ELG 2138 Cheat Sheet

1 Introduction

Power is the rate at which something gives energy. It is the time derivative of work.

We have independant current sources, and independant voltage sources.

Current is the amount of charge per second, and voltage is the potential energy difference.

Anything that consumes energy is a resistor with a resistance in Ohms (Ω).

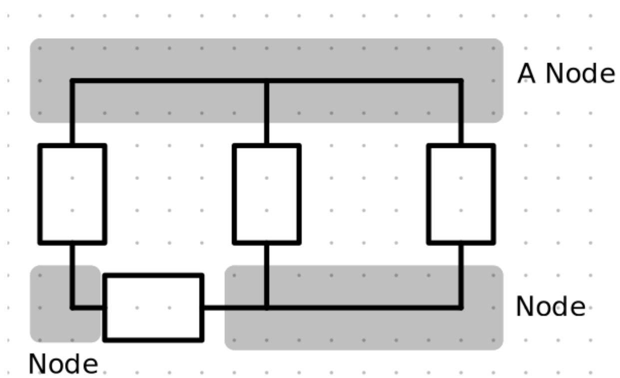
A resistor can be read using the colored bands on the resistor. I will not include a chart here, but it is a simple internet search away.

Electrical sources can be either Direct Current (DC), where the voltage is constant with time, or Alternating Current (AC) where the voltage varies (along a sine wave) with time.

2 Resistive Circuits

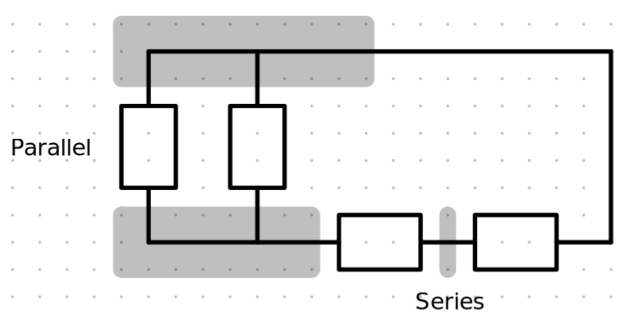
These are circuits with **only** resistors, and independant current/voltage sources.

Circuit elements are directly connected when they share the same terminal. This common terminal is called the node.



A series connection is when 2 elements share a single node. The current is the same.

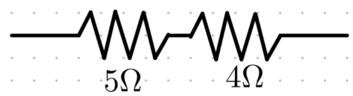
A parallel connection is when there are 2 terminals shared between 2 or more elements. If 2 elements are in parallell (//), then they must both share a node on each side of the elements.



2.1 Adding Resistors

To add resistors in series, we just add up their value.

Ex. What is the equivalent resistance to these resistors?

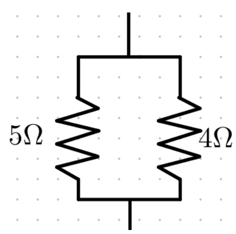


Since they are in series, we just add the value to get $5 + 4 = 9\Omega$.

To add resistors in parallel, we say:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...} \qquad \text{For 2 resistances: } R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$$

Ex. What is the equivalent resistance to these resistors?



Since they are in parallel, we need to do some calculations.

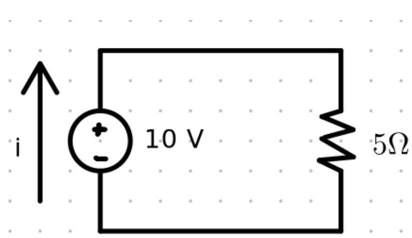
$$R_{eq} = \frac{1}{\frac{1}{4} + \frac{1}{5}} = \frac{4 \times 5}{4 + 5} = \frac{20}{9}\Omega$$

2.2 Ohms Law

Ohms law relates resistance to current and voltage.

$$v = iR$$

Ex. Find the current i through the voltage source.



We cannot use Ohms law directly since we do not know the resistance of the source.

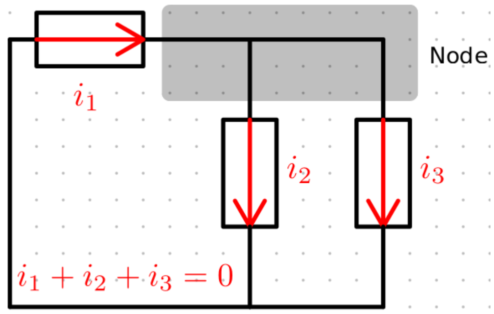
However, we know that the source and resistance are in parallel, so the voltage at the resistor is also 10 V. We know that it is also in series with the 10 V source, so the current everywhere is the same.

$$v = iR \implies i = \frac{v}{R} = \frac{10}{5} = 2A$$

Therefore the current in the resistor is 2A, so the current in the source is 2A.

