ACTIVITY NO. 10

GRAPHS

Course Code: CPE010	**Program:** Computer Engineering
Course Title: Data Structures and Algorithms	**Date Performed:**
Section:	**Date Submitted:**
Name:	**Instructor:** Engr. Roman M. Richard

1. Objective(s)

Create C++ code for implementation of Graph ADT.

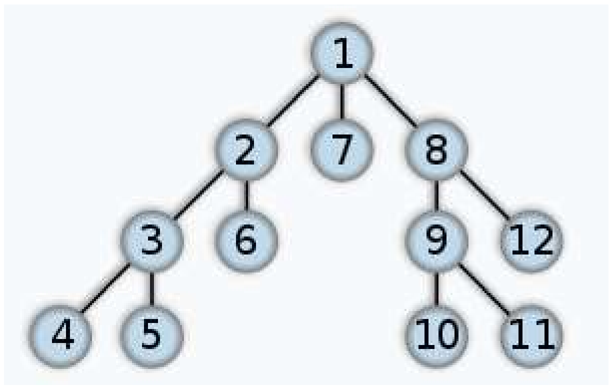
2. Intended Learning Outcomes (ILOs)

After this activity, the student should be able to:

- Create C++ code for graph implementation utilizing adjacency matrix and adjacency list.
- Create C++ code for implementing graph traversal algorithms such as Breadth-First and Depth-First Search.
- Demonstrate an understanding of graph implementation, operations and traversal methods.

3. Discussion

For a given vertex v, the depth-first search algorithm proceeds along a path from v as deeply into the graph as possible before backing up. That is, after visiting a vertex v, the depth-first search algorithm visits (if possible) an unvisited adjacent vertex to vertex v. The depth-first traversal algorithm does not completely specify the order in which it should visit the vertices adjacent to v. We may visit the vertices adjacent to v in sorted order.



The DFS algorithm is a recursive algorithm that uses the idea of backtracking. It involves exhaustive searches of all the nodes by going ahead, if possible, else by backtracking. Here, the word backtrack means that when you are moving forward and there are no more nodes along the current path, you move backwards on the same path to find nodes to traverse. All the nodes will be visited on the current path till all the unvisited nodes have been traversed after which the next path will be selected.

This recursive nature of DFS can be implemented using stacks. The basic idea is as follows:

- Pick a starting node and push all its adjacent nodes into a stack.
- Pop a node from stack to select the next node to visit and push all its adjacent nodes into a stack.

- Repeat this process until the stack is empty. However, ensure that the nodes that are visited are marked. This will prevent you from visiting the same node more than once. If you do not mark the nodes that are visited and you visit the same node more than once, you may end up in an infinite loop.

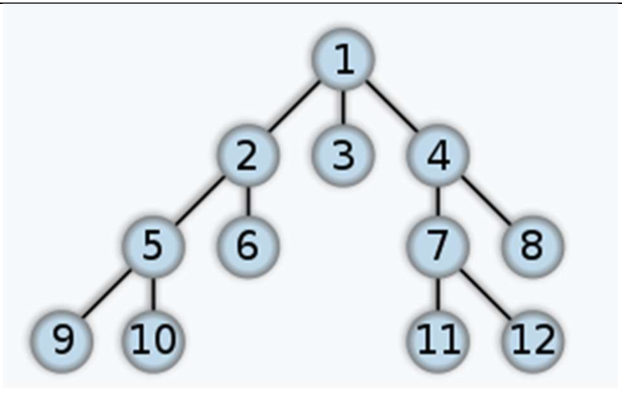
Recursive Depth-First Search Algorithm

```
dfs(in v:Vertex) {
// Traverses a graph beginning at vertex v
// by using depth-first strategy
// Recursive Version
      Mark v as visited;
      for (each unvisited vertex u adjacent to v)
            dfs(u)
}
```

Iterative Depth-First Search Algorithm

```
dfs(in v:Vertex) {
// Traverses a graph beginning at vertex v
// by using depth-first strategy: Iterative Version
      s.createStack();
      // push v into the stack and mark it
      s.push(v);
      Mark v as visited;
      while (!s.isEmpty()) {
         if (no unvisited vertices are adjacent to the vertex on
             the top of stack)
            s.pop();  // backtrack
         else {
            Select an unvisited vertex u adjacent to the vertex
               on the top of the stack;
            s.push(u);
            Mark u as visited;
         }
      }
}
```

After visiting a given vertex v, the breadth-first search algorithm visits every vertex adjacent to v that it can before visiting any other vertex. The breadth-first traversal algorithm does not completely specify the order in which it should visit the vertices adjacent to v. However, we may visit the vertices adjacent to v in sorted order.



