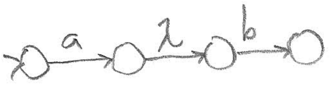
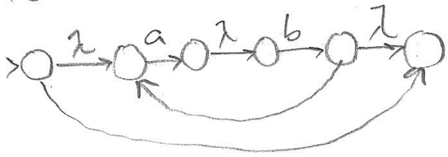
CNC Winter
Formal Languages
Professor Sherri Shulman
HomeWork 1
Owen Meyer

1.) $(ab)^*ba$

ab →



$(ab)^*$ →



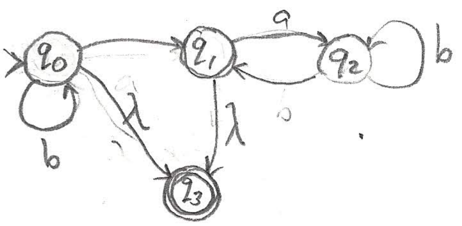
ba →

$(ab)^*ba$ →

2.) using Haskell : (--code included in submission)

w/ e standing in for λ as the empty string and + as union

A.)

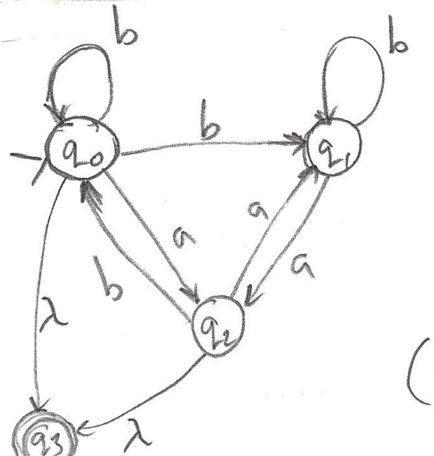


$b^*a(ab^*b)^* + e$ > remove 2 then 1

or

$b^*aa((ba+b))^*b+(a+e)$ > remove 1 then 2

B.)

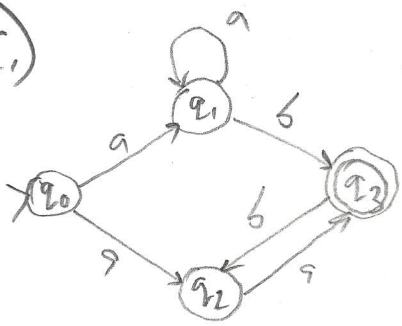


$((bb^*a+a)(ab^*a)^*b+b)^*((bb^*a+a)(ab^*a)^*+e)$

or

$((ab+b)+(aa+b)((aa+b))^*ab)^*((a+e)+(aa+b)((aa+b))^*a)$

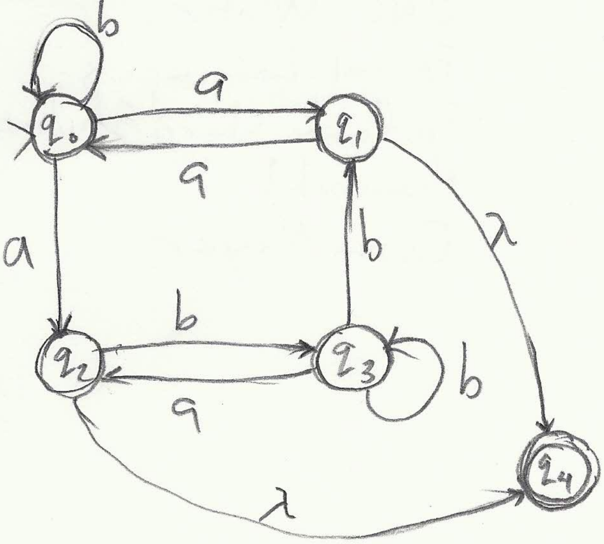
C.)



$(aa^*b+aa)(ba)^*$

in this case, the order of state removal does not produce differing forms of equivalent set describing regular expressions.

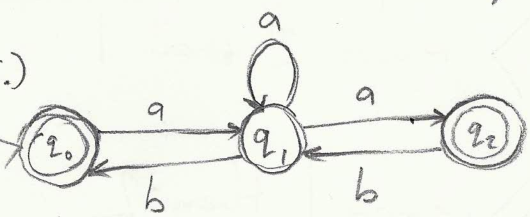
D.)



$((ab(ab+b)^*b+a)a+b)^*((ab(ab+b)^*b+a)+(ab(ab+b)^*a+a))$

and/or many others

5.)



let $q_0 = S$ $\quad S \rightarrow aA \mid \lambda$

$q_1 = A$ $\quad A \rightarrow aA \mid bS \mid aB$

$q_2 = B$ $\quad B \rightarrow bA \mid \lambda$

$(a((ab+a)^*b)^*a((a+b)^*a+e)$

7.) where L is regular over $\{a,b,c\}$, show that the following are also regular.

a.) $L_w = \{w \mid w \in L \text{ and } w \text{ ends with } aa\}$

let $L_a = (a|b|c)^*aa$, then $\overline{L_a}$ (the set of all strings over $\{a,b,c\}$ that don't end in aa) is also regular.

