PHY 2323 Cheat Sheet

1 Coordinate Systems

There are 3 main coordinate systems:

1. Cartesian (x, y, z)
2. Cylindrical (ρ, ϕ, z)
3. Spherical (r, ϕ, θ)

2 Electric Fields

2.1 Coulombs Law

Coulomb's law is used to sum up all the charges in a location which will give an electric field E.

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_S \frac{\rho(\vec{r\prime})(\vec{r}-\vec{r\prime})}{|\vec{r}-\vec{r\prime}|^3}\mathrm{d}l$$

This can also be extended to a surface with ds and 2 integrals, or volume with dv

Note that anything with the $\prime$ means that it is related to the **surface of charge**, and anything without the prime is related to the **observation point.**

2.2 Gausses Law

Gausses Law can be used on a **closed surface** where we make a guassian surface (such as a sphere, or cylander) at the point of interest.

$$\int_S \vec{E}\mathrm{d}\vec{s} = \frac{Q_{enc}}{\epsilon}$$

We also have the $\vec{D}$ field which is the *Electric Flux Density.*

$$\int_S \vec{D}\mathrm{d}\vec{s} = Q_{enc}$$

Finally, we have the flux ψ, a scalar.

$$\psi = \epsilon\int_s \vec{E}\mathrm{d}\vec{s} = \int \vec{D}\mathrm{d}\vec{s}$$

This is useful in 3 main cases.

1. Spherical Symmetry is present
2. Cylindrical symmetry is present (long line of charge with uniform ρ or cylander with no angular dependance)
3. Planar Symmetry (Long 2D surface of charge)

A useful piece of information is if we want to find the Q_{enc}, we can often just integrate the charge density in a volume V.

$$Q_{enc} = \iiint_V \rho \mathrm{d}V$$

Also, the $\vec{D}$ field is just the $\vec{E}$ field times a factor of ϵ ($\vec{D} = \epsilon\vec{E}$)

3 Electric Potential

This is the potential energy per unit charge. AKA the voltage. This is a **scalar field**. This is *independant of the path chosen.*

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{l\prime} \frac{\rho_l \mathrm{d}l\prime}{|\vec{r} - \vec{r}\prime|}$$

This can be extended into 2d or 3d space by changing the $l\prime$ and $\mathrm{d}l\prime$ for $s\prime, \mathrm{d}s\prime$ or $v\prime, \mathrm{d}v\prime$.

Then we can relate the change in voltage to the electric field:

$$\vec{E} = -\nabla V \qquad \nabla V = -\int \vec{E} \cdot \mathrm{d}\vec{l}$$

3.1 Electric Dipole

A **dipole** is a pair of equal and opposite charges that are very close to each other relative to the point of observation.

This means that at the point of observation, they seem as one charge.

We have an equation that relates the charge of each end of the dipole q, the distance between the charges d, and the vector between the dipole and the observation point $\vec{r}$. This vector must be large compared to d.

$$V(\vec{r}) = \frac{(qd)\hat{z} \cdot \hat{r}}{4\pi\epsilon|r|^2}$$

3.2 Capacitors

The capacitence C can be calculated using the following formula:

$$C = \frac{Q}{\Delta V}$$

In practice, we use the following 3 step procedure:

1. Find E
2. Find ΔV
3. Find C using $C = \frac{Q}{\Delta V}$

4 Materials in Electric Fields

There are 3 types of materials:

1. Conductors
2. Insulators
3. Semiconductors

We have rules concerning the normal and tangential part of the boundary between 2 materials.

To obtain the normal part, we take the unit vector of the boundary, and this is our normal vector ($\hat{n} = \frac{\vec{n}}{|n|}$).

If we then want to get the **normal part** of E, we dot product it with $\hat{n}$, then append n to keep direction ($\vec{E}_n = (\vec{E} \cdot \hat{n})\hat{n}$)

The **tangential part** is just $\vec{E}_t = \vec{E} - \vec{E}_n$

4.1 Boundary between 2 Dielectrics

If we have 2 electric fields between 2 **dielectric** (insulators) surfaces, we have the following formulas for the bounds:

$$E_{1t} = E_{2t} \qquad \epsilon_1 E_{1n} - \epsilon_2 E_{2n} = \rho_s$$

4.2 Surface of a Conductor

If we have an electric field at the **surface** of a **conductive** surface (note that inside the surface $E = 0$), we have the following formulas for the bounds:

$$E_{1t} = E_{2t} = 0 \qquad E_n = \frac{\rho_s}{\epsilon_0}$$

5 Energy Stored in an Electric Field

We say that W is the energy stored in an electric field, or the energy required to assemble a charge distribution.