Using Solar Panels to Minimize Home Power Costs

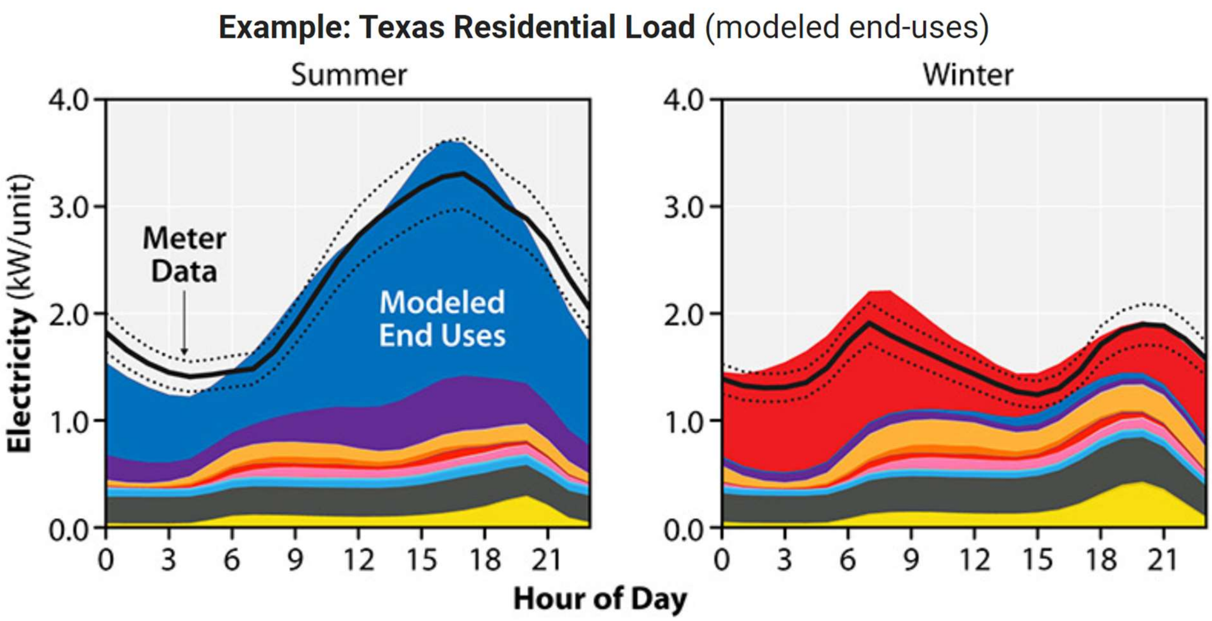
Owen Frausto

Note: this isn't a formal write up of a project, it's all of my thoughts and scratch work for the back end of my optimizing project. I'm meaning to LaTeX this up at some point once the project has moved along a bit more.

