CNC 2012
Languages and Machines
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Homework 1
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1.) $X = \{1,2,3,4\}$ $y = \{0,2,4,6\}$

a. $x \cup y = \{1,2,3,4,6\}$

b. $x \cap y = \{2,4\}$

c. $x - y = \{1,3\}$

d. $y - x = \{0,6\}$

e. $P(x) = \{ \{\}, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\},$
$\{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\}$

4. $X = \{n^3+3n^2+3n \mid n \geq 0\}$ $y = \{n^3-1 \mid n > 0\}$ prove $X = Y$

$$\begin{aligned} y = n^3-1 &= (m+1)^3-1 \\ &= n^3+3n^2+3n+1-1 \\ &= n^3+3n^2+3n \\ &= x \end{aligned}$$

$x \subseteq y$

$$\begin{aligned} x &= (n_0)^3+3(n_0)^2+3(n_0) \\ &= (n_0+1)^3-1 \\ &= y \end{aligned}$$

$y \subseteq x$

6.

a. $f(x) = x+1$

b. $f(x) = \begin{cases} 0 & \text{if } n=0 \\ x-1 & \text{o/w} \end{cases}$

c. $f(x) = x \div 1$

d. $f(x) = x-1$

10.)

(i) reflexive $a \equiv a$ $(a = a)$

(ii) symmetric $a \equiv b$ & $b \equiv a$ $(a = b)$ $(b = a)$

(iii) transitive $a \equiv b;$ $b \equiv c;$ $c \equiv a$ $(a = b)$ $(b = c)$ $(c = a)$

22.) suppose a list of all monotone increasing functions:

$f_0, f_1, \ldots$

then the function : $g(i) = 1 + \max(g(i-1), f_i(i))$

cannot be among these listed.

29.).

(i) $[0,0] \in Eq$

(ii) if $[m,n] \in Eq$ then $[s(m), s(n)] \in Eq$

(iii) $[m,n] \in Eq$ if it can be obtained by a finite # of application of (ii) to (i).

38.) $2+5+8+\ldots+(3n-1) = n(3n+1)/2$

Base Case: $3 \cdot 1 - 1 = 2$ $1(3 \cdot 1 + 1)/2 = 2$

$$\frac{n(3n+1)}{2} + 3(n+1) - 1 =$$

$$\frac{3n^2+n}{2} + \frac{6n+4}{2} =$$

$$\frac{3n^2+7n+4}{2}$$

$(n+1)(3(n+1)+1)/2$

$(n+1)(3n+4)/2$

$3n^2+4n+3n+4/2$

$$\frac{3n^2+7n+4}{2}$$

$$\frac{3n^2+7n+4}{2} = \frac{3n^2+7n+4}{2}$$

42.)

a.) E_0 : $\{A, B\}$

E_1 : $\{(A \wedge B), (A \vee B), A, B\}$

E_2 : $\{((A \wedge B) \wedge (A \vee B)), ((A \wedge B) \vee (A \vee B)), (A \wedge (A \wedge B)), (A \vee (A \wedge B)), A, B,$
$(A \wedge (A \vee B)), (A \vee (A \vee B)), (B \wedge (A \wedge B)), (B \vee (A \wedge B)), (B \wedge (A \vee B)), (B \vee (A \vee B))\}$

b.) $n_p(u) = \#$ of prop variables $\quad$ prove $n_o(u) = n_p(u) - 1$
$n_o(u) = \#$ of operators

Base Case $\quad n_p(u) = 1$
$(E_0) \quad n_o(u) = 0$

let w be an expression generated by $n+1$ applications of the recursive step (ii)

then $w = (u \wedge v)$, $w = (u \vee v)$

by the inductive hypothesis : $\begin{cases} n_o(u) = n_p(u) - 1 \\ n_o(v) = n_p(v) - 1 \end{cases}$

if $w = u \wedge v$ or $w = u \vee v$ then

$n_o(w) = n_o(u) + n_o(v) + 1 \xrightarrow{\text{I.H.}} n_o(w) = (n_p(u) - 1) + (n_p(v) - 1) + 1$

