APPM 4600 — HOMEWORK # 11

For all homeworks, you should use Python. **Do not use** symbolic software such as Maple or Mathematica.

1. (a) Write a code to approximate $\int_{-5}^{5} \frac{1}{1+s^2} ds$ using a composite Trapezoidal rule. To do this, partition the interval $[-5, 5]$ into equally spaced points $t_0, t_1, \ldots, t_n$.

   Write another code to approximate $\int_{-5}^{5} \frac{1}{1+s^2} ds$ using a composite Simpson's rule. To do this, partition the interval $[-5, 5]$ into equally spaced points $t_0, t_1, \ldots, t_n$ where $n = 2k$ is even. The even indexed points should be the endpoints of your subintervals.

   You may combine the two into one code that selects the desired method if you wish.

   Turn in a listing of your code(s).

   (b) Use the error estimates derived in class to choose n so that

$$\left| \int_{-5}^{5} \frac{1}{1+s^2} ds - T_n \right| < 10^{-4} \quad \text{and} \quad \left| \int_{-5}^{5} \frac{1}{1+s^2} ds - S_n \right| < 10^{-4},$$

   where T_n is the result of the composite Trapezoidal rule and where S_n is the result of the composite Simpson's rule. Be sure to explain your reasoning for choosing n in both cases (these n values will be different in the two cases).

   (c) Run your code with the predicted values of n and compare your computed values S_n and T_n with that of `SCIPY`'s `quad` routine on the same problem. Run the built in quadrature twice, once with the default tolerance of 10^{-6} and another time with the set tolerance of 10^{-4}. Report the number of function evaluations required in both cases and compare these to the number of function values your codes (both S_n and T_n) required to meet the tolerance

   Turn in your codes and the results of this test.

2. Use the transformation $t = x^{-1}$ and Composite Simpson's rule with 5 nodes to approximate

$$\int_{1}^{\infty} \frac{\cos(x)}{x^3} dx.$$

3. Assume the error in an integration formula has the asymptotic expansion

$$I - I_n = \frac{C_1}{n\sqrt{n}} + \frac{C_2}{n^2} + \frac{C_3}{n^2\sqrt{n}} + \frac{C_4}{n^3} + \cdots$$

   Generalize the Richardson extrapolation process to obtain an estimate of I with an error of order $\frac{1}{n^2\sqrt{n}}$. Assume that three values I_n, $I_{n/2}$ and $I_{n/4}$ have been computed.