National University of Computer and Emerging Sciences
Lahore Campus

Electromagnetic Theory (EE3005)

Final Exam

Date: January 01, 2025

Course Instructor(s)

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Total Time (Hrs):	**3**
Total Marks:	**100**
Total Questions:	**5**

Roll No ____________ Section ____________ Student Signature ____________

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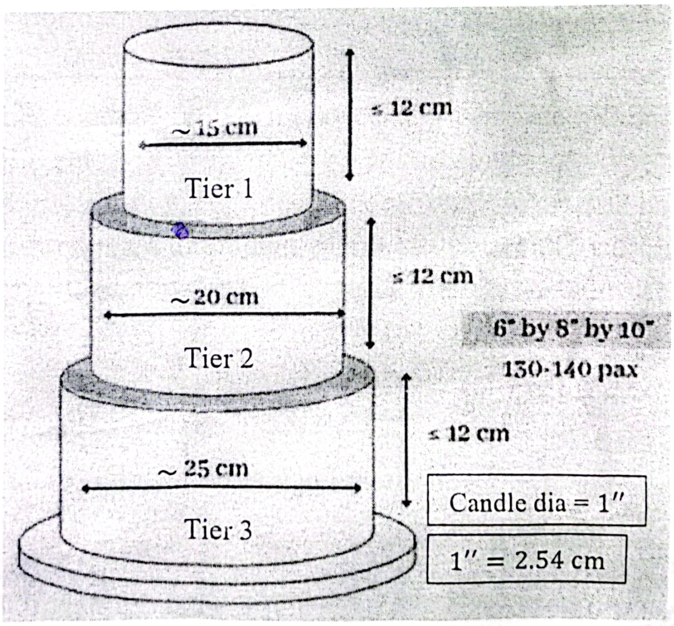
1. **Attempt all questions and remember to solve parts of the same question together.**
2. **Final answers should be correct up to two decimal places with proper SI units.**
3. **Show all the steps with the help of diagrams and equations.**

CLO # 01: Demonstrate the use of 3D orthogonal coordinate system and vector analysis tools in problem solving

Q1. Sister Ayesha is good in baking cakes but she is often confused about the dimensions of wrapping and the count of candles to lit on the cake. Now, she is up to making a big 3-tier cake for an upcoming event. The diameter of each tier in inches are mentioned as $6'' \times 8'' \times 10''$. The height of each tier is 12 cm. Help her in decorating by answering the following. **[20 marks]**



(a) **Identify** the coordinate system and **determine** the total volume (in cm^3) of the 3-tier cake using *differential method* only. **[8]**

(b) **Determine** the surface area (in cm^2) and the length (in cm) of the ribbon required to wrap each tier up to height of 10 cm using *differential method* only. **[6]**

(c) **Determine** the count of candles required to cover the sides (*shaded as gray*) of Tier 2 and Tier 3 top surfaces without gap. **[6]**

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CLO # 02: Formulate electrostatic fields and/or its properties governed by Coulomb's / Gauss's law for a given charge distribution in free space and / or dielectrics.

Q2: A precision sensor system for a high-tech device, operating in a region with *spatially varying dielectric properties* is to be designed. The electric potential in the field is given by: **[20 marks]**

$$V = 20x^2yz - 10z^2 \quad \text{(in free space)}$$

However, the region is partially filled with a dielectric material where the permittivity ϵ_r is function of the space and varies as:

$$\epsilon_r(y, z) = 1 + 0.5y^2z$$

(a) **Derive** the equations for the conducting surfaces when $V = 0$ and $V = 60$ V. **[3]**

(b) At point $P(2, y, 1)$, **determine** 'y' when the surface potential is $V = 60$ V. **[2]**

(c) **Formulate** for electric field intensity **E** [V/m] and the electric flux density **D** [nC/m^2] at $P(2, y, 1)$ considering the *dielectric's spatial variation*. Use value of 'y' as already found in part **(b)** in calculating $\mathbf{E}_P$ and $\mathbf{D}_P$. **[8]**

(d) **Determine** a unit vector '$\boldsymbol{n}$' normal to the $V = 60\,V$ equipotential surface at $P(2, y, 1)$ and directed towards the $V = 0$. **[3]**

(e) Knowing the property of a perfect conductor, we can safely assume that the $V = 60\,V$ surface is a perfect conductor. **Determine** the surface charge density, ρ_S [nC/m^2] at $P(2, y, 1)$, using two different methods. **[4]**

CLO # 03: Formulate the capacitance with one dimensional potential variation using direct integration.

