On the Modelling and Strategy Optimisation of Solar Racing

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***Abstract*—In this work, a driving strategy to compete in the World Solar Challenge was developed and optimised. Five strategies were developed to take advantage of the changing environmental conditions encountered throughout the race, including two novel strategies that respond to the yaw angle and velocity of the oncoming flow.**

Utilising two models of differing complexity, the developed strategies were first optimised and evaluated separately. By finding the optimal linear combination of strategies, a final strategy that takes into account the multiple changing environmental factors was then developed. The optimised driving strategy resulted in a gain of 0.4 kWh of energy, and showed that the two novel strategies were able to provide more significant gains compared to other previously proposed strategies.

To support this work, a python package capable of assembling high resolution weather data from multiple sources, and performing race simulations in parallel was created.

***Index Terms*—Optimisation, race strategy, solar car.**

NOMENCLATURE

$c_d A$	Drag area of the solar car.
g	Acceleration due to gravity.
m	Mass of the solar car.
SoC	Battery state of charge.
ρ	Air density.

I. INTRODUCTION

SOLAR RACING consists of racing highly efficient prototype vehicles over long distances using power from the sun.

Various races exist around the world, the World Solar Challenge (WSC) and the American Solar Challenge take place on public roads and teams compete to cover the race distance in the shortest amount of time. Other races like the European Solar Challenge and the Formula Sun Grand Prix take place at a track where teams compete by covering the most distance in a set amount of time.

This work is based around WSC, a 3021 km race across the Australian Outback. The race takes place over 5 days where teams drive from 08:00 to 17:00, with 9 control stops along the route where teams are mandated to stop for 30 minutes. This work aims to develop driving strategies in response to the changing environmental conditions.

The optimisation of solar racing strategy involves two distinct elements: 1) modelling of the vehicle and operating conditions, where the aim is to recreate an accurate representation of the real world so different strategies can be tested; and 2) optimising the driving strategy, where the velocity profile during the race is adjusted to complete the race in the shortest amount of time.

II. LITERATURE REVIEW

A. Race Modelling

In the context of solar racing, the modelling of the race focuses on the power flow between different systems in the car and its surrounding environment.

Figure 1 illustrates, with some simplification, the flow of energy between these components. Approximately 90% of the energy throughout the race is produced by the solar panels, with the remaining 10% present in the battery at the start of the race. The battery also acts as an energy buffer when there is an excess of energy.

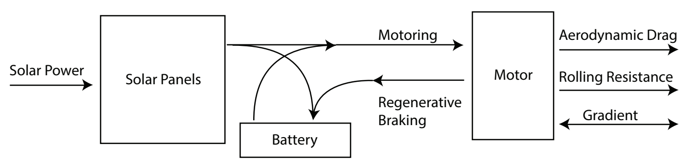

Fig. 1. Simplified flow of energy during a solar race.

1) Governing Equations: While the car is in motion, there are a number of forces acting against the forward motion. Aerodynamic drag varies in relation to the oncoming wind velocity and direction, while a component of the gravitational force acts upon the car while it is on an incline, and the deformation of the tyres contributes to energy loss. A well defined model should capture all of these effects and allow the strategist to make data-driven decisions to achieve the best results.

While Mocking [1] noted the yaw dependency in his scaled wind tunnel test, he argues that the values were a rough optimistic estimation and the yaw dependency did not represent real behaviour. Along with [2]–[5], most authors ignored the effect of yaw angle, ψ, on the aerodynamic drag, D. On the other hand, Sims-Williams [6] recognised that a crosswind does reduce the drag area on the solar car and includes a yaw dependent $c_d A$ in the simulation model. Following that model, the equation modelling the aerodynamic drag acting on the solar car is

$$D = \frac{1}{2}\rho v^2 c_d A(\psi). \tag{1}$$

Suspension setup and tyre slip are less relevant in the context of WSC. Unlike other circuit racing events, there are no tight corners on the route, therefore, suspension setup and tyre slip are generally ignored by most authors [1]–[6]. The rolling resistance is modelled by two coefficients,

$$R = (c_{rr1} + c_{rr2} v) mg, \tag{2}$$

and the component of weight acting opposite to the motion of the car on a incline with gradient G is given by

$$F_{grad} = mgG(x). \tag{3}$$

2) System Efficiencies: Apart from forces resisting the motion of the car, further inefficiencies exist in components such as the solar panels, batteries, and motors.

Both [6] and [2] recognised that solar cells operate less efficiently as the cell temperature increases. By including a Simplified Thermal Model presented by [7] on horizontal solar cells, Sims-Williams [6] took into account the heat transfer and efficiency drop in his model. Mocking [1] did not consider the temperature increase and assumed that the cell efficiency remains constant throughout the simulation.

Pudney [2] considered mechanical and electrical drive losses as a function of output power and vehicle velocity, while Mocking [1] argues that " although the motor efficiency varies with the motor input power, it is fairly constant for values less than 100 W " , and used a constant motor efficiency along with [4].

Several authors have adopted different battery models: Pudney [2] discussed various complex models but due to the limited computing power at the time of writing, choose to employ the Ohmic Battery Model, while Mocking [1], employed an empirically determined equilibrium curve at low pack charge and extrapolated in the linear region.

3) Environmental Conditions: Modelling the rapidly changing weather conditions still remains a challenge, and various levels of simplifications were made.

To keep their analytical models relatively simple, [5], [8] assumed a sine variation of irradiance, while [2] used a Truncated Fourier Series to allow a more realistic representation of the irradiance. Less attention was paid to other environmental factors. Pudney [2] along with [5], [8] simply ignored wind variation in their model. Not bound by the need to express environmental conditions in terms of exact equations, [1], [6] were able to capture more realistic variation in these environmental conditions and both include irradiation and wind variation in their models.

B. Optimisation of Strategy

With a model formulated, it can then be used to simulate and evaluate different strategies.

One relatively simple driving strategy is to drive at a constant velocity. By comparing the energy cost between a constant velocity strategy and a varying velocity driving strategy, and showing that the resistance forces are convex, Pudney [2] showed analytically that driving at a constant velocity could be a viable strategy under a few assumptions. To create more complex strategies, the common approach was to formulate and solve different forms of optimisation problems.

