National University of Computer & Emerging Sciences, Lahore
Department of Electrical Engineering
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Name :
Agha Abdul Wahab
Rollno: 22L-7729
Section: 5A1

EE3005 – Electromagnetic Theory (EMT)
Assignment # 4 on CLO # 04 (Total Marks = 20)

Date of Submission: 10th Dec 2024 in your Class

Problem # 01: Suppose that $\mathbf{H} = 0.2z^2\ \mathbf{a_x}$ for $z > 0$, and $\mathbf{H} = 0$ elsewhere, as shown in Figure 7.15. Calculate $\mathbf{H} \cdot d\mathbf{L}$ about a square path with side d, centered at $(0, 0, z1)$ in the $y = 0$ plane where $z1 > d/2$.

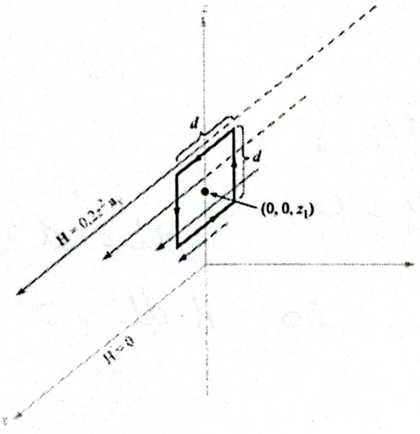

Figure 7.15 A square path of side d with its center on the z axis at $z = z_1$ is used to evaluate $\oint \mathbf{H} \cdot d\mathbf{L}$ and find curl $\mathbf{H}$.

Problem # 02: (a) Evaluate the closed line integral of **H** about the rectangular path $P_1(2, 3, 4)$ to $P_2(4, 3, 4)$ to $P_3(4, 3, 1)$ to $P_4(2, 3, 1)$ to P_1, given $\mathbf{H} = 3z\mathbf{a}_x - 2x^3\mathbf{a}_z$ A/m. (b) Determine the quotient of the closed line integral and the area enclosed by the path as an approximation to $(\nabla \times \mathbf{H})_y$. ($c$) Determine $(\nabla \times \mathbf{H})_y$ at the center of the area.

Problem 1: H along the four segments beginning at top

$$\oint H \cdot dl = 0.2\left(z_1 + \frac{1}{2}d\right)^2 d + 0 - 0.2\left(z_1 - \frac{1}{2}d^2\right)d + 0$$

$$= 0.4\, z_1 d^2$$

Area approaches 0:

$$(\nabla \times H)_y = \lim_{d \to 0} \frac{\oint H \cdot dl}{d^2} = \lim_{d \to 0} \frac{0.4 z_1 d^2}{d^2} = 0.4 z_1$$

$$\nabla \times H = 0.4 z_1\, a_y$$

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ d/dx & d/dy & d/dz \\ 0.2z^2 & 0 & 0 \end{vmatrix} = \frac{d}{dz}(0.2z^2)\, a_y = 0.4 z\, a_y$$

$$z = z_1$$

→ Segment 1:

$x = -d/2, d/2$

$z = z_1 + d/2$

$y = 0$ $\qquad dL = dx\, a_x$

$H = 0.2 z^2 a_z$ $\qquad$ where $z = z_1 + d/2$

$$\int_{-d/2}^{d/2} (0.2(z_1+d/2)^2)\, dx = 0.2(z_1+d/2)^2 \cdot d$$

→ Segment 2

$z = [z_1 + d/2, z_1 - d/2]$

$x = d/2 \quad y = 0$

$dL = dz\, a_z$

H is along a_x $\quad$ so $\quad H \cdot dL = 0$

→ Segment 3

$x = [d/2, -d/2]$

$z = z_1 - d/2 \quad , y = 0$

$dL = dx\, a_x$

$H = 0.2 z^2 a_z$ $\qquad z = z_1 - d/2$

$$\int H \cdot dL = \int_{d/2}^{-d/2} (0.2(z_1 - d/2)^2\, dx$$

$$= -0.2(z_1 - d/2)^2 \cdot d$$

→ Segment 4

$z = z_1 - d/2, z_1 + d/2$ $\quad , x = -d/2 \quad , y = 0$

$dL = dz\, a_z$

$H \cdot dL = 0$

Sum all segmets

$$\oint H\cdot dl = 0.2\,(z_1+d/2)^2 d - 0.2\,(z_1-d/2)^2 d$$

$$= 0.2d\;2z_1 d = 0.4 z_1 d^2$$

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ d/dx & d/dy & d/dz \\ H_x & H_y & H_z \end{vmatrix}$$

$$H = 0.2z^2 a_z$$

$$\nabla \times H = \left[+\frac{d0}{dz} - \frac{d(0.2z^2)}{dx} \right] a_y$$

$$\nabla \times H = 0.4z\, a_y$$

Problem 2

→ Segment 1:

$P_1(2,3,4)$ to $P_2(4,3,4)$

$x \in [2,4]$ $y=3$, $z=4$

$$dL = dx a_x$$

$$H = 3z a_x - 2x^3 a_z$$

$$H\cdot dl = 3z\,dx$$

$z=4$

$$H\cdot dl = 12dx$$

Integerate $[2,4]$

$$\int_{P_1-P_2} H\cdot dl = \int_2^4 12dx = 12(4-2) = 24A$$

→ Segment 2 $P_2 (4,3,4)$

$z [4,1]$ $P_3 (4,3,1)$

$x=4$, $y=3$

$dl = dz\, a_z$

$H = 3z a_x - 2x^3 a_z$

$H \cdot dl = -2x^3 dz$

$x=4$

$H \cdot dl = 128 dz$

$z [4,1]$

$$\int_{P_2-P_3} H \cdot dl = \int_4^1 -128\, dz = 384A$$

→ Segment 3

$P_3 (4,3,1)$ to $P_4 (2,3,1)$

$x [4,2]$ $y=3$ $z=1$

$dl = dx\, a_x$

$H = 3z a_x - 2x^3 a_z$

$H \cdot dl = 3z\, dx$

$z=1$

$H \cdot dl = 3 dx$

$$\int_{P_3-P_4} H \cdot dl = \int_4^2 3 dx = 3(2-4) = -6A$$

→ Segment 4

$P_4 (2,3,1)$ to $P (2,3,4)$

$z [1,4]$ $x=2$, $y=3$

$dl = dz\, a_z$

$$H = 3z\,a_x - 2x^3 a_z$$

$$H \cdot dl = -2x^3\,dz$$

$$x = 2$$

$$H \cdot dl = -2(2)^3\,dz$$
$$= -16dz$$

$$\int_{P_4-P_1} H \cdot dl = \int_1^4 16\,dz = -48A$$

→ Sum of all segmets

$$\oint H \cdot dl = 24 + 384 - 6 - 48 = 354A$$

→ Approximation to $(\nabla \times H)_y$

$$Area = (x_2 - x_1)(z_1 - z_3)$$
$$= (4-2)(4-1) = 6m^2$$

$$(\nabla \times H)_y = \frac{\oint H \cdot dl}{Area}$$
$$= \frac{354}{6} = 59A/m^2$$

→ $\nabla \times H_y$

Curl formula:

$$(\nabla \times H)_y = \frac{dH_x}{dz} - \frac{dH_z}{dx}$$

1. $H_x = 3z$

$$\frac{dH_x}{dz} = 3$$

2. $H_z - 2x^3$

$$\frac{dH_z}{dx} = -6x^2$$

Center of rectangle

$\cancel{(\nabla \times H)_y = 3}$

$x = (2+4)/2 = 3$

$z = (1+4)/2 = 2.5$

$(\nabla \times H)_y = 3 - (-6(3)^2)$

$= 3 + 54$

$(\nabla \times H)_y = 57 \ A/m^2$