CNC 2012
Formal Languages
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Assignment 2
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14.) Let $X_1, \ldots, X_n$ be a partition of a set X. Then an equivalence relation $R_\equiv$ whose EQ classes are the sets $X_1, \ldots, X_n$ can be defined as:
$R_\equiv = X_1 \times X_1 \cup X_2 \times X_2 \cup \ldots \cup X_n \times X_n$

20.) Prove uncountability of functions from $\mathbb{N}$ to $\{0,1\}$
Let $f_0, f_1, f_2, \ldots$ be the enumerations of said functions. Define a function $g()$ where
$g(n) = \begin{cases} 1 & \text{if } f_n(n) = 0 \\ 0 & \text{otherwise} \end{cases}$

30.) Recursive Definition of GT on $\mathbb{N} \times \mathbb{N}$ given operator (S)
(i) $\{(0,1)\}$
(ii) if $(n, m) \in GT$ then
$(s(n), s(m)) \in GT$ and
$(n, s(m)) \in GT$
(iii) closure

33.) Recursive definition of mult. of $\mathbb{N}$ given operators $(+), (S)$
(i) if $n=0$ then $m \cdot n = 0$
if $n=1$ then $m \cdot n = m$
(ii) $m \cdot S(n) = m + m \cdot n$
(iii) closure

40.) Prove $1+2^n < 3^n$ for all $n > 2$

base case $1+2^3 < 3^3$

$9 < 27$

inductive case

assume : $1+2^n < 3^n$

show : $1+2^{(n+1)} < 3^{(n+1)}$

$1+2^n \cdot 2 < 3^n \cdot 3$

$\frac{a \cdot 2}{2} < \frac{b \cdot 3}{2}$

$a < b \cdot \frac{3}{2}$

fizzle

let $a = .5 + 2^n$

$b = 3^n$

assert $a < b$: $.5+2^n < 1+2^n$

$-2^n \quad -2^n$

$.5 < 1$

since $a < 1+2^n$, by the inductive hypothosis

$a < b$

assert $b < b \cdot \frac{3}{2}$: any positive Rational times 1.5 is greater than said number

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47.)

Let T_n be a strictly Binary Tree with n leaves.

Prove $P(T_n)$ = number of nodes in $T_n = 2n-1$

Base Case : T_1 $\quad P(T_1) = 2 \cdot 1 - 1$

$= 1$

Inductive Case

assume $P(T_k) = 2k-1$

show $P(T_{k+1}) = 2(k+1)-1$

let T_w be some tree constructed by a root node with children T_k

the number of leaves in $T_w = 2 \cdot T_k$ $\quad P(T_w) = 2(2k) - 1$

$= 4k-1$

the number of nodes in $T_w = (2k-1)+(2k-1)+1$

$4k-1$

Prove for any tree of depth d for which each node has at most n children that the number of leaves is at most n^d. Induction on depth of tree.

Base Case: depth 0 (root only)

$$n^0 = 1$$

assume $P(T_{nd})$ = max number of leaves for tree $T_{nd} = n^d$

$$P(T_{n(d+1)})$$

Let T be a tree built from a root node with children $(t_1, \ldots, t_n)$ where n is max number of children per node.

$$P(T_{n(d+1)}) \leq (P(t_1) + \ldots + P(t_n)) \leq n^{(d+1)}$$

by the inductive hyp. $(n^d + \ldots + n^d)$

$$n^d \cdot n$$

$$n^{(d+1)} \leq n^{(d+1)}$$