Exercise Sheet 2: Tree Diagrams, Bayes Theorem & Expected Value

1) 70% of houses in a residential area have an alarm. It is found that 35% of the owners of the alarmed houses have a dog and 42% of the owners of the non-alarmed houses have a dog.

   a) Construct a tree diagram to show all possible events and probabilities.

   b) A house is selected at random. Calculate the probability that the house does not have a dog.

   c) If a randomly selected house is one of the houses that does not have a dog, what is the probability that the house has an alarm?

2) 65% of CIT students live in Cork City. 80% of the CIT students who live in Cork City travel to college by bus each day. 40% of CIT students who live elsewhere travel to college by bus each day.

   a) Construct a tree diagram to show all possible events and probabilities.

   b) If one CIT student is selected at random, calculate the probability that he/she travels to college by bus.

   c) At randomly selected student stated that she travelled to college by bus. Calculate the probability that she does not live in Cork City.

3) On any given day, the probability that an employee will drive to work is 0.5, the probability that he/she will cycle to work is 0.3 and the probability that he/she will walk to work is 0.2. If he/she drives, the probability that he/she will arrive on time is 0.6 If he/she cycles, the probability that he/she will arrive on time is 0.8. If he/she walks, the probability that he/she will arrive on time is 0.9. An employee is selected at random.

   a) Calculate the probability that the employee will arrive on time for work.

   b) If the employee has arrived on time for work yesterday. Calculate the probability that he/she walked to work yesterday.

   c) The employee has not arrived on time for work yesterday. Calculate the probability that he/she walked to work yesterday.

4) A game consists of throwing two dice and adding the scores.

- If you score more than 10, you win €50
- If you score a total of 6, you win €24.
- If you score 7, you lose €27.
- For all other scores, you neither win nor lose.

a) Calculate the expected value of this game.
b) If you pay €2 to play this game, could you say it was a fair game? Explain your answer.

5) A card is drawn from a normal pack of cards.

- If the ace of spades is drawn, you win €50.
- If a diamond is drawn, you win €12.
- If an even numbered club is drawn, you lose €12.
- If you draw any other card, you neither win nor lose.

How much is a player expected to win if they pay €2.50 to play the game?

6) A basketball game at a carnival costs €7 to play. In the game, the player gets three shots. If the player

- Scores a basket, they win €5
- Hit the rim but do not score, they win €1.50
- Miss all everything, they get €0.

The probability a player scores is 0.25, hits the rim is 0.5 and misses altogether is 0.25.

a) What is the expected value of one shot?
b) What is the expected value of a game (i.e. 3 shots)?

7) When undergraduates are asked to write a 250-word essay without spell-checking, the number, X, of spelling mistakes has the following distribution:

Number of Spelling Mistakes in an Essay	Probability
0	0.1
1	0.3
2	0.3
3	0.2
4	0.1

If one essay is written by an undergraduate is selected at random,

a) What is the probability it will contain at least one spelling mistake?
b) What is the expected number of spelling mistakes that it is likely to contain?

Solutions

1)

     b) 0.629

     c) 0.7234

2)

     b) 0.66

     c) 0.2121

3)

     a) 0.72

     b) 0.25

     c) 0.0714

4)

     a) $E(X) = €3$

     b) This is not a fair game as the input (€2) is not equal to the output (€3)

5) €0.31

6)

     a) $E(X = 1 \text{ shot}) = €2$

     b) $E(X = 3 \text{ shots}) = €6$

7)

     a) 0.9

     b) 1.9 spelling mistakes