

## EXPERIMENT 1

### Study of first order systems

**Objective:**

- To design first order RL and RC circuits and Analyze their Frequency responses.

**Equipment Required:**

- Oscilloscope with probes
- Function generator
- Digital multi-meter
- Power supply
- Bread Board
- Capacitor
- Inductor
- Resistor

**Theory:**

First order systems are, by definition, systems whose input-output relationship is a first order differential equation. A first order differential equation contains a first order derivative but no derivative higher than first order – the order of a differential equation is the order of the highest order derivative present in the equation.

The first order system has only one pole as shown

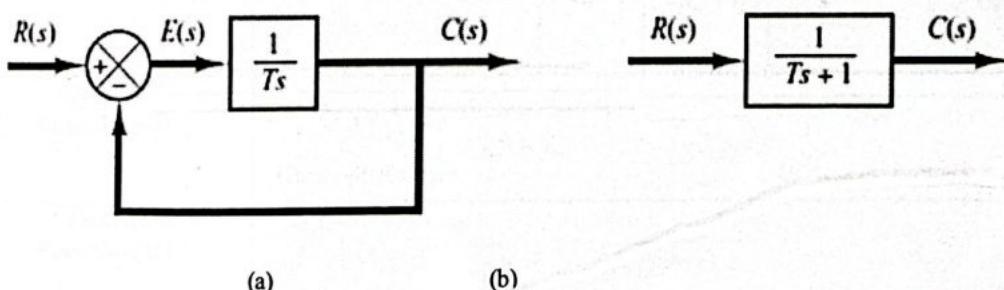


Figure 1:(a) Block Diagram of a first order system (b) Simplified block Diagram

$$\frac{C(S)}{R(S)} = K \frac{1}{TS + 1}$$

Where  $K$  is the DC gain and  $\tau$  is the time constant of the system.

# Lab Manual of Feedback Control Systems

7. Compare the calculated and measured time constants.
8. Find the transfer functions for both circuits using K and  $\tau$ .
9. Using the transfer function found in step(8), draw the asymptotic Bode plots on given semi log paper in Figure 6 and Figure7.

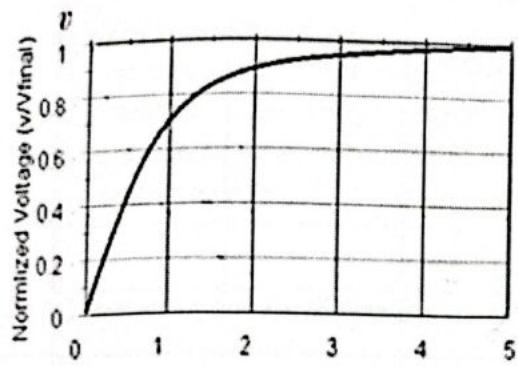


Figure4(Normalized time  $t/RC$ )

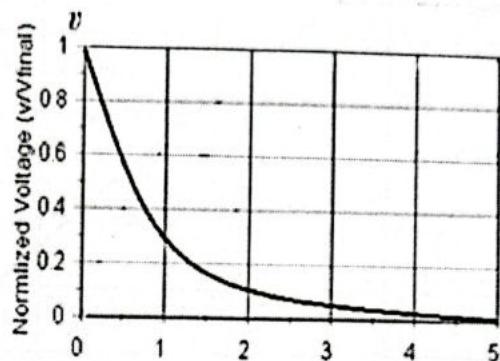


Figure5(Normalized time  $t/R/L$ )

Note that the time constant ( $t = \tau = RC$ ) occurs at  $0.632V_{in}$

TABLE-1: Calculated Circuit Parameters:

Time Constant	$R=2.2\text{ k}\Omega$	$C= 10\text{ nF}$	$\tau = 22\text{ }\mu\text{s}$
Time Constant	$R=4.7\text{ k}\Omega$	$L= 150\text{ mH}$	$\tau = 31.91\text{ }\mu\text{s}$
Transfer Function(RC)	$H(s) = \frac{R}{1+RCs} = \frac{1}{1+(22\mu)s}$		
Transfer Function(RL)	$H(s) = \frac{R}{1+\frac{L}{R}s} = \frac{1}{1+(31.91\mu)s}$		

Part(b): Frequency Response of First order circuit.

**Procedure:**

1. Construct RC and RL circuit on breadboard as shown in Figure2 and Figure3.
2. Connect the function generator at input. Adjust the function generator to produce 2Vpp Square wave.
3. Increase the frequency from 500Hz to 10 KHz.
4. Measure all the voltage across capacitor and Inductor.
5. Plot the frequency response of each system on the given semi log paper (Take logarithmic frequency on x-axis and voltage in dBs on y-axis).

TABLE-2: Observations and Calculations

Frequency (Hz)	$V_{in}$ (V <sub>pp</sub> )	System1(RC)		System2(RL)	
		$V_{out}$	$V_{out}(\text{dB})$	$V_{out}$	$V_{out}(\text{dB})$
500	2	0.99	-0.087	0.25	-12.09
1000	2	0.97	-0.264	0.35	-9.112
2000	2	0.92	-0.724	0.49	-6.196
3000	2	0.86	-1.31	0.60	-4.437
4000	2	0.81	-1.83	0.69	-3.223
5000	2	0.76	-2.383	0.75	-2.499
6000	2	0.71	-2.975	0.81	-1.83
7000	2	0.65	-3.742	0.84	-1.514
8000	2	0.61	-4.293	0.87	-1.209
9000	2	0.57	-4.883	0.89	-1.012
10,000	2	0.53	-5.514	0.91	-0.819

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Compare asymptotic and actual frequency response of both circuits?

Asymptotic graph is composed from of frequency response because it grows logarithmically.

Post Lab:

Q1. For the first order system given below find

$$G(s) = \frac{3}{s+5} \Rightarrow 3 \left( \frac{1}{s+5} \right) \Rightarrow 3 e^{-5t}$$

- DC gain K and time constant  $\tau$
- Draw the step response of above first order system.

Q2. Comment on the shape of frequency response of each system. Why is it different from the expected shape?

Q3. Construct the bode plot on a semilog paper whose transfer function is given by

$$\frac{10(s+1)}{s+10}$$

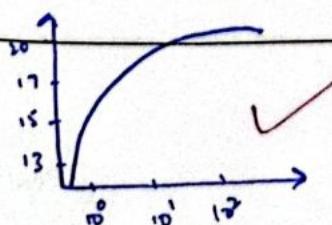
(Q1) General eq. of 1<sup>st</sup> order system  $\frac{C(s)}{R(s)} = \frac{K(s)}{s+1}$ , when compared to given system  $\Rightarrow K=3, T=\frac{1}{5}$

Step-response:  $\frac{0.3}{0.25} t$

$$\begin{aligned} s+5 \\ \frac{s}{5} + \frac{1}{5} \\ \frac{s}{5} + 1 \\ \downarrow T = \frac{1}{5} \end{aligned}$$

(Q2) We are plotting graphs in dB, which results in freq. response, that is not the expected graph. Instrumental errors can also cause the measured values to differ from expected value.

(Q3)  $\Rightarrow \frac{10(s+1)}{s+10}$



**EXPERIMENT 2****Frequency sweep of second order systems****Objective:**

Observe and plot frequency sweep of second order systems.

**Equipment Required:**

- Oscilloscope with probes
- Function generator with probes
- Digital multi-meter
- Dual Power supply
- Breadboard
- Capacitor 1nF
- Inductor 150mH
- Resistor 1KΩ

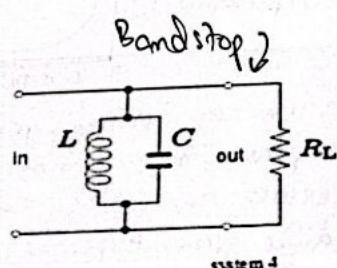
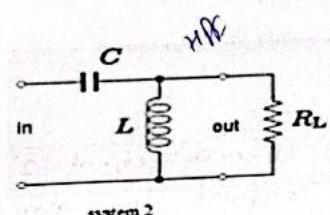
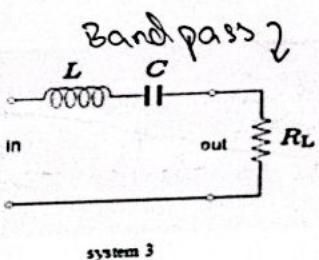
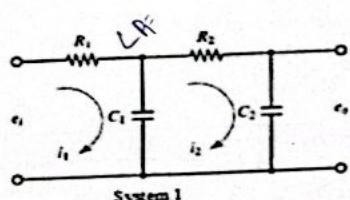


Figure 1(Second Order Systems)

**Procedure:**

1. Measure the actual values of capacitor, inductor and resistor using DMM and LCR meter and record them in table 1.
2. Connect the circuit as shown in Figure1.
3. Connect the function generator at input. Adjust the function generator to produce 4Vpp sine wave.
4. Increase the frequency from 100Hz to 40 KHz and complete Table-2.
5. Plot the frequency response of each system on the given semi log paper in Fig2 (Take logarithmic frequency on x-axis and voltage in dBs on y-axis).

**DATA FOR EXPERIMENT:**

TABLE-1 (MEASURED VALUES)

Resistor	1.01K
Capacitor	1.06nF
Inductor	162mH

TABLE-2(OBSERVATIONS AND CALCULATIONS)

Frequency (Hz)	V <sub>in</sub> V <sub>pp</sub>	System 1		System 2	
		V <sub>out</sub>	V <sub>out</sub> (dB)	V <sub>out</sub>	V <sub>out</sub> (dB)
100	4	2.973	9.463	13.08m	-37.667
500	4	2.972	9.4609	65.32m	-23.699
1000	4	2.9718	9.4604	160.95m	-15.866
5000	4	2.9559	9.4137	439.75m	-7.1358
10,000	4	2.9101	9.2782	0.755	-2.4410
15,000	4	2.8413	9.0703	0.9213	-0.7119
20,000	4	2.7540	8.7993	1.0307	0.2126
25,000	4	2.6527	8.4770	1.1825	1.456
30,000	4	2.5507	8.1332	1.2946	2.2427
35,000	4	2.4393	7.7453	1.3956	2.8952
40,000	4	2.3271	7.3363	1.4895	3.4608

TABLE-3 (DATA FOR EXPERIMENT)

Frequency (Hz)	$V_{in}$ $V_{pp}$	System 3		System 4	
		$V_{out}$	$V_{out}(\text{dB})$	$V_{out}$	$V_{out}(\text{dB})$
100	4	69.28m	-23.197	3.43	10.7058
500	4	154.55m	-16.2186	3.8872	11.79
1000	4	233.90m	-12.6193	3.977	11.9934
5000	4	367.93m	-8.684	3.4012	12.0672
10,000	4	0.616V	-4.208	3.998	12.0368
15,000	4	0.9677	-0.2952	3.9802	12.0111
20,000	4	348.76m	-9.149	3.975	11.9867
25,000	4	217.93m	-13.232	3.9637	11.9620
30,000	4	159.28m	-15.956	3.9605	11.955
35,000	4	124.49m	-19.097	3.9506	11.9332
40,000	4	101.10m	-19.904	3.9402	11.9103

