ELG 2136 Cheat Sheet

1 Physics

Electric Conduction can happen due to 2 different mechanisms.

1. Forced Drift (When an Electric Field is applied)
2. Natural Diffusion (Free charges tend to move to less densely charged areas)

We define J as the current density which is just $\frac{\text{total current}}{\text{area}}$. J can be calculated for Drift, or Diffusion but we need a few more things defined.

We define n_i as the density of electrons/holes, which depends on the boltzmann constant k, the temperature T, and the material specific bandgap energy E_g.

$$n_i = 5.2 \times 10^{15} T^{\frac{3}{2}} e^{\frac{-E_g}{2kT}}$$

We define q as the charge of a single electron/hole, μ_n, μ_p as the mobility constants of the electrons (n) and holes (p), D_n, D_p as the diffusion constants, n and p are the electron and hole concentrations, and E is the electric field applied.

$$J_{drift} = q(\mu_n n + \mu_p p)E \qquad J_{diffusion} = qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx}$$

1.1 Doping

While pure silicon is neutral, we can dope the silicon with boron (extra proton) or phosphorus (extra electron).

We define n_n, p_p as the number of free electrons and protons in a material at thermal equilibrium.

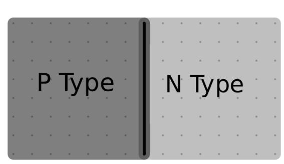
If the material has majority positive (p-type), we say: $n_p, p_p, p_p \cdot n_p = n_i^2$

If the material has majority negative (n-type), we say: $n_n, p_n, p_n \cdot n_n = n_i^2$

We can put a p-type and n-type material adjacent (PN Junction), and that creates a **diode**.

1.2 At a PN Junction

This is a junctin of two materials, specifically a positive (P) and negative (N) material.



We have constants of L_p and L_n for the spacial exponential decay of the P or N charges.

Then the numbers N_A and N_D are porportional to the number of positive and negative charges respectively. They are the number of *acceptor* or *donor* atoms.

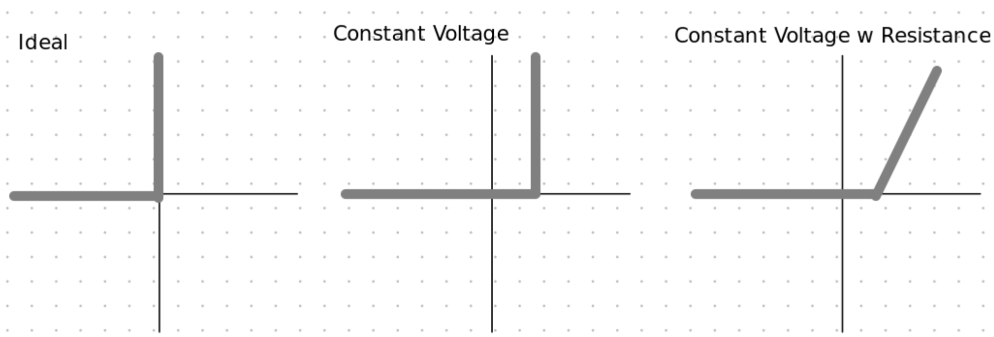
If A is the area, then we say that the total diffusion current density is:

$$J_{diff\_tot} = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)\left(e^{\frac{v}{V_t}} - 1\right)$$

This is also the total current since the drift current in a PN junction is small due to the generated electric field.

2 Circuit Analysis with Diodes

We have three models for diodes. There is the ideal, constant voltage (ideal with voltage source), and constant voltage with resistance (ideal with voltage source and resistance). All of these model the real diode.



An ideal diode can be either in the ON state (where current passes through in the correct direction) or the OFF state (where it acts as an open circuit). The other models just have this ideal diode with the voltage and resistance in series.

To determine the state of the diode, we need to assume a state (OFF or ON) and then check if it makes sense.

- If it is ON, then current MUST go from anode to cathode.
- If it is OFF, then voltage at cathode must be HIGHER than voltage at anode.

