = -g_m R'_L$$

where

$$R'_L = R_L \parallel r_o$$

$$= 40 \parallel 20 = 20 \text{ k}\Omega$$

$$A_M = -5 \times 20 = -100 \text{ V/V}$$

$$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{sig}$$

$$= 2 \times 20 = 40 \text{ ns}$$

$$R_{gd} = R_{sig}(1 + g_m R'_L) + R'_L$$

$$= 20(1 + 5 \times 20) + 20$$

$$= 2040 \text{ k}\Omega$$

$$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 2040$$

$$= 204 \text{ ns}$$

$$\tau_{CL} = C_L R'_L$$

$$= 1 \times 20 = 20 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 40 + 204 + 20 = 264 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 264 \times 10^{-9}} = 603 \text{ kHz}$$

$$\text{GB} = 100 \times 603 = 60.3 \text{ MHz}$$

(b) With  $R_s = 400 \text{ }\Omega$ ,

$$G_m = \frac{g_m}{1 + g_m R_s}$$

$$= \frac{5}{1 + 5 \times 0.4} = 1.67 \text{ mA/V}$$

$$R_o = r_o(1 + g_m R_s)$$

$$= 40(1 + 5 \times 0.4) = 120 \text{ k}\Omega$$

$$R'_L = R_L \parallel R_o = 40 \parallel 120 = 30 \text{ k}\Omega$$

$$A_M = -G_m R'_L$$

$$= -1.67 \times 30 = -50 \text{ V/V}$$

$$R_{gd} = R_{sig}(1 + G_m R'_L) + R'_L$$

$$= 20(1 + 1.67 \times 30) + 30$$

$$= 1050 \text{ k}\Omega$$

$$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 1050 = 105 \text{ ns}$$

$$\tau_{CL} = C_L R_{CL}$$

$$= C_L R'_L$$

$$= 1 \times 30 = 30 \text{ ns}$$

$$R_{gs} = \frac{R_{sig} + R_s + R_{sig} R_s / (r_o + R_L)}{1 + g_m R_s \left( \frac{r_o}{r_o + R_L} \right)}$$

$$= \frac{20 + 0.4 + 20 \times 0.4 / (40 + 40)}{1 + 5 \times 0.4 \left( \frac{40}{40 + 40} \right)}$$

$$= 10.25 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs} R_{gs} = 2 \times 10.25 = 20.5 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 20.5 + 105 + 30 = 155.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 155.5 \times 10^{-9}} = 1.02 \text{ MHz}$$

$$\text{GB} = 51.2 \text{ MHz}$$

$$9.88 \text{ } G_m = \frac{g_m}{1 + g_m R_s} = \frac{g_m}{1 + k}$$

$$R'_L = R_L \parallel R_o$$

$$= r_o \parallel r_o(1 + g_m R_s)$$

$$= r_o \parallel r_o(1 + k)$$

$$= \frac{r_o \times r_o(1 + k)}{r_o + r_o(1 + k)}$$

$$= r_o \frac{1 + k}{2 + k}$$

$$A_M = -G_m R'_L = -\frac{g_m r_o}{2 + k}$$

Thus,

$$A_M = \frac{-A_0}{2 + k} \quad \text{Q.E.D.}$$

$$R_{gs} = \frac{R_{sig} + R_s + R_{sig}R_s/(r_o + R_L)}{1 + g_m R_s \left( \frac{r_o}{r_o + R_L} \right)}$$

$$= \frac{R_{sig} + R_s + R_{sig}R_s/2r_o}{1 + \frac{1}{2}g_m R_s}$$

For  $R_{sig} \gg R_s$ ,

$$R_{gs} \simeq \frac{R_{sig}(1 + R_s/2r_o)}{1 + (k/2)}$$

For  $r_o \gg R_s$ ,

$$R_{gs} \simeq \frac{R_{sig}}{1 + (k/2)}$$

$$\tau_{gs} = C_{gs} R_{gs} = \frac{C_{gs} R_{sig}}{1 + (k/2)}$$

$$R_{gd} = R_{sig}(1 + G_m R'_L) + R_L$$

Utilizing the expressions for  $R'_L$  and  $G_m R'_L$  derived earlier, we obtain

$$R_{gd} = R_{sig} \left[ 1 + \frac{A_0}{2+k} \right] + r_o \left( \frac{1+k}{2+k} \right)$$

$$\begin{aligned} \tau_{gd} &= C_{gs} R_{gd} = \\ &C_{gd} R_{sig} \left( 1 + \frac{A_0}{2+k} \right) + C_{gd} r_o \left( \frac{1+k}{2+k} \right) \end{aligned}$$

$$\tau_{CL} = C_L R'_L$$

$$= C_L r_o \frac{1+k}{2+k}$$

Thus,

$$\begin{aligned} \tau_H &= \tau_{gs} + \tau_{gd} + \tau_{CL} \\ &= \frac{C_{gs} R_{sig}}{1 + (k/2)} + C_{gd} R_{sig} \left( 1 + \frac{A_0}{2+k} \right) \\ &\quad + C_{gd} r_o \left( \frac{1+k}{2+k} \right) + C_L r_o \left( \frac{1+k}{2+k} \right) \\ &= \frac{C_{gs} R_{sig}}{1 + (k/2)} + C_{gd} R_{sig} \left( 1 + \frac{A_0}{2+k} \right) \\ &\quad + (C_L + C_{gd}) r_o \left( \frac{1+k}{2+k} \right) \quad \text{Q.E.D.} \end{aligned}$$

9.89 Substituting the given numerical values in the expressions for  $A_M$  and  $\tau_H$  given in the statement for Problem 9.88 and noting that  $A_0 = g_m r_o = 5 \times 40 = 200$ , we obtain

$$f_H = \frac{1}{2\pi \tau_H}$$

and

$$GB = |A_M| f_H$$

The results we obtain appear in the following table.

k	A <sub>M</sub>  , V/V	$\tau_H$ ns	f <sub>H</sub> (MHz)	GB (MHz)
0	100	264	0.603	60.3
1	66.7	191.3	0.832	55.6
2	50	155	1.03	51.5
3	40	133.2	1.19	47.6
4	33.3	118.7	1.34	44.6
5	28.6	108.3	1.47	42.0
6	25	100.5	1.58	39.5
7	22.2	94.4	1.69	37.5
8	20	89.6	1.78	35.6
9	18.2	85.7	1.86	33.9
10	16.7	82.3	1.93	32.2
11	15.4	79.6	2.00	30.8
12	14.3	77.2	2.06	29.5
13	13.3	75.1	2.12	28.2
14	12.5	73.3	2.17	27.1
15	11.8	71.6	2.22	26.2

To obtain f<sub>H</sub> = 2 MHz, we see from the table that

$$k = 11$$

Thus,

$$1 + g_m R_s = 11$$

$$\Rightarrow R_s = \frac{10}{2} = 5 \text{ k}\Omega$$

The gain achieved is

$$|A_M| = 15.4 \text{ V/V}$$

9.90 (a) Refer to Fig. P9.90(a). Since the total resistance at the drain is r<sub>o</sub>, we have

$$A_M = -g_m r_o \quad \text{Q.E.D.}$$

$$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{sig}$$

$$R_{gd} = R_{sig}(1 + g_m R'_L) + R'_L$$

$$= R_{sig}(1 + g_m r_o) + r_o$$

$$\tau_{gd} = C_{gd} R_{gd} = C_{gd} [R_{sig}(1 + g_m r_o) + r_o]$$

$$\tau_{CL} = C_L R'_L = C_L r_o$$

Thus,

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= C_{gs} R_{sig} + C_{gd} [R_{sig}(1 + g_m r_o) + r_o]$$

$$+ C_L r_o \quad \text{Q.E.D.}$$

This figure belongs to Problem 9.90, part (b).

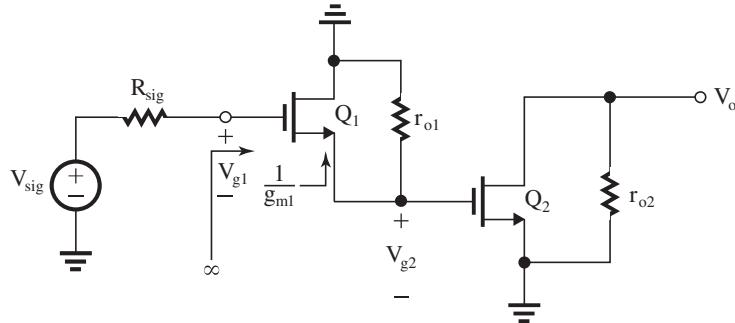


Figure 1

For the given numerical values,

$$A_M = -1 \times 20 = -20 \text{ V/V}$$

$$\begin{aligned} \tau_H &= 20 \times 20 + 5[20(1 + 1 \times 20) + 20] + 10 \times 20 \\ &= 400 + 2200 + 200 = 2800 \text{ ps} = 2.8 \text{ ns} \end{aligned}$$

$$\begin{aligned} f_H &= \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 2.8 \times 10^{-9}} \\ &= 56.8 \text{ MHz} \end{aligned}$$

$$GB = 20 \times 56.8 = 1.14 \text{ GHz}$$

(b) From Fig. 1 we see that

$$\frac{V_{g1}}{V_{sig}} = 1$$

$$\frac{V_{g2}}{V_{g1}} = \frac{r_{o1}}{\frac{1}{g_{m1}} + r_{o1}}$$

$$\frac{V_o}{V_{g2}} = -g_{m2}r_{o2}$$

Thus,

$$\begin{aligned} A_M &= 1 \times \frac{r_{o1}}{\frac{1}{g_{m1}} + r_{o1}} \times -g_{m2}r_{o2} \\ &= -\frac{r_{o1}}{1/g_{m1} + r_{o1}} (g_{m2}r_{o2}) \quad \text{Q.E.D.} \end{aligned}$$

Next we evaluate the open-circuit time constants.  
Refer to Fig. 2.

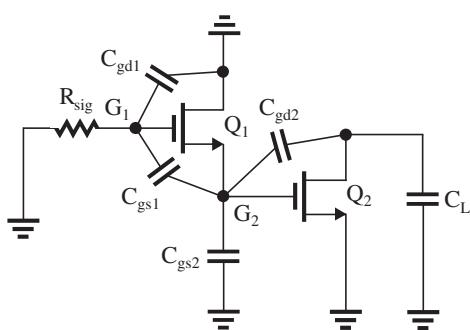


Figure 2

\$C\_{gd1}\$: Capacitor \$C\_{gd1}\$ is between \$G\_1\$ and ground and thus sees the resistance \$R\_{sig}\$,

$$R_{gd1} = R_{sig}$$

$$\tau_{gd1} = C_{gd1}R_{sig}$$

\$C\_{gs1}\$: To find the resistance \$R\_{gs1}\$ seen by capacitor \$C\_{gs1}\$, we replace \$Q\_1\$ with its hybrid-\$\pi\$ equivalent circuit with \$V\_{sig}\$ set to zero, \$C\_{gd1} = 0\$, and \$C\_{gs1}\$ replaced by a test voltage \$V\_x\$. The resulting equivalent circuit is shown in Fig. 3.

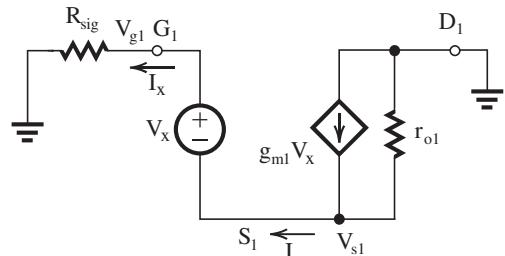


Figure 3

Analysis of the circuit in Fig. 3 proceeds as follows:

$$V_{g1} = I_x R_{sig}$$

$$V_{s1} = V_{g1} - V_x = I_x R_{sig} - V_x$$

Node equation at \$S\_1\$,

$$\begin{aligned} I_x &= g_{m1}V_x - \frac{V_{s1}}{r_{o1}} \\ &= g_{m1}V_x - \frac{I_x R_{sig} - V_x}{r_{o1}} \\ I_x \left( 1 + \frac{R_{sig}}{r_{o1}} \right) &= V_x \left( g_{m1} + \frac{1}{r_{o1}} \right) \end{aligned}$$

Thus,

$$R_{gs1} \equiv \frac{V_x}{I_x} = \frac{R_{sig} + r_{o1}}{1 + g_{m1}r_{o1}}$$

$$\tau_{gs1} = C_{gs1}R_{gs1}$$

$$= C_{gs1} \frac{R_{sig} + r_{o1}}{1 + g_{m1}r_{o1}}$$

$C_{gs2}$ : Capacitor  $C_{gs2}$  sees the resistance between  $G_2$  and ground, which is the output resistance of source follower  $Q_1$ ,

$$R_{gs2} = \frac{1}{g_{m1}} \parallel r_{o1}$$

Thus,

$$\tau_{gs2} = C_{gs2} \left( \frac{1}{g_{m1}} \parallel r_{o1} \right)$$

$C_{gd2}$ : Transistor  $Q_2$  operates as a CS amplifier with an equivalent signal-source resistance equal to the output resistance of the source follower  $Q_1$ , that is,  $\left( \frac{1}{g_{m1}} \parallel r_{o1} \right)$  and with a gain from gate to drain of  $g_{m2}r_{o2}$ . Thus, the formula for  $R_{gd}$  in a CS amplifier can be adapted as follows:

$$R_{gd2} = \left( \frac{1}{g_{m1}} \parallel r_{o1} \right) (1 + g_{m2}r_{o2}) + r_{o2}$$

and thus,

$$\tau_{gd2} = C_{gd2} \left[ \left( \frac{1}{g_{m1}} \parallel r_{o1} \right) (1 + g_{m2}r_{o2}) + r_{o2} \right]$$

$C_L$ : Capacitor  $C_L$  sees the resistance between  $D_2$ , and ground which is  $r_{o2}$ ,

$$\tau_{CL} = C_L r_{o2}$$

Summing  $\tau_{gd1}$ ,  $\tau_{gs1}$ ,  $\tau_{gs2}$ ,  $\tau_{gd2}$  and  $\tau_{CL}$  gives  $\tau_H$  in the problem statement. Q.E.D.

For the given numerical values:

$$A_M = -\frac{20}{1+20}(1 \times 20)$$

$$= -19 \text{ V/V}$$

$$\tau_{gd1} = C_{gd1} R_{sig} = 5 \times 20 = 100 \text{ ps}$$

$$\begin{aligned} \tau_{gs1} &= C_{gs1} \frac{R_{sig} + r_{o1}}{1 + g_{m1}r_{o1}} \\ &= 20 \frac{20 + 20}{1 + 1 \times 20} = 38 \text{ ps} \end{aligned}$$

$$\tau_{gs2} = C_{gs2} \left( \frac{1}{g_{m1}} \parallel r_{o1} \right)$$

$$= 20 \times (1 \parallel 20) = 19 \text{ ps}$$

$$\tau_{gd2} = C_{gd2} \left[ \left( \frac{1}{g_{m1}} \parallel r_{o1} \right) (1 + g_{m2}r_{o2}) + r_{o2} \right]$$

$$= 5[(1 \parallel 20)(1 + 20) + 20]$$

$$= 200 \text{ ps}$$

$$\tau_{CL} = C_L r_{o2} = 10 \times 20 = 200 \text{ ps}$$

$$\tau_H = 100 + 38 + 19 + 200 + 200 = 557 \text{ ps}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 557 \times 10^{-12}}$$

$$= 286 \text{ MHz}$$

$$GB = 19 \times 286 = 5.43 \text{ GHz}$$

Thus, while the dc gain remained approximately the same both  $f_H$  and GB increased by a factor of about 5!

9.91 At an emitter bias current of 0.1 mA,  $Q_1$  and  $Q_2$  have

$$g_m = 4 \text{ mA/V}$$

$$r_e = 250 \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.1} = 1000 \text{ k}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{4 \times 10^{-3}}{2\pi \times 200 \times 10^6} = 3.2 \text{ pF}$$

$$C_\mu = 0.2 \text{ pF}$$

$$C_\pi = 3 \text{ pF}$$

To determine  $R_{in}$  and the voltage gain  $A_M$ , refer to the circuit in Fig. 9.40(a). Here, however,  $R_L$  is  $r_{o2}$ .

$$R_{in2} = r_\pi = 25 \text{ k}\Omega$$

$$R_{in} = (\beta_1 + 1)[r_{e1} + (r_{o1} \parallel R_{in2})]$$

$$= 101[0.25 + (1000 \parallel 25)]$$

$$\simeq 2.5 \text{ M}\Omega$$

$$\frac{V_{b1}}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{2.5 \text{ M}\Omega}{2.5 \text{ M}\Omega + 10 \text{ k}\Omega} \simeq 1 \text{ V/V}$$

$$\frac{V_{b2}}{V_{b1}} = \frac{(R_{in2} \parallel r_{o1})}{(R_{in2} \parallel r_{o1}) + r_{e1}}$$

$$= \frac{25 \parallel 1000}{(25 \parallel 1000) + 0.25} = 0.99 \simeq 1 \text{ V/V}$$

$$\frac{V_o}{V_{b2}} = -g_{m2}r_{o2} = -4 \times 1000 = -4000 \text{ V/V}$$

Thus,

$$A_M = \frac{V_o}{V_{sig}} = -4000 \text{ V/V}$$

To determine  $f_H$  we use the method of open-circuit time constants. Figure 9.40(b) shows the circuit with  $V_{sig} = 0$  and the four capacitances indicated. Again, recall that here  $R_L = r_{o2}$ . Also, in our present circuit there is a capacitance  $C_L$  at the output.

Capacitance  $C_{\mu 1}$  sees a resistance  $R_{\mu 1}$ ,

$$R_{\mu 1} = R_{sig} \parallel R_{in}$$

$$= 10 \text{ k}\Omega \parallel 2.5 \text{ M}\Omega \simeq 10 \text{ k}\Omega$$

To find the resistance  $R_{\pi 1}$  we refer to the circuit in Fig. 9.40(c) where  $R_{in2}$  is considered to include  $r_{o2}$ ,

$$R_{in2} = 25 \text{ k}\Omega \parallel 1000 \text{ k}\Omega = 24.4 \text{ k}\Omega$$

We use the formula for  $R_{\pi 1}$  given in Example 9.13:

$$R_{\pi 1} = \frac{R_{sig} + R_{in2}}{1 + \frac{R_{sig}}{r_{\pi 1}} + \frac{R_{in2}}{r_{e1}}}$$

$$R_{\pi 1} = \frac{10 + 24.4}{1 + \frac{10}{25} + \frac{24.4}{0.25}} = 347 \text{ }\Omega$$

Capacitance  $C_{\pi 2}$  sees a resistance  $R_{\pi 2}$ :

$$R_{\pi 2} = R_{in2} \parallel R_{out1}$$

$$= r_{\pi 2} \parallel r_{o1} \parallel \left[ r_{e1} + \frac{R_{sig}}{\beta_1 + 1} \right]$$

$$= 25 \parallel 1000 \parallel \left[ 0.25 + \frac{10}{101} \right]$$

$$= 344 \text{ }\Omega$$

Capacitance  $C_{\mu 2}$  sees a resistance  $R_{\mu 2}$ :

$$R_{\mu 2} = (1 + g_{m2}r_{o2})(R_{in2} \parallel R_{out1}) + r_{o2}$$

$$= (1 + 4 \times 1000) \times 0.344 + 1000$$

$$= 2376 \text{ k}\Omega$$

We can determine  $\tau_H$  from

$$\tau_H = C_{\mu 1}R_{\mu 1} + C_{\pi 1}R_{\pi 1} + C_{\mu 2}R_{\mu 2}$$

$$+ C_{\pi 2}R_{\pi 2} + C_Lr_o$$

$$= 0.2 \times 10 + 3 \times 0.347 + 0.2 \times 2376$$

$$+ 3 \times 0.344 + 1 \times 1000$$

$$\tau_H = 2 + 1 + 475.2 + 1 + 1000$$

$$= 1479.2 \text{ ns}$$

Observe that there are two dominant capacitances: the most significant is  $C_L$  and the second most significant is  $C_{\mu 2}$ .

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 1479.2 \times 10^{-9}} = 107.6 \text{ kHz}$$

$$9.92 \text{ g}_m = \frac{2I_D}{V_{OV}} = \frac{2(I/2)}{V_{OV}}$$

$$= \frac{I}{V_{OV}} = \frac{0.2 \text{ mA}}{0.2 \text{ V}} = 1 \text{ mA/V}$$

$$\frac{V_o}{V_{sig}} = \frac{R_D}{2/g_m}$$

$$= \frac{1}{2}g_mR_D = \frac{1}{2} \times 1 \times 50 = 25 \text{ V/V}$$

The high-frequency analysis can be performed in an analogous manner to that used in the text for the bipolar circuit. Refer to Fig. 9.42(b) and adapt the circuit for the MOS case. Thus,

$$f_{P1} = \frac{1}{2\pi R_{sig} \left( \frac{C_{gs}}{2} + C_{gd} \right)}$$

$$= \frac{1}{2\pi \times 100 \times 10^3 \left( \frac{4}{2} + 0.5 \right) \times 10^{-12}}$$

$$= 637 \text{ kHz}$$

and

$$f_{P2} = \frac{1}{2\pi R_D C_\mu}$$

$$= \frac{1}{2\pi \times 50 \times 10^3 \times 0.5 \times 10^{-12}}$$

$$= 6.37 \text{ MHz}$$

Since  $f_{P2} \simeq 10f_{P1}$ , the pole at  $f_{P1}$  will dominate and

$$f_H \simeq f_{P1} = 637 \text{ kHz}$$

9.93 Using an approach analogous to that utilized for the BJT circuit (Fig. 9.42), we see that there is a pole at the input with frequency  $f_{P1}$ :

$$f_{P1} = \frac{1}{2\pi R_{sig} \left( \frac{C_{gs}}{2} + C_{gd} \right)}$$

$$f_{P1} = \frac{1}{2\pi \times 20 \times 10^3 \left( \frac{2}{2} + 0.1 \right) \times 10^{-12}}$$

$$= 7.2 \text{ MHz},$$

and a pole at the output with frequency  $f_{P2}$ ,

$$f_{P2} = \frac{1}{2\pi (C_{gd} + C_L)R_L}$$

$$= \frac{1}{2\pi \times (0.1 + 1) \times 10^{-12} \times 20 \times 10^3}$$

$$= 7.2 \text{ MHz}$$

Thus,

$$f_{P1} = f_{P2} = 7.2 \text{ MHz}$$

The midband gain  $A_M$  is obtained as

$$A_M = \frac{R_L}{2/g_m} = \frac{1}{2}g_mR_L$$

$$= \frac{1}{2} \times 5 \times 20 = 50 \text{ V/V}$$

Thus, the amplifier transfer function is

$$\frac{V_o(s)}{V_{sig}(s)} = \frac{50}{\left( 1 + \frac{s}{2\pi \times 7.2 \times 10^6} \right)^2}$$

$$\left| \frac{V_o}{V_{sig}} \right| = \frac{50}{1 + \left( \frac{\omega}{2\pi \times 7.2 \times 10^6} \right)^2}$$

At  $\omega = \omega_{3dB}$ ,  $\left| \frac{V_o}{V_i} \right| = \frac{50}{\sqrt{2}}$ , thus

$$\sqrt{2} = 1 + \left( \frac{\omega_{3dB}}{2\pi \times 7.2 \times 10^6} \right)^2$$

$$f_{3dB} = \sqrt{\sqrt{2} - 1} \times 7.2 \text{ MHz} \\ = 4.6 \text{ MHz}$$

$$9.94 \ g_m = \frac{I_C}{V_T} \simeq \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_e \simeq 25 \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{40} = 3 \text{ k}\Omega$$

$$R_{in} = 2r_\pi = 6 \text{ k}\Omega$$

$$\frac{V_o}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} \frac{\alpha R_L}{2r_e}$$

$$\simeq \frac{6}{6+12} \times \frac{10}{2 \times 0.025} = 66.7 \text{ V/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6} = 12.7 \text{ pF}$$

$$C_\mu = 0.5 \text{ pF}$$

$$C_\pi = 12.2 \text{ pF}$$

$$f_{p1} = \frac{1}{2\pi R_{sig} \left( \frac{C_\pi}{2} + C_\mu \right)}$$

$$= \frac{1}{2\pi \times 12 \times 10^3 \left( \frac{12.2}{2} + 0.5 \right) \times 10^{-12}}$$

$$= 2 \text{ MHz}$$

$$f_{p2} = \frac{1}{2\pi R_L C_\mu}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 0.5 \times 10^{-12}} = 31.8 \text{ MHz}$$

Thus,  $f_{p1}$  is the dominant pole and

$$f_H \simeq f_{p1} = 2 \text{ MHz}$$

9.95 (a) For each of  $Q_1$  and  $Q_2$ ,

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_o = \frac{|V_A|}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_m r_o = 100$$

$$\frac{V_o}{V_{sig}} = (g_m r_o)^2 = 10,000 \text{ V/V}$$

$$(b) \tau_{gs1} = C_{gs} R_{sig}$$

$$= 20 \times 10 = 200 \text{ ps}$$

$$R_{gd1} = R_{sig}(1 + g_{m1} r_{o1}) + r_{o1}$$

$$= 10(1 + 100) + 100$$

$$= 1110 \text{ k}\Omega$$

$$\tau_{gd1} = C_{gd1} R_{gd1} = 5 \times 1110 = 5550 \text{ ps}$$

At the drain of  $Q_1$  we have  $(C_{db1} + C_{gs2})$  and the resistance seen is  $r_o$ :

$$\tau_{d1} = (C_{db1} + C_{gs2}) r_o$$

$$= (5 + 20) \times 100 = 2500 \text{ ps}$$

$$R_{gd2} = r_{o1}(1 + g_{m2} r_{o2}) + r_{o2}$$

$$= 100(1 + 100) + 100 = 1110 \text{ k}\Omega$$

$$\tau_{gd2} = C_{gd2} R_{gd2} = 5 \times 1110 = 5550 \text{ ps}$$

$$\tau_{d2} = C_{db2} r_{o2}$$

$$= 5 \times 100 = 500 \text{ ps}$$

$$\tau_H = \tau_{gs1} + \tau_{gd1} + \tau_{d1} + \tau_{gd2} + \tau_{d2}$$

$$= 200 + 5550 + 2500 + 5550 + 500$$

$$= 14,300 \text{ ps} = 14.3 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 14.3 \times 10^{-9}} = 11.1 \text{ MHz}$$

9.96 (a)

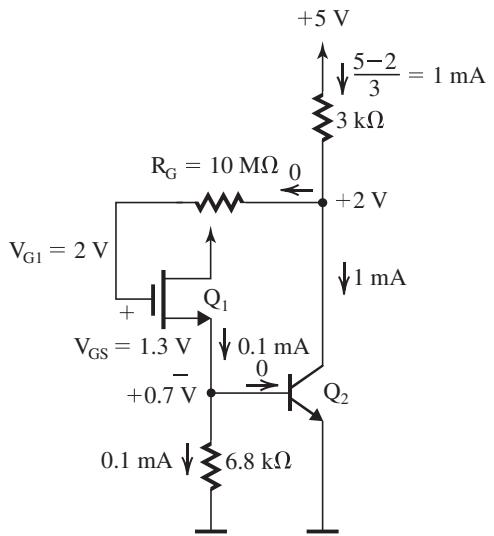


Figure 1

The dc analysis is shown in Fig. 1. It is based on  $V_{S1} = V_{BE2} = 0.7$  V. Neglecting  $I_{B2}$ , we obtain

$$I_{D1} = \frac{0.7 \text{ V}}{6.8 \text{ k}\Omega} \simeq 0.1 \text{ mA} \quad \text{Q.E.D.}$$

$$I_{D1} = \frac{1}{2} k'_n (W/L) V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$$\Rightarrow V_{OV} \simeq 0.3 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 1 + 0.3 = 1.3 \text{ V}$$

$$V_{G1} = 0.7 + 1.3 = 2 \text{ V}$$

$$V_{C1} = V_{G1} = 2 \text{ V}$$

$$I_{C2} = \frac{5 - 2}{3} = 1 \text{ mA} \quad \text{Q.E.D.}$$

$$(b) g_{m1} = \frac{2I_{D1}}{V_{OV}} = \frac{2 \times 0.1}{0.3} = 0.67 \text{ mA/V}$$

$$C_{gs} = C_{gd} = 1 \text{ pF}$$

$$g_{m2} = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{200}{40} = 5 \text{ k}\Omega$$

$$C_{\pi 2} + C_{\mu 2} = \frac{g_{m2}}{2\pi f_{T2}}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 600 \times 10^6} = 10.6 \text{ pF}$$

$$C_{\mu 2} = 0.8 \text{ pF}$$

$$C_{\pi 2} = 9.8 \text{ pF}$$

(c)  $Q_1$  acts as a source follower, thus

$$\frac{V_{b2}}{V_i} = \frac{6.8 \text{ k}\Omega \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + (6.8 \text{ k}\Omega \parallel r_{\pi 2})}$$

$$= \frac{(6.8 \parallel 5)}{1.5 + (6.8 \parallel 5)} = 0.66 \text{ V/V}$$

Neglecting  $R_G$ , we obtain

$$\frac{V_o}{V_{b2}} = -g_{m2}(3 \text{ k}\Omega \parallel 1 \text{ k}\Omega)$$

$$= -40(3 \parallel 1) = -30 \text{ V/V}$$

Thus,

$$\frac{V_o}{V_i} = 0.66 \times -30$$

$$\simeq -20 \text{ V/V}$$

Using Miller's theorem, the input resistance  $R_{in}$  is found as

$$R_{in} = \frac{R_G}{1 - \frac{V_o}{V_i}} = \frac{10 \text{ M}\Omega}{1 - (-20)}$$

$$= 476 \text{ k}\Omega$$

$$\frac{V_i}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}}$$

$$= \frac{476}{476 + 100} = 0.83 \text{ V/V}$$

$$\frac{V_o}{V_{sig}} = 0.83 \times 20 = 16.5 \text{ V/V}$$

(c) The pole due to  $C_1$  has a frequency  $f_l$ :

$$f_l = \frac{1}{2\pi C_1 (R_{sig} + R_{in})}$$

$$= \frac{1}{2\pi \times 0.1 \times 10^{-6} (100 + 476) \times 10^3}$$

$$= 2.8 \text{ Hz}$$

The pole due to  $C_2$  has a frequency  $f_2$ :

$$f_2 = \frac{1}{2\pi C_2 (3 + 1) \times 10^3}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6} \times 4 \times 10^3} = 40 \text{ Hz}$$

Since  $f_2 \gg f_l$ , the lower 3-dB frequency  $f_L$  will be  $f_L \simeq f_2 = 40 \text{ Hz}$

$$(d) \tau_{gd1} = C_{gd1} (R_{in} \parallel R_{sig})$$

$$= 1 \times 10^{-12} (476 \parallel 100) \times 10^3$$

$$= 82.6 \text{ ns}$$

To determine the resistance  $R_{gs}$  seen by  $C_{gs}$ , refer to Fig. 2.

This figure belongs to Problem 9.96, part (d).

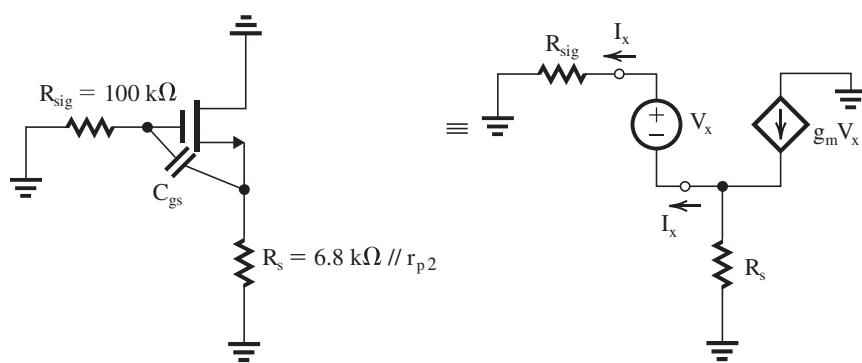


Figure 2

We can show that

$$R_{gs} \equiv \frac{V_x}{I_x} = \frac{R_{sig} + R_s}{1 + g_m R_s}$$

where

$$R_s = 6.8 \text{ k}\Omega \parallel r_{\pi 2}$$

$$= 6.8 \parallel 5 = 2.88 \text{ k}\Omega$$

$$R_{gs} = \frac{100 + 2.88}{1 + 0.67 \times 2.88} = 35.1 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs} R_{gs} = 1 \times 10^{-12} \times 35.1 \times 10^3 = 35.1 \text{ ns}$$

$$C_{\pi 2} = C_{\pi 2}(r_{\pi 1} \parallel 6.8 \text{ k}\Omega)$$

$$= 9.8 \times 10^{-12} \times 2.88 \times 10^3$$

$$= 28.2 \text{ ns}$$

$$R_{\mu 2} = \left( \frac{1}{g_m} \parallel 6.8 \text{ k}\Omega \right) [1 + g_m(3 \parallel 1)] + (3 \parallel 1)$$

$$= (1.5 \parallel 6.8) \left( 1 + 40 \times \frac{3}{4} \right) + 0.75$$

$$= 38.8 \text{ k}\Omega$$

$$\tau_{\mu 2} = C_{\mu 2} R_{\mu 2} = 0.8 \times 38.8 = 31.1 \text{ ns}$$

$$\tau_H = \tau_{gd} + \tau_{gs} + \tau_{\pi 2} + \tau_{\mu 2}$$

$$= 82.6 + 35.1 + 38.8 + 31.1$$

$$= 187.6 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 187.6 \times 10^{-9}} = 848 \text{ kHz}$$

9.97 All transistors are operating at  $I_E = 0.5 \text{ mA}$ . Thus,

$$g_m \simeq 20 \text{ mA/V}$$

$$r_e \simeq 50 \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$r_o$  = very high (neglect)

$r_x$  = very small (neglect)

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{20 \times 10^{-3}}{2\pi \times 400 \times 10^6}$$

$$= 8 \text{ pF}$$

$$C_\mu = 2 \text{ pF}$$

$$C_\pi = 6 \text{ pF}$$

(a) CE amplifier

$$A_M = -\frac{r_\pi}{r_\pi + R_{sig}} g_m R_L$$

$$= -\frac{5}{5 + 10} \times 20 \times 10 = -66.7 \text{ V/V}$$

Since  $C_L = 0$ , we can obtain a good estimate of  $f_H$  using the Miller approximation:

$$C_{in} = C_\pi + C_\mu (g_m R_L + 1)$$

$$= 6 + 2(20 \times 10 + 1)$$

$$= 408 \text{ pF}$$

$$f_H = \frac{1}{2\pi R_{sig} C_{in}}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 408 \times 10^{-12}}$$

$$= 39 \text{ kHz}$$

(b) This is a cascode amplifier. Refer to Fig. 9.30 for the analysis equations.

$$A_M = -\frac{r_\pi}{r_\pi + R_{sig}} g_m (\beta r_o \parallel R_L)$$

$$\simeq -\frac{r_\pi}{r_\pi + R_{sig}} g_m R_L$$

$$= -66.7 \text{ V(V)} \text{ (same as the CE in (a))}$$

$$R'_{sig} = r_\pi \parallel R_{sig} = 5 \parallel 10 = 3.33 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{sig}$$

$$\tau_{\pi 1} = C_{\pi 1} R_{\pi 1} = 6 \times 3.33 = 20 \text{ ns}$$

$$R_{c1} = r_{e2} = 50 \Omega$$

$$R_{\mu 1} = R'_{sig} (1 + g_m R_{c1}) + R_{c1}$$

$$= 3.33(1 + 40 \times 0.05) + 0.05$$

$$= 3.33(1 + 2) + 0.05 = 10.05 \text{ k}\Omega$$

$$\tau_{\mu 1} = C_{\mu 1} R_{\mu 1} = 2 \times 10.05 = 20.1 \text{ ns}$$

$$\tau_{c1} = C_{\pi 2} R_{c1} = 6 \times 0.05 = 0.3 \text{ ns}$$

$$\tau_{\mu 2} = C_{\mu 2} R_L = 2 \times 10 = 20 \text{ ns}$$

$$\tau_H = 20 + 20.1 + 0.3 + 20 = 60.4 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 60.4 \times 10^{-9}}$$

$$= 2.6 \text{ MHz}$$

(c) This is a CC-CB cascade similar to the circuit analyzed in Fig. 9.42. There are two poles: one at the input,

$$f_{p1} = \frac{1}{2\pi (R_{sig} \parallel 2r_\pi) \left( \frac{C_\pi}{2} + C_\mu \right)}$$

$$f_{p1} = \frac{1}{2\pi (10 \parallel 10) \times 10^3 (3 + 2) \times 10^{-12}}$$

$$= \frac{1}{2\pi \times 5 \times 5 \times 10^{-9}}$$

$$= 6.4 \text{ MHz}$$

and one at the output,

$$\begin{aligned} f_{p2} &= \frac{1}{2\pi R_L C_\mu} \\ &= \frac{1}{2\pi \times 10 \times 10^3 \times 2 \times 10^{-12}} \\ &= 8 \text{ MHz} \end{aligned}$$

Since the two poles are relatively close to each other, we use the root-sum-of-the-squares formula to obtain an estimate for  $f_H$ :

$$\begin{aligned} f_H &= 1/\sqrt{\frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2}} \\ &= 1/\sqrt{\frac{1}{6.4^2} + \frac{1}{8^2}} = 5 \text{ MHz} \\ A_M &= \frac{R_L}{2r_e + \frac{R_{sig}}{\beta + 1}} \\ &= \frac{10}{2 \times 0.05 + \frac{10}{101}} \simeq 50 \text{ V/V} \end{aligned}$$

(d) This is a CC-CE cascade similar to the circuit analyzed in Example 9.13.

$$\begin{aligned} R_{in} &= (\beta_1 + 1)(r_{e1} + r_{\pi2}) \\ &= 101(0.05 + 5) = 510 \text{ k}\Omega \\ \frac{V_{b1}}{V_{sig}} &= \frac{R_{in}}{R_{in} + R_{sig}} = \frac{510}{510 + 10} \\ &= 0.98 \text{ V/V} \\ \frac{V_{b2}}{V_{b1}} &= \frac{r_{\pi2}}{r_{\pi2} + r_{e1}} = \frac{5}{5 + 0.05} = 0.99 \text{ V/V} \\ \frac{V_o}{V_{b2}} &= -g_{m2}R_L = -20 \times 10 = -200 \text{ V/V} \\ A_M &= \frac{V_o}{V_{sig}} = -0.98 \times 0.99 \times 200 = -194 \text{ V/V} \end{aligned}$$

$$\begin{aligned} R_{\mu1} &= R_{sig} \parallel R_{in} \\ &= 10 \parallel 510 = 9.81 \text{ k}\Omega \end{aligned}$$

$$\tau_{\mu1} = C_{\mu1}R_{\mu1} = 2 \times 9.81 = 19.6 \text{ ns}$$

$$\begin{aligned} R_{\pi1} &= \frac{R_{sig} + R_{in2}}{1 + \frac{R_{sig}}{r_{\pi1}} + \frac{R_{in2}}{r_{e1}}} \\ &= \frac{10 + 5}{1 + \frac{10}{5} + \frac{5}{0.05}} = 0.15 \text{ k}\Omega \end{aligned}$$

$$\tau_{\pi1} = C_{\pi1}R_{\pi1} = 6 \times 0.15 = 0.9 \text{ ns}$$

$$\begin{aligned} R_{\pi2} &= R_{in2} \parallel R_{out1} \\ &= r_{\pi2} \parallel \left( \frac{R_{sig}}{\beta_1 + 1} + r_{e1} \right) \\ &= 5 \parallel \left( \frac{10}{101} + 0.05 \right) = 0.15 \text{ k}\Omega \end{aligned}$$

$$\tau_{\pi2} = C_{\pi2}R_{\pi2} = 6 \times 0.15 = 0.9 \text{ ns}$$

$$\begin{aligned} R_{\mu2} &= (1 + g_{m2}R_L)(R_{in2} \parallel R_{out1}) + R_L \\ &= (1 + 20 \times 10) \left[ 5 \parallel \left( \frac{10}{101} + 0.05 \right) \right] + 10 \\ &= 39.1 \text{ k}\Omega \end{aligned}$$

$$\tau_{\mu2} = C_{\mu2}R_{\mu2} = 2 \times 39.1 = 78.2 \text{ ns}$$

$$t_H = 19.6 + 0.9 + 0.9 + 78.2 = 99.6 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 99.6 \times 10^{-9}} = 1.6 \text{ MHz}$$

(e) This is a folded cascode amplifier. The analysis is identical to that for (b) above.

$$A_M = -66.7 \text{ V/V}$$

$$f_H = 2.6 \text{ MHz}$$

(f) This is a CE-CB cascade. The analysis is identical to that for case (c) above.

$$A_M = 50 \text{ V/V}$$

$$f_H = 5 \text{ MHz}$$

### Summary of Results

Case	Configuration	$A_M$ (V/V)	$f_H$ (MHz)
a	CE	-66.7	0.039
b	Cascode	-66.7	2.6
c	CC-CB	50	5
d	CC-CE	-194	1.6
e	Folded Cascode	-66.7	2.6
f	CC-CB	50	5

### Exercise 10-1

Ex: 10.1 (c)  $A = 100 \text{ V/V}$  and  $A_f = 10 \text{ V/V}$

Since  $A$  is not much greater than  $A_f$ , we shall use the exact expression to determine  $\beta$  and hence  $R_2/R_1$ ,

$$A_f = \frac{A}{1 + A\beta}$$

$$10 = \frac{100}{1 + 100\beta}$$

$$\Rightarrow \beta = 0.09 \text{ V/V}$$

Now,

$$\frac{R_1}{R_1 + R_2} = 0.09$$

$$\frac{R_2}{R_1} = \frac{1}{0.09} - 1 = 10.11$$

$$(d) A\beta = 100 \times 0.09 = 9$$

$$1 + A\beta = 10$$

$$\Rightarrow 20 \text{ dB}$$

$$(e) V_o = A_f V_s = 10 \times 1 = 10 \text{ V}$$

$$V_f = \beta V_o = 0.09 \times 10 = 0.9 \text{ V}$$

$$V_i = \frac{V_o}{A} = \frac{10}{100} = 0.1 \text{ V}$$

$$(f) A \rightarrow 80 \text{ V/V}$$

$$A_f = \frac{80}{1 + 80 \times 0.09} = 9.756$$

$$\text{a change of } \frac{9.756 - 10}{10} \times 100 = -2.44\% \text{ or a reduction of } 2.44\%.$$

Ex. 10.2 (c)  $A = 10^4 \text{ V/V}$  and  $A_f = 10^3 \text{ V/V}$

$$A_f = \frac{A}{1 + A\beta}$$

$$10^3 = \frac{10^4}{1 + 10^4\beta}$$

$$\Rightarrow \beta = 9 \times 10^{-4} \text{ V/V}$$

$$\frac{R_1}{R_1 + R_2} = 9 \times 10^{-4}$$

$$\frac{R_2}{R_1} = \frac{1}{9 \times 10^{-4}} - 1 = 1110.1$$

$$(d) A\beta = 10^4 \times 9 \times 10^{-4} = 9$$

$$1 + A\beta = 10$$

$$\Rightarrow 20 \text{ dB}$$

$$(e) V_s = 0.01 \text{ V}$$

$$V_o = A_f V_s = 10^3 \times 0.01 = 10 \text{ V}$$

$$V_f = \beta V_o = 9 \times 10^{-4} \times 10 = 0.009 \text{ V}$$

$$V_i = \frac{V_o}{A} = \frac{10}{10^4} = 0.001 \text{ V}$$

$$(f) A \Rightarrow 0.8 \times 10^4 \text{ V/V}$$

$$A_f = \frac{0.8 \times 10^4}{1 + 0.8 \times 10^4 \times 9 \times 10^{-4}}$$

$$= 975.6 \text{ V/V}$$

$$\text{which is a change of } \frac{975.6 - 1000}{1000} \times 100$$

$$= -2.44\% \text{ or a reduction of } 2.44\%.$$

Ex. 10.3 To constrain the corresponding change in  $A_f$  to 0.1%, we need an amount-of-feedback of at least

$$1 + A\beta = \frac{10\%}{0.1\%} = 100$$

Thus the largest obtainable closed-loop gain will be

$$A_f = \frac{A}{1 + A\beta} = \frac{1000}{100} = 10 \text{ V/V}$$

Each amplifier in the cascade will have a nominal gain of 10 V/V and a maximum variability of 0.1%; thus the overall voltage gain will be  $(10)^3 = 1000 \text{ V/V}$  and the maximum variability will be 0.3%.

$$\text{Ex. 10.4 } \beta = \frac{R_1}{R_1 + R_2} = \frac{1}{1 + 9} = 0.1$$

$$A\beta = 10^4 \times 0.1 = 1000$$

$$1 + A\beta = 1001$$

$$A_f = \frac{A}{1 + A\beta}$$

$$A_f = \frac{10^4}{1 + 10^4 \times 0.1} = 9.99 \text{ V/V}$$

$$f_{Hf} = f_H(1 + A\beta)$$

$$= 100 \times 1001 = 100.1 \text{ kHz}$$

$$\text{Ex. 10.5 Signal at output} = V_s \frac{A_1 A_2}{1 + A_1 A_2 \beta}$$

$$= 1 \times \frac{1 \times 100}{1 + 1 \times 100 \times 1} = 1 \times \frac{100}{101} \simeq 1 \text{ V}$$

$$\text{Interference at output} = V_n \frac{A_1}{1 + A_1 A_2 \beta}$$

$$= 1 \times \frac{1}{1 + 1 \times 100 \times 1} \simeq 0.01 \text{ V}$$

Thus S/I at the output becomes 1/0.01

$$= 100 \text{ or } 40 \text{ dB}$$

Since S/I at the input is 1/1=1 or 0 dB, the improvement is 40 dB.



### Exercise 10-3

These figures belong to Exercise 10.8.

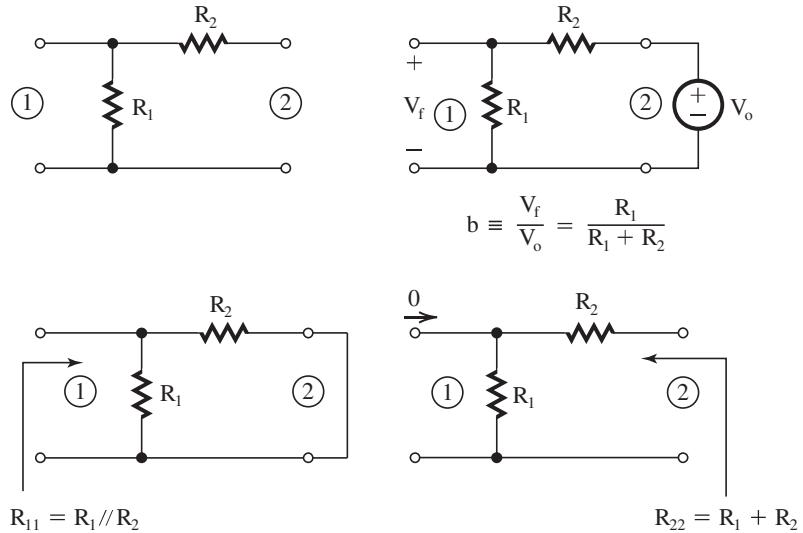


Figure 1

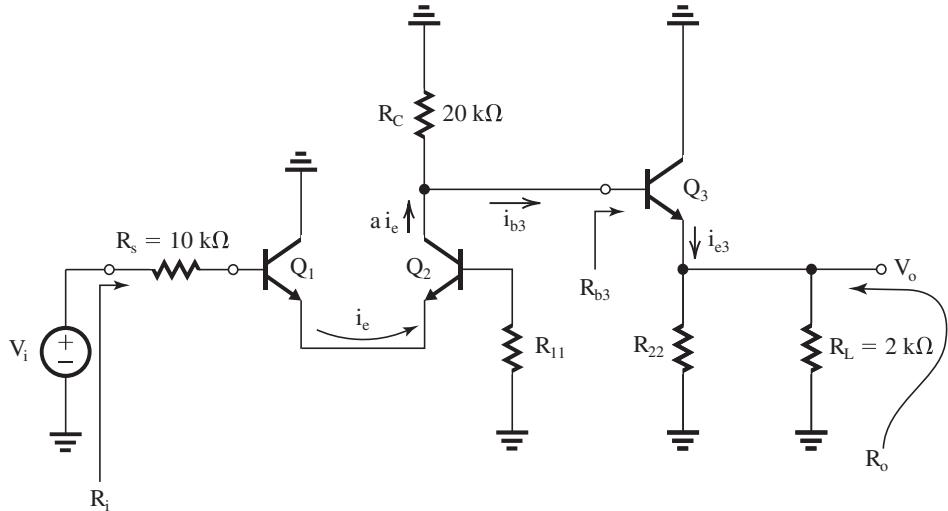


Figure 2

$$\begin{aligned} i_{b3} &= \alpha i_e \frac{R_C}{R_C + R_{b3}} \\ &= 0.99 i_e \frac{20}{20 + 168.84} \\ \Rightarrow i_{b3} &= 0.105 i_e \quad (2) \\ V_o &= i_{e3} (R_{22} \parallel R_L) \\ &= (\beta + 1) i_{b3} (R_{22} \parallel R_L) \\ &= i_{b3} \times 101 (10 \parallel 2) \\ \Rightarrow V_o &= 168.33 i_{b3} \quad (3) \end{aligned}$$

Combining (1)–(3), we obtain

$$\begin{aligned} A &\equiv \frac{V_o}{V_i} = 85 \text{ V/V} \\ \beta &= 0.1 \text{ V/V} \\ A\beta &= 8.5 \\ 1 + A\beta &= 9.5 \\ A_f &= \frac{85}{9.5} = 8.95 \text{ V/V} \\ \text{From the A circuit, we have} \\ R_i &= R_s + R_{l1} + (\beta + 1)(r_{e1} + r_{e2}) \\ &= 10 + 0.9 + 101 \times 0.1 \\ &= 21 \text{ k}\Omega \end{aligned}$$

### Exercise 10-4

$$R_{if} = R_i(1 + A\beta)$$

$$= 21 \times 9.5 = 199.5 \text{ k}\Omega$$

$$R_{in} = R_{if} - R_s = 199.5 - 10 = 189.5 \text{ k}\Omega$$

From the A circuit, we have

$$R_o = R_L \parallel R_{22} \parallel \left( r_{e5} + \frac{R_C}{\beta + 1} \right)$$

$$= 2 \parallel 10 \parallel \left( 0.005 + \frac{20}{101} \right)$$

$$= 181 \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{181}{9.5} = 19.1 \Omega$$

$$R_{of} = R_L \parallel R_{out}$$

$$19.1 = 2 \text{ k}\Omega \parallel R_{out}$$

$$\Rightarrow R_{out} = 19.2 \Omega$$

Ex. 10.9 Figure 1 shows the  $\beta$  circuit together with the determination of  $\beta$ ,  $R_{11}$  and  $R_{22}$ .

$$\beta = \frac{R_1}{R_1 + R_2}$$

$$R_{11} = R_1 \parallel R_2$$

$$R_{22} = R_1 + R_2$$

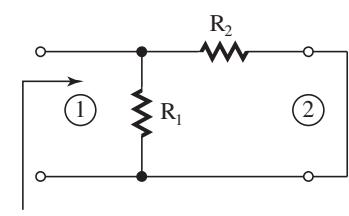
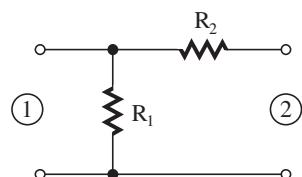
Figure 2 shows the A circuit. We can write

$$V_o = g_m(R_D \parallel R_{22})V_i$$

Thus,

$$A \equiv \frac{V_o}{V_i} = g_m[R_D \parallel (R_1 + R_2)]$$

This figure belongs to Exercise 10.9.



$$R_{11} = R_1 \parallel R_2$$

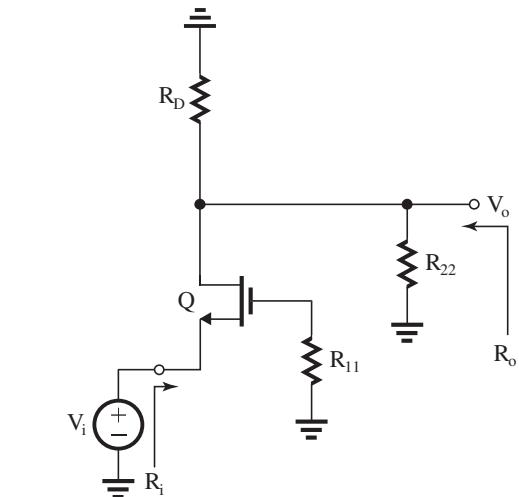


Figure 2

$$\beta = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{A}{1 + A\beta}$$

From A circuit, we have

$$R_i = \frac{1}{g_m}$$

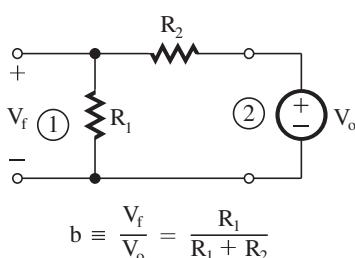
$$R_o = R_D \parallel R_{22}$$

$$R_{in} = R_{if} = R_i(1 + A\beta)$$

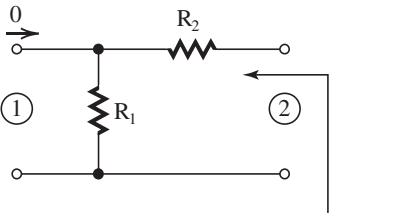
$$R_{in} = \frac{1}{g_m}(1 + A\beta)$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$R_{out} = \frac{R_D \parallel (R_1 + R_2)}{1 + A\beta}$$



$$b \equiv \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2}$$



$$R_{22} = R_1 + R_2$$

Figure 1

### Exercise 10–5

Comparison with the results of Exercise 10.6 shows that the expressions for  $A$  and  $\beta$  are identical. However,  $R_{in}$  and  $R_{out}$  cannot be determined using the method of Exercise 10.6.