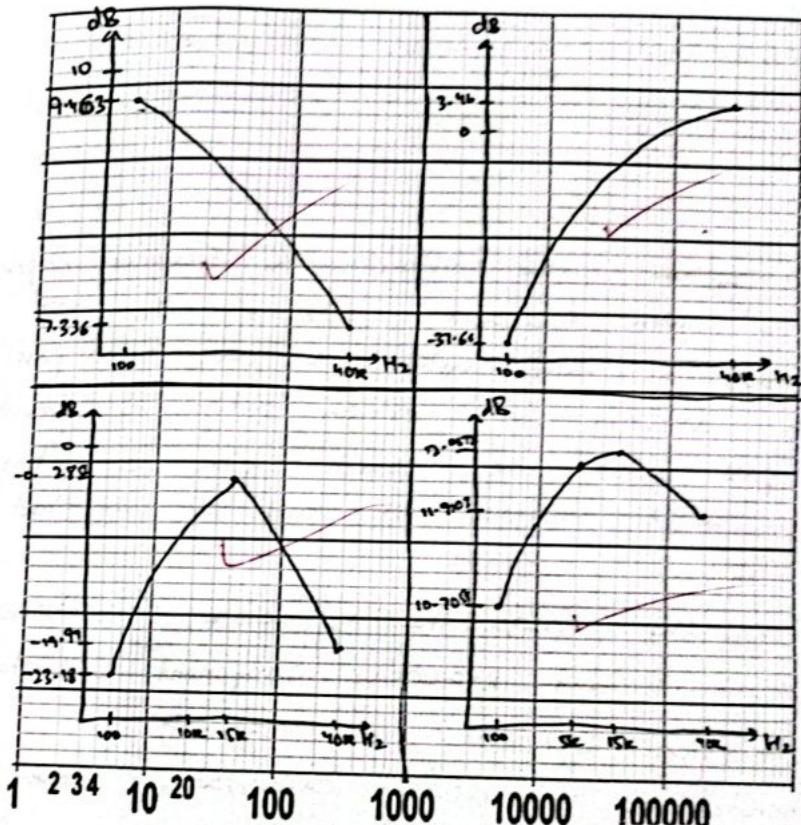


Figure 2 (Frequency response of 2<sup>nd</sup> order system)

Post Lab:

Q1. Explain the observations made during the performance of the lab experiment.

Q2. Would the frequency sweep of each second order system be altered if the value of inductor or capacitor was changed? If so, why?

(Q1) All systems worked differently. The frequency response of 1<sup>st</sup> system decreased while of the 2<sup>nd</sup> increased. The third system is a band-pass filter allowing a certain range of frequencies to pass while the fourth system is a band-stop filter. It decreases in such a way that it appears constant.

(Q2) Yes, the freq, the effective inductance of almost any practical component will change. The device's behavior will typically be dominated by inter-winding capacitance above a certain freq resulting in negative and decreasing reactance

12/02/2025  
Suresh

**EXPERIMENT 3****Design of System's Transfer Function & Response and Introduction to MATLAB SIMULINK****Objective:**

1. To understand the MATLAB functions used to define the transfer function and response of a system and to solve complicated polynomials.
2. To find the inverse Laplace transform and to compute partial fraction expansion of the ratio of two polynomials.
3. To understand MATLABSIMULINK and implement system's transfer function using it.
4. To solve the system equations and obtain the response of the system for different inputs.

**3.1 Transfer Function with MATLAB**

Use the following MATLAB commands in all the in-Lab tasks. Use MATLAB 'help' to find the purpose of these commands

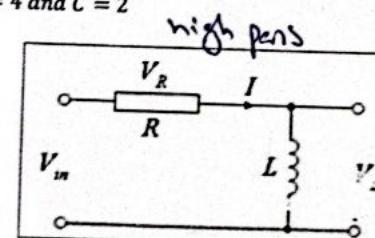
- a. `sys=tf(num, den)`
- b. `bode(sys)`
- c. `step(sys)`
- d. `impulse(sys)`

**Tasks:**

1. For the following circuits shown in Figure 3.1, Find

- i. Transfer Function
- ii. Identify whether it is low pass, high pass, band pass or band reject filter
- iii. Step response
- iv. Impulse response

Use  $R = 3$ ,  $L = 4$  and  $C = 2$



(a)

$$\frac{V_o}{V_i} = \frac{\frac{Ls}{R}}{\frac{Ls}{R} + 1} = \frac{4s}{4s+3}$$

### 3.2 Partial Fraction Expansion with MATLAB

When trying to find the inverse Laplace transfer (or inverse z transform), it is helpful to be able to break a complicated ratio of two polynomials into forms that exist on the Laplace Transform (or z transform) table. The "residue" function of MATLAB can be used to compute the partial fraction expansion (PFE) of the ratio of two polynomials. It takes numerator and denominator vectors as input which are coefficients of the numerator and denominator polynomials respectively. The "tf2zp" function returns poles, zeros and K of a transfer function. The "zp2tf" function returns the numerator and denominator coefficients when zeros, poles and K are sent as input parameters.

- a.  $[r,p,k] = \text{residue}(\text{num},\text{den})$
- b.  $[\text{z},\text{p},\text{K}] = \text{tf2zp}(\text{num},\text{den})$
- c.  $[\text{num},\text{den}] = \text{zp2tf}(\text{z},\text{p},\text{K})$

#### Exercise 1:

Obtain the inverse Laplace transform of the following  $F(s)$ . [Use MATLAB to find the partial fraction expansion of  $F(s)$ ]. Write the inverse Laplace transform in the text box below

$$F(s) = \frac{s^5 + 8s^4 + 23s^3 + 35s^2 + 28s + 3}{s^3 + 6s^2 + 8s}$$

$$Z = 0.3750 \quad 0.2500 \quad 0.3750$$

$$P = -4 \quad -2 \quad 0$$

$$K = 1 \quad 2 \quad 3$$

$$\sum_i \frac{z_i}{s-p_i} + K \Rightarrow \frac{0.375}{s+4} + \frac{0.25}{s+2} + \frac{0.375}{s} + s^2 + 2s + 3$$

$$\Rightarrow 0.375e^{-4t} + 0.25e^{-2t} + 0.375er(t) + r(t) + 2\ddot{r}(t) + 3\ddot{r}(t)$$

#### Exercise 2:

Given the zero(s), pole(s), and gain K of  $B(s)/A(s)$ , obtain the function  $B(s)/A(s)$  using MATLAB. Consider the three cases below. Write the transfer function of each in the text box below:

- There is no zero. Poles are at  $-1+2j$  and  $-1-2j$ ,  $K=10$
- A zero is at 0. Poles are at  $-1+2j$  and  $-1-2j$ ,  $K=10$
- A zero is at -1. Poles are at  $-2, -4$  and  $-8$ ,  $K=12$ .

$$\rightarrow TF = \frac{10}{s^2 + 2s + 5}$$

$$\rightarrow TF = \frac{10s}{s^2 + 2s + 5}$$

$$\rightarrow TF = \frac{12s + 12}{s^3 + 14s^2 + 56s + 64}$$

#### Exercise 3:

A function  $B(s)/A(s)$  consists of the following zeros, poles, and gain K:

- Zeros at  $s=-1, s=-2$
- Poles at  $s=0, s=-4, s=-6$
- Gain  $K=5$

Obtain the expression for  $B(s)/A(s) = \text{num}/\text{den}$  with MATLAB and write it in the space below:

$$TF = \frac{5s^2 + 15s + 10}{s^3 + 10s^2 + 24s}$$

#### Exercise 4:

Obtain the partial fraction expansion of the following function with MATLAB:

$$F(s) = \frac{10(s+2)(s+4)}{(s+1)(s+3)(s+5)^2}$$

Then, obtain the inverse Laplace transform of F(s). Write the inverse Laplace transform in the space below:

$$F(s) = \frac{10s^2 + 60s + 80}{s^4 + 14s^3 + 68s^2 + 130s + 75}$$

$$\sum \frac{x_i}{s-p} + 1 \Rightarrow \frac{-2.1875}{s+5} + \frac{3.75}{s+5} + \frac{1.25}{s+3} + \frac{0.9375}{s+1}$$

$$\Rightarrow -2.1875e^{-st} + 3.75e^{-st} + 1.25e^{-3t} + 0.9375e^{-t}$$

#### Exercise 5:

Consider the following function F(s):

$$F(s) = \frac{s^4 + 5s^3 + 6s^2 + 9s + 30}{s^4 + 6s^3 + 21s^2 + 46s + 30}$$

Using MATLAB, obtain the partial fraction expansion of F(s). Then, obtain the inverse Laplace transform of F(s) and write it in the box below:

$$\frac{-1.08 + 1.75j}{s + (-1 + 3j)} + \frac{-1.08 - 1.75j}{s + (-1 - 3j)} + \frac{-0.1154 + 0j}{s + (-3 + 0j)} + \frac{1.2748 + 0j}{s + (-1 - 0j)}$$

### Getting started with MATLABSIMULINK

#### 3.3 SIMULINK Tutorial

SIMULINK is the Graphical User Interface (GUI) for MATLAB. This section presents a brief tutorial on how to use SIMULINK to create an open-loop block diagram.

1. Start MATLAB and type "simulink" (all lower case) in the command window
2. If installed, the SIMULINK Library Browser will soon pop up as shown in Figure 3.3.

**Exercise 2:**

a. Obtain the unit impulse response of the following system using SIMULINK.

$$\frac{B(s)}{A(s)} = \frac{1}{s^2 + 0.2s + 1}$$

b. Obtain the unit step response of the following system using SIMULINK.

$$\frac{B(s)}{A(s)} = \frac{s}{s^2 + 0.2s + 1}$$

Explain why the results in 'a' and 'b' are same.

In a, When we apply unit impulse having Laplace ' $\frac{1}{s}$ ', given function is multiplied by 1.  
 In b, when we apply unit step response having Laplace ' $\frac{1}{s}$ ', given function is multiplied by  $\frac{1}{s}$  where  $s$  and  $\frac{1}{s}$  are cancelled out giving same function as of 'a'.

