Oppgave 1

MAT 101 Oblig 3 Ole Kristian Westby

Oppgave 1)

Grenseverdier:

a) $\lim_{x\to 0} \frac{2e^{-3x}-2}{e^{4x}-1}$

$\lim_{x\to 0} \frac{2\frac{1}{e^{3x}}-2}{e^{4x}-1}$

$\lim_{x\to 0} \frac{\frac{2}{e^{3x}}-2}{e^{4x}-1}$

$\lim_{x\to 0} \frac{\frac{2}{e^{3x}}-\frac{2e^{3x}}{e^{3x}}}{e^{4x}-1}$

$\lim_{x\to 0} \frac{\frac{2-2e^{3x}}{e^{3x}}}{e^{4x}-1}$

$\lim_{x\to 0} \frac{2-2e^{3x}}{(e^{4x}-1)e^{3x}}$

a^2-b^2

$(a-b)(a+b)$

$\lim_{x\to 0} \frac{2(1-e^{3x})}{(e^{4x}-1)(e^{3x})}$

a^3-b^3

$(a-b)(a^2+ab+b^2)$

$\lim_{x\to 0} \frac{2(1-e^{3x})}{(e^{2x}-1)(e^{2x}+1)(e^{3x})} = \lim_{x\to 0} \frac{2(1-e^{x})(1+e^{x}+e^{2x})}{e^{3x}(e^{x}+1)(e^{x}-1)(e^{2x}+1)}$

$\lim_{x\to 0} \frac{2(-\cancel{(e^{x}-1)})(1+e^{x}+e^{2x})}{e^{3x}\cancel{(e^{x}-1)}(e^{x}+1)(e^{2x}+1)}$

$\lim_{x\to 0} \frac{2(-1)(1+e^{x}+e^{2x})}{e^{3x}(e^{2x}+1)(e^{x}+1)}$

$\lim_{x\to 0} -\left(\frac{2+2e^{x}+2e^{2x}}{(e^{4x}+e^{3x})(e^{2x}+1)}\right)$

$\lim_{x\to 0} -\left(\frac{2+2e^{x}+2e^{2x}}{e^{6x}+e^{4x}+e^{5x}+e^{3x}}\right)$

$-\frac{2+2e^{0}+2e^{2\cdot 0}}{e^{6\cdot 0}+e^{4\cdot 0}+e^{5\cdot 0}+e^{3\cdot 0}}$

$-\frac{2+2+2}{4} = -\frac{6}{4} = \underline{\underline{-\frac{3}{2}}}$

1b) $\lim_{x\to 2} \frac{\cos\left(\frac{\pi}{4}x\right)}{3x-6} = \lim_{x\to 2} \frac{\cos\left(\frac{\pi}{4}x\right)}{3(x-2)}$

$= \frac{\lim_{x\to 2} \cos\left(\frac{\pi}{4}x\right)}{\lim_{x\to 2} 3(x-2)} \quad \frac{0}{0}$ ✓ ok.

$\lim_{x\to n} \left(\frac{f(x)}{g(x)}\right) = \lim_{x\to n} \left(\frac{f'(x)}{g'(x)}\right)$

$\lim_{x\to 2} \frac{\frac{d}{dx}\left(\cos\left(\frac{\pi}{4}x\right)\right)}{\frac{d}{dx}(3x-6)} =$

$-\sin\left(\frac{\pi x}{4}\right)\left(\frac{\pi}{4}\right) = -\frac{\pi \sin\left(\frac{\pi x}{4}\right)}{4}$

$-\frac{\frac{\pi \sin\left(\frac{\pi x}{4}\right)}{4}}{3} = -\frac{\pi \sin\left(\frac{\pi x}{4}\right)}{12} = -\lim_{x\to 2} \frac{\pi \sin\left(\frac{\pi x}{4}\right)}{12}$

$= -\frac{\lim_{x\to 2}\left(\pi \sin\left(\frac{\pi x}{4}\right)\right)}{\lim_{x\to 2}(12)} = -\frac{\pi \lim_{x\to 2}\left(\sin\left(\frac{\pi x}{4}\right)\right)}{12}$

