5.7: Introduction to Dielectric Materials

- A **dielectric** is a material that does not conduct electricity but can store electrical energy in the presence of an electric field.
- It contains **bound charges**, which are distinct from free charges found in conductors. These charges cannot flow freely but can shift slightly in response to an external electric field.
- The ability of dielectric materials to store electric energy is due to the displacement of bound charges within the material, akin to stretching a spring, which results in **potential energy storage**.

Polarization in Dielectrics

1. **Definition**:
   - **Polarization** $\mathbf{P}$ is the **electric dipole moment per unit volume** of the dielectric material:

   $$\mathbf{P} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{i=1}^{n} \mathbf{p}_i$$

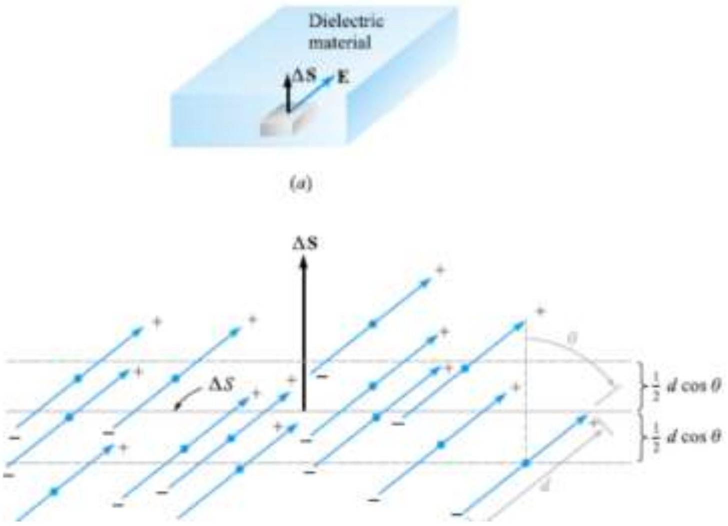
   where $\mathbf{p}_i$ is the dipole moment of each molecule in the volume Δv, and n is the number of dipoles per unit volume.

2. **Dipole Moment**:
   - A dipole moment $\mathbf{p}$ is defined as:

   $$\mathbf{p} = Q\mathbf{d}$$

   where Q is the magnitude of the bound charges and $\mathbf{d}$ is the displacement vector from the negative to the positive charge.

3. **Types of Molecules**:
   - **Polar Molecules**: Have a permanent dipole moment due to an inherent charge separation (e.g., water molecules). In an electric field, the dipoles tend to align with the field.
   - **Nonpolar Molecules**: Do not have an inherent dipole moment but develop one under the influence of an electric field due to induced charge separation.



Electric Susceptibility and Permittivity

1. **Electric Susceptibility:**
   - Polarization $\mathbf{P}$ is linearly related to the electric field intensity $\mathbf{E}$ in linear dielectrics:

   $$\mathbf{P} = \chi_e \epsilon_0 \mathbf{E}$$

   where χ_e is the **electric susceptibility**, a dimensionless constant that indicates the ease with which a material can be polarized.

2. **Permittivity and Relative Permittivity:**
   - **Permittivity** ϵ:

   $$\epsilon = \epsilon_0(1 + \chi_e)$$

   - **Relative Permittivity** ϵ_r:

   $$\epsilon_r = 1 + \chi_e$$

   ϵ_r (also called the dielectric constant) is a dimensionless quantity representing the ratio of the permittivity of the material to that of free space (ϵ_0).

3. **Electric Flux Density:**
   - The **electric flux density** $\mathbf{D}$ accounts for the effect of polarization:

   $$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

   - Substituting $\mathbf{P}$:

   $$\mathbf{D} = \epsilon \mathbf{E}$$

Gauss's Law in Dielectrics

1. **Generalized Form:**
    - For materials other than free space:
    $$\nabla \cdot \mathbf{D} = \rho_f$$
    where ρ_f is the **free charge density**.
    - In integral form:
    $$\int_S \mathbf{D} \cdot d\mathbf{S} = Q_{\text{free}}$$
2. **Polarization Charge:**
    - The **bound charge density** ρ_b arises from polarization:
    $$\nabla \cdot \mathbf{P} = -\rho_b$$
3. **Total Charge Density:**
    - The total charge density is the sum of free and bound charges:
    $$\rho_T = \rho_f + \rho_b$$

