EE3104C Introduction to RF and Microwave Systems & Circuits

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1 Transmission Line

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0, V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$V(z,t) = |V_o^+| \cos(\omega t - \beta z + \phi^+) + |V_0^-| \cos(\omega t + \beta z + \phi^-)$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}, \lambda = \frac{2\pi}{\beta}, v_p = \frac{\omega}{\beta}$$

α is attenuation constant, β is phase constant.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, I_0^+ = \frac{V_0^+}{Z_0}, I_0^- = -\frac{V_0^-}{Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V_0^-}{V_0^+}, \ \Gamma(-l) = \Gamma_L e^{-j2\beta l} = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}}$$

$$Z_{in}(-l) = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

$$SWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \quad P_{av} = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1 - |\Gamma|^2)$$

Lossless transmission line $R = G = 0, \alpha = 0$

2 Smith Chart & Scattering Parameters

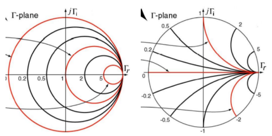


Figure 1: (L) Const resistance (r) circles, (R) Const reactance (x) circles

Normalized impedance $z = r + jx$
Admittance smith chart is same but rotated 180°

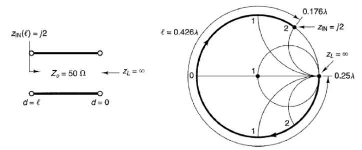


Figure 2: Rotate clockwise to obtain z_{in} from z_L by multiples of λ

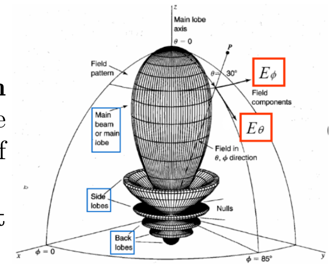
2.1 Scattering Matrix

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}, \quad S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0 \text{ for } k \neq j}$$

For reciprocal network, $S = S^T$

Lossless, $\sum_{n=1}^{N} S_{ni} S_{ni}^* = 1$ and $\sum_{n=1}^{N} S_{ni} S_{nj}^* = 0$ for $i \neq j$

3 Antennae



Far-Field Region Practical definition where maximum phase error of 22.5°. $R \geq \frac{2D^2}{\lambda}$
Reflection Coefficient $\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$
VSWR voltage standing wave ratio is $VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$. 1 for perfectly matched, ∞ for completely unmatched.
Radiated Power $P_{rad} = \oiint_S \frac{1}{2} \Re[E \times H^*] \cdot ds$
Directivity The ratio of maximum power density to average value.

$$D = \frac{4\pi}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi}$$

Gain $G = \eta_{rad} D, 0 < \eta_{rad} < 1$ is antenna efficiency
$D_{max} = \frac{4\pi A_P}{\lambda^2}, D = \eta_{ap} D_{max} = \frac{4\pi A_e}{\lambda^2}$, for aperture efficiency η_{ap}, aperture area A_P and effective aperture area $A_e = \eta_{ap} A_P$.
Beam area $\Omega_A = \iint_{4\pi} P_n(\theta, \phi) d\Omega$

Brightness Temp $T_b = \frac{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} T_B(\theta,\phi) D(\theta,\phi) \sin\theta d\theta d\phi}{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} D(\theta,\phi) \sin\theta d\theta d\phi}$
T_B background noise temp. D radiation pattern.
Antenna Noise Temp $T_A = e_{rad} T_b + (1 - e_{rad}) T_p$
e_{rad} radiation efficiency, 1 for lossless antenna. T_p is physical temperature of antenna.

