MAE 8 - Winter 2018 Project

Satellite Trajectories with Time-varying Orbits

1 Introduction

In this project, you are to simulate the trajectory of several satellites as they maneuver into their new orbits. At initial time, the satellite trajectories are produced by the gravitational force from Earth. An engine will be turned on for a short period of time to produce a thrust needed for the satellites to move to higher orbits. You will simulate and analyze the trajectories of the satellites before, during and after the maneuver.

From Newton's second law, the orbital motion of the satellites can be described by the following differential equations:

$$
\begin{aligned}
\frac{\partial U}{\partial t} &= \frac{Th^x}{m} - GM_e\frac{X}{(X^2+Y^2+Z^2)^{3/2}},\\
\frac{\partial V}{\partial t} &= \frac{Th^y}{m} - GM_e\frac{Y}{(X^2+Y^2+Z^2)^{3/2}},\\
\frac{\partial W}{\partial t} &= \frac{Th^z}{m} - GM_e\frac{Z}{(X^2+Y^2+Z^2)^{3/2}},\\
\frac{\partial X}{\partial t} &= U,\\
\frac{\partial Y}{\partial t} &= V,\\
\frac{\partial Z}{\partial t} &= W, \qquad (1)
\end{aligned}
$$

where t is time (in seconds); X, Y, and Z are position (in meters) of the satellites relative to the center of Earth in rectilinear coordinate. U, V, and W are the velocity components (in meters per second) in the X, Y and Z directions, respectively. Th^x, Th^y and Th^z are three components of thrust (in N) produced by the engine. Earth is assumed to be stationary in this project.

The parameters in the equation 1 above are given as follows:

- Radius of Earth: $R_e = 6.37 \times 10^6\,(m)$
- Mass of the Earth: $M_e = 5.97 \times 10^{24}\,(kg)$
- Gravitational constant : $G = 6.67408 \times 10^{-11}\,(m^3\,kg^{-1}\,s^{-2})$
- Mass of the satellites: $m = 1500\,(kg)$:

Furthermore, the following quantities will be of interest while analyzing the satellite orbits:

- Altitude of the satellites (m): $h = \sqrt{X^2+Y^2+Z^2} - R_e$
- Speed of the satellites $(m\,s^{-1})$: $V_{mag} = \sqrt{U^2+V^2+W^2}$
- Acceleration of the satellites $(m\,s^{-2})$: $Acc = d(V_{mag})/dt$

2 Approach

Orbits of six satellites will be simulated. The difference among the orbits are due to the initial position, the initial velocity of the satellites and the maximum thrust produced by the engine. The initial position and velocity components are stored in text file: **satellite_data.txt**. In the file, you can also find the start and end times of the engine as well as the maximum thrust. When the engine is turned on, the instantaneous values of the three components of thrust (Th^x, Th^y, and Th^z) are produced by function **engine**. Download **satellite_data.txt** and **engine.m** from TritonED.

Using Euler-Cromer method, equations 1 can be transformed into the following algebraic form:

$$\begin{aligned}
U_{n+1} &= U_n + \left[\frac{Th_n^x}{m} - GM_e\frac{X_n}{(X_n^2+Y_n^2+Z_n^2)^{3/2}}\right]\Delta t,\\
V_{n+1} &= V_n + \left[\frac{Th_n^y}{m} - GM_e\frac{Y_n}{(X_n^2+Y_n^2+Z_n^2)^{3/2}}\right]\Delta t,\\
W_{n+1} &= W_n + \left[\frac{Th_n^z}{m} - GM_e\frac{Z_n}{(X_n^2+Y_n^2+Z_n^2)^{3/2}}\right]\Delta t,\\
X_{n+1} &= X_n + U_{n+1}\Delta t,\\
Y_{n+1} &= Y_n + V_{n+1}\Delta t,\\
Z_{n+1} &= Z_n + W_{n+1}\Delta t,
\end{aligned} \tag{2}$$

where subscript n denotes variables at current time, subscript n+1 denotes variables at time that is Δt ahead.

3 Tasks to perform

To simulate and analyze the orbits, you are to write three MATLAB files: **satellite.m**, **read_input.m** and **project.m**. The descriptions of these files are given below.

1, File satellite.m:

This is the function that solves the equations 2 for a given set of initial conditions. Use $\Delta t = 1\,s$. The function should have the following header: **function [T, X, Y, Z, U, V, W] = satellite(Xo, Yo, Zo, Uo, Vo, Wo, tstart, tend, maxthrust)** where the inputs are components of the initial position (Xo, Yo, Zo) and initial velocity (Uo, Vo, Wo). The engine is turned on at time (tstart) and off at time (tend) with a maximum value of thrust (maxthrust). The outputs are quantities discussed above. All inputs are scalars while all outputs are vectors. You need to simulate the orbits for a total travel distance of $4.2 \times 10^8\,m$ around Earth.

