National University of Computer & Emerging Sciences, Lahore
Department of Electrical Engineering
(FALL 2024)

EE3005 – Electromagnetic Theory (EMT)
Assignment # 1 on CLO # 01 (Total Marks = 10)

Notes:

1. **Submit the hardcopy in the very start of the lecture. Use A4 sheets for the answers.**
2. **No late submissions.**
3. **Any help from internet, solution manuals, or any other source is strictly prohibited. Violation may lead to serious consequences.**
4. **Draw the diagrams where necessary.**

Date of Submission: Thursday 19th September 2024 in your Class

For practice, solve all the drills and end problems of Chapter 1 of Hayt and Buck. Submit the solution of the following problem set only.

Problem # 01: Given two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, the dot product (also called inner product) is defined by

$$\boldsymbol{u} \cdot \boldsymbol{v} = u\, v \cos\theta$$

and the cross product is defined by

$$\boldsymbol{u} \times \boldsymbol{v} = u\, v \sin\theta\, \boldsymbol{a}_n$$

Which of the above expressions may be used to determine the angle between two vectors and why? **Demonstrate**.

Problem # 02: Using *differential method*, **determine** the **i)** volume and the **ii)** surface area oriented along $\boldsymbol{a}_z$ and $\boldsymbol{a}_{-\phi}$ of the 3D shape shown in the Fig. 1. The range of variables are;

$$0 \le \rho \le 1, \qquad 0 \le \phi \le \frac{7\pi}{4}, \qquad 0 \le z \le 2$$

Hint: *For the surface area part, out of the total six faces; you are required to find only 2 of them.*

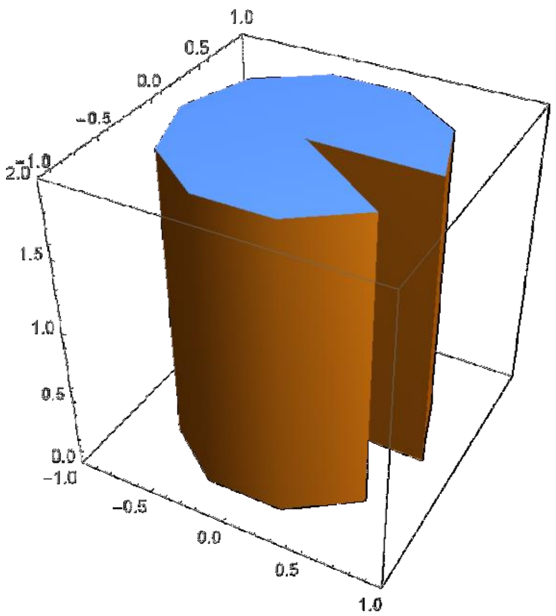

Figure 1. 3D cylindrical object

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Problem # 03: Using *differential method*, **determine** the **i)** volume and the **ii)** surface area oriented along $\boldsymbol{a}_\rho$ and $\boldsymbol{a}_\theta$ of the 3D shape shown in the Fig. 2. Also, **iii) compute** the length of the circular arc formed at the intersection of sphere and cone as shown in the top view of the same shape. The range of variables are;

$$0 \leq r \leq 1, \qquad 0 \leq \theta \leq \frac{\pi}{4}, \qquad 0 \leq \phi \leq \frac{7\pi}{4}$$