There are many ways to traverse graphs. BFS is the most commonly used approach. BFS is a traversing algorithm where you should start traversing from a selected node (source or starting node) and traverse the graph layer-wise thus exploring the neighbor nodes (nodes which are directly connected to source node). You must then move towards the next-level neighbor nodes.

As the name BFS suggests, you are required to traverse the graph breadthwise as follows:

1. First move horizontally and visit all the nodes of the current layer
2. Move to the next layer

Iterative Breadth-First Search Algorithm

```
bfs(in v:Vertex) {
// Traverses a graph beginning at vertex v
// by using breath-first strategy: Iterative Version
      q.createQueue();
      // add v to the queue and mark it
      q.enqueue(v);
      Mark v as visited;
      while (!q.isEmpty()) {
         q.dequeue(w);
         for (each unvisited vertex u adjacent to w) {
            Mark u as visited;
            q.enqueue(u);
         }
      }
}
```

4. Materials and Equipment

Personal Computer with C++ IDE
Recommended IDE:

- CLion (must use TIP email to download)
- DevC++ (use the embarcadero fork or configure to C++17)

5. Procedure

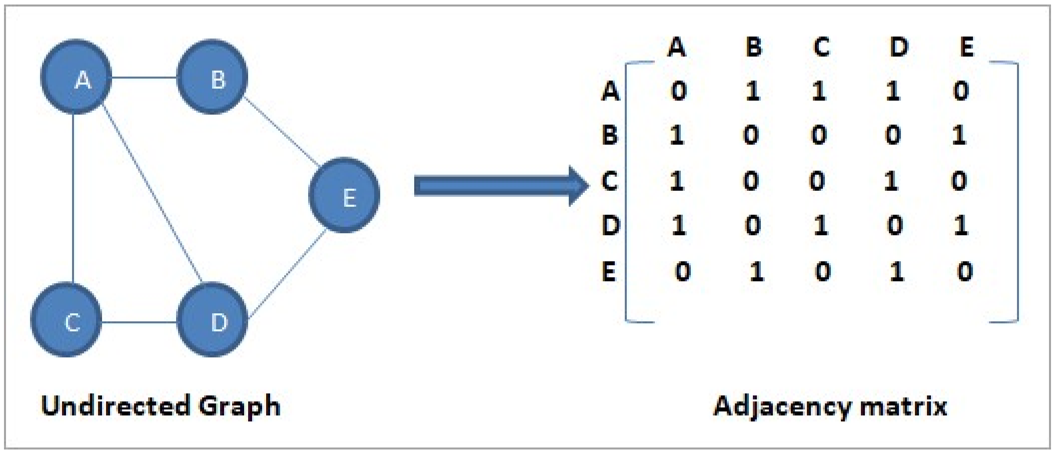
<u>ILO A: Create C++ code for graph implementation utilizing adjacency matrix and adjacency list</u>

A.1. Create a Graph

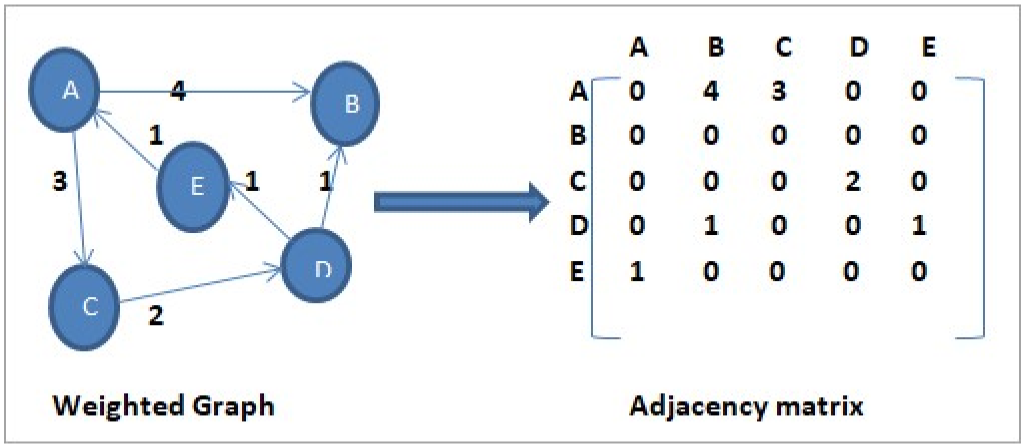
Following are the basic operations that we can perform on the graph data structure:

- **Add a vertex:** Adds vertex to the graph.
- **Add an edge:** Adds an edge between the two vertices of a graph.
- **Display the graph vertices:** Display the vertices of a graph.

A.2. Adjacency Matrix

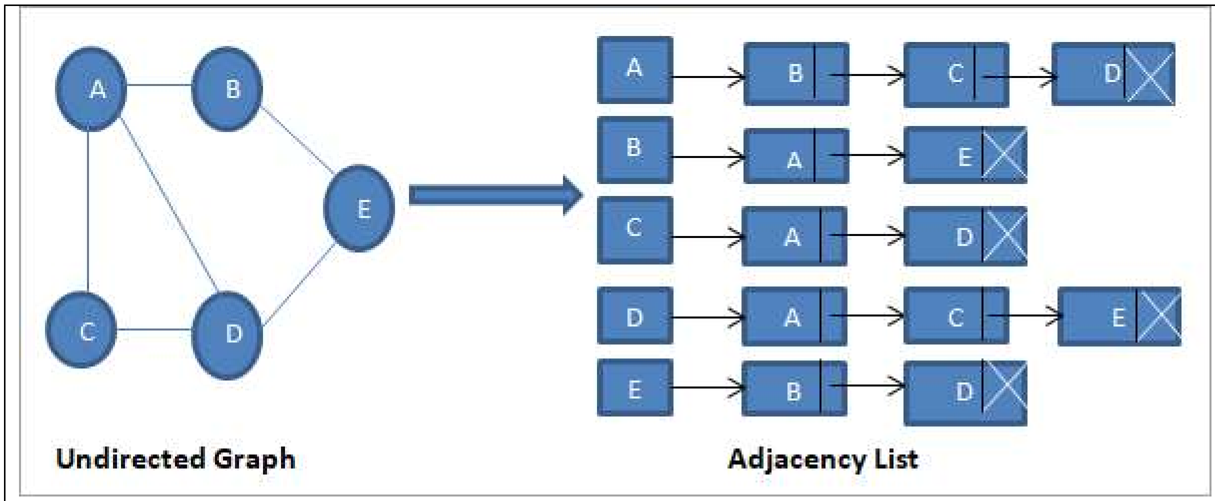


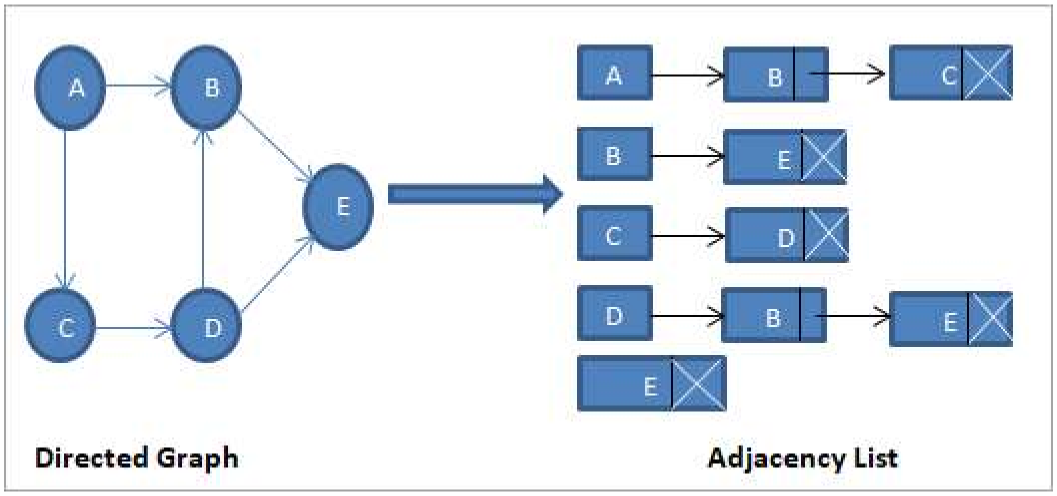
$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 1 & 1 & 0 \\ B & 1 & 0 & 0 & 0 & 1 \\ C & 1 & 0 & 0 & 1 & 0 \\ D & 1 & 0 & 1 & 0 & 1 \\ E & 0 & 1 & 0 & 1 & 0 \end{array}$$