If either of these criteria are wrong, then our assumption is on, and we need to try another one.

NOTE: There can only be **one** combination that works with a circuit. So if we have 2 diodes, and the first try it works, then it MUST be ONLY that combination.

Ex.

2.1 Plotting

For these problems, we have a circuit, and we have an input voltage (x axis) and another value such as output voltage, or output current, and we need to plot the output as a function of the inputs.

This is a lot of work since we have diodes, and when each diode changes, as will the relation between the input and the output.

This is the procedure:

1. Replace all diodes with their model (Ideal, Constant Voltage, CV with Resistance)
2. Assume very low value for X (input) and find the state of each diode using this low value.
3. Analyse the circuit to find Y (output) as a function of X (input)
4. Find out which diode will switch its state **first** (let ON diode current=0, or OFF diode voltage=0)
5. Replace the diode whose state changes first with its new state and *return to step 3.*
6. STOP analysis once no more diodes will change state (diodes only change states at most 1 time).

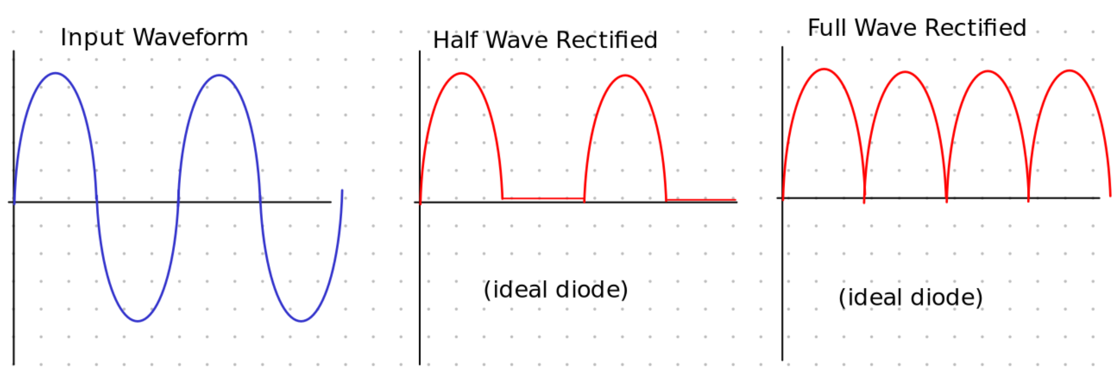
Ex.

3 DC Power Supply

This is a very important application of diodes. This will convert an AC time dependant source to a DC constant source.

3.1 Rectifier

The rectifyer will take an AC source, and make the output be positive.



A Half wave rectifier (1 diode) will just kill the negative part of the wave, while a Full wave rectifier (2 diodes) will keep the positive part, and invert the negative part.

A Bridge rectifyer is a special type of full wave rectifier that has 4 diodes instead of 2.

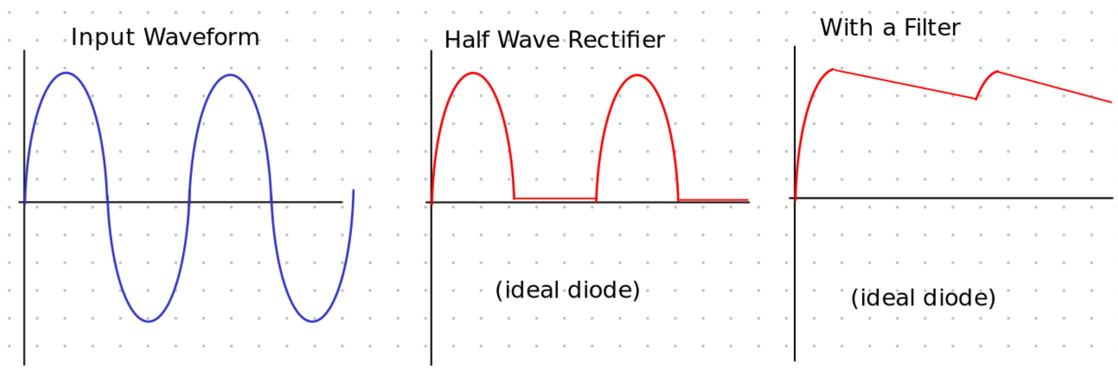
A useful statistic is the PIV (Peak Inverse Voltage) across a diode. To find this, we take a diode that we want to find the PIV across, then we find what would be the maximum negative voltage across this diode (note the diode will be OFF) when the source is at its inverse peak.