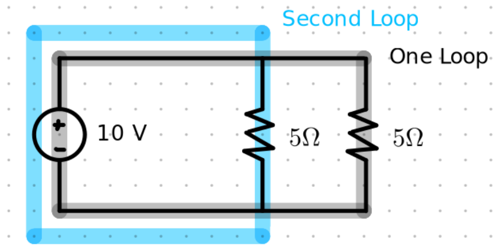
2.3 KCL (Kirchoffs Current Law)

This states that the sum of all currents at any node is 0.



2.4 KVL (Kirchoffs Voltage Law)

This states that the sum of voltages in any loop is 0.



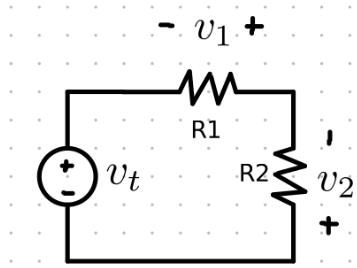
So we go around a loop, getting the voltages of each element (recall v=iR) and then solve. We can create multiple KVLs to get a system of equations.

2.5 Voltage Divider

If we have a voltage that goes through 2 resistors in series, the voltage gets divided between both of them.

$$v_1 = \frac{R_1}{R_1 + R_2} v_t$$

where v_t is the total voltage going in.

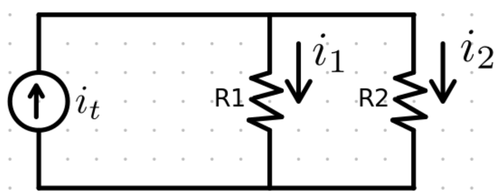


2.6 Current Divider

If we have current that goes though 2 resistors in parallel, the current gets divided between both of them.

$$i_1 = \frac{R_2}{R_2 + R_1} i_t$$

where i_t is the total current going in.



2.7 Power

If the power is **positive**, an element **consumes** power.

If the power is **negative**, an element **produces** power.

We say that:

$$P = vi$$

The net power in a circuit is always 0.

3 Methods to Analyse Resistive Circuits

These methods are ways to easily find all values in a circuit no matter the complexity.

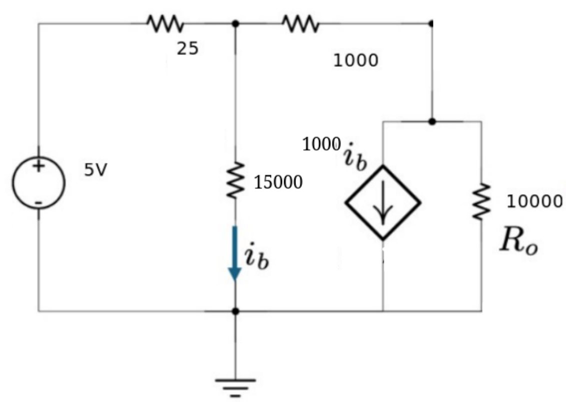
3.1 Node Voltage Analysis (NVA)

With this method, we need a reference node (Ground node). This node can usually be any node.

This method works on using KCL at each node of the circuit, and then we will get all the currents in terms of node voltage, and then solve the system of equations.

1. Identify the number of nodes in the circuit and designate the reference node
2. If there is an independant voltage source that is not connected to ground, make that a supernode and get it an equation.
3. Write the node voltage KCLs for all nodes. Note that if the node is a voltage source connected to ground, we know the voltage.
4. Solve the KCLs to get all voltages.
5. Get the information we need

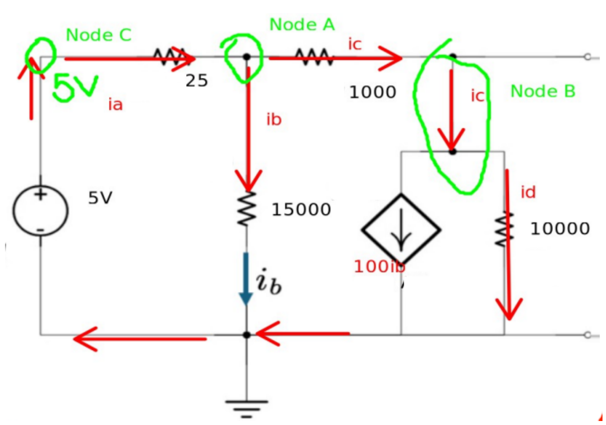
Ex. Find the node voltages in this circuit.



We already have a ground. We notice that there is an independant source in this circuit, as well as 3 nodes.

I will draw in the nodes as well as the current directions I assume (it does not matter what I assume as long as I am consistent).

Note that the leftmost node is 5v.



Now I create equations for nodes A and B. We use the vact that $i = \frac{V}{R}$.

For A:

$$\text{currentIn} = \text{currentOut} \implies \frac{5 - V_A}{25} = \frac{V_A}{15000} + \frac{V_A - V_B}{1000}$$

For B it is a bit harder since we do not know the dependant source, except we actually do since $i_b = \frac{V_A}{15000}$.

$$\text{currentIn} = \text{currentOut} \implies \frac{V_A - V_B}{1000} = 100 \cdot \frac{V_A}{15000} + \frac{V_B}{10000}$$

Now I have 2 equations and 2 unknowns so I solve to get $V_A = 4.33, V_B = -22.3$.