19/02/2025

Hanif

**Post Lab****Question 1:**

Solve the following differential equation using MATLAB:

$$\ddot{x} + 2\dot{x} + 10x = e^{-t}, x(0) = 0, \dot{x}(0) = 0$$

The function  $e^t$  is given at  $t=0$  when the system is at rest.

$$\ddot{x}(t) = 1$$

$$x = t^2$$

$$a = 1, b = 2, c = 10$$

$$\lambda^2 + 2\lambda + 10 = 0$$

$$\lambda = -1 \pm 3i$$

$$x = A e^{(-1+3i)t} + B e^{(-1-3i)t} + \frac{1}{10} e^{-t}$$

$$x(0) = \dot{x}(0) = 0$$

$$A(-1+3i) + B(-1-3i) = 0 \Rightarrow x = \left( -\frac{3}{20} + \frac{1}{60}i \right) e^{(-1+3i)t} + \left( \frac{1}{20} - \frac{1}{60}i \right) e^{(-1-3i)t} + \frac{1}{10} e^{-t}$$

**Question 2:** Find the transfer function of following circuit shown in Figure 3.12:

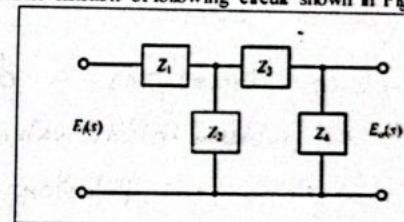


Figure 3.12

$$(Z_1 + Z_2)I_1 - Z_2 I_2 = E_i \quad \text{--- (1)}$$

$$-Z_1 I_1 + (Z_2 + Z_3 + Z_4)I_2 = E_o \quad \text{--- (2)}$$

$$\Delta = Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_4 + Z_2 Z_3 + Z_2 Z_4$$

$$\frac{I_2}{\Delta} = \frac{|Z_1 Z_2 \quad E_i|}{|Z_2 \quad 0|} = \frac{Z_2}{Z_1 (Z_2 + Z_3 + Z_4) + Z_2 (Z_3 + Z_4)}$$

$$\frac{E_o}{E_i} = \frac{Z_2 Z_4}{Z_1 (Z_2 + Z_3 + Z_4) + Z_2 (Z_3 + Z_4)}$$

**Question 3:**

- a) How can LTI filters be uniquely identified by their impulse response?
- b) How can you identify the order of a system from its differential equation?
- c) How can Laplace transform offer a convenient method for the solution of linear, time-invariant differential equations?

a) Any LTI system can be completely characterized by a single function, known as system impulse response according to LTI system theory.

Simply combining the system impulse response  $h(t)$  with input to system  $x(t)$  creates output of system  $y(t)$ .

b) Highest derivatives in a differential eq. determines equation order.

c) When Laplace transforms a diff. eq into an algebraic problem, initial value is taken care during algebraic manipulation.

In the end, after resolving algebraic problem, we use inverse Laplace transform to precisely determine what we wanted.

## EXPERIMENT 4

## Mathematical Modeling of Physical Systems

## Objective:

1. To understand the role of mathematical models of physical systems in design and analysis of control systems.
2. To learn MATLAB functions in solving and simulating such models.

## 4.1 Mechanical System: Mass-Spring System Model

Consider the following Mass-Spring system shown in the Figure 4.1 below.

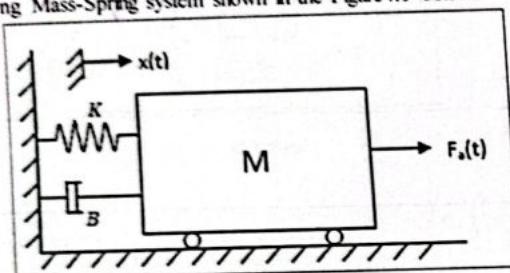


Figure 4.1

K is spring constant.

B is friction coefficient

x(t) is the displacement

F\_a(t) is the applied force

## Exercise 1:

- a) Derive the second order differential equation of the mass-spring system shown in Figure 4.1 and write it down in the space below:

$$M \frac{d^2x}{dt^2} + B \frac{dx}{dt} + Kx = F_a(t)$$

- b) Write the transfer function of the system:

$$X(s)/F(s) = \frac{1}{M s^2 + B s + K}$$

- c) Write the state space equation of the above system in the space below:

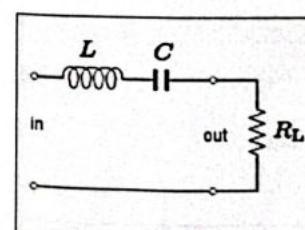
$$\begin{aligned} A &= \begin{bmatrix} -2 & -8 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0.5 \end{bmatrix}, \quad D = [0] \\ \dot{x} &= \begin{bmatrix} -2 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U(t) \\ y &= \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] U(t) \end{aligned}$$

- d) Construct a SIMULINK diagram to calculate the response of the Mass-Spring system. The input force increases from 0 to 8 N at t = 1 s. The parameter values are M = 2 kg, K = 16 N/m, and B = 4 N.s/m (Draw a block diagram to represent this equation and draw the corresponding SIMULINK diagram before implementing it on SIMULINK).

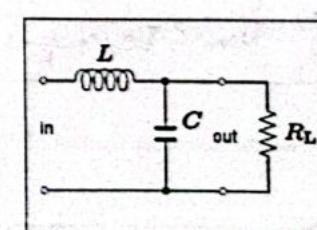
## 4.2 Electrical System: RLC circuit

## Exercise 2:

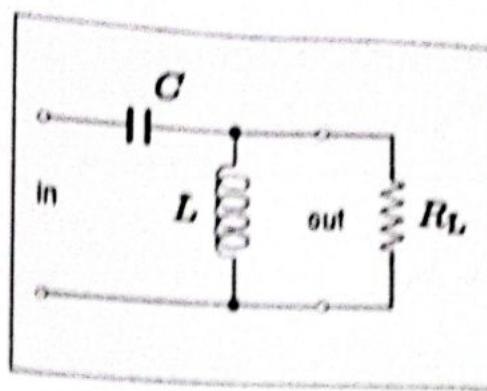
Consider the electrical circuits shown in the Figure 4.2 below.



(a)



(b)



(c)

Figure 4.2

- a) Derive the differential equation of the above systems and write it down in the space below, also state its order:

a)  $\frac{dV_{in}}{dt} = LC \frac{d^2i}{dt^2} + RC \frac{di}{dt} + V_{in}$

b)  $\frac{dV_{in}}{dt} = LC \frac{dV_C}{dt} + \frac{L}{R} \frac{dV_C}{dt} + V_{in}$

c)  $\frac{dV_{in}}{dt} = \frac{V_L}{LC} + \frac{d}{dt} \frac{V}{RC} + \frac{d}{dt} V_L$

- b) Write the transfer function of the systems in the space below:

a)  $\frac{V_{out}}{V_{in}} = \frac{RCs}{LCs^2 + RCs + 1}$

c)  $\frac{V_C}{V_{in}} = \frac{s^2}{\frac{1}{LC} + \frac{1}{RC}s + s^2}$

b)  $\frac{V_C}{V_{in}} = \frac{1}{LCs^2 + \frac{L}{R}s + 1}$

- c) Write the state space equation of the above system in the space below:

## EXPERIMENT 5

### Block Diagrams Reduction using MATLAB

#### Objective:

1. To obtain transfer functions of complex block diagrams through MATLAB.
2. To plot the responses of systems

#### 5.1 Transfer Functions of Block Diagrams and Step Response

In control systems analysis, we frequently need to simplify a network of interconnected transfer functions into a single transfer function which is then used in subsequent calculations for analysis purposes. There are three different types of connections between transfer function that are usually encountered in practice: cascade-connected, parallel-connected and feedback-connected (closed-loop) transfer functions. MATLAB has convenient commands to obtain these transfer functions. To obtain the transfer functions of the cascaded, parallel, feedback and unity feedback systems, the following commands are used, respectively:

`[num, den] = series (num1, den1, num2, den2)`

`[num, den] = parallel (num1, den1, num2, den2)`

`[num, den] = feedback (num1, den1, num2, den2)`

`[num, den] = cloop (num1, den1, -1)`

#### Exercise 1:

Obtain  $Y(s)/X(s) = \text{num} / \text{den}$  for each of the arrangement of  $G_1(s)$  and  $G_2(s)$  as shown in Figure 5.1 and write the transfer function in front of each arrangement

$$1. G_1(s) = \frac{10}{s^2 + 2s + 10}$$

(a)

$$\frac{50}{s^3 + 7s^2 + 20s + 50}$$

(b)

$$\frac{s^2 + 20s + 100}{s^3 + 7s^2 + 20s + 50}$$

$$2. G_2(s) = \frac{5}{s+5}$$

(c)  $\frac{10s + 50}{s^2 + 7s^2 + 20s + 100}$

$G_2(s) = \frac{10}{s^2 + 2s + 20}$

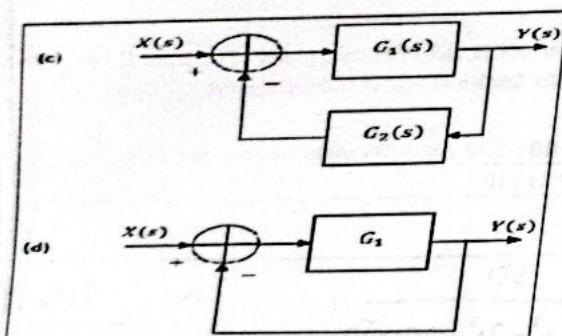
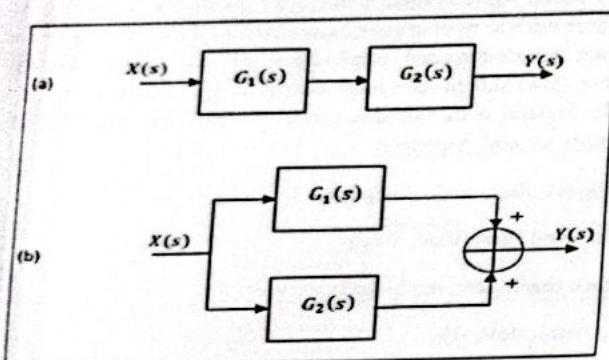


Figure 5.1

**Exercise 2:**

Evaluate the transfer function of the feedback system shown in the Figure 5.2 using MATLAB.  $G_1(s) = 4$ ,  $G_2(s) = 1/(s+2)$ ,  $H(s) = 5s$ . Write the code and the transfer function below.

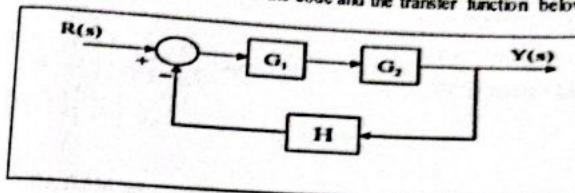


Figure 5.2

Transfer function:

$$\frac{4}{21s + 2}$$

**Exercise 3:**

Determine the transfer function of the system shown in Figure 5.3. Check your answer with MATLAB

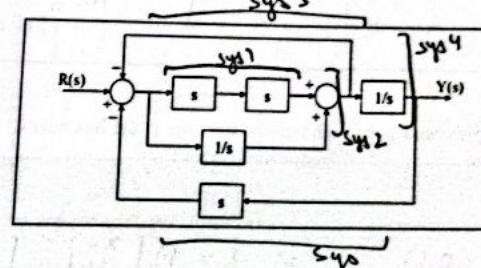


Figure 5.3

Transfer function:

$$\begin{aligned} sys1 &= s^2 & sys3 &= \frac{s^3 + 1}{s^3 + s + 1} \\ sys2 &= \frac{s^3 + 1}{s} & sys4 &= \frac{s^3 + 1}{s^4 + s^2 + s} \end{aligned}$$

$$sys = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$

## 5.2 Transformation of Mathematical Models with MATLAB

 $[A, B, C, D] = tf2ss(num, den)$  $[num, den] = ss2tf (A, B, C, D)$  $[z, p, k] = ss2zp (A, B, C, D, i)$  $[num, den] = tfdata (T, V)$ 

## Exercise 4:

Transform the transfer function into the state space representation using MATLAB

(a)  $Y(s)/U(s) = \frac{s}{(s+10)(s^2+4s+16)}$

$$A = \begin{bmatrix} -14 & -56 & -160 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = [0 \ 1 \ 0] \quad D = [0]$$

(b)  $Y(s)/U(s) = \frac{s^2+7s+2}{s^3+9s^2+26s+24}$

$$A = \begin{bmatrix} -9 & -26 & -24 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = [1 \ 7 \ 2] \quad D = [0]$$

Write the state space representation of each transfer function in the box below.

