A Review of Solar Car Vehicle Modelling and Strategy Optimisation

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***Abstract*—The World Solar Challenge poses an interesting problem in determining the optimal strategy to collect and expend energy from the sun. Numerous methods have been proposed since the 90s with varying levels of success. A review of the modelling approach and optimisation method was conducted. It is found that the modelling assumptions and approaches are widely agreed on but the optimisation method have not been fully developed, with a wide range of approaches presented. There is also a lack of consideration in executing the strategy, such as reacting to unexpected disruptions and discrepancies in the model during the race.**

***Index Terms*—Optimisation, race strategy, solar vehicle, vehicle simulation**

I. Introduction

SOLAR RACING consist of racing highly efficient prototype vehicles over long distances using only the power from the sun. Various races exist around the world, the World Solar Challenge and the American Solar Challenge take place on public roads and teams compete to cover the race distance in the shortest amount of time. Other races like the European Solar Challenge and the Formula Sun Grand Prix take place at a track where teams compete by covering the most distance in a set amount of time. This project will focus on the race strategy competing in the Challenger Class of the World Solar Challenge.

The World Solar Challenge is held biannually in Australia where around 50 teams from around the world compete and showcase the capabilities of solar powered vehicles. The Challenger Class is a race focused on building and racing the most efficient single occupant vehicle to complete the full course of $3021\,\mathrm{km}$ from Darwin to Adelaide.

Although developing race strategies for a solar race may seem to be quite specific, these strategies can be applied to wider applications such as electric vehicle ranging and renewable integration.

The optimisation of solar racing strategy can be describe in two distinct parts. 1) Modelling of the vehicle and operating conditions, which the aim is to recreate an accurate representation of the real world so different strategies can be tested. 2) Optimising the strategy, where the velocity profile during the race is adjusted to complete the race in the shortest amount of time.

II. Literature Review

A. Vehicle Modelling

Vehicle modelling is a reasonably developed topic for circuit motorsport. [1]–[3] have all considered factors such as the aerodynamic forces, weight distribution and tyre slip angles, with [1], [2] created multi-body models which included full suspensions and complex tyre and track simulation.

However, in the case of solar racing, not all of them can be simply transferred. The main objective of vehicle dynamics for circuit motorsport is to determine the optimal setup for the vehicle to maximise factors such as cornering speed or vehicle handling. Due to the race being mostly a straight line, vehicle set up have little impact to race time compared to factors such as gradient, and solar irradiance.

Therefore, [4]–[8] all ignored vehicle set up and employed a model focused on energy balance with varying complexity. This involved taking in account the resistance forces acting on the car, the system efficiencies of the components, the environmental conditions and the race regulations.

1) Resistance forces: Consider a free body diagram of all the forces acting on the solar car, there are numerously forces acting against the thrust provided by the motor. Forces such as drag varies with the relative oncoming wind velocity, gravitational forces act on the car while it is going up and down an incline, and the deformation of the tyres contribute to energy loss. A well defined model should capture all of these effects and allow the strategist to make data-driven decisions to achieve the best result.

While [5] did note the yaw dependence of drag force in his scaled wind tunnel test, he argues that the values are a rough optimistic estimation and the yaw dependency only apply to the scale model. Along with [4], [6]–[8], they all ignored the effect of yaw angle ψ on the aerodynamic drag, and the only variable that changes in (1) is the velocity term v.

$$D = \frac{1}{2}\rho v^2 C_d(\psi) A \tag{1}$$

As the race is mostly a straight and therefore slip angles and tyre wear are not significant, all literature used a rather simple model for rolling resistance of

$$R = (c_{rr1} + c_{rr2} v) mg \tag{2}$$

and the gradient force is given by

$$G = mg\sin(\theta) \tag{3}$$

where θ is positive when going uphill.

Pudney [4] also took in account the change in rotational energy of the wheels by using a modified effective mass.

Combining all the above factors, the equation of motion can be written as

$$\frac{dv}{dt} = \frac{1}{m}\left[F_{Motor} - R - D - G\right] \tag{4}$$

2) System efficiencies: Pudney [4] considered mechanical and electrical drive losses as a function of output power and velocity, while [5] argues that "Although the motor efficiency varies with the motor input power, it is fairly constant for values of > 1000 W", and used a constant motor efficiency along with [7].

Some literature choose to employ an ideal battery model, but Pudney [4] after discussing various complex battery models choose to employ a simple ohmic battery model as it was difficult to model given the limited computing power in 2000. Mocking [5] noted the difficulty of measuring the state of charge as keeping track of the time-integral of the battery current was sensitive to error, and the output voltage depends on the discharge current. He end up extrapolating from a empirically determined curve [5] in his model.

