A few formulas

Euler

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Summations, geometric series

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \quad |a| < 1 \;, \quad \sum_{k=0}^{n_1} a^k = \frac{1-a^{n_1+1}}{1-a} \quad \begin{matrix} a \neq 1 \\ n_1 \geq 0 \end{matrix}$$

$$\sum_{k=n_1}^{\infty} a^k = \frac{a^{n_1}}{1-a} \quad |a| < 1$$

$$\sum_{k=n_1}^{n_2} a^k = \frac{a^{n_1} - a^{n_2+1}}{1-a} \quad \begin{matrix} a \neq 1 \\ n_2 \geq n_1 \\ |a| < 1 \text{ if } |n_1| \text{ or } |n_2| \to \infty \end{matrix}$$

Even and odd parts

$$x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x^*(-t) \quad x_o(t) = \frac{1}{2}x(t) - \frac{1}{2}x^*(-t)$$

$$x_e[n] = \frac{1}{2}x[n] + \frac{1}{2}x^*[-n] \quad x_o[n] = \frac{1}{2}x[n] - \frac{1}{2}x^*[-n]$$

Convolutions

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

LTI systems and eigenfunctions

$$e^{j\omega t} \xrightarrow{LTI(cont.)} H(j\omega)e^{j\omega t} = |H(j\omega)|e^{j(\omega t + \angle H(j\omega))}$$

$$e^{j\omega n} \xrightarrow{LTI(discr.)} H(e^{j\omega})e^{j\omega n} = |H(e^{j\omega})|e^{j(\omega n + \angle H(e^{j\omega}))}$$

$$\cos(\omega t) \xrightarrow{LTI(cont.)} |H(j\omega)|\cos(\omega t + \angle H(j\omega))$$

$$\cos(\omega n) \xrightarrow{LTI(discr.)} |H(e^{j\omega})|\cos(\omega n + \angle H(e^{j\omega}))$$

$$e^{st} \xrightarrow{LTI(cont.)} H(s)e^{st}$$

$$z^n \xrightarrow{LTI(discr.)} H(z)z^n$$

Standard first and second order low-pass systems, continuous time

$$H(j\omega) = \frac{1}{1 + j\omega\tau} \qquad H(j\omega) = \frac{{\omega_n}^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + {\omega_n}^2}$$

$$\tau > 0 \qquad \omega_n > 0, \zeta > 0$$

Standard first and second order recursive systems, discrete time

$$H(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}} \quad |a| < 1$$

$$H(e^{j\omega}) = \frac{1}{1 - 2r\cos\theta e^{-j\omega} + r^2 e^{-j2\omega}} \quad 0 \leq r < 1,\; 0 < \theta < \pi$$

Continuous time sampling

$$x_p(t) = x(t) \times p(t) \qquad x_d[n] = x(nT) = x(t)\big|_{t=nT}$$

$$X_p(j\omega) = f_s \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s)) \qquad f_s = \frac{1}{T}, \omega_s = \frac{2\pi}{T}$$

$$X_d(e^{j\omega}) = X_p(j\omega f_s)$$

$$H_0(j\omega) = e^{-j\omega T/2}\sin(\omega T/2)/(\omega/2) \text{ (sample \& hold)}$$

$$X_r(j\omega) = X(e^{j\omega/f_s})H_0(j\omega)H_r(j\omega) \quad (H_r(j\omega) \text{ low-pass})$$

Other formulas

$$\int_{t_1}^{t_2} te^{at}dt = \left[\frac{e^{at}}{a^2}(at-1)\right]_{t_1}^{t_2}$$

$$\frac{d\,\mathrm{atan}(u(\omega))}{d\omega} = \frac{d\tan^{-1}(u(\omega))}{d\omega} = \frac{1}{1+u^2(\omega)}\frac{du(\omega)}{d\omega}$$

Partial fractions

$$G(x) = \frac{b_M x^M + b_{M-1}x^{M-1} + \ldots + b_1 x + b_0}{a_N(x-\rho_1)^{\sigma_1}(x-\rho_2)^{\sigma_2}\ldots(x-\rho_r)^{\sigma_r}} = \sum_{i=1}^{r}\sum_{k=1}^{\sigma_i}\frac{A_{ik}}{(x-\rho_i)^k}$$

$$A_{ik} = \frac{1}{(\sigma_i - k)!}\left[\frac{d^{\sigma_i - k}}{dx^{\sigma_i - k}}\left[(x-\rho_i)^{\sigma_i}G(x)\right]\right]\Bigg|_{x=\rho_i}$$

$$G(x) = \frac{b_0 + b_1 x + \ldots + b_{M-1}x^{M-1} + b_M x^M}{a_0(1-\rho_1^{-1}x)^{\sigma_1}(1-\rho_2^{-1}x)^{\sigma_2}\ldots(1-\rho_r^{-1}x)^{\sigma_r}} = \sum_{i=1}^{r}\sum_{k=1}^{\sigma_i}\frac{A_{ik}}{(1-\rho_i^{-1}x)^k}$$

$$A_{ik} = \frac{1}{(\sigma_i - k)!(-\rho_i^{-1})^{\sigma_i - k}}\left[\frac{d^{\sigma_i - k}}{dx^{\sigma_i - k}}\left[(1-\rho_i^{-1}x)^{\sigma_i}G(x)\right]\right]\Bigg|_{x=\rho_i}$$