Q3: A parallel-plate capacitor is made using two circular plates each of radius a, with the bottom plate on the xy plane, centred at the origin. The top plate is located at $z = 4\pi$, with its center on the z axis. Given the potential; $V_o = 100\,e^{\pi}$ is on the top plate while the bottom plate is grounded. Dielectric having *radially dependent* permittivity fills the region between plates is given by: **[20 marks]**

$$\epsilon(\rho) = \epsilon_0\left(1 + \frac{\rho^2}{a^2}\right)$$

Formulate for the capacitance of the system by utilizing one-dimensional change in potential and direct integration method by answering the following:

(i) Draw the diagram and solve for the potential; $V(z)$ using initial boundary conditions. **[2+6]**

(ii) Solve for the electric field intensity; **E**. And the electric flux density; **D**. **[4]**

(iii) Solve for the surface charge density, ρ_S and the total charge; Q **[6]**

(iv) Compute the capacitance, C. **[2]**

*Note: Solve the problem using **Laplace's Equation in Cylindrical Coordinates** exclusively. No other method or approach will be considered for marking.*

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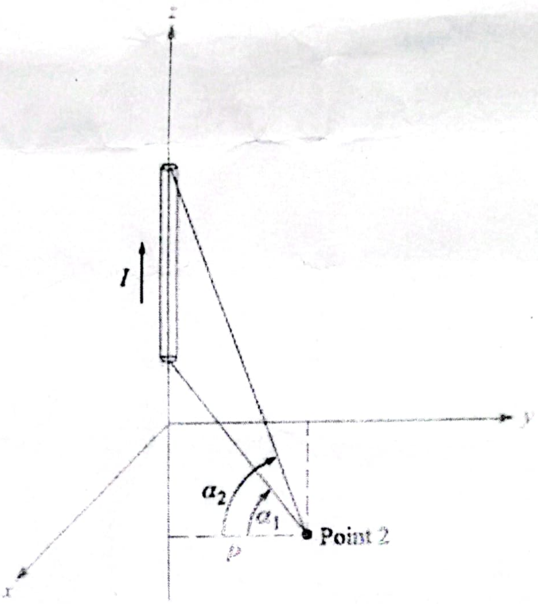
CLO # 04: Formulate magnetic fields and its effects for a given distribution of moving charges using laws of magnetostatics.

Q4: **[20 marks]**

(a) **State** Biot-Savart's law and mathematically express it using diagram. Also write different forms of the law for line, surface, and volume current. **[4]**

(b) **Formulate** magnetic field intensity at Point 2 shown below. **[6]**



(c) Suppose we have three current sheets, $\mathbf{K}_1 = 2.7\ \boldsymbol{a}_x$ [A/m] at $y = 0.1$, $\mathbf{K}_2 = -1.4\ \boldsymbol{a}_x$ [A/m] at $y = 0.15$, and $\mathbf{K}_3 = -1.3\ \boldsymbol{a}_x$ [A/m] at $y = 0.25$. **Determine** the value of **H** in RCS everywhere by showing complete working with the help of a suitable diagram. **[10]**

Regions →	$-\infty < y \leq 0.1$	$0.1 \leq y \leq 0.15$	$0.15 \leq y \leq 0.25$	$y > 0.25$

CLO # 04: Formulate magnetic fields and its effects for a given distribution of moving charges using laws of magnetostatics.

Q5: **[20 marks]**

(a) A cylindrical conductor of radius $a = 0.05$ m carries a uniformly distributed current $I = 10$ A in the +ive $\boldsymbol{a}_z$ direction. The magnetic field intensity **H** inside the conductor $(0 \leq \rho \leq a)$ is given as $H_\phi = \frac{I\rho}{2\pi a^2}$. **[12]**

- (i) **Verify** Ampere's circuital law by calculating $\oint \mathbf{H} \cdot d\mathbf{L}$ for a circular path of radius $\rho = 0.03$ m.
- (ii) **Compute** the curl, $\nabla \times \mathbf{H}$ at $\rho = 0.03$ m and **verify** that it matches the current density **J** there.

(b) A circular loop located on $x^2 + y^2 = 16,\ z = 0$ carries a direct current of 5 A along $\boldsymbol{a}_\varphi$. **Formulate H** at $(0, 0, 5)$ and $(0, 0, -5)$. **[8]**

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Formula Sheet

$$\mathbf{F}_1 = -\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{21} = -\frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12} \qquad \mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2}\mathbf{a}_R \qquad \mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n}\frac{Q_m}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_m|^2}\mathbf{a}_m \qquad \mathbf{E} = \frac{\rho_S}{2\epsilon_0}\mathbf{a}_N$$

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\mathbf{a}_\rho \qquad \mathbf{D} = \int_{vol}\frac{\rho_v dv}{4\pi R^2}\mathbf{a}_R \qquad \Psi = \oint_S \mathbf{D}_S \cdot d\mathbf{S} = \text{charge enclosed} = Q \qquad \mathbf{J} = \rho_v \mathbf{v} \qquad \text{div}\,\mathbf{D} = \rho_v$$

$$\text{div}\,\mathbf{D} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \qquad \text{div}\,\mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z} \quad \text{(cylindrical)} \qquad \boxed{\mathbf{E} = -\nabla V}$$

$$\text{div}\,\mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi} \quad \text{(spherical)} \qquad W = -Q\int_{init}^{final}\mathbf{E}\cdot d\mathbf{L} \qquad \nabla^2 V = 0$$

