MAT101 Oblig 2 Ole Westby

Oppgave 1) Finn grenseverdiene

a) $\lim\limits_{t \to 2} \frac{t-2}{t^2-4}$

$\lim\limits_{t \to 2} \left(\frac{\cancel{t-2}}{(t+2)\cancel{(t-2)}}\right)$

$\lim\limits_{t \to 2} \left(\frac{1}{t+2}\right)$ $t = 2$

$\frac{1}{2+2} = \underline{\underline{\frac{1}{4}}}$

b) $\lim\limits_{x \to -2} \frac{x^2 - x - 6}{x^2 + 3x + 2}$

$\frac{\cancel{(x+2)}(x-3)}{(x+1)\cancel{(x+2)}}$

$\lim\limits_{x \to -2} \frac{x-3}{x+1}$ $x = -2$

$\frac{-2-3}{-2+1} = \frac{-5}{-1} = \underline{\underline{5}}$

c) $\lim_{x \to 3} \frac{2x^2 - x + 3}{x+3}$ $x = 3$

$\frac{2 \cdot 3^2 - 3 + 3}{6} = \frac{2 \cdot 9 - 3 + 3}{6} = \frac{18}{6} = \underline{\underline{3}}$

d) $\lim_{x \to \infty} \frac{3x^3 + 1}{6x^3 + 2x^2 + 4x}$

$\lim_{x \to \infty} \frac{3 + \frac{1}{x^3}}{6 + \frac{2}{x} + \frac{4}{x^2}}$ $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

$\frac{\lim_{x \to \infty} 3 + \frac{1}{x^3}}{\lim_{x \to \infty} 6 + \frac{2}{x} + \frac{4}{x^2}} = \frac{\lim_{x \to \infty} \frac{1}{x^2} + \lim_{x \to \infty} 3 = 3 + \lim_{x \to \infty} \frac{1}{x^2} = 3 +}{\lim_{x \to \infty} 6 + \lim_{x \to \infty} \frac{2}{x} + \lim_{x \to \infty} \frac{4}{x^2}}$

$= \frac{3}{6 + \lim_{x \to \infty} \frac{2}{x} + \lim_{x \to \infty} \frac{4}{x^2}} = 6 + 2 \lim_{x \to \infty} \frac{1}{x}$

$= \frac{3}{6 + \lim_{x \to \infty} \frac{4}{x^2} + 2\left(\frac{\lim_{x \to \infty} 1}{\lim_{x \to \infty} x}\right)}$

$= \frac{3}{6 + \lim_{x \to \infty} \frac{4}{x^2} + \cancel{2(0)}} = 6 + 4 \cancel{\lim_{x \to \infty} \frac{1}{x^2}}$

$= \frac{3}{6} = \underline{\underline{\frac{1}{2}}}$

e) $\lim\limits_{x \to 1} \frac{x-1}{|x-1|}$

$\lim\limits_{x \to 1^-} \frac{x-1}{1-x} = -1$

De er ikke det samme

$\neq$ dvs. $\lim\limits_{x \to 1} \frac{x-1}{|x-1|}$ Stemmer ikke

$\lim\limits_{x \to 1^+} \frac{x-1}{1-x} = 1$

Oppgave 2)

Finn vertikale og horisontale asymptoter til

$$f(x) = \frac{x^2 + 2x - 1}{x^2 - 1}$$

V. asymptote: $x^2 - 1 = 0$

$x^2 = 1$

$\underline{\underline{x = 1}} \vee \underline{\underline{x = -1}}$

H. asymptote: $\lim\limits_{x \to \infty} \frac{x^2 + 2x - 1}{x^2 - 1}$

teller og nevner har samme denominator,

$x: \frac{1}{1} = \underline{\underline{1}}$

Oppgave 3)

Finn sum:

a) $2 + 3 + \frac{9}{2} + \frac{27}{4} + \dots + \frac{3^{20}}{2^{19}}$ $S = a \cdot \frac{1-k^n}{1-k}$

$S = 2\left(\frac{1-1{,}5^{21}}{1-1{,}5}\right) \approx \underline{\underline{19\ 947}} \approx \underline{\underline{2 \cdot 10^4}}$

b)

