Properties – Continuous time Fourier series (C.T.F.S.)

Definitions:

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt \qquad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_0 = \frac{1}{T}\int_T x(t)dt$$

$x(t)$ periodic with period T sec.,

Fundam. angular frequency $\omega_0 = 2\pi f_0 = 2\pi/T$ rad./sec.

$x(t) \xleftrightarrow{C.T.F.S.} a_{x,k} \qquad y(t) \xleftrightarrow{C.T.F.S.} a_{y,k}$

If $x(t) \xrightarrow{LTI} y(t)$ then $a_{y,k} = a_{x,k} \, H(j\omega)\big|_{\omega = k\omega_0}$

Linearity: $Ax(t) + By(t) \xleftrightarrow{C.T.F.S.} A\, a_{x,k} + B\, a_{y,k}$

Shifting: $x(t - t_0) \xleftrightarrow{C.T.F.S.} e^{-jk\omega_0 t_0} a_k$

Scaling: $x(\alpha t) \xleftrightarrow{C.T.F.S.} a_k$ ($\alpha > 0$, period T/α)

Flipping: $x(-t) \xleftrightarrow{C.T.F.S.} a_{-k}$

Conjugate: $x^*(t) \xleftrightarrow{C.T.F.S.} a_{-k}^*$

$x^*(-t) \xleftrightarrow{C.T.F.S.} a_k^*$

Symmetries:

if $x(t)$ is real: $a_k = a_{-k}^*$, $|a_k| = |a_{-k}|$, $\angle a_k = -\angle a_{-k}$

$x_e(t) = (x(t) + x^*(-t))/2 \xleftrightarrow{CTFS} \mathrm{Re}\{a_k\}$

$x_o(t) = (x(t) - x^*(-t))/2 \xleftrightarrow{CTFS} j\,\mathrm{Im}\{a_k\}$

Periodic convolution:

$\int_T x(\tau)y(t-\tau)d\tau \xleftrightarrow{C.T.F.S.} T\, a_k\, b_k$

Modulation: $x(t)y(t) \xleftrightarrow{C.T.F.S.} a_k * b_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$

$e^{jm\omega_0 t}x(t) \xleftrightarrow{C.T.F.S.} a_{k-m}$

Differentiation: $\frac{dx(t)}{dt} \xleftrightarrow{C.T.F.S.} jk\omega_0 a_k$

Parseval: $\frac{1}{T}\int_T |x(t)|^2\, dt = \sum_{k=-\infty}^{\infty} |a_k|^2$

Table of continuous time Fourier series (C.T.F.S.)

$x(t)$ periodic, period $T = \frac{2\pi}{\omega_0} = \frac{1}{f_0}$ sec.	Fourier series coefficients a_k
$e^{j\omega_0 t}$	$a_1 = 1$ $a_k = 0$ elsewhere
$\cos(\omega_0 t)$	$a_1, a_{-1} = 1/2$ $a_k = 0$ elsewhere
$\sin(\omega_0 t)$	$a_1 = 1/(2j)$ $a_{-1} = -1/(2j)$ $a_k = 0$ elsewhere
$\begin{cases} 1 & \lvert t\rvert < T_1 \\ 0 & T_1 < \lvert t\rvert < T/2 \end{cases}$ (periodic T)	$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi} \quad k \neq 0$ $a_0 = \frac{T_1\omega_0}{\pi} = \frac{2T_1}{T}$
1	$a_0 = 1$ $a_k = 0$ elsewhere
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$a_k = \frac{1}{T}$

Properties – Discrete time Fourier series (D.T.F.S.)