Ex. 10.10 From the solution to Example 10.6, we have

$$A\beta = 653.6$$

$$1 + A\beta = 654.6$$

A decrease in the op amp gain by 10% results in a decrease in  $A$  by 10% and a corresponding decrease in  $A_f$  by

$$\frac{10\%}{1 + A\beta} = \frac{10\%}{654.6} = 0.015\%$$

A more exact solution (not using differential) is as follows:

The open-loop gain  $A$  becomes

$$A = 0.9 \times 653.6 = 588.24 \text{ mA/V}$$

$$\beta = R_F = 1 \text{ k}\Omega$$

$$A_f = \frac{588.24}{1 + 588.24 \times 1} = 0.9983 \text{ mA/V}$$

$$\text{Change in } A_f = 0.9983 - 0.9985$$

$$= -0.0002$$

$$\text{Percentage change in } A_f = \frac{-0.0002}{0.9983} \times 100 = -0.02\%$$

Ex. 10.11 For a nominal closed-loop transconductance of 2 mA/V, we have

$$R_F = \beta = \frac{1}{2 \text{ mA/V}} = 0.5 \text{ k}\Omega$$

From the solution to Example 10.6, we obtain

$$A = \frac{\mu}{R_F} \frac{g_m(R_F \parallel R_{id} \parallel r_{o2})}{1 + g_m(R_F \parallel R_{id} \parallel r_{o2})}$$

$$A = \frac{1000}{0.5} \frac{2(0.5 \parallel 100 \parallel 20)}{1 + 2(0.5 \parallel 100 \parallel 20)}$$

$$A = 985.2 \text{ mA/V}$$

$$A_f \equiv \frac{I_o}{V_s} = \frac{985.2}{1 + 985.2 \times 0.5} = 1.996 \text{ mA/V}$$

Ex. 10.12  $A_f \simeq 5 \text{ mA/V}$

$$\beta \simeq \frac{1}{A_f} = 0.2 \text{ k}\Omega = 200 \text{ }\Omega$$

$$R_F = 200 \text{ }\Omega$$

Using Eq. (10.36), we obtain

$$A_f \simeq \frac{A_l g_{m2}}{1 + A_l g_{m2} R_F} = \frac{200 \times 2}{1 + 200 \times 2 \times 0.2} = 4.94 \text{ mA/V}$$

From Eq. (10.32), we have

$$R_i = R_s + R_{id} + R_F \\ \simeq R_{id} + R_F \\ = 100 + 0.2 = 100.2 \text{ k}\Omega$$

From Eq. (10.35), we get

$$A\beta \simeq A_l g_{m2} R_F = 200 \times 2 \times 0.2 = 80$$

$$1 + A\beta = 81$$

$$R_{of} = (1 + A\beta) R_i \\ = 81 \times 100.2 \simeq 8.1 \text{ M}\Omega$$

From Eq. (10.33), we have

$$R_o = r_{o2} + R_L + R_F \\ \simeq r_{o2} + R_F \\ = 20 + 0.2 = 20.2 \text{ k}\Omega$$

$$R_{of} = R_o(1 + A\beta) = 20.2 \times 81 = 1.64 \text{ M}\Omega$$

If  $g_{m2}$  drops by 50%,  $A$  drops by 50% to

$$A = A_l g_{m2} = 200 \times 1 = 200 \text{ mA/V}$$

and  $A_f$  becomes

$$A_f = \frac{200}{1 + 200 \times 0.2} = 4.878 \text{ mA/V}$$

Thus,

$$\Delta A_f = 4.94 - 4.878 = -0.062$$

$$\frac{\Delta A_f}{A_f} \times 100 = -\frac{0.062}{4.94} \times 100 = -1.25\%$$

Ex. 10.13 See figure on next page. Figure 1 shows the circuit for determining the loop gain. The figure also shows the analysis. We start by finding the current in the drain of  $Q_2$  as  $g_{m2} V_{g2}$  (this excludes the current in  $r_{o2}$ ). Since  $r_{o2} \gg R_L + R_F$ , most of  $g_{m2} V_{g2}$  will flow through  $R_L$  and  $R_F \parallel (R_{id} + R_s)$ . Since  $R_F \ll R_{id} + R_s$ , the voltage across  $R_F$  will be approximately  $-g_{m2} R_F V_{g2}$ . This voltage is amplified by  $A_l$  which provides at its output

$$V_r = -A_l g_{m2} R_F V_{g2}$$

Thus, we find  $A\beta$  as

$$A\beta \equiv -\frac{V_r}{V_{g2}} = A_l g_{m2} R_F$$

### Exercise 10-6

This figure belongs to Exercise 10.13.

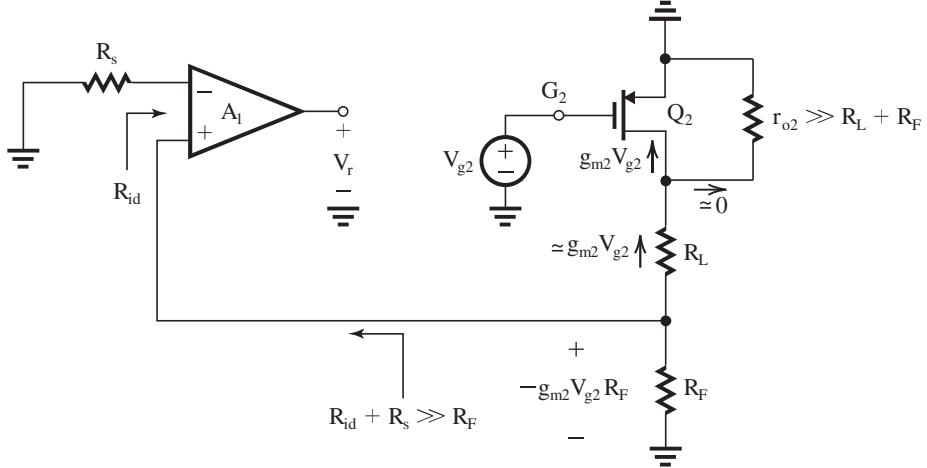


Figure 1

Ex. 10.14 From Eq. (10.34), we obtain

$$A\beta = (A_1 g_{m2} R_F) \left( \frac{R_{id}}{R_{id} + R_s + R_F} \right) \left( \frac{r_{o2}}{r_{o2} + R_L + R_F} \right)$$

From Eq. (10.32), we obtain

$$R_i = R_s + R_{id} + R_F$$

$$R_{if} = R_i(1 + A\beta)$$

$$= R_i + A\beta R_i$$

$$= R_s + R_{id} + R_F + (A_1 g_{m2} R_F) R_{id} \left( \frac{r_{o2}}{r_{o2} + R_L + R_F} \right)$$

$$R_{in} = R_{if} - R_s$$

$$R_{in} = R_{id} + R_F + (A g_{m2} R_F) R_{id} \left( \frac{r_{o2}}{r_{o2} + R_L + R_F} \right)$$

For  $R_F \ll R_{id}$  and  $r_{o2} \gg R_L + R_F$ , we have

$$R_{in} \simeq R_{id} + A g_{m2} R_F R_{id}$$

$$= R_{id}(1 + A g_{m2} R_F) \quad \text{Q.E.D.}$$

Ex. 10.15 From Eq. (10.34), we obtain

$$A\beta = (A_1 g_{m2} R_F) \left( \frac{R_{id}}{R_{id} + R_s + R_F} \right) \left( \frac{r_{o2}}{r_{o2} + R_L + R_F} \right)$$

From Eq. (10.33), we get

$$R_o = r_{o2} + R_L + R_F$$

$$R_{of} = R_o(1 + A\beta)$$

$$= R_o + A\beta R_o$$

$$= r_{o2} + R_L + R_F + (A_1 g_{m2} R_F) \left( \frac{R_{id}}{R_{id} + R_s + R_F} \right) r_{o2}$$

$$R_{out} = R_{of} - R_L$$

$$= r_{o2} + R_F + (A_1 g_{m2} R_F) \left( \frac{R_{id}}{R_{id} + R_s + R_F} \right) r_{o2}$$

For  $R_F \ll r_{o2}$  and  $R_{id} \gg R_s + R_F$ , we have

$$R_{out} \simeq r_{o2}(1 + A_1 g_{m2} R_F) \quad \text{Q.E.D.}$$

Ex. 10.16 To obtain

$$A_f \equiv \frac{I_o}{V_s} \simeq 100 \text{ mA/V}$$

when the loop gain is large, we use

$$\beta \simeq \frac{1}{A_f} = 10 \Omega$$

But,

$$\beta = R_{E1} \times \frac{R_{E2}}{R_{E2} + R_F + R_{E1}}$$

For  $R_{E1} = R_{E2} = 100 \Omega$ ,

$$10 = \frac{100 \times 100}{200 + R_F}$$

$$\Rightarrow R_F = 800 \Omega$$

$$\frac{V_o}{V_s} = \frac{-I_o R_{C1}}{V_s}$$

$$= -A_f R_{C1} = -100 \times 0.6 = -60 \text{ V/V}$$

Ex. 10.17 See figure on next page. Figure 1 shows the circuit prepared for the determination of the loop gain,

$$A\beta \equiv -\frac{V_r}{V_t}$$

### Exercise 10-7

This figure belongs to Exercise 10.17.

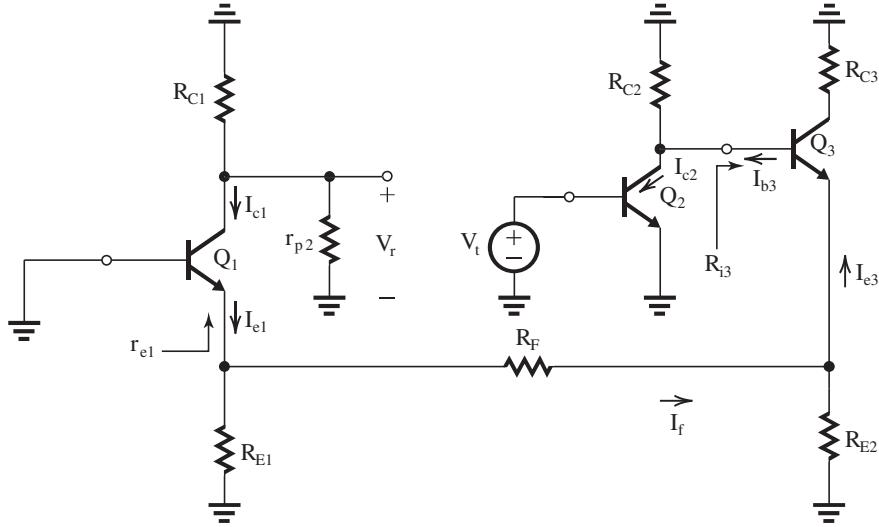


Figure 1

We shall trace the signal around the loop as follows:

$$I_{c2} = g_m V_t \quad (1)$$

$$I_{b3} = I_{c2} \frac{R_{C2}}{R_{C2} + R_{i3}} \quad (2)$$

where

$$R_{i3} = (\beta + 1) \{ r_{e3} + [R_{E2} \parallel (R_F + (R_{E1} \parallel r_{e1}))] \} \quad (3)$$

$$I_{e3} = (\beta + 1) I_{b3} \quad (4)$$

$$I_f = I_{e3} \frac{R_{E2}}{R_{E2} + R_F + (R_{E1} \parallel r_{e1})} \quad (5)$$

$$I_{e1} = I_f \frac{R_{E1}}{R_{E1} + r_{e1}} \quad (6)$$

$$I_{c1} = \alpha I_{e1} \quad (7)$$

$$V_r = -I_{c1} (R_{C1} \parallel r_{\pi 2}) \quad (8)$$

Combining (1)–(7) gives  $V_r$  in terms of  $V_t$  and hence  $A\beta \equiv -V_r/V_t$ . We shall do this numerically using the values in Example 10.8:

$$g_m = 40 \text{ mA/V}, R_{C2} = 5 \text{ k}\Omega, \beta = 100,$$

$$r_{e3} = 6.25 \Omega, R_{E1} = R_{E2} = 100 \Omega, R_F = 640 \Omega,$$

$$r_{e1} = 41.7 \Omega, \alpha_1 = 0.99, R_{C1} = 9 \text{ k}\Omega, \text{ and } r_{\pi 2} = 2.5 \text{ k}\Omega$$

$$R_{i3} = 101 \{ 0.00625 + [0.1 \parallel (0.64 + (0.1 \parallel 0.0417))] \}$$

$$= 9.42 \text{ k}\Omega$$

$$I_{c2} = 40 V_t \quad (9)$$

$$I_{b3} = 0.347 I_{c2} \quad (10)$$

$$I_{e3} = 101 I_{b3} \quad (11)$$

$$I_f = 0.13 I_{e3} \quad (12)$$

$$I_{e1} = 0.706 I_f \quad (13)$$

$$I_{c1} = 0.99 I_{e1} \quad (14)$$

$$V_r = -1.957 I_{c1} \quad (15)$$

Combining (9)–(15), we obtain

$$A\beta = -\frac{V_r}{V_t} = 249.3$$

Ex. 10.18

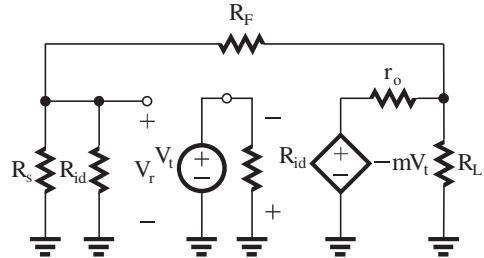


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain

$$A\beta \equiv -\frac{V_r}{V_t}$$

Using the voltage-divider rule, we can write by inspection

$$V_r = -\mu V_t \frac{R_L \parallel [R_F + (R_s \parallel R_{id})]}{r_o + \{R_L \parallel [R_F + (R_s \parallel R_{id})]\}} \frac{(R_s \parallel R_{id})}{R_F + (R_s \parallel R_{id})}$$

### Exercise 10-8

$$V_r = -\mu V_t \frac{R_L(R_s \parallel R_{id})}{r_o[R_L + R_F + (R_s \parallel R_{id})] + R_L[R_F + (R_s \parallel R_{id})]}$$

Thus,

$$A\beta = -\frac{V_r}{V_t} = \frac{\mu R_L(R_{id} \parallel R_s)}{r_o[R_L + R_F + (R_{id} \parallel R_s)] + R_L[R_F + (R_{id} \parallel R_s)]}$$

Q.E.D.

Using the numerical values in Example 10.9, we get

$$A\beta = \frac{10^4 \times 1 \times 1}{0.1(1+10+1) + 1(10+1)} = 819.7$$

Ex. 10.19 See figure on next page. Figure 1(a) shows the feedback amplifier circuit. The  $\beta$  circuit is shown in Fig. 1(b), and the determination of  $\beta$  is shown in Fig. 1(c),

$$\beta = -\frac{1}{R_F}$$

(a) For large loop gain, we have

$$A_f \simeq \frac{1}{\beta} = -R_F$$

(b) The determination of  $R_{11}$  and  $R_{22}$  is illustrated in Figs. 1(d) and (e), respectively:

$$R_{11} = R_{22} = R_F$$

Finally, the A circuit is shown in Fig. 1(f). We can write by inspection

$$R_i = R_s \parallel R_{11} = R_s \parallel R_F$$

$$R_o = r_o \parallel R_{22} = r_o \parallel R_F$$

$$V_{gs} = I_i R_i$$

$$V_o = -g_m V_{gs} (r_o \parallel R_{22})$$

Thus,

$$A \equiv \frac{V_o}{I_i} = -g_m (R_s \parallel R_F) (r_o \parallel R_F)$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$A_f = -\frac{g_m (R_s \parallel R_F) (r_o \parallel R_F)}{1 + g_m (R_s \parallel R_F) (r_o \parallel R_F) / R_F}$$

Q.E.D.

$$(c) R_{if} = \frac{R_i}{1 + A\beta}$$

$$R_{if} = \frac{R_s \parallel R_F}{1 + g_m (R_s \parallel R_F) (r_o \parallel R_F) / R_F}$$

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_F} + \frac{g_m (r_o \parallel R_F)}{R_F}$$

But,

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_{in}}$$

thus,

$$\frac{1}{R_{in}} = \frac{1}{R_F} [1 + g_m (r_o \parallel R_F)]$$

$$\Rightarrow R_{in} = \frac{R_F}{1 + g_m (r_o \parallel R_F)}$$

Q.E.D.

$$(d) R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{r_o \parallel R_F}{1 + g_m (R_s \parallel R_F) (r_o \parallel R_F) / R_F}$$

$$\frac{1}{R_{out}} = \frac{1}{r_o} + \frac{1}{R_F} + \frac{g_m (R_s \parallel R_F)}{R_F}$$

$$\Rightarrow R_{out} = r_o \parallel \frac{R_F}{1 + g_m (R_s \parallel R_F)}$$

Q.E.D.

$$(e) A = -5(1 \parallel 10)(20 \parallel 10)$$

$$A = -30.3 \text{ k}\Omega$$

$$\beta = -\frac{1}{R_F} = -\frac{1}{10} = -0.1 \text{ mA/V}$$

$$A\beta = 3.03$$

$$1 + A\beta = 4.03$$

$$A_f = \frac{A}{1 + A\beta} = \frac{30.3}{4.03} = -7.52 \text{ k}\Omega$$

(Compare to the ideal value of  $-10 \text{ k}\Omega$ ).

$$R_i = R_s \parallel R_F = 1 \parallel 10 = 909 \Omega$$

$$R_o = r_o \parallel R_F = 20 \parallel 10 = 6.67 \text{ k}\Omega$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{909}{4.03} = 226 \Omega$$

$$R_{in} = 1 \left/ \left[ \frac{1}{R_{if}} - \frac{1}{R_s} \right] \right. = 291 \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{6.67}{4.03} = 1.66 \text{ k}\Omega$$

$$R_{out} = R_{of} = 1.66 \text{ k}\Omega$$

Ex. 10.20 From Eq. (10.54), we obtain

$$A = -\mu \frac{R_i}{R_1 \parallel R_2} \frac{R_1 \parallel R_2 \parallel r_{o2}}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})}$$

For  $\mu = 100$ ,  $R_1 = 10 \text{ k}\Omega$ ,  $R_2 = 90 \text{ k}\Omega$ ,  $g_m = 5 \text{ mA/V}$ ,  $r_{o2} = 20 \text{ k}\Omega$ , we have

$$R_i = R_s \parallel R_{id} \parallel (R_1 + R_2)$$

$$= \infty \parallel \infty \parallel 100 = 100 \text{ k}\Omega$$

$$A = -100 \frac{100}{10 \parallel 90} \frac{10 \parallel 90 \parallel 20}{0.2 + (10 \parallel 90 \parallel 20)}$$

$$= -1076.4 \text{ A/A}$$

$$\beta = -\frac{R_1}{R_1 + R_2} = -\frac{10}{10 + 90} = -0.1 \text{ A/A}$$

$$A_f = -\frac{1076.4}{1 + 107.64} = -9.91 \text{ A/A}$$

### Exercise 10-9

This figure belongs to Exercise 10.19.

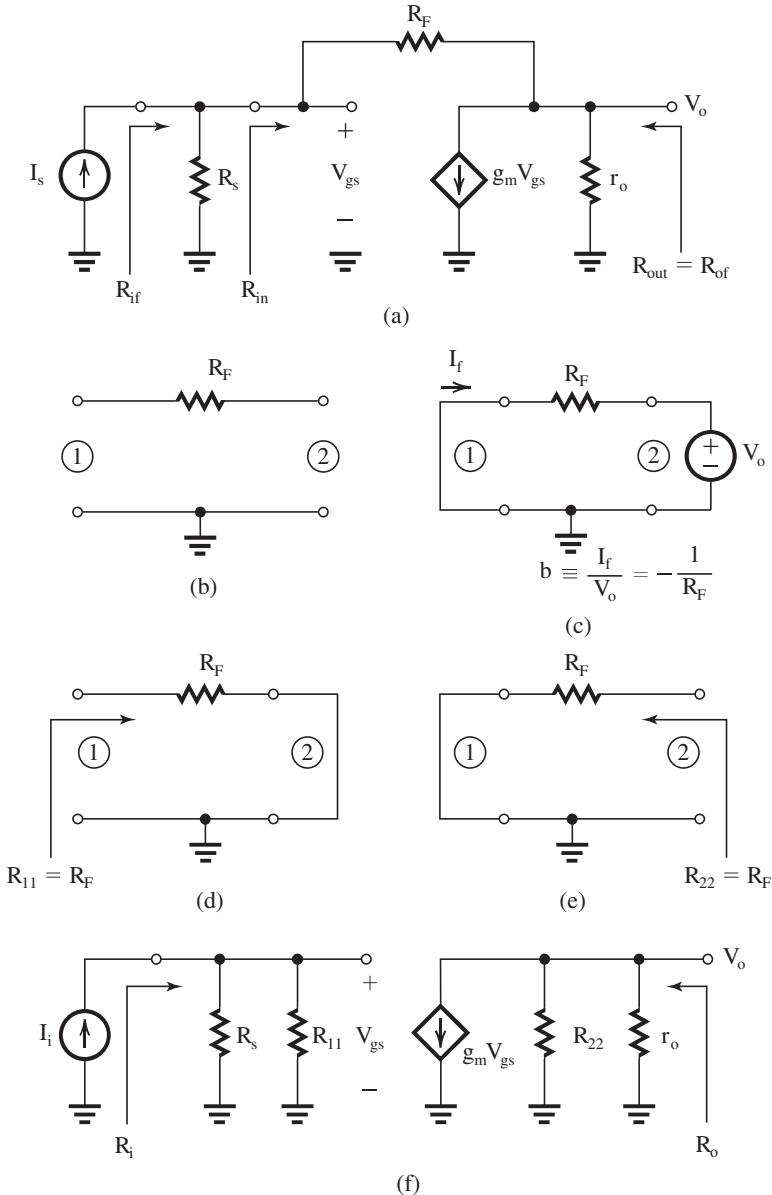


Figure 1

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{100 \text{ k}\Omega}{108.64} = 920 \Omega$$

$$R_{in} = R_{if} = 920 \Omega$$

$$R_{out} = R_{of} = R_o(1 + A\beta)$$

$$\begin{aligned} R_o &= r_{o2} + (R_1 \parallel R_2) + g_m r_{o2} (R_1 \parallel R_2) \\ &= 929 \text{ k}\Omega \end{aligned}$$

$$R_{out} = 929 \times 108.64 = 101 \text{ M}\Omega$$

Ex. 10.21 With  $R_2 = 0$ , Eq. (10.48) gives

$$\beta = -1$$

$$A_f = -1 \text{ A/A}$$

Substituting  $R_2 = 0$  and  $R_s = R_{id} = \infty$  in Eq. (10.50), we obtain

$$R_i = R_l$$

and in Eq. (10.55), we obtain

$$R_o = r_{o2}$$

and in Eq. (10.53), we obtain

$$A = -\mu \frac{R_l}{1/g_m} = -\mu g_m R_l$$

Now,

$$A_f = \frac{A}{1 + A\beta}$$

### Exercise 10–10

$$A_f = -\frac{\mu g_m R_i}{1 + \mu g_m R_i}$$

$$R_{in} = R_{if} = R_i / (1 + A\beta)$$

$$= \frac{R_i}{1 + \mu g_m R_i}$$

For  $\mu g_m R_i \gg 1$ , we have

$$R_{in} \simeq 1/\mu g_m$$

$$R_{out} = R_{of} = (1 + A\beta)R_o$$

$$= (1 + \mu g_m R_i)R_o$$

$$\simeq \mu (g_m R_o) R_i$$

Ex. 10.22 Total phase shift will be  $180^\circ$  at the frequency  $\omega_{180}$  at which the phase shift of each amplifier stage is  $60^\circ$ . Thus,

$$\tan^{-1} \frac{\omega_{180}}{10^4} = 60^\circ$$

$$\omega_{180} = \tan 60^\circ \times 10^4$$

$$= \sqrt{3} \times 10^4 \text{ rad/s}$$

At  $\omega_{180}$ , we have

$$|A| = \left( \frac{10}{\sqrt{1+3}} \right)^3$$

$$= 125$$

Thus, the loop gain magnitude will be

$$|A\beta| = 125\beta$$

For stable operation, we require

$$125\beta_{cr} < 1$$

$$\Rightarrow \beta_{cr} = \frac{1}{125} = 0.008$$

$\beta \geq \beta_{cr}$  will result in oscillations.

Correspondingly, the minimum closed-loop gain for stable operation will be

$$A_f = \frac{10^3}{1 + 10^3 \beta_{cr}}$$

$$= \frac{10^3}{1 + 1000 \times 0.008} = \frac{1000}{9} = 111.1$$

Ex. 10.23 The feedback shifts the pole by a factor equal to the amount of feedback:

$$1 + A_0\beta = 1 + 10^5 \times 0.01 = 1001$$

The pole will be shifted to a frequency

$$f_{pf} = f_p(1 + A_0\beta)$$

$$= 100 \times 1001 = 100.1 \text{ kHz}$$

If  $\beta$  is changed to a value that results in a nominal closed-loop gain of 1, then we obtain

$$\beta \simeq 1$$

and

$$1 + A_0\beta = 1 + 10^5 \times 1 \simeq 10^5$$

then the pole will be shifted to a frequency

$$f_{pf} = 10^5 \times 100 = 10 \text{ MHz}$$

Ex. 10.24 From Eq. (10.68), we see that the poles coincide when

$$(\omega_{p1} + \omega_{p2})^2 = 4(1 + A_0\beta)\omega_{p1}\omega_{p2}$$

$$(10^4 + 10^6)^2 = 4(1 + 100\beta) \times 10^4 \times 10^6$$

$$\Rightarrow 1 + 100\beta = 25.5$$

$$\Rightarrow \beta = 0.245$$

The corresponding value of  $Q = 0.5$ . This can also be verified by substituting in Eq. (10.70).

A maximally "at response is obtained when  $Q = 1/\sqrt{2}$ . Substituting in Eq. (10.70), we obtain

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{(1 + 100\beta) \times 10^4 \times 10^6}}{10^4 + 10^6}$$

$$\Rightarrow \beta = 0.5$$

In this case, the low-frequency closed-loop gain is

$$A_f(0) = \frac{A_0}{1 + A_0\beta}$$

$$= \frac{100}{1 + 100 \times 0.5} = 1.96 \text{ V/V}$$

Ex. 10.25 The closed-loop poles are the roots of the characteristic equation

$$1 + A(s)\beta = 0$$

$$1 + \left( \frac{10}{1 + \frac{s}{10^4}} \right)^3 \beta = 0$$

### Exercise 10–11

To simplify matters, we normalize  $s$  by the factor  $10^4$ , thus obtaining the normalized complex-frequency variable  $S = s/10^4$ , and the characteristic equation becomes

$$(S + 1)^3 + 10^3\beta = 0 \quad (1)$$

This equation has three roots, a real one and a pair that can be complex conjugate. The real pole can be found from

$$\begin{aligned} (S + 1)^3 &= -10^3\beta \\ \Rightarrow S &= -1 - 10\beta^{1/3} = -(1 + 10\beta^{1/3}) \end{aligned} \quad (2)$$

Dividing the characteristic polynomial in (1) by  $(S + 1 + 10\beta^{1/3})$  gives a quadratic whose two roots are the remaining poles of the feedback amplifier. After some straightforward but somewhat tedious algebra, we obtain

$$\begin{aligned} S^2 + (10\beta^{1/3} - 2)S + (1 + 100\beta^{2/3} - 10\beta^{1/3}) \\ = 0 \end{aligned} \quad (3)$$

The pair of poles can now be obtained as

$$S = (-1 + 5\beta^{1/3}) \pm j5\sqrt{3}\beta^{1/3} \quad (4)$$

Equations (1) and (3) describe the three poles shown in Fig. E10.25.

From Eq. (2) we see that the pair of complex poles lie on the  $j\omega$  axis for the value of  $\beta$  that makes the coefficient of  $S$  equal to zero, thus

$$\beta_{cr} = \left(\frac{2}{10}\right)^3 = 0.008$$

Note that this is the same value found in the solution of Exercise 10.22.

Ex. 10.26

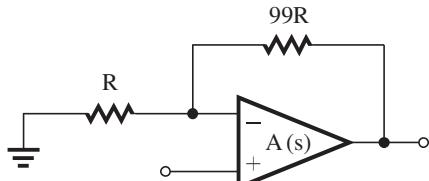


Figure 1

From Fig. 1, we can easily obtain the loop gain as

$$\begin{aligned} A\beta &= A(s) \times 0.01 \\ &= \frac{10^5}{1 + \frac{s}{2\pi \times 10}} \times 0.01 \\ &= \frac{1000}{1 + \frac{s}{2\pi \times 10}} \end{aligned}$$

From this single-pole response (low-pass STC response) we can find the unity-gain frequency by inspection as

$$\begin{aligned} f_l &= f_p \times 1000 \\ &= 10^4 \text{ Hz} \end{aligned}$$

The phase angle at  $f_l$  will be  $-90^\circ$  and thus the phase margin is  $90^\circ$ .

Ex. 10.27 From Eq. (10.82), we obtain

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + e^{-j\theta}|$$

$$= 1/|1 + \cos \theta - j \sin \theta|$$

(a) For  $PM = 30^\circ$ ,  $\theta = 180 - 30 = 150^\circ$ , thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 150^\circ - j \sin 150^\circ|$$

$$= 1.93$$

(b) For  $PM = 60^\circ$ ,  $\theta = 180 - 60 = 120^\circ$ , thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 120^\circ - j \sin 120^\circ|$$

$$= 1$$

(c) For  $PM = 90^\circ$ ,  $\theta = 180 - 90 = 90^\circ$ , thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 90^\circ - j \sin 90^\circ|$$

$$= 1/\sqrt{2} = 0.707$$

Ex. 10.28 See figure on next page. To obtain guaranteed stable performance, the maximum rate of closure must not exceed 20 dB/decade. Thus we utilize the graphical construction in Fig. 1 to obtain the

### Exercise 10–12

This figure belongs to Exercise 10.28.

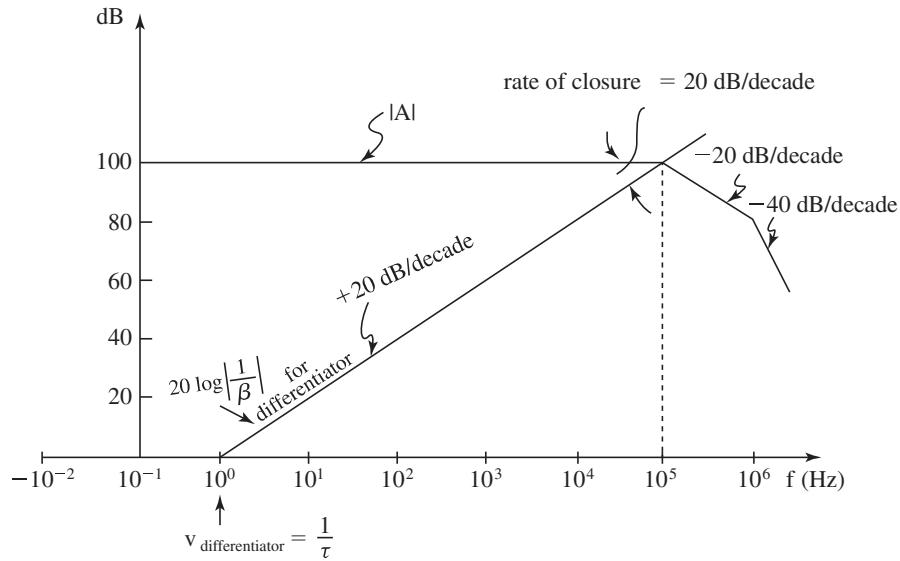


Figure 1

maximum value of the differentiator frequency as 1 Hz. Thus,

$$\frac{1}{\tau} \leq 2\pi \times 1 \text{ Hz}$$

$$\tau \geq \frac{1}{2\pi} \text{ s} = 159 \text{ ms}$$

Ex. 10.29 To obtain stable performance for closed-loop gains as low as 20 dB (which is 80 dB below  $A_0$ , or equivalently  $10^4$  below  $A_0$ ), we must place the new dominant pole at  $1 \text{ MHz}/10^4 = 100 \text{ Hz}$ .

Ex. 10.30 The frequency of the first pole must be lowered from 1 MHz to a new frequency

$$f'_D = \frac{10 \text{ MHz}}{10^4} = 1000 \text{ Hz}$$

that is, by a factor of 1000. Thus, the capacitance at the controlling node must be increased by a factor of 1000.

## Chapter 10–1

$$10.1 \quad A_f = \frac{A}{1 + A\beta}$$

$$200 = \frac{10^4}{1 + 10^4\beta}$$

$$\Rightarrow \beta = 4.9 \times 10^{-3}$$

If  $A$  changes to  $10^3$ , then we get

$$A_f = \frac{1000}{1 + 10^3 \times 4.9 \times 10^{-3}}$$

$$= \frac{1000}{5.9} = 169.5$$

$$\text{Percentage change in } A_f = \frac{169.5 - 200}{200} \times 100$$

$$= -15.3\%$$

10.2 (a) Because of the infinite input resistance of the op amp, the fraction of the output voltage  $V_o$  that is fed back and subtracted from  $V_s$  is determined by the voltage divider ( $R_1, R_2$ ), thus

$$\beta = \frac{R_1}{R_1 + R_2}$$

(b) (i)  $A = 1000 \text{ V/V}$

$$A_f = \frac{A}{1 + A\beta}$$

$$10 = \frac{1000}{1 + 1000\beta}$$

$$\Rightarrow \beta = 0.099 \text{ V/V}$$

$$\frac{R_1}{R_1 + R_2} = 0.099$$

$$1 + \frac{R_2}{R_1} = \frac{1}{0.099}$$

$$R_2 = R_1 \left( \frac{1}{0.099} - 1 \right)$$

$$= 10 \left( \frac{1}{0.099} - 1 \right) = 91 \text{ k}\Omega$$

(ii)  $A = 200 \text{ V/V}$

$$10 = \frac{200}{1 + 200\beta}$$

$$\Rightarrow \beta = 0.095 \text{ V/V}$$

$$R_2 = R_1 \left( \frac{1}{0.095} - 1 \right)$$

$$= 10 \left( \frac{1}{0.095} - 1 \right) = 95.3 \text{ k}\Omega$$

(iii)  $A = 15 \text{ V/V}$

$$10 = \frac{15}{1 + 15\beta}$$

$$\Rightarrow \beta = 0.033 \text{ V/V}$$

$$R_2 = 10 \left( \frac{1}{0.033} - 1 \right)$$

$$= 290 \text{ k}\Omega$$

$$(c) (i) \quad A = 1000(1 - 0.2) = 800 \text{ V/V}$$

$$A_f = \frac{800}{1 + 800 \times 0.099}$$

$$= 9.975 \text{ V/V}$$

Thus,  $A_f$  changes by

$$= \frac{9.975 - 10}{10} \times 100 = -0.25\%$$

$$(ii) \quad A = 200(1 - 0.2) = 160 \text{ V/V}$$

$$A_f = \frac{160}{1 + 160 \times 0.095} = 9.877 \text{ V/V}$$

Thus,  $A_f$  changes by

$$= \frac{9.877 - 10}{10} \times 100 = -1.23\%$$

$$(iii) \quad A = 15(1 - 0.2) = 12 \text{ V/V}$$

$$A_f = \frac{12}{1 + 12 \times 0.033} = 8.574$$

Thus,  $A_f$  changes by

$$= \frac{8.575 - 10}{10} \times 100 = -14.3\%$$

We conclude that as  $A$  becomes smaller and hence the amount of feedback  $(1 + A\beta)$  is lower, the desensitivity of the feedback amplifier to changes in  $A$  decreases. In other words, the negative feedback becomes less effective as  $(1 + A\beta)$  decreases.

10.3 The direct connection of the output terminal to the inverting input terminal results in  $V_f = V_o$  and thus

$$\beta = 1$$

If  $A = 1000$ , then the closed-loop gain will be

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{1 + 1000 \times 1} = 0.999 \text{ V/V}$$

$$\text{Amount of feedback} = 1 + A\beta$$

$$= 1 + 1000 \times 1 = 1001$$

or 60 dB

For  $V_s = 1 \text{ V}$ , we obtain

$$V_o = A_f V_s = 0.999 \times 1 = 0.999 \text{ V}$$

$$V_i = V_s - V_o = 1 - 0.999$$

$$= 0.001 \text{ V}$$

## Chapter 10–2

If  $A$  becomes  $1000(1 - 0.1) = 900$  V/V, then we get

$$A_f = \frac{900}{1 + 900 \times 1} = 0.99889$$

Thus,  $A_f$  changes by

$$= \frac{0.99889 - 0.999}{0.999} \times 100 = -0.011\%$$

$$10.4 \text{ A} = \frac{V_o}{V_i} = \frac{5 \text{ V}}{10 \text{ mV}} = 500 \text{ V/V}$$

$$V_f = V_s - V_i = 1 - 0.01 = 0.99 \text{ V}$$

$$\beta = \frac{V_f}{V_o} = \frac{0.99}{5} = 0.198 \text{ V/V}$$

$$10.5 \text{ (a)} \quad A_f = \frac{A}{1 + A\beta}$$

Ideally,

$$A_f = \frac{1}{\beta}$$

$$A_f|_{\text{ideal}} - A_f = \frac{1}{\beta} - \frac{A}{1 + A\beta} \\ = \frac{1 + A\beta - A\beta}{(1 + A\beta)\beta} = \frac{1}{(1 + A\beta)\beta}$$

Expressed as a percentage of the ideal gain  $1/\beta$ , we have

$$\frac{\text{Difference}}{\text{Ideal}} = \frac{1}{1 + A\beta} \times 100\%$$

For  $A\beta \gg 1$ ,

$$\frac{\text{Difference}}{\text{Ideal}} \approx \frac{100}{A\beta}\%$$

(b) For  $A_f$  to be within:

(i) 0.1% of ideal value, then

$$\frac{100}{A\beta} \leq 0.1$$

$$\Rightarrow A\beta \geq 1000$$

(ii) 1% of ideal value, then

$$\frac{100}{A\beta} \leq 1$$

$$\Rightarrow A\beta \geq 100$$

(iii) 5% of ideal value, then

$$\frac{100}{A\beta} \leq 5$$

$$\Rightarrow A\beta \geq 20$$

$$10.6 \quad A_{\text{nominal}} = 1000$$

$$A_{\text{low}} = 500$$

$$A_{\text{high}} = 1500$$

If we apply negative feedback with a feedback factor  $\beta$ , then

$$A_{f, \text{nominal}} = \frac{1000}{1 + 1000\beta}$$

$$A_{f, \text{low}} = \frac{500}{1 + 500\beta}$$

$$A_{f, \text{high}} = \frac{1500}{1 + 1500\beta}$$

It is required that

$$A_{f, \text{low}} \geq 0.99A_{f, \text{nominal}} \quad (1)$$

and

$$A_{f, \text{high}} \leq 1.01A_{f, \text{nominal}} \quad (2)$$

If we satisfy condition (1) with equality, we can determine the required value of  $\beta$ . We must then check that condition (2) is satisfied. Thus,

$$\frac{500}{1 + 500\beta} = 0.99 \times \frac{1000}{1 + 1000\beta}$$

$$\Rightarrow \beta = 0.098$$

For this value of  $\beta$ , we obtain

$$A_{f, \text{nominal}} = \frac{1000}{1 + 1000 \times 0.098}$$

$$= 10.101$$

$$A_{f, \text{low}} = \frac{500}{1 + 500 \times 0.098} = 10$$

$$A_{f, \text{high}} = \frac{1500}{1 + 1500 \times 0.098} = 10.135$$

Thus, the low value of the closed-loop gain is 0.101 below nominal or  $-1\%$ , and the high value is 0.034 above nominal or  $0.34\%$ . Thus, our amplifier meets specification and the nominal value of closed-loop gain is 10.1. This is the highest possible closed-loop gain that can be obtained while meeting specification.

Now, if three closed-loop amplifiers are placed in cascade, the overall gain obtained will be

$$\text{Nominal Gain} = (10.1)^3 = 1030$$

$$\text{Lowest Gain} = 10^3 = 1000$$

$$\text{Highest Gain} = (10.135)^3 = 1041$$

Thus, the lowest gain will be approximately 3% below nominal, and the highest gain will be 1% above nominal.

$$10.7 \text{ A} = \frac{5 \text{ V}}{2 \text{ mV}} = 2500 \text{ V/V}$$

$$A_f = \frac{5 \text{ V}}{100 \text{ mV}} = 50 \text{ V/V}$$

Amount of feedback  $\equiv 1 + A\beta$

$$= \frac{A}{A_f} = \frac{2500}{50} = 50$$

or 34 dB

$A\beta = 49$

$$\beta = \frac{49}{2500} = 0.0196 \text{ V/V}$$

we get

$$\begin{aligned} A &= \mu \frac{R_1 \parallel R_{id}}{(R_1 \parallel R_{id}) + R_2} \frac{R_1 + R_2}{R_1} \\ &= \mu \frac{R_{id}/(R_1 + R_{id})}{R_1 R_{id}/(R_1 + R_{id}) + R_2} (R_1 + R_2) \\ &= \mu \frac{R_{id}(R_1 + R_2)}{R_1 R_{id} + R_2 R_{id} + R_1 R_2} \end{aligned}$$

Thus,

$$A = \mu \frac{R_{id}}{R_{id} + (R_1 \parallel R_2)} \quad \text{Q.E.D.}$$

10.8

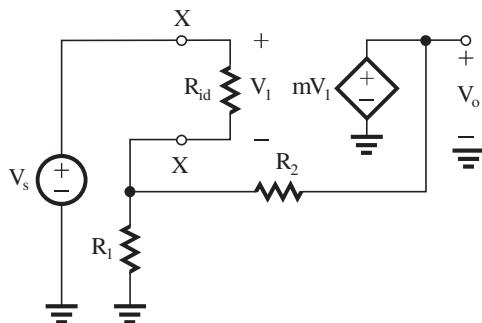


Figure 1

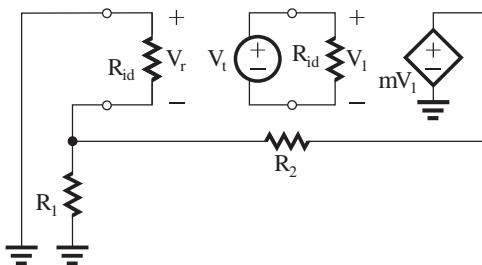


Figure 2

Figure 1 shows the given circuit with the op amp replaced with its equivalent circuit model. To determine the loop gain  $A\beta$ , we short circuit  $V_s$  and break the loop at the input terminals of the op amp. To keep the circuit unchanged, we must place a resistance equal to  $R_{id}$  at the left-hand side of the break. This is shown in Fig. 2, where a test signal  $V_t$  is applied at the right-hand side of the break. To determine the returned voltage  $V_r$ , we use the voltage-divider rule as follows:

$$V_r = -\mu V_t \frac{R_1 \parallel R_{id}}{(R_1 \parallel R_{id}) + R_2}$$

Substituting  $V_1 = V_t$  and rearranging, we obtain

$$A\beta \equiv -\frac{V_r}{V_t} = \mu \frac{R_1 \parallel R_{id}}{(R_1 \parallel R_{id}) + R_2}$$

Since

$$\beta = \frac{R_1}{R_1 + R_2}$$

10.9 From Eq. (10.10), we have

$$\frac{dA_f/A_f}{dA/A} = \frac{1}{1 + A\beta}$$

Since  $-40 \text{ dB}$  is 0.01, we have

$$0.01 = \frac{1}{1 + A\beta}$$

$$\Rightarrow A\beta = 99$$

For

$$\frac{dA_f/A_f}{dA/A} = \frac{1}{5}$$

we have

$$1 + A\beta = 5$$

$$\Rightarrow A\beta = 4$$

10.10  $A_f = 10 \text{ V/V}$

$$1 + A\beta = \frac{\pm 10\%}{\pm 0.1\%} = 100$$

$$10 = \frac{A}{100}$$

$$\Rightarrow A = 1000 \text{ V/V}$$

$$\beta = \frac{100 - 1}{1000} = 0.099 \text{ V/V}$$

10.11 For  $A = 1000 \text{ V/V}$ , we have

$$A_f = 10 = \frac{1000}{1 + A\beta}$$

$\Rightarrow$  Densensitivity factor  $\equiv 1 + A\beta = 100$

$$A\beta = 99$$

$$\beta = \frac{99}{1000} = 0.099 \text{ V/V}$$

For  $A = 500 \text{ V/V}$ , we have

$$A_f = 10 = \frac{500}{1 + A\beta}$$

$\Rightarrow$  Densensitivity factor  $\equiv 1 + A\beta = 50$

$$\beta = \frac{49}{500} = 0.098 \text{ V/V}$$

If the  $A = 1000$  amplifiers have a gain uncertainty of  $\pm 10\%$ , the gain uncertainty of the closed-loop amplifiers will be

$$= \frac{\pm 10\%}{100} = \pm 0.1\%$$

If we require a gain uncertainty of  $\pm 0.1\%$  using the  $A = 500$  amplifiers, then

$$\pm 0.1\% = \frac{\text{Gain uncertainty of } A = 500 \text{ amplifiers}}{50}$$

$$\Rightarrow \text{Gain uncertainty} = \pm 5\%$$

10.12 Let the gain of the ideal (nonvarying) driver amplifier be denoted  $\mu$ . Then, the open-loop gain  $A$  will vary from  $2\mu$  to  $12\mu$ . Correspondingly, the closed-loop gain will vary from 95 V/V to 105 V/V. Substituting these quantities into the closed-loop gain expression, we obtain

$$95 = \frac{2\mu}{1 + 2\mu\beta} \quad (1)$$

$$105 = \frac{12\mu}{1 + 12\mu\beta} \quad (2)$$

Dividing Eq. (2) by Eq. (1) yields

$$1.105 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$$1.105 + 1.105 \times 12\mu\beta = 6 + 12\mu\beta$$

$$\Rightarrow \mu\beta = 3.885$$

Substituting in Eq. (1) yields

$$\mu = \frac{95(1 + 2 \times 3.885)}{2} = 416.6 \text{ V/V}$$

$$\beta = \frac{3.885}{416.6} = 9.33 \times 10^{-3} \text{ V/V}$$

If  $A_f$  is to be held to within  $\pm 0.5\%$ , Eqs. (1) and (2) are modified to

$$99.5 = \frac{2\mu}{1 + 2\mu\beta} \quad (3)$$

$$100.5 = \frac{12\mu}{1 + 12\mu\beta} \quad (4)$$

Dividing (4) by (3) yields

$$1.01 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$$\Rightarrow \mu\beta = 49.92$$

Substituting into (3) provides

$$\begin{aligned} \mu &= \frac{99.5(1 + 2 \times 49.92)}{2} \\ &= 5016.8 \text{ V/V} \end{aligned}$$

which is more than a factor of 10 higher than the gain required in the less constrained case. The value of  $\beta$  required is

$$\beta = \frac{49.92}{5016.8} = 9.95 \times 10^{-3} \text{ V/V}$$

Repeating for  $A_f = 10$  V/V (a factor of 10 lower than the original case):

(a) For  $\pm 5\%$  maximum variability, Eqs. (1) and (2) become

$$9.5 = \frac{2\mu}{1 + 2\mu\beta} \quad (5)$$

$$10.5 = \frac{12\mu}{1 + 12\mu\beta} \quad (6)$$

Dividing (6) by (5) yields

$$1.105 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$$\Rightarrow \mu\beta = 3.885$$

which is identical to the first case considered, and

$$\mu = \frac{9.5(1 + 2 \times 3.885)}{1 + 12\mu\beta} = 41.66 \text{ V/V}$$

which is a factor of 10 lower than the value required when the gain required was 100. The feedback factor  $\beta$  is

$$\beta = \frac{3.885}{41.66} = 9.33 \times 10^{-2} \text{ V/V}$$

which is a factor of 10 higher than the case with  $A_f = 10$ .

