APPM 4600 Homework #7

Owen O'Connor

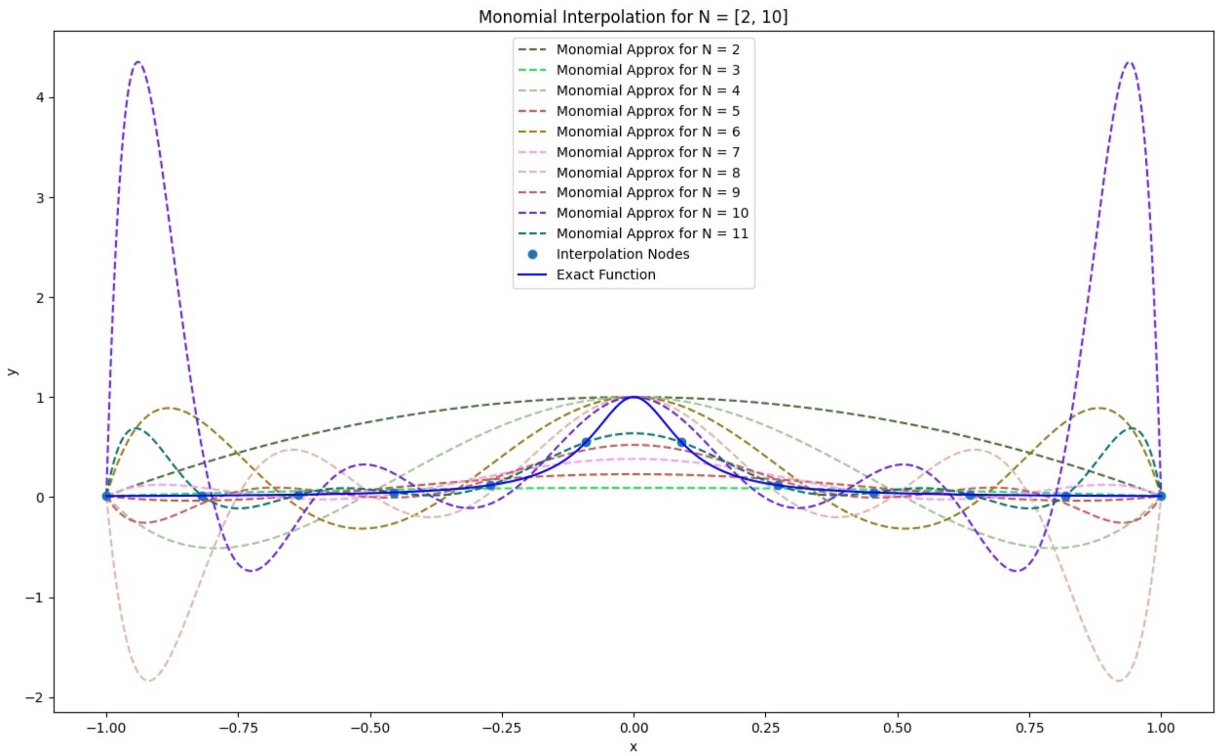
1a) $V\underline{c} = \underline{y}$

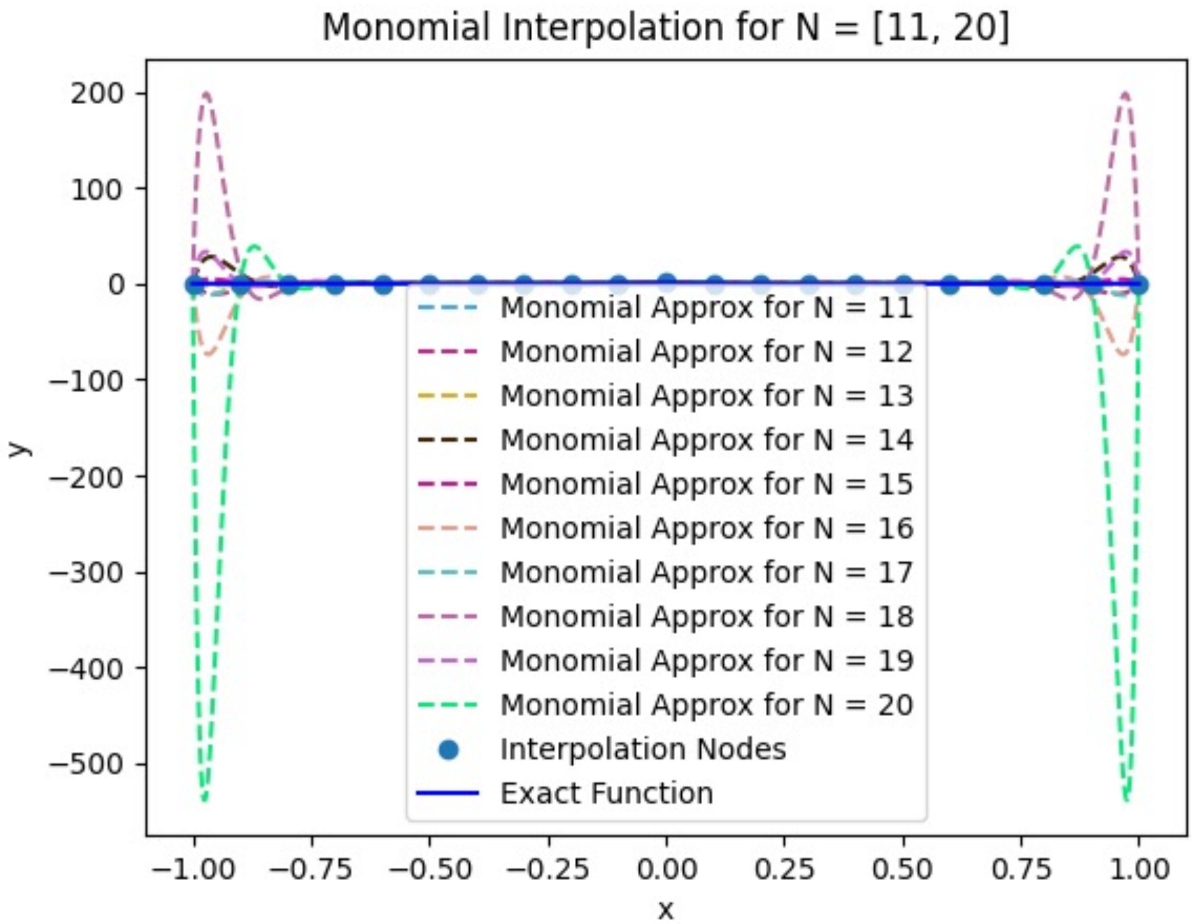
$$\begin{pmatrix} & & \\ & V & \\ & & \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$V = \begin{pmatrix} x_1^{n-1} & x_1^{n-2} & \cdots & 1 \\ x_2^{n-1} & x_2^{n-2} & \cdots & 1 \\ \vdots & & & \vdots \\ x_j^{n-1} & x_j^{n-2} & \cdots & 1 \end{pmatrix}$$

