Report on PCA and Fast Map Algorithms

by Ruijie Rao

Import Libraries

In [3]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

Data Import

In [4]:

```
with open("pca-data.txt", "r") as file:
    content = file.readlines()
pca_data = np.array([line.strip("\n").split("\t") for line in content] ).astype(floa
```

In [5]:

```
pca_data
```

Out[5]:

```
array([[  5.90626285,  -7.72946458,   9.14494487],
       [ -8.64032311,   1.72426044, -10.69680519],
       [  0.25854061,   0.23062224,   0.76743916],
       ...,
       [ -3.69142791,  -0.474338  ,   0.55020057],
       [  7.63831529,  -4.47583291,   8.15392291],
       [  9.72207756,  -8.50135442,   8.8424068 ]])
```

In [6]:

```
with open("fastmap-data.txt", "r") as file:
    content = file.readlines()
fm_data = np.array([line.strip("\n").split("\t") for line in content] ).astype(float
with open("fastmap-wordlist.txt", "r") as file:
    content = file.readlines()
fm_wordlist = np.array([line.strip("\n") for line in content] )
```

In [7]:

```
fm_wordlist
```

Out[7]:

```
array(['acting', 'activist', 'compute', 'coward', 'forward',
       'interaction', 'activity', 'odor', 'order', 'international'],
      dtype='<U13')
```

PCA Algorithm

Brief Description

The goal of PCA is to lower the dimension of high-dimensional data by capturing the attributes with most variability, which will help us in visualization and acceleration in other model trainings.

1. Calculate covariance matrix of the data points.
2. Compute the eigenvalues and their eigenvectors.
3. Pick the largest N eigenvalues and their eigenvectors.
4. Generate a new data set using the original data and the new eigenvectors.

Algorithm Construct

In [21]:
```
n = 2 # Reduce to 2D
pca_data_meaned = pca_data-np.mean(pca_data,axis=0)
cov_mat = np.cov(pca_data_meaned,rowvar=False) # Covariance Matrix 3x3
e_val, e_vec = np.linalg.eigh(cov_mat) # eval are 3 scalars, evec is 3x3 mat
e_vec_reduced = e_vec[np.argsort(e_val)][::-1][:,:n] # Reduced to 3x2
pca_data_reduced = pca_data_meaned@e_vec_reduced
```

Test

In [22]:
```
pca_data_reduced
```

Out[22]:
```
array([[-3.73636464, -3.47164008],
       [-2.68767077,  1.73739336],
       [ 0.30249399,  0.3288875 ],
       ...,
       [-2.41984582,  2.69798271],
       [-0.01802045, -3.59947044],
       [-2.29884773, -6.73009149]])
```

Result

Directions of the first two principal components:

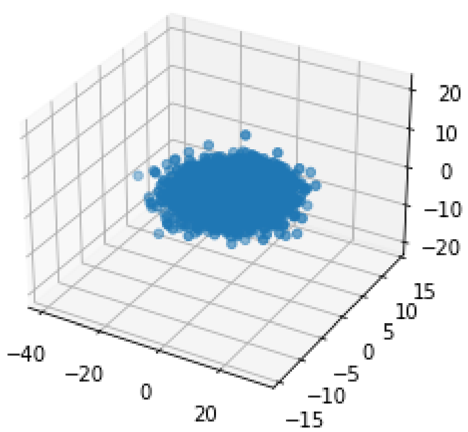
In [36]:
```
e_vec_reduced.T
```

Out[36]:
```
array([[ 0.54233352,  0.83862076, -0.0508879 ],
       [-0.71496368,  0.4924792 ,  0.4962773 ]])
```

Following is a graph in 3D of the original coordinates.

In [10]:
```
fig = plt.figure()
ax = fig.add_subplot(projection='3d')
ax.scatter(pca_data[:,0], pca_data[:,1], pca_data[:,2])
plt.show
```

Out[10]:
```
<function matplotlib.pyplot.show(close=None, block=None)>
```

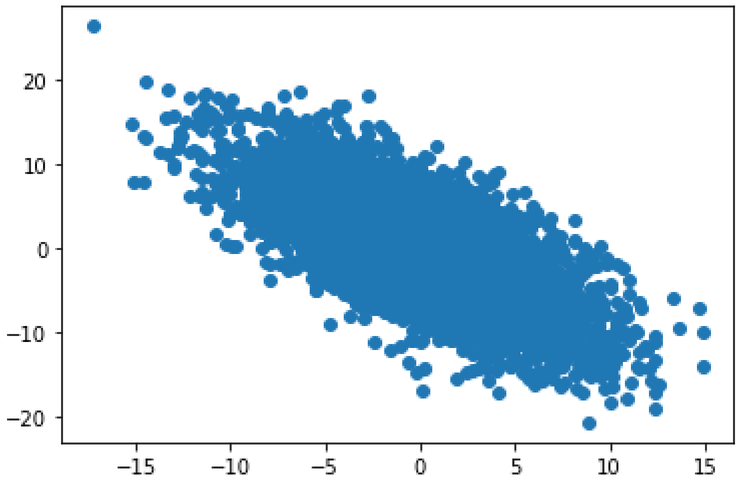


Following is a graph in 2D of the reduced coordinates.