$\dot{x} = Ax + Bu$

$y = cx + du$

$\dot{x} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$

$y = [0 \ 1 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0] v$

$\dot{x} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$

$y = [1 \ 7 \ 2] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0] v$

## Exercise 5:

Obtain the transfer function of the state space equations using MATLAB

(a)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = [8 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$s+8$

$\checkmark \frac{s^2+8s+6}{s^2+s+6}$

(b)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$10s^2 + 30s + 20$

$\checkmark \frac{s^3 + 3s^2 + 2s + 1}{s^3 + 3s^2 + 2s + 1}$

**Exercise 6:**

Obtain the closed-loop transfer function and the function to obtain the closed-loop state space model of the system shown below, using MATLAB.

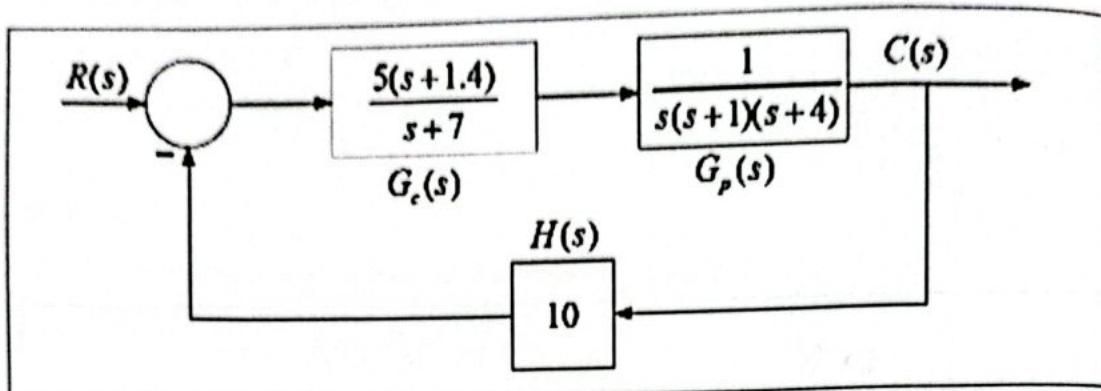


Figure 5.4

**Transfer function:**

$$\frac{5s+7}{s^4 + 12s^3 + 39s^2 + 28s + 70}$$

**State-space Equation:**

$$A = \begin{bmatrix} -12 & -39 & -78 & -70 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 5 & 7 \end{bmatrix} \quad D = [0]$$

12/03/2025  
Hawige

# Lab Manual of Feedback Control Systems

## EXPERIMENT 6

### Second Order System Analysis using MATLAB

#### Objective:

1. To implement the systems in MATLAB
2. To understand the system transient and steady-state responses

The steady-state response means the manner in which the system output behaves as  $t$  approaches infinity. The transient response means that which goes from the initial state to the final state.

#### 6.1 Transient Response

Exercise 1: Write MATLAB program to obtain the unit-step response curves for the following systems:

(a).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y = [8 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

b).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u(t)$$
$$y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

→ Damping condition

Exercise 2: For  $\zeta=0.4$  and  $\omega_n=5$  rad/sec. Obtain the standard second order system transfer function and write it in the box below: → natural frequency

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{25}{s^2 + 4s + 25}$$



**Exercise 3:** For the system found in Question 2, write a MATLAB program to find the rise time, peak time, maximum overshoot and the settling time. Use bview() command.

Peak Time: 0.6908  
Maximum Overshoot: 2.53741  
Rise Time: 0.910  
Settling Time: 1.6319

bview (sp)  
stepinfo (sp)

**Exercise 5:** When the closed-loop system involves a numerator dynamics, the unit-step response curve may exhibit a large overshoot. Comment on this statement after obtaining the unit-step response and the unit-ramp response of the following system with MATLAB.

$$\frac{C(s)}{R(s)} = \frac{10s + t}{s^2 + 4s + 4}$$

$$C(s) = \frac{10s + t}{s^2 + 4s + 4}$$

$$= \frac{10s + t}{s^2 + 4s + 4}$$

## 6.2 Steady State Response

**Exercise 4:** Obtain the partial fraction expansion of C(s) using MATLAB when R(s) is a unit step function. Show the system response c(t) curve using bview() command. Also write down the time response c(t) in the space below:

$$\frac{C(s)}{R(s)} = \frac{3s^3 + 25s^2 + 72s + 80}{s^4 + 8s^3 + 40s^2 + 96s + 80}$$

$$\sum \frac{Y_i}{s - P_i} + R$$

$$(1) \Rightarrow \frac{-0.281 - 0.1719i}{s - (-2+4i)} + \frac{-0.281 + 0.1719i}{s - (-2-4i)} + \frac{-0.4375}{s - (-2)}$$

$$+ \frac{-0.375}{s - (-2)}$$

$$(2) = (-0.281 - 0.1719i)e^{(-2+4i)t} + (-0.281 + 0.1719i)e^{(-2-4i)t}$$

$$+ -0.4375e^{-2t} - 0.375e^{-2t}$$

**Exercise 6:** Using MATLAB, obtain the unit-ramp response of the closed-loop control system whose closed-loop transfer function is given below. Also, obtain the response of this system when the input is given by:  $r = e^{-0.5t}$

$$\frac{C(s)/R(s)}{s^3 + 6s^2 + 9s + 10} = \frac{s + 10}{s^3 + 6s^2 + 9s + 10}$$

$$C(s) = \frac{s + 10}{(s^2 + 6s + 9)^2 + 0.5s^2} (s + 0.5)$$

$$= \frac{s + 10}{s^4 + 6s^3 + 9s^2 + 0.5s^2 + 3s^2 + 4.5s + 5}$$

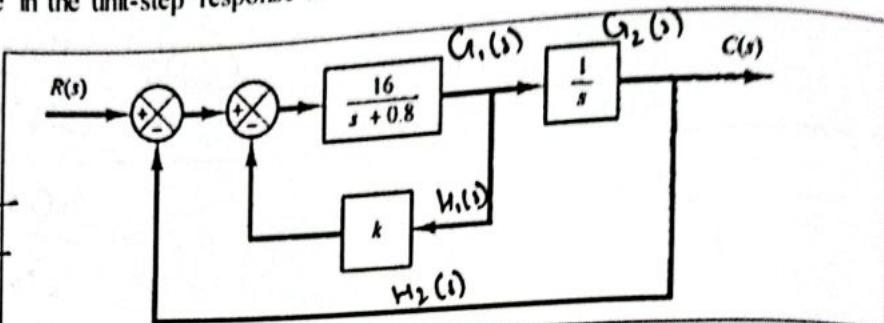
$$= \frac{s + 10}{s^4 + 6s^3 + 12s^2 + 14.5s + 5}$$

**Exercise 7:** Consider the system shown in Figure 4.1 below. Determine the value of  $k$  such that damping ratio is 0.5. Using MATLAB obtain the rise time, peak time, maximum overshoot, and settling time in the unit-step response and fill the following table.

$$tf_1 = \frac{G_1(s)}{1 + H_1(s)G_1(s)}$$

$$= \frac{\frac{16}{s+0.8}}{1 + \frac{16k}{s+0.8}}$$

$$= \frac{16}{s+0.8+16k}$$



$$G(s) = tf_1 \times G_2(s) \xrightarrow{\text{Figure 6}} \left( \frac{16}{s+0.8+16k} \right) \left( \frac{1}{s} \right) = \frac{16}{s^2 + 16ks + 0.8s}$$

$$\omega_n^2 = 16$$

$$\Rightarrow \omega_n = 4$$

$$16k + 0.8 = 2$$

$$16k = 2(0.5)(4) - 0.8$$

$$k = \frac{3.2}{16}$$

$$= 0.2$$

Rise Time:	0.4098
Peak Time:	0.8980
Maximum overshoot	16.2929
Settling Time	2.0190
Value of $k$	0.2

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + H_2(s)G(s)} = \frac{16}{s^2 + 16ks + 0.8s + 16}$$

$$= \frac{16}{s^2 + (16k + 0.8)s + 16}$$

**Q1:** Using MATLAB, obtain the unit-step response, unit-ramp response, and unit impulse response of the system defined below. Where  $R(s)$  and  $C(s)$  are LAPLACE transform of the input  $r(t)$  and output  $c(t)$  respectively.

$$C(s)/R(s) = \frac{10}{s^2 + 2s + 10}$$

5/03/2025  
Hanyu

**EXPERIMENT 7**  
**Modeling and System Identification of a DC Motor**

**Objective:**

- Study the characteristics of step response of first order servo system
- Measure the steady state gain  $K$  and time constant  $\tau$
- Familiarize with QNET DC motor trainer

**7.1 DC Motor Modeling**

Modeling the dynamical properties of a system is an important step in analysis and design of control systems. In this experiment, you will study the steady-state and dynamic characteristics of a DC motor. Here a motor-generator combination serves as a first-order linear system. It is important from a practical point of view as many real-world systems can be modeled in this manner. The DC motor considered in this experiment is just one such example.

The electrical equations describing the open-loop response of the DC motor are

$$V_m(t) - R_m I_m(t) - E_{emf}(t) = 0 \quad [1]$$

and  $E_{emf}(t) = K_m \omega_m(t)$  [2]

The mechanical equations describing the torque of the motor are

$$T_m(t) = J_{eq} \left[ \frac{d}{dt} \omega_m(t) \right] \quad [3]$$

and  $T_m(t) = K_t I_m(t)$  [4]

Where  $T_m$ ,  $J_{eq}$ ,  $\omega_m$ ,  $K_t$ ,  $K_m$ , and  $I_m$  are described in Table 7.1. This model does not take into account friction or damping.