4 Amplifiers

4.1 Noise Power of White Noise Source

$N_0 = KTB = (1.38 \times 10^{-23} J/K)(290K)(\text{BW, Hz})$
Equivalent Noise Temperature $T_e = \frac{N_0}{kB}$

4.2 Noise Figure of a Component

$$F = \frac{S_i/N_i}{S_o/N_o} \geq 1$$

$$T_e = (F - 1)T_0, \ F = 1 + \frac{T_e}{T_0}, \ T_0 = 290K$$

$$N_0 = FkGBT_0 = kGB(T_0 + T_e)$$

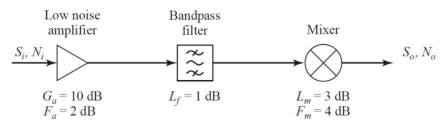
4.2.1 Cascaded Amplifiers

$$T_{cas} = T_{e1} + \frac{T_{e2}}{G_1} \quad T_{cas} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2}$$

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} \quad F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

$$G = G_1 G_2 \quad G = G_1 G_2 G_3$$

4.2.2 Example



$F_{cas} = F_a + \frac{F_f - 1}{G_a} + \frac{F_m - 1}{G_a G_f} = 10^{0.2} + \frac{10^{0.1} - 1}{10^1} + \frac{10^{-0.3} - 1}{10^1 \times 10^{-0.1}}$
$T_{cas} = (F_{cas} - 1)T_0 = (1.80 - 1) \times 290 = 232K$
$N_o = k(T_A + T_{cas})BG, SNR = \frac{S_o}{N_o}, S_o = GS_i$

4.3 Nonlinearities

Suppose $v_o = c_o + c_1 v_i + c_2 v_i^2 + c_3 v_i^3$ and $v_i = A\cos(\omega t)$
$v_o = \left(c_o + \frac{c_2 A^2}{2}\right) + \left(c_1 A + \frac{3c_3 A^3}{4}\right)\cos\omega t$
$\quad + \frac{c_2 A^2}{2}\cos 2\omega t + \frac{c_3 A^3}{4}\cos 3\omega t$
1dB Compression Point $OP_{1dB} = IP_{1dB} + G - 1$

4.3.1 IM3 (Third-order inter-modulation) products

For $v_i = A(\cos\omega_1 t + \cos\omega_2 t)$
$\frac{3}{4}c_3 A^3(\cos(2\omega_1+\omega_2)t + \cos(2\omega_1-\omega_2)t$
$\quad + \cos(2\omega_2+\omega_1)t + \cos(2\omega_2-\omega_1)t)$
Extrapolate 1st-order and 3rd-order output to find $IIP3$, for small A, $|c_1 A| = \left|\frac{3}{4}c_3 A^3\right| \implies A^2 = \frac{4}{3}\left|\frac{c_1}{c_3}\right|$

5 Propagation

Equivalent isotropically radiated power: to get the same power either increase input power or increase directivity, both can achieve the same radiated power.
U is power density.
Downlink is satellite to earth transmission
In system design always consider frequency, consider time varying channel Some estimation of the link in the power point of view

5.1 Friis Transmission Equation

Gain $G = \max G(\theta,\phi)$

$$G(\theta,\phi) = \frac{U(\theta,\phi)}{U_{ave}} = e_{rad}\frac{U(\theta,\phi)}{U_{rad,ave}} = e_{rad}D(\theta,\phi)$$

Total radiated power $P_{rad} = \oiint_{4\pi} U(\theta,\phi)d\Omega$
$U_{rad,ave} = e_{rad}U_{ave} = e_{rad}\frac{P_{input}}{4\pi}$
Equivalent Isotropically Radiated Power $EIRP = G_t P_t \quad (W)$
Effective Aperture P_r available power, A_e effective area, U_{inc} power density of incident plane wave

$$A_e = \frac{G\lambda^2}{4\pi} \quad P_r = A_e U_{inc} = A_e\frac{G_t P_t}{4\pi R^2} = \frac{G_t G_r \lambda^2 P_t}{(4\pi R)^2} \quad (W)$$

$$A_{ei} = \frac{\lambda^2}{4\pi}\ (m^2) \quad G = \frac{P_r}{P_{ri}} = \frac{A_e}{A_{ei}}$$

$$e_{ap}(\theta,\phi) = \frac{A_e(\theta,\phi)}{A_{phy}} \quad A_e(\theta,\phi) = G_r A_{ei}$$