Definitions: $a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-j(k\frac{2\pi}{N})n}$ $\quad x[n] = \sum_{k=<N>} a_k e^{j(k\frac{2\pi}{N})n}$ $a_0 = \frac{1}{N}\sum_{n=<N>} x[n]$ $x[n]$ periodic with period N samples (fundamental angular frequency $\omega_0 = \frac{2\pi}{N}$ rad./sample) $x[n] \xleftrightarrow{D.T.F.S.} a_{x,k}$ $\quad y[n] \xleftrightarrow{D.T.F.S.} a_{y,k}$ If $x[n] \xrightarrow{LTI} y[n]$ then $a_{y,k} = a_{x,k} H(e^{j\omega})\big\vert_{\omega=k\frac{2\pi}{N}}$
Periodicity: $x[n] \xleftrightarrow{D.T.F.S.} a_k = a_{k+N}$
Linearity: $Ax[n] + By[n] \xleftrightarrow{D.T.F.S.} Aa_{x,k} + Ba_{y,k}$
Shifting: $x[n-n_0] \xleftrightarrow{D.T.F.S.} e^{-jk\frac{2\pi}{N}n_0} a_k$
Flipping: $x[-n] \xleftrightarrow{D.T.F.S.} a_{-k}$
Conjugate: $x^*[n] \xleftrightarrow{D.T.F.S.} a^*_{-k}$ $x^*[-n] \xleftrightarrow{D.T.F.S.} a^*_k$
Symmetries: if $x[n]$ is real : $a_k = a^*_{-k}$, $\vert a_k\vert = \vert a_{-k}\vert$, $\angle a_k = -\angle a_{-k}$ $x_e[n] = (x[n] + x^*[-n])/2 \xleftrightarrow{DTFS} \mathrm{Re}\{a_k\}$ $x_o[n] = (x[n] - x^*[-n])/2 \xleftrightarrow{DTFS} j\,\mathrm{Im}\{a_k\}$
Periodic convolution: $\sum_{m=<N>} x[m]y[n-m] \xleftrightarrow{D.T.F.S.} N\, a_k\, b_k$
Modulation: $x[n]y[n] \xleftrightarrow{D.T.F.S.} \sum_{l=<N>} a_l b_{k-l}$ $e^{jm\frac{2\pi}{N}n} x[n] \xleftrightarrow{D.T.F.S.} a_{k-m}$
Parseval: $\frac{1}{N}\sum_{n=<N>} \vert x[n]\vert^2 = \sum_{k=<N>} \vert a_k\vert^2$
Duality : if $x[n] \xleftrightarrow{DTFS} a_k$ then $a[n] \xleftrightarrow{DTFS} \frac{1}{N} x_{-k}$

Table of discrete time Fourier series (D.T.F.S.)

$x[n]$ periodic, period N samples	Fourier series coefficients a_k (periodic with period N)
$e^{j\omega_0 n}$	if $e^{j\omega_0 n}$ periodic with $\omega_0 = \frac{2\pi m}{N}$: $a_k = 1 \quad k = m$ $a_k = 0$ elsewhere (except that a_k is also periodic N)
$\cos(\omega_0 n)$	if $\cos(\omega_0 n)$ periodic $\omega_0 = \frac{2\pi m}{N}$: $a_k = 1/2 \quad k = m, -m$ $a_k = 0$ elsewhere (except that a_k is also periodic N)
$\sin(\omega_0 n)$	if $\sin(\omega_0 n)$ periodic $\omega_0 = \frac{2\pi m}{N}$: $a_k = 1/(2j) \quad k = m$ $a_k = -1/(2j) \quad k = -m$ $a_k = 0$ elsewhere (except that a_k is also periodic N)
$\begin{cases} 1 & \vert n\vert \le N_1 \\ 0 & N_1 < \vert n\vert \le N/2 \end{cases}$ (periodic N, N even)	$a_k = \dfrac{\sin\left(\frac{2\pi}{N}k(N_1 + 1/2)\right)}{N\sin(\frac{\pi}{N}k)}$ $-N/2 \le k \le N/2$, except for $k = 0$ $a_k = (2N_1 + 1)/N \quad k = 0$ (but a_k is also periodic N))
1	$a_k = 1 \quad k = 0$ $a_k = 0$ elsewhere (except that a_k is also periodic N, and for any N value chosen!)
$\sum_{m=-\infty}^{\infty} \delta[n - mN]$	$a_k = \frac{1}{N} \quad \forall k$

Properties – Continuous time Fourier transform (C.T.F.T.)

Definitions:

$$X(j\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt \qquad x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)e^{j\omega t}d\omega$$

ω in rad./sec.

$$X(j\omega)=\sum_{k=-\infty}^{+\infty}2\pi\ a_k\delta(\omega-k\omega_0)\ \text{ if } x(t) \text{ periodic}$$

Linearity: $ax(t)+by(t)\xleftrightarrow{CTFT}aX(j\omega)+bY(j\omega)$

Shifting: $x(t-t_0)\xleftrightarrow{CTFT}e^{-j\omega t_0}X(j\omega)$

Scaling: $x(at)\xleftrightarrow{CTFT}\dfrac{1}{|a|}X(\dfrac{j\omega}{a})$

Flipping: $x(-t)\xleftrightarrow{CTFT}X(-j\omega)$

Conjugate: $x^*(t)\xleftrightarrow{CTFT}X^*(-j\omega)$

$x^*(-t)\xleftrightarrow{CTFT}X^*(j\omega)$

Symmetries:

if $x(t)$ is real-valued: $X(j\omega)=X^*(-j\omega)$,

$|X(j\omega)|=|X(-j\omega)|$, $\angle X(j\omega)=-\angle X(-j\omega)$

$x_e(t)=(x(t)+x^*(-t))/2\xleftrightarrow{CTFT}\mathrm{Re}\{X(j\omega)\}$

$x_o(t)=(x(t)-x^*(-t))/2\xleftrightarrow{CTFT}j\,\mathrm{Im}\{X(j\omega)\}$

Convolution:

$$x(t)*y(t)=\int_{-\infty}^{\infty}x(\tau)y(t-\tau)d\tau\overset{CTFT}{\leftrightarrow}X(j\omega)Y(j\omega)$$