$d\mathbf{L} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z$ (rectangular)

$d\mathbf{L} = d\rho\,\mathbf{a}_\rho + \rho\,d\phi\,\mathbf{a}_\phi + dz\,\mathbf{a}_z$ (cylindrical)

$d\mathbf{L} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi$ (spherical)

$D = \epsilon E$

$$V(\mathbf{r}) = \int_{vol}\frac{\rho_v(\mathbf{r}')\,dv'}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|}$$

$$\boxed{\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z} \qquad \nabla V = \frac{\partial V}{\partial\rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial\phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z \qquad \nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial\theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\mathbf{a}_\phi$$

$$I = \int_S \mathbf{J}\cdot d\mathbf{S} \qquad (\nabla\cdot\mathbf{J}) = -\frac{\partial\rho_v}{\partial t} \qquad W_E = \frac{1}{2}\int_{vol}\mathbf{D}\cdot\mathbf{E}\,dv = \frac{1}{2}\int_{vol}\epsilon_0 E^2\,dv \qquad W_E = \frac{1}{2}\int_{vol}\rho_v V\,dv \qquad \mathbf{J} = \sigma\mathbf{E}$$

$$\mathbf{E}\times\mathbf{n}\big|_s = 0$$

$$\sigma = -\rho_e\mu_e \qquad \mathbf{D}\cdot\mathbf{n}\big|_s = \rho_s \qquad \frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2} \qquad \mathbf{P} = \lim_{\Delta v\to 0}\frac{1}{\Delta v}\sum_{i=1}^{n\Delta v}\mathbf{p}_i \qquad \mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} \qquad \mathbf{P} = \chi_e\epsilon_0\mathbf{E}$$

$$\mathbf{H} = \oint\frac{I\,d\mathbf{L}\times\mathbf{a}_R}{4\pi R^2} \qquad W_E = \frac{1}{2}CV_0^2 = \frac{1}{2}QV_0 = \frac{1}{2}\frac{Q^2}{C}$$

$$(\mathbf{D}_1 - \mathbf{D}_2)\cdot\mathbf{n} = \rho_s \qquad (\mathbf{E}_1 - \mathbf{E}_2)\times\mathbf{n} = 0$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\epsilon} \qquad \nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial\phi^2}\right) + \frac{\partial^2 V}{\partial z^2} \qquad \oint\mathbf{H}\cdot d\mathbf{L} = I \qquad \Phi = \int_S\mathbf{B}\cdot d\mathbf{S}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} \qquad I\,d\mathbf{L} = \mathbf{K}\,dS = \mathbf{J}\,dv$$

$$\nabla\times\mathbf{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial\phi} - \frac{\partial H_\phi}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial\rho}\right)\mathbf{a}_\phi + \left(\frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial\phi}\right)\mathbf{a}_z \quad \text{(cylindrical)}$$

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z\cdot$	$\cos\theta$	$-\sin\theta$	0

$$\nabla\times\mathbf{H} = \frac{1}{r\sin\theta}\left(\frac{\partial(H_\phi\sin\theta)}{\partial\theta} - \frac{\partial H_\theta}{\partial\phi}\right)\mathbf{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial(rH_\phi)}{\partial r}\right)\mathbf{a}_\theta + \frac{1}{r}\left(\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial\theta}\right)\mathbf{a}_\phi \quad \text{(spherical)}$$

$$V = \frac{Q}{4\pi\epsilon_0 r} \qquad \mathbf{B} = \mu_0\mathbf{H}$$

$$\mathbf{H} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\mathbf{a}_\phi$$

$$\mathbf{F} = \oint I\,d\mathbf{L}\times\mathbf{B} \qquad \mathbf{F} = Q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad \oint\mathbf{H}\cdot d\mathbf{L} = \int_S(\nabla\times\mathbf{H})\cdot d\mathbf{S} \qquad \frac{dW_E}{dv} = \frac{1}{2}\mathbf{D}\cdot\mathbf{E}$$

$$\int\frac{1}{(a^2\pm x^2)^{3/2}}dx = \frac{\pm x}{a^2\sqrt{a^2\pm x^2}} \qquad \int\frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$dS_\rho = \rho d\varphi dz\,\boldsymbol{a}_\rho,\; dS_\varphi = d\rho dz\,\boldsymbol{a}_\varphi,\; dS_z = \rho d\varphi d\rho\,\boldsymbol{a}_z,\; dv = \rho d\rho d\varphi dz$$

$$dS_r = r^2\sin\theta\,d\theta d\varphi\,\boldsymbol{a}_r,\; dS_\theta = r\sin\theta\,dr d\varphi\,\boldsymbol{a}_\theta,\; dS_\varphi = r dr d\theta\,\boldsymbol{a}_\varphi$$

$$dv = r^2\sin\theta\,dr d\theta d\varphi$$