In [23]:
```
fig = plt.figure()
```

```
ax = fig.add_subplot()
ax.scatter(pca_data_reduced[:,0], pca_data_reduced[:,1])
plt.show
```

Out[23]: <function matplotlib.pyplot.show(close=None, block=None)>



Implementation of Library Function

Going to use SKLearn PCA on the training data to see the difference.

In [1]:

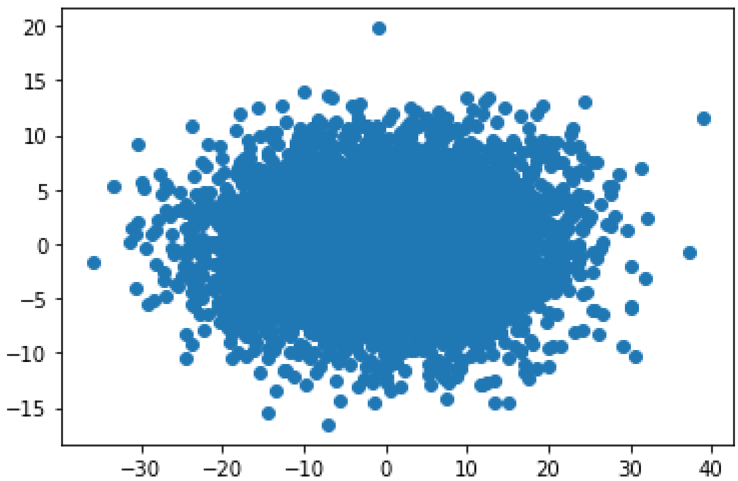
```
from sklearn.decomposition import PCA
```

In [11]:

```
pca = PCA(n_components=2)
sk_pca_data_reduced = pca.fit_transform(pca_data)
print(sk_pca_data_reduced)
fig = plt.figure()
ax = fig.add_subplot()
ax.scatter(sk_pca_data_reduced[:,0], sk_pca_data_reduced[:,1])
plt.show
```

```
[[-10.87667009   7.37396173]
 [ 12.68609992  -4.24879151]
 [ -0.43255106   0.26700852]
 ...
 [  2.92254009   2.41914881]
 [-11.18317124   4.20349275]
 [-14.2299014    5.64409544]]
```

Out[11]: <function matplotlib.pyplot.show(close=None, block=None)>



Fast Map Algorithm

Brief Description

The goal of Fast Map is to map objects into euclidean space.

1. According to distance between each pair of objects, iterate and find the furthest pair (as O_a and O_b).
2. O_a and O_b forms a line that defines the first dimension of the resulted points. Calculate the first dimension of every point by projecting O_i onto O_aO_b.
3. Find a new dimension that is perpendicular to the former dimension. Find the distance between each pair of points using the function:$D_{new}(O'_i, O'_j)^2 = D(O_i, O_j)^2 - (x_i, x_j)^2$
4. Iterate.

Algorithm Construct

Data Preparation

Going to build a distance matrix every iteration and inherit the matrix to the next iteration for new distance calculation. Distance matrix is going to be a dictionary, with *index* and *value* 2 lists as following. *Index* will be a list of sets that contains a and b of the pair. *Value* will be a list of distance values between p_a and p_b.

In [75]:

```
dist_mat = {"index":[],"value":[]}
print("index: value:")
for row in fm_data:
    a = int(row[0])-1
    b = int(row[1])-1
    d = row[2]
    dist_mat["index"].append({a,b})
    dist_mat["value"].append(d)
    print(f" {a},{b}     {d}")
```

```
index: value:
 0,1     4.0
 0,2     7.0
 0,3     6.0
 0,4     7.0
 0,5     7.0
 0,6     4.0
 0,7     6.0
 0,8     6.0
 0,9     10.0
 1,2     7.0
 1,3     7.0
 1,4     8.0
 1,5     9.0
 1,6     2.0
 1,7     8.0
 1,8     8.0
 1,9     11.0
 2,3     5.0
 2,4     6.0
 2,5     10.0
 2,6     6.0
 2,7     6.0
```

```
2,8     6.0
2,9     12.0
3,4     2.0
3,5     10.0
3,6     7.0
3,7     4.0
3,8     5.0
3,9     12.0
4,5     10.0
4,6     8.0
4,7     5.0
4,8     4.0
4,9     11.0
5,6     9.0
5,7     10.0
5,8     9.0
5,9     4.0
6,7     8.0
6,8     8.0
6,9     11.0
7,8     2.0
7,9     12.0
8,9     11.0
```

Finding the distance between p_a and p_b will simply be slicing in the value list using index pair.