One approach was to apply different previously proposed search algorithms to the problem to find the velocity for each segment of the route. The Big Bang-Big Crunch (BB-BC) algorithm was first used by [9] in the context of solar race strategy optimisation. Following that, [10] compared the use of BB-BC with Exhaustive Search and Genetic Algorithms, finding that: even with a very simplistic model, Exhaustive Search is still not practical due to the expensive running time, and the use of BB-BC or Genetic Algorithms failed to find the global optimum despite conducted 28000-38000 simulations. Considering each day of the race individually and taking a multi-objective approach, Jakkidi [4] used the Strength Pareto technique to optimise the end of day battery charge, distance covered, and the battery used per unit distance. These global optimisation techniques perform well when the functions were not convex, but are very computationally heavy, as a sizeable number of function evaluations were needed at each iteration, requiring tens of thousands of simulations to be performed for each optimisation. This is one reason some authors [1], [3], [4], [11] took a hierarchical approach where the race was split into subdivisions which can be optimised independently in parallel and the optimisation process can be sped up. Pudney [2] also notes that these search algorithms rarely give insight into the actual science behind the decisions.

The alternative approach was to analytically obtain an equation for the optimal driving velocity. Pudney [2] formulated an optimal control problem to minimise the time to complete the race, while [5] used the Euler–Lagrange equation to minimise the energy consumption. By inspecting the analytical results, these techniques give a good insight into the mechanism behind the strategy, however, the analytical nature limits the complexity of the models evaluated, and strategies were focused only on one driver at a time. Both solutions independently agree with the strategy based on changing irradiance conditions first proposed by [12]. However, due to the lack of high resolution and accurate irradiance forecast, this strategy can only be applied at a limited capacity during the race.

Gradient dependent strategies were also proposed by [2], [13], both noting that maintaining a constant velocity strategy when going uphill will cause an increase in I^2R losses as more thrust is required to drive the car forward.

C. Literature Review Summary

Numerous models have been used to simulate solar racing, with the general approach being widely agreed on. However, the level of detail captured varies from model to model. The lack of capability to accurately model the environmental conditions means that all strategies proposed thus far were mostly qualitative and could only be applied during a race in a limited capacity.

On the optimisation of strategies, there were two main approaches, one that employs existing optimisation algorithms, and the other based on reacting to the selected environmental conditions. Techniques that relied entirely on optimisation algorithms are computationally expensive as a large number of variables need to be optimised at the same time, and they rarely give insight into the science behind it. Techniques that respond to changing environmental conditions have only been conducted in response to irradiance and gradient, and were limited to responding to one environmental condition only.

III. Project Outline

This project proposes a new computationally efficient method to produce a driving strategy that responds to multiple changing environmental conditions utilising optimisation techniques.

In Section IV, suitable sources of data were identified and assembled to provide a high quality representation of the changing environmental conditions from which the strategies were developed.

Strategies in response to irradiance and gradient were redeveloped, and extended to respond to changing wind conditions in Section V. Optimisation techniques were used in Section VI to determine the trade off of strategies in response to environmental conditions, and the performance gain was evaluated employing *SolarSim*, the model created by Sims-Williams [6].

To allow easier comparison between the optimisation methods, the different strategies aim to complete the race in the same duration and the objective was to maximise the amount of energy at the end of the race.

IV. Weather Data Assembly

As this project focuses on developing strategies in response to changing weather conditions, the input weather file need to capture sufficient detail to enable the strategies to be evaluated. Irradiance and atmospheric data shown in Table I were described in terms of time and distance along the route.

TABLE I
DATA ENCODED IN A SOLARSIM WEATHER FILE

Data Category	Data
Irradiance Data	Direct Irradiance Diffuse Irradiance Azimuth Elevation
Atmospheric Data	Air Temperature Air Pressure Wind Velocity Wind Direction

A preexisting set of weather data obtained from the Australia Bureau of Meteorology [14] was available for the 2015 World Solar Challenge period. It contains irradiance data at 1 minute intervals and daytime averages for atmospheric data for 5 location along the route, each corresponds to one of the observation stations operated by the Bureau. Although the irradiance data have a high temporal resolution, the lack of temporal variation in wind and the low spatial resolution make it unsuitable to test the proposed strategies.

In order to produce an input file with high data resolution and accuracy, multiple sources were considered. A new python package, *SolarSim Strategy Support Software* (S5) was created to assemble and convert the various sources of weather data into a form compatible with SolarSim.

Data representing the conditions during the 2019 World Solar Challenge were used. The irradiance data were purchased from Solcast [15] at 10 minute intervals available as comma-separated values (CSV) files. The data were derived from satellite observations and correlates well with measured irradiance [16].

The atmospheric data were extracted from the ERA5-Land dataset [17] at 1 hour intervals, available in the form of General Regularly-distributed Information in Binary (GRIB) files. The dataset is a reanalysis result, produced through a combination of real life observations and meteorological models which creates fixed grid data with high spatial and temporal resolution.

To keep the cost of obtaining the data down while capturing as much detail as possible, the updated weather input file includes 200 points in space: at 10km intervals around Darwin, Alice Springs, and Adelaide; and 20km in the Outback. This combination of spatial resolution was selected as more spatial variation in weather conditions was expected around those areas.

Although not used in this project, S5 also have the capability to produce input files from weather forecasts in order to support strategy development during a race.

V. Strategy Development

In this section, potential strategies were developed in response to different changing environmental conditions in isolation. Taking into account the aerodynamic drag and using the parameters listed in Table II, the potential strategies were evaluated with a step input. The magnitude of the velocity profiles in response were varied from 0 km/h (constant velocity) up to 20 km/h, and the difference in energy cost for the race compared to a baseline of constant velocity is presented in Figure 2.

TABLE II
MODEL PARAMETERS

$c_d A$ (m^2)	0.089
Mass (kg)	250
c_{rr1}	0.008
c_{rr2} ($\mathrm{s\,m}^{-1}$)	0
Nominal Battery Pack Voltage (V)	140
Battery Internal Resistance (Ω)	0.036

A. Aerodynamic Drag

Aerodynamic drag is the force that acts opposite to the motion of an object while it is moving through a fluid. As shown by Equation 1, this force is quadratically related to the relative velocity of the oncoming flow. By varying the driving velocity, the power loss due to aerodynamic drag can be minimised.

