GNG 1105 Cheat Sheet

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Whole Year Cheat Sheet

Forces and Moment

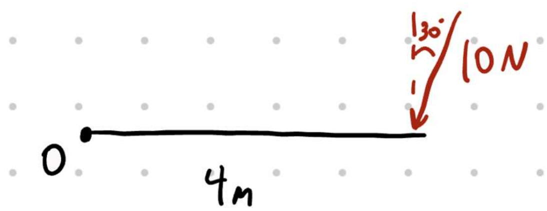
2D Moment

The moment force is the rotational force around a point o. It is produced by having a perpendicular distance between an applied force, and a point o.

The units of Moment are Newton Meters ($N \cdot m$)

$$M_o = F_x d_y + F_y d_x$$

Find the moment about point o.



In this example, we can ignore the x direction of the force, since the perpendicular distance between it and o is 0.

$$\sum M_o = (10\cos(30°)) \cdot (4) = 34.63\ N \cdot m$$

Cross Product

The cross product is a way to find a vector product of 2 vectors in 3d space.

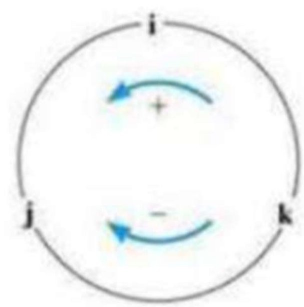
It has a long and complicated formula of:

$$\vec{u} \times \vec{v} = (u_y v_z - u_z v_y, u_z v_x - u_x v_z, u_x v_y - u_y v_x)$$

But we can use a graphic to make it easier.

This shows that if we do an i component times a j component, then it will give a positive k component.

$$(x, y, z) = (1,2,3) = i + 2j + 3k$$



3D Systems

When we are given a 3D system, we end up generally end up having 6 equations available.

We have the three force direction equations, and the three moment direction equations.

To get these three equations is a bit harder.

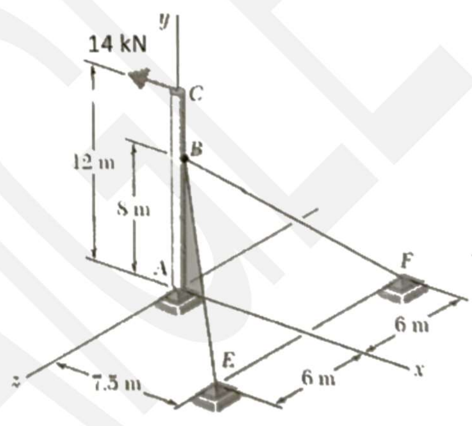
First, we need to get the coordinates of all important points. Then, we can find the vectors and then unit vectors $\left(\frac{vector}{\sqrt{vector^2}}\right)$ of each force we have. Also, we find the positional vectors of each force (how to get from the origin to any point on the force vector). Once we have those, we can multiply the unit vectors of each force by their magnitudes. Finally, we can solve by moment by using the equation $M_o = \vec{r} \times \vec{F}$ where $\vec{r}$ are the positional vectors, and $\vec{F}$ are the force vectors.

Then, we will have 6 equations, and therefore, we can solve for up to 6 unknowns.

Find the reaction forces at A.

To find the reaction forces, we will need to take the summation of all forces. Unfortunately, we do not know BE or BF, so we will have too many unknowns.

First, we can find the coordinates.



$$\begin{aligned}&A(0,0,0)\\&B(0,8,0)\\&C(0,12,0)\\&E(7.5,0,6)\\&F(7.5,0,-6)\end{aligned}$$

Now, we can find the unit vectors.

$$\begin{aligned}BF &= F - B = (7.5,-8,-6)\\BE &= E - B = (7.5,-8,6)\\AB &= B - A = (0,8,0)\\AC &= C - A = (0,12,0)\end{aligned}$$

I also know that the 14kN force is $(-14,0,0)$.

We can see that AB is the positional vector for BF and BE, and AC is the positional vector for the 14kN force.