In [45]:
```
def find_distance(a,b,dist_mat):
    return dist_mat["value"][dist_mat["index"].index({a,b})]
```

Config

In [65]:
```
N = len(fm_wordlist) # Number of objects
k = 2 # 2D dimension
```

Generate New Distance Matrix

$D_{new}(O_i', O_j')^2 = D(O_i, O_j)^2 - (x_i, x_j)^2$

In [68]:
```
def gen_distance_matrix(dist_mat,x):
    for i in range(len(dist_mat["index"])):
        a,b = dist_mat["index"][i]
        dist_mat["value"][i] = np.sqrt(np.square(dist_mat["value"][i])-np.square(x[a]
    return dist_mat
```

Find Furthest Pair

Going to find the furthest pair of points by finding the max distance among all distances of all pairs.

In [61]:
```
def find_furthest_pair(dist_mat):
    a,b = dist_mat["index"][np.argmax(dist_mat["value"])]
    return a,b
```

Generate New Dimension Coordinates

Except for $i = a, b$, $xi = \frac{(d_{ai}^2 + d_{ab}^2 - d_{ib}^2)}{(2d_{ab})}$

In [57]:

```
def gen_x(a,b,dist_mat):
    x = np.zeros(N)
    dab = find_distance(a,b,dist_mat)
    x[a] = 0 # P_a to itself
    x[b] = dab # Furthest distance
    for i in range(N):
        if i==a or i==b:
            continue
        dai = find_distance(a,i,dist_mat)
        dib = find_distance(i,b,dist_mat)
        x[i] = (dai**2+dab**2-dib**2)/(2*dab)
    return x
```

Loop Construct

In [66]:

```
def Fast_Map(dist_mat,k=2): # k dimension
    result = np.zeros((N,k))
    for c in range(k):
        if c != 0:
            dist_mat = gen_distance_matrix(dist_mat,x)
        a,b = find_furthest_pair(dist_mat)
        x = gen_x(a,b,dist_mat)
        result[:,c] = x
    return result
```

Test

In [74]:

```
Fast_Map(dist_mat,k=2)
```

Out[74]:

```
array([[ 8.125     ,  6.0625    ],
       [ 9.        ,  7.75      ],
       [12.        ,  4.        ],
       [10.95833333,  1.1875    ],
       [ 9.54166667,  0.        ],
       [ 2.5       ,  5.1875    ],
       [ 9.54166667,  8.        ],
       [10.5       ,  1.5625    ],
       [ 9.54166667,  1.        ],
       [ 0.        ,  4.        ]])
```

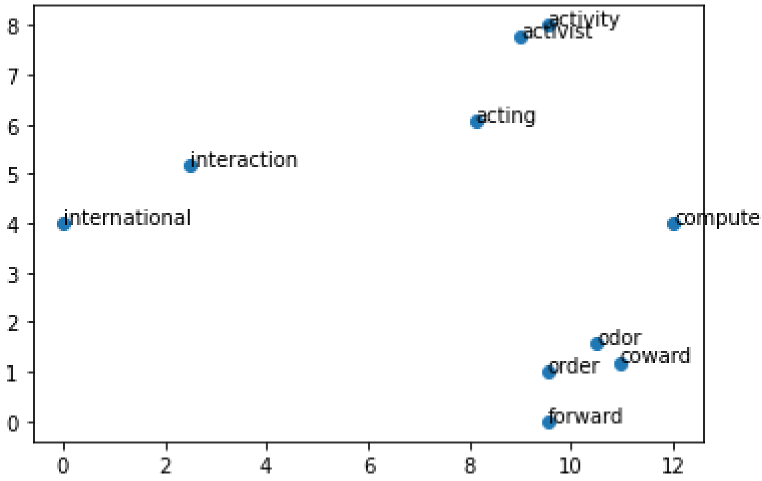
Plot Result

In [76]:

```
map_result = Fast_Map(dist_mat,k=2)
fig = plt.figure()
ax = fig.add_subplot()
ax.scatter(map_result[:,0], map_result[:,1])
for (label,x,y) in zip(fm_wordlist,map_result[:,0],map_result[:,1]):
    plt.annotate(label, (x, y))
plt.show
```

Out[76]:

```
<function matplotlib.pyplot.show(close=None, block=None)>
```


activity
activist
acting
interaction
international
compute
odor
coward
order
forward
0
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0
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12