Beginning 5/27/23

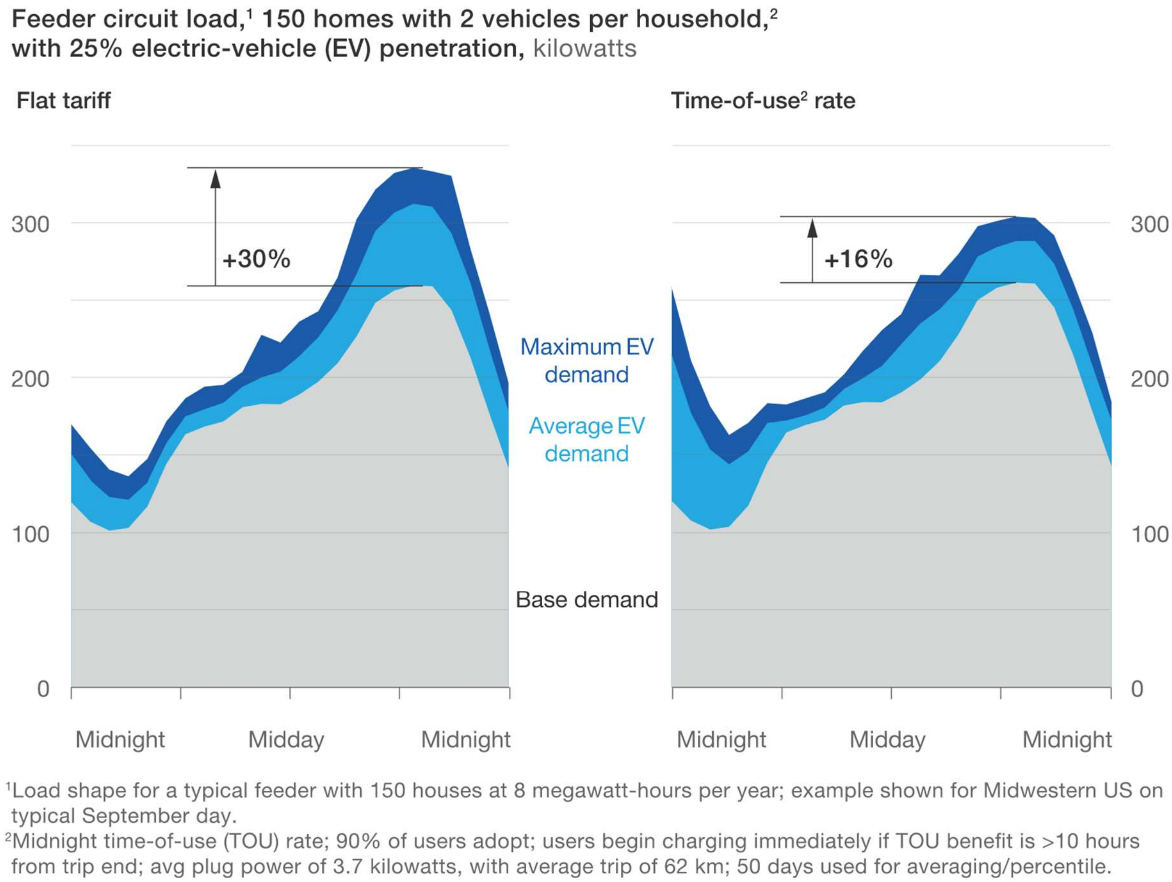
Solar PV is increasingly becoming a source of power as the country shifts to renewable energy. This brings the issue that daily power production from Solar PV doesn't follow daily power demand trends.

Simulated daily demand curves:



[(Source: NREL)]

Electric vehicles will exaggerate the issue, leading to even higher drains at night. For example, an increase in residential electric vehicle charging:



[Source: Mckinsey & Company]()

Existing Literature

I did a quick google scholar search and skimmed through seemingly relevant articles

[Efficient operation of residential solar panels with determination of the optimal tilt angle and optimal intervals based on forecasting model]() (Akhlaghi et al. 2017)

They optimized the tilt angle of solar panels periodically throughout the day

[Data-driven load profiles and the dynamics of residential electricity consumption]()

Work

Variable definitions (As stated in Duffie and Beckman)

ϕ – Latitude, the angular location north or south of the equator; $-90° \leq \phi \leq 90°$

δ – Declination, the angular position of the sun at solar noon (i.e. when the sun is on the local meridian) with respect to the plane of the equator, north positive; $-23.45° \leq \delta \leq 23.45$

β – Slope, the angle between the plane of the surface in question and the horizontal; $0° \leq \beta \leq 180°$

γ – Surface azimuth angle, the deviation of the projection on a horizontal plane of the normal to the surface from the local meridian, with zero south, east negative, and west positive. $-180° \leq \gamma \leq 180°$

ω – Hour angle, the angular displacement of the sun east or west of the local meridian due to the rotation of the earth on its axis at $15°$ per hour; morning negative, afternoon positive

θ – Angle of incidence, the angle between the beam radiation on a surface and the normal from that surface

Additional angles the defined the position of the sun in the sky:

θ_z – Zenith angle, the angle between the vertical and the line to the sun, that is, the angle of incidence of beam radiation on a horizontal surface.

α_s – Solar altitude angle, the angle between the horizontal and the line to the sun, that is, the complement of the zenith angle

γ_s – Solar azimuth angle, the angular displacement from the south of the projection of beam radiation on the horizontal plane. Displacements east of south are negative and west of south are positive.

