Obligatorisk innlevering 2 Stat 110

Oppgave 1)

$E(x) = 4\%$, $V(x) = 0.49$, $E(y) = 6\%$, $V(y) = 0.64$

$$W = pX + (1-p)Y \qquad p(X,Y) = 0.3$$

$$W = 0.4x + 0.6Y$$

$$E(W) = 0.4 \cdot 0.04 + 0.6 \cdot 0.06 = \underline{0.052}$$

$$V(W) = (0.4^2 \cdot V(x)) + (0.6^2 \cdot V(y)) + (2 \cdot 0.4 \cdot 0.6 \cdot p \cdot V(x) \cdot V(y))$$

$$= 0.4^2 \cdot 0.49 + 0.6^2 \cdot 0.64 + 2 \cdot 0.4 \cdot 0.6 \cdot 0.3 \cdot \sqrt{0.49} \cdot \sqrt{0.64}$$

$$= 0.389 \approx \underline{\underline{0.39}}$$

Forventningen til $W = 0.052 \rightarrow 5.2\%$.

Den er høyere enn $E(x)$, men fortsatt lavere enn $E(y)$

Variansen $V(W) = 0.39$, dette er lavere enn $V(x)$ og $V(y)$

Oppgave 2)

$p(x,y) = P(X = x \cap Y = y)$

gitt ved:

x\y	0	1	2
0	0.2	0.1	0.1
1	0.05	0.05	0.15
2	0.2	0	0.15

$p_X(x) = \sum_y p(x,y)$

x/y	0	1	2	$p_X(X)$
0	0.2	0.1	0.1	0.4
1	0.05	0.05	0.15	0.25
2	0.2	0	0.15	0.35
$p_Y(Y)$	0.45	0.15	0.4	

a) $p(Y=2) = 0.4$

$p(X=2 \wedge Y=2) = 0.15$

$p(X=2 \mid Y=2) = \frac{P(X=2 \cap Y=2)}{P(Y=2)} = \frac{0.15}{0.4} = \underline{\underline{0.375}}$

$p(X=2 \mid Y=2) = \underbrace{0.375 \neq 0.35}$ $= P(X=2)$

X og Y, ikke uavhengige fordi

b) $E(x) = 0.95$, $E(Y) = 0.95$

$V(y) = 0.847$, $V(x) = 0.747$

$E(xy) = 0 \cdot 0 \cdot 0.2 + 0 \cdot 1 \cdot 0.1 + 0 \cdot 2 \cdot 0.1 + 1 \cdot 0 \cdot 0.05 + \overset{1 \cdot 1 \cdot 0.05}{1 \cdot 2} \cdot 0.15 + 2 \cdot 0 \cdot 0.2 + 2 \cdot 1 \cdot 0 + 2 \cdot 2 \cdot 0.15$

$E(xy) = \underline{0.95}$

$Cov(x,y) = E(xy) - E(x)E(y)$

$= 0.95 - (0.95 \cdot 0.95) = \underline{0.0475}$

$p = \frac{cov(x,y)}{\sqrt{V(x) \cdot V(y)}} = \frac{0.0475}{\sqrt{0.747 \cdot 0.847}} \approx \underline{0.0597}$

c) $E(Y|x=2)$, $V(Y|x=2)$

$E(Y|x=2) = \frac{E(x=2 \wedge Y)}{E(x=2)}$

$= \frac{2 \cdot 0 \cdot 0.2 + 2 \cdot 1 \cdot 0 + 2 \cdot 2 \cdot 0.15}{2 \cdot 0.2 + 2 \cdot 0 + 2 \cdot 0.15}$

$= \underline{\underline{0.857}}$

$V(Y|x=2) = E(y^2|x^2=2^2) - E(Y|x=2)^2$

$E(Y^2|x^2=2^2) = \frac{2^2 \cdot 0^2 \cdot 0.2 + 2^2 \cdot 1^2 \cdot 0 + 2^2 \cdot 2^2 \cdot 0.15}{2^2 \cdot 0.2 + 2^2 \cdot 0 + 2 \cdot 0.5}$

$= \underline{\underline{1.714}}$

$V(Y|x=2) = 1.714 - 0.857^2$

$= 0.9795 \approx \underline{0.98}$

3) $f(t) = 3e^{-3t}$, $t \geq 0$

$f(t) = \lambda \cdot e^{-\lambda x}$, $\lambda = 3$

$E(x) \frac{1}{\lambda} = E(t) \frac{1}{\lambda} = \underline{\underline{\frac{1}{3}}}$

$V(x) = \frac{1}{\lambda^2}$

$V(t) = \underline{\underline{\frac{1}{9}}}$

$f(x) = P(x \leq y) = 1 - e^{-\lambda x}$

$P(t > 1) = 1 - P(T < 1) = 1(1 - e^{-\frac{1}{3}})$

$= 0.7165 \approx \underline{\underline{0.717}}$

Sannsynlighet for $T > 1 = \underline{\underline{0.717}}$

Både T_1 og T_2 er uavhengige og da er $P(T_1) = P(T_2) = 0.717$

Sannsynligheten for at T_1 og T_2 er større enn 1 er,

$0.717 \cdot 0.717 = \underline{\underline{0.514}}$