National University of Computer & Emerging Sciences, Lahore
Department of Electrical Engineering
(FALL 2024)

Notes:

1. **Submit the hardcopy at the very start of the lecture. Use A4 sheets for the answers.**
2. **No late submissions. Also submit a scan of the assignment in GCR.**
3. **Any help from the internet, solution manuals, or any other source is strictly prohibited. Violation may lead to serious consequences.**
4. **Draw the diagrams where necessary.**

Date of Submission: Monday 24th November 2024

1. Let D = 4xyax + 2(x2 + z2)ay + 4yzaz nC/m2 and evaluate surface integrals to find the total charge enclosed in the rectangular parallelepiped 0 < x < 2, 0 < y < 3, 0 < z < 5 m.

2. In free space, a volume charge of constant density ρv = ρ0 exists within the region −∞ < x < ∞,−∞ < y < ∞, and −d/2 < z < d/2. Find D and E everywhere.

3. Calculate ∇ ·D at the point specified if (a) D = (1/z2)[10xyz ax + 5x2z ay + (2z3 − 5x2y) az] at P(−2,3,5); (b) D = 5z2 aρ + 10ρz az at P(3,−45◦,5); (c) D = 2r sinθ sinφ ar +r cosθ sinφ aθ +r cosφ aφ at P(3, 45◦, −45◦).

4. In a region in free space, electric flux density is found to be
D= ρ0(z+2d)az C/m2 (−2d≤z≤0) −ρ0(z−2d)az C/m2 (0≤z≤2d)
Everywhere else, D = 0. (a) Using ∇ · D = ρv , find the volume charge density as a function of position everywhere. (b) Determine the electric flux thatpassesthroughthesurfacedefinedbyz=0, −a≤x≤a, −b≤y≤b. (c) Determine the total charge contained within the region −a ≤ x ≤ a, −b ≤ y ≤ b, −d ≤ z ≤ d. (d) Determine the total charge contained within theregion−a≤x≤a, −b≤y≤b, 0≤z≤2d.

5. Let D = 5.00r2ar mC/m2 for r ≤ 0.08 m and D = 0.205 ar/r2 μC/m2 for r ≥0.08m.(a)Findρv forr =0.06m.(b)Findρv forr =0.1m.(c)What surface charge density could be located at r = 0.08 m to cause D = 0 for r > 0.08 m?

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ELECTROMAGNETIC THEORY

Assignment # 2 on CLO # 02

Name:	**Roll Number:**
Date of Submission: November 24th, 2024	**Section:**

Do not write below this line. Do not write below this line.

Instructor: Dr. Huzaifa Rauf

Instructions:

1. Assignment must be handed over to the instructor at the very start of the class.
2. Late submission will not be accepted.
3. Only submit your own work. Do not copy from others. Plagiarism will be dealt with seriously and it will be treated according to the University disciplinary rules.
4. Submit assignment on **A4 size paper**.
5. Use blue or black ink only. Figures may be in color.
6. Handwriting must be legible. Otherwise, your assignment may be returned ungraded.

Problems ↓ [Marks]	**Score**	**CLO # 02**				
		Exemplary (5)	**Proficient (4)**	**Developing (3)**	**Beginning (2)**	**Novice (1)**
P1 [10]						
P2 [10]						
P3 [10]						
P4 [10]						

Oath: *I have solved all the questions by myself and taken no help from the internet, solution manuals, colleagues, or any other unfair means. It is my work only.*

Signature:________________________

Attach this sheet to the front of submitted work.

Q. No. 1

$D = 4xy\,\hat{a}_x + 2(x^2+z^2)\,\hat{a}_y + 4yz\,\hat{a}_z \quad nC/m^2$ — ①

$0<x<2$ m; $0<y<3$ m; $0<z<5$ m

As total charge enclosed

$$Q = \Psi = \oint D_s \cdot dS \quad — ②$$

From eq ①, put $x=0$
hence, $D_x = 0$; $x=0$
Similarly, from eq ①, put $z=0$
hence, $D_z = 0$; $z=0$
From eq ①, $D_y = 2(x^2+z^2)$
Here, at $y=0$ and $y=3$ have same charge but in opposite directions eventually cancelling out eachother.

