APPM 4600 — HOMEWORK # 12

For all homeworks, if coding is required, you should use Python. **Do not use** symbolic software such as Maple or Mathematica.

1. Consider the linear system

$$\begin{aligned} 6x + 2y + 2z &= -2 \\ 2x + 2/3y + 1/3z &= 1 \\ x + 2y - z &= 0 \end{aligned}$$

   (a) Verify that $(x, y, z) = (2.6, -3.8, -5)$ is the exact solution.
   (b) Using 4 digit floating point arithmetic with rounding, solve the system via Gaussian elimination without pivoting.
   (c) Repeat part (a) with partial pivoting.
   (d) Which method is more accurate? i.e. stable.

   (Remember to do the rounding to 4 significant digits as the machine would.)

2. By a similarity transform using a Householder matrix, bring the symmetric matrix

$$\mathsf{A} = \begin{bmatrix} 12 & 10 & 4 \\ 10 & 8 & -5 \\ 4 & -5 & 3 \end{bmatrix}$$

   to tridiagonal form.

3. (a) Implement the power method for finding the dominant eigenvalue λ_1 $|\lambda_1| > |\lambda_2| \geq \ldots \geq |\lambda_n|$. Use the power method to find the dominant eigenvalue and corresponding eigenvector for the Hilbert matrix with entries

$$\mathsf{A}_{i,j} = \frac{1}{i+j-1}$$

   for $i, j = 1, \ldots, n$. (Note the dominant eigenvalue is well-separated in this case.) Test for $n = 4:4:20$. How many iterations are needed?
   (b) Use the power method (with the necessary modifications) to find the smallest eigenvalue of the matrix with $n = 16$. How accurate is the eigenvalue?
   (c) Is the error estimate in (b) consistent with the estimate $\min_{\lambda \in \sigma(\mathsf{A})} |\lambda - \mu| \leq \|\mathsf{E}\|_2$ where μ is the eigenvalue of the perturbed matrix $\mathsf{A} + \mathsf{E}$?

   Thm: (Bauer-Fiske) If μ is an eigenvalue of $\mathsf{A} + \mathsf{E} \in \mathbb{C}^{n \times n}$ and A is diagonalizable, i.e.

$$\mathsf{A} = \mathsf{P}^{-1}\mathsf{A}\mathsf{P} = \mathsf{D} = \text{diag}(\lambda_1, \ldots, \lambda_n)$$

   then $\min_{\lambda \in \sigma(\mathsf{A})} |\lambda - \mu| \leq \|\mathsf{P}^{-1}\|_p \|\mathsf{P}\|_p \|\mathsf{E}\|_p$ for any p-norm.
   (d) Construct a numerical example where the power method does not work.