Modulation:

$$x(t)y(t)\overset{CTFT}{\leftrightarrow}\frac{1}{2\pi}X(j\omega)*Y(j\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\theta)Y(j(\omega-\theta))d\theta$$

$$e^{j\omega_0 t}x(t)\xleftrightarrow{CTFT}X(j(\omega-\omega_0))$$

$$\cos(\omega_0 t)x(t)\xleftrightarrow{CTFT}\frac{1}{2}X(j(\omega-\omega_0))+\frac{1}{2}X(j(\omega+\omega_0))$$

Differentiation: $\dfrac{dx(t)}{dt}\xleftrightarrow{CTFT}j\omega\, X(j\omega)$

Integration: $\int_{-\infty}^{t}x(\tau)d\tau\xleftrightarrow{CTFT}\dfrac{1}{j\omega}X(j\omega)+\pi X(j0)\delta(\omega)$

Differentiation in freq.: $tx(t)\xleftrightarrow{CTFT}j\dfrac{dX(j\omega)}{d\omega}$

Integration in freq.:

$$-\frac{1}{jt}x(t)+\pi x(0)\delta(t)\xleftrightarrow{CTFT}\int_{-\infty}^{\omega}X(j\eta)d\eta$$

Parseval: $\int_{-\infty}^{+\infty}|x(t)|^2\,dt=\dfrac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2\,d\omega$

Duality : if $x(t)\xleftrightarrow{CTFT}X(j\omega)$ then

$X(t)\xleftrightarrow{CTFT}2\pi\ x(-j\omega)$

Table of continuous time Fourier transforms (C.T.F.T.)

signal $x(t)$ typ. aperiodic	$X(j\omega)$ (ω in rad./sec.)
if $x(t)$ is periodic, with period $T=\dfrac{2\pi}{\omega_0}=\dfrac{1}{f_0}$ sec.	$2\pi\sum_{k=-\infty}^{+\infty}a_k\delta(\omega-k\omega_0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0 t)$	$\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$
$\sin(\omega_0 t)$	$\dfrac{\pi}{j}\delta(\omega-\omega_0)-\dfrac{\pi}{j}\delta(\omega+\omega_0)$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_s\sum_{k=-\infty}^{+\infty}\delta(\omega-k\omega_s)\quad \omega_s=\dfrac{2\pi}{T}$
1	$2\pi\delta(\omega)$
$\begin{cases}1 & \lvert t\rvert<T_1\\ 0 & \lvert t\rvert>T_1\end{cases}$	$\dfrac{2\sin(\omega T_1)}{\omega}$
$\dfrac{\sin(\omega_c t)}{\pi t}\qquad \omega_c>0$	$\begin{cases}1 & \lvert\omega\rvert\le\omega_c\\ 0 & \lvert\omega\rvert>\omega_c\end{cases}$
$\delta(t)$	1
$u(t)$	$\dfrac{1}{j\omega}+\pi\delta(\omega)$
$e^{-at}u(t)\qquad \mathrm{Re}\{a\}>0$	$\dfrac{1}{j\omega+a}$
$-e^{-at}u(-t)\qquad \mathrm{Re}\{a\}<0$	$\dfrac{1}{j\omega+a}$
$\dfrac{t^{r-1}}{(r-1)!}e^{-at}u(t)\quad \mathrm{Re}\{a\}>0$	$\dfrac{1}{(j\omega+a)^r}\quad r\ge 1$
$-\dfrac{t^{r-1}}{(r-1)!}e^{-at}u(-t)\ \mathrm{Re}\{a\}<0$	$\dfrac{1}{(j\omega+a)^r}\quad r\ge 1$
$\dfrac{1}{\mathrm{Im}\{a\}}e^{-\mathrm{Re}\{a\}t}\sin(\mathrm{Im}\{a\}t)u(t)$ $\mathrm{Re}\{a\}>0$, $\mathrm{Im}\{a\}\ne 0$ or $\dfrac{-1}{\mathrm{Im}\{a\}}e^{-\mathrm{Re}\{a\}t}\sin(\mathrm{Im}\{a\}t)u(-t)$ $\mathrm{Re}\{a\}<0$, $\mathrm{Im}\{a\}\ne 0$	$\dfrac{1}{(j\omega+a)(j\omega+a^*)}$ $=\dfrac{1}{(j\omega)^2+2\,\mathrm{Re}\{a\}(j\omega)+\lvert a\rvert^2}$
$2\lvert A\rvert e^{-\mathrm{Re}\{a\}t}\times$ $\cos(\mathrm{Im}\{a\}t-\angle A)u(t)$ $\mathrm{Re}\{a\}>0$	$\dfrac{A}{j\omega+a}+\dfrac{A^*}{j\omega+a^*}$
$-2\lvert A\rvert e^{-\mathrm{Re}\{a\}t}\times$ $\cos(\mathrm{Im}\{a\}t-\angle A)u(-t)$ $\mathrm{Re}\{a\}<0$	$\dfrac{A}{j\omega+a}+\dfrac{A^*}{j\omega+a^*}$

