APPM 4600 Homework # 5

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1) See code on Github

a) Converges on

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -276.8 \\ 167.7 \end{bmatrix}$$

b) approximates the Jacobian to speed up iteration.

c) $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -276.8 \\ 167.7 \end{bmatrix}$$

2) $G = \begin{cases} x_{n+1} = \frac{1}{\sqrt{2}}\sqrt{1+(x_n+y_n)^2} - \frac{2}{3} \\ y_{n+1} = \frac{1}{\sqrt{2}}\sqrt{1+(x_n-y_n)^2} - \frac{2}{3} \end{cases}$

Calculate partial derivatives:

$$g_1(x,y) = x_{n+1} = \frac{1}{\sqrt{2}}\sqrt{1+(x_n+y_n)^2} - \frac{2}{3}$$

$$g_2(x,y) = y_{n+1} = \frac{1}{\sqrt{2}}\sqrt{1+(x_n-y_n)^2} - \frac{2}{3}$$

$$\frac{\partial g_1}{\partial x} = \frac{2(x_n+y_n)}{2\sqrt{2}\sqrt{1+(x_n+y_n)^2}}$$

$$\frac{\partial g_1}{\partial y} = \frac{2(x_n+y_n)}{2\sqrt{2}\sqrt{1+(x_n+y_n)^2}} = \frac{\partial g_1}{\partial x}$$

$$\frac{\partial g_2}{\partial x} = \frac{2(x_n-y_n)}{2\sqrt{2}\sqrt{1+(x_n-y_n)^2}}$$

$$\frac{\partial g_2}{\partial y} = \frac{-2(x_n - y_n)}{2\sqrt{2}\sqrt{1+(x_n-y_n)^2}} = -\frac{\partial g_2}{\partial x}$$

all must be $\leq \frac{k}{n} = \frac{1}{2}$

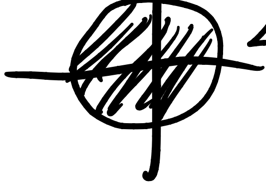
$$\left| \frac{(x_n + y_n)}{\sqrt{2}\sqrt{1+(x_n+y_n)^2}} \right| \leq \frac{1}{2}$$

$$4(x_n + y_n)^2 \leq 2(1 + (x_n + y_n)^2)$$

$$2(x_n + y_n)^2 \leq 1 + (x_n + y_n)^2$$

$$(x_n + y_n)^2 \leq 1$$

So our region D is defined by this circle of radius 1:



D

3)

a) We want to move towards curve so update should be proportional to partial derivative of respective variable

$$x_{n+1} \propto x_n - f_x$$

$$y_{n+1} \propto y_n - f_y$$

divide by $|\nabla f|$ for each step to ensure consistent steps of reasonable magnitude:

$$x_{n+1} \propto x_n - \frac{f_x}{f_x^2 + f_y^2}$$

$$y_{n+1} \propto y_n - \frac{f_y}{f_x^2 + f_y^2}$$

size of update also proportional to $f(x,y)$:

$$x_{n+1} = x_n - \frac{f f_x}{f_x^2 + f_y^2} = x_n - d f_x$$

$$\underline{\underline{y_{n+1} = y_n - \frac{f f_y}{f_y^2 + f_x^2} = y_n - d f_y}}$$

b) See code on Github

$x^2 + 4y^2 + 4z^2 - 16 = 0$

Starting with

$x_0 = y_0 = z_0 = 1$:

$$\boxed{\begin{array}{l} x = 1.09 \text{ to 2 d.p.} \\ y = 1.36 \text{ to 2 d.p.} \\ z = 1.36 \text{ to 2 d.p.} \end{array}}$$

order of convergence ≈ 2

$\hookrightarrow$ quadratic convergence as expected