Assuming perfectly efficient systems and ignoring the effects of other environmental factors. Consider a first case where the the velocity is constant at $\overline{v}$, and a second case where the velocity varies in space with the mean velocity still being $\overline{v}$, such that both cases take the same time to complete the race with distance L.

The energy required to overcome the drag force over the whole race would be

$$E = \int_0^L \frac{1}{2}\rho v^2 c_d A \, dx, \tag{4}$$

and the difference in energy cost between the two cases is

$$\Delta E = \frac{1}{2}\rho c_d A \int_0^L \left(v(x)^2 - \overline{v}^2\right) \, dx. \tag{5}$$

Expressing the velocity in the second case as a distance averaged component $\overline{v}$ and distance varied component $v'(x)$,

$$v(x) = \overline{v} + v'(x), \tag{6}$$

and substituting it in Equation 5 gives

$$\Delta E = \frac{1}{2}\rho c_d A \int_0^L \overline{v}^2 + 2\overline{v}v'(x) + v'(x)^2 - \overline{v}^2 \, dx. \tag{7}$$

Inspecting Equation 7, the first and last term cancel out. Within the second term, $\overline{v}$ is a constant, and as $v'(x)$ is the time varying component, by definition this term will integrate to 0. As the third term is squared, it takes the minimum value when v' is 0. Therefore, from the perspective of aerodynamic drag, driving at a constant velocity consumes less energy.

B. Spatially Varying Irradiance

As the car travels further away from the equator, the effect of atmospheric attenuation is more pronounced. This is because the sunlight travels through a thicker atmosphere and the amount of solar irradiance decreases. Local cloud coverage could also reduce the amount of irradiance available.

Ignoring all losses, the strategy to maximise power input would be to travel as fast as possible to the spot with the most solar irradiance and charge up for as long as possible.

In reality, there exists an optimal trade off between this extra energy gain from the sun and the extra aerodynamic losses from travelling at high speed, further constrained by factors such as the legal speed limit along the route.

C. Temporally Varying Irradiance

The amount of solar irradiance varies as the sun's position and cloud coverage change in the sky through the day. By varying the velocity of the car, the power output can be matched with the power input. Lower net power flow into or out of the battery can minimise the battery I^2R losses.

With a temporally varying irradiance $I(t)$, the net power flowing out of the battery at any point at time t is

$$P_{bat} = \frac{1}{2}\rho v(t)^3 c_d A + Rv(t) - I(t). \tag{8}$$

Assuming that the change in battery pack voltage is negligible, the I^2R loss can be modelled as

$$P_{i2r} = \frac{R}{V_{nom}^2} P_{bat}^2, \tag{9}$$

where V_{nom} is the nominal pack voltage, and R is the pack internal resistance.

Finding the root for Equation 8 gives the driving velocity profile such that there is no net power flow. However, this does not ensure that the mean velocity will remain at $\overline{v}$, so the temporal variation is preserved and the mean velocity adjusted to $\overline{v}$ by increasing the driving velocity uniformly.

D. Gradient

In order to maintain a constant velocity while going uphill, the thrust provided by the motor would have to be increased to balance the gravitational force acting against the motion of the car. This additional power output increases the I^2R losses. A strategy to minimise this loss would be to slow down going uphill to minimise the net power output.

Assuming that on average the power from solar irradiance, P_{ird}, equals to the power required to drive the car on level ground,

$$P_{ird} = \frac{1}{2}\rho c_d A \overline{v}^3 + c_{rr1}\overline{v}mg + c_{rr2}\overline{v}^2 mg, \tag{10}$$

combining this assumption with Equations 1 to 3 gives the net power coming out of the battery,

$$\begin{aligned} P_{bat} =& P_{drive} - P_{ird} && (11)\\ =& \frac{1}{2}\rho c_d A(v^3 - \overline{v}^3) + c_{rr1}(v - \overline{v})mg \\ &+ c_{rr2}(v^2 - \overline{v}^2)mg + mgGv. && (12) \end{aligned}$$

Similar to the temporally varying irradiance strategy, the root of Equation 12 does not guarantee that the mean velocity will remain at $\overline{v}$. As such, the spatial variation in driving velocity was preserved and the mean velocity adjusted to $\overline{v}$ by increasing the driving velocity uniformly.

E. Yaw Angle

The drag area for a solar car decreases as the yaw angle of the oncoming flow increases [18]. By driving faster when the yaw angle is favourable, energy can be used more efficiently.

Consider a relatively simple case with constant wind velocity and spatially varying $c_d A$,

$$c_d A(x) = \begin{cases} \overline{c_d A} + c_d A' & \text{for } 0 \le x < \frac{L}{2} \\ \overline{c_d A} - c_d A' & \text{for } \frac{L}{2} \le x < L \end{cases}, \tag{13}$$

and assume that $v_{air} \approx v_{car} - \cos\psi \times v_{wind}$, the energy loss due to aerodynamic drag throughout the race is given by:

$$E = \int \frac{1}{2}\rho c_d A(x) v(x)^2 \, dx. \tag{14}$$

The energy saved by running a constant velocity strategy is therefore

$$\begin{aligned} \Delta E =& \int_0^{\frac{L}{2}} \frac{1}{2}\rho(\overline{c_d A} + c_d A') \\ &\quad \left({v'_{car}}^2 + 2v'_{car}\left[\overline{v_{car}} - \cos\psi v_{wind}\right]\right) dx \\ &+ \int_{\frac{L}{2}}^{L} \frac{1}{2}\rho(\overline{c_d A} - c_d A') \\ &\quad \left({v'_{car}}^2 + 2v'_{car}\left[\overline{v_{car}} - \cos\psi v_{wind}\right]\right) dx \end{aligned} \tag{15}$$

$$\begin{aligned} \Delta E =& L\frac{1}{2}\rho\overline{c_d A}{v'_{car}}^2 \\ &+ L\frac{1}{2}\rho c_d A'\left[-2v'_{car}(\overline{v_{car}} - \cos\psi v_{wind})\right]. \end{aligned} \tag{16}$$

As the dominant term, ${v'_{car}}^2$, is positive, there exists a minimum ΔE. Differentiating w.r.t. v'_{car} to obtain the optimum v'_{car} gives

$$\frac{\partial \Delta E}{\partial v'_{car}} = L\rho \left(\overline{c_d A} v'_{car} - c_d A' \left[(\overline{v_{car}} - \cos\psi v_{wind}) \right] \right) \tag{17}$$

$$v'_{car} = \frac{\overline{v_{car}} - \cos\psi v_{wind}}{\overline{c_d A}} c_d A'. \tag{18}$$

F. Wind Velocity

As the drag force is quadratically related to the relative oncoming flow velocity, driving faster while the wind velocity is low could provide some energy saving.