Properties – Discrete time Fourier transform (D.T.F.T.)

Definitions: $x[n] = x(nT) = x(t)\big\|_{t=nT}$, where $T = 1/f_s = 2\pi/\omega_s$ $X(e^{j\omega})$ is the DTFT of $x[n]$ defined as: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \qquad x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(e^{j\omega})e^{j\omega n}d\omega$ Relative to $x(t)$ and its CTFT $X(j\omega)$, the relation is: $X_p(j\omega) = f_s \sum_{k=-\infty}^{\infty} X(j(\omega - k\,\omega_s)), \qquad X(e^{j\omega}) = X_p(j\omega f_s)$
Periodicity: $x[n] \xleftrightarrow{DTFT} X(e^{j\omega}) = X(e^{j(\omega+2\pi)})$
Linearity: $ax[n] + by[n] \xleftrightarrow{DTFT} aX(e^{j\omega}) + bY(e^{j\omega})$
Shifting: $x[n-n_0] \xleftrightarrow{DTFT} e^{-j\omega n_0}X(e^{j\omega}) \quad n_0$ integer
Expansion, insertion of zeros ("upsampling"): $x_{(k)}[n] = x[n/k]$ if n is a multiple of k $x_{(k)}[n] = 0$ elsewhere $x_{(k)}[n] \xleftrightarrow{DTFT} X(e^{jk\omega})$ where k is a positive integer
Discrete time sampling ("downsampling"): $x_d[n] = x[Mn] \xleftrightarrow{DTFT} X_d(e^{j\omega}) = \frac{1}{M}\sum_{m=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M} - m\frac{2\pi}{M}\right)}\right)$
Flipping: $x[-n] \xleftrightarrow{DTFT} X(e^{-j\omega})$
Conjugate: $x^*[n] \xleftrightarrow{DTFT} X^*(e^{-j\omega})$ $x^*[-n] \xleftrightarrow{DTFT} X^*(e^{j\omega})$
Symmetries: if $x[n]$ is real : $X(e^{j\omega}) = X^*(e^{-j\omega})$, $\left\|X(e^{j\omega})\right\| = \left\|X(e^{-j\omega})\right\|, \ \angle X(e^{j\omega}) = -\angle X(e^{-j\omega})$ $x_e[n] = (x[n] + x^*[-n])/2 \xleftrightarrow{DTFT} \mathrm{Re}\{X(e^{j\omega})\}$ $x_o[n] = (x[n] - x^*[-n])/2 \xleftrightarrow{DTFT} j\,\mathrm{Im}\{X(e^{j\omega})\}$
Convolution: $x[n]*y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k] \overset{DTFT}{\leftrightarrow} X(e^{j\omega})Y(e^{j\omega})$
Modulation: $x[n]y[n] \xleftrightarrow{DTFT} \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$ $e^{j\omega_0 n}x[n] \xleftrightarrow{DTFT} X(e^{j(\omega-\omega_0)})$
Accumulation: $\sum_{m=-\infty}^{n} x[m] \overset{DTFT}{\leftrightarrow} \frac{1}{1-e^{-j\omega}} X(e^{j\omega}) + \pi X(e^{j0}) \sum_{m=-\infty}^{+\infty} \delta(\omega - m2\pi)$
Differentiation in freq.: $nx[n] \xleftrightarrow{DTFT} j\frac{dX(e^{j\omega})}{d\omega}$
Parseval: $\sum_{n=-\infty}^{+\infty} \left\|x[n]\right\|^2 = \frac{1}{2\pi}\int_{2\pi} \left\|X(e^{j\omega})\right\|^2 d\omega$
Duality : If $x[n] \xleftrightarrow{DTFT} X(e^{j\omega})$ then $X(t) \xleftrightarrow{CTFS} x_{-k}$