3) Weather conditions: The irradiance function in **atmaca_energy_nodate**, [8] was modelled by as a sin function, while [4] modelled irradiance as a truncated Fourier series. An Artificial Neural Network is used by [7] by feeding information such as the location of the sun, cloud condition and altitude to obtain the solar irradiance.

Both [5], [9] obtained wind velocities from forecast services and a mobile weather station with the convoy during the race while [10] use weather forecast services available online.

4) Control stops: Control stops were taken in account by [5], [7] during the modelling process, but had not suggested any optimisation technique or strategy related to it. Teshima [8] considered control stops by slowly increasing the number of possible arrival times and evaluated the optimal of the set of arrival times at each iteration, until the number of possible arrival times exceed a threshold value.

5) SolarSim: SolarSim is a race simulation software developed in Durham primarily used for simulating a solar car racing in the World Solar Challenge [11]. SolarSim is implemented in C and similar to [5], [8], employs a model based on energy balance running on constant time steps, with one key difference that it does consider $C_d A$ in (1) as an function of yaw angle ψ. Efficiency curves for components such as solar cell, motor, MPPT, and the battery pack are presented to the software as a set of data points in Tecplot compatible format and SolarSim linearly interpolates (in 2 or 3 dimension) between data points to obtain the required values. This is done similarly for the weather and the route data. The weather file contains direct and diffuse irradiance, temperature, pressure and wind velocity at fixed time interval at multiple locations along the route. SolarSim uses a simplified 1 dimension track model where the track is described in terms of altitude and the heading to the next data-point as a function of distance. Modelling of control stops and unexpected stops during the race is possible and the solar charging at overnight stops is modelled.

6) Implementation: Table I lists the programming languages that different literature used to implement their respective methods.

TABLE I
IMPLANTATION METHODS BY DIFFERENT LITERATURE

Implementation method	Literature
Python	[8], [10]
Matlab	[5]
C	[9], SolarSim

B. Optimisation of Strategy

With a model formulated, it can then be used to simulate and evaluate different strategies. There is no clear definite solution to the optimisation problem, and a wide range of methods have been attempted in the past.

1) Deterministic Approach: Selin et al. [6] identified speed and torque values as control variables and the objective is to minimise the race time. They proposed an hierarchical optimisations workflow, by splitting the race into large segments and optimising the race assuming the parameters remain constant in each segment. This is similar to [5], [12] where they employs long term (whole race), mid-term (1 day) and short-term (few hours) strategies. Mocking [5] also considered the problem to be a time optimal problem with two objectives, minimising the time required to travel the race distance; and maximising the efficiency in using the available energy. The optimal strategy is determined using Newton and Quasi-Newton optimization techniques, in particular, the hessian is approximated by the Davidon-Fletchler-Powell (DFP) method.

Pudney [4] on the other hand, formulated it as an optimal control problem and maximised the Hamiltonian and analytically obtained the optimum solution in the form of differential equations which was solved using the simple Euler method. He was only able to deal with relatively simple inputs, such as modelling the irradiance as a truncated Fourier series or a Markov transition matrix (see Section II-B2). The optimial strategies are found analytically as partial differential equations which are solve numerically to implement the strategies.

Another analytical approach was demonstrated by [8], the Euler-Lagrange equation of the model was derived and they aimed to minimise the energy consumption. An optimal solution is produced in the form of a differential equation which was then solved by two different numerical methods.

2) Stochastic Approach: Pudney [4] has evaluated the problem with a stochastic model of solar power in the context of solar racing. By examining the residuals after subtracting the deterministic truncated Fourier series model from the observed time series, he decided the residuals are best modelled by a Markov Chain with two state, the residuals greater than, and less than zero. The benefit of the method is the model gives a probability of achieving a certain distance in the remaining time, which depending on the level of risk the team is ready to accept, the target distance for the day can be adjusted accordingly.

Not straightly stochastic, [13] acknowledged that there are uncertainties in the model and described the problem in fuzzy logic where there are "levels of truth". One of the key differences in this is approach is that it provides the opportunity to transfer existing knowledge in race strategy. However the literature provided little detail on the actual implementation of this fuzzy engine.

The lack of research in stochastic approach to solar racing strategy is in some level surprising as stochastic methods such as Monty Carlo are not uncommon in other forms of motorsport [14], [15].

3) Heuristic Approach: Betancur, Osorio-Gómez and Rivera [16] tested three optimisation technique, namely Exhaustive search, Genetic Alogrithims, and Big Bang-Big Crunch (BB-BC), which was first proposed by [17]. Jakkidi [7] took a multi-objective approach with the Strength Pareto technique. He considered the optimal strategy daily and optimised the end of the day battery charge, the distance covered, and the battery used per distance.

One of the advantage of these approaches are they will more likely to give the global optimum in the case when the functions are not convex, and more simple method may converge on local minimums. However these approaches are very computational heavy as a sizeable population were needed to be evaluated at each iteration (36000 simulations ran in Betancur's BB-BC optimisation), [4] also noted that search algorithms rarely give any insights in the actual science behind the decisions.