$= -\frac{\pi \sin\left(\frac{\pi}{4} \cdot \lim_{x\to 2}(x)\right)}{12} = -\frac{\pi \sin\left(\frac{\pi}{2}\right)}{12} = \underline{\underline{-\frac{\pi}{12}}}$

1c) $\lim_{x \to 0} \dfrac{4^x - e^x}{\pi^x - \cos(4x)}$

deriverer oppe og nede

$\lim_{x \to 0} \dfrac{\ln(2) \cdot 2^{2x+1} - e^x}{\ln(\pi)\,\pi^x + 4\sin(4x)}$

$\lim_{x \to 0} \dfrac{\ln(2) \cdot 2^{2\cdot 0+1} - e^0}{\ln(\pi)\,\pi^0 + 4\sin(4\cdot 0)}$

$\dfrac{\ln(2) \cdot 2 - 1}{\ln(\pi) + 4 \cdot 0}$

$\dfrac{2\ln(2) - 1}{\ln(\pi)}$

1d) $\lim\limits_{x\to\infty} \frac{6^x+4^x}{x^2-2\cdot 6^x}$

$$\lim_{x\to\infty} \frac{\frac{6^x}{6^x}+\frac{4^x}{6^x}}{\frac{x^2}{6^x}-\frac{2\cdot 6^x}{6^x}} = \lim_{x\to\infty} \frac{1+\left(\frac{2}{3}\right)^x}{\frac{x^2}{6^x}-2}$$

$$\frac{\lim\limits_{x\to\infty}\left(1+\left(\frac{2}{3}\right)^x\right)}{\lim\limits_{x\to\infty}\left(\frac{x^2}{6^x}-2\right)} = \frac{\lim\limits_{x\to\infty} 1 + \lim\limits_{x\to\infty}\left(\frac{2}{3}\right)^x}{\lim\limits_{x\to\infty}\left(\frac{x^2}{6^x}\right) + \lim\limits_{x\to\infty}(-2)} = \frac{1}{-2} = \underline{\underline{-\frac{1}{2}}}$$

Oppgave 2

Oppgave 2)

$$P_n(x) = \sum_{k=0}^{n} f^{(k)}(x_0)(x-x_0)^k$$

$f(x) = e^{2x}$

a) $T_2(x)$, Taylorpolynomet grad 2 til f om $x=0$

Finn tilnærmet verdi til $e^{0,2}$

e^{2x} $\quad x = 0,1$

$e^{2\cdot 0,1} = e^{0,2} \approx \underline{\underline{1,22}}$

b) $T_2(x)$ til f om $x=1$

Tilnærmet verdi til $e^{2,2}-4$

$2 \cdot 1,1 = 2,2$

$T_2(1,1) = e^{2\cdot 1,1} - 4 = \underline{\underline{5.02}}$

Oppgave 3

Oppgave 3)

a) Volum Kule:

$$V = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi r^3 = V$$

$$4\pi r^3 = 3V$$

$$r^3 = \frac{3V}{4\pi}$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

b) $r'(t)$ $V = 1000t$

$$3V = 3000t$$

$$\frac{dr}{dt}\sqrt[3]{\frac{3000t}{4\pi}} = \frac{d}{du}\left(\sqrt[3]{u}\right)\frac{d}{dt}\left(\frac{3000t}{4\pi}\right)$$

$$\boxed{\sqrt[n]{a} = a^{\frac{1}{n}}}$$

$$\boxed{\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}}$$

$$= \frac{d}{du}\left(u^{1/3}\right) = \frac{1}{3}u^{\frac{1}{3}-1} = \frac{1}{3u^{2/3}}$$

$$(a \cdot f)' = a \cdot f'$$

$$= \frac{1}{3u^{2/3}}\frac{d}{dt}\left(\frac{3000t}{4\pi}\right) \rightarrow \frac{d}{dt}\left(\frac{3000t}{4\pi}\right) = \frac{3000}{4\pi}\frac{d}{dt}(t)$$