Table of discrete time Fourier transforms (D.T.F.T.)

signal $x[n]$ typ. aperiodic	$X(e^{j\omega})$ (periodic 2π, ω in rad./sample)
if $x[n]$ is periodic, with period N samples	$2\pi \sum_{k=<N>} a_k \delta(\omega - k\frac{2\pi}{N})$ (also periodic 2π)
$e^{j\omega_0 n}$	$2\pi\delta(\omega - \omega_0)$ (also periodic 2π)
$\cos(\omega_0 n)$	$\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$ (also periodic 2π)
$\sin(\omega_0 n)$	$\frac{\pi}{j}\delta(\omega-\omega_0) - \frac{\pi}{j}\delta(\omega+\omega_0)$ (also periodic 2π)
$\sum_{m=-\infty}^{\infty} \delta[n-mN]$	$\frac{2\pi}{N}\sum_{k=<N>} \delta(\omega - k\frac{2\pi}{N})$ (also periodic 2π)
1	$2\pi\delta(\omega)$ (also periodic 2π)
$\begin{cases} 1 & \|n\| \le N_1 \\ 0 & \|n\| > N_1 \end{cases}$	$\sin(\omega(N_1 + \frac{1}{2}))/\sin(\frac{\omega}{2})$
$\frac{\sin(\omega_c n)}{\pi n} \quad 0 < \omega_c < \pi$	$\begin{cases} 1 & 0 \le \|\omega\| \le \omega_c \\ 0 & \omega_c < \|\omega\| \le \pi \end{cases}$ (also periodic 2π)
$\delta[n]$	1
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \pi\delta(\omega)$ (also periodic 2π)
$a^n u[n] \quad \|a\| < 1$	$1/(1-ae^{-j\omega})$
$-a^n u[-n-1] \quad \|a\| > 1$	$1/(1-ae^{-j\omega})$
$\frac{(n+r-1)!}{n!(r-1)!}a^n u[n] \quad \|a\| < 1$	$1/(1-ae^{-j\omega})^r \quad r \ge 1$
$\frac{-(n+r-1)!}{n!(r-1)!}a^n u[-n-1] \quad \|a\| > 1$	$1/(1-ae^{-j\omega})^r \quad r \ge 1$
$\frac{r^n}{\sin(\theta)}\sin(\theta(n+1))u[n]$ $0 \le r < 1 \quad 0 < \theta < \pi$ or $-\frac{r^n}{\sin(\theta)}\sin(\theta(n+1))u[-n-1]$ $r > 1 \quad 0 < \theta < \pi$	$\frac{1}{(1-re^{j\theta}e^{-j\omega})(1-re^{-j\theta}e^{-j\omega})}$ $= \frac{1}{1-2r\cos\theta e^{-j\omega} + r^2 e^{-j2\omega}}$
$2\|A\|r^n\cos(\theta n + \angle A)u[n]$ $0 \le r < 1 \quad 0 \le \theta \le \pi$	$\frac{A}{1-re^{j\theta}e^{-j\omega}} + \frac{A^*}{1-re^{-j\theta}e^{-j\omega}}$
$-2\|A\|r^n\cos(\theta n + \angle A)u[-n-1]$ $r > 1 \quad 0 \le \theta \le \pi$	$\frac{A}{1-re^{j\theta}e^{-j\omega}} + \frac{A^*}{1-re^{-j\theta}e^{-j\omega}}$