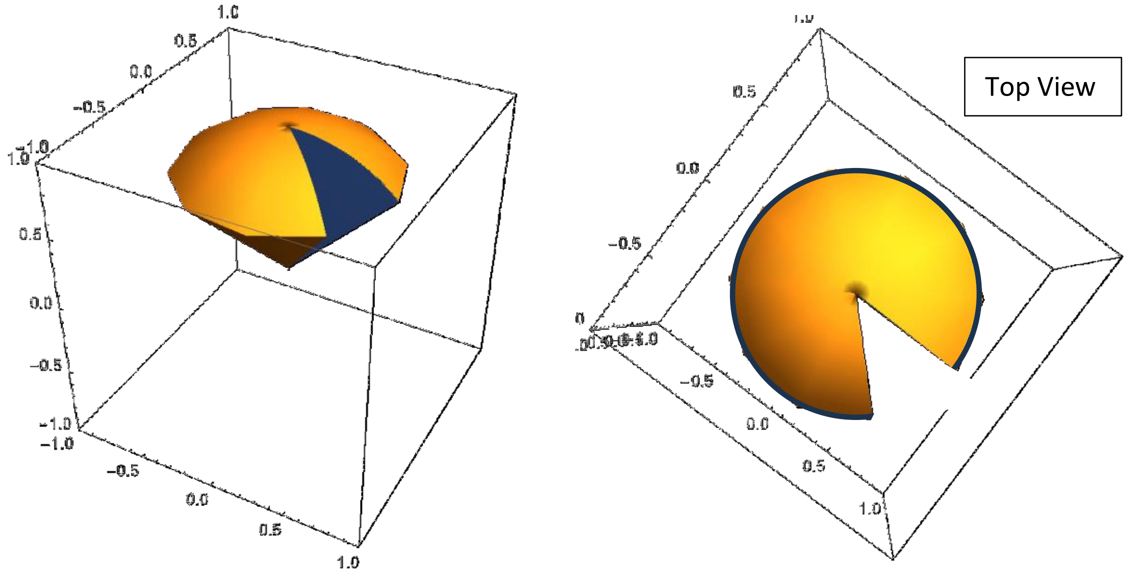

Figure 2. 3D Spherical Object

Problem # 04: Determine the unit vector $\boldsymbol{a}_x$ in spherical components at the point defined by:

(a) $P(r = 2, \theta = 1\,rad, \varphi = 0.8\,rad)$

(b) $R(\rho = 2.5, \varphi = 0.7\,rad, z = 1.5)$

Problem # 05: Draw the closed surface formed by the range of variables given below and then **find** the **(i)** length of all the edges, **(ii)** the areas of all surfaces, **(iii)** the volume of the closed surface and **(iv)** the length of the longest straight line that lies entirely within the surface defined by:

(a) $1 \leq x \leq 5, \quad 2 \leq y \leq 8, \quad 0 \leq z \leq 3$

(b) $2 \leq r \leq 4, \quad 30° \leq \theta \leq 50°, \quad 0 \leq \varphi \leq 60°$

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EE3005 – ELECTROMAGNETIC THEORY (EMT)

Assignment # 1 on CLO # 01

Name:	**Roll Number:**
Date of Submission: 19th September 2024	**Section: BEE-5A**

Instructor: Mohsin Yousuf

Instructions:

1. Assignment must be handed over to the instructor at the very start of the class.
2. Late submission will not be accepted.
3. Only submit your own work. Do not copy from others. Plagiarism will be dealt seriously and it will be treated according to the University disciplinary rules.
4. Submit assignment on **A4 size paper**.
5. Use blue or black ink only. Figures may be in color.
6. Handwriting must be legible. Otherwise, your assignment may be returned un-graded.

Problems ↓ [Marks]	Score	CLO # 01				
		Exemplary (5)	Proficient (4)	Developing (3)	Beginning (2)	Novice (1)
P1 [3]						
P2 [5]						
P3 [7]						
P4 [4]						
P5 [11]						
Total [30]						

Oath: *I have solved all the questions by myself and taken no help from internet, solution manuals, colleagues, or any other unfair means. It is my work only.*

Signature:________________________

Attach this sheet to the front of submitted work.

Q. No. 1

⇒ The dot product of two vectors $\vec{u}$ and $\vec{v}$ is given by

$$\vec{u} \cdot \vec{v} = |u|\ |v| \cos\theta$$

where $|u|$ and $|v|$ are magnitudes of the vectors and θ is the angle between them.

To find the angle θ, we rearrange the formula

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|u|\ |v|}$$

$$\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{|u|\ |v|}\right)$$

⇒ Similarly, the cross product of two vectors $\vec{u}$ and $\vec{v}$ is given by

$$\vec{u} \times \vec{v} = |u|\ |v| \sin\theta\, \hat{a}_n$$

where $\hat{a}_n$ is unit vector perpendicular to both $\vec{u}$ and $\vec{v}$ and θ is angle between them.

Although, the magnitude of cross product

$$|\vec{u} \times \vec{v}| = |u|\ |v| \sin\theta \quad \Rightarrow \theta = \sin^{-1}\left(\frac{|\vec{u} \times \vec{v}|}{|u|\ |v|}\right)$$

gives us θ but it is not correct in all cases.