(b) Finally, for the case  $A_f = 10 \pm 0.5\%$  we can write by analogy

$$\mu\beta = 49.92$$

$$\mu = 5016.8 \text{ V/V}$$

$$\beta = 9.95 \times 10^{-3} \text{ V/V}$$

10.13 If the nominal open-loop gain is  $A$ , then we require that as  $A$  drops to  $(A/2)$  the closed-loop gain drops from 10 to a minimum of 9.8.

Substituting these values in the expression for the closed-loop gain, we obtain

$$10 = \frac{A}{1 + A\beta} \quad (1)$$

$$9.8 = \frac{A/2}{1 + \frac{1}{2}A\beta} \quad (2)$$

Dividing Eq. (1) by Eq. (2) yields

$$1.02 = \frac{2 \left( 1 + \frac{1}{2}A\beta \right)}{1 + A\beta}$$

$$\begin{aligned} 1.02 &= \frac{2 + A\beta}{1 + A\beta} \\ &= 1 + \frac{1}{1 + A\beta} \\ \Rightarrow 1 + A\beta &= \frac{1}{0.02} = 50 \end{aligned}$$

Substituting in Eq. (1) gives

$$A = 10 \times 50 = 500 \text{ V/V}$$

and

$$\beta = \frac{50 - 1}{500} = 0.098 \text{ V/V}$$

If  $\beta$  is accurate to within  $\pm 1\%$ , to ensure that the minimum closed-loop gain realized is 9.8 V/V, we have

$$\begin{aligned} 9.8 &= \frac{A/2}{1 + \frac{1}{2}A \times 0.098 \times 1.01} \\ \Rightarrow A &= 653.4 \text{ V/V} \end{aligned}$$

10.14 If we use one stage, the amount of feedback required is

$$1 + A\beta = \frac{A}{A_f} = \frac{1000}{100} = 10$$

Thus the closed-loop amplifier will have a variability of

$$\text{Variability of } A_f = \frac{\pm 30\%}{10} = \pm 3\%$$

which does not meet specifications. Next, we try using two stages. For a nominal gain of 100, each stage will be required to have a nominal gain of 10. Thus, for each stage the amount of feedback required will be

$$1 + A\beta = \frac{1000}{10} = 100$$

Thus, the closed-loop gain of each stage will have a variability of

$$= \frac{\pm 30\%}{100} = \pm 0.3\%$$

and the cascade of two stages will thus show a variability of  $\pm 0.6\%$ , well within the required  $\pm 1\%$ . Thus two stages will suffice.

We next investigate the design in more detail. Each stage will have a nominal gain of 10 and thus

$$\begin{aligned} 1 + A\beta &= \frac{1000}{10} = 100 \\ \Rightarrow A\beta &= 99 \\ \Rightarrow \beta &= 0.099 \end{aligned}$$

Since  $A$  ranges from 700 V/V to 1300 V/V, the gain of each stage will range from

$$A_{f, \text{low}} = \frac{700}{1 + 700 \times 0.099} = 9.957 \text{ V/V}$$

and a high value of

$$A_{f, \text{high}} = \frac{1300}{1 + 1300 \times 0.099} = 10.023 \text{ V/V}$$

Thus, the cascade of two stages will have a range of

$$\text{Lowest gain} = 9.957^2 = 99.14 \text{ V/V}$$

$$\text{Highest gain} = 10.023^2 = 100.46 \text{ V/V}$$

which is  $-0.86\%$  to  $+0.46\%$  of the nominal 100 V/V gain, well within the required  $\pm 1\%$ .

$$10.15 \quad A_f = \frac{A}{1 + A\beta}$$

$$100 = \frac{A}{1 + A\beta} \quad (1)$$

$$99 = \frac{0.1A}{1 + 0.1A\beta} \quad (2)$$

Dividing Eq. (1) by Eq. (2) gives

$$1.01 = \frac{10(1 + 0.1A\beta)}{1 + A\beta}$$

$$= \frac{10 + A\beta}{1 + A\beta}$$

$$= 1 + \frac{9}{1 + A\beta}$$

$$\Rightarrow \frac{9}{1 + A\beta} = 0.01$$

$$1 + A\beta = 900$$

$$A\beta = 899$$

Substituting  $(1 + A\beta) = 900$  into Eq. (1) yields

$$A = 100 \times 900 = 90,000 \text{ V/V}$$

The value of  $\beta$  is

$$\beta = \frac{899}{90,000} = 9.989 \times 10^{-3} \text{ V/V}$$

If  $A$  were increased tenfold, i.e.,  $A = 900,000$ , we obtain

$$A_f = \frac{900,000}{1 + 899} = 100.1 \text{ V/V}$$

If  $A$  becomes infinite, we get

$$\begin{aligned} A_f &= \frac{A}{1 + A\beta} \\ &= \frac{1}{\frac{1}{A} + \beta} = \frac{1}{\beta} \\ &= \frac{1}{9.989 \times 10^{-3}} = 100.11 \text{ V/V} \end{aligned}$$

$$\begin{aligned}
 10.16 \quad A &= A_M \frac{s}{s + \omega_L} \\
 A_f &= \frac{A}{1 + A\beta} \\
 &= \frac{A_M s / (s + \omega_L)}{1 + A_M \beta s / (s + \omega_L)} \\
 &= \frac{A_M s}{s + \omega_L + s A_M \beta} \\
 &= \frac{A_M s}{s(1 + A_M \beta) + \omega_L} \\
 &= \frac{A_M}{1 + A_M \beta} \frac{s}{s + \omega_L / (1 + A_M \beta)}
 \end{aligned}$$

Thus,

$$\begin{aligned}
 A_{Mf} &= \frac{A_M}{1 + A_M \beta} \\
 \omega_{Lf} &= \frac{\omega_L}{1 + A_M \beta}
 \end{aligned}$$

Thus, both the midband gain and the 3-dB frequency are lowered by the amount of feedback,  $(1 + A_M \beta)$ .

$$10.17 \quad 1 + A_M \beta = \frac{1000}{10} = 100$$

Thus,

$$\begin{aligned}
 f_{Hf} &= (1 + A_M \beta) f_H \\
 &= 100 \times 10 = 1000 \text{ kHz} = 1 \text{ MHz} \\
 f_{Lf} &= \frac{f_L}{1 + A_M \beta} \\
 &= \frac{100}{100} = 1 \text{ Hz}
 \end{aligned}$$

10.18 To capacitively couple the output signal to an 8-Ω loudspeaker and obtain  $f_L = 100$  Hz, we need a coupling capacitor  $C$ ,

$$\begin{aligned}
 C &= \frac{1}{2\pi f_L \times 8} \\
 &= \frac{1}{2\pi \times 100 \times 8} = 198.9 \mu\text{F} \simeq 200 \mu\text{F}
 \end{aligned}$$

If closed-loop gain  $A_{Mf}$  of 10 V/V is obtained from an amplifier whose open-loop gain  $A_M = 1000$  V/V, then

$$1 + A_M \beta = \frac{1000}{10} = 100$$

and

$$f_{Lf} = \frac{f_L}{100} = \frac{100}{100} = 1 \text{ Hz}$$

If the required  $f_{Lf}$  is 50 Hz, then

$$\begin{aligned}
 f_L &= 50 \times (1 + A_M \beta) \\
 &= 50 \times 100 = 5000 \text{ Hz},
 \end{aligned}$$

and the coupling capacitor  $C$  will have a value of

$$C = \frac{1}{2\pi \times 5000 \times 8} \simeq 4 \mu\text{F}$$

10.19 Let  $s$  first try  $N = 2$ . The closed-loop gain of each stage must be

$$A_f = \sqrt{1000} = 31.6 \text{ V/V}$$

Thus, the amount-of-feedback in each stage must be

$$1 + A\beta = \frac{A}{A_f} = \frac{1000}{31.6} = 31.6$$

The 3-dB frequency of each stage is

$$\begin{aligned}
 f_{3dB}|_{\text{stage}} &= (1 + A\beta) f_H \\
 &= 31.6 \times 20 = 632 \text{ kHz}
 \end{aligned}$$

Thus, the 3-dB frequency of the cascade amplifier is

$$f_{3dB}|_{\text{cascade}} = 632\sqrt{2^{1/2} - 1} = 406.8 \text{ kHz}$$

which is less than the required 1 MHz.

Next, we try  $N = 3$ . The closed-loop gain of each stage is

$$A_f = (1000)^{1/3} = 10 \text{ V/V}$$

and thus each stage will have an amount-of-feedback

$$1 + A\beta = \frac{1000}{10} = 100$$

which results in a stage 3-dB frequency of

$$\begin{aligned}
 f_{3dB}|_{\text{stage}} &= (1 + A\beta) f_H \\
 &= 100 \times 20 = 2000 \text{ kHz} \\
 &= 2 \text{ MHz}
 \end{aligned}$$

The 3-dB frequency of the cascade amplifier will be

$$\begin{aligned}
 f_{3dB}|_{\text{cascade}} &= 2\sqrt{2^{1/3} - 1} \\
 &= 1.02 \text{ MHz}
 \end{aligned}$$

which exceeds the required value of 1 MHz. Thus, we need three identical stages, each with a closed-loop gain of 10 V/V, an amount-of-feedback of 100, and a loop gain

$$A\beta = 99$$

Thus,

$$\beta = 0.099 \text{ V/V}$$

$$10.20 \quad A_f = \frac{A_1 A_2}{1 + A_1 A_2 \beta}$$

$$100 = \frac{10 A_2}{1 + A_1 A_2 \beta}$$

$$(1 + A_1 A_2 \beta) \times 8 = 40 \text{ kHz}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 5$$

Substituting in (1) gives

$$A_2 = \frac{100 \times 5}{10} = 50 \text{ V/V}$$

$$1 + 10 \times 50 \times \beta = 5$$

$$\Rightarrow \beta = 0.008 \text{ V/V}$$

$$f_{Lf} = \frac{80}{1 + A_1 A_2 \beta}$$

$$= \frac{80}{5} = 16 \text{ Hz}$$

$$10.21 \quad V_o \text{ ripple} = V_n \frac{A_1}{1 + A_1 A_2 \beta}$$

To reduce  $V_o$  ripple to 100 mV,

$$0.1 = 1 \times \frac{0.9}{1 + A_1 A_2 \beta}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 9$$

$$A_f = \frac{A_1 A_2}{1 + A_1 A_2 \beta}$$

$$10 = \frac{0.9 A_2}{9}$$

$$\Rightarrow A_2 = 100 \text{ V/V}$$

$$\beta = \frac{8}{0.9 \times 100} = 0.089 \text{ V/V}$$

To reduce  $V_o$  ripple to 10 mV,

$$0.01 = 1 \times \frac{0.9}{1 + A_1 A_2 \beta}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 90$$

$$A_f = \frac{A_1 A_2}{1 + A_1 A_2 \beta}$$

$$10 = \frac{0.9 A_2}{90}$$

$$A_2 = 1000 \text{ V/V}$$

$$\beta = \frac{89}{0.9 \times 1000} = 0.099 \text{ V/V}$$

To reduce  $V_o$  ripple to 1 mV,

$$0.001 = 1 \times \frac{0.9}{1 + A_1 A_2 \beta}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 900$$

$$\begin{aligned} 10 &= \frac{0.9 A_2}{900} \\ \Rightarrow A_2 &= 10,000 \text{ V/V} \\ \beta &= \frac{899}{0.9 \times 10,000} = 0.0999 \text{ V/V} \end{aligned}$$

10.22

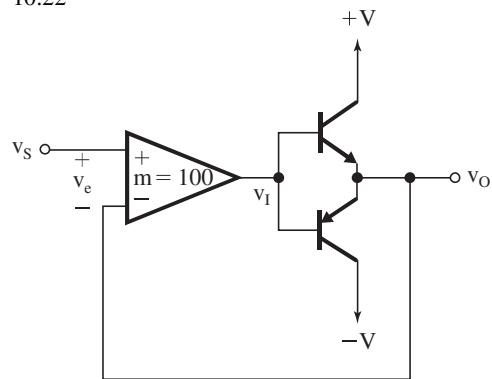


Figure 1

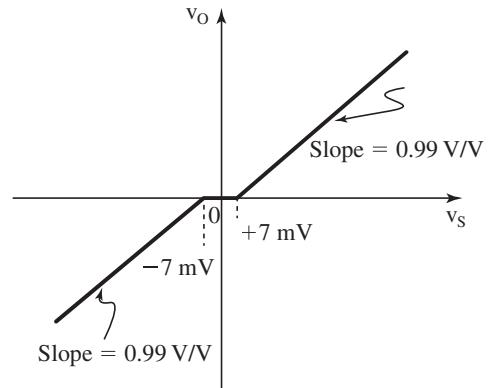


Figure 2

Refer to Fig. 1. For  $v_I = +0.7 \text{ V}$ , we have  $v_O = 0$  and

$$v_e = \frac{v_I}{\mu} = \frac{+0.7}{100} = +7 \text{ mV}$$

Similarly, for  $v_I = -0.7 \text{ V}$ , we obtain  $v_O = 0$  and

$$v_e = \frac{v_I}{\mu} = \frac{-0.7}{100} = -7 \text{ mV}$$

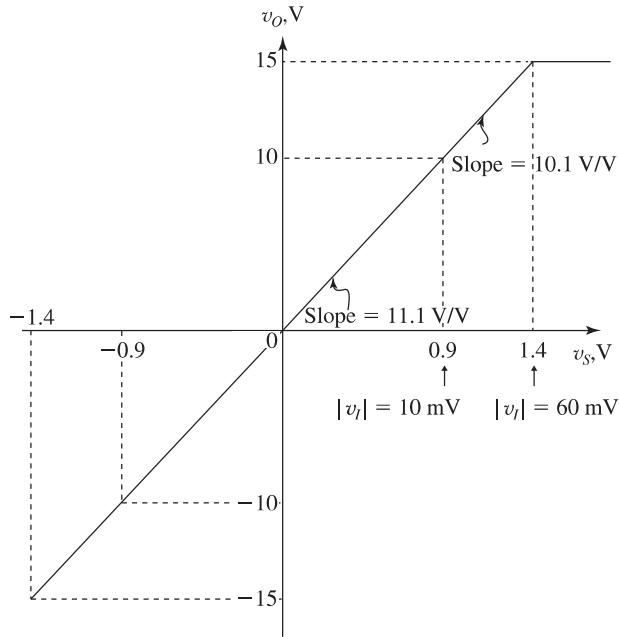
Thus, the limits of the deadband are now  $\pm 7 \text{ mV}$ . Outside the deadband, the gain of the feedback amplifier, that is,  $v_O/v_e$ , can be determined by

noting that the open-loop gain  $A \equiv v_O/v_e = 100 \text{ V/V}$  and the feedback factor  $\beta = 1$ , thus

$$\begin{aligned} A_f &\equiv \frac{v_O}{v_S} = \frac{A}{1 + A\beta} \\ &= \frac{100}{1 + 100 \times 1} = 0.99 \text{ V/V} \end{aligned}$$

The transfer characteristic is depicted in Fig. 2.

This figure belongs to Problem 10.23.



10.23 The closed-loop gain for the first (high-gain) segment is

$$A_{f1} = \frac{1000}{1 + 1000\beta} \quad (1)$$

and that for the second segment is

$$A_{f2} = \frac{100}{1 + 100\beta} \quad (2)$$

We require

$$\frac{A_{f1}}{A_{f2}} = 1.1$$

Thus, dividing Eq. (1) by Eq. (2) yields

$$1.1 = 10 \frac{1 + 100\beta}{1 + 1000\beta}$$

$$1.1 + 1100\beta = 10 + 1000\beta$$

$$\Rightarrow \beta = 0.089$$

$$A_{f1} = \frac{1000}{1 + 1000 \times 0.089} = 11.1 \text{ V/V}$$

$$A_{f2} = \frac{100}{1 + 100 \times 0.089} = 10.1 \text{ V/V}$$

The first segment ends at

$|v_o| = 10 \text{ mV} \times 1000 = 10 \text{ V}$ . This corresponds to

$$v_s = \frac{10 \text{ V}}{A_{f1}} = \frac{10}{11.1} = 0.9 \text{ V}$$

The second segment ends at  $|v_o| = 10 + 0.05 \times 100 = 15 \text{ V}$ . This corresponds to

$$v_s = 0.9 + \frac{15 - 10}{A_{f2}}$$

$$= 0.9 + \frac{5}{10.1} = 1.4 \text{ V}$$

Thus, the transfer characteristic of the feedback amplifier can be described as follows:

For  $|v_s| \leq 0.9 \text{ V}$ ,  $v_o/v_s = 11.1 \text{ V/V}$

For  $0.9 \text{ V} \leq |v_s| \leq 1.4 \text{ V}$ ,  $v_o/v_s = 10.1 \text{ V/V}$

For  $|v_s| \geq 1.4 \text{ V}$ ,  $v_o = \pm 15 \text{ V}$

The transfer characteristic is shown in the above figure.

10.24 Because the op amp has an infinite input resistance and a zero output resistance, this circuit is a direct implementation of the ideal feedback structure and thus

$$A = 1000 \text{ V/V}$$

and

$$\beta = \frac{R_1}{R_1 + R_2}$$

The ideal closed-loop gain is

$$A_f = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

Thus,

$$10 = 1 + \frac{R_2}{10}$$

$$\Rightarrow R_2 = 90 \text{ k}\Omega$$

$$\beta = \frac{10}{10 + 90} = 0.1 \text{ V/V}$$

$$A\beta = 1000 \times 0.1 = 100$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{1 + 100} = 9.9 \text{ V/V}$$

To obtain  $A_f$  that is exactly 10, we use

$$10 = \frac{1000}{1 + A\beta}$$

$$\Rightarrow A\beta = 99$$

$$\beta = 0.099$$

$$0.099 = \frac{R_1}{R_1 + R_2}$$

$$0.099 = \frac{10}{10 + R_2}$$

$$\Rightarrow R_2 = 91 \text{ k}\Omega$$

10.25 Refer to Fig. 10.11.

(a) The ideal closed-loop gain is given by

$$A_f = \frac{1}{\beta} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

$$10 = 1 + \frac{R_2}{10}$$

$$\Rightarrow R_2 = 90 \text{ k}\Omega$$

(b) From Example 10.3, we obtain

$$A\beta = \mu \frac{R_L \parallel [R_2 + R_1 \parallel (R_{id} + R_s)]}{[R_L \parallel [R_2 + R_1 \parallel (R_{id} + R_s)]] + r_o}$$

$$\times \frac{R_1 \parallel (R_{id} + R_s)}{[R_1 \parallel (R_{id} + R_s)] + R_2} \times \frac{R_{id}}{R_{id} + R_s}$$

$$A\beta = 1000 \frac{10 \parallel [90 + 10 \parallel (100 + 100)]}{\{10 \parallel [90 + 10 \parallel (100 + 100)]\} + 1}$$

$$\times \frac{10 \parallel (100 + 100)}{[10 \parallel (100 + 100)] + 90} \times \frac{100}{100 + 100}$$

$$= 1000 \times 0.9009 \times 0.0957 \times 0.5$$

$$= 43.11$$

$$A = \frac{A\beta}{\beta} = \frac{43.11}{0.1} = 431.1 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{431.1}{1 + 43.11} = 9.77 \text{ V/V}$$

(c) To obtain  $A_f = 9.9 \text{ V/V}$ , we use

$$9.9 = \frac{A}{1 + A\beta}$$

$$= \frac{A}{1 + A \times 0.1}$$

$$\Rightarrow A = 1010 \text{ V/V}$$

Thus  $\mu$  must be increased by the factor

$$\frac{1010}{431.1} = 2.343 \text{ to become}$$

$$\mu = 2343 \text{ V/V}$$

10.26 Refer to Fig. 10.10.

$$(a) \beta = \frac{R_1}{R_1 + R_2}$$

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$5 = 1 + \frac{R_2}{1}$$

$$\Rightarrow R_2 = 4 \text{ k}\Omega$$

(b) From Example 10.2, we have

$$A\beta = (g_{m1}R_{D1})(g_{m2}R_{D2}) \frac{1}{1 + g_{m1}R_1} \times$$

$$\frac{R_1}{R_{D2} + R_2 + \left( R_1 \parallel \frac{1}{g_{m1}} \right)}$$

$$= (4 \times 10)(4 \times 10) \frac{1}{1 + 4 \times 1} \times \frac{1}{10 + 4 + (1 \parallel 0.25)}$$

$$= 22.54$$

$$A = \frac{A\beta}{\beta} = \frac{22.54}{0.2} = 112.7 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{112.7}{1 + 22.54} = 4.79 \text{ V/V}$$

10.27 (a) The feedback network consists of the voltage divider ( $R_1 R_2$ ), thus

$$\beta = \frac{R_1}{R_1 + R_2}$$

If the loop gain is large, the closed-loop gain approaches the ideal value

$$A_f = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$= 1 + \frac{10}{1} = 11 \text{ V/V}$$

(b)

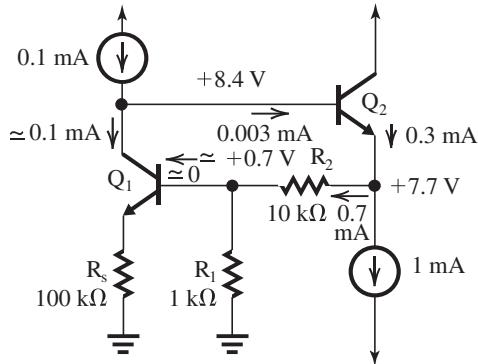


Figure 1

The dc analysis is shown in Fig. 1 from which we see that

$$I_{E1} \approx 0.1 \text{ mA}$$

$$I_{E2} \approx 0.3 \text{ mA}$$

$$V_{E2} = +7.7 \text{ V}$$

(c) Setting  $V_s = 0$  and eliminating dc sources, the feedback amplifier circuit simplifies to that shown in Fig. 2.

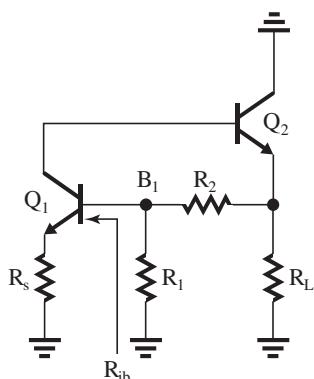


Figure 2

Now, breaking the feedback loop at the base of  $Q_1$  while terminating the right-hand side of the circuit (behind the break) in the resistance  $R_{ib}$ ,

$$R_{ib} = (\beta_1 + 1)(r_{e1} + R_s)$$

results in the circuit in Fig. 3 which we can use to determine the loop gain  $A\beta$  as follows:

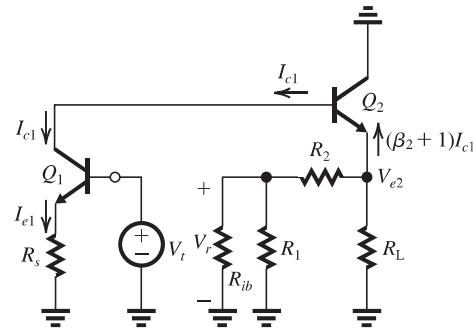


Figure 3

$$I_{e1} = \frac{V_t}{r_{e1} + R_s} \quad (1)$$

$$I_{c1} = \alpha_1 I_{e1} \quad (2)$$

$$V_{e2} = -(\beta_2 + 1) I_{c1} \{ R_L \parallel [R_2 + (R_1 \parallel R_{ib})] \} \quad (3)$$

$$V_r = V_{e2} \frac{R_1 \parallel R_{ib}}{(R_1 \parallel R_{ib}) + R_2} \quad (4)$$

Combining (1) to (4), we can determine  $A\beta$  as

$$\begin{aligned} A\beta &\equiv -\frac{V_r}{V_t} \\ &= \alpha_1 \frac{(\beta_2 + 1) \{ R_L \parallel [R_2 + (R_1 \parallel R_{ib})] \}}{r_{e1} + R_s} \\ &\quad \times \frac{R_1 \parallel R_{ib}}{(R_1 \parallel R_{ib}) + R_2} \end{aligned}$$

Substituting

$$\alpha_1(\beta_2 + 1) = \alpha(\beta + 1) = \beta = 100$$

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \Omega$$

$$R_s = 100 \Omega$$

$$R_L = 1 \text{ k}\Omega$$

$$R_1 = 1 \text{ k}\Omega$$

$$R_2 = 10 \text{ k}\Omega$$

$$R_{ib} = 101(0.25 + 0.1) = 35.35 \text{ k}\Omega$$

we obtain

$$\begin{aligned} A\beta &= \frac{100 \{ 1 \parallel [10 + (1 \parallel 35.35)] \}}{0.25 + 0.1} \\ &\quad \times \frac{1 \parallel 35.35}{(1 \parallel 35.35) + 10} \\ &= 23.2 \end{aligned}$$

$$(d) A = \frac{A\beta}{\beta} = \frac{23.2}{(1/11)} = 255.2 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{255.2}{1 + 23.2} = 10.5 \text{ V/V}$$

10.28 (a) The feedback network consists of the voltage divider ( $R_F, R_{S1}$ ). Thus,

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

and the ideal value of the closed-loop gain is

$$A_f = \frac{1}{\beta} = 1 + \frac{R_F}{R_{S1}}$$

$$10 = 1 + \frac{R_F}{0.1}$$

$$\Rightarrow R_F = 0.9 \text{ k}\Omega$$

(b)

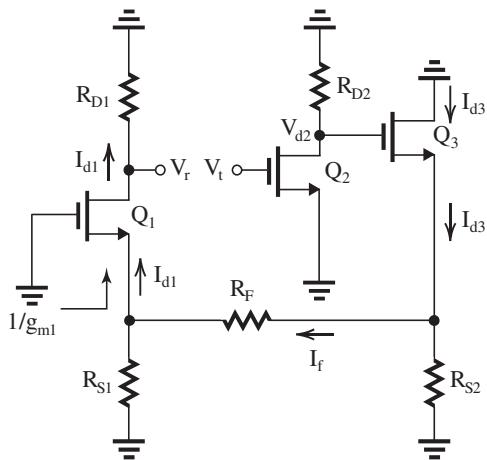


Figure 1

Figure 1 shows the circuit for determining the loop gain. Observe that we have broken the loop at the gate of  $Q_2$  where the input resistance is infinite, obviating the need for adding a termination resistance. Also, observe that as usual we have set  $V_s = 0$ . To determine the loop gain

$$A\beta \equiv -\frac{V_r}{V_t}$$

we write the following equations:

$$V_{d2} = -g_{m2}R_{D2}V_t \quad (1)$$

$$I_{d3} = \frac{V_{d2}}{\frac{1}{g_{m3}} + \left\{ R_{S2} \parallel \left[ R_F + \left( R_{S1} \parallel \frac{1}{g_{m1}} \right) \right] \right\}} \quad (2)$$

$$I_f = I_{d3} \frac{R_{S2}}{\left[ R_F + \left( R_{S1} \parallel \frac{1}{g_{m1}} \right) \right] + R_{S2}} \quad (3)$$

$$I_{d1} = I_f \frac{R_{S1}}{R_{S1} + \frac{1}{g_{m1}}} \quad (4)$$

$$V_r = I_{d1}R_{D1} \quad (5)$$

Substituting the numerical values in (1)–(5), we obtain

$$V_{d2} = -4 \times 10V_t = -40V_t \quad (6)$$

$$I_{d3} = \frac{V_{d2}}{\frac{1}{4} + \left\{ 0.1 \parallel \left[ 0.9 + \left( 0.1 \parallel \frac{1}{4} \right) \right] \right\}} \quad (7)$$

$$I_{d3} = 2.935V_{d2} \quad (7)$$

$$I_f = I_{d3} \frac{0.1}{\left[ 0.9 + \left( 0.1 \parallel \frac{1}{4} \right) \right] + 0.1} \quad (8)$$

$$I_f = 0.0933I_{d3} \quad (8)$$

$$I_{d1} = I_f \frac{0.1}{0.1 + \frac{1}{4}} = 0.286I_f \quad (9)$$

$$V_r = 10I_{d1} \quad (10)$$

Combining (6)–(10) gives

$$V_r = -31.33V_t$$

$$\Rightarrow A\beta = 31.33$$

$$A = \frac{A\beta}{\beta} = \frac{31.33}{0.1} = 313.3 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{313.3}{1 + 313.3\beta} = 9.7 \text{ V/V}$$

Thus,  $A_f$  is 0.3 V/V lower than the ideal value of 10 V/V, a difference of  $-3\%$ . The circuit could be adjusted to make  $A_f$  exactly 10 by changing  $\beta$  through varying  $R_F$ . Specifically,

$$10 = \frac{313.3}{1 + 313.3\beta}$$

$$\Rightarrow \beta = 0.0968$$

But,

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$



$$I_{e1} = I_{e3} \frac{50}{50 + 25} = 0.667 I_{e3} \quad (9)$$

$$V_r = -0.99(2 \parallel 1.25)I_{e1}$$

$$V_r = -0.7615I_{e1} \quad (10)$$

Combining (6)–(10) results in

$$A\beta = 33.13$$

$$A = \frac{A\beta}{\beta} = \frac{33.13}{1/25} = 828.2 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{828.2}{1 + 33.13} = 24.3 \text{ V/V}$$

10.30 All MOSFETs are operating at  $I_D = 100 \mu\text{A} = 0.1 \text{ mA}$  and  $|V_{ov}| = 0.2 \text{ V}$ , thus

$$g_{m1,2} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

All devices have

$$r_o = \frac{|V_A|}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

(a)

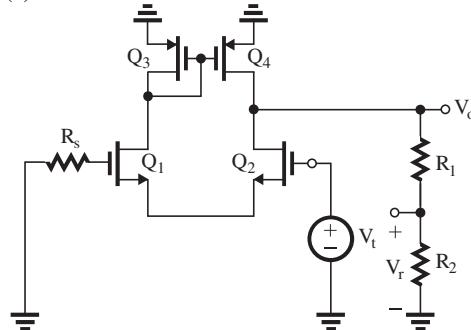


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain  $A\beta$ .

$$V_o = -g_{m1,2}[r_{o2} \parallel r_{o4} \parallel (R_1 + R_2)]V_t \quad (1)$$

$$V_r = \frac{R_2}{R_1 + R_2}V_o = \beta V_o$$

Thus,

$$A\beta \equiv -\frac{V_r}{V_t} = g_{m1,2}[r_{o2} \parallel r_{o4} \parallel (R_1 + R_2)]\beta$$

$$= 1(100 \parallel 100 \parallel 1000)\beta$$

$$= 47.62\beta$$

Thus,

$$A = 47.62 \text{ V/V}$$

$$(b) A_f = \frac{A}{1 + A\beta}$$

$$5 = \frac{47.62}{1 + 47.62\beta}$$

$$\Rightarrow \beta = 0.179 \text{ V/V}$$

$$\frac{R_2}{R_1 + R_2} = 0.179$$

$$\Rightarrow R_2 = 179 \text{ k}\Omega$$

$$R_1 = 821 \text{ k}\Omega$$

$$10.31 R_i = 2 \text{ k}\Omega$$

$$R_o = 2 \text{ k}\Omega$$

$$A = 1000 \text{ V/V}$$

$$\beta = 0.1 \text{ V/V}$$

$$\text{Loop Gain} \equiv A\beta = 1000 \times 0.1 = 100$$

$$1 + A\beta = 101$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{101} = 9.9 \text{ V/V}$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 2 \times 101 = 202 \text{ k}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{2}{101} = 19.8 \Omega$$

10.32 Since the output voltage is sampled, the resistance-with-feedback is lower. The reduction is by the factor  $(1 + A\beta)$ , thus

$$1 + A\beta = 200$$

$$A\beta = 199$$

$$R_{of} = \frac{R_o}{200}$$

$$\Rightarrow R_o = 200 \times 100 = 20,000 \Omega$$

$$= 20 \text{ k}\Omega$$

$$10.33 \ A = 1000 \text{ V/V}$$

$$R_i = 1 \text{ k}\Omega$$

$$R_{if} = 10 \text{ k}\Omega$$

Thus, the connection at the input is a series one, and

$$1 + A\beta = \frac{10}{1} = 10$$

$$\begin{aligned} A_f &= \frac{A}{1 + A\beta} \\ &= \frac{1000}{10} = 100 \text{ V/V} \end{aligned}$$

To implement a unity-gain voltage follower, we use  $\beta = 1$ . Thus the amount of feedback is

$$1 + A\beta = 1 + 1000 = 1001$$

and the input resistance becomes

$$R_{if} = (1 + A\beta)R_i$$

$$= 1001 \times 1 = 1001 \text{ k}\Omega = 1.001 \text{ M}\Omega$$

$$10.34 \ (a) \ \beta = 1$$

$$A_f|_{ideal} = 1 \text{ V/V}$$

(b) Substituting  $R_1 = \infty$  and  $R_2 = 0$  in the expression for  $A$  in Example 10.4, we obtain

$$A = \mu \frac{R_L}{R_L + r_o} \frac{R_{id}}{R_{id} + R_s}$$

$$A\beta = A \times 1 = A$$

$$(c) \ A = 10^4 \times \frac{2}{2+1} \times \frac{100}{100+10}$$

$$= 6060.6 \text{ V/V}$$

$$A\beta = 6060.6$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{6060.6}{1 + 6060.6} = 0.9998 \text{ V/V}$$

From Example 10.4 with  $R_i = \infty$  and  $R_L = 0$ , we have

$$R_i = R_s + R_{id} = 10 + 100 = 110 \text{ k}\Omega$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 110 \times 6061.6 = 667 \text{ M}\Omega$$

$$R_{in} = R_{if} - R_s \simeq 667 \text{ M}\Omega$$

$$R_o = r_o \parallel R_L = 1 \parallel 2 = 0.67 \text{ k}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{0.67 \text{ k}\Omega}{6061.6} = 0.11 \text{ }\Omega$$

$$R_{of} = R_{out} \parallel R_L$$

$$R_{out} \simeq 0.11 \text{ }\Omega$$

10.35 Refer to the solution to Problem 10.27.

$$(a) \ \beta = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$= 1 + \frac{10}{1} = 11 \text{ V/V}$$

(b) From the solution to Problem 10.27, we have

$$I_{E1} \simeq 0.1 \text{ mA}$$

$$I_{E2} \simeq 0.3 \text{ mA}$$

$$V_{E2} = +7.7 \text{ V}$$

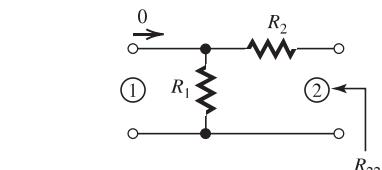
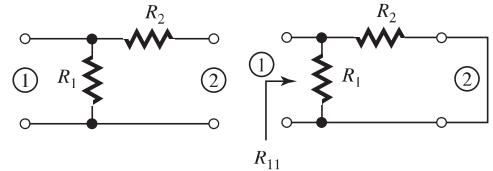
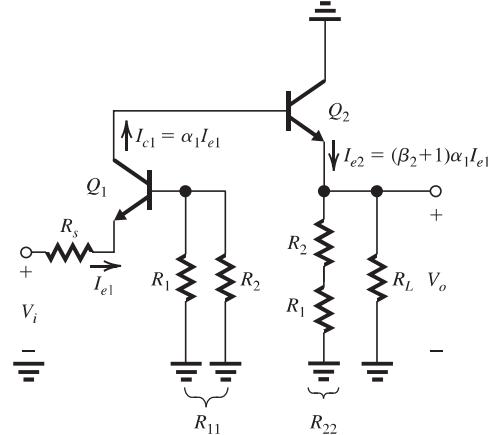


Figure 1

(c) The A circuit is shown in Fig. 1.

$$I_{e1} = \frac{V_i}{R_s + r_{e1} + \frac{R_L \parallel R_2}{\beta_1 + 1}}$$

$$V_o = I_{e2}[R_L \parallel (R_1 + R_2)]$$

$$= (\beta_2 + 1)\alpha_1 V_i \frac{R_L \parallel (R_1 + R_2)}{R_s + r_{e1} + \frac{R_L \parallel R_2}{\beta_1 + 1}}$$

Since  $\beta_1 = \beta_2 = \beta$  and  $\alpha = \frac{\beta}{\beta + 1}$ , we have

$$A \equiv \frac{V_o}{V_i} = \beta \frac{R_L \parallel (R_1 + R_2)}{R_s + r_{e1} + \frac{R_L \parallel R_2}{\beta_1 + 1}}$$

Substituting  $\beta = 100$ ,  $R_L = 1 \text{ k}\Omega$ ,  $R_1 = 1 \text{ k}\Omega$ ,  $R_2 = 10 \text{ k}\Omega$ ,  $R_s = 0.1 \text{ k}\Omega$ , and  $r_{e1} = 0.25 \text{ k}\Omega$  gives

$$A = 100 \frac{1 \parallel 11}{0.1 + 0.25 + \frac{1 \parallel 10}{101}}$$

$$= 255.3 \text{ V/V}$$

$$R_i = R_s + r_{e1} + \frac{R_L \parallel R_2}{\beta_1 + 1}$$

$$= 0.1 + 0.25 + \frac{1 \parallel 10}{101} = 0.359 \text{ k}\Omega$$

$$R_o = R_L \parallel (R_1 + R_2)$$

$$= 1 \parallel 11 = 0.917 \text{ k}\Omega$$

$$(d) \beta = \frac{R_1}{R_1 + R_2}$$

$$= \frac{1}{1 + 10} = \frac{1}{11}$$

$$(e) \frac{V_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$1 + A\beta = 1 + \frac{255.3}{11} = 24.21$$

$$A_f = \frac{255.3}{24.21} = 10.5 \text{ V/V}$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 0.359 \times 24.21 = 8.69 \text{ k}\Omega$$

$$R_{in} = R_{if} - R_s$$

$$= 8.69 - 0.1 = 8.59 \text{ k}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{0.917 \text{ k}\Omega}{24.21} = 37.9 \text{ }\Omega$$

$$R_{of} = R_{out} \parallel R_L$$

$$37.9 = R_{out} \parallel 1000$$

$$\Rightarrow R_{out} = 39.4 \text{ }\Omega$$

The value of  $A_f$  (10.5 V/V) is 0.5 less than the ideal value of 11, which is 4.5%.

10.36 (a) Let  $V_s$  increase by a small increment. Since  $Q_1$  is operating in effect as a CS amplifier, a negative incremental voltage will appear at its drain. Transistor  $Q_2$  is also operating as a CS amplifier; thus a positive incremental voltage will appear at its drain. Transistor  $Q_3$  is operating as a source follower; thus the signal at its source (which is the output voltage) will follow that at its gate and thus will be positive. The end result is that we are feeding back through the voltage divider ( $R_2$ ,  $R_1$ ) a positive incremental signal that will appear across  $R_1$  and thus at the source of the  $Q_1$ . This signal, being of the same polarity as the originally assumed change in the signal at the gate of  $Q_1(V_s)$ , will subtract from the original change, causing a smaller signal to appear across the gate-source terminals of  $Q_1$ . Hence, the feedback is negative.

$$(b) \beta = \frac{R_1}{R_1 + R_2}$$

Thus,

$$\beta = \frac{2}{2 + 18} = 0.1 \text{ V/V}$$

If the loop gain is large, the closed-loop gain approaches the ideal value

$$A_f|_{ideal} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

Thus,

$$A_f|_{ideal} = 1 + \frac{18}{2} = 10 \text{ V/V}$$

$$(c) V_{G1} = 0.9 \text{ V}$$

$$V_{S1} = V_{G1} - V_{GS1}$$

$$= V_{G1} - V_{t1} - V_{OV1}$$

$$= 0.9 - 0.5 - 0.2 = 0.2 \text{ V}$$

$$V_{G2} = V_{DD} - V_{SG2}$$

$$= V_{DD} - |V_{t2}| - |V_{OV2}|$$

$$= 1.80 - 0.5 - 0.2 = 1.1 \text{ V}$$

Thus, current source  $I_1$  will have 0.7-V drop across it, more than sufficient for its proper operation. Since  $V_{S1} = 0.2$  V the dc current through  $R_1$  will be

$$I_{R1} = \frac{V_{S1}}{R_1} = \frac{0.2 \text{ V}}{2 \text{ k}\Omega} = 0.1 \text{ mA}$$

Now, a node equation at  $S_1$  reveals that because  $I_{D1} = 0.1$  mA and  $I_{R1} = 0.1$  mA, the dc current in  $R_2$  will be zero. Thus, it will have a zero voltage drop across it and

$$V_{S3} = V_{S1} = 0.2 \text{ V}$$

Thus, current source  $I_3$  will have across it, the minimum voltage required to keep it operating properly. Finally,

$$\begin{aligned} V_{G3} &= V_{S3} + V_{GS3} \\ &= V_{S3} + V_{t3} + V_{OV3} \\ &= 0.2 + 0.5 + 0.2 = 0.9 \text{ V} \end{aligned}$$

Thus, current source  $I_2$  will have across it a voltage more than sufficient to keep it operating properly.

(d)

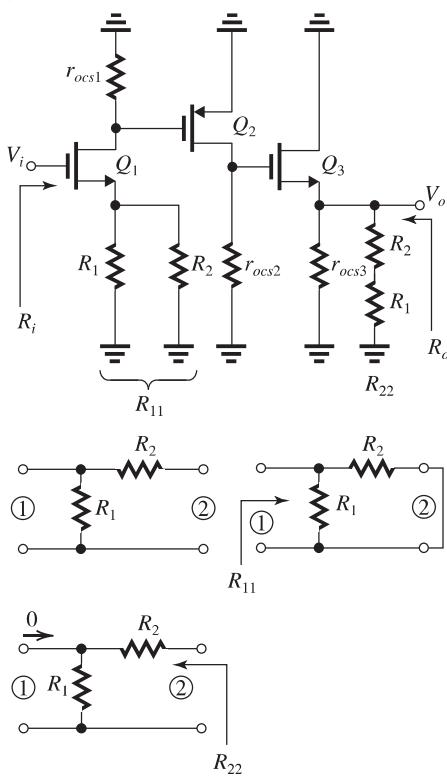


Figure 1

Figure 1 shows the A circuit as well as the  $\beta$  circuit and how the loading-effect resistances  $R_{11}$  and  $R_{22}$  are determined.

To determine A, let's first determine the small-signal parameters of all transistors as well as  $r_o$  of each of the three current sources.

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o1} = \frac{|V_A|}{I_{D1}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs1} = \frac{|V_A|}{I_1} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_{m2} = \frac{2I_{D2}}{V_{OV2}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = \frac{|V_A|}{I_{D2}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs2} = \frac{|V_A|}{I_2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_{m3} = \frac{2I_{D3}}{V_{OV3}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o3} = \frac{|V_A|}{I_{D3}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs3} = \frac{|V_A|}{I_3} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

Refer to the A circuit.

Transistor  $Q_1$  is a CS amplifier with a resistance  $R_{11}$  in its source:

$$R_s = R_{11} = R_1 \parallel R_2 = 2 \parallel 18 = 1.8 \text{ k}\Omega$$

Transistor  $Q_1$  will have an effective transconductance:

$$G_{m1} = \frac{g_{m1}}{1 + g_{m1}R_s} = \frac{2}{1 + 2 \times 1.8} = 0.43 \text{ mA/V}$$

The output resistance of  $Q_1$  will be

$$R_{o1} = (1 + g_{m1}R_s)r_{o1}$$

$$= (1 + 2 \times 1.8) \times 100 = 460 \text{ k}\Omega$$

The total resistance at the drain of  $Q_1$  is

$$R_{d1} = r_{ocs1} \parallel R_{o1}$$

$$= 100 \parallel 460 = 82.1 \text{ k}\Omega$$

Thus, the voltage gain of the first stage is

$$\begin{aligned} A_1 &= -G_m R_{d1} \\ &= -0.43 \times 82.1 = -35.3 \text{ V/V} \end{aligned}$$

The gain of the second stage is

$$\begin{aligned} A_2 &= -g_{m2}(r_{ocs2} \parallel r_{o2}) \\ &= -1(100 \parallel 100) = -50 \text{ V/V} \end{aligned}$$

To determine the gain of the third stage, we first determine the total resistance between the source of  $Q_3$  and ground:

$$\begin{aligned} R_{s3} &= r_{ocs3} \parallel r_{o3} \parallel (R_1 + R_2) \\ R_{s3} &= 100 \parallel 100 \parallel 20 \\ &= 14.3 \text{ k}\Omega \end{aligned}$$

Thus,

$$\begin{aligned} A_3 &= \frac{R_{s3}}{R_{s3} + \frac{1}{g_{m3}}} \\ &= \frac{14.3}{14.3 + \frac{1}{1}} = 0.935 \text{ V/V} \end{aligned}$$

The overall voltage gain  $A$  can now be found as

$$\begin{aligned} A &= A_1 A_2 A_3 \\ &= -35.3 \times -50 \times 0.935 = 1650 \text{ V/V} \end{aligned}$$

(e) We already found  $\beta$  in (b) as

$$\beta = 0.1 \text{ V/V}$$

$$(f) 1 + A\beta = 1 + 1650 \times 0.1 = 166$$

$$A_f = \frac{A}{1 + A\beta} = \frac{1650}{166} = 9.94 \text{ V/V}$$

which is lower by 0.06 or 0.6% than the ideal value obtained in (b).

$$(g) R_{of} = \frac{R_o}{1 + A\beta}$$

To obtain  $R_o$  refer to the output part of the  $A$  circuit.

$$\begin{aligned} R_o &= (R_1 + R_2) \parallel r_{ocs3} \parallel r_{o3} \parallel \frac{1}{g_{m3}} \\ &= 20 \parallel 100 \parallel 100 \parallel 1 \\ &= 935 \Omega \end{aligned}$$

$$R_{of} = \frac{935}{166} = 5.6 \Omega$$

Note: This problem, though long, is extremely valuable as it exercises the student's knowledge in many aspects of amplifier design.

10.37 (a) Refer to Fig. P10.37. Assume that for some reason  $v_s$  increases. This will increase the differential input signal ( $v_s - v_o$ ) applied to the differential amplifier. The drain current of  $Q_1$  will increase, and this increase will be mirrored in the drain current of  $Q_4$ . The increase in  $i_{D4}$  will cause the voltage at the gate of  $Q_5$  to rise. Since  $Q_5$  is operating as a source follower, the voltage at its source,  $v_o$ , will follow and increase. This will cause the differential input signal ( $v_s - v_o$ ) to decrease, thus counteracting the originally assumed change. Thus, the feedback is negative.

(b) Figure 1 on the next page shows the circuit prepared for dc analysis. We see that

$$I_{D1} = I_{D2} = 100 \mu\text{A}$$

$$I_{D3} = 100 \mu\text{A}$$

$$I_{D4} = 300 \mu\text{A}$$

$$I_{D5} = 0.8 \text{ mA}$$

For  $Q_1$  and  $Q_2$ , use

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) V_{OV}^2$$

$$100 = \frac{1}{2} \times 120 \times \frac{20}{1} V_{OV1,2}^2$$

$$\Rightarrow V_{OV1,2} = 0.29 \text{ V}$$

For  $Q_3$ , use

$$I_D = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right) |V_{OV3}|^2$$

$$100 = \frac{1}{2} \times 60 \times \frac{40}{1} |V_{OV3}|^2$$

$$\Rightarrow |V_{OV3}| = 0.29 \text{ V}$$

Since  $V_{SG4} = V_{SG3}$ , we have

$$|V_{OV4}| = |V_{OV3}| = 0.29 \text{ V}$$

Finally for  $Q_5$ , use

$$800 = \frac{1}{2} \times 120 \times \frac{20}{1} \times V_{OV5}^2$$

$$\Rightarrow V_{OV5} = 0.82 \text{ V}$$

This figure belongs to Problem 10.37, part (b).

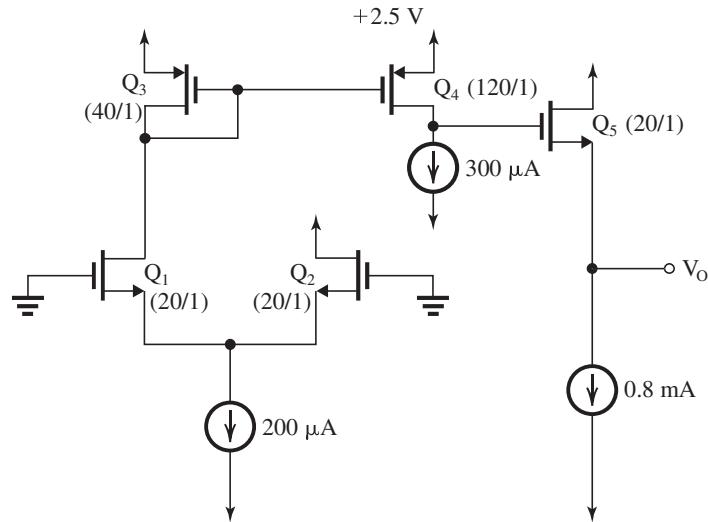


Figure 1

This figure belongs to Problem 10.37, part (d).