Electrical area $\frac{A_e}{\lambda^2}$
Polarization $e_{pm} = |a_i \cdot a_r|^2 = |a_\theta \cdot a_z|^2 = |\cos\theta|^2$
Pathloss

$$P_r = U_{inc}A_{er} = G_t\frac{P_t}{4\pi r^2}G_r\frac{\lambda^2}{4\pi} = P_t G_t G_r(\frac{\lambda}{4\pi r})^2$$

$$\frac{P_r}{P_t} = G_t G_r(\frac{\lambda}{4\pi r})^2|a_t \cdot a_r|^2$$

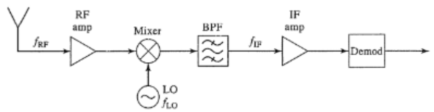
Freespace pathloss $(\frac{4\pi r}{\lambda})^2$ or $20\log(\frac{4\pi R}{\lambda})$ dB (>0)
G/T ratio Gain of the receiving antenna divided by antenna noise temperature

$$SNR = \frac{S_{input}}{N_{input}} = \frac{G_r}{T_A}\frac{G_t P_t \lambda^2}{kB(4\pi r)^2} \propto \frac{G_r}{T_A}$$

5.2 Pathloss, Attenuation and Fading

Receive power = Transmit Power - Transmit antenna line loss + Transmit antenna gain - Path loss (free space) - Atmospheric attenuation + Receive antenna gain - Receive antenna line loss

6 Receiver Architectures



Noise temp of antenna $T_A = e_{rad}T_b + (1-e_{rad})T_p$
Minimum detectable input signal
$S_{i,min} = \frac{S_{o,min}}{G} = \frac{N_o}{G}\frac{S_o}{N_o}_{min} = kB(T_A + T_e)\frac{S_o}{N_o}_{min}$
$V_{i,min} = \sqrt{2Z_o S_{i,min}}$
$DR_r = \frac{s_{i,max}}{s_{i,min}}$, $s_{i,max}$ determined by 3rd-order intercept point, $s_{i,min}$ determined above.
Automatic gain control gain depending on input level to meet dynamic range requirements.
Down-conversion to IF frequency
$f_{IF} = |f_{RF} - f_{LO}|, f_{LO} = f_{RF} - f_{IF}$
$f_{IM} = f_{RF} - 2f_{IF}$ or $f_{RF} + 2f_{IF}$
Intermodulation products are at $f = |mf_{RF} - nf_{LO}|$, should not be less than the IF bandwidth.

7 Radar

Radar power density $S_t(\theta,\phi) = \frac{P_t G}{4\pi R^2}\ W/m^2$
Radar cross section $\sigma(\theta,\phi) = \frac{P_s}{S_t}\ m^2$ is ratio of power scattered in the direction to incident power density
Received scattered power $S_r(\theta,\phi) = \frac{P_s}{4\pi R^2}$
Radar equation $P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}\ W$
Max detection range $R_{max} = \sqrt[4]{\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 P_{min}}}$
Radar mechanisms continuous wave (cw) for velocity, pulsed for range, pulsed-Doppler for range and velocity, frequency-modulated continuous wave for velocity and range
Doppler shift, transmitted frequency f_0, target velocity v, then doppler frequency shift is $f_d = \frac{2vf_0}{c}$, received frequency is $f_0 \pm f_d$, + for approaching target and - for receding target

8 Extras

1 in = 2.54 cm
$k = 10^3, M = 10^6, G = 10^9, T = 10^{12}, P = 10^{15}, E = 10^{18}$
$m = 10^{-3}, \mu = 10^{-6}, n = 10^{-9}, p = 10^{-12}, f = 10^{-15}, a = 10^{-18}$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\cos 3x = 4\cos^3 x - 3\cos x, \sin 3x = 3\sin x - 4\sin^3 x$$

On a sphere, $ds = (rd\theta)(r\sin\theta d\phi) = r^2\sin\theta d\theta d\phi$
Sphere area $\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} r^2\sin\theta d\theta d\phi = 4\pi r^2$
$ds = r^2 d\Omega, d\Omega = ds/r^2 = \sin\theta d\theta d\phi$ (solid angle)