Setter alt inn for u:

$$= \frac{3000}{4\pi}\cdot 1 = \frac{750}{\pi}$$

$$\frac{1}{3\left(\frac{3000t}{4\pi}\right)^{\frac{2}{3}}}\cdot\left(\frac{750}{\pi}\right)$$

$$= ?$$

$$\frac{1}{3\left(\frac{3000t}{4\pi}\right)^{2/3}} \cdot \frac{750}{\pi}$$

$$\left(\frac{3000t}{4\pi}\right)^{2/3} \quad :4$$

$$\left(\frac{750t}{\pi}\right)^{2/3} = \frac{(750t)^{2/3}}{\pi^{2/3}} \qquad \left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

$$= \frac{750^{2/3} \cdot t^{2/3}}{\pi^{2/3}} = \qquad 750 = 2 \cdot 3 \cdot 5^3$$

$$\frac{(2 \cdot 3 \cdot 5^3)^{2/3} \cdot t^{2/3}}{\pi^{2/3}} = \frac{((2 \cdot 3 \cdot 5^3)^{1/3})^2 \cdot t^{2/3}}{\pi^{2/3}}$$

$$\frac{\left(\left(5^{\cancel{3}}\right)^{\cancel{1/3}} \cdot 6^{1/3}\right)^2 t^{2/3}}{\pi^{2/3}} = \frac{1}{3 \cdot \frac{5^2 \cdot 6^{2/3} \cdot t^{2/3}}{\pi^{2/3}}}$$

$$= \frac{1}{\frac{75 \cdot 6^{2/3} \cdot t^{2/3}}{\pi^{2/3}}} = \frac{\pi^{2/3}}{75 \cdot 6^{2/3} t^{2/3}} \qquad \frac{1}{\frac{b}{c}} = \frac{c}{b}$$

$$\frac{\pi^{2/3} \cdot 750}{6^{2/3} \pi^{1/3} t^{2/3}} = \frac{10\pi^{2/3}}{6^{2/3} \pi t^{2/3}} \qquad \frac{x^a}{x^b} = \frac{1}{x^{b-a}} \quad :75$$

$$\downarrow$$

$$\frac{\pi^{2/3}}{\pi} = \frac{1}{\pi^{1-2/3}}$$

$$= \frac{10}{6^{2/3} \pi^{1/3} t^{2/3}} = \frac{2 \cdot 5}{2^{2/3} \cdot 3^{2/3} \pi^{1/3} t^{2/3}} \qquad 10 = 2 \cdot 5$$

$$6^{2/3} = 2^{2/3} \cdot 3^{2/3}$$

$$= \frac{5 \cdot 2^{1/3}}{3^{2/3} \pi^{1/3} \cdot t^{2/3}}$$

3c) $\dfrac{5\cdot 2^{1/3}}{3^{2/3}\pi^{1/3}4^{2/3}}$ $\qquad 4 - 2$

$$\frac{2^{1/3}}{2^{2/3}} = \frac{1}{2^{2/3-1/3}} \qquad \frac{x^a}{x^b} = \frac{1}{x^{b-a}}$$

$$\frac{5}{3^{2/3}\pi^{1/3}2^{1/3}} \approx \underline{\underline{1{,}3\ \text{cm/s}}}$$

Oppgave 4

Oppgave 4) $h(t) = 29.7^\circ + 23.4^\circ \cdot \cos\left(\frac{2\pi}{365}(t-172)\right)$

a) 20. juni.