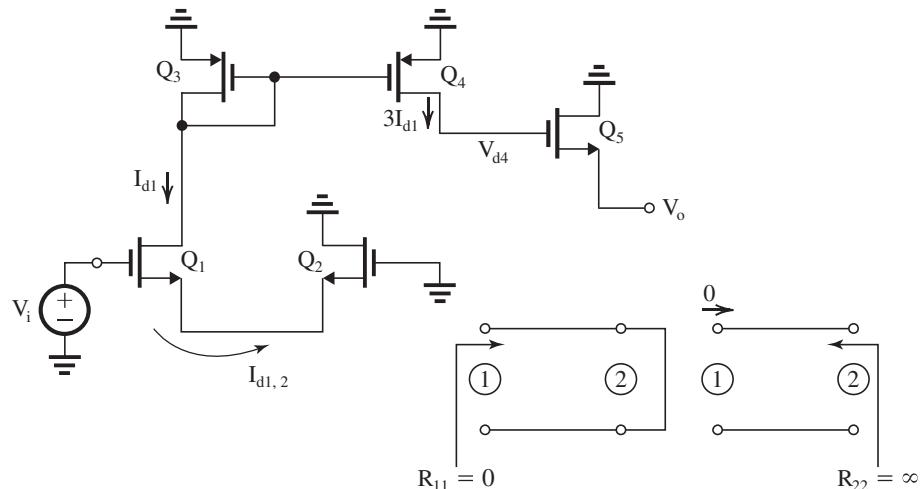


Figure 2

If perfect matching pertains, then

$$V_{D4} = V_{D3} = V_{DD} - V_{SG3}$$

$$= 2.5 - |V_t| - |V_{OV3}|$$

$$= 2.5 - 0.7 - 0.29 = 1.51 \text{ V}$$

$$V_O = V_{D4} - V_{GS5}$$

$$= V_{D4} - V_t - V_{OV5}$$

$$= 1.51 - 0.7 - 0.82 = -0.01 \text{ V}$$

which is approximately zero, as stated in the Problem statement.

$$(c) g_{m1} = g_{m2} = g_{m3} = \frac{2I_D}{|V_{ov}|}$$

$$= \frac{2 \times 0.1}{0.29} = 0.7 \text{ mA/V}$$

$$g_{m4} = \frac{2 \times 0.3}{0.29} \simeq 2 \text{ mA/V}$$

$$g_{m5} = \frac{2 \times 0.8}{0.82} \simeq 2 \text{ mA/V}$$

$$r_{o1} = r_{o2} = r_{o3} = \frac{|V_A|}{I_D} = \frac{|V'_A| \times L}{I_D}$$

$$= \frac{24 \times 1}{0.1} = 240 \text{ k}\Omega$$

$$r_{o4} = \frac{24}{0.3} = 80 \text{ k}\Omega$$

$$r_{o5} = \frac{24}{0.8} = 30 \text{ k}\Omega$$

(d) Figure 2 on the previous page shows the A circuit. Observe that since the  $\beta$  network is simply a wire connecting the output node to the gate of  $Q_2$ , we have  $R_{11} = 0$  and  $R_{22} = \infty$ . To determine  $A$ , we write

$$I_{d1,2} = \frac{V_i}{2/g_{m1,2}} = \frac{1}{2}g_{m1,2}V_i$$

Since  $\left(\frac{W}{L}\right)_4 = 3\left(\frac{W}{L}\right)_3$ , the drain current of  $Q_4$  will be

$$I_{d4} = 3I_{d1} = \frac{3}{2}g_{m1,2}V_i$$

The voltage at the drain of  $Q_4$  will be

$$V_{d4} = I_{d4}r_{o4}$$

$$= \frac{3}{2}g_{m1,2}r_{o4}V_i$$

Finally,  $V_o$  is related to  $V_{d4}$  as

$$\frac{V_o}{V_{d4}} = \frac{r_{o5}}{r_{o5} + \frac{1}{g_{m5}}}$$

Thus,

$$A \equiv \frac{V_o}{V_i} = \frac{3}{2}g_{m1,2}r_{o4} \frac{r_{o5}}{r_{o5} + \frac{1}{g_{m5}}}$$

Substituting numerical values, we obtain

$$\begin{aligned} A &= \frac{3}{2} \times 0.7 \times 80 \times \frac{30}{30 + 0.5} \\ &= 82.6 \text{ V/V} \end{aligned}$$

The output resistance  $R_o$  is

$$R_o = r_{o5} \parallel \frac{1}{g_{m5}}$$

$$= 30 \parallel 0.5 = 0.492 \text{ k}\Omega$$

$$= 492 \text{ }\Omega$$

$$(e) A_f = \frac{A}{1 + A\beta}$$

$$= \frac{82.6}{1 + 82.6} = 0.988 \text{ V/V}$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{492}{1 + 82.6} = 5.9 \text{ }\Omega$$

$$R_{out} = R_{of} = 5.9 \text{ }\Omega$$

(f) To obtain a closed-loop gain of 5 V/V, we connect a voltage divider in the feedback loop, as shown in Fig. 3.

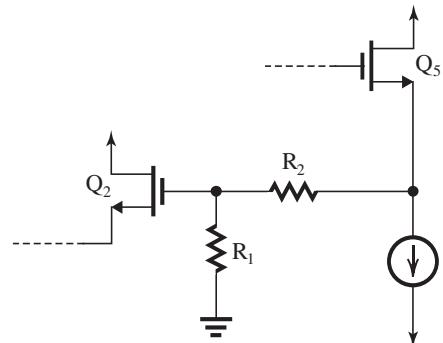


Figure 3

$$\beta = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$5 = \frac{82.6}{1 + 82.6\beta}$$

$$\Rightarrow \beta = 0.188$$

Selecting  $R_1 = 1 \text{ M}\Omega$ , we obtain

$$0.188 = \frac{1}{1 + R_2}$$

$$\Rightarrow R_2 = 4.319 \text{ M}\Omega$$

Note that by selecting large values for  $R_1$  and  $R_2$ , we have ensured that their loading effect on the A circuit would be negligible. The output resistance will be  $R_{out} = 5 \times 5.9 = 29.5 \text{ }\Omega$ .

10.38 (a) Refer to Fig. P10.30. Let  $V_s$  increase by a positive increment. This will cause the drain current of  $Q_1$  to increase. The increase in  $I_{d1}$  will be fed to the  $Q_3 - Q_4$  mirror, which will provide a corresponding increase in the drain current of  $Q_4$ . The latter current will cause the voltage at the output node to rise. A fraction of the increase in  $V_o$  is applied through the divider ( $R_1, R_2$ ) to the

This figure belongs to Problem 10.38, part (c).

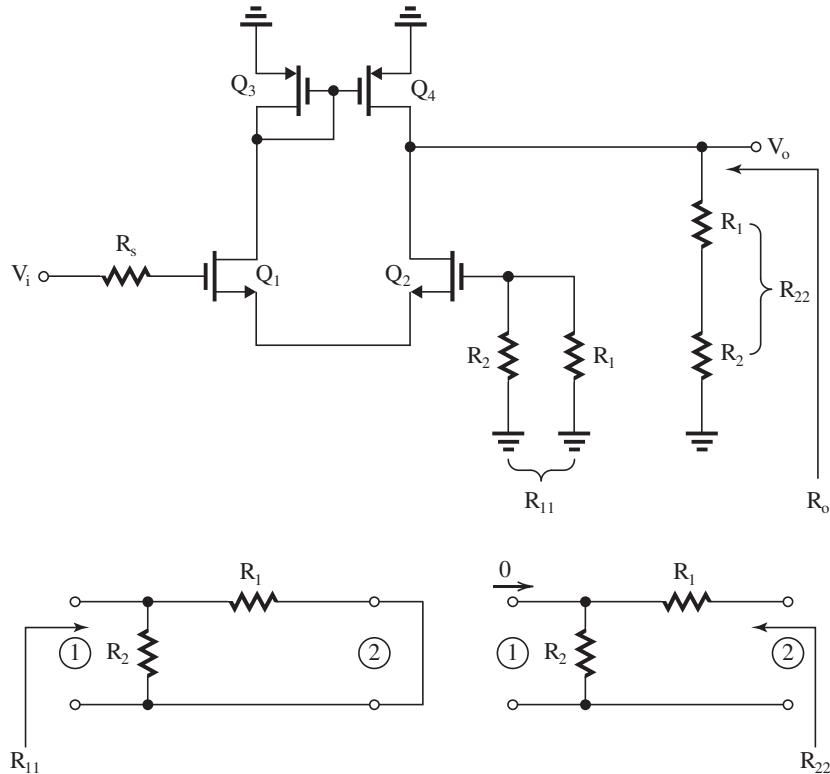


Figure 1

gate of  $Q_2$ . The increase in the voltage of the gate of  $Q_2$  will subtract from the initially assumed increase of the voltage of the gate of  $Q_1$ , resulting in a smaller increase in the differential voltage applied to the  $(Q_1, Q_2)$  pair. Thus, the feedback counter acts the originally assumed change, verifying that it is negative.

(b) The negative feedback will cause the dc voltage at the gate of  $Q_2$  to be approximately equal to the dc voltage at the gate of  $Q_1$ , that is, zero. Now, with  $V_{G2} \simeq 0$ , the dc current in  $R_2$  will be zero and similarly the dc current in  $R_1$  will be zero, resulting in  $V_O = 0$  V dc.

(c) Figure 1 shows the A circuit. It also shows how the loading effect of the  $\beta$  network on the A circuit, namely  $R_{11}$  and  $R_{22}$ , are found. The gain of the A circuit can be written by inspection as

$$A = g_{m1,2}(r_{o2} \parallel r_{o4} \parallel R_{22})$$

where

$$g_{m1,2} = \frac{2I_{D1,2}}{V_{Ov1,2}}$$

$$= \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I_{D3,4}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$R_{22} = R_1 + R_2 = 1 \text{ M}\Omega$$

$$A = 1(100 \parallel 100 \parallel 1000) = 47.62 \text{ V/V}$$

This is identical to the value found in the solution to Problem 10.30.

$$(d) \frac{V_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$5 = \frac{47.62}{1 + 47.62\beta}$$

$$\Rightarrow \beta = 0.179$$

Thus,

$$\frac{R_2}{R_1 + R_2} = 0.179$$

$$R_2 = 0.179 \text{ M}\Omega = 179 \text{ k}\Omega$$

$$R_i = 1000 - 179 = 821 \text{ k}\Omega$$

Again, these values are identical to those found in Problem 10.30.

(e) Refer to Fig. 1 on the preceding page.

$$\begin{aligned} R_o &= R_{22} \parallel r_{o2} \parallel r_{o4} \\ &= 1000 \parallel 100 \parallel 100 = 47.62 \text{ k}\Omega \end{aligned}$$

$$\begin{aligned} R_{\text{out}} &= R_{\text{of}} = \frac{R_o}{1 + A\beta} \\ &= \frac{47.62}{1 + 47.62 \times 0.179} \\ &= 5 \text{ k}\Omega \end{aligned}$$

This value cannot be found using the loop-gain analysis method of Problem 10.30.

(f) With  $R_L = 10 \text{ k}\Omega$ ,

$$\frac{V_o}{V_s} = 5 \times \frac{R_L}{R_L + R_{\text{out}}}$$

$$5 \times \frac{10}{10 + 5} = 3.33 \text{ V/V}$$

(g) As an alternative to (f), we shall redo the analysis of the A circuit in (c) above with  $R_L = 10 \text{ k}\Omega$  included:

$$\begin{aligned} A &= g_{m1,2}(r_{o2} \parallel r_{o4} \parallel R_{22} \parallel R_L) \\ &= 1(100 \parallel 100 \parallel 1000 \parallel 10) \\ &= 8.26 \text{ V/V} \end{aligned}$$

This figure belongs to Problem 10.39, part (a).

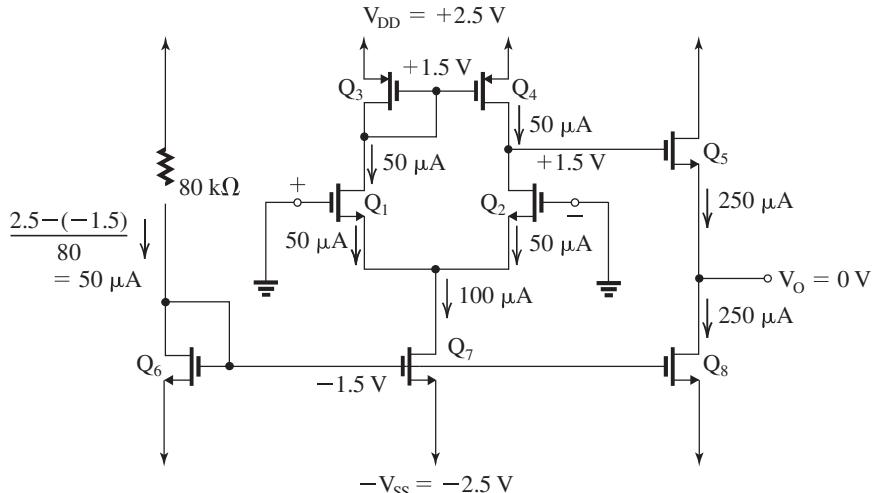


Figure 1

Using  $\beta = 0.179$ , we obtain

$$A_f = \frac{8.26}{1 + 8.26 \times 0.179} = 3.33 \text{ V/V}$$

which is identical to the value found in (f) above.

10.39 All transistors have  $L = 1 \mu\text{m}$ , thus all have  $|V_A| = |V'_A| \times L = 10 \times 1 = 10 \text{ V}$ . Also, all have  $|V_t| = 0.75 \text{ V}$ .

(a) Figure 1 shows the circuit prepared for dc design. We have also indicated some of the current and voltage values. We now find the ( $W/L$ ) ratios utilizing

$$I_D = \frac{1}{2} \mu_n C_{\text{ox}} \left( \frac{W}{L} \right) V_{\text{ov}}^2$$

for the NMOS transistors, and

$$I_D = \frac{1}{2} \mu_p C_{\text{ox}} \left( \frac{W}{L} \right) |V_{\text{ov}}|^2$$

for the PMOS devices.

For  $Q_6$ ,

$$50 = \frac{1}{2} \times 100 \times \left( \frac{W}{L} \right)_6 \times 0.25^2$$

$$\Rightarrow \left( \frac{W}{L} \right)_6 = 16$$

For  $Q_7$ ,

$$\frac{(W/L)_7}{(W/L)_6} = \frac{100 \mu\text{A}}{50 \mu\text{A}} = 2$$

$$\Rightarrow (W/L)_7 = 2 \times 16 = 32$$

For Q<sub>8</sub>,

$$\frac{(W/L)_8}{(W/L)_6} = \frac{250 \mu A}{50 \mu A} = 5$$

$$\Rightarrow \left(\frac{W}{L}\right)_8 = 5 \times 16 = 80$$

For Q<sub>1</sub> and Q<sub>2</sub>,

$$50 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{1,2} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 16$$

For Q<sub>3</sub> and Q<sub>4</sub>,

$$50 = \frac{1}{2} \times 50 \times \left(\frac{W}{L}\right)_{3,4} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 32$$

Finally, since V<sub>G3</sub> = V<sub>D4</sub> = V<sub>D3</sub> = 1.5 V and we require V<sub>O</sub> = 0 V, we have

$$V_{GS5} = 1.5 \text{ V}$$

$$V_{OV5} = 1.5 - 0.75 = 0.75 \text{ V}$$

$$250 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_5 \times 0.75^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 8.9$$

(b) The maximum value of V<sub>ICM</sub> is limited by Q<sub>1</sub> leaving the saturation region,

$$V_{ICM\max} = V_{D1} + V_t$$

$$= 1.5 + 0.75 = 2.25 \text{ V}$$

The minimum value of V<sub>ICM</sub> is limited by the need to keep Q<sub>7</sub> in saturation. This is achieved by keeping V<sub>D7</sub> at a minimum voltage of

$$-2.5 + |V_{OV7}| = -2.5 + 0.25 = -2.25 \text{ V}$$

Thus,

$$V_{ICM\min} = -2.25 + V_{GS1}$$

$$= -2.25 + 1 = -1.25 \text{ V}$$

Thus,

$$-1.25 \text{ V} \leq V_{ICM} \leq +2.25 \text{ V}$$

$$(c) g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}}$$

$$= \frac{2 \times 0.05}{0.2} = 0.5 \text{ mA/V}$$

$$g_{m5} = \frac{2I_D}{|V_{OV5}|} = \frac{2 \times 0.25}{0.75} = 0.67 \text{ mA/V}$$

$$(d) r_{o1} = r_{o2} = r_{o3} = r_{o4} = r_{o6} = \frac{|V_A|}{I_D} = \frac{10}{0.05} = 200 \text{ k}\Omega$$

$$r_{o7} = \frac{10}{0.01} = 100 \text{ k}\Omega$$

$$r_{o5} = r_{o8} = \frac{10}{0.25} = 40 \text{ k}\Omega$$

(e) Figure 2 on the next page shows the A circuit, the  $\beta$  circuit, and how the loading effects of the  $\beta$  circuit on the A circuit, namely R<sub>11</sub> and R<sub>22</sub>, are determined.

$$\frac{V_{g5}}{V_i} = g_{m1,2}(r_{o2} \parallel r_{o4})$$

$$= 0.5(200 \parallel 200) = 50 \text{ V/V}$$

$$\frac{V_o}{V_{g5}} = \frac{R_s}{R_s + \frac{1}{g_{m5}}}$$

where

$$R_s = r_{o8} \parallel r_{o5} \parallel (R_1 + R_2) \parallel R_L$$

$$= 40 \parallel 40 \parallel 100 \parallel 100 = 14.3 \text{ k}\Omega$$

Thus,

$$\frac{V_o}{V_{g5}} = \frac{14.3}{14.3 + (1/0.67)} = 0.905 \text{ V/V}$$

$$A = \frac{V_o}{V_i} = \frac{V_{g5}}{V_i} \times \frac{V_o}{V_{g5}}$$

$$= 50 \times 0.905 = 45.3 \text{ V/V}$$

$$A_f = 10 = \frac{A}{1 + A\beta}$$

$$10 = \frac{45.3}{1 + 45.3\beta}$$

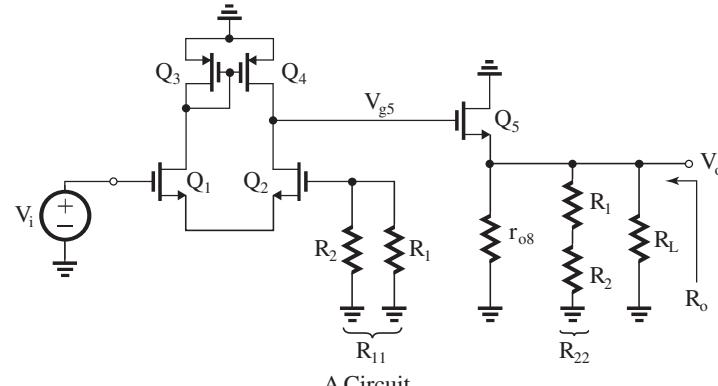
$$\Rightarrow \beta = 0.078$$

$$\frac{R_2}{R_1 + R_2} = 0.078$$

$$R_2 = 7.8 \text{ k}\Omega$$

$$R_1 = 100 - 7.8 = 92.2 \text{ k}\Omega$$

This figure belongs to Problem 10.39, part (e).



A Circuit

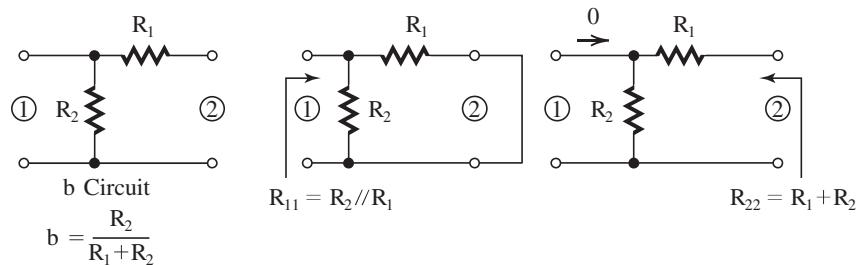


Figure 2

(f) Refer to Fig. 2.

$$R_o = R_L \parallel (R_1 + R_2) \parallel r_{o8} \parallel r_{o5} \parallel \frac{1}{g_{m5}}$$

$$= R_s \parallel \frac{1}{g_{m5}} = 14.3 \parallel (1/0.67)$$

$$= 1.36 \text{ k}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{1.36 \text{ k}\Omega}{1 + 45.3 \times 0.078} \simeq 300 \Omega$$

$$R_{out} \parallel R_L = R_{of}$$

$$\Rightarrow R_{out} \simeq 300 \Omega$$

10.40 (a) Refer to Fig. P10.40. If \$V\_s\$ increases, the output of \$A\_1\$ will decrease and this will cause the output of \$A\_2\$ to increase. This, in turn, causes the output of \$A\_3\$, which is \$V\_o\$, to increase. A portion of the positive increment in \$V\_o\$ is fed back to the positive input terminal of \$A\_1\$ through the voltage divider \$(R\_2, R\_1)\$. The increased voltage at the positive input terminal of \$A\_1\$ counteracts the

originally assumed increase at the negative input terminal, verifying that the feedback is negative.

$$(b) A_f|_{ideal} = \frac{1}{\beta}$$

where

$$\beta = \frac{R_1}{R_1 + R_2}$$

Thus, to obtain an ideal closed-loop gain of 5 V/V we need \$\beta = 0.2\$:

$$0.2 = \frac{20}{20 + R_2}$$

$$\Rightarrow R_2 = 80 \text{ k}\Omega$$

(c) Figure 1 on the next page shows the small-signal equivalent circuit of the feedback amplifier.

(d) Figure 2 on the next page shows the A circuit and the \$\beta\$ circuit together with the determination of its loading effects, \$R\_{l1}\$, and \$R\_{22}\$. We can write

$$\frac{V_1}{V_i} = -\frac{82}{82 + 9 + 16} = -0.766 \text{ V/V}$$

$$V_2 = 20V_1 \times \frac{5}{3.2 + 5} = 12.195V_1$$

$$V_3 = -20V_2(20 \parallel 20) = -200V_2$$

Chapter 10-24

This figure belongs to Problem 10.40, part (c).

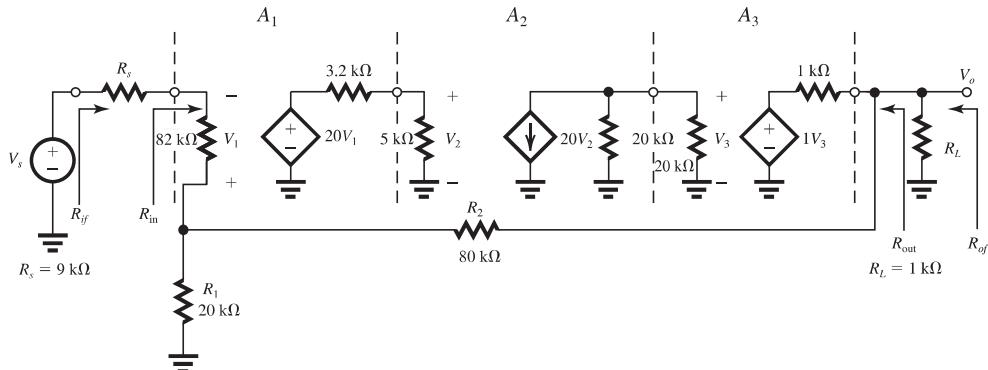


Figure 1

This figure belongs to Problem 10.40, part (d).

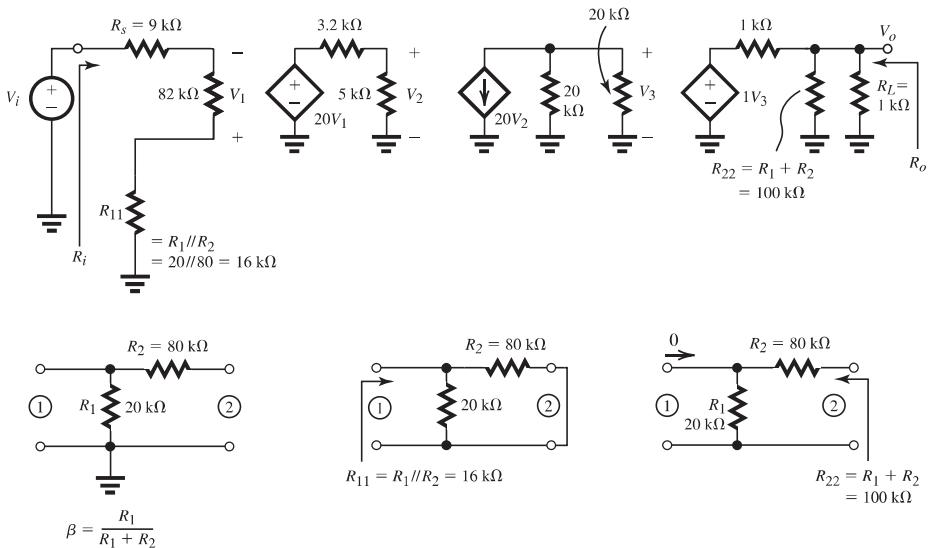


Figure 2

$$V_o = V_3 \frac{1 \parallel 100}{(1 \parallel 100) + 1} = 0.497V_3$$

Thus,

$$A \equiv \frac{V_o}{V_i} = 0.497 \times -200 \times 12.195 \times -0.766$$

$$= 928.5 \text{ V/V}$$

$$(e) \beta = \frac{20}{20 + 80} = 0.2 \text{ V/V}$$

$$1 + A\beta = 1 + 928.5 \times 0.2 = 186.7$$

$$(f) A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{928.5}{186.7} = 4.97 \text{ V/V}$$

which is nearly equal to the ideal value of 5 V/V.

(g) From the A circuit,

$$R_i = 9 + 82 + 16 = 107 \text{ k}\Omega$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 107 \times 186.7 = 19.98 \text{ M}\Omega$$

$$R_{in} = R_{if} - R_s \simeq 19.98 \text{ M}\Omega$$

(h) From the A circuit,

$$R_o = R_L \parallel R_{22} \parallel 1 \text{ k}\Omega$$

$$= 1 \parallel 100 \parallel 1 = 497.5 \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{497.5}{186.7} = 2.66 \Omega$$

$$R_{out} \parallel R_L = R_{of}$$

$$R_{out} \parallel 1000 = 2.66 \Omega$$

$$R_{out} \simeq 2.66 \Omega$$

$$(i) f_{Hf} = f_H(1 + A\beta)$$

$$= 100 \times 186.7$$

$$= 18.67 \text{ kHz}$$

(j) If  $A_l$  drops to half its nominal value,  $A$  will drop to half its nominal value:

$$A = \frac{1}{2} \times 928.5 = 464.25$$

and  $A_f$  becomes

$$A_f = \frac{464.25}{1 + 464.25 \times 0.2} = 4.947 \text{ V/V}$$

Thus, the percentage change in  $A_f$  is

$$= \frac{4.947 - 4.97}{4.97} = -0.47\%$$

10.41 (a) Figure 1 on the next page shows the A circuit and the circuit for determining  $\beta$  as well as the determination of the loading effects of the  $\beta$  circuit.

(b) If  $A\beta$  is large, then

$$A_f \equiv \frac{V_o}{V_s} \simeq \frac{1}{\beta}$$

Since

$$\beta = \frac{R_E}{R_F + R_E}$$

we have

$$A_f = \frac{R_F + R_E}{R_E} \quad \text{Q.E.D.}$$

$$(c) 25 = 1 + \frac{R_F}{50 \Omega}$$

$$\Rightarrow R_F = 1.2 \text{ k}\Omega$$

(d) Refer to the A circuit in Fig. 1. The voltage gain of  $Q_1$  is given by

$$\frac{V_{c1}}{V_i} = -\alpha_1 \frac{R_{C1} \parallel r_{\pi 2}}{r_{e1} + R_{l1}}$$

where

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \Omega$$

$$R_{l1} = R_E \parallel R_F = 50 \Omega \parallel 1200 \Omega = 48 \Omega$$

$$g_{m2} = \frac{I_{C2}}{V_T} \simeq \frac{I_{E2}}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ mA}} = 80 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta_2}{80} = \frac{100}{80} = 1.25 \text{ k}\Omega$$

$$\alpha_1 = 0.99 \simeq 1$$

$$\frac{V_{c1}}{V_i} = -10 = -\frac{R_{C1} \parallel 1.25}{0.025 + 0.048}$$

$$\Rightarrow R_{C1} = 1.75 \text{ k}\Omega$$

Next consider the second stage composed of the CE transistor  $Q_2$ . The load resistance of the second stage is composed of  $R_{C2}$  in parallel with the input resistance of emitter-follower  $Q_3$ . The latter resistance is given by

$$R_{i3} = (\beta_3 + 1)(r_{e3} + R_{22})$$

where

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5 \Omega$$

$$R_{22} = R_F + R_E = 1.2 + 0.05 = 1.25 \text{ k}\Omega$$

Thus,

$$R_{i3} = 101 \times 1.25 = 126.3 \text{ k}\Omega$$

$$A_2 \equiv \frac{V_{c2}}{V_{b2}} = -g_{m2}(R_{C2} \parallel R_{i3})$$

$$-50 = -80(R_{C2} \parallel 126.3)$$

$$\Rightarrow R_{C2} = 628 \Omega$$

$$(e) A = A_l A_2 A_3$$

where

$$A_3 = \frac{R_{22}}{R_{22} + r_{e3}} = \frac{1.25}{1.25 + 0.005} = 0.996 \text{ V/V}$$

$$A \equiv -10 \times -50 \times 0.996$$

$$= 498 \text{ V/V}$$

$$A_f \equiv \frac{V_o}{V_s} = \frac{498}{1 + 498 \times \frac{50}{1250}}$$

$$= 23.8 \text{ V/V}$$

(f) Refer to the A circuit in Fig. 1.

$$R_i = (\beta_1 + 1)(r_{e1} + R_{l1})$$

$$R_i = 101(0.025 + 0.048)$$

$$= 7.37 \text{ k}\Omega$$

This figure belongs to Problem 10.41, part (a).

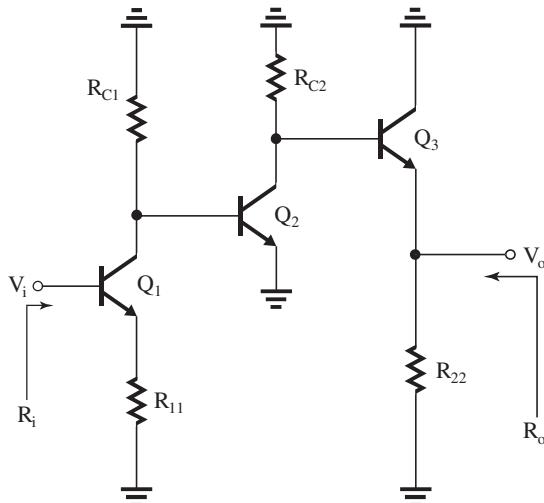
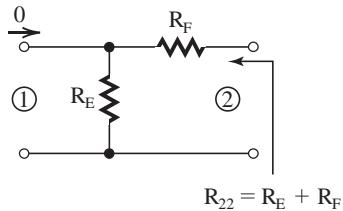
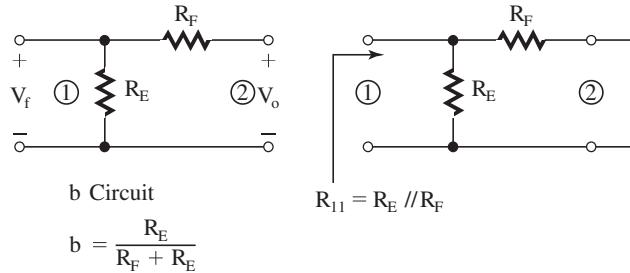


Figure 1

$$R_{if} = R_i(1 + A\beta)$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

where

$$= \frac{11.1}{20.92} = 0.53 \Omega$$

$$1 + A\beta = 1 + \frac{498}{25} = 20.92$$

$$R_{if} = 7.37 \times 20.92 = 154 \text{ k}\Omega$$

10.42 To obtain  $A_f \equiv \frac{I_o}{V_s} \simeq 10 \text{ mA/V}$ , we select

$$R_o = R_{22} \parallel \left[ r_{e3} + \frac{R_{C2}}{\beta_3 + 1} \right]$$

$$R_F = \beta = \frac{1}{A_f} = 100 \Omega$$

$$= 1.25 \parallel \left[ 0.005 + \frac{0.628}{101} \right]$$

$$A = \frac{\mu}{R_F} \frac{g_m(R_F \parallel R_{id} \parallel r_{o2})}{1 + g_m(R_F \parallel R_{id} \parallel r_{o2})}$$

$$= 11.1 \Omega$$

$$\equiv \frac{1000}{0.1 \text{ k}\Omega} \frac{2(0.1 \parallel 100 \parallel 20)}{1 + 2(0.1 \parallel 100 \parallel 20)}$$

$$From Example 10.6, we obtain$$

$$= 10,000 \times 0.1658$$

$$= 1.658 \text{ A/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1658}{1 + 1658 \times 0.1} = 9.94 \text{ mA/V}$$

10.43 (a)

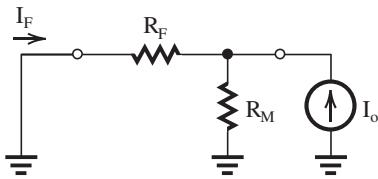


Figure 1

Figure 1 shows the  $\beta$  network with the input port short-circuited. Thus,

$$\beta \equiv \frac{I_F}{I_o} = -\frac{R_M}{R_M + R_F}$$

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = -\left(1 + \frac{R_F}{R_M}\right)$$

(b) Figure 2 below shows the circuit for determining the loop gain  $A\beta$ ,

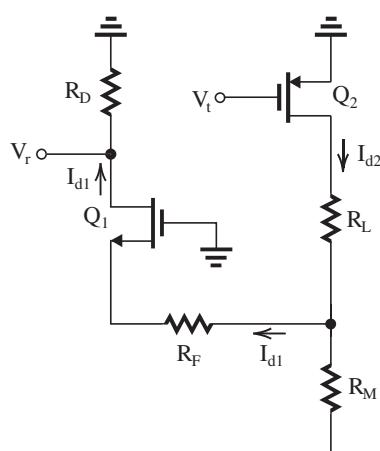


Figure 2

$$A\beta = -\frac{V_r}{V_t}$$

First, we express  $I_{d2}$  in terms of  $V_t$ :

$$I_{d2} = -g_{m2}V_t \quad (1)$$

Then we determine  $I_{d1}$ :

$$I_{d1} = I_{d2} \frac{R_M}{R_M + R_F + \frac{1}{g_{m1}}} \quad (2)$$

The returned voltage  $V_r$  can now be obtained as

$$V_r = I_{d1}R_D \quad (3)$$

Combining Eqs. (1)–(3), we find  $V_r/V_t$ :

$$\frac{V_r}{V_t} = -\frac{g_{m2}R_D R_M}{R_M + R_F + \frac{1}{g_{m1}}}$$

Thus,

$$A\beta = \frac{g_{m2}R_D R_M}{R_M + R_F + \frac{1}{g_{m1}}}$$

Dividing the expression for  $A\beta$  by

$$\beta = -\frac{R_M}{R_M + R_F}$$

$$A = -\frac{g_{m2}R_D}{1 + 1/[g_{m1}(R_M + R_F)]} \quad \text{Q.E.D.}$$

$$(c) A = -\frac{4 \times 10}{1 + 1/[4 \times 1]}$$

$$= -32 \text{ A/A}$$

$$A_f = -5 = -\frac{32}{1 - 32 \times \beta}$$

$$\beta = -0.169 \text{ A/A}$$

$$-\frac{R_M}{R_M + R_F} = -0.169$$

$$R_M = 0.169 \times 1 = 0.169 \text{ k}\Omega$$

$$= 169 \Omega$$

10.44 (a) Refer to Fig. P10.44(b).

$$\beta \equiv \frac{I_F}{V_o} = -\frac{1}{R_F}$$

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = -R_F$$

For  $A_f|_{\text{ideal}} = -1 \text{ k}\Omega$ , we have

$$R_F = 1 \text{ k}\Omega$$

(b)

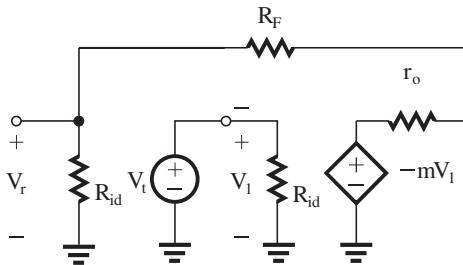


Figure 1

Figure 1 shows the circuit for determining the loop gain  $A\beta$ :

$$A\beta \equiv -\frac{V_r}{V_t}$$

Writing  $V_r$  in terms of  $V_i = V_t$  yields

$$V_r = -\mu V_t \frac{R_{id}}{R_{id} + R_F + r_o}$$

Thus,

$$A\beta \equiv -\frac{V_r}{V_t} = \mu \frac{R_{id}}{R_{id} + R_F + r_o} \quad \text{Q.E.D.}$$

$$(c) A\beta = 1000 \frac{100}{100 + 1 + 1}$$

$$= 980.4$$

$$A = \frac{980.4}{\beta} = \frac{980.4}{-1/R_F}$$

$$= -980.4 \text{ k}\Omega$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= -\frac{980.4}{1 + 980.4} = -0.999 \text{ k}\Omega$$

10.45

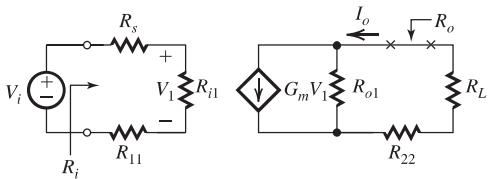


Figure 1

Figure 1 shows the A circuit where

$$R_{i1} = 10 \text{ k}\Omega$$

$$R_{o1} = 100 \text{ k}\Omega$$

$$\beta = 200 \Omega$$

$$R_{22} = 200 \Omega$$

$$R_{11} = 10 \text{ k}\Omega$$

$$R_s = 10 \text{ k}\Omega$$

$$R_L = 10 \text{ k}\Omega$$

To determine A,

$$A \equiv \frac{I_o}{V_i}$$

we write

$$\begin{aligned} V_1 &= V_i \frac{R_{i1}}{R_{i1} + R_s + R_{11}} \\ &= V_i \frac{10}{10 + 10 + 10} = \frac{1}{3} V_i \end{aligned} \quad (1)$$

$$\begin{aligned} I_o &= G_m V_1 \frac{R_{o1}}{R_{o1} + R_L + R_{22}} \\ &= 0.6 \times \frac{100}{100 + 10 + 0.2} V_i \\ &= 0.544 V_i \end{aligned} \quad (2)$$

Combining (1) and (2), we obtain

$$I_o = 0.544 \times \frac{1}{3} V_i = 0.1815 V_i$$

$$A = 0.1815 \text{ A/V}$$

$$A_f = \frac{I_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{0.1815}{1 + 0.1815 \times 200} = \frac{0.1815}{1 + 36.2} = 4.88 \text{ mA/V}$$

$$R_{if} = R_i(1 + A\beta)$$

$R_i$  is obtained from the A circuit as

$$R_i = R_s + R_{i1} + R_{11}$$

$$= 10 + 10 + 10 = 30 \text{ k}\Omega$$

Thus,

$$R_{if} = 30 \times 37.2 = 1.116 \text{ M}\Omega$$

$$R_{in} = R_{if} - R_s$$

$$= 1.116 - 0.010 = 1.006 \text{ M}\Omega$$

$$\simeq 1 \text{ M}\Omega$$

$$R_{of} = R_o(1 + A\beta)$$

where  $R_o$  is obtained from the A circuit as

$$\begin{aligned} R_o &= R_L + R_{o1} + R_{22} \\ &= 10 + 100 + 0.2 = 110.2 \text{ k}\Omega \end{aligned}$$

$$R_{of} = 110.2 \times 37.2 = 4.1 \text{ M}\Omega$$

$$R_{out} = R_{of} - R_L = 4.1 - 0.01 = 4.09 \text{ M}\Omega$$

10.46 (a)

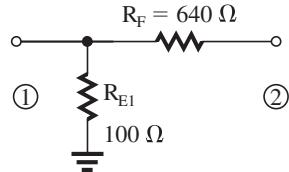


Figure 1

Figure 1 shows the  $\beta$  circuit from which we obtain

$$\begin{aligned} \beta &= \frac{R_{E1}}{R_{E1} + R_F} \\ &= \frac{100}{100 + 640} = 0.135 \text{ V/V} \end{aligned}$$

(b) For  $A\beta \gg 1$ ,

$$\frac{V_{e3}}{V_s} = A_f \Big|_{ideal} = \frac{1}{\beta} = 7.4 \text{ V/V}$$

(c)

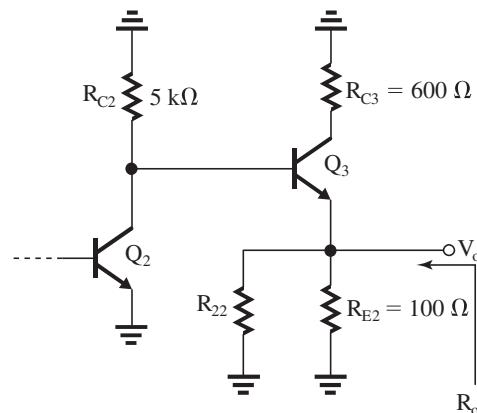


Figure 2

Figure 2 shows the portion of the A circuit relevant for calculating  $R_o$ :

$$R_o = R_{E2} \parallel R_{22} \parallel \left[ r_{e3} + \frac{R_{C2}}{\beta_3 + 1} \right]$$

where  $R_{E2} = 100 \Omega$ ,  $R_{22}$  (from  $\beta$  circuit) =  $740 \Omega$ ,  $r_{e3} = 5 \Omega$ ,  $R_{C2} = 5 \text{ k}\Omega$ ,  $\beta_3 = 100$ ; thus,

$$\begin{aligned} R_o &= 100 \parallel 740 \parallel \left[ 5 + \frac{5000}{101} \right] \\ &= 33.7 \Omega \\ R_{of} &= \frac{R_o}{1 + A\beta} \\ &= \frac{33.7}{1 + 246.3} = 0.14 \Omega \end{aligned}$$

10.47 (a)

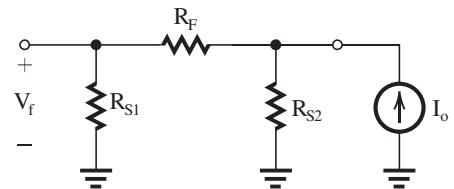


Figure 1

Figure 1 shows the  $\beta$  network. The value of  $\beta$  can be obtained from

$$\begin{aligned} \beta &\equiv \frac{V_f}{I_o} \\ &= \frac{R_{S1}R_{S2}}{R_{S2} + R_F + R_{S1}} \end{aligned}$$

If  $A\beta \gg 1$ , then

$$A_f \approx \frac{1}{\beta} = \frac{1}{R_{S1}} + \frac{1}{R_{S2}} + \frac{R_F}{R_{S1}R_{S2}} \quad (1)$$

For  $A_f \approx 100 \text{ mA/V}$ ,

$$\begin{aligned} 100 &= \frac{1}{0.1} + \frac{1}{0.1} + \frac{R_F}{0.1 \times 0.1} \\ \Rightarrow R_F &= 0.8 \text{ k}\Omega \end{aligned}$$

(b) Figure 2 on the next page shows the A circuit and the determination of the loading effects of the  $\beta$  circuit, namely  $R_{11}$  and  $R_{22}$ ,

$$\begin{aligned} R_{11} &= R_{S1} \parallel (R_F + R_{S2}) \\ &= 100 \parallel (800 + 100) = 80 \Omega \end{aligned}$$

$$R_{22} = R_{S2} \parallel (R_F + R_{S1}) = 80 \Omega$$

The value of A is determined as follows:

$$\begin{aligned} \frac{V_{d1}}{V_i} &= -\frac{R_{D1}}{(1/g_{m1}) + R_{11}} \\ &= -\frac{10}{(1/4) + 0.08} = -30.3 \text{ V/V} \end{aligned}$$

This figure belongs to Problem 10.47, part (b).

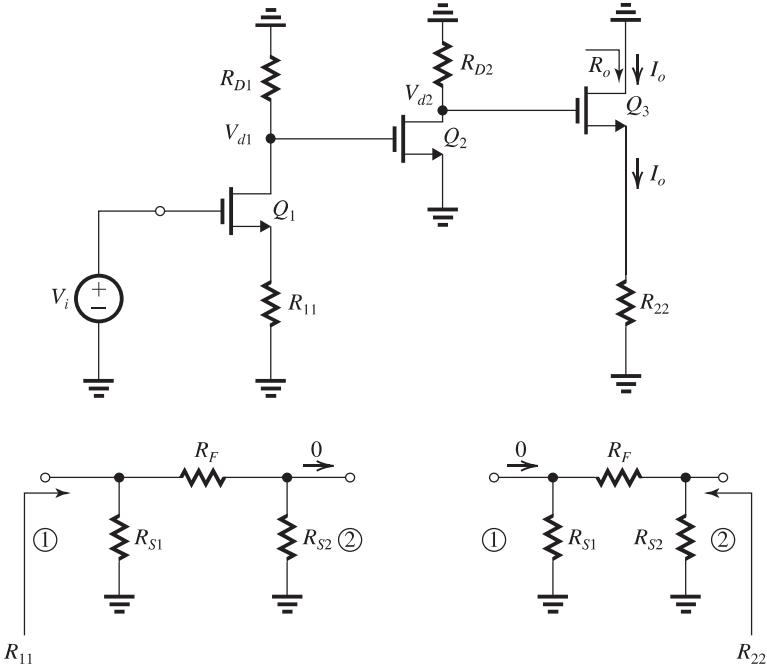


Figure 2

$$\frac{V_{d2}}{V_{d1}} = -g_{m2}R_{D2} = -4 \times 10 = -40 \text{ V/V}$$

$$\frac{I_o}{V_{d2}} = \frac{1}{1/g_{m3} + R_{22}}$$

$$= \frac{1}{0.25 + 0.08} \simeq 3 \text{ mA/V}$$

Thus,

$$A = \frac{I_o}{V_i} = 3 \times -40 \times -30.3 = 3636 \text{ mA/V}$$

(c)  $\beta = 0.01 \text{ k}\Omega$

$$1 + A\beta = 1 + 3636 \times 0.01$$

$$= 37.36$$

$$A_f = \frac{I_o}{V_i} = \frac{3636}{37.36} = 97.3 \text{ mA/V}$$

Difference from design value

$$= \frac{97.3 - 100}{100} \times 100$$

$$= -2.7\%$$

To make  $A_f$  exactly 100 mA/V, we can increase  $R_F$  (see Eq. (1) to appreciate why we need to increase  $R_F$ ).

(d) From the A circuit in Fig. 2, we have

$$R_o = r_{o3} + R_{22} + g_{m3}r_{o3}R_{22}$$

$$= 20 + 0.08 + 4 \times 20 \times 0.08$$

$$= 26.48 \text{ k}\Omega$$

$$R_{out} = R_{of} = R_o(1 + A\beta)$$

$$= 26.48 \times 37.36 = 989.3 \text{ k}\Omega$$

(e)

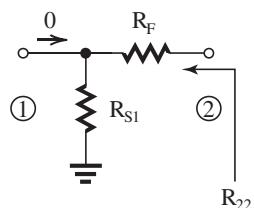
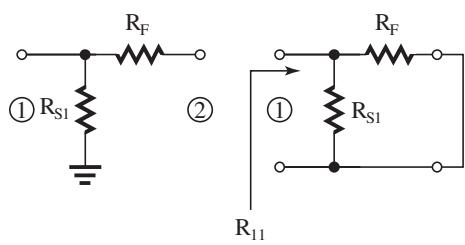


Figure 3

Figure 3 on the preceding page shows the  $\beta$  circuit for the case the output is  $V_o$ .

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

$$= \frac{100}{100 + 800} = \frac{1}{9}$$

Also shown is how  $R_{11}$  and  $R_{22}$  are determined in this case:

$$R_{11} = R_{S1} \parallel R_F = 100 \parallel 800 = 88.9 \Omega$$

$$R_{22} = R_F + R_{S1} = 800 + 100 = 900 \Omega$$

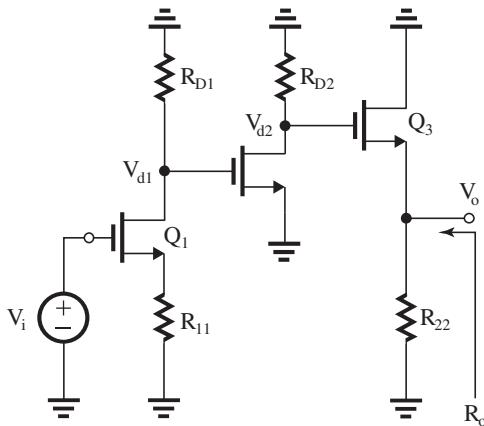


Figure 4

Figure 4 shows the A circuit for this case. To determine A, we write

$$\frac{V_{d1}}{V_i} = -\frac{R_{D1}}{1/g_{m1} + R_{11}}$$

$$\frac{V_{d1}}{V_i} = -\frac{10}{0.25 + 0.0889} = -29.5 \text{ V/V}$$

$$\frac{V_{d2}}{V_{d1}} = -g_{m2}R_{D2} = -4 \times 10 = -40 \text{ V/V}$$

$$\frac{V_o}{V_{d2}} = \frac{R_{22}}{R_{22} + \frac{1}{g_{m3}}} = \frac{88.9}{88.9 + 250} = 0.26 \text{ V/V}$$

Thus,

$$A \equiv \frac{V_o}{V_i} = 0.26 \times -40 \times -29.5 = 306.9 \text{ V/V}$$

$$1 + A\beta = 1 + 306.9 \times \frac{1}{9} = 35.1$$

which is a little lower than the value (37.36) found when we analyzed the amplifier as a transconductance amplifier.

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{306.9}{35.1} = 8.74 \text{ V/V}$$

(f) From the A circuit in Figure 4, we have

$$R_o = R_{22} \parallel \frac{1}{g_{m3}}$$

$$R_o = 900 \Omega \parallel 250 \Omega$$

$$= 195.7 \Omega$$

$$R_{out2} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{195.7}{35.1} = 5.6 \Omega$$

10.48 (a)

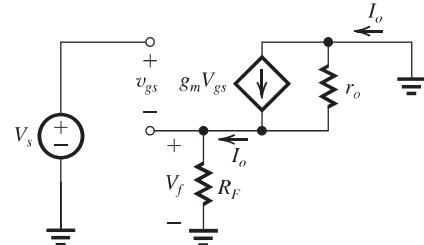


Figure 1

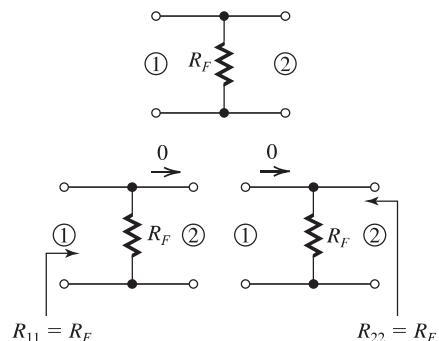


Figure 2

Figure 1 shows the small-signal equivalent circuit of the feedback amplifier. Observe that the resistance  $R_F$  senses the output current  $I_o$  and provides a voltage  $I_oR_F$  that is subtracted from  $V_s$ . Thus the feedback network is composed of the resistance  $R_F$ , as shown in Fig. 2. Because the feedback is of the series-series type, the loading resistances  $R_{11}$  and  $R_{22}$  are determined as indicated in Fig. 2,

$$R_{11} = R_F$$

$$R_{22} = R_F$$

(b) The  $\beta$  circuit is shown in Fig. 2 and  
 $\beta = R_F$

Figure 3 shows the A circuit.

(c)

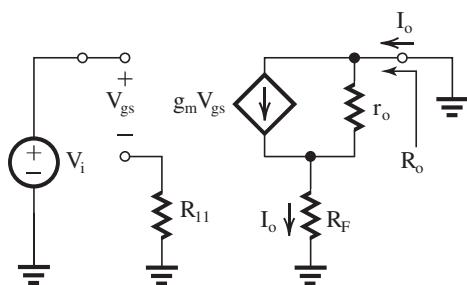


Figure 3

To determine  $A = I_o / V_i$ , we write

$$V_{gs} = V_i$$

$$I_o = g_m V_{gs} \frac{r_o}{r_o + R_F}$$

$$A \equiv \frac{I_o}{V_i} = g_m \frac{r_o}{r_o + R_F}$$

$$1 + A\beta = 1 + \frac{g_m r_o R_F}{r_o + R_F}$$

$$\begin{aligned} \frac{I_o}{V_s} &= A_f = \frac{A}{1 + A\beta} \\ &= \frac{g_m r_o / (r_o + R_F)}{1 + g_m r_o R_F / (r_o + R_F)} \end{aligned}$$

This figure belongs to Problem 10.49, part (a).

