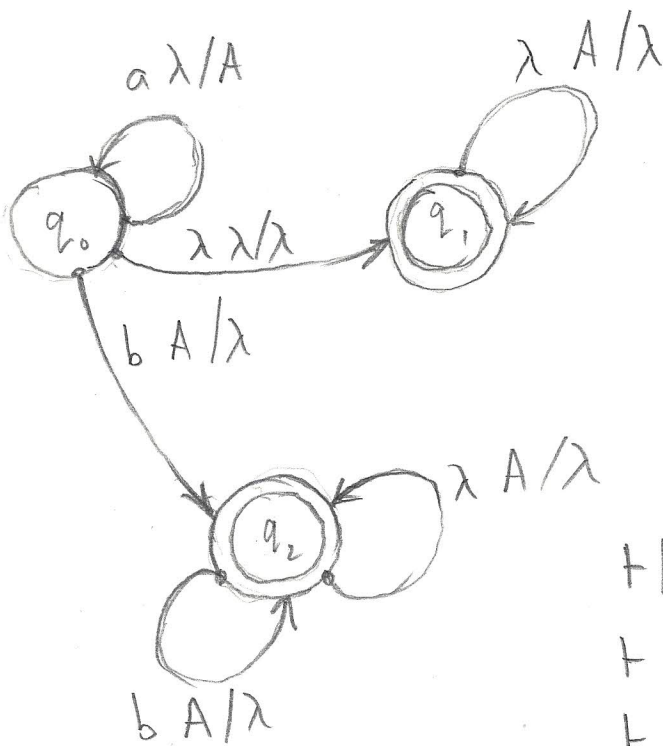


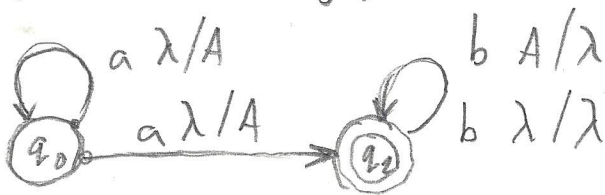
1.)



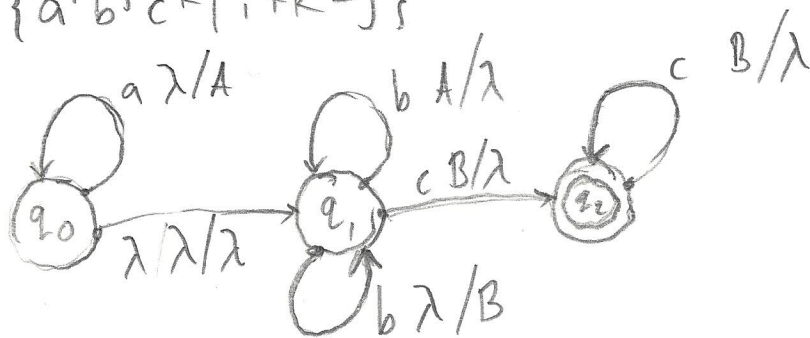
$$\{a^*\} \cup \{a^i b^j \mid i > 0, j > 0, i > j\}$$

$\vdash [q_0, aab, \lambda]$
 $\vdash [q_0, ab, A], [q_1, aab, \lambda]$
 $\vdash [q_0, b, AA], [q_2, ab, \lambda]$
 $\vdash [q_2, \lambda, A]$
 $\vdash [q_2, \lambda, \lambda]$

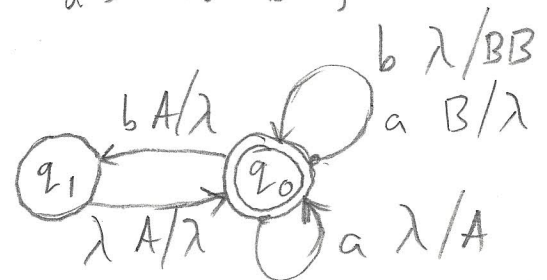
3.) $\{a^i b^j \mid 0 \leq i \leq j\}$



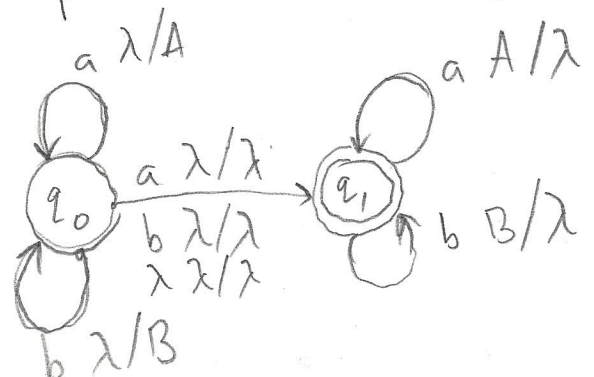
$$\{a^i b^j c^k \mid i + k = j\}$$



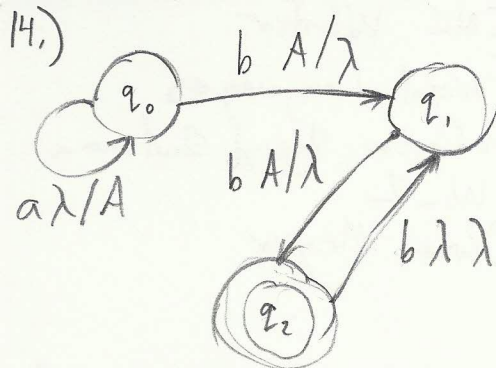
$\{w \mid w \in \{a, b\}^* \text{ where } w \text{ contains twice as many } a\text{'s as } b\text{'s}\}$



palindromes over $\{a, b\}$



(14.)