3.2 Mesh Current Analysis (MCA)

With this method, we use meshes (loops) in a circuit and do the KVL around those loops. Same idea as the NVA.

1. Divide the circuit into meshes and assign each mesh a name.
2. If there is an independant current source common to 2 meshes, form a supermesh and get it an equation.
3. Write the mesh current KVLs for all meshes. Note that if there is an independant current source only common to 1 mesh, we know that meshes current.
4. Solve the KVLs to get all currents.
5. Get the information we need.

Ex. Example found in Module 5.2

4 Circuit Theorums

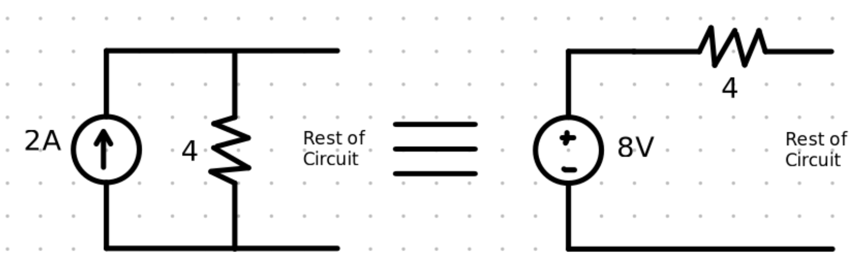
These are things that help make a circuit simpler.

4.1 Source Transformations

We can turn a voltage source and resistor in series into a current source and resistor in parallel.

We can turn a current source and resistor in parallel to a voltage source and resistor in series.

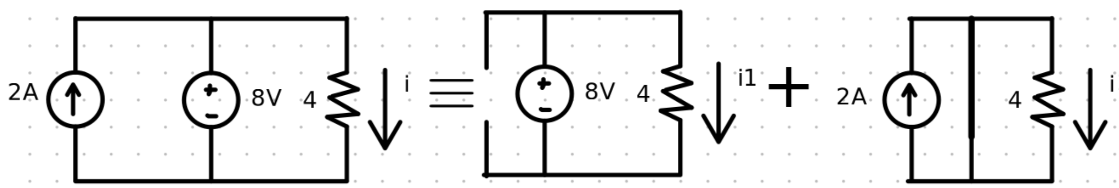
For both of these we just use Ohms law.



4.2 Superposition

This states that if we have 2 circuits with current and voltage sources only, then the value of any current or voltage can be gotten by adding the values obtained from each individual source with other sources deactivated.

Deactivating means replace a voltage source with a short circuit (SC), and a current source with an open circuit (OC).



$$i = i_1 + i_2$$

4.3 Thevanin

Any circuit **connected between 2 terminals** can be simplified down to an independant voltage source in series with a resistor.

This voltage source is called the Open Circuit Voltage (V_{OC}) and the resistance is called the Thevanin resistance (R_t)

To find V_{OC} between 2 terminals, we create an open circuit between those 2 terminals and solve for the voltage across them. This is the V_{OC}.

To find R_t, we can deactivate all independant sources (Voltage becomes SC, Current becomes OC), then if there are no dependant sources, combine resistors until we have one left. If there are still dependant sources, then we connect a dependant current source with 1A to the terminals, then find the voltage across the terminals, then find R_t knowing 1A current, and voltage.

4.4 Norton

Any circuit **connected between 2 terminals** can be simplified down to an independant current source in parallel with a resistor.

This current source is called the Short Circuit Current (I_{SC}) and same as the Thevanin circuit, the resistance is R_t.

To find I_{SC} between 2 terminals, we create a short circuit between the terminals and find the current flowing through that wire. This is the I_{SC}.

If we know both I_{SC} and V_{OC}, then we can find $R_t = \frac{V_{OC}}{I_{SC}}$

5 Energy Storage Elements

5.1 Capacitors and Inductors

Capacitors and Inductors are circuit elements that we usually work with in the complex domain. For this module though, we only work in the real domain.

A capacitor has a **capacitance C** which is measured in Farads, and an inductor has an **inductance L** measured in Henrys.

The current and voltage through a capacitor is:

$$i(t) = C\frac{\mathrm{d}V}{\mathrm{d}t} \qquad v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)\mathrm{d}\tau$$

The current and voltage through an inductor is:

$$v(t) = L\frac{\mathrm{d}i}{\mathrm{d}t} \qquad i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(\tau)\mathrm{d}\tau$$

Inductors are added together similar to **resistors** (sum in series, inverse sum in //) while **capacitors** are added together the **opposite of resistors** (sum in //, inverse sum in series).

The current in the inductor and voltage in the capacitor can **never** change instantly.