For a spatially varying wind velocity

$$v_{wind}(x) = \begin{cases} \overline{v_{wind}} + v'_{wind} & \text{for } 0 \le x < \frac{L}{2} \\ \overline{v_{wind}} - v'_{wind} & \text{for } \frac{L}{2} \le x < L \end{cases}, \tag{19}$$

the extra energy cost by driving a spatially varying velocity, $v_{car}(x) = \overline{v_{car}} + v'_{car}(x)$, is

$$\begin{aligned} \Delta E = \int_0^{\frac{L}{2}} & \frac{1}{2}\rho c_d A \Big({v'_{car}}^2 + \\ & 2v'_{car} \left[\overline{v_{car}} - \cos\psi(\overline{v_{wind}} + v'_{wind}) \right] \Big) \, dx \\ + \int_{\frac{L}{2}}^{L} & \frac{1}{2}\rho c_d A \Big({v'_{car}}^2 + \\ & 2v'_{car} \left[\overline{v_{car}} - \cos\psi(\overline{v_{wind}} + v'_{wind}) \right] \Big) \, dx \end{aligned}. \tag{20}$$

Substitute in a strategy of $v'_{car} = -cv'_{wind}$, where $c \in \mathbb{R}$, the change in energy is then:

$$\Delta E = \frac{1}{2}\rho c_d A L c^2 {v'_{wind}}^2 - \rho c_d A L c v'_{wind} \cos\psi v'_{wind}. \tag{21}$$

As the dominating term of c^2 is positive, there exists a minimum. Differentiate w.r.t. c to obtain the optimal,

$$\frac{\partial \Delta E}{\partial c} = 0 = \rho c_d A L c {v'_{wind}}^2 - \rho c_d A L v'_{wind} \cos\psi v'_{wind} \tag{22}$$

$$c = \frac{\rho c_d A L v'_{wind} \cos\psi v'_{wind}}{\rho c_d A L {v'_{wind}}^2} \tag{23}$$

$$c = \cos\psi \tag{24}$$

Therefore the optimal strategy ignoring other environmental conditions is

$$v'_{car} = -\cos\psi v'_{wind}. \tag{25}$$

G. Discussion

Figure 2 shows the energy savings from the different strategies given a step input representing typical variation of the environmental conditions. For the strategies in response to temporally varying irradiance and gradient, which were related to I^2R losses, the gains were minimal compared to those in response to spatial irradiance and wind. It is worth noting that in practice, two strategies with the same standard deviation may not necessarily have the same velocity profile, as they are following different environmental conditions, and these individual energy savings cannot necessarily be superimposed.

This shows that ignoring other factors, the strategies developed could provide performance gains.

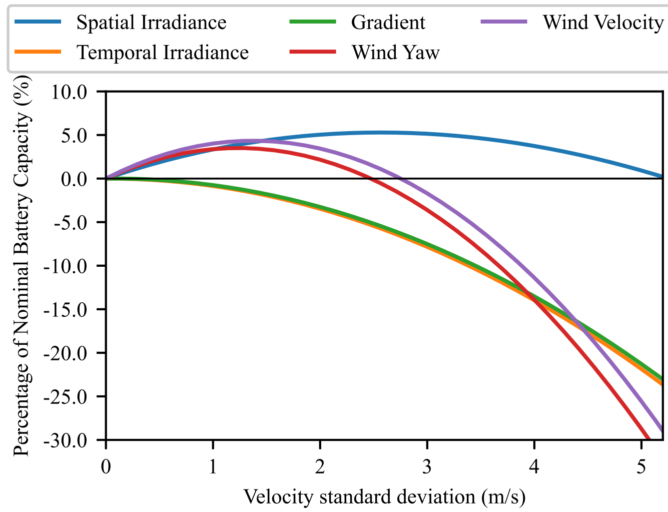

Fig. 2. Energy savings from different strategies in response to a step input.

VI. Strategy Optimisation

This section evaluates the developed strategies with more complex models, finds the optimal trade-off when following the strategies, and proposes a combined strategy driven by multiple changing environmental conditions simultaneously. The model parameters shown in Table II was used.

A. Methodology

SolarSim analysis of the constant velocity strategy at various $\overline{v}$ was first conducted. This was used to determine the $\overline{v}$ for subsequently proposed strategies and to set a baseline to compare performance gain.

It was assumed that when applying the strategies they will not place the solar car significantly far away from its baseline position such that the environmental conditions are completely different. Consequently, the conditions encountered during the constant velocity simulation were used to derive the conditions which drive the strategies developed in Section V.

Using the clear sky irradiance model by [19], the daily variation at Alice Springs was assumed to be the temporal variation throughout the route and the spatial variation was calculated by taking the difference of the encountered irradiation and the temporal irradiance.

The $c_d A$ which drives the yaw angle strategy and the headwind velocity were retrieved from the output of the constant velocity simulation, and the gradient was calculated from the input road file.

Figure 3 shows the environmental conditions encountered and the corresponding velocity profile in response to each of the environmental conditions was calculated as presented in Section V.