Dielectric Behavior and Applications

1. **Energy Storage:**
    - Dielectric materials store energy by displacing bound charges against atomic and molecular forces, analogous to stretching a spring.
2. **Nonlinear and Anisotropic Behavior:**
    - In some materials (e.g., **ferroelectrics** like barium titanate), the relationship between $\mathbf{P}$ and $\mathbf{E}$ is nonlinear and may exhibit **hysteresis**.
    - In **anisotropic materials**, the permittivity varies with direction, leading to a tensor relationship:
    $$D_x = \epsilon_{xx} E_x + \epsilon_{xy} E_y + \epsilon_{xz} E_z$$
    where ϵ_{ij} are components of the permittivity tensor.
3. **Applications:**
    - **Capacitors:** Dielectrics increase the capacitance by reducing the electric field for a given charge.
    - **Insulators:** Dielectrics prevent current flow while allowing energy storage.
    - **Electronics:** High permittivity dielectrics are used in DRAM and other memory devices.

$$D_x = \epsilon_{xx} E_x + \epsilon_{xy} E_y + \epsilon_{xz} E_z$$
$$D_y = \epsilon_{yx} E_x + \epsilon_{yy} E_y + \epsilon_{yz} E_z$$
$$D_z = \epsilon_{zx} E_x + \epsilon_{zy} E_y + \epsilon_{zz} E_z$$

D5.8. A slab of dielectric material has a relative dielectric constant of 3.8 and contains a uniform electric flux density of 8 nC/m^2. If the material is lossless, find: (*a*) E; (*b*) P; (*c*) the average number of dipoles per cubic meter if the average dipole moment is 10^{-29} C · m.

(a) Electric Field Intensity (E):

We use $D = \epsilon E$, where $\epsilon = \epsilon_0 \epsilon_r$.

Thus,

$$E = \frac{D}{\epsilon}$$

Substituting values:

$$E = \frac{8 \times 10^{-9}}{3.8 \times 8.854 \times 10^{-12}}$$

(b) Polarization (P):

$$P = D\left(1 - \frac{1}{\epsilon_r}\right)$$

Substituting:

$$P = 8 \times 10^{-9}\left(1 - \frac{1}{3.8}\right)$$

(c) Number of Dipoles per Cubic Meter (n):

$$n = \frac{P}{p}$$

Substituting:

$$n = \frac{P}{10^{-29}}$$

(a) Electric Field Intensity E

$$E = \frac{D}{\epsilon} = \frac{8 \times 10^{-9}}{3.8 \times 8.854 \times 10^{-12}}$$

$$\epsilon = 3.8 \times 8.854 \times 10^{-12} = 3.36452 \times 10^{-11}$$

$$E = \frac{8 \times 10^{-9}}{3.36452 \times 10^{-11}} = 238\ \mathrm{V/m}$$

(b) Polarization P

$$P = D\left(1 - \frac{1}{\epsilon_r}\right)$$

$$P = 8 \times 10^{-9}\left(1 - \frac{1}{3.8}\right) = 8 \times 10^{-9}\,(1 - 0.26316)$$

$$P = 8 \times 10^{-9} \times 0.73684 = 5.89 \times 10^{-9}\,\mathrm{C/m^2}$$

(c) Number of Dipoles per Cubic Meter n

$$n = \frac{P}{p} = \frac{5.89 \times 10^{-9}}{10^{-29}}$$

$$n = 5.89 \times 10^{20}\,\mathrm{dipoles/m^3}$$

Boundary Conditions for Perfect Dielectric Materials

When dealing with two different dielectric materials or a dielectric-conductor interface, it is important to understand the behavior of the electric field ($\mathbf{E}$) and electric flux density ($\mathbf{D}$) at the boundary. The conditions at the boundary determine how fields transition between regions with different permittivities (ε_1 and ε_2).

Tangential Components of Electric Field (E_{tan})

The tangential component of the electric field is continuous across the boundary between two dielectrics. This is derived using the integral form of Faraday's law:

$$\oint \mathbf{E} \cdot d\mathbf{L} = 0$$

For a small rectangular loop at the boundary, the contributions from the tangential components across the interface dominate as the loop height (Δh) approaches zero:

$$E_{\text{tan},1} = E_{\text{tan},2}$$

This continuity implies that the potential difference between two points at the boundary is the same on either side of the interface.