Sine function gives the same value for θ and $180° - \theta$ due to periodicity.

Moreover, cross product is zero when vectors are parallel i.e $\theta = 0°$ or $\theta = 180°$, giving no value of angle in such case.

⇒ Therefore, dot product is more reliable to find angle between two vectors.

Let $\vec{u} = \vec{u}_1 \hat{i} + \vec{u}_2 \hat{j} + \vec{u}_3 \hat{k}$

$\vec{v} = \vec{v}_1 \hat{i} + \vec{v}_2 \hat{j} + \vec{v}_3 \hat{k}$

⇒ $|u| = \sqrt{u_1^2 + u_2^2 + u_3^2}$

⇒ $|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

$$\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{|u|\ |v|}\right)$$

$$\theta = \cos^{-1}\left(\frac{u_1 v_1 + u_2 v_2 + u_3 v_3}{\sqrt{u_1^2 + u_2^2 + u_3^2}\ \sqrt{v_1^2 + v_2^2 + v_3^2}}\right)$$

Q. No. 2

$0 \le \rho \le 1 \quad , \quad 0 \le \phi \le \frac{7\pi}{4} \quad , \quad 0 \le z \le 2$

(i)

Volume

As volume in cylindrical coordinates is given by

$$dV = \rho \, d\rho \, d\phi \, dz$$

$\Rightarrow$ Total volume : $V = \int_0^2 \int_0^{\frac{7\pi}{4}} \int_0^1 \rho \, d\rho \, d\phi \, dz$

$$= \int_0^2 \int_0^{\frac{7\pi}{4}} \left[\frac{\rho^2}{2}\right]_0^1 d\phi \, dz$$

$$= \int_0^2 \int_0^{\frac{7\pi}{4}} \frac{(1)^2 - (0)^2}{2} \, d\phi \, dz$$

$$= \int_0^2 \int_0^{\frac{7\pi}{4}} \frac{1}{2} \, d\phi \, dz$$

$$= \int_0^2 \frac{1}{2} \left[\phi\right]_0^{\frac{7\pi}{4}} dz$$

$$= \int_0^2 \left(\frac{1}{2}\right)\left(\frac{7\pi}{4}\right) dz$$

$$= \frac{7\pi}{8} \left[z\right]_0^2$$

$$= \frac{7\pi}{\cancel{8}4} (\cancel{2})$$

$$= \frac{7\pi}{4} \text{ cubic units}$$

(ii)

Surface area along a_z

As $dS_z = \rho \, d\rho \, d\phi$

Therefore, $S_z = 2\int_0^{\frac{7\pi}{4}}\int_0^1 \rho \, d\rho \, d\phi$ (for top and bottom surface area)

$= 2\int_0^{\frac{7\pi}{4}} \frac{(1)^2-(0)^2}{2} \, d\phi$

$= 2\int_0^{\frac{7\pi}{4}} \frac{1}{2} \, d\phi$

$= \cancel{2}\left(\frac{1}{\cancel{2}}\right)\left[\phi\right]_0^{\frac{7\pi}{4}}$

$= \frac{7\pi}{4}$ square units

Surface area along $a_{-\phi}$

As $dS_{\phi} = d\rho \, dz$

Therefore, $S_{-\phi} = \int_0^2\int_0^1 d\rho \, dz$

$= \int_0^2 (1) \, dz$

$= \left[z\right]_0^2$

$= 2$ square units

Q. No. 3

$0 \le r \le 1$, $0 \le \theta \le \frac{\pi}{4}$, $0 \le \phi \le \frac{7\pi}{4}$

(i)

Volume

As ~~d~~ volume in spherical coordinates in given by

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

Therefore, $$V = \int_0^{\frac{7\pi}{4}} \int_0^{\frac{\pi}{4}} \int_0^{1} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_0^{\frac{7\pi}{4}} \int_0^{\frac{\pi}{4}} \left[\frac{r^3}{3}\right]_0^1 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{\frac{7\pi}{4}} \int_0^{\frac{\pi}{4}} \frac{1}{3} \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{\frac{7\pi}{4}} \left(\frac{1}{3}\right) \left[-\cos\theta\right]_0^{\frac{\pi}{4}} d\phi$$

$$= \int_0^{\frac{7\pi}{4}} \left(\frac{1}{3}\right) \left(-\cos\frac{\pi}{4} + \cos 0\right) d\phi$$

$$= \int_0^{\frac{7\pi}{4}} \left(\frac{1}{3}\right) \left(1 - \frac{1}{\sqrt{2}}\right) d\phi$$

$$= \left(\frac{1}{3}\right) \left(\frac{2-\sqrt{2}}{2}\right) \left(\frac{7\pi}{4}\right)$$

$$= \frac{14 - 7\sqrt{2}}{24}\pi \text{ cubic units}$$

$$= 0.1709\pi \text{ cubic units}$$

$$= 0.5367 \text{ cubic units}$$

(ii)