Therefore, from eq ②

$$Q = \Psi = \oint D\big|_{x=2} \cdot dS_x + \oint D\big|_{z=5} \cdot dS_z$$

$$= \int_0^5\int_0^3 4xy\big|_{x=2}\,\hat{a}_x\,dy\,dz + \int_0^3\int_0^2 4yz\big|_{z=5}\,\hat{a}_z\,dx\,dy$$

$$= \int_0^5\int_0^3 8y\,dy\,dz + \int_0^3\int_0^2 20y\,dx\,dy$$

$$= \int_0^5 8\left(\frac{y^2}{2}\right)\Big|_0^3 dz + \int_0^3 20y\,(x)\Big|_0^2\,dy$$

$$= 36\,(y)\Big|_0^5 + (20)(2)\left(\frac{y^2}{2}\right)\Big|_0^3$$

$$= (36)(5) + (20)(9)$$

$$\boxed{Q = 360\ C}$$

Q. No. 2

$\rho_v = \rho_0$ within $-\infty < x < \infty$; $-\infty < y < \infty$; $-\frac{d}{2} < z < \frac{d}{2}$

As we know

$$\Psi = Q$$

$$\oint_S D_s \cdot dS = \int_{vol.} \rho_v \, dV \quad — ①$$

To find D and E, lets consider a gaussian surface defined by ~~...~~ $|x| < 2$, $|y| < 2$. ~~and $|z| < \frac{d}{2}$~~

To find flux inside the gaussian surface, $|z| < \frac{d}{2}$. For two parallel surfaces at $z = -\frac{d}{2}$ and $z = \frac{d}{2}$, eq ① becomes

$$\oint D_z \cdot dS_z = \int_{vol.} \rho_v \, dV$$

$$\int_{-2}^{2}\int_{-2}^{2} D_z \, dx dy + \int_{-2}^{2}\int_{-2}^{2} D_z \, dx dy = \int_{-2}^{2}\int_{-2}^{2}\int_{-2}^{2} \rho_0 \, dx dy dz$$

$$2 D_z \int_{-2}^{2} (x)_{-2}^{2} \, dy = \rho_0 \int_{-2}^{2}\int_{-2}^{2} (x)_{-2}^{2} \, dy dz$$

$$2 D_z (4) (y)_{-2}^{2} = \rho_0 (4) \int_{-2}^{2} (y)_{-2}^{2} \, dz$$

$$8 D_z \cancel{(4)} = 4 \rho_0 \cancel{(4)} (2z)$$

$$\cancel{8} D_z = \cancel{8} \rho_0 z$$

$$D_z = \rho_0 z$$

$$\Rightarrow \boxed{D_{in} = \rho_0 z \hat{a}_z \, C/m^2 \; ; |z| < \frac{d}{2}}$$

As $D = E \epsilon_0$

$$E = \frac{D}{\epsilon_0} = \frac{\rho_0 z}{\epsilon_0} \hat{a}_z$$

$$\Rightarrow \boxed{E_{in} = \frac{\rho_0 z}{\epsilon_0} \hat{a}_z \; V/m \; ; |z| < \frac{d}{2}}$$

Similarly, to find flux outside the gaussian surface, $|z| > \frac{d}{2}$. For two parallel surfaces at $z = \frac{d}{2}$ and $z = -\frac{d}{2}$.

eq ① becomes

$$2 \oint_S D_z \cdot dS_z = \int_{vol} \rho_v \, dV$$

$$2 \int_{-2}^{2}\int_{-2}^{2} D_z \, dx\,dy = \int_{-2}^{2}\int_{-2}^{2}\int_{-2}^{2} \rho_0 \, dx\,dy\,dz$$

$$(\cancel{2}) D_z (\cancel{4})(\cancel{4}) = \rho_0 (\cancel{4})(\cancel{4})(\cancel{2}z)$$

$$D_z = \rho_0 z$$

$$\Rightarrow \boxed{D_{out} = \rho_0 z \, \hat{a}_z \ C/m^2 \ ; |z| > \frac{d}{2}}$$

Similarly, $\boxed{E_{out} = \frac{\rho_0 z}{\epsilon_0} \hat{a}_z \ V/m \ ; |z| > \frac{d}{2}}$