2, File read_input.m:

This function reads the parameters stored in the file **satellite_data.txt** into MATLAB. The function should have the following declaration: **function [Xo, Yo, Zo, Uo, Vo, Wo, tstart, tend, maxthrust] = read_input(inputfile, sat_id)** where **inputfile** is a string denoting the name of the file to be read and **sat_id** is an integer indicating the satellite number. The outputs are the initial position (**Xo**, **Yo**, **Zo**), components of initial velocity (**Uo**, **Vo**, **Wo**), start and end times of the engine (**tstart**, **tend**) and the maximum thrust produced (**maxthrust**). When the input **sat_id** is not available in the file, the function should set all outputs to **NaN** and display an error warning to screen.

3, File project.m:

This is the main script to perform the tasks (and subtasks) described below.

Task 1: Here, you simulate satellite orbits. There are six satellites with different initial values of position and velocity. Call function **satellite** to get the orbits. For each orbit, the outputs should include time (T), three components of position (X, Y, Z) and three components of velocities (U, V, W), and all of these variables should be vectors with the same length.

Task 2: Here, you will create 3 figures based on the results from Task 1. All figures should have title, axis labels with correct units and legends. Use different colors to indicate the orbits. Please make sure that all 3 figures are plotted when your **project.m** is executed.

- Create figure 1 to plot the six trajectories in six different panels. Also include the final positions of the satellite and Earth's surface in each panel. Use different colors for different satellites. Use function **plot3** and **subplot**. Sample script to include the Earth surface is given on TritonED, see the file **plot_earth.m**.
- Create figure 2 which includes 3 panels. In the top panel, plot altitude versus time for all satellites. Plot speed versus time in the middle panel and acceleration versus time in the bottom panel. Use function **subplot** to create different panels of the figure.
- Create figure 3 to plot acceleration versus speed for all satellites. Put all plots together in a single panel.

Task 3: Create a 6-element data structure named **stat** with the following fields:

- **sat_id**: to include a number ranging from 1 to 6 to indicate the satellite ID.
- **end_time**: to include the total travel time of each satellite.
- **final_position**: to include a 3-element vector indicating the final position of the satellites.
- **final_velocity**: to include a 3-element vector indicating the velocity component (U, V, W) at final time.
- **max_speed**: to include the maximum speed during the simulation
- **min_speed**: to include the minimum speed during the simulation
- **time_lmax_altitude**: to include the times at which the satellites are farthest from Earth (e.g. times at which the altitude h shows a local maximum).
- **orbital_period_before**: to include the time that the satellites take to revolve around Earth for one time *before* the engine is turned on. *Hint: Subtract the times of the first two local maxima of altitude h.*
- **orbital_period_after**: to include the time that the satellites take to revolve around Earth for one time *after* the engine is turned off. *Hint: Subtract the times of the last two local maxima of altitude h.*

Task 4: Create a file named **report.txt** to report the following:

- Your name on the first line
- Your PID on the second line
- A string **sat_id max_speed min_speed orbital_period_before orbital_period_after** on the third line
- Corresponding values of satellite ID, maximum speed, minimum speed, orbital periods before and after the change in orbits for each of the six satellites from line fourth to line nineth. Use single digit for the satellite ID and format %15.9e for others.

Task 5: At the end of your **project.m** script, set the following:

```
p1a =evalc('help read_input ';
p1b =evalc('help satellite ';
p1c ='See figure 1';
p1d ='See figure 2';
p1e ='See figure 3';
p2a = stat(1);
p2b = stat(2);
p2c = stat(3);
p2d = stat(4);
p2e = stat(5);
p2f = stat(6);
p3 = evalc('type report.txt');
```

and answer the following questions using figure 2:

Q1. The satellites move fastest when they are closest or farthest from Earth? Put the answer in string **p4a = '...'**.

Q2. As the satellites travel away from earth, does their velocity increase or decrease? Put the answer in string **p4b = '...'**.

4 Instructions to submit

Follow the homework solution template. Remember to **clear all**, **close all**, **clc**, and fill in your name and PID. Set **hw_num = 'project'**. Create a zip archive named **project.zip**. The zip archive should include the following files: **project.m**, **satellite.m**, **read_input.m**, **engine.m**, **satellite_data.txt**, **report.txt**, **earth_topo.mat** and other scripts / functions that you have written for the project. Make sure that you include all necessary files so that your **project.m** will run properly. Submit **project.zip** through TritonED before 9 PM on 03/16/2018. Use double precision unless otherwise stated.