$$\Rightarrow y_j = c_1 x_j^{n-1} + c_2 x_j^{n-2} + \cdots + c_n$$

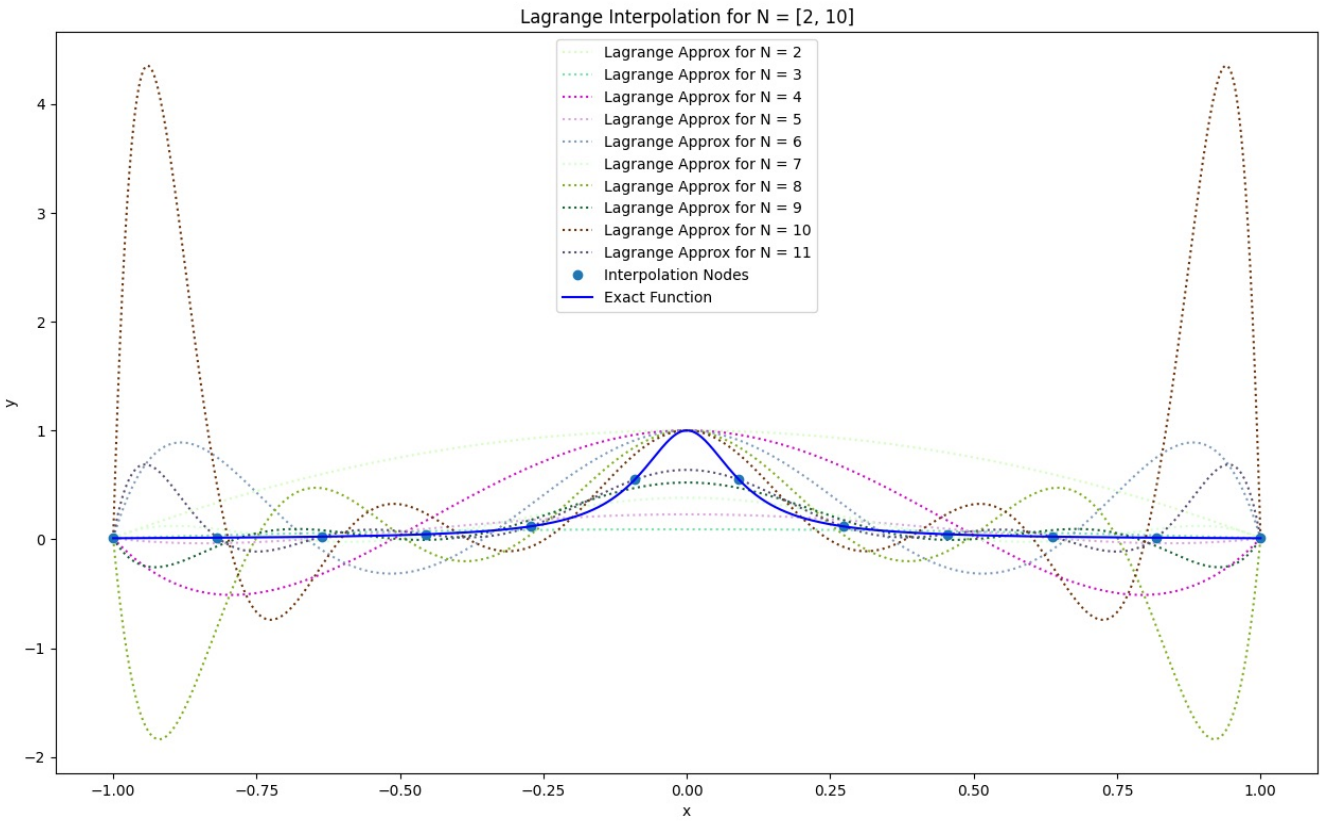
b) See code on GitHub:

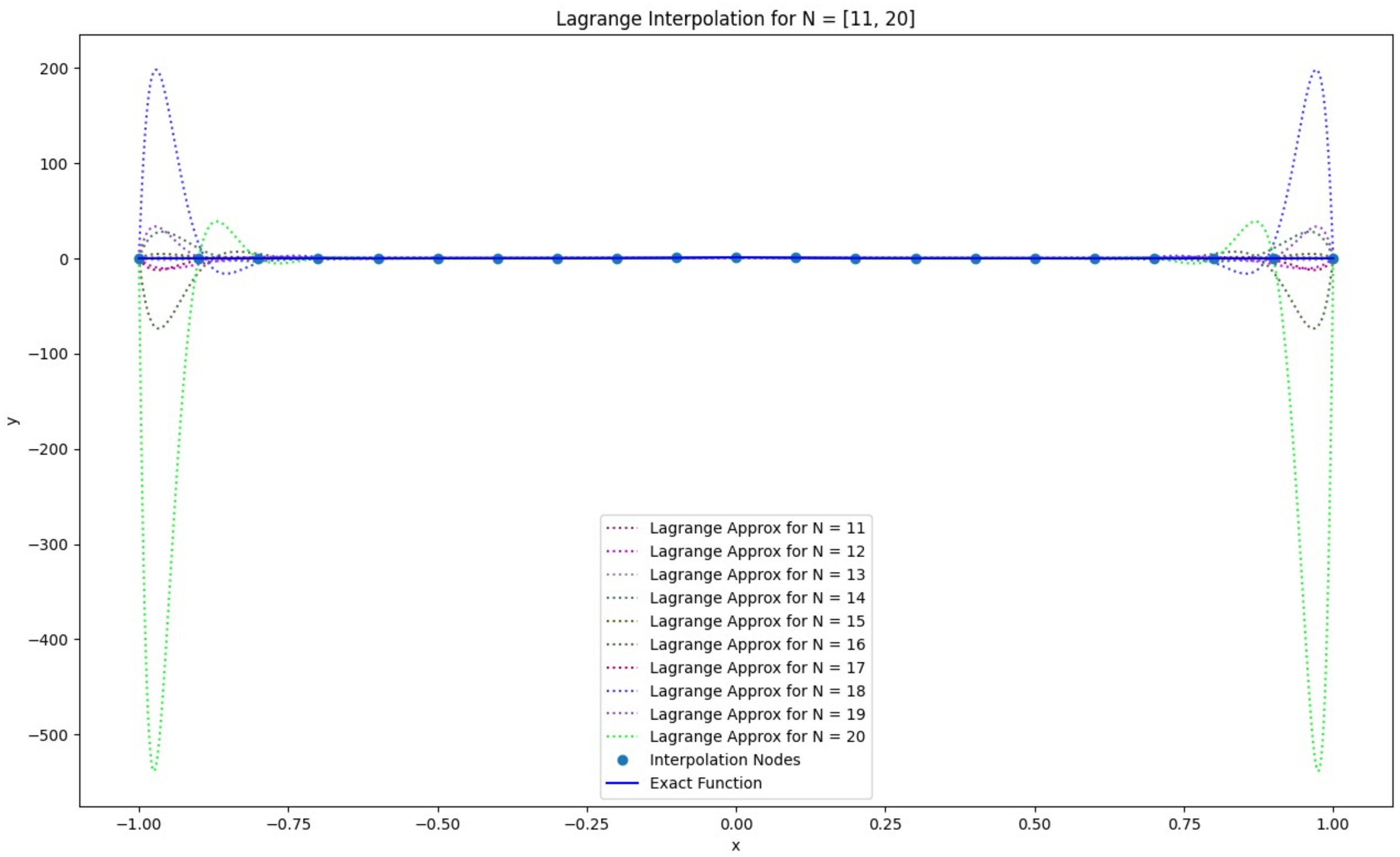




as seen in plots above, monomial interpolation is a great approximation for small x, especially for large N, but at interval endpoints, the error blows up (Runge phenomena)

2) See code on GitHub:



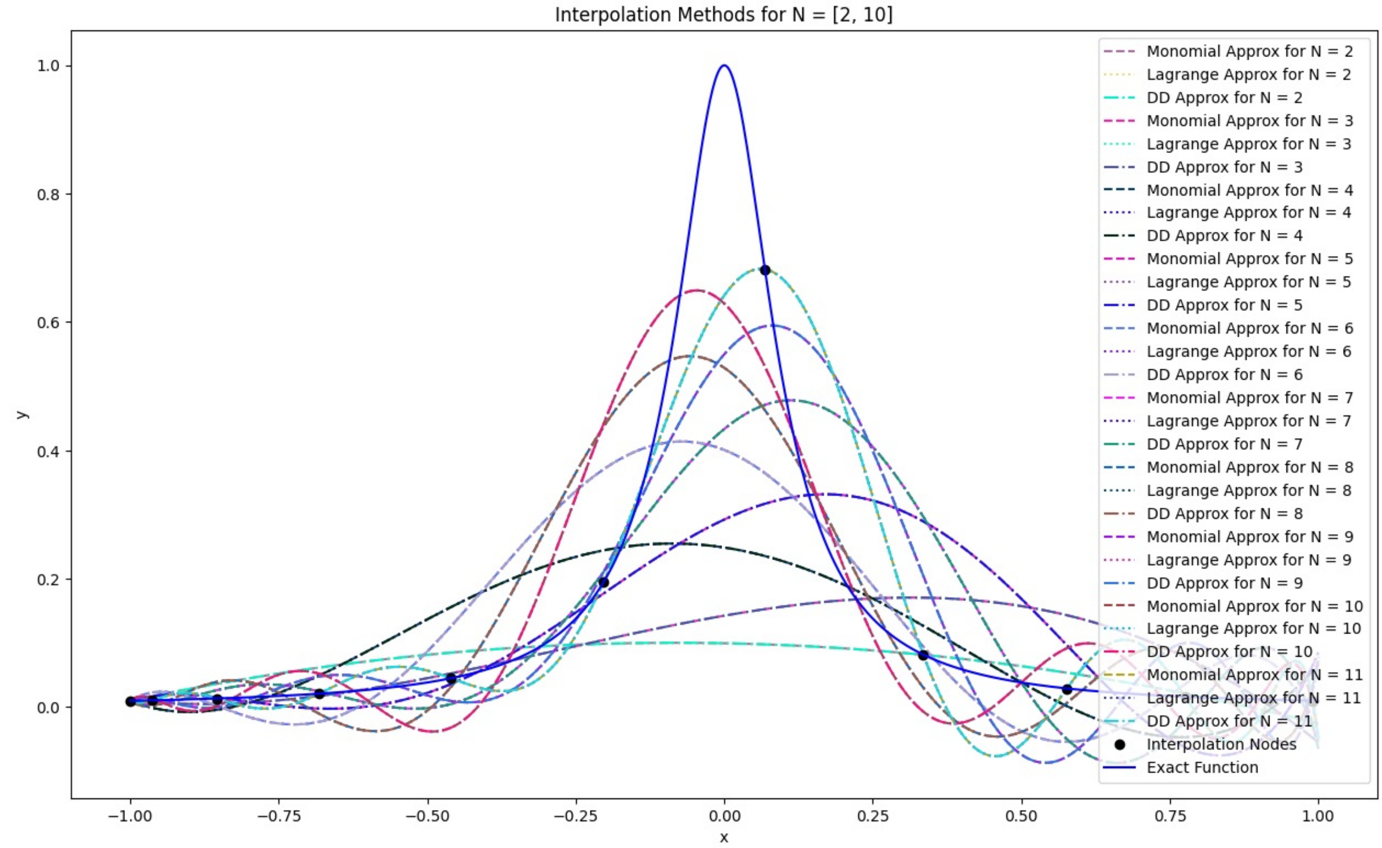


as seen in plots above, lagrange int suffers same problem as monomial. In fact due to Weierstrass approx theorem, polynomials from mono. and lagrange int. are identical.