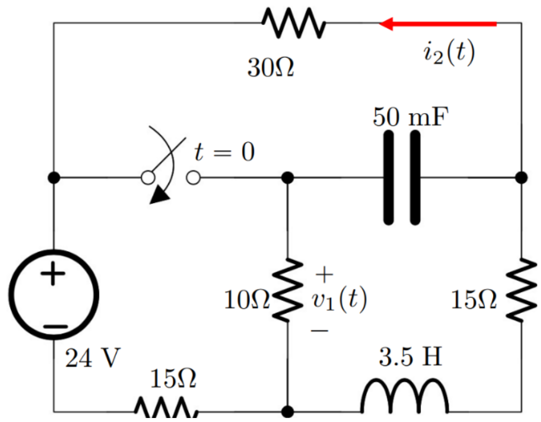
5.2 Switches and Initial Conditions

A **switching circuit** has a switch that changes at a certain time. We say that before the switch changes, the circuit has been **stable** for a very long time (infinity).

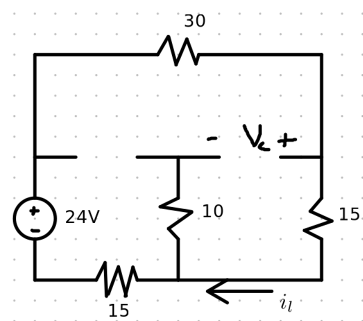
When working with switching circuits, we use the following procedure:

1. Before the switch changes
   Capacitor is an **open circuit** and inductor is a **short circuit**
   Find the voltage across the capacitor or/and the current through the inductor.
2. Right after the switch changes
   Voltage across the capacitor and current in inductor are the same as before.
   Solve for what we need treating the capacitor as voltage source and inductor as current source.
3. (Optional) At time is infinity, consider capacitor as open circuit, and inductor as closed circuit.

Ex. Find $v_1(t^+), v_1(t^-)$ in the following circuit.

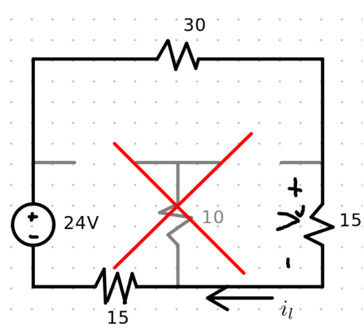


When looking before the switch is closed, we draw the capacitor as an OC and inductor as SC.



We can cancel out the 10 Ω resistor since only one side is connected. This means that the voltage across is is 0. $v_1(0^-) = 0$.

The voltage across the capacitor is the same as the voltage across the 15 Ω resistor.



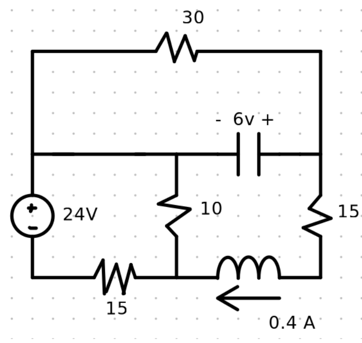
So I will do a KVL across the whole loop: $-24 + 30i + 15i + 15i \implies i = 0.4A$

Then I will use Ohms law to get the voltage across the 15 Ohm resistor: $v = 14 \cdot 0.4 = 6$

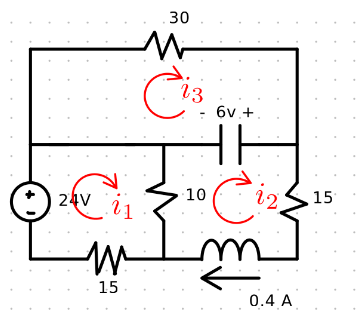
So we know that $v_c(0^-) = v_c(0^+) = 6V$

Also the current in the inductor is: $i_l(t^-) = i_l(t^+) = 0.4A$

Now we look at the circuit after the switch is clicked.



Now this is a simple circuit if we consider the cap a voltage source, and the inductor a current source. I will use the mesh current analysis to solve.



We know that i2 is 0.4A due to the current source in that mesh.

Do a KVL for mesh 1.

$$-24 + 10(i_1 - 0.4) + 15i_1 = 0 \implies i_1 = 1.12A$$

Do a KVL for mesh 2.

$$30i_3 + 6 = 0 \implies i_3 = -0.2$$

NOTE *that in general for Mesh Current we will now have 2 equations with 2 unknowns, this was just a really easy case.*

Now that we know the currents, we can find the voltage across the 10 Ohm resistor is just $V = iR = (i_2 - i_1)10 = 7.2V$

The circuit was verified using multisim as found in Appendix B.

6 Complete Responce of RL and RC

This module is very similar to module 5 except we find the v(t) or i(t) for any t using an equation.

We follow the same steps of:

1. **Before** the switch changes
   Capacitor is an **open circuit** and inductor is a **short circuit**
   Find the voltage across the capacitor or/and the current through the inductor.
2. **Right after** the switch changes
   Voltage across the capacitor and current in inductor are the same as before.
   Solve for Norton and Thevanin circuits treating the capacitor as voltage source and inductor as current source.
3. Sub values into the current or voltage in inductor or capacitor equation.