The magnitude of v' for each strategy was scaled according to how strongly the respective strategies should be followed. Instead of metrics such as the standard deviation or the maximum amplitude, the value of the 68th percentile of v' (v'_{68th}) of each velocity profile was used to provide a comparable metric on how strongly the strategy should be followed while

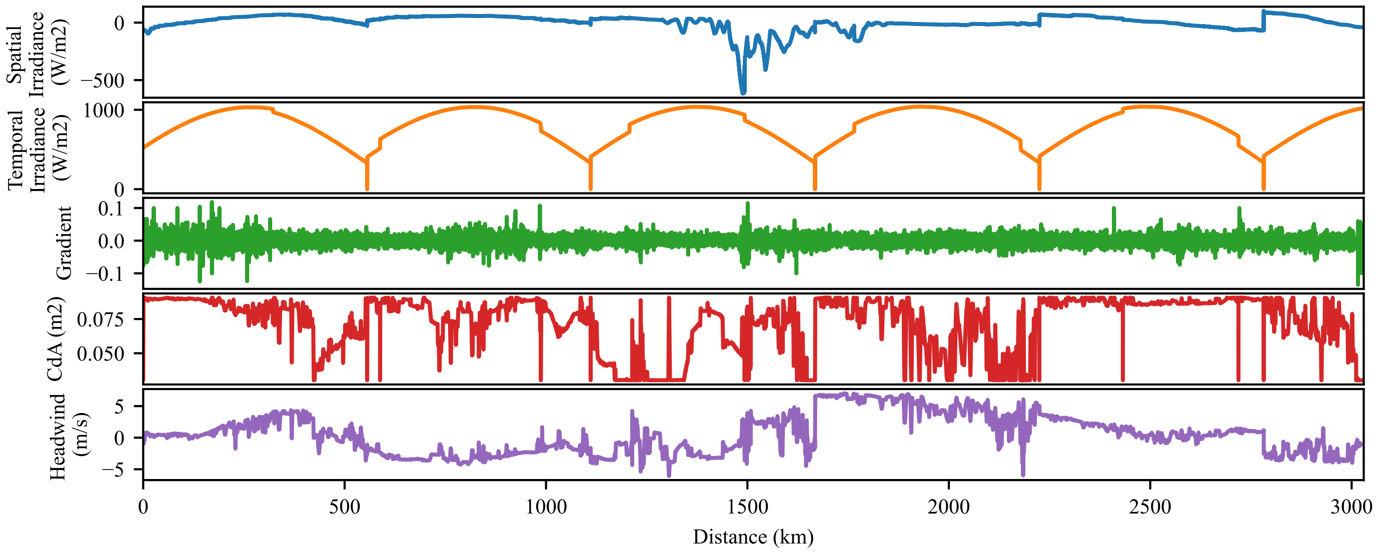

Fig. 3. Environmental conditions encountered in the constant velocity simulation

being robust against noise and outliers. The target velocity profiles were limited to values between 10 and 130 km/h to prevent non-physical results. During the SolarSim simulation, this was further limited by the legal speed limit along the route.

A simplified model and the SolarSim model were used to simulate the race. The simplified model was based on the equations presented in Section II-A, and calculates the change in energy at the end of the race using array programming techniques to allow quick comparison between proposed strategies. SolarSim on the other hand provides a much more detailed simulation of the car and its interaction with the environment at fine intervals in the order of seconds. Details such as battery discharge curve, canopy shading, legal speed limit, and control stops were modelled, at the cost of a significantly longer computation time of approximately 20 minutes instead of approximately 30 seconds using the simplified model.

Table III summarises the simulations conducted. The results were assumed to be locally convex, and were optimised by the Limited-memory Broyden–Fletcher–Goldfarb–Shanno (L-BFGS) algorithm. As the focus of this work is not on algorithm selection, L-BFGS from the SCIPY.OPTIMISE package [20] was chosen as it is robust [21].

Proposed strategies were in response to either one of the changing environmental conditions (Isolated Strategy Optimisation), or all of them at the same time (Combined Strategy Optimisation). For one set of simplified simulations, idealised weather was used. In this case, only the environmental condition the strategy driving the strategy was changing, with all other conditions remaining constant. The optimisation for the combined strategy utilised the v'_{68th} metric to determine how strongly the combined strategy should be influenced by each of the environmental factors, and the resulting strategy was a linear combination of the isolated strategies.

The objective of the optimisation was to maximise the ending SoC and avoid reaching the battery capacity or running out of charge. As such, a deadzone linear penalty was applied when the maximum SoC race reached above 98%, and a log barrier was applied on the minimum SoC starting from 5% which goes to infinity when it falls below 1%.

TABLE III
LIST OF SIMULATIONS CONDUCTED

Model	Proposed Strategies	Environmental Conditions
Simplified	Isolated	Ideal
Simplified	Isolated	All
SolarSim	Isolated	All
Simplified	Combined	All
SolarSim	Combined	All

B. Constant Velocity Strategy Results

Figure 4 shows the average velocity and the SoC summary for constant velocity strategies with $\overline{v}$ between 60 km/h and 80 km/h. *Max* and *min* denote the maximum and minimum SoC reached throughout the race, which is not necessarily the starting and ending SoC, which are denoted by *starting* and *ending* respectively.

At target velocities less than 70 km/h, the battery ends up with more charge than it started the race with. This excess of energy means that there was still potential to run faster and the performance was not maximised. Reaching 100% SoC at any point during the race is also undesirable as that means the battery cannot act effectively as a buffer if there is an excess of energy.

At target velocities between 70 km/h and 73 km/h, the maximum battery SoC during the race is higher than the starting SoC, with the ending SoC lower than the starting SoC. This is the desired behaviour as the battery acts as a buffer, temporarily storing energy when the conditions were favourable.

At target velocities above 73 km/h, the ending SoC is 0, meaning the battery was depleted before the end of the race,

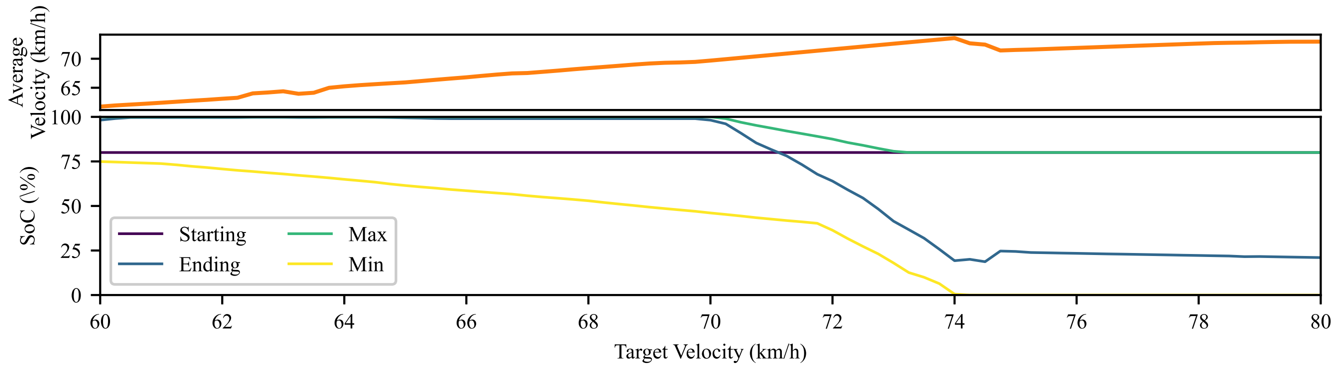

Fig. 4. Average velocity and state of charge summary for constant velocity strategies.

