Senario based sample question:

Book: 1- John Methew

2- Steven chapra

3-website

http://numericalmethods.eng.usf.edu

21.11 The following data were collected for the distance traveled versus time for a rocket:

t, s	0	25	50	75	100	125
y, km	0	32	58	78	92	100

Use numerical differentiation to estimate the rocket's velocity and acceleration at each time.

21.12 A jet fighter's position on an aircraft carrier's runway was timed during landing:

t, s	0	0.52	1.04	1.75	2.37	3.25	3.83
x, m	153	185	208	249	261	271	273

where x is the distance from the end of the carrier. Estimate **(a)** velocity (dx/dt) and **(b)** acceleration (dv/dt) using numerical differentiation.

21.13 Use the following data to find the velocity and acceleration at $t = 10$ seconds:

Time, t, s	0	2	4	6	8	10	12	14	16
Position, x, m	0	0.7	1.8	3.4	5.1	6.3	7.3	8.0	8.4

Use second-order correct **(a)** centered finite-difference, **(b)** forward finite-difference, and **(c)** backward finite-difference methods.

17.17 You measure the voltage drop V across a resistor for a number of different values of current i. The results are

i	0.25	0.75	1.25	1.5	2.0
V	−0.45	−0.6	0.70	1.88	6.0

Use first- through fourth-order polynomial interpolation to estimate the voltage drop for $i = 1.15$. Interpret your results.

17.18 The current in a wire is measured with great precision as a function of time:

t	0	0.1250	0.2500	0.3750	0.5000
i	0	6.24	7.75	4.85	0.0000

Determine i at $t = 0.23$.

17.19 The acceleration due to gravity at an altitude y above the surface of the earth is given by

y, m	0	30,000	60,000	90,000	120,000
g, m/s^2	9.8100	9.7487	9.6879	9.6278	9.5682

Compute g at $y = 55{,}000$ m.

To infer the surface shape of an object from images taken of a surface from three different directions, one needs to solve the following set of equations.

$$\begin{bmatrix} 0.2425 & 0 & -0.9701 \\ 0 & 0.2425 & -0.9701 \\ -0.2357 & -0.2357 & -0.9428 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 247 \\ 248 \\ 239 \end{bmatrix}$$

The right hand side values are the light intensities from the middle of the images, while the coefficient matrix is dependent on the light source directions with respect to the camera. The unknowns are the incident intensities that will determine the shape of the object.

Find the values of x_1, x_2, and x_3 using the Gauss-Seidel method. Use

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$$

as the initial guess and conduct two iterations.

19.4 Evaluate the following integral:

$$\int_{-2}^{4} (1 - x - 4x^3 + 2x^5)\, dx$$

(a) analytically, **(b)** single application of the trapezoidal rule, **(c)** composite trapezoidal rule with $n = 2$ and 4, **(d)** single application of Simpson's 1/3 rule, **(e)** Simpson's 3/8 rule, and **(f)** Boole's rule. For each of the numerical estimates **(b)** through **(f)**, determine the true percent relative error based on **(a)**.

19.5 The function

$$f(x) = e^{-x}$$

can be used to generate the following table of unequally spaced data:

x	0	0.1	0.3	0.5	0.7	0.95	1.2
$f(x)$	1	0.9048	0.7408	0.6065	0.4966	0.3867	0.3012

Evaluate the integral from $a = 0$ to $b = 0.6$ using **(a)** analytical means, **(b)** the trapezoidal rule, and **(c)** a combination of the trapezoidal and Simpson's rules wherever possible to attain the highest accuracy. For **(b)** and **(c)**, compute the true percent relative error.

Exercises for Composite Trapezoidal and Simpson's Rule

1. (i) Approximate each integral using the composite trapezoidal rule with $M = 10$.
   (ii) Approximate each integral using the composite Simpson rule with $M = 5$.

(a) $\int_{-1}^{1}(1 + x^2)^{-1}\,dx$ (b) $\int_0^1(2 + \sin(2\sqrt{x}))\,dx$ (c) $\int_{0.25}^{4} dx/\sqrt{x}$
(d) $\int_0^4 x^2e^{-x}\,dx$ (e) $\int_0^2 2x\cos(x)\,dx$ (f) $\int_0^{\pi}\sin(2x)e^{-x}\,dx$

2. *Length of a curve.* The arc length of the curve $y = f(x)$ over the interval $a \le x \le b$ is

$$\text{length} = \int_a^b \sqrt{1 + (f'(x)^2)}\,dx.$$

(i) Approximate the arc length of each function using the composite trapezoidal rule with $M = 10$.
(ii) Approximate the arc length of each function using the composite Simpson rule with $M = 5$.

(a) $f(x) = x^3$ for $0 \le x \le 1$
(b) $f(x) = \sin(x)$ for $0 \le x \le \pi/4$
(c) $f(x) = e^{-x}$ for $0 \le x \le 1$

3. *Surface area.* The solid of revolution obtained by rotating the region under the $y = f(x)$, where $a \le x \le b$, about the x-axis has surface area given by

$$\text{area} = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2}\,dx.$$

(i) Approximate the surface area using the composite trapezoidal rule with $M = 10$.
(ii) Approximate the surface area using the composite Simpson rule with $M = 5$.

(a) $f(x) = x^3$ for $0 \le x \le 1$
(b) $f(x) = \sin(x)$ for $0 \le x \le \pi/4$
(c) $f(x) = e^{-x}$ for $0 \le x \le 1$

A robot arm with a rapid laser scanner is doing a quick quality check on holes drilled in a $15'' \times 10''$ rectangular plate. The centers of the holes in the plate describe the path the arm needs to take, and the hole centers are located on a Cartesian coordinate system (with the origin at the bottom left corner of the plate) given by the specifications in Table 2.

Table 2 The coordinates of the holes on the plate.

x (in.)	y (in.)
2.00	7.2
4.25	7.1
5.25	6.0
7.81	5.0
9.20	3.5
10.60	5.0

If the laser is traversing from $x = 2.00$ to $x = 4.25$ to $x = 5.25$ in a quadratic path, what is the value of y at $x = 4.00$ using a second order Lagrange polynomial? Find the absolute relative approximate error for the second order polynomial approximation.

If the laser is traversing from $x = 2$ to $x = 4.25$ in a linear path, what is the value of y at $x = 4.00$ using the Lagrangian method and a first order polynomial?