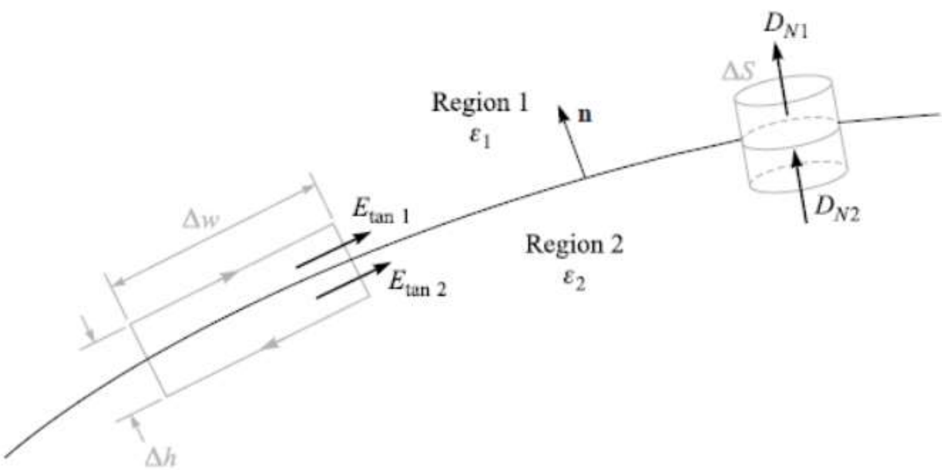


Figure 5.10 The boundary between perfect dielectrics of permittivities ϵ_1 and ϵ_2. The continuity of D_N is shown by the gaussian surface on the right, and the continuity of $E_{\tan}$ is shown by the line integral about the closed path at the left.

Tangential Components of Electric Flux Density (D_{tan})

The tangential components of $\mathbf{D}$ are not continuous due to the relationship between $\mathbf{D}$ and $\mathbf{E}$ in materials with different permittivities:

$$D_{\text{tan},1} = \varepsilon_1 E_{\text{tan},1}, \quad D_{\text{tan},2} = \varepsilon_2 E_{\text{tan},2}$$

From $E_{\text{tan},1} = E_{\text{tan},2}$:

$$\frac{D_{\text{tan},1}}{D_{\text{tan},2}} = \frac{\varepsilon_1}{\varepsilon_2}$$

This relationship highlights the discontinuity in $\mathbf{D}$ due to differing permittivities.

Normal Components of Electric Flux Density (D_N)

The normal component of $\mathbf{D}$ is governed by Gauss's law. For a small pillbox enclosing the boundary:

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q_{\text{free enclosed}}$$

For perfect dielectrics with no free charge at the interface ($\rho_S = 0$):

$$D_{N,1} = D_{N,2}$$

This continuity indicates that the normal component of $\mathbf{D}$ does not change across the boundary.

Normal Components of Electric Field (E_N)

The relationship between $\mathbf{D}$ and $\mathbf{E}$ provides:

$$D_{N,1} = \varepsilon_1 E_{N,1}, \quad D_{N,2} = \varepsilon_2 E_{N,2}$$

Since $D_{N,1} = D_{N,2}$:

$$\frac{E_{N,1}}{E_{N,2}} = \frac{\varepsilon_2}{\varepsilon_1}$$

The normal component of the electric field is inversely proportional to the permittivity.

General Boundary Conditions

Using vector notation, the boundary conditions can be expressed as:

1. **Continuity of $\mathbf{D}_N$:**

$$(\mathbf{D}_1 - \mathbf{D}_2) \cdot \mathbf{n} = \rho_S$$

   For perfect dielectrics ($\rho_S = 0$):

$$\mathbf{D}_1 \cdot \mathbf{n} = \mathbf{D}_2 \cdot \mathbf{n}$$

2. **Continuity of $\mathbf{E}_{\text{tan}}$:**

$$(\mathbf{E}_1 - \mathbf{E}_2) \times \mathbf{n} = 0$$

   This implies $\mathbf{E}_{\text{tan},1} = \mathbf{E}_{\text{tan},2}$.

Field Refraction at the Interface

The refraction of $\mathbf{D}$ and $\mathbf{E}$ at the boundary is characterized by their angles with the normal:

1. **Normal Components:**

$$D_{N,1} = D_{N,2}, \quad \varepsilon_1 E_{N,1} = \varepsilon_2 E_{N,2}$$

2. **Tangential Components:**

$$\frac{D_{\tan,1}}{D_{\tan,2}} = \frac{\varepsilon_1}{\varepsilon_2}, \quad E_{\tan,1} = E_{\tan,2}$$

3. **Angles of Refraction:** Using $\tan\theta = \frac{D_{\tan}}{D_N}$:

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

Field Magnitudes in Region 2

Given $\mathbf{D}_1$ and θ_1, the magnitude of $\mathbf{D}_2$ is:

$$D_2 = D_1\sqrt{\cos^2\theta_1 + \left(\frac{\varepsilon_2}{\varepsilon_1}\sin\theta_1\right)^2}$$

The magnitude of $\mathbf{E}_2$ is:

$$E_2 = E_1\sqrt{\sin^2\theta_1 + \left(\frac{\varepsilon_1}{\varepsilon_2}\cos\theta_1\right)^2}$$

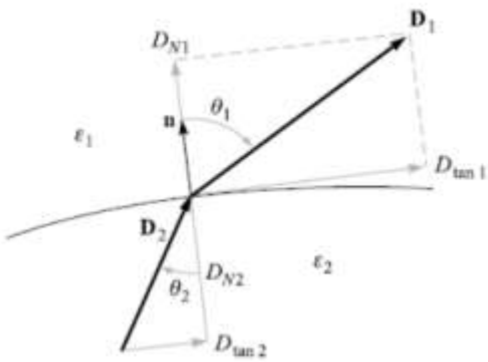


Figure 5.11 The refraction of D at a dielectric interface. For the case shown, $\epsilon_1 > \epsilon_2$; $\mathbf{E}_1$ and $\mathbf{E}_2$ are directed along $\mathbf{D}_1$ and $\mathbf{D}_2$, with $D_1 > D_2$ and $E_1 < E_2$.

Solution to Example 5.4: Fields Within and Outside a Teflon Slab

We are tasked to calculate the electric flux density ($\mathbf{D}$), the electric field intensity ($\mathbf{E}$), and the polarization ($\mathbf{P}$) everywhere for a slab of Teflon ($\varepsilon_r = 2.1$) located in the region $0 \le x \le a$, with free space on either side ($x < 0$ and $x > a$). The external field outside the Teflon slab is given as:

$$\mathbf{E}_{\text{out}} = E_0 \mathbf{a}_x \quad \text{in free space.}$$

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1. Outside the Teflon Slab ($x < 0$ and $x > a$)

- Electric Field ($\mathbf{E}_{\text{out}}$):

$$\mathbf{E}_{\text{out}} = E_0 \mathbf{a}_x$$

- Electric Flux Density ($\mathbf{D}_{\text{out}}$): Using $\mathbf{D} = \varepsilon_0 \mathbf{E}$ in free space:

$$\mathbf{D}_{\text{out}} = \varepsilon_0 E_0 \mathbf{a}_x$$

- Polarization ($\mathbf{P}_{\text{out}}$): Since there is no dielectric material in free space:

$$\mathbf{P}_{\text{out}} = 0$$

2. Inside the Teflon Slab ($0 \le x \le a$)

- Electric Flux Density ($\mathbf{D}_{\text{in}}$): By the continuity of D_N at the boundary, the normal component of $\mathbf{D}$ is continuous across the interface:

$$\mathbf{D}_{\text{in}} = \mathbf{D}_{\text{out}} = \varepsilon_0 E_0 \mathbf{a}_x$$

- Electric Field ($\mathbf{E}_{\text{in}}$): Using the relationship $\mathbf{D} = \varepsilon \mathbf{E}$ and $\varepsilon = \varepsilon_r \varepsilon_0$:

$$\mathbf{E}_{\text{in}} = \frac{\mathbf{D}_{\text{in}}}{\varepsilon} = \frac{\varepsilon_0 E_0 \mathbf{a}_x}{\varepsilon_r \varepsilon_0} = \frac{E_0}{\varepsilon_r} \mathbf{a}_x$$

  Substituting $\varepsilon_r = 2.1$:

$$\mathbf{E}_{\text{in}} = 0.476 E_0 \mathbf{a}_x$$

- Polarization ($\mathbf{P}_{\text{in}}$): Using the relationship $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$, we find:

$$\mathbf{P}_{\text{in}} = \mathbf{D}_{\text{in}} - \varepsilon_0 \mathbf{E}_{\text{in}}$$

  Substituting $\mathbf{D}_{\text{in}} = \varepsilon_0 E_0 \mathbf{a}_x$ and $\mathbf{E}_{\text{in}} = 0.476 E_0 \mathbf{a}_x$:

$$\mathbf{P}_{\text{in}} = \varepsilon_0 E_0 \mathbf{a}_x - \varepsilon_0 (0.476 E_0) \mathbf{a}_x$$

$$\mathbf{P}_{\text{in}} = (1 - 0.476) \varepsilon_0 E_0 \mathbf{a}_x = 0.524 \varepsilon_0 E_0 \mathbf{a}_x$$