APPM 4600 — HOMEWORK # 8

1. Consider the task of interpolating the function $f(x) = \frac{1}{1+x^2}$ on the interval $[-5, 5]$. Using equispaced nodes with $n = 5, 10, 15$ and 20, interpolate the function using the methods below:

   (a) Lagrange interpolation.

   (b) Hermite interpolation.

   (c) Natural Cubic spline.

   (d) Clamped Cubic spline.

   Which method performs best? Do you have an intiution why?

2. Repeat the experiment from the previous problem but with Chebychev nodes. How does this impact the performance of the different interpolation techniques?

3. Consider the task of approximating a periodic function such as $f(x) = \sin(10x)$ on the interval $[0, 2\pi]$ using the cubic spline. How do you modify the end point conditions on the coefficients so that the spline is naturally periodic?