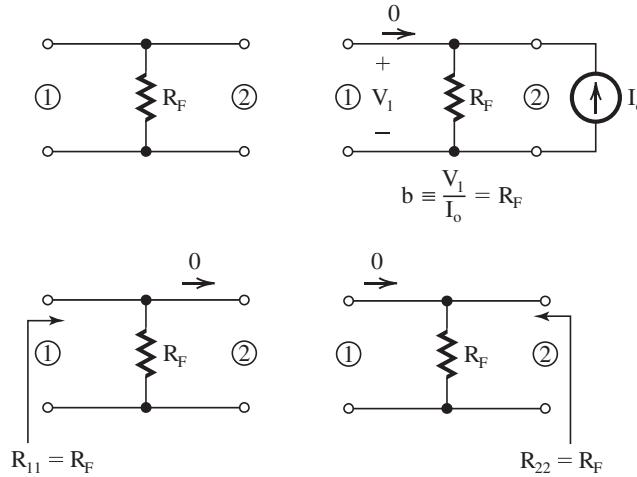


Figure 1

$$= \frac{g_m}{1 + g_m R_F + \frac{R_F}{r_o}}$$

From the A circuit in Fig. 3, we have

$$R_o = r_o + R_F$$

$$R_{of} = (1 + A\beta)R_o$$

$$= \left(1 + \frac{g_m r_o R_F}{r_o + R_F}\right)(r_o + R_F)$$

$$= r_o + R_F + g_m r_o R_F$$

which is a familiar relationship!

10.49 (a) The  $\beta$  circuit is shown in Fig. 1:

$$\beta = R_F$$

For  $A\beta \gg 1$ ,  $A_f \equiv I_o / V_s$  approaches the ideal value

$$A_f|_{\text{ideal}} = \frac{1}{\beta} = \frac{1}{R_F}$$

To obtain  $A_f \approx 5 \text{ mA/V}$ , we use

$$R_F = \frac{1}{5} = 0.2 \text{ k}\Omega = 200 \Omega$$

(b) Determining the loading effects of the  $\beta$  network is illustrated in Fig. 1:

$$R_{11} = R_{22} = R_F$$

Figure 2 (next page) shows the A circuit. An expression for  $A \equiv I_o / V_i$  can be derived as follows:

$$V_1 = V_i \quad (1)$$

This figure belongs to Problem 10.49, part (b).

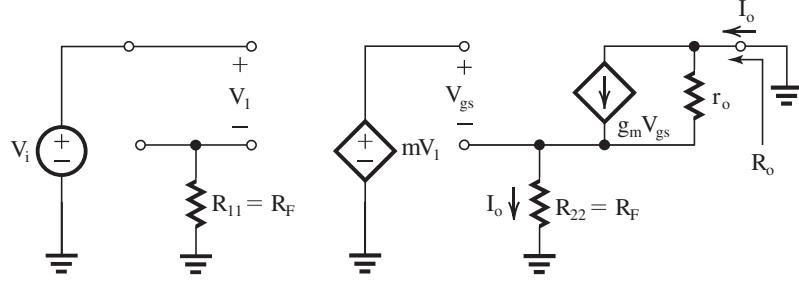


Figure 2

$$V_{gs} = \mu V_i - I_o R_F \quad (2)$$

$$I_o = g_m V_{gs} \frac{r_o}{r_o + R_F} \quad (3)$$

Combining Eqs. (1)–(3) yields

$$A \equiv \frac{I_o}{V_i} = \frac{\mu g_m r_o}{r_o + R_F + g_m r_o R_F}$$

For  $\mu = 1000 \text{ V/V}$ ,  $g_m = 2 \text{ mA/V}$ ,  $r_o = 20 \text{ k}\Omega$ , and  $R_F = 0.2 \text{ k}\Omega$ , we have

$$A = \frac{1000 \times 2 \times 20}{20 + 0.2 + 2 \times 20 \times 0.2}$$

$$= 1418.4 \text{ mA/V}$$

$$(c) A\beta = \frac{\mu g_m r_o R_F}{r_o + R_F + g_m r_o R_F}$$

$$A\beta = 283.7$$

$$1 + A\beta = 284.7$$

$$(d) A_F \equiv \frac{I_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{1418.4}{284.7} = 4.982 \text{ mA/V}$$

which is very close to the ideal value of 5 mA/V.

(e) From the A circuit in Fig. 2, we have

$$R_o = r_o + R_F + g_m r_o R_F$$

$$1 + A\beta = 1 + \frac{\mu g_m r_o R_F}{r_o + R_F + g_m r_o R_F}$$

$$R_{of} = (1 + A\beta)R_o$$

$$= r_o + R_F + g_m r_o R_F + \mu g_m r_o R_F$$

$$= r_o + R_F + (\mu + 1)g_m r_o R_F$$

$$\simeq \mu g_m r_o R_F$$

$$R_o = 20 + 0.2 + 2 \times 20 \times 0.2$$

$$= 28.2 \text{ k}\Omega$$

$$R_{of} = 20 + 0.2 + 1001 \times 2 \times 20 \times 0.2$$

$$= 20 + 0.2 + 8008 = 8028.2 \text{ k}\Omega$$

$$\simeq 8 \text{ M}\Omega$$

10.50 Figure 1 on the next page shows the equivalent circuit with  $V_s = 0$  and a voltage  $V_x$  applied to the collector for the purpose of determining the output resistance  $R_o$ ,

$$R_o \equiv \frac{V_x}{I_x}$$

Some of the analysis is displayed on the circuit diagram. Since the current entering the emitter node is equal to  $I_x$ , we can write for the emitter voltage

$$V_e = I_x [R_e \parallel (r_\pi + R_b)] \quad (1)$$

The base current can be obtained using the current-divider rule applied to  $R_e$  and  $(r_\pi + R_b)$  as

$$I_b = -I_x \frac{R_e}{R_e + r_\pi + R_b} \quad (2)$$

The voltage from collector to ground is equal to  $V_x$  and can be expressed as the sum of the voltage drop across  $r_o$  and  $V_e$ ,

$$V_x = (I_x - \beta I_b) r_o + V_e$$

Substituting for  $V_e$  from (1) and for  $I_b$  from (2), we obtain

$$\begin{aligned} R_o &= \frac{V_x}{I_x} = r_o + [R_e \parallel (r_\pi + R_b)] \\ &\quad + \frac{R_e \beta r_o}{R_e + r_\pi + R_b} \\ &= r_o + [R_e \parallel (r_\pi + R_b)] \left[ 1 + r_o \frac{\beta}{r_\pi + R_b} \right] \end{aligned}$$

Since  $\beta = g_m r_\pi$ , we obtain

$$R_o = r_o + [R_e \parallel (r_\pi + R_b)] \left[ 1 + g_m r_o \frac{r_\pi}{r_\pi + R_b} \right] \quad \text{Q.E.D.}$$

This figure belongs to Problem 10.50.

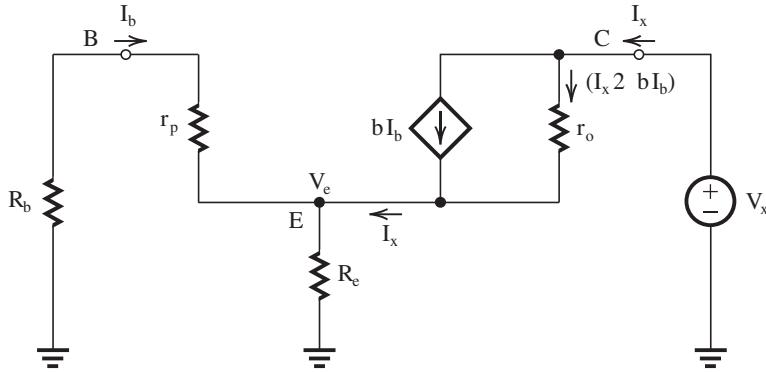


Figure 1

For  $R_b = 0$ ,

$$R_o = r_o + (R_e \parallel r_\pi)(1 + g_m r_o)$$

The maximum value of  $R_o$  will be obtained when  $R_e \gg r_\pi$ . If  $R_e$  approaches infinity (zero signal current in the emitter),  $R_o$  approaches the theoretical maximum:

$$\begin{aligned} R_{o\max} &= r_o + r_\pi(1 + g_m r_o) \\ &= r_o + r_\pi + \beta r_o \\ &\simeq \beta r_o \end{aligned} \quad (3)$$

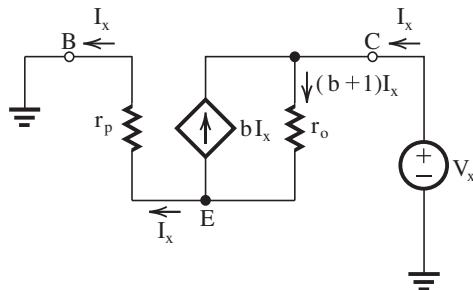


Figure 2

The situation that pertains in the circuit when  $R_e = \infty$  is illustrated in Fig. 2. Observe that since the signal current in the emitter is zero, the base current will be equal to the collector current ( $I_x$ ) and in the direction indicated. The controlled-source current will be  $\beta I_x$ , and this current adds to  $I_x$  to provide a current  $(\beta + 1)I_x$  in the output resistance  $r_o$ . A loop equation takes the form

$$V_x = (\beta + 1)I_x r_o + I_x r_\pi$$

and thus

$$R_o \equiv \frac{V_x}{I_x} = r_\pi + (\beta + 1)r_o$$

which is identical to the result in Eq. (3).

10.51

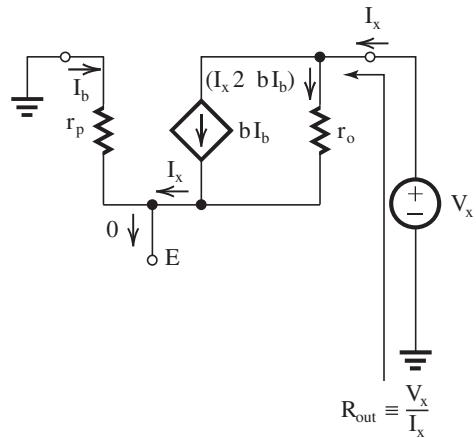


Figure 1

Figure 1 shows the situation that pertains in the transistor when  $\mu$  is so large that  $V_b \simeq 0$  and  $I_e \simeq 0$ . Observe that

$$I_b = -I_x$$

Writing a loop equation for the C-E-B, we obtain

$$V_x = (I_x - \beta I_b)r_o - I_b r_\pi$$

Substituting  $I_b = -I_x$ , we obtain

$$R_{out} = \frac{V_x}{I_x} = r_\pi + (\beta + 1)r_o$$

or if  $\beta$  is denoted  $h_{fe}$ ,

$$R_{out} = r_\pi + (h_{fe} + 1)r_o \quad Q.E.D.$$

Thus, for large amounts of feedback,  $R_{out}$  is limited to this value, which is approximately  $h_{fe}r_o$  independent of the amount of feedback. This phenomenon does not occur in MOSFET circuits where  $h_{fe} = \infty$ .

10.52

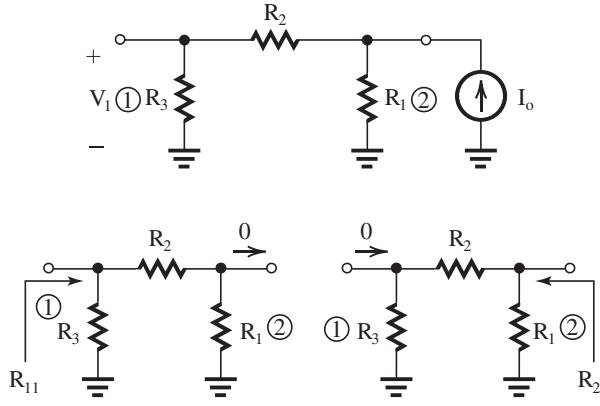


Figure 1

Figure 1 shows the feedback network fed with a current  $I_o$  to determine  $\beta$ :

$$\beta = \frac{V_f}{I_o} = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

For  $A\beta \gg 1$ ,

$$A_f = \frac{I_o}{V_s} \approx \frac{1}{\beta}$$

Thus,

$$A_f = \frac{1}{R_3} + \frac{R_2}{R_1 R_3} + \frac{1}{R_1}$$

For  $R_1 = R_3 = 0.1 \text{ k}\Omega$  and  $A_f = 100 \text{ mA/V}$ ,

$$100 = 10 + \frac{R_2}{0.01} + 10$$

$$\Rightarrow R_2 = 0.8 \text{ k}\Omega$$

To obtain the loading effects of the feedback network, refer to Fig. 1.

$$\begin{aligned} R_{11} &= R_3 \parallel (R_2 + R_1) \\ &= 100 \Omega \parallel (800 + 100) \Omega = 90 \Omega \end{aligned}$$

$$\begin{aligned} R_{22} &= R_1 \parallel (R_2 + R_3) \\ &= 100 \parallel (800 + 100) = 90 \Omega \end{aligned}$$

The A circuit is shown in Fig. 2. We can write

$$V_g = \mu V_i \quad (1)$$

$$I_o = \frac{V_g}{(1/g_m) + R_{22}} \quad (2)$$

Thus,

$$A = \frac{I_o}{V_i} = \frac{\mu}{(1/g_m) + R_{22}}$$

Since  $\beta = 0.01$ , we have

$$\begin{aligned} A\beta &= \frac{0.01\mu}{1/g_m + R_{22}} \\ &= \frac{0.01\mu}{1 + 0.09} = 9.17 \times 10^{-3}\mu \end{aligned}$$

For a 60-dB amount of feedback,

$$1 + A\beta = 1000$$

$$A\beta = 999$$

$$9.17 \times 10^{-3}\mu = 999$$

$$\Rightarrow \mu = 1.09 \times 10^5 \text{ V/V}$$

$$R_{\text{out}} = R_{\text{of}} = (1 + A\beta)R_o = 1000R_o$$

where  $R_o$  can be obtained from the A circuit as

$$R_o = r_o + R_{22} + g_m r_o R_{22}$$

$$= 50 + 0.09 + 1 \times 50 \times 0.09$$

$$= 54.6 \text{ k}\Omega$$

Thus,

$$R_{\text{out}} = 1000 \times 54.6 = 54.6 \text{ M}\Omega$$

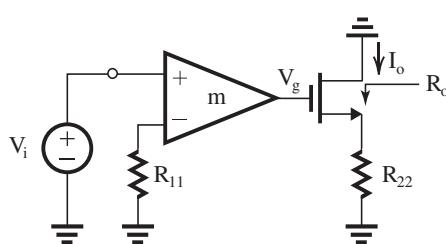


Figure 2

10.53 (a) Since  $V_s$  has a zero dc component, the gate of  $Q_1$  is at zero dc voltage. The negative feedback will force the gate of  $Q_2$  to be approximately at the same dc voltage as that at the gate of  $Q_1$ , thus

$$V_O = 0$$

$$\begin{aligned} V_{D1} &= 1.2 - V_{SG3} \\ &= 1.2 - |V_t| - |V_{OV3}| \\ &= 1.2 - 0.4 - 0.2 = +0.6 \text{ V} \\ V_{D2} &= V_O + V_{GS5} \\ &= 0 + V_t + V_{OV5} \\ &= 0.6 \text{ V} \end{aligned}$$

(b)

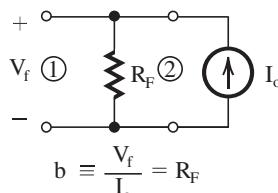


Figure 1

The feedback network is shown in Fig. 1, from which we find

$$\beta = R_F = 10 \text{ k}\Omega$$

For  $A\beta \gg 1$ ,

$$A_f \approx \frac{1}{\beta} = \frac{1}{R_F}$$

$$A_f = \frac{1}{10 \text{ k}\Omega} = 0.1 \text{ mA/V}$$

(c) From the  $\beta$  circuit in Fig. 1 and noting that the feedback topology is series-series, the loading effects of the feedback network are

$$R_{11} = R_{22} = R_F = 10 \text{ k}\Omega$$

Figure 2 shows the A circuit. We can write

$$\frac{V_{g5}}{V_i} = -g_{m1,2}(r_{o2} \parallel r_{o4}) \quad (1)$$

$$I_o = \frac{V_{g5}}{(1/g_{m5}) + R_{22}}$$

Thus,

$$A \equiv \frac{I_o}{V_i} = \frac{g_{m1,2}(r_{o2} \parallel r_{o4})}{(1/g_{m5}) + R_{22}}$$

$$g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}}$$

$$= \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I_{D2,4}}$$

$$= \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$g_{m5} = \frac{2I_{D5}}{V_{OV5}} = \frac{2 \times 0.8}{0.2} = 8 \text{ mA/V}$$

$$A = \frac{1 \times (200 \parallel 200)}{0.125 + 10}$$

$$= 9.88 \text{ mA/V}$$

$$\frac{I_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$= \frac{9.88}{1 + 9.88 \times 10} = 0.099 \text{ mA/V}$$

This figure belongs to Problem 10.53, part (b).

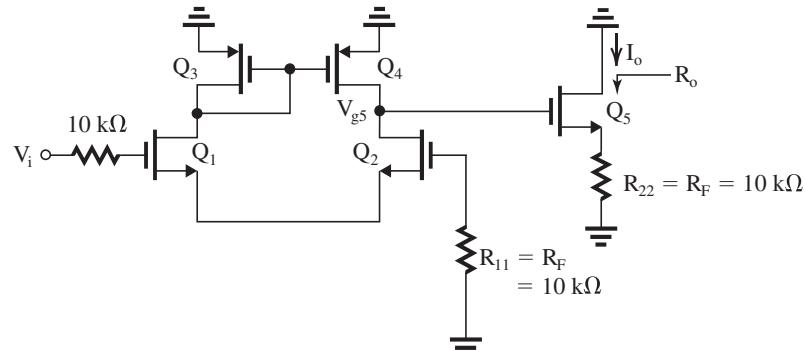


Figure 2

(d) From the A circuit, we have

$$R_o = r_{o5} + R_{22} + g_{m5}r_{o5}R_{22}$$

where

$$r_{o5} = \frac{|V_A|}{I_{D5}} = \frac{20}{0.8} = 25 \text{ k}\Omega$$

$$R_o = 25 + 10 + 8 \times 25 \times 10 = 2035 \text{ k}\Omega$$

$$R_{out} = R_{of} = R_o(1 + A\beta)$$

$$= 2.035 \times (1 + 9.88 \times 10)$$

$$= 203 \text{ M}\Omega$$

$$(e) V_o = I_o R_F$$

$$= A_f V_s R_F$$

$$\frac{V_o}{V_s} = A_f R_F = 0.099 \times 10 = 0.99 \text{ V/V}$$

$$R_{out} = \frac{\text{Output resistance at source of } Q_5}{1 + A\beta}$$

$$\approx \frac{1/g_{m5}}{1 + A\beta}$$

$$= \frac{125 \text{ }\Omega}{1 + 9.88 \times 10} = 1.25 \text{ }\Omega$$

10.54

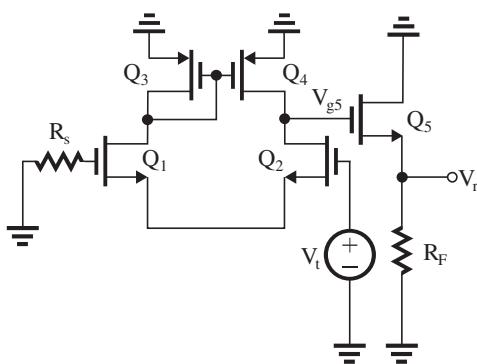


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain  $A\beta$ :

$$A\beta \equiv -\frac{V_r}{V_t}$$

First we write for the gain of differential amplifier

$$\frac{V_{g5}}{V_t} = -g_{m1,2}(r_{o2} \parallel r_{o4}) \quad (1)$$

Next we write for the source follower,

$$\frac{V_r}{V_{g5}} = \frac{R_F \parallel r_{o5}}{(R_F \parallel r_{o5}) + (1/g_{m5})} \quad (2)$$

Combining (1) and (2) yields

$$A\beta = -\frac{V_r}{V_t} = g_{m1,2}(r_{o2} \parallel r_{o4}) \frac{R_F \parallel r_{o5}}{(R_F \parallel r_{o5}) + (1/g_{m5})}$$

Q.E.D.

$10.55 \mu = 10^3 \text{ V/V}$ ,  $R_{id} = \infty$ ,  $r_o = 100 \text{ }\Omega$ ,  $R_F = 10 \text{ k}\Omega$ , and  $R_s = R_L = 1 \text{ k}\Omega$ . From Example 10.9 Eqs. (10.37) and (10.41),

$$\beta = -\frac{1}{R_F} = -\frac{1}{10 \text{ k}\Omega} = -0.1 \text{ mA/V}$$

$$A = -\mu R_i \frac{(R_F \parallel R_L)}{r_o + (R_F \parallel R_L)}$$

where

$$R_i = R_{id} \parallel R_F \parallel R_s$$

$$R_i = \infty \parallel 10 \parallel 1 = 0.909 \text{ k}\Omega$$

$$A = -10^3 \times 0.909 \times \frac{(10 \parallel 1)}{0.1 + (10 \parallel 1)}$$

$$= -818.9 \text{ k}\Omega$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{818.9}{1 - 818.9 \times -0.1}$$

$$= -\frac{818.9}{1 + 81.89} = -9.88 \text{ k}\Omega$$

$$R_{if} = R_i/(1 + A\beta)$$

$$= \frac{0.909 \text{ k}\Omega}{1 + 81.89} = 11 \text{ }\Omega$$

$$R_{if} = R_{in} \parallel R_s$$

$$= R_{in} \parallel 1 \text{ k}\Omega$$

$$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{1000}} = \frac{1}{\frac{1}{11} - \frac{1}{1000}}$$

$$= 11.1 \text{ }\Omega$$

From Eq. (10.42), we have

$$R_o = r_o \parallel R_F \parallel R_L$$

$$= 0.1 \parallel 10 \parallel 1 = 90.1 \text{ }\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{90.1}{1 + 81.89} = 1.1 \text{ }\Omega$$

$$R_{of} = R_{out} \parallel R_L$$

$$= R_{out}/1 \text{ k}\Omega$$

$$\Rightarrow R_{out} \approx 1.1 \text{ }\Omega$$

Comparison to the values in Example 10.9:

	$\mu = 10^4 \text{ V/V}$	$\mu = 10^3$
$A_f$	-9.99 k $\Omega$	-9.88 k $\Omega$
$R_{in}$	1.11 $\Omega$	11.1 $\Omega$
$R_{out}$	0.11 $\Omega$	1.1 $\Omega$

$$\beta = -\frac{1}{R_F} = -0.1 \text{ mA/V}$$

$$A\beta = 3.23$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{32.3}{1 + 3.23} = -7.63 \text{ k}\Omega$$

Compare these results to those found in Exercise 10.19:  $A = -32.3 \text{ k}\Omega$  ( $-30.3 \text{ k}\Omega$ ),  $\beta = -0.1 \text{ mA/V}$  ( $-0.1 \text{ mA/V}$ ),  $A\beta = -3.23$  ( $-3.03$ ), and  $A_f = -7.63 \text{ k}\Omega$  ( $-7.52 \text{ k}\Omega$ ). The slight differences are due to the approximation used in the systematic analysis method.

10.56

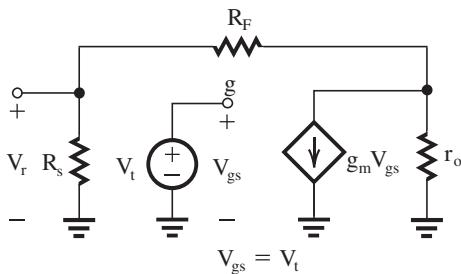


Figure 1

Figure 1 shows the circuit prepared for the determination of the loop gain  $A\beta$ :

$$A\beta = -\frac{V_r}{V_t}$$

An expression for  $V_r$  can be written by inspection as

$$V_r = -g_m V_t [r_o \parallel (R_s + R_F)] \frac{R_s}{R_s + R_F}$$

Thus,

$$A\beta = g_m [r_o \parallel (R_s + R_F)] \frac{R_s}{R_s + R_F} \quad (1)$$

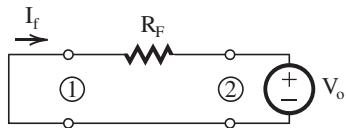


Figure 2

The feedback network ( $\beta$  circuit) is shown in Fig. 2 fed with  $V_o$  at port 2 and with port 1 short-circuited to determine  $\beta$ :

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F} \quad (2)$$

Equations (1) and (2) can now be used to determine  $A$ :

$$A = -g_m [r_o \parallel (R_s + R_F)] (R_s \parallel R_F)$$

Using the numerical values given in Exercise 10.19(e), we obtain

$$A = -5[20 \parallel (1 + 10)][1 \parallel 10]$$

$$= -32.3 \text{ k}\Omega$$

10.57 (a)

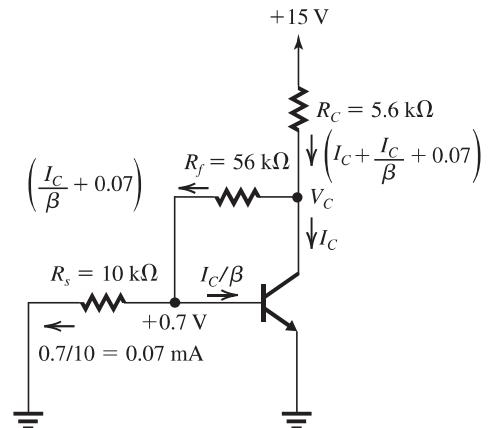


Figure 1

Figure 1 illustrates the dc analysis. We can express the dc collector voltage  $V_C$  in two alternative ways:

$$V_C = +15 - R_C \left( I_C + \frac{I_C}{\beta} + 0.07 \right)$$

and

$$V_C = 0.7 + R_f \left( \frac{I_C}{\beta} + 0.07 \right)$$

Equating these two expressions yields

$$15 - 5.6(I_C + 0.01I_C + 0.07)$$

$$= 0.7 + 56(0.01I_C + 0.07)$$

$$\Rightarrow I_C = 1.6 \text{ mA}$$

$$V_C \approx 5.5 \text{ V}$$

This figure belongs to Problem 10.57, part (b).

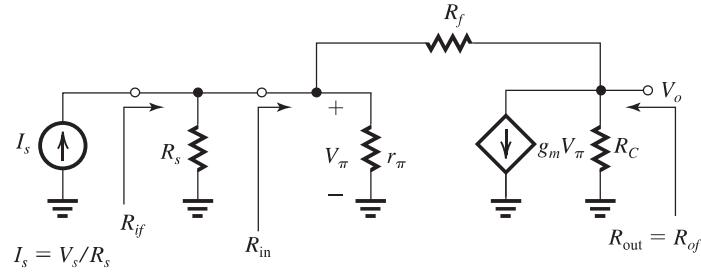


Figure 2

The below two figures belong to Problem 10.57, part (c).

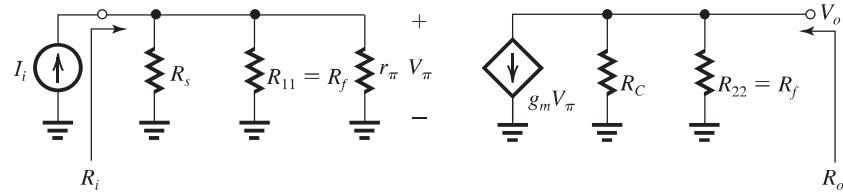


Figure 3

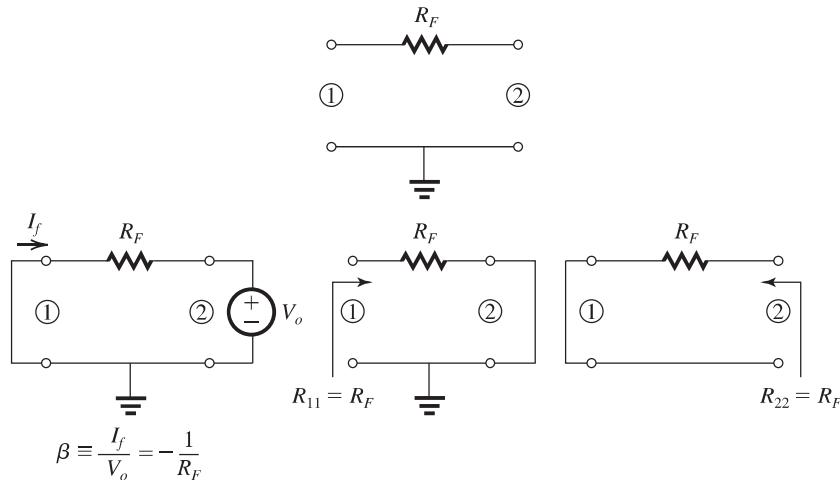


Figure 4

(b) Figure 2 above shows the small-signal equivalent circuit of the amplifier where

$$g_m = \frac{1.6 \text{ mA}}{0.025 \text{ V}} = 64 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{64} = 1.56 \text{ k}\Omega$$

(c) Figure 3 above shows the A circuit. It includes the loading effects of the feedback network:

$$R_{11} = R_{22} = R_f$$

Also, observe that

$$\beta = -\frac{1}{R_f}$$

From the A circuit in Fig. 4, we have

$$R_i = R_s \parallel R_f \parallel r_\pi \quad (1)$$

$$V_\pi = I_i R_i \quad (2)$$

$$V_o = -g_m V_\pi (R_C \parallel R_f) \quad (3)$$

Combining (1), (2) and (3) gives

$$A \equiv \frac{V_o}{I_i} = -g_m(R_s \parallel R_f \parallel r_{\pi})(R_C \parallel R_f)$$

$$R_i = 10 \text{ k}\Omega \parallel 56 \text{ k}\Omega \parallel 1.56 \text{ k}\Omega$$

$$= 1.32 \text{ k}\Omega$$

$$A = -64 \times 1.32 \times (5.6 \parallel 56)$$

$$= -429 \text{ k}\Omega$$

From the A circuit, we have

$$R_o = R_C \parallel R_f$$

$$= 5.6 \text{ k}\Omega \parallel 56 \text{ k}\Omega$$

$$= 5.1 \text{ k}\Omega$$

$$(d) \beta = -\frac{1}{R_f} = -\frac{1}{56 \text{ k}\Omega}$$

$$A\beta = \frac{429}{56} = 7.67$$

$$1 + A\beta = 8.67$$

$$(e) A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{429}{8.67} = -49.5 \text{ k}\Omega$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{1.32 \text{ k}\Omega}{8.67} = 152 \text{ }\Omega$$

$$R_{if} = R_s \parallel R_{in}$$

$$152 \text{ }\Omega = 10 \text{ k}\Omega \parallel R_{in}$$

$$R_{in} = 155 \text{ }\Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{5.1 \text{ k}\Omega}{8.67} = 588 \text{ }\Omega$$

$$(f) \frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s} = -\frac{49.5}{10} = -4.95 \text{ V/V}$$

Refer to Fig. P10.57 and assume the gain of the BJT to be infinite so that the signal voltage at its base is zero (virtual ground). In this case, we have

$$\frac{V_o}{V_s} = -\frac{R_f}{R_s} = -\frac{56}{10} = -5.6 \text{ V/V}$$

Thus, the actual gain magnitude ( $\simeq 5 \text{ V/V}$ ) is only about 12% below the ideal value; not bad for a single transistor inverting op amp!

10.58

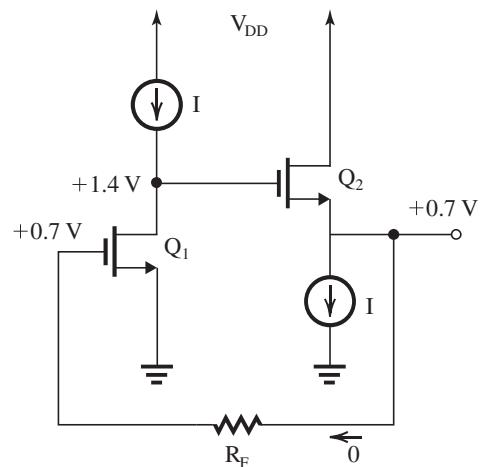


Figure 1

(a) See Figure 1.

$$V_{G1} = V_{GS1} = V_t + V_{OV}$$

$$= 0.5 + 0.2 = +0.7 \text{ V}$$

(because the dc voltage across  $R_F$  is zero)

$$V_O = V_{G1}$$

$$V_O = +0.7 \text{ V}$$

$$V_{D1} = V_O + V_{GS2}$$

$$= 0.7 + 0.5 + 0.2$$

$$= +1.4 \text{ V}$$

$$(b) g_{m1,2} = \frac{2I}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4 \text{ mA/V}$$

$$r_{o1,2} = \frac{V_A}{I} = \frac{16 \text{ V}}{0.4 \text{ mA}} = 40 \text{ k}\Omega$$

(c) Figure 2 on the next page shows the  $\beta$  circuit and the determination of its loading effects,

$$R_{11} = R_{22} = R_F$$

Figure 2 shows also the A circuit. We can write

$$V_{g1} = I_i R_i \quad (1)$$

where

$$R_i = R_{11} = R_F \quad (2)$$

$$V_{d1} = -g_{m1} r_{o1} V_{g1} \quad (3)$$

This figure belongs to Problem 10.58, part (c).

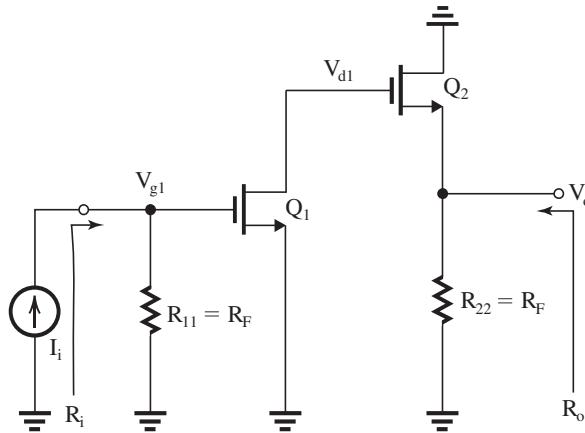
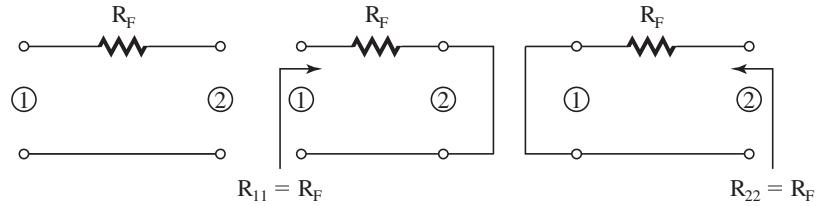


Figure 2

$$\frac{V_o}{V_{d1}} = \frac{R_{22} \parallel r_{o2}}{(R_{22} \parallel r_{o2}) + \frac{1}{g_{m2}}} \quad (4)$$

Combining Eqs. (1)–(4) results in

$$A \equiv \frac{V_o}{I_i} = -g_{m1}r_{o1}R_F \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

(d)

$$(e) \quad A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{g_{m1}r_{o1}R_F(R_F \parallel r_{o2})}{(R_F \parallel r_{o2}) + 1/g_{m2} + (g_{m1}r_{o1})(R_F \parallel r_{o2})}$$

$$(f) \quad R_i = R_F$$

$$R_{in} = R_{if} = R_i / (1 + A\beta)$$

$$= R_F \left/ \left[ 1 + g_{m1}r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}} \right] \right.$$

$$R_{out} = R_{of} = R_o / (1 + A\beta)$$

where from the A circuit we have

$$R_o = R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$R_{out} = \left( R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}} \right) \left/ \left[ 1 + g_{m1}r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}} \right] \right.$$

$$(g) \quad A = -4 \times 40 \times 10 \frac{10 \parallel 40}{(10 \parallel 40) + 0.25}$$

$$= -1551.5 \text{ k}\Omega$$

$$\beta = -\frac{1}{R_F} = -\frac{1}{10 \text{ k}\Omega} = -0.1 \text{ mA/V}$$

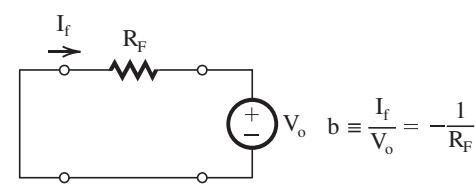


Figure 3

From Fig. 3 we see that

$$\beta = -\frac{1}{R_F}$$

$$A\beta = g_{m1}r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

$$1 + A\beta = 1 + g_{m1}r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

$$A\beta = 155.15$$

$$1 + A\beta = 156.15$$

$$A_f = -\frac{1551.5}{156.15} = -9.94 \text{ k}\Omega$$

$$R_i = R_F = 10 \text{ k}\Omega$$

$$R_{in} = R_{if} = \frac{R_F}{1 + A\beta} = \frac{10,000 \Omega}{156.15} = 64 \Omega$$

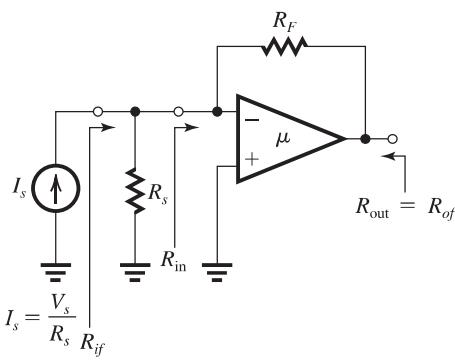
$$R_o = R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$R_o = 10 \parallel 40 \parallel 0.25 = 242 \Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{242}{156.15} = 1.55 \Omega$$

10.59 (a)



Refer to the feedback network shown in Fig. 10.24(b) and to the determination of  $\beta$  illustrated in Fig. 10.24(c). Thus,

$$\beta = -\frac{1}{R_F}$$

If  $A\beta \gg 1$ , then we have

$$A_f = \frac{V_o}{I_s} \simeq \frac{1}{\beta} = -R_F$$

and the voltage gain realized will be

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} \simeq -\frac{R_F}{R_s}$$

If  $R_s = 2 \text{ k}\Omega$ , to obtain  $V_o/V_s \simeq -10 \text{ V/V}$ , we required

$$R_F = 10 \times R_s = 20 \text{ k}\Omega$$

(b) Refer to the solution to Example 10.9.

$$\beta = -\frac{1}{R_F} = -\frac{1}{20 \text{ k}\Omega} = -0.05 \text{ mA/V}$$

Using Eq. (10.39), we obtain

$$R_i = R_{id} \parallel R_F \parallel R_s$$

$$= 100 \parallel 20 \parallel 2 = 1.786 \text{ k}\Omega$$

Using Eq. (10.41) with  $R_L = \infty$ , we get

$$A \equiv \frac{V_o}{I_i} = -10^3 \times 1.786 \times \frac{20}{20+2} \\ = -1623.6 \text{ k}\Omega$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

where

$$1 + A\beta = 1 + 1623.6 \times 0.05$$

$$= 82.18$$

$$A_f = \frac{V_o}{I_s} = -\frac{1623.6}{82.18}$$

$$= -19.76 \text{ k}\Omega$$

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s} = -\frac{19.76}{2}$$

$$= -9.88 \text{ V/V}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{1.786}{82.18} = 21.7 \Omega$$

$$R_{if} = R_s \parallel R_{in}$$

$$21.7 \Omega = 2000 \Omega \parallel R_{in}$$

$$R_{in} \simeq 21.7 \Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

where from Eq. (10.42) with  $R_L = \infty$  we get

$$R_o = r_o \parallel R_F = 2 \parallel 20 = 1.818 \text{ k}\Omega$$

$$R_{out} = R_{of} = \frac{1.818}{82.18} = 22.1 \Omega$$

$$(c) f_{HF} = f_H(1 + A\beta)$$

$$= 1 \times 82.18$$

$$= 82.18 \text{ kHz}$$

This figure belongs to Problem 10.60.

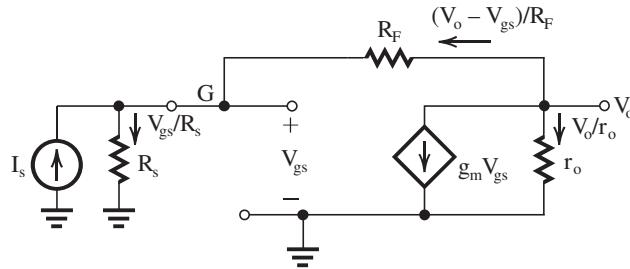


Figure 1

This figure belongs to Problem 10.61, part (a).

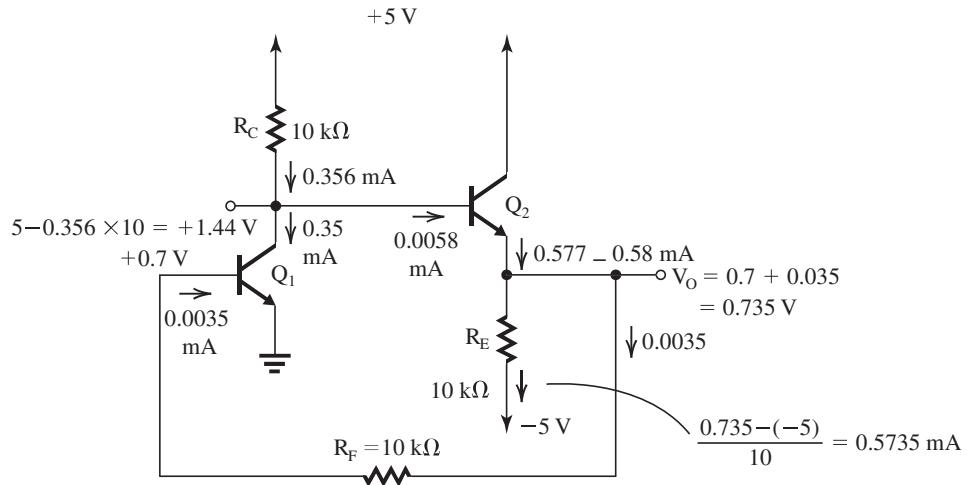


Figure 1

10.60 Figure 1 shows the small-signal-equivalent circuit of the feedback amplifier of Fig. E10.19.

Analysis to determine  $V_o/I_s$  proceeds as follows:

Writing a node equation at the output node provides

$$\begin{aligned} g_m V_{gs} + \frac{V_o}{r_o} + \frac{V_o - V_{gs}}{R_F} &= 0 \\ \Rightarrow V_{gs} &= -V_o \frac{r_o + \frac{1}{R_F}}{g_m - \frac{1}{R_F}} \\ \Rightarrow V_{gs} &= -V_o \frac{1}{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)} \end{aligned} \quad (1)$$

Writing a node equation at node G provides

$$\begin{aligned} I_s - \frac{V_{gs}}{R_s} + \frac{V_o - V_{gs}}{R_F} &= 0 \\ I_s - \frac{V_{gs}}{(R_s \parallel R_F)} + \frac{V_o}{R_F} &= 0 \end{aligned} \quad (2)$$

Substituting for  $V_{gs}$  from (1) into (2), we obtain

$$\begin{aligned} I_s + V_o \frac{1}{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)} + \frac{V_o}{R_F} \\ = 0 \Rightarrow \frac{V_o}{I_s} \\ = \frac{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)}{1 + \left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)/R_F} \end{aligned}$$

For the feedback analysis to be reasonably accurate, we use

$$g_m \gg \frac{1}{R_F}$$

10.61 (a) Figure 1 (see figure above) shows the dc analysis. We assumed  $I_{C1} = 0.35$  mA and found that  $I_{C2} = 0.58$  mA, thus verifying the given values. The dc voltage at the output is

$$V_O = +0.735 \text{ V}$$

(b)

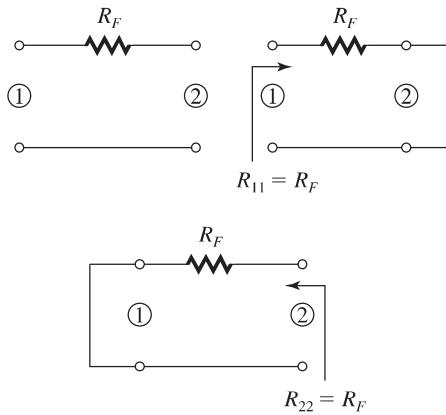


Figure 2

Figure 2 shows the  $\beta$  circuit and the determination of its loading effects on the A circuit:

$$R_{11} = R_{22} = R_F = 10 \text{ k}\Omega$$

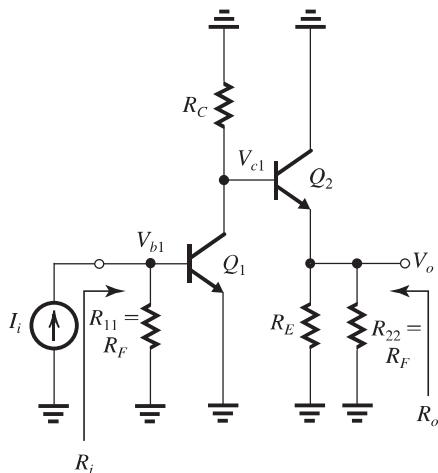


Figure 3

Figure 3 shows the A circuit. The input resistance is given by

$$R_i = R_F = R_F \parallel r_{\pi 1}$$

where

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.35}{0.025} = 14 \text{ mA/V}$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{14} = 7.14 \text{ k}\Omega$$

Thus,

$$R_i = 10 \text{ k}\Omega \parallel 7.14 \text{ k}\Omega = 4.17 \text{ k}\Omega$$

The input voltage  $V_{b1}$  is given by

$$V_{b1} = I_i R_i = 4.17 I_i \quad (1)$$

The collector voltage of  $Q_1$  is given by

$$V_{c1} = -g_{m1} V_{b1} \{ R_C \parallel (\beta_2 + 1) [r_{e2} + (R_E \parallel R_F)] \}$$

where

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25 \text{ mV}}{0.58 \text{ mA}} = 43.1 \text{ }\Omega$$

$$V_{c1} = -14 V_{b1} \{ 10 \parallel 101 [0.0431 + (10 \parallel 10)] \}$$

$$= -137.3 V_{b1} \quad (2)$$

The gain of the emitter-follower  $Q_2$  is given by

$$\frac{V_o}{V_{c1}} = \frac{R_E \parallel R_F}{(R_E \parallel R_F) + r_{e2}}$$

$$= \frac{5}{5 + 0.0431} = 0.99 \text{ V/V} \quad (3)$$

Combining (1)–(3) gives

$$A \equiv \frac{V_o}{I_i} = -0.99 \times 137.3 \times 4.17$$

$$= -567.6 \text{ k}\Omega$$

(c) The value of  $\beta$  can be obtained from the  $\beta$  circuit as shown in Fig. 4:

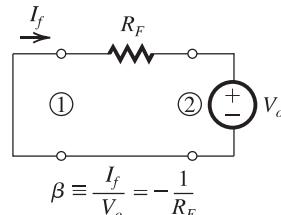


Figure 4

$$\beta = -\frac{1}{R_F} = -\frac{1}{10 \text{ k}\Omega} = -0.1 \text{ mA/V}$$

$$A\beta = -567.6 \times -0.1$$

$$= 56.76$$

$$1 + A\beta = 57.76$$

$$(d) A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$A_f = -\frac{567.6}{57.76} = -9.83 \text{ k}\Omega$$

$$R_{in} = R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{4.17 \text{ k}\Omega}{57.76} = 72.2 \text{ }\Omega$$

This figure belongs to Problem 10.62.

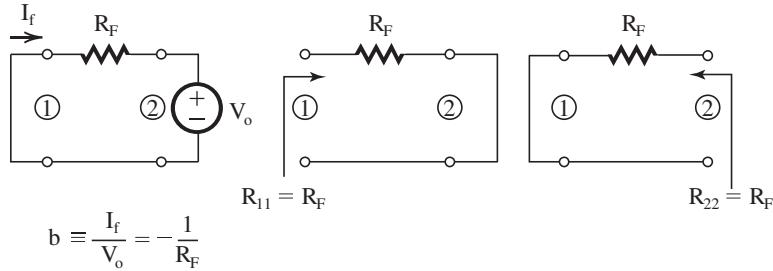


Figure 1

From the A circuit, we have

$$\begin{aligned} R_o &= R_F \parallel R_E \parallel \left[ r_{e2} + \frac{R_C}{\beta_2 + 1} \right] \\ &= 10 \parallel 10 \parallel \left[ 0.0431 + \frac{10}{101} \right] \\ &= 138.2 \Omega \end{aligned}$$

$$\begin{aligned} R_{out} &= R_{of} = \frac{R_o}{1 + A\beta} \\ &= \frac{138.2}{57.76} = 2.4 \Omega \end{aligned}$$

10.62 Figure 1 shows the feedback network with a voltage  $V_o$  applied to port 2 to determine  $\beta$ :

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F}$$

For  $A\beta \gg 1$ , we have

$$\frac{V_o}{I_s} \equiv A_f \simeq \frac{1}{\beta} = -R_F$$

Thus, for  $\frac{V_o}{I_s} \simeq -10 \text{ k}\Omega$ , we select

$$R_F = 10 \text{ k}\Omega$$

The loading of the feedback network on the A circuit can be determined as shown in Fig. 1:

$$R_{11} = R_{22} = R_F$$

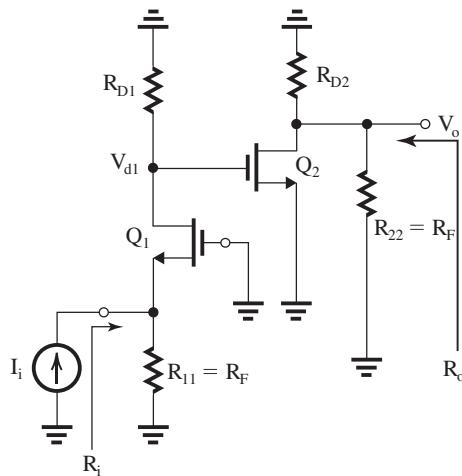


Figure 2

Figure 2 shows the A circuit. For the CG amplifier  $Q_1$ , we can write

$$R_i = R_F \parallel \frac{1}{g_{m1}} \quad (1)$$

$$V_{sg} = I_i R_i \quad (2)$$

$$V_{d1} = g_{m1} V_{sg} R_{D1} \quad (3)$$

Combining (1)–(3) yields

$$V_{d1} = (g_{m1} R_{D1}) \left( R_F \parallel \frac{1}{g_{m1}} \right) I_i \quad (4)$$

For the CS stage  $Q_2$ , we can write

$$\frac{V_o}{V_{d1}} = -g_{m2}(R_{D2} \parallel R_F) \quad (5)$$

Combining (4) and (5), we obtain the open-loop gain  $A$ :

$$A \equiv \frac{V_o}{I_s} = -(g_{m1} R_{D1}) \left( R_F \parallel \frac{1}{g_{m1}} \right) g_{m2}(R_{D2} \parallel R_F)$$

Substituting  $g_{m1} = g_{m2} = 4 \text{ mA/V}$ ,  $R_{D1} = R_{D2} = 10 \text{ k}\Omega$ , and  $R_F = 10 \text{ k}\Omega$  gives

$$A = -(4 \times 10) \times (10 \parallel 0.25) \times 4 \times (10 \parallel 10)$$

$$A = -195 \text{ k}\Omega$$

$$A\beta = \frac{195}{10} = 19.5$$

$$1 + A\beta = 20.5$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{195}{20.5} = -9.52 \text{ k}\Omega$$

$$R_{in} = R_{if} = \frac{R_i}{1 + A\beta}$$

From Eq. (1), we obtain

$$R_i = 10 \parallel 0.25 = 244 \Omega$$

$$R_{in} = \frac{244}{20.5} = 11.9 \Omega$$

From the A circuit,

$$R_o = R_{D2} \parallel R_F$$

$$= 10 \parallel 10 = 5 \text{ k}\Omega$$

$$\begin{aligned} R_{\text{out}} &= R_{\text{of}} = \frac{R_o}{1 + A\beta} \\ &= \frac{5000}{20.5} = 244 \text{ }\Omega \end{aligned}$$

10.63 (a) Converting the signal source to its Norton's form, we obtain the circuit shown in Fig. 1(a).