Equation relating the incidence of the beam radiation on a surface θ, to the other angle:

$$\cos(\theta) = \sin(\delta)\sin(\phi)\cos(\beta) - \sin(\delta)\cos(\phi)\sin(\beta)\cos(\gamma) + \cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega) + \cos(\delta)\sin(\phi)\sin(\beta)\cos(\gamma)\cos(\omega) + \cos(\delta)\sin(\beta)\sin(\gamma)\sin(\omega)$$

Daily power production as a function of angle

δ can be approximately found using Cooper (1969):

$$\delta = 23.45\sin\left(360 \cdot \frac{284+n}{365}\right)$$

Where n is the day of the year (doesn't hold for $|\phi| > 66.5°$)

ω_s, the sunrise/sunset angle occurs when $|\theta_z| < 90°$ (Beyond this point the sun is not visible). Using

$$\cos(w_s) = -\tan(\phi)\tan(\delta)$$

Picking up the project again on 7/23/2023

List of things I'm not considering:

- NSC price changing through the year (assumed constant)
- Energy loss due to air mass based on sun's position.
- How wavelengths of light change temporally and spatially with sun's position, and the effect this has on production.
- Clouds/other weather conditions throughout the year.
- Behavior across time zones (assumes you're in the center of your time zone).

Picking up again on 10/2/23

Initial Objective Definition

I'm going to accept that it's highly nonlinear and try to go from here.

Assumption:

- Given a pricing profile $p(h)$ and usage profile $u(h)$.
- Excess net metering gets fraction μ of price added to bank
- N solar panels, each generates P power in an hour with perpendicular sunlight
- $\gamma, \beta_i \in [0,2\pi]$

$$\underset{\gamma_1,\beta_1,\ldots,\gamma_N,\beta_N}{\operatorname{argmin}} \quad Amount\ Homeowner\ Pays$$

$$= \underset{\gamma_1,\beta_1,\ldots,\gamma_N,\beta_N}{\operatorname{argmin}} \quad (Positive\ Net\ Energy\ Usage \times Cost) - \mu(Excess\ Generation)$$

At each day n and hour h, there will be an excess $u(h) - g(\gamma_1, \beta_1, \ldots, \gamma_N, \beta_N, n, h)$. At a given hour, if the excess is greater than zero, we get μ times the price times the excess at that point. If the excess is less than zero, we pay the price times the excess. We'll reframe as a maximization process:

$$= \underset{\gamma_1,\beta_1,\ldots,\gamma_N,\beta_N}{\operatorname{argmax}} \sum_{n=1}^{365} \sum_{h=1}^{24} p(h)(\min\{excess, 0\} + \mu \max\{excess, 0\})$$

In data science, the rectified linear unit (ReLU) is often used as a nonlinearity function. Notice that our formulation aligns with a scaling of the "leaky relu" LReLU. We'll rewrite to reflect that:

$$\underset{\gamma_1,\beta_1,\ldots,\gamma_N,\beta_N}{\operatorname{argmax}} \sum_{n=1}^{365} \sum_{h=1}^{24} -p(h) \times \mathrm{LReLU}_{\mu}(-excess)$$

Defining the excess

Defining $g(\gamma_1, \beta_1, \ldots, \gamma_N, \beta_N, n, h)$. The generation is an amount that comes from the solar panel angles as well as the dynamics of the sun for the given time of day/day of year.

$$g(\gamma_1, \beta_1, \ldots, \gamma_N, \beta_N, n, h) = P \times \sum_{i=1}^{N} \mathbb{1}\{\text{Sun visible}\} \times \cos(\theta_d)$$

The indicator function ensures that the sun isn't past the horizon (no sunlight then), and the cosine comes from incident amount of sunlight hitting the panel ($Power\ Generated = P\cos(\theta_d)$).

$$g = P \times \sum_{i=1}^{N} \mathbb{1}\left\{-\frac{\pi}{2} \leq \theta_d^i \leq \frac{\pi}{2}\right\} \times \cos(\theta_d^i)$$

Duffie and Beckman outline the following formula:

$$\sin(\delta)\sin(\phi)\cos(\beta) - \sin(\delta)\cos(\phi)\sin(\beta)\cos(\gamma) + \cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega) + \cos(\delta)\sin(\phi)\sin(\beta)\cos(\gamma)\cos(\omega) + \cos(\delta)\sin(\beta)\sin(\gamma)\sin(\omega)$$

Where ω is the hour angle,

$$\omega = -15(h - 12)[°]$$

and δ is the declination angle (angle of sun in sky with respect to time of year)

$$\delta = 23.45\sin\left(360 \cdot \frac{284 + n}{365}\right)\ [°]$$

Optimization

Because of the discontinuous nature of the loss function, we'll use particle swarm optimization, an algorithm which doesn't rely on the computation of gradients.