Symbol	Description	Units
$V_m$	Motor terminal voltage	V
$R_m$	Motor terminal resistance	$\Omega$
$I_m$	Motor armature current	A
$K_t$	Motor torque constant	N.m/A
$K_m$	Motor back-electromotive force constant	V/(rad/s)
$\omega_m$	Motor shaft angular velocity	rad/s
$T_m$	Torque produced by the motor	N.m
$J_{eq}$	Motor armature moment of inertia and load moment of inertia	kg. m <sup>2</sup>

Table 7.1:DC Motor Model Parameter

# Lab Manual of Feedback Control Systems

Frequency = 0.40 Hz  
Offset = 3.0 V

5. Once you have collected a step response, click on the Stop Button to stop running the VI.
6. Select the Measurement Graphs tab to view the measured response.
7. Use the responses in the speed (rad/s) and Voltage (V) graphs to compute the steady-state gain of the DC motor and fill out the values in table 7.5. See the Bumptest Method section for details on how to find the steady-state gain from a step response. Finally you can use the Graph Palette for zooming functions and the Cursor Palette to measure data.

Description	Symbol	Value	Unit
Steady-state motor speed	$\omega_{ss}$	118	rad/s
Initial step motor speed	$\omega_0$	0	rad/s
Input step amplitude	$A_v$	5-1=4	V
Measured steady-state gain using bump test	$K_{a,b}$	104.5	rad(V.s)

Table 7.5: Finding steady-state gain using bump test

8. Based on the Bumptest Method, find the time constant. Make sure you complete table 5.6.

Description	Symbol	Value	Unit
Decay speed	$\omega_{ss}(t_1)$	264.17	rad/s
Initial step time	$t_0$	1.26	s
Decay step time	$t_1$	1.299	s
Measured time constant using bump test	$\tau_{a,b}$	0.039	s

Table 7.6: Finding time constant using bump test

9. Enter the steady-state gain and time constant values found in this section in table 5.7. These are called the bump test model parameters.

Description	Symbol	Value	Unit
Open-Loop steady state Gain	$K_{a,b}$	104.5	rad(V.s)
Open-Loop Time Constant	$\tau_{a,b}$	0.039	s

Table 7.7: Bump test Modeling

## Model Validation

When the modeling is complete it is validated by running the model and the actual process in open loop. That is, open loop voltage is fed to both the model and the actual device such that both the simulated and measured response can be viewed on the same scope. The model can then be adjusted to fit the measured motor speed by fine-tuning the modeling parameters.

1. Open the QNET\_DCMCT\_Modeling.vi.
2. Ensure the correct Device is Chosen.
3. Run the QNET\_DCMCT\_Modeling.vi. You should hear the DC motor begin running.
4. In the Signal Generator section set:  
Amplitude = 2.0 V

Frequency = 0.40 Hz

Offset = 3.0 V

5. In the Model Parameters section of the VI, enter the bump test model parameters,  $K$  and  $\tau$ . The blue simulation should match the red measured motor speed more closely. How well does your model represent the actual system? If they do not match, name one possible source of discrepancy

Our model doesn't match with the actual system. It can be due to human error or we may have not zoomed exactly at the respective points giving irregular values.



6. Tune the steady-state gain,  $K$ , and the time constant  $\tau$  in the model Parameters section so the simulation matches the actual system better. Enter both the bump test and tuned model parameters in table 7.8.

Description	Symbol	Value	Unit
<b>Bump Test Modeling</b>			
Open-Loop steady state Gain	$K_{a,b}$	104.5	rad(V.s)
Open-Loop Time Constant	$\tau_{a,b}$	0.039	s
<b>Tuned Model Parameters</b>			
Open-Loop steady state Gain	$K_{a,v}$	10.45	rad(V.s)
Open-Loop Time Constant	$\tau_{a,v}$	0.039	s

Table 7.8: QNET DCMCT modeling results summary



03/2025

Hanjie

## EXPERIMENT 8

### QNET DC Motor Speed Control

Objective:

- Design a PI controller to regulate the speed of the DC motor

#### 8.1 DC Motor Speed Control

The experiment is designed in order to provide full understanding of PI controllers. The aim is to give you the knowledge to simulate and validate the controllers.

The speed of the DC motor is controlled using proportional integral control system. The PI control also includes set point weight. The transfer function representing the DC-motor speed voltage relation is used to design the PI controller.

The mathematical model of a DC motor has developed and its physical parameters are identified in previous experiment. Once the model is verified it is used to design a proportional-integral, or PI, controller that must meet certain given specifications.

The block diagram of the closed-loop system is shown in Figure 8.1.

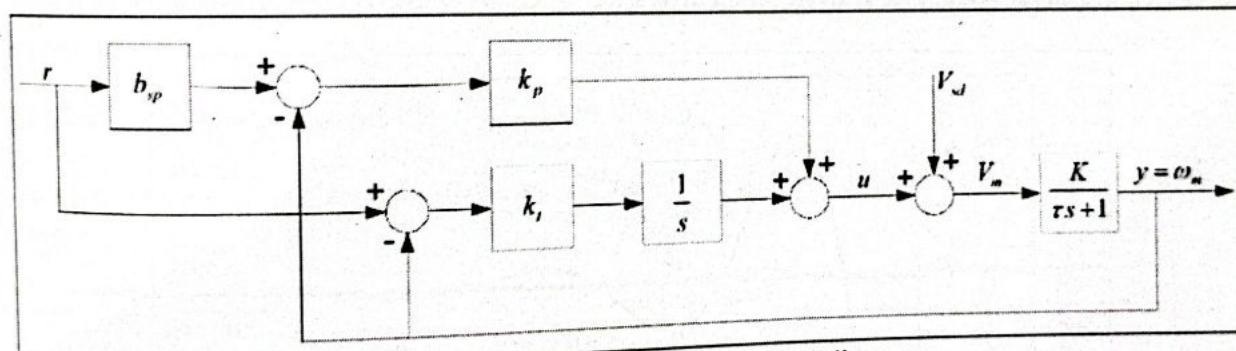


Figure 8.1: DC Motor PI closed-loop block diagram

The transfer function representing the DC motor speed-voltage relation in Equation 1 is used to design the PI controller. The input-output relation in the time-domain for a PI controller with set-point weighting is

$$u = k_p(b_{sp}r - y) + \frac{k_i(r-y)}{s} \quad [1]$$

Where  $k_p$  is the proportional gain,  $k_i$  is the integral gain, and the  $b_{sp}$  is the set point weight. The closed loop transfer function from the speed reference,  $r$ , to the angular motor speed output,  $\omega_m$ , is

$$G_{\omega,r}(s) = \frac{K(k_p b_{sp} s + k_i)}{\tau s^2 + (Kk_p + 1)s + Kk_i} \quad [2]$$

**8.4 Procedure****8.4.1 Qualitative PI Control**

1. Open the QNET\_DCMCT\_Speed\_Control.vi
2. Ensure the correct Device is chosen.
3. Run the QNET\_DCMCT\_Speed\_Control.vi. The motor should begin rotating.

**4. In the Signal Generator section set:***Signal type = 'square wave'**Amplitude = 25.0 rad/s**Frequency = 0.40 Hz**Offset = 100.0 rad/s*

5. In the Control Parameters section set:

 *$k_p = 0.050 \text{ V.s/rad}$*  *$k_i = 1.00 \text{ V/rad}$*  *$b_{sp} = 0.00$* 

6. Examine the behavior of the measured speed, shown in red, with respect to the reference speed, shown in blue, in the Speed (rad/s) scope. Explain what is happening.

System experiences a slight overshoot and also a small time delay when speed is increased or decreased,

Description	Symbol	Value	Unit
SLD specifications	$\omega_n$	16.0	rad/s
Damping ratio	$\zeta$	0.75	
Peak time	$t_p$	0.79	s
Percentage overshoot	PO	2.827	%

Table 8.2: Expected peak time and overshoot

7. Increment and decrement  $k_p$  by steps of 0.005 V.s/rad.
8. Look at the changes in the measured signal with respect to the reference signal. Explain the performance difference of changing  $k_p$ .

When  $k_p$  is increased, overshoot decreases and vice versa. Hence inverse relation.

9. Set  $k_p$  to 0 V.s/rad and  $k_i$  to 0 V/rad. The motor should stop spinning.
10. Increment the integral gain,  $k_i$ , by steps of 0.05 V/rad.
11. Examine the response of the measured speed in the Speed (rad/s) scope and compare the result with  $k_i$  is set low to when it is set high. Explain what is happening?

Q) You make system spinning  
w) The overshoot increases as we increase  $K_i$  and time delay decreases.

12. Stop the VI by clicking on the Stop button.

**8.4.2 PI Control According to Specifications**

1. Using the equations 8.9 and 8.10. Record the expected peak time,  $t_p$ , and percentage overshoot, PO in table 8.1, given the following Speed Lab Designs (SLD) specifications:  $\zeta = 0.75$

 $\omega_n = 16.0 \text{ rad/s}$ 

Description	Symbol	Value	Unit
SLD specifications	$\omega_n$	16.0	rad/s
Damping ratio	$\zeta$	0.75	
Peak time	$t_p$	0.79	s
Percentage overshoot	PO	2.827	%

Table 8.3: PI speed control design

- The motor should begin spinning.
3. Run the QNET\_DCMCT\_Speed\_Control.vi. The motor should begin spinning.
  4. In the Signal Generator set
    - Signal type = 'square wave'
    - Amplitude = 25.0 rad/s
    - Frequency = 0.40 Hz
    - Offset = 100.0 rad/s
  5. In the Control Parameters section, enter the SLD PI control gains found in table 8.2 and make sure bsp = 0.00.
  6. Stop the VI when you obtain two sample cycles by clicking on the Stop button.
  7. Capture the measured SLD speed response. Make sure you include both the Speed (rad/s) and the control signal Voltage (V) scopes.
  8. Measure the peak time and percentage overshoot of the measured SLD response. Are the specifications satisfied? Peak time is satisfied whereas overshoot has slight error due to better  $k_i$ .
  9. Write down the effect of increasing the specification $\zeta$  have on the measured speed response in Table 8.3? How about on the control gains?

Description	Symbol	Behaviour	Unit
Peak time	$t_p$	Inverse	s
Percent overshoot	$PO$	Inverse	%
Proportional gain	$k_p$	Direct	V.s/rad
Integral gain	$k_i$	No effect	V/rad

Table 8.4: Effect of increasing damping ratio specification in speed control

10. Write down the effect of increasing specification  $\omega_n$  have on the measured speed response and the generated control gains in Table 8.4.

Description	Symbol	Behaviour	Unit
Peak time	$t_p$	Inverse	s
Percent overshoot	$PO$	No effect	%
Proportional gain	$k_p$	Direct	V.s/rad
Integral gain	$k_i$	Direct	V/rad

Table 8.5: Effect of increasing natural frequency specification in speed control

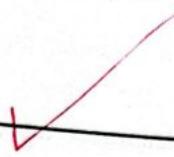
#### 8.4.3 Effect of Set-Point Weight

1. Run the QNET\_DCMCT\_Speed\_Control.vi. The motor should begin rotating.
2. In the Signal Generator section set:
  - Signal type = 'square wave'
  - Amplitude = 25.0 rad/s
  - Frequency = 0.40 Hz
  - Offset = 100.0 rad/s
3. In the Control Parameters section set:

$$k_p = 0.050 \text{ V.s/rad}$$

4. Increment the set-point weight parameter  $bsp$  in steps of 0.05. Vary the parameter between 0 and 1.
5. Examine the effect that raising  $bsp$  has on the shape of the measured speed signal in the Speed (rad/s) scope. Explain what the set-point weight parameter is doing.