Q. No. 3

(a)

$$D = \left(\frac{1}{z^2}\right)\left[10xyz\,\hat{a}_x + 5x^2z\,\hat{a}_y + (2z^3 - 5x^2y)\hat{a}_z\right] \quad ; P = (-2,3,5)$$

In RCS,

$$\nabla \cdot D = \frac{\partial}{\partial x}(D_x) + \frac{\partial}{\partial y}(D_y) + \frac{\partial}{\partial z}(D_z)\Big|_P$$

$$= \frac{\partial}{\partial x}\left(\frac{10xyz}{z^2}\right) + \frac{\partial}{\partial y}\left(\frac{5x^2z}{z^2}\right) + \frac{\partial}{\partial z}\left(\frac{2z^3 - 5x^2y}{z^2}\right)\Big|_P$$

$$= \frac{10y}{z} + 0 + 2 + \frac{10x^2y}{z^3}\Big|_{P(-2,3,5)}$$

$$= \frac{10(3)}{(5)} + 2 + \frac{10(-2)^2(3)}{(5)^3}$$

$$\boxed{\nabla \cdot D = 8.96}$$

(b)

$$D = 5z^2\,\hat{a}_\rho + 10\rho z\,\hat{a}_z \quad ; P = (3, -45^\circ, 5)$$

In CCS,

$$\nabla \cdot D = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(D_\phi) + \frac{\partial}{\partial z}(D_z)\Big|_P$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(5\rho z^2) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(0) + \frac{\partial}{\partial z}(10\rho z)\Big|_P$$

$$= \frac{1}{\rho}(5z^2) + 10\rho\Big|_{P(3,-45^\circ,5)}$$

$$= \frac{1}{3}(5)(5)^2 + 10(3)$$

$$\boxed{\nabla \cdot D = 71.667}$$

(C)

$$D = 2r\sin\theta\sin\phi\,\hat{a}_r + r\cos\theta\sin\phi\,\hat{a}_\theta + r\cos\phi\,\hat{a}_\phi \quad ; P = (3, 45^\circ, 45^\circ)$$

In SCS,

$$\nabla\cdot D = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta D_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(D_\phi)\Big|_P$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(2r^3\sin\theta\sin\phi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r\sin\theta\cos\theta\sin\phi) + \frac{1}{r\sin\theta}(r\cos\phi)\Big|_P$$

$$= \frac{1}{\cancel{r^2}}(6\cancel{r^2}\sin\theta\sin\phi) + \frac{2}{r\sin\theta}\Big\{\frac{\partial}{\partial\theta}(2r\sin\theta\cos\theta\sin\phi) - \frac{1}{\cancel{r}\sin\theta}(-\sin\phi\,\cancel{r})\Big|_P$$

$$= 6\sin\theta\sin\phi + \frac{2}{r\sin\theta}\frac{\partial}{\partial\theta}(r\sin 2\theta\sin\phi) - \frac{\sin\phi}{\sin\theta}\Big|_P$$

$$= 6\sin\theta\sin\phi + \frac{\cancel{2}}{\cancel{r}\sin\theta}\left(\frac{\cancel{r}\cos 2\theta\sin\phi}{\cancel{2}}\right) - \frac{\sin\phi}{\sin\theta}\Big|_{P(3,45^\circ,-45^\circ)}$$

$$= 6\sin(45^\circ)\sin(-45^\circ) + \frac{\cos(2(45^\circ))\sin(-45^\circ)}{\sin(45^\circ)} - \frac{\sin(-45^\circ)}{\sin(45^\circ)}$$

$$= -3 + 0 - (-1)$$

$$\boxed{\nabla\cdot D = -2}$$

Q. No. 4

$$D = \begin{cases} \rho_o (z+2d)\,\hat{a}_z & ; \; -2d \le z \le 0 \\ -\rho_o (z-2d)\,\hat{a}_z & ; \; 0 \le z \le 2d \end{cases} \quad C/m^2$$