Properties – bilateral (two-sided) Laplace transform

Definitions:	$X(s)=\int_{-\infty}^{+\infty} x(t)e^{-st}dt \quad ROC_x\ \sigma_l<\mathrm{Re}\{s\}<\sigma_r$ $x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds=\frac{e^{\sigma t}}{2\pi}\int_{\omega=-\infty}^{\omega=\infty} X(\sigma+j\omega)e^{j\omega t}d\omega$ $\sigma\in ROC_x \qquad (\sigma=0)\in ROC_x$ $x(t)\xleftrightarrow{LT}X(s)\quad ROC_x \qquad y(t)\xleftrightarrow{LT}Y(s)\quad ROC_y$
Linearity:	$ax(t)+by(t)\xleftrightarrow{LT}aX(s)+bY(s)$ $ROC_x\cap ROC_y$
Shifting:	$x(t-t_0)\xleftrightarrow{LT}e^{-st_0}X(s)$ ROC_x unchanged
Scaling:	$x(at)\xleftrightarrow{LT}\frac{1}{\lvert a\rvert}X(\frac{s}{a})$ a is real valued $\lvert a\rvert ROC_x$, $s_k\to as_k$, $a\sigma_l<\mathrm{Re}\{s\}<a\sigma_r\quad a>0$ $a\sigma_r<\mathrm{Re}\{s\}<a\sigma_l\quad a<0$
Flipping:	$x(-t)\xleftrightarrow{LT}X(-s)$ ROC_x inversed
Conjugate:	$x^*(t)\xleftrightarrow{LT}X^*(s^*)$ ROC_x unchanged, $s_k\to s_k^*$
Symmetry:	if $x(t)$ real: $X(s)=X^*(s^*)$, $\lvert X(s)\rvert=\lvert X(s^*)\rvert$ and if s_k is a zero or a pole then s_k^* is also a zero or a pole.
Convolution:	$x(t)*y(t)=\int_{-\infty}^{\infty}x(\tau)y(t-\tau)d\tau\xleftrightarrow{LT}X(s)Y(s)$ $ROC_x\cap ROC_y$
Modulation:	$e^{s_0t}x(t)\xleftrightarrow{LT}X(s-s_0)$ ROC_x shifted right by $\mathrm{Re}\{s_0\}$ $s_k\to s_k+s_0$, $\sigma_l+\mathrm{Re}\{s_0\}<\mathrm{Re}\{s\}<\sigma_r+\mathrm{Re}\{s_0\}$
Differentiation:	$\frac{dx(t)}{dt}\xleftrightarrow{LT}s\,X(s)$ ROC_x unchanged
Integration:	$\int_{-\infty}^{t}x(\tau)d\tau\xleftrightarrow{LT}\frac{1}{s}X(s)$ $ROC_x\cap(\mathrm{Re}\{s\}>0)$
Differentiation in freq.:	$-tx(t)\xleftrightarrow{LT}\frac{dX(s)}{ds}$ ROC_x unchanged
Initial value theorem:	if $x(t)=0$ $t<0$ with no impulse $\delta(t)$ or singularity at $t=0$: $x(0^+)=\lim_{s\to\infty}sX(s)$
Final value theorem:	if $x(t)=0$ $t<0$ has a finite value at $t\to\infty$: $\lim_{t\to\infty}x(t)=\lim_{s\to 0}sX(s)$

Table of bilateral (two-sided) Laplace transforms

Signal $x(t)$	Laplace transform $X(s)$	ROC
$\delta(t)$	1	$\forall s$
$u(t)$	$1/s$	$\mathrm{Re}\{s\}>0$
$-u(-t)$	$1/s$	$\mathrm{Re}\{s\}<0$
$\begin{cases}1 & \lvert t\rvert<T_1\\ 0 & \lvert t\rvert>T_1\end{cases}$	$\frac{e^{T_1s}-e^{-T_1s}}{s}$	$\forall s$
$e^{-at}u(t)$	$1/(s+a)$	$\mathrm{Re}\{s\}>\mathrm{Re}\{-a\}$
$-e^{-at}u(-t)$	$1/(s+a)$	$\mathrm{Re}\{s\}<\mathrm{Re}\{-a\}$
$\frac{t^{r-1}}{(r-1)!}e^{-at}u(t)$	$1/(s+a)^r \quad r\ge 1$	$\mathrm{Re}\{s\}>\mathrm{Re}\{-a\}$
$-\frac{t^{r-1}}{(r-1)!}e^{-at}u(-t)$	$1/(s+a)^r \quad r\ge 1$	$\mathrm{Re}\{s\}<\mathrm{Re}\{-a\}$
$\frac{1}{\mathrm{Im}\{a\}}e^{-\mathrm{Re}\{a\}t}\sin(\mathrm{Im}\{a\}t)u(t)$ $\mathrm{Im}\{a\}\neq 0$	$\frac{1}{(s+a)(s+a^*)}$ $=\frac{1}{s^2+2Re\{a\}s+\lvert a\rvert}$	$\mathrm{Re}\{s\}>\mathrm{Re}\{-a\}$
$\frac{-1}{\mathrm{Im}\{a\}}e^{-\mathrm{Re}\{a\}t}\sin(\mathrm{Im}\{a\}t)u(-t)$ $\mathrm{Im}\{a\}\neq 0$	$\frac{1}{(s+a)(s+a^*)}$ $=\frac{1}{s^2+2Re\{a\}s+\lvert a\rvert}$	$\mathrm{Re}\{s\}<\mathrm{Re}\{-a\}$
$2\lvert A\rvert e^{-\mathrm{Re}\{a\}t}\times$ $\cos(\mathrm{Im}\{a\}t-\angle A)u(t)$	$\frac{A}{s+a}+\frac{A^*}{s+a^*}$	$\mathrm{Re}\{s\}>\mathrm{Re}\{-a\}$
$-2\lvert A\rvert e^{-\mathrm{Re}\{a\}t}\times$ $\cos(\mathrm{Im}\{a\}t-\angle A)u(-t)$	$\frac{A}{s+a}+\frac{A^*}{s+a^*}$	$\mathrm{Re}\{s\}<\mathrm{Re}\{-a\}$