When  
decreases       $bsp$  increases      overshoot  
                        and      vice versa.



6. Stop the VI by clicking on the Stop button

#### 8.4.4. Tracking Triangular Signals

1. Run the QNET\_DCMCT\_Speed\_Controlvi. The motor should begin rotating.  
2. In the Signal Generator section set:

Signal type = 'triangular wave'

Amplitude = 50.0 rad/s

Frequency = 0.40 Hz

Offset = 100.0 rad/s

7. In the Control Parameters section set:

$$k_p = 0.020 \text{ V.s/rad}$$

$$k_i = 0.00 \text{ V/rad}$$

$$bsp = 1.00$$

3. Compare the measured speed and the reference speed. Explain why there is a tracking error

There is tracking error because  
 $k_i$  equals to zero.



*Elaborate*

4. Increase  $k_i$  to 0.1 V/rad and examine the response. Vary  $k_i$  between 0.1 V/rad and 1.0 V/rad.
5. What effect does increasing  $k_i$  have on the tracking ability of the measured signal? Explain using the observed behavior in the scope.

As  $k_i$  is increased, tracking error decreases and signal is measured fully.

26/3/2015  
Hariy

6. Stop the VI by clicking on the Stop button.

#### Post Lab

- a) What do you understand by inertia, friction and dampening?

- b) What kind of controller is used to control the speed of the motor? Give reasons to support your answer.

- c) Explain the purpose of proportional, integral and derivative gains and how do they affect the speed of the motor.

#### EXPERIMENT 9

#### QNET DC Motor Position Control

##### Objective:

- Design a PD controller to regulate the speed of the DC motor
- #### 9.1 DC Motor Position Control

The aim of this is to make you understand the system, commission the system, run and evaluate it. Control of motor position is a natural way to introduce the benefits of derivative action. In this experiment performance of proportional-derivative controller is observed.

The mathematical model of a DC motor has developed and its physical parameters are identified previously. Once the model is verified it is used to design a proportional-derivative, or PD, controller that must meet certain given specifications.

The block diagram of the closed-loop system is shown in Figure 7.1.

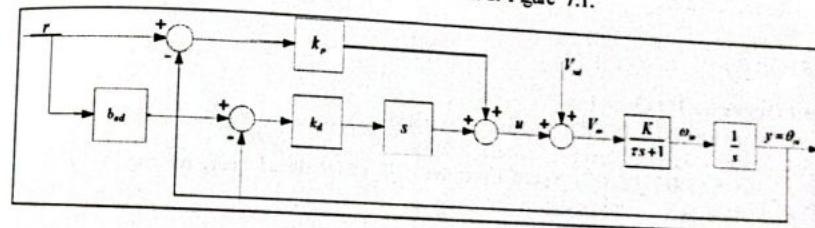


Figure 9.1: DC Motor PD closed-loop block diagram

The transfer function representing the DC motor angular position-voltage relation in Equation 9.1 is used to design the PD controller. The input-output relation in the time-domain for a PD controller is

$$u = k_p(r - y) + k_d(b_{sd}r - y) \quad [1]$$

Where  $k_p$  is the proportional gain,  $k_d$  is the derivative gain. Combining the position process model

$$\frac{\theta_m(s)}{V_m(s)} = \frac{K}{s(rs+1)} \quad [2]$$

with the PD control equation 7.1 gives the closed loop transfer function of the motor position system

$$G_{\theta,r}(s) = \frac{K(k_p + b_{sd}k_d s)}{rs^2 + (1 + Kk_d)s + Kk_p} \quad [3]$$

## Lab Manual of Feedback Control Systems

When  $R_p$  increases, overshoot ~~in~~ decreases whereas  
the steady state error decreases.

9. Increment the derivative gain,  $kd$ , by steps of 0.01 V.s/rad.
10. Look at the changes in the measured position with respect to the desired position. Explain what is happening.

When  $R_d$  is increased, overshoot and settling time decreased. Steady state error increases.

### 9.3.2 PD Control according to Specifications

1. Using the equations in 8.9 and 8.10 (Experiment 8) calculate and record the expected peak time,  $t_p$ , and percentage overshoot,  $PO$  in Table 9.1, given

$$\text{Zeta} = 0.60$$

$$\omega_0 = 25.0 \text{ rad/s}$$

$$p0 = 0.0$$

Description	Symbol	Value	Unit
Natural frequency specification	$\omega_n$	25.0	rad/s
Damping ratio specification	$\zeta$	0.6	
Peak time	$t_p$	0.15707	s
Percentage overshoot	$PO$	2.478	%

Table 9.1: Expected peak time and overshoot

2. Calculate the proportional,  $k_p$ , and derivative,  $kd$ , control gains according to the model parameters found in previous experiment and the required specifications and write down these values in Table 9.2

Description	Symbol	Value	Unit
Natural frequency specification	$\omega_n$	25.0	rad/s
Damping ratio specification	$\zeta$	0.6	
Steady-state model gain	$K$	32	rad/(V.s)
Model time constant	$\tau$	0.06	s
Proportional gain	$k_p$	1.17167	V.s/rad
Derivative gain	$k_d$	-0.5725	V/rad

Table 9.2: PD speed control design

3. Run the QNET\_DCMCT\_Position\_Control.vi. You should see the DC motor rotating back and forth.
4. In the Signal Generator section set:

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$PO = e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} \times 100$$

# Lab Manual of Feedback Control Systems

*Amplitude = 2.00 rad*

*Frequency = 0.4. Hz*

*Offset = 0.00 rad*

5. In the control Parameters section set the PD gains found in Table 9.2.
6. Measure the peak time and percentage overshoot of the measured position response. Are the specifications satisfied? If they are not, then give on possible reason why there would be discrepancy.

$$T_p = T_{max} - T_0 = 8.4 - 1.25 = 0.15s$$

$$OS = \frac{100(y_{max} - R_0)}{R_0} = \frac{100(2.2 - 1)}{2} = 10\%$$

Specifications are almost satisfied.

7. Write down the changing the specification  $\zeta$  have on the measured position response and the generated control gains in Table 9.3

Description	Symbol	Behaviour	Unit
Peak time	$t_p$	inverse	s
Percent overshoot	$PO$	inverse	%
Proportional gain	$k_p$	direct	V.s/rad
Derivative gain	$k_d$	direct	V/rad

Table 9.3: Effect of increasing damping ratio specification in position control

8. Write down the effect of increasing the specification  $\omega_n$  have on the measured position response and the generated control gains in Table 9.4

Description	Symbol	Behaviour	Unit
Peak time	$t_p$	inverse	s
Percent overshoot	$PO$	no relation	%
Proportional gain	$k_p$	direct	V.s/rad
Derivative gain	$k_d$	direct	V/rad

Table 9.4: Effect of increasing natural frequency specification in position control

9. Stop the VI by Clicking on the *Stop* button.

## Post Lab

**EXPERIMENT 10****QNET Rotary Pendulum Trainer****Objective:**

Rotary pendulum system is chosen for you to understand a task-based controller. The experiment begins by modeling the system and determines strategies to dampen the oscillations of the system. Furthermore, it makes you understand the relationship between stability and inertia of the system.

**10.1 Modeling**

The QNET-ROTPENT Simple Modeling runs the DC motor connected to the pendulum arm (as shown in Figure 10.1) in open-loop and plots the corresponding pendulum arm and link angles as well as the applied input motor voltage.

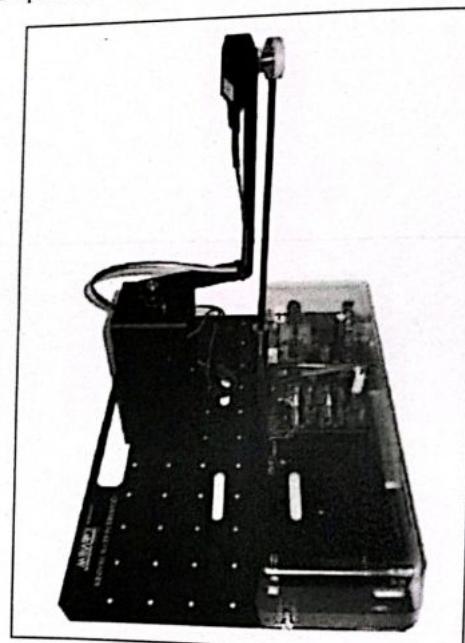


Figure 10.1(Pendulum)

**10.1.1. Dampening**

1. Open the QNET\_ROTPENT\_Simple\_Modeling.vi.
2. Ensure the correct Device is chosen as shown in Figure 10.2.

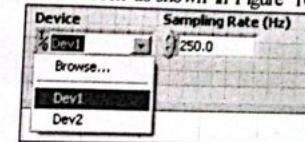
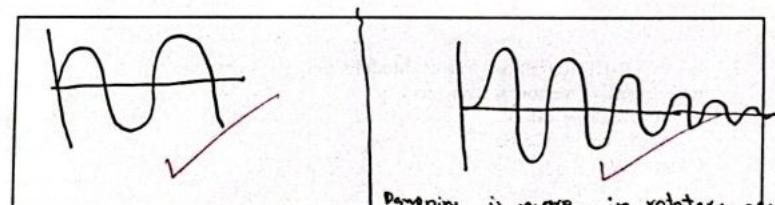


Figure 10.2

3. Run the QNET\_ROTPENT\_Simple\_Modeling.vi.
4. Hold the arm of the rotary pendulum system stationary and manually perturb the pendulum.
5. While still holding the arm, examine the response of Pendulum Angle (deg) in the Angle (deg) scope. This is the response from the pendulum system.
6. Repeat Step 3 above and release the arm after several swings.
7. Examine the Pendulum Angle (deg) response when the arm is not fixed. This is the response from the rotary pendulum system. Given the response from these two systems pendulum and rotary pendulum – which converges faster towards angle zero? Why does one system dampen faster than the other?



8. Stop the VI by clicking on the Stop button.

**10.1.2. Friction**

1. Run the QNET\_ROTPENT\_Simple\_Modeling.vi.
2. In the Signal Generator section set:  
Amplitude = 0.00 V  
Frequency = 0.25 Hz  
Offset = 0.00 V111
3. Change the Offset in steps of 0.10 V until the pendulum begins moving. Record the voltage at which the pendulum moved.

4. Repeat Step 3 above for steps of -0.10 V.
5. Enter the positive and negative voltage values needed to get the pendulum moving in Table 10.1. Why does the motor need a certain amount of voltage to get the motor shaft moving?