6.1 RC Circuits (Resistance and Capacitor)

We use the formula:

$$v_c(t) = V_{OC} + (v_c(0^+) - V_{OC})e^{-\frac{t}{R_tC}}$$

6.2 RL Circuits (Resistor and Inductor)

We use the formula:

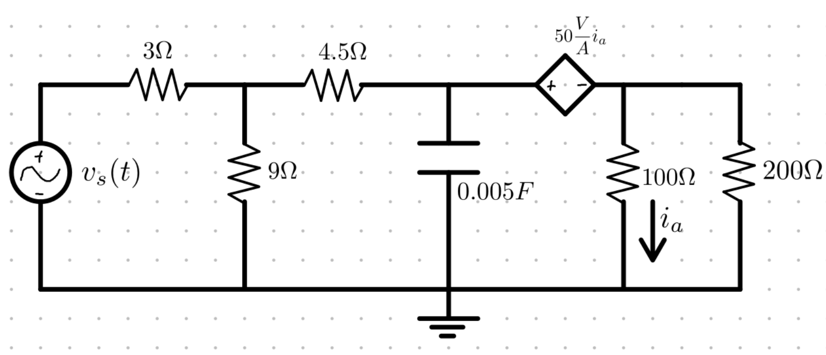
$$i_l(t) = I_{SC} + (i_l(0^+) - I_{SC})e^{-\frac{tR_t}{L}}$$

6.3 Multiple Periods

We can have **multiple periods where we need a different equation.**

This is very simple in that there is a seperate I_{SC} or V_{OC} for each time period, and the initial part ($v_c(0^+)$ or $i_l(0^+)$) is just the final of the previous time period.

Ex. Find the voltage in the capacitor for the following circuit, wheree $v_s(t) = 1 \times u(t) - u(t-0.1)$ and $u(t)$ is the unit step function. *Note that the unit step function is 0 before t0, and 1 after t0.*



So in this case, there are 3 timezones. I will make a timeline to show:

—— Before t0 ———— After t0 ————— After t0+0.1 ——

v(t) —— 0 ——————— 10 —————————— 0 —————

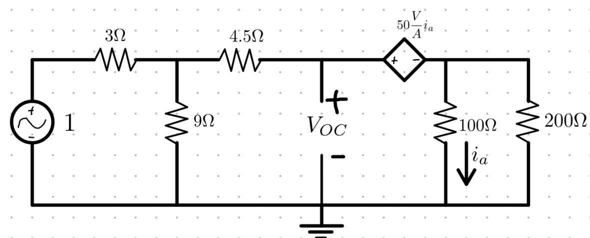
We start with **Before t0**:

Here the voltage source is 0V, which means that i_a is 0A, which means that there is no voltage/current in the circuit.

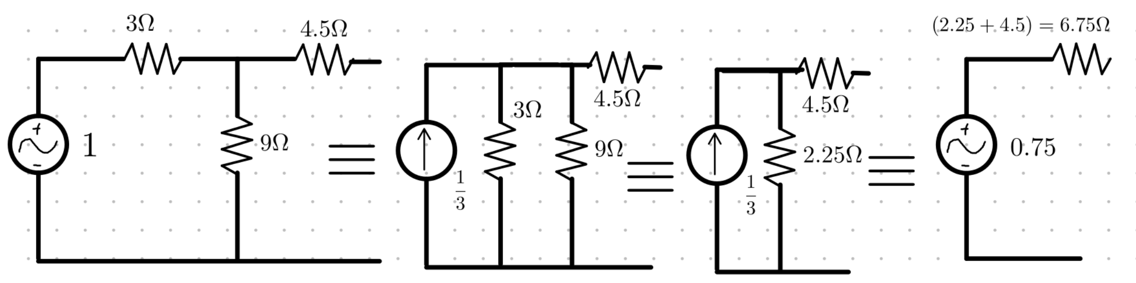
$V_{OC} = 0, v(t_0) = 0 \implies v(t) = 0\{t < t_0\}$

Now we go to **After t0, before t0+0.1**

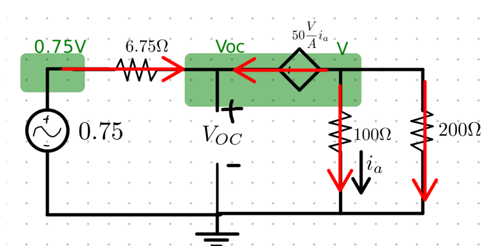
The voltage is given by $v(t) = V_{OC} + (v(t0^+) - V_{OC})e^{\frac{-t}{R_t C}}$ and we know $v(0^-) = v(t^+) = 0$. We need the open circuit voltage, and the thevanin resistance. So we create thevanin circuit.



We can simplify the left half of the circuit using source transformations.