A few properties – unilateral (one-sided) Laplace transform

Definitions:	$X(s)=\int_{0^-}^{+\infty}x(t)e^{st}dt \qquad x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}X(s)e^{st}ds$ $x(t)\xleftrightarrow{ULT}X(s) \qquad y(t)\xleftrightarrow{ULT}Y(s)$
Linearity:	$ax(t)+by(t)\xleftrightarrow{ULT}aX(s)+bY(s)$
Differentiation:	$\frac{dx(t)}{dt}\xleftrightarrow{ULT}s\,X(s)-x(0^-)$ $\frac{d^kx(t)}{dt^k}\xleftrightarrow{ULT}s^kX(s)-s^{k-1}x(0^-)-\ldots-s^0\left.\frac{dx^{k-1}(t)}{dt^{k-1}}\right\rvert_{t=0^-}$
Integration:	$\int_{0^-}^{t}x(\tau)d\tau\xleftrightarrow{ULT}\frac{1}{s}X(s)$

Properties – Bilateral (two-sided) z-transform

Definitions: $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n} \quad ROC_x \quad r_i<|z|<r_o$

$$x[n]=\frac{1}{2\pi j}\oint X(z)z^{n-1}dz=\frac{r^n}{2\pi}\int_{\omega=-\pi}^{\pi} X(re^{j\omega})e^{j\omega n}d\omega$$

$z\in ROC_x \qquad (r=1)\in ROC_x$

Linearity: $ax[n]+by[n]\overset{z.t.}{\leftrightarrow} aX(z)+bY(z) \quad ROC_x\cap ROC_y$

Shifting: $x[n-n_0]\overset{z.t.}{\longleftrightarrow} z^{-n_0}X(z) \quad ROC_x$ unchanged

Conjugate: $x^*[n]\overset{z.t.}{\longleftrightarrow} X^*(z^*) \quad ROC_x$ unchanged, $z_k\to z_k^*$

Symmetry: if $x[n]$ real: $X(z)=X^*(z^*)$, $|X(z)|=|X(z^*)|$

if z_k is a zero or a pole then z_k^* is also a zero or a pole

Flipping: $x[-n]\overset{z.t.}{\leftrightarrow} X(z^{-1}) \quad 1/ROC_x$, $z_k\to 1/z_k$, $1/r_o<|z|<1/r_i$

Upsampling, expansion, insertion of zeros:

$x_{(M)}[n]=x[n/M]$ if n is a multiple of M, else $x_{(M)}[n]=0$

$x_{(M)}[n]\overset{z.t.}{\longleftrightarrow} X(z^M)$ where M is a positive integer

$ROC_x^{1/M}$, $z_k\to z_k^{1/M}$, $r_i^{1/M}<|z|<r_o^{1/M}$

Downsampling:

$x_p[n]=x[n]$ if n is a multiple of M, else $x_p[n]=0$

$$x_p[n]\overset{z.t.}{\longleftrightarrow} X_p(z)=\frac{1}{M}\sum_{m=0}^{M-1} X(e^{-jk\frac{2\pi}{M}}z)\text{, } M \text{ positive integer}$$

ROC_x unchanged, $z_i\to e^{jk\frac{2\pi}{M}}z_i$

$$x_d[n]=x_p[Mn]=x[Mn]\overset{z.t.}{\leftrightarrow} X_d(z)=\frac{1}{M}\sum_{m=0}^{M-1} X(e^{-jk\frac{2\pi}{M}}z^{1/M})$$

ROC_x^M, $z_k\to z_k^M$, $r_i^M<|z|<r_o^M$

Convolution: $\sum_{k=-\infty}^{\infty} x[k]y[n-k]\overset{z.t.}{\leftrightarrow} X(z)Y(z) \quad ROC_x\cap ROC_y$