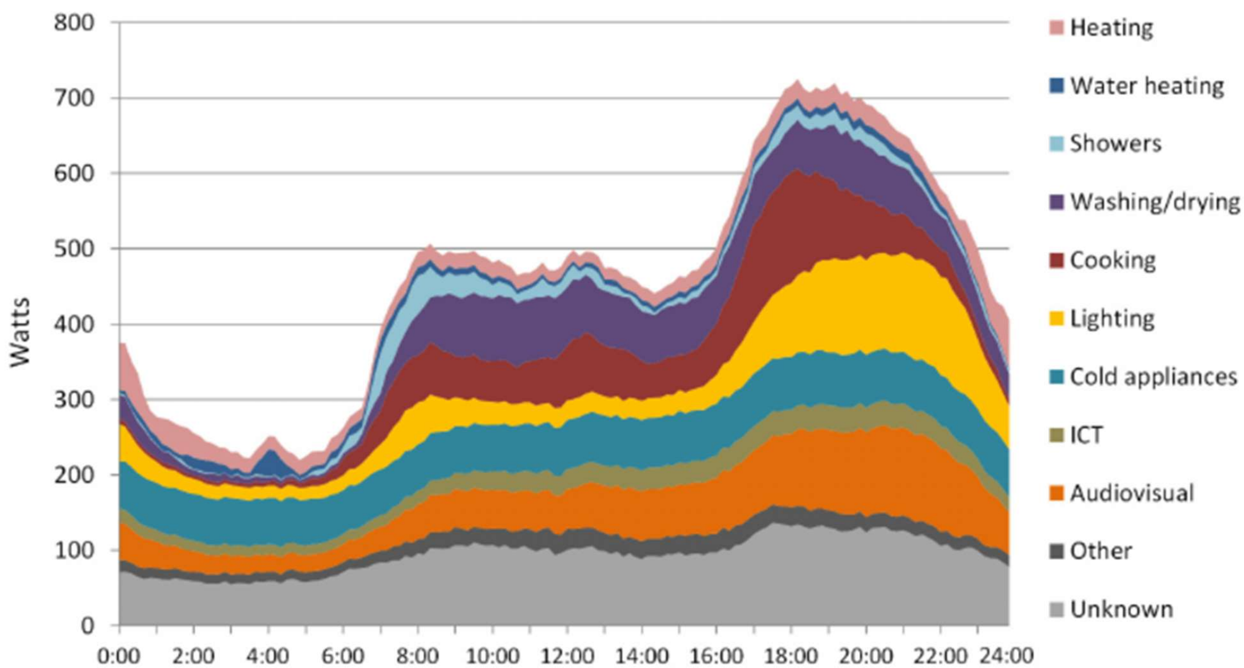
Clerc 2012 outlines the basic algorithm (as summarized on Wikipedia):

```
for each particle i = 1, ..., S do
    Initialize the particle's position with a uniformly distributed random vector: xi ~ U(blo, bup)
    Initialize the particle's best known position to its initial position: pi ← xi
    if f(pi) < f(g) then
        update the swarm's best known position: g ← pi
    Initialize the particle's velocity: vi ~ U(-|bup-blo|, |bup-blo|)
while a termination criterion is not met do:
    for each particle i = 1, ..., S do
        for each dimension d = 1, ..., n do
            Pick random numbers: rp, rg ~ U(0,1)
            Update the particle's velocity: vi,d ← w vi,d + φp rp (pi,d-xi,d) + φg rg (gd-xi,d)
        Update the particle's position: xi ← xi + vi
        if f(xi) < f(pi) then
            Update the particle's best known position: pi ← xi
            if f(pi) < f(g) then
                Update the swarm's best known position: g ← pi
```

In this case, the particle positions are $x = [\gamma_1, \beta_1, \dots, \gamma_N, \beta_N] \in \mathbb{R}^{2N}$

Default Usage Profile

Based on
https://www.researchgate.net/publication/290105581_Residential_Demand_Response_Scheduling_with_Consideration_of_Consumer_Preferences



Default TOU Profile

Based on PECO
https://www.peco.com/SmartEnergy/InnovationTechnology/Pages/TimeOfUsePricing.aspx