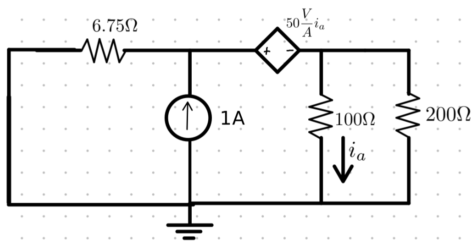
Then we can do a node voltage analysis to solve this. We have 1 known node, and 1 supernode. So we have 1 KCL and 1 supernode equation.



Voc Node: $\dfrac{0.75 - V_{OC}}{6.75} - \dfrac{V}{100} - \dfrac{V}{200} = 0$ Supernode: $V_{OC} = V + 50i_a = V + 50\dfrac{V}{100}$

$$\implies V_{OC} = 0.703$$

Now I just need to get R_t. I will deactivate all sources in the circuit, but since there is a dependant source, I need to add an independant 1A source.



Doing the mesh current analysis, we get the current though the 6.75 Ohm resistor which gives us the voltage there which is the same as the voltage over the 1A source. This gives us: $R = V/I = 6.32\Omega$

Now I can make the equation for the second part of the voltage.

$$v(t) = 0.702 + (0 - 0.702)e^{\frac{-t}{6.32 \cdot 0.005}}$$

Now I need the third period of **After t0+0.1**

For this, I see that the voltage source will be 0. This means that no current as well. However there is a resistance. I will find this resistance to be 6.32 Ohms again. The initial condition will be $v(0.1^-) = 0.676$ so the equation is: $v(t) = 0 + 0.676e^{\frac{-(t+0.1)}{6.32 \cdot 0.005}}$.

I added the 0.1 to t to adjust for this interval starting after 0.1 seconds.

The entire voltage for the whole interval is:

$$v(t) = \begin{cases} 0 t < t_0 \\ 0.702 + (0 - 0.702)e^{\frac{-t}{6.32 \cdot 0.005}} t_0 \le t \le t_0 + 0.1 \\ 0.676e^{\frac{-(t+0.1)}{6.32 \cdot 0.005}} t > t_0 + 0.1 \end{cases}$$

7 AC Analysis of RLC Circuits using Phasors

When working in the complex domain, we say we are working in the **phasor domain or frequency domain**.

We are given the voltage source as a sinusoidial in the form of $v(t) = A\cos(\omega t + \phi)$ where A is the amplitude of the voltage, ω is the period, and ϕ is the phase angle. We can also write this in **phasor form** as $v(t) = A\angle\phi$

To work in the phasor domain, we do the following steps:

1. Represent all independant sources in their phasor form.

2. Represent all unknown currents and voltages as phasors (Inductors, Capacitors, Resistors, etc.)
3. Treat the circuit as a convential circuit and solve using conventional means such as KVL, KCL, Source Transformations, NVA, etc.
4. Once we have the piece of information we want, **put it back into the time domain.**

7.1 Inductance

In the phasor domain, resistors still are measured in Ohms. We do not have to change anything.

Capacitors need to be converted into their **impedance** (complex form of resistance) using the equation:

$$Z = \frac{1}{j\omega C}\Omega = \frac{-j}{\omega C}\Omega$$

Inductors also have to be converted into their **impedance** using the equation:

$$Z = j\omega L\Omega$$

7.2 Admittance

The admittance Y is just the inverse of the impedance Z.

$$Y = Z^{-1} = \frac{1}{Z}$$

8 Power Calculations in AC Circuits

This module mostly just has to do with using the correct equations and subbing in the correct values. These equations are located on the formula sheet in Appendix A.

8.1 RMS

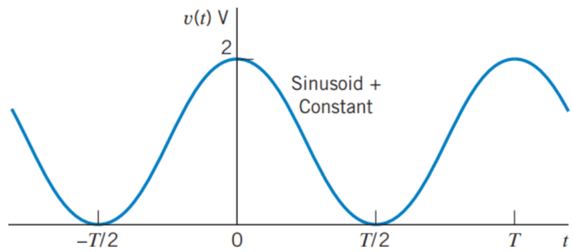
If we have an AC current, and we want to find the equivalent DC current (RMS Value, or Effective Value) we use the following equation:

$$I_{RMS-DC} = \sqrt{\frac{1}{T}\int_0^T i(\tau)^2 \mathrm{d}\tau} \qquad I_{RMS-AC} = \frac{I_m}{\sqrt{2}}$$

where I_m is the amplitude of the current.

It is the same idea with voltage.

Ex. Find the RMS value for the following waveform.



We know that this sinusoidial wave has a constant DC voltage of 1V, and a sinusoidial AC voltage with amplitude of 1V.