Surface Area along a_r

As differential surface area along a_r in SCS is given by

$$dS_r = r^2 \sin\theta \, d\theta \, d\phi$$

Therefore,

$$S_r = \int_0^{\frac{7\pi}{4}} \int_0^{\frac{\pi}{4}} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{\frac{7\pi}{4}} r^2 \left[-\cos\theta\right]_0^{\frac{\pi}{4}} d\phi$$

$$= r^2 \int_0^{\frac{7\pi}{4}} \left(1 - \frac{1}{\sqrt{2}}\right) d\phi$$

$$= r^2 \left(\frac{2-\sqrt{2}}{2}\right)\left(\frac{7\pi}{4}\right)$$

$$= 1.6102\, r^2 \text{ square units}$$

Surface Area along a_θ

Similarly, diff. surface area along a_θ in SCS is given by

$$dS_\theta = r \sin\theta \, dr \, d\phi$$

Therefore,

$$S_\theta = \int_0^{\frac{7\pi}{4}} \int_0^{1} r \sin\theta \, dr \, d\phi$$

$$= \int_0^{\frac{7\pi}{4}} \left[\frac{r^2}{2}\right]_0^1 \sin\theta \, d\phi$$

$$= \int_0^{\frac{7\pi}{4}} \left(\frac{1}{2}\right) \sin\theta \, d\phi$$

$$= \left(\frac{1}{2}\right)(\sin\theta)\left(\frac{7\pi}{4}\right)$$

$$= \frac{7\pi}{8} \sin\theta \text{ sq. units}$$

$$= 2.7489 \sin\theta \text{ sq. units}$$

Q. No. 4

(a)

$P(r=2, \theta=1 \text{ rad}, \phi=0.8 \text{ rad})$

$P_{scs} = 2a_r + 1a_\theta + 0.8a_\phi$

As $a_x = (a_r \cdot a_x)a_r + (a_\theta \cdot a_x)a_\theta + (a_\phi \cdot a_x)a_\phi$

$= \sin\theta\cos\phi \, a_r + \cos\theta\cos\phi \, a_\theta + (-\sin\phi)\, a_\phi$

$= \sin(1)\cos(0.8)a_r + \cos(1)\cos(0.8)a_\theta - \sin(0.8)a_\phi$

$= \sin(57.2958°)\cos(45.8366°)a_r + \cos(57.2958°)\cos(45.8366°)a_\theta$
$- \sin(45.8366°)a_\phi$

$= 0.5863a_r + 0.3764a_\theta - 0.7174a_\phi$

(b)

$R(\rho=2.5, \phi=0.7 \text{ rad}, z=1.5)$

In spherical coordinates,

$r = \sqrt{\rho^2 + z^2}$
$= \sqrt{2.5^2 + 1.5^2}$
$= \sqrt{8.5}$
$= 2.9155$

$\theta = \cos^{-1}\left(\frac{z}{r}\right)$
$= \cos^{-1}\left(\frac{1.5}{2.9155}\right)$
$= 59.0365°$

$\phi = 0.7 \text{ rad}$
$= 40.1070°$

Similarly $a_x = (\sin\theta\cos\phi)a_r + (\cos\theta\cos\phi)a_\theta + (-\sin\phi)a_\phi$

$= \sin(59.0365°)\cos(40.1070°)a_r + \cos(59.0365°)\cos(40.1070°)a_\theta$
$- \sin(40.1070°)a_\phi$

$= 0.6558a_r + 0.3935a_\theta - 0.6442a_\phi$

Q. No. 5

(a)

$1 \le x \le 5 \ , \ 2 \le y \le 8 \ , \ 0 \le z \le 3$

(i)