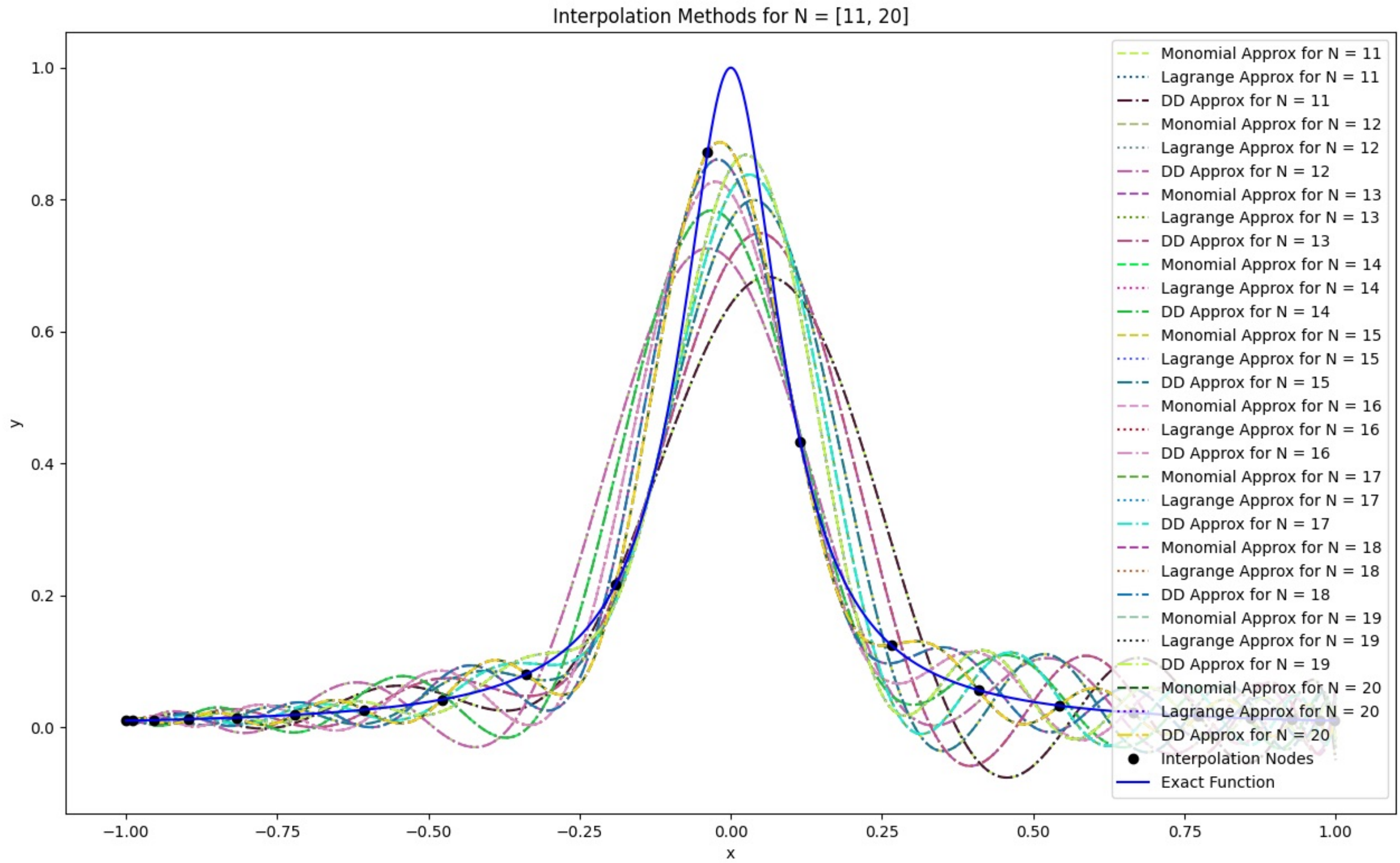
3) See code on GitHub:

I changed $2j-1$ to $2j+1$ to get