and the car has to run purely on solar energy. This is reflected in the average velocity, which is significantly less than the target velocity. In reality this would be further constrained by the regulations [22] stating that teams could be penalised for driving at less than 50 km/h in areas with a speed limit of 100 km/h or greater.

The average driving velocity was mostly linear at target velocities below 73 km/h. The average velocity in the simulation deviates slightly from the target velocity due to factors such as the legal speed limit and the acceleration limit of the car.

C. Isolated Strategy Optimisation Results

The simulation results using the simplified model are presented in Figure 5. In general, the results produced using idealised and all changing environmental conditions were similar, with most gain provided by wind related strategies. The spatial irradiance strategy suggests it would yield a small amount of gain, and the temporal irradiance strategy suggests driving opposite to the developed strategy, speeding up, instead of slowing down when there is less sun which is slightly unexpected. Gradient strategy suggests no gain at all.

Reviewing the spatial irradiance profile used to drive the strategy shown in Figure 3, there was only a small amount of spatial variation and that may explain the lack of performance gain by driving faster when there is bright sun.

Attempting to explain the unexpected result during the temporal irradiance simulation, the different sources of energy output shown in Figure 6 were inspected. It can be seen that following the strategy does provide a reduction in I^2R loss, however, the reduction in loss is three orders of magnitude less than the energy change due to aerodynamic losses or irradiance gain. Therefore in general, strategies aiming to reduce I^2R losses were not worth pursuing.

The increase in irradiance gain by driving opposite to the temporal irradiance strategy, essentially applying the spatial irradiance strategy, does provide a gain in irradiance input, further confirming that the lack of performance gain in the spatial irradiance strategy was only due to its lack of variation.

The optimisation process was repeated using the SolarSim model, and the results were compared with the optimum suggested by the simplified model evaluated in SolarSim. Both

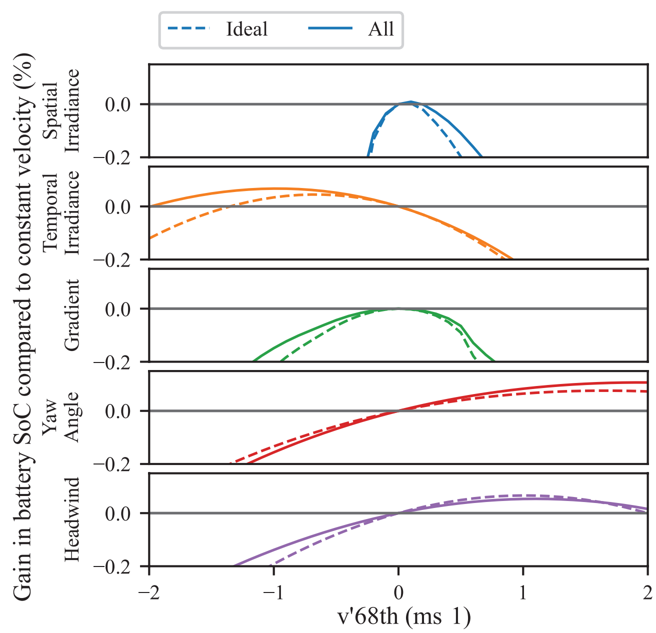

Fig. 5. Simplified isolated simulation results.

sets of results evaluated with all the changing environmental conditions were presented in Table IV.

TABLE IV
COMPARISON BETWEEN OPTIMAL STRATEGY SUGGESTED BY THE SIMPLIFIED MODEL AND SOLARSIM

Strategy	Simplified		SolarSim	
	V'_{68th}	SoC Gain	V'_{68th}	SoC Gain
Gradient	-0.01	0.01	-1.76	0.74
Spatial Irradiance	0.05	0.02%	-0.01	0.14%
Temporal Irradiance	-1.01	0.25%	-1.08	0.31%
Yaw	1.94	2.09%	3.00	2.45%
Headwind	1.05	4.70%	2.51	7.60%

The optimised result from both models successfully yielded a gain in ending SoC. However, the more complex SolarSim model shifted the optimal point to follow the strategies more strongly.

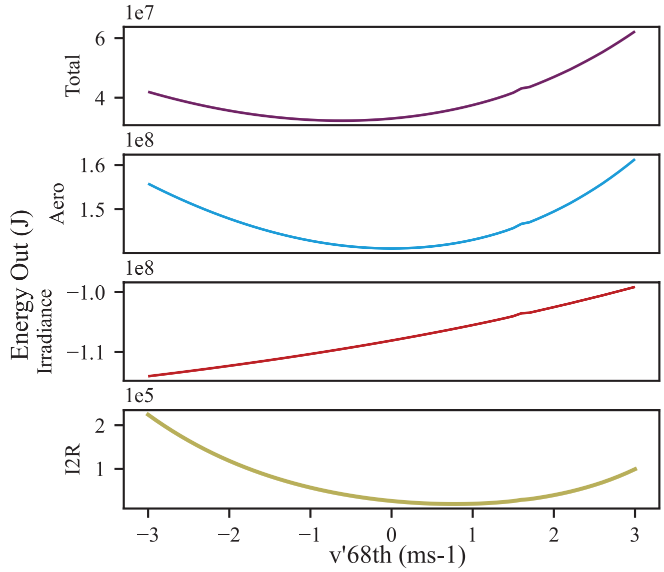

Fig. 6. Energy expenditure during the temporal irradiance strategy simulation with the simplified model.

