Obligatorisk oppgave 03 STAT110

1)

a) $\bar{x}_1 = 120 \quad s_1 = 120$

$\bar{x}_2 = 140 \quad s_2 = 70$

$\bar{x}_3 = 240 \quad s_3 = 140$

$\bar{x}_{ref} = 354$

$s_{ref} = 210$

b)

Bruker formel + t-tabell

$n_1 = 18$

$n_2 = 20$

konfidensint. $= \bar{x} \pm 2 \frac{s}{\sqrt{n}}$

$n_3 = 13$

$n_{ref} = 17$

μ_1: $\frac{(1-.95)}{2} = 0.025$

$CI = 120 - 2.110 \cdot \frac{120}{\sqrt{18}} , 120 + 2.110 \cdot \frac{120}{\sqrt{18}}$

$CI = 60.3 , 179,6$

μ_2:

$CI = 140 - 2.093 \cdot \frac{70}{\sqrt{20}} , 140 + 2.093 \frac{70}{\sqrt{20}}$

$CI = 107.2 , 172,7$

μ_3:

$$CI = 240 - 2.179\frac{140}{\sqrt{13}},\ 240 + 2.179\frac{140}{\sqrt{13}}$$

$$CI = 155.3,\ 321.6$$

c) Statistisk hypotese:

Vi formulerer nullhypotese og alternativ hypotese

null hypotese $H_0: \mu_{ref}$

Å der stoppet det ö

2) X og Y, $X \sim \text{Poisson}(\lambda)$ og $Y \sim \text{Poisson}(2\lambda)$

$x = 2$, $y = 5$

a) Vis at log-likelihood funksjon er gitt ved:

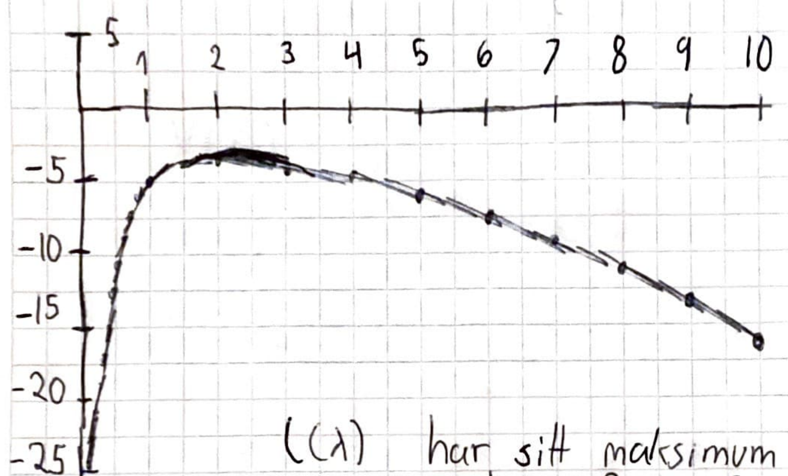
$$l(\lambda) = [5\ln(2) - \ln(2!) - \ln(5!)] + 7\ln\lambda - 3\lambda$$

$$\frac{e^{-\lambda}\lambda^2}{2!} \cdot \frac{e^{-2\lambda}(2\lambda)^5}{5!} \rightarrow$$ likelihood funksjon:

$$L(\lambda) = \frac{e^{-\lambda-2\lambda} \cdot \lambda^{2+5} \cdot 2^5}{2! \cdot 5!}$$

Ta $\ln(L(\lambda))$ og du får

at $l(\lambda) = [5\ln(2) - \ln(2!) - \ln(5!)] + 7\ln\lambda - 3\lambda$



$l(\lambda)$ har sitt maksimum et sted mellom 2 og 3.

$[2{,}333, -3{,}084]$

b) $\hat{\lambda}$ for x, y

$$\rightarrow \frac{\partial}{\partial\lambda}\ln(l(\lambda)) = \frac{7}{\lambda} - 3 = 0$$

$$\rightarrow \frac{7}{\lambda} = 3$$

$$\rightarrow \hat{\lambda} = \frac{7}{3} = 2{,}33...$$

3)

Sannsynlighet for gevinst:

første gang = θ

andre gang = 2θ , $\theta < \max(1/2, 1/5)$

tredje gang = 5θ

a) $E(X_1) = \theta$

$E(X_2) = 2\theta$ $j = 1,2,3$

$E(X_3) = 5\theta$

$V(X_1) = \theta(1-\theta)$

$V(X_2) = 2\theta(1-2\theta)$

$V(X_3) = 5\theta(1-5\theta)$

X_1, X_2, X_3 er binomisk fordelte

med $n=1$

$$\boxed{\begin{aligned} E(X) &= np = p \\ V(X) &= np(1-np) \end{aligned}}$$

b)

to forskjellige estimatorer

$$\hat{\theta}_1 = \frac{1}{8}(X_1 + X_2 + X_3), \quad \hat{\theta}_2 = \frac{1}{3}\left(X_1 + \frac{X_2}{2} + \frac{X_3}{5}\right)$$

$$E(\hat{\theta}_1) = E\left(\frac{1}{8}(X_1 + X_2 + X_3)\right) = \frac{1}{8}(E(X_1) + E(X_2) + E(X_3))$$

$$= \frac{1}{8}(\theta + 2\theta + 5\theta)$$

$$= \frac{1}{8}(8\theta) = \frac{\cancel{8}\theta}{\cancel{8}} = \underline{\underline{\theta}}$$