This is a shunt-shunt feedback amplifier with the feedback network consisting of the resistor  $R_f$ . To determine  $\beta$ , we use the arrangement shown in Fig. 1(b),

$$\beta = -\frac{1}{R_f}$$

Now, for  $A\beta \gg 1$ , the closed-loop gain becomes

$$A_f \equiv \frac{V_o}{I_s} \simeq \frac{1}{\beta} = -R_f$$

The voltage gain  $\frac{V_o}{V_s}$  is obtained as

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s}$$

Thus,

$$\frac{V_o}{V_s} \simeq -\frac{R_f}{R_s} \quad \text{Q.E.D.}$$

(b) To obtain a closed-loop voltage gain of approximately  $-100 \text{ V/V}$ , we use

$$-100 = -\frac{R_f}{R_s}$$

For  $R_s = 10 \text{ k}\Omega$ , we obtain

$$R_f = 1 \text{ M}\Omega$$

Now consider the amplifier stage shown in Fig. P10.63(b). First, we determine the dc bias point as follows:

$$I_E = \frac{15 \times \frac{10}{10 + 15} - 0.7}{4.7 + \frac{10 \parallel 15}{101}} = 1.11 \text{ mA}$$

$$I_C = 1.11 \times 0.99 = 1.1 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1.1}{0.025} = 44 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{44} = 2.27 \text{ k}\Omega$$

$$R_{\text{in}} = 10 \parallel 15 \parallel 2.27 = 1.65 \text{ k}\Omega$$

$$R_{\text{out}} = 7.5 \text{ k}\Omega$$

$$A_{v0} = -g_m \times 7.5 \text{ k}\Omega$$

$$= -44 \times 7.5 = -330 \text{ V/V}$$

Figure 2(a) (next page) shows the equivalent circuit of the amplifier stage. Figure 2(b) shows the A circuit of the feedback amplifier made up of the cascade of three stages. Observe that we have included  $R_s$  and  $R_L$  as well as  $R_{11}$  and  $R_{22}$ . The overall gain  $A \equiv V_o/I_s$  can be obtained as follows:

$$R_i = R_s \parallel R_F \parallel 1.65 \text{ k}\Omega$$

$$= 10 \parallel 1000 \parallel 1.65 = 1.414 \text{ k}\Omega$$

$$V_1 = I_i R_i = 1.414 I_i \quad (1)$$

$$V_2 = -330 V_1 \times \frac{1.65}{1.65 + 7.5} = -59.5 V_1 \quad (2)$$

$$V_3 = -330 V_2 \times \frac{1.65}{1.65 + 7.5} = -59.5 V_2 \quad (3)$$

$$V_o = -330 V_3 \times \frac{1 \parallel 1000}{(1 \parallel 1000) + 7.5}$$

$$= -38.8 V_3 \quad (4)$$

This figure belongs to Problem 10.63, part (a).

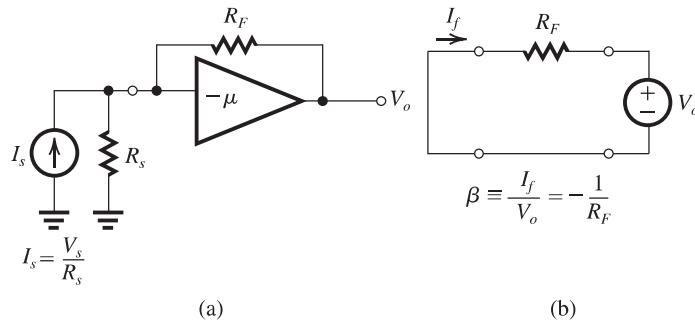


Figure 1

This figure belongs to Problem 10.63, part (b).

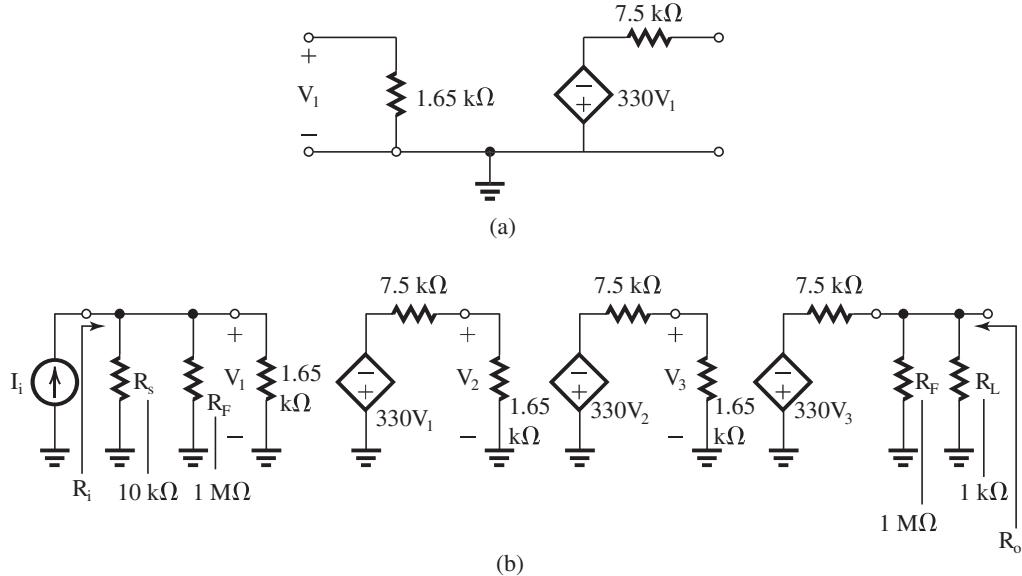


Figure 2

Combining (1)–(4) gives

$$A \equiv \frac{V_o}{I_i} = -1.94 \times 10^5 \text{ k}\Omega$$

Since

$$\beta = -\frac{1}{R_f} = -\frac{1}{1 \text{ M}\Omega}$$

we have

$$A\beta = 194$$

and

$$1 + A\beta = 195$$

Thus,

$$A_f \equiv \frac{V_o}{I_s} = -\frac{1.94 \times 10^5}{195}$$

$$= -995 \text{ k}\Omega$$

and the voltage gain realized is

$$\frac{V_o}{V_s} = \frac{A_f}{R_s} = \frac{-99.5}{10} = -99.5 \text{ V/V}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{1414 \text{ }\Omega}{195} = 7.3 \text{ }\Omega$$

$$R_{if} = R_s \parallel R_{in}$$

$$R_{in} \simeq 7.3 \text{ }\Omega$$

From the A circuit, we have

$$R_o = R_L \parallel R_F \parallel 7.5 \text{ k}\Omega$$

$$R_o = 1 \parallel 1000 \parallel 7.5 = 881.6 \text{ }\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{881.6}{195} = 4.5 \text{ }\Omega$$

$$R_{of} = R_{out} \parallel R_L$$

$$\Rightarrow R_{out} = 4.5 \text{ }\Omega$$

10.64 (a)

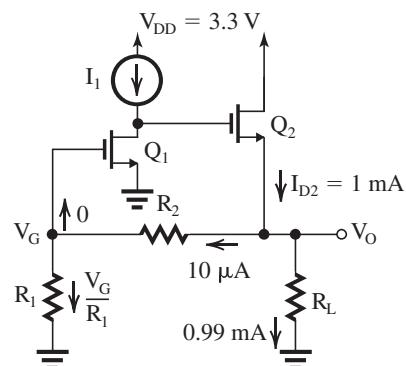


Figure 1

Figure 1 shows the circuit for the purpose of performing a dc design.

$$I_{D1} = 100 \mu\text{A} \Rightarrow I_1 = 100 \mu\text{A}$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 V_{OV1}^2$$

$$100 = \frac{1}{2} \times 200 \times \left(\frac{W}{L}\right)_1 \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 25$$

$$V_{G1} = V_{tn} + V_{OV1}$$

$$= 0.6 + 0.2 = 0.8 \text{ V}$$

Since  $I_{R2,R1} = 10 \mu\text{A}$ , we have

$$R_1 = \frac{0.8 \text{ V}}{0.01 \text{ mA}} = 80 \text{ k}\Omega$$

$$I_{RL} = I_{D2} - I_{R2,R1}$$

$$= 1 - 0.01 = 0.99 \text{ mA}$$

$$V_O = 0.99 \times 2 = 1.98 \text{ V}$$

$$I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 V_{OV2}^2$$

$$1 = \frac{1}{2} \times 0.2 \times \left(\frac{W}{L}\right)_2 \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 250$$

$$R_2 = \frac{V_O - V_G}{0.01 \text{ mA}}$$

$$= \frac{1.98 - 0.8}{0.01} = 118 \text{ k}\Omega$$

$$V_{GS2} = V_{tn} + V_{OV2} = 0.8 \text{ V}$$

$$V_{DI} = V_{G2} = 1.98 + 0.8 = 2.78 \text{ V}$$

(b) The  $\beta$  circuit consists of resistance  $R_2$ . The value of  $\beta$  can be determined as shown in Fig. 2.

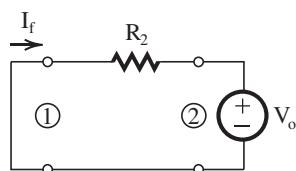


Figure 2

$$\beta = \frac{I_f}{V_o} = -\frac{1}{R_2} = -\frac{1}{118 \text{ k}\Omega}$$

$$= -8.47 \times 10^{-3} \text{ mA/V}$$

Thus,

$$A_f|_{\text{ideal}} \equiv \frac{1}{\beta} = -118 \text{ k}\Omega$$

(c) Converting the signal source to its Norton form, the feedback amplifier takes the form shown in Fig. 3.

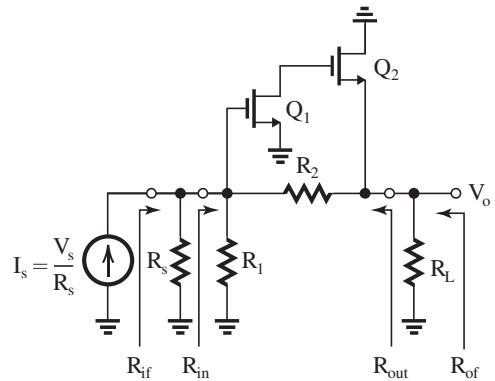


Figure 3

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s}$$

Thus,

$$-6 = -\frac{118}{R_s}$$

$$\Rightarrow R_s = 19.7 \text{ k}\Omega$$

(d)

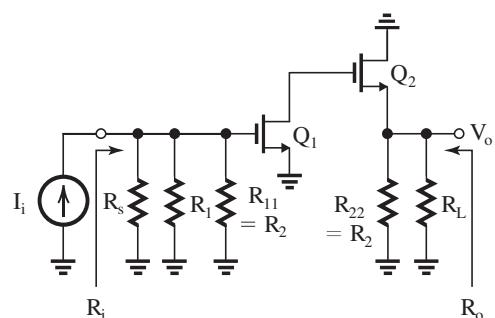


Figure 4

The A circuit is shown in Fig. 4.

$$\begin{aligned} R_i &= R_s \parallel R_1 \parallel R_2 \\ &= 19.7 \parallel 80 \parallel 118 = 13.92 \text{ k}\Omega \end{aligned}$$

$$V_{gs1} = I_i R_i = 13.92 I_i \quad (1)$$

$$V_{di} = -g_{mi} V_{gs1} r_{o1}$$

where

$$g_{mi} = \frac{2I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o1} = \frac{V_A}{I_{D1}} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

Thus,

$$V_{d1} = -200V_{gs1} \quad (2)$$

$$\frac{V_o}{V_{d1}} = \frac{R_L \parallel R_2 \parallel r_{o2}}{(R_L \parallel R_2 \parallel r_{o2}) + 1/g_{m2}}$$

where

$$g_{m2} = \frac{2I_{D2}}{V_{OV2}} = \frac{2 \times 1}{0.2} = 10 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I_{D2}} = \frac{20}{1} = 20 \text{ k}\Omega$$

Thus,

$$\frac{V_o}{V_{d1}} = \frac{(2 \parallel 118 \parallel 20)}{(2 \parallel 118 \parallel 20) + 0.1} = 0.947 \text{ V/V} \quad (3)$$

Combining (1)–(3), we obtain

$$A = \frac{V_o}{I_i} = -13.92 \times 200 \times 0.947$$

$$= -2636.7 \text{ k}\Omega$$

$$R_o = R_L \parallel R_2 \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$= 2 \parallel 118 \parallel 20 \parallel 0.1$$

$$= 94.7 \Omega$$

$$(e) A_f = \frac{V_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{2636.7}{1 + (2636.7/118)}$$

$$= -\frac{2636.7}{23.34} = -113 \text{ k}\Omega$$

$$\frac{V_o}{V_s} = \frac{A_f}{R_s} = -\frac{113}{19.7} = -5.73 \text{ V/V}$$

$$(f) R_{if} = \frac{R_i}{1 + A\beta} = \frac{13.92}{23.34} = 0.596 \text{ k}\Omega$$

$$R_{if} = R_s \parallel R_{in}$$

$$0.596 = 19.7 \parallel R_{in}$$

$$\Rightarrow R_{in} = 615 \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{94.7}{23.34} = 4.06 \Omega$$

$$R_{of} = R_{out} \parallel R_L$$

$$4.06 = R_{out} \parallel 2000 \Rightarrow R_{out} = 4.1 \Omega$$

10.65

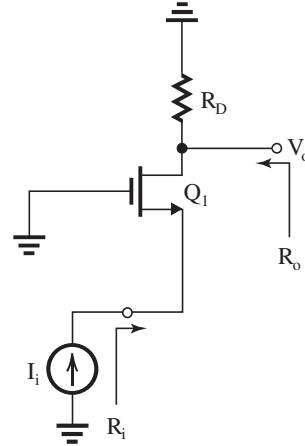


Figure 1

The A circuit is shown in Fig. 1.

$$R_i = \frac{1}{g_{m1}} = \frac{1}{5} = 0.2 \text{ k}\Omega$$

$$A \equiv \frac{V_o}{I_i} = R_D = 10 \text{ k}\Omega$$

$$R_o = R_D = 10 \text{ k}\Omega$$

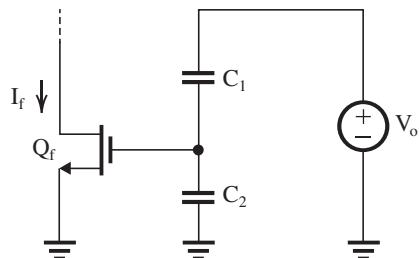


Figure 2

The β circuit is shown in Fig. 2.

$$\beta \equiv \frac{I_f}{V_o} = \frac{C_1}{C_1 + C_2} g_{mf} = \frac{0.9}{0.9 + 0.1} \times 2$$

$$= 1.8 \text{ mA/V}$$

$$A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{10}{1 + 10 \times 1.8} = 0.53 \text{ k}\Omega$$

$$R_{in} = R_{if} = \frac{R_i}{1 + A\beta} = \frac{200 \Omega}{19} = 10.5 \Omega$$

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta} = \frac{10 \text{ k}\Omega}{19} = 526 \Omega$$

These figures belong to Problem 10.66, part (a).

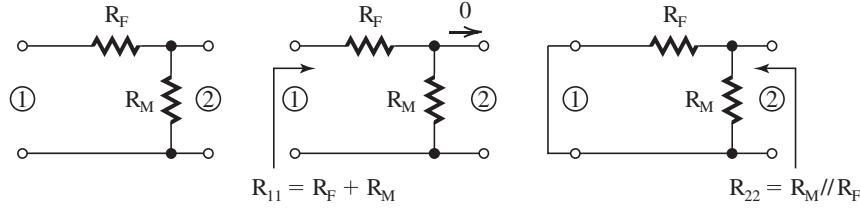


Figure 1

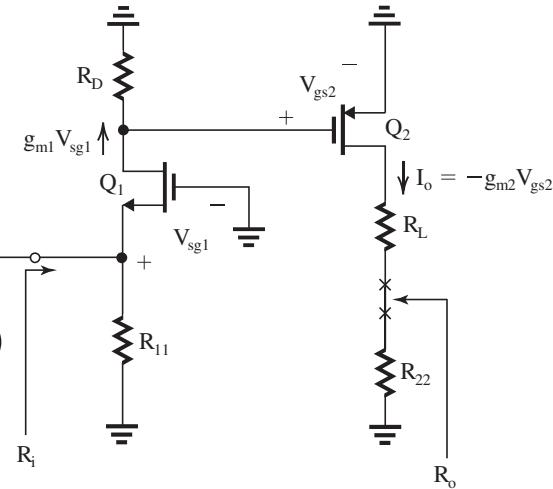


Figure 2

10.66 (a) Figure 1 shows the  $\beta$  network as well as the determination of its loading effects on the A circuit:

$$R_{11} = R_F + R_M$$

$$R_{22} = R_M \parallel R_F$$

Figure 2 shows the A circuit. Some of the analysis is shown on the diagram.

$$R_i = R_{11} \parallel \frac{1}{g_{m1}} \quad (1)$$

$$V_{sg1} = I_i R_i \quad (2)$$

$$V_{gs2} = g_{m1} V_{sg1} R_D \quad (3)$$

$$I_o = -g_{m2} V_{gs2} \quad (4)$$

Combining (1)–(4) gives

$$A \equiv \frac{I_o}{I_i} = -\left(R_{11} \parallel \frac{1}{g_{m1}}\right)(g_{m1} R_D) g_{m2} \quad (5)$$

(b)

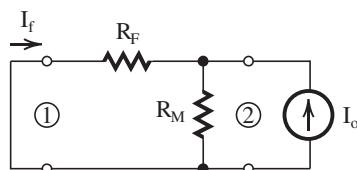


Figure 3

The  $\beta$  circuit prepared for the determination of  $\beta$  is shown in Fig. 3.

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_M}{R_M + R_F} \quad (6)$$

(c) From (5)–(6), we obtain

$$A\beta = \frac{R_M}{R_M + R_F} \left( R_{11} \parallel \frac{1}{g_{m1}} \right) (g_{m1} R_D) g_{m2}$$

$$(d) g_{m1} = g_{m2} = 5 \text{ mA/V}, \quad R_D = 20 \text{ k}\Omega$$

$$R_M = 10 \text{ k}\Omega, \text{ and } R_F = 90 \text{ k}\Omega, \text{ thus}$$

$$R_{11} = R_F + R_M = 90 + 10 = 100 \text{ k}\Omega$$

$$R_{22} = R_M \parallel R_F = 10 \parallel 90 = 9 \text{ k}\Omega$$

$$A = -(100 \parallel 0.2) \times (5 \times 20) \times 5 = -99.8 \text{ A/A}$$

$$\beta = -\frac{10}{10 + 90} = -0.1 \text{ A/A}$$

$$A\beta = 9.98$$

$$1 + A\beta = 10.98$$

$$A_f \equiv \frac{I_o}{I_s} = \frac{A}{1 + A\beta} = -\frac{99.8}{10.98} = -9.1 \text{ A/A}$$

$$R_{in} = R_{if} = \frac{R_i}{1 + A\beta}$$

where

$$R_i = 100 \text{ k}\Omega \parallel 0.2 \text{ k}\Omega \simeq 0.2 \text{ k}\Omega$$

$$R_{in} = \frac{200 \text{ }\Omega}{10.98} = 18.2 \text{ }\Omega$$

(e) Breaking the output loop of the A circuit between XX, we find

$$R_o = R_{22} + R_L + r_{o2}$$

$$= (R_M \parallel R_F) + R_L + r_{o2}$$

$$= (10 \parallel 90) + 1 + 20$$

$$= 30 \text{ k}\Omega$$

$$R_{of} = R_o(1 + A\beta) = 30 \times 10.98$$

$$= 329.4 \text{ k}\Omega$$

$$R_{out} = R_{of} - R_L = 328.4 \text{ k}\Omega$$

10.67

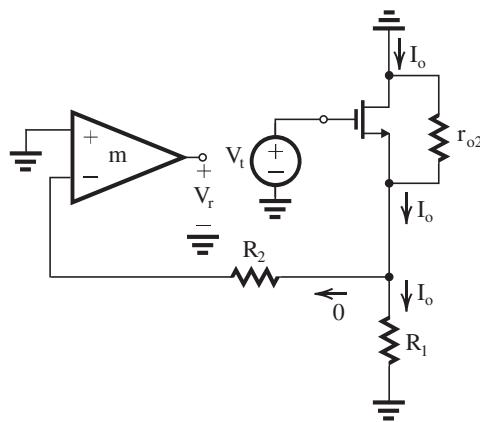


Figure 1

Figure 1 shows the shunt-series feedback amplifier circuit of Fig. 10.27(a) prepared for determining the loop gain,

$$A\beta \equiv -\frac{V_r}{V_t}$$

observe that here  $R_s = R_{id} = \infty$ . Thus  $I_o$  can be obtained as

$$I_o = \frac{V_t}{\frac{1}{g_m} + (r_{o2} \parallel R_1)} \frac{r_{o2}}{r_{o2} + R_1} \quad (1)$$

The voltage  $V_r$  can be obtained as

$$V_r = I_o R_1 \times -\mu = -\mu R_1 I_o \quad (2)$$

Combining Eqs. (1) and (2), we obtain

$$A\beta \equiv -\frac{V_r}{V_t} = \mu \frac{R_1}{\frac{1}{g_m} + (r_{o2} \parallel R_1)} \frac{r_{o2}}{r_{o2} + R_1}$$

For

$$\mu = 1000 \text{ V/V}, R_1 = 10 \text{ k}\Omega,$$

$$g_m = 5 \text{ mA/V}, \text{ and } r_{o2} = 20 \text{ k}\Omega$$

we obtain

$$\begin{aligned} A\beta &= 1000 \times \frac{10}{0.2 + (20 \parallel 10)} \frac{20}{20 + 10} \\ &= 970.9 \end{aligned}$$

which is slightly lower than the value found in Example 10.10 (1076.4), the difference being about  $-10\%$ . This is a result of the assumptions and approximations made in the general feedback analysis method.

From Example 10.10 (or directly from the  $\beta$  circuit) we have

$$\beta = -0.1 \text{ A/A}$$

Thus,

$$A = -9709$$

and

$$A_f = -\frac{9709}{971.9} = -9.99 \text{ A/A}$$

which is identical to the value obtained in Example 10.10. Thus while  $A\beta$  and  $A$  differ slightly for the earlier results,  $A_f$  is identical; an illustration of the power of negative feedback!

10.68 Refer to Fig. 10.27(c), which shows the determination of  $\beta$ ,

$$\beta = \frac{I_f}{I_o} = -\frac{R_1}{R_1 + R_2} \quad (1)$$

Refer to Fig. 10.27(e), which shows the A circuit. The input resistance  $R_i$  is given by

$$R_i = R_s \parallel R_{id} \parallel (R_1 + R_2)$$

For our case here,  $R_s = R_{id} = \infty$ , thus

$$R_i = R_1 + R_2$$

For  $R_{in} = R_{if} = 1 \text{ k}\Omega$ , we have

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$\Rightarrow R_i = R_{if}(1 + A\beta)$$

Thus

$$R_1 + R_2 = 1 \text{ k}\Omega \times (1 + A\beta)$$

Since  $1 + A\beta$  is 40 dB, that is,

$$1 + A\beta = 100$$

we have

$$R_1 + R_2 = 1 \times 100 = 100 \text{ k}\Omega \quad (2)$$

Now,

$$A_f = \frac{A}{1 + A\beta}$$

$$-100 = \frac{A}{100}$$

$$\Rightarrow A = -10^4 \text{ A/A}$$

$$\beta = \frac{A\beta}{A} = \frac{99}{-10^4} = -0.0099$$

Using Eqs. (1) and (2), we obtain

$$-0.0099 = -\frac{R_1}{R_1 + R_2}$$

$$\Rightarrow R_1 = 0.0099 \times 100 = 0.99 \text{ k}\Omega$$

$$R_2 = 100 - 0.99 = 99.01 \text{ k}\Omega$$

Now, using Eq. (10.53), (page 859), we obtain

$$A = -\mu \frac{R_i}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \parallel R_2)}$$

$$-10^4 = \frac{100}{-\mu \frac{100}{0.2 + (0.99 \parallel 99.01 \parallel 20)} \frac{20}{20 + (0.99 \parallel 99.01)}} \frac{20}{20 + (0.99 \parallel 99.01)}$$

$$\mu = 119 \text{ V/V}$$

From Example 10.10, we have

$$R_o = r_{o2} + (R_1 \parallel R_2) + g_m r_{o2} (R_1 \parallel R_2)$$

$$R_o = 20 + (0.99 \parallel 99.01)(1 + 5 \times 20)$$

$$= 119 \text{ k}\Omega$$

$$R_{out} = R_o(1+A\beta) = 119 \times 100 = 11.9 \text{ M}\Omega$$

10.69 (a)

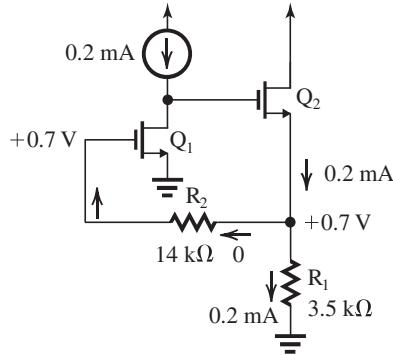


Figure 1

Figure 1 shows the dc analysis. It starts by noting that  $I_{D1} = 0.2 \text{ mA}$ . Thus  $V_{OV1} = 0.2 \text{ V}$  and

$$V_{G1} = V_{GS1} = V_t + V_{OV1} = 0.5 + 0.2$$

$$= 0.7 \text{ V}$$

Since the dc current through  $R_2$  is zero, the dc voltage drop across it will be zero, thus

$$V_{S2} = +0.7 \text{ V}$$

and

$$I_{R1} = \frac{0.7 \text{ V}}{3.5 \text{ k}\Omega} = 0.2 \text{ mA}$$

Thus,  $Q_2$  is operating at

$$I_D = 0.2 \text{ mA} \quad \text{Q.E.D.}$$

$$(b) g_{m1} = g_{m2} = \frac{2I_D}{V_{OV}}$$

$$= \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{10}{0.2} = 50 \text{ k}\Omega$$

(c)

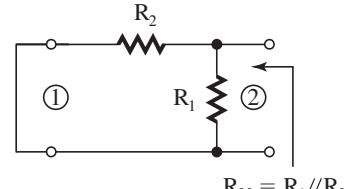
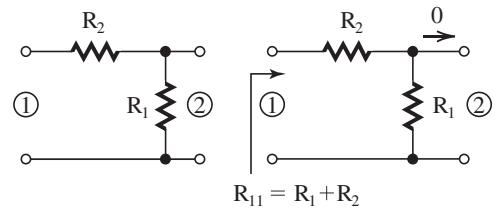


Figure 2

Figure 2 shows the  $\beta$  circuit and the determination of its loading effects,  $R_{11}$  and  $R_{22}$ ,

$$R_{11} = R_1 + R_2 = 3.5 + 14 = 17.5 \text{ k}\Omega$$

$$R_{22} = R_1 \parallel R_2 = 3.5 \parallel 14 = 2.8 \text{ k}\Omega$$

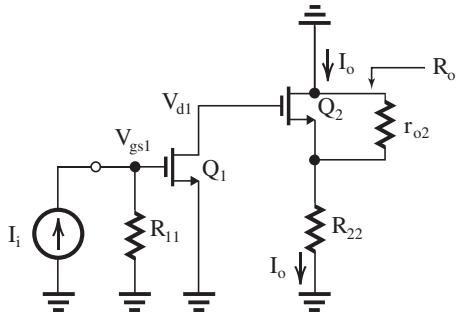


Figure 3

Figure 3 shows the A circuit. To obtain  $A = I_o/I_i$ , we write

$$V_{gs1} = I_i R_{11} \quad (1)$$

$$V_{d1} = -g_{m1} r_{o1} V_{gs1} \quad (2)$$

$$I_o = \frac{V_{d1}}{\frac{1}{g_{m2}} + (r_{o2} \parallel R_{22})} \frac{r_{o2}}{r_{o2} + R_{22}} \quad (3)$$

Combining (1)–(3) yields

$$A = \frac{I_o}{I_i} = -\frac{R_{11}}{\frac{1}{g_{m2}} + (r_{o2} \parallel R_{22})} (g_{m1} r_{o1}) \frac{r_{o2}}{r_{o2} + R_{22}}$$

$$A = -\frac{17.5}{0.5 + (50 \parallel 2.8)} \times 2 \times 50 \times \frac{50}{50 + 2.8}$$

$$A = -525.8 \text{ A/A}$$

$$R_i = R_{11} = 17.5 \text{ k}\Omega$$

$$R_o = r_{o2} + R_{22} + g_{m2} r_{o2} R_{22}$$

$$= 50 + 2.8 + 2 \times 50 \times 2.8$$

$$= 332.8 \text{ k}\Omega$$

(d)

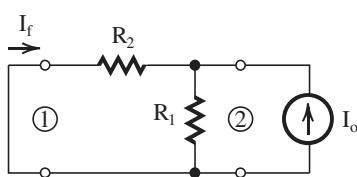


Figure 4

Figure 4 shows the determination of the value of  $\beta$ :

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_1}{R_1 + R_2}$$

Thus,

$$\beta = -\frac{3.5}{3.5 + 14} = -0.2 \text{ A/A}$$

$$(e) A\beta = -525.8 \times -0.2$$

$$= 105.16$$

$$1 + A\beta = 106.16$$

$$A_f = -\frac{525.8}{106.16} = -4.95 \text{ A/A}$$

$$(f) R_{in} = R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{17.5 \text{ k}\Omega}{106.16} = 164.8 \Omega$$

$$R_{out} = R_{of} = R_o(1 + A\beta)$$

$$= 332.8 \times 106.16 = 35.3 \text{ M}\Omega$$

10.70 (a) If  $\mu$  is a very large, a virtual ground will appear at the input terminal. Thus the input resistance  $R_{in} = V_-/I_i = 0$ . Since no current "ows in  $R_s$ , or into the amplifier input terminal, all the current  $I_s$  will "ow in the transistor source terminal and hence into the drain, thus

$$I_o = I_s$$

and

$$\frac{I_o}{I_s} = 1$$

(b) This is a shunt-series feedback amplifier in which the feedback circuit consists of a wire, as shown on the next page in Fig. 1. As indicated,

$$R_{11} = \infty$$

$$R_{22} = 0$$

The A circuit is shown in Fig. 2 (next page) for which we can write

$$V_{id} = -I_i(R_s \parallel R_{id})$$

$$\simeq -I_i R_s \quad (1)$$

(since  $R_{id}$  is very large)

$$V_{gs} = \mu V_{id} \quad (2)$$

$$I_o = g_{m2} V_{gs} \quad (3)$$

Combining (1)–(3), we obtain

$$A \equiv \frac{I_o}{I_i} = -\mu g_{m2} R_s$$

These figures belong to Problem 10.70, part (b).

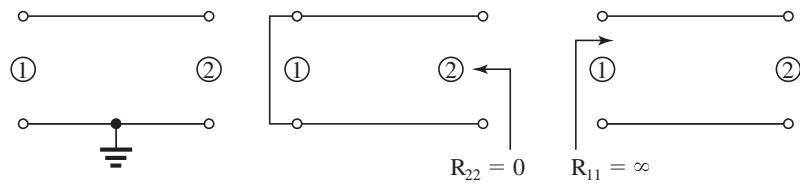


Figure 1

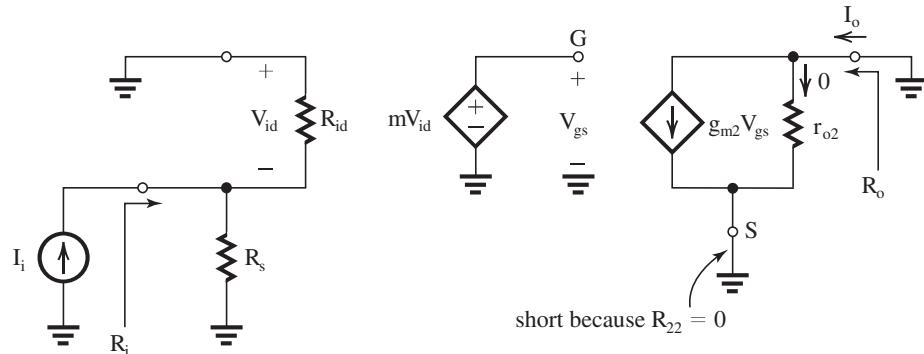


Figure 2

$$R_i = R_s$$

$$R_o = r_{o2}$$

(c)

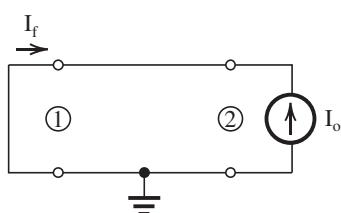


Figure 3

From Fig. 3 we find

$$\beta \equiv \frac{I_f}{I_o} = -1$$

$$(d) A\beta = \mu g_{m2} R_s$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= -\frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s}$$

Note that the negative sign is due to our assumption that  $I_s$  "ows into the input node (see Fig. 2 for the way  $I_i$  is applied). If instead  $I_s$  is "owing out of the input node, as indicated in Fig. P10.70, then

$$A_f \equiv \frac{I_o}{I_s} = \frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s}$$

If  $\mu$  is large so that  $\mu g_{m2} R_s \gg 1$ ,

$$A_f \simeq 1$$

$$(e) R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{R_s}{\mu g_{m2} R_s}$$

$$R_{if} = R_{in} \parallel R_s$$

$$\frac{1}{R_{if}} = \frac{1}{R_{in}} + \frac{1}{R_s}$$

$$\frac{1}{R_s} + \mu g_{m2} = \frac{1}{R_{in}} + \frac{1}{R_s}$$

$$\Rightarrow R_{in} = \frac{1}{\mu g_{m2}}$$

$$R_{out} = R_{of} = R_o(1 + A\beta)$$

$$= r_{o2}(1 + \mu g_{m2} R_s)$$

(f)

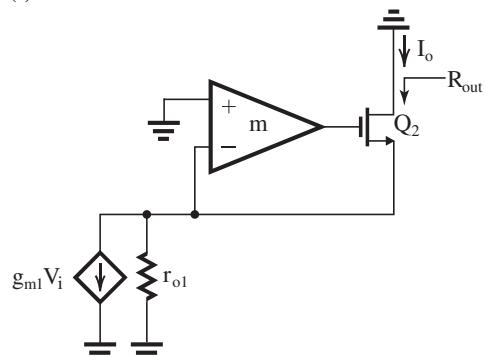


Figure 4



These figures belong to Problem 10.72.

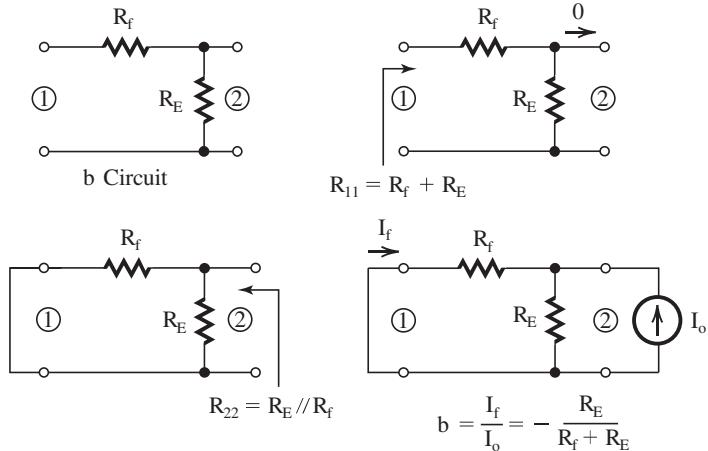


Figure 2

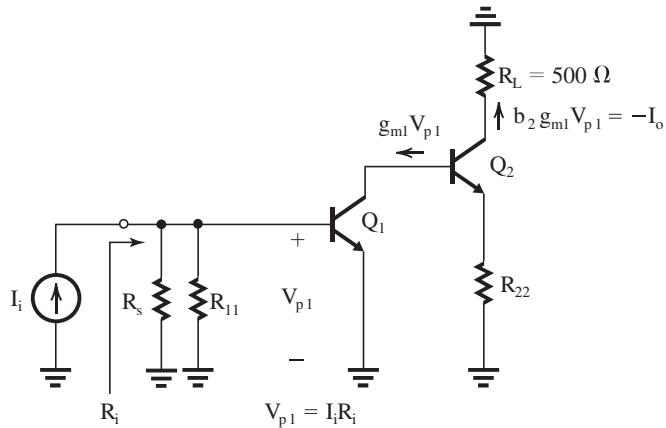


Figure 3

$$V_{\pi 1} = I_i R_i$$

$$I_o = -\beta_2 g_{ml} V_{\pi 1}$$

$$\Rightarrow A = \frac{I_o}{I_i} = -\beta_2 g_{ml} R_i$$

$$A = -100 \times 4 \times 4.19 = -1676 \text{ A/A}$$

$$A\beta = -1676 \times -0.0138 = 23.13$$

$$1 + A\beta = 24.13$$

$$A_f = \frac{I_o}{I_s} = \frac{A}{1 + A\beta}$$

where

$$I_s = \frac{V_s}{R_s}$$

$$A_f = -\frac{1676}{24.13} = -69.5 \text{ A/A}$$

$$\frac{V_o}{V_s} = \frac{-I_o R_L}{-I_s R_s} = 69.5 \times \frac{0.5}{10} = 3.47 \text{ V/V}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{4190 \Omega}{24.13} = 173.6 \Omega$$

$$R_{if} = R_s \parallel R_{in}$$

$$173.6 \Omega = 10,000 \Omega \parallel R_{in}$$

$$\Rightarrow R_{in} = 176.7 \Omega$$

10.73 See figures on the next two pages.

(a) Refer to Fig. 1.

(b) Refer to Fig. 2.

(c)  $R_{outz} = r_{o2} \parallel r_{o4}$

These figures belong to Problem 10.73, part (a).

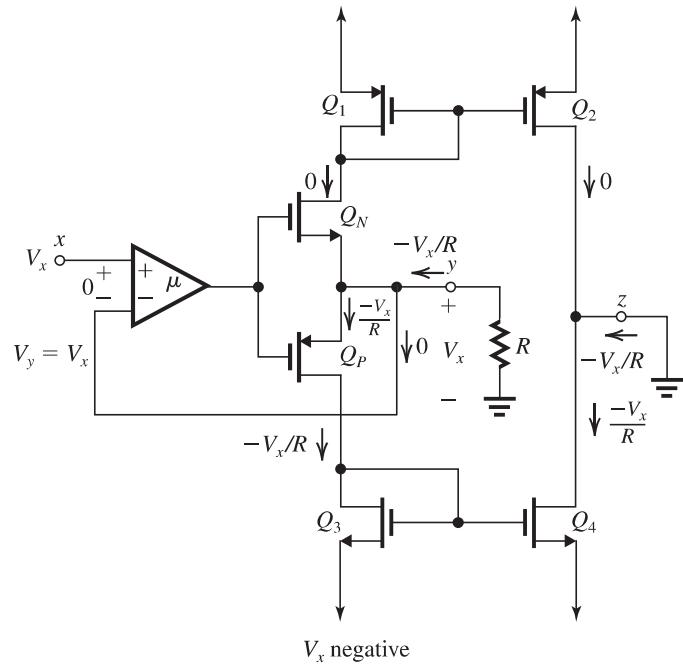
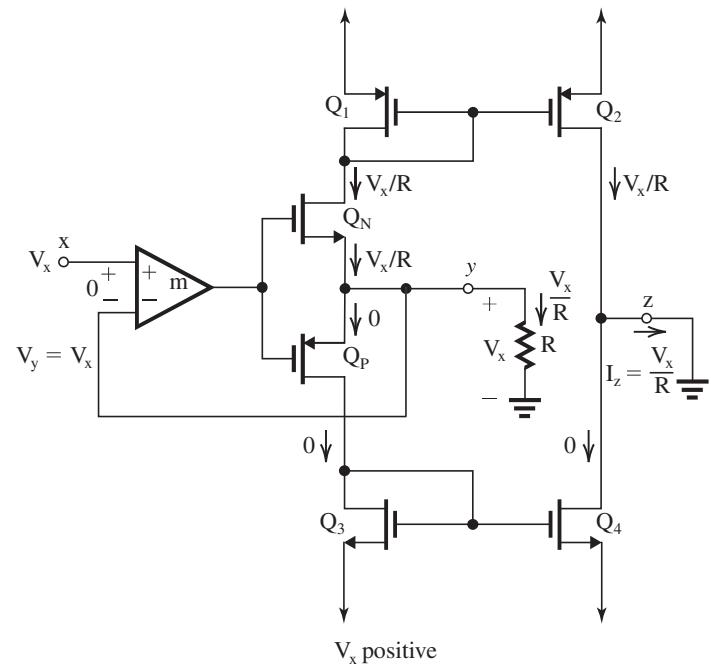


Figure 1

These figures belong to Problem 10.73, part (b).

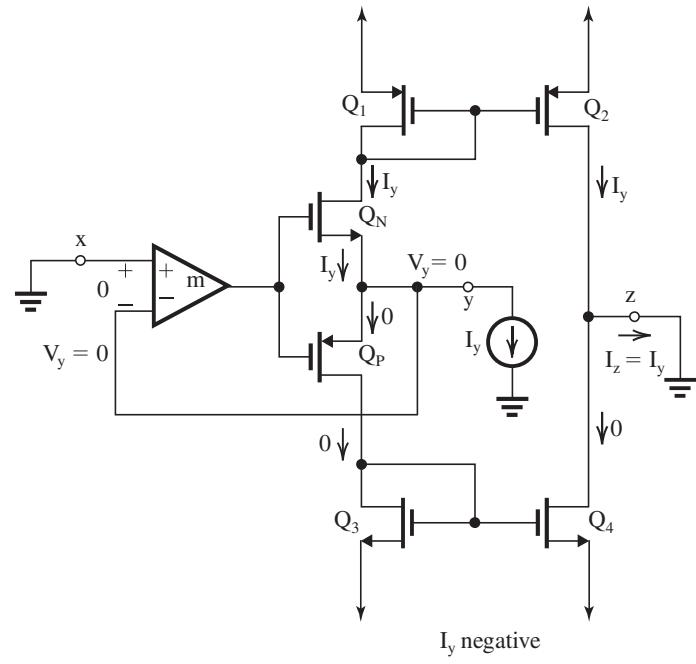
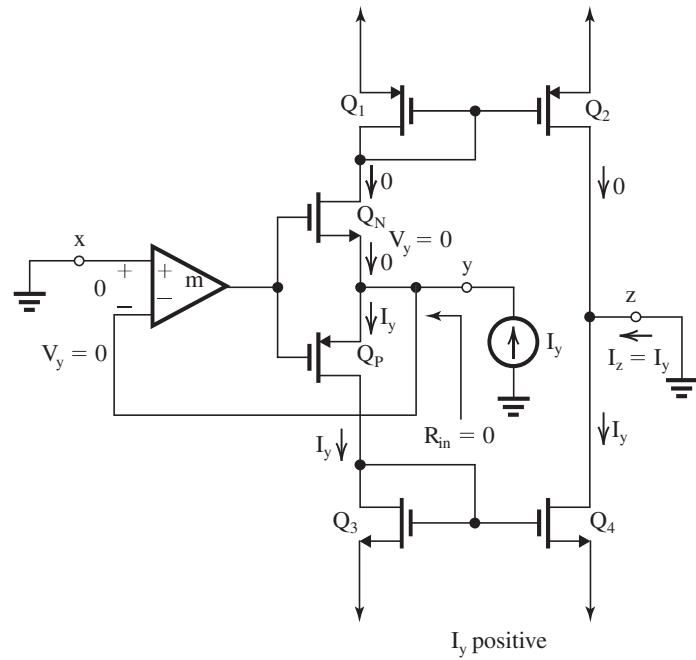


Figure 2

10.74 (a) Refer to the circuit in Fig. P10.74. Observe that the feedback signal is capacitively coupled and so are the signal source and  $R_L$ ; thus, these do not enter into the dc bias calculations and the feedback does not affect the bias. The dc emitter current in  $Q_1$  can be determined from

$$I_{E1} = \frac{12 \times \frac{15}{100+15} - 0.7}{0.870 + \frac{100 \parallel 15}{101}} = 0.865 \text{ mA}$$

$$I_{C1} = 0.99 \times 0.865 = 0.86 \text{ mA}$$

Next consider  $Q_2$  and let its emitter current be  $I_{E2}$ . The base current of  $Q_2$  will be  $I_{E2}/(\beta + 1) \approx 0.01 I_{E2}$ . The current through  $R_{C1}$  will be  $(I_{C1} + I_{B2}) = (0.86 + 0.01 I_{E2})$ . We can thus write the following equation:

$$12 = (0.86 + 0.01 I_{E2}) \times 10 + 0.7 + 3.4 \times I_{E2}$$

$$\Rightarrow I_{E2} = \frac{12 - 8.6 - 0.7}{3.4 + 0.1} = \frac{2.7}{3.5} = 0.77 \text{ mA}$$

$$I_{C2} = 0.76 \text{ mA}$$

The small-signal parameters of  $Q_1$  and  $Q_2$  can now be obtained as

$$g_{m1} = \frac{0.86}{0.025} = 34.4 \text{ mA/V}$$

$$r_{\pi 1} = \frac{100}{34.4} = 2.91 \text{ k}\Omega$$

$$g_{m2} = \frac{0.76}{0.025} = 30.4 \text{ mA/V}$$

$$r_{\pi 2} = \frac{100}{30.4} = 3.3 \text{ k}\Omega$$

(b) The equivalent circuit of the feedback amplifier is shown in Fig. 1, where

$$R_s = 10 \text{ k}\Omega$$

$$I_s = \frac{V_s}{R_s}$$

$$R_B = R_{B1} \parallel R_{B2} = 13 \text{ k}\Omega$$

(c) See figure on the next page. The determination of the loading effects of the  $\beta$  circuit on the A circuit is shown in Fig. 2:

$$R_{11} = R_f + R_{E2} = 10 + 3.4 = 13.4 \text{ k}\Omega$$

$$R_{22} = R_{E2} \parallel R_f = 3.4 \parallel 10 = 2.54 \text{ k}\Omega$$

The A circuit is shown in Fig. 3 on the next page.

Analysis of the A circuit to determine  $A \equiv I_o/I_s$  proceeds as follows:

$$R_i = R_s \parallel R_{11} \parallel R_B \parallel r_{\pi 1}$$

$$= 10 \parallel 13.4 \parallel 13 \parallel 2.91 = 1.68 \text{ k}\Omega$$

$$V_{\pi 1} = I_i R_i \quad (1)$$

$$I_{b2} = -g_{m1} V_{\pi 1} \frac{R_{C1}}{R_{C1} + r_{\pi 2} + (\beta + 1) R_{22}} \quad (2)$$

$$I_o = I_{e2} = (\beta + 1) I_{b2} \quad (3)$$

Combining Eqs. (1)–(3) results in

$$A \equiv \frac{I_o}{I_i} = -\frac{(\beta + 1) R_i g_{m1} R_{C1}}{R_{C1} + r_{\pi 2} + (\beta + 1) R_{22}}$$

$$= \frac{101 \times 1.68 \times 34.4 \times 10}{10 + 3.3 + 101 \times 2.54}$$

$$= -216.3 \text{ A/A}$$

Breaking the emitter loop of  $Q_2$  at XX gives

$$R_o = R_{22} + \frac{r_{\pi 2} + R_{C1}}{\beta + 1}$$

$$= 2.54 + \frac{3.3 + 10}{101} = 2.67 \text{ k}\Omega$$

This figure belongs to Problem 10.74, part (b).

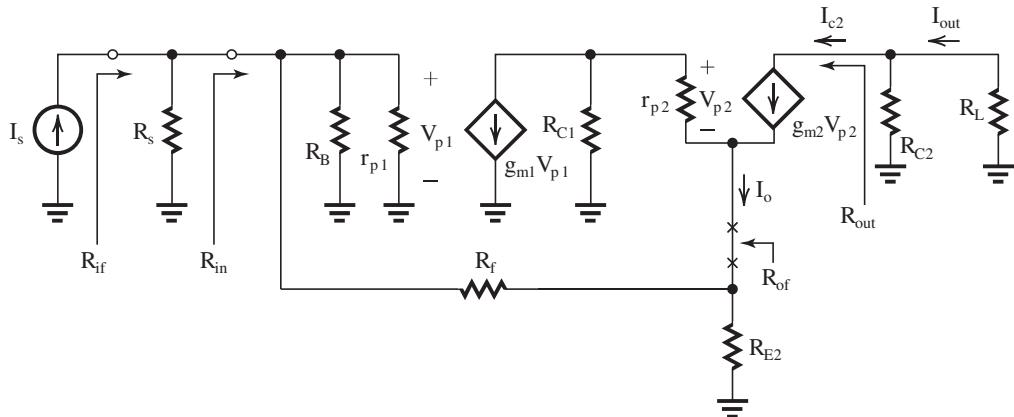


Figure 1

Chapter 10–60

These figures belong to Problem 10.74, part (c).

