Computability 2012
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Formal Languages; HW 3
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1.) recursive def. of length of a string over Σ

(i) if $w == \lambda$ then $\text{length}(w) = 0$

(ii) if $wa \in \Sigma^*$ then $\text{length}(wa) = 1 + \text{length}(w)$

(iii) Closure

5.) Let L be the set of strings over $\{a,b\}$ generated by:

(i) $b \in L$

(ii) if u is $\in L$ then $\{ub, uab, uba, bua\} \in L$ (1) (2) (3) (4)

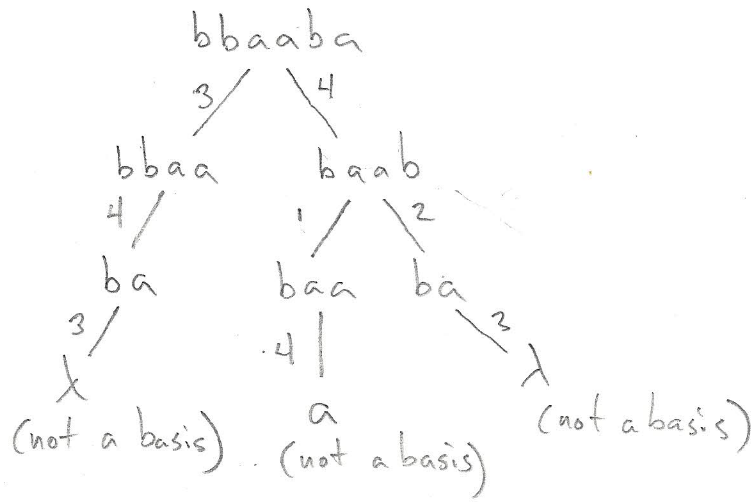
a.)

$L_0 = \{b\}$

$L_1 = \{bb, bab, bba\}$

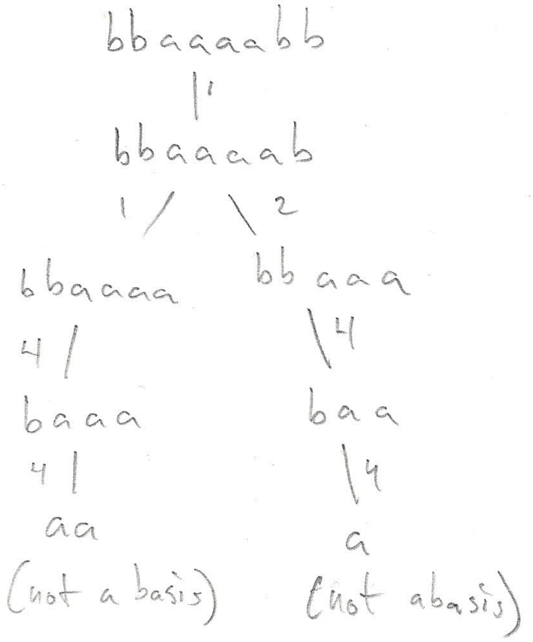
$L_2 = \{bbb, bbab, bbba, babb, babab, babba, bbaba, bbaab, bbbaa\}$

b.) is $bbaaba \in L$? No : reverse derivation tree:



c.) is $bbaaaabb \in L$? No

reverse derivation tree:



6.) recursive def. for strings over $\{a, b\}$ w/ at least one b and an even # of a's preceeding 1st b.

(i) $b \in L$

(ii) if $w \in L$ then $\{aaw, wb, wa\} \in L$

(iii) Closure

10.) Let L be the set of strings over $\{a, b\}$ generated by:

(i) λ

(ii) if $w \in L$ then, $\{wa, wab\} \in L$

prove that for all $u \in L$ there are at least as many a's as b's

$P_a(u) = $ # of a's $P_b(u) = $ # of b's

base case:

$u = \lambda$

$P_a(u) = 0$

$P_b(u) = 0$

Inductive case: Let u be some string generated by n applications of (ii)

assume : $P_a(u) \geq P_b(u)$

prove : $P_a(w) \geq P_b(w)$ where w is some string generated by $(n+1)$ apps of (ii)

then $w = \{ua \mid uab\}$

$P_a(w) = P_a(u) + 1$

$P_b(w) = P_b(u)$ or $P_b(w) = P_b(u) + 1$

$$P_a(w) \geq P_b(w)$$

$$P_a(u) + 1 \geq P_b(u) + 1$$

$$-1 \qquad\qquad -1$$

$$P_a(u) \geq P_b(u)$$

or $P_b(u) - 1$

Inductive Hypothesis

12.) Let palindromes = L where L is the set of strings over Σ generated by:

(i) $\{\lambda, a\}$ for all $a \in \Sigma$

(ii) If $w \in L$ then $awa \in L$ where $a \in \Sigma$

(iii) closure

Prove ~~L ==~~ $W = \{w \mid w = w^R\}$; Show A : $L \subseteq W$ & B : $W \subseteq L$

A: all strings $x \in L$ have the property $x = x^R$; induction on length(x)

Base case : $\lambda = \lambda^R$; $a \in \Sigma$, $a = a^R$ by def. of reverse

Inductive case: Let u be a string generated by $k \geq 1$ applications of (ii). Let w be a string generated by $k+1$ applications of (ii)

assume $u = u^R$

show $w = w^R$

$w = aua$ where $a \in \Sigma$; let $au = x$

$$\begin{aligned}(xa)^R &= a^R x^R \\ &= a x^R \\ &= a(au)^R \\ &= a(u^R a^R) \\ &= aua \quad \text{inductive hypoth. / basis}\end{aligned}$$

B: show that all strings $w \in \Sigma^*$ having the property $w = w^R$ can be generated by (i) (ii) (iii) (Rec.)

if $w = \lambda$ or a where $a \in \Sigma$ then $w = w^R$ and w can be generated by Rec.

if length$(w) > 1$ and $w \in \Sigma^+$ and $w = w^R$ then $w = aua$ where $a \in \Sigma$ and $u = u^R$ matching the pattern of (ii)

13.) Let $L_1 = \{aaa\}^*$; $L_2 = \{a,b\}\{a,b\}\{a,b\}\{a,b\}$; $L_3 = L_2^*$

L_1 is the set of strings where the length(w) mod 3 == 0

L_2 is the set of strings of a's & b's of length 4

L_3 is the set all strings of a's & b's where length(w) mod 4 == 0

$L_1 \cap L_3$ is the set of strings of only a's where length(w) % 12 == 0

14.) $\Sigma = \{a,b,c\}$; the set of strings $w \in \Sigma^*$ where all a's preceed all the b's which preceed all the c's:

$W = a^* b^* c^*$

23.) $\Sigma = \{a,b,c\}$; the set of strings $w \in \Sigma^*$ that begin with an a, contain 2 b's, and end with cc:

$w = a(a,c)^* b (a,c)^* b (a,c)^* cc$