2020 er så skuddår (+1)

I geogebra → a= Max(h, 0, 365)

$\underline{\underline{a = 172}}$ som er den 21 juni

358 = julaften (24. december)

b) på julaften er $h(358) \underline{= 6{,}34^\circ}$

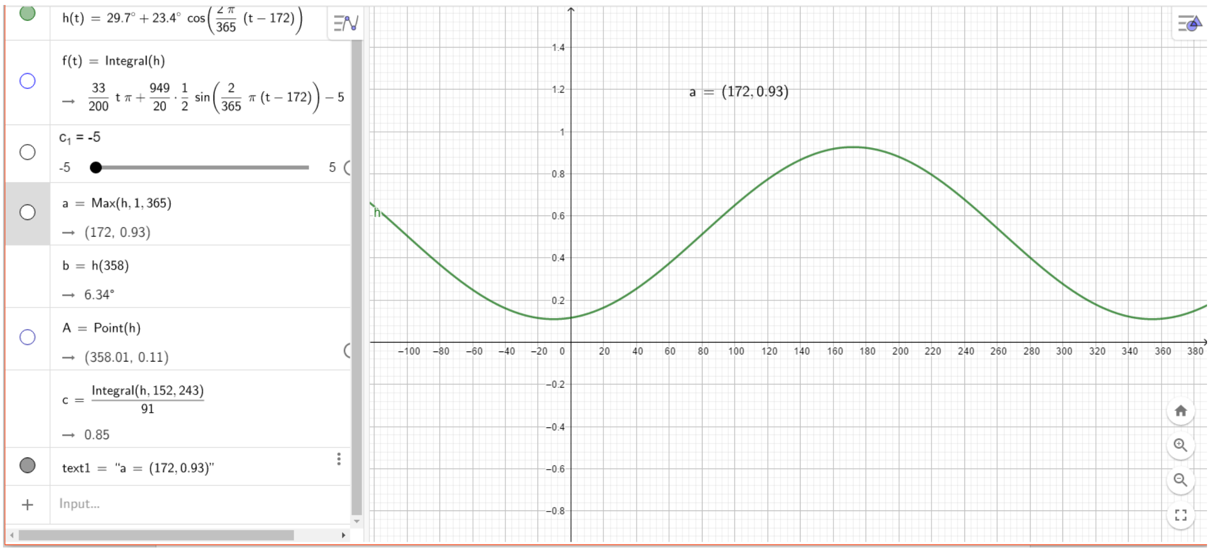
$h(358) = 29{,}7^\circ + 23{,}4^\circ \cdot \cos\left(\frac{2\pi}{365}(358-172)\right) \approx 0{,}11 \underline{\underline{= 6{,}34^\circ}}$