Modulation: $a^n x[n]\overset{z.t.}{\longleftrightarrow} X(\frac{z}{a})$

$|a|ROC_x$, $z_k\to az_k$, $|a|r_i<|z|<|a|r_o$

$e^{j\omega_0 n}x[n]\overset{z.t.}{\longleftrightarrow} X(e^{-j\omega_0}z) \quad ROC_x$ unchanged

Accumulation: $\sum_{m=-\infty}^{n} x[m]\overset{z.t.}{\leftrightarrow}\frac{1}{1-z^{-1}}X(z) \quad ROC_x\cap(|z|>1)$

Differentiation in freq.: $nx[n]\overset{z.t.}{\longleftrightarrow} -z\frac{dX(z)}{dz}$ same ROC_x

Initial value theorem: if $x[n]=0 \quad n<0 \quad x[0]=\lim_{z\to\infty} X(z)$

Final value theorem: if $x[n]=0 \quad n<0$, and poles of $(z-1)X(z)$ inside unit circle: $\lim_{n\to\infty} x[n]=\lim_{z\to 1}(z-1)X(z)$

Table of bilateral (two-sided) z-transforms

Signal $x[n]$	z-transform $X(z)$	ROC
$\delta[n]$	1	$\forall z$
$u[n]$	$1/(1-z^{-1})$	$\lvert z\rvert>1$
$-u[-n-1]$	$1/(1-z^{-1})$	$\lvert z\rvert<1$
$a^n u[n]$	$1/(1-az^{-1})$	$\lvert z\rvert>\lvert a\rvert$
$-a^n u[-n-1]$	$1/(1-az^{-1})$	$\lvert z\rvert<\lvert a\rvert$
$\frac{(n+r-1)!}{n!(r-1)!}a^n u[n]$	$1/(1-az^{-1})^r \quad r\ge 1$	$\lvert z\rvert>\lvert a\rvert$
$\frac{-(n+r-1)!}{n!(r-1)!}a^n u[-n-1]$	$1/(1-az^{-1})^r \quad r\ge 1$	$\lvert z\rvert<\lvert a\rvert$
$\frac{r^n}{\sin(\theta)}\sin(\theta(n+1))u[n]$ $0<\theta<\pi$	$\frac{1}{(1-re^{j\theta}z^{-1})(1-re^{-j\theta}z^{-1})}$ $=\frac{1}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert>r$
$-\frac{r^n}{\sin(\theta)}\sin(\theta(n+1))u[-n-1]$ $0<\theta<\pi$	$\frac{1}{(1-re^{j\theta}z^{-1})(1-re^{-j\theta}z^{-1})}$ $=\frac{1}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert<r$
$2\lvert A\rvert r^n\cos(\theta n+\angle A)u[n]$ $0\le\theta\le\pi$	$\frac{A}{1-re^{j\theta}z^{-1}}+\frac{A^*}{1-re^{-j\theta}z^{-1}}$ $=\frac{2\mathrm{Re}\{A\}-2r\lvert A\rvert\cos(\angle A-\theta)z^{-1}}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert>r$
$-2\lvert A\rvert r^n\cos(\theta n+\angle A)u[-n-1]$ $0\le\theta\le\pi$	$\frac{A}{1-re^{j\theta}z^{-1}}+\frac{A^*}{1-re^{-j\theta}z^{-1}}$ $=\frac{2\mathrm{Re}\{A\}-2r\lvert A\rvert\cos(\angle A-\theta)z^{-1}}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert<r$
$a^n \quad 0\le n\le N-1$; 0 elsewhere	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert>0$

A few properties – unilateral (one-sided) z transform

Definitions:

$$X(z)=\sum_{n=0}^{\infty} x[n]z^{-n} \qquad x[n]=\frac{1}{2\pi j}\oint X(z)z^{n-1}dz$$

Linearity: $ax[n]+by[n]\overset{u.z.t.}{\longleftrightarrow} aX(z)+bY(z)$

Shifting:

$$x[n-1]\overset{u.z.t.}{\longleftrightarrow} z^{-1}\left[X(z)+x[-1]z\right]=z^{-1}X(z)+x[-1]$$

$$x[n-k]\overset{u.z.t.}{\longleftrightarrow} z^{-k}\left[X(z)+\sum_{l=1}^{k}x[-l]z^{l}\right] \quad k>0$$

$$x[n+1]\overset{u.z.t.}{\longleftrightarrow} z\left[X(z)-x[0]\right]=zX(z)-zx[0]$$

$$x[n+k]\overset{u.z.t.}{\longleftrightarrow} z^{k}\left[X(z)-\sum_{l=0}^{k-1}x[l]z^{-l}\right] \quad k>0$$