$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 4 & 3 & 0 & 0 \\ B & 0 & 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 & 2 & 0 \\ D & 0 & 1 & 0 & 0 & 1 \\ E & 1 & 0 & 0 & 0 & 0 \end{array}$$

A.3. Adjacency List





Sample Code:

```
#include <iostream>

// stores adjacency list items
struct adjNode {
    int val, cost;
    adjNode* next;
};
// structure to store edges
struct graphEdge {
    int start_ver, end_ver, weight;
};
class DiaGraph{
    // insert new nodes into adjacency list from given graph
    adjNode* getAdjListNode(int value, int weight, adjNode* head)    {
        adjNode* newNode = new adjNode;
        newNode->val = value;
        newNode->cost = weight;

        newNode->next = head;   // point new node to current head
```

```
            return newNode;
        }
        int N;  // number of nodes in the graph
    public:
        adjNode **head;                //adjacency list as array of pointers
        // Constructor
        DiaGraph(graphEdge edges[], int n, int N)  {
            // allocate new node
            head = new adjNode*[N]();
            this->N = N;
            // initialize head pointer for all vertices
            for (int i = 0; i < N; ++i)
                head[i] = nullptr;
            // construct directed graph by adding edges to it
            for (unsigned i = 0; i < n; i++)  {
                int start_ver = edges[i].start_ver;
                int end_ver = edges[i].end_ver;
                int weight = edges[i].weight;
                // insert in the beginning
                adjNode* newNode = getAdjListNode(end_ver, weight, head[start_ver]);

                // point head pointer to new node
                head[start_ver] = newNode;
            }
        }
        // Destructor
        ~DiaGraph() {
            for (int i = 0; i < N; i++)
                delete[] head[i];
            delete[] head;
        }
    };
    // print all adjacent vertices of given vertex
    void display_AdjList(adjNode* ptr, int i)
    {
        while (ptr != nullptr) {
            std::cout << "(" << i << ", " << ptr->val
                 << ", " << ptr->cost << ") ";
            ptr = ptr->next;
        }
        std::cout << std::endl;
    }
    // graph implementation
    int main()
    {
        // graph edges array.
        graphEdge edges[] = {
                // (x, y, w) -> edge from x to y with weight w
                {0,1,2},{0,2,4},{1,4,3},{2,3,2},{3,1,4},{4,3,3}
        };
        int N = 6;      // Number of vertices in the graph
        // calculate number of edges
        int n = sizeof(edges)/sizeof(edges[0]);
        // construct graph
        DiaGraph diagraph(edges, n, N);
        // print adjacency list representation of graph
        std::cout<<"Graph  adjacency  list  "<<std::endl<<"(start_vertex,  end_vertex,
    weight):"<<std::endl;
```

```
          for (int i = 0; i < N; i++)
          {
              // display adjacent vertices of vertex i
              display_AdjList(diagraph.head[i], i);
          }
          return 0;
      }
```

Implement the given code and indicate your output as a table in section 6.