Length of all edges

$$\text{Length (total)} = 4\left[\int_1^5 dx + \int_2^8 dy + \int_0^3 dz\right]$$

$\because$ Each edge is 4 time on each axis

$$= 4\left[x\Big|_1^5 + y\Big|_2^8 + z\Big|_0^3\right]$$

$$= 4\big((5-1) + (8-2) + (3-0)\big)$$

$$= 16 + 24 + 12$$

$$= 52 \text{ units}$$

(ii)

Area of all Surfaces

As $dA_{(xy)} = dx\,dy$ and $dA_{(xz)} = dx\,dz$ and $dA_{(yz)} = dy\,dz$

Moreover, each face is twice.

Therefore, $A_{total} = 2A_{xy} + 2A_{xz} + 2A_{yz}$

$$= 2\left[\int_2^8\int_1^5 dx\,dy + \int_0^3\int_1^5 dx\,dz + \int_0^3\int_2^8 dy\,dz\right]$$

$$= 2\left[(5-1)(8-2) + (5-1)(3-0) + (8-2)(3-0)\right]$$

$$= 2[24 + 12 + 18]$$

$$= 2(54)$$

$$= 108 \text{ sq. units.}$$

(iii)
Volume

As $dV = dx\,dy\,dz$

Therefore, $V = \int_0^3 \int_2^8 \int_1^5 dx\,dy\,dz$

$= \int_0^3 \int_2^8 (5-1)\,dy\,dz$

$= \int_0^3 (4)(8-2)\,dz$

$= (4)(6)(3)$

$= 72$ cubic units

(iv)
Length of Longest Straight line

The longest straight line inside the surface would be diagonally.
Using distance formula,

$L = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$= \sqrt{(5-1)^2 + (8-2)^2 + (3-0)^2}$

$= \sqrt{4^2 + 6^2 + 3^2}$

$= \sqrt{16 + 36 + 9}$

$= \sqrt{61}$

$= 7.8102$ units

(b)

$2 \le r \le 4, \quad 30^\circ \le \theta \le 50^\circ, \quad 0 \le \phi \le 60^\circ$

(iii)

Volume

As $dV = \cancel{\int_0^{60^\circ}\int_{30^\circ}^{50^\circ}\int_2^4 dr\, d\theta\, d\phi} \quad \int_0^{60^\circ}\int_{30^\circ}^{50^\circ}\int_2^4 r^2 \sin\theta \, dr\, d\theta\, d\phi$

$= \cancel{\int_0^{60^\circ}\int_{30^\circ}^{50^\circ} (4-2)\, d\theta\, d\phi} \quad \int_0^{60^\circ}\int_{30^\circ}^{50^\circ} \left[\frac{r^3}{3}\right]_2^4 \sin\theta \, d\theta\, d\phi$

$= \cancel{\int_0^{60^\circ} (2)(50} \quad \int_0^{60^\circ} \left(\frac{64-8}{3}\right)\left[-\cos\theta\right]_{30^\circ}^{50^\circ} d\phi$

$= \left(\frac{56}{3}\right)(-0.6428 + 0.8660)(60^\circ)$

$= \left(\frac{56}{3}\right)(0.2232)\left(\frac{\pi}{3}\right)$

$= 4.3630$ cubic unit

(iv)

Length of Longest Straight line

Let, $A = (r = 2, \theta = 30^\circ, \phi = 0^\circ)$

and $B = (r = 4, \theta = 50^\circ, \phi = 60^\circ)$

$\Rightarrow x = r\sin\theta\cos\phi \quad \Rightarrow y = r\sin\theta\sin\phi \quad \Rightarrow z = r\cos\theta$

$\Rightarrow A = ((2)\sin(30^\circ)\cos(0^\circ), (2)\sin(30^\circ)\sin(0^\circ), (2)\cos(30^\circ))$

$= (1, 0, \sqrt{3})$

$\Rightarrow B = ((4)\sin(50^\circ)\cos(60^\circ), (4)\sin(50^\circ)\sin(60^\circ), (4)\cos(50^\circ))$

$= (1.5321, 2.6536, 2.5711)$

$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(1.5321 - 1)^2 + (2.6536 - 0)^2 + (2.5711 - \sqrt{3})^2}$

$= \sqrt{8.0287}$

$= 2.8335$ units