As the simplified model previously suggested I^2R related strategies would only provide negligible gain, the optimal gradient result suggested by SolarSim seems to contradict this statement. Figure 7 shows the car was unable to closely match the target velocity as it cannot accelerate fast enough. The speed limit enforced by the SolarSim model further reduced the average driving velocity during the simulation.

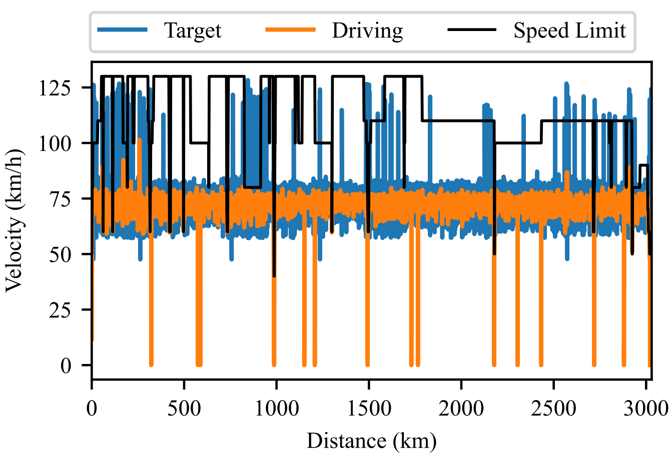

Fig. 7. Speed limit, target velocity, and driving velocity in the gradient strategy simulation using SolarSim.

This lower average driving velocity of 72.1 km/h in turn reduces the aerodynamic loss and misleadingly appears as an improvement. This reinforces the idea of not blindly following results suggested by optimisation algorithms without careful inspection. To minimise the deviation between the target velocity and the average driving velocity in subsequent optimisations, a quadratic penalty function was applied.

To confirm that the gains obtained by following each of the strategy drivers were legitimate, Pearson correlation coefficients between the variables were calculated and shown in Table V.

TABLE V
CORRELATION BETWEEN DIFFERENT STRATEGY DRIVERS

	Spatial Irradiance	Temporal Irradiance	Gradient	CdA	Headwind
Spatial Irradiance		-0.001	0.086	0.120	-0.143
Temporal Irradiance	-0.001		0.022	0.317	0.008
Gradient	0.086	0.022		0.036	-0.023
CdA	0.120	0.317	0.036		0.224
Headwind	-0.143	0.008	-0.023	0.224	

Generally the strategy drivers were not correlated, with the exception of being weakly correlated between c_dA and temporal irradiance, and between c_dA and headwind.

Recall that following the temporal strategy does not yield any gain due to the insignificance in the I^2R gain, therefore despite the weak correlation between c_dA and temporal irradiance, there is no danger that the gain in the yaw strategy was due to the influence of temporal irradiance.

This however may not be true for c_dA and headwind. With the weak correlation between the two strategy drivers, the gains by following one of the two drivers may be slightly inflated. This can be clearly seen in Figure 5 where gain by following the yaw angle increases when switching from ideal environmental conditions to include all changing conditions. However, this does not invalidate the statement that the wind related strategy provides the most gain as this is still the case in the simplified simulation using the ideal conditions.

D. Combined Strategy Optimisation Results

Table VI shows the optimal linear combination of strategies and the performance gain suggested by the optimisation algorithm.

TABLE VI
COMBINED OPTIMISATION RESULTS

	Optimal v'_{68th} for each strategy					SoC Gain
	Gradient	Spatial Irradiance	Temporal Irradiance	Yaw	Headwind	
Simplified	-0.19	-0.13	-0.82	1.65	1.59	6.82%
SolarSim	-1.13	-0.02	-0.21	3.10	2.33	8.07%

Similar to previous investigations, both models suggest that wind related strategies provide more significant gains than the other strategies. With 96 simulations performed in 24 hours, the optimisation results using the SolarSim model were able to provide an 8% gain in ending SoC.

The aerodynamic power output and the irradiance power input while following the optimised driving strategy are shown in Figure 8, which demonstrates that the optimised driving strategy was able to respond to cloud coverage and minimise the aerodynamic losses at the same time.

Figure 9 shows the solar car location and wind velocity along the route at different times. Comparing the optimised strategy to the constant velocity strategy, it can be seen that the strategy sped up when low wind velocity was encountered and slowed down when high wind velocity was encountered.

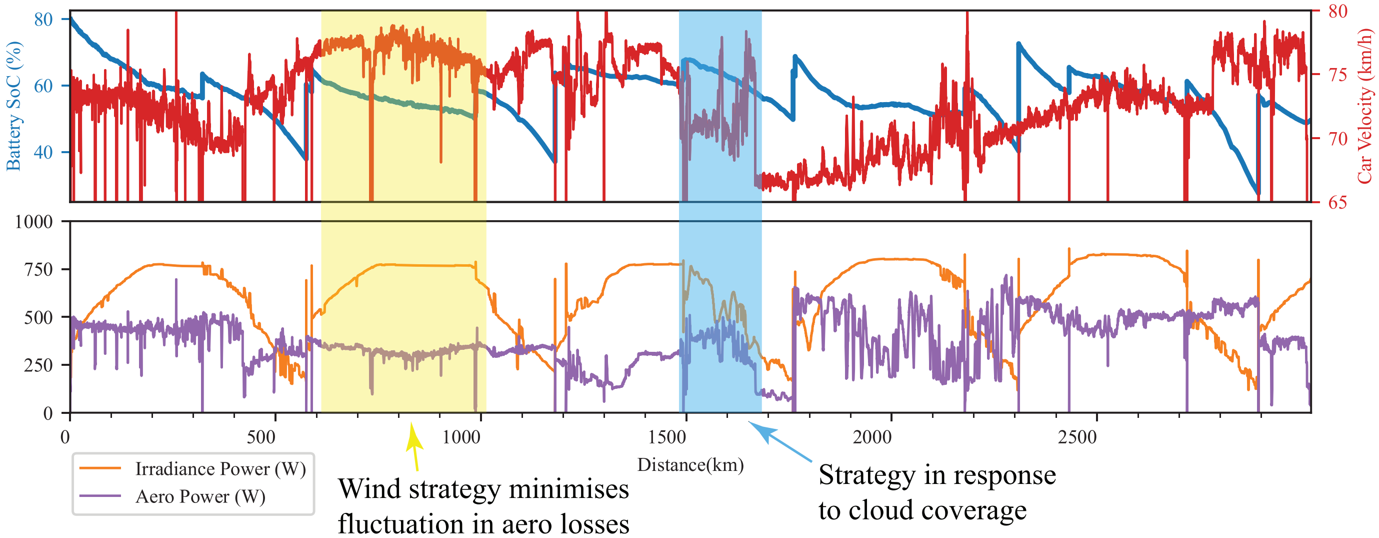

Fig. 8. Battery SoC, irradiance power in and aerodynamic power while following the optimised driving strategy.