$$\{a^i b^j \mid i > 0, j = 2i\}$$

$$S \rightarrow C$$

$$C \rightarrow aF$$

$$E \rightarrow bH \mid aEN \mid aFK$$

$$F \rightarrow bI \mid aEO \mid aFL$$

$$H \rightarrow \lambda$$

$$I \rightarrow bR$$

$$K \rightarrow bH$$

$$L \rightarrow bI$$

$$N \rightarrow bK$$

$$O \rightarrow bL$$

$$R \rightarrow \lambda$$

$$[q_0, aabbbb, \lambda]$$

$$\vdash [q_0, abbbb, A], \dots$$

$$\vdash [q_0, bbbb, AA], \dots$$

$$\vdash [q_1, bbb, A]$$

$$\vdash [q_2, bb, A]$$

$$\vdash [q_1, b, \lambda]$$

$$\vdash [q_2, \lambda, \lambda]$$

$$\begin{aligned}
 &S \\
 &\downarrow \\
 &C \\
 &\downarrow \\
 &aF \\
 &\downarrow \\
 &aaEO \\
 &\downarrow \\
 &aabHO \\
 &\downarrow \\
 &aabO \\
 &\downarrow \\
 &aabbbL \\
 &\downarrow \\
 &aabbbbI \\
 &\downarrow \\
 &aabbbbR \\
 &\downarrow \\
 &aabbbb
 \end{aligned}$$

17.)

a) Let $z = a^{k^2} = uv^2wx^2y$; $\text{length}(vwx) \leq k$

$$\text{length}(uv^2wx^2y) = k^2 + \text{length}(v) + \text{length}(x) \\ \leq k^2 + k$$

$$< k^2 + 2k + 1 = (k+1)^2$$

$$\text{length}(uv^2wx^2y) > k^2 \text{ and } < (k+1)^2$$

c) $\{a^i b^i c^i \mid i \geq 0\}$

Let $z = a^k b^{2k} c^k = uv^2wx^2y$

Since vwx can have at most two symbols in it, when v, x are pumped the symmetry will be thrown.

d) $\{a^i b^j c^k \mid 0 \leq i \leq j \leq k \leq 2i\}$

Let $z = a^k b^{k+1} c^{2k-1} = uv^2wx^2y$

same as above; $uvwx$ can be $\{a^*, b^*, c^*, a^*b^*, b^*c^*\}$

since the pumping takes place within one of these RE's the symbol(s) not in the RE will disarrange

: if a^* then pumping makes $i > k+1$

: if b^* $i > 2k-1$ in $b^i w b^i$

: if a^*b^* i or $j > 2k-1$ in $a^i w b^i$

: if b^*c^* $k > 2i$

for $a^i w a^i$

$$19) \{a^i b^j c^j d^i \mid i, j \geq 0\} = L_1$$

$$\left. \begin{array}{l} S \rightarrow XY \mid \lambda \\ X \rightarrow aXb \mid \lambda \\ Y \rightarrow cYd \mid \lambda \end{array} \right\} \text{context free grammar equivalent to } L_1$$

$$\{a^i b^j c^i d^k \mid i, j, k \geq 0\} = L_2$$

$$\left. \begin{array}{l} S \rightarrow AXD \mid \lambda \\ A \rightarrow aA \mid \lambda \\ D \rightarrow Dd \mid \lambda \\ X \rightarrow bXc \mid \lambda \end{array} \right\} \text{context free grammar equivalent to } L_2$$

$$\{a^i b^j c^i d^j \mid i \geq 0\}$$

$$\text{Let } z = a^k b^k c^k d^k = uvwxy$$

Since $vwxy$ can have at most two symbols in it, when v and x are pumped only one or two symbols will be changing numeracy.

No ww strings =

states $\{q_0, q_1\}$

sigma $\{a, b\}$

gamma $\{S, A, B, X, Y, \lambda\}$

start q_0

finals $\{q_1\}$

same # of a's, b's

$$\delta(q_0, a, \lambda) = \{[q_1, AXB]\}$$

(there's 32 rules; I'll enclose a haskell file that contains this PDA..)

$$Q = \{q_0\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{A, B, \lambda\}$$

$$S = q_0$$

$$F = \{q_1\}$$

$$\delta(q_0, a, \lambda) = \{[q_0, A]\}$$

$$\delta(q_0, a, B) = \{[q_0, \lambda]\}$$

$$\delta(q_0, b, \lambda) = \{[q_0, B]\}$$

$$\delta(q_0, b, A) = \{[q_0, \lambda]\}$$

Last problem:

same as Homework 2 last quarter