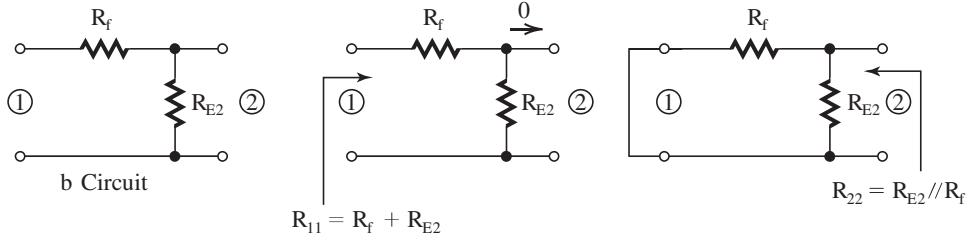


Figure 2

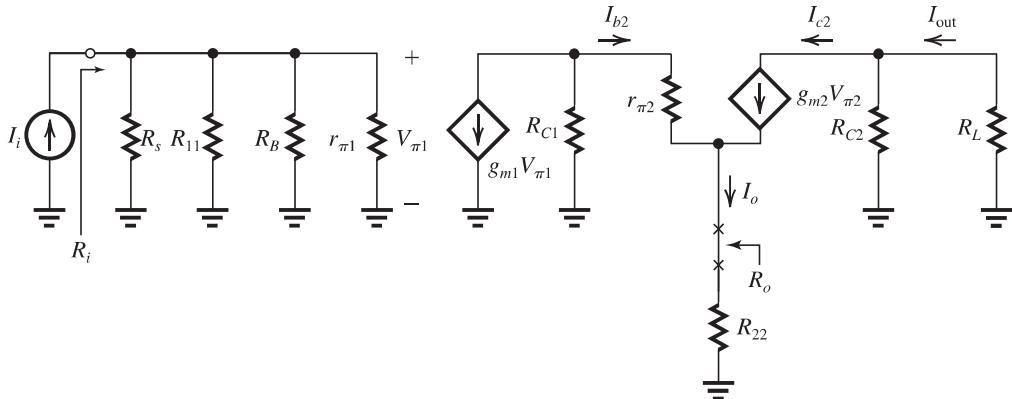


Figure 3

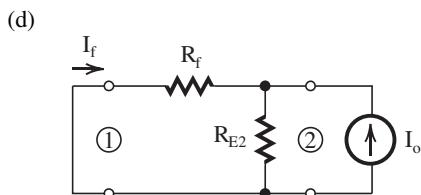


Figure 4

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_{E2}}{R_{E2} + R_f}$$

$$= -0.254 \text{ A/A}$$

$$(e) A\beta = -216.3 \times -0.254 = 54.88$$

$$1 + A\beta = 55.88$$

$$A_f = \frac{I_o}{I_s} = -\frac{216.3}{55.88} = -3.87 \text{ A/A}$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{1.68 \text{ k}\Omega}{55.88} = 30.1 \Omega$$

$$R_{of} = R_o(1 + A\beta) = 2.67 \times 55.88$$

$$= 149.2 \text{ k}\Omega$$

$$(f) R_{if} = R_s \parallel R_{in}$$

$$30.1 \Omega = 10 \text{ k}\Omega \parallel R_{in}$$

$$R_{in} = 30.2 \Omega$$

$$I_{in} = I_s \frac{R_s}{R_s + R_{in}} \simeq I_s$$

$$I_{out} = I_{C2} \frac{R_{C2}}{R_{C2} + R_L} = \alpha I_o \frac{R_{C2}}{R_{C2} + R_L}$$

$$\frac{I_{out}}{I_{in}} \simeq \frac{I_{out}}{I_s} = \frac{I_o}{I_s} \times \alpha \frac{R_{C2}}{R_{C2} + R_L}$$

$$\Rightarrow \frac{I_{out}}{I_{in}} = -3.87 \times 0.99 \times 0.99 \times \frac{8}{8+1}$$

$$= -3.41 \text{ A/A}$$

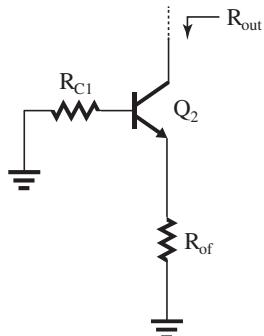


Figure 5

To determine  $R_{out}$ , consider the circuit in Fig. 5. Using the formula given at the end of Example 10.8, adapted to our case here, we get

$$R_{out} = r_{o2} + [R_{of} \parallel (r_{pi2} + R_{C1})] \left[ 1 + g_{m2} r_{o2} \frac{r_{pi2}}{r_{pi2} + R_{C1}} \right]$$

where

$$r_{o2} = \frac{75 \text{ V}}{0.76 \text{ mA}} = 98.7 \text{ k}\Omega$$

$$\begin{aligned} R_{\text{out}} &= 98.7 + [149.2 \parallel \\ &(3.3 + 10)] \left[ 1 + 30.3 \times 98.7 \times \frac{3.3}{3.3 + 10} \right] \\ &= 98.7 + 12.21 \times 743 = 9.17 \text{ M}\Omega \end{aligned}$$

Check:

$$\text{Maximum possible } R_{\text{out}} \simeq \beta r_o \simeq 10 \text{ M}\Omega$$

So, our result is reasonable.

$$10.75 \text{ A(s)} = \frac{10^5}{\left(1 + \frac{s}{100}\right) \left(1 + \frac{s}{20,000}\right)^2}$$

$$\phi = -\tan^{-1} \frac{\omega}{100} - 2 \tan^{-1} \frac{\omega}{20,000}$$

$$180^\circ = \tan^{-1} \frac{\omega_{180}}{100} + 2 \tan^{-1} \frac{\omega_{180}}{20,000}$$

Since  $\omega_{180}$  will be much greater than 100 rad/s, we can assume that at  $\omega_{180}$ ,  $\tan^{-1}(\omega_{180}/100)$  is approximately  $90^\circ$ , thus

$$2 \tan^{-1} \frac{\omega_{180}}{20,000} = 90^\circ$$

$$\Rightarrow \tan^{-1} \frac{\omega_{180}}{20,000} = 45^\circ$$

$$\Rightarrow \omega_{180} = 20,000 \text{ rad/s}$$

which is indeed much greater than 100 rad/s, justifying our original assumption.

At  $\omega = \omega_{180}$ , the magnitude of A becomes

$$|A(j\omega_{180})| = \frac{10^5}{\sqrt{\left[1 + \left(\frac{20,000}{100}\right)^2\right] \left[1 + \left(\frac{20,000}{20,000}\right)^2\right]^2}}$$

$$|A(j\omega_{180})| = \frac{10^5}{200 \times 2} = 250 \text{ V/V}$$

$$|A(j\omega_{180})|\beta_{\text{cr}} = 1$$

$$\beta_{\text{cr}} = \frac{1}{250} = 4 \times 10^{-3} \text{ V/V}$$

Correspondingly,

$$A_f = \frac{10^5}{1 + 10^5 \times 4 \times 10^{-3}}$$

$$= \frac{10^5}{1 + 400} \simeq 250 \text{ V/V}$$

$$10.76 \text{ A(s)} = \frac{10^5}{\left(1 + \frac{s}{100}\right) \left(1 + \frac{s}{20,000}\right)^2}$$

$$A(j\omega) = \frac{10^5}{\left(1 + j \frac{\omega}{100}\right) \left(1 + j \frac{\omega}{20,000}\right)^2}$$

$$\phi(\omega) = -\tan^{-1} \left( \frac{\omega}{100} \right) - 2 \tan^{-1} \left( \frac{\omega}{20,000} \right) \quad (1)$$

$$|A(j\omega)| = \frac{10^5}{\sqrt{1 + \left(\frac{\omega}{100}\right)^2} \left[1 + \left(\frac{\omega}{20,000}\right)^2\right]} \quad (2)$$

Using Eqs. (1) and (2), we can obtain the data required to construct Nyquist plots for the two cases:  $\beta = 1$  and  $\beta = 10^{-3}$ . The results are given in the following table.

This table belongs to Problem 10.76.

$\omega$ rad/s	$-\tan^{-1} \left( \frac{\omega}{100} \right)$	$-2 \tan^{-1} \left( \frac{\omega}{20,000} \right)$	$\phi$	$ A $	$ A\beta $ $\beta = 1$	$ A\beta $ $\beta = 10^{-3}$
0	0	0	0	$10^5$	$10^5$	100
$10^2$	$-45^\circ$	$-0.6^\circ$	$-45.6^\circ$	$0.7 \times 10^5$	$0.7 \times 10^5$	70
$10^3$	$-84.3^\circ$	$-5.7^\circ$	$-90^\circ$	$10^4$	$10^4$	10
$10^4$	$-89.4^\circ$	$-53.1^\circ$	$-142.5^\circ$	800	800	0.8
$2 \times 10^4$	$-89.7^\circ$	$-90^\circ$	$\simeq -180^\circ$	250	250	0.25
$\infty$	$-90^\circ$	$-180^\circ$	$-270^\circ$	0	0	0

This figure belongs to Problem 10.76.

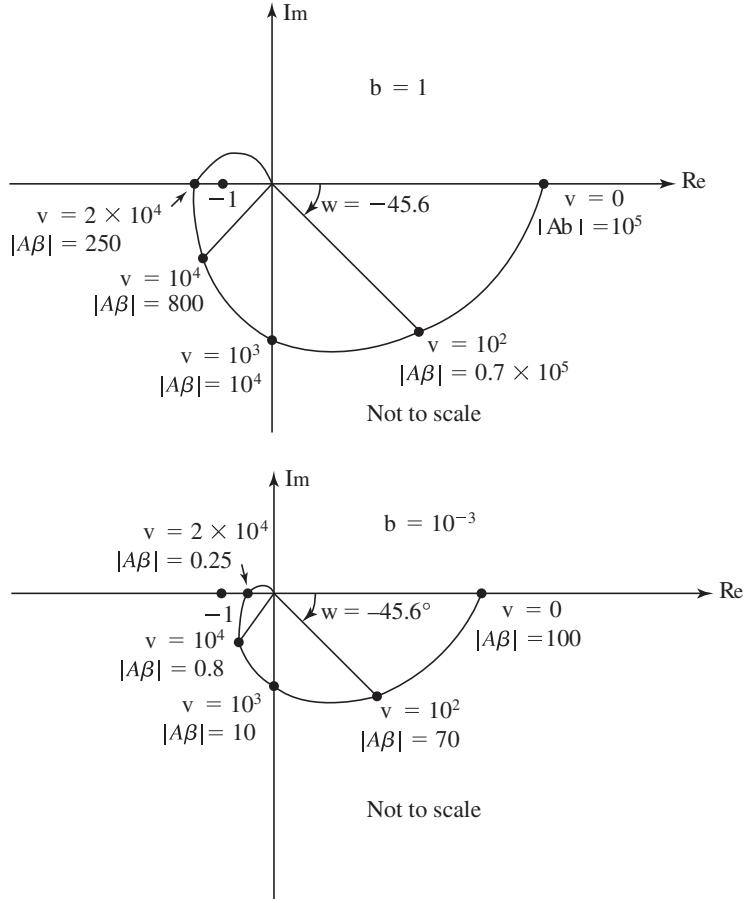


Figure 1

Using these data, we obtain the two Nyquist plots shown in Fig. 1.

We observe that the amplifier with  $\beta = 1$  will be unstable and that with  $\beta = 10^{-3}$  will be stable.

$$10.77 \quad A(s) = \frac{10^4}{\left(1 + \frac{s}{10^4}\right) \left(1 + \frac{s}{10^5}\right)^2}$$

$$A(j\omega) = \frac{10^4}{\left(1 + j\frac{\omega}{10^4}\right) \left(1 + j\frac{\omega}{10^5}\right)^2}$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{10^4}\right) - 2 \tan^{-1}\left(\frac{\omega}{10^5}\right)$$

$$-180 = -\tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) - 2 \tan^{-1}\left(\frac{\omega_{180}}{10^5}\right)$$

By trial and error we find

$$\omega_{180} = 1.095 \times 10^5 \text{ rad/s}$$

At this frequency,

$$|A| = \frac{10^4}{\sqrt{1 + 10.95^2} \sqrt{1 + 1.095^2}} \\ = 413.6$$

For stable operation,

$$|A|\beta_{cr} < 1 \\ \beta_{cr} < 2.42 \times 10^{-3}$$

Thus, oscillation will commence for  $\beta \geq 2.42 \times 10^{-3}$

$$10.78 \quad A(s)\beta(s) = \frac{10^4 k}{\left(1 + \frac{s}{10^3}\right)^3}$$

$$A(j\omega)\beta(j\omega) = \frac{10^4 k}{\left(1 + j\frac{\omega}{10^3}\right)^3}$$

$$|A(j\omega)\beta(j\omega)| = \frac{10^4 k}{\left(1 + \frac{\omega^2}{10^6}\right)^{3/2}}$$

$$\phi(\omega) = -3 \tan^{-1} \left( \frac{\omega}{10^3} \right)$$

$$-180^\circ = -3 \tan^{-1} \frac{\omega_{180}}{10^3}$$

$$\Rightarrow \omega_{180} = 10^3 \tan 60 = 1732 \text{ rad/s}$$

$$|A(j\omega_{180})\beta(j\omega_{180})| = \frac{10^4 k}{\left(1 + \frac{1732^2}{10^6}\right)^{3/2}}$$

$$= 0.125 \times 10^4 k$$

For stable operation,

$$|A(j\omega_{180})\beta(j\omega_{180})| < 1$$

$$\Rightarrow 0.125 \times 10^4 k < 1$$

$$k < 8 \times 10^{-4}$$

$$10.79 A_f(0) = \frac{A_0}{1 + A_0 \beta}$$

where

$$A_0 = \frac{1 \text{ MHz}}{10 \text{ Hz}} = 10^5 \text{ V/V}$$

Thus,

$$A_f(0) = \frac{10^5}{1 + 10^5 \times 0.1} \simeq 10 \text{ V/V}$$

$$f_{3dB} = 10(1 + A_0 \beta)$$

$$= 10(1 + 10^5 \times 0.1) \simeq 10^5 \text{ Hz}$$

$$\begin{aligned} \text{Unity-gain frequency of closed-loop amplifier} \\ = A_f(0) \times f_{3dB} \end{aligned}$$

$$= 10 \times 10^5 = 10^6 \text{ Hz} = 1 \text{ MHz}$$

Thus, the pole shifts by a factor equal to the amount-of-feedback,  $(1 + A_0 \beta)$ .

$$10.80 A_0 = 10 \text{ V/V}$$

$$f_p = 100 \text{ Hz}$$

$$f_{Hf} = f_p(1 + A_0 \beta) = 10 \text{ kHz}$$

$$\Rightarrow 1 + A_0 \beta = \frac{10 \times 10^3}{100} = 100$$

$$\Rightarrow \beta = \frac{99}{10^4} = 0.0099 \text{ V/V}$$

$$A_f(0) = \frac{A_0}{1 + A_0 \beta}$$

$$= \frac{10^4}{100} = 100 \text{ V/V}$$

$$A_f(s) = \frac{A_f(0)}{1 + \frac{s}{\omega_{Hf}}}$$

$$A_f(s) = \frac{100}{1 + s/2\pi \times 10^4}$$

$$10.81 A_f(0) = \frac{A_0}{1 + A_0 \beta}$$

$$10 = \frac{1000}{1 + 1000\beta}$$

$$\Rightarrow \beta = 0.099$$

To obtain a maximally "at response,

$$Q = 0.707$$

Using Eq. (10.70), we obtain

$$0.707 = \frac{\sqrt{100 \times 1 \times f_{p2}}}{1 + f_{p2}}$$

$$\frac{1}{2} = \frac{100 f_{p2}}{(1 + f_{p2})^2}$$

$$f_{p2}^2 + 2 f_{p2} + 1 = 200 f_{p2}$$

$$f_{p2}^2 - 198 f_{p2} + 1 = 0$$

$$f_{p2} \simeq 198 \text{ kHz}$$

(the other solution is a very low frequency which obviously does not make physical sense).

The 3-dB frequency of the closed-loop amplifier is  $f_0$ , which can be obtained from Eq. (10.68) and the graphical construction of Fig. 10.32:

$$\frac{f_0}{2Q} = \frac{1}{2}(f_{p1} + f_{p2})$$

$$f_0 = \frac{1}{\sqrt{2}}(1 + 198) = 140.7 \text{ kHz}$$

10.82 (a) The closed-loop poles become coincident when  $Q = 0.5$ . Using Eq. (10.70), we obtain

$$Q = \frac{\sqrt{(1 + A_0 \beta) \omega_{p1} \omega_{p2}}}{\omega_{p1} + \omega_{p2}}$$

$$0.5 = \frac{\sqrt{(1 + A_0 \beta) \omega_{p1} \omega_{p2}}}{\omega_{p1} + \omega_{p2}}$$

$$\Rightarrow 1 + A_0 \beta = 0.5^2 \frac{(\omega_{p1} + \omega_{p2})^2}{\omega_{p1} \omega_{p2}}$$

$$= 0.5^2 \times \frac{(2\pi)^2 (10^4 + 10^5)^2}{(2\pi)^2 \times 10^4 \times 10^5}$$

$$= 0.5^2 \times \frac{11^2}{10} = 3.025$$

$$\beta = 2.025 \times 10^{-4}$$

$$\omega_c = \frac{1}{2}(\omega_{p1} + \omega_{p2})$$

$$= \frac{1}{2} \times 2\pi(f_{p1} + f_{p2})$$

$$f_c = \frac{1}{2} \times (10^4 + 10^5) = 5.5 \times 10^4 \text{ Hz}$$

$$(b) A_f(0) = \frac{A_0}{1 + A_0\beta}$$

$$= \frac{10^4}{1 + 2.205 \times 10^{-4} \times 10^4} = 3306 \text{ V/V}$$

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta}$$

where

$$A(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right)}$$

$$A_f(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right) + A_0\beta}$$

$$= \frac{A_0}{(1 + A_0\beta) + s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}}}$$

$$A_f(j\omega) = \frac{A_0}{(1 + A_0\beta) + j\left(\frac{\omega}{\omega_{p1}} + \frac{\omega}{\omega_{p2}}\right) - \left(\frac{\omega}{\omega_{p1}}\right)\left(\frac{\omega}{\omega_{p2}}\right)}$$

$$A_f(j\omega_c) = \frac{10^4}{3.025 + j(5.5 + 0.55) - 5.5 \times 0.55}$$

$$A_f(j\omega_c) = \frac{10^4}{j 6.05}$$

$$|A_f|(j\omega_c) = \frac{10^4}{6.05} = 1653 \text{ V/V}$$

(c)  $Q = 0.5$ .

(d) If  $\beta = 2.025 \times 10^{-3}$  V/V. Using Eq. (10.68), we obtain

$$s = -\frac{1}{2} \times 2\pi(10^4 + 10^5)$$

$$\pm \frac{1}{2} \times$$

$$2\pi\sqrt{(10^4 + 10^5)^2 - 4(1 + 10^4 \times 2.025 \times 10^{-3}) \times 10^4 \times 10^5}$$

$$\frac{s}{2\pi} = -5.5 \times 10^4$$

$$\pm 0.5\sqrt{121 \times 10^8 - 4 \times 21.25 \times 10^9}$$

$$= -5.5 \times 10^4 \pm 0.5 \times 10^4 \sqrt{121 - 40 \times 21.25}$$

$$= -5.5 \times 10^4 \pm j0.5 \times 10^4 \times 27$$

$$= (-5.5 \pm j13.25) \times 10^4 \text{ Hz}$$

Using Eq. (10.70), we obtain

$$Q = \frac{\sqrt{(1 + 10^4 \times 2.025 \times 10^{-3})10^4 \times 10^5}}{10^4 + 10^5}$$

$$= 1.325$$

$$10.83 \quad A_f = 10$$

Maximally "at response with  $f_{sdB} = f_0 = 1$  kHz.

Let each stage in the cascade have a dc gain  $K$  and a 3-dB frequency  $f_p$ , thus

$$A(s) = \frac{K^2}{\left(1 + \frac{s}{\omega_p}\right)^2}$$

Using the expression for  $Q$  in Eq. (10.70), we get

$$Q = \frac{\sqrt{(1 + A_0\beta)\omega_{p1}\omega_{p2}}}{\omega_{p1} + \omega_{p2}}$$

and substituting  $Q = 1/\sqrt{2}$ ,  $\omega_{p1} = \omega_{p2} = \omega_p$ , and  $A_0 = K^2$ , we obtain

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{1 + K^2\beta}}{2}$$

$$1 + K^2\beta = 2$$

and

$$K^2\beta = 1$$

Now,

$$A_{f0} = \frac{A_0}{1 + A_0\beta}$$

$$10 = \frac{K^2}{1 + K^2\beta}$$

$$= \frac{K^2}{2}$$

$$\Rightarrow K^2 = 20 \Rightarrow K = \sqrt{20} = 4.47 \text{ V/V}$$

$$\beta = \frac{1}{20} = 0.05 \text{ V/V}$$

Using Eq. (10.68), we obtain

$$\frac{\omega_0}{Q} = \frac{1}{2}(\omega_{p1} + \omega_{p2})$$

$$\frac{f_0}{1/\sqrt{2}} = \frac{1}{2} \times 2 f_p$$

$$\Rightarrow f_p = \sqrt{2}f_0 = \sqrt{2} \times 1 = 1.414 \text{ kHz}$$

$$A_f(s) = \frac{10\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$A_f(s) = \frac{10(2\pi \times 10^3)^2}{s^2 + s\frac{2\pi \times 10^3}{1/\sqrt{2}} + (2\pi \times 10^3)^2}$$

10.84 Let each stage have the transfer function

$$T(s) = \frac{K}{1 + \frac{s}{\omega_p}}$$

where  $\omega_p = 2\pi \times 100 \times 10^3$  rad/s

$$A(s) = \left( \frac{K}{1 + \frac{s}{\omega_p}} \right)^3$$

$$\beta = 1$$

Thus the characteristic equation is given by

$$1 + A(s)\beta = 0$$

$$1 + \frac{K^3}{\left(1 + \frac{s}{\omega_p}\right)^3} = 0$$

To simplify matters, let  $\frac{s}{\omega_p} = S$ , where  $S$  is a normalized frequency variable, thus

$$(1 + S)^3 + K^3 = 0 \quad (1)$$

This equation has three roots, which are the poles of the feedback amplifier. One of the roots will be real and the other two can be complex conjugate depending on the value of  $K$ . The real pole can be directly obtained from Eq. (1) as

$$(1 + S_1)^3 = -K^3$$

$$(1 + S_1) = -K$$

$$S_1 = -(1 + K) \quad (2)$$

Now we need to obtain the two other poles. The characteristic equation in (1) can be written as

$$S^3 + 3S^2 + 3S + (1 + K^3) = 0 \quad (3)$$

Equivalently it can be written as

$$(S + 1 + K)(S^2 + aS + b) = 0 \quad (4)$$

Equating the coefficients of corresponding terms in (3) and (4), we can find  $a$  and  $b$  and thus obtain the quadratic factor

$$S^2 + (2 - K)S + (1 - K + K^2) = 0 \quad (5)$$

This equation can now be easily solved to obtain the pair of complex conjugate poles as

$$S_{2,3} = -1 + \frac{K}{2} \pm j \frac{\sqrt{3}}{2} K$$

Figure 1 shows the root locus of the three poles in the normalized s plane (normalized relative to  $\omega_p = 2\pi \times 10^5$ ). Observe that as  $K$  increases the real pole  $S_1$  moves outwardly on the negative real axis. The pair of conjugate poles move on straight

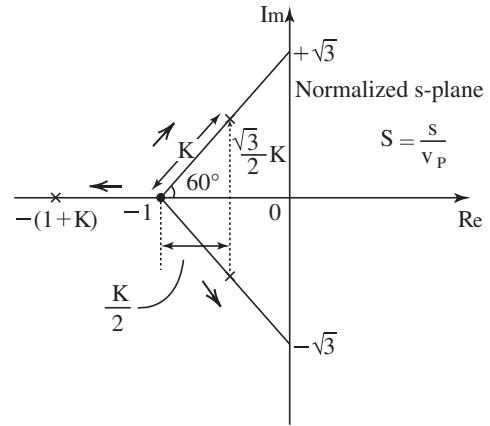


Figure 1

lines with  $60^\circ$  angles to the horizontal. These two poles reach the  $j\omega$  axis (which is the boundary for stable operation) at  $K = 2$  at which point

$$S_{2,3} = \pm j\sqrt{3}$$

Thus the minimum value of  $K$  from which oscillations occur is  $K = 2$ . Oscillations will be at

$$\omega = \sqrt{3} \times 2\pi f_p$$

or

$$f = \sqrt{3} \times 100 \text{ kHz}$$

$$= 173.2 \text{ kHz}$$

10.85

$$A(s) = \frac{10^5}{\left(1 + \frac{s}{2\pi \times 10}\right)\left(1 + \frac{s}{2\pi \times 10^3}\right)}$$

$$\beta = 0.01$$

$$A\beta(j\omega) = \frac{1000}{\left(1 + j\frac{\omega}{2\pi \times 10}\right)\left(1 + j\frac{\omega}{2\pi \times 10^3}\right)} \quad (1)$$

$$|A\beta| = \frac{1000}{\sqrt{\left[1 + \left(\frac{\omega}{2\pi \times 10}\right)^2\right]\left[1 + \left(\frac{\omega}{2\pi \times 10^3}\right)^2\right]}} \quad (2)$$

Figure 1 (next page) sketches the Bode plot for  $|A\beta|$ . The unity-gain frequency will occur on the  $-40 \text{ dB/decade}$  line. The value of  $f_l$  can be obtained from the Bode plot as follows: The  $-40 \text{ dB/decade}$  line represents gain drop proportional to  $10^3/f^2$ . For a drop by only 20 dB (a factor of 10) the change in frequency is

$$\frac{10^3}{f_l^2} = \frac{1}{10}$$

$$f_l = \sqrt{10} \times 10^3 \text{ Hz} = 3.16 \times 10^3 \text{ Hz}$$

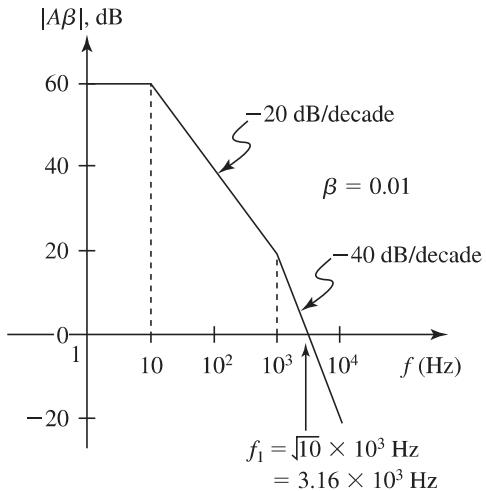


Figure 1

However, the Bode plot results are usually approximate. If we require a more exact value for  $f_1$  we need to iterate a couple of times using the exact equation in (2). The result is

$$f_1 = 3.085 \times 10^3 \text{ Hz}$$

The phase angle can now be obtained using (1) as follows:

$$\begin{aligned} \phi &= -\tan^{-1} \frac{\omega_1}{2\pi \times 10} - \tan^{-1} \frac{\omega_1}{2\pi \times 10^3} \\ &= -\tan^{-1} \frac{f_1}{10} - \tan^{-1} \frac{f_1}{10^3} \\ &= -\tan^{-1}(3.085 \times 10^2) - \tan^{-1}(3.085) \\ &= -89.81^\circ - 72.03^\circ = -161.84^\circ \end{aligned}$$

Thus,

$$\text{Phase margin} = 180^\circ - 161.84^\circ = 18.15^\circ$$

To obtain a phase margin of  $45^\circ$ :

The phase shift due to the first pole will be  $\approx 90^\circ$ . Thus, the phase shift due to the second pole must be  $\approx -45^\circ$ , thus

$$-45^\circ = -\tan^{-1} \frac{f_1}{10^3}$$

$$f_1 \approx 10^3 \text{ rad/s.}$$

Since  $f_1$  is two decades above  $f_p$  we need  $A_0\beta$  to be 100. Thus,  $\beta$  will now be

$$\beta = 100/10^5 = 10^{-3}$$

Figure 2 shows a sketch of the Bode plot for  $|A\beta|$  in this case.

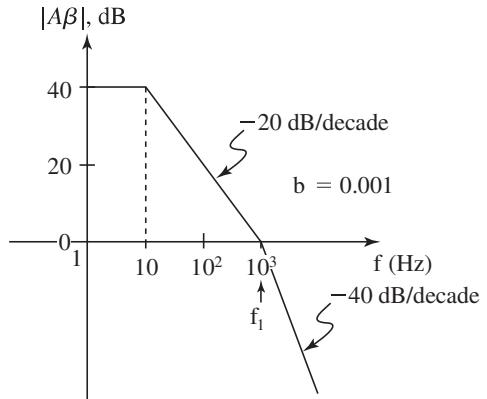


Figure 2

10.86  $A_0 = 10^5$  with a single pole at  $f_p = 10 \text{ Hz}$

For a unity-gain buffer,  $\beta = 1$ , thus

$$A_0\beta = 10^5 \text{ and } f_p = 10 \text{ Hz}$$

Since the loop gain rolls off at a uniform slope of  $-20 \text{ dB/decade}$ , it will reach the 0 dB line ( $|A\beta| = 1$ ) five decades beyond 10 Hz. Thus the unity-gain frequency will be

$$f_1 = 10^5 \times 10 = 10^6 \text{ Hz} = 1 \text{ MHz}$$

The phase shift will be that resulting from a single pole,  $-90^\circ$ , resulting in a phase margin:

$$\text{Phase margin} = 180^\circ - 90^\circ = 90^\circ$$

10.87 Using Eq. (10.82), we obtain

$$|A_f(j\omega_1)| = \frac{1/\beta}{|1 + e^{-j\theta}|}$$

where

$$\theta = 180^\circ - \phi$$

$$\phi \equiv \text{Phase margin}$$

$$\text{Peaking, } P \equiv \frac{|A_f(j\omega_1)|}{1/\beta}$$

Thus,

$$\begin{aligned} P &= 1/|1 + e^{-j\theta}| \\ &= 1/|1 + \cos \theta - j \sin \theta| \\ &= 1/\sqrt{(1 + \cos \theta)^2 + \sin^2 \theta} \\ &= 1/\sqrt{2 + 2 \cos \theta} \\ &= 1/\sqrt{2 + 2 \cos(180^\circ - \phi)} \\ P &= 1/\sqrt{2(1 - \cos \phi)} \\ \Rightarrow \phi &= \cos^{-1}\left(1 - \frac{1}{2P^2}\right) \end{aligned}$$

We use this equation to obtain the following results:

P	$\phi$
1.05	56.9°
1.10	54.1°
0.1 dB ≡ 1.0115	59.2°
1.0 dB ≡ 1.122	52.9°
3 dB ≡ 1.414	41.4°

10.88 Figure 1 below shows magnitude and phase Bode plots for the amplifier specified in this problem. From the phase plot we find that  $\theta = -135^\circ$  (which corresponds to a phase margin of 45°) occurs at

$$f = 3.16 \times 10^5 \text{ Hz}$$

At this frequency,  $|A|$  is 70 dB. The  $\beta$  horizontal straight line drawn at 70-dB level gives

$$20 \log\left(\frac{1}{\beta}\right) = 70 \text{ dB}$$

$$\Rightarrow \beta = 3.16 \times 10^{-4}$$

Correspondingly,

$$A_f = \frac{10^4}{1 + 10^4 \times 3.16 \times 10^{-4}} = 2.4 \times 10^3 \text{ V/V}$$

or 67.6 dB

10.89 Figure 1, on the next page, is a replica of Fig. 10.37 except here we locate on the phase plot the points at which the phase margin is 90° and 135°, respectively. Drawing a vertical line from each of those points and locating the intersection with the  $|A|$  line enables us to determine the maximum  $\beta$

This figure belongs to Problem 10.88.

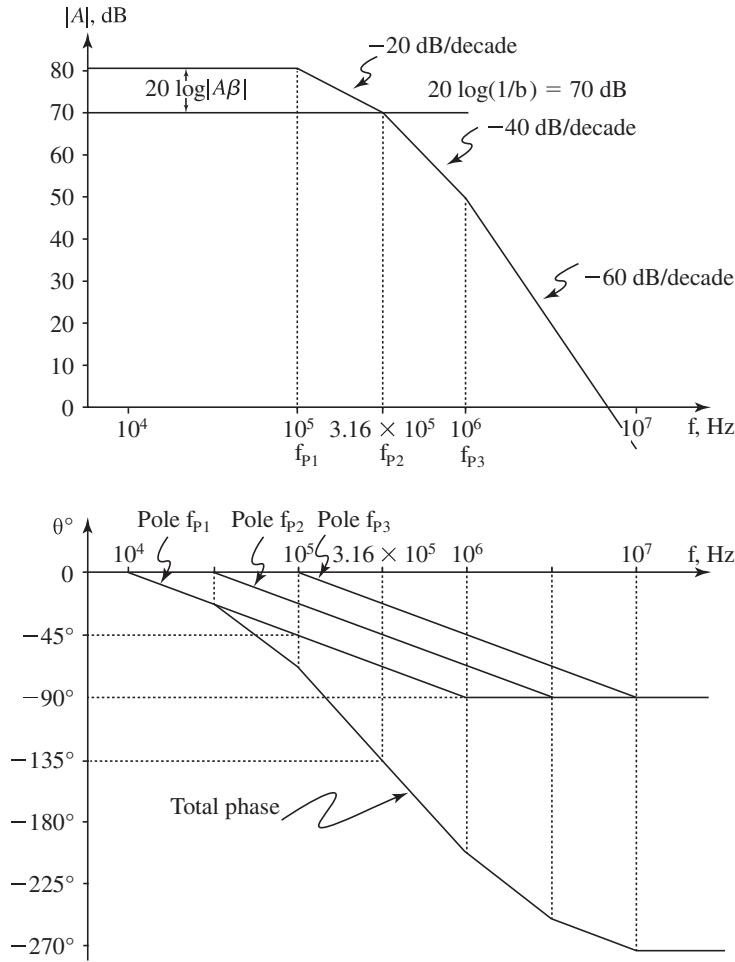


Figure 1

This figure belongs to Problem 10.89.

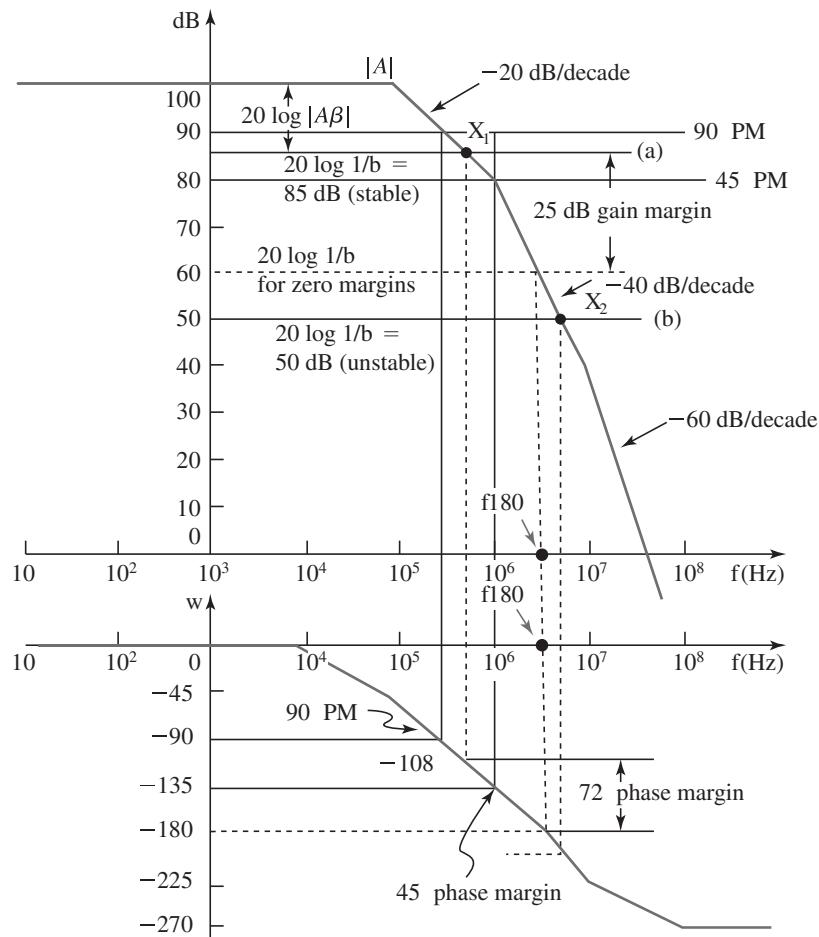


Figure 1

that can be used in each case. Thus, for  $\text{PM} = 90^\circ$ :

$$20 \log \frac{1}{\beta} = 90 \text{ dB}$$

$$\Rightarrow \beta = 3.16 \times 10^{-5}$$

and the corresponding closed-loop gain is

$$A_f = \frac{A_0}{1 + A_0 \beta} = \frac{10^5}{1 + 10^5 \times 3.16 \times 10^{-5}} = 2.4 \times 10^4 \text{ V/V or } 87.6 \text{ dB}$$

and for  $\text{PM} = 45^\circ$ , we have

$$20 \log \frac{1}{\beta} = 80 \text{ dB}$$

$$\Rightarrow \beta = 10^{-4} \text{ V/V}$$

and the corresponding closed-loop gain is

$$A_f = \frac{A_0}{1 + A_0 \beta} = \frac{10^5}{1 + 10^5 \times 10^{-4}} = 9.09 \times 10^3 \text{ V/V}$$

or 79.2 dB

10.90 Figure 1 below shows the Bode plot for the amplifier gain and for a differentiator. Observe that following the rate-of-closure rule the intersection of the two graphs is arranged so that the maximum difference in slopes is 20 dB/decade.

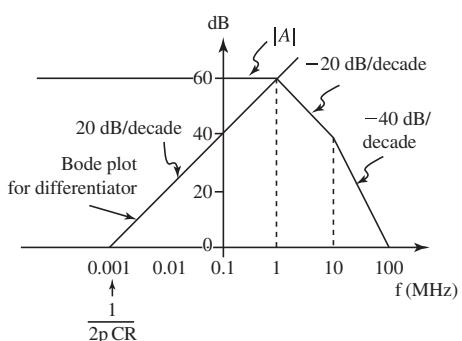


Figure 1

Thus, to guarantee stability,

$$\frac{1}{2\pi CR} \leq 0.001 \text{ MHz or } 1 \text{ kHz}$$

$$\Rightarrow CR \geq \frac{1}{2\pi \times 10^3} = 0.159 \text{ ms}$$

10.91 The new pole must be placed at

$$f_D = \frac{1 \text{ MHz}}{10^4} = 100 \text{ Hz}$$

In this way the modified open-loop gain will decrease at the uniform rate of  $-20 \text{ dB/decade}$ , thus reaching  $0 \text{ dB}$  in four decades, that is at  $1 \text{ MHz}$  where the original pole exists. At  $1 \text{ MHz}$ , the slope changes to  $-40 \text{ dB/decade}$ , but our amplifier will be guaranteed to be stable with a closed-loop gain at low as unity.

10.92 We must move the  $1 \text{ MHz}$  pole to a new location,

$$f_D = \frac{20 \text{ MHz}}{10^4} = 2 \text{ kHz}$$

This reduction in frequency by a factor of  $\frac{1 \text{ MHz}}{2 \text{ kHz}} = 500$  will require that the total capacitance at the controlling node must become 500 times what it originally was.

10.93 Refer to Fig. 10.38.

(a) For  $\beta = 0.001$ ,

$$20 \log \frac{1}{\beta} = 60 \text{ dB}$$

A horizontal line at the  $60\text{-dB}$  level will intersect the vertical line at  $f_{p2} = 10^6 \text{ Hz}$  at a point  $Z_1$ . Drawing a line with a slope of  $-20 \text{ dB/decade}$  from  $Z_1$  will intersect the  $100\text{-dB}$  horizontal line at a frequency two decade lower than  $f_{p2}$ , thus the frequency to which the 1st pole must be moved is

$$f'_D = \frac{f_{p2}}{100} = \frac{10^6}{100} = 10 \text{ kHz}$$

(b) For  $\beta = 0.1$ ,

$$20 \log \frac{1}{\beta} = 20 \text{ dB}$$

Following a process similar to that for (a) above, the first pole must be lowered to

$$f'_D = \frac{10^6}{10^4} = 100 \text{ kHz}$$

10.94 The fourth, dominant pole must be at

$$\begin{aligned} f_D &= \frac{f_{p1}}{A_0} \\ &= \frac{10^6}{10^5} = 10 \text{ Hz} \end{aligned}$$

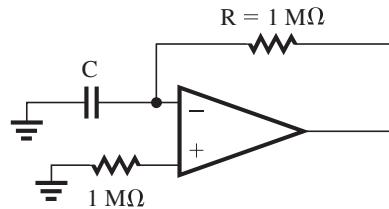


Figure 1

Refer to Fig. 1.

$$\begin{aligned} f_D &= \frac{1}{2\pi RC} \\ 10 &= \frac{1}{2\pi \times 1 \times 10^6 \times C} \\ \Rightarrow C &= 15.9 \text{ nF} \end{aligned}$$

$$\begin{aligned} 10.95 \quad f_{p1} &= \frac{1}{2\pi C_1 R_1} \\ 10^5 &= \frac{1}{2\pi \times 150 \times 10^{-12} \times R_1} \\ \Rightarrow R_1 &= 10.61 \text{ k}\Omega \end{aligned}$$

$$\begin{aligned} f_{p2} &= \frac{1}{2\pi C_2 R_2} \\ 10^6 &= \frac{1}{2\pi \times 5 \times 10^{-12} \times R_2} \\ \Rightarrow R_2 &= 31.83 \text{ k}\Omega \end{aligned}$$

First, we determine an approximate value of  $f'_{p2}$  from Eq. (10.94)

$$f'_{p2} = \frac{g_m C_f}{2\pi |C_1 C_2 + C_f(C_1 + C_2)|}$$

Assume that  $C_f \gg C_2$ , then

$$\begin{aligned} f'_{p2} &\simeq \frac{g_m}{2\pi(C_1 + C_2)} \\ &= \frac{40 \times 10^{-3}}{2\pi(150 + 5) \times 10^{-12}} \\ &= 41.1 \text{ MHz} \end{aligned}$$

which is much greater than  $f_{p3}$ . Thus, we use  $f_{p3}$  to determine the new location of  $f'_{p1}$ ,

$$f'_{p1} = \frac{2 \times 10^6}{10^4} = 200 \text{ Hz}$$

Using Eq. (10.93), we obtain

$$f'_{p1} = \frac{1}{2\pi g_m R_2 C_f R_1}$$

$$200 = \frac{1}{2\pi \times 40 \times 10^{-3} \times 31.83 \times 10^3 \times C_f \times 10.61 \times 10^3}$$

$$\Rightarrow C_f = 58.9 \text{ pF}$$

Since  $C_f$  is indeed much greater than  $C_2$ , the pole at the output will have the frequency already calculated:

$$f'_{P2} \simeq 41.1 \text{ MHz}$$

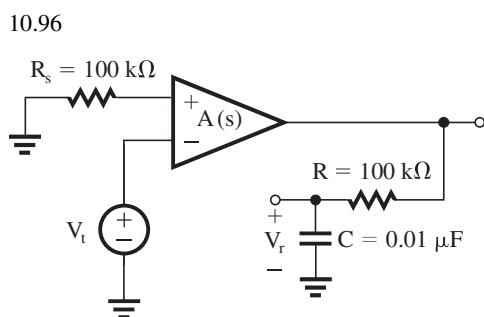


Figure 1

$$A(s)\beta(s) = -\frac{V_r}{V_t}$$

$$= A(s) \frac{1/sC}{R + 1/sC}$$

$$A(s)\beta(s) = \frac{10^5}{1 + \frac{s}{10}} \frac{1}{1 + sCR}$$

$$CR = 0.01 \times 10^{-6} \times 100 \times 10^3 = 10^{-3} \text{ s}$$

$$A(s)\beta(s) = \frac{10^5}{\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{10^3}\right)}$$

(a) Bode plots for the magnitude and phase of  $A\beta$  are shown in Fig. 2. From the magnitude plot we find the frequency  $f_l$  at which  $|A\beta| = 1$  is

$$f_l = 3.16 \times 10^4 \text{ Hz}$$

(b) From the phase plot we see that the phase at  $f_l$  is  $180^\circ$  and thus the phase margin is zero. A more exact value for the phase margin can be obtained as follows:

This figure belongs to Problem 10.96, part (a).

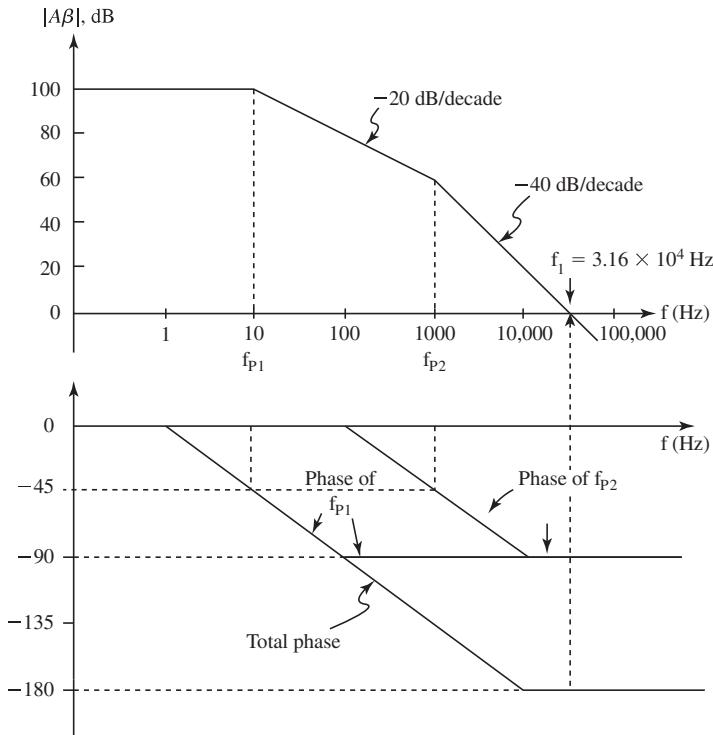


Figure 2

$$\theta(f_1) = -\tan^{-1} \frac{3.16 \times 10^4}{10} - \tan^{-1} \frac{3.16 \times 10^4}{10^3}$$

$$= -89.98 - 88.19 = -178.2$$

Thus the phase margin is  $1.8^\circ$ .

$$(c) A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)}$$

$$= \frac{10^5 / \left(1 + \frac{s}{10}\right)}{1 + \frac{10^5}{\left(1 + \frac{s}{10}\right) \left(1 + \frac{s}{10^3}\right)}}$$

$$= \frac{10^5 \left(1 + \frac{s}{10^3}\right)}{10^5 + \left(1 + \frac{s}{10}\right) \left(1 + \frac{s}{10^3}\right)}$$

$$= \frac{\left(1 + \frac{s}{10^3}\right)}{1 + 10^{-5}(1 + 0.101s + 0.0001s^2)}$$

At  $s = 0$ ,

$$A_f \approx 1$$

The transmission zero is

$$s_z = -10^3 \text{ rad/s}$$

The poles are the roots of

$$10^{-9}s^2 + 1.01 \times 10^{-6}s + 1 = 0$$

which are

$$s = (-0.505 \pm j31.62) \times 10^3 \text{ rad/s}$$

The poles and zero are shown in Fig. 3.

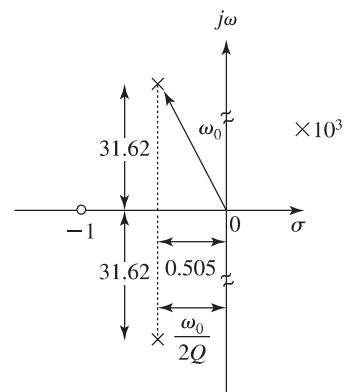


Figure 3

The pair of complex-conjugate poles have

$$\omega_0 \approx 31.62 \text{ krad/s}$$

$$Q = 31.3$$

Thus, the response is very peaky, as shown in Fig. 4.