stable behaviour (I was getting singular V with

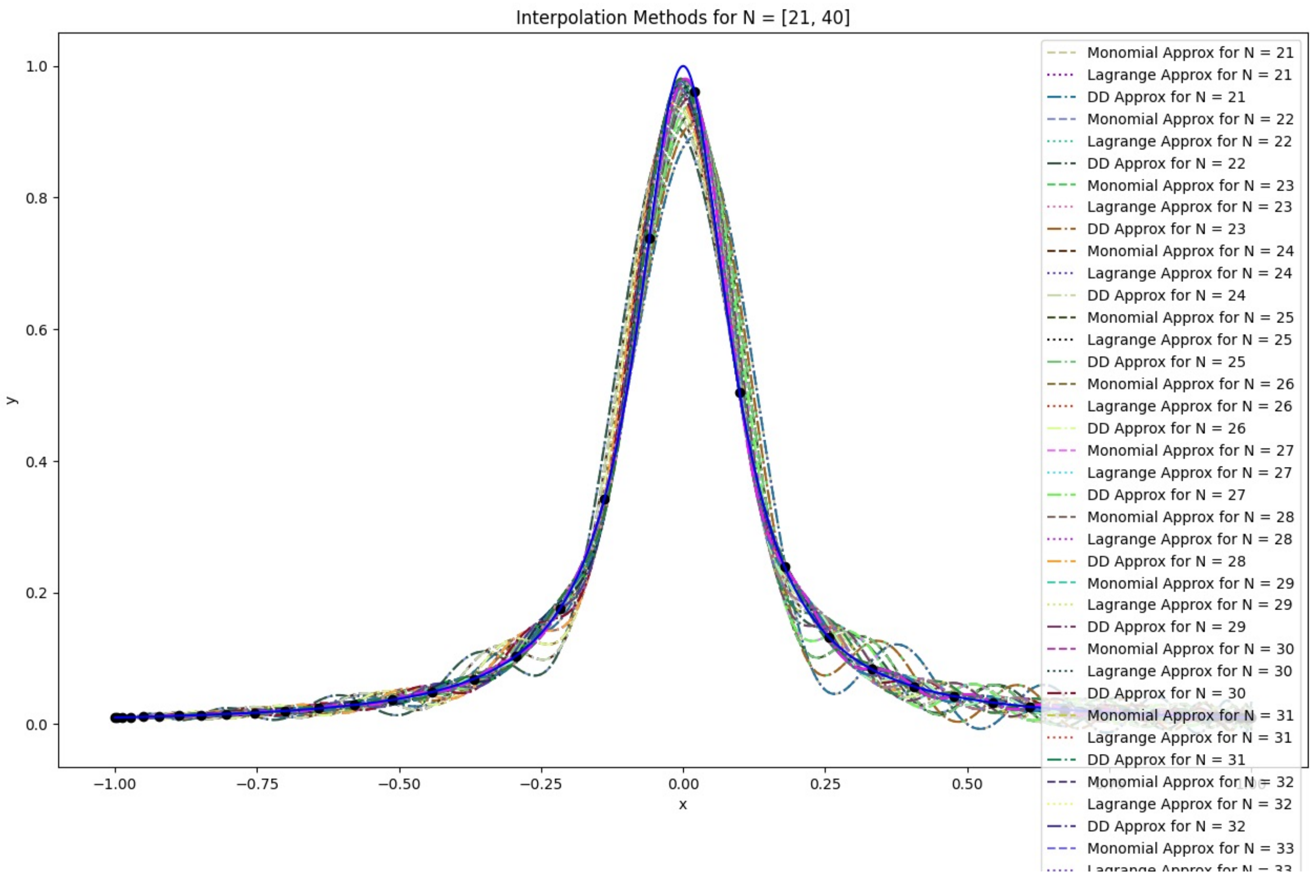
$(2j-1)$.