Description	Symbol	Value	Unit
Positive Constant Encoder Voltage	$V_2$	1.0	V
Negative Constant Encoder Voltage	$V_1$	-0.30	V

Table 10.1

6. Stop the VI by clicking on the Stop button.

#### 10.1.5. Moment of Inertia

1. Using references QNET User Manual and QNET Practical Control Guide, calculate the moment of inertia acting about the pendulum pivot.

$$J_p = 1.70 \times 10^{-4} \text{ kg m}^2$$

2. Run the QNET\_ROTPEND\_Simple\_Modeling.vi

3. In the Signal Generator section set:

Amplitude = 1.00 V

Frequency = 0.25 Hz

Offset = 0.00 V

4. Click on the Disturbance toggle switch to perturb the pendulum and measure the amount of time it takes for the pendulum to swing back-and-forth in a few cycles (e.g. 4 cycles).

5. Find the frequency and moment of inertia of the pendulum using the observed results. See QNET Practical Control Guide to see how to calculate the inertia experimentally and make sure you fill the following Table 10.2.

Description	Symbol	Value	Unit
Cycles	$n_{cyc}$	4	
Duration	$\Delta t$	1.6	s
Frequency	$f$	2.5	Hz
Pendulum moment of inertia	$J_{p_{exp}}$	$1.64 \times 10^{-4}$	$\text{kg m}^2$

$$f = \frac{n_{cyc}}{\Delta t}$$

$$J_{p_{exp}} = \frac{1}{4} \frac{(M_1 \theta_1)}{\pi^2 f^2}$$

6. Compare the moment of inertia calculated analytically in step 1 and the moment of inertia found experimentally. Is there a large discrepancy between them?

Due to air resistance, friction between surfaces and possibility of human error.

21/4/2025

Hawais

#### Post Lab

- a) What is rotary inverted pendulum system?

- b) Define dampening and friction?

## EXPERIMENT 11

## QNET-ROTPENT Balance Control Design

**Objective:**

The experiment illustrates one of the important methods for finding the parameters of control strategies. It will give you deeper understanding of LQR technique that is suitable for finding the parameters of the balance controller.

## 11.1 Model Analysis

1. Open the QNET\_ROTPENT\_Control\_Design.vi.
2. Run the QNET\_ROTPENT\_Control\_Design.vi.
3. Select the Symbolic Model tab.
4. The Model Parameters array includes all the rotary pendulum modeling variables that are used in the state-space matrices A, B, C, and D.
5. Select the Open Loop Analysis tab.
6. This shows the numerical linear state-space model and a pole-zero plot of the open-loop inverted pendulum system. What do you notice about the location of the open-loop poles? Recommended: In the Model Parameters section, it is recommended to enter the pendulum moment of inertia,  $J_p$ , determined experimentally in previous experiment.

One of the pole is in right half plane  
Thus it is an unstable system.

Poles  
 $\begin{matrix} 1 \\ -11.3 \\ -0.346 \end{matrix}$

7. In the Symbolic Model tab, set the pendulum moment of inertia,  $J_p$ , to  $1.0e-5 \text{ kg.m}^2$ .
8. Select the Open Loop Analysis tab. How did the locations of the open-loop poles change with the new inertia? Enter the pole locations of each system with a different moment of inertia in the following table 11.1. Are the changes of having a pendulum with a lower inertia as expected?

Poles moves further away in the right half planes & in left half plane.

Description	Symbol	Value	Unit
System w/ $J_p = 1.7e-4$	$p_0$	9.03	rad/s
	$p_1$	-9.25	rad/s
	$p_2$	-0.346	rad/s
	$p_3$	0	rad/s
System w/ $J_p = 1.00e-5$	$p_0$	11	rad/s
	$p_1$	-11.3	rad/s
	$p_2$	-0.346	rad/s
	$p_3$	0.00	rad/s

Table 11.1

9. Reset the pendulum moment of inertia,  $J_p$ , back to  $1.77e-4$ .
10. Stop the VI by clicking on the Stop button.

## 11.2 Control Design and Simulation

1. Open the QNET\_ROTPENT\_Control\_Design.vi.
2. Select the Simulation tab.
3. Run the QNET\_ROTPENT\_Control\_Design.vi.
4. In the Signal Generator section set:  
 Amplitude = 45.0 deg  
 Frequency = 0.20 Hz  
 Offset = 0.0 deg
5. Set the Q and R LQR weighting matrices to the following:  
 $Q(1, 1) = 10$ , ie. set first element of Q matrix to 10.  
 $R = 1.00$
6. Changing the Q matrix generates a new control gain.
7. The arm reference (in red) and simulated arm response (in blue) are shown in the Arm (deg) scope. How did the arm response change? How did the pendulum response change in the Pendulum (deg) scope?

Settling time decreased while overshoot increased.

8. Set the third element in the Q matrix to 0, i.e.  $Q(3, 3) = 0$ .  
 9. Examine and describe the change in the Arm (deg) and Pendulum (deg) scope.

*settling time reduced  
overshoot increased*

10. By varying the diagonal elements of the Q matrix, design a balance controller that adheres to the following specifications:

Arm peak time less than 0.75 seconds:  $t_p \leq 0.75 s$

Motor voltage peak less than  $\pm 12.5 V$ :  $|V_m| \leq 12.5 V$

Pendulum angle less than 10.0 degrees:  $|\alpha| \leq 10.0 \text{ deg}$

$$|d| = 0 \rightarrow 7.5$$

$$|V_m| = 9$$

$$t_p = 0.64 s$$

11. Enter the Q and R matrices along with and control gain used to meet the specifications in Table 11.2.

Description	Symbol	Value	Unit
$Q(1,1)$	$Q_{1,1}$	10	
$Q(2,2)$	$Q_{2,2}$	15	
$Q(3,3)$	$Q_{3,3}$	20	
$Q(4,4)$	$Q_{4,4}$	25	
R	R	1	
K(1)	$k_{p,\theta}$	-3.16	V/rad
K(2)	$k_{p,\alpha}$	189.32	V/rad
K(3)	$k_{d,\theta}$	-5.46	V.s/rad
K(4)	$k_{d,\alpha}$	26.58	V.s/rad

Table 11.2

## Post Lab

- a) How is inertia related to stability? What does the pole-zero plot tells you about stability of the inverted pendulum system?

- b) How is the arm response of pendulum related to proportional and derivative gain?

**EXPERIMENT 12****Design of Light Intensity Control Using Hardware Based PID Controller(Open Ended Lab)****Problem Statement**

Design and implement a hardware based PID controller which will control the intensity of light based upon the reference value that has been provided by the user. You are available with the LM741 operational amplifiers, BJT and some assorted resistors and capacitors. For sensing the intensity of light, you have Light Dependent Resistor (LDR) and you have to think carefully that how can you sense the intensity variations in terms of voltage. As an output device, you have a filament-based DC bulb which can bear a maximum voltage of 5V and draws almost 500mA. Display your results on oscilloscope.

**EXPERIMENT 13****Root Locus Analysis****Objective:**

1. To plot the root locus for a given transfer function of the system using MATLAB
2. To use the root locus techniques to analyze the effect of loop gain upon system's transient response and stability

**13.1 Design and analysis using Root Locus**

Root locus, a graphical presentation of the closed-loop poles as a system parameter is varied. Root loci are used to study the effects of varying feedback gains on closed-loop pole locations. In turn, these locations provide indirect information on the time and frequency response. The root locus of an open-loop transfer function  $KG(s)H(s)$  is the plot of all possible closed loop pole locations with proportional gain K, which varies from zero to infinity.

In order to plot the root locus entire s-plane is required to be traversed but this is very tedious job. We can plot the root locus using computer program e.g. MATLAB. The open loop transfer function of the system shown in Figure 13.1 is  $KG(s)H(s)$ .

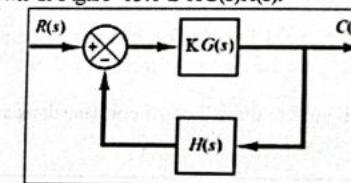


Figure 13.1: Closed Loop System with gain K  
The command used to plot root locus in MATLAB is

```

sys = tf(num,den)
rlocus(sys) //calculates and plots the root locus of the open-loop SISO model sys

```

Root locus of a given transfer function can be plotted using following set of commands in MATLAB:

```

clc;
closeall;
clearall;
num=[2 1]; % Define numerator of Open Loop Transfer Function
den=[2 -3 5]; % Define denominator of Open Loop Transfer Function
sys=tf (num,den)
rlocus (sys) % Draw root locus
title ('Root Locus')
Note: rlocus(A,B,C,D) command can be used when system is defined in State Space.

```

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Constant zeta and constant natural frequency lines can be omitted by using empty brackets in sgrid command.  
For example: `sgrid(z, [])` omits the constant  $\omega_n$  lines.

## Exercise 1

Consider the unity feedback system in figure 13.3:

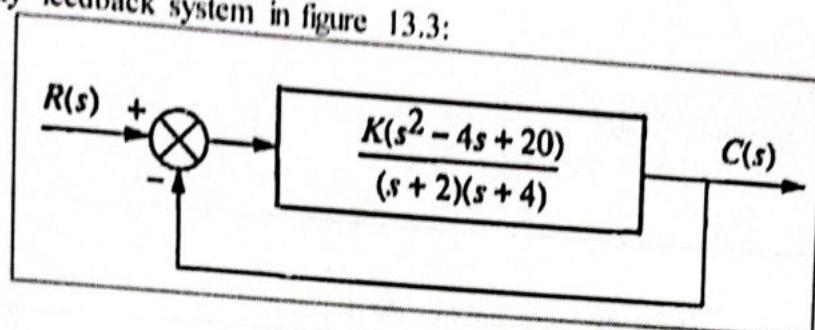


Figure 13.3: Control system

Use MATLAB to find following:

- Location of closed loop poles at  $K=0$  and  $K=\infty$
- Find the break points
- The point and gain where the locus crosses the  $j\omega$ -axis
- Find close loop pole at gain  $K=1$
- The range of  $K$  within which the system is stable
- Find the value of closed loop poles which yield 0.45 damping ratio. Also determine the value of gain  $K$  at this point.

a)  $K=0 \Rightarrow P = -4, -2$

~~$K=\infty \Rightarrow P = 2+4i, 2-i$~~

b)  $K = 0.248 \quad P = 2$

c)  $0.00 + 3.9099i$

d)  $P = -0.5 + 3.7081i$

~~$-0.5 - 3.7081i$~~

e)  $0.0248 - 1.5$

f)  $K = 0.4176$

~~$P = -1.5270 + 3.0237$~~

~~$-1.5270 - 3.0237$~~

## Exercise 2

Consider the unity feedback system where:

$$G(s) = \frac{K(s+5)(s+4)}{s^2}$$

Using MATLAB perform the following tasks.