<u>ILO B: Create C++ code for implementing graph traversal algorithms such as Breadth-First and Depth-First Search</u>

B.1. Depth-First Search

Step 1. Include the required header files, as follows:

```
      #include <string>
      #include <vector>
      #include <iostream>
      #include <set>
      #include <map>
      #include <stack>

      template <typename T>
      class Graph;
```

Step 2. Write the following struct in order to implement an edge in our graph:

```
      template <typename T>
      struct Edge
      {
          size_t src;
          size_t dest;
          T weight;
          // To compare edges, only compare their weights,
          // and not the source/destination vertices
          inline bool operator<(const Edge<T> &e) const
          {
              return this->weight < e.weight;
          }
          inline bool operator>(const Edge<T> &e) const
          {
              return this->weight > e.weight;
          }
      };
```

Step 3. Next, overload the << operator for the graph so that it can be printed out using the following function:

```
      template <typename T>
      std::ostream &operator<<(std::ostream &os, const Graph<T> &G)
      {
          for (auto i = 1; i < G.vertices(); i++)
          {
              os << i << ":\t";
              auto edges = G.outgoing_edges(i);
              for (auto &e : edges)
                  os << "{" << e.dest << ": " << e.weight << "}, ";
              os << std::endl;
          }
          return os;
      }
```

Step 4. Implement the graph data structure that uses an edge list representation as follows:

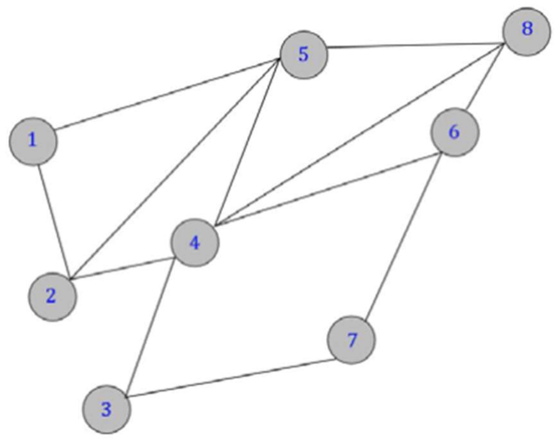
```
    template <typename T>
    class Graph
    {
    public:
        // Initialize the graph with N vertices
        Graph(size_t N) : V(N)
        {
        }
        // Return number of vertices in the graph
        auto vertices() const
        {
            return V;
        }
        // Return all edges in the graph
        auto &edges() const
        {
            return edge_list;
        }

        void add_edge(Edge<T> &&e)
        {
            // Check if the source and destination vertices are within range
            if (e.src >= 1 && e.src <= V &&
                e.dest >= 1 && e.dest <= V)
                edge_list.emplace_back(e);
            else
                std::cerr << "Vertex out of bounds" << std::endl;
        }
        // Returns all outgoing edges from vertex v
        auto outgoing_edges(size_t v) const
        {
            std::vector<Edge<T>> edges_from_v;
            for (auto &e : edge_list)
            {
                if (e.src == v)
                    edges_from_v.emplace_back(e);
            }
            return edges_from_v;
        }
        // Overloads the << operator so a graph be written directly to a stream
        // Can be used as std::cout << obj << std::endl;
        template <typename U>
        friend std::ostream &operator<<(std::ostream &os, const Graph<U> &G);
```

```
private:
    size_t V; // Stores number of vertices in graph
    std::vector<Edge<T>> edge_list;
};
```

Step 5. Now, we need a function to perform DFS for our graph. Implement it as follows:

```
template <typename T>
auto depth_first_search(const Graph<T> &G, size_t dest)
{
    std::stack<size_t> stack;
    std::vector<size_t> visit_order;
    std::set<size_t> visited;
    stack.push(1); // Assume that DFS always starts from vertex ID 1
    while (!stack.empty())
    {
        auto current_vertex = stack.top();
        stack.pop();
        // If the current vertex hasn't been visited in the past
        if (visited.find(current_vertex) == visited.end())
        {
            visited.insert(current_vertex);
            visit_order.push_back(current_vertex);
            for (auto e : G.outgoing_edges(current_vertex))
            {
                // If the vertex hasn't been visited, insert it in the stack.
                if(visited.find(e.dest) == visited.end())
                {
                    stack.push(e.dest);
                }
            }
        }
    }
    return visit_order;
}
```

Step 6. We shall test our implementation of the DFS on the graph shown here:



Use the following function to create and return the graph:

```
template <typename T>
auto create_reference_graph()
{
    Graph<T> G(9);
    std::map<unsigned, std::vector<std::pair<size_t, T>>> edges;
    edges[1] = {{2, 0}, {5, 0}};
    edges[2] = {{1, 0}, {5, 0}, {4, 0}};
    edges[3] = {{4, 0}, {7, 0}};
    edges[4] = {{2, 0}, {3, 0}, {5, 0}, {6, 0}, {8, 0}};
    edges[5] = {{1, 0}, {2, 0}, {4, 0}, {8, 0}};
    edges[6] = {{4, 0}, {7, 0}, {8, 0}};
    edges[7] = {{3, 0}, {6, 0}};
    edges[8] = {{4, 0}, {5, 0}, {6, 0}};
    for (auto &i : edges)
        for (auto &j : i.second)
            G.add_edge(Edge<T>{i.first, j.first, j.second});
    return G;
}
```

Note the use of null values for edge weights since DFS does not require edge weights. A simpler implementation of the graph could have omitted the edge weights entirely without affecting the behavior of our DFS algorithm.

Step 7. Finally, add the following test and driver code, which runs our DFS implementation and prints the output:

```
template <typename T>
void test_DFS()
{
    // Create an instance of and print the graph
    auto G = create_reference_graph<unsigned>();
    std::cout << G << std::endl;
    // Run DFS starting from vertex ID 1 and print the order
    // in which vertices are visited.
    std::cout << "DFS Order of vertices: " << std::endl;
    auto dfs_visit_order = depth_first_search(G, 1);
    for (auto v : dfs_visit_order)
        std::cout << v << std::endl;
}
int main()
{
    using T = unsigned;
    test_DFS<T>();
    return 0;
}
```

Step 8. Compile and run the preceding code. **Include your output as a table in section 6.**

B.2. Breadth-First Search

Step 1: Include the required header files and declare the graph as follows:

```
#include <string>
#include <vector>
#include <iostream>
#include <set>
#include <map>
#include <queue>

template <typename T>
class Graph;
```

Step 2: Write the following struct, which represents an edge in our graph:

```
template <typename T>
struct Edge
{
    size_t src;
    size_t dest;
    T weight;

    inline bool operator<(const Edge<T> &e) const
    {
        return this->weight < e.weight;
    }

    inline bool operator>(const Edge<T> &e) const
    {
        return this->weight > e.weight;
    }
};
```

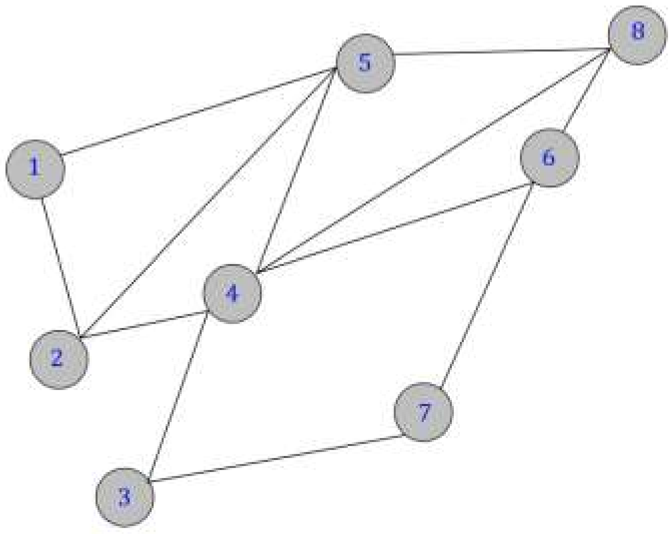
Step 3: Next, overload the << operator for the Graph data type in order to display the contents of the graph:

```
template <typename T>
std::ostream &operator<<(std::ostream &os, const Graph<T> &G)
{
    for (auto i = 1; i < G.vertices(); i++)
    {
        os << i << ":\t";

        auto edges = G.outgoing_edges(i);
        for (auto &e : edges)
            os << "{" << e.dest << ": " << e.weight << "}, ";

        os << std::endl;
    }
}
```