So we need to get both the DC and AC parts of the RMS value.

$$I_{rmsDC} = \sqrt{\frac{1}{T}\int_0^T i(\tau)^2 \mathrm{d}\tau} = \sqrt{\frac{1}{T}\int_0^T 1^2 \mathrm{d}\tau} = 1$$

$$I_{rmsAC} = \frac{1}{\sqrt{2}}$$

Then we need the magnitude of the rms value. This is just the square root of the squares.

$$I_{rms} = \sqrt{1^2 + \frac{1}{\sqrt{2}}^2} = 1.225A$$

8.2 Complex Power

In the frequency domain, Power has 2 parts. The **real part P** (average power), and the **imaginary part Q** (reactive power).

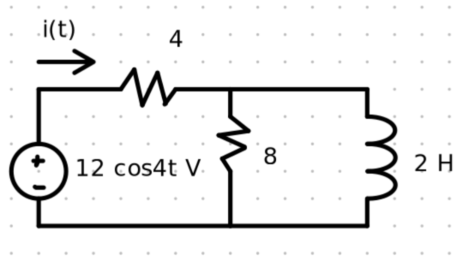
The **average power** of inductors and capacitors is **always 0**.

Complex power is measured in VA, the real part in W, and the reactive part in VAR.

Z is the impedance of an element. We can find the complex power of a circuit by knowing the current, and the impedance.

$$S = \frac{I_m^2}{2}\Re\{Z\} + \jmath\frac{I_m^2}{2}\Im\{Z\}$$

Ex. Find the complex power delivered by the source.



We start by getting the circuit into the complex domain. We can only work with it in that domain.

The inductor becomes $\jmath\omega L$ $\jmath 4 \cdot 2 = \jmath 8$

Then just combine them together to get: $4 + \frac{8 \cdot 8\jmath}{8+\jmath 8} = 8 + \jmath 4$

Now I can find the current so I can use the equation $S = \frac{I_m^2}{2}\Re\{Z\} + \jmath\frac{I_m^2}{2}\Im\{Z\}$

$$I = \frac{V}{Z} = \frac{12\angle 0}{8+\jmath 4} = 1.341\angle -26.6 \implies I_m = 1.341$$

Now I can find the complex power using the complex and real components of the impedance, and the current magnitude.

$$S = \frac{1.341^2}{2} \cdot 8 + \jmath\frac{1.341^2}{2} \cdot \jmath 4 = 7.2 + \jmath 3.6$$

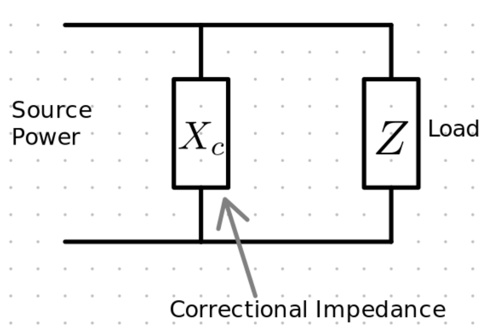
8.3 Power Factor

The **apparent power** is just the **magnitude** of the complex power S so $|S| = \sqrt{\Re\{S\}^2 + \Im\{S\}^2}$.

The power factor (pf) is the **cosine of the difference in phase angles** of the voltage and current ($\cos(\theta_V - \theta_I)$).

The power factor is between **0 and 1**. A power factor closer to 1 means less power wasted, closer to 0 means more power wasted.

We can connect a correctional impedance in parallel with the load to get that power factor closer to 1.



We have the equation of:

$$X_c = \frac{R^2 + X^2}{R\tan(\arccos(pfc)) - X}$$

where R is the real part of the load, X is the imaginary part of the load (X of $1 + \jmath 2$ would be 2), and pfc is the power factor we are aiming for.

If we have a **leading** power factor, then the arccos of the power factor is **-**. If we have **leading** pf, then arccos(pf) is $+$.

8.4 Maximum Power Transfer Theorum

This is just a simple formula.

$$P_{max} = \frac{|V_{OC}|^2}{8R_t}$$

This is the maximum average power that a load impedance can receive from the circuit.

Appendix

A Formula Sheet

Module 8

$I_{\text{eff}} = I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i(\tau)^2 \mathrm{d}\tau}$ $V_{\text{eff}} = V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v(\tau)^2 \mathrm{d}\tau}$ Non-sinusoidal

$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$ $I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$ sinusoidal

$\mathbf{S} = \frac{\mathbf{V}\mathbf{I}^\star}{2}$ $\mathbf{S} = P + \jmath Q$ $P = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I)$ $Q = \frac{V_m I_m}{2}\sin(\theta_V - \theta_I)$

$\mathbf{S} = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I) + \jmath\frac{V_m I_m}{2}\sin(\theta_V - \theta_I)$ $|\mathbf{S}| = \frac{V_m I_m}{2} = V_{\text{rms}} I_{\text{rms}}$

$\mathbf{S} = |\mathbf{S}| \times \left(\text{pf} + \jmath\sin(\pm\cos^{-1}(\text{pf}))\right)$ $\mathbf{S} = P \times \left(1 + \jmath\tan(\pm\cos^{-1}(\text{pf}))\right)$