Typically we di a KVL through the circuit. Note that we find the voltage across the whole diode (including voltage and resistance if using that model).

Ex.

3.2 Filter

The Filter is just adding a capacitor in parallel with the load resistor. This will smooth out the big inflections, but there will still be a ripple.



We call V_p the peak voltage of the wave, V_r as the ripple voltage (if it fluctuates between 9 and 11 V, then $V_4 = 2V$), and V_k as the minimum voltage.

$$V_p = V_k + V_r$$

We can make some approxmiations (with ideal diodes) to see that:

$$V_r = \frac{V_p}{fRC} \text{ for a half wave rectifier or}$$

$$V_r = \frac{V_p}{2fRC} \text{ for a full wave}$$

$$\omega \Delta t = \sqrt{\frac{2V_r}{V_p}} \text{ for both types of rectifiers}$$

$$\text{Current through diode average } i_{Davg} = I_L(1 + \pi\sqrt{\frac{2V_p}{V_r}}) \text{ for half wave}$$

$$\text{or for full wave } i_{Davg} = I_L(1 + \pi\sqrt{\frac{V_p}{2V_r}})$$

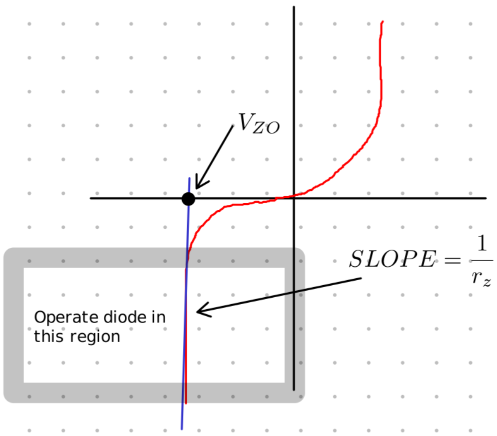
$$\text{Current through diode MAX } i_{Dmax} = I_L(1 + 2\pi\sqrt{\frac{2V_p}{V_r}}) \text{ for half wave}$$

$$\text{or for full wave } i_{Dmax} = I_L(1 + 2\pi\sqrt{\frac{V_p}{2V_r}})$$

If we are working with a different model, such as the constant voltage model, then we will need to modify the equations such as increasing the peak voltage by 0.7V. The one exception is the conduction angle formula $\omega\Delta t$ since that is not affected by te extra 0.7V.

3.3 Regulator

The regulator is a function that takes in a voltage source with unwanted ripples, and greatly reduces these ripples. It does this with a diode working in the breakdown region (zener diode).



r_z is the internal resistance of the diode. V_{ZO} is the internal voltage source of the diode.

We need to ensure that the zener operates only in the breakdown region for no fluctuations in the output voltage. We do this by finding the minimum and maximum current allowed for the diode to stay in the breakdown region (straight line part).

We want to maintain the output voltage to be as constant as possible, and this is measured using the line and load regulation.

$$\text{Line Regulation} = \frac{\Delta V_O}{\Delta V_s} = \frac{r_z}{r_z + R} \qquad \text{Load Regulation} = \frac{\Delta V_O}{\Delta I_L} = -\frac{r_z \times R}{r_z + R} = -r_z//R$$

Ex.

