Rules of Unit Analysis (aka Dimensional Analysis)

Why It's Good to Pay Attention to Units:

- It's universal to physics. All physics equations need to balance units
- You can use it to figure out problems.

Rules/Tools for using Units:

- We will use "SI units" (French for International System of Units)
- These are an extension of "mks" units, which are easier to remember:
  - Stands for "meters – kilograms – seconds"
  - Symbols: m = meters, kg = kilograms, s = seconds

1. Units in an equation must be the same to be compared:
   - E.g. A = B + C
   - The Units of A, B and C must all be the same
     - Good example: 9 m = 6 m + 3 m
     - Bad example: 9 m = 6 m + 3 kg (doesn't make sense)
2. Units can be factored, commuted and cancelled just like normal numbers and variables:
   - Factor example: 9 m = 6 m + 3 m = (6 + 3) m = 9 m
   - Commute and cancel example: 9 m = 3 m/s * 3s = (3*3) (m/s * s) = 9 m
3. Most units let you use a factor-label method to convert:

$$\frac{10 \cancel{\text{miles}}}{\cancel{\text{hour}}} * \frac{1609 \text{ meters}}{1 \cancel{\text{mile}}} * \frac{1 \cancel{\text{hour}}}{3600 \text{ seconds}} = \left(\frac{10}{1} * \frac{1609}{3600}\right) \frac{\text{meters}}{\text{second}} = 4.47 \frac{\text{meters}}{\text{second}}$$

   - Search for "dimensional analysis factor label method" for more examples
4. A few units cannot use this method (its any conversion where you add a term, not just multiply, e.g. converting temperature – Celsius, Fahrenheit, Kelvin):
   - 50 °C = (50 * 9/5 + 32) °F = 122 °F, 100 °F = (100 – 32) * (5/9) °C = 37.78 °C
5. Google's web search will convert units for you: "50 degrees celsius to fahrenheit"
6. One last oddity: angles in radians – "dimensionless units"
   - Radians are defined by the notion of arclength:
     - $l = r\theta$, Units: [meters] = [meters][radians]
   - In order for this to make sense according to rule 1, radians are considered "dimensionless units".
   - Radians make sense here and degrees do NOT!
     - If the $\theta = 2\pi$, and the radius is 1m, we get the full circumference of a 1m-radius circle is 2π meters ~ 6.28m. If you straightened this circle, and measured its length, that's what you get.
     - If you assume degrees are the same and use them without converting, degrees are defined as 1/360 of the angle swept by a full circle, you get the arclength as (1m)*(360 degrees) = 360 meters?! Which is silly.