C. Previously proposed strategy

1) Constant Velocity: This is probably the most mentioned strategy, and can be proven analytically given a few assumptions, [4] proved it by comparing the energy cost and arguing that the power consumption due to resistance forces is convex. However this is not very easy to understand. Hence, an alternative prove done by decomposing the velocity into the distance averaged component and the distance varying component is presented below.

Assuming perfectly efficient systems and ignoring the effect of environmental factors. We consider a first case where the the velocity is constant at $\bar{v}$ with energy cost $\mathcal{E}$, and a second case where the velocity varies but starts and ends at velocity $\bar{v}$ which took the same time to complete the journey. x_0 and x_1 denotes the start and end distance of the journey.

Using (4), the energy required would be

$$E = \int_{x_0}^{x_1} mv\frac{dv}{dx} + \frac{1}{2}\rho v^2 C_d A + R + G \, dx \tag{5}$$

Notice that the first term integrates to the kinetic energy at the start and end of the journey. As the velocities are the same, it will evaluate to 0. Rolling resistance and gradient forces can also be cancelled out as they are the same in both cases. Therefore the difference in energy cost between the two cases is simply

$$E - \mathcal{E} = \frac{1}{2}\rho C_d A \int_{x_0}^{x_1} v(x)^2 - \bar{v}^2 \, dx \tag{6}$$

Decomposing the velocity in to the distance averaged $\bar{v}$ and varying components $v'(x)$, we get

$$v(x) = \bar{v} + v'(x) \tag{7}$$

Substituting (7) back into (6) and expanding gives

$$E - \mathcal{E} = \frac{1}{2}\rho C_d A \int_{x_0}^{x_1} \bar{v}^2 + 2\bar{v}v'(x) + v'(x)^2 - \bar{v}^2 \, dx \tag{8}$$

As we defined $\bar{v}$ as the distance averaged component of v, it will cancel out with $\bar{v}$. In the second term $\bar{v}$ is a constant and as $v'(x)$ is the time varying component so it will integrate to 0, and the third term is squared, so it will always evaluate to be larger than or equal to 0.

Therefore, ignoring environmental conditions and system efficiencies, driving at a constant velocity is be more efficient from an aerodynamics perspective.

2) Irradiance dependent: [18] have first tested the idea in simulation and proposed the idea that to optimise energy collected from the sun, solar car should speed up under the cloud. This is shown by [4] analytically.

[8] modelled two different case of cloud coverage, one temporal invariant case where the cloud are at fixed distance along the route and a case where the cloud slowly develops at a fix point along the route and disappears.

3) Gradient dependent: Pudney [4] and Wright [19] both noted a constant velocity strategy is not the most optimal solution, with [19] noting that a constant velocity will cause a increase in I^2R losses as the motor try to maintain the constant velocity as the car climb the gradient. The suggested strategy is therefore to reduce the velocity going uphill.

4) Conclusions: It is worth noting that through analytical analysis, [4, p. 156] noted "all of these strategies will get you to the finish line a couple of minutes earlier than if you had travelled at a constant speed". So it is questionable whether deriving more complex strategy is worth the gain or effort would be better spend on designing a more efficient car in the first place.

D. Execution of strategy

In a real race, the energy consumption of the car may not match what is modelled and there might be unexpected disruptions to the strategy plan. Weather forecasts may not accurately predict local variations, and model parameters may not be totally accurate. Events like tyre punctures, vehicle breakdowns, overtaking and radio issues are not unlikely to happen and the strategy will have to be adapted to accommodate for that.

Mocking [5] considered monitoring different variables during the race and suggested actions to deal with errors in the initial model parameters, and proposed alternative strategy in the event of any unexpected disruption to the strategy plan. Pudney [4] briefly suggested matching the discharge curve to compensate for modelling error but it is not supported by much analysis. This is a relatively simple to ensure all the energy stored will be expended at the end of the race by discharging the same amount as the strategy suggests, albeit not necessarily travelling at the same speed as initially intended. He subsequently won the 1999 World Solar Challenge with the Aurora Solar Team. [4], [5], [9] have all included a day to day diary during their race in the World Solar Challenge which gave some insight into how they have dealt with deviations to the strategy plan.

III. Conclusion

A wide range of research being conducted on different aspects of solar car race strategy, with the modelling approach being widely agreed on. However, the are still gaps in the domain to be covered. In particular, although there are quite a few literature proposing strategies on varying the target velocity in relation to variations in solar irradiance, there are little to no literature proposing a strategy that varies the target velocity in relation to the wind conditions, or strategy related to control stops. Most literature assumes that the conditions of the race are know, which is rarely the case. Some further research into reacting to unexpected disruptions to the strategy plan, and the discrepancy of the modelling parameters could be beneficial.

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