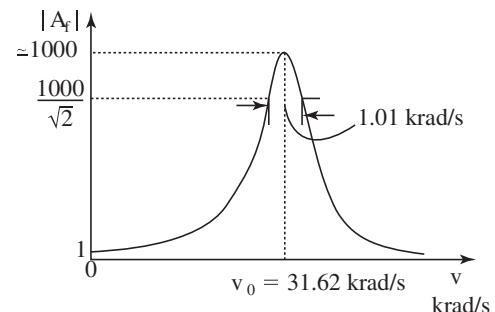


Figure 4

### Exercise 11-1

Ex: 11.1 To allow for  $v_o$  to reach  $-V_{CC} + V_{CEsat} = -15 + 0.2 = -14.8$  V, with  $Q_1$  just cutting off (i.e.  $i_{E1} = 0$ ),

$$I = \frac{14.8 \text{ V}}{R_L} = \frac{14.8}{1 \text{ k}\Omega} = 14.8 \text{ mA}$$

The value of R can now be found from

$$I = \frac{V_R}{R} = \frac{V_{CC} - V_D}{R}$$

$$14.8 = \frac{15 - 0.7}{R}$$

$$\Rightarrow R = \frac{14.3}{14.8} = 0.97 \text{ k}\Omega$$

The resulting output signal swing will be  $-14.8$  V to  $+14.8$  V. The minimum current in  $Q_1 = 0$ . The maximum current in  $Q_1 = 14.8 + 14.8 = 29.6$  mA

Ex: 11.2 At  $v_o = -10$  V, we have

$$i_L = \frac{-10}{1} = -10 \text{ mA}$$

$$i_{E1} = I + i_L = 14.8 - 10 = 4.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{4.8}{1}\right)$$

$$= 0.64 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= -10 + 0.64 = -9.36 \text{ V}$$

At  $v_O = 0$  V, we have

$$i_L = 0 \text{ mA}$$

$$i_{E1} = I = 14.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{14.8}{1}\right)$$

$$= 0.67 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= 0 + 0.67 = 0.67 \text{ V}$$

At  $v_O = 10$  V

$$i_L = \frac{10}{1} = 10 \text{ mA}$$

$$i_{E1} = I + i_L = 14.8 + 10 = 24.8 \text{ mA}$$

$$v_{BE1} = 0.6 + 0.025 \ln\left(\frac{24.8}{1}\right)$$

$$= 0.68 \text{ V}$$

$$v_I = v_O + v_{BE1}$$

$$= 10 + 0.68 = 10.68 \text{ V}$$

At  $v_O = -10$  V, we have

$$i_{E1} = 4.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{4.8 \text{ mA}} = 5.2 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.0052} = 0.995 \text{ V/V}$$

At  $v_O = 0$  V,

$$i_{E1} = 14.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{14.8 \text{ mA}} = 1.7 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.0017} = 0.998 \text{ V/V}$$

At  $v_O = +10$  V,

$$i_{E1} = 24.8 \text{ mA}$$

$$r_{e1} = \frac{25 \text{ mV}}{24.8 \text{ mA}} = 1.0 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}} = \frac{1}{1 + 0.001} = 0.999 \text{ V/V}$$

Ex: 11.3

$$a. P_L = \frac{\left(\hat{V}_o/\sqrt{2}\right)^2}{R_L} = \frac{(8/\sqrt{2})^2}{100} = 0.32 \text{ W}$$

$$P_S = 2V_{CC} \times I = 2 \times 10 \times 100 \times 10^{-3}$$

$$= 2 \text{ W}$$

$$\text{Efficiency } \eta = \frac{P_L}{P_S} \times 100$$

$$= \frac{0.32}{2} \times 100$$

$$= 16\%$$

$$Ex: 11.4 (a) P_L = \frac{1}{2} \frac{\hat{V}_o^2}{R_L}$$

$$= \frac{1}{2} \frac{(4.5)^2}{4} = 2.53 \text{ W}$$

$$(b) P_{S+} = P_{S-} = V_{CC} \times \frac{1}{\pi} \frac{\hat{V}_o}{R_L}$$

$$= 6 \times \frac{1}{\pi} \times \frac{4.5}{4} = 2.15 \text{ W}$$

$$(c) \eta = \frac{P_L}{P_S} \times 100 = \frac{2.53}{2 \times 2.15} \times 100$$

$$= 59\%$$

$$(d) \text{Peak input currents } = \frac{1}{\beta + 1} \frac{\hat{V}_o}{R_L}$$

$$= \frac{1}{51} \times \frac{4.5}{4}$$

$$= 22.1 \text{ mA}$$

(e) Using Eq. (11.22), we obtain

$$P_{DNmax} = P_{DPmax} = \frac{V_{CC}^2}{\pi^2 R_L}$$

$$= \frac{6^2}{\pi^2 \times 4} = 0.91 \text{ W}$$

## Exercise 11–2

Ex: 11.5 (a) The quiescent power dissipated in each transistor =  $I_Q \times V_{CC}$

$$\begin{aligned} & \text{Total power dissipated in the two transistors} \\ &= 2I_Q \times V_{CC} \\ &= 2 \times 2 \times 10^{-3} \times 15 \\ &= 60 \text{ mW} \end{aligned}$$

(b)  $I_Q$  is increased to 10 mA

At  $v_O = 0$ , we have  $i_N = i_P = 10 \text{ mA}$

From Eq. (11.31), we obtain

$$R_{out} = \frac{V_T}{i_P + i_N} = \frac{25}{10 + 10} = 1.25 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{out}} = \frac{100}{100 + 1.25}$$

$$\frac{v_o}{v_i} = 0.988 \text{ at } v_O = 0 \text{ V}$$

At  $v_O = 10 \text{ V}$ , we have

$$i_L = \frac{10 \text{ V}}{100 \Omega} = 0.1 \text{ A} = 100 \text{ mA}$$

Use Eq. (11.27) to calculate  $i_N$ :

$$i_N^2 - i_N i_L - I_Q^2 = 0$$

$$i_N^2 - 100 i_N - 10^2 = 0$$

$$\Rightarrow i_N = 101.0 \text{ mA}$$

Using Eq. (11.26), we obtain

$$i_P = \frac{I_Q^2}{i_N} \simeq 1 \text{ mA}$$

$$R_{out} = \frac{V_T}{i_N + i_P} = \frac{25}{101.0 + 1} \simeq 0.2451 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{out}} = \frac{100}{100 + 0.2451} \simeq 1$$

$$\% \text{ change} = \frac{1 - 0.988}{1} \times 100 = 1.2\%$$

In Example 11.3,  $I_Q = 2 \text{ mA}$ , and for  $v_O = 0$

$$R_{out} = \frac{V_T}{i_N + i_P} = \frac{25}{2 + 2} = 6.25 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{out}} = \frac{100}{100 + 6.25} = 0.94$$

$$v_O = 10 \text{ V}$$

$$i_L = \frac{10 \text{ V}}{100 \Omega} = 100 \text{ mA}$$

Again calculate  $i_N$  (for  $I_Q = 2 \text{ mA}$ ) using Eq. (11.27) ( $i_N = 100.04 \text{ mA}$ ):

$$i_P = \frac{I_Q^2}{i_N} = \frac{2^2}{100.04} = 0.04 \text{ mA}$$

$$R_{out} = \frac{V_T}{i_N + i_P} = \frac{25}{100.04 + 0.04} = 0.25 \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{out}} \simeq 1$$

$$\% \text{ Change} = \frac{1 - 0.94}{1} \times 100 = 6\%$$

For  $I_Q = 10 \text{ mA}$ , change is 1.2%

For  $I_Q = 2 \text{ mA}$ , change is 6%

(c) The quiescent power dissipated in each transistor =  $I_Q \times V_{CC}$

$$\begin{aligned} & \text{Total power dissipated} = 2 \times 10 \times 10^{-3} \times 15 \\ &= 300 \text{ mW} \end{aligned}$$

Ex: 11.6 From Example 11.4, we have  $V_{CC} = 15 \text{ V}$ ,  $R_L = 100 \Omega$

$Q_N$  and  $Q_P$  matched and  $I_S = 10^{-13} \text{ A}$  and  $\beta = 50$ ,  $I_{Bias} = 3 \text{ mA}$

$$\text{For } v_O = 10 \text{ V}, \text{ we have } i_L = \frac{10}{100} = 0.1 \text{ A}$$

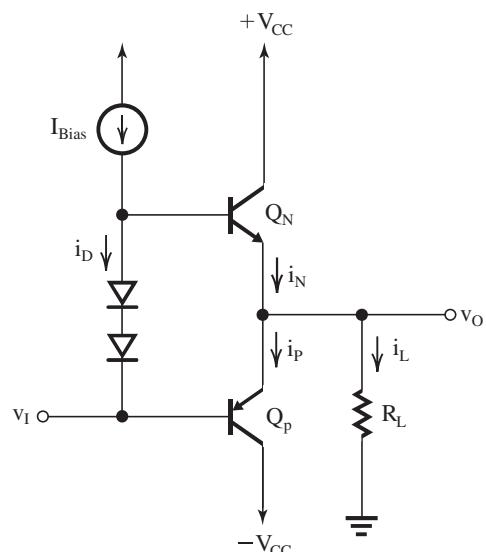
As a first approximation,  $i_N \simeq 0.1 \text{ A}$ ,

$$i_P = 0, i_{BN} \simeq \frac{0.1 \text{ A}}{50 + 1} \simeq 2 \text{ mA}$$

$$i_D = I_{Bias} - i_{BN} = 3 - 2 = 1 \text{ mA}$$

$$V_{BB} = 2V_T \ln \left( \frac{10^{-3}}{\frac{1}{3} \times 10^{-13}} \right) \quad (1)$$

This  $\frac{1}{3}$  is because biasing diodes have  $\frac{1}{3}$  area of the output devices.



### Exercise 11-3

But  $V_{BB} = V_{BEN} + V_{BEP}$

$$\begin{aligned} &= V_T \ln\left(\frac{i_N}{I_S}\right) + V_T \ln\left(\frac{i_N - i_L}{I_S}\right) \\ &= V_T \ln\left[\frac{i_N(i_N - i_L)}{I_S^2}\right] \end{aligned} \quad (2)$$

Equating Eqs. 1 and 2, we obtain

$$\begin{aligned} 2V_T \ln\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) &= V_T \ln\left[\frac{i_N(i_N - i_L)}{I_S^2}\right] \\ \left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)^2 &= \frac{i_N(i_N - 0.1)}{(10^{-13})^2} \end{aligned}$$

$$i_N(i_N - 0.1) = 9 \times 10^{-6}$$

If  $i_N$  is in mA, then

$$i_N(i_N - 100) = 9$$

$$i_N^2 - 100 i_N - 9 = 0$$

$$\Rightarrow i_N = 100.1 \text{ mA}$$

$$i_P = i_N - i_L = 0.1 \text{ mA}$$

$$\text{For } v_O = -10 \text{ V and } i_L = \frac{-10}{100} = -0.1 \text{ A}$$

$$= -100 \text{ mA:}$$

As a first approximation assume  $i_P \cong 100 \text{ mA}$ ,

$$\begin{aligned} i_N &\cong 0. \text{ Since } i_N = 0, \text{ current through diodes} \\ &= 3 \text{ mA} \end{aligned}$$

$$\therefore V_{BB} = 2V_T \ln\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) \quad (3)$$

$$\begin{aligned} \text{But } V_{BB} &= V_T \ln\left(\frac{i_N}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right) \\ &= V_T \ln\left(\frac{i_P + i_L}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right) \end{aligned} \quad (4)$$

$$\text{Here } i_L = -0.1 \text{ A}$$

Equating Eqs. (3) and (4), we obtain

$$\begin{aligned} 2V_T \ln\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) &= \\ V_T \ln\left(\frac{i_P - 0.1}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right) & \\ \left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)^2 &= \frac{i_P(i_P - 0.1)}{(10^{-13})^2} \\ i_P(i_P - 0.1) &= 81 \times 10^{-6} \end{aligned}$$

Expressing currents in mA, we have

$$i_P(i_P - 100) = 81$$

$$i_P^2 - 100 i_P - 81 = 0$$

$$\Rightarrow i_P = 100.8 \text{ mA}$$

$$i_N = i_P + i_L = 0.8 \text{ mA}$$

Ex: 11.7  $\Delta I_C = g_m \times 2 \text{ mV}/^\circ\text{C} \times 5^\circ\text{C}$ , mA

where  $g_m$  is in mA / mV

$$g_m = \frac{10 \text{ mA}}{25 \text{ mV}} = 0.4 \text{ mA/mV}$$

$$\text{Thus, } \Delta I_C = 0.4 \times 2 \times 5 = 4 \text{ mA}$$

Ex: 11.8 Refer to Fig. 11.15.

(a) To obtain a terminal voltage of 1.2 V, and since  $\beta_1$  is very large, it follows that  $V_{R1} = V_{R2} = 0.6 \text{ V}$ .

$$\text{Thus } I_{C1} = 1 \text{ mA}$$

$$I_R = \frac{1.2 \text{ V}}{R_1 + R_2} = \frac{1.2}{2.4} = 0.5 \text{ mA}$$

$$\text{Thus, } I = I_{C1} + I_R = 1.5 \text{ mA}$$

(b) For  $\Delta V_{BB} = +50 \text{ mV}$ :

$$V_{BB} = 1.25 \text{ V } I_R = \frac{1.25}{2.4} = 0.52 \text{ mA}$$

$$V_{BE} = \frac{1.25}{2} = 0.625 \text{ V}$$

$$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = e^{0.025/0.025}$$

$$= 2.72 \text{ mA}$$

$$I = 2.72 + 0.52 = 3.24 \text{ mA}$$

For  $\Delta V_{BB} = +100 \text{ mV}$ , we have

$$V_{BB} = 1.3 \text{ V}, \quad I_R = \frac{1.3}{2.4} = 0.54 \text{ mA}$$

$$V_{BE} = \frac{1.3}{2} = 0.65 \text{ V}$$

$$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = 1 \times e^{0.05/0.025}$$

$$= 7.39 \text{ mA}$$

$$I = 7.39 + 0.54 = 7.93 \text{ mA}$$

For  $\Delta V_{BB} = +200 \text{ mV}$ :

$$V_{BB} = 1.4 \text{ V}, \quad I_R = \frac{1.4}{2.4} = 0.58 \text{ mA}$$

$$V_{BE} = 0.7 \text{ V}$$

$$I_{C1} = 1 \times e^{0.1/0.025} = 54.60 \text{ mA}$$

$$I = 54.60 + 0.58 = 55.18 \text{ mA}$$

### Exercise 11-4

For  $\Delta V_{BB} = -50 \text{ mV}$ :

$$V_{BB} = 1.15 \text{ V}, \quad I_R = \frac{1.15}{2.4} = 0.48 \text{ mA}$$

$$V_{BE} = \frac{1.15}{2}$$

$$= 0.575$$

$$I_{C1} = 1 \times e^{-0.025/0.025} = 0.37 \text{ mA}$$

$$I = 0.48 + 0.37 = 0.85 \text{ mA}$$

For  $\Delta V_{BB} = -100 \text{ mV}$ :

$$V_{BB} = 1.1 \text{ V} \quad I_R = \frac{1.1}{2.4} = 0.46 \text{ mA}$$

$$V_{BE} = 0.55 \text{ V}$$

$$I_{C1} = 1 \times e^{-0.05/0.025} = 0.13 \text{ mA}$$

$$I = 0.46 + 0.13 = 0.59 \text{ mA}$$

For  $\Delta V_{BB} = -200 \text{ mV}$ :

$$V_{BB} = 1.0 \text{ V} \quad I_R = \frac{1}{2.4} = 0.417 \text{ mA}$$

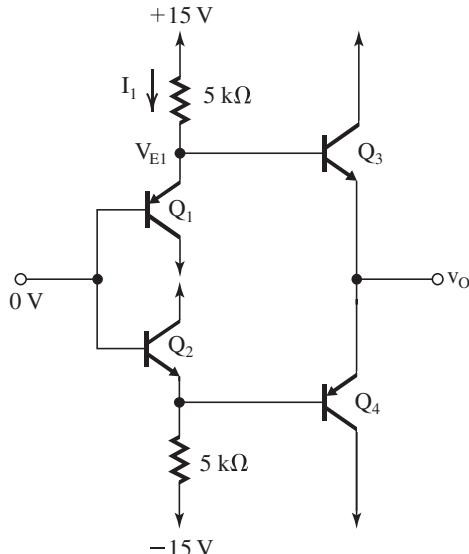
$$V_{BE} = 0.5 \text{ V}$$

$$I_{C1} = 1 \times e^{-0.1/0.025} = 0.018 \text{ mA}$$

$$I = 0.43 \text{ mA}$$

Ex: 11.9 (a) From symmetry we see that all transistors will conduct equal currents and have equal  $V_{BE}$ 's. Thus,

$$v_O = 0 \text{ V}$$



If  $V_{BE} \simeq 0.7 \text{ V}$ , then

$$V_{E1} = 0.7 \text{ V} \text{ and } I_1 = \frac{15 - 0.7}{5} = 2.86 \text{ mA}$$

If we neglect  $I_{B3}$ , then

$$I_{C1} \simeq 2.86 \text{ mA}$$

At this current,  $|V_{BE}|$  is given by

$$|V_{BE}| = 0.025 \ln\left(\frac{2.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right) \simeq 0.63 \text{ V}$$

Thus  $V_{E1} = 0.63 \text{ V}$  and  $I_1 = 2.87 \text{ mA}$

No more iterations are required and

$$i_{C1} = i_{C2} = i_{C3} = i_{C4} \simeq 2.87 \text{ mA}$$

(b) For  $v_I = +10 \text{ V}$ :

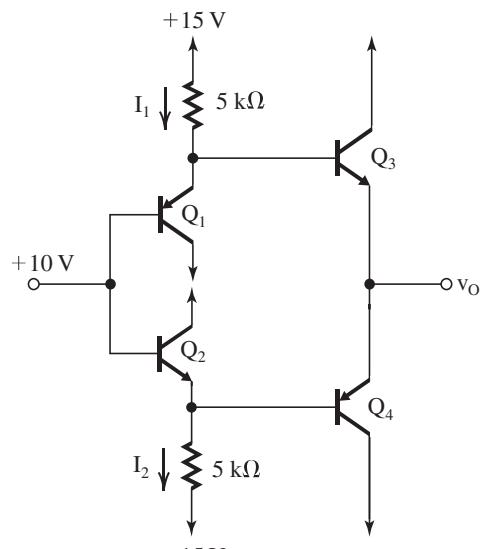
To start the iterations, let  $V_{BE1} \simeq 0.7 \text{ V}$

Thus,

$$V_{E1} = 10.7 \text{ V}$$

and

$$I_1 = \frac{15 - 10.7}{5} = 0.86 \text{ mA}$$



Neglecting  $I_{B3}$ , we obtain

$$I_{C1} \simeq I_{E1} \simeq I_1 = 0.86 \text{ mA}$$

But at this current

$$|V_{BE1}| = V_T \ln\left(\frac{I_{C1}}{I_S}\right)$$

$$= 0.025 \ln\left(\frac{0.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$$= 0.6 \text{ V}$$

Thus,  $V_{E1} = +10.6 \text{ V}$  and  $I_1 = 0.88 \text{ mA}$ . No further iterations are required and  $I_{C1} \simeq 0.88 \text{ mA}$ .

To find  $I_{C2}$ , we use an identical procedure:

$$V_{BE2} \simeq 0.7 \text{ V}$$

$$V_{E2} = 10 - 0.7 = +9.3 \text{ V}$$

$$I_2 = \frac{9.3 - (-15)}{5} = 4.86 \text{ mA}$$

### Exercise 11-5

$$V_{BE2} = 0.025 \ln\left(\frac{4.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$$= 0.643 \text{ V}$$

$$V_{E2} = 10 - 0.643 = +9.357$$

$$I_2 = 4.87 \text{ mA}$$

$$I_{C2} \simeq 4.87 \text{ mA}$$

Finally,

$$I_{C3} = I_{C4} = 3.3 \times 10^{-14} e^{V_{BE}/V_T}$$

where

$$V_{BE} = \frac{V_{E1} - V_{E2}}{2} = 0.62 \text{ V}$$

$$\text{Thus, } I_{C3} = I_{C4} = 1.95 \text{ mA}$$

The symmetry of the circuit enables us to find the values for  $v_I = -10 \text{ V}$  as follows:

$$I_{C1} = 4.87 \text{ mA } I_{C2} = 0.88 \text{ mA}$$

$$I_{C3} = I_{C4} = 1.95 \text{ mA}$$

$$\text{For } v_I = +10 \text{ V, we have } v_O = V_{E1} - V_{BE3}$$

$$= 10.6 - 0.62 = +9.98 \text{ V}$$

$$\text{For } v_I = -10 \text{ V, we have } v_O = V_{E1} - V_{BE3}$$

$$= -9.357 - 0.62 = -9.98 \text{ V}$$

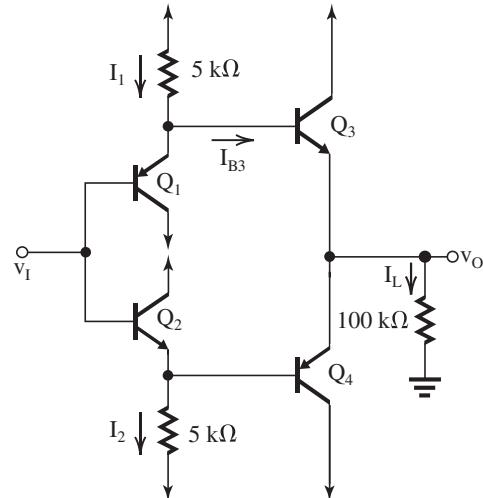
(c) For  $v_I = +10 \text{ V}$ , we have

$$v_O \simeq 10 \text{ V}$$

$$I_L \simeq 100 \text{ mA}$$

$$I_{C3} \simeq 100 \text{ mA}$$

$$I_{B3} = \frac{100}{201} \simeq 0.5 \text{ mA}$$



Assuming that  $|V_{BE1}|$  has not changed much from 0.6 V, then

$$V_{E1} \simeq 10.6 \text{ V}$$

$$I_1 = \frac{15 - 10.6}{5} = 0.88 \text{ mA}$$

$$I_{E1} = I_1 - I_{B3} = 0.88 - 0.5 = 0.38 \text{ mA}$$

$$I_{C1} \simeq 0.38 \text{ mA}$$

$$|V_{BE1}| = 0.025 \ln\left(\frac{0.38 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$$= 0.58 \text{ V}$$

$$V_{E1} = 10.58 \text{ V}$$

$$I_1 = \frac{15 - 10.58}{5} = 0.88 \text{ mA}$$

$$\text{Thus, } I_{C1} \simeq 0.38 \text{ mA}$$

Now for  $Q_2$  we have

$$V_{BE2} = 0.643 \text{ V}$$

$$V_{E2} = 10 - 0.643 = 9.357$$

$$I_2 = 4.87 \text{ mA}$$

$$I_{B4} \simeq 0$$

$$I_{C2} \simeq 4.87 \text{ mA (as in (b))}$$

Assuming that  $I_{C3} \simeq 100 \text{ mA}$ , we have

$$V_{BE3} = 0.025 \ln\left(\frac{100 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$$= 0.72 \text{ V}$$

$$\text{Thus, } v_O = V_{E1} - V_{BE3}$$

$$= 10.58 - 0.72 = +9.86 \text{ V}$$

$$|V_{BE4}| = v_O - V_{E2}$$

$$9.86 - 9.36 = 0.5 \text{ V}$$

$$\text{Thus, } I_{C4} = 3.3 \times 10^{-14} e^{0.5/0.025}$$

$$\simeq 0.02 \text{ mA}$$

From symmetry we find the values for the case

$v_I = -10 \text{ V}$  as:

$$I_{C1} = 4.87 \text{ mA}, \quad I_{C2} = 0.38 \text{ mA}$$

$$I_{C3} = 0.02 \text{ mA}, \quad I_{C4} = 100 \text{ mA}$$

$$v_O = -9.86 \text{ V.}$$

Ex: 11.10 For  $Q_1$ :

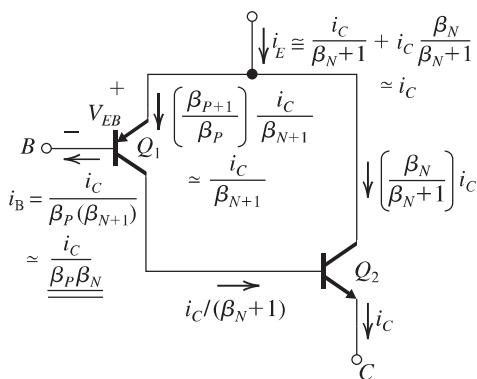
$$i_{C1} = I_{SP} e^{v_{EB}/V_T}$$

$$\frac{i_C}{\beta_N + 1} = I_{SP} e^{v_{EB}/V_T}$$

$$i_C \simeq \beta_N I_{SP} e^{v_{EB}/V_T}$$

### Exercise 11–6

Thus, effective scale current =  $\beta_N I_{SP}$



$$\begin{aligned}
 \text{(b) Effective current gain} &\equiv \frac{i_C}{i_B} = \beta_P \beta_N \\
 &= 20 \times 50 = 1000 \\
 100 \times 10^{-3} &= 50 \times 10^{-14} e^{v_{EB}/0.025} \\
 v_{EB} &= 0.025 \ln(2 \times 10^{11}) \\
 &= 0.651 \text{ V}
 \end{aligned}$$

Ex: 11.11 See Figure 11.21

$$\begin{aligned}
 \text{When } V_{BE5} &= 150 \times 10^{-3} \times R_{E1}, \text{ then } I_{CS} = I_{Bias} \\
 &= 2 \text{ mA} \\
 V_{BE5} &= V_T \ln\left(\frac{I_{CS}}{I_S}\right) \\
 &= 25 \times 10^{-3} \ln\left(\frac{2 \times 10^{-3}}{10^{-14}}\right) \\
 &= 0.651 \text{ V}
 \end{aligned}$$

$$150 \times 10^{-3} R_{E1} = 0.651$$

$$R_{E1} = 4.34 \Omega$$

If peak output current = 100 mA

$$\begin{aligned}
 V_{BE5} &= R_{E1} \times 100 \text{ mA} = 4.34 \times 100 \times 10^{-3} \\
 &= 0.434 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 i_{CS} &= I_S e^{V_{BE5}/V_T} \\
 &= 10^{-14} e^{0.434/25 \times 10^{-3}}
 \end{aligned}$$

$$\simeq 0.35 \mu \text{A}$$

Ex: 11.12 Using Eq. (11.43), we obtain

$$I_Q = I_{Bias} \frac{(W/L)_n}{(W/L)_1}$$

$$1 = 0.2 \frac{(W/L)_n}{(W/L)_p}$$

$$\frac{(W/L)_n}{(W/L)_1} = 5$$

$$Q_1: I_{Bias} = \frac{1}{2} k'_n \left(\frac{W}{L}\right)_1 (V_{GS} - V_{tn})^2$$

$$0.2 = \frac{1}{2} \times 0.250 \left(\frac{W}{L}\right)_1 (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 40$$

$$Q_2: I_{Bias} = \frac{1}{2} k'_p \left(\frac{W}{L}\right)_2 (V_{GS} - |V_t|)^2$$

$$0.2 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_2 \times (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 100$$

$$Q_N: I_Q = \frac{1}{2} k'_n \left(\frac{W}{L}\right)_N (V_{GS} - V_t)^2$$

$$1 = \frac{1}{2} \times 0.250 \times \left(\frac{W}{L}\right)_n 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_n = 200$$

$$Q_P: I_Q = \frac{1}{2} k'_p \left(\frac{W}{L}\right)_p (V_{GS} - |V_t|)^2$$

$$1 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_p \times 0.2^2$$

$$\left(\frac{W}{L}\right)_p = 500$$

$$\text{Now } V_{GG} = V_{GS1} + V_{GS2}$$

$$= (V_{ov1} + V_t) + (V_{ov2} + |V_t|)$$

$$= (0.2 + 0.5) + (0.2 + 0.5)$$

$$= 1.4 \text{ V}$$

Ex: 11.13  $I_N = i_{Lmax} = 10 \text{ mA}$

$$\therefore 10 = \frac{1}{2} k'_n \left(\frac{W}{L}\right)_n V_{ov}^2$$

$$10 = \frac{1}{2} \times 0.250 \times 200 \times V_{ov}^2$$

$$\Rightarrow V_{ov} = 0.63 \text{ V}$$

Using equation 11.46, we obtain

$$V_{Omax} = V_{DD} - V_{ov}|_{Bias} - V_{tn} - V_{OVN}$$

$$= 2.5 - 0.2 - 0.5 - 0.63$$

$$= 1.17 \text{ V}$$

Ex: 11.14 New values of W/L are

$$\left(\frac{W}{L}\right)_p = \frac{2000}{2} = 1000$$

$$\left(\frac{W}{L}\right)_n = \frac{800}{2} = 400$$

### Exercise 11-7

$$I_Q = \frac{1}{2} k_p \left( \frac{W}{L} \right)_p V_{OV}^2$$

$$1 \times 10^{-3} = \frac{1}{2} \times 0.1 \times 10^{-3} \times 1000 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.14 \text{ V}$$

Gain error

$$= -\frac{V_{OV}}{4\mu I_Q R_L} = -\frac{0.14}{4 \times 10 \times 1 \times 10^{-3} \times 100}$$

$$= -0.035$$

Gain error  $= -0.035 \times 100 = -3.5\%$

$$g_{mn} = g_{mp} = \frac{2I_Q}{V_{OV}} = \frac{2 \times 1 \times 10^{-3}}{0.14}$$

$$= 14.14 \text{ mA/V}$$

$$R_{out} = \frac{1}{\mu(g_{mp} + g_{mn})}$$

$$= \frac{1}{10 \times (14.14 + 14.14) \times 10^{-3}}$$

$$\simeq 3.5 \Omega$$

$$\text{Ex: 11.15 Total current into node B} = \frac{2v_i}{R_3} + \frac{v_o}{R_2}$$

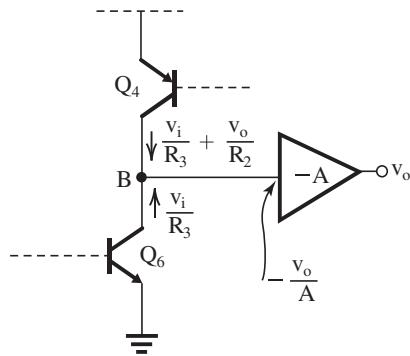
Thus

$$\left( \frac{2v_i}{R_3} + \frac{v_o}{R_2} \right) R = -\frac{v_o}{A}$$

$$\Rightarrow v_o \left( \frac{1}{A} + \frac{R}{R_2} \right) = -\frac{2R}{R_3} v_i$$

$$\frac{v_o}{v_i} = \frac{-2R}{\frac{1}{A} + \frac{R}{R_2}}$$

$$= \frac{-2R_2 / R_3}{1 + (R_2 / AR)} \quad \text{Q.E.D.}$$



For  $AR \gg R_2$ , we have

$$\frac{v_o}{v_i} \simeq -\frac{2R_2}{R_3}$$

Ex: 11.16 From Fig. 11.31 we see that for  $P_{dissipation}$  to be less than 2.9 W, a maximum supply voltage of 20V is called for. The 20-V-supply curve intersects the 3% distortion line at a point for which the output power is 4.2 W. Since

$$P_L = \frac{(\hat{V}_o / \sqrt{2})^2}{R_L}$$

$$\text{we have } \hat{V}_o = \sqrt{4.2 \times 2 \times 8} = 8.2 \text{ V}$$

or 16.4 V peak-to-peak

Ex: 11.17 Voltage gain = 2 K

$$\text{where } K = \frac{R_4}{R_3} = 1 + \frac{R_2}{R_1} = 1.5$$

Thus,  $A_v = 3 \text{ V/V}$

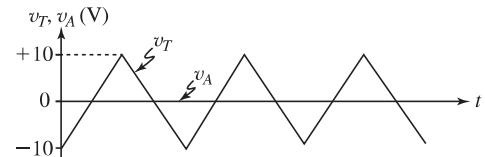
Input resistance =  $R_3 = 10 \text{ k}\Omega$

Peak-to-Peak  $v_o = 3 \times 20 = 60 \text{ V}$

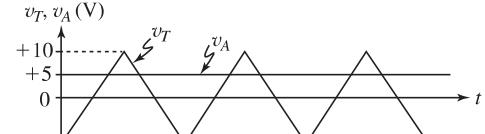
$$\text{Peak load current} = \frac{30 \text{ V}}{8 \Omega} = 3.75 \text{ A}$$

$$P_L = \frac{(30/\sqrt{2})^2}{8} = 56.25 \text{ W}$$

Ex: 11.18 See Fig. 1.



(a)  $v_A = 0$ ;  $D = 50\%$ ; Average = 0



(b)  $v_A = +5 \text{ V}$ ;  $D = 75\%$ ; Average = +5 V

Figure 1 continued

### Exercise 11-8

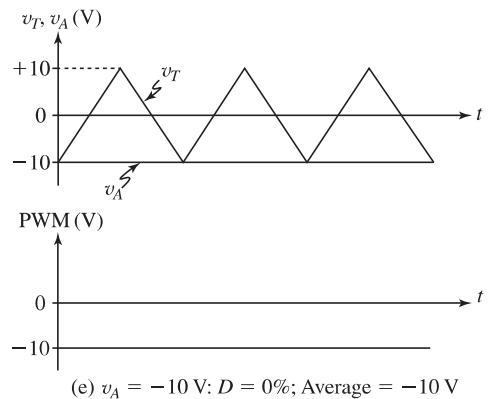
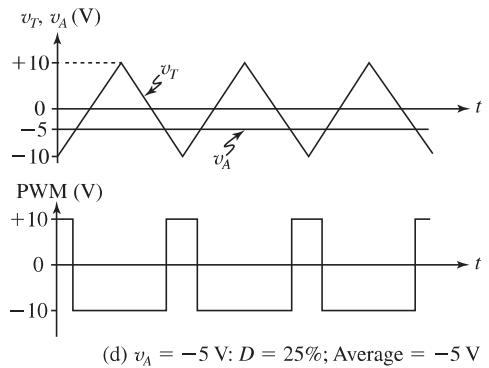
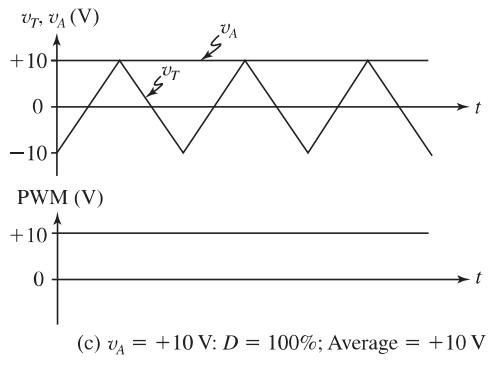


Figure 1

Ex: 11.19

$f_s = 10 \times \text{highest frequency in audio signal}$

$$= 10 \times 20 = 200 \text{ kHz}$$

Since  $f_s$  is a decade higher than  $f_p$ , the gain will have fallen by 40 dB. Thus the PWM component at  $f_s$  will be attenuated by 40 dB.

Ex: 11.20 Maximum peak amplitude =  $V_{DD}$

$$\begin{aligned} \text{Maximum power delivered to } R_L &= \frac{(V_{DD}/\sqrt{2})^2}{R_L} \\ &= \frac{V_{DD}^2}{2R_L} \end{aligned}$$

For  $V_{DD} = 35 \text{ V}$  and  $R_L = 8 \Omega$ :

Peak amplitude =  $35 \text{ V}$

$$\text{Maximum power} = \frac{35^2}{2 \times 8} = 76.6 \text{ W}$$

Power delivered by power supplies

$$= \frac{P_L}{\eta} = \frac{76.6}{0.9} = 85.1 \text{ W}$$

Ex: 11.21  $T_J - T_A = \theta_{JA} P_D$

$$200 - 25 = \theta_{JA} \times 50$$

$$\theta_{JA} = \frac{175}{50} = 3.5^\circ\text{C/W}$$

But,  $\theta_{JA} = \theta_{JC} + \theta_{CS} + \theta_{SA}$

$$3.5 = 1.4 + 0.6 + \theta_{SA}$$

$$\Rightarrow \theta_{SA} = 1.5^\circ\text{C/W}$$

$$T_J - T_C = \theta_{JC} \times P_D$$

$$T_C = T_J - u_{JC} \times P_D$$

$$= 200 - 1.4 \times 50$$

$$= 130^\circ\text{C}$$

$$11.1 \quad I = \frac{0 - (-V_{CC}) - V_D}{R}$$

$$= \frac{10 - 0.7}{1} = 9.3 \text{ mA}$$

Upper limit on  $v_O = V_{CC} - V_{CEsat}$

$$= 10 - 0.3 = 9.7 \text{ V}$$

Corresponding input =  $9.7 + 0.7 = 10.4 \text{ V}$

Lower limit on  $v_O = -IR_L = -9.3 \times 1$

$$= -9.3 \text{ V}$$

Corresponding input =  $-9.3 + 0.7 = -8.6 \text{ V}$

If the EBJ area of  $Q_3$  is twice as large as that of  $Q_2$ , then

$$I = \frac{1}{2} \times 9.3 = 4.65 \text{ mA}$$

There will be no change in  $v_{Omax}$  and in the corresponding value of  $v_I$ . However,  $v_{omin}$  will now become

$$v_{omin} = -IR_L$$

$$= -4.65 \times 1 = -4.65 \text{ V}$$

and the corresponding value of  $v_I$  will be

$$v_I = -4.65 + 0.7 = -3.95 \text{ V}$$

If the EBJ area of  $Q_3$  is made half as big as that of  $Q_2$ , then

$$I = 4 \times 9.3 = 18.6 \text{ mA}$$

There will be no change in  $v_{Omax}$  and in the corresponding value of  $v_I$ . However,  $v_{omin}$  will now become

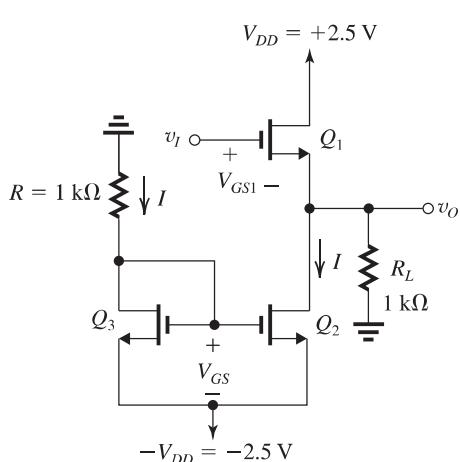
$$v_{omin} = -V_{CC} + V_{CEsat}$$

$$= -10 + 0.3 = -9.7 \text{ V}$$

and the corresponding value of  $v_I$  will be

$$v_I = -9.7 + 0.7 = -9 \text{ V}$$

11.2 First we determine the bias current  $I$  as follows:



$$I = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (V_{GS} - V_t)^2$$

But

$$V_{GS} = 2.5 - IR = 2.5 - I$$

Thus

$$I = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (2.5 - I - V_t)^2$$

$$= \frac{1}{2} \times 20(2.5 - I - 0.5)^2$$

$$I = 10(2 - I)^2$$

$$\Rightarrow I^2 - 4.1I + 4 = 0$$

$$I = 1.6 \text{ mA and } V_{GS} = 0.9 \text{ V}$$

The upper limit on  $v_O$  is determined by  $Q_1$  leaving the saturation region (and entering the triode region). This occurs when  $v_I$  exceeds  $V_{DD}$  by  $V_t$  volts:

$$v_{Imax} = 2.5 + 0.5 = 3 \text{ V}$$

To obtain the corresponding value of  $v_O$ , we must find the corresponding value of  $V_{GS1}$ , as follows:

$$v_O = v_I - V_{GS1}$$

$$i_L = \frac{v_O}{R_L} = \frac{v_I - V_{GS1}}{R_L} = \frac{v_I - V_{GS1}}{1}$$

$$i_L = 3 - V_{GS1}$$

$$i_{D1} = i_L + I = 3 - V_{GS1} + 1.6$$

$$= 4.6 - V_{GS1}$$

But,

$$i_{D1} = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (V_{GS1} - V_t)^2$$

$$4.6 - V_{GS1} = \frac{1}{2} \times 20(V_{GS1} - 0.5)^2$$

$$\Rightarrow V_{GS1}^2 - 0.9V_{GS1} - 0.21 = 0$$

$$V_{GS1} = 1.09 \text{ V}$$

$$v_{Omax} = v_{Imax} - V_{GS1}$$

$$= 3 - 1.09 = +1.91 \text{ V}$$

The lower limit of  $v_O$  is determined either by  $Q_1$  cutting off,

$$v_O = -IR_L = -1.6 \times 1 = -1.6 \text{ V}$$

or by  $Q_2$  leaving saturation,

$$v_O = -V_{DD} + V_{OV2}$$

where

$$V_{OV2} = V_{GS2} - V_t = 0.9 - 0.5 = 0.4 \text{ V}$$

Thus,

$$v_O = -2.5 + 0.4 = -2.1 \text{ V}$$

We observe that  $Q_1$  will cut off before  $Q_2$  leaves saturation, thus

$$v_{O\min} = -1.6 \text{ V}$$

and the corresponding value of  $v_I$  will be

$$\begin{aligned} v_{I\min} &= v_{O\min} + V_t \\ &= -1.6 + 0.5 = -1.1 \text{ V} \end{aligned}$$

11.3 Refer to Fig. 11.2. For a load resistance of  $100 \Omega$  and  $v_O$  ranging between  $-5 \text{ V}$  and  $+5 \text{ V}$ , the maximum current through  $Q_1$  is

$$I + \frac{5}{0.1} = I + 50, \text{ mA and the minimum current is } I - \frac{5}{0.1} = I - 50, \text{ mA.}$$

For a current ratio of 15, we have

$$\frac{I + 50}{I - 50} = 15$$

$$\Rightarrow I = 57.1 \text{ mA}$$

$$R = \frac{9.3 \text{ V}}{57.1 \text{ mA}} = 163 \Omega$$

The incremental voltage gain is  $A_v = \frac{R_L}{R_L + r_{e1}}$

For  $R_L = 100 \Omega$ ;

At  $v_O = +5 \text{ V}$ ,  $i_{E1} = 57.1 + 50 = 107.1 \text{ mA}$

$$r_{e1} = \frac{25}{107.1} = 0.233 \Omega$$

$$A_v = \frac{100}{100 + 0.233} = 0.998 \text{ V/V}$$

At  $v_O = 0 \text{ V}$ ,  $i_{E1} = 57.1 \text{ mA}$

$$r_{e1} = \frac{25}{57.1} = 0.438 \Omega$$

$$A_v = \frac{100}{100.438} = 0.996 \text{ V/V}$$

At  $v_O = -5 \text{ V}$ ,  $i_{E1} = 57.1 - 50 = 7.1 \text{ mA}$

$$r_{e1} = \frac{25}{7.1} = 3.52 \Omega$$

$$A_v = \frac{100}{103.52} = 0.966 \text{ V/V}$$

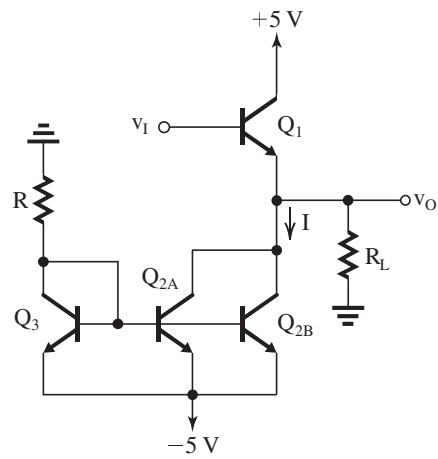
Thus the incremental gain changes by  $0.998 - 0.966 = 0.032$  or about 3% over the range of  $v_O$ .

11.4 Refer to Fig. 11.2. With  $V_{CC} = +5 \text{ V}$ , the upper limit on  $v_o$  is  $4.7 \text{ V}$ , which is greater than the required value of  $+3 \text{ V}$ . To obtain a lower limit of  $-3 \text{ V}$ , we select  $I$  so that

$$IR_L = 3$$

$$\Rightarrow I = 3 \text{ mA}$$

Since we are provided with four devices, we can minimize the total supply current by paralleling two devices to form  $Q_2$  as shown below.



The resulting supply current will be  $3 \times \frac{I}{2}$  rather than  $2I$  which is the value obtained in the circuit of Fig. 11.2. Then the supply current is  $4.5 \text{ mA}$ . The value of  $R$  is found from

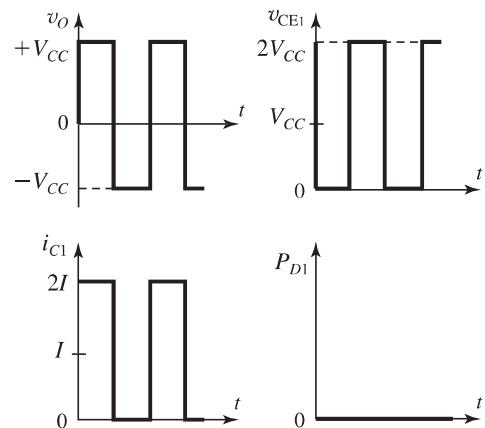
$$R = \frac{4.3 \text{ V}}{1.5 \text{ mA}} = 2.87 \text{ k}\Omega$$

In a practical design we would select a standard value for  $R$  that results in  $I$  somewhat larger than  $3 \text{ mA}$ . Say,  $R = 2.7 \text{ k}\Omega$ . In this case  $I = 3.2 \text{ mA}$ .

$$\begin{aligned} \text{Power from negative supply} &= 3 \times 1.6 \times 5 \\ &= 24 \text{ mW.} \end{aligned}$$

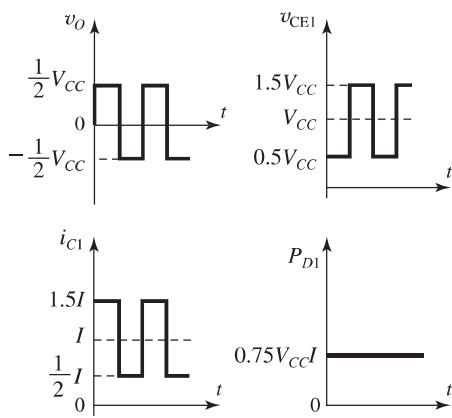
11.5 Refer to Figs. 11.2 and 11.4.

For  $v_O$  being a square wave of  $\pm V_{CC}$  levels:



$P_{D1}|_{\text{average}} = 0$ . For the corresponding sine wave curve [Fig. 11.4], we have  $P_{D1}|_{\text{avg}} = \frac{1}{2}V_{CC}I$ .

For  $v_o$  being a square wave of  $\pm V_{CC}/2$  levels:



$$P_{D1}|_{\text{average}} = 0.75V_{CC}I$$

For a sine-wave output of  $V_{CC}/2$  peak amplitude:

$$v_o = \frac{1}{2} V_{CC} \sin \theta$$

$$i_{C1} = I + \frac{\frac{1}{2}V_{CC}}{R_L} \sin \theta = I + \frac{1}{2}I \sin \theta$$

$$v_{CE1} = V_{CC} - \frac{1}{2}V_{CC} \sin \theta$$

$$P_{D1} = \left( V_{CC} - \frac{1}{2}V_{CC} \sin \theta \right) \left( I + \frac{1}{2}I \sin \theta \right)$$

$$= V_{CC}I - \frac{1}{4}V_{CC}I \sin^2 \theta$$

$$= V_{CC}I - \frac{1}{4}V_{CC}I \times \frac{1}{2}(1 - \cos 2\theta)$$

$$= \frac{7}{8}V_{CC}I + \frac{1}{8}V_{CC}I \cos 2\theta$$

$$P_{D1}|_{\text{average}} = \frac{7}{8}V_{CC}I$$

11.6 In all cases, the average voltage across  $Q_2$  is equal to  $V_{CC}$ . Thus, since  $Q_2$  conducts a constant current  $I$ , its average power dissipation is  $V_{CC}I$ .

11.7 The minimum required value of  $V_{CC}$  is

$$V_{CC} = \hat{V}$$

and the minimum required value of  $I$  is

$$I = \frac{\hat{V}}{R_L}$$

From Eq. (11.10),

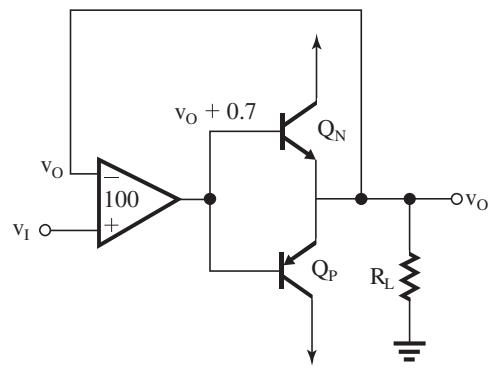
$$\eta = \frac{1}{4} \left( \frac{\hat{V}_o}{IR_L} \right) \left( \frac{\hat{V}_o}{V_{CC}} \right)$$

$$= \frac{1}{4} \left( \frac{\hat{V}}{\hat{V}} \right) \left( \frac{\hat{V}}{\hat{V}} \right) = 0.25$$

or 25%

11.8 Refer to Figs. 11.6 and 11.7. A 10% loss in peak amplitude is obtained when the amplitude of the input signal is 5 V.

11.9



With  $v_i$  sufficiently positive so that  $Q_N$  is conducting, the situation shown obtains. Then,

$$(v_i - v_o) \times 100 = v_o + 0.7$$

$$\Rightarrow v_o = \frac{1}{1.01}(v_i - 0.007)$$

This relationship applies for  $v_i \geq 0.007$ .

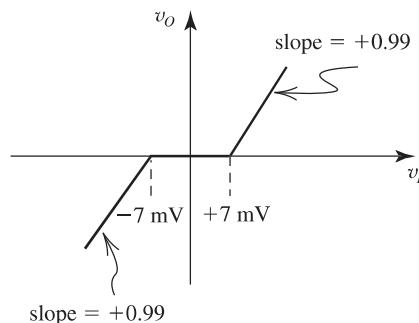
Similarly, for  $v_i$  sufficiently negative so that  $Q_P$  conducts, the voltage at the output of the amplifier becomes  $v_o - 0.7$ , thus

$$(v_i - v_o) \times 100 = v_o - 0.7$$

$$\Rightarrow v_o = \frac{1}{1.01}(v_i + 0.007)$$

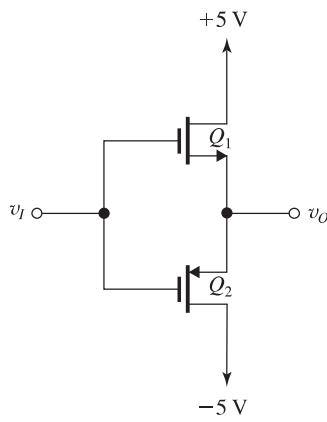
This relationship applies for  $v_i \leq -0.007$ .

The result is the transfer characteristic



Without the feedback arrangement, the deadband becomes  $\pm 700$  mV and the slope change a little (to nearly  $+1$  V/V).

11.10



Devices have  $|V_t| = 0.5$  V

$$\mu C_{ox} \frac{W}{L} = 2 \text{ mA/V}^2$$

For  $R_L = \infty$ , the current is normally zero, so

$$V_{GS} = V_t$$

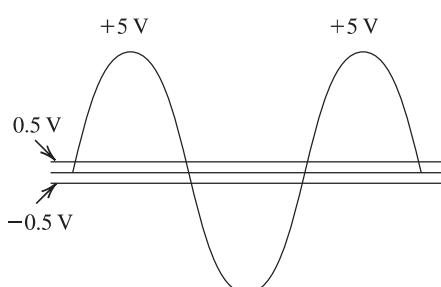
$$\therefore v_o = v_i - V_{GS1} = 5 - 0.5 = 4.5 \text{ V}$$

The peak output voltage will be 4.5 V

$$\sin \theta = \frac{0.5}{5} \Rightarrow \theta = 5.74^\circ$$

Crossover interval =  $4\theta = 22.968$

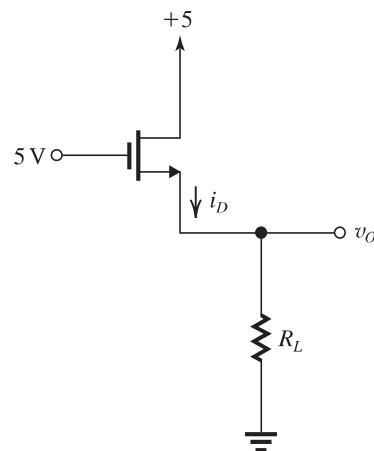
$$\begin{aligned} &= \frac{22.96}{360} \times 100 \\ &= 6.4\% \end{aligned}$$



For  $v_i = 5$  V,  $v_o = 2.5$  V:

$$\therefore V_{GS} = 5 - 2.5 = 2.5 \text{ V}$$

$$\begin{aligned} i_D &= \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 \\ &= \frac{1}{2} \times 2 \times (2.5 - 0.5)^2 \end{aligned}$$



$$i_D = 4 \text{ mA} \text{ and } R_L = \frac{2.5 \text{ V}}{4 \text{ mA}} = 625 \Omega$$

11.11 For  $V_{CC} = 10$  V and  $R_L = 8 \Omega$ , the maximum sine-wave output power occurs when

$$\hat{V}_o = V_{CC} \text{ and is } P_{Lmax} = \frac{1}{2} \frac{V_{CC}^2}{R_L}$$

$$= \frac{1}{2} \times \frac{100}{8} = 6.25 \text{ W}$$

Correspondingly,

$$P_{S+} = P_{S-} = \frac{1}{\pi} \frac{\hat{V}_o}{R_L} V_{CC}$$

$$= \frac{1}{\pi} \times \frac{10}{8} \times 10 = 3.98 \text{ W}$$

for a total supply power of

$$P_S = 2 \times 3.98 = 7.96 \text{ W}$$

The power conversion efficiency  $\eta$  is

$$\eta = \frac{P_L}{P_S} \times 100 = \frac{6.25}{7.96} \times 100 = 78.5\%$$

For  $\hat{V}_o = 5$  V,

$$P_L = \frac{1}{2} \frac{\hat{V}_o^2}{R_L} = \frac{1}{2} \times \frac{25}{8} = 1.56 \text{ W}$$

$$P_{S+} = P_{S-} = \frac{1}{\pi} \frac{\hat{V}_o}{R_L} V_{CC}$$

$$= \frac{1}{\pi} \times \frac{5}{8} \times 10 = 2 \text{ W}$$

$$P_S = 4 \text{ W}$$

$$\eta = \frac{1.56}{4} \times 100 = 39\%$$

Thus, the efficiency reduces to half its maximum value.

$$11.12 P_L = \frac{1}{2} \frac{\hat{V}_o^2}{R_L}$$

$$50 = \frac{1}{2} \frac{\hat{V}_o^2}{8}$$

$$\Rightarrow \hat{V}_o = 28.3 \text{ V}$$