4 Bipolar Junction Transistor (BJT)

4.1 DC Analysis

4.2 AC Analysis

4.3 Early Effect

5 Metal Oxide Semiconductor Field Effect Transistor (MOSFET)

5.1 DC Analysis

5.2 AC Analysis

5.3 Channel Modulation

6 Complementary MOSFET (CMOS)

6.1 Voltage Transfer Characteristic (VTC) of CMOS

6.2 Delay in the CMOS

7 Appendix

7.1 Short Formula Sheet

This is just a summary of all the important formulas:

Semiconductor Physics:

$$n_i = 5.2 \times 10^{15} T^{\frac{3}{2}} e^{\frac{-E_g}{2kT}}$$

$$J_{drift} = q(\mu_n n + \mu_p p)E \qquad J_{diffusion} = qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx}$$

$$p_p \propto N_A \qquad n_n \propto N_D$$

$$p_p n_p = n_n p_n = n_i^2$$

$$J_{diff\_tot} = Aqn_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) \left(e^{\frac{v}{V_t}} - 1 \right) \text{ For PN Junction}$$

Rectifyer with Filter: Half wave rectifyer with single ideal diode

$$V_r = \frac{V_p}{fRC} \qquad \omega \Delta t = \sqrt{\frac{2V_r}{V_p}}$$

$$i_{Davg} = I_L(1 + \pi \sqrt{\frac{2V_p}{V_r}}) \qquad i_{Dmax} = I_L(1 + 2\pi \sqrt{\frac{2V_p}{V_r}})$$

Zener Diode

$$\text{Line Regulation} = \frac{\Delta V_O}{\Delta V_s} = \frac{r_z}{r_z + R} \qquad \text{Load Regulation} = \frac{\Delta V_O}{\Delta I_L} = -\frac{r_z \times R}{r_z + R} = -r_z // R$$

BJT

$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{\beta}{g_m} \qquad r_0 = \frac{V_A}{I_C} \qquad r_e = \frac{\alpha}{g_m} \qquad \alpha = \frac{\beta}{\beta + 1}$$

NMOS ($V_{TH} > 0$)

$$V_{GS} < V_{TH} \qquad I_D = I_S = 0 \qquad \text{Cutoff}$$

$$V_{GS} > V_{TH} \qquad V_{DS} < V_{GS} - V_{TH} \qquad \text{Triode}$$

$$I_D = I_S = \mu_n C_{ox} \frac{W}{L} \left((V_{GS} - V_{TH})V_{DS} - \frac{1}{2} V_{DS}^2 \right) \qquad \text{Triode}$$

$$V_{GS} > V_{TH} \qquad V_{DS} \geq V_{GS} - V_{TH} \qquad \text{Saturation}$$

$$I_D = I_S = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS}) \qquad \text{Saturation}$$

PMOS ($V_{TH} < 0$)

$$V_{GS} > V_{TH} \qquad I_D = I_S = 0 \qquad \text{Cutoff}$$

$$V_{GS} < V_{TH} \qquad V_{DS} \geq V_{GS} - V_{TH} \qquad \text{Triode}$$

$$I_D = I_S = \mu_p C_{ox} \frac{W}{L} \left((V_{SG} - |V_{TH}|)|V_{DS}| - \frac{1}{2} V_{DS}^2 \right) \qquad \text{Triode}$$

$$V_{GS} < V_{TH} \qquad V_{DS} < V_{GS} - V_{TH} \qquad \text{Saturation}$$

$$I_D = I_S = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (V_{SG} - |V_{TH}|)^2 (1 + \lambda |V_{DS}|) \qquad \text{Saturation}$$

Small Signal Parameters for NMOS or PMOS

$$g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T H) \qquad r_0 = \frac{1}{\lambda I_D}$$

CMOS Inverter

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t) \qquad V_{IL} = \frac{1}{8}(3V_{DD} + 2V_t)$$

$$t_{PHL} = \frac{a_n C}{\mu_n Cox \frac{W}{L}_n V_{DD}} \qquad \alpha_n = \frac{2}{\frac{7}{4} - \frac{3V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2}$$

$$t_{PHL} = \frac{a_p C}{\mu_p Cox \frac{W}{L}_p V_{DD}} \qquad \alpha_p = \frac{2}{\frac{7}{4} - \frac{3|V_{tp}|}{V_{DD}} + \left(\frac{V_{tp}}{V_{DD}}\right)^2}$$