(a)

$$\nabla \cdot D = \rho_v$$

As $\rho_v = \nabla \cdot D = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$

$$= \frac{\partial}{\partial z}\left(\begin{cases} z\rho_o + 2d\rho_o \\ -z\rho_o + 2d\rho_o \end{cases} \right)$$

$$\boxed{\rho_v = \begin{cases} \rho_o & ; \; -2d \le z \le 0 \\ -\rho_o & ; \; 0 \le z \le 2d \end{cases}}$$

(b)

$z = 0 \;;\; -a \le x \le a \;;\; -b \le y \le b$

At $z = 0$ in x-y plane,

$$D = \begin{cases} \rho_o (z+2d)\,\hat{a}_z \\ -\rho_o (0-2d)\,\hat{a}_z \end{cases}$$

$$\Rightarrow D = 2d\rho_o\,\hat{a}_z \; C/m^2$$

As ~~...~~ $\psi = \oint_s D_s \cdot ds = \int_s D_z \cdot dS_z = \int_{-b}^{b}\int_{-a}^{a} 2d\rho_o \, dx\, dy$

$$= 2d\rho_o (2a)(2b)$$

$$= 8d\rho_o ab$$

$$\boxed{\psi = 8\rho_o abd \; C}$$

(c)

$-a \le x \le a$; $-b \le y \le b$; $-d \le z \le d$

The given region at $|z| < d$ lies within the region $|z| < 2d$. From part a, the volume charge density is the same but in equal direction in x-y plane. Hence, total charge

$\boxed{Q = 0}$

(d)

$-a \le x \le a$; $-b \le y \le b$; $0 \le z \le 2d$

As $Q = \Psi = \oint_S D \cdot dS = \int_{vol} \rho_v \, dV$

For $0 \le z \le 2d$, $D = -\rho_0 (z - 2d)\, \hat{a}_z \; C/m^2$ and $\rho_v = -\rho_0$

$$Q = \int_0^{2d}\int_{-b}^{b}\int_{-a}^{a} -\rho_0 \, dx\, dy\, dz$$

$$= -\rho_0 (2a)(2b)(2d)$$

$$\boxed{Q = -8\rho_0 abd \; C}$$

Q. No. 5

$\Rightarrow D = 5r^2 \hat{a}_r \; mC/m^2$ for $r \leq 0.08m$ — ①

$\Rightarrow D = \dfrac{0.205 \, \hat{a}_r}{r^2} \; \mu C/m^2$ for $r \geq 0.08m$ — ②

(a)

ρ_v at $r = 0.06m$

From ①

As $$\rho_v = \nabla \cdot D = \cancel{\frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y} \; \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta \, D_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} D_\phi \Big|_r$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \, 5r^2) + 0 + 0 \Big|_r$$

$$= \frac{1}{\cancel{r^2}}(20 r^{\cancel{3}})\Big|_{r=0.06}$$

$$= 20(0.06)$$

$$\boxed{\rho_v = 1.2 \; mC/m^2}$$

(b)

ρ_v at $r = 0.1m$

From ②

As $$\rho_v = \nabla \cdot D = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + 0 + 0$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(\cancel{r^2} \frac{0.205}{\cancel{r^2}}\right)$$

$$= \frac{1}{r^2}(0)$$

$$\boxed{\rho_v = 0}$$

(C)

ρ_s at $r=0.08m$ to cause $D=0$ for $r>0.08m$

For $D=0$ for $r>0.08m$, ρ_s should be equal but opposite to total volume charge.

As $Q = \Psi = \int_{vol} \rho_v \, dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{0.08} 20r \, r^2 \sin\theta \, dr \, d\theta \, d\phi$

$= \int_0^{2\pi}\int_0^{\pi} 20\sin\theta \left[\frac{r^4}{4}\right]_0^{0.08} d\theta \, d\phi$

$= \int_0^{2\pi} [-\cos\theta]_0^{\pi} \, d\phi \, (0.2048\times10^{-3})$

$= (2\pi)(1+1)(0.2048\times10^{-3})$

$= 2.574\times10^{-3}$ mC

$Q = 2.574 \, \mu C$

As $Q = 4\pi r^2 \rho_s$

$\Rightarrow \rho_s = -\left[\frac{Q}{4\pi r^2}\right] = -\frac{2.574\mu}{4\pi(0.08)^2}$

$$\boxed{\rho_s = -32.005 \, \mu C/m^2}$$