Oblig 4 MAT101 Ole Kristian Wrolby

Oppgave 1) a)

generelle løsning til $\frac{dy}{dt} = -5y$

$\frac{dy}{dt} = -5y$ sub y' inn

$y' = -5y$ $\quad :y$

$\frac{y'}{y} = -5 = \frac{1}{y} y' = -5$

$= \int \frac{1}{y} dy = \int -5 dt$

$\int \frac{1}{y} dy = -5t + c$

$\ln(y) = -5t + c$

$e^{\ln(y)} = e^{-5t+c}$ $\quad$ ^e

$y = e^{-5t+c} = e^{c_1} \cdot e^{-5t}$

$\underline{\underline{y = ce^{-5t}}}$

1b) $y(0) = 5$

$t = 0$
$y = 5$

$5 = ce^{0}$

$5 = c \quad \Rightarrow \quad \underline{\underline{y = 5e^{-5t}}}$

Oppgave 2) a)

$$\int_0^1 x^2 \cos\left(\frac{x^3}{4}\right) dx$$

$u = x^3$

$u' = 3x^2$

$$\int x^2 \cos\left(\frac{u}{4}\right) dx$$

$du = 3x^2 dx$

$dx = \frac{1}{3x^2} du$

$$x^2 \cos\left(\frac{u}{4}\right) \frac{1}{3x^2} du$$

$$\int \frac{\cancel{x^2} \cos\left(\frac{u}{4}\right)}{3\cancel{x^2}} du$$

$u = x^3$

$x = \sqrt[3]{u}$

$$\int \frac{\cos\left(\frac{u}{4}\right)}{3} du = \int \frac{u \cos\left(\frac{u^{2/3}}{4}\right)}{2x} du$$

$u = x^3$	$u = x^3$
$u = 0^3$	$u = 1^3$
$u = 0$	$u = 1$

$$\int_0^1 \frac{\cos\left(\frac{u}{4}\right)}{3} du$$

$v = \frac{u}{4}$

$$\frac{1}{3} \int_0^1 \cos\left(\frac{u}{4}\right) du$$

$\frac{dv}{du} = \frac{1}{4}$

$dv = \frac{1}{4} du$

$du = 4 dv$

$v = \frac{u}{4}$	$v = 0$
	$v = \frac{1}{4}$

$$\rightarrow \frac{1}{3} \int_0^{1/4} \cos(v) \cdot 4 dv$$

2a)

$\int \cos(v)\,dv = \sin(v)$

$\frac{1}{3}\int_0^{1/4} \cos(v) \cdot 4\,dv$

$\frac{4}{3}\int_0^{1/4} \cos(v)\,dv$ $\int \cos(v)\,dv = \sin(v)$

$\frac{4}{3}\left[\sin(v)\right]_0^{1/4}$

$\sin\frac{1}{4}$ og $\sin 0$

$\sin\frac{1}{4} \cancel{-0}$

$\frac{4}{3}\sin\frac{1}{4}$

2b) $\int (x-1)\sin(2x)\,dx$

$a(b-c) = ab - ac$

$\sin(2x)(x-1)$

$\sin(2x)x - \sin(2x)\cdot 1$

$x\sin(2x) - 1\cdot\sin(2x)$

$\int x\sin(2x) - \sin(2x)\,dx$

$\int uv' = uv - \int u'v$

① $\int x\sin(2x)\,dx$

$u = x$

$v' = \sin(2x)$

$u' = 1$

$v = -\frac{1}{2}\cos(2x)$

$x\left(-\frac{1}{2}\cos(2x)\right) - \int 1\left(-\frac{1}{2}\cos(2x)\right)dx$

$-x\frac{1}{2}\cos(2x) - \int -1\cdot\frac{1}{2}\cos(2x)\,dx$

$-x\frac{1}{2}\cos(2x) - \int -\frac{1}{2}\cos(2x)\,dx$

③ $\int -\frac{1}{2}\cos(2x)\,dx$

$-\frac{1}{2}\int \cos(2x)\,dx$ $\quad u = 2x$

$-\frac{1}{2}\int \cos(u)\frac{1}{2}\,du$

$-\frac{1}{2}\cdot\frac{1}{2}\sin(2x)$

② $\int \sin(2x)\,dx$

$\int \sin(u)\frac{1}{2}\,du$

$\frac{1}{2}\int \sin(u)\,du$

$\frac{1}{2}(-\cos(2x)) + c$

$$= -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) - \left(-\frac{1}{2}\cos(2x)\right)$$

$$= \underline{\underline{-\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + \frac{1}{2}\cos(2x) + C}}$$

Oppgave 3)

a) $y = 1 - x^2$

$y = x^2$

Skjæringspunkt når $1 - x^2 = x^2$

$1 - x^2 = x^2$

$1 = 2x^2$

$x^2 = \frac{1}{2}$

Skjæringspunkt er:

$\underline{x_1 = \sqrt{\frac{1}{2}} \quad , \quad x_2 = -\sqrt{\frac{1}{2}}}$

Arealet:

$$\iint dx\,dy = \int_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} \int_{x^2}^{1-x^2} dy\,dx = \int_{-\sqrt{\frac{1}{2}}}^{\sqrt{1/2}} \left[y\right]_{x^2}^{1-x^2} dx$$

$$\int_{-\sqrt{\frac{1}{2}}}^{\sqrt{1/2}} \left[1 - x^2 - x^2\right] dx = \int_{-\sqrt{\frac{1}{2}}}^{\sqrt{1/2}} 1 - 2x^2\,dx =$$

$$A = \left[x - \frac{2x^3}{3}\right]_{-\sqrt{1/2}}^{\sqrt{1/2}} = \left[\sqrt{\frac{1}{2}} - \frac{2}{3}\left(\sqrt{\frac{1}{2}}\right)^3\right] - \left[-\sqrt{\frac{1}{2}} - \frac{2}{3}\left(-\sqrt{\frac{1}{2}}\right)^3\right]$$

$$A = \left[\sqrt{\frac{1}{2}} - \frac{2}{3(2\sqrt{2})}\right] - \left[-\sqrt{\frac{1}{2}} - \frac{2}{3(2\sqrt{2})}\right] = \frac{2}{\sqrt{2}} - \frac{4}{6\sqrt{2}}$$

$$= \sqrt{2} - \frac{2}{8\sqrt{2}} = \sqrt{2} - \frac{\sqrt{2}}{3} = \frac{3(\sqrt{2}) - (\sqrt{2})}{3} = \underline{\underline{\frac{2\sqrt{2}}{3}}}$$

3b) $\int \frac{(\ln(5t))^2}{t} dt$

$\ln(5t) = u$

$= \frac{1}{5t} \cdot 5\, dt = du$

$\int u^2 du = \frac{u^3}{3} + C$

$\frac{1}{t} dt = du$ $\qquad (\ln 5t)' = \frac{1}{t}$

$= \frac{(\ln(5t))^3}{3} + C$

3c) $\displaystyle\int_0^{\pi/3} x\cos(3x)\,dx$

$u = x$

$v' = \dfrac{\sin(3x)}{3}$

$\dfrac{x\sin(3x)}{3} - \displaystyle\int \dfrac{\sin(3x)}{3}\,dx$

$\dfrac{x\sin(3x)}{3} - \dfrac{1}{3}\displaystyle\int \sin(3x)\,dx$

$u = 3x$

$-\dfrac{1}{3}\cdot\dfrac{1}{3}\displaystyle\int \sin u\,du$

$\left[x\dfrac{\sin(3x)}{3} - \dfrac{1}{9} - \cos(3x)\right]_0^{\pi/3}$

$\left[x\right]_a^b = xa - xb$

$\dfrac{\frac{\pi}{3}\sin\left(\cancel{3}\frac{\pi}{\cancel{3}}\right)}{3} + \dfrac{1}{9}\,\dfrac{\cos\left(\cancel{3}\frac{\pi}{\cancel{3}}\right)}{9} - \cancel{\dfrac{0\sin(3\cdot 0)}{3}} + \dfrac{\cos(3\cdot 0)}{9}$

$\cancel{\dfrac{\frac{\pi}{3}\sin(\pi)}{3}} + \dfrac{\cos(\pi)}{9} - \dfrac{\cos(0)}{9} = \dfrac{\frac{\pi}{3}\sin\pi}{3} + \dfrac{\cos\pi}{9}$

$= \dfrac{-1}{9} - \dfrac{-1}{9} = \dfrac{-2}{9}$

4a) $\frac{dx}{dt} = 3 - 0,1x \qquad x(0) = 5$

x'

$x' = 3 - 0,1x$

$\frac{x'}{3-0,1x} = 1 \quad = \quad \frac{1}{3-0,1x} x' = 1$

$\int \frac{1}{3-0,1x} dx = \int 1 dt = 1t + c_1$

$\frac{du}{dx} = -0,1 \qquad u = 3 - 0,1x$

$du = -0,1 dx \qquad \int \frac{1}{u}\left(-\frac{1}{0,1}\right) du$

$dx = \left(-\frac{1}{0,1}\right) du$

$\int \frac{-1}{0,1u} du$

$-10 \ln(3 - 0,1x) = t + c_1$

$-10 \int \frac{1}{u} du$

$t=0$, $x=5$: $-10 \ln(3 - 0,1 \cdot 5) = 0 + c_1$

$c_1 = -10 \ln(25) \qquad -10 \ln(u) + C_2$

$-10 \ln(3 - 0,1x) + C_2 = 1t + c_1$

$x = t + c_1 \qquad -10 \ln(3 - 0,1x) = t - 10 \ln(25)$

løse for x

gir $\underline{\underline{x = -25e^{-0,1t} + 30}}$

4) $\frac{dx}{dt} = 3 - 0{,}1x$

4a) $\frac{dx}{dt} + 0{,}1x = 3$

$x(t) = ce^{-0{,}1t} + \frac{3}{0{,}1} = ce^{-0{,}1t} + 30$

$x(0) = ce^{-0{,}1 \cdot 0} + 30 = 5$

$ce^{0} = -25$

$c = -25$

$x(t) = -25e^{-0{,}1t} + 30$

$\underline{\underline{x(t) = 30 - 25e^{-0{,}1t}}}$

5) $x(t) = 10$

$30 - 25e^{-0{,}1t} = 10$

$-25e^{-0{,}1t} = -20$

$e^{0{,}1t} = \frac{4}{5}$

$\ln e^{-0{,}1t} = \ln\frac{4}{5}$

$-0{,}1t = \ln\frac{4}{5} = \quad t = \frac{\ln\frac{4}{5}}{-0{,}1} \approx \underline{\underline{2{,}23}}$

4c) $x(t) = -25e^{-0{,}1t} + 30$

$\lim\limits_{t\to\infty} x(t)$

$\lim\limits_{t\to\infty} 30 + \lim\limits_{t\to\infty} -25e^{-0{,}1t}$

Ingen t^{n}

$30 + \left(g(t) \cdot f(t) \right)$

$-25 \lim\limits_{t\to\infty} -0{,}1t = -\infty \quad , \quad \lim\limits_{t\to\infty} e^{u} = 0$

$30 + (-25 \cdot 0)$

$= \underline{\underline{30}}$