- Determine the closed loop poles for  $K=0$  and  $K=\infty$

$K=0$

$P > \infty$

$P = 0$

$P = -5$



$$Z = 1$$

- b. Find the value of K that yields closed loop critically damped poles.

$$K = 80$$



- c. The point and gain where the locus crosses the  $j\omega$ -axis

No crossing

- d. Find the range of gain K for which system is stable

$$\text{Range} \Rightarrow 0 \rightarrow \infty$$



- e. Find the value of gain K and location of closed loop poles that yields a damping ratio of 0.75

$$K = 1.2372$$

$$P = -2.4885 + 2.2062i$$

$$-2.4885 - 2.2062i$$



- f. Find the point (closed loop pole) on root locus that yields zeta 0.667 and natural frequency 3 rad/sec.

$$K = 0.8010$$

$$P = -2.0015 + 2.112i$$

$$-2.0015 - 2.112i$$



### Exercise 3

For the following unity feedback system

$$G(s) = \frac{K(s+3)}{s(s+2)(s^2 + 4s + 5)}$$

Using MATLAB perform the following task.

- a. Determine the closed loop poles for K=0 and K= $\infty$

$$K = 0$$

$$P = 0 + 0i$$

$$-2 + 0i$$

$$-2 - i$$

$$-2$$

$$K = \infty$$

$$1.0e+03 *$$

$$0.499 + 0.866i$$

$$0.499 - 0.866i$$

$$-1.001 + 0i$$

$$-0.0030$$

- b. Find the break points and value of K at break point.

$$K_p = 34$$

$$P = -3.54 + 5.72e-08i$$

$$K_p = 1.07$$

$$P = -0.75 - 0.0363i$$



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- c. Find the location of closed loop poles and the value of K at which system is marginally stable?

$$K = 11.3$$

$$P = -0.00803 + 1.89i$$



- d. Find the value of K for which dominant closed loop poles have damping ratio 0.35

$$K = 3.4403$$

$$P = -2.5798 + 0.7632i$$

$$-2.5798 - 0.7632i$$

$$P = -0.4202 + 1.1178i$$

$$-0.4202 - 1.1178i$$



~~Hanif  
30/4/2025~~

Note: Save the MATLAB graphs in your PC in order to get verified from your instructor.

## EXPERIMENT 14

### Compensator Design using Root Locus

**Objective:**

To successfully design a compensator to meet the transient response and steady state error performance specifications

The experiment is designed to understand the graphical procedure for determining the stability of a control system based on root locus. MATLAB Control system tool box contains two root locus design GUI, sisotool and rtool. The main aim of this experiment is to use SISO toolbox to understand and design the compensators using root locus design.

**14.1 SISO Toolbox Tutorial**

SISOTOOL opens a SISO Design GUI for interactive compensator design. This GUI allows you to design a single-input/single-output (SISO) compensator using root locus.

By default, the SISO Design Tool

- Opens the Control and Estimation Tools Manager with a default SISO Design Task node.
- Opens the Graphical Tuning editor with root locus and open-loop Bode diagrams.
- In current architecture, compensatorC is placed in the forward path in series with the plant G.
- Assumes the prefilter, F, and the sensor, H, are unity gains. Once you specify G and H, they are fixed in the feedback structure.

To make the plant model and start the SISO Design Tool, at the MATLAB prompt type

`>>sisotool`

An empty SISO design tool opens as shown in figure 14.1.

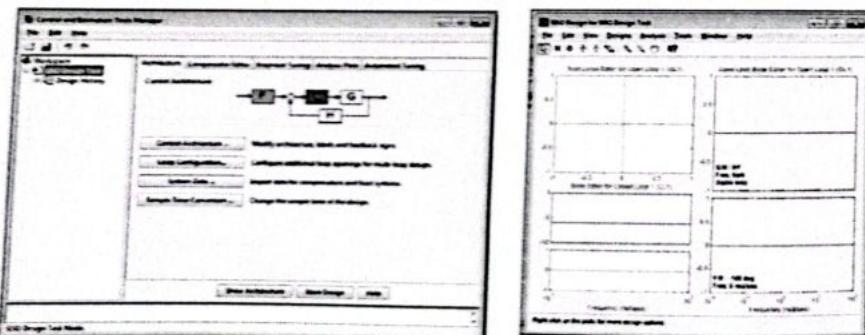


Figure 14.1: SISO Design Tool

The default control architecture is shown in the current architecture of SISO design tool.

**Example 1:**

In the Control System shown in the figure 14.2,  $G(s)$  is a proportional controller, K.

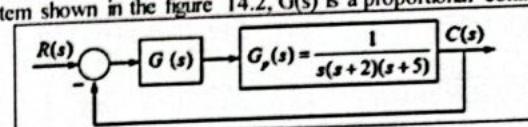


Figure 14.2: Control System

Using sisotool, determine the following:

a) Range of K for system stability

~~$M_{\min} = 0.001071$~~

~~$M_{\max} = 72.424$~~

b) Value of K for the complex dominant poles damping ratio 0.6. For this value of K obtain the step response and time domain specifications.

~~$K = 9.0063$~~

To start the SISO Design Tool, at the MATLAB prompt type

```
>>Gp = tf(1, [1 7 10 0])
>>sisotool
```

An empty SISO Design Tool opens. The SISO Design Tool by default assumes that the compensator is in the forward path. Select System Data to Import Model. This opens the System Data dialog box, which is shown in figure 14.3.

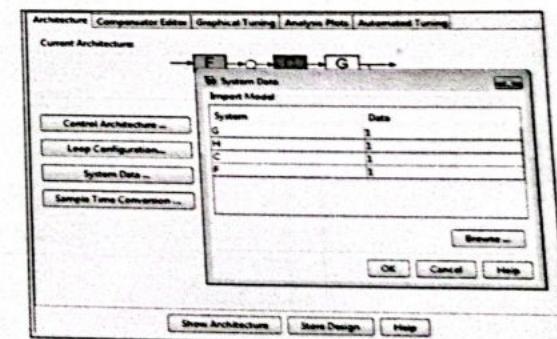


Figure 14.3: Import Model

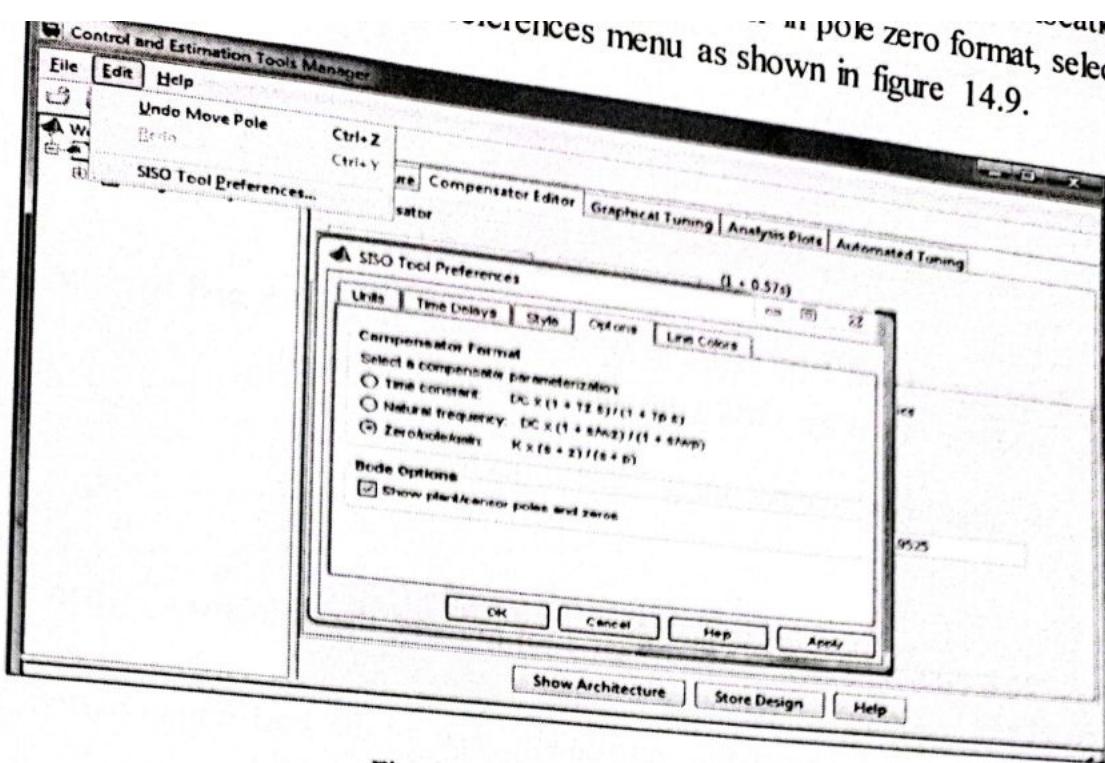


Figure 14.9: Compensator format

Also verify the systems time response using step response plot and Record these values in Table 14.1.

#### Observations:

Transfer function of Lead Compensator	$1 + 12.5s$
Transfer function of Compensated System	$1 + 12.5s$
Damping ratio	0.7071
Settling time	1.56 ✓

Table 14.1: Compensated system and its performance

$$C = 13.581 \times \frac{(1 + 0.57s)}{(1 + 0.13s)}$$

Exercise 1: Reevaluate the system's performance when the lead compensator's zero is placed at -2 in example 2. Record the values in Table 14.2

Transfer function of Lead Compensator	
Transfer function of Compensated System	
Damping ratio	0.7070
Settling time	2.23

Table 14.2: Compensated system and its performance

$$17.288 \times \frac{(1+0.5s)}{(1+0.0s)}$$

Exercise 2: A unity feedback system has open loop transfer function

$$G(s)H(s) = \frac{4}{s(s+2)}$$

$$K_v = 40$$

$$K_w = \lim_{s \rightarrow 0} s G(s) H(s)$$

It is desired that dominant closed loop poles provide damping ratio=0.5 and have an un-damped natural frequency 4rad/sec. Velocity error constant is required to be 40.

Verify that only gain adjustment cannot meet these objectives.

$$= \frac{8K_v K_c}{s(s+2)}$$

### Design 1

$$K_w = 2K_c$$

$$\frac{40}{2} = K_c$$

$$K_c = 20$$

- Design a lead compensator using SISO design tool to meet the objectives, when compensator zero is placed at -1.
- Using GUI, Record the peak overshoot and settling time of the lead-compensated system in Table 14.3.

Transient response	Value	Units
Peak Overshoot	5.61 %	%
Settling Time	2.48 s	s
Peak Time	0.97 s	s

Table 14.3: Transient response analysis of compensated system

### Design 2

- Design a lead compensator using SISO design tool to meet the objectives, when compensator zero is placed at -2.
- Using GUI, Record the peak overshoot and settling time of the lead-compensated system in Table 14.4.

Transient response	Value	Units
Peak Overshoot	16.11 %	%
Settling Time	2.02 s	s
Peak Time	0.896 s	s

Table 14.4: Transient response analysis of compensated system

## Lab Manual of Feedback Control Systems

Which design is preferable in exercise 2, in order to achieve the performance specifications and why?

Design 2 is preferable in order to achieve performance specifications because its settling time is less.

Hariya  
5/5/202

### POST LAB

Use the following plant:

$$P(s) = \frac{8.96}{0.00147s^2 + 0.01455s + 1}$$

Simulate its step response. Using a Lead Compensator, try to get the settling time to less than 0.5 sec (approx. 0.1 sec) while keeping the %OS lesser than 10%. Justify your design in the report.