$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1024} + \dots$

$\sum_{n=0}^{\infty} ak^n = \frac{a}{1-k}$ $a + ak + ak^2 + ak^3, \dots$

$\frac{1}{4} + \frac{1}{4} \cdot \frac{1}{4}$

$= \frac{\frac{1}{4}}{1-\frac{1}{4}} =$ $\boxed{\frac{\frac{a}{b}}{c} = \frac{a}{c \cdot b}}$

$= \underline{\underline{\frac{1}{3}}}$

c) $-1-\frac{2+3}{5}-\frac{2^2+3^2}{5^2}-\frac{2^3+3^3}{5^3}-\dots$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad \text{dersom } |a|<1$$

$$S_{n,1} = 1+\frac{2}{5}+\left(\frac{2}{5}\right)^2+\dots+\left(\frac{2}{5}\right)^{n-1}, \quad n \geq 1$$

$$S_{n,2} = 1+\frac{3}{5}+\left(\frac{3}{5}\right)^2+\dots+\left(\frac{3}{5}\right)^{n-1}, \quad n \geq 1$$

S_{n_1} og S_{n_2} konvergerer fordi $|k|<1$

$$S = S_1+S_2 = \frac{a_{1,1}}{1-k_1}+\frac{a_{1,2}}{1-k_2} = \frac{1}{1-\frac{2}{5}}+\frac{1}{1-\frac{3}{5}} = \frac{5}{3}+\frac{5}{2} = \frac{25}{6}$$

$$\text{Sum} = 1-(S_1+S_2) = 1-\frac{25}{6} = \underline{\underline{-\frac{19}{6}}}$$

Oppgave 4)

a) $M(t) = C_0 + C\cos\frac{2\pi}{T}(t - t_0)$

Bestem C_0, C, T og t_0

$C_0 = 150$

$C = 50$

$T = 12$

$t_0 = 8$

b) Bestem a, b, og ω

$M(t) = C_0 + a\cos\omega t + b\sin\omega t$

$150 + 50\cos\left(\frac{\pi}{6}\right)(t-8)$

$\cos(a-b) = \cos a \cdot \cos b + \sin a + \sin b$

$150 + 50\left(\cos\frac{\pi \cancel{t}}{6}\cos\frac{8\pi}{6} + \sin\frac{\pi t}{6}\sin\frac{8\pi}{6}\right.$

$150 + 50\left(\cos\frac{4\pi}{3} \cdot \cos\frac{\pi}{6}t + \sin\frac{4\pi}{3} \cdot \sin\frac{\pi}{6} \cdot t\right)$

$a = 50\cos\frac{4\pi}{3}$

$b = 50\sin\frac{4\pi}{3}$

$\omega = \frac{\pi}{6}$

g

$M(t) = 150 = 150 + 50\cos\left(\frac{\pi}{6}(t-8)\right)$

$0 = 50\cos\left(\frac{\pi}{6}(t-8)\right)$

$\frac{\pi}{2} \vee \frac{3\pi}{2}$ gir cosinus verdi $= 0$

$\frac{\pi}{6}(t-8) = \frac{\pi}{2} + 2\pi \cdot n$

og

$\frac{\pi}{6}(t-8) = \frac{3\pi}{2} + 2\pi \cdot n$

Som gir $t = 11$ og $\cancel{t = 17}$ går ikke

Om vi setter det inn i geogebra får vi også $t = 5$

<u>Dermed når $t = 5$ og 11 er $M = 150$</u>

Oppgave 5)

