APPM4600 Homework #3

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1) $2x - 1 = \sin(x)$

a) $2x - 1 - \sin(x) = 0$

let $f(x) = 2x - 1 - \sin(x)$

root r when $f(r) = 0$

try interval $[0, 1]$

$f(0) = 0 - 1 - 0 = -1$

$f(1) = 2 - 1 - \sin(1) = 1 - \sin(1)$

$\sin(1) \approx 0.84$

so $f(1) \approx 0.16 > 0$

since $f(0) = -1 < 0$

$f(1) \approx 0.16 > 0$

there is a change of sign so
root r exists in interval $[0, 1]$.

b) Calculate derivative of $f(x)$:

$f'(x) = 2 - \cos(x)$

$f'(0) = 2 - \cos 0 = 1 > 0$

$f'(1) = 2 - \cos 1 \approx 1.45 > 0$

so function is always increasing in interval
$[a, b]$ which means there is only 1 root in $[a, b]$

c) See code on Github

$r = 0.88786222$

2) See code on Github

a) result for approx root = 5.0000073242187501

b) result for approx root = 5.1117772460937501

c) In b), precision is lost because in expanded form, polynomial subtracts large numbers close in value to each other when x is close to 5.

3) theorem 2.1: $\underbrace{|p_n - p|}_{\varepsilon = 10^{-3}} \leq \frac{b-a}{2^n}$

$$2^n \leq \frac{b-a}{\varepsilon} \qquad [1,4)$$

$$\log_2(2^n) \leq \log_2\left(\frac{b-a}{\varepsilon}\right)$$

$$n \leq \log_2\left(\frac{b-a}{\varepsilon}\right) = \log_2\left(\frac{3}{10^{-3}}\right)$$

$$n \leq 11.55$$

so upper bound is $\underline{n = 12 \text{ iterations}}$

b) —— ran out of time submitted late

4)

a) $x_{n+1} = -16 + 6x_n + \frac{12}{x_n}, \quad x_* = 2$

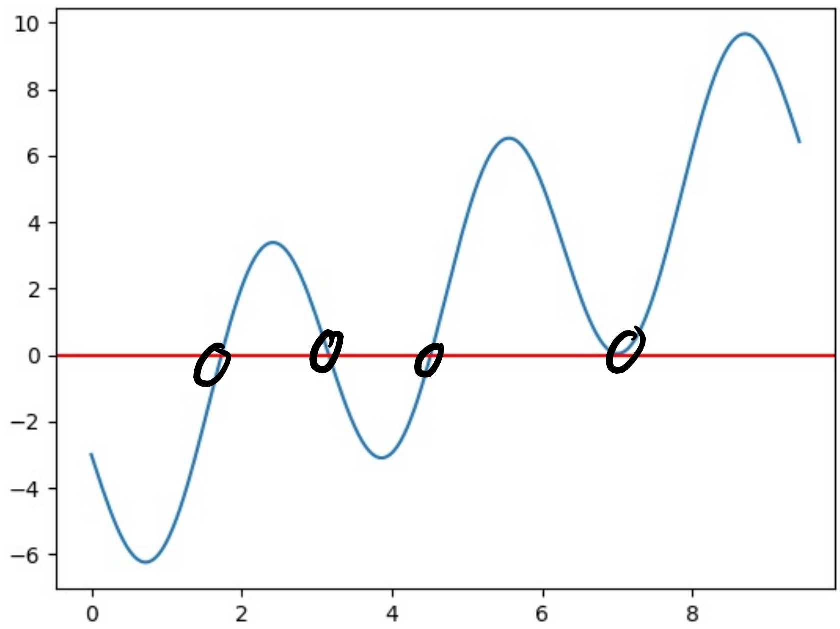
b) /

c) /

ran out of time submitted late

5)

a) See code on Github



4 zero crossings