c) Integral av h fra 152, 243

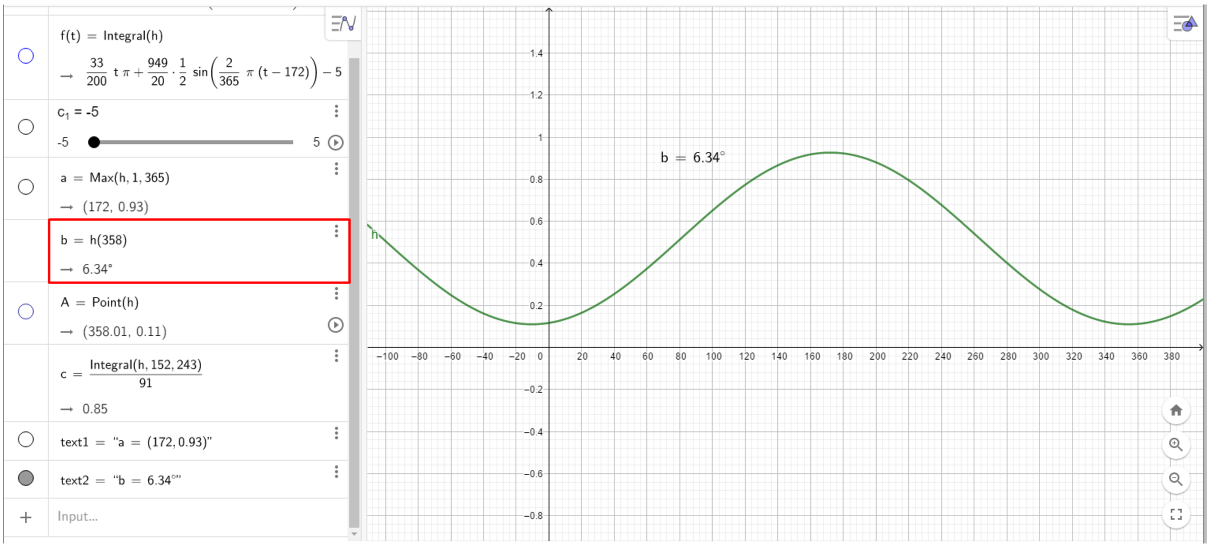
Delt på 243 − 152 gir 0,85

$\underline{\underline{\text{eller } 48{,}70^\circ}}$

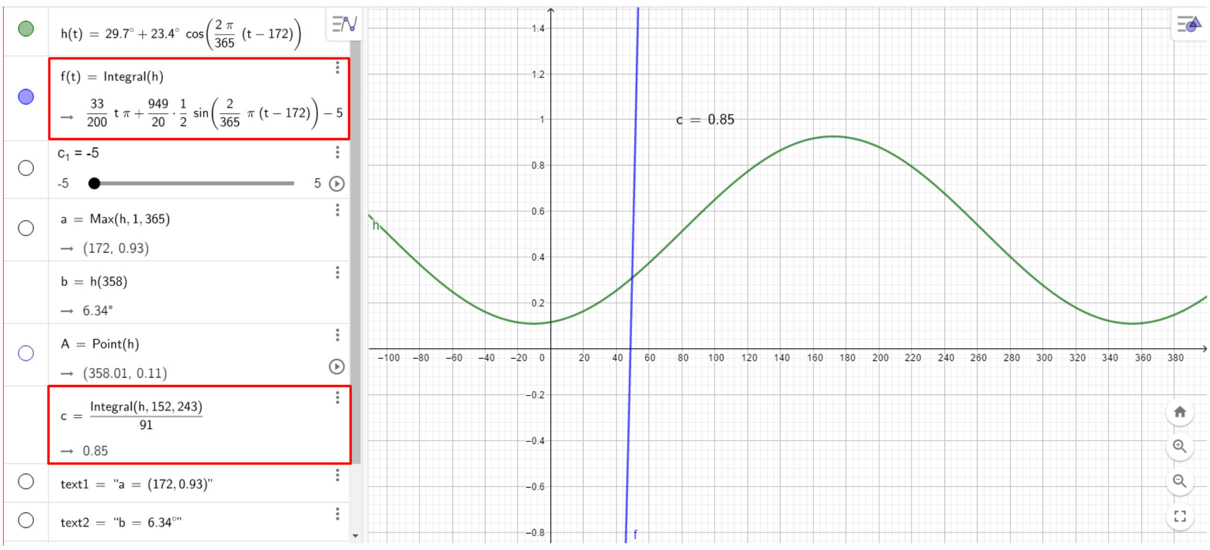
a)



b)



c)

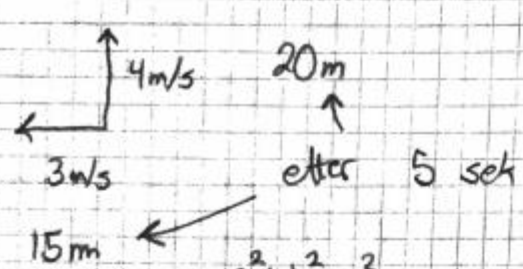


0.85 radianer er 48.70 grader.

Oppgave 5

Oppgave 5)

a)



$a^2+b^2=c^2$

$15^2+20^2=c^2$

$c^2=225+400$

$c^2=625$

$\underline{\underline{c=25}}$ ✓ $-\cancel{25}$

Kan ikke være − meter

b) De løper fra hverandre 3 m/s og 4 m/s,

Pytagoras, $3^2+4^2=c^2$

$25=c^2$

$\underline{\underline{c=5 \text{ m/s}}}$