and $L_w = L - \overline{L_a}$ which is regular under set difference (closure) (see #8)

b.) $L_w = \{w \mid w \in L \text{ or } w \text{ contains an } a\}$

let $L_a = (a|b|c)^*a(a|b|c)^*$ (the set of all strings over $\{a,b,c\}$ that contain at least one a)

then $L_w = L_a \cup L$ which is regular under set union.

c.) $L_w = \{w \mid w \notin L \text{ and } w \text{ does not contain an } a\}$

let $L_x = (a|b|c)^*a(a|b|c)^*$ (the set of all strings over $\{b,c,a\}$ that contain at least one a)

$L_w = \overline{L} - L_x$, regular under complement and difference.

d.) $L_w = \{uv \mid u \in L \text{ and } v \notin L\}$

$L_w = L\overline{L}$, regular under complement and concatenation.

8.) $L - M = L \cap \overline{M}$ where L, M are regular languages, set difference is equivalent to the intersection of the complement. (set properties)

and regularity is closed under intersection and complement.

11.) let L be some regular language, the following are regular:

a.) $L^R = \{w^R \mid w \in L\}$

Basis: if w is a single symbol, then $w = w^R$

Induction: if w is

$u + v$ then $w^R = u^R + v^R$

uv then $w^R = v^R u^R$

u^* then $w^R = (u^R)^*$

b.) $E = \{uv \mid v \in L\}$

let $M = \Sigma^*$

then ME is regular; closure under concatenation

14.) show that the following are not regular:

a.) palindromes over $\{a, b\}$

let $z = a^k b a^k$

$u = a^{k-m}$, $v = a^n$, $w = ba^k$ $\Big\}$ uv^iw: v cannot be pumped to produce palis.

b.) $\{a^n b^m \mid n < m\}$

let $z = a^k b^{k+1}$

$u = a^{k-n}$, $v = a^n$, $w = b^{k+1}$ $\Big\}$ uv^iw pumping v produces $n > m$

c.) $\{a^i b^j c^{2j} \mid i \geq 0, j \geq 0\}$

d.) the set of initial sequences of $abaab \ldots ba^n ba^{n+1} \ldots$

let $z = abaab \ldots ba^{k-1} ba^k b$

then is now way to decompose z into uvw that won't throw off the pattern by pumping v. if $v = a^i$ then pumping i throws off the a count. if v contains b's then pumping it produces repetitions which are not in the original sequence.

f.) the set of strings over $\{a,b\}$ where the number of a's is a perfect cube. let $z = a^{k^3}ba$ then $u = a^{k^3-n}$

$v = a^n$

$w = ba$

pumping produces: $\{v^0, v^1, v^2, v^3 \ldots\}$

cubes are : $\{r^{n^3}, (r+1)^{n^3}, (r+2)^{n^3} \ldots\}$

~~let Gr be some right-linear grammar~~

20.) show that right-linear grammars produce precisely the regular sets.

(i) the regular grammars $\subseteq$ right-linear grammars as each rule of the regular grammars is also a legitamate right-linear rule.

(ii) Theorem 6.3.1 can be modified to produce a DFA from any right-linear grammar

A.) $Q = \begin{cases} V \cup \{Z\} & \text{where } Z \notin V, \text{ if P contains } A \to u \\ V & \text{otherwise} \end{cases}$

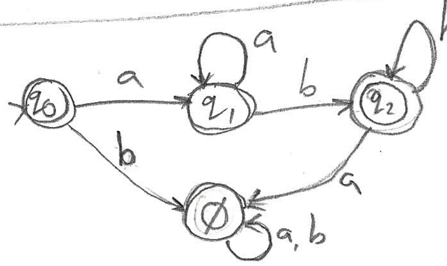
B.) $\hat{\delta}(A, u) = B$ iff $\exists\ A \overset{*}{\underset{Gr}{\Rightarrow}} uB$

$\hat{\delta}(A, u) = Z$ iff $\exists\ A \overset{*}{\underset{Gr}{\Rightarrow}} u$

C.) $F = \begin{cases} \{A \mid A \to \lambda \in P\} \cup \{Z\} & \text{if } Z \in Q \\ \{A \mid A \to \lambda \in P\} & \text{otherwise.} \end{cases}$

The $\hat{\delta}$'s can be transformed into DFA sequences using Theorem 5.5.3 for concatenation

24.) $\equiv_L$ for a^+b^+



$[\lambda]_\equiv \quad \lambda$

$[a]_\equiv \quad aa^+$

$[ab]_\equiv \quad aa^*bb^* = a^+b^+$

$[b]_\equiv \quad (aa^*bb^*a \mid b)(a \mid b)^*$

27.) show that $L = \{a^i \mid i$ is a perfect square$\}$ is not regular using Myhill-Nerode.

if a^{i^2}, a^{j^2} where $i \neq j$ are $\in L$ then concatenating $a^{3(i^2)}$ onto each string produces $a^{i^2}a^{3(i^2)} \in L$ and $a^{j^2}a^{3(i^2)} \notin L$, thus $a^{i^2} \not\equiv_L a^{j^2}$ and an infinite number of equivalence classes $\equiv_L$ can produced with the same technique.