a) Finn alle løsninger som oppfyller $-2\pi \leq t \leq 2\pi$

i) $\cos(t) = -\frac{\sqrt{3}}{2}$

mulige løsninger = 1. $\frac{7\pi}{6}$, 2. $\frac{5\pi}{6}$

3. $-\frac{7\pi}{6}$, 4. $-\frac{5\pi}{6}$

ii) $\sin(t) = \cos(t)$ 1. $\frac{5\pi}{4}$ 2. $\frac{\pi}{4}$

3. $-\frac{5\pi}{4}$ 4. $-\frac{\pi}{4}$

iii) $\sin(t) = \frac{1}{2}$ 1. $\frac{5\pi}{6}$ 2. $\frac{\pi}{6}$

3. $-\frac{5\pi}{6}$ 4. $-\frac{\pi}{6}$

b) $\cos(3t) + \sin(3t)$ på formen $C\cos(\omega(t - t_0))$

- $\cos(u - v) = \cos u \cdot \sin v + \sin u \cdot \cos v$

- $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

løsning:

$$\cos(3t) + \sin(3t) = \sqrt{2}\left(\cos(3t)\cdot\frac{1}{\sqrt{2}} + \sin(3t)\cdot\frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}\left(\cos(3t)\sin\frac{\pi}{4} + \sin(3t)\cos\frac{\pi}{4}\right)$$

$$= \sqrt{2}\cos\left(3t - \frac{\pi}{4}\right)$$

$$= \underbrace{\sqrt{2}}_{C}\cos \underbrace{3}_{\omega}\left(t - \underbrace{\frac{\pi}{12}}_{t_0}\right)$$

Oppgave 6)

a) $10^{x^2-1} = 1000$

$10^{x^2-1} = 10^3$

$\left[a^{f(x)} = a^{g(x)}\right]$

$\downarrow$

$f(x) = g(x)$

$x^2 - 1 = 3$

$x^2 = 4$

$\underline{\underline{x = 2 \lor x = -2}}$

b) $\ln(x^2 - 5) = 2\ln(x-1)$

$2\ln(x-1) = \ln((x-1)^2)$

$\ln(x^2-5) = \ln((x-1)^2)$

Stryker ln på begge sider

$x^2 - 5 = (x-1)^2$

$x^2 - 5 = x^2 - 2x + 1$

$\cancel{x^2} = \cancel{x^2} - 2x + 6$

$2x = 6$

$\underline{\underline{x = 3}}$

c) Finn a og c i

$y = ca^x$ når $x = \frac{1}{2}$ gir $y = 6$ og

$x = 2$ gir $y = 162$

II: $6 = ca^{1/2}$

$a^{1/2} = \frac{6}{c}$

$9^{1/2} = \frac{6}{c}$

$9^{1/2}c = 6$

$3c = 6$

$\underline{\underline{c = 2}}$

I:

$162 = \left(\frac{6}{a^{1/2}}\right) \cdot a^2$

$162 = \frac{6a^2}{a^{1/2}}$

$162 = 6a^{2-\frac{1}{2}}$

$27 = a^{3/2}$

$\left(a^{3/2}\right)^2 = 27^2$

$a^3 = 729$

$a = \sqrt[3]{729}$

$\underline{\underline{a = 9}}$

Oppgave 7)

$$D(t) = ca^{t}$$

t ↗ tid (s)

↓ Temp. diff

egget er 100 grader ut av kokende vann

rommet er 20 grader

$$D(0) = 100 - 20 = 80$$

Temperatur differansen $= 80$.

$$\underline{\underline{C = 80}}$$

a: $D(120) = 80 - 20 = 60$

$$ca^{t} = 60$$

$$80a^{120}$$

$$a^{120} = \frac{60}{80}$$

$$\underline{\underline{a = \sqrt[120]{\frac{3}{4}}}}$$

tid når egget er 40°C:

$$ca^{t} = 20$$

$$80\left(\sqrt[120]{\frac{3}{4}}\right)^{t} = 20$$

$$\sqrt[120]{\frac{3}{4}}^{\,t} = \frac{1}{4} = \ln \sqrt[120]{\frac{3}{4}}^{\,t} = \ln\frac{1}{4}$$

$$t \ln \sqrt[120]{\frac{3}{4}} = \ln\frac{1}{4} = t = \frac{\ln\frac{1}{4}}{\ln\sqrt[120]{\frac{3}{4}}}$$

eller også $\underline{x = -120 \cdot \frac{\ln(4)}{\ln(3) - \ln(4)}}$

simplified $= \underline{-240 \log_{\frac{3}{4}}(2)}$