Step 4: Write a class to define our graph data structure, as shown here:

```
template <typename T>
class Graph
{
public:
    Graph(size_t N) : V(N) {}

    auto vertices() const
    {
        return V;
    }

    auto &edges() const
    {
        return edge_list;
    }

    void add_edge(Edge<T> &&e)
    {
        if (e.src >= 1 && e.src <= V &&
            e.dest >= 1 && e.dest <= V)
            edge_list.emplace_back(e);
        else
            std::cerr << "Vertex out of bounds" << std::endl;
    }

    auto outgoing_edges(size_t v) const
    {
        std::vector<Edge<T>> edges_from_v;
        for (auto &e : edge_list)
        {
            if (e.src == v)
            {
                edges_from_v.emplace_back(e);
            }
        }
        return edges_from_v;
    }

    template <typename T>
    friend std::ostream &operator<<(std::ostream &os, const Graph<T> &G);

private:
    size_t V;
    std::vector<Edge<T>> edge_list;
};
```

Step 5: For this exercise, we shall test our implementation of BFS on the following graph:



We need a function to create and return the required graph. Note that while edge weights are assigned to each edge in the graph, this is not necessary since the BFS algorithm does not need to use edge weights. Implement the function as follows:

```
template <typename T>
auto create_reference_graph()
{
    Graph<T> G(9);

    std::map<unsigned, std::vector<std::pair<size_t, T>>> edges;
    edges[1] = {{2, 2}, {5, 3}};
    edges[2] = {{1, 2}, {5, 5}, {4, 1}};
    edges[3] = {{4, 2}, {7, 3}};
    edges[4] = {{2, 1}, {3, 2}, {5, 2}, {6, 4}, {8, 5}};
    edges[5] = {{1, 3}, {2, 5}, {4, 2}, {8, 3}};
    edges[6] = {{4, 4}, {7, 4}, {8, 1}};
    edges[7] = {{3, 3}, {6, 4}};
    edges[8] = {{4, 5}, {5, 3}, {6, 1}};

    for (auto &i : edges)
        for (auto &j : i.second)
            G.add_edge(Edge<T>{i.first, j.first, j.second});

    return G;
}
```

Step 6: Implement the breadth-first search like so:

```
template <typename T>
auto breadth_first_search(const Graph<T> &G, size_t dest)
{
    std::queue<size_t> queue;
```

```
        std::vector<size_t> visit_order;
        std::set<size_t> visited;
        queue.push(1); // Assume that BFS always starts from vertex ID 1
        while (!queue.empty())
        {
            auto current_vertex = queue.front();
            queue.pop();
            // If the current vertex hasn't been visited in the past
            if (visited.find(current_vertex) == visited.end())
            {
                visited.insert(current_vertex);
                visit_order.push_back(current_vertex);
                for (auto e : G.outgoing_edges(current_vertex))
                    queue.push(e.dest);
            }
        }
        return visit_order;
    }
```

Step 7: Add the following test and driver code that creates the reference graph, runs BFS starting from vertex 1, and outputs the results:

```
    template <typename T>
    void test_BFS()
    {
        // Create an instance of and print the graph
        auto G = create_reference_graph<unsigned>();
        std::cout << G << std::endl;
        // Run BFS starting from vertex ID 1 and print the order
        // in which vertices are visited.
        std::cout << "BFS Order of vertices: " << std::endl;
        auto bfs_visit_order = breadth_first_search(G, 1);
        for (auto v : bfs_visit_order)
            std::cout << v << std::endl;
    }

    int main()
    {
        using T = unsigned;
        test_BFS<T>();
        return 0;
    }
```

Include the out and observation as a table in section 6.

6. Output
7. Supplementary Activity
ILO C: Demonstrate an understanding of graph implementation, operations and traversal methods. 1. A person wants to visit different locations indicated on a map. He starts from one location (vertex) and wants to visit every vertex until it finishes from one vertex, backtracks, and then explore other vertex from same vertex. Discuss which algorithm would be most helpful to accomplish this task. 2. Identify the equivalent of DFS in traversal strategies for trees. To efficiently answer this question, provide a graphical comparison, examine pseudocode and code implementation. 3. In the performed code, what data structure is used to implement the Breadth First Search? 4. How many times can a node be visited in the BFS?
8. Conclusion
Provide the following: • Summary of lessons learned • Analysis of the procedure • Analysis of the supplementary activity • Concluding statement / Feedback: How well did you think you did in this activity? What are your areas for improvement?
9. Assessment Rubric