

Introduction to Modern Astrophysics Thesis Notes

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Contents

1	Stellar Radiation for Cloudy Input	3
1.1	Stellar Blackbodies, ch 3.4	3
1.2	Formation of Spectral Lines, ch 8.1	3
1.3	The Boltzmann Equation, relative occupation of energy levels	3
1.3.1	The Saha Equation, percentage of ionization equation	3

Stellar Radiation for Cloudy Input

1.1 STELLAR BLACKBODIES, CH 3.4

“Stars and planets are blackbodies, at least to a rough first approximation”. Luminosity is related to temperature by:

$$L = 4\pi R^2 \sigma T_e^4 \quad (1)$$

1.2 FORMATION OF SPECTRAL LINES, CH 8.1

The **Maxwell-Boltzman Distribution** gives statistical gas partial velocities. The **Boltzmann Equation** gives velocity distribution of particles in a gas as a function of T . The **Saha Equation** gives probabilities of the different energy level occupation configuration. Boltzmann and Saha require thermodynamic equilibrium.

Figure 8.1 is a diagram of the strength of spectral lines as a function of temperature. Could be useful.

1.3 THE BOLTZMANN EQUATION, RELATIVE OCCUPATION OF ENERGY LEVELS

$$\frac{N_b}{N_a} = \frac{g_b e^{-E_b/kT}}{g_a e^{-E_a/kT}} = \frac{g_b}{g_a} e^{-(E_b-E_a)/kT} \quad (2)$$

The Boltzmann Equation relates the number of atoms in each energy level, where a and b are both energy levels of an atom. This gives the relative strengths of emission and absorption lines within an atom. g represents the degeneracy of the energy level. $g \in \mathbb{Z}$. E is energy level and N is number in that state.

1.3.1 The Saha Equation, percentage of ionization equation

Gas must be in thermal equilibrium and have a density $< 1 \text{ kg m}^{-3}$

$$\frac{N_{i+1}}{N_i} = \frac{2 Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{\frac{3}{2}} e^{-\chi_i/kT} \quad (3)$$

This describes the ratio of higher-level atoms to lower level atoms (example: HII to HI) as a function of temperature, T , energy required for ejection of an electron from the ground state, χ , and electron properties of the ionized gas, m_e , n_e .

Note: This is also formulated with electron pressure, P_e , whose determination is explained in section 9.5. $P_e = n_e kT$.

This equation also depends on the **partition functions**, Z , for the initial and final atoms:

$$Z = \sum_{j=1}^{\infty} g_j e^{-(E_j - E_1)/kT} \quad (4)$$

Where E_j is the energy of the j th energy level and g_j is the degeneracy of that level.