$n_p(w) = n_p(u) + n_p(v)$

$= n_p(u) - 1 + n_p(v) - 1 + 1$

$= \underbrace{n_p(u) + n_p(v)} - 1$

$n_o(w) = n_p(w) - 1$

c. prove $n_\ell(u) = n_r(u)$ $\quad$ # of left parens = # of right parens

Base Case : $n_\ell(u) = 0$
$(E_0) \quad n_r(u) = 0$

let w be an expression generated by $n+1$ applications of the recursive step (ii)

then $w = (u \wedge v)$, $w = (u \vee v)$ $\quad$ inductive hypothesis: $n_\ell(u) = n_r(u)$
$n_\ell(v) = n_r(v)$

$n_\ell(w) = n_\ell(u) + n_\ell(v) + 1$
$n_r(w) = n_r(u) + n_r(v) + 1$

$n_\ell(w) = n_r(u) + n_r(v) + 1$
$= n_r(w)$

46.)

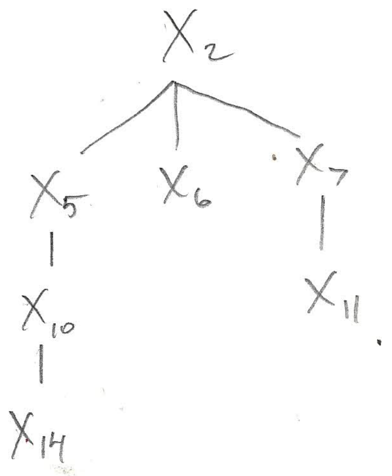
a. depth = 4

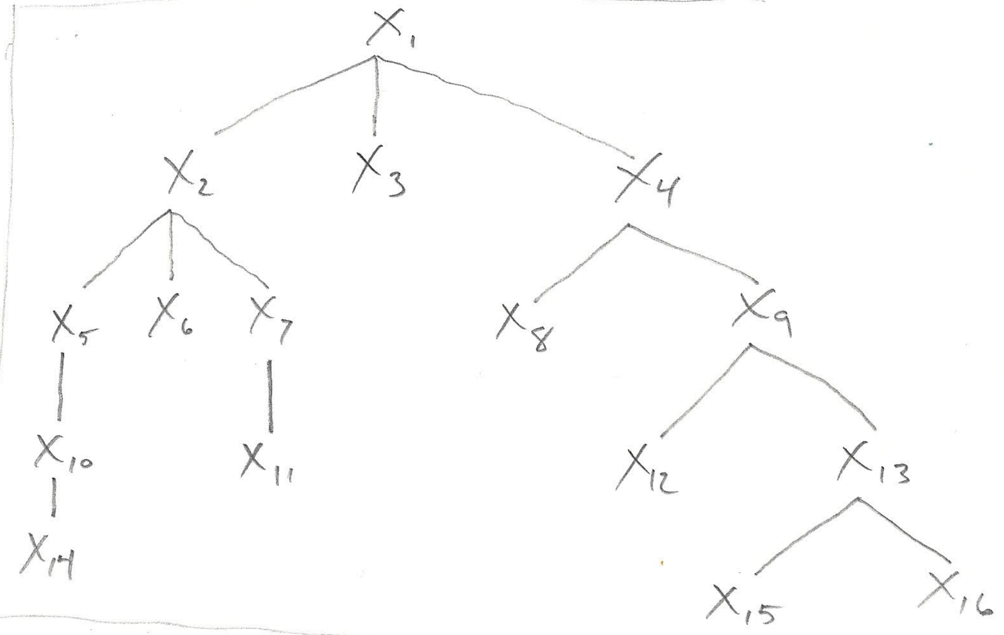
b. ancestors of $X_{11} = \{X_7, X_2, X_1\}$

c. minimal common ancestor of $X_{14}, X_{11} = X_2$

$X_{15}, X_{11} = X_1$

d. subtree by X_2





e. frontier of tree is:

$(X_{14}, X_6, X_{11}, X_3, X_8, X_{12}, X_{15}, X_{16})$