Now, I need the force vectors for BF and BE. I do not know the magnitude of the force for now, so I will call it T_{BE} *and* T_{BF}. *To find this, I need the unit vectors.*

$$n_{BE} = \frac{7.5i - 8j + 6k}{\sqrt{7.5^2 + 8^2 + 6^2}} = 0.6i - 0.64j + 0.48k$$
$$n_{BF} = \frac{7.5i - 8j - 6k}{\sqrt{7.5^2 + 8^2 + 6^2}} = 0.6i - 0.64j - 0.48k$$

If I add in the unknown tensions, it becomes:

$$\overrightarrow{T_{BE}} = 0.6iT_{BE} - 0.64jT_{BE} + 0.48kT_{BE}$$
$$\overrightarrow{T_{BF}} = 0.6iT_{BF} - 0.64jT_{BF} - 0.48kT_{BF}$$

I see that I still have too many unknowns. So, I can introduce moment equations.

$$\sum M_A = \big(AC \times (-14i)\big) + (AB \times T_{BE}) + (AB \times T_{BF}) = 0$$

Now, I can just do the cross product between all these values to get:

$$\sum M_A = (168k + (60k - 48i)T_{BE} + (60k + 48i)T_{BF}) = 0$$

This is basically 2 different equations, with 2 unknowns.

$$\sum M_{A_x} = -48T_{BE} + 48T_{BF} = 0$$
$$\sum M_{A_z} = 168 + 60T_{BE} + 60T_{BF} = 0$$

Now I can solve to get $T_{BE} = 1.4kN$ *and* $T_{BF} = 1.4kN$.

Finally, I have everything I need to calculate the sum of forces in all directions. Using the equations for the tension components, I can sub them and the reactions into the final equations.

$$\sum F_x = A_x + 0.6T_{BF} + 0.6T_{BE} = 0$$
$$A_x = 12.32kN$$

$$\sum F_y = -0.64T_{BF} - 0.64T_{BE} + A_y = 0$$
$$A_y = 1.792kN$$

$$\sum F_z = -0.48T_{BF} + 0.48T_{BE} + A_z = 0$$
$$A_z = 0\ kN$$

And now it is done.

Trusses

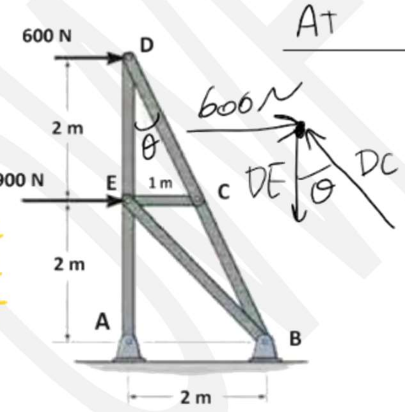
Method of Joints

If we have a truss, we can find the forces at each joint by analyzing each joint separately.

We start at a joint where we only have one unknown in either the x or y direction, and then we can solve from there.

1- Determine the force in DE and DC of the truss and state if the members are in tension or compression. (Hint: No need to calculate reactions at A and B)

$\theta = \arctan\frac{1}{2} = 26.57°$



600N : C
DC : C
DE = T

At D

$\Sigma F_x = 0 = 600 - DC \sin 26.57°$

$\boxed{DC = 1341.41N \quad C}$

DC is pushing into D to oppose the 600N, so compression

$\Sigma F_y = 0 = 1341.41 \cos 26.57 - DE$

$\boxed{DE = 1199.74N \quad T}$

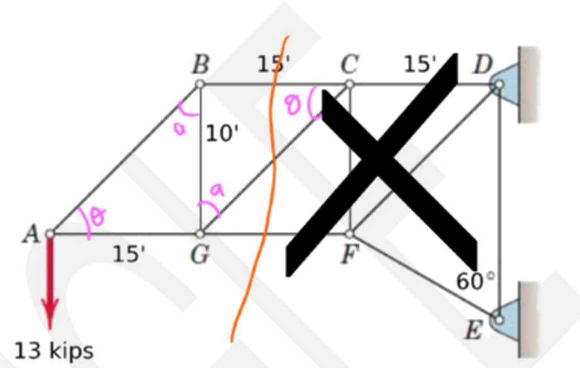
DE is pulling away from D to counteract DC, so tension.

Method of Sections

The method of sections is another way to calculate the forces in a truss. This is useful since it allows us to ignore a large part of a truss by making a “cut” through many members. We usually try to cut through the members that we want to find.

Determine the forces in members BC and CG.

Since we need BC and CG, we can cut through those 2 members. Then, we can only pay attention to the left side of the truss.



First, I can calculate the angles θ and α.

$$\alpha = \arctan\left(\frac{15}{10}\right) = 56.31°$$

$$\theta = \arctan\left(\frac{10}{15}\right) = 33.69°$$

Next, I will calculate the forces at joint A since we have a known force there.

$$\sum F_y = AB\sin(33.69) - 13$$
$$AB = 23.44\ kips$$

$$\sum F_x = AB\cos(33.69) - AG$$
$$AG = 19.50\ kips$$

Now, I can do the same thing at points B, and then G. Then at point G, I will be able to solve for BC and CG.