$$E(\hat{\theta}_2) = E\left(\frac{1}{3}\left(X_1 + \frac{X_2}{2} + \frac{X_3}{5}\right)\right)$$

$$= \frac{1}{3}\left(\theta + \frac{\cancel{2}\theta}{\cancel{2}} + \frac{\cancel{5}\theta}{\cancel{5}}\right)$$

$$= \frac{1}{3}(3\theta) = \frac{\cancel{3}\theta}{\cancel{3}} = \underline{\underline{\theta}}$$

$$V(\hat{\theta}_1) = V\left(\frac{1}{8}(X_1 + X_2 + X_3)\right) = \frac{1}{8^2}(V(X_1) + V(X_2) + V(X_3))$$

$$= \frac{1}{8^2}(\theta(1-\theta) + 2\theta(1-2\theta) + 5\theta(1-5\theta))$$

$$\underline{\underline{\frac{1}{8^2}(-30\theta^2 + 8\theta)}}$$

②

$$V(\hat{\theta}_2) = V\left(\frac{1}{3}\left(X_1 + \frac{X_2}{2} + \frac{X_3}{5}\right)\right)$$

$$= \frac{1}{3^2}\left(V(X_1) + \frac{1}{2^2}V(X_2) + \frac{1}{5^2}V(X_3)\right)$$

$$= \frac{1}{3^2}\left(\theta(1-\theta) + \frac{1}{2^2}2\theta(1-2\theta) + \frac{1}{5^2}5\theta(1-5\theta)\right)$$

$$= \frac{1}{3^2}\left(\theta - \theta^2 + \frac{\theta(1-2\theta)}{2} + \frac{\theta(1-5\theta)}{5}\right)$$

$$\frac{-30\theta^2 + 17\theta}{90}$$

Vi anser den beste estimatoren som den med minst varians/nærmest null!

$$V(\hat{\theta}_1) = \frac{1}{8^2}(-30\theta^2 + 8\theta)$$

$$= \frac{\theta(-15\theta+4)}{32} \quad \theta = 0.2 \text{ gir } V(\hat{\theta}_1) = \underline{0.00625}$$

vs.

$$V(\hat{\theta}_2) = \frac{-30\theta^2 + 17\theta}{90} \quad \theta = 0.2 \text{ gir } V(\hat{\theta}_2) = \underline{0.0244}$$

Vi velger $V(\hat{\theta}_1)$ fordi minst varians.

4)

Vekten av moden kål, X antas å være normaltfordelt med forventningsverdi $\mu = 2.4$ kg og standardavvik $\sigma = 1.4$

a) i) P(veie mindre enn 1.5k)

$$= P\left(\frac{X-\mu}{\sigma} < \frac{1.5-\mu}{\sigma}\right)$$

$$= P\left(Z < \frac{1.5-2.4}{1.4}\right)$$

$$= P(Z < \underset{-0.64}{\cancel{}}) = \underline{\underline{0.2611}}$$

0.6 og 0.04

ii) $P(2 < X < 2.5)$

$$P(X < 2.5) - P(X < 2)$$

$$P\left(Z < \frac{2.5-\mu}{\sigma}\right) - P\left(Z < \frac{2-\mu}{\sigma}\right)$$

$$P(Z < 0.071) - P(Z < -0.286)$$

$$\underset{0.5279}{\cancel{0.}} \quad -0.38\overset{59}{\cancel{}} \quad = \cancel{0.08}\ \underline{\underline{0.142}}$$

iii) $Y = X_1 - X_2$ $\quad$ X_1 og X_2 er uavhengige

$E(Y) = E(X_1) - E(X_2)$

$= 2.4 - 2.4 = 0$

$V(Y) = V(X_1) + V(X_2) = 2 \cdot 1,96 = 3,92$

$P(|X_1 - X_2| > 1) = P(|Y| > 1)$

$= P(|Y/1,98| > 1/1,98)$

$= P(|Z| > 1/1,98)$

$= 2P(Z < -1/1,98) = \underline{\underline{0.305}}$

iv) $P(2 < X < 2,5 \mid X > 1,5)$

$\dfrac{P(2 < X < 2.5 \cap X > 1,5)}{P(X > 1,5)}$ $\quad \because P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$\dfrac{P(2 < X < 2,5)}{P(X > 1,5)} = \dfrac{0.084}{1 - P(X < 1,5)}$

$= \dfrac{0,0824}{0.7389} = \underline{\underline{0,192}}$

b)

Rimelig estimator $\hat{\mu}_Y = \bar{Y}$

estimatet $\hat{\mu}_Y = \bar{y} = \frac{27,8}{10} = \underline{2,78}$

$(\bar{y} - z_{0.05}\sigma_Y/\sqrt{n}, \ \bar{y} + z_{0,05} \sigma_Y/\sqrt{n})$

$(2,78 - 1,833 \cdot 1,4/\sqrt{10}, \ 2,78 + 1,833 \cdot 1,42/\sqrt{10}$

$\underline{\underline{(1,968 , \ 3,591)}}$

Lengden av intervallet

$L = 2 \cdot 1,833 \cdot 1,4/\sqrt{10} = \underline{\underline{1,623}}$

antall, n, kålplanter for at L skal være mindre enn 0.2kg

$2 \cdot 1,833 \frac{1,4}{\sqrt{n}} < 0.2$

$n > \left(\frac{2 \cdot 1,833 \cdot 1,4}{0,2}\right)^2 \approx \underline{\underline{659}}$

dersom $n = 659$ så blir bredden $< 0,2$kg