Interpolation Methods for N = [2, 10]
Monomial Approx for N = 2
Lagrange Approx for N = 2
DD Approx for N = 2
Monomial Approx for N = 3
Lagrange Approx for N = 3
DD Approx for N = 3
Monomial Approx for N = 4
Lagrange Approx for N = 4
DD Approx for N = 4
Monomial Approx for N = 5
Lagrange Approx for N = 5
DD Approx for N = 5
Monomial Approx for N = 6
Lagrange Approx for N = 6
DD Approx for N = 6
Monomial Approx for N = 7
Lagrange Approx for N = 7
DD Approx for N = 7
Monomial Approx for N = 8
Lagrange Approx for N = 8
DD Approx for N = 8
Monomial Approx for N = 9
Lagrange Approx for N = 9
DD Approx for N = 9
Monomial Approx for N = 10
Lagrange Approx for N = 10
DD Approx for N = 10
Monomial Approx for N = 11
Lagrange Approx for N = 11
DD Approx for N = 11
Interpolation Nodes
Exact Function
y
x



Interpolation Methods for N = [11, 20]
Monomial Approx for N = 11
Lagrange Approx for N = 11
DD Approx for N = 11
Monomial Approx for N = 12
Lagrange Approx for N = 12
DD Approx for N = 12
Monomial Approx for N = 13
Lagrange Approx for N = 13
DD Approx for N = 13
Monomial Approx for N = 14
Lagrange Approx for N = 14
DD Approx for N = 14
Monomial Approx for N = 15
Lagrange Approx for N = 15
DD Approx for N = 15
Monomial Approx for N = 16
Lagrange Approx for N = 16
DD Approx for N = 16
Monomial Approx for N = 17
Lagrange Approx for N = 17
DD Approx for N = 17
Monomial Approx for N = 18
Lagrange Approx for N = 18
DD Approx for N = 18
Monomial Approx for N = 19
Lagrange Approx for N = 19
DD Approx for N = 19
Monomial Approx for N = 20
Lagrange Approx for N = 20
DD Approx for N = 20
Interpolation Nodes
Exact Function
y
x



As seen in plots above, using Chebyshev nodes concentrates nodes closer to endpoints. For small N this fixes Runge's phenomena but sacrifices precision for small x. However, increasing N now yields only more and more precise approximations around both the peak and the endpoint. I can't get the interpol. to fail.