Centroids

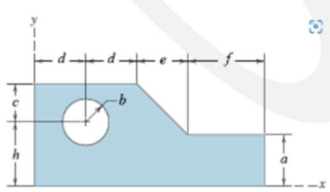
2D Systems

To find the centroid of a 2D system (no mass for the structure), we break it up into the x and y components.

$$x = \frac{\sum Ax}{\sum A}, \qquad y = \frac{\sum Ay}{\sum A}$$

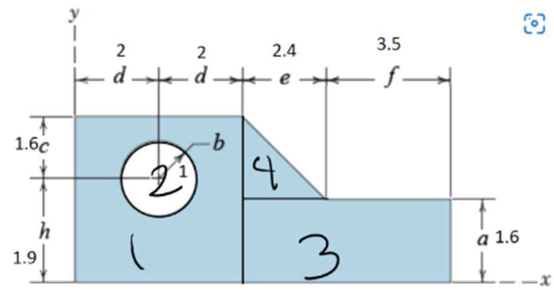
x, and y are the x and y coordinates of the centroid. The sum of Ax is the sum of the individual areas, times the individual y coordinates of the centroids. Same with y. A is the whole area of the shape.

Determine the x and y coordinates of the centroid. Assume $a = 1.6, c = 1.6, d = 2, e = 2.4, f = 3.5, h = 1.9$.



We will end up needing the sums of the areas of all shapes, and then the sums of all the $(area \times x)$ and $(area \times y)$.

First, we can break up the shape into smaller more manageable shapes and assign them numbers.



Now, we can create a table with all the information we need, and have.

Part	A	x	y	Ax	Ay
1	$3.5 \times 4 = 14$	2	1.75	28	24.5
2	$-\pi$	2	1.9	-2π	-1.9π
3	9.44	6.95	0.8	65.608	7.552
4	$1.9 \times 2.4 \times 0.5 = 2.28$	4.8	2.33	10.944	5.092
Sum	22.578	/	/	98.268	31.1750

Now that we have all that information, we can go about calculating the final x and y coordinates.

$$x = \frac{Ax}{A} = \frac{98.268}{22.578} = 4.352$$

$$y = \frac{Ay}{A} = \frac{31.1750}{22.578} = 1.381$$

3D Systems

With 3D systems, it is similar, except we do:

$$x = \frac{\sum Wx}{\sum W}, \qquad y = \frac{\sum Wy}{\sum W}, \qquad z = \frac{\sum Wz}{\sum W}$$

Where W is the weight of each individual component.

Friction

Friction Coefficients

We use μ as the friction coefficient. If the object in question is moving, then we use μ_k, the kinetic coefficient of friction. If the object is static, we use μ_s, the static coefficient of friction.

Friction and Normal Force Method

If we are not sure whether the object is kinetic or static, we assume that it is static, and then if the required force of friction is greater than the maximum allowed force of friction, then we go back and use the kinetic force of friction.

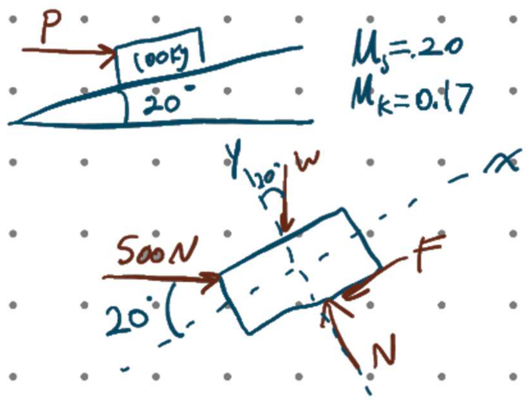
$$F_{max} = \mu_s N$$

$$F_k = \mu_k N$$

To calculate the required friction force to counteract the applied force, we create a FBD and then calculate the summation of forces.

Determine the magnitude and direction of the friction force acting on the $100\ kg$ block shown if, $P = 500\ N$. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17.

We can draw a FBD to see what is going on, and then we can find the summation of forces in the x and y directions. Then, we will need to check if this force is allowable based on F_{max}.



$\Sigma F_y = 0 = -500\sin 20 - 981\cos 20 + N$

$N = 1093 N$

$\Sigma F_x = -F - 981\sin 20 + 500\cos 20$

$F = 134.3 N$

Find F_{max} to check if 134.3N is allowable.

$F_{max} = \mu_s N = 0.20 \cdot 1093 = 291 N$

Since $291 > 134.3 N$, $F = 134.3 N$ is allowed.

Angle Method

If we do not want to deal with μ and the normal, we can change it into a force R, and an angle that R makes with the vertical called ϕ where $\phi = \arctan(\mu)$. Then, we can calculate the system like regular.

Dynamics

x and y motion

Circular Motion