$\mathbf{S} = \frac{V_m^2}{2}\Re\{\mathbf{Y}\} - \jmath\frac{V_m^2}{2}\Im\{\mathbf{Y}\}$ $\mathbf{S} = \frac{I_m^2}{2}\Re\{\mathbf{Z}\} + \jmath\frac{I_m^2}{2}\Im\{\mathbf{Z}\}$ Complex power

$\theta_V - \theta_I > 0$ **lagging** PF

$\theta_V - \theta_I < 0$ **leading** PF

$X_C = \frac{R^2 + X^2}{R\tan(\pm\cos^{-1}(\text{pfc})) - X}$ PF correction

For maximum **average** power transfer $\mathbf{Z}_L = \mathbf{Z}_t^\star$

$P_{\max} = \frac{|\mathbf{V}_{\text{OC}}|^2}{8R_t} = \frac{|\mathbf{I}_{\text{SC}}|^2 R_t}{8}$ Maximum Power Transfer in Phasor circuits

Module 5

$i_c(t) = C\frac{\mathrm{d}v_c(t)}{\mathrm{d}t}$ $v_C(t)$, $i_C(t)$

$v_L(t) = L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t}$ $i_L(t)$, $v_L(t)$

Module 4

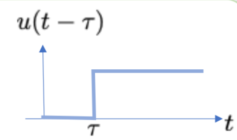
For maximum power transfer $R_L = R_t$

$p_{\max} = \frac{V_{\text{OC}}^2}{4R_t} = \frac{I_{\text{SC}}^2 R_t}{4}$

Module 6

$v_C(t) = V_{\text{OC}} + (v_C(0^+) - V_{\text{OC}})\, e^{-\frac{t}{R_t C}}$ $v_C(t)$

$i_L(t) = I_{\text{SC}} + (i_L(0^+) - I_{\text{SC}})\, e^{-\frac{R_t}{L}t}$ $i_L(t)$



$$u(t-\tau) = \begin{cases} 0 & t < \tau \\ 1 & t \geqslant \tau \end{cases}$$

Module 7

$\mathbf{Z}_C = \frac{1}{\jmath\omega C}\ \Omega$

$\mathbf{Z}_L = \jmath\omega L\ \Omega$

Module 2

$p = +v \times i$ + - Power absorbed

$p = -v \times i$ + - Power supplied

$p = \frac{v^2}{R} = i^2 \times R$ Power absorbed

Complex Analysis

$e^{\jmath\theta} = \cos(\theta) + \jmath\sin(\theta)$

$\cos(\theta) = \frac{1}{2}\left(e^{\jmath\theta} + e^{-\jmath\theta}\right)$

$\sin(\theta) = \frac{1}{2\jmath}\left(e^{\jmath\theta} - e^{-\jmath\theta}\right)$

$\jmath = \sqrt{-1}$, $\frac{1}{\jmath} = -\jmath$, $\jmath^2 = -1$, $\jmath^3 = -\jmath$, $\jmath^4 = 1$

$(x + \jmath y)(x - \jmath y) = x^2 + y^2$

$z = x + \jmath y$ Cartesian Representation

$z = \sqrt{x^2 + y^2}\quad e^{\jmath\tan^{-1}(y/x)}$ Polar Representation

$z = \sqrt{x^2 + y^2}\quad \angle\phi^\circ,\quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\mathbf{z}e^{\jmath\theta} + \mathbf{z}^\star e^{-\jmath\theta} = 2\Re\{z\}\cos(\theta) - 2\Im\{z\}\sin(\theta)$

$\frac{1}{\mathbf{z}} = \frac{1}{x + \jmath y} = \frac{x - \jmath y}{x^2 + y^2}$

$$\frac{\mathbf{z}_1}{\mathbf{z}_2} = \frac{x_1 + \jmath y_1}{x_2 + \jmath y_2} = \sqrt{\frac{x_1^2 + y_1^2}{x_2^2 + y_2^2}}\, e^{\jmath\left(\tan^{-1}(y_1/x_1) - \tan^{-1}(y_2/x_2)\right)}$$

$$\mathbf{z}_1 \times \mathbf{z}_2 = (x_1 + \jmath y_1) \times (x_2 + \jmath y_2) = \sqrt{(x_1^2 + y_1^2) \times (x_2^2 + y_2^2)}\quad e^{\jmath\left(\tan^{-1}(y_1/x_1) + \tan^{-1}(y_2/x_2)\right)}$$

$$a\cos(\theta) \pm b\sin(\theta) = \sqrt{a^2 + b^2}\cos\left(\theta \mp \tan^{-1}\left(\frac{b}{a}\right)\right)$$

$$a\cos(\theta) \pm b\sin(\theta) = \frac{\sqrt{a^2 + b^2}}{2}\left[e^{\jmath\left(\theta \mp \tan^{-1}\left(\frac{b}{a}\right)\right)} + e^{-\jmath\left(\theta \mp \tan^{-1}\left(\frac{b}{a}\right)\right)}\right]$$

B Multisim Verification for 5.2