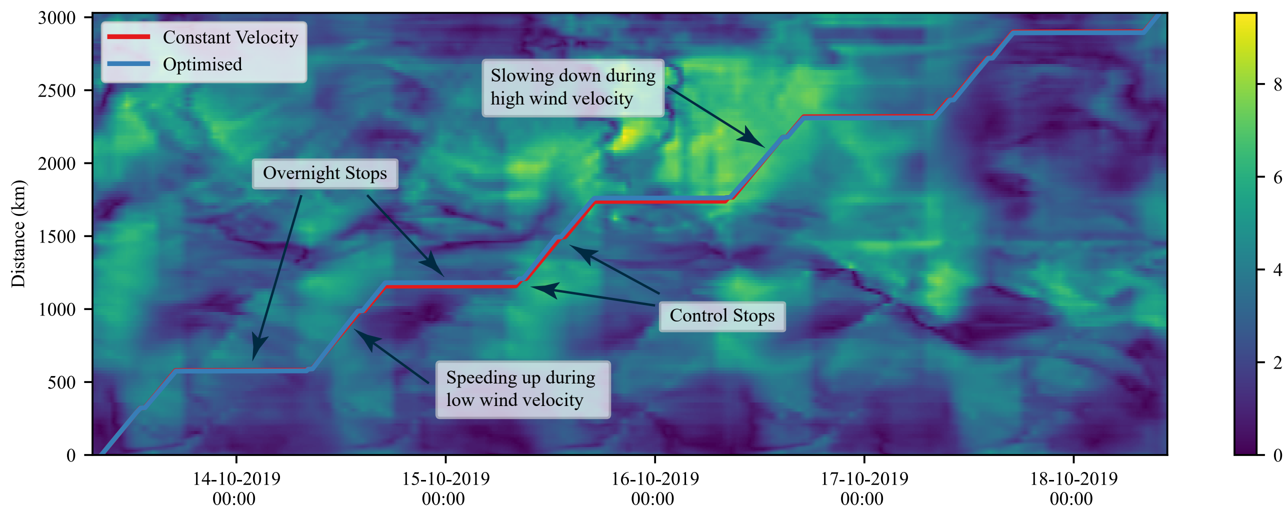

Fig. 9. Constant and optimised driving strategy overlaid on contours of wind velocity.

E. Discussion

Optimisation methods of varying complexity were tested, resulting in different levels of gain in ending SoC. The most significant gain was given by performing a combined optimisation using SolarSim. With a typical battery capacity of 5 kWh, the gain of 8% ending SoC can roughly translate to an extra 0.4 kWh of energy available.

The collection of simulations shows in strong agreement that attempts to reduce I^2R loss were not fruitful, and the gains from reducing aerodynamic losses were significant.

This method of finding the optimal linear combination of analytically derived strategies was able to give a strategy that takes advantage of multiple changing environmental conditions with significantly fewer simulations. This method only requires 96 simulations to optimise the 5 variables, compared to the 36000 simulations conducted by [10] who applied the BB-BC algorithm to optimise the velocity of the route split into 10 segments.

While the optimisations herein were performed to a high precision to allow comparison between the minute difference in gain between strategies, in practice the uncertainties in the model parameters or unexpected events such as tyre punctures or overtaking of another convoy would mean excessively precise optimisation results would not translate to a performance gain in real life. In addition, the strategist while travelling in one of the support vehicles during the race, would only have access to their laptop, and limited access to the internet.

As a result, a balance should be struck between finding small performance gains and to more efficiently utilise the limited resources available. Due to the lack of gains shown in the results, the gradient and temporal irradiance strategy should not be considered during the race, and the strategist should respond to all changing irradiance encountered using what was presented as the spatial irradiance strategy above.

Table VII shows the cost of conducting the different optimisation methods presented in this section.

During the day, simplified combined optimisation should be

TABLE VII
COST AND PERFORMANCE GAIN FROM DIFFERENT OPTIMISATION METHODS

Model	Strategy	SoC Gain	CPU Time	Elapsed Time	CPU Core
SolarSim	Constant Velocity	Baseline	12 hr	30 min	24
Simplified	Isolated	4.70	8 min	8 min	1
SolarSim	Isolated	7.60	26 hr	6 hr	5
Simplified	Combined	6.82	22 min	22 min	1
SolarSim	Combined	8.07	24 hr	24 hr	1

used to quickly adjust to the strategy to react to unexpected changes.

Parallel computing techniques were implemented in the S5 package created by the author to speed up the optimisation of the constant velocity strategy. This significant reduction of computing time serves as a proof of concept and shows that there could be potential for more efficient use of computing power.

VII. CONCLUSIONS

With the aim of taking advantage of the changing environmental conditions during the race, five different strategies were developed analytically, and have shown to be able to provide performance gain when other factors were ignored.

Evaluating the five strategies using the two models in isolation highlights the significant gains made possible by the novel strategies developed in response to the yaw angle and velocity of the oncoming flow. It also reveals that strategies aiming to reduce I^2R losses are not worth pursuing. By finding the optimal linear combination of the five strategies, a strategy that takes in multiple changing environmental conditions was created, resulting in an extra 0.4 kWh of energy available. This method is also more computationally efficient than previously proposed optimisation methods.

The software created for this work has made high definition weather data available. Along with the optimisation method presented, quantitative driving strategies can be created during the race to maximise race performance.

Further improvement could be made on the selection of the specific optimisation algorithm and tuning of the hyperparameters. There is also potential to extend this work to be applied to autonomous control in transportation such as trains or trucks to improve their fuel efficiency.

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