APPM4600 Homework #12

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1) $6x + 2y + 2z = -2$ ①

$2x + \frac{2y}{3} + \frac{z}{3} = 1$ ②

$x + 2y - z = 0$ ③

a) verify $(2.6, -3.8, -5)$

①: $6(2.6) + 2(-3.8) + 2(-5) = -2$

$15.6 - 7.6 - 10 = -2$

$-2 = -2$ ✓✓

②: $2(2.6) + \frac{2(-3.8)}{3} + \frac{(-5)}{3} = 1$

$$5.2 - \frac{7.6}{3} - \frac{5}{3} = 1$$

$$\frac{15.6 - 7.6 - 5}{3} = 1$$

$$\frac{3}{3} = 1 \quad \checkmark\checkmark$$

(3): $2.6 + 2(-3.8) - (-5) = 0$

$$2.6 - 7.6 + 5 = 0$$

$$0 = 0 \quad \checkmark\checkmark$$

b) augmented matrix:

$$\begin{array}{c} \begin{bmatrix} 6 & 2 & 2 & -2 \\ 2 & 2/3 & 1/3 & 1 \\ 1 & 2 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$R_1' = \frac{1}{6} R_1 = \begin{bmatrix} 1 & 0.3333 & 0.3333 & -0.3333 \end{bmatrix}$$

$$R_2' = R_2 - 2R_1'$$

$$= \begin{bmatrix} 0 & 0 & -0.3333 & 1.6667 \end{bmatrix}$$

$$R_3' = R_3 - R_1'$$

$$= \begin{bmatrix} 0 & 1.6667 & -1.3333 & 0.3333 \end{bmatrix}$$

$$\begin{array}{cc} \begin{bmatrix} 1 & 0.3333 & 0.3333 & -0.3333 \\ 0 & \underline{0} & -0.3333 & 1.6667 \\ 0 & 1.6667 & -1.3333 & 0.3333 \end{bmatrix} & \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \end{array}$$

due to 0 in R_2 second column, pivot R_2/R_3:

$$\begin{array}{cc} \begin{bmatrix} 1 & 0.3333 & 0.3333 & -0.3333 \\ 0 & 1.6667 & -1.3333 & 0.3333 \\ 0 & 0 & -0.3333 & 1.6667 \end{bmatrix} & \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \end{array}$$

$R_2' = \frac{3}{5} R_2 = [0 \quad 1 \quad -0.8 \quad 0.2]$

from R_3: $-0.3333z = 1.6667$

$\Rightarrow \underline{\underline{z = -5}}$

R_2': $y - 0.8(z) = 0.2$

$y = 0.2 + 0.8(-5)$

$y = 0.2 - 4$

$\underline{\underline{y = -3.8}}$

R_1: $x + 0.3333(-3.8) + 0.3333(-5) = -0.3333$

$\frac{x}{0.3333} = -1 + 3.8 + 5$

$x = 0.3333[7.8]$

$x = 2.6$

c) use same method as above since pivoting was only used at end.

Looking at original matrix, 6 is the largest first column value, so no pivoting on first step.

d) same method, same stability

2) $\begin{bmatrix} 12 & 10 & 4 \\ 10 & 8 & -5 \\ 4 & -5 & 3 \end{bmatrix}$

look at $\begin{bmatrix} 10 \\ 4 \end{bmatrix} = x$

$|x| = \sqrt{116} \approx 10.8$

$$r = x - |x| \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$r = \begin{bmatrix} 10 \\ 4 \end{bmatrix} - 10.8 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$r = \begin{bmatrix} -0.8 \\ 4 \end{bmatrix}$$ normalize $|v| = \sqrt{16 + 0.64} \approx 4.1$

$$v = \frac{1}{4.1} \begin{bmatrix} -0.8 \\ 4 \end{bmatrix}$$

first Householder Matrix:

$$H_1 = I - 2rr^T$$

$$vr^T = \frac{1}{4.1^2} \begin{bmatrix} -0.8 \\ 4 \end{bmatrix} \begin{bmatrix} -0.8 & 4 \end{bmatrix}$$

$$= \frac{1}{16.8} \begin{bmatrix} 0.64 & -0.32 \\ -0.32 & 16 \end{bmatrix} = \begin{bmatrix} 0.034 & -0.019 \\ -0.019 & 0.95 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2 \begin{bmatrix} 0.034 & -0.019 \\ -0.019 & 0.95 \end{bmatrix}$$

Projected into 3x3:

$$H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.932 & 0.038 \\ 0 & 0.038 & -0.9 \end{bmatrix}$$

$$A = H_1 A H_1^T$$

Computing in python:

should = 0

$$A = \begin{bmatrix} 12 & 9.472 & \underline{-3.22} \\ 9.472 & 6.599 & 4.3675 \\ \underline{-3.22} & 4.367 & 2.784 \end{bmatrix}$$

→ this should be 0 so I must have <u>made a rounding error.</u>

3) See code on GitHub

a)

Matrix Size	Iterations
4	4
8	6
12	5
16	6
20	7

b) smallest eigenvalue:

$$\lambda = \sim 5 \times 10^{-18}$$

c) I'm not sure

d) Power method won't work when there are eigenvalues of the same